Advanced Modeling of Distortion Effects in Bipolar Transistors Using the Mextram Model

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Abstract—The modeling of distortion effects in bipolar transistors due to the onset of quasi-saturation is considered. Computational results obtained using the Mextram and Gummel-Poon models as implemented in a harmonic balance simulator are compared with measured results.

I. INTRODUCTION

In mobile telecommunication receivers and transmitters there are several constraints on the power consumption and the supply voltage. These will, in general, lead to a limit on the collector emitter voltage. It is clear that at higher current levels or at a relatively low collector emitter voltage, the voltage drop over the epilayer of the collector can lead to forward biasing of the internal junction. This effect is called quasi-saturation (q.s.) and has been the subject of several publications: [1], [4]–[7], [9]–[11]. Quasi-saturation leads to current gain ($\beta$) and cut-off frequency ($f_T$) fall-off at higher current levels. Less well known to designers is the fact that the onset of q.s. also has a dominant influence on the distortion behavior of the bipolar transistor [9]. In conventional transistor models the modeling of q.s. is limited to the voltage drop over the fixed internal collector resistance ($R_c$), which can lead to "unexpected" results when realized circuits are measured. Quasi-saturation effects are of particular interest under high drive conditions, as in power stages or mixers.

When the internal base collector junction becomes forward biased, the injection of minority carriers in the epilayer leads to a strong reduction of the epilayer resistance and the build up of storage charge. This reduction of the epilayer resistance is described by Kull et al. [11] and more completely by de Graaff and Kloosterman [14], [18] who included a more precise description of the influence of space charge modulation due to hot carriers and current spreading. The latter leads to a more accurate description of the epilayer behavior in all operating modes [18]. We can model this reduction of the epilayer resistance by a current source which is controlled by two voltages, namely the internal junction voltage $V_{IJC}$ and the external voltage $V_{BE}$ (see Fig. 1). The equations for $I_{EPI}$ depend on the chosen epilayer model: Kull or Mextram.

Fig. 2 shows the dc epilayer resistance ($R_{EPI}=V_{EPI}/I_{EPI}$) behavior for the Kull Model (see Appendix). As can be noted from this figure the epilayer resistance is initially equal to $RCV$ and increases due to the space charge modulation (Kirk effect) until the internal junction voltage exceeds the built-in voltage $V_{BC}$ and q.s. sets in. It must be noted that the Kull model becomes inaccurate for a collector current $I_C \geq I_{HC} = q \cdot A \cdot N_{Qmax} \cdot N_c$.

The Mextram model release 503 [19] also includes hot carrier behavior and current spreading. The derivation of the epilayer current is given in [18]. The equations involved are given in the Appendix. Fig. 3 shows the epilayer resistance $R_{EPI}$ as function of the collector current for different external junction voltages $V_{BE}$. In this figure, the epilayer resistance once again is initially equal to the value of $RCV$; when the collector current reaches the hot carrier current the epilayer resistance will increase to a maximum value. This maximum is given by the Mextram parameter $SCR_{CV}$ which represents the space-charge limited epilayer resistance [18]. Note that the epilayer resistance decreases when the internal junction voltage exceeds the built-in voltage $V_{BC}$. The parameters used in Figs. 2 and 3 are the same.

II. THE MODELING OF QUASI-SATURATION IN TRANSISTOR MODELS

In conventional transistor models such as the Spice implementation of the Gummel–Poon (GP) model [12] the voltage drop in the collector is modelled by a single ohmic resistor $R_C$ (see Fig. 4). In the Mextram and Kull model the voltage drop...
in the collector region is modelled by the ohmic buried layer resistor $R_{CC}$ and a voltage-controlled current source $I_{CC2}$ (see Fig. 5). The controlled source is used to account for the voltage drop across the epilayer resistance, modulated by the injected storage charge in the collector region [14], [18]. The extra voltage drop in the Mextram model leads to earlier forward biasing of the internal base collector junction than in the Gummel-Poon model. This is best illustrated by considering the simulation results obtained using the Gummel–Poon and Mextram models for the same transistor. We have chosen the BFR520 transistor commonly employed in discrete RF circuits. Computed $I_C(V_{CE})$ characteristics are compared with measurement data in Fig. 6. The q.s. region ($V_{BB2} \geq 0.7$ V) is much larger for Mextram than for GP. Both models gives a reasonable fit of the $I_C(V_{CE})$ characteristic, but GP does so by manipulating the Early voltages; GP however, fails to describe distortion behavior at higher current levels or at low collector emitter voltages, as will be explained in Section III.

III. HIGH CURRENT LOW FREQUENCY DISTORTION EFFECTS

Distortion effects at low current levels in bipolar transistors are well understood [2], [3], [8]. Conventional transistor models like Gummel–Poon [12] as well as the Mextram model can model most of the distortion effects for moderate drive conditions. At high current levels or at lower collector emitter voltages q.s. will set in at a certain point for a given device. The onset of q.s. will lead to an increase in the third order distortion component [9].

**Distortion Increase Due to q.s.**

This increase in distortion is caused by the internal base collector junction becoming forward biased. When this happens the injection of minority carriers in the epilayer (related to the built-in base collector junction voltage) will lead to a substantial reduction of the epilayer resistance and the build up of storage charge. This, combined with the increase of the reverse component of the main current, will lead to a significant change in the small-signal transfer of the device under consideration. The distortion behavior can be understood by studying the small-signal transfer of the circuit in Fig. 7. At sufficiently low frequencies (ignoring reactive elements) the higher harmonics in the output of the transistor will be related to the input voltage by a Taylor series [12]. Consequently, the distortion behavior of a device at low frequencies can be studied for low driving conditions by investigating the
The influence of RCV on the fundamental and third harmonic derivatives of the ac transfer characteristics with respect to the input voltage \( V_{b2e1} \).

The small-signal definitions of the epilayer current and the main current in the Mextram implementation are

\[
\begin{align*}
    i_n &= g_x \cdot V_{b2e1} + g_y \cdot V_{b2c2} \\
    i_{epi} &= g_{epi} \cdot V_{b2c2} + g_{epix} \cdot V_{b2e1}
\end{align*}
\]

where

\[
\begin{align*}
    g_x &= \frac{\partial I_n}{\partial V_{b2e1}}, \\
    g_y &= \frac{\partial I_n}{\partial V_{b2c2}}, \\
    g_{epi} &= \frac{\partial I_{epi}}{\partial V_{b2c2}}, \\
    g_{epix} &= \frac{\partial I_{epi}}{\partial V_{b2e1}}.
\end{align*}
\]

ac short circuiting the output for the given network topology yields: \( V_{b2c1} = V_{b2e1} \) and \( i_n = i_{epi} \). Using these conditions we can solve for the small-signal transconductance \( i_{epi}/V_{b2e1} \), leading to

\[
\frac{i_{epi}}{V_{b2e1}} = g_x + g_y \left( \frac{g_{epix} - g_x}{g_y - g_{epix}} \right).
\]

Although appearing simple in its representation the conductances used in (4) represent very complicated functions. As we are interested in the influence of the epilayer parameters on the distortion behavior at the onset of q.s, we ignore (for this analysis only) the influence of the normalized base charge component (high injection, Early effect) and simplify the conductances to their first order approximation.

**No Saturation:** In this situation the main current is given by

\[
I_n = I_f = I_s \cdot \exp \left( \frac{V_{b2e1}}{V_t} \right)
\]

consequently \( g_x = I_f/V_t \) (linear with the collector current) and \( g_y \approx 0 \).

**Quasi-Saturation** \((V_{b2c2} > 0)\): In this situation the main current is given by

\[
I_n = I_f - I_s \cdot \left( \exp \left( \frac{V_{b2e1}}{V_t} \right) - \exp \left( \frac{V_{b2c2}}{V_t} \right) \right)
\]

consequently \( g_x = I_f/V_t, g_y = -I_s/V_t \), and \( g_x + g_y = I_n/V_t \).

Considering the transconductance of (4) as function of the dc collector current we note that \( i_{epi}/V_{b2e1} \) in the non-saturated region equals \( g_x \) and is linear with the dc collector current. When entering q.s. \( g_y \) is no longer zero and the transconductance no longer increases linearly (see curves marked with “fund”(-amental) in Figs. 8 and 9). The higher harmonics (2nd and 3rd) can be determined from the derivatives of the transconductance with respect to \( V_{b2e1} \).
The Influence of the Parameters VDC and RCV

For reasons of simplicity we will use the Kull model [11] with a purely ohmic epilayer description, neglecting hot carrier effects (the Kull model in any case becomes inaccurate when approaching IHC). This simplified Kull model is equal to the Mextram model for low current values and has the advantage for this analysis that $I_{q}$ is only affected by two parameters (see Appendix): RCV, which represents the epilayer resistance, and VDC (the built-in base collector junction voltage). RCV will influence the value of the collector current at which the internal junction becomes forward biased for a given external base collector voltage (see Fig. 8). In contrast, the built-in voltage VDC hardly affects the current at which the transistor enters q.s, but it will determine how much the internal base collector junction will become forward biased and thus directly determines $g_{y}$. The value of $g_{y}$ acts as weighting factor for the term between brackets in (4) and it affects the slope of the decrease of the transconductance. This is of major importance for the increase in distortion when the device enters the q.s. region. In summary, we note that a device with a high built-in junction voltage will result in more distortion when it enters q.s. than a similar device with a lower built-in voltage.

In the standard Mextram parameter extraction the epilayer parameters RCV, VDC, IHC, SFH, and SCRCV are found by fitting both the $f_{T}(I_{c})$ fall-off and the q.s. region of the $I_{c}(V_{ce})$ characteristics (see shaded area Fig. 6). This will give a good first order approximation of the epilayer parameter values, fine tuning for accurate harmonic description can be performed by concentrating on the fit of the transconductance as a function of $I_{c}$ for several base collector voltages in the frequency range of interest.

IV. SIMULATION AND MEASUREMENTS

RESULTS AT LOW FREQUENCIES

Simulations: The calculated distortion data for the BFR520 has been obtained using our Mextram implementation in Hewlett Packard's harmonic balance simulator MDS. This has proven to be a particularly valuable tool in this investigation.

Measurements: Although pulsed bias voltages are applied to minimize self heating effects, the device under test (DUT) will generally heat-up within 0.1 ms, leading to an increase in the collector current. To avoid corruption of the distortion data the LF measurement should be short in time and take place at exactly the same moment when the bias current $I_{c}$ is measured. At lower current levels distortion measurements can be carried out using a spectrum analyzer (SPA). At higher current levels self heating of the DUT will give temperature-related errors due to the minimum sweep time (e.g., 20 ms) inherent to the instrument. In principle, faster measurements are possible by setting the SPA to zero bandwidth and locking the signal source to the SPA. In practice, SPA (hp8566A) firmware related problems made it impossible to achieve the desired measurement time (e.g., $t < 0.1$ ms).
Alternatively, distortion measurements can be carried out using a 100 MHz, 500 megasamples/s data acquisition scope to maintain the phase information. An illustration of this measurement setup is given in Fig. 10. The network analyzer is used as a 10 MHz signal source. At a constant $V_{cc}$, the $V_{be}$ of the transistor is pulsed for 1 ms. The amplified signal will stabilize at the output of the DUT within 50 ps and the sample scope triggered. Following a delay of approximately 0.7 ms, the dc collector current as well as the ac voltage over the 50 ohm load resistance during a time span of 0.1 ms are measured and read by HP’s data acquisition program VEE TEST. For each bias point, a trace of the load resistance voltage is taken and a fast Fourier transformation (FFT) is performed to find the coefficients of the distortion components. Since a large number of periods are measured the FFT will work as an averaging filter, leading to improved accuracy. The measurement results obtained using a SPA and a sample scope are compared in Fig. 11. The results of the SPA drift away at higher collector base voltages as the DUT consumes more power.

Results: In comparing Mextram to Gummel–Poon simulations, we note that the third order distortion maximum is manifest when approaching the $V_{SC2} \approx 0.7$ V lines of Fig. 6. Since in the case of Gummel–Poon this line is reached much later, bipolar transistor distortion at higher current levels is improperly modelled (see Figs. 12–14 for increasing drive voltages). In these figures the amplitude of the fundamental frequency (10 MHz) as well as the second and third order distortion components of the collector voltage are plotted as a function of $I_c$ for $V_{be} = 0, -1, \text{ and } -3$ V. (The ac collector voltage is directly related to the ac collector current via a 50 ohm load resistor).

In general, the distortion of larger signals is more easily modelled because small details in the device characteristics are then of reduced significance.

V. DISCUSSION AND LOW FREQUENCY RESULTS

Comparison of the measured and simulated low frequency (LF) distortion results leads to the conclusion that Mextram is far more accurate than GP at all power levels. At higher power

levels the fit for GP seems to improve somewhat, due to the fact that the transistor now enters hard saturation increasing
the third order distortion. In Fig. 14 we note that although there is a better fit for the third order distortion, the minima are still dislocated with respect to the collector current. Second order distortion products as predicted by the GP model fail as well to describe the measured results.

VI. HIGH CURRENT HIGH FREQUENCY DISTORTION EFFECTS

In analyzing high frequency signal distortion in bipolar transistors, charge functions have to be taken into account. The distortion is attributable to an interaction of contributions caused by the nonlinear current sources (as described previously) and charges. In the previous analysis it was evident that the built-in voltage \( V_{DC} \) has a major influence on the distortion behavior at low frequencies. The same parameter \( V_{DC} \) is also very important in modeling the \( f_T \) fall-off at high current levels. In bipolar transistor models with an epilayer model based on a voltage-controlled current source, \cite{11, 18} the choice of the parameter \( V_{DC} \) is very critical. A somewhat lower built-in voltage (e.g., 650 mV) can easily cause non-monotonic behavior of the \( f_T(I_c) \) fall-off characteristic. In practice, one compromises during parameter extraction to avoid nonmonotonic behavior by choosing a somewhat higher value for the built-in voltage (e.g., \( V_{DC} = 700 \) mV). Doing so, however, increases the apparent distortion at the onset of q.s. and leads to improper modeling at high current levels.

To circumvent the problem above, an improvement to the modeling of the collector charge function, which enforces monotonic \( f_T \) fall-off and Early voltages for all parameter sets has been formulated. Implementation of this new collector charge expression in the Mextram model leads to better modeling of the distortion at the onset of quasi-saturation at higher frequencies. The improved collector charge description will be discussed in \cite{20}, in this paper we restrict ourselves to a summary of the final results.

Proper verification of distortion modeling requires comparative data. Obtaining accurate, reliable RF distortion data, while avoiding self-heating, proved, however, to be very troublesome with the equipment available. We have, therefore, in lieu of measurements used a 2-D device simulator Medici \cite{21} to generate data as suggested in \cite{17} for very large signal (almost switching) conditions. In our simulations we have used the transistor structure described in \cite{17} which is based on SIMS measurements and is representative of present day bipolar

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**Fig. 14.** Simulated and measured distortion components in the collector voltage of the BFR520 at a driving input voltage of 62 mV.

**Fig. 15.** Configuration used in the Medici calculations.

**Fig. 16.** Simulated distortion components in the collector current at a driving input voltage of 10 mV at 1 GHz.
transistor technology. The configuration that has been used in the calculations is given in Fig. 15. Distortion results for a 10 mV, 1 GHz input signal, as obtained with the 2-D device simulator Medici and the modified Mextram model are given in Fig. 16.

VII. CONCLUSIONS

Incorporation of the latest developments in the formulation of epilayer behavior in Mextram, has provided a compact transistor model capable of accurately describing the distortion behavior of a transistor when operation extends into the region of quasi-saturation. Implementation of Mextram in the simulator package MDS has resulted in a very powerful combination facilitating the use of harmonic balance techniques in strongly non-linear circuit design. The results of this paper are verified by experiments for LF conditions. When considering RF excitation the charge functions must also be taken into account. A modification to the (standard 503) Mextram model ensuring $f_T$ fall-off monotonicity, and concomitantly improved modeling of distortion under RF conditions, will be published shortly [20].

APPENDIX

EPILAYER MODELS

The Epilayer Current Formulation in the MEXTRAM Model

The derivation of the MEXTRAM epilayer model may be found in [18]. The epilayer current is given by

$$I_{\text{epi}} = I_{\text{low}} + \text{SFH} \cdot \frac{V_{\text{cl2}} - I_{\text{low}} \cdot R_{\text{CV}} \cdot \left(1 - \frac{X_i}{W_{\text{epi}}}\right)}{SC_{\text{RCV}} \cdot \left(1 - \frac{X_i}{W_{\text{epi}}}\right)^2}$$  \hspace{1cm} (A.1)

with

$$I_{\text{low}} = \frac{I_{\text{HC}} \cdot V_{\text{cl2}}}{V_{\text{cl1}} + \text{IHC} \cdot R_{\text{CV}} \cdot \left(1 - \frac{X_i}{W_{\text{epi}}}\right)}$$  \hspace{1cm} (A.2)

$$X_i/W_{\text{epi}} = \frac{E_c}{I_{\text{epi}} \cdot R_{\text{CV}}}$$  \hspace{1cm} (A.3)

$E_c$ is defined as

$$E_c = V_f \left(K_O - K_W - \ln \frac{K_O + 1}{K_W + 1}\right)$$  \hspace{1cm} (A.4)

with

$$K_O(V_{\text{cl2}}) = \sqrt{1 + 4 \cdot \exp \left([V_{\text{cl2}} - V_{\text{DC}}]/V_f\right)}$$  \hspace{1cm} (A.5)

$$K_W(V_{\text{cl2}}) = \sqrt{1 + 4 \cdot \exp \left([V_{\text{cl2}} - V_{\text{DC}}]/V_f\right)}$$  \hspace{1cm} (A.6)

where

$X_i/W_{\text{epi}}$ Normalized thickness of the injected region of the epilayer.

$\text{SFH}$ Factor for the epilayer current spreading.

$\text{IHC}$ Critical current density for hot carriers.

$R_{\text{CV}}$ Ohmic epilayer resistance.

$SC_{\text{RCV}}$ Space-charge limited epilayer resistance.

$V_{\text{DC}}$ Built-in voltage of the base collector

$V_f$ Thermal voltage.

$\text{SFH, IHC, RCV, and SC}_{\text{RCV}}$ are Mextram model parameters [18], [19]. Substitution of (A.2) and (A.3) in (A.1) and solving for $I_{\text{epi}}$ will lead to a cubic equation. Complete implementation of the Mextram model is described in [19].

The Epilayer Current According to the Kull Model

Neglecting hot carriers the Mextram epilayer current model reduces to the simplified Kull model for the ohmic case. This can be found by letting $I_{\text{HC}}$ go to infinity in (A.2) and by substitution of (A.2) into (A.1), leading to $I_{\text{epi}} = I_{\text{low}}$. With the help of (A.3) we find

$$I_{\text{epi}} = \frac{E_c + V_{\text{cl2}}}{R_{\text{CV}}}.$$  \hspace{1cm} (A.7)

The complete Kull model [11] including hot carrier effects has an additional term in the denominator

$$I_{\text{epi}} = \frac{E_c + V_{\text{cl2}}}{R_{\text{CV}} + \frac{[V_{\text{cl2}}]}{I_{\text{HC}}}}.$$  \hspace{1cm} (A.8)

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