AES/PE/12-35  Effect of Overdisplacement of Proppant in Hydraulic Fracturing Treatments on the Productivity of Shale Gas Reservoirs

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Abstract

Overdisplacement of proppant in hydraulic fracturing treatments is an operational consequence dependent on the type of completion. It is described as the injection of clean fluid to remove residual proppant from the wellbore lateral and/or to transport plugging elements after the pumping of the designed fracturing treatment is finished. This thesis investigates the effect of overdisplacement of proppant on the productivity of Shale gas reservoirs stimulated with transverse fractures. In tight gas reservoirs, where permeabilities are higher if compared with Shale gas reservoirs, overdisplacement is considered detrimental for the productivity, but studies of this effect are scarce in the literature neither for tight nor Shale gas reservoirs. In the case of Shale gas applications, where many treatments are required in the same well, there is a large economic benefit due to time savings when treatments can be overdisplaced. In Shale gas wells, where the gas flow rates per propped fracture are low, it has been proposed that overdisplacing the treatments does not impair productivity. The effect of overdisplacement is strongly related to formation strength (influencing fracture opening) and completion type, where for the latter, the perforate and plug systems, the treatments are normally overdisplaced. For modern ball drop systems, it is claimed that overdisplacement is not required, and it has been suggested that this can potentially improve the production compared to wells where the treatments have been overdisplaced.

This work presents an analytical model that quantifies what is the erosion effect of overdisplacement on the proppant bank. The model is based on the physics that describe the slurry transport in pipes. A rock mechanics approach was adopted to calculate the geometry of the fracture after overdisplacement assuming the formation of an arch at the top of the proppant bank. Productivity calculations were done based on analytical solutions for gas wells including the effect of reducing fracture connectivity to the perforations due to overdisplacement.

The results showed that even in low permeability formations, the reduction of the proppant bank height due to overdisplacement will impair fracture productivity. The height of the open fracture and the permeability of the different fracture zones, specially the closed fracture, will define the impact of overdisplacing the treatment. An analysis about the impact of changing overdisplacement operational parameters, such as viscosity, proppant size and concentration, showed that significant erosion rates occurred when using low viscosity fluids and larger proppant sizes. The results presented in this work calculated the impact of overdisplacement for a set of specific treatment and reservoir parameters and constitute a first step in the modeling of overdisplacement effect. Accurate prediction of the overdisplacement effect may certainly lead to benefits on production and costs reduction for future treatments. Further modeling of proppant distribution and fracture mechanics in Shale reservoirs will be the next step to have a better understanding of the effect and relevance of overdisplacement in fracturing treatments.
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1 Introduction

Shale reservoirs are defined as unconventional reservoir type, characterized by having very large coverage areas and hydrocarbons initially in place, but also very low matrix permeability and low final recovery factor. Opposite to conventional hydrocarbon resources, these types of reservoirs are not affected by hydrodynamic pressures to produce; therefore gas often must flow through natural or artificial fractures in order to make production profitable.

Shale gas formations are composed of sedimentary rock formations that act as both the source and the reservoir rock. In this type of reservoir there is no migration path for the hydrocarbon and the top of the reservoir lacks the presence of a trapping mechanism. These types of reservoirs are usually rich in organic material (0.5% to 25%) and mature petroleum source rocks. Characterized by continues gas accumulations, these reservoirs have gas-bearing layers not stratified by density and do not contain a gas/water contact.

Shale gas reservoirs are characterized by small grain size (average grain size below 0.0025in) and pores, resulting in formations with low or ultra-low permeability and low porosity. Typical permeabilities in this type of reservoir are in the range of hundreds of nano darcies and porosities between 2-10%. With the application of conventional methods the recovery factor in this type of reservoirs ranges around 2% which makes this type of reservoir uneconomic (Arogundade & Sohrabi, 2012).

The gas found in Shale reservoirs is contained by different mechanisms: trapping of the gas in organic matter (adsorption), the non-organic matrix porosity, the micro-fracture porosity, gas contained in Hydraulic fractures or/and existing in the pore network within the organic matter (Aguilera, 2010).

Why Shale Gas?

Shale gas reservoirs have been known since the beginnings of gas production. However due to their very low permeabilities, flow from the rock to the wellbore was insufficient and unless the reservoir contained natural fractures. For these reason for long time Shale reservoirs did not represented an economically viable target for the oil and gas industry. In fact, the first commercial gas well in the USA was a Shale gas well. It was drilled in 1821 many years before the first oil wells and was producing from shallow natural fractured reservoir (Arogundade & Sohrabi, 2012).

Similarly conventional gas reservoirs the production in Shale gas reservoirs starts with an initial peak production and quickly decreases thereafter as gas desorption replenishes the natural-fracture system. During the exploration stage however, opposite to conventional sources, Shale gas reservoirs represent a lower exploration risk due to the abundance of hydrocarbons in large coverage areas; however due to this vast area of Shale gas reservoirs, the real challenge in order to find the hydrocarbons is not to find the hydrocarbons itself but rather to
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find the areas that can be produced commercially. Shale gas reservoirs properties are very variable along the depth and laterally, making it difficult to do an adequate reservoir characterization (Jenkins & II, 2008).

The continuous developments in horizontal drilling, hydraulic fracturing and stimulation completions have turned Shale gas reservoirs into an important success in the attempt to find new energy resources. The increasing trend to explore Shale gas reservoirs and the attention it has received in the last few years is mainly due to these technological developments that improved reservoir connectivity and made it possible to achieve economically significant production rates. The availability of these new technologies encouraged the oil and gas companies to increase exploration of Shale gas resources generating new discoveries or re-assessing old reservoirs.

*Shale gas playing a worldwide role in the energy supply market*

Many of the Shale gas reservoirs exist in geological environments where a source rock exists but no conventional reservoir rock is present for the gas to permeate. Shale gas formations are composed mainly by fluid in gaseous form and condensates.

Shale reservoirs often have been found nearby to other conventional reservoirs and it is expected to have plenty of this type of gas reservoirs around the world inclusive places where no significant conventional gas resources have been discovered. Since the beginnings of natural gas production, large gas volumes from conventional reservoirs such as sandstones and siltstones represented almost the entire contribution of gas to the energy market. However during last decade great amount of wells were developed in Shale gas reservoirs in the North American basins converting Shale gas in an important source of energy. These developments and success of Shale gas exploration have converted this type of resources to be of interest for the world oil and gas industry.

According to the Council (2010) in the United States the share of Shale gas in gas production grew nearly 8% from 1996 to 2008. In the same years important discoveries increased the gas reserves estimates of US gas from 21.7tcf to 32.8tcf. By the end of 2008 Shale gas represented the 13.4% of US proved reserves of natural gas, an increment of almost 5%. Part of the excitement and fast evolution of Shale gas exploration was due to the success in the Barnett Shale where it’s fast growth (8000 wells are producing gas) has made it one of the most profitable gas Shale plays in the United States. The success in the Barnett Shale is mainly due to horizontal drilling and multi-stage vertical fracture stimulation.

North America has been the focus of Shale gas production during last years; however exploration and production prospects have been increasing all around the world. Recent studies estimate the gas in-place of five major Shale gas basins in the USA at 3,760 tcf, of which 475 tcf is considered to be economically recoverable, while two Canadian basins are estimated to hold 1,380 tcf, with about 240 tcf recoverable.

The potential of Shale gas resources create high expectation in the energy industry since the estimation of reserves has been quantified in just few amount of countries. With the exception
of North America, Shale gas production around the world has been impaired by the shortage of geological studies and reservoir knowledge at these locations. The increasing demand for new energy resources and the current gas market conditions plays a favorable role for Shale gas exploration; in Europe for example there are important basin developments undergoing such as the Alum Shale in Sweden, the Silurian Shale in Poland and the Austrian Mikulov Shale. It is estimated that around 1,000tcf of gas are contained in this reservoirs, which about 140tcf is considered to be recoverable (Council, 2010).

The reserves prospects in Europe are promising however the development of Shale gas will depend in great extent on the public policies dedicated to control and guarantee safe production. There are many doubts between Europeans about the overall safety of Shale gas development regarding water contamination. In contrast with USA, where landowners have rights and get direct profit from the minerals found at their land, in Europe the mineral rights are owned by the governments and the combination of these factors adds challenges to the Shale gas industry (Jenkins & II, 2008).

Other countries where Shale gas developments are increasing are China, Australia and India. Important exploration has been done in these countries increasing reserves in the last couple of years. Major Chinese companies merged with Shale gas experienced companies, facilitating the transfer of technology and experience in the Asian market. The demand for domestic production is growing in Asian countries encouraging companies to explore and expand operations into Shale gas prospects. An example is in India where gas production is low and most of the gas supply comes from offshore conventional reservoirs, making it costly. As consequence additional plans to build pipelines and export gas from neighbor countries are being considered. The future advances in Shale gas exploration may represent the changing point for of India’s energy supply strategy.

The declining production of oil, the increasing demand of energy worldwide, the dynamic in technology developments and the current gas market conditions, set up a favorable stand point for the nonconventional resources market. Undoubtedly Shale gas will contribute significantly to accomplish the world needs of gas supplies in the near future.
2 Literature study

Shale gas reservoirs may naturally produce gas at really low rates, however almost all the wells producing gas from Shale reservoirs rely on well stimulation in order to be economically feasible. Due to the recent developments in down hole technologies associated to increasing benefit-cost ratios, most of Shale gas reservoirs developments are done based on long horizontal wells stimulated by multi-staged hydraulic fracture treatments with tightly spaced transverse fractures.

The main role of hydraulic fracturing is to increase reservoir inflow area by creating fractures and connecting the fracture network to the wellbore. Most of Shale gas wells are cased, cemented and perforated with multiple fracture treatments performed along the length of the horizontal well, but a significant number are also completed with open-hole wells, using packers to isolate the different intervals to be fractured.

The complexity of performing hydraulic fracture treatments is related to the uniqueness of each Shale gas reservoir and its variability, as result Shale gas wells characteristics will vary from basin to basin but also within the same field.

When performing hydraulic fracturing treatments, the main objective is to generate fractures into the reservoir that improve the effective well inflow area and increase productivity. To achieve effective results the fractures must be placed precisely in the zones of interest, minimizing the stimulation of non-productive reservoir rock and maximizing the potential of hydrocarbon bearing layers.

In Shale gas reservoirs, due to the low permeabilities and variability of the rock properties, significant production is dependent on the generation of several fractures along the lateral, which is achieved by stage fracturing, where the reservoir target is divided into sections and hydraulically fractured in sequential stages (Economides & Nolte, 2000)

The main principle to perform a hydraulic fracturing treatment is to isolate the different target zones and propagate a fracture into the formation by injecting fluid through perforations previously shot at this interval. The main issue when performing a treatment is then to successfully isolate the different stages and depending on the technical characteristics of the well and economic constraints different type of completions may be selected to perform this isolation.

There are several combinations of technologies and developments available in the industry to perform multi-stage fracturing treatments. However there are two main completion methods: plug-and-perf method and multi-stage ball drop systems. Although both methods aim to increase connectivity between the reservoir and the well along the lateral they have significant differences from an operational perspective (Edwards, Braxton, & Smith, 2010).
2.1 Plug-and-perf fracturing method

In Shale reservoirs where matrix permeability is low and production from reservoir without stimulation is negligible, open-hole completions are not required and cemented completions are often preferred. Perf-and-plug completion is the most common method used in multi-stage fracturing of low permeability reservoirs with cemented liners.

In these cemented completions, the casing and cement will provide the required zone isolation. The main purpose in plug-and-perf fracturing is to provide mechanical isolation inside the liner by the setting of bridge plugs using pump down wire line or coiled tubing (Figure 1). Often the pump-down plug will carry into the lateral the perforating gun so that, after setting the plug, the next target zone can be perforated and the fracturing process performed. This process is then repeated for the number of stimulations desired for the horizontal wellbore. After all the stages have been completed, CT is used to drill out the composite plugs, thus re-establishing access to the toe of the horizontal wellbore (Edwards et al., 2010).

For low permeability formations the plug-and-perf system is used for sand/water or slick-water treatments. When performing a treatment with perf and plug completion, the adequate positioning of the plug will depend on having a solids-free horizontal section, this condition will guarantee that the plug will be set without sticking before reaching the intended location and will guarantee a complete sealing fit between the plug and the liner, avoiding leakages during the pumping of the next stage (zone isolation).

In order to fulfill the solids-free condition, residual proppant particles must be removed or cleaned from the bottom of the lateral bed. The cleaning of the wellbore is often performed by injecting additional fluid at certain critical flow rates that generate turbulence and will transport the proppant sediments along the lateral into the fracture.

The practice of injecting additional fluid, once the planned treatment is performed, to clean the wellbore is called overdisplacement. In general terms treatments are over displaced to have a clean wellbore, however the fluid required to transport the plug and gun assembly into the lateral is also considered part of the overdisplacement (Gijtenbeek, Shaoul, & Pater, 2012).

Figure 1- Plug-and-perf completion (Thompson, Rispler, Stadnyk, Hock, & McDaniel, 2009)
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2.2 Multi-stage ball drop fracturing systems

Multi-stage ball drop fracturing systems are often performed in open-hole completions, the main goal being to perform multi-stage fracturing in a more time and cost efficient manner. Since there is no cement at the annulus this type of completion use external packers instead of bridge plugs to isolate sections of the wellbore. This adds a degree of completion risk over cemented-liner completions, since there is a need to achieve complete installation of the liner to obtain a liner seal and ensure that the liner can be pressured adequately to activate inflatable packers (Mcdaniel & Rispler, 2009).

These packers typically expand to seal against the wellbore and do not need to be drilled out to initiate production. Since there is no cement isolating the formation, instead of perforating the casing to allow fracturing, these systems have sliding sleeve tools to create ports in between the packers (isolated zone). These sliding sleeves can be opened hydraulically (at a specific pressure) or by dropping size-specific activation balls into the system to shift the sleeve and expose the injecting port (Figure 2).

The balls create internal isolation from stage to stage, eliminating the need for bridge plugs. In order to catch the proper ball but allow some to pass through, each sleeve tool must have a properly sized baffle, with the smallest baffle in the tool just above the toe and progressively larger baffles moving toward the heel.

This system permits to perform treatments in a continuous and single pumping operation without the need for a drilling rig. Once stimulation treatment is complete, the well can be immediately flowed back and production brought on line saving time and money (Edwards et al., 2010).

Since Ball drop systems need to pre select the intervals to be treated before running into hole (contrary to perf and plug where intervals can be chosen after the casing has been cemented), the fracturing target is fixed and cannot be modified once the fracturing job has initiated, adding uncertainty and less control to the fracture output if compared with plug-and-perf completions.

In this system, there is no need to overdisplace the treatments, and this will not occur if the balls seat correctly. However if the Ball sealers get eroded due to the erosive nature of proppant or there is communication between zones due to leakage or loss of differential pressure the ball sealer may become unseated and zone isolation will be lost. If ball drop systems must be used, several guidelines have been established for their application (Economides, Martin, & Company, 2007).

Development of best fracturing practices over the last few decades would include: not overdisplacing proppant, ensuring near-wellbore conductivity, promoting immediate flow back, optimizing load recovery, keeping breakdown pressures low, minimizing fracture tortuosity and minimizing fluid loading (Snyder, Seale, & Hollingsworth, 2010).

A comparison between the main pros and cons of each type of completion are presented in Table 1:
Table 1 - Completion comparison

<table>
<thead>
<tr>
<th>COMPLETION COMPARISON TABLE</th>
</tr>
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<tbody>
<tr>
<td><strong>Ball Drop Systems</strong></td>
</tr>
<tr>
<td>Fast execution of several stages</td>
</tr>
<tr>
<td>Does not require cemented completion</td>
</tr>
<tr>
<td>Limited number of stages depending on ball sizes</td>
</tr>
<tr>
<td>Higher installation and operational risk</td>
</tr>
<tr>
<td>High direct cost</td>
</tr>
<tr>
<td>Intervals must be preselected and cannot be modified once the completion is down hole</td>
</tr>
</tbody>
</table>

2.3 The Overdisplacement

In a hydraulic fracturing treatment, proppant is pumped with fracturing fluids to create a large fracture and maintain a conductive, high permeable connection between the well and the reservoir. After the designed amount of proppant has been pumped, a final stage of clean fluid is pumped in to clean out any proppant from the wellbore before isolating and perforating the next interval. This clean fluid injection stage is called overdisplacement of proppant used in the hydraulic fracture treatments.
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Overdisplacement is often considered an undesirable practice, it is thought that it will flush out the proppant from the near-wellbore area damaging the objective of having a highly conductivity area inside the created propped fracture (McLennan, Green, & Bai, 2008). In perf-and-plug completions, where overdisplacement is a common practice, typically an entire wellbore worth of fluid is overdisplaced before a bridge plug is pumped down (Snyder et al., 2010).

Due to its low matrix permeability, fracturing treatments performed in Shale gas reservoirs are usually based on low viscosity fracturing fluids (water). The use of this type of fluids plays an important role in determining the distribution of the connectivity between proppant bank in the fracture and the well, since its transport capacity is low and proppant will settle fast compared with gels.

In higher permeability formations, where cross linked gels are injected and a more spread distribution of proppant along the fracture height is expected, overdisplacing proppant can seriously compromise wellbore-to-fracture connectivity (Gijtenbeek et al., 2012). The concept of overdisplacing serves as a platform for reviewing the role of proppant in these low permeability complex systems.

In horizontal wells fractures propagate perpendicularly to the direction of the minimum horizontal stress. Therefore depending on the direction towards the horizontal well was drilled the fractures will be longitudinal or transverse to the axis of the horizontal section. In Shale gas reservoirs, where large areas and the creation of fracture networks are often preferable, several transverse fractures (stages) along the well are commonly the chosen option. Compared with longitudinal fractures, in transverse fractures the contact area between the well and the fracture is small and limited to the well circumference times the width. This condition makes well bore-fracture connectivity very important and if proppant is flushed away from the wellbore (overdisplaced) at the end of a treatment the fracture can completely close - choking flow into the wellbore (McLennan et al., 2008).

In general an undesirable situation for wells stimulated with transverse fractures is a closed, un-propped fracture at the near wellbore area. Therefore, in transverse fractures any effort to improve fluid flow by means of wider fractures, cleaner fluids, and better or improved proppant placement should provide better production.

In low viscosity treatments and transverse fracture wells, proppant will tend to settle fast in the fracture and conductivity in the near wellbore is the critical factor to production. It may be then beneficial to pack the proppant off in the vertical direction to ensure that the entire thickness of the formation of interest is propped open at the near wellbore area.

Even thought, for these type of treatments it is recognized in practice that the creation of very narrow proppant-free gaps will occur at the closed fracture zone, and It has even been under discussion that having an un-propped zone at the top of the fracture may be beneficial to production if a significant residual width or conductivity remains at this region (Gijtenbeek et al., 2012). In fact, certain low permeability treatments have been explicitly overdisplaced to try and guarantee that there will be an un-propped zone at the top of the open fracture (McLennan et al., 2008). Even has been discussed that for treatments performed in high modulus, high strength formations a transition zone between the proppant bank and this un-propped closed portion of the fracture, will remain open without proppant support after the injection has stopped and the fracture has closed onto the settled proppant bank.
In this case, additionally to the un-propped closed fracture at the wellbore, this in between zone if located at the near wellbore area would deliver infinite conductivity just outside the perforations and would be of enormous benefit in conductivity. In fact, this situation is comparable to what is thought to occur after proppant flow back in moderate-permeability reservoirs, where a cavity (or channels) would be created in the proppant pack outside the perforations (Gijtenbeek et al., 2012). The benefits of creating a channels or arch, as we refer to it in this document, have lead operators to review and consider the overdisplacement of treatments as a method to physically ensure the creation of such channels at the end of the injection. Since the conductivity of an open channel trends to infinity the arch height does not need to be large to have a major effect on well productivity (McLennan et al., 2008).

It is well known that for slick water fracturing treatments in Shale gas reservoirs the poor proppant transport conditions may result in insufficient near wellbore conductivity even without overdisplacement. For this reason successful Shale-gas stimulation must guarantee a minimum amount of residual fracture width to keep the fracture connected to the wellbore. The discussion is then focused on studying under which circumstances this can be achieved even when the treatment is overdisplaced (Gijtenbeek et al., 2012). Apparently under specific conditions of proppant distribution and reservoir properties, overdisplacement may not cause a total failure of the propped-fracture stimulation, and even improve production by the creation of un-propped productive zones at the top of the propped fracture.
3 Overdisplacement model

This chapter presents the equations and concepts used to calculate fracture treatment parameters and productivity for an overdisplaced fracture treatment. For certain pre-defined fracture geometry, proppant concentration, reservoir properties and well dimensions, the output of the calculations will be the required pressure and flow rate to perform the treatment, the overdisplacement flow rates, and the effective fracture geometry and productivity.

3.1 Hydraulic fracture treatment design

In practice, there are several computational packages and software types that are used to design and model hydraulic fracture treatment. However in order to calculate analytically the effect of overdisplacement, an analytical fracture design model was used and explained in this section.

Previous to the execution of a fracturing job, and especially in very low permeability formations, it is common practice to perform several tests and measurements to gather information about the formation (e.g. in situ stresses, mechanical properties, permeability) and fracture propagation behavior (reservoir pressure, the pressure decline curve). One useful test that is often performed is the mini-frac or mini-falloff test. The mini-frac test consists of creating a small fracture by injecting fluid without proppant into the formation and observing pressure decline during fracture closure. Fluid efficiency is one of several important parameters that can be obtained from mini-frac tests, and which gives an estimate of the proppant treatment efficiency. Fluid efficiency η is defined as (Economides & Nolte, 2000).

\[ \eta = \frac{V_f}{V_i} \]  

(1)

Where \( V_f \) and \( V_i \) are the fracture and injected volumes before pumping is shut down. From pressure diagnostics theory, and making use of the relationship between fluid efficiency and dimensionless closure time (Figure 3), it is possible to estimate fluid efficiency. The dimensionless closure time (Economides & Nolte, 2000) is the ratio between the shut in time \( \Delta t \) (fracture closure time after pumping stops), and injection or pumping time \( t_i \).

\[ \Delta t_D = \frac{\Delta t}{t_i} \]  

(2)

The dimensionless functions attached to the derivation of the relationship shown in Figure 3 (Economides & Nolte, 2000), are described by power law expressions with exponents \( \alpha \) and are evaluated on the boundaries of these \( \alpha \) value. As the difference in results at the boundaries is small, averaged efficiency values were selected for this work. The relationship presented in Figure 3 is independent of the propagation model.
The concept of fluid efficiency is fundamental to fracture design calculations and for a fracture with dimensions width $w$, half-length $x_f$ and height $h_f$ the injected volume required to create the fracture is (Economides & Nolte, 2000):

$$V_i = \frac{w_f h_f x_f}{\eta}$$

(3)

The most recent models developed for the modeling of fracture propagation in 2D are the KGD and PKN models. Both models assume fully confined fractures but differ in their conversion method from 3D to 2D. The KGD model assumes a rectangular vertical cross section and equal horizontal cross sections along the fracture. The KGD model is valid if the ratio $x_f/h_f << 0.5$. The PKN model assumes that pressure at any vertical section is height dominated, thus generating an elliptical vertical cross section. The PKN model is valid if the ratio $x_f/h_f > 3/2$. The literature also describes the Radial model, which is a modification of the KGD model in the case $x_f/h_f \approx 1$.

The main assumptions considered in the 2D models are:

- Fracture is planar; however the error due to planar strain assumption is negligible.
- Fluid flow is in one dimension along the fracture length
- Rock is continuous, homogeneous, isotropic and elastic.
- The Stress difference between layers is enough to confine the fracture.
- Fracture propagation is dominated by tip effects in the KGD model
- The PKN model neglects fracture mechanics (tip effect).

In low permeability reservoirs, fracture length is critical to improve dimensionless conductivity and longer fractures are desirable. In such treatment fracture length is the dominant dimension, making the PKN model appropriate to simulate fracture propagation. The PKN model calculates the maximum width at the wellbore as (Economides & Nolte, 2000):

$$w_w(\text{max}) = 3 \left[ \frac{\mu q_i x_f}{E r} \right]^{1/4}$$

(4)
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Where $W_{w\text{max}}$ is the maximum fracture width at the wellbore (mt), $\mu$ is the fluid viscosity (Pa*s), $q_i$ is one wing flow rate (m$^3$/s), $x_f$ is the half-fracture length (mt) and $E'$ is the plane strain modulus defined as $E' = E/(1-\nu^2)$ where $\nu$ is the Poisson’s ratio and $E$ is the young’s modulus.

In Equation (4) fracture width is controlled by fracture length, fluid viscosity, flow rate (which is linked to pressure gradient and fluid velocity) and the stiffness of the rock. By combining equation (4) and the Ostwald–de Waele relationship (power-law fluid equation), a more generalized solution was developed to include the viscosity behavior of non-Newtonian fluids (Economides & Nolte, 2000):

$$w_{w\text{max}} = 9,15 \left(\frac{1}{2n+2}\right)^{3,98} \left(\frac{n}{2n+2}\right) \left[1 + 2,14n\right]^{\frac{n}{2n+2}} K \left(\frac{1}{2n+2}\right) \left[\frac{q_i n h_f (1-n) x_f}{E'}\right]^{\frac{1}{2n+2}}$$

(5)

Where $n$ is the flow behavior index and $K$ is the flow consistency index (Pa*s$^n$). Equations (4) and (5) determine the maximum width at the wellbore $W_{w\text{max}}$, however in the case of fracture conductivity calculations the average width may be more realistic. For a fracture with an elliptical cross section the average width at the wellbore is (Economides & Nolte, 2000):

$$w_{w\text{avg}} = 0,628 w_{w\text{max}}$$

(6)

The essence of hydraulic fracturing models is to predict the fracture geometry and estimate fracture dimensions for given treatment parameters such as pressure, flow rates, fluid properties and rock properties. In practice, following the inverse process makes it possible to calculate some of these treatment parameters for certain pre-defined fracture geometry. By arranging equations (4) and (5) and assuming certain fluid and rock properties, it is possible to calculate the pump rate required to create a specified fracture. The injection time is then obtained from the definition of flow rate $q = V/t$.

![Figure 4 - Injection flow rate vs. Max width for different fluid viscosity](image-url)
Figure 4 presents the required injection rate to generate a fracture of certain maximum width at the wellbore for different values of viscosity of the fracturing fluid. This graph validates the calculations and shows the importance of selecting and adequate fracturing fluid. As expected, high viscosity fluids required lower injection rates to propagate a fracture of certain with. In the case of Shale gas reservoirs, where slick water treatments are often performed, the injection rates are extremely high and require high pumping capacity. The injection rates for water injection may rise around 100 times higher than those for high viscosity fluids.

It is important to notice than for low permeability formations high viscosity injection fluids are not effective in proppant transport and lead to reservoir damage. The selection of injection fluids is then limited to low viscosity fluids, however these types of fluids have poor proppant transport capacity and sedimentation of proppant particles are expected to settle fast both in the lateral section of the well and in the fracture.

Shale gas reservoirs are characterized by the variability of their properties between basins and even inside a field. The young modulus defines the stiffness of the rock to be deformed elastically. As expected for higher young modulus, which means a stiffer rock, higher pressures are needed to open the fracture and therefore higher injection rates are needed to generate the higher pressure. In some Shale gas formations, where any improvement in fracture connectivity may generate a significant production improvement, successful results may be achieve by creating narrow fractures. However, the pressures is a given, determined by factors like friction. In narrow, branching fracture systems the pressure may be very high. If this is the case, the required injection rates may be low and independent from the young modulus. This can have a counter effect if due to the low flow rates there is not enough turbulence to transport the proppant in the lateral section of the well. Additional attention must be then put into the selection of proppant size, assuring low settling velocities and fulfilling proppant pack permeability requirements.

Figure 5 presents required injection rate to generate a fracture of certain maximum width at the wellbore for different type of reservoir young modulus.

![Figure 5 - Injection flow rate vs. width for different reservoirs young modulus](image-url)
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The adequate selection of viscosity and characterization of young modulus is critical to guarantee that for any selected maximum width a realistic flow rate is calculated. It is important to notice that the flow rate described in previous figures do not correspond to the flow rate required to clean the lateral or to well production, it is simply the injection rate required to propagate the fracture for different type of fluids and reservoir conditions.

Quantification of fluid loss into the formation during fracture treatment is fundamental to treatment success. Fluid loss is controlled by the leakoff coefficient $C_L$, which includes three leakoff contributing mechanisms acting as resistors in series: wall building, filtrate effect and formation fluid leakoff resistance (Economides & Nolte, 2000). In addition to the leakoff coefficient, for wall building fluid materials, any losses at the beginning of the treatment before the filter cake is formed are characterized by the spurt loss coefficient $S_p$.

Economides and Nolte (2000) presented a simple mathematical equation in which it was assumed that the combined effect of loss mechanisms could be seen as a material property, and presented a general form for an overall material balance along the fracture:

$$V_i = V_f + 2A_f r_p (kC_L \sqrt{t} + S_p) \quad (7)$$

Where $V_i$ is the injected volume (m$^3$), $V_f$ is the fracture volume (m$^3$), $A_f$ is one wing fracture face area defined as $A_f = h_f x_f$ (m$^2$), $r_p$ is the ratio of permeable area to total area defined as $h_p/h_f$ (dimensionless), $\kappa$ is the opening time distribution factor (dimensionless), $C_L$ the leakoff coefficient (m/s$^{0.5}$), $t$ is the injection time (s) and $S_p$ is the spurt loss coefficient, m.

Equation (7) is the basis for the derivation of the fluid efficiency-dimensionless time relationship. As we have mentioned it considers the leakoff behavior by means of the $C_L$, $S_p$ and $\kappa$. The value of $\kappa$ depends on the surface opening during the first pumping time and represents the ratio of fluid loss with and without spurt. It reaches a maximum value of 2 if the entire surface opens at the first moment and a value of 1 for the no spurt loss case. The term $r_p$ represents the ratio between permeable (fluid loss) and total fracture area and for simple calculations it may be assumed to be 1. By substituting $V_i$ and $V_f$ by their equations into equation (7) and rearranging the resulting expression it is possible to calculate the fluid loss coefficient:

$$\left(\frac{q_i}{h_f x_f}\right) t - \left(2kC_L \sqrt{t} - (w_{W_{av}}) + 2S_p\right) = 0 \quad (8)$$

An estimation of the proppant schedule (pad volumes and proppant addition) may be calculated using the analytical approximation presented by Nolte (1986) based on fluid efficiency (Economides & Nolte, 2000). If spurt loss is neglected, the fraction of clean or pad volume required to propagate the fracture is calculated by means of the fluid efficiency as:

$$f_{pad} \approx (1 - \eta)^2 \equiv \frac{1 - \eta}{1 + \eta} \quad (9)$$
The $f_{pad}$ calculation using both equalities in equation (9) may differ, if it does, an average value is then recommended. The pad volume is defined as $V_{pad} = f_{pad} * V_i$ and the pad injection time is $t_{pad} = V_{pad}/q_i$.

The required proppant concentration $c_p(t)$, defined as proppant weight $W_{ppt}$ per slurry volume $V_i$ at time $t > t_{pad}$ is (Economides & Nolte, 2000):

$$c_p(t) = c_f \left( \frac{t - t_{pad}}{t_i - t_{pad}} \right)^\epsilon$$

(10)

Where $c_p$ is the required Proppant concentration (kg/m$^3$), $c_f$ is the final proppant concentration at the fracture (kg/m$^3$), $t_{pad}$ is the pad injection time (s), $t_i$ is the total injection time (s) and $\epsilon$ is the pad fraction or $f_{pad}$.

Equation (10) defines the concentration at time $t$ which is required to achieve a spatially uniform concentration $c_f$ along the fracture at the end of pumping. The average proppant concentration $c_{p,avg}$ and proppant mass $M_{ppt}$ are obtained by integrating equation (10) and are presented in equations (11) and (12).

$$c_{p,avg} = \frac{c_f}{1 + \epsilon}$$

(11)

$$M_{ppt} = c_{p} V_i$$

(12)

From equation (10) it is possible to calculate the amount of proppant that has to be added to clean fluid volume:

$$c_{added} = \frac{c_p}{1 - \frac{c_p}{\rho_{ppt}}}$$

(13)

Where $\rho_{ppt}$ is proppant density expressed in kg/m$^3$.

When designing a hydraulic fracturing treatment the main aim is to obtain optimal dimensionless fracture conductivity $C_{Df}$ to improve production. Dimensionless fracture conductivity depends on reservoir permeability, fracture length and fracture conductivity (Economides & Nolte, 2000):

$$C_{Df} = \frac{k_f w_{f,ppt}}{k_{res} x_f}$$

(14)

Where $k_f$ is the proppant pack permeability (mD), $w_{f,ppt}$ is the propped fracture width (m), $k_{res}$ is the reservoir permeability (mD) and $x_f$ is the fracture half-length (m).

With the exception of reservoir permeability, all parameters in equation (14) are provided as part of the hydraulic treatment design. Propped width and fracture half-length are outputs of the stimulation process and proppant pack permeability depends on proppant and fluid selection, in-situ stress and residual damage caused by additives.
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The Propped width (Economides & Nolte, 2000) depends on the proppant concentration required to determine how much from the hydraulic width will remain after fracture closure. An estimation of the propped width \( w_{f,\text{ppt}} \) is obtained by:

\[
w_{f,\text{ppt}} = F w_f
\]  
(15)

Where \( F \) is the filling fraction defined as,

\[
F = \frac{c_f}{(8.33 \gamma_{\text{ppt}} + c_f) (1 - \phi)}
\]  
(16)

And \( c_f \) is final proppant concentration at shut in (ppg), \( \gamma_{\text{ppt}} \) is proppant specific gravity and \( \phi \) is proppant pack porosity.

As part of the iterative process of designing hydraulic fracture treatment, it is necessary to compare the propped width with the proppant admittance criteria, ensuring enough perforation and fracture width to admit proppant particles. The proppant admittance criterion (Economides, 2000) is based on proppant volume fraction \( f_{\text{ppt}} \) and expressed as:

\[
f_{\text{ppt}} < 0.17 \text{ then } w_{f,\text{avg.}} > \left( \frac{1 + \frac{2 f_{\text{ppt}}}{0.17}}{3 d_{\text{ppt}}} \right) d_{\text{ppt}}
\]  
(17)

Where \( f_{\text{ppt}} \) is the proppant volume fraction \( f_{\text{ppt}} = c_f / \rho_{\text{ppt}} \) and \( d_{\text{ppt}} \) is the average proppant particle diameter (mt).

In order to carry out hydraulic fracture treatment and as an important part of the economic analysis, it is necessary to quantify the required amount of equipment to execute the job. In practice, equipment is a decisive design factor and limitation, and one of the controlling components is treatment pressure \( P_{\text{pump}} \). Treatment or pumping pressure (Economides & Nolte, 2000) defines the power required to execute a job and it is composed by the contribution of hydrostatic pressure, friction pressure, net pressure and minimum horizontal stress.

The pressure in the fracture is defined as:

\[
P_f = P_{\text{pump}} + P_{\text{hydrostatic}} - P_{\text{friction}}
\]  
(18)

Hydrostatic pressure is proportional to fracture fluid density and vertical depth, while friction pressure represents pressure losses due to pipe and perforations friction.

The pressure required to propagate a fracture or Net pressure \( P_{\text{net}} \), is defined by the difference between fracturing pressure and closure stress, which for an ideal homogeneous formation is assumed to be the minimum in situ stress. In deep wells closure stress is the minimum horizontal stress. Combining equation (18) with the definition of net pressure we obtain:

\[
P_{\text{net}} = P_f - \sigma_{h,\min} = P_{\text{pump}} + P_{\text{hydrostatic}} - P_{\text{friction}} - \sigma_{h,\min}
\]  
(19)

By using fluid and fracture mechanics and including the fracture tip effect, it is possible to get an estimation of the net pressure:
Where \( q \) is the total flow rate in m\(^3\)/s and \( P_{\text{tip}} \) represents the effect of formation toughness in fracture propagation. The effect of \( P_{\text{tip}} \) varies depending on the formation properties and it is often neglected for a wide range of reservoir types.

By combining equations (19) and (20) it is possible to estimate \( P_{\text{pump}} \) pressure and the required power:

\[
P_{\text{pump}} = P_{\text{net}} - P_{\text{hydrostatic}} + P_{\text{friction}} + \sigma_{h,\text{min}}
\]

\[
\text{Power} = \left( P_{\text{pump}} \times q \right)
\]

The previous concepts represent the simple relations to describe single planar fractures, applicable to conventional fracturing. The analytical model described in this work assumes the generation of a single planar fracture at the proximity of the well, however Shale gas reservoirs are only economic when fracture networks are generated and then the simple relations may be invalid.

3.2 Calculation of overdisplacement volumes

After executing a fracture stage during treatment, the required placement of plugs or ball dropping, to fracture the next stage implies the injection of additional fluid to clean the lateral wellbore section or displace the ball (overdisplacement). This additional fluid volume is then defined as the volume required to remove the residual proppant in the pipe bed in order to avoid leakages once the plugs have been placed (for perf-and-plug completion) or the fluid volume to transport a ball to the completion mechanism (drop ball completion).

Perf and plug completion has been widely used around the world in fracturing practice, which is the most common type of completion. The overdisplacement volume during execution of perf and plug completion is considerably higher compared to ball drop completion. The following calculations focus on estimating overdisplacement volumes for perf-and-plug types of completion.

The movement of particles being transported in the pipes is mainly controlled by fluid velocity density and viscosity and particle concentration and size. For certain pipes, fluid and particle parameters, it is possible to estimate the fluid critical velocities defined as the minimum velocities in which suspended particles will form a sediment bed at the bottom of the pipe. Durand and Condolios derived an equation to calculate deposition critical velocity for sand and gravel (Abulnaga, 2002).

\[
v_{\text{DC}} = F_L \left[ 2gD \left( \frac{\rho_{\text{ppt}} - \rho_l}{\rho_l} \right) \right]^{1/2}
\]

Where \( F_L \) is Durand velocity factor, \( g \) gravity acceleration (9,81 m/s\(^2\)), \( \rho_{\text{ppt}} \) is the density of solids or proppant particles (kg/m\(^3\)), \( \rho_l \) is the liquid or fluid density (kg/m\(^3\)) and \( D \) is the pipe diameter (mt).

Since its development, Durand and Condolios equation has been refined and new and more accurate correlations have been developed; nevertheless the results obtained from equation...
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(23) may be used as a reference to validate velocities from other models. For low concentrations the Durand velocity factor had been tabulated. Schiller and Herbich also presented an equation to calculate $F_L$ (Abulnaga, 2002).

$$F_L = 1.3C_f^{0.125}[1 - e^{-6.9d_{50}}]$$

(24)

Where $C_f$ is the proppant fraction (dimensionless) and $d_{50}$ is the $P_{50}$ particle diameter (mm).

Critical deposition velocity is the minimum flow velocity required to avoid the settlement of particles on the pipe bed and it defines the minimum pump rate for minimum particle sedimentation in heterogeneous flow (existence of concentration gradient along the pipe cross section). Re-suspension critical velocity is the minimum velocity required to re-initiate suspended particle motion after interrupted flow.

On the basis of energy balance and energy dissipation, Oroskar and Turian (1980) developed a more accurate correlation to estimate critical velocities for slurry transport. This correlation has been modified by several authors, calculating critical velocities for the different set up conditions (e.g. non-newtonian fluids, smaller particle diameters, etc.) that were not considered in the original correlation. For the purposes of this work, Oroskar and Turian (1980) approach offers a fair estimation of the critical deposition velocity. Calculations of re-suspension critical velocity or critical velocity for non-newtonian fluids are based on the contribution made by Shah and Lord (1991).

The Oroskar and Turian (1980) correlation for calculating critical deposition velocity $v_{DC}$ is:

$$v_{DC} = \left(1.85c^{0.1536}(1 - c)^{0.3564}\left(\frac{dp}{D}\right)^{-0.378}N'_{Re}^{0.09}X^{0.30}\right)\sqrt{\frac{gd_p(P_{ppt} - 1)}{\rho_l - 1}}$$

(25)

- $N'_{Re}$: Modified Reynolds number:

$$N'_{Re} = \frac{D\rho_l\sqrt{gd_p(P_{ppt} - 1)}}{\mu}$$

(26)

Where $c$ is the proppant concentration (dimensionless), $d_p$ is the particle diameter (m), $D$ is the pipe diameter (m), $X$ is the energy dissipation fraction (It is assumed as 0.95), $g$ is gravity acceleration ($9.81m/s^2$), $\rho_{ppt}$ is the density of solids or proppant particles (kg/m$^3$), $\rho_l$ is liquid or fluid density (kg/m$^3$) and $\mu$ is the fluid viscosity or fluid apparent viscosity for Newtonian and non-Newtonian fluid respectively.

Equation (25) is based on an analysis of the energy balance and energy dissipation required to suspend particles in flow. The coefficients and exponents were obtained from regression analysis gained from experimental investigation.

Shah and Lord (1991) proposed a generalized form of equation (25) to estimate critical velocities for specific operational conditions.
To calculate re-suspension critical velocity in sand/water slurries, Shah and Lord (1991) altered coefficient $A_1$ and exponent $A_3$ in equation (27), to obtain the following correlation:

$$v_{Sc} = \left( 0.963 c^{0.1536} (1 - c)^{0.3564} \left( \frac{d_p}{D} \right)^{0.378} N'_{Re}^{0.181} X^{0.30} \right) \sqrt{g d_p \left( \frac{\rho_{ppt}}{\rho_l} - 1 \right)} \quad (28)$$

For non-newtonian fluid Subhas and Lord proposed replacing the viscosity term in the Reynolds number calculation with the apparent viscosity calculated from power law fluid equation.

The critical velocities obtained from equations (25), (27) and (28) have limited application in certain operational conditions and flow parameters; the accuracy of the results under different assumptions must be thus carefully reviewed.

When converting velocity to flow rate by $q = v \cdot A$, being $v$ velocity and $A$ the pipe cross section area it is possible to compare pumping flow rates to critical flow rate. Since the described fluid velocities define the flow rate necessary to re-suspend and transport particles in a pipe were the main criteria to calculate the overdisplacement volumes. The cross section area of the tubular will define the flow rates, and the overdisplacement volume will be proportional to the liner diameter. Figure 6 shows the variation of the critical sedimentation flow rate with respect the lateral pipe diameter for different viscosities.

![Figure 6 - Sedimentation Flow rates](image)
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Figure 7 presents the variation of overdisplacement volume for different pipe diameters and the comparison of these volumes with respect the wellbore volume.

As we can observe in Figure 6 and Figure 7, overdisplacement volumes are proportional to the pipe diameter and present an almost straight slope. This shows that the effect of pipe diameter on equations (25), (27) and (28) is not significant and the flow rates are mostly dependent on cross section area. Pipe diameter however it is not considered as a main controlling parameter since in practice, for a specific well, the liner diameter and well dimensions are often constant and are not easily modified.

Figure 7 - Overdisplacement Volume vs. Pipe diameter

Estimation of Overdisplacement Volumes for plug-and-perf completion

To estimate overdisplacement volumes a simple transport model is proposed. If we assume that a thin bed of proppant particles settles at the pipe bottom along the lateral and after the injection of proppant slurry, the overdisplacement volume for a fracturing stage will be proportional to the required distance that proppant particles must travel to remove the proppant bed in certain sections of the lateral.

If one considers the first particle of proppant to be deposited at the heel of the lateral, and the proppant bed to extend from this point to the perforations, then it is required that by injecting fluid at a certain flow rate, such a particle will move forward and settle beyond the position for the next stage plug (Figure 4). The previous statement assumes that the movement of particles occurs simultaneously, thus meaning that particles at the heel travel a longer distance and once they have been deposited at the target position, all other particles will have already been transported, so resulting in a clean wellbore from the heel to the target position (after the plug).
Figure 8 shows a horizontal well with a proppant bed along the lateral. The travel distance of particle “A” then is described by:

\[ d_o = x_{plug} + L_{plug} + x_{plug-perf} \]  \hspace{1cm} (29)

Where \( d_o \) is the travel distance required to clean (transport) particle “A” (mt), \( x_{plug} \) is the plug position (m), \( L_{plug} \) is the plug length (mt) and \( x_{plug-perf} \) is the distance between plug-and-perforations or target position (mt).

\[ \text{Figure 8 - Cleaning and plug positioning.} \]

The time to clean the wellbore is defined by the travel time of particle A as:

\[ t_o = \frac{d_o}{V_c} \]  \hspace{1cm} (30)

For water fracturing treatments, \( V_c \) is the critical re-suspension velocity \( V_{crit} \), otherwise \( V_c \) should be considered as the critical deposition velocity \( V_{cd} \).

The required volume to clean the wellbore for certain fracturing stage \( n \) will be then:

\[ V_o = q_o t_o = v_c A_{pipe} t_o \]  \hspace{1cm} (31)

Where \( V_o \) is the overdisplacement volume at fracturing stage \( n \) (m\(^3\)), \( q_o \) is the cleaning injection rate (m\(^3\)/s) and \( A_{pipe} \) is the liner cross section area (m\(^2\)).
The calculated volume expressed in equation (31) does not include the placement of the plug after the cleaning procedure is performed (Figure 8). To estimate the total volume of overdisplacement it is assumed that the required volume to place the plug will be equivalent to the volume required to remove the proppant since the distance that the plug must travel will be the same as the distance required to clean the particles from the lateral. The previous assumption implies that the total volume of clean fluid injected will be twice the calculated by equation (31).

3.3 Proppant distribution in the fracture

The objective of this section is to give an estimation or method to approximate a probable proppant bank height once the designed fracture treatment is performed. The calculation of the proppant bank height will allow an estimate of the probable effect of overdisplacement on the proppant distribution near the wellbore and it will be the input to calculate the fracture conductivities in the next chapter.

Hydraulic fracturing treatments that are performed in Shale induce complex fracture networks. The complexity of the fracture system makes it difficult to predict in detail what the output of a treatment will be, however it is a reasonable assumption that for a long fracture system, a planar bi wing fracture will form in the vicinity (long enough to assume a PKN model) of the well, followed by a fracture network as the fracture system extents deep inside the reservoir. Also, most of the proppant will end up in a few back-bone fractures in the network.

Slick water is commonly used as fracturing fluid in the treatments in Shale reservoirs. Slick water viscosity is low hence its proppant transport capacity is poor, which usually results in an accumulation of proppant at the bottom of the fracture (proppant bank), affecting the fracture conductivity in the near wellbore area. Low viscosity fluids however can place proppant far into the fracture network and then increase the connectivity along the reservoir. The focus of this work is to study the conductivity in the near wellbore area.

As proppant is being injected into the fracture, the proppant falls down to the bottom of the fracture, and as it settles, the height of the proppant bank increases. The cross section area in the fracture will decrease as result of the rise in the proppant bank height. The fluid velocity will increase as the fracture cross section area decreases, and it will reach a critical value for which the particles at the top of the proppant bank will be removed and transported further into the fracture.

Depending on the flow rate and particle properties, the transport of particles will occur at or above this critical velocity. The height of the proppant bank will fluctuate then as the critical velocity is reached. Eventually equilibrium between the height and the velocity will occur and the particles will be transported without changes in the proppant bank height.

Figure 9 is an example of how the dynamic of proppant transport would be. As the proppant is injected a proppant bed of certain height is formed and the particles at the top are removed and transported further into the fracture when the critical velocity is reached.
When a fracture treatment is over displaced, it means that additional clean fluid is injected after the designed volumes of slurry were injected. This additional injection will have an impact on proppant distribution, and if one assume that the top of the proppant bank height was placed at or above the perforations, it is probable that overdisplacement volumes will flush out part of the proppant height and the fracture will close at the top of the proppant bank, causing a choke effect at the perforations when production starts.

As overdisplacement fluid is injected to clean proppant sediments from the wellbore, one can foresee that the required injection rates to remove particles from the bottom of the lateral are higher than the critical velocity required to flush particles from the top of the proppant bank once the fluid gets into the fracture.

There are several formulations to calculate the critical velocity for proppant transport in fractures, and these may be used to make a comparison between the velocities generated by overdisplacement flow rates and the critical velocity, yielding an estimate of the occurrence of proppant flushing.

McLennan et al. (2008) presented Equations 32 and 33 in oil units to estimate the equilibrium velocity at which the proppant bank height will be stable and proppant will be transported continuously. Equation 32 is derived from the concept of flow rate divided by cross sectional area.

\[ v_e = \frac{0.5615(H - h)Q\eta}{w} \]  \hspace{1cm} (32)

Equation 33 is based in empirical data for experiments with Newtonian fluids and gives another indication of the equilibrium velocity:

\[ v_e = \frac{kd^{0.32}w^{0.443}\rho_s^{0.82}}{\mu} \left( \frac{\rho_p - \rho_l}{\rho_l} \right) \]  \hspace{1cm} (33)

Where \( H \) is the fracture height (ft), \( H \) is the proppant bank height (ft), \( w \) is the average fracture width (ft), \( Q \) is the total flow rate (BPM), \( \eta \) is the fluid efficiency, \( d \) is the proppant diameter (in), \( P_p \) is proppant density (g/cm\(^3\)), \( \rho_l \) is the fluid density (g/cm\(^3\)), \( \mu \) is the fluid viscosity (cp) and \( K \) is a dimensionless empirical constant.
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The height of the proppant bank is a fundamental parameter required to analyze the different scenarios of the effect of the overdisplacement. For horizontal wells with transverse fractures, the contact area between the fractures and the well is small compared with longitudinal fractures, making possible to assume that for transverse fractures production will occur at a source point where the conductivity at the perforations is fundamental to guarantee the success of the treatment and the productivity improvements.

It is important to determine if a propped and open fracture will be connected to the perforations before overdisplacement, making feasible to calculate the effect of injecting the additional fluid and the removal of proppant from this near wellbore area. In the case where, before overdisplacement, proppant spreads deeply along the fracture resulting in insufficient height at the wellbore and no conductivity between fracture and well, the effect of overdisplacement may be neglected and the analysis should be focused on the calculation of the geometry of the fracture.

Another scenario to be considered is when fracturing treatments are performed with high viscosity fluids. In this case proppant is transported more efficiently along the fracture cross section area increasing proppant bank height. The effect of overdisplacement in this situation is relevant, especially for transverse fractures, since the additional volumes may remove proppant from the perforations and affecting negatively the conductivity in the near wellbore area, even though the rest of the fracture remains propped. The analysis presented in this chapter can be applied to high viscosity treatments, assuming that when flushing the proppant the fracture will close at its top and a new proppant bank will be formed due to the lack of support at the perforation; fracture geometry and new conductivities may then be calculated.

Volume integration approach to calculate the height of the proppant bank

An estimation of the proppant bank height was done based on a volume balance. The fracture volume and the required proppant volume were already calculated from the treatment design stage. From the PKN model a volume function dependent of the height was formulated. The proppant bank height was calculated with the volume that the settled proppant will fill at the bottom of the fracture. This approach gives a fair estimate however in practice it is known that proppant will not spread evenly along the fracture and the height of the bank will decrease when moving along the fracture length.

By definition the PKN model assumes a fracture with elliptical fracture cross section and constant height. The shape of the fracture is defined by the equation of an ellipse and is dependent of the net pressure which varies with the fracture length. As a result, at the wellbore the ellipse will have its maximum width, decreasing along the fracture until reach the fracture tip where the width is zero. The equation that defines the fracture width along the length is (Economides & Nolte, 2000):

\[
W(x) = 3 \left(\frac{q_{\mu}(L - z)}{E'}\right)^{1/4}
\]  

(34)
Where \( w \) is fracture width (mt), \( q_i \) is the injection flow rate (m\(^3\)/s), \( L \) is the fracture length (mt), \( E' \) is the plane strain modulus (Pa), \( \mu \) is the fluid viscosity (Pa*s) and \( z \) is the coordinate in direction of the fracture length (mt).

In order to calculate the volume of the fracture as a function of its height, we considered the equation that describes the area of an ellipse:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]  
(35)

Where \( a \) and \( b \) are the half minor and half major axis respectively and \( h \) and \( w \) are the height and width at the fracture.

\[
a = \frac{h}{2}
\]  
(36)

\[
b = \frac{w}{2}
\]  
(37)

Figure 10 defines the coordinate system for the fracture cross section area calculations.

![Figure 10 - Ellipse coordinate system](image)

In order to calculate the area of the ellipse we divided the ellipse in four parts (quadrants) and got advantage of the symmetry. To solve the integrals a change in of variable is done where the height of the ellipse is defined as:

\[
|y| = a \sin \theta
\]  
(38)

Then the area of the shadowed region in figure B. as a function of the angle \( \theta \) is:

\[
Area 1(\theta) = ab \left( \theta - \frac{\sin 2\theta}{2} \right)
\]  
(39)

Combining equations 34, 38 and 39 we obtained:

\[
Area 1(z, \theta) = \left[ \frac{3}{2} \left( \frac{q_i \mu (L - z)}{E'} \right)^{1/4} \right] \left( \frac{h}{2} \right) \left( \theta - \frac{\sin 2\theta}{2} \right)
\]  
(40)
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The variable $z$ is the dimension along the fracture length. Integrating equation 40 along the distance $z$ from 0 to $x_f$ (fracture length) the result was:

$$Volume\ 1(\theta) = 0.6 \left(\frac{q_i H}{E'}\right)^{1/4} h \left(\theta - \frac{\sin 2\theta}{2}\right)\left(x_f x_f^{1/4}\right)$$  \hspace{1cm} (41)$$

Equation 41 calculates the volume of the fracture for certain height $y$ from the heel to the tip of the fracture. Solving for $y$ it is possible to calculate what would be the height for certain volume. For certain proppant volume and due to the symmetry of the ellipse, the use of equation 36 must fulfill the following conditions:

For $V_{ppt} > V_{total}/2$,

$$h_{ppt} = \frac{h}{2} + y\left(V_{ppt} - \frac{V_{total}}{2}\right)$$  \hspace{1cm} (42)$$

For $V_{ppt} < V_{total}/2$

$$h_{ppt} = \frac{h}{2} - y\left(\frac{V_{total}}{2} - V_{ppt}\right)$$  \hspace{1cm} (43)$$

Where,

$$V_{total} = 2 \times Volume\ 1\left(\frac{y}{2}\right)$$  \hspace{1cm} (44)$$

As we mentioned previously, equations 42 and 43 calculate an estimate of the proppant bank height, however the final result will depend on the assumptions and considerations that are made to fit the characteristics of the problem. The method we present aim to add subjectivity to the calculation of the proppant height bank but the result is not an exact prediction of what will be this height in practice.

**Volume balance approach to calculate fracture volume**

This section presents a direct approach used to calculate the height of the proppant bank. Since the PKN model and the calculations for designing a treatment are based on the average value of the width, the volume of the fracture may be approximated as explained in Chapter 1.

For low viscosity treatments where proppant will settle after being injected in the fracture, it is possible to assume that for transverse fractures the turbulence at the wellbore and the small contact area at the well will increase particles velocity and the proppant will be transported along the fracture until reaching the fracture tip. If we assume a uniform distribution of the proppant along the fracture, then making use of a simple volume balance it becomes easy to calculate the height of the proppant bank since the other dimensions of the proppant bank are constant. Equation 45 calculates the height of the proppant bank and was the selected approach used in the results in chapter 4.
where \( V_{ppt} \) is the total proppant volume, \( x_f \) is the fracture half-length and \( w_{avg} \) is the average width along the fracture.

The complexity of proppant distribution modeling makes the calculation of the exact proppant height at the wellbore difficult. The changes in concentration, the fracture homogeneity, and the rock stress gradients, among others are factors that will add uncertainty to any calculation about proppant distribution. However this method makes possible to estimate certain height that may be adjusted and calculate the effect on production if a small gap forms on the top of the propped area when part of the fracture closes after proppant flushing (over displacement).

In practice it is well known that for slick water treatments proppant will fall down and accumulate below the perforations. However, the discussion about overdisplacement is focused on the fact that sometimes treatments are pumped and over displaced attempting to form the gap above the fracture, arguing that the production due to this open channel will be more beneficial than the one in a propped fracture. This analysis is presented in the next chapter.

3.4 Erosion Equation and turbulence length

The productivity in a fracture is defined by the permeability and geometry of the different fracture zones. The characteristics of the propped region will depend of the resulting proppant distribution and constitutes a complex problem defined by the flow turbulent conditions, the proppant transport capacity and the geometry of the fracture. In order to relate the effect of overdisplacement with the resulting geometry of the propped zone, it was assumed that the injection of clean fluid will erode the top of the proppant bank and reduce the propped fracture height at the well.

The ability of the overdisplacement flow rate to erode the proppant bank was calculated based on the equations to predict sediment transport in rivers and coastal marine environments. These formulas, which are widely used for river applications, are all based on the concept that the sediment transport rate can be related to the shear stress exerted on the bed by the fluid above (Ribberink, 1998).

Ribberink (1998) presented a non-dimensional parameter to quantify the bed-load transport represented as:

\[
\Phi_b = \frac{q_b}{\sqrt{\Delta g D_{50}^3}}
\]  

(46)

Where \( q_b \) is the bed load transport rate in volume per unit time and width \((m^3/s*m)\), \( D_{50} \) is the median grain diameter \((mt)\), \( \Delta \) is the relative density defined as \( (\rho_s - \rho_f)/\rho_f \) and \( g \) is the gravity acceleration \((m/s^2)\).
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The bed-transport non dimensional parameter is related to the Shields parameter which is defined as a non-dimensional sediment forcing parameter and represented by (Ribberink, 1998):

\[
\theta = \frac{\tau_b}{(\rho_s - \rho_f)gD_{50}} \tag{47}
\]

A threshold of motion for sand grains is included by establishing the critical Shields parameter which was described by Van Rijn (1993) as a function of the non-dimensional grain size \(D^*\) and represented by the following equations (Ribberink, 1998):

\[
\theta_c = \frac{\tau_b}{(\rho_s - \rho_f)gD_{50}}
\]

\[
\theta_c = 0.24D_*^{-1} \quad \text{for } 1 < D_* < 4
\]
\[
\theta_c = 0.14D_*^{-0.64} \quad \text{for } 4 < D_* < 10
\]
\[
\theta_c = 0.04D_*^{-0.1} \quad \text{for } 10 < D_* < 20
\]
\[
\theta_c = 0.013D_*^{0.29} \quad \text{for } 20 < D_* < 150
\]
\[
\theta_c = 0.055 \quad \text{for } D_* > 150
\]

Where the dimensionless grain size is defined as,

\[
D_* = D_{50}\left(\frac{g\Delta}{u^2}\right)^{\frac{1}{3}} \tag{49}
\]

Where \(\tau_b\) is critical bed-shear stress and \(\nu\) is the kinematic viscosity of water (1E-6m²/s).

For steady flows presented an equation to calculate the non dimensionless bed-load as:

\[
\Phi_b = m(\theta - \theta_c)^n \tag{50}
\]

The effective Shields parameter \(\theta'\) is the part of the total bed-shear used for the bed-load transport process. The coefficients in equation 50 were found by Fernandez Luque and van Beek (1976) based on bed-load transport experiments for small size particles \((D>0.9\text{mm})\) (Ribberink, 1998).

Based on the depth-averaged current velocity, which was assumed as the overdisplacement critical velocity, it is possible to estimate the bed-shear stress:

\[
\tau_b = \rho_f g \frac{V^2}{C'^2} \tag{51}
\]

Where the \(C'\) is defined as the The Chezy friction coefficient and is based on the skin friction of the bed, using a rough-wall friction formulation and a Nikuradse grain-roughness height \(k\) (Ribberink, 1998):

\[
C' = 18 \log\left(\frac{12h}{k_s}\right) \tag{52}
\]

Where \(H\) represents the water depth (mt) and for purposes of our work it was assumed an average size of the arch on the top of the proppant bank. \(K_s\) is the roughness height (mt).
The previous equations provide an estimate of the amount of proppant removed or eroded from the top of the proppant bank due to the injection of the clean fluid. To calculate the reduction of proppant bank height it was assumed that the effect of proppant erosion will occur in the nearby wellbore area where the flow is turbulent. For flow in parallel plates the entrance region or turbulent length is defined as (Lautrup, 2009):

\[ L_{turbulence} = 0.063 \, N_{Re} \, (2b) \]  \hspace{1cm} (53)

Where \( N_{Re} \) is the Reynolds number defined as:

\[ N_{Re} = \frac{V(2b)}{\mu \rho_f} \] \hspace{1cm} (54)

Where \( b \) is the separation between fracture faces or width (mt), \( V \) is the overdisplacement velocity (m/s) and \( \mu \) is the fluid viscosity (Pa*s).

The reduction in proppant bank height will be defined then as:

\[ h_{erosion} = q_b \, w_{ppt} \, L_{turb} \, t_{inj, over} \] \hspace{1cm} (55)

Where \( h_{erosion} \) represents the reduction in proppant bank height due to overdisplacement (mt), \( q_b \) is the erosion rate per propped fracture width (m³/s*mt), \( w_{ppt} \) is the width at the top of the proppant bank (mt), \( L_{turb} \) is the length of the turbulence regime (mt), \( t_{inj, over} \) is the overdisplacement injection time (s).

The effective height of the proppant bank after overdisplacement will be then:

\[ h_{ppt} = h_{reference} - L_{turb} - h_{erosion} \] \hspace{1cm} (56)

Where \( h_{ppt} \) is the effective height of the proppant bank after overdisplacement (mt), \( h_{reference} \) is the height of proppant bank for a non-overdisplaced treatment (mt), \( L_{turb} \) is the length of the turbulence regime and defines the eroded area and the effect of turbulence on proppant distribution (mt) and \( h_{erosion} \) is the reduction of proppant height due to removal of proppant particles at the top of the proppant bank (mt).

### 3.5 Calculation of fracture geometry and arch size

The procedures to calculate a fracture treatment and the additional injected volumes when a fracturing treatment is over displaced were previously explained. The method presented in this chapter will create a relationship between the calculation of these overdisplacement volumes and the effect they may have on well productivity. This method is based on the equations and concepts presented by Warpinski (2009), which calculates the proppant stress in a fracture when poor proppant transport occurred.

The quantification of the effect that overdisplacement may have on productivity is related to how affected was the proppant distribution at the well due to this practice, and consequently to the change it caused on fracture conductivity in the near wellbore area. Proppant transport and distribution models had been developed making use of complex numerical methods. For purposes of this work, some assumptions were made in order to simplify our approach and
create a simple method to link the different concepts and calculate the overdisplacement effect.

The selected approach will find the geometry of a fracture when the proppant has settled during transport and formed a proppant bank at the bottom of the fracture. This proppant distribution implies that just part of the fracture will remain open and propped, while the rest of the fracture will close, or partly close (un-propped). The output of the method will be the dimensions of the open fracture, which together with the proppant parameters that were defined in the fracture treatment design, will make possible to calculate an average weighted fracture conductivity and hence to calculate the effective productivity index, which is our final objective.

The new fracture geometry generated by the settling of proppant at the bottom of the fracture will change the stress configuration into the fracture. The approach presented by Warpinski (2009) calculates a pressure profile that describes this new state of stresses and guarantees a smooth and continuous transition in the geometry of the fracture between the propped and the un-propped region. This transition region is characterized by the formation of an un-propped arch. At this arch the rock must support the additional stresses in order to keep the rock open and the arch will perform as an open channel during production.

Warpinski (2009) considered that for a slick-water fracturing treatment the proppant transport conditions are poor and therefore once the proppant is placed into the fracture it will fall out of suspension to the bottom of the fracture (Figure 11b). The transportation of the proppant occurs due to saltation, a transport phenomenon where the inflow particles will remove the particles at the top of the proppant bank by rolling them along the fracture length, hence extending the length of the proppant bed but decreasing its height. This saltation process implies that the proppant will not be evenly distributed along the fracture profile and the fracture will close at its top (Figure 11c).

The objective of the method presented by Warpinski (2009) described the stress configuration and fracture geometry when performing a treatment with a poor transport fracturing fluid, more precisely, slick water fluid. This method will be used for our purposes by assuming that the height of the propped region will be affected by the overdisplacement volume, and then will change the fracture width profile. The fracture conductivity and productivity index will be calculated based on the new width profile and the effective fracture heights respectively.

If we assume that the rock behaves linear elastically, then an arch region will form above the proppant bank, allowing a smooth transition of the rock deformation between the propped and un-propped region. This deformation will generate high stress on the proppant at the top of the dune. This arch can be seen as an open channel with high conductivity. Warpinski (2009) method calculates the dimensions of the arch region and the pressure profile along the fracture.
For a fully propped fracture, the width profile can be calculated straight forward for long fractures by using the PKN model:

\[ w = \frac{4(1 - \nu^2)}{E} P_{\text{net}} \sqrt{a^2 - y^2} \]  

(57)

Where \( w \) is the fracture width (ft), \( \nu \) is the Poisson’s ratio, \( E \) is the Young’s modulus (psi), \( P_{\text{net}} \) is the net pressure defined as \( P_{\text{net}} = P_f - \sigma_{\text{min}} \) (psi), \( a \) is the Fracture half height (ft), \( y \) is the height coordinate (ft), \( P_f \) is the pressure into the fracture (psi) and \( \sigma_{\text{min}} \) is the minimum closure stress (psi).

Equation 57 will be used to calculate the fully propped fracture profile. The method that will be explained below attempts to find a width profile that matches the profile expressed by equation 57. The accuracy of the results will depend of how good is the match between the width profiles.

For a pre calculated fracture profile (equation 57) under the assumption of perfect proppant transport and for certain rock properties, the method we present here will define a pressure distribution into the fracture and will allow to calculate a new width profile for the case of poor proppant transport or proppant removal. This method is based on the following assumptions:

- Rock is linearly elastic.
- The fracture is described by the PKN model.
- The fracture is constrained at the bottom and will not propagate downwards.
- The stress intensity factor (Kic) at the top of the fracture (top of the arch) is zero.
- The pressure in the arch region is the initial flowing pressure (reservoir pressure).
- The strength of the proppant is high enough to avoid proppant failure.

In order to obtain a realistic representation of the pressure into the fracture, the solution to the problem will be restrained by two conditions: 1). the fracture width obtained from the
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pressure distribution must match the already known width profile and 2). The stress intensity factor at the top of the fracture must be zero in order to prevent the fracture from opening at the un-propped region. Each of the previous conditions will be represented by a mathematical expression describing the mechanics of the problem.

To match the width profiles Warpinski (2009) used a method provided by England and Green in which by assuming certain pressure profile it is possible to calculate the width for a 2D fracture. The zero fracture toughness condition at the top of the fracture is based on the solution proposed by Rise to calculate the stress intensity factor in a pressurized fracture.

Making use of the England and Green method, Warpinski (2009) divided the problem by parts and presented a pressure profile for each region along the fracture. For a fracture composed by a proppant bank, an arch and an un-propped region, the pressure profile for each region is described by Figure 12 and defined as:

- From the bottom of the proppant bed to the half fracture height \( a \), the pressure is assumed constant and defined as \( p \). The pressure at this region will be higher than the pressure calculated for a perfectly propped fracture; this is due to the reduction in the effective fracture height.
- The pressure from the half of the fracture height \( a \), to the top of the proppant bed \( b \), is defined by a power law equation and will have a maximum at the top and defined by the relationship \( p(1+k) \), where \( k \) is a pressure intensification factor. The exponent of the power law approximation will define how well the match between width profiles is and it will be selected as part of the analysis and results presented by Warpinski (2009).
- The pressure at the arch is constant and defined as \( \Delta \). It is calculated as the initial flowing pressure \( P_{wfi} \) minus the minimum closure stress, \( \sigma_{min} \), resulting in a negative value.
- Assuming that the fracture is closed above the arch region, the pressure along this region is zero.

![Figure 12 - Pressure profile with arch formation](image-url)
England and Green method allows describing the pressure distribution by means of even and odd functions. Warpinski (2009) derived the equations to calculate the width profile making use of linear, quadratic and cubic power law exponents and divided the odd and even equations for the limit \( Y>b \) and \( Y<b \), where \( Y \) is the axis coordinate in the height direction with its origin at the center of the fracture. The results showed that the best match between width profiles occurred when using a cubic power law approximation however this method increases complexity in the calculations and mainly affects the stress on proppant but in less extent the arch size. For purposes of our work the linear equation will be used and is described by the following equations:

For \( |y| \geq b \),

\[
\begin{align*}
\frac{w(1-v^2)}{E} &= \frac{p + \Delta}{8} + \frac{k p}{8 \pi b} \left( a - \sqrt{a^2 - b^2} \right) \\
&+ \frac{1}{4 \pi} \left( \frac{k p y}{2 b} + p - \Delta \right) \sin^{-1} \left( \frac{b}{a} \right) \sqrt{a^2 - y^2} \\
&+ \frac{k p y^2}{8 \pi b} \ln \left( \frac{a + \sqrt{a^2 - y^2}}{|y|} \right) \\
&+ \frac{1}{4 \pi} \left( p - \Delta \right) (b - y)
\end{align*}
\]

(58)

For \( |y| \leq b \),

\[
\begin{align*}
\frac{w(1-v^2)}{E} &= \frac{p + \Delta}{8} + \frac{k p}{8 \pi b} \left( a - \sqrt{a^2 - b^2} \right) \\
&+ \frac{1}{4 \pi} \left( \frac{k p y}{2 b} + p - \Delta \right) \sin^{-1} \left( \frac{b}{a} \right) \sqrt{a^2 - y^2} \\
&+ \frac{k p y^2}{8 \pi b} \ln \left( \frac{a + \sqrt{a^2 - y^2}}{|y|} \right) \\
&+ \frac{1}{4 \pi} \left( p - \Delta \right) (b - y)
\end{align*}
\]

(59)

Where \( w \) is the Fracture width at height \( y \) (ft), \( v \) is the Poisson’s ratio, \( E \) is Young’s modulus (Psi), \( P \) is the fracture pressure at the constant region (psi), \( \Delta \) is the pressure at the arch
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defined as the minimum horizontal stress minus the reservoir pressure (psi), \( K \) is the fracture intensification factor, \( Y \) is the height coordinate being zero at half fracture height (ft), \( a \) is half fracture height including arch (ft) and \( b \) is the distance from the center of the fracture to the top of the proppant bank and is defined as \( b = (H_p - a) \).

To include the zero fracture toughness condition into the analysis, Warpinski (2009) used the method presented by Rice to calculate the fracture toughness at the top of the fracture. For a cubic pressure profile the solution is described by the following equation:

\[
K_I = \frac{1}{\sqrt{\pi}a} \left\{ p a \left[ \frac{\pi}{2} - \frac{\sqrt{a^2 - b^2}}{a} + \sin^{-1} \left( \frac{b}{a} \right) \right] \right.
+ \Delta \left[ \sqrt{a^2 - b^2} + \frac{\pi a}{2} - a \sin^{-1} \left( \frac{b}{a} \right) \right] \\
- kp \left[ \frac{a}{b} + \frac{1}{2} \sqrt{a^2 - b^2} \right]
+ \frac{k p a^2}{b} \left[ 1 + \frac{1}{2} \sin^{-1} \left( \frac{b}{a} \right) \right] \right\} \tag{60}
\]

By adjusting the fracture height \( a \), the pressure \( p \) and the factor \( k \) in Equations 58, 59 and 60, the calculated width profile should matches adequately the width profile for a fully propped fracture at the propped region and guarantee the zero fracture toughness at the top of the fracture. The process must be done by iteration making use of the non-linear square approach or manually by programming the equation in a spread sheet.

The solution may be done straight forward in a spread sheet by observing the match between profiles for an initial assumed fracture height \( a \). To start the iteration one can select a fracture half height \( a \) slightly higher than half of the propped height and calculate the width profile and the stress intensity factor. The procedure must continue by adjusting the pressure parameters until get a fair match between profiles.

Once the new width profile is calculated, it is possible to obtain the fracture conductivities for each fracture region. The selection of permeabilities will be based on proppant characteristics and assumptions about the conditions at the un-propped and arch region.

For each part of the fracture the conductivity will be chosen according to the following assumptions:

- At the arch and closed fracture region conductivity will be calculated based on the parallel plate flow conditions. This calculation assumes an open channel flow, opposite to the Darcy’s flow calculations for porous media. The permeability of these regions will depend of the separation between fracture faces and described as (Cook, 2003):

\[
k = \frac{w^2}{12} \tag{61}
\]

Where \( k \) is the permeability (m\(^2\)) and \( w \) is the width of the average width of the fracture at these zones.
The propped region permeability will depend on the proppant permeability defined in the treatment design.

The permeabilities at each part of the fracture will be used as an input in the PI calculation presented in the next chapter.

3.6 Calculation of productivity index

As any other stimulation treatment, the main goal when performing a Hydraulic fracturing treatment is to improve well profitability by increasing the production or reducing the pressure drawdown. The performance of the treatment will be measured by calculating the well productivity increment determined by the stimulation method.

This chapter presents the basic concepts about turbulent gas flow in porous media and couple of methods to calculate the productivity in horizontal fractured gas wells. Further on these calculations will be used to compare the effect of overdisplacement on well productivity for different well settings and reservoir characteristics.

The theory of Reservoir flow modeling is based on Darcy’s law. Its application will depend on the validity of several assumptions such as laminar flow regime or fluid incompressibility. In the case of gas reservoirs, and contrary to oil wells, the effect of the inertial and viscous forces becomes relevant. This effect is especially observed in the near well bore region where the flow area will be reduced and the velocity will increase. As a consequence, the Reynolds number at this region will also increase and the flow will become turbulent. When turbulent flow occurs, the pressure change will rise at a higher rate than the flow rate change, thus resulting in non-Darcy flow conditions. In reservoirs with low permeability and for low production wells, the fluid velocity will remain below the turbulence threshold and the flow regime will be laminar, making valid the use of Darcy’s law.

The occurrence of turbulence flow in gas reservoirs will affect negatively the well performance. The flow impairment will be proportional to the potential of the flow, meaning that for high flow rates the detriment in production will be higher. To account for turbulence effects in the inflow performance calculations, a turbulence term was included in the model (equation 51). The turbulence effect term may be seen as a flow dependent skin effect. Hydraulic fracturing treatments will reduce turbulence which will result in a more negative skin.

Depending on the depletion state of the reservoir, certain flow behavior will be assumed when modeling inflow performance in a well. In the case where there is constant pressure at the reservoir boundaries, the difference between the pressure at the well and the pressure at these boundaries will be constant. This condition allows flow modeling assuming a steady state condition. In comparison, a pseudo-steady state condition will occur when the reservoir has no flow boundaries, and the difference between the average pressure (declining trend) at the reservoir and the pressure at the well will approach to a constant value. At first glance, both approaches may look similar, however the difference lies on the pressure consideration, where for pseudo-steady state the average pressure is not a constant value, but instead it is related to reservoir depletion.
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The estimation of well production is fundamental to evaluate the viability of any gas or oil project. The previous concepts make possible to predict well performance under certain pressure and reservoir conditions. An important parameter that has been used to evaluate well performance is the Productivity Index.

The productivity index makes possible to evaluate and compare well performance during his productive life, and assess the impact on production when stimulation treatments or modifications are applied. The productivity index is defined as the ratio of flow rate and pressure drawdown and for constant reservoir conditions it will remain constant. The definition of productivity index is defined by Equation 62:

\[ q = J * \Delta p \]  

(62)

Where \( J \) is the productivity Index and it has units of flow rate over pressure.

For flow calculations in vertical or horizontal wells is common to assume that the flow from the reservoir to the well is radial. There are however special cases such as in horizontal wells with transverse fractures, where linear flow occurs from the reservoir to the fracture and radial flow from the fracture to the well. During production most of the pressure losses occur in the near wellbore area and any disturbance at this region will have a high impact on pressure drawdown. As we can observe in Equation 62, for a constant flow rate, an increase on pressure drawdown will decrease the productivity index (Economides & Nolte, 2000).

The damage at the wellbore area will be represented by the dimensionless skin factor, \( s \). The skin can be seen as a measure of the quality of the well and it is a fact that any well will have certain degree of damage or skin effect. For any degree of near wellbore damage the skin factor will be a positive value and well production will be less if compared with production in an equivalent zero skin or un-damaged well.

The application of any stimulation method has as main objective to improve well productivity or in other words, to decrease the skin factor. Depending on the success of the treatment it is possible to achieve negative skin factors which will mean that the stimulation method not only removed the damage but also improved the reservoir inflow area connected to the well.

The skin effect due to turbulence has a high impact on inflow performance considerations however several other conditions in a well can also cause skin. For purposes of our work will focus on turbulence effects and not take into account any skin generated by drilling, completions, permeability anisotropy, etc.

For the case of pseudo-steady state Diyashev and Economides (2006) presented the basic equation that describes productivity index for a gas well:

\[ J = \frac{q}{p^2 - p_{wf}^2} = \frac{kh}{\alpha \mu ZT} J_{D} \]  

(63)

Where \( J \) is the Productivity Index, \( q \) is the gas flow rate, \( P \) is the average reservoir pressure, \( P_{wf} \) is the bottom hole pressure, \( k \) is the reservoir permeability, \( h \) is the reservoir pay thickness, \( \alpha \) is a conversion factor that apply just if non-consistent units are used, \( \mu \) is the fluid viscosity, \( Z \) is the gas compressibility factor, \( T \) is the reservoir temperature and \( J_{D} \) is the dimensionless Productivity Index.
The units to be used in Equation 63 must be carefully selected. In case of using oil field units, $\alpha$ is equal to 1424.

Equation 63 introduced the dimensionless productivity index term $J_d$. It is a useful tool to compare well performance under different reservoir conditions and independently from the type of completion, type of well, geometry or type of stimulation treatment.

Economides et al. (2007) presented the following equations to approximate vertical well gas inflow for steady and pseudo steady state including turbulence effects:

For Steady state:

\[
q = \frac{kh\left(p_e^2 - p_{wf}^2\right)}{1424\mu ZT \left[\ln \left(\frac{r_e}{r_w}\right) + s + Dq\right]}
\]  
(64)

For pseudo-steady state:

\[
q = \frac{kh\left(p_e^2 - p_{wf}^2\right)}{1424\mu ZT \left[\ln \left(\frac{0.472r_e}{r_w}\right) + s + Dq\right]}
\]  
(65)

Where $q$ is the gas flow rate (Mscf/d), $k$ is the reservoir permeability (mD), $h$ is the reservoir thickness (ft), $\mu$ is the gas viscosity (cp), $Z$ is the gas compressibility factor, $T$ is the reservoir temperature (Rankine), $r_e$ is the reservoir radius (ft), $r_w$ is the wellbore radius (ft), $s$ is the skin factor, $D$ is the turbulence effect factor (1/(Mscf/d)), $p_e$ is the reservoir pressure (psi) and $p_{wf}$ is the flowing bottom hole pressure (psi).

As an illustration for the case of pseudo steady state flow in a vertical well and according with equations 64 and 65, the dimensionless productivity index is defined as:

\[
J_d = \frac{1}{\ln \left(\frac{0.472r_e}{r_w}\right) + s + Dq}
\]  
(66)

In the case of undamaged well or skin factor equal to zero the dimensionless productivity index in equation 66 would have a value around 0.1. Higher values of $J_d$ would mean improvements due to stimulation while lower values will mean wellbore damage. For our purposes the dimensionless productivity index will be an indicator of what is the effect of overdisplacement on the quality of the well.

Economides et al. (2007) also presented analogue inflow equations for horizontal wells:

\[
q = \frac{k_hh\left(p_e^2 - p_{wf}^2\right)}{1424\mu ZT \left[A_o + \frac{1}{L} \left(\ln \left(\frac{L_{ani}}{r_w L_{ani + 1}}\right) + Dq\right)\right]}
\]  
(67)
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\[
q = \frac{k_H h (\bar{p}^2 - p_w^2)}{1424 \mu T \left[ A_u + \frac{l_{ani} h}{L} \left( \ln \left( \frac{l_{ani} h}{r_w (l_{ani} + 1)} \right) - \frac{3}{4} + Dq \right) \right]}
\]

(68)

Where,

\[
A_u = \ln \left( a + \sqrt{a^2 - \left( \frac{L}{2} \right)^2} \right) / L/2
\]

(69)

\[
l_{ani} = \sqrt{\frac{k_h}{k_v}}
\]

(70)

\[
a = \frac{L}{2} \left\{ 0.5 + \left[ 0.25 + \left( \frac{r_{eh}}{L/2} \right)^4 \right]^{0.5} \right\} \text{ for } L/2 < 0.9r_{eh}
\]

(71)

- \( r_{eh} \): Equivalent radial flow drainage radius, ft.

Hydraulically fractured gas wells

When stimulating a well through hydraulic fracturing the goal is to improve production by improving connectivity between the reservoir and the wellbore. There are several methods to calculate the inflow performance of fractured wells and to include the stimulation effect (beneficial skin) in the productivity index calculation. A common method selected by different authors is to convert certain well characteristics into an equivalent skin effect. This also applies to account for the hydraulic fracturing stimulation effects.

There are two main parameters that will define the performance of a well and were used to derive the dimensionless productivity index relationships (Oligney, Economides, & Valkó, 2002): The penetration ratio \( I_x \) (equation 72) and the dimensionless fracture conductivity \( C_{fd} \) (equation 73).

\[
I_x = 2x_f / x_e
\]

(72)

\[
C_{fd} = \frac{k_f w}{k x_f}
\]

(73)

In accordance with the definition of the previous parameters, if considering a fracturing system in a vertical well composed by a fully penetrated fracture in a reservoir with net thickness \( h \) and certain drainage geometry with radius \( r_e \) or length \( x_e \) (depending if the selected shape is circular or rectangular respectively) the reservoir drainage area is:

\[
A = r_e^2 \pi = x_e^2
\]

(74)

Cinco (1978) related the dimensionless conductivity and the fracture length with an equivalent skin effect \( s_f \) and presented his results in the well known graph \( C_{fd} \) vs. \( s_f + ln(x_f / r_w) \). Based on Cinco development an empirical equation for the dimensionless productivity index in vertical fractured wells was developed (Diyashev & Economides, 2006):
\[ J_D = \frac{1}{\ln(r_{r_2}) - 0.75 - 0.5 \ln \left( \frac{k_f V_f}{k h} \right) + 0.5 \ln \left( C_{FD} \right) + \ln \left( \frac{x_f}{r_w} \right) + s_f} \]  

(75)

In equation 75, the maximum dimensionless productivity index \( J_D \), will occur when the dimensionless conductivity \( C_{FD} \) has a value of 1.6. For any other value of \( C_{FD} \), the corresponding \( J_D \) will be lower than the optimum.

In order to optimize the dimensions of a fracture treatment to get the maximum \( J_D \) and for certain reservoir and proppant properties already defined, Economides and Valkó introduced a new parameter called the Proppant Number \( N_{prop} \). This new parameter is a combination of the penetration radio and dimensionless conductivity, and represents a way to handle the propped volume constrain between the two former parameters. The \( N_{prop} \) is defined as:

\[ N_{prop} = l_x z C_{FD} = \frac{4k_f x_f w}{k x_e^2} = \frac{4k_f x_f w h}{k x_e^2 h} = \frac{2k_f V_p}{k V_{res}} \]  

(76)

Where \( k_f \) is effective proppant pack permeability, \( K \) is the reservoir permeability, \( x_f \) is the reservoir length, \( W \) is the propped fracture width, \( x_e \) is the well drainage dimension, \( H \) is the pay reservoir thickness, \( V_p \) is the propped volume (two wings) and \( V_{res} \) is the Reservoir drainage volume.

All the variables in Equation 76 must be applied in the same unit system.

Oligney et al. (2002) also presented some relationships to calculate maximum dimensionless productivity index and maximum dimensionless conductivity as a function of the proppant number:

\[ J_{D_{max}}(N_{prop}) = \begin{cases} 
\frac{1}{0.990 - 0.5 \ln(N_{prop})} & \text{if } N_{prop} \leq 0.1 \\
6 - \exp \left[ \frac{0.423 - 0.311 N_{prop} - 0.089 (N_{prop})^2}{1 + 0.142 \ln N_{prop}} \right] & \text{if } N_{prop} > 0.1 
\end{cases} \]  

(77)

\[ C_{FD_{opt}}(N_{prop}) = \begin{cases} 
1.6 & \text{if } N_{prop} < 0.1 \\
1.6 + \exp \left[ \frac{-0.583 + 1.48 \ln N_{prop}}{1 + 0.142 \ln N_{prop}} \right] & \text{if } 0.1 \leq N_{prop} \leq 10 \\
N_{prop} & \text{if } N_{prop} > 10 
\end{cases} \]  

(78)

From equation 77 and 78, Oligney et al. (2002) found that for certain proppant number, there is optimum fracture conductivity where the dimensionless productivity index is maximum. For proppant numbers less than 0.1 the maximum \( J_D \) will occur for \( C_{FD} \) equal to 1.6 and will have a value of 1.9.

For \( N_{prop} \) higher than 1 the maximum \( J_D \) will occur at higher values of \( C_{FD} \) due to the \( l_x \) restriction (\( l_x < 1 \)). For medium and high permeability formations (\( k > 50 \text{ md} \)) it is difficult to achieve proppant numbers higher than 0.1. In practice, for tight gas reservoirs proppant numbers larger than 1 are also difficult to get. At this proppant number the maximum \( J_D \) will be 0.9.
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After calculating the maximum dimensionless conductivity for certain proppant number it is possible to optimize fracture geometry and calculate an optimum fracture length and width (Oligney et al., 2002):

\[ x_{f,\text{opt}} = \left[ \frac{k_f V_f}{C_{f0,\text{opt}} k h} \right]^{0.5} \]  
\[ w_{\text{opt}} = \left[ C_{f0,\text{opt}} k V_f / k_f h \right]^{0.5} \]  

Where \( V_p \) is the proppant volume and \( V_f \) is half wing propped Volume \( (V_f = V_p/2) \).

For most of the common hydraulic fracturing activities, including stimulation in Shale gas reservoirs, the proppant number will be less than 0.1, which is considered a moderate proppant number. For these range of proppant numbers \( (N_{\text{prop}} < 0.1) \), Diyashev and Economides (2006) presented relationships developed by Cinco-ley and Samaniego and Prats that relate dimensionless productivity index, proppant number and fracture design parameters to a pseudo skin function and an equivalent wellbore radius.

According to the definition of dimensionless productivity index and including the effect of the fracture treatment as an equivalent skin effect we can write the following equation:

\[ J_D = \frac{1}{\ln \left( \frac{0.472 r_e}{r_w} \right) + s_f} \]  

Where \( s_f \) represents the skin effect generated by the fracture treatment.

Prats reorganized equation 81 and included the concept of equivalent wellbore radius \( r_w' \) (Diyashev & Economides, 2006), which will define what would be the equivalent well radius in a non-stimulated well (as result of the fracture skin) to obtain the same dimensionless productivity than the fractured well.

\[ J_D = \frac{1}{\ln \left( \frac{0.472 r_e}{r_w' f} \right)} \]  

Cinco-ley related the dimensionless productivity to the equivalent skin effect through the following skin function:

\[ f = \frac{1,65 - 0.328 \ln C_{fD} + 0.116 (\ln C_{fD})^2}{1 + 0.18 \ln C_{fD} + 0.064 (\ln C_{fD})^2 + 0.005 (\ln C_{fD})^3} \]  

As we can observe the skin function presented in equation 83 only depends on dimensionless conductivity and is just valid for moderate proppant numbers. The dimensionless productivity index may be written then as:

\[ J_D = \frac{1}{\ln \left( 0.472 \frac{r_e}{x_f} + f \right)} \]  

The equivalent relationships between skin factor, skin and wellbore radius are defined then as:
For large values of dimensionless conductivity (above 100) the skin function reaches a maximum value and the fracture behaves like an infinite conductivity fracture.

Productivity in transverse fractures in gas reservoirs

For all hydraulic fracturing applications, the state of stress in the reservoir will define the direction of fracture propagation. Fractures normally propagate vertically and perpendicular to the minimum horizontal stress and depending on the orientation of the well the fracture will be longitudinal or transverse to the well axis. Longitudinal and vertical fractures will perform similarly for similar fractures lengths and conductivities. This similarity allows applying vertical well analysis to calculate productivity in longitudinal wells. For similar fracture volumes practice has shown that transverse fractures will perform considerably better compared with longitudinal/vertical fractured wells in reservoirs with very low permeability, because the fractures can be placed arbitrarily close together, allowing for a much larger effective reservoir contact area from the same length of horizontal wellbore.

In the case of transverse fractures the contact area between the wellbore and the fracture is reduced and the flow regime will change from linear (reservoir-fracture flow path), to radial (fracture-well flow path). The methods presented in this chapter assumed that there is no flow from the reservoir to the well through the walls of the lateral.

Due to the small contact area between the fracture and the well (Figure 13) a choke effect will occur at the near wellbore area increasing the pressure drop and turbulence. The effect of this choke effect in transverse fracture limits its application to reservoirs with small permeabilities (< 1mD) in order to be profitable. The viability of transverse fractures is then attached to the generation of multiple fractures, since a single transverse fracture will have less production than a vertical fractured well.

The choke effect in transverse fractures can be calculated using the equation presented by Wei and Economides (2005), which considers the pressure decrement as a skin effect:

\[
\frac{r_w'}{r_w} = x_f \exp[-f] \tag{85}
\]

\[
\frac{r_w'}{r_w} = r_w \exp[-s_f] \tag{86}
\]
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\[ s_c = \frac{kh}{k_f w} \left[ \ln \left( \frac{h}{2r_w} \right) - \frac{\pi}{2} \right] \]  

(87)

Where \( s_c \) is the skin due to choke effect, \( K \) is the reservoir permeability (md), \( K_f \) is the fracture permeability (md), \( H \) is the reservoir thickness (mt), \( w \) is the propped fracture width (mt) and \( r_w \) is the well radius (mt).

As we mentioned before, in high rate gas wells, the increment in fluid velocity at the wellbore will generate turbulence. When turbulent regime occurs, the flow become non-Darcy, production is negatively affected and corrections to the fracture (proppant pack) permeability must be done to account for this turbulent effect. The effect of turbulence in oil wells is small and can be neglected.

For high rate gas wells the fracture permeability become then function of the flow. From now on we will refer to the permeability at the fracture as the effective fracture or proppant permeability, whereas the nominal proppant permeability will be the initial permeability provided by the proppant manufacturers.

Economides et al. (2007) presented a method to correct fracture permeability with base on the Reynolds number:

\[ k_{f,e} = \frac{k_{f,n}}{1 + N_{Re}} \]  

(88)

Where the Reynolds number for non-Darcy flow is calculated as:

\[ N_{Re} = \frac{\beta k_{f,n} v \rho}{\mu} \]  

(89)

\[ \beta = 1 \times 10^8 \frac{b}{k_{f,n} a} \]  

(90)

Where \( k_{f,e} \) is the effective fracture permeability (mD), \( k_{f,n} \) is the nominal fracture permeability (mD), \( N_{Re} \) is the Reynolds number, \( v \) is the fluid velocity (m/s), \( \rho \) is the fluid density (kg/m3), \( \mu \) is the fluid viscosity (Pa*s), \( a \) and \( b \) are Cook Constants (Table 2).

The parameters \( a \) and \( b \) are constants and depend on the proppant size. Economides et al. (2007) presented these parameters for some commonly used proppant sizes that are presented in Table 2:

<table>
<thead>
<tr>
<th>Proppant Size (mesh)</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 to 12</td>
<td>1.24</td>
<td>17.423</td>
</tr>
<tr>
<td>10 to 20</td>
<td>1.34</td>
<td>27.539</td>
</tr>
<tr>
<td>20 to 40</td>
<td>1.54</td>
<td>110.470</td>
</tr>
<tr>
<td>40 to 60</td>
<td>1.60</td>
<td>69.405</td>
</tr>
</tbody>
</table>

Wei and Economides (2005) presented an equation to calculate the dimensionless productivity index for horizontal gas wells with transverse fractures:
\[ J_{DTH} = \frac{1}{\frac{1}{J_{DV}} + s_c} \]  

(91)

As a single transverse fracture will have less performance than a vertical fractured well, the comparison between both options should consider the performance of the vertical well versus the performance of the transverse fracture times the number of isolated zones.

Demarchos, Porcu, and Economides (2006) presented an iterative method to calculate the optimum fracture dimensions in a treatment to maximize the dimensionless productivity index. This method may also be used in combination with Cinco developments to calculate the dimensionless productivity index for a treatment with already defined dimensions. The calculations steps for both methods are described below.

**Optimum design**

- For an assumed Reynolds number \( N_{re} \), calculate the effective permeability \( K_{fe} \) making use of equation 88 and further on calculate the Proppant number \( N_{prop} \) making use of the already calculated effective permeability. For the optimization case the \( N_{prop} \) should be calculated based on the proppant mass required to perform the job.

- The \( J_{D_{max}} \) and \( C_{dopt} \) from equations 86 and 87 can be calculated now with the proppant number \( N_{prop} \). Once the \( C_{dopt} \) is calculated the optimum dimensions from equations 77 and 78 can be calculated.

- With the optimum fracture width calculate the choke skin factor. Calculate the \( J_{D_{th}} \) making use of the \( J_{D_{max}} \) and the skin factor.

- Re writing the equation 64 for gas flow, and using the previous dimensionless productivity index for horizontal well with transverse fracture we can calculate the flow rate at the fracture:

\[ q = \frac{kh(p^2 - p_{wf}^2)}{1424\mu ZT} J_{DTH} \]  

(92)

- In order to start a new iteration and calculate a new Reynolds number, the fluid velocity can be calculated from the definition of volumetric flow:

\[ v = \frac{q}{A} = \frac{q}{h w_{opt}} \]  

(93)

- With the new Reynolds number a new effective permeability is calculated and the process is repeated. The iteration ends when the difference between Reynolds numbers in two iterations is small.

**Productivity for a specified design (not optimum)**

The previous method allows calculating the maximum dimensionless productivity index for an optimum conductivity; however the optimum fracture dimensions are not always physically or economically feasible. An additional constrain in the iterative method is related then to the net
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

pressure. Since the net pressure is proportional to the fracture width, the selected design must guarantee an adequate value of net pressure ($P_{net}<1000$psi).

In order to calculate the dimensionless productivity index for an already known fracture dimensions or to guarantee that the designed treatment will fulfill the net pressure constrain, Wei and Economides (2005) combined the Unified Fracture design, as the previous iterative method is known, with the skin factor calculated by Cinco.

The iterative steps to calculate the dimensionless productivity index are described below:

- For an assumed Reynolds number $N_{re}$, calculate the effective permeability $K_{fe}$ making use of equation 88 and further on calculate the Proppant number $N_{prop}$ making use of the already calculated effective permeability. The $N_{prop}$ should be calculated based on the fracture dimensions.

  For the optimization case where it is required to fulfill the $P_{net}$ constrain, the fracture half-length should be reduced by a factor. The width must be adjusted according to the fracture volume balance. The $P_{net}$ is then calculated and the dimensions adjusted again until obtaining an adequate pressure value.

- With the effective permeability $K_{fe}$ it is possible to calculate dimensionless conductivity and the skin effect function described by equations 73 and 83 respectively.

- The dimensionless productivity index $J_d$ for vertical well can be calculated making use of equations 91.

- With the fracture dimensions and the effective fracture permeability calculate the choke skin factor.

- Calculate the $J_{dth}$ making use of the $J_d$ and skin choke factor.

- With the equation 92 and using the previous dimensionless productivity index for horizontal well with transverse fracture we can calculate the flow rate at the fracture.

- In order to start a new iteration and calculate a new Reynolds number (equation 89), the fluid velocity can be calculated from equation 93.

- With the new Reynolds number a new effective permeability is calculated and the process is repeated. The iteration ends when the difference between Reynolds numbers in two iterations is small.

The methods described previously account for the turbulence effect and give a fair estimate of the dimensionless productivity index; however they have not direct link with the calculation of the overdisplacement effect. The methods presented in this chapter will be used to calculate the dimensionless productivity index based on input data from the methods described in previous chapters. As mentioned in chapter 5, the fracture conductivity will be the main input to the productivity calculations. Modeling different scenarios of overdisplacement will result in conductivity changes that will affect productivity, making possible to do an analysis of the effect of overdisplacement on well performance.
4 Results

The different concepts and calculations presented in the previous chapters described the selected approaches to analytically quantify the effect of overdisplacement on the productivity of a hydraulically fractured horizontal well in Shale gas reservoirs. The result was an analytical model that was initialized with several input parameters typical of hydraulic fracturing treatments in Shale gas reservoirs. The model development was made in Matlab and the code is presented in Appendix A.

This chapter presents the calculations of fracture height and productivity for a specific set of treatment parameters and for different scenarios of overdisplacement variables. The discussion of the results, the conclusions and the recommendations are presented in the following chapters.

The input parameters that were selected to model the different treatment scenarios do not correspond to real data; however their selection was consistent with examples found in the literature and consulted to experts in this field. The input parameters that were used to calculate the base case are showed in the Appendix A – Input Parameters, where each parameter is described by a Matlab variable, the numerical value, the name and the unit used as input in the Matlab code.

After developing the model and selecting and adequate set of input parameters, an identification of the variables that have a significant impact on the model output was done. These results present the effect of overdisplacement on productivity and fracture geometry and its dependence to the variability of such high impact variables. Taking into account the wide range of possible combinations of input variables and as consequence the variety of results, the input parameters used to calculate these results were selected specifically to identify the effect of the most relevant operational variables on overdisplacement.

To add simplicity to the calculations, and considering that although in practice well production depends on the sum of gas flow from the different fracture stages, the calculations presented in this work are focused on the performance of a single fracture and the selected input variables were chosen based on this assumption. The edges of the reservoir were assumed equal in length resulting in a square coverage area. For a fully penetrated horizontal well and a fully propagated fracture, Figure 14 presents a description of the well and fracture dimensions. To give an idea of the size of the treatment, Table 3 presents the required treatment volumes for the selected fracture dimensions.

<table>
<thead>
<tr>
<th>Fracture Injection and Pad Volumes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vf (Fracture volume)</td>
<td>184 m³</td>
</tr>
<tr>
<td>Vi (Injected Volume)</td>
<td>438 m³</td>
</tr>
<tr>
<td>Vpad (Pad Volume)</td>
<td>163 m³</td>
</tr>
</tbody>
</table>
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

Making use of the PKN theory for Newtonian fluids it was possible to calculate the required injection flow rate and net pressure to generate a fracture with the specified dimensions. The results are presented in Table 4 together with the required power to pump the treatment. These values are important since they worked as a reference to verify if the input parameters represent a coherent and viable design.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SI units</th>
<th>Field units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i$ (Injection Flow rate)</td>
<td>0.45 m$^3$/s</td>
<td>171 bbl/min</td>
</tr>
<tr>
<td>$P_{net}$ (Net Pressure)</td>
<td>2.88 MPa</td>
<td>417 psi</td>
</tr>
<tr>
<td>$P_{pump}$ (Pumping pressure)</td>
<td>32.2 MPa</td>
<td>4.677 psi</td>
</tr>
<tr>
<td>Power (Pumping power)</td>
<td>14.64 kNm/s</td>
<td>19.625 Hp</td>
</tr>
</tbody>
</table>

Based on the width equations of the PKN model and in accordance with the profile obtained in the arch calculations, Figure 15 and Figure 16 present the fracture width profile along the length and height directions respectively.

Figure 15 – Fracture width profile along the fracture length direction.
The fracture profiles presented in Figure 15 and Figure 16 were calculated under the assumption of perfect proppant transport and represent the case for a fully load supported fracture. Figure 15 was used as a reference and was compared to the non-fully propped profile (arch occurrence).

The prediction of proppant distribution in fractures is a complex problem. In Shale gas reservoirs this prediction is even more difficult due to the creation of complex fracture networks. As explained in chapter 3, for the analytical model a simple fracture volume balance approach was selected to predict proppant bank height at the fracture in a non-overdisplaced scenario. In this approach the main variable to define proppant bank height is the proppant concentration, which defines the proppant mass injected in the fracture.

Due to the fast proppant settling in low viscosity treatments and to avoid the occurrence of fracture screen out (consequence of high concentrations) a maximum concentration was assumed. Since the well is located at the center of the fracture, the maximum proppant bank height was assumed to be half of the total fracture height ($h_f/2$). The concentration and proppant mass required to generate this maximum height is presented in Table 5.

Table 5 - Proppant mass and end concentration base case

<table>
<thead>
<tr>
<th></th>
<th>SI units</th>
<th>Field Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proppant mass</td>
<td>244.000 kg</td>
<td>537.930 lb</td>
</tr>
<tr>
<td>$c_p$ end</td>
<td>784 kg/m3</td>
<td>6.27 ppg</td>
</tr>
</tbody>
</table>

For the base case the critical velocities and their corresponding critical flow rates (required to overdisplace the proppant along the lateral) are presented in Table 6. Even though two critical velocities calculations were presented in Chapter 3, the use of $V_{dc}$ was preferred since it...
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

applies to any Newtonian fluid independently of its viscosity. The use of $V_{sc}$ is limited to treatments that are overdisplaced with water. Table 6 presents the critical velocities calculated for the base case scenario:

Table 6 - Critical Velocities

<table>
<thead>
<tr>
<th>Critical Flow rate and Velocity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{dc}$</td>
<td>2,23</td>
</tr>
<tr>
<td>$q_{dc}$</td>
<td>0,084</td>
</tr>
<tr>
<td>$V_{sc}$</td>
<td>2,04</td>
</tr>
<tr>
<td>$q_{sc}$</td>
<td>0,077</td>
</tr>
</tbody>
</table>

Table 7 presents the overdisplacement volumes injected for each stage and the corresponding fraction of wellbore volume. The first stage is assumed to be located at the toe of the horizontal well and the last stage located at the heel of the lateral. The overdisplacement volumes were calculated for every stage of the fracturing treatment (the base case assumes 8 stages) however the analytical model estimates the overdisplacement effect on a single fracture. The analysis presented in this thesis was done for the first stage where maximum overdisplacement occurs.

Table 7 - Overdisplacement Volumes

<table>
<thead>
<tr>
<th>Stage</th>
<th>m$^3$</th>
<th>Gallons</th>
<th>% Wellbore Vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26,5</td>
<td>7008</td>
<td>0,24</td>
</tr>
<tr>
<td>2</td>
<td>11,4</td>
<td>3006</td>
<td>0,10</td>
</tr>
<tr>
<td>3</td>
<td>9,5</td>
<td>2509</td>
<td>0,09</td>
</tr>
<tr>
<td>4</td>
<td>7,6</td>
<td>2011</td>
<td>0,07</td>
</tr>
<tr>
<td>5</td>
<td>5,7</td>
<td>1513</td>
<td>0,05</td>
</tr>
<tr>
<td>6</td>
<td>3,8</td>
<td>1015</td>
<td>0,03</td>
</tr>
<tr>
<td>7</td>
<td>2,0</td>
<td>518</td>
<td>0,02</td>
</tr>
<tr>
<td>8</td>
<td>0,1</td>
<td>20</td>
<td>0,00</td>
</tr>
<tr>
<td>Total Vol.</td>
<td>66,6</td>
<td>17600,9</td>
<td>0,6</td>
</tr>
</tbody>
</table>

Since overdisplacement is an operational consequence due to the need of cleaning the well for certain types of completions, the overdisplacement volumes will vary depending on the well geometry, well design (length, liner diameter, etc.) and the number of stages (fracture treatment design). It is important to mention then that for a specific well geometry and design
the overdisplacement volume is constant and the effect of overdisplacement will mainly be sensitive to the variables that define the flow and erosion rates.

The effect on productivity of the overdisplacement will be defined by the final geometry and permeability of the un-propped region. Figure 17 presents the width profile when the formation of an arch at the top of the proppant bank occurs. The method to calculate the length of the arch assumes that the fracture will close above the arch. For this reason the Y axis in Figure 17 uses as reference the effective open fracture and do not show the closed length of the fracture. The effective open fracture is then the sum of the proppant bank height and the arch length. For convenience the zero coordinate was selected as half of the total open fracture length. Since just the open fracture profile is being plotted, the red line in the figure represents the width profile of Figure 16 but plotted until the height of the proppant bank. In comparison with the fracture profile presented in Figure 16, the x axis in Figure 17 corresponds to total width.

![Fracture Height vs. Width](image)

**Figure 17 - Fracture Profile Including Arch**

The calculations of permeability and average widths that are presented in Table 8 corresponds to the fracture geometry presented in Figure 17, where the length of the closed fracture is the total fracture height minus the open fracture length.
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Table 8 - Fracture zone permeability and dimension

<table>
<thead>
<tr>
<th>Frac Zone</th>
<th>Geometry</th>
<th>Height (mt)</th>
<th>Permeability (mD)</th>
<th>Width (mt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed Fracture</td>
<td></td>
<td>25.6</td>
<td>2,11E+05</td>
<td>5,00E-05</td>
</tr>
<tr>
<td>Arch</td>
<td></td>
<td>2.9</td>
<td>6,88E+08</td>
<td>2,88E-03</td>
</tr>
<tr>
<td>Propped Fracture</td>
<td></td>
<td>21.4</td>
<td>5,00E+04</td>
<td>4,50E-03</td>
</tr>
<tr>
<td>Open Fracture</td>
<td></td>
<td>24.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The dimensions presented in Table 8 correspond to the fracture geometry after overdisplacing the treatment (proppant bank eroded). The propped fracture height is then less than the one calculated based on maximum concentration. The open fracture term corresponds to the sum between the arch and the propped fracture and will define the connectivity between the fracture and the well.

Production and productivity index were calculated for the fracture dimensions presented in Table 8. It is important to notice that these calculations correspond to a specific proppant concentration, overdisplacement condition and a resulting proppant bank height. Sensitivity analysis for different proppant concentrations and overdisplacement variables are presented further in this chapter. Table 9 presents the total production and productivity and the corresponding contribution from each fracture region. The calculation of production will be impaired by the choke skin of the zone intersecting the well. For this specific case the open fracture height does not reach the well and production is choked through the closed fracture.

Table 9 - Production and Productivity Index Base case

<table>
<thead>
<tr>
<th>Fracture Zone</th>
<th>Production (m3/s)</th>
<th>PI (m3/s/MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed Fracture</td>
<td>1,14E-03</td>
<td>1,10E-04</td>
</tr>
<tr>
<td>Arch</td>
<td>9,77E-05</td>
<td>9,44E-06</td>
</tr>
<tr>
<td>Propped Fracture</td>
<td>1,55E-03</td>
<td>1,50E-04</td>
</tr>
<tr>
<td>Total Production</td>
<td>2,79E-03</td>
<td>2,69E-04</td>
</tr>
</tbody>
</table>

Under the assumption that the wellbore is located at the center of the fracture and has a radius $r_w$ specified as an input parameter, the range of fracture height where the fracture will have a connection to the wellbore is very narrow. It is important to notice that due to the nature of transverse fractures, this range of connectivity between the wellbore and the fracture is limited to the well radius, making production very sensitive to any change in proppant height around this range.

Figure 18 illustrates the importance of the propped height and its effect on well-fracture connectivity in transverse fractures. It presents the variation of proppant bank height and arch versus proppant concentration for a non overdisplaced treatment. The narrow range of height where well-fracture connectivity occurs is represented by the red colored area. For any open fracture height in this range the productivity of the well is considerable higher as we will
discuss further on. The total open fracture height is the sum of both, the arch (green line) and the proppant bank (blue line) heights.

![Proppant Bank height vs. Proppant concentration](image)

Figure 18 - Proppant Bank height vs. Proppant concentration

For different overdisplacement parameter scenarios, certain degree of erosion at the top of the proppant bank will occur, affecting well-fracture connectivity. It is understood then that the effect of overdisplacement will depend on its impact on the height of the proppant bank which is the main parameter to ensure fracture connection to the well from a 2D approach. The height of the proppant bank after overdisplacing a treatment will depend on the erosion effect generated by the cleaning rates, however reservoir parameters will play a role in the formation of the arch and by means will also affect the final open fracture height.

The dimension of the fracture is defined by the height of the proppant bank and the arch, and the calculation of the latter was based on the method presented by Warpinski (2009), where the calculation of the width profile is mainly defined by three reservoir parameters: the effective pressure, the young modulus and Poisson’s ratio. These parameters will define the “fracability” of the rock and the size of the arch. It is known that a brittle Shale gas reservoir will have a high young modulus and low Poisson’s ratio and will create a fracture network when fracturing stimulation treatments are performed. On the other hand, a ductile Shale reservoir is characterized by low young modulus and high Poisson’s ratio, meaning that fractures will tend to close and will result in poor fracture network.

Additionally to the formation properties, the occurrence of an arch will depend on the load carrying capacity of the proppant. Since the stress on proppant is not being studied in this work, it is assumed that proppant strength will support the intensified pressure at the arch-proppant bank interface.

Figure 19 presents the variation of arch size versus the proppant height for different young modulus. For the highest young modulus (5E6 psi or 34500MPa), the arch sizes are four times higher than for the lower case (2E6psi or 13800MPa) at the well. The brittle characteristic of
high young modulus rocks will prevent the fracture from plastic deformations and will keep the fracture open.

Figure 19 - Arch height vs. Proppant bank height for different young modulus

The effective pressure is defined as the pressure inside the arch, which for initial production may be assumed as the reservoir pressure minus the minimum horizontal stress or sigma min. Figure 20 present the variation of the arch size versus proppant bank height for different values of effective pressure. Since the effective pressure is dependent on the reservoir pressure, the arch size will be reduced during depletion.

Figure 20 - Arch Height vs. Proppant bank height for different Effective pressure
The Poisson’s ratio will play a main role if rock failure analysis is considered. The formation of the arch is of course possible under the assumption that the rock will support the peak stresses and will not break and close the fracture. This analytical model however is a 2D approximation of the fracture deformation and will not consider what would be the implications of the arch formation for the different strain directions.

The previous results showed the influence of reservoir properties on the open fracture height and its importance for well-fracture connection, however with the exception of concentration (normally limited to economic and fracture propagation issues), previous variables cannot be controlled and do not influence the reduction of proppant bank height due to overdisplacement.

The overdisplacement effect is mainly controlled by the velocity at which the fluid enters the fracture and erodes the proppant bank. Turbulence effects, that to a less extent affect the height of the bank due to the high injection rates during low viscosity treatments, are also considered.

Treatments in Shale gas reservoirs are characterized by being performed with low viscosity fluids, and changes in viscosity will strongly influence the proppant transport capacity and the flow regime. Low viscosity fluids will require higher injection rates in order to clean the wellbore, and due to the limited contact area between the well and the fracture high fluid velocities will occur inside the fracture increasing the erosion effect. Figure 21 presents the variation of erosion rate per width to viscosity of overdisplacement fluid. As it was expected high erosion rates occur at low viscosity treatments.

![Figure 21 - Erosion rate per width unit vs Fluid viscosity](image-url)
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

Figure 22 presents the eroded height rate at the top of the proppant bank. It is observed that contrary to the erosion rate presented in Figure 21, the eroded height increases for high viscosity fluids. This is explained by the reduction of turbulence length for high viscosity fluid resulting in a smaller eroded area (Figure 23). To compensate the reduction of eroded area in the mass balance the eroded height will increase for smaller turbulent lengths.

Figure 22 – Eroded height rate vs. Overdisplacement Fluid Viscosity

Figure 23 – Turbulence length vs. viscosity
In practice, due to the fast settling velocity of low viscosity fluids will not allow to reach proppant bank heights and even for higher viscosities problems such as screen outs may occur. The ideal fully propped fracture considered as reference in this problem is then a fracture propped until the bottom of the wellbore (i.e. proppant in the fracture up to the perforations). Overdisplacement and turbulent effects will account as a reduction of proppant height from this reference height case. Figure 24 presents the variation of open fracture height and proppant bank for different values of viscosity.

![Fracture Height vs. Overdisplacement Fluid Viscosity](image)

**Figure 24 - Fracture Height vs. Overdisplacement fluid viscosity**

![Non-connected Fracture, Low viscosity treatment](image)

**Figure 25 - Non-connected Fracture, Low viscosity treatment**
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

The 2D approach considered in this model and the fact that the fractures are transverse to the lateral, result in a very narrow range of proppant heights to connect the well. It is expected than in reality the connectivity between the fracture and the well, and the opening of the fracture along the length is not uniform, and from a 3D perspective, different geometries such as interspersed channels along the fracture height, pinch points or closed regions may occur and affect productivity.

Figure 25 and Figure 26 show an illustration of a non-connected and connected fracture for a treatment performed with low and high viscosity fluid respectively.

The calculation of productivity for certain specific fracture geometry was based on the assumption that production with almost no choke skin effect will occur just in the case where the open fracture reaches the well. It was assumed that the horizontal well is located at the center of the fracture and, for transverse fractures from a 2D approach; the range of connectivity between the fracture and the well will be half fracture height +/- the well radius. Since the specific production results cannot be compared with real data the performance of the fracture will be evaluated by the productivity index ratio defined as the ration between the calculated PI for certain proppant height and the ideal case where the proppant bank reaches the bottom of the well.

Figure 27 presents the variation of productivity index ratio for different values of viscosity. The production jump observed in the graph indicates that at that viscosity the fracture height reaches the well resulting in a dramatically reduction of choke skin effect.
From the productivity ratio plot it is possible to quantify the effect of overdisplacement since it represents a comparison between the productivity for an overdisplaced scenario and the reference case. The productivity plot additionally facilitates several sensitivities of the effects of the different operational variables.

Another important operational parameter that influences the overdisplacement effect is the size of the proppant particles. The selection of the proppant size is part of the treatment design and it has to be in accordance with the expected permeability, type of reservoir, proppant transport capacity, fracture width and several other conditions.

Figure 28 and Figure 29 present the variation of erosion rate per width unit and turbulence length for different proppant diameters.
Effect of overplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

Figure 28 - Erosion rate per width unit vs. Proppant particle diameter

![Graph showing erosion rate vs. proppant particle diameter](image)

Figure 29 - Turbulent length vs. Proppant particle diameter

![Graph showing turbulent length vs. proppant particle diameter](image)

Figure 30 presents the variation of open fracture height and proppant bank height for different proppant diameters. Observing the ending points in the fracture height plots it is possible to observe that the proppant diameter has less influence in fracture height if compared with the
viscosity effect. Figure 30 shows the threshold value at which the arch will intersect the well and will reduce skin effect.

![Figure 30 - Fracture Height vs. Proppant diameter](image)

Figure 30 presents the changes in productivity index ratio for different proppant diameters. The jump in production occurs at the size which the arch will reach the well as it may be observed in Figure 30.

![Figure 31 - Productivity Ratio vs. Proppant particle diameter](image)

Figure 31 - Productivity Ratio vs. Proppant particle diameter
Figure 32 shows the variation of eroded height rate versus proppant concentration. The turbulence length is not affected significantly by changes in the concentration. The eroded height rate is mainly dependent on the erosion rate. Figure 33 presents the change of fracture height for different values of concentration. The fracture height is proportional to the concentration. Figure 33 presents the fracture height affected by the overdisplacement and for the selected input parameters it is observed that the open fracture does not reach the well.
Figure 34 presents the production profile versus the proppant concentration making use of the base case input parameters. As a consequence of the no well-fracture connectivity the production is impaired by the choke skin effect at any concentration.

To illustrate the contribution from the closed fracture, Figure 35 presents the productivity profile for different closed fracture widths for the fixed parameters showed in the input data for the non overdisplaced scenario.
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

The productivity jumps in Figure 35 indicate the values of viscosity where the open fracture will be connected to the well. As expected for the low value of residual width (10μm) the permeability of the closed fracture is small if compared with the permeability of the open fracture zones and there is a significant impairment in productivity when the fracture is not connected to the well. For higher values of residual width (50μm and 100μm) the contribution to production from the closed fracture considerably increases and in the case of 100μm it even equal the productivity from the propped region, making overdisplacement irrelevant.
5 Discussion

As in any other model the validity of the results depends on the adequate selection of the design variables. The output of the fracture design calculations was used to gain an initial understanding of the sensitivity of the input variables and confirm the viability and coherence of the fracturing design. As was expected the model confirmed that for low viscosity treatments, where slick water is commonly used as fracturing fluid, there is a need of high fluid injection rates to guarantee the required fracture width. Fractures in low permeability formations, such as Shale reservoirs, are commonly targeted to be long and narrow, and even though this leads to more achievable operational requirements (pumping and power capacity) the low viscosity of the fluids will still require very high injection rates. It is important to notice that, since for narrow fractures the contact area between the well and the fracture will be small, high injection rates may result in removal of proppant even if the fracture has not been overdisplaced.

The base case presented is an example of the high operational requirements to perform a low viscosity fracturing job. For a relatively wide fracture and a fluid viscosity of 10cp, the require pumping power was approximately 20k horse-power and flow rates close to 0,45m3/s (170bbl/min). These values are relevant if we compare them with the critical flow rates required to clean the wellbore. For the base case scenario the required critical velocity was 0,084m3/s, meaning that the required injection rate to open the fracture was almost 5 times higher than the overdisplacement critical flow rate. This difference in flow rates lead to reconsidering the need of overdisplacing treatments for certain high rate treatments, since the high injection rates may already guarantee that proppant is transported and not deposited at the bottom of the lateral.

In the case of plug-and-perf completions the positioning of the plug already implies to overdisplace any treatment. Although this is a requirement to perform the job there was no reference in the literature about a specific procedure or method to calculate the amount of liquid to be injected to place the plug and eventually to clean the wellbore. Accurate calculation of the volumes implies savings in money and time; especially when in practice it seems that the overdisplacement volumes are determined mainly empirically. When performing several fracturing stages the overdisplacement volume may be reduced for the stages closer to the well as the cleaning distance is reduced (table 4).

Once the treatment has been overdisplaced the discussion about its effect is focused on determining if there is any connection between the fracture and the well or contrarily the fracture closes at the well and chokes production. As discussed further on, it is believed that overdisplacement will erode the proppant bank reducing the height of the propped fracture and impairing the connection to the well. The question is then if even for a reduced proppant bank the fracture will remain open beyond of the top of the proppant bank due to the creation of an un-propped arch. Based on an analytical method, the model developed in this work calculates the size of the arch for a given proppant bank height and reservoir parameters and the results showed that the height of this arch may be significant if compared with the total
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

fracture height. The formation of this arch may be seen as an extension of the open fracture above the proppant bank. Since there is no proppant at this arch, it acts like an open channel with infinite permeability. As we can see in table 8, the permeability of the arch exceeds by four orders of magnitudes the proppant pack permeability. The effect of removing the proppant bank from the well may be compensated if this arch reaches the well.

When the creation of an arch occurs, the formation strength is keeping the fracture open and an increase of pressure on the proppant pack will occur. The calculations of the arch were based on a 2D approach and a linear pressure distribution on the proppant which decreases the accuracy of the width profile prediction. More accurate pressure distributions are possible, resulting in a more accurate match between width profiles at the top of the proppant bank; however this will also increase the peak pressure on the proppant pack and increase the probability of proppant failure. It is important to mention then that the occurrence of an arch is related to the assumption of proper proppant pack and rock strength.

The calculation of the arch length and the effective proppant bank height (height reduced by the overdisplacement) allow determining the width of the fracture by making use of the width profile equations. It was assumed that the rock at the closed fracture, which is the zone that remain un-propped and above the arch, will keep certain width due to irregularities in the faces and roughness of the rock generated by the fracturing. For the base calculations, a residual width of 50 micrometers was considered.

The determination of the open fracture height and width allows estimating the productivity for each fracture zone. As we can observe in table 9, in these low permeability formations the flow from the closed fracture may be significant if compared with the open fracture contribution. For the open fracture region, the flow from the arch is reduced due to the low reservoir permeability and most of the production is attributable to the propped region. The production of the arch increase significantly for reservoirs with higher permeability (100 micro darcies).

For transverse fractures, where the contact between the well and the fracture is limited to the well circumference, the success of the fracturing treatment will be determined by the proper distribution of proppant along the height. Under the assumption of arch occurrence, a connection between the fracture and the well may occur even if the proppant pack was eroded by overdisplacement fluids. The calculation of the productivity was based on the assumption that production will increase for the cases where open fracture height reaches the well due to the reduction of choke skin effect. Figure 36 represent the situation where the fracture is connected to the well for certain proppant bank and arch height.

The productivity calculations are function of the fracture geometry, and before discussing the productivity profiles it was necessary to understand the variables that affect the height of the proppant bank when overdisplacing a treatment.

From the several parameters that may influence the effect of overdisplacement and that are part of the design of the treatment, the viscosity of the overdisplacement fluid and the size of the proppant particles have important relevance since it will affect the fluid cleaning capacity, the turbulence regime and the erosion rate.
Erosion rate

Figure 22 presented the variation of the eroded height rate for different overdisplacement fluid viscosities. The calculation of the eroded height is based on the erosion rate which defined the removed volume and the turbulence length which defines the area from where this volume is removed. As we can observe in Figure 23 the turbulence length will be reduced at higher viscosities and will reduce the eroded area. This effect will balance the reduction in erosion rates and result in an increasing tendency of eroded height with viscosity. As it will be observed however, the increasing tendency of erosion rates will still affect the connectivity of the well.

Figure 21 presents the variation of the erosion rate per width unit versus viscosity. As expected, for higher viscosities the need of low critical velocities at the lateral will decrease the high velocities at the fracture. Since the erosion rate is dependent on both the fluid velocity at the perforation and the viscosity, low viscosity fluids will have a higher impact on proppant erosion and open fracture height. Although the erosion rate will decrease for high viscosities, the benefits of increasing fluid viscosity to reduce overdisplacement may result in additional costs. Figure 22 shows that for low viscosity values there is a high slope in the reduction of erosion rate meaning that at this range of viscosities the impact of increasing viscosity may be higher.
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

It is important to mention that the calculation of erosion rates were based on erosion models for sediment transport in rivers which may lead to differences regarding the flow regime. Due to the small area of the fracture if compared with natural channels, it may be the case that turbulent effects during overdisplacement not considered here, will generate a re-falling of proppant particles on the proppant bank top thereby decreasing the erosion effect.

Figure 28 and Figure 29 showed the changes in erosion rate and turbulence length for different proppant diameter size. It is observed that in comparison with the viscosity, changes in the proppant diameter may reduce the erosion rates significantly. As we mentioned decreasing the velocity in the fracture will affect the erosion effect and for small diameters the transport of particles requires lower critical velocities and by means lowers cleaning injection rates. The proppant diameter will impact also the erosion calculation since the threshold of motion for the particles will decrease if the proppant diameter increases. This effect however is compensated by the need of higher critical velocities to transport the bigger particles in the lateral. The turbulence length changes due to proppant size do not affect the overdisplacement effect considerably however they are taking into account to calculate the effective proppant bank height.

As we observed in the figures the selection of the proppant size may have an effect on the fracture connectivity to the well; however there are other fundamental considerations that are made when selecting proppant size. The permeability and strength of the proppant bank will be affected by changing the proppant size. As an example small particles may lead to compaction and reduction of proppant pack pores affecting the permeability and the strength of the propped zone.

Figure 32 presented the variation of erosion rate for different values of proppant concentration. The proppant concentration is proportional to the proppant fraction which describes the amount of proppant per volume of fluid. The critical velocity to clean the lateral are dependent on the proppant fraction and as it is showed in the plot increments in proppant concentration will lead to higher erosion rates. Since the proppant bank height is proportional to the amount of proppant injected in the fracture, reductions in proppant concentration will reduce overdisplacement effect but also will decrease proppant bank height, which may lead to un-connected fractures.

*Fracture height and connectivity with the well*

In Figure 24, Figure 30 and Figure 33 the variation of open fracture height and proppant bank height is shown for different values of viscosity, proppant diameter and concentration. For the case presented in these results it was assumed that the well is located at the centre of the fracture and its position is indicated in the plots by the red lines. The distance between the red lines is the well radius and corresponds to the well-fracture connection threshold.

From Figure 24 we can observe that at low viscosities the production of the well will be choked by the closed fracture. Above 70cp the reduction in erosion rate will result in sufficient open fracture height and the arch will be connected to the well. In practice the use of low viscosity fluids will generate proppant settling and for high concentrations the occurrence of screen out may be expected. For this computation it was considered that the maximum un-affected
proppant bank height will not exceed the position of the well, which for this case is half total fracture height. Under this assumption Figure 24 shows that the proppant bank will not reach the well for any viscosity. At higher viscosities however better proppant distribution and transport is expected and higher concentrations may apply.

It is important to notice that the erosion rate is more sensitive to changes at low viscosities, showing the advantage of viscosity increments at that range. For non-connected wells the closed fracture choke skin effect will impair production however any increase in open fracture height will contribute to improve gas flow.

Figure 30 presented the variation of fracture height affected by overdisplacement for different values of proppant diameter. The figure showed that for proppant diameters above 0.4mm there is no connection between the open fracture and the well. This is in agreement with the erosion plot where the critical velocities and erosion rate increased for larger particles. Observing Figure 30 and Figure 24 it is possible to notice that the difference in open fracture height, at the end points in the viscosity and particle diameter plots, is larger for the case of the viscosity. This is attributable to the sensitivity of the critical velocities and erosion equations to viscosity changes.

Figure 33 showed the effective open fracture height for different concentrations. As the proppant volumes is proportional to the concentration there is a linear dependence between the height and the concentration. Under the conditions of the base case scenario the assumed maximum concentration will not generate sufficient fracture height to connect it with the well. It is necessary then to increase viscosity or decrease proppant size to reduce the erosion effect and increase the proppant bank height.

Productivity

Any reduction of proppant bank height due to overdisplacement will affect the fracture connectivity to the well and depending of where the perforations are located it will decrease the productivity. For each of the operational parameters studied in the results, production flow rates were calculated and compared with a reference case. It was assumed that a fracture with a proppant bank reaching the bottom of the well and an arch intersecting the well will represent the reference case of a non overdisplaced fracture. Figure 27, Figure 31 and Figure 34 showed the productivity for certain range of parameters as a fraction of the reference productivity.

Figure 27 showed that for viscosity values below 70cp the open fracture zone will not be connected to the well and the productivity will be choked. There is 37% productivity reduction due to overdisplacement. For viscosities above 70cp the open fracture height will reach the well and since the choke skin will be reduced close to zero. The connection to the fracture will occur through the arch and due to turbulent effects during fluid injection the proppant bank will reach the maximum value resulting in a maximum productivity of 95% with respect to the reference scenario.

Figure 31 presented the productivity ratio for different proppant diameters. The overdisplacement effect will have higher impact on productivity for proppant sizes larger than
Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs

0.45mm where the productivity was reduced 35%. For small particle size the fracture will be connected to the well and the effect of overdisplacement will reduce productivity by 4% if compared with the reference case.

Figure 34 showed the productivity ratio profile for different concentrations. As mentioned earlier in the discussion, the viscosity and other input parameters selected for the base case will result in a non-connected fracture. The production for this case will be choked by the closed fracture for any proppant concentration and the variation in fracture dimensions will determine the production. Production for low concentration and low overdisplacement effect (erosion rate) will result in a 40% productivity reduction while for the maximum concentration the reduction will be around 35% with respect to the reference case.

In summary the productivity ratio profiles showed clearly two trends:

- A low productivity index ratio zone where production from the different fracture zones is choked by the closed fracture resulting in a positive choke skin effect. For this range of open fracture heights the production from the closed fracture is close in magnitude to the proppant bank production.

- There is a second trend that corresponds to the production when the fracture is connected to the well through the arch and is characterized by a fast rise in the productivity index ratio. Production will be increased dramatically once the arch reaches the well due to the reduction to zero of the choke skin. The size of the arch increases with the proppant bank height however for higher values the arch height change rate is almost constant (see Figure 19) resulting in small productivity improvements from arch growing.

Residual width

Figure 35 presents the productivity profile for different viscosities and different residual widths. For the calculations of the closed fracture width it was assumed that the faces of the formation are parallel and smooth. The permeability of the closed fracture is proportional the separation between fracture faces (width) and it will have a significant impact on the productivity. The width of the closed fracture will depend on the degree of rock deformation or the occurrence of irregularities on the rock due to shear stresses after the fracturing treatment is performed. For certain values of residual width the choke skin effect will not have a significant impact on productivity and production from the closed fracture may equal the flow from the arch and the proppant bank independently from the height of the proppant bank. For these cases the effect of overdisplacement is irrelevant. In Figure 35 it is possible to observe that at the 100μm residual width curve there is no considerable change in productivity when the open fracture height reaches the well as it is for the cases with lesser residual width. In low permeability formations the effect of overdisplacement on productivity will not be relevant in the cases where the conditions of the reservoir rock make possible to obtain sufficient residual width after the fracture treatment is performed.

The degree of residual width and the formation of the arch are related to the strength of the rock and the reservoir conditions. Figure 19 and Figure 20 presented the variation of the arch
size for different reservoir young modulus and effective pressures. Although this work does not consider the influence of the rock properties on the residual width, it may be useful to observe the effect of these parameters on the formation of the arch and assume that the capacity of the rock to generate an arch is also related to the tendency of the rock to remain certain residual width.

As expected, Figure 19 indicated that for a high young modulus of 5E6 psi (34.5 GPa) the size of the arch was around 4 times higher than for the low young modulus case of 2E6 psi (13.8 GPa). It may be the case then that for high young modulus rocks, the strength and stiffness of the rock will also keep certain separation between fracture faces at the closed fracture region. In low young modulus formation the importance of keeping a propped fracture increases and the effect of overdisplacing the treatment will have a higher impact on productivity. As the young modulus increases, also does the probability of having higher residual widths and the size of the arch. In this case the effect of overdisplacement will depend on the closed fracture dimensions and for certain conditions it may be not cause a significant impairment on productivity.

Figure 20 presented the variation of the arch size for different effective pressures. The effective pressure is the pressure inside the arch and was assumed as the difference between the minimum horizontal stress and the reservoir pressure. If a constant minimum horizontal stress is assumed, the size of the arch will decrease as the reservoir pressure decreases and therefore the effective pressure increases. The influence of the reservoir pressures in the arch formation may be seen as equivalent stress acting in the opposite direction to the fracture closure. As was discussed previously, for lower effective pressures it may be a higher tendency in the rock to keep sufficient residual width. As the formation depletes the size of the arch will decrease and it may be the case that the residual width also decreases reducing productivity considerably. The effect of overdisplacement on productivity may increase with time as the reservoir pressure depletes. For lower reservoir pressures the size of the arch will decrease and depending on the residual width and proppant bank dimensions is possible to loss well-fracture connectivity and increase choke skin effect.
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6 Conclusions

The development of Shale gas reservoirs is based on the implementation of horizontal wells and hydraulic fracturing technologies. There are different types of completions that may be used to perform a hydraulic fracturing treatment. For the case of treatments that are performed with the perf and plug completion, it is a common practice to overdisplace the treatments in order to clean the residual proppant bed from the horizontal section of the well and place the plug. The cleaning or overdisplacement will assure the adequate placing and sealing of the plug. Overdisplacement of fracturing treatments in Shale gas reservoirs with transverse fractures may impair productivity. When overdisplacing a treatment the small contact area between the fracture and the well will generate high fluid velocities into the fracture that may erode the proppant bank and result in loss of fracture connectivity to the perforations.

Hydraulic fracturing treatments in Shale gas formations are often performed with low viscosity fluids (slick water) which need high injection rate to transport the sand into the fracture. In these cases the use of overdisplacement may be avoided if the high injection rates considerably exceed the critical flow rate for erosion of the proppant bank.

The overdisplacement volumes are dependent on the size of the well and the number of fracturing stages (treatment design). The adequate determination of overdisplacement volumes for each fracturing stage may lead to cost optimization and reduction on the proppant bank erosion.

In partially filled propped fractures it is believed that the creation of an arch may occur at the top of the proppant bank. The creation of an arch will have a considerable impact in higher permeability formations (in the order of hundreds of micro darcies) since it acts as an open channel with infinite permeability and zero choke skin effect. In low permeability formations (hundreds of nano darcies) the gas flow rates are very limited and production from the arch will not be significant if compared with the propped or closed fracture region.

The main effect of overdisplacing a treatment is to lose the connection between the fracture and the perforations due to erosion on the proppant bank and reduction in the open fracture height. In this case the production will occur through the closed fracture zone and the flow will be affected by a choke skin effect. It may happen however that even for an overdisplaced treatment the formation of the arch (un-propped open fracture) will allow connection between the fracture and the perforations.

The results showed that for low viscosity treatments, increasing the overdisplacement fluid viscosity will considerably reduce the erosion of proppant bank and reduce risk of losing fracture connectivity in a connected fracture.

Regarding the size of the proppant particles, the results showed that larger proppant particles required higher flow rates in order to be transported, therefore increasing the effect of overdisplacement. The erosion plots also showed that for larger proppant sizes erosion on the
proppant bank increases. The selection of proppant size must be an optimization process between the optimum design and minimum overdisplacement effect (erosion rate).

Increments of proppant concentration will increase the overdisplacement critical velocities and erosion rates. Reduction in proppant concentration should consider the advantages of decreasing erosion rates but also the impact of reducing proppant bank height (proppant volume).

For the described base case and input parameters presented in the appendix the effect of overdisplacing a treatment caused a reduction of approx. 40% of the fracture productivity if compared with the case where no overdisplacement occurred and the proppant bank height is reaching the bottom of the perforations.

In a non-connected fracture the degree of residual width will considerably affect the choke skin effect and the productivity. For certain values of residual width the skin effect will not be significant and the production from the closed fracture will be comparable to the one in the propped fracture, making the effect of overdisplacement irrelevant.

If it is assumed that the ability of the rock to form an arch is related to the width of the closed fracture after pumping the treatment, it is believed that the effect of overdisplacement may be compensated by larger open fracture height and residual widths. Depending on the closed fracture conditions the overdisplacement effect may be neglected. For low young modulus formations the effect of overdisplacement may be more relevant due to the reduction in the open fracture height (shorter arch size) and the difficulty to obtain sufficient residual width (assuming lower modulus yields smaller residual width).

The height of the arch will be reduced if the effective pressure increases. The effect of overdisplacement will then increase during depletion due to the reduction in reservoir pressure. In this case the productivity may decrease with time due to the reduction of open fracture height and the possible progressive reduction of the residual width. The latter will result in a larger choke skin effect.
7 Recommendations

The work presented in this thesis attempts to present an initial quantification of the effect of overdisplacing fracturing treatments on productivity. The analytical model that was developed approaches the problem from a 2D perspective and uses simplified solutions for the description of proppant distribution in the fracture before and after overdisplacement. The following recommendations are suggested to validate the results obtained from the analytical model and go further on the study of the overdisplacement effect on productivity:

- As it was discussed, the residual width at the closed fracture zone will have a high impact on fracture productivity in low permeability formations. It is recommended to identify under what rock conditions and for which fracturing variables there will be sufficient residual width.

- The effect of overdisplacement was presented as the reduction of proppant bank height due to erosion on the top of the proppant bank. The complexity of fracture networks and proppant distribution may lead to additional turbulence effects or proppant transport mechanisms inside the fracture that are not accounted in this work. The more accurate prediction of the proppant distribution after the injection of the overdisplacement volume will definitely lead to have a better understanding on its effect of productivity.

- In order to validate the analytical model and further studies it is recommended to compare the results with field production data.

- Modeling of production making use of reservoir engineering simulators including the change of overdisplacement as a function of time.
8 References

Effect of overdisplacement of proppant in hydraulic fracturing treatments on the productivity of shale gas reservoirs


Appendix A – Input Parameters

% Reservoir Properties
Y = 5E6; % Young modulus, psi.
Poisson = 0.2; % Poisson’s ratio.
Yps = Y/(1-Poisson^2); % Plain strain modulus, psi.
G = Y/(2*(1+Poisson)); % Shear modulus, psi.
Pres = 4500; % Initial reservoir Pressure, psi.
T = 200; % Reservoir temperature, F.
kres = 0.0001; % Reservoir permeability in milidarcy, mD.
porosity = 0.05; % Reservoir Porosity.
sigma_min = 6000; % Min. Horizontal stress, psi.

% Input Fracturing geometry
hf = 50; % fracture height, m.
wmax = 0.008; % maximum width at wellbore, m.
xf = 400; % fracture half length, m.

% Mini frac test parameters
tp = 20; % Injection or pumping time, min.
delta_t = 20; % Shut in time, min.
dimen_t = delta_t/tp; % Dimensionless time.
n = 0.42; % Fluid efficiency from Figure 3.
kloss = 1; % Distribution factor.
sp = 0; % Spurt loss, m.

% Reservoir Dimensions
a = xf*2*3.28; % Reservoir wide, ft.
b = a; % Reservoir length, ft. This value should be equal to Lateral. Assuming a fully penetrated well model.

% Horizontal Well dimensions
TVD = 6000; % True Vertical Depth, ft.
lateral = b; % Lateral length, ft. Fully penetrated well.
MD = TVD+lateral; % Measure depth, ft.

% Completion and Pipes parameters
rw = (8+(5/8))/2; % Wellbore radius in the lateral, in.
D.pipe = rw*2; % Pipe or liner inside diameter, in.
D_casing = 9.375; % Production Casing inside diameter, in.
stages = 8; % Number of fracturing stages.
friction_loss = 100; % Friction losses, psi/1000ft.
Pwf = 3000; % Bottom hole pressure, Psi.

% Fracturing Fluid and Proppant Properties
rho_fluid = 1000; % Fracturing Fluid density, kg/m3.
rho_ppt = 2650; % Proppant density, kg/m3.
dppt = 0.03; % Proppant particle diameter, in.
porosity_ppt = 0.4; % Proppant pack porosity.
kfrac = 50000; % Proppant pack permeability, mD.
visc = 10; % Fluid viscosity, Cp.

%Parameters to calculate erosion rate
visc_over = 50; % Cleaning Fluid viscosity, cp.
h_free = 2; % Free height equivalent to water depth in erosion model. Assumed as an approximated arch height.
ks = 0.5*dppt*0.0254; % Roughness height, mt.