AUTOMATED
DIGITAL PROCESSING
OF INTERFEROGRAMS
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PROEFSCHRIFT

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AMAR CHOUDRY
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'Mysteries of the invisible wind
are written on the clouds.
If I could read the Cloud-Patterns
I will see the invisible wind'.

From MEGHDOET (The Cloud-Messenger) by
KALIDAS, Sanskrit poet 4-th century A.D.
CHAPTER 1

INTRODUCTION

A large number of experiments and industrial processes yield information in the form of images. Mostly these are not the usual photographs of objects but functional images i.e. the image is related to the distribution of some physical parameter, HAUS(1980). Thus an infrared photograph gives a temperature distribution, a moiré pattern may give a displacement distribution etc. In the majority of the cases the images are evaluated qualitatively since a quantitative evaluation is too complex to be carried out with the tools commonly available. Bubble and spark chamber pictures from high energy physics experiments were perhaps the only class of pictures systematically evaluated quantitatively. The complexity and the intensity of effort required in this quantitative analysis can be judged by the fact that in any bubble chamber experiment the analysis of pictures is the most costly and time consuming part of the experiment and in spite of an intense effort sustained over a couple of decades it has not yet been fully automated, CERN(1979). In part, this is the reason for developing alternate filmless detectors e.g. wire proportional chambers, acoustic chambers etc. Quantitative analysis of bubble chamber pictures, though highly advanced, is dedicated to the very special problems and aims of high energy physics and the methods developed are not suitable for general applications. Analysis of astronautic functional imagery (Landsat, Weather maps etc.) is also in a very advanced stage, SWAIN(1978), but is again highly dedicated and thus unsuitable for general use. Analysis of other types of functional images such as interferograms or X-ray images, is done on an ad hoc basis.

On the other hand, in the last two decades digital image processing has evolved into a highly developed discipline embodying sophisticated techniques and it has been successfully applied to diverse fields. However, most of the digital image processing activity is directed to pattern recognition (PR) and allied problems. Here much of the emphasis is placed on image restoration and enhancement, feature extraction and classification etc. whereas metrologic aspects such as parameter estimation, numerical accuracy etc. are seldom considered. Thus a large number of digital image processing operations have been developed and much is known about their qualitative effects on images. The metric properties of these operations, i.e. how does a given digital image processing operation alter the value of a certain physical parameter to be measured from a functional image, are rarely studied. There are, of course, industrially oriented digital image processing software packages for quality control and alignment check that are based on elementary numerical operations, ENGLE(1982). The general question of metrology and PR has been discussed by VERHAGEN(1982).

Functional image metrology, defined as the art and science of measuring physical quantities from functional images, has not yet developed into a uniform and coherent discipline of general appeal. For reasons of computational facility and potential automation digital image processing has to be an integral part of functional image metrology. This calls for developing digital image processing techniques that aid in extracting quantitative features, e.g. maxima-minima, zero crossing points etc. Questions of noise management, numerical accuracy, discretization errors, and parameter estimation in the context of digital image data should be studied, DORST(1982).
The general question of functional image metrology and digital image processing is much too broad to be discussed here and instead we shall confine ourselves (though not exclusively) to the development and application of digital image processing techniques to the analysis of optical interferometric patterns. Our emphasis here will be on eliciting metrologic information from such patterns and accordingly the technique developed here could be designated as Digital Interferographic Metrology, abbreviated to DIM.

Historically, optical interferometry has been one of the most precise metrologic techniques. The very origin of modern physics owes a debt to the precision of the interferometric experiment of Michelson and Morley. By the middle of this century interferometry had made great strides, however as an industrial tool it eluded wide scale exploitation. Its applications remained largely confined to the esoteric domain of high precision optical engineering or industrial research and development. There were several factors that prevented interferometry from becoming a routine industrial practice. First of all it required optical components of a high quality as also exacting tolerances in the optical instrumentation and control of environmental conditions. Moreover, the light from sources then available had very small coherence length and thus severely limited the possible applications. Last but not least a quantitative analysis of interferograms is an arduous task. The interferograms not originating from surfaces of the simplest geometry are very complex. Most often in such cases their quantitative analysis is very difficult if not altogether impossible. As a result most of the industrial applications of interferometry are largely as go/no-go quality controls rather than as a measuring tool. Fizeau rings for the analysis of spherical lens surfaces or gravity-induced distortions produced in large telescope mirrors are just two notable examples of industrial applications of interferometry.

The development of the laser and holography, LEITH(1980), THOMPSON(1980), in the early 60's opened up new prospects for interferometry. The laser provided the bright light of long coherence length so essential for many problems. Holography made it possible to record the complete wave front from which the original object could be reconstructed. In contrast, the traditional interferometry does not allow this reconstruction. Laser and holography allowed the possibility of studying objects complex enough to be of practical interest. However the information density in a hologram is far higher than that in an interferogram, although much of this information may be metrologically redundant. A quantitative analysis of a hologram is thus even more difficult than that of the erstwhile interferogram, since a much larger amount of information has to be processed. In this sense holography itself is not, metrologically speaking, more attractive and one seldom attempts a direct quantitative analysis of a hologram. The development of holographic interferometry solved several of these problems.

HORMAN(1965) was the first one to suggest the application of holography to interferometry. Stetson and Powell were the first to report the vibration analysis of diffusely reflecting objects, POWELL(1966), and shortly thereafter double-exposure and real-time holographic interferometry was developed in several places. This includes the work of BURCH(1965), COLLIER(1965), and HAINES(1966). In holographic interferometry, as the name implies, two optical fields are made to interfere. One field is obtained from the unperturbed state of the object and the second one with one or more physical variables (temperature,
strain, etc.) changed. Using diverse optical schemes (single reference beam, multiple reference beam etc.) these two states of the object are reconstructed and made to interfere. This can be likened to an analog-parallel subtraction of two very complex patterns. The resulting pattern, namely the holographic interferogram superposed over the image(s) of the object, contains information which is directly related to the changes in the states of the object. The analysis of an interferogram is thus much simpler than that of the hologram itself. In particular if the optical configurations were identical when the holograms were taken then the complications due to optical imperfections would be largely absent from the interferogram. This allows one to somewhat relax the quality of the optics and at the same time rendering the analysis easier to perform. In spite of these advantages holographic interferometry is still not widely accepted in engineering practice. There are several reasons for this. Among others, the degree of vibration-free stability demanded is still incompatible with the realistic industrial environment, ABRAMSON (1977). Further the quantitative analysis of even a simple holographic interferogram remains a slow and tedious process. Other problems besides, for holographic interferometry to be widely accepted it is necessary that electronic data acquisition and digital data processing be incorporated into it.

Introduction of digital techniques to holography has become a subfield in its own right, YAROSLAVSKII (1980). However, much of this activity is devoted to optical computing and computer generated holograms. Digital analysis of holographic interferograms has also been a subject of active interest and impressive results have been obtained by BELLANI (1974), BIEDERMANN (1977) and DANDLIKER (1980). However, in these techniques before embarking upon digital analysis the interferogram is first subjected to an optical data processing step. This step entails a nontrivial modification of the commonly employed optical setup. It is an exciting area of research but still far removed from the industrial routine. On the other hand the simple single reference beam off-axis holography is frequently used in industrial research and development and has become a valuable NDT (non-destructive testing) tool, VEST (1979), (1980). Quantitative analysis of interferograms obtained from such work can be greatly facilitated and to a large extent automated if techniques of digital image processing could be employed, TAKEDA (1982). Our aim is to contribute to the development of a technique for an automated analysis of such interferograms. Ideally one should begin with a single interferogram and carry out automatically all the relevant data processing steps to calculate the numerical value of the physical parameter. In general this would involve an interferogram reader (to be called a sensor), a general purpose digitizer and data processor and a dedicated software package to carry out the calculations specific to the physical problem at hand.

Keeping in with the advances in solid state physics and processing technologies, electronic video sensors have experienced a significant evolution in the last decade. A wide variety of sensor characteristics is now available to meet diverse needs. However the photographic film remains the single predominant medium for holographic recording. Several attempts have been made to replace the traditional film in holography and allied fields e.g. thermoplastic film based holography has received much attention, HANDOJO (1979). However, if digital processing of the image is envisaged, a video sensor is favored on account of its ready compatibility with electronic data processing. Numerous attempts at various levels, with varying degree of success, have been made in this direction. On the one hand one has the commercially available electronic speckle pattern
interferometer (ESPI) based on analog techniques of video signal recording and subtraction, BUTTERS(1978). On the other hand one has exploratory smart sensors for image analysis, VAHEY(1979), INIGO(1981). However, holography with its demands on high resolution (about 10,000 pixels per line) remains the most intractable. A good quality commercial TV-sensor has a limit of about 1000 pixels per line which is adequate for many interferometric applications but not enough for holography. A linear array with mechanical scan can deliver up to 4k pixels per line, SPRAGUE(1982). This is still no match for the high resolution film albeit the extra complication due to the mechanical scan. In any case, the fact remains that the general problem of interferogram analysis by direct TV-sensor has not received the attention it is due, SMIGIELSKI(1979). As a matter of fact the choice of an optimal sensor would remain a key element in the future of holography, particularly in view of the emerging smart sensors, ROBERTS(1977), and BARBE(1980). Important though this question is we shall, however, not address it directly.

Interferometric patterns possess, at least in principle, a well-defined structure. The noise present in these patterns has also certain characteristics e.g. speckle, parasitic rings etc. Both the structural systematics and the noise characteristics form a substantial pool of a priori knowledge which can be used to great advantage in designing some general purpose data processing algorithms. Several methods have been developed to deal with particular cases, KREITLOW(1979a), KREITLOW(1979b), HOT(1979a), HOT(1979b), NAKDATE(1980a), NAKDATE(1980b). These employ various combinations of hardware-software techniques to arrive at the desired result. A general discussion of either the structural systematics or the noise characteristics in the digital context is however lacking. Just as workers in Interferometry have not considered the interferogram analysis from a general digital image processing point of view, the quantitative analysis of these patterns has also escaped the attention of workers in the highly developed fields of Pattern Recognition and Digital Image Processing. A comprehensive review of this field is given by Verhagen(1980). Under possible applications of digital image processing one rarely, if ever, sees holographic interferometry as a potential candidate, HUANG(1979), PRESTON(1979), NIEMAN(1979). Edge detection, texture determination, noise filtering etc are very well established techniques in digital image processing, however they are seldom translated into the interferometric context to help determine fringe maxima, speckle pattern analysis, speckle noise reduction etc. In developing DIM, our task will therefore be twofold. We shall first consider the general problem of structure in digitized interferometric patterns and the digital signature of the noise commonly encountered in such patterns. Secondly we shall develop some digital techniques which can be used for processing such patterns.
Fig. (2,1) Functional Image Formation.
CHAPTER 2

THEORETICAL BACKGROUND

2.1 'INTRODUCTION

Functional images that require quantitative analysis differ widely in their origin and characteristics. With the current techniques it is not practicable to devise a single universal program to analyse such a diverse variety of patterns. However, it may become possible to develop a limited number of subprograms which can be combined in different ways to analyse different types of patterns. In general, functional images can be classified according to the physical phenomenon being studied and the imaging technique and interpretation employed. In Fig. (2,1) several examples of physical parameters and imaging techniques are shown. Functional images originating from a given combination of physical phenomenon and imaging technique would share common features and a broad scheme for the analysis of this particular type of functional image could be developed. Analysis of a given functional image, as carried out here, starts with the realization that the physics underlying the phenomenon and the imaging technique is understood well enough to allow the development of a mathematical model that can, at least in principle, describe the intensity distribution in the functional image. This set of mathematical steps or ALGORITHM, which translates the physical parameter distribution into a functional image intensity distribution will be called the STRUCTURAL ALGORITHM (SA) of the functional image. Thus if we scan a functional image along a given line then the SA gives the grey-value distribution that can be expected on theoretical grounds. Only one-dimensional SA's have been studied here, though their extension to two dimensions is quite feasible, SCHUESSLER(1979).

From the SA one has to derive another algorithm which does the opposite, namely, it translates the functional image intensity distribution into the physical parameter distribution. This algorithm will be called the RECONSTRUCTIVE ALGORITHM (RA). Construction of an RA places some special demands and constraints. First of all quite generally, the RA has to be compatible with digital processing so as to avoid excessive computing time and/or accumulation of numerical errors. Perhaps a more serious theoretical problem is posed by the fact that some essential parameters used in the SA are 'trapped'. These trapped parameters are indispensable in calculating the intensity distribution in the functional image, however, their values are, in principle, not extricable from the functional image. This may include such parameters as position of the instrumental axes with respect to the functional image, absolute fringe order, phase shifts, magnification factors, photographic film processing influence, etc. KLEIN(1979). A lack of knowledge about some of these factors could sometimes be fatal, but if enough information about the underlying factors is available then in most of the cases a suitable RA can be constructed. Needless to say that there is no general strategy to tackle such problems and each new RA has to be devised on a case by case basis. Some examples of RA's so constructed will be discussed in Chapters 3, and 4.

Even if a suitable RA for a particular functional image has been devised its analysis cannot commence directly since almost all functional images are degraded by noise. This noise can originate in the physical phenomenon, the imaging technique, and in the image digitization by the
Fig. (2,2) Interference of Two Point Sources.

Fig. (2,3) Interferometric Comparison of Two Optical Systems.
TVS. These encompass a very large variety and thus a general statement about noise in functional images cannot be made. Since the present study has been limited largely to the analysis of interferometric functional images, only noise encountered in such images, which is mostly related to the coherence of light used, will be discussed.

Quantitative analysis of an interferometric functional image (henceforward to be called an image or a pattern), as carried out here, is almost always based on position and order of the fringes. An accurate determination of these two becomes a major goal of the analysis. Noise that hinders this step has to be managed somehow, whereas other types of noise that degrade the cosmetic quality of the image can largely be tolerated. The two most serious types of noise are SPECKLE and MISSING FRINGE. The former can influence the apparent position of a fringe. On the whole, missing fringes occur much less frequently, however, when they do occur, perturbation in the fringe order is generally quite serious and some sort of action becomes mandatory. To further discuss the details of the nature and sources of noise it is essential to recall some elements of interferometry. Since most of the concepts basic to optical interferometry are not an integral part of the digital image processing lore, a brief review of the former will be attempted. Thus the following review of interferometry is intended merely to establish the notation and introduce some concepts relevant to the analysis. For a more detailed introduction to the subject one of the several standard texts could be consulted, GOODMAN(1968), KLEIN(1970), HECHT(1979), FRANKENA(1979).

2.2 OPTICAL BACKGROUND

Interferometry, as practiced currently, is a highly developed field including several specialized sub-fields e.g. holographic interferometry, speckle interferometry etc. In spite of this great diversity in interferometric techniques it is possible to introduce most of the relevant concepts by examining a two-dimensional Young's experiment on the interference of light originating from two point sources. Other specific details such as those related to holography will be discussed as needed.

In Fig.(2,2) P and Q are two monochromatic point sources of light of wave number k, surrounded by an isotropic, homogeneous medium. Consider the electric field from these sources at a point S whose position along the line OX is given by x = OS. Let the field due to P as received at S at time t be given by E(P)cos k(p-ct+u(P)) where

- E(P) = amplitude of the field due to P,
- p = distance SP,
- c = velocity of light,
- u(P) = arbitrary phase factor.

We assume that the distance p is large enough and thus one can neglect the variation in E(P) due to the variation in p. It is also assumed that E(P) is constant in time.

Similarly the field due to Q at S can be written as E(Q)cos k(q-ct+u(Q)). Omitting polarization effects the total field at S, E(x), is
\[ E(x) = E(P)\cos k(p-ct+u(P)) + E(Q)\cos k(q-ct+u(Q)). \] (2.1)

The experimentally measured intensity \( I(x) \) is directly proportional to the time average of \( E(x)^2 \), where \( E(x)^2 \) is the square of \( E(x) \). Ignoring a numerical factor the intensity at \( x \) is

\[ I(x) = \langle E(x)^2 \rangle, \] (2.2)

where \( \langle \rangle \) denotes a time average over an interval much larger than \( 1/(\text{light frequency}) \). From Eq. (2.1) and Eq. (2.2) we get

\[ I(x) = \langle (E(P)\cos k(p-ct+u(P)) + E(Q)\cos k(q-ct+u(Q)))^2 \rangle. \] (2.3)

This gives

\[ I(x) = I(P)\langle \cos k(p-ct+u(P)) \rangle^2 + I(Q)\langle \cos k(q-ct+u(Q))^2 \rangle + R, \]

where \( I(P) = \langle E(P)^2 \rangle \) and \( I(Q) = \langle E(Q)^2 \rangle \) and \( R \) is the interference term

\[ R = E(P)E(Q)\langle \cos k(p-ct+u(P))\cos k(q-ct+u(Q)) \rangle. \]

With

\[ \langle \cos kct \rangle^2 = \langle \sin kct \rangle^2 = 1/2 \]

and

\[ \langle \cos kct \rangle = \langle \sin kct \rangle = 0, \]

we have

\[ I(x) = A + B\cos k(p-q+u), \] (2.4a)

where \( A = (I(P)+I(Q))/2 \) and \( B = E(P)E(Q) \), both of these are constant here and \( (p-q) \) is the optical path difference. \( u(P)-u(Q) \) is the constant phase factor difference between the two sources. In practical situations this difference is always present and it is seldom, if ever, known and the intensity distribution of an interferometric pattern contains this unknown phase factor difference as in Eq. (2.4a). Henceforward, unless otherwise stated, this unknown phase factor will be designated by \( u \). Thus Eq. (2.4a) can be written as

\[ I(x) = A + B\cos k(p-q+u). \] (2.4)

The intensity maxima, henceforward called FRINGES, occur where the following is valid

\[ (p-q) + u = n\lambda. \] (2.5)

Here \( \lambda \) is the light wavelength and \( n \) is an integer and is called the FRINGE ORDER. The path difference \( (p-q) \) is dependent on the position \( x \) and to make it explicit we introduce a phase function \( P(x) \) defined as
\[ P(x) = \frac{(p-q)}{\lambda}. \]  

(2.6)

If the \( n \)-th fringe occurs at \( x(n) \) then

\[ P(x(n)) + \frac{u}{\lambda} = n. \]  

(2.7)

The visibility of the fringes, \( V \), is given by

\[ V = \frac{(I_{\text{max}} - I_{\text{min}})}{(I_{\text{max}} + I_{\text{min}})}. \]  

(2.8)

For the fringes discussed here, Eq.(2.4), we obtain

\[ V = \frac{B}{A}. \]  

(2.9)

The metrologic merit of this interference pattern can be appreciated by examining Fig.(2,3) where two systems \( S_1 \) and \( S_2 \) are inserted as shown. All other things being equal, if \( S_1 \) and \( S_2 \) are identical then the pattern from Fig.(2,2) is identical to the one from Fig.(2,3), except perhaps for a change in intensity. However, if \( S_1 \) and \( S_2 \) cause different changes in the optical paths \( p \) and \( q \), then the two patterns are different. This difference is quantitatively related to the differences in the optical properties of \( S_1 \) and \( S_2 \). As a further step consider a 'subtraction' of two patterns, one of which corresponds to the state \( S_1 \) being identical to \( S_2 \) and the other when they are different. The signal in the 'subtracted' pattern is related only to the difference between \( S_1 \) and \( S_2 \). By choosing a proper optical technique e.g. holographic interferometry, one does not need two separate systems \( S_1 \) and \( S_2 \) as shown but the difference between two states of the same system is observed. The pattern so obtained is analysed to evaluate the quantitative difference between the two states of the system. For this analysis, equations similar to Eq.(2.4) serve as the basis. Taken literally Eqs.(2.4) and (2.9) give an infinite pattern of constant visibility, since \( A \) and \( B \) are assumed to be constants. In practice such is never the case and the reasons therefore are manifold as discussed below.

For the pattern to be stationary in time the phase factor difference \( u \) must be constant i.e. the two light sources must be mutually coherent. Two separate optical sources do not satisfy this condition. In practice the two sources are derived from a single source through amplitude or wave front division, using a realistic finite extent light source, light beams of non-uniform profile, optical components such as lenses, slits, stops etc. All of these contribute complications through integration over finite sources and apertures. Furthermore, if the optical path difference \( (p-q) \) is large, partial temporal coherence effects (through the finite line-width of the source) become important. As a result both \( A \) and \( B \) become functions of numerous optical parameters and \( x \). A detailed discussion of these effects can be found in THOMPSON(1957), MANDEL(1965), LEVI(1980). For our purpose it suffices to say that since \( A \) and \( B \) in Eq.(2.4) are in general not independent of \( x \), the visibility is therefore also not constant as it seemed earlier. In a realistic situation \( V \) depends on so many different parameters that a detailed calculation of it is impractical and one has to start with some reasonable assumption about \( V \). Unless extreme conditions prevail one can assume that the visibility \( V \) changes only gradually and fractionally from one fringe to the next. To make these observations explicit we rewrite Eq.(2.4) and Eq.(2.6) as

\[ I(x) = A(x) + B(x) \cos k( P(x)+u), \]  

(2.10)
where $A(x)$ and $B(x)$ are generally unknown but assumed to be slowly varying as mentioned above. Further details about their nature will be discussed later. $I(x)$ represents the grey values measured by scanning the pattern along a line in the $x$-direction. $P(x)$ contains the information relevant to the physical state under study and finding it is the goal of further analysis. $P(x)$ may have an arbitrarily complicated form but maxima of the cosine-term occur at $x(n)$ which satisfy Eq.(2.7). In general, one tries to find $x(n)$ from the data and then interpolates a physically acceptable form of $P(x)$ and this is termed as determination of $P(x)$. For reasons of computational expediency and if physically acceptable, linear and quadratic forms of $P(x)$ are tried first. In Eq.(2.10) to eliminate the unknown $u$ we note that for two adjoining fringes of order $n$ and $n+1$

$$P(x(n+1)) - P(x(n)) = 1, \quad (2.11)$$

or in general for fringes of order $n$ and $n+m$

$$P(x(n+m)) - P(x(n)) = m. \quad (2.12)$$

The right hand side of Eq.(2.12) depends only on $m$, the relative fringe order, and $n$ can have an arbitrary value. This will be used for determination of $P(x)$. In Eq.(2.12) along with $u$, the $x$-independent term (dc-term) in $P(x)$ is eliminated and cannot be determined from this relation. In all the practical situations encountered in this study, the absolute phase, and hence the dc-term, could not be determined. This has to be accepted even though a lack of its knowledge may cause some difficulties e.g. in determining the position of the hot plate as will be discussed in Chapter 3.

The local intensity extrema are given by

$$dI(x)/dx = 0. \quad (2.13)$$

These extrema can be determined from $I(x)$ but due to the possible influence of $A(x)$ and $B(x)$ the physically interesting $x(n)$, from Eq.(2.7), may not be identical to those of Eq.(2.10). Before discussing such details the review of interferometric elements will be continued with a word about speckle as noise.

Coherent light incident on a rough surface, with irregularities greater than the light wavelength, produces a mottled or speckled pattern. Light scattered from different parts of the surface is mutually coherent but with a random modification in phase and amplitude. Interference of this light produces speckle. In laser based interferometry it is always present as noise and was considered as enemy number one. Its properties have been exhaustively studied, GABOR (1970), DAINTY (1975), GOODMAN (1976), ARSENAULT (1976), FROEHLY (1979). Here, only a brief description of some of the salient features of speckle patterns will be discussed. For our purpose the three most important characteristics of speckle are its irradiance, contrast and spatial frequency. The following is not supposed to be a full or rigorous treatment of these factors but is intended to merely give a general idea about their origin and nature.

In Fig.(2.4) laser light, of amplitude $b$ and intensity $B$, is scattered by a rough surface. The scattered light is received by a point detector $D$. The rough surface can be assumed to be an ensemble of scatterers each of which modifies the phase and the amplitude of the incident light.
Fig. (2,4) Speckle Formation.
independent of each other. Thus the light scattered from two different scatterers has a fixed but arbitrary phase difference. Light received by D is an ensemble average over all the scattering elements of the surface. Let $U(j,x)$ be the complex amplitude of the light scattered by the jth scatterer and received at x, the position of D. $U(j,x)$ is given by

$$U(j,x) = a(j,x)b \exp(-iu(j,x)), \quad (2.14)$$

where $b$ is the real amplitude of the incident light and assuming linear effects, the scattered amplitude is directly proportional to the incident amplitude $b$. Let $a(j,x)b$ be the amplitude and $u(j,x)$ the phase of the light scattered by j and received at x. The total amplitude $U(x)$ at D is a sum over all the scatterers as given by

$$U(x) = U(1,x) + U(2,x) + U(3,x) + \ldots \quad (2.15)$$

Both $a(j,x)$ and $u(j,x)$ are random and $U(j,x)$ is a complex random vector. From Eq.(2.15) $U(x)$ is a sum of complex random vectors. It can be shown that this problem is the same as a random walk in the complex plane and it is best described by an irradiance probability density function $R(I)$. The speckle intensity $I = U^*U$, where $U^*U$ is the product of $U$ with its complex conjugate, is directly proportional to $B$, the intensity of the incident light. $R(I)$ is an exponential distribution as follows

$$R(I) = \left(\frac{1}{<I>}\right) \exp\left(-\frac{I}{<I>}\right). \quad (2.16)$$

Here $<I>$ is the average intensity. The most probable speckle intensity is $I = 0$ i.e. a dark spot. This implies that the speckle shifts the pattern histogram to darker grey values. Based on this fact there are several techniques to enhance the quality of speckled patterns, GUENTHER(1978), LIM(1981). Note that from Eq.(2.16) the standard deviation of the intensity and the average intensity are both $<I>$. The speckle contrast $C$ defined as the ratio of the standard deviation to the average intensity is evaluated as follows:

$$C = \frac{<I>}{<I>} = 1. \quad (2.17)$$

Thus we find that the speckle contrast is unity.

Spatial frequency of speckle can be estimated by recalling that $f$, the spatial (angular) frequency of the fringes formed by two sources separated by a distance $d$ and observed at a distance $z$ is given by

$$f = kd/z. \quad (2.18)$$

This has to be summed over all $d$'s, the pair separations. The number of pairs with a given $d$ is inversely proportional to $d$ in the range $0 \leq d \leq L$ and its average is $L/3$, where $L$ is the overall dimension of the scattering surface. The average spatial frequency is given by

$$f = kL/3z. \quad (2.19)$$

Thus scanning a speckle pattern along a line the intensity distribution $I(x)$ is expected to be

$$I(x) = B + Bo\cos k(u + Lx/3z). \quad (2.20)$$
Here $u$ is an arbitrary phase factor. In a realistic situation different surfaces (different L's) situated at different distances (different z's) contribute to the speckle field and thus one cannot expect to predict the precise nature of the speckle field in a simple manner as given by Eq.(2.20). However, one can get some idea about the average speckle size and compare it with the fringe separation expected from Eq.(2.12). In general the optical system is so designed that the speckle size is appreciably smaller than the fringe separation. This is vital to the speckle removal through filtering as will be discussed later. For common experimental conditions this indicates that speckle may appear in the intensity distribution as narrow peaks of high contrast, to be called SPIKES. It is quite possible that these spikes may be mistaken for the 'real' signal peaks (henceforward unless otherwise stated PEAKS would imply 'real' signal peaks). Special provisions, to be discussed later, must be provided to minimize this misidentification. It might be emphasized here that our interest in speckle is only in as much as it affects or hinders the determination of $x(n)$'s, whereas the cosmetic degradation of the image caused by it is of no great concern. In brief, speckle as noise is expected to be high-frequency, high-contrast and multiplicative (signal dependent).

A MISSING FRINGE is not noise in the usual sense of the term. It is more of a pattern pathology of unknown etiology since there is no unique well defined mechanism that causes it. It could be a shadow, an accidental speck of dust, inhomogeneity in photographic processing etc. However it may creep in, a missing fringe can seriously jeopardize the quantitative analysis. Suppose that the fringe next to the j-th order fringe is missing i.e. the (j+1)-th fringe is missing. If unchecked, the fringe of rightful order j+2 will be assigned the order j+1. This discrepancy can lead to severe numerical errors. There is no general method to correct for it except that in some cases it is possible to detect a missing fringe, JONES(1968), as will be discussed in the next section. Once detected, appropriate steps, depending on the physical nature of the problem under study, could be taken to secure the numerical integrity of the calculation.

A few general conclusions that can be drawn from this cursory review of the optical fundamentals, are

- almost all interferometric SA's have the general form of Eq.(2.10),
- determination of $x(n)$'s and the corresponding n's is the major goal of the analysis,
- speckle as noise is to be expected in laser based experiments and its influence on $x(n)$ determination should be carefully studied,
- effects of missing fringes as unavoidable accidentals should be anticipated and if possible measures should be taken to forestall its serious impact on numerical calculations.

The general scheme to analyse a pattern now proceeds along the following lines
- scan a line to get $I(x)$,
- manage noise and try to determine maxima, $x(n)$,
determine \( P(x) \) by fitting appropriate forms to \( n \) and \( x(n) \).

The last step cannot be discussed in general since it depends on the particular problem under study. Scanning a line to get \( I(x) \), with appropriate hardware as discussed by PALMIERI(1971) and HITOSHI(1981), is a routine operation and needs no further elaboration. The bulk of the effort is directed to the general step of NOISE MANAGEMENT and MAXIMA DETERMINATION as discussed in the next section.

2.3 MAXIMA DETERMINATION

An accurate determination of fringe positions, through intensity maxima determination, is essential to interferometry. This accuracy, for a given method, is expressed as a fraction of the fringe distance. Thus already in 1887 an accuracy of 0.02 (a fiftieth of a fringe) was attained in the Michelson-Morley experiment. Accuracy of 0.01 was also achieved in multiple-beam interferometers e.g. Fabry-Perot, at the turn of the century. Recently impressive advances in new techniques such as heterodyne interferometry have pushed this limit, even in single-beam interferometers, to beyond 0.01, DANDLIKER(1976), (1977), MASSIE(1979), (1980). Most of these techniques are real time since one looks at a given fringe and measures its relative displacement as the state of the system changes, DANDLIKER(1981). Here the problem is slightly different. One gets a single pattern with noise and the physics is related to the distance between fringes. Measuring the relative shift of a given fringe and measuring the distance between fringes are two different problems that call for different techniques.

To formulate the problem more precisely a brief recapitulation of some of the points discussed earlier follows. Physically interesting maxima \( x(n) \) are the cosine-maxima, Eq.(2.12) which have to be determined from the data \( I(x) \) of Eq.(2.10). The local intensity extrema of \( I(x) \), Eq.(2.10), can be determined automatically and unambiguously, and without a priori knowledge. These are designated by \( X(n) \) and are the zero gradient points of \( I(x) \). In general the difference between \( x(n) \) and \( X(n) \), \( s(n) \), defined as

\[
s(n) = X(n) - x(n),
\]

is nonzero. \( x(n) \)'s cannot be obtained from \( X(n) \) automatically or unambiguously without making some assumptions and/or deliberate choice in the subsequent analysis. The general problem can now be stated as: for a given experiment is it possible to estimate \( s(n) \) and then derive \( x(n) \) from \( X(n) \)? Since \( s(n) \) depends on so many different parameters and noise, it is not possible to develop a general technique. In the following some aspects of this problem are studied. First consider the difference between \( x(n) \) and \( X(n) \) as caused by 'systematic' factors such as the \( x \)-dependence of the shading \( A(x) \) and the modulation \( B(x) \) in Eq.(2.10) which affect the fringe visibility. As a first step we study a pure sine pattern of constant amplitude \( B \). Let \( q \) be the spatial wave length of the pattern (the constant fringe separation), and \( K \) the corresponding 'wave number' of the pattern. Assuming zero arbitrary phase, analogous to Eq.(2.10), we have

\[
I(x) = A(x) + B \cos (Kx)
\]

and the fringe visibility \( V \), from Eq.(2.9), is

\[
V = B/A(x).
\]
With Eq.(2.22) the 'zero gradient' maxima $X(n)$ are given as solutions of

$$A'(x) = KB\sin(Kx), \quad (2.24)$$

where prime(') denotes differentiation with respect to $x$. Eq.(2.24) with Eq.(2.23) immediately leads to

$$\sin(Kx) = L(A)/VK, \quad (2.25)$$

where $L$ is the log-differential operator

$$L(A) = \frac{d(\ln(A))}{dx} = \frac{A'}{A}. \quad (2.26)$$

In practice quite frequently $A(x)$ is a Gaussian function, say with a mean $c$ and variance $v$, then $L(A) = -(x-c)/v$. This is a very simple form for interpreting shading effects. The cosine maxima $x(n)$ are given by

$$x(n) = nq. \quad (2.27)$$

Here $n$, the fringe order, is an integer. Eq.(2.25) is a transcendental equation whose multiple roots are the $X(n)$'s, with coordinates $(X(n), L(A)/VK)$. The $x(n)$'s are the 'zero crossing' points of the 'sine term', with coordinates $(x(n), 0)$. At these points the slope is $K$. From Eqs.(2.23) and (2.25) and making the approximation that the line connecting a $X(n)$ with the corresponding $x(n)$ has a slope of $K$ then the difference between $X(n)$ and $x(n)$ can be shown to be

$$s(n) = X(n) - x(n) = L(A)/(VK^2). \quad (2.28)$$

Thus with a given background $A(x)$, low visibility fringes, $x(n)$'s, differ (are shifted) from $X(n)$'s more than the high visibility ones. Furthermore, this shift has the sign of $L(A)$ and thus it can be both positive and negative within the same pattern. The fact that the shift can vary within the same pattern is important since a constant shift of all the fringes is not relevant.

Consider now the general case where the linear argument $Kx$ is replaced by a function $K(x)$ in Eq.(2.22). We find that the zero gradient points are given as solutions of

$$\sin(K(x)-b) = (\cos b)L(A)/(VK'), \quad (2.29)$$

where $\tan(b) = L(B)/K'$. In Fig.(2,5), using arbitrary units and an arbitrary form of $K(x)$, the thin line is $\sin K(x)$ and thus its zero at $P$ gives an $x(n)$. The medium line is $\sin (K(x)-b)$ i.e. $\sin K(x)$ phase shifted by $b$, Eq.(2.29). The crossing point of $\sin (K(x)-b)$ and the thick line, $(\cos b)L(A)/(VK')$, gives $Q$. This gives $X(n)$, the absolute intensity maximum corresponding to the cosine-maximum $x(n)$. The 'maxima shift' $s(n)$ is thus $PQ$. $Q$ will be measured as the intensity maximum whereas $P$ is the desired maximum. In principle one could calculate $P$ from $Q$ if one knows $A(x)$, $B(x)$ and $K(x)$. Such a detailed knowledge if available would render the entire pattern analysis superfluous. In practice, for want of such detailed knowledge, all what one can do is to make 'intelligent guesses' about these and estimate the shift and then decide upon a proper action e.g. background-estimation and subtraction. An example of this will be discussed in Chapter 4.
FIG. (2.5) INTENSITY EXTREMA SHIFT.

\[ \sin K(x) \]

\[ (\cos b)L(A)/VK' \]

\[ \sin (K(x)-b) \]

FIG. (2.6) VISIBILITY CHANGE DUE TO A FILTER.

- SIGNAL
- FILTERED SIGNAL
Sometimes \( I(x) \) has constant \( V \) (visibility) overall but may have some small sharp peaks. It is quite common and acceptable to use a 'smoothing-filter' e.g. moving-average, to filter out these peaks. Besides smoothing, such filters can change the visibility over an otherwise constant visibility pattern. In Fig.(2,6) the thin line shows the constant-\( V \) \( I(x) \) and the thick line the same after a moving-average filter has been applied to it. In the filtered signal \( A(x) \), the shading, is constant but the modulation \( B(x) \) is no longer so. This change in \( B(x) \) depends on the filter window used and the nature of the signal. For a given filter window, in a variable frequency signal the modulation of the 'higher frequency' part of the signal is affected more than that of the 'lower frequency' part as seen in Fig.(2,6). In principle, if the nature of the signal is known then one can calculate the modulation of the filtered signal for a given filter window. Since \( A(x) \) is constant \( L(A) \) is zero and the right hand side of Eq.(2.29) is also zero. The shift of the maximum is now governed by the phase shift \( b \) only. Of course, since \( \tan(b) = L(B)/K \), \( b \) depends not only on \( K \) but also on the changed \( B \). Note, that before filtering \( L(B) = 0 \), and after filtering the change in \( B \) depends on the filter used. Again to calculate this shift one has to know \( K(x) \) but this is exactly what one is after and hence only an 'intelligent-guess' can be made about the influence of filters.

The shifts caused by these 'systematic-factors' can at least be estimated or foreseen and perhaps some action can be taken. For example in an interactive program one could filter out the real signal and thus get an approximation to \( A(x) \). This can be fitted with a Gaussian to find \( L(A) \). This approximate \( A(x) \) can also be subtracted from the signal to keep only the \( B \)-term. To this signal one can try to fit an envelope to get an approximate form of the modulation \( B(x) \). From the above \( B(x) \) one can 'correct' the modulation and get a signal 'without' shading and modulation. Maxima of this signal could be taken for \( x(n) \) and compared with \( X(n) \). Of course, if shading and modulation are serious then perhaps the best solution is to redesign the optics. Now onwards we shall assume that such corrections for shading and modulation are applied when necessary and accordingly all the maxima would be assumed to be 'cosine maxima' unless explicitly specified otherwise.

Even in a well-designed optical system the effect of spikes and missing fringes, as discussed earlier, can remain important and if these are anticipated some preventive measure, to be called NOISE MANAGEMENT, becomes essential.

Noise management comprises of the following steps

- determination of \( x(n) \),
- classification of \( x(n)'s \) into peaks or spikes,
- check for missing fringe(s),
- remedial action e.g. filtering.

Details of executing these steps will depend on the pattern but the general outline universally followed is discussed here. Determination of \( x(n) \) is fairly straightforward and begins with writing \( I(x) \) in digital form. In digital image processing maxima determination is not as extensively studied as, for example, edge detection. Maxima determination does not even merit a mention in many of the standard texts on the
subject, MEISEL(1972), ROSENFELD(1976), GONZALES(1976). Sometimes in the context of texture analysis 'hysteresis' methods based on 'gear-backlash' are discussed, MASON(1968), MITCHELL(1978), DUDA(1978). These are essentially 'differential smoothing' techniques where successive grey value differences (positive or negative), as produced by say 'small' spikes, are 'smoothed' to zero. The resulting signal is 'flatter' and 'shifted'. For texture analysis this is not a serious perturbation but for metrologic work this is unacceptable. This calls for developing other techniques which are more compatible with quantitative analysis as discussed below.

First we replace the continuous intensity function \( I(x) \) by a discrete function \( I(k) \) at discrete pixels \( k=1,2,\ldots \). We define \( D(k) = I(k+1) - I(k) \). For the moment we assume that \( D(k) \) is nonzero which implies that there are no flat regions in \( I(k) \). The case of \( D(k)=0 \) will be discussed shortly. Next we define an operator \( \text{SIGN}(p,q) \) operating on two numbers \( p \) and \( q \) such that \( \text{SIGN}(p,q) \) is a number whose magnitude is equal to that of \( p \) and whose sign is that of \( q \). From this we define

\[
S(k) = \text{SIGN}(1, D(k)). \tag{2.30}
\]

In Eq.(2.30) \( S(k) \) is +1 or -1. We define another variable \( M(i) \) as

\[
M(k) = S(k)S(k-1). \tag{2.31}
\]

In Fig.(2.7) at points C and D (single pixel extrema)

\[
M(k) = -1, \tag{2.32}
\]

and elsewhere

\[
M(k) = +1. \tag{2.33}
\]

We further define

\[
G(k) = S(k)(M(k)-1)/2. \tag{2.34}
\]

From the above equations it follows that \( G(k) = +1 \) at C and \( = -1 \) at D and \( = 0 \) elsewhere, see Fig.(2.7). Values of \( k \) at which \( G(k) = 1 \) are the local maxima \( x(n) \).

Due to grey level quantization \( I(x) \) may not have single pixel maxima but instead 'flat-tops', having left and right edges, Fig.(2.8). Such quantization effects are well known in optical research, GOODMAN(1979). In principle in such cases it is not possible to find the 'true-maxima' without taking some deliberate action e.g. fitting a parabola, BARNAL(1977). Obviously there is no general rule for this and it has to be discussed individually. Whatever action is taken here it is possible to determine both the left and the right edges. In Fig.(2.8) C and D are 'leading-extrema' or right edges. In order to determine the left and right edges a little modification of the above technique is needed since this involves the case of \( D(k) = 0 \) in the 'flat regions' as discussed earlier. We define a function \( f(k) \) as

\[
f(k) = E(1/(1+\text{abs}D(k))).
\]

Here \( E \) is the ENTIER.
**FIG. (2.7) SINGLE PIXEL INTENSITY EXTREMA.**

**FIG. (2.8) FLAT TOP EXTREMA AND LEADING EDGES.**
Note that \( f(k) = 1 \) only if \( D(k) = 0 \), otherwise \( f(k) = 0 \). In other words \( f(k) = 1 \) only in the 'flat regions' and zero elsewhere. Now consider

\[
F(k) = f(k)S(k-1) + (1 - f(k))D(k).
\]

(2.35)

In Eq. (2.35) recalling that \( f(k) = 1 \) only in 'flat regions', and zero elsewhere we see that \( F(k) = S(k-1) \) in 'flat regions' where \( f(k) = 0 \), and elsewhere \( F(k) = D(k) \). Now we define \( S(k) \) as

\[
S(k) = \text{SIGN}(1, f(k)).
\]

(2.36)

Eq. (2.36) now replaces Eq. (2.30) as the definition of \( S(k) \). This new definition should be used subsequent to Eq. (2.30). This has the effect that if \( D(k) = 0 \), as in the 'flat regions', then \( S(k) = S(k-1) \). The program starts by finding the first 'non flat region' i.e. the first nonzero value of \( D(k) \) is found and the corresponding \( S(k) \) is found. Now the recursive relation \( S(k) = S(k-1) \) can be used, without ambiguity, whenever \( D(k) \) is zero. All of these steps leading to the calculation of \( G(k) \) can be combined into a single operation EXTrema which operates on the grey values \( I(k) \) and its output is +1 at the right edge of a maximum, -1 at the right edge of a minimum, and 0 elsewhere. For \( k = 1, 2, \ldots \)

\[
\text{EXTI}(k) = +1 \text{ at max right edge},
\]

and

\[
\text{EXTI}(k) = -1 \text{ at min right edge},
\]

(2.37)

and zero elsewhere. These values of \( k \) at which \( \text{EXTI}(k) \) is nonzero are stored in an array \( R(n) \). The array \( R(n) \) is thus the set of pixel values of the right edges.

Now we reverse \( I(k) \) into \( J(k) \) (henceforward, \( J(k) \) will denote transformed \( I(k) \), e.g. reversed, filtered etc.). Thus we have

\[
J(k) = I(k_{\text{max}} - k + 1),
\]

(2.38)

where \( k_{\text{max}} \) is the total number of pixels in a scan line i.e. \( k_{\text{max}} = 256 \) or 512 in our case. Now the left edges \( L(n) \) are given by considering the values of \( k \) at which \( \text{EXTJ}(k) \) is nonzero.

By using EXT on the 'direct' intensity \( I(k) \) and on the 'reversed' intensity \( J(k) \) one gets the entire 'range' of the extrema. From these one can define the 'true' maxima \( x(n) \) by choosing a certain rule. For example

\[
x(n) = (R(n) + L(n))/2,
\]

(2.39)

or

\[
x(n) = \text{CG}(R(n), L(n)),
\]

(2.40)

where \( \text{CG} \) is the 'center of gravity' of the region between \( R(n) \) and \( L(n) \). Of course one can define other rules at this stage.

Having determined \( x(n) \)'s in one or the other manner our task now is to check if all the \( x(n) \)'s are real peaks (and not spikes) and, if an otherwise expected peak is missing. This is noise management as discussed next.
2.4 NOISE MANAGEMENT

A given set of maxima, \( x(k) \), has real peaks along with noise in the form of spikes and missing fringe(s). To classify each maximum into a peak or spike and to locate a missing fringe requires some a priori knowledge about the function \( K(x) \). This could be in the form of a general assumption that can be made about \( I(x) \). For example an assumption that the signal \( I(x) \), analogous to Eq.(2.10), is a pure 'sine' of constant visibility i.e. constant \( A(x) \) and \( B(x) \), and arbitrary phase factor \( u \),

\[
I(x) = A + B \cos K(x + u),
\]

would be sufficient because a pure 'sine' pattern has equidistant fringes and one could check the fringe separation to detect noise. Let \( q \) be the constant fringe separation, then for this pattern one could define a phase function \( P(x) \) as follows,

\[
P(x) = (x+u)/q.
\]

For the maxima at \( x(n) \) we have

\[
P(x(n)) = (x(n) + u)/q = n,
\]

where \( n \) is an integer. For a 'sine' pattern the analysis is quite simple but we shall use it to develop the notation for later use. Let \( N \) be the total number of fringes in the pattern then we have

\[
x(n+1)-x(n) = q,
\]

where \( n = 1,2,...,N-1 \), is the fringe order. We define the first-order finite difference

\[
D(1,n) = x(n+1)-x(n).
\]

Further at each fringe we define the right and left distances to its neighbors \( R(n) \) and \( L(n) \) as

\[
R(n) = D(1,n)
\]

and

\[
L(n) = D(1,n-1).
\]

Note that \( R(n) \) is defined for \( n = 1,2,...,N-1 \), whereas \( L(n) \) is defined for \( n = 2,3,...,N \).

For \( n = 2,3,...,N-1 \), we define the second-order finite difference

\[
D(2,n) = x(n+1)-2x(n)+x(n-1) = R(n)-L(n).
\]

Thus for pure 'sine' fringes

\[
D(1,n) = q
\]

and

\[
D(2,n) = 0.
\]
For each \( n \), \( D(2,n) \) is calculated. For pure 'sine' these should be zero. A non-zero \( D(2,n) \) implies the presence of a spike or a missing fringe, to be collectively called 'noise'. In practice one seldom encounters pure 'sine' fringes and Eqs.(2.47) and (2.48) are not applicable. A more common occurrence is that the fringe separation slowly increases or decreases. This implies that \( P(x) \) is not a linear function as assumed in the above illustration. An appropriate form of \( P(x) \) should be chosen to correspond to the actual pattern.

To make a simple model of the changing fringe separation we borrow the concept of CHIRP from radar, BARTON (1975). A fringe system is defined to have a LINEAR CHIRP \( m \) if

\[
D(1,n) = q + nm \quad (2.49)
\]

and

\[
D(2,n) = m. \quad (2.50)
\]

This gives a signal whose wavelength increases by \( m \) over each period, and the position of the \( n \)-th peak is given by

\[
x(n) = -u + n(q + m(n-1)/2). \quad (2.51)
\]

From the above we see that the relationship between the fringe order and fringe position is quadratic. Similarly by introducing higher order terms in Eq. (2.50) we can get higher order polynomial relationships. In the present study we shall confine ourselves to the linear chirp and thus in the following, unless otherwise stated, a 'pure signal' would imply a signal with a linear chirp. It is further assumed that the noise is not pathological, i.e. the speckle pattern itself is not a chirped signal and that the missing fringes occur sparingly. Pure 'sine', being a signal of chirp \( m = 0 \), will not be accorded special attention except as an illustration. We shall now develop a simple technique to detect spikes and missing fringes in a pure signal.

Consider a pure signal, as shown in Fig.(2,9). From its maxima, \( x(n) \), one can generate a scattergram in the \( D(1,n)/D(2,n) \) plane as shown in Fig.(2,10), (henceforward, without any loss of generality we shall assume \( m > 0 \)). Transforming the maxima \( x(n) \) into a scattergram in the \( D(1,n)/D(2,n) \) plane could be called a CHIRP TRANSFORM. However, in the Fast Fourier Transform there already exists a 'Chirp Transform' and therefore the present transform would simply be called C-TRANSFORM. Similarly the \( D(1,n)/D(2,n) \) plane will be called the C-PLANE. In Fig.(2,9) the maxima are numbered 1 to 12. From Eq.(2.44) and Eq.(2.46) we see that the first and the last maximum, i.e. \( n = 1 \) and 12, cannot be represented in the C-plane and thus only maxima 2 to 11, of Fig.(2,9) are represented in the C-plane. These maxima lie on a line \( D(2,n) = m \) as shown in Fig.(2,10). If \( I(x) \) is a pure 'sine' of wavelength \( q \) then in the C-plane it will be entirely given by a single point \( (q,0) \). We shall now study the effect of adding noise (spike and missing fringe) to the pure signal. This is done as follows.

- Fig.(2,11A) a 'trailing' spike \( W \) between 8 and 9,
- Fig.(2,11B) a 'leading' spike \( V \) between 9 and 10,
- Fig.(2,11C) a missing fringe between 4 and 5,
FIG. (2,10) C-TRANSFORM OF FIG. (2,9).
Fig. (2.11D) a missing fringe between 4 and 5 and spikes between 7 and 11.

At this stage one does not know if a given intensity maximum is a peak or a spike and thus all maxima are numbered indiscriminately.

The C-plane configurations, corresponding to the above 'noise', are Figs. (2, 12A-D). These figures are drawn in arbitrary units as normalized automatically by the plotting program. The C-Transform needs only the relative order of points in the C-Plane to classify the maxima. The absolute values of $D(1,n)$ and $D(2,n)$ are retained and used in numerical calculations as described below. A comparison of Fig. (2, 10) and Fig. (2, 12A-D) reveals that with respect to their position in the pure signal

- spikes can move the maxima only 'backwards', i.e. to smaller $D(1,n)$,
- missing fringes can move the maxima only 'forwards', i.e. to larger $D(1,n)$,
- maxima move 'upwards' and 'downwards', i.e. $D(2,n)$ changes,
- as points move under the influence of noise, 'gaps' appear in the erstwhile continuous set $(q+nm,m)$.

The above can be used to develop a technique to identify spikes and missing fringes. However, to use the above one needs to know $q$ and $m$ which are of course not known. The following illustrates how one can achieve this goal without the prior knowledge of $q$ and $m$.

From $x(n)$ calculate $D(1,n)$ and $D(2,n)$. If $D(2,n)$ is a constant, we have a pure signal and no further action is taken. Next a check for 'gaps' is made to detect noise. A non-constant $D(2,n)$ with no gaps indicates 'no noise', but only a local chirp fluctuation and again no further action is taken.

To check for the 'gap' we define a GAP INDEX $G$. This will now be used to detect irregularities. There are several ways of doing it, one of which is discussed here. Consider $N$ maxima of a pure signal. We define $C$ as the sum

$$C = \text{abs}D(2,2) + \text{abs}D(2,3) + \ldots + \text{abs}D(2,N-1),$$

then from Eqs. (2.49) and (2.50) we get

$$\text{max}D(1,n)-\text{min}D(1,n) = (N-1)m = C.$$  \hfill (2.52)

We now define

$$G = C/(\text{max}D(1,n)-\text{min}D(1,n)).$$  \hfill (2.53)

For a signal without noise, Fig. (2, 9), $G = 1$. The occurrence of a spike or a missing fringe, as discussed below, would give $G < 1$.

To study the effect of local chirp fluctuations and noise on $G$ consider the $n$-th maximum of a pure signal of known $q$ and $m$. From
FIG. (2,11A) TRAILING SPIKE.
FIG. (2.12A) C-TRANSFORM OF FIG. (2.11A).
FIG. (2, 11B) LEADING SPIKE.
FIG. (2,12B) C-TRANSFORM OF FIG. (2,11B).
FIG. (2,11C) MISSING FRINGE.
FIG. (2,12C)
C-TRANSFORM OF FIG. (2,11C).
FIG. (2, 11D) MISSING FRINGE AND SPIKES.
FIG. (2.12D) C-TRANSFORM OF FIG. (2.11D).
Eq. (2.45) we have
\[ D(2, n-1) = D(2, n) = D(2, n+1) = m. \] (2.54)

Introduce a 'window-sum' operator \( S(w) \) \((w = 1, 2, ...)\), such that upon operating on a function \( f(n) \) we have
\[ S(w)f(n) = f(n-w)+f(n-w+1)+...+f(n+w). \] (2.55)

From Eq. (2.54) and Eq. (2.55) we get
\[ S(1)D(2, n) = 3m. \] (2.56)

Now change the position of only the \( n \)-th maximum by \( d(n) > 0 \), i.e. \( x(n) \) becomes \( x(n)+d(n) \). From Eq. (2.46) the new values of \( D(2, n) \), defined as \( H(2, n) \), are
\[ H(2, n-1) = m+d(n), \]
\[ H(2, n+1) = m+d(n), \]
\[ H(2, n) = m-2d(n), \] (2.57)
and thus again
\[ S(1)H(2, n) = S(1)D(2, n) = 3m. \] (2.58)

This is self-evident since a local position change cannot change the total chirp. Now sum the absolute chirps as in Eq. (2.53) and let
\[ S(1)\text{abs}H(2, n) = M. \] (2.59)

From Eq. (2.57) and Eq. (2.59) follows \( M = 3m \) only if \( d(n) < m/2 \). In other words if the change in the local maximum is less than half the chirp, the absolute sum, and hence \( G \), does not change. Now consider the case \( d(n) > m/2 \) and put
\[ d(n) = e(n)+m/2, \] (2.60)
where \( e(n) > 0 \). With Eq. (2.57) and Eq. (2.59) this gives
\[ M = 3(m+e(n)) > 3m. \] (2.61)

Now that \( M > 3m \) for \( d(n) > m/2 \), the value of \( G \) will increase. A similar argument can be made for \( d(n) < 0 \). From this follows, that \( G = 1 \) for \(-m < d(n) < m/2 \). If \( d(n) \) is outside these bounds 'order-reversal' occurs in the C-plane i.e. the \( n \)-th maximum comes either 'behind' the \((n-1)\)-th, or 'ahead' of the \((n+1)\)-th maximum. Thus, if \( G > 1 \) the order of the pure signal (no-gaps) is not preserved and gaps in the \((q, n, m)\) set occur.

Once noise (gap) is detected through \( G \), remedial action is started by inspecting the C-plane. In practice, speckle as noise is predominant and an isolated missing fringe unencumbered by speckle, as shown in Fig. (2.11D), is seldom encountered. Spikes are thus the first targets for remedial action, which entails using a smoothing filter. As has already been discussed, using filters is not without its consequences and one should use it only when necessary. There are several types of smoothing
filters, OPPENHEIM(1978), BEAUCHAMPS(1979), SHLIEN(1981). The moving average filter, or its optical equivalent, the finite slit aperture, is quite common and will be discussed here.

Let \( I(k), k = 1, 2, \ldots \) be the grey values to be smoothed into a new set of 'smoothed' grey values \( J(k) \) using a moving average window of half width \( w \), then

\[
J(k) = S(w)I(k)/(2w+1).
\] (2.62)

In practice an initial inspection of the pattern is made and if speckle is serious, i.e. if most of the peaks are influenced by it then the first step is to use the above filter. This calls for choosing a proper filtering window \( w \). Through an interactive program different windows are tried and the smallest window that gives appropriate relief from speckle is chosen. This filter is applied to the entire pattern. It is assumed that after this step the larger part of the signal is free of spikes and only isolated cases of spikes remain. Although this is not a prerequisite for C-Transform analysis, however, if excessive speckle is present this step could considerably reduce the time and memory requirements of the C-Transform. Assuming that such general smoothing, if necessary, has been employed and that only a few spikes are left, a C-Transform carried out now will reveal the presence of spikes for which additional smoothing is called for. Instead of filtering the entire pattern, only local smoothing is carried out by local filtering as discussed below.

Once a spike has been identified, its elimination (smoothing) can be accomplished in several different ways. One of the simplest methods is to restrict the range of \( k \) in Eq.(2.62). This can be done by specifying 'Begin' and 'End' values, \( B \) and \( E \), of \( k \) so that \( B < k < E \). To make it explicit we extend the definition of \( S(w) \) to \( S(B,E,p) \) as follows

\[
J(k) = S(B,E,p)I(k)/(2p+1).
\] (2.63)

Since local filtering, Eq.(2.63), is carried out under special circumstances only, henceforward, unless explicitly stated, filtering would imply global filtering, Eq.(2.62).

The LOCAL FILTER OPERATOR \( S(B,E,p) \) will filter the region \( B < k < E \) with a window (half width) \( p \). The C-Transform should deliver these three parameters \( (B,E,p) \) for each spike removal. This is illustrated by the following example.

In Fig.(2,13A) a signal with a missing fringe and spikes is shown. To help visual identification, begin and end parts of the signal are simulated to have no noise. The following steps are now taken.

1. Find \( x(n), D(1,n) \) and \( D(2,n) \) as done earlier. If \( D(2,n) \) is constant then the signal has no noise and no further action is taken.

2. Calculate \( G \). If \( G > 1 \) then order-reversal (noise) is present and carry out C-Transform, Fig.(2,14A).

3. In C-plane seek \( \min D(1,n) \) that has \( D(2,n) < 0 \). If more than one maximum are at \( \min D(1,n) \) then select \( \max(-D(2,n)) \), the highest negative chirp. Let this be at \( n' \). In Fig.(2,14A) \( n' = 16 \).
FIG. (2,13A) SCAN OF A MISSING FRINGE AND SPIKES.
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FIG. (2.14A) C-TRANSFORM OF FIG. (2.13B).
FIG. (2,13B) THE RESULT OF FILTERING SOME SPIKES.
FIG. (2.13C) MISSING FRINGE.
At \( n' \) there is a 'trailing' peak, e.g. \( W \) in Fig.(2,11A) and \( 9 \) in Fig.(2,11B).

A trailing spike is identified at \( n' \) if

\[
\min D(1,n) = D(1,n')
\]

and

\[
D(2,n') < 0.
\]  \tag{2.64}

Now calculate the local filter parameters, Eq.(2.63),

\[
2p+1 = D(1,n'), \tag{2.65}
\]

\[
B = x(n') - p, \tag{2.66}
\]

\[
E = x(n') + p. \tag{2.67}
\]

The region \( n' \) to \( n'+1 \) is now locally filtered. In filtering care should be taken to avoid discontinuities at the begin and end of the filter range. This can be accomplished by 'weighting' the filter. In Fig.(2,14A), the C-Transform of Fig.(2,13A), \( n = 16 \) is found in the third step above. Thus 16 and 17 will be filtered locally. In Fig.(2,13B) the result of a few initial local filters is shown. From Fig.(2,14B) the next maximum to be locally filtered is the trailing spike 6. In Fig.(2,13B) spike 6 trails spike 7 and thus 6 and 7 are now locally filtered. As can be seen, the speckle noise within the missing fringe had masked the missing fringe in the C-plane, in Fig.(2,14B). After a few such steps of filtering out spikes the missing fringe becomes apparent, in the C-plane, Fig.(2,13C) and Fig.(2,14C). During the program it is tested as shown below.

We define the average chirp \(<m>\) as

\[
<m> = \text{avg}(|D(2,n)|), \tag{2.68}
\]

let \( \max D(1,n) = D(1,n') \) and if

\[
D(2,n') > <m>
\]

and

\[
D(2,n'+1) < 0, \tag{2.69}
\]

then there is a missing fringe between \( n' \) and \( n'+1 \), Fig.(2,13C) and Fig.(2,14C). This may now be removed by creating an ad hoc maximum at

\[
x = x(n') + (D(1,n'-1)+D(1,n'+1))/2, \tag{2.70}
\]

or a flag may be generated for possible alternate action.

In general one keeps checking for spikes, Eq.(2.64), until at \( \min D(1,n), D(2,n) > 0 \). Satisfying this condition enables one to detect missing fringe(s). After this, returning to the beginning should give either a constant \( D(2,n) \) i.e. a pure signal or, if \( D(2,n) \) is not constant, \( G \) should be unity indicating a fluctuation in the chirp but no noise. The \( x(n)'s \) so obtained can now be used for calculating \( P(x) \).
CHAPTER 3

ANALYSIS OF HEAT TRANSPORT PATTERNS

The technique developed in the previous chapter for finding the fringe position and noise management can be used for the analysis of different types of interferograms. With this technique several types of interferograms have been investigated at various levels of detail. Real time holographic interferograms for investigating convective heat transfer have been analysed in sufficient detail to yield quantitative information about the convective heat transport in solar cells. A discussion of this analysis follows.

3.1 PHYSICAL BACKGROUND

Natural convection of air in enclosures is an important problem in heat transport. A theoretical study of heat transport by convection, CONVECTIVE HEAT TRANSFER, is made difficult by both mathematical and physical problems. Convective heat transport can be mathematically described by the full Navier-Stokes equations, the energy equation and the continuity equation. Since air is a compressible fluid the mechanisms of mass transfer and heat transport are not independent of each other. For the mathematical analysis of this problem one gets a system of coupled partial differential equations which have to be solved with appropriate boundary conditions. Furthermore, the gas flow can become turbulent. The physics underlying this transition i.e. from a two-dimensional laminar flow to a three-dimensional turbulent flow is not well understood. Without invoking simplifying assumptions this problem remains mathematically intractable. In view of such problems experimental investigations of this phenomenon become essential. The temperature gradient in a volume of air, experiencing convective heat transport, is an important physical quantity and it sheds much light on the physics of this process. An accurate measurement of the temperature gradient, under conditions of convective heat transport, thus becomes an important experimental undertaking. Since the temperature has to be measured in 'flowing' air, physical thermometers e.g. thermocouples, semiconductors etc. cannot be used lest they might disturb the flow conditions. A non-tactile 'remote-sensing' method for measuring temperature is thus called for. Holographic interferometry offers an excellent method for carrying out these measurements.

In a gas the refractive index is related to the temperature (and pressure). Changes in the refractive index and hence in temperature (assuming constant pressure) can be very accurately measured by optical techniques e.g. interferometry. The heat transport group at the Delft University of Technology has carried out a series of such measurements using real time holographic interferometry to measure temperature gradients in solar cells under experimentally controlled conditions. These experiments are discussed in detail by SCHINKEL(1980). Holographic interferograms originating from these experiments have been analysed by the technique described earlier. A brief description of the optical technique used in the experiment follows.

The basic geometry of the solar collector is shown in Figs.(3,1A,B). Two flat plates of length and width L (both \( \sim 40 \)cm), are separated by a distance D (\( \sim 5 \)cm). These two form the basic collector. The entire length along the y-axis is partitioned into sections and the interferograms from each section are analysed separately. Furthermore, some of these
Fig. (3,1A) Solar Cell Plates.

Fig. (3,1B) Solar Cell.

Fig. (3,2) Interferometer.
dimensions can be changed to study convection under different geometries. In a solar cell the plate lying in the \( x = D \) plane is of transparent material and is called the 'cold plate'. The plate in the \( x = 0 \) plane is the absorber plate and hence it is called the 'hot plate'. The collector is rotated around the \( z \)-axis so that the sun's rays are parallel to the \( x \)-axis. The cell is placed in an arm of an interferometer as shown in Fig. (3,2). A laser beam is split into a reference beam \( R \) and a measurement beam \( M \) which traverses a length \( L \) through the cell along the \( z \)-axis. \( R \) and \( M \) interfere at \( H \). To start the experiment the entire cell is kept at a uniform 'cold plate' temperature, \( T(c) \). This state of the system is designated as the zero state. A hologram of the system in this state is made at \( H \). As usual, it is processed and replaced at its original position at \( H \). Now the cold plate is kept at \( T(c) \) while the hot plate is heated to a temperature \( T(h) \). This is called the steady state. In the steady state a temperature gradient exists in the cell which produces a refractive index gradient. Variations of the refractive index within the cell modulate the phase of \( M \). The reference beam \( R \) and the hologram \( H \) reconstruct the original zero state of the system. Phase modulated \( M \) and the reconstructed state interfere to produce a holographic interferogram which is photographed at \( P \). This interferogram contains the information about the changes in the refractive index as caused by the change in the thermal state of the system. It is processed and suitably enlarged for digital analysis. Fig. (3,3A) shows a typical interferogram obtained in this manner. In Fig. (3,3B) a small part of the pattern next to the hot plate is enlarged for analysis.

The physical information about the refractive index, the temperature gradient and the holographic process can all be combined to construct a mathematical model, or the Structural Algorithm (SA), that describes the relationship between the intensity distribution in Fig. (3,3) and the temperature distribution in the cell. Our task thus is to develop an SA by studying the mathematical relation between the pattern intensity distribution and the physical variables such as temperature gradient, refractive index gradient, phase modulation etc. A discussion of developing an SA along these lines is given below.

First we define the necessary variables. Distances along the \( x \)-axis are normalized to the plate separation \( D \) so that at the hot plate \( x = 0 \) and at the cold plate \( x = 1 \). Further, \( T(c) \) = cold plate temperature at \( x = 1 \), and \( T(h) \) = hot plate temperature at \( x = 0 \). Assume that the cell contains an ideal gas of molecular weight \( M \), at a pressure \( p \), at a temperature \( T \), and of a density \( g(T) \). The ideal gas law gives

\[
M_p = g(T)RT, \tag{3.1}
\]

where \( R \) is the universal gas constant. From the kinetic theory of gasses the refractive index of a gas, \( n(T) \), is a function of the temperature as given by the Gladstone-Dale relation

\[
(n(T)-1)/(Rg(T)) = K \quad \text{(constant)}. \tag{3.2}
\]

From Eq. (3.1) and Eq. (3.2) we obtain

\[
T(n(T)-1) = KM_p = \text{constant}. \tag{3.3}
\]

We further assume that for a given \( y \) the temperature (and thus \( n \)) depends only on \( x \). Now consider a ray of light at a distance \( x \) passing through the cell as shown in Fig. (3,2). Its optical path length is \( n(x)L \).
**Fig. (3.3A) Typical Interferogram.**

![Typical Interferogram](image1)

**Fig. (3.3B) Enlarged Section.**

![Enlarged Section](image2)

**Fig. (3.4) Pattern Analysis Schematic**

![Pattern Analysis Schematic](image3)
In the zero state \( n(x) = \text{constant} \) since \( T(x) = T(c) \). When the hot plate temperature is \( T(h) \) the steady state interferes with the holographically reconstructed zero state. The intensity distribution in the resulting interferogram depends on the optical path length differences between the zero state and the steady state. From Eq. (3.3), the optical path length for the zero state is

\[
n(x)L = L(1+K\beta \mu /T(c)) = \text{constant}, \tag{3.4}
\]

for the steady state

\[
n(x)L = L(1+K\beta \mu /T(x)). \tag{3.5}
\]

The path difference \( dL \), between the two states, at a given \( x \), is

\[
dL = K\beta \mu (1/T(c) - 1/T(x)),
\]

or

\[
dL = C(1/T(c) - 1/T(x)), \tag{3.6}
\]

where \( C = K\beta \mu \text{constant} \).

Analogous to Eq. (2.4) the intensity distribution in the pattern can be expressed as

\[
I(x) = A(x) + B(x)\cos k(dL + u), \tag{3.7}
\]

where \( k \) is the wavenumber of the laser light and \( u \) is an arbitrary phase factor. \( A(x) \) and \( B(x) \) are slowly varying functions of \( x \) as discussed earlier. The relationship between the pattern intensity and the temperature is given by the following expression:

\[
I(x) = A(x) + B(x)\cos k(C(1/T(c)-1/T(x)) + u). \tag{3.8}
\]

By introducing the inverse temperature \( R(x) = 1/T(x) \), Eq. (3.8) becomes,

\[
I(x) = A(x) + B(x)\cos k(C(R(c)-R(x)) + u). \tag{3.9}
\]

This is the desired SA of the pattern i.e. given the value of the variables one can calculate the intensity distribution. We shall now analyse the fringes (maxima) resulting from Eq. (3.9).

In the following, a constant with a prime(') implies division by the light wavelength \( \lambda \) and thus

\[
C' = C/\lambda, \text{ and } u' = u/\lambda.
\]

The 'cosine maxima' of Eq. (3.9) occur when

\[
C'(R(c)-R(x)) + u' = j, \tag{3.10}
\]

where \( j \) is an integer. Let \( R(j) \) be the inverse temperature at the \( j \)-th maximum. Considering Eq. (3.10) for the \( j \)-th and \((j+1)\)-th maxima we get

\[
R(j+1) - R(j) = 1/C'. \tag{3.11}
\]

In general for the \( i \)-th and the \((i+j)\)-th maxima we have
(3.12)

This implies that the inverse temperature difference between any two adjoining fringes is a constant \((1/C')\), Eq.(3.11), and that the inverse temperature difference between any two fringes is directly proportional to the relative fringe order of one fringe with respect to the other, Eq.(3.12). Note that Eq.(3.12) depends only on the relative fringe order \(j\). This is valid for 'cosine maxima' of Eq.(3.12), however, the intensity maxima of Eq.(3.9) may be different from the 'cosine maxima' as discussed in Chapter 2. It turns out that this is not a problem in this case, as shown below.

The aim of the pattern analysis is to start with a given \(I(x)\) and then determine the temperature distribution \(T(x) = 1/R(x)\). In practice, for studying convective heat transport one does not need the entire temperature distribution but only its first derivative at the hot plate, i.e. \(dT/dx\) at \(x = 0\). From this, one calculates the Nusselt number \(NU\) which is (except for a numerical factor) given by

\[
NU = \frac{-D}{(T(h) - T(c))} \left(\frac{dT}{dx}\right) \text{ at } x = 0. \tag{3.13}
\]

Thus one needs to determine \(T(x)\) only over a small region near \(x = 0\). Furthermore, there is another physical factor that limits the zone of analysis. Away from the immediate neighborhood of the hot plate the pattern tends to become erratic due to the onset of turbulence and it is thus not amenable to the above analysis. In practice the typical range over which the pattern is analysed is \(0 < x < 0.2\). This offers a big advantage since over such a small region one could assume \(A(x)\) and \(B(x)\) to be constants. Validity of this assumption is amply borne out by practical observations. The SA could thus be simplified to the following form

\[
I(x) = A + B \cos k(C(R(c) - R(x)) + u). \tag{3.14}
\]

To determine \(NU\), Eq.(3.13), one starts from the hot plate where \(x = 0\) and \(R(x) = R(h)\). However, on a given pattern there is no clearcut location marked as \(x = 0\). The only landmark that can be unambiguously identified is the first intensity maximum next to the hot plate. If one could measure the intensity \(I(0)\) at \(x = 0\) as given by

\[
I(0) = A + B \cos k(C(R(c) - R(h)) + u), \tag{3.15}
\]

then locating this intensity at values of \(x\) smaller than that of the first maximum could locate the hot plate. From a given pattern it is easy to determine \(A\) and \(B\) (assumed constants), however, \(I(0)\) is still not known since the arbitrary phase \(u\) in Eq.(3.15) is not known as discussed earlier. This remains a problem and operator judgement is required in locating the position of the hot plate on the pattern. The best one can do is to begin with the first fringe and from there estimate the location \(x = 0\). Further details of this will be discussed in the following section on the analysis of patterns.
3.2 ANALYSIS

The general scheme followed in analysing an interferogram (pattern) is shown in Fig. (3,4). The pattern \( I(x,y) \) is uniformly illuminated and scanned by a TV Scanner TVS, VIDICON COHU Corp., and digitized into 256 (8-bit) grey levels. Spatial digitization is done on a grid of 512x512. Thus an entire TV frame consists of 512 lines with 512 pixels per line. This frame is stored in a 256k byte memory, GRINNELL 273, which is under the general control of a minicomputer, HP-1000. Once the pattern has been stored in the memory the scanner (TVS) is disconnected and the entire analysis is carried out on the stored TV frame.

The first time a given type of interferogram is analysed an interactive program is activated. A general discussion of interactive programs is given in Chapter 5. The first step in this program is to choose a line parallel to the y-axis for carrying out various operations. From Fig. (3,3) a line is chosen and its grey value (intensity) distribution is displayed on a graphic terminal as shown in Fig. (3,5). This grey value distribution serves as the input signal for further analysis. An inspection of this intensity distribution reveals the amount of noise present in the signal and the variation in the fringe visibility. These factors dictate the form of Signal Processing (SP), Fig. (3,4), needed. SP is a collection of general purpose operations which can be called upon to perform various tasks which include, smoothing filters, shading and modulation 'corrections' as discussed earlier, maxima determination operation EXT Eq. (2.37) and rules Eq. (2.35), Eq. (2.36) etc. From Fig. (3,5) it is clear that the visibility is almost constant as anticipated above and thus Eq. (2.29) does not have to be invoked here. However, spikes do occur everywhere and some form of smoothing applied to the entire signal is necessary. As discussed earlier there are several types of filters that can be used for smoothing. In the following a filter is assumed to be moving average unless otherwise stated. A filter based on the window-sum, Eq. (2.55) with a window of 9 pixels, \( S(4) \), is applied to the data in Fig. (3,5). The 'smoothed' data is shown in Fig. (3,6). If the signal has asymmetric peaks then a moving average filter shifts the position of the intensity maxima. Also, the visibility of the smoothed signal is no more constant and from Eq. (2.29) the maxima are now shifted from their 'true' positions. However, the variation is not severe enough to warrant a corrective measure for this. The filter has removed all spikes but one. This is removed by repeated local filtering. In Fig. (3,7) the thin line gives the locally filtered spike-less signal. If this local filter were applied to the entire signal the resulting signal would be given by the thick line in Fig. (3,7). The 'overall' filter has changed the visibility of the pattern as discussed earlier. If strong spikes, that require large window filtering, are occasionally expected then local, instead of global, filtering is recommended since the application of a large window filter might distort the rest of the signal undesirably.

The filtered signal is now used to determine the positions of the 'true' intensity extrema as discussed earlier. Operationally the input to SP is a line scan and the output a set of pixel numbers that correspond to the intensity maxima of the smoothed signal. The next step in the analysis is parameter estimation which depends on the physical problem. Even though physical parameter estimation is not an integral part of digital image processing, the former has to be carried out to judge the soundness of the entire scheme.
FIG. (3.5) TYPICAL SIGNAL.
FIG. (3,6) FILTERED SIGNAL.
FIG. (3,7) SPIKE REMOVAL.
As discussed earlier, to determine the relevant physical parameter(s), temperature gradient in this case, one has to construct an RA. The input to the RA is the set of maxima pixel numbers, as delivered by SP, and the output is the physical parameter(s).

For the convective heat transport problem the RA development begins with the intensity distribution $I(x)$, Eq.(3.14), whose maxima at $x(j)$, $j=1,2,...,N$, have been determined by SP. The RA should be so developed that starting from the given $x(j)$ one arrives at the desired result, $\text{NU}$ Eq.(3.13), while keeping within the bounds of experimental constraints such as unknown phases, ill defined positions etc. It also incorporates the assumptions and approximations needed for the analysis. We begin by defining the dimensionless inverse temperature distribution $r(x)$ as

$$r(x) = \frac{(R(x) - R(h))/(R(c) - R(h))}{(3.16)}.$$  

For $0< x < 1$ we have $0 < r(x) < 1$ and Eq.(3.14) gives

$$I(x) = A + B\cos k(C(R(c) - R(h))(1 - r(x)) + u).$$  

(3.17)

Let $r(j)$ be the inverse temperature at the $j$-th maximum then similar to Eq.(3.12) we get

$$r(i+j) - r(i) = j/(C'(R(c) - R(h))) = Wj,$$  

(3.18)

where $W=1/(C'(R(c) - R(h)))$. Now we build into the RA the physical assumption that $r(x)$ is quadratic near $x=0$, i.e. $r(x)$ is of the form

$$r(x) = x(ax + b) + c,$$  

(3.19)

where $a, b$ and $c$ are constants. If $a$ and $b$ can be determined then the desired Nusselt number $\text{NU}$ can be calculated from Eq.(3.13). Note that $c=0$ due to the boundary condition $r(0) = 0$. Using Eq.(3.19) at the two maxima $j$ and $i$ we get

$$r(j) = x(j)(ax(j) + b)$$

and

$$r(i) = x(i)(ax(i) + b).$$  

(3.20)

Now consider a function $f(j)$ of integer $j$. Corresponding to this we define a lower case double-index variable $f(j,i)$ so that

$$f(j,i) = f(j) - f(i).$$  

(3.21)

From Eq.(3.20) and Eq.(3.21) we get

$$r(j,i) = ax(j,i)(x(j) + x(i)) + bx(j,i),$$  

(3.22)

where $r(j,i)$ and $x(j,i)$ are defined according to Eq.(3.21). Using Eq.(3.21) to rewrite Eq.(3.18) also, we get

$$r(j,i) = (j-i)W = n(j,i)W,$$  

(3.23)

where the integer number $n(j,i)$ is the difference $j-i$. Physically $n(j,i)$ is the relative order of the $j$-th fringe with respect to the $i$-th fringe. Combining Eq.(3.22) and Eq.(3.23) we get
ax(j,i)(x(j) + x(i)) + bx(j,i) = n(j,i)W.  \hspace{1cm} (3.24)

Define

\[ Y(j,i) = \frac{n(j,i)}{x(j,i)}. \hspace{1cm} (3.25) \]

From Eq. (3.24) and Eq. (3.25) follows

\[ a(x(j) + x(i)) + b = Y(j,i)W. \hspace{1cm} (3.26) \]

We further define an upper case double-index variable \( X(j,i) \) corresponding to a lower case single-index variable \( x(j) \) as

\[ X(j,i) = x(j) + x(i). \hspace{1cm} (3.27) \]

Note that this definition does not apply to \( Y(j,i) \) since we have not defined the single index variable \( y(j) \).

From Eqs. (3.26, 27) we get

\[ aX(j,i) + b = Y(j,i)W, \hspace{1cm} (3.28) \]

Eq. (3.28) is a linear relation between the variables \( X(j,i) \) and \( Y(j,i) \). It involves only the maxima positions \( x(j) \) in \( X(j,i) \) and the relative fringe orders \( j-i \) and \( x(j,i) \) in \( Y(j,i) \). The determination of \( Y(j,i) \) introduces no more error than in \( x(j,i) \) since \( n(j,i) \) is without error.

In reality one does not know the \( x \)-values of the maxima but only the pixel numbers \( p \). The relation between the two, assuming a linear magnification \( M \), is

\[ p - w = Mx, \hspace{1cm} (3.29) \]

where \( w \) is the pixel number of the hot plate at \( x=0 \). To express the linear relation Eq. (3.28) in terms of the experimentally measured maxima pixel numbers \( p(j)'s \) we note that from Eq. (3.27) and Eq. (3.29)

\[ X(j,i) = \frac{(P(j,i) - 2w)}{M}, \hspace{1cm} (3.30) \]

where

\[ P(j,i) = p(j) + p(i). \hspace{1cm} (3.31) \]

Similarly with

\[ p(j,i) = p(j) - p(i) \hspace{1cm} (3.32) \]

and Eq. (3.25) we get

\[ Y(j,i) = \frac{Mn(j,i)}{p(j,i)}. \hspace{1cm} (3.33) \]

The linear relation Eq. (3.28) now becomes

\[ a(P(j,i) - 2w)/M + b = Mn(j,i)/p(j,i). \hspace{1cm} (3.34) \]

With
Eq. (3.34) becomes

$$aP(j,i) + (Mb-2aw) = MQ(j,i).$$  \hspace{1cm} (3.36)

This linear relation between $P(j,i)$ and $Q(j,i)$, Eq. (3.36), is the desired RA. It depends only on pixel numbers and relative fringe orders. These quantities are measured from the data. $P(j,i)$ and $Q(j,i)$ are calculated as shown above and with these a least square error linear fit is made in the $P$-$Q$ plane. The slope of the line, $a'$, is $a/M$ and its intercept, $q$, on the $Q$-axis is $(b-2aw/M)$. We note that the constant '$a$' of Eq. (3.19) can thus be determined without the knowledge of $w$, however, to find '$b$' one needs to know $w$ since

$$b = q + 2aw/M = q + 2a'w.$$  \hspace{1cm} (3.37)

From the pattern displayed on the TV screen an estimate of $w$, the hot plate position ($x=0$), in pixel number is made. This and other experimental parameters such as $T(h)$, $T(c)$ and $M$ become input to the program.

To calculate the Nusselt number $NU$ from Eq. (3.13), we note that

$$dT/dx = (dT/dR)(dR/dx).$$  \hspace{1cm} (3.38)

From Eq. (3.18) we have

$$1/WC' = R(c) - R(h).$$  \hspace{1cm} (3.39)

Differentiating Eq. (3.16) and Eq. (3.19) and equating the result we get

$$dR/dx = (2ax + b)/WC'.$$  \hspace{1cm} (3.40)

At the hot plate $x=0$, we have $T(x)=T(h)$ and thus

$$dT/dx = -bT(h)((T(h)/T(c) - 1), at x=0.$$  \hspace{1cm} (3.41)

From Eq. (3.13) and Eq. (3.41) we get

$$NU = bDT(h)/T(c),$$  \hspace{1cm} (3.42)

and from Eq. (3.37)

$$NU = D(q + 2a'w)T(h)/T(c).$$  \hspace{1cm} (3.43)

Assuming that $D$, $T(h)$ and $T(c)$ are known with great accuracy then the error in $NU$ is due to the error in estimating $w$ and the errors in $q$ and $m$ as obtained by the linear fit in the RA, Eq. (3.36). In general a linear fit, as contrasted to a higher order polynomial fit, is computationally more efficient. A general discussion of polynomial fits is given in Forsythe (1977). It is to achieve this efficiency that the development of this RA has been based on a linear rather than a polynomial fit.

The above discussed linear fit was possible only because Eq. (3.22) has just two variables i.e. $a$ and $b$. There are other quadratic forms of the inverse temperature distribution that do not allow such simplification i.e. based on only two variables. For such cases attempt should be made to
arrive at alternate efficient forms of the algorithm. An example of such an alternate algorithm is given below.

In the convective heat transport problem there are two special features that can be exploited to develop an alternate algorithm. First we note that the analysis has to be restricted to the immediate neighborhood of $x=0$. This was noted earlier also in justifying Eq. (3.19), however, no explicit use of this was made since the ensuing RA, Eq. (3.36), can be fitted to an arbitrarily large number of maxima and thus large values of $x$. Secondly, in the $R-x$ plane the maxima are equally spaced along the $R$-axis, Eq. (3.11). This fact has also been used to advantage above in Eq. (3.24). These two features allow one to fit a 'five-point parabola', SCHEID(1968). In doing so it will entail changing the physical assumption about the temperature distribution, Eq. (3.19), as discussed below.

The physical assumption Eq. (3.19) implies that the temperature distribution is a parabola in the $r-x$ plane and that the axis of the parabola is parallel to the $r$-axis. A similar 'quadratic' nature of the temperature distribution can also be given by a parabola whose axis is parallel to the $x$-axis. There is no a priori reason to choose one or the other i.e. should one choose Eq. (3.19) or the following form

$$x = r(x)(ar(x) + b) + c. \quad (3.44)$$

Unless there are some overriding physical reasons one should be prepared to investigate both the alternatives. This second form can not be put in a linear form as the previous RA since all the three unknowns, including the dc-term $c$, are essential. However it can be put in a form that is easy to calculate. We start by defining a variable $q(j)$ such that from Eq. (3.18)

$$q(j) = (r(i+j) - r(i))/W = j, \quad (3.45)$$

where $i=1,2,\ldots$, is the fringe number (order) and $j$ is the relative fringe number (order) with respect to $i$. Fringe number $i$ will be called the 'pivot'. Let $p(i)$ be the position of the $i$-th maximum as measured in pixel number $p$. Consider the pivot $i$ and two fringes on each side of it i.e. $j=-1,0,1,\ldots$. The corresponding pixel numbers are $p(i-2), p(i-1), p(i+1), p(i+2)$. For these fringes, from Eq. (3.45), $q(j)=-2,-1,0,1,2$. Thus, as $p$ varies from $p(i-2)$ to $p(i+2)$, $q$ varies from $-2$ to $+2$. We can generalise that $q$ is a function of $p$, $q(p)$,

$$q(p) = (r(p) - r(i))/W, \quad (3.45a)$$

where $r(p)$ is the inverse temperature at pixel $p$. We now assume that instead of Eq. (3.19) the temperature distribution in pixels is given by the following quadratic relation,

$$p = q(p)(aq(p) + b) + c. \quad (3.46)$$

Constants $a, b$ and $c$ can now be determined by the standard least square method. The deviation, $d(i)$, at the $i$-th maximum is defined as

$$d(i) = p(i) - q(i)(aq(i) + b) - c. \quad (3.47)$$

Taking $i$ as the pivot point, the total squared error $V$ is given by

$$V = S(2)(d(i)**2). \quad (3.48)$$
where \( S \) is the window-sum operator introduced earlier. \( V \) can be minimized by the following conditions

\[
\begin{align*}
\frac{dV}{da} &= 0, \\
\frac{dV}{db} &= 0, \\
\frac{dV}{dc} &= 0. 
\end{align*}
\]

(3.49a)

With Eq. (3.47) and Eq. (3.48) the above leads to the normal equations

\[
\begin{align*}
10c + 34a &= S(2)(p(i)q(i))^{2}, \\
10b &= S(2)(p(i)q(i)), \\
5c + 10a &= S(2)p(i).
\end{align*}
\]

(3.49b)

For \( j = i-2, \ldots, i+2, q(j) = -2, -1, \ldots, +2 \), the solution of the normal equations is found to be

\[
\begin{align*}
a &= \frac{(2p(i-2)-p(i-1)-2p(i)+p(i+1)+2p(i+2))}{14}, \\
b &= \frac{(-2p(i-2)+p(i-1)+p(i+1)+2p(i+2))}{10}, \\
c &= \frac{p(i)-3(p(i-2)-4p(i-1)+6p(i)-4p(i+1)+p(i+2))}{35}.
\end{align*}
\]

(3.50)

Note that contrary to the previous RA, Eq. (3.19), the 'dc term' \( c \) is not zero here. From Eq. (3.46) at \( q(p) = 0 \), \( p = c \) i.e. \( c \) is the pivot pixel in the fitted curve. The above solution, Eq. (3.50), is simple since it involves only the most elementary arithmetic operations on the five maxima pixels. The experimental values \( p(i) \) are integers and added (subtracted) in multiples. This is purely an integer operation and totally error free. In each equation there is only one division which might give some round-off error. This makes this RA very attractive since it is fast and relatively immune to numerical errors.

To determine the Nusselt number, Eq. (3.13), we note that

\[
\frac{dT}{dx} = \frac{dT}{dR}\frac{dR}{dr}\frac{dr}{dq}\frac{dq}{dp}\frac{dp}{dx}.
\]

(3.51)

From Eq. (3.29) we obtain

\[
\frac{dp}{dx} = M; \quad (3.52)
\]

from Eq. (3.46)

\[
\frac{dq}{dp} = \frac{1}{2aq(p)+b}; \quad (3.53)
\]

from Eq. (3.45a)

\[
\frac{dr}{dq} = W, \quad (3.54)
\]

and from Eq. (3.16)

\[
\frac{dR}{dr} = R(c) - R(h). \quad (3.55)
\]
Note that p is considered as a continuous variable and hence the above relations are exact. The fact that experimentally p has only integer values does not change the above equations.

Combining Eqs. (3.51-55,39) we can calculate $dT/dx$ at $x=0$, where $p=w$. Using this in Eq. (3.13) the Nusselt number is given by

$$\nu = \frac{(T(h)/T(c))WMD/(2aq(w) + b)}{\gamma},$$  \hspace{1cm} (3.56)

where $q(w)$ is the value of $q$ at the hot plate, i.e. for $p=w$. Thus $q(w)$ is the root of Eq. (3.46) with $p=w$:

$$w-c = q(aq + b).$$  \hspace{1cm} (3.57)

Let $q(w)$ be the physically interesting root of Eq. (3.57) where

$$q(w) = b'(-1 + \sqrt{1-c'})$$  \hspace{1cm} (3.58)

with

$$b' = b/2a$$

and

$$c' = 2(c-w)/bb'.$$  \hspace{1cm} (3.59)

Again the error in estimating $w$ will cause an error in $\nu$. By a straightforward differentiation using Eqs. (3.56,58,59), it can be shown that the error $d\nu$ due to an error $dw$ is

$$d\nu/\nu = -(dw)\nu/H(aq(w) + b'),$$  \hspace{1cm} (3.60)

where $H=T(h)WMD/T(c)$. From this one can estimate the error in $\nu$ for a given error in $w$. However, it does not lend itself to a simple interpretation as was the case with the linear fit, Eq. (3.43). The attractive feature of this algorithm is that it is easy to implement. Furthermore, by choosing different pivot points, the best individual fits in different sections of the data can be obtained. Of course, here most of the interest lies in the pivot at 3 i.e. a fit over the first five maxima nearest to the hot plate. This is also equivalent to limiting the entire analysis to the first few maxima. The best way to realize this would be to scan only the desired region of the pattern. In other words the magnification factor M should be large enough so that the entire pattern length of 512 pixels covers not more than the desired region e.g. not more than five fringes in a frame. However, sometimes one would like to have the flexibility to examine more fringes. In such cases a single frame does contain more fringes but the extent of the analysis is governed by parameters chosen by the operator. These points will be discussed in Sec. 3.3 but for the moment we continue with the development of different types of RA.

There is a physically interesting assumption, different from the above two, that leads to yet another form of RA. Near $x=0$ one may assume pure conduction and thus a linear temperature distribution as follows
\[ T(x) = T(h) - ax, \quad (3.61) \]

where 'a' is a constant and the object of the RA now is to determine 'a'. From 'a' NU can be calculated, following Eq.(3.13)

\[ NU = aD/(T(h) - T(c)). \quad (3.62) \]

To find 'a' we rewrite Eq.(3.61) in terms of the maxima pixels \( p(i) \) and the corresponding temperatures \( T(i) \). With Eq.(3.29) we have

\[ T(i) = T(h) - a(p(i) - w)/M. \quad (3.63) \]

With \( a' = a/M \) this becomes

\[ T(i) = T(h) + a'w - a'p(i). \quad (3.64) \]

In Eq.(3.64) we know everything except the \( T(i)'s \) and thus to fit a straight line one has to first calculate the temperatures at the maxima \( p(i)'s \). All the information we have about \( T(i)'s \) is contained in Eq.(3.12) and Eq.(3.61) and the boundary condition that at \( x=0 \), or \( p=w \) Eq.(3.29), \( T(x)=T(h) \). This is not sufficient information to calculate \( T(i)'s \). We have to make another assumption, namely, that \( R(=1/T) \) is linear in the small region between \( x=0 \) and the second maximum \( p(2) \). This is clearly contrary to the physical assumption Eq.(3.61). Note that Eq.(3.61) makes the temperature linear whereas now we assume the inverse temperature \( R(x) \) to be linear. Its only justification is based on the observation that over a small domain all 'non-pathological' functions can be assumed to be linear. With Eq.(3.11) this assumption gives

\[ C'(R(1) - R(h)) = (p(1) - w)/(p(2) - p(1)). \quad (3.65) \]

Once \( T(1)=1/R(1) \), the temperature at the first maximum, is known the subsequent \( T(i)'s \) are found from Eq.(3.11) as

\[ T(i) = C'T(1)/(C' + T(1)(i-1)). \quad (3.66) \]

Having thus found the \( T(i)'s \) one can now fit a straight line to the set of points \( (p(i), T(i)) \), Eq.(3.64). The slope of this line, \( a' \), immediately gives the desired Nusselt number \( NU \), Eq.(3.62),

\[ NU = a'MD/(T(h) - T(c)). \quad (3.67) \]

Again the error in NU is directly proportional to the error in \( a' \). However, the error in \( a' \) stems from the experimental error in \( w \) as also from the inconsistency between Eq.(3.61) and Eq.(3.65).

Except for the inconsistency mentioned above this RA, Eq.(3.64), based on the linear temperature distribution, is statistically preferable since here again a straight line fit is made. Furthermore it has a simple physical basis, namely, that the heat flow near \( x=0 \) is modelled as a purely conductive process which leads to a linear temperature distribution. It is for such reasons that this is used quite often in practice. Evidently, while employing this RA one does not fit over more than a few of the fringes next to \( x=0 \). As discussed in the context of the five-point parabola RA, this can be accomplished either by scanning a limited region or through operator choice during the execution of the program.
In the foregoing discussion we have seen how the physics of a problem and the corresponding experimental constraints can be synthesized into different forms of RA. In Chapter 4 a different type of experiment will be analysed. There we shall consider the development of an RA for the analysis of patterns originating from the interference of a plane wave and a spherical wave. This type of interference, leading to the formation of the well known ring patterns, is often encountered in optical research. Its analysis will be carried out in the context of studying crystal growth, MISCHGOFSKY(1977).

To execute the various steps in SP and RA as outlined above, a comprehensive program has to be written. This program should offer various options in filtering, curve fitting, limiting the extent of analysis etc. Such and similar questions are discussed in the following section on program structure.

### 3.3 PROGRAM STRUCTURE

The theoretical analysis carried out thus far should enable us to identify the various data processing steps and options that are desirable for the analysis of a particular experiment. Based on this insight the design of an appropriate program proceeds in two major stages. In the first stage an interactive program is developed. In this interactive program a large number of data processing options, intermediate result interrogation, diverse I/O possibilities etc., are included. Not much attention is directed to optimizing computer time and memory requirements. This interactive program is named PUSTF.

With the interactive program different options are tried on patterns with varying amounts of noise and corresponding to different physical conditions. Based on this experience a selection of the most appropriate options is made. Options so selected are combined into a single program which has very few possibilities of interrogating and displaying the intermediate results. This automated version of the program is rather inflexible, however, once started it arrives at the final result without operator intervention. This version is named PUESL. Once the development stage is over, PUESL is used for the routine analysis of patterns. These programs are dedicated to the heat transport problem and a more general description of the program structure is given in Chapter 5. In the following a brief description of PUSTF and PUESL is given.

A general outline of the data processing for the heat transport problem is shown in Fig.(3,8). Reading-in the pattern is a standard operation and is done under the control of general purpose software. The first step in the program is the choice of a scanline for processing. Normally it is one of the standard 'horizontal' sweeps of the TV frame. However, in some other applications it is desirable to choose a skew line. This calls for special operations that are not included in the standard program but have been developed separately and will be published elsewhere.

The choice of a line delivers an intensity distribution $I(x)$. After suitable filtering its maxima pixel numbers, $p(j)$, are determined. Following this, the corresponding temperatures $T(j)$'s or the inverse temperatures $R(j)$'s are found. The selected RA now fits the appropriate curve (linear or quadratic) to the set of points $(p(j), T(j)$ or $R(j))$. From this curve one finds the temperature gradient at $x=0$ to calculate the Nusselt number. If desired, it is stored away to eventually generate a
Fig. (3,8) General Schematic of Data Analysis
Fig. (3,9) Structure of Interactive Package
Nusselt number distribution along the y-axis, as is quite often the case in practice.

As mentioned earlier the program PUSTF allows one to check the intermediate output at various stages of execution and alter its course if so desired. A simplified block diagram of PUSTF is shown in Fig. (3,9). To illustrate the functioning of PUSTF we shall consider a simple example in the following. A more detailed discussion of this program is given by BOOGH (1982).

In Fig. (3,10) an interferogram as projected on the TV screen is shown. The calibrated markings as seen on the left edge of Fig. (3,3A), are also present in Fig. (3,10) but far to the left to be in the field of view of the TVS. These markings are photographically printed on each interferogram. They are brought into the field of view and are used for measuring the magnification factor M, Eq. (3.29). On this photograph the known width of the cell, 54 mm, is measured in terms of these markings to find the magnification. Once this frame is stored in image memory PUSTF is activated. Aided by a MENU, as discussed in Chapter 5, the operator chooses various operations by typing an integer number, indicated as K or N etc. in Fig. (3,9). These are merely mnemonic labels to guide the MENU. In Fig. (3,9) blocks 1 to 6 ask input parameters which include,

- hot and cold plate temperatures,
- estimated hot plate position w (to be called wall pixel),
- analysis domain that can be specified by an end pixel p(E) such that operations are carried out only on pixel numbers between w and p(E),
- global filter window width,
- type of RA (curve fitting) to be used,
- Noise management (CTF) option.

From block 7 to 11 an option to select a line number L and plot the corresponding I(x) is offered. In Fig. (3,11) a typical I(x), signal, is shown in thin line. Blocks 12-13 allow a plot of the filtered I(x). The thick line in Fig. (3,11) shows the filtered I(x). Blocks 14 to 16, if chosen, perform noise management by carrying out the C-TransForm (CTF) as discussed in Chapter 2. Blocks 17 to 19 determine p's, the I(x) extrema pixels, and the corresponding T's or R's as dictated by the RA selected in the MENU. A scattergram (a plot of points without a 'fitted' or connecting curve) of points (p,R) or (p,T) can be selected. In Fig. (3,12) a scattergram of (p,R) points is shown. Both the intensity maxima and minima are shown in these plots. For numerical calculations the operator can choose one or both of these. Note that the pixel numbers p are plotted along the x-axis. These curves are drawn in arbitrary units as automatically normalised by the plotter. The chosen RA is now executed and the resulting Nusselt number appears on the screen, block 20. If so desired, a continuous R or T distribution is calculated and plotted, block 21-22. In Fig. (3,13) the thick line is a five-point parabola fitted to the first five maxima, Eq. (3.46). The thin line in Fig. (3,13) is the global parabola, Eq. (3.19). It is easy to see that the five-point parabola does not fit the data well at large pixel numbers as expected. The corresponding Nusselt numbers appear on the screen for inspection. In this case the five-point parabola Nusselt number was 2.69 and the global
The parabola Nusselt number was 3.06. In Fig. (3,14) a scattergram of (X,T)
points in the pixel-temperature plane is shown. This corresponds to the
linear temperature RA of Eq. (3,61). The relationship between the
temperature and pixel number is clearly not linear as assumed in
Eq. (3,61). The thin straight line is fitted to the entire range as shown
in Fig. (3,15). The thick line is limited to the first five maxima. The
Nusselt numbers are 2.45 for the entire range fit and 2.76 for the
limited range fit. Given the almost linear variation in the beginning it
is advisable to choose a limited range linear X-T fit or a five point
parabola. Considering such factors, at this point a decision can be made
to limit the fit to only a few points in the beginning. This can be
accomplished by reducing the value of the end pixel p(E).

In the above illustration, blocks 14 to 16, which call the C-Transform
(CTF) to handle 'noise features' (missing fringe and/or local spikes) were
not activated. Since this step does slow down the program, it is activated
by choosing a certain paramater in the beginning, only if 'noise features'
are expected in a given frame. If so chosen, the C-Transform is activated
to check for such situations. In Fig. (3,10), around the black scan lines
missing fringes are expected and thus blocks 14 to 16 are activated for
this frame, whereas, for Fig. (3,3B) this was used for the local filtering
of spikes as shown in Fig. (3,6) and Fig. (3,7). In Fig. (3,16), I(x) scanned
'through' the missing fringe is shown by the thin line and the thick line
is the filtered signal. Fig. (3,17) shows the X-T scattergram. It can be
easily contrasted with the 'normal' scattergram of Fig. (3,14). If no
'corrective' measure (blocks 14 to 16) for the missing fringe were taken
and one of the above discussed RA's was used the fit will be very
unsatisfactory as shown by the thin line in Fig. (3,18). Here we have used
the limited linear X-T fit, since this RA is used often in routine use.
The fit is limited to the first 6 extrema. This gives a Nusselt number of
1.86. The thick line in Fig. (3,18) shows the results obtained by using the
option of noise management provided by blocks 14 to 16. The 'corrected'
Nusselt number is 2.88. This difference is quite significant and shows the
necessity of noise management. It may be emphasised here that this option
is chosen for each frame in the beginning and once chosen the 'corrective'
measure is applied automatically.

As mentioned earlier, Fig. (3,9) is one of the many possible
configurations of the program PUSTF. It is built in modules which can be
organized in different ways to yield other configurations. The routine-use
program PUELS contains almost the same modules as PUSTF but in a fixed
order, and no plotting and/or display of intermediate results is allowed.
Once activated, PUELS asks to type-in the experimental parameters and then
it analyses a (chosen) sequence of lines from a given frame and prints out
the final results, Nusselt numbers, and it plots a graph (off-line) of the
Nusselt number distribution. In PUELS a single line is typically processed
in less than one second. Some results, obtained by using this technique
and conclusions drawn on the performance of the program(s) will be
discussed in the next section.

3.4 RESULTS

In the previous sections we have discussed the development of a
digital data processing technique for the quantitative analysis of
interferograms, abbreviated to DIM for Digital Interferograhic Metrology.
Particular attention was paid to the analysis of real time holographic
interferograms, originating from a study of convective heat transport in	solar cells. These are now routinely analysed by DIM. Based on this
FIG. (3,10) PATTERN WITH MISSING FRINGE.

FIG. (3,11) SIGNAL WITHOUT MISSING FRINGE.
- SIGNAL
- FILTERED SIGNAL
FIG. (3,12) X-R SCATTERGRAM.
FIG. (3.13) X-R PLOT.
- GLOBAL PARABOLA
- 5-POINT PARABOLA
FIG. (3,14) X-T SCATTERGRAM.
FIG. (3,15) X-T STRAIGHT LINE FIT.

- ENTIRE RANGE FIT
- LIMITED RANGE FIT
FIG. (3,16) MISSING FRINGE SIGNAL.

SIGNAL

FILTERED SIGNAL
FIG. (3,17) MISSING FRINGE
X-T SCATTERGRAM.
FIG. (3,18) MISSING FRINGE
X-T PLOT.
— WITHOUT C-TRANSFORM
— WITH C-TRANSFORM
Fig. (3,19) Pattern used for comparison.

Fig. (3,20) Comparison of techniques.
- PREVIOUS TECHNIQUE
- PRESENT TECHNIQUE
experience some general results can be stated.

First of all, a satisfactory degree of automation and ease of operation has been achieved. Both the programs, the interactive program PUSTF and the routine use program PUELS are MENU-directed i.e. the program provides, at appropriate stages of execution, a list of options and instructions to aid in selecting an option. Thus the training required in executing a program is not excessive. One has to become familiar only with the MENU instructions, whereas a detailed knowledge of the inner workings of the program is not essential. In particular, PUELS requires operator intervention only in the beginning to supply the relevant input parameters and data processing options e.g. plate temperatures, filter window, hot plate position etc. Once these are supplied the rest of the program execution, including noise management, is automatic. As compared to the technique used previously, BOS(1980), the turn around time has been reduced by at least an order of magnitude.

An important aspect of all numerical techniques is the precision that can be expected in the final result. In the problem studied by us, the important result is the Nusselt number distribution along the y-axis. Thus the precision with which DIM can calculate the Nusselt number distribution is an important question. Since there is no 'standard' pattern of known Nusselt number distribution, it has not been possible to 'calibrate' DIM, nor the previous technique. However, we did make a comparison of DIM with the previous technique. The pattern shown in Fig.(3,19) had been measured earlier by an independent operator using the previous technique. This technique used an RA similar to the linear fit of Eq.(3.61) and used by DIM for this comparison. The results are shown in Fig.(3,20). The dots are from the previous technique whereas the solid curve represents a collection of about 80 DIM measurements along the y-axis. There is general agreement in the trend of the results. In the central part the numerical values agree within about 5% which is an acceptable margin in such studies. Near the side walls where perturbations due to the edge-effects are expected the situation is less well defined and the discrepancy between the two is large. Note that the choice of an RA, linear in this case, also influences the final result. This is particularly so, if a linear fit is attempted with a strongly quadratic trend, as seems to be the case near the side walls. In the following we shall try to study some of the probable sources of error in DIM.

At certain points, DIM requires operator judgement as is true of any other technique. This is prone to cause an error or subjective bias. In as much this is unavoidable, an estimate of these 'errors' was made by simulating controlled 'misjudgements'. The two most important parameters subject to operator bias are the choice of the global filter window and the hot plate position. Effects of 'misjudgements' in these two parameters were studied as follows.

The pattern shown in Fig.(3,3B) was analysed using three different filter windows of width=5, 9 and 13 pixels. The resulting Nusselt number distributions are shown in Fig.(3,21). There are three curves in Fig.(3,21) with thin, medium and thick lines, respectively for the three filters. The difference among the three distributions is less than 1%, whereas the change in the filter width is rather extreme. Thus, within 'reasonable' limits, the filter window can be chosen with impunity.

A similar check on the effect of wall pixel 'error' was made on Fig.(3,3B). The 'best' value of the wall pixel (x=0) for Fig.(3,3B) was
FIG. (3.21) EFFECT OF FILTER WINDOW.

Thin line ———— Width 5
Medium line ———— Width 9
Thick line ———— Width 13

FIG. (3.22) EFFECT OF WALL POSITION.

Thin line ——— Wall pixel 10
Medium line ——— Wall pixel 14
Thick line ——— Wall pixel 18
estimated to be 14. An error of four pixels, on either side of the 'best' value, is a 'reasonable' amount of error. Accordingly, Fig.(3,3B) was analysed for wall pixel values of 10,14 and 18. The ensuing Nusselt number distributions are shown in Fig.(3,22). Again there are three curves with thin, medium and thick lines, respectively for wall pixel values of 10,14 and 18. A 'higher' estimate of the wall pixel (18 in this case), implies a higher temperature gradient near x=0 and hence a larger value of the Nusselt number. This trend is clearly visible in Fig.(3,22). Of course, to confirm this trend was not the purpose of this check since this could be directly elicited from the various RA's e.g. Eq.(3.43), Eq.(3.60) etc. It was, however, not a straightforward matter to theoretically ascertain the magnitude of this variation in a realistic pattern. As is clear from Fig.(3,22), an error of eight pixels (from 10 to 18) causes less variation than the local fluctuations in the Nusselt number. A 'reasonable' error in estimating the wall pixel (4 to 8 pixels) would change the Nusselt number by not more than 1%.

To check the overall performance of DIM in relation to pattern noise a further test was made by adding noise to Fig.(3,3B). An experimentally obtained speckle pattern was added to Fig.(3,3B). Fig.(3,23) shows the pattern of Fig.(3,3B) with noise. This is an extreme amount of noise and none of the patterns studied so far have exhibited a severity of noise to this extent. Indeed, the 'noise-less' pattern of Fig.(3,3B) is a typical example of the experimental patterns 'with noise'. DIM analysed both the patterns, Fig.(3,3B) and Fig.(3,23), with identical parameters i.e. the same filter, wall pixel etc. The two Nusselt number distributions are shown in Fig.(3,24). The thin line is for Fig.(3,3B) and the thick line for the noisy pattern of Fig.(3,23). In the noisy pattern, thick line Fig.(3,24), Nusselt number fluctuations are easily discernible. The magnitude of these fluctuations is given by dNU/NU, where dNU is the difference between the thick line and the thin line. This fluctuation seldom exceeds 2%.

In an ideal situation with exactly plane walls, perfectly 'steady state' flow etc., a gradual variation of the Nusselt number along the y-axis is expected. The deviations from such a 'fluency trend', as evidenced in Figs.(3,21,22) and elsewhere, may result from a departure from the ideal conditions. The various factors that may contribute to such a departure include, thermal inhomogeneities, optical reflections, parasitic rings etc. The deviations from the 'fluency trend' (which sometimes suggest a more or less periodic course, Fig.(3,21)) are also of the order of few percents.

The choice of a particular RA does change the result as expected. Fitting a straight line and a parabola to the same set of data is bound to give different results. The difference between these two fits will largely depend on the nature of the data. If the data were 'strongly quadratic' the discrepancy between the two fits is expected to be large. In Fig.(3,25) we compare the results of a straight line , the thick line and the five point parabola fit, given by the thin line. The difference between these two fits is quite noticeable and again the physics of the problem dictates which of the two is the appropriate fit. The five point parabola fits display a greater amount of fluctuations since they are more sensitive to deviations in the data.

Except for the choice of the RA, one can say with due optimism, that even with the loosely defined factors within DIM, e.g. filter width, wall pixel etc., and in the presence of the usually encountered pattern noise,
FIG. (3.23) PATTERN WITH NOISE.

FIG. (3.24) EFFECT OF NOISE.
- WITH NOISE
- WITHOUT NOISE

FIG. (3.25) COMPARISON OF RA'S.
- 5-POINT PARABOLA
- LIMITED LINEAR FIT
FIG. (3.26) EFFECT OF VARIATION IN TEMPERATURE DIFFERENCE OF 10--50 K.
the error in the final result is not expected to be more than 5%. This claim was further fortified by carrying out measurements on a series of patterns that correspond to identical experimental conditions except a known variation in the temperature difference $T(h)-T(c)$. Under such circumstances the variation in the Nusselt number distribution from pattern to pattern follows a known course. The results are shown in Fig. (3,26). Results of DIM faithfully follow the physical expectations i.e. where the Nusselt number was supposed to be a constant it was so found by DIM, and where it was supposed to vary, DIM found the variation within the bounds of the accuracy mentioned above.
CHAPTER 4

DIVERSE APPLICATIONS

We have discussed the development of a general framework for the analysis of interferograms. Detailed working of these general principles has been discussed in the context of the heat transport problem. The same broad principles have been applied to several other problems originating from widely different physical phenomena. These investigations were carried out as feasibility studies only. The detail and depth of analysis, as evidenced in the heat transport problem, is lacking in these studies. The following discussion of some of these feasibility studies is necessarily incomplete and is provided here merely as an illustration of the principles. Subsequent research may alter the further course of analysis in these problems.

4.1 ANALYSIS OF CIRCULAR FRINGES

A spherical wave and a plane wave interfere to produce a system of concentric circular fringes. We shall first study the formation of such fringes and then discuss how an analysis of such a pattern can aid in the investigation of concentration profiles in crystal growth. In Fig. (4,1) a point source at the origin emits a spherical wave $\sin(kr)/r$ of wavenumber $k$ (and wavelength $\lambda$). This interferes with a plane wave $\sin kz$. Consider the interference pattern formed at the plane $z=b$. The phases of the two waves at $B$ (in plane $z=b$) are

Phase of spherical wave = $k(r+u(s))$, \hspace{1cm} (4.1)

Phase of plane wave = $k(b+u(p))$. \hspace{1cm} (4.2)

$u(p)$ and $u(s)$ are the arbitrary phase factors. The phase difference between the two waves at $B$ is

Phase difference = $k(r-b-u)$, \hspace{1cm} (4.3)

where $u=u(p)-u(s)$. Assuming unit visibility this leads to the following SA

$I(x,y) = 1 + \cos k(r-b-u)$. \hspace{1cm} (4.4)

Here $r$ is symmetric about the z-axis and the fringes form a system of concentric rings. For the $i$-th fringe (intensity maximum) we have

$r(i) - b - u = i\lambda$, \hspace{1cm} (4.5)

where $r(i)$ is the value of $r$ for the $i$th fringe. Eq.(4.5) is a relation between the fringe 'position' $r(i)$ and its order $i$.

In the actual experiment of crystal growth, as carried out by MISCHOFSKY(1977), the plane wave, before reaching the plane $z=b$, passes through the crystal growth melt. This is a supersaturated solution, forming a layer of thickness $t$ between the planes $z=b$ and $z=b+t$. If the concentration in the solution is uniform then the refractive index and hence the optical path traversed by the plane wave is constant over the plane $z=b$. Since the thickness $t$ of the melt is very small it is assumed that the melt is uniform in $z$-direction. Under such conditions the 'true'
Fig. (4,1) Formation of fringes.

Fig. (4,2) Analysis of fringes.
concentric rings will appear and their radii will follow Eq. (4.5). A nonuniform concentration in the melt will result in non-circular fringes which will manifest itself in the nonvalidity of Eq. (4.5). The task of pattern analysis now is to carry out measurements on the pattern to check the validity of Eq. (4.5) and thereby the uniformity of the melt.

Due to the various experimental constraints it is not possible to measure \( r(i)'s \) (or \( b \)). The best one can do is to measure the radii \( AB, AC \) etc., in the plane \( z=b \), in Fig. (4.1). These are denoted by \( a(i) \). From the SA, Eq. (4.4) or Eq. (4.5), one has to now develop a RA which checks the uniformity of the melt without invoking the knowledge of the 'trapped' variables \( r \) and \( b \). To develop this we again introduce the double-index notation for a general, single indexed variable \( g(i) \)

\[
g(i, j) = g(i) - g(j) \tag{4.6}
\]

and

\[
G(i, j) = g(i) + g(j), \tag{4.7}
\]

where \( i \) and \( j \) are integers. We further define

\[
f(i) = i\lambda. \tag{4.8}
\]

With this notation and considering two rings of order \( i \) and \( m \), such that from Fig. (4.1)

\[
r(i) = OB \tag{4.9}
\]

and

\[
r(m) = OC. \tag{4.10}
\]

We get from Eq. (4.5) and Eq. (4.8)

\[
r(i, m) = f(i, m). \tag{4.11}
\]

In Fig. (4.1) OAB and OAC form right triangles with the common base \( b \). From this follows

\[
a(i, m)A(i, m) = r(i, m)R(i, m). \tag{4.12}
\]

Note that from Eq. (4.6), Eq. (4.7) and Eq. (4.11) we get

\[
R(i, m) = r(i, m) + 2r(m) = f(i, m) + 2r(m) \tag{4.13}
\]

From Eq. (4.12) follows

\[
a(i, m)A(i, m) = f(i, m)(f(i, m) + 2r(m)). \tag{4.14}
\]

Eq. (4.14) can serve as a RA since its left hand side has pattern measurable quantities. In the pattern if one chooses two rings of known order \( i \) and \( m \) and measures the respective radii \( a(i) \) and \( a(m) \), then all the quantities in Eq. (4.14), except \( r(m) \), are known. However, \( r(m) \) does not vary with \( i \) and this fact could be used for verifying Eq. (4.14), and thereby check if there is uniform concentration in the melt. A particular value of \( m \) is chosen and kept fixed and \( a(m) \) is measured. Now different rings i.e. different values of \( i \) are chosen and the corresponding \( a(i)'s \)
are measured. Using Eq. (4.14) \( r(m) \) can now be calculated for each value of \( i \). If \( r(m) \) does not change with \( i \) then the melt has uniform concentration.

In practice the above procedure becomes unwieldy since one has to know the absolute value of the radii \( a(i) \) and \( a(m) \) to calculate \( A(i,m) \). This entails first finding the center, which is an additional step prone to errors. With a little manipulation of Eq. (4.14) one can derive another RA which is based only on the differences between the radii. Eq. (4.14) is rewritten as follows

\[
a(i,m)(a(i,m) + 2a(m)) = f(i,m)(f(i,m) + 2r(m)).
\]

Let

\[
q(i) = f(i,m)/a(i,m).
\]

Note that \( q(i) \) is the relative fringe order \( (i-m) \) divided by the difference in radii (in units of \( \), from Eq. (4.8)). Eq. (4.15) now becomes

\[
a(i,m) + 2a(m) = (f(i,m) + 2r(m))q(i).
\]

Now consider a fringe of order \( j \)

\[
a(j,m) + 2a(m) = (f(j,m) + 2r(m))q(j).
\]

By subtracting Eq. (4.16) from Eq. (4.17) we get

\[
f(i,m)(1/q(i) - q(i)) - f(j,m)(1/q(j) - q(j)) = 2r(m)(q(i) - q(j)).
\]

In Eq. (4.19) all the pattern-measurable variables i.e. \( f \)'s and \( q \)'s are relative quantities. One starts again with a fixed value of \( m \) and then selects two other rings of orders \( i \) and \( j \). In Fig. (4,2) we have

\[
a(i,m) = BC
\]

and

\[
a(j,m) = BD.
\]

Note that the position of the center \( A \) does not have to be found. The only requirement is that the scan line passes through the center i.e. the scan be radial. By measuring these two distances and \( m, i \) and \( j \) all the variables in Eq. (4.19) can be calculated (\( r(m) \) is presumably not a variable). To simplify the RA note that

\[
q(i,j) = q(i) - q(j).
\]

Define

\[
p(i) = f(i,m)(1/q(i) - q(i)).
\]

With Eq. (4.22) and Eq. (4.23) and the convention of Eq. (4.6), Eq. (4.19) becomes

\[
p(i,j) = 2r(m)q(i,j).
\]
This can serve as a simple RA since the relationship between \( p(i,j) \) and \( q(i,j) \) is linear. To test for the uniform concentration one chooses a ring of order \( m \) and then measures from this the distances to various rings i.e. different values of \( i \) and \( j \). From these, one calculates \( p(i,j) \) and \( q(i,j) \). If the melt has a uniform concentration a straight line will pass through these points in the \( q(i,j)-p(i,j) \) plane.

The RA represented by Eq.(4.24) does not explicitly require the position of the center, but it is still burdened by the requirement that the scan line AD, in Fig.(4.2), must pass through the center. Note that the actual value of \( m \) is not required. This RA can operate only with radial scans. This is an undesirable feature of Eq.(4.24). To test other parts of the pattern one has to scan radially in other directions. This calls for a departure from horizontal scans. An algorithm for generating such non-horizontal (skew) scans has been developed and will be published elsewhere. This step can introduce further errors and also makes the program slower. In the following we develop another RA which is not limited to radial scans.

In Fig.(4.2) consider a non-radial scan line PG. Let \( s(i) \) be the position (with respect to an arbitrary origin) of the \( i \)th fringe on this line, then

\[
s(i,m) = EF \tag{4.25}
\]

and

\[
s(j,m) = EG. \tag{4.26}
\]

Also note that

\[
AB = AE = a(m). \tag{4.27}
\]

By an argument similar to the one used earlier, the right triangles APE and APF share the same base AP and hence similar to Eq.(4.12) we have

\[
s(i,m)s(i,m) = a(i,m)A(i,m) = r(i,m)R(i,m). \tag{4.28}
\]

This implies that if, instead of Eq.(4.16), \( q(i) \) is defined as;

\[
q(i) = f(i,m)/s(i,m), \tag{4.29}
\]

then we get the same RA as Eq.(4.24). The method for checking uniform concentration thus remains the same as described above except that now a non-radial scan line can be chosen.

Fig.(4.3) shows a typical pattern obtained in such an experiment. Since the number of pixels in a scan line is fixed (512), a higher accuracy results if, at a time, only a small portion of this is selected and scanned by the TVS. Here we have chosen roughly the top right quadrant of Fig.(4.3). In Fig.(4.4) a typical grey value distribution along a scan line is shown. The thin line in Fig.(4.4) shows the scan data and the thick one after filtering it globally with a window of 7 pixels. Using maxima determination techniques discussed earlier the fringe positions are determined and the last RA, with Eq.(4.29), is used to fit a straight line. Fig.(4.5) shows a straight line fit to four scans. A more detailed analysis of departures from this fit, accuracy and stability of the fit have to be further investigated in conjunction with an analysis of the
FIG. (4,3) CIRCULAR FRINGE PATTERN.

FIG. (4,4) CIRCULAR FRINGE SIGNAL.

- SIGNAL
- FILTERED SIGNAL
FIG. (4.5) LINEAR FIT TO CIRCULAR FRINGE DATA.
4.2 DISPLACEMENT MEASUREMENT

A technique to calibrate holographic interferometry as a means of displacement measurement has been developed by VAN BEEK (1982). Precise displacements are produced to serve as measurement standards. A given displacement, in this case a pure in-plane translation, produces a fringe pattern consisting of parallel, equidistant straight lines. The fringe separation is a measure of the displacement. In Fig.(4,6), the pattern originating from a displacement of 100 micrometers is shown. The object of pattern analysis is to measure the fringe separation by finding the intensity extrema. As usual, a scan line is chosen for processing the intensity distribution. A typical intensity distribution along such a line is shown by the thin line in Fig.(4,7). A very prominent background, due to the laser beam profile is discernible. Before attempting to find the extrema, this background has to be removed. This is done by first finding the absolute maxima and then, through the C-Transform, the average frequency of the pattern is estimated. From this a filter window width is selected (equal to average fringe separation). With this filter the 'signal' is smoothed out and only the background remains. The thick line in Fig.(4,7) shows the background. By subtracting the background from the full signal (and adding a constant level and multiplying by a factor of two) the 'corrected' signal, as shown in Fig.(4,8), is obtained. Since the C-Transform can operate automatically this 'correction' can be implemented in an automatic method for the analysis of such patterns.

For quantitative analysis the positions of the intensity maxima can be determined. A straight line fit to these extrema positions and fringe order was made before and after the background correction, as shown in Fig.(4,9). The thin line is for the raw data and the thick line is for the corrected data. The straight line fit parameters thus obtained can be used for quantitative comparison of different displacements as discussed below.

We consider three patterns, numbered 1 to 3 by an index J (J=1,2,3), corresponding to displacements Q(J) of 100, 150 and 200 micrometers. Each Q(J) has an experimental error E(J) which depends on the optical configuration. This is expressed as a percentage of the displacement and is shown in the table below. For each pattern, choosing an arbitrary unit of length, one can define a periodicity P(J) as the number of fringes per unit length. Next, one defines a ratio R(J)=P(J)/Q(J). In general, R(J), for a given point on the measured surface is a constant as long as the optical configuration does not change. The magnitude of this constant is a function of, among others, the illumination and observation directions. If we assume that for all patterns:

- the illumination and observation directions are constant over the entire object, and that
- these patterns are produced by a pure translation,

then the value of R(J) is constant over the entire surface for all the patterns.

Each of these patterns was processed and a straight line was fitted, as discussed above. The slope of each line is directly proportional to the periodicity P(J). As a measure of goodness of fit the standard deviation...
Fig. (4,6) Displacement pattern.

Fig. (4,7) Signal and background from displacement pattern.

- SIGNAL
- BACKGROUND
FIG. 4.8 DISPLACEMENT SIGNAL WITHOUT BACKGROUND.

FIG. 4.9 STRAIGHT LINE FIT.
- WITH BACKGROUND
- WITHOUT BACKGROUND
S(J) was also calculated for each fit. This deviation is calculated as a fraction of the fringe separation. From Q(J) and P(J), R(J) was calculated. These results are shown below.

<table>
<thead>
<tr>
<th>J</th>
<th>Q(J)</th>
<th>E(J)</th>
<th>P(J)</th>
<th>R(J)</th>
<th>S(J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>5.0%</td>
<td>.0278</td>
<td>.000278</td>
<td>.071</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>3.8%</td>
<td>.0429</td>
<td>.000286</td>
<td>.057</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>2.5%</td>
<td>.0558</td>
<td>.000279</td>
<td>.064</td>
</tr>
</tbody>
</table>

First of all we note that the spread of the data points, as measured by S(J), is less than a tenth of a fringe in all the patterns. Since the experimental error, E(J), is the smallest for J=3, one can take R(3) as a standard and calculate Q(1) and Q(2). With R(3) as standard, Q(1)=99.82 micrometers, and Q(2)=153.78 micrometers. These values are well within the experimental errors E(J). A maximal variation of about 2.5% in R(J) is also well within the experimental limits. Thus within these limits the above results have established that the displacements were pure translations of the specified Q(J).

4.3 SHAPE ANALYSIS BY C-SKELETON

The analysis of interferometric patterns carried out so far has been based on maxima positions and fringe orders. These remain as important variables, but in certain applications other parameters of the fringe shape, for example the local fringe curvature, play an important role. To carry out such analysis a method to look at the local 'shape' of the fringe was devised. This method is illustrated by considering the intensity distribution of Fig. (4,7). First the 'background', the thick line in Fig. (4,7), is determined. The points of intersection of the 'background' and the intensity curve, 'thin' line in Fig. (4,7), are called the 'zero crossing' points and designated by p(j). Thus, p(j) is the pixel number of the j-th zero crossing point. The center of gravity of the intensity between each pair of adjoining p(j)'s is found. Let c(j) be the pixel number of the center of gravity and m(j) the 'mass' of the fringe between p(j) and p(j+1). We define the 'left extent' of the fringe as L(j) = c(j) - p(j) and the 'right extent' as R(j) = p(j+1) - c(j). Now at pixel number c(j) the mass m(j) is stored and at pixel number c(j+1), R(j) is stored and at c(j-1), L(j) is stored. Thus the region between any two zero crossing points is now replaced by three pixels. On repeating this for each fringe and each scan line the entire pattern of fringes is replaced by a skeleton of 3-pixel wide lines. This is called the C-Skeleton (short for Clad-Skeleton). The C-Skeleton contrasts with the more familiar single pixel skeleton which could, in this context, justifiably be called a B-Skeleton (short for Bare-Skeleton). The B-Skeleton once obtained has lost all information about the lateral extent of the object. The C-Skeleton, however, contains enough information to calculate local curvature, gradient etc. Some possible applications, both interferometric and non-interferometric, of this method are discussed below.

High performance aircraft wings are made of a composite of a honeycomb core epoxied to fiber-laminates. Faults in the epoxy bond between the core and the laminates are considered unacceptable and non-destructive methods for detecting such faults are developed. One such method is to thermally strain the wing and study its surface by holographic interferometry. The interferometric pattern resulting from such a study contains a system of 'free flowing' fringes if there are no epoxy faults. Faults appear as regions of high local curvature. A limited effort was made to use...
Fig. (4.10) Flash X-ray of fragments. Fig. (4.11) C-Skeleton of fragments.

Fig. (4.12) Acoustic polar C-Scan.
C-Skeleton to detect these faults. The results are promising enough to warrant further investigation into this problem.

In several industrial applications the object of pattern analysis is to determine some physical parameter from the image of an irregular shape. In a certain application flash X-ray of a moving metallic fragment was taken with the intent to determine the mass and the center of gravity of the fragment while in motion. In Fig.(4,10) a flash X-ray of this type is shown. The grey value at a given point is directly related to the density and the thickness of the fragment at that point. To determine the total mass and the center of gravity of the fragment a special program was written to 'track' each fragment. The grey values along each line were converted into mass distribution by taking into account the X-ray absorption and the density. Mass and center of gravity of each line was found. Further, the distances of the left and right boundaries from the center of gravity (L and R) were also found. With these a C-Skeleton of each fragment was determined, as discussed above. The C-Skeleton, contains enough information to calculate not only the mass and the center of gravity, as was the original intention, but from it one can calculate other dynamic aspects also e.g. rotation and flexure of the fragment. In Fig.(4,11) C-Skeletons of fragments in Fig.(4,10) are shown.

Another application of the C-Skeleton has been in the fingerprinting of carbon fiber cross-laminates. For reasons of strength to weight ratio certain structural members of an aircraft are made of carbon fiber laminates. In these, layers of carbon fibres are glued together, with the 'run' of the fibres in adjacent layers being orthogonal. Such composite materials show marked anisotropy in their elastic moduli. This anisotropy can be measured by acoustic polar C-Scan as discussed by VAN DREUMEL(1981). A sample of the composite is rotated between a transducer and a detector. The acoustic impedance of the sample is measured as a function of the angle of rotation. The signal from the detector is coupled to an oscilloscope where it traces a pattern characteristic of the material as shown in Fig.(4,12). One can determine the C-Skeleton of this pattern quite easily. Since the C-Skeleton is a quantitative representation of the original pattern it is possible to quantitatively 'fingerprint' the composite by its C-Skeleton.
CHAPTER 5

SOFTWARE STRUCTURE

5.1 INTRODUCTION

In our development of software packages for DIM we have tried to follow the general guidelines of good-programming. These guidelines refer to such features as

- modularity,
- ease of operation,
- transportability.

Modularity of a software package divides the task into smaller tasks which are carried out by separate and independent sub-units of the package. This allows a flexibility in organizing the entire task in different manners as the occasion demands. Diagnosing and debugging becomes easier since intermediate results can be easily monitored. The price one pays for this is in speed and memory. The independence of various sub-units results in some duplication of tasks, and thereby slowing down the entire package. The linkages necessary to connect the various sub-units create a certain amount of bookkeeping that burdens the memory and slows down the package. In general, a highly modular, task-differentiated package is perhaps not the optimal package for any task, in terms of speed and memory. In developing a package a balance between optimality and modularity is sought. There is no hard and fast rule for achieving this balance. A rule of thumb is to first consider the range of problems relevant to the package and then try to find the most frequently changed variables and then make separate sub-units for them. As an illustration, in our experience, although it is more economical to filter and find extrema in the same sub-unit, it was most desirable to keep them in separate sub-units.

The speed and ease of operation is one of the most attractive features of a DIM software package and is an important consideration in its development. From the user's point of view the possibility of a dual system i.e. an 'interactive package and a 'production' package, proved to be quite desirable. Before embarking upon a long production run, involving an analysis of several patterns, a dry run with a typical pattern is made with the interactive package to determine the optimal values of parameters and to check the overall performance. To make the packages operable without encumbering the user with the details of programming a MENU-directed format was chosen. Even though this is not the optimal program structure qua speed and memory, some sacrifice of these was accepted to attain a degree of ease of operation. In general, I/O of a software package can become unwieldy for users not familiar with the computing environment and for this reason special attention was directed to make this aspect simple. In brief, special effort was given to the MENU and I/O organization to acquire a level of ease of operation.

The possibility of using our software at industrial sites, with different computers, was envisaged. Different computers differ widely in their I/O organization and image processing is quite I/O intensive. An image processing package developed at one computer cannot be run at a
different computer without further ado. Unfortunately the computing industry has not agreed upon a standard format to facilitate such tasks. In our case, the interactive packages are heavily I/O oriented and to make them operational at a different computer will require reorganization of the I/O structure. In a production package, a lot of work is numerical and such packages are more easily transportable than the former. An attempt was made to separate the numerical tasks from the I/O tasks and to assign them to different sub-units. On the whole, without modifications in the I/O organization, the packages developed here are not transportable.

To optimize speed and memory, production packages are highly dedicated and cannot be discussed in a general context. To illustrate the points made above we shall discuss a few details of a general purpose interactive package PUOPT developed by us.

5.2 INTERACTIVE PACKAGE

PUOPT is a supervisory program that consists of a MENU and calls to various subroutines as directed by the MENU. A library of about 50 subroutines was specially developed for this purpose. To illustrate its use we shall follow a typical run. To begin the analysis a pattern is first read-in with the TVS. This is done outside PUOPT by using the standard system commands. PUOPT requires two image memories, to be called A and B. Each of the memories has either 64 kbytes for an image of 256x256 pixels or 256 kbytes for an image of 512x512 pixels. In A the pattern to be analysed is stored. For all subsequent work A is used as a ROM i.e. it can be only read and no information is written on it. This preserves the original image. B is used for displaying intermediate results e.g. a filtered signal, positions of extrema etc. On starting PUOPT, a MENU appears on the terminal, as shown below

!!! MENU LIST !!!
0 STOP —————— 0
1 CHOOSE LINE— ——— 1000+LN
2 PLOT —————— 20+FL
3 FILTER —————— 30+FL
4 MAX-MIN— ————— 40+FL
5 OPERATIONS ———— 50
6 C-SKELETON ———— 60
7 ABS MAXIN ———— 70+FL
8 VRSTC PLOT ———— 80+FL
9 SIMULATE ———— 90+FL

!!! N.B. C-SKLTN 60 OUT.ADD %PUXRY & %PUCGM TO $PUOPT !!!

The first MENU choice is to select a line. This is done by typing 1000+line number. Typing 1234 will read line number 234 from A. The grey values of line 234 are stored in an array. PUOPT uses four such arrays,
to be called files, which appear as FL in the MENU. These files are as
to large as the line i.e. 256 or 512 locations and are numbered 1 to 4. Line
234 is stored both in FL 1 and FL 2, the other two files are used for
intermediate storage.

After reading the line the main MENU returns and the operator may
elect to plot the grey values on the terminal. This is done by typing
20+FL number to be plotted, e.g. typing 21 plots FL 1 on the terminal.
After plotting, the main MENU returns on the terminal. If it is desired to
have a hard copy of the terminal plot the operator types 81. An off-line
plot of FL 1, similar to Fig.(3,5), is made on the VERSATEC plotter. The
main MENU returns again and now the operator may wish to filter FL 2 (it
is customary to keep FL 1 as it is). There are several filter subroutines
such as moving average, weighted average etc. To save on memory only one
at a time is put in PUOPT. Moving average filter is used quite frequently
and is assumed to be the current filter in PUOPT. To filter, one types
30+FL number i.e. 32 for filtering FL 2. A filter instruction now appears
on the terminal as shown below

** TYPE FLTR WINDOW FULL WDTH. EXT=0. **

After typing in the window width the file is filtered. Upon typing 0
for the window width the subroutine EXits and no filtering is done. It may
be noted here that PUOPT is divided into small tasks and each task takes a
fraction of a second. For all practical purposes the operator is not
aware of the execution time. After filtering, the filtered signal is
returned in FL 2 and the main MENU returns. To inspect the filtered
signal, now in FL 2, 22 is typed in. This results in a plot of FL 2 on the
terminal. Again for a hard copy of the filtered signal 82 is typed and a
plot similar to Fig.(3,6) is available.

Typing 50 in the main MENU brings OPERATIONS into action. This is a
grab-bag of common arithmatic operations as seen in the following MENU.

!!! TYPE OP-CODE !!!

STOP - - - - - - - =0
ADD - - - - - - =1
SUB - - - - - - =2
MULT - - - - - - =3
DIV - - - - - - =4
LOG10 - - - - - =5
COPY - - - - - - =6

By choosing the proper code one can carry out the corresponding
arithmatic operation or, through 6, copy one file into another. For
example, typing 2 for the OPeration CODE an instruction, as shown below,
appears:

!! TYPE 3 DGT FL SEQ. 213= 2.OP.1=3. !!

A 3-digit file sequence is asked. Typing 213 will carry out
2.Operation. and put the result in FL 3. In this case FL 2 – FL 1 = FL 3. The main MENU reappears now.

On typing 90+FL number, a function will be simulated and returned in the specified file. Again different types of algebraic functions can be written into this subroutine. Currently chirped signals can be simulated. The following MENU appears for this

$$ $\text{TYPE WAVE LENGTH \& THEN CHIRP}$$ $$

After choosing the desired wavelength and chirp a new instruction appears as shown below.

$$ $\text{!! TYPE AVG. \& AMPLTD. \& THEN PHASE IN PXL NRS. !!}$$ $$

Here one chooses the average value (d-c level) and the amplitude as also the absolute phase. A chirped signal as shown in Fig.(2,9) is returned in the specified file.

The MAX-MIN option allows one to determine grey value extrema with a specified rule, which is chosen by annexing the appropriate subroutine. Currently a center of gravity rule subroutine is annexed, and the extrema are determined according to Eq.(2.40). Since absolute extrema are used quite frequently, a separate subroutine, based on Eq.(2.39), for determining these was written and comes under the ABS MAXIN option.

Both, MAX-MIN and ABS MAXIN, make CALLs to C-Transform subroutine if so chosen. On typing 42 or 72, the position of extrema appears on the terminal and then the C-Transform option as shown below:

$$ $\text{%% TYPE 1 FOR CTF. EXT=0.}$$ $$

If 1 is typed here a C-TranForm on the maxima, as discussed in Chapter 2, is performed. Values of the gap index and the average chirp are found and an OUTPUT option, as shown below, appears:

$$ $\text{$$$ TYPE 1=PL0T, 2=VRSTC, EXT=0. $$}$

C-Transform plots appearing in Chapter 2 have been obtained in this manner.

At the end of the main MENU a message may appear to give the status. In the example shown above the C-Skeleton option, as discussed in Chapter 4, has been deleted from PUOPT to save memory allocation. There is also an instruction on how one can restore the C-Skeleton. This restoring can be accomplished in a few minutes and involves no programming but does require a rudimentary knowledge of general system commands such as COMPILE and LOAD. %PUXRY and %PUCGM are two RELOCATABLE FILES that have to be inserted into the LOADER COMMAND FILE $PUOPT and LOADED. Sometimes, to save the memory %PUXRY is not stored but it can be retrieved through COMPILE. This illustrates the modularity of the package. One can create a pool of subroutines for PUOPT and add (or delete) them to PUOPT to suit particular needs.

As can be seen from the above description PUOPT is quite flexible but it is no paragon of speed and efficiency. After each step the operator must prod the program, via MENU, to execute the next step. Upon the execution of a given step all the results are stored, since the next step,
as yet unknown, may require these intermediate results. This creates a lot of internal bookkeeping and memory allocations making the entire package profligate in memory and speed. In its agility and utility PUOPT is truly a train on a milk-run and it would never serve as a production package. However, using PUOPT one can determine the optimal mix of operations and parameter values for a given task and then use these to construct a production package. In our experience this strategy has served us well.

In general, the use of PUOPT by persons not steeped into the arcaneness of programming, has not provoked unfavorable response and thus a modicum of ease of operation can be claimed. Modularity of PUOPT is almost entirely governed by the structure of the modules or subroutines. These subroutines have to be written to be compatible with the general requirements of PUOPT. Once properly written and debugged they can be easily included in PUOPT. As for the transportability we have tried to keep the I/O subroutines, the most nettlesome of the lot qua transportability, apart from the main stream. The subroutines for PLOT and VRSTC options are relatively independent of the main stream and have to be completely rewritten for some other computing system. Since a real transplant of this package to some other computer has not been tried it is not possible to divine exactly what else has to change.

5.3 PRODUCTION PACKAGE

A production package is geared to rapid number crunching tasks and has a much simpler but at the same time a very specialised structure. In contrast to an interactive package it offers no possibility for intermediate step control. The usual strategy in designing a production package is to first probe the problem with the interactive package and determine the operations and their sequence for analysing the problem at hand. Then one starts with the respective subroutines in the interactive package and strips them off the extraneous I/O burden. The resulting subroutine is almost entirely a numerical algorithm. These 'stripped-off' subroutines are stacked together according to the sequence determined through interactive package. It is quite possible that the problem may require special procedures to handle very specific demands e.g. the properties of a particular optical configuration. Subroutines to handle such specific tasks are normally not part of a general purpose interactive package. In such cases one either makes a special purpose interactive package and then the production package is designed or a few extra subroutines, to handle the peculiarities, are written. In Chapter 3 we dicussed a production package, PUELS and the special purpose interactive package, PUSTF. PUELS is in many ways similar to PUOPT but it has some very specific task oriented subroutines such as temperature determination at the maxima.

Nearly all physical problems have a certain individuality and it is quite unlikely that the production package of some problem could totally be made out of the general purpose interactive package PUOPT. For any project that is expected to run for any length of time it becomes almost mandatory to have its own production package and interactive package, as was the case with PUELS and PUSTF in Chapter 3.

5.4 I/O ORGANIZATION

A program like PUOPT makes use of several I/O devices and the I/O communications are at various levels. The reading of the image is supervised through JIM, which is a software system developed specially for
the hardware used here. Once the image has been read-in and PUOPT is
started, special I/O calls developed in the group are used to open image
files. Once an image file is opened it can be read in many ways. We use
both systems library and other CALLS developed for this purpose to read
the image file. Once the image data is in the program, the output of
results is carried out in several different ways. Plotting of graphs is
done through a program developed by us for this special purpose. This plot
program is based on the special HP-Plotter commands. Writing on the TVS is
done by WRITE calls developed for this purpose. For hard copy print-outs
special system commands are used to communicate with various I/O devices.

Off-line plotting of VERSATEC plots calls for many levels of I/O
CALLS. First within the program local I/O CALLS open a data file and store
the data with local WRITE. After the termination of PUOPT, REMAT (Remote
Node Link) is used to send the data at the VERSATEC NODE (Central
Computer). At the VERSATEC NODE a specially developed program opens the
local data file and activates a specially developed subroutine to
translate the data into PLOTTER FORMAT and creates special PLOTTER FILES.
These PLOTTER FILES are used by another subroutine which has the PLOTTER
COMMANDS to plot the data.
CHAPTER 6

SUMMARY AND CONCLUSIONS

In the previous chapters we have discussed the development of a technique for the quantitative analysis of interferograms using digital data processing (DIM). Although the original aim was to analyse interferograms, it has been possible to use DIM for the analysis of patterns of non-interferometric origin also. In the course of this study some problem areas were identified and some possible solutions were developed.

In Chapter 1 some general aspects of quantitative image analysis of functional images were considered. Special attention was given to interferometric image analysis. As there was a dearth of guidance from the available literature in formulating a systematic approach to this problem, we divided the task into the development of the Structural Algorithm (SA), the necessary image processing and the Reconstructive Algorithm (RA). To translate these into practice required the introduction of the Signal Processor (SP). In general this task division has proved to be useful and should be used in the future.

In Chapter 2, some aspects of fringe intensity maxima determination and noise management were considered. To a large extent, the ultimate success of the analysis depends on an accurate determination of the fringe positions (maxima). There are two main factors that threaten this endeavor. First of all, a measured intensity maximum need not unambiguously be identified with the sought after fringe position. Due to noise, shading, modulation and/or flat-tops resulting from digitisation, it becomes essential to devise a rule for extracting the desired fringe position from the intensity maximum. This step is not unique and would always be subject to question. Secondly, noise management, essential in nearly all practical situations, influences the determination of the fringe position. This is again an unavoidable bane of signal processing and precious little can be done to counter its undesirable side effects, except be aware of it and approach it with caution. The concept of chirp and its relation to noise management, as embodied in the C-Transform, was found to be useful. Given the limited experience this deserves further scrutiny.

In Chapter 3, details of analysing interferograms originating from convective heat transport were discussed. The role, played by the physics of the experiment, remains a dominant one and to a large extent guides the development of the method. Ambiguities about the physics are dutifully reflected in the structure of the method. The goal here is to offer the experimenter an easy possibility of trying alternate choices as dictated by physics. This called for developing different RA's and making them readily accessible. To make such a method practically acceptable, certain criteria, other than the normal ones of accuracy, consistency etc., have to be met. This included speed and ease of analysis. This may not be the intellectual challenge of the problem but if not enough effort is devoted to it the entire exercise may become moot in a practical sense. This object should remain in sight all through the development of the technique. The relative numerical accuracy of DIM is well within the bounds of acceptability for the heat transport problem. The question of absolute numerical accuracy remains a ticklish one for want of a suitable calibration standard. In any event some suspected sources of error were
identified and their influence on the final result was studied. On the whole, an accuracy of about 5% and the speed of analysing a frame giving about 64 values, in about 50 seconds seems to have been well worth the effort.

In Chapter 4, the possibility of analysing other types of patterns was considered. Broadening the technique used in the analysis of the heat transport problem, it was possible to carry out feasibility studies for several unrelated industrial problems that included,

- analysis of circular fringe systems to study concentration gradients in crystal growth,
- analysis of interferograms due to precisely controlled displacements,
- analysis of real time holographic interferograms to detect delamination faults in thermally strained aircraft wings made of honeycomb composites,
- analysis of flash X-rays to study the dynamics of high velocity fragment motion,
- analysis of acoustic C-Scans to fingerprint fiber-epoxy composite laminates.

For the crystal growth problem an RA was developed to meet the special constraints of the experiment and initial results indicate the possibility of detecting nonuniformities of the crystal melt in an automatic manner. However, further work in collaboration with the experimenter is needed to interpret the results and refine the technique.

An analysis of patterns corresponding to three known displacements was carried out. Although a check of the absolute measurement of the displacement could not be carried out, due to the constraints of the interferometric experiment, an internal consistency among the three patterns was found. To fully exploit DIM, such future experiments may have to be modified.

The last three applications mentioned above called for the development of a skeletonization method. This method replaces a fringe, an irregular shape etc., by a three pixel wide skeleton, called the C-Skeleton. There is enough information in the C-Skeleton to permit calculation of several interesting properties of the original object. It was possible to detect delamination faults by calculating, through the C-Skeleton, the local fringe curvature. It was also possible to analyse the motion of the fragments. In particular the mass and the center of gravity could be determined with relative ease. There is a possibility that along with the fringe position and fringe order the C-Skeleton may also become important in the analysis of interferograms.

In Chapter 5, some aspects of programming were discussed. The importance of having an interactive package which is used by the experimenter, was emphasised. The development of a production package at the earliest stages was important in accruing physical feedback as soon as possible. The availability of a general purpose digital image analysis system together with an extended library of algorithms, both local and global, proved to be most desirable. This considerably expedites the development of a sequence of operations along with the suitable parameters.
for a given application. Special problems require special programs, but quite often a library of specific subroutines e.g. extrema determination, would meet this demand effectively. This is especially welcome for the feasibility studies as carried out in Chapter 4.

The development of a DIM package requires a close cooperation with the expert in the particular field (field-expert). In particular, the SA development calls for the input from the field-expert as regards the sensitivity for various parameters and the nature and strength of noise sources. An image analysis expert must understand these factors well enough to synthesise them into an effective algorithm. A good working knowledge of the underlying physics becomes mandatory in achieving this goal, and a close cooperation with the field-expert is most desirable. The development of the RA is often the responsibility of the image analyst since he/she has to put it into action. However, in doing this, enough flexibility must be exercised to accommodate the alternatives desired by the field-expert. Since the image analyst is quite likely to be the last person in the data-processing chain, it becomes his/her responsibility to assure the numerical and logical integrity of the entire analysis.

With an adequate image analysis system and library as discussed above, the field-expert and the image analyst may be able to demonstrate, in a relatively short time, if a given metrologic problem is compatible with automated image processing. If this seems to be true and the frequency of its use warrants, a dedicated hardware system may be developed for the problem at hand. The advent of the microprocessor together with advanced dedicated computer architecture, has made it possible to develop such systems, GERRITSEN(1982), NAKDATE(1982).

The facility and speed of DIM, as evidenced in the heat transport problem, is not an end itself. It may become useful in certain research techniques. One of such possibilities could be the study of time varying phenomena. Dynamic aspects of heat flow (as of other processes) are interesting and important, however, frequently, these cannot be studied due to the delay and volume of analysis. It may be quite possible that with the speed offered by DIM some dynamic experiments, hitherto impractical, could now be undertaken, and a deeper look into the physics of the phenomenon could be realised. Perhaps DIM, the acronym notwithstanding, might help shed some light on such aspects of physical research.
In dit proefschrift wordt de ontwikkeling van een digitale beeldbewerkingsmethode beschreven die het mogelijk maakt op automatische wijze interferogrammen kwantitatief te analyseren; enige toepassingen worden behandeld. Interferometrie is een hulpmiddel om op nauwkeurige wijze diverse fysische grootheden te meten, zoals die van belang zijn voor wetenschappelijk onderzoek en praktische toepassingen. De komst van laser vergemakkelijkte het verkrijgen van interferogrammen aanzienlijk; de kwantitatieve analyse ervan bleef echter een moeilijke en omslachtige taak. Dit heeft de praktische toepassingen van de interferometrie niet bevorderd. Digitale beeldbewerking geeft nu een mogelijkheid deze kwantitatieve analyse te vergemakkelijken en aanzienlijk te versnellen. Dit proefschrift beschrijft de ontwikkeling van een methode -met bijbehorende rekenprogramma's- voor een automatisering van de analyse en bespreekt tevens een aantal zeer uiteenlopende toepassingen.

Hoofdstuk 1 geeft een kort overzicht van enige gegevens uit de literatuur over holografische interferometrie en over daarmee in verband staande digitale beeldbewerking. Lacunes ten aanzien van een automatische digitale analyse komen daarbij naar voren.

In Hoofdstuk 2 worden enige theoretische achtergronden besproken. Vooreerst wordt een onderscheid gemaakt tussen een "structureel" en een "reconstructie" algorithme. Een "structureel" algorithme maakt het mogelijk op basis van een mathematisch model van een fysisch systeem, verschijnselen veroorzaakt door dat systeem te beschrijven in samenhang met parameters van het systeem: aan die verschijnselen worden de metingen verricht. De verschijnselen zijn hier de holografische interferogrammen die samenhangen met een parameter van het beschouwde systeem. Bij het "reconstructie" algorithme gaat het er om, uitgaande van metingen aan de verschijnselen, de fysische parameters te vinden.

Vervolgens wordt kort de achtergrond van holografische interferometrische verschijnselen besproken. Het intensiteitsverloop in een interferogram wordt beschreven door het structurele algorithme. Dit gebeurt voor het ideale geval zowel als voor praktische situaties waarbij zowel de gemiddelde intensiteit als de amplitude van de interferentiellijn constant zijn. Er wordt aandacht gegeven aan de invloed van spikkelruis (speckle noise) en aan het feit dat in de praktijk soms een interferentiellijn door storingen (plaatselijk) niet aanwezig is (missing fringe). Tenslotte wordt uitvoerig nagegaan hoe digitale beeldbewerking gebruikt kan worden om automatisch de plaats van interferentiellijnen te bepalen, rekening houdend met spikkelruis. Hiervoor is een z.g. "C-transformatie" ontwikkeld ("C" in verband met de hier van belang zijnde "Chirped signals", dat wil zeggen, in spatiale frequentie varie rende signalen). Deze transformatie maakt het mogelijk automatisch de plaatsen op te sporen waar ruispieken aanwezig zijn en de daarna op passende wijze weg te filteren. Tevens kunnen niet-aanwezige interferentiellijnen worden gecoreerd.

Een toepassing van Hoofdstuk 2 geeft de in Hoofdstuk 3 beschreven analyse van interferogrammen die bij een warmtetransportprobleem in zon necellen optreden. De temperaturen in het gas van de cel worden met behulp van holografische interferometrie bepaald. Uitgaande van een mathematisch model van warmtetransport wordt een structureel algorithme aangeduid dat verband legt tussen interferogrammen en het warmtetransport.
De foto's van de interferogrammen, met interferentielijnen die min of meer evenwijdig aan de warme wand van de cel verlopen, zijn met de eerder genoemde beeldbewerkingsmethoden geanalyseerd. Om de temperatuurgradient nabij de wand te berekenen worden enige reconstructie algorithmen ontwikkeld en toegepast, uitgaande van verschillende veronderstellingen over het temperatuurverloop in de buurt van de warme wand. De invloed van ruis en van fouten in de schatting van enige grootheden wordt nagegaan. De resultaten zijn in behoorlijke overeenstemming met die welke niet automatisch werden verkregen. Het is nu echter mogelijk de resultaten aanzienlijk sneller te produceren waarom een veel meer gegevens over het warmtetransport voorhanden kunnen komen. De programmastructuur voor deze toepassing wordt besproken.

In tegenstelling tot de betrekkelijk uitvoerige wijze waarop in Hoofdstuk 3 het warmtetransportprobleem wordt uitgewerkt, geeft Hoofdstuk 4 drie vrij korte beschrijvingen van toepassingen op andere problemen. Dit betreft:

- het bepalen van concentratieprofielen bij kristalgroei. Hierbij ontstaan min of meer cirkelvormige interferentielijnen; een test op het cirkelvormig zijn kan automatisch worden uitgevoerd.

- het meten van een lineaire verplaatsing; er ontstaan equidistante evenwijdige interferentielijnen met vrij veel varierende gemiddelde intensiteit.

- het analyseren van de vorm van interferentielijnen; een hiervoor ontworpen skelet algorithme is ook te gebruiken voor het bepalen van de vorm van voorwerpen.

Hoofdstuk 5 bespreekt de algemene structuur van de ontwikkelde rekenmachineprogramma's. Er zijn twee typen structuren. Voor eerst is er een interactieve structuur. Hiermee kunnen aan de hand van diverse "menu's" allerlei soorten subroutines voor beeldbewerking e.d. worden opgeroepen. Het resultaat van een bewerking kan door een onderzoeker worden beoordeeld waarna eventueel de bewerking wordt vervangen door een met andere waarden van de parameters of door een ander soort bewerking, te putten uit een omvangrijke bibliotheek van beeldbewerkingsprogramma's. De onderzoeker kan op deze wijze een volgorde van wenselijke bewerkingen samenstellen die voor een bepaald probleemgebied van belang is. Vervolgens is er een operationele structuur. Hierbij wordt, zonder verdere ingreep van de onderzoeker, de eerder bepaalde volgorde van bewerkingen automatisch afgewerkt en daagwenst voor vele situaties na elkaar. Voor alle situaties wordt de fysische parameter berekend waar alles om begonnen is.

Tot slot benadrukt Hoofdstuk 6 de voornaamste resultaten van de beschreven ontwikkeling samen.
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ABBREVIATIONS AND NOTATION

Abbreviations

DIM     Digital Interferographic Metrology
RA      Reconstructive Algorithm
SA      Structural Algorithm
TVS     Television Scanner

Notation

A brief summary of the major notation is given below.
A given symbol may represent different entities in different contexts.

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>COMMENT</th>
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<tbody>
<tr>
<td>A</td>
<td>Shading</td>
</tr>
<tr>
<td>B</td>
<td>Modulation</td>
</tr>
<tr>
<td>C</td>
<td>Numerical constant.</td>
</tr>
<tr>
<td>D</td>
<td>Plate separation or a finite difference</td>
</tr>
<tr>
<td>E</td>
<td>Electric field or ENTIER</td>
</tr>
<tr>
<td>G</td>
<td>Gap index</td>
</tr>
<tr>
<td>I</td>
<td>Intensity</td>
</tr>
<tr>
<td>J</td>
<td>Smoothed or reversed intensity</td>
</tr>
<tr>
<td>i,j,n</td>
<td>Integer fringe order</td>
</tr>
<tr>
<td>k</td>
<td>Pixel number or wavenumber</td>
</tr>
<tr>
<td>L</td>
<td>Solar cell length</td>
</tr>
<tr>
<td>M</td>
<td>Magnification factor</td>
</tr>
<tr>
<td>m</td>
<td>Fringe order or chirp</td>
</tr>
<tr>
<td>P</td>
<td>Phase function</td>
</tr>
<tr>
<td>p</td>
<td>Pressure or pixel number</td>
</tr>
<tr>
<td>q</td>
<td>Pattern wavelength</td>
</tr>
<tr>
<td>R</td>
<td>Inverse temperature</td>
</tr>
<tr>
<td>r</td>
<td>Dimensionless inverse temperature or radius</td>
</tr>
<tr>
<td>S</td>
<td>Summation operator</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>u</td>
<td>Arbitrary phase (factor)</td>
</tr>
<tr>
<td>V</td>
<td>Visibility or a variable</td>
</tr>
<tr>
<td>W</td>
<td>Constant</td>
</tr>
<tr>
<td>w</td>
<td>Filter window or hot plate position in pixels</td>
</tr>
</tbody>
</table>
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