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SOARING FLIGHT OPTIMIZATION THEORY AND AN APPLICATION IN SAILPLANE DESIGN

by

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INTRODUCTION

In a soaring contest the object is to travel a specified distance, for instance a triangle, within the shortest possible time. Another challenge is to maximize the flight distance, but when the time is limited, for instance because of the sunny hours that thermals are produced, the pilot's aim is again to fly such as to realize the highest possible cross-country speed.

The first mathematical approach with respect to thermalling flight optimization was published in 1938 and has since been elaborated by several investigators. A significant practical result was the "speed ring" introduced in 1948 by P.B. MacCready, which enables the pilot to fly with optimum inter-thermal speed, provided that the rate of climb in the next thermal, which has to be estimated in advance, is realized. The fact that pilots flying high-performance sailplanes in a dolphin mode occasionally reached unexpectedly high cross-country speeds stimulated the interest and research to optimize this mode of flying.

A conveniently arranged collection of such studies based on ordinary calculus can be found in reference 1; the analyses of references 2-4 differ mathematically by utilizing the calculus of variation, but after all lead to the same conclusions.

Finally, in references 5 and 6 the optimal flight procedure is considered which yields maximum climbing performance in straight flight under favourable atmospheric conditions.

In these analyses of the dolphin mode and optimal climb procedure, frequently a square-wave model is adopted to describe the atmospheric conditions. In consequence, practical application of the results is very difficult, as will be shown.

Based on a lecture presented to the Netherlands Association of Aeronautical Engineers on May 18, 1978.

4.1
The first chapter of this paper is intended to show that the well-known classical theory - which is based on a rather simple atmospheric model - if only slightly extended and properly interpreted, is adequate to cover all the previous optimal solutions. This approach, formulated in terms familiar to most sailplane pilots, provides insight into the various optimal flight modes, and ultimately leads to a simple plot which might be useful in actual cross-country flying.

As an example of the application of soaring flight optimization theory in sailplane design, the second chapter of the paper deals with the problem which aspect ratio is optimal for a Standard Class glass-fiber sailplane (span 15 m) considering various atmospheric conditions, practical minimum weights and the use of water ballast. Several investigators studied this question before (Refs. 7-11). Their results vary from an optimum aspect ratio of 15 to 20, which is lower than the aspect ratio of most of the current high-performance Standard Class sailplanes. Questions with respect to the implications of several assumptions applied in these studies gave rise to study the problem again, using a computer program developed for parametric sailplane performance optimization as described in reference 16. The assumptions applied at present indicate again that the aspect ratio should be lower than those of most of the current Standard Class sailplanes; a sailplane with an aspect ratio of about 18, built as light as possible but capable to carry a large amount of ballast (graphite-reinforced composites?) is the best compromise under most atmospheric conditions.

SOARING FLIGHT OPTIMIZATION THEORY

In classical theory an element of the flight (Fig. 1a) is considered, where a straight glide through a region with uniform vertical velocity of the air w is followed by a climb in a thermal up to the initial altitude. Wind and limitations with respect to altitude (ground, cloud base) are not considered, and the effects of transitions between different conditions of flight are neglected. The sailplane horizontal velocity V (which is approximately equal to its airspeed), the sailplane rate of sink C and the rate of climb in the thermal RC are constant. Upwards directions are considered positive.
The average cross-country speed $\bar{V}$ according to equation 1 is shown graphically in figure 1b; the horizontal axis of the sailplane speedpolar is shifted over an amount $w$, thus presenting the sailplane performance during penetration relative to the ground, and the straight line between $(0, RC)$ and the relevant point on the sailplane speedpolar crosses the horizontal axis of the translated coordinate system at $\bar{V}$. The oblique line is divided into parts which lengths are proportional to the elapsed time for the climb and the glide phase. The optimal inter-thermal flight speed $V_{opt}$ which gives maximum average cross-country speed $\bar{V}_{max}$ for this flight element is found by differentiating equation 1 with respect to $V$ and setting the derivative to zero. The result is the well-known equation 2 upon which the speed ring is based, and substituting this result into equation 1 gives equation 3; the resulting best speeds are attained when the oblique line in figure 1b is the tangent to the speedpolar (Fig. 1c).

Figure 2 illustrates the construction and application of the speed ring, using the speedpolar of the well-known Standard Class
sailplane St. Cirrus (based on measurements taken from references 12-14). By drawing the tangent to the speedpolar from several points on the vertical axis, the so-called "MacCready curve" can be constructed.

Appropriate values of the flight speed \( V \) are indicated on a moving ring around a linear scale variometer at corresponding values on the variometer (inner) scale, given by the vertical difference between the MacCready curve and the speedpolar. The flight speed at the sailplane minimum rate of sink, opposite the zero reading on the variometer scale, is clearly marked.

If for instance a climb rate of 1.5 m/s in the next thermal is expected, the triangular index is set at this value, and penetration takes place in such manner that the reading of the flight speed indicator corresponds to the indication of the variometer needle on the outer scale. At a downward velocity of the air during penetration of -0.5 m/s, a flight speed of 130 km/h is read on the outer scale, corresponding to a total sink rate of -1.9 m/s as shown in the example. Consequently, the flight is optimal, provided that the estimated rate of climb in the next thermal is realized.

A slight extension of the theory implies the assumption of a non-zero net altitude change between the start and the end of the flight element, expressed in the net flight path angle \( \varepsilon \) (Refs. 15 and 1). This gives rise to the determination of the average climbing or sinking speed \( \bar{W} \) over the flight element, in addition to the average cross-country speed \( \bar{V} \) (Fig. 3a). Applying the maximizing condition \( \tan \varepsilon = \text{constant} \) leads to the same equation for the optimal penetration speed \( V_{\text{opt}} \) as in the previous case, implying that the speed ring remains applicable since
$V_{\text{opt}}$ is independent of $\varepsilon$. The resulting maximum cross-country speed and maximum average climbing or sinking speed over the element (both a specified gain and a specified loss in altitude are realized in the shortest possible time) can be graphically constructed as well (Fig. 3b). Again the tangent is divided into parts which lengths are proportional to the elapsed time for the climb and the glide phase. Particular cases occur when

$$t_1 = 0; \quad \varepsilon = 90^{\circ}, \quad \bar{W}_{\text{max}} = RC (>0), \quad \bar{V}_{\text{max}} = 0 \quad \text{i.e. climbing in a thermal only}$$

$$t_2 = 0; \quad \varepsilon = \frac{C+w}{V_{\text{opt}}}, \quad \bar{W}_{\text{max}} = C+w, \quad \bar{V}_{\text{max}} = V_{\text{opt}} \quad \text{i.e. penetrating only}$$

In a latter case, penetration takes place according to the command of the speed ring, set at a pre-selected value. If a circling climb in a thermal is involved, this value corresponds to the climb rate $RC$ in the thermal.

Before going into further detail some important remarks have to be made. It should be noted that the equation for the optimal speed-to-fly remains valid when the vertical velocity of the air during penetration is varying; the flight can be regarded as being composed of infinitely small segments with constant vertical velocity of the air and each segment can be optimized separately (in practice, the pilot brings the flight speed into agreement with the variometer reading on the pre-set speed ring, thus performing a series of dolphin motions). However, in the appendix of reference 16 it is elucidated that, for estimating cross-country performance in predefined meteorological conditions, the spatial average of the vertical air velocity distribution $\bar{\omega}$ may be taken, provided that the variation of the vertical air velocity is within a few meters per second. Consequently, the constant vertical velocity of the air $\bar{w}$, used before, should be interpreted accordingly.

With this interpretation of $w$ in mind, figure 4 ($w = 0 \text{ m/s}$) and figure 5 ($w = 2 \text{ m/s}$) provides insight in the various optimal flight modes and corresponding performance respectively ring settings according to the previous theory. Again the speedpolar of the St. Cirrus has been used.
FIG. 4 ILLUSTRATION OF OPTIMAL FLIGHTS, BASED ON ST. CIRRUS SPEEDPOLAR (w = 0 m/s)

FIG. 5 ILLUSTRATION OF OPTIMAL FLIGHTS, BASED ON ST. CIRRUS SPEEDPOLAR (w = 2 m/s)

4.6
The limiting cases "penetrating only" and "climbing in a thermal only" are indicated in the figures, and all the curves in between represent the conventional thermalling mode of cross-country flying, where a penetration phase is followed by a climb phase in a thermal up to a specific height expressed by $\epsilon$. Since these flights are optimal it is assumed that penetration takes place according to the command of the speed ring, and that the zero-setting of the ring $RS$ corresponds to the rate of climb $RC$ achieved in the thermal. The increase of the maximum average cross-country speed and the maximum average climbing or sinking speed with increasing rate of climb for any possible value of $\epsilon$ explains the pilot's intention to select the strongest thermals on course and to penetrate accordingly.

When the sum of $w$ and the sailplane minimum sink rate ($-0.63 \text{ m/s}$) is negative, as in figure 4, there is always a loss of height at the end of the penetration phase, and a circling climb in a thermal is needed to obtain height (thermalling mode). However, when this sum is positive, as in figure 5, the net change of height at the end of the penetration phase can be negative, zero or positive, depending on the applied zero-setting of the speed ring. The value of $RS$ which results in a horizontal flight (on the average), i.e. the straight dolphin mode of flying, can be easily determined by the graphical construction sketched in figure 6a.

The maximum attainable average climbing speed over the element by flying just straight and according to the command of the speed ring, indicated by "optimal straight climb" in figure 5, is attained when the ring is set at a value equal to $w$ plus the (negative) sailplane minimum sink rate. In this case the average climbing speed is equal to the ring setting, as shown by the graphical construction in figure 6b. When this straight climb is followed by a circling climb in

**FIG. 6** \textsc{Graphical construction for some particular straight flight cases}
a thermal with the same climb rate, $\tilde{W}_{\text{max}}$ does not change, being the reason for the knot of the thermalling mode lines for $\epsilon > 3.65^\circ$ in the left figure of figure 5.

Finally, the maximum climb angle in straight flight, 4.03 degrees in figure 5, is attained when the speed ring is set at zero. Figure 6c illustrates this case.

(Similarly, but not shown here, if the sum of $w$ and the sailplane minimum sink rate is negative, as in figure 4, the minimum attainable average sinking speed over the element in straight flight is reached at a negative value of $RS$ equal to $w$ plus the sailplane minimum sink rate. The minimum flight path angle under these circumstances is realized with the speed ring set at zero.)

Based on these considerations a simple plot can be composed, (Fig. 7) which might be useful to the pilot for making an estimate of the proper ring-setting when entering a region where straight dolphinning or optimal climbing can be utilized, as for instance under a cloud street. A reasonable estimate of the average vertical velocity of the air $\tilde{w}$ - analogous to estimating the expected rate of climb in the next thermal - should be made in advance and, depending on the tactical decision concerning the flight mode which is preferable under the given circumstances, the proper ring-setting is read from the plot.

Realizing that this ring-setting is relevant too when an additional circling climb with corresponding climb rate could be performed, it is clear that, when a higher climb rate could be achieved in a localized area with strong updraft (thermal), the flight should be performed accordingly to reduce the flight time further.

An extension of the atmospheric model implies the adoption of two regions (instead of one) with arbitrary constant vertical velocity of the air within each region, to be interpreted again as spatial average values.
Such a model, frequently used in soaring flight optimization studies, provides a better approximation of actual vertical air velocity distributions and hence cross-country performance than in the previous case. However, application of the results in practice is, to put it mildly, difficult, as will be shown.

Considering a flight through the atmospheric square-wave model of figure 8, specified by the (average) vertical air velocities \( w_1 \) and \( w_2 \) and the region length ratio \( \beta \), the ring-setting which leads to a straight dolphin flight \( (\Delta H = 0) \) or optimal climb over the flight element - provided that these flight modes are possible - can be calculated in an iterative way.

However, in order to generate a plot in which the optimal ring-setting is shown as a function of \( \beta \), \( w_1 \) and \( w_2 \), it is easier to determine the optimal speeds-to-fly (and corresponding sailplane sink rates) for given values of \( w_1 \), \( w_2 \) and \( RS \), and then calculate the relevant value of \( \beta \) by using the equations given in figure 8.

Some results are shown in figure 9, their interpretation is analogous to figure 7. Thus, in case \( w_1 = 0 \) m/s and \( w_2 = 1.5 \) m/s, the thermalling flight mode (with the highest attainable rate of climb and corres-
ponding ring-setting) is required when $\beta < 0.38$. A straight dolphin flight and an optimal climb can be performed when $\beta \geq 0.38$. The appropriate ring-setting follows from the plot, unless a climb rate higher than this value of the ring-setting could be achieved by circling somewhere in the region considered; in that case the flight should be performed accordingly. (In this respect it is reasonable to assume that the climb rate is at least equal to $w_2$ plus the (negative) sailplane rate of sink in turning flight.)

Anyhow, it is obvious that, even when an electronic device should provide the proper ring-setting, practical application of these results is very difficult considering the nature and number of quantities which should be estimated in advance. Apart from the many other considerations which may affect soaring tactics too, figure 7 might be more appropriate for estimating the proper ring-setting when entering a region of favourable atmospheric conditions.

APPLICATION OF SOARING FLIGHT OPTIMIZATION THEORY IN SAILPLANE DESIGN; AN EXAMPLE

During the last years at the Department of Aerospace Engineering (Delft University of Technology), a computer program has been developed for the analysis and synthesis of a sailplane speedpolar and subsequent calculation of cross-country performance in predefined atmospheric conditions.

To facilitate aerodynamic design studies, several design parameters (geometric and aerodynamic data, airplane gross weight) can be varied continuously, and computer plots can be produced to show the effects on the spanwise lift distribution and local lift coefficient at maximum wing lift coefficient (to assess stalling characteristics), the sailplane drag polar (in which several drag contributions are shown), and the speedpolar. Subsequent calculation of cross-country performance is based on the previous theory, assuming that optimum flight techniques are employed in given atmospheric conditions. In order to get a suitable basis for comparison, the usual assumption is made that the altitude at the end of each element considered is the same as the altitude at the start; $\epsilon = 0$. (It is noted that this assumption rules out the optimal climb mode described before.)

Computer plots of cross-country speeds can be generated for systematic variation of atmospheric conditions (fixed sailplane configuration and weight), atmospheric conditions and weight (fixed sail-
plane configuration) or weight and aspect ratio (fixed atmospheric conditions). A detailed review of methods and basic considerations, illustrated by some calculation results, is given in reference 16.

As an example of the application of soaring flight optimization theory in sailplane design, the program is used to find the optimum aspect ratio for a Standard Class glass-fiber sailplane, considering various atmospheric conditions, practical minimum weight and the use of water ballast. As previously mentioned, several investigators studied this problem before (Refs. 7-11). Questions with respect to the implications of several assumptions applied in these studies gave rise to study the problem again. In brief, the present treatment generally follows the comprehensive approach of reference 7, extended by taking maximum weight figures into account. However, thermal updraft profiles have the linear character proposed in reference 11, and analogous to reference 9 the best possible climb rate is used for comparison purposes. Practical minimum weight figures are estimated using the expression given in reference 17. And finally, all calculations start from the measured speedpolar of the Astir CS. These propositions will be considered into more detail below.

According to measurements (Refs. 11 and 18) (type B thermal which "seems to represent the original element of convection"), the best fit to thermal velocity distributions in the radius of turn region of interest is obviously a straight line. As shown in figure 10, the gradient of this distribution defines the point on the circling flight polar where, at the flight technique considered, the best climbing speed is attained. As elucidated in reference 19, flight techniques which result in minimum rate of sink at any radius of turn respectively any angle of bank are

FIG. 10 CLIMBING PERFORMANCE AND THE GRADIENT OF LINEAR THERMAL VELOCITY DISTRIBUTIONS
relevant in practice. For twelve sailplanes the gradient of the circling flight polars were plotted against the corresponding angle of bank; the results are within the shaded areas of figure 10. It is noted that the average values of the gradients at 30 degrees and 45 degrees are close to the average of the values proposed in reference 11 representing wide respectively narrow thermals, while the average value at 40 degrees corresponds to the value proposed in reference 20 for this angle of bank.

For present purposes thermals are considered which are specified by discrete values of the thermal central strength \( w_{th} \) and average values of the gradients at 25, 35 and 45 degrees angle of bank, being typical values in the angle of bank range applied in practice. (It should be noted that \( w_{th} \) is an extrapolated value as shown in figure 10; the rates of climb achieved are much lower.)

Since optimal flight techniques are assumed, the best possible climbing speed is used in calculating and comparing cross-country performance of different sailplanes flying in equal atmospheric conditions. Analogous to previous studies it is assumed that penetration takes place through air which is - on the average - at rest. Consequently, only the most practised mode of cross-country flying, the thermal mode, is considered.

Practical minimum all-up weights are estimated from an expression for the empty weight of glass-fiber sailplanes given in reference 17 and actual empty weight data of 15 meter span production type glass-fiber sailplanes, taking about 100 kgf into account for the weight of the pilot, parachute and instruments: \( W_{\text{min}} \approx 100 + 1600/A^{2/3} \) kgf. To cover a wide range of ballasted weights, the maximum allowable weight is assumed 150 kgf higher.

In reference 16 it is shown that the parasitic drag (total drag less wing drag) greatly affects the speedpolar of a sailplane and that the - frequently applied - assumption of a constant value of the parasitic drag coefficient at all values of the lift coefficient may lead to an unsatisfactory prediction of the speedpolar. A fraction of this drag, the tailplane drag, can be estimated using measured aerodynamic airfoil data and some theory. However, calculation of the remaining part, called miscellaneous drag, containing the drag of the real fuselage shape, wing-fuselage interference effects, trim drag etc. is problematic or even impossible. For present purposes, realistic miscellaneous drag data were obtained by subtract-
ing the wing and tailplane drag from the total drag derived from the measured speedpolar of the Astir CS. This commercially successful sailplane has been chosen because it applies a new airfoil (E603) in a relatively low aspect ratio wing (A=18.15), and because measured aerodynamic airfoil data (Laminar Windtunnel of the University of Stuttgart) and a measured speedpolar of high quality (Ref. 12) were available. For present purposes, the weight of this sailplane is varied from 200 to 600 kgf, and the aspect ratio from 15 to 30. The drag contributions of the wing and the tailplanes, adjusted if necessary, are calculated (including Reynolds number effects) while the original miscellaneous drag area is taken constant at equal values of the lift coefficient. Further details are given in reference 16, where the same sailplane is used to illustrate the features and some capabilities of the computer program.

Coming to the computed results now, a typical plot is presented in figure 11. It shows the effect of weight on cross-country speed for the configuration with aspect ratio 25 climbing in thermals with gradient .002 s\(^{-1}\) (25 degrees angle of bank in figure 10) and various thermal (central) strengths. At each thermal strength there is an optimum weight, indicated by the oblique line, at which maximum cross-country speed is attained. This weight rapidly changes with thermal strength. Weight limitations, however, make realization of the maximum cross-country speed at all thermal strengths impossible. At lower and higher thermal strengths the maximum attainable cross-country speed is determined by the minimum practical respectively maximum weight.
Results of similar plots, generated for aspect ratios 15, 20 and 30, are summarized in figure 12, showing for each thermal strength the maximum attainable or possible cross-country speed at the aspect ratio and corresponding practical minimum, theoretical optimum or maximum weight considered. It strikes that, when the sailplanes fly at their proper weight, aspect ratio has little effect on cross-country performance. At the theoretical optimum weights the optimum aspect ratio (22.5) which gives the maximum possible cross-country speed is obviously independent of thermal strength. At the lower thermal strength, where practical minimum weights are relevant, the optimum aspect ratio decreases as the thermal strength decreases. At the higher thermal strength, where maximum weights determine the maximum attainable cross-country speed, the opposite is true. The same procedure was followed for thermals with gradients .009 s⁻¹ and .027 s⁻¹ (35 respectively 45 degrees angle of bank in figure 10). The results show effects similar to those described above; however, the optimum aspect ratio curve shifts to lower values when the gradient increases (Fig. 13). In figure 14 the relative loss in cross-country speed with respect to the maximum attainable or possible cross-country speed is shown for sailplanes with aspect ratio 15, 19 and 22.5 flying at their practical
minimum, theoretical optimum or maximum weight in all atmospheric conditions considered. When these sailplanes fly at their theoretical optimum weight, the loss in cross-country speed is nearly constant (or zero), and due to the upper and lower weight limits the loss in cross-country speed departs from this value. The aspect ratio of 22.5, being optimal at gradient .002 s⁻¹ and thermal strengths between 1.9 m/s and 2.5 m/s, is always at a disadvantage at lower values of the thermal strength. At gradient .027 s⁻¹ and thermal strengths below 4.9 m/s, aspect ratios lower than 15 are required. However, these sailplanes will have inconveniently high empty weights and show, when ballasted to their proper weights, considerable losses in cross-country speed at low thermal gradients. Moreover, the degradation of the best glide ratio, as shown in references 7 and 11, is disadvantageous when regions with poor thermal activity have to be overpassed.

Taking it all in all, a sailplane with an aspect ratio of about 18, built as light as possible but capable to carry a large amount of water ballast (graphite-reinforced composites?), is the best compromise under most atmospheric conditions.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Author(s)</th>
<th>Title</th>
<th>Source</th>
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### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>A</td>
<td>Wing aspect ratio</td>
<td>(-)</td>
</tr>
<tr>
<td>C</td>
<td>Sailplane sink rate</td>
<td>(m/s)</td>
</tr>
<tr>
<td>H</td>
<td>Altitude relative to datum</td>
<td>(m)</td>
</tr>
<tr>
<td>L</td>
<td>Length of flight element</td>
<td>(m)</td>
</tr>
<tr>
<td>RC</td>
<td>Rate of climb</td>
<td>(m/s)</td>
</tr>
<tr>
<td>RS</td>
<td>Ring-setting</td>
<td>(m/s)</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>(s)</td>
</tr>
<tr>
<td>V</td>
<td>Forward speed</td>
<td>(km/h)</td>
</tr>
<tr>
<td>V&lt;sub&gt;opt&lt;/sub&gt;</td>
<td>Optimal speed-to-fly</td>
<td>(km/h)</td>
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<tr>
<td>(\bar{V})</td>
<td>Average cross-country speed</td>
<td>(km/h)</td>
</tr>
<tr>
<td>w</td>
<td>Vertical velocity of the air</td>
<td>(m/s)</td>
</tr>
<tr>
<td>(\bar{w})</td>
<td>Spatial average vertical velocity of the air</td>
<td>(m/s)</td>
</tr>
<tr>
<td>(w_{th \text{ max}})</td>
<td>Thermal central strength</td>
<td>(m/s)</td>
</tr>
<tr>
<td>W</td>
<td>Weight</td>
<td>(kgf)</td>
</tr>
<tr>
<td>(\bar{W})</td>
<td>Average climbing or sinking speed over the element</td>
<td>(m/s)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Region length ratio</td>
<td>(-)</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Net flight path angle</td>
<td>(degree)</td>
</tr>
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### Subscripts

- **C**: Optimal climb
- **D**: Straight dolphin flight
- **max**: maximum
- **min**: minimum