SOME MEASUREMENTS IN THE WAKE BEHIND
A CIRCULAR CYLINDER

by

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DELFIT - THE NETHERLANDS

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SUMMARY.

Measurements are reported in the wake behind a 2 mm diameter circular cylinder at a free stream velocity of 50 m/sec. Mean velocities, longitudinal turbulence intensity, and turbulent shear stresses were measured by means of linearized hot wire equipment, up till 1000 cylinder diameters downstream.

Mean velocity profiles are shown to collapse into a universal curve for $x \geq 100d$ when plotted as $(U_{\infty} - \bar{U})/U_o$ vs. $y/b$ where $U_o$ is the maximum velocity defect and $b$ the half width to half depth. Turbulence data, when plotted as $\overline{uu}/u_t^2$ and $u'/u_t$ vs. $y/b$, where $u_t^2 = |\overline{uv}|_{\text{max}}$, also show a universal distribution for $x \geq 100d$.

In an appendix a correction procedure is given for linearized hot wire calibration drift during measurements.
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SYMBOLS.

$a_1, a_2$ universal constants associated with mean velocity profile, defined on page 6

$b$ measure for width of mean velocity wake; distance between wake center and position of half maximum velocity defect; defined in fig. 1.

$b_t$ measure for width of turbulence; distance between positions of maximum and minimum in turbulent shear stress

$f$ non-dimensional mean velocity profile

$q$ free stream dynamic pressure

$R_d$ cylinder Reynolds number based on free stream velocity and cylinder diameter

$\bar{U}$ mean streamwise velocity in wake

$U_\infty$ free stream velocity

$U_0$ maximum velocity defect in wake, defined in fig. 1; mean velocity scale

$u'$ root mean square of streamwise velocity fluctuations; longitudinal turbulence intensity

$\overline{u_t} = \sqrt{\overline{u'v'}}_{\text{max}}$; turbulent velocity scale

$\overline{uv}$ kinematic turbulent shear stress

$x$ streamwise coordinate; origin at cylinder axis

$y$ coordinate perpendicular to free stream direction and cylinder axis; origin at station of maximum velocity defect

$y'$ coordinate in y direction as used during measurements, arbitrary origin

$z$ coordinate along cylinder axis
\( \delta^* \)  
**displacement thickness of wake flow**

\( \theta \)  
**momentum thickness of wake flow**

\( \theta_\infty \)  
**value of \( \theta \) for \( x \to \infty \)**
1. INTRODUCTION.

The plane wake behind a circular cylinder in a constant pressure flow is a well known example of a turbulent free shear flow. It has been studied extensively by for instance Schlichting [1], Townsend [2,3,4,5], Uberoi and Freyimuth [6]. In these experimental investigations many questions concerning the development of mean flow properties and the structure of turbulence have been elucidated. Yet, it is difficult to obtain a clear picture of the development of a particular flow from the available literature. For instance, if one wishes to know, for the purpose of comparison with a calculation method, the development of mean flow parameters together with the development of some relevant turbulence properties like the turbulent normal and shear stresses, it is difficult to obtain these data in sufficient detail for one particular flow.

The present report is intended to provide such data, in order to be able to compare them with a calculation method for turbulent wakes developed by the present author [7]. The main features of this method relevant to the present report are: similarity of mean velocity defect profiles, and similarity of the turbulence properties. The mean velocity profiles are scaled with the maximum velocity defect $U_o$ and the half width to half depth $b$. (For definitions of $U_o$ and $b$ see fig. 1). The turbulence properties are scaled with the square root of the maximum (kinematic) turbulent shear stress $u_t = \sqrt{\overline{uv}_{\text{max}}}$ and the mean velocity scale $b$.

The quantities calculated are these three scales: $U_o$, $u_t$ and $b$.

The measurements described in the present report were limited by the hot-wire equipment available at the time. Only the mean velocity, the longitudinal turbulence intensity and the turbulent shear stress could be measured. However, this is sufficient to compare the calculation method with these results, and to check certain assumptions made in the calculation method.
2. MEASUREMENTS.

2.1. Description of experimental set-up.

All measurements were performed in the low speed low turbulence wind tunnel of the Department for Aerospace Engineering at Delft University of Technology. This tunnel is of the closed return type; its test section has a cross-section of 1.80 x 1.25 m$^2$ and a length of 2.60 m. The turbulence level in the free stream is about .03% at a speed of 50 m/sec. Further details may be found in ref. [8].

A sketch of the experimental set-up is shown in fig. 2. Near the entrance of the test section a circular cylindrical wire of 1.975 mm diameter was stretched vertically across the test section. The mean velocity and turbulence properties in the wake were measured with a hot-wire probe, supported from one of the side walls by a microscrew in a streamlined strut. The microscrew had a resolution of .01 mm. The hot-wire was connected with a DISA D00 system, comprising an anemometer (DISA 55 D01), auxiliary unit (DISA 55 D25), digital voltmeter (DISA 55 D30), a linearizer (DISA D10) and an RMS meter (DISA 55 D35) (see fig. 2). The hot-wires were 5μm diameter tungsten wires, with an active length of 1 mm and a total length of 4 to 5 mm, the remainder being copper plated. Normal wires placed perpendicular to the main stream and parallel to the cylinder axis, were used for the measurement of mean velocity and longitudinal turbulence intensity; 45° slant wires, placed in the x-y plane were used to measure the turbulent shear stress. All reported measurements were performed with one single probe of each kind.

The dynamic pressure was measured at the same x-station as the hot-wire, about 120-150 mm outside the wake, with a 3 mm diameter pitot tube, supported from the opposite wall by a streamlined strut. This set-up was chosen, firstly to have a constant influence of the pressure disturbance caused by the hot-wire probe support, if any, on the reference dynamic pressure, and, secondly to have a symmetric disturbance on the wake. The pitot tube was connected with a Texas Instruments quartz-tube manometer (see fig. 2).
The wake producing cylinder was kept at constant tensile stress by means of a screw during measurements with one kind of probe. But it was probably different during the normal and slant wire traverses, due to the fact that there was an interval of several months between the two sets of measurements. This probably had some influence on the final results, as will be discussed later on.

2.2. **Test procedure.**

Some preliminary spectrum measurements showed that a 20 kHz low-pass filter should be inserted to cut off amplifier noise produced by the anemometer, linearizer and auxiliary unit. Unfortunately there existed a second source of spurious signals: it was discovered that with the microscrew fully, or almost fully extended the prongs started vibrating with a frequency of about 2000 Hz, corresponding to their own frequency. Apparently this was induced by vibration of the strut. The vibration was worse for the normal wire probe, but also present for the slant wire probe, though to a lesser degree. The latter was due to the use of a different streamlined strut and a better probe design. The error caused by the vibration could not be filtered out. It was accepted because only the longitudinal turbulence data were seriously influenced, as will be shown later on.

The hot-wires were calibrated frequently against velocity (at least twice a day), at a position well outside the wake. It turned out that the calibration curve remained linear, although it might change appreciably during a wake traverse. Also there was no zero velocity output drift. Therefore, these calibrations were not used for processing the results of the measurements, but only to check the above mentioned linearity. The results of the measurements were processed using one point calibrations outside the wake, before and after the traverse as described below. The slant wire probes were also calibrated against yaw angle. For this the wire was placed in a plane parallel to the x-z-
plane, well outside the wake, as in this position the yaw angle could be varied easily by rotating the microscrew. It appeared that, if plotted in a certain way (see Appendix), the yaw calibration did not change, probably because the yaw sensitivity is mainly determined by the geometry of the probe.

With the normal wire, traverses were made at several distances behind the cylinder, midway between top and bottom walls, to measure the mean velocity and the longitudinal turbulence intensity. At station $x = 1000$ mm traverses were also made 50 and 100 mm above and below the usual z-wise measuring station. To measure the turbulent shear stress two traverses with the slant wire are needed, one with the wire at about $+45^\circ$ and the other at $-45^\circ$ with respect to the main stream. These two traverses were made immediately after each other. During a traverse the dynamic pressure was kept constant within .02 to .05% at a value near $q = 1450$ N/m$^2$, corresponding to a free stream velocity of about 50 m/sec.

First a hot-wire calibration point was taken well outside the wake, about 40 mm from the wake axis. At the largest x-station this was still some 7 mm outside the edge of the wake. Then for different y-stations the mean and r.m.s. output of the anemometer system were read. The total number of measuring points was 20 to 30. The free stream dynamic pressure, the barometric pressure and the tunnel temperature were also recorded several times during a traverse. At the end of the traverse another calibration point was taken. For the normal wire measurements this point was at the opposite side of the wake, while for the slant wire measurements the probe was always returned to the starting position where the first calibration point was taken. Of course the latter procedure is the best. The slant wires were rotated through $180^\circ$ after the first traverse, and a second traverse was made, following the same procedure.

Typically calibration changes during a traverse were about .1 to .2% of the output at the same dynamic pressure for the normal wire measurements, and about .5% for the slant wire measurements. The difference between the two sets of measurements was caused by a defective tunnel.
cooling system during the slant wire runs, resulting in larger temperature changes. Although these may seem quite slight changes, and have indeed little influence on the r.m.s. results, the influence on the non-dimensional velocity profile is considerable: a typical value of the maximum velocity defect, the scaling velocity, is about 6% of the free stream velocity. This means that a calibration change of .5% results in changes of 8% in the non-dimensional representation.

2.3. Processing of the data.

The hot-wire readings can be corrected for calibration drift, using the calibration points before and after the traverse. The appendix gives the details of the correction. In short it is assumed that the velocity calibration as well as the yaw calibration remain linear, and that changes in calibration occur linearly. For the normal wire measurements corrections were applied using a linear interpolation in y-values between the two calibration points. For the slant wire measurements all points were numbered consecutively and the readings were corrected using a linear interpolation in reading index, which corresponds roughly with time. The normal wire readings were thus reduced to values of $\bar{U}/U_\infty$ and $u'/U_\infty$, according to formulas (A.17) and (A.18) and the slant wire data to $\bar{U}/U_\infty$ and $\overline{uv}/U_\infty$ according to formulas (A.21) and (A.22).

The mean velocity data were then plotted as $\bar{U}/U_\infty$ versus $y'$ to obtain the values of the maximum velocity defect $U_o/U_\infty$, the half width to half depth $b$, and also the position of the maximum defect in the $y'$-coordinate. Then the velocity profiles were non-dimensionalized as:

$$(1 - \bar{U}/U_\infty) / (U_o/U_\infty)$$

and

$$y/b$$

Alternatively $U_o$ and $b$ were determined from the measured values of the momentum thickness $\theta$ and the displacement thickness $\delta^*$, assuming a universal velocity profile:
\( \frac{(U_\infty - \bar{U})}{U_\infty} = f(\eta) \), where \( \eta = \frac{y}{b} \)

Then

\[ U_\infty \delta^2 = b \frac{U_\infty}{U_\infty} (U_\infty a_1 - U_\infty a_2) \]

and

\[ U_\infty \delta^\infty = b U_\infty a_1 \]

where

\[ a_1 = \int_\infty \frac{f(\eta)}{\delta^\infty} d\eta \]

and

\[ a_2 = \int_\infty \frac{f^2(\eta)}{\delta^\infty} d\eta \]

\( U_\infty \) and \( b \) may be calculated from the above formulas. The values of the supposedly universal constants \( a_1 \) and \( a_2 \) depend somewhat on the particular choice of the velocity profile. Here the well-known exponential profile was chosen:

\[ f(\eta) = \exp(-\lambda \eta^2), \text{ where } \lambda = \log_e 2 \]

\( \delta^\infty \) and \( \theta \) were obtained by integrating the \( \bar{U}/U_\infty \) data, using the trapezoid rule. From plots of \( u'v'/U_\infty^2 \) versus \( y' \) the maximum value of \( |u'v'| \) was determined. This value, indicated by \( u'y' \), was used for non-dimensionalizing the shear stress data. The \( y' \)-coordinate of the measurements was reduced to \( y/b \), using the position of the wake centre and the half width to half depth \( b \) obtained from the slant wire mean velocity data. The longitudinal turbulence intensity data were non-dimensionalized in the same way, using \( u' \) from the shear stress data (evaluated from a faired curve through the measured points at the appropriate \( x/\theta_\infty \) value, as explained in the next section), and the \( b \) from the mean velocity data measured with the normal wire.
3. DISCUSSION OF RESULTS.

3.1. Mean velocity data.

The values of \( \frac{U_o}{U_\infty} \) and \( b \), measured with the normal and slant wire probes are given in figs. 3 and 4 respectively. In fig. 3 the results obtained from the momentum thickness \( \theta \) and the displacement thickness \( \delta^b \) are also plotted. For high values of \( x \) this procedure seems to underestimate \( b \) and overestimate \( U_o \) systematically. This is due to the values of \( a_1 \) and \( a_2 \) used in the calculation, which are based on the exponential profile. As will be seen from the mean velocity profiles in figs. 10 to 22 this profile is not correct near the edge of the wake. The correct profile yields slightly different values for \( a_1 \) and \( a_2 \), which would undoubtedly give better results. For low values of \( x \) the difference is reversed. Here the difference is due to the departure of the velocity profile from a universal curve. It could be maintained that the values of \( b \) and \( U_o \) derived from \( \theta \) and \( \delta^b \) are more suitable for comparison with a calculation method, as these fictitious values are a kind of weighed \( b \) and \( U_o \).

Comparing figs. 3 and 4 we notice a consistent difference of about 10% between the normal and slant wire results. This must be caused by a difference in cylinder drag coefficient for the two measurements. The existence of this difference can be concluded from figs. 5 and 6, where the development of the momentum thickness for the two measurements is plotted. In fig. 5, which shows the results of the normal wire, we observe that for \( x > 200 \text{ mm} \) \( \theta \) is essentially constant within 2%. If \( \theta_\infty \) is taken as the arithmetical average of all points for which \( x > 200 \text{ mm} \), we have \( \theta_\infty = 1.045 \). For \( x < 200 \text{ mm} \) a sharp rise to this value is seen. As the longitudinal pressure gradient is zero, this rise is probably mainly due to the presence of a transverse static pressure gradient and, for a smaller part, to the normal Reynolds stresses.

Fig. 6 gives the results of the slant wire probe. Firstly the region where \( \theta \) rises to its value at large \( x \) is much larger than in fig. 5,
secondly the scatter is much larger, and finally the values for large x are greater. At first it was thought that the larger region of 0-rise and the scatter were caused by the presence of a large wave-length bubble on one of the side wall plexiglass windows of the test section. As this was only noticed after the tests were finished, and the position of that particular window had been varied during the course of the measurements, the exact influence could not be determined. Later on the influence of the bubble on the pressure distribution was measured, but the disturbance would be responsible for a fall followed by a rise in 0 (all very slight). Although the varying position of the window could obscure this effect, it seems at least improbable that the pressure disturbance was the cause of the observed behaviour. Also it must be borne in mind that all mean velocity data measured with the slant wire are the result of combining two traverses, so these measurements are inherently less accurate than the normal wire measurements. This could be responsible for the larger scatter. However, it clearly cannot explain any systematic deviation.

Yet it can be observed that the momentum thickness for the slant wire measurements also reaches a more or less constant value. Now taking \( \theta_\infty \) as the arithmetical average of the points for which \( x > 800 \) mm we obtain \( \theta_\infty = 1.22 \) (i.e. about 17\% higher than the normal wire case). Except for the inexplicable point at \( x = 1800 \) mm the variation is about 3\%.

The difference in drag coefficient may be caused by a difference in Reynolds number or in tension of the cylinder wire. Now for the normal wire measurements the Reynolds number based on cylinder diameter and free stream velocity was \( R_d = 6700 \), slightly higher than for the slant wire tests: \( R_d = 6450 \). According to Schlichtings fig. 1.4 [9] this would cause the opposite effect. Remains the wire tension of the cylinder as a possible cause of the difference. As mentioned earlier the tension was kept constant during all runs with one kind of probe, but it was probably different for runs with different probes. A few measurements at \( x = 1000 \) mm with different tensions were made, but the results failed to show a consistent change with tension, although in
some cases differences of 3% in $\theta$ and $b$ were observed. Still it is conceivable that at certain discrete values of the cylinder wire tension small vibrations occur in the wire, causing differences in drag coefficient.

Whatever the cause of the difference in momentum thickness, the fact remains that the two sets of measurements cannot be compared directly. Now it may be argued that far from the wake producing object only the total force exerted on the flow is important and not the details of the flow in the neighbourhood of this object. It is therefore illogical to compare the two sets of measurements on cylinders with different drag coefficients directly at the same streamwise position.

As suggested by Townsend [10] an appropriate scaling length should be the momentum thickness, as this is a measure of the force exerted on the flow. The free stream velocity should then be the appropriate scaling velocity. This procedure has been followed in fig. 7, where $b/\theta_* \text{ and } U_o/U_* \text{ are plotted versus } x/\theta_*$ for the two sets of measurements. It is seen that in general this is quite successful. For large $x$ both sets of measurements show good agreement. In the range of $50 < x < 800$ there is still a difference, presumably due to the same causes as the slow rise in $\theta$ for the slant wire measurements.

The mean velocity and longitudinal turbulence intensity, measured with a normal wire at $x = 1000 \text{ mm}$ at different $z$-stations, are given in figs. 8 and 9. The results indicate a reasonable two-dimensionality of the flow. The mean velocity profiles measured with the normal wire at different $x$-stations, are presented in non-dimensional form in figs. 10 to 17. The mean velocity profiles obtained from the slant wire measurements are given in figs. 18 to 22. The well known exponential velocity profile is drawn in each figure for mutual comparison only, not to suggest that the measurements should follow this curve.

From $x = 200 \text{ mm}$ onwards all velocity distributions show a similarity, which near the tails differs slightly from the exponential profile, a well established fact [11,12]. For smaller values of $x$ the velocity profiles differ from the universal distribution, though not extremely.
From these results, especially the ones at large $x$, it may be concluded that the correction procedure for the hot-wire mean velocity measurements is successful. The calibration changes for the normal wire were about .1 to .2%, in some cases even .3 or almost .5%. The maximum velocity defect $U_0$ at the largest $x$-station is about 4% of the free stream velocity, which means that the correction is about 2 to 5, or sometimes 8 to 10% of the non-dimensional velocity. For the slant wire measurements the calibration drift was even larger, about .5% (with values up to 1% and even one case (at $x = 300$ mm) of 1.4%). This is about 10% (in extreme cases 15%) of the maximum velocity defect. Furthermore it must be borne in mind that the results for the slant wire are obtained by processing two traverses. The symmetry with respect to $y = 0$ in the final results indicates that the correction procedure was largely successful. In a few cases the apparent asymmetry may be explained by a bad choice of the centre position of the wake, rather than by a failure of the correction procedure. For comparison a corrected and an uncorrected measurement are shown together in fig. 23. A compilation of mean-velocity parameters may be found in tables 1 and 2.

3.2. Turbulence data.

In figs. 24 to 28 the results of the shear stress measurements are presented in non-dimensional form. In the figures the average of all measurements in the range $x > 200$ mm is indicated by a full line; this distribution is regarded as the universal curve. The results indicate that this way of non-dimensionalizing the shear stress distribution is indeed a valid one. The use of $U_0$ as a scaling velocity must lead to a less satisfactory similarity, as indicated in fig. 29, middle curve, where $u_t/U_0$ is plotted versus $x/\theta_\infty$. This shows a consistent change of this variable with distance. Only very far downstream a constant value would be reached, indicating self-preservation of the flow. The use of the half width to half depth $b$ of the mean velocity, is apparently
quite correct. If the turbulence properties can be described by one length scale, this scale has a constant ratio to the mean velocity width. Further indication of this is given in fig. 29, upper curve, where \( b_t/b \) is plotted versus \( x/\theta_0 \), \( b_t \) is the distance between maximum and minimum of the shear stress distribution. Although, due to the uncertainty of the results, the conclusion of a certain change with distance cannot be ruled out, the essential constancy of \( b_t/b \) seems rather more probable, in contrast with the \( u_t/U_0 \) change. The above conclusion was also reached by Narasimha and Prabhu [11].

In fig. 30 the change of \( u_t/U_\infty \) with \( x/\theta_\infty \) is plotted. The faired curve through the measuring points has been used as an interpolation curve for the determination of \( u_t/U_\infty \) to non-dimensionalize the longitudinal turbulence data.

The longitudinal turbulence intensities are given in figs. 31 to 38, non-dimensionalized with \( u_t \). The \( y \)-values were non-dimensionalized with the mean velocity width \( b \). These results are far from satisfactory. It must be borne in mind that these data were obtained with only a 20 kHz filter inserted in the circuit, and plotted without any further correction for possible noise. It is quite certain that, at least for the highest values of \( y/b \), especially at large \( x \)-positions (corresponding to the microscrew almost fully extended), vibrations of the prongs occurred, induced by vibrations of the whole strut. It must be assumed that the influence of these vibrations is present in all measurements, perhaps to a varying degree. The magnitude of \( u'/u_t \) at the tails also seems to indicate that some apparent intensity not caused by the wake turbulence plays an important role.

Incidentally, the same remarks could be made about the shear stress measurements, though a different streamlined strut and a different probe design gave less vibrations. Yet here too, high values of the intensity measured during a traverse were observed in the tails. However, if it is assumed that this apparent intensity is independent of the wake turbulence, it is uncorrelated with the turbulence and may be substractioned from the total signal. For the normal wire we do not know the magnitude of the apparent intensity but in the case of the
shear stress measurements they cancel automatically by the subtraction of two traverses, as can be seen from eq. (A.22).

In figs. 31 to 38 a curve is drawn representing the average of measurements at intermediate x-values, which may be regarded as the universal distribution. The deviation from this curve at large x, where similarity is also expected, can be explained as the influence of the (unknown) apparent intensity, which of course becomes increasingly important for larger x, where the turbulent intensity decreases. Fig. 29 gives the values of $u'_c/u_t$ where $u'_c$ is the centre line intensity. It seems that this parameter is essentially constant for $x > 200$ mm, although a slight rise is observed for large x, due to the deficiency of the measurements mentioned above.

A compilation of all major results is given in tables 1 and 2 for the slant wire and normal wire experiments respectively.
4. CONCLUSIONS.

The mean velocity profiles show similarity for \( x/\theta_\infty > 200 \), if plotted as \((U_\infty - \bar{U})/U_0\) vs. \( y/b \).

The turbulence properties \( u' \) and \( uv \) show similarity for \( x/\theta_\infty > 200 \), if non-dimensionalized with \( u'_c = \sqrt{\langle uv \rangle_{\text{max}}} \) and plotted versus \( y/b \). For \( u' \) this was somewhat obscured by the presence of spurious vibrations during the measurements.

This means that the velocity scales for the mean velocity and the turbulence field are different, while the length scale is the same.

It is essential that the mean flow measurements in small velocity defect wakes are corrected for small hot wire calibration changes. The correction procedure for mean velocity measurements described in the present report, is quite successful.

The measurement of the longitudinal turbulence intensity requires a more careful approach than was used here.

On the whole, a useful picture of the wake flow behind a circular cylinder has been obtained, suitable for comparison with a calculation method.
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APPENDIX.

The correction of linearized hot-wire signals for calibration drift.

We recall the experimentally established fact that the linearized hot-wire output $E$ remained linear with velocity when a calibration change occurred, and that there was no zero velocity output drift. This means:

$$E = \mathcal{L}(\phi, c) U$$

or

$$\frac{\partial E}{\partial U} = \mathcal{L}(\phi, c)$$

(A.1)

The variable $\phi$ denotes the angle between the hot-wire axis and the flow velocity vector in the $x$-$y$-plane (see fig. A1). In (A.1) $c$ is a variable along which change in calibration occurs. It may be interpreted as a change in fluid temperature or a change in time. In the present report it is interpreted either as a change in the $y'$-variable during a traverse (normal wire probe measurements) or a change in measuring point index (slant wire probe measurements).

Also we make use of an experimentally established yaw calibration property. It appeared that:

$$\frac{\partial E}{\partial \phi} = k_0 E_\phi (U, c)$$

(A.2)

Here $k_0$ is a constant (for a particular wire) and $E_\phi$ is defined in fig. A2; it may be interpreted as the output of the hot-wire system in zero cross flow.

In the following we shall also suppose that changes in calibration occur linearly in $c$, i.e.:

$$\frac{\partial E}{\partial c} = m(U, \phi)$$

(A.3)

In general we may write for the output of a hot-wire system in two-dimensional flow:

$$E = E + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\Delta U \partial / \partial U \right)_o + \Delta \phi \partial / \partial \phi \right)_o + \Delta C \partial / \partial c \right)_o \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \r
The index \( o \) denotes the state characterized by:

\[
U = U_o
\]

\[
\phi = \phi_o
\]

\[
c = c_o
\]

From the above equations (A.1), (A.2), (A.3) it is clear that:

\[
0 = \frac{\partial^2 E}{\partial U^2} = \frac{\partial^2 E}{\partial \phi^2} = \frac{\partial^2 E}{\partial c^2} = \frac{\partial^3 E}{\partial U^2 \partial \phi} = \text{etc.}
\]

We then find for eq. (A.4):

\[
E = E_o + (\frac{\partial E}{\partial U})_o \Delta U + (\frac{\partial E}{\partial \phi})_o \Delta \phi + (\frac{\partial E}{\partial c})_o \Delta c + (\frac{\partial^2 E}{\partial U \partial \phi})_o \Delta U \Delta \phi + (A.5)
\]

\[
+ (\frac{\partial^2 E}{\partial U \partial c})_o \Delta U \Delta c + (\frac{\partial^2 E}{\partial \phi \partial c})_o \Delta \phi \Delta c + (\frac{\partial^3 E}{\partial U \partial \phi \partial c})_o \Delta U \Delta \phi \Delta c
\]

From this we find, by putting \( \Delta \phi = 0 \):

\[
k_o \frac{E_o}{\phi_o} = k_o E_o + k_o \xi_o \Delta U + k_o m_o \Delta c + k_o \left(\frac{\partial^2 E}{\partial U \partial c}\right)_o \Delta U \Delta c \quad (A.6)
\]

where

\[
\xi_o = (\frac{\partial E}{\partial U})_o
\]

and

\[
m_o = (\frac{\partial E}{\partial c})_o
\]

By differentiating (A.5) with respect to \( \phi \):

\[
\frac{\partial E}{\partial \phi} = (\frac{\partial E}{\partial \phi})_o + (\frac{\partial^2 E}{\partial U \partial \phi})_o \Delta U + (\frac{\partial^2 E}{\partial \phi \partial c})_o \Delta c + (\frac{\partial^3 E}{\partial U \partial \phi \partial c})_o \Delta U \Delta \phi \Delta c \quad (A.7)
\]

According to eq. (A.2), the eqs. (A.6) and (A.7) should be equivalent. This implies:

\[
(\frac{\partial E}{\partial \phi})_o = k_o \xi_o
\]

\[
(\frac{\partial^2 E}{\partial E \partial \phi})_o = k_o \xi_o \quad (A.8)
\]
\[ (\partial^2 E/\partial \phi \partial c)_o = k_o m_o \]  \hspace{1cm} (A.8)

\[ (\partial^3 E/\partial U \partial \phi \partial c)_o = k_o (\partial^2 E/\partial U \partial c)_o = k_o r_o \]

where

\[ r_o = (\partial^2 E/\partial U \partial c)_o \]

By differentiating (A.5) with respect to \( \Delta U \) and substituting (A.8) we find:

\[ \partial E/\partial U = \ell = \ell_o + k_o \ell_o \Delta \phi + r_o \Delta c + k_o r_o \Delta \phi \Delta c \]  \hspace{1cm} (A.9)

By differentiating (A.5) with respect to \( c \):

\[ m = m_o + r_o \Delta U + k_o m_o \Delta \phi + k_o r_o \Delta U \Delta \phi \]  \hspace{1cm} (A.10)

We now recall the calibration procedure used during the measurements. The yaw calibration (A.2) was established independent of a traverse and \( k_o \) can be regarded as a known quantity. For the velocity calibration a calibration point before and after a traverse was taken at a station outside the wake, where \( U = U_\infty, \phi = \phi_o \) and \( c = c_o \) and \( c_n \) respectively. We then have, denoting the two calibrations with indices \( o \) and \( n \) respectively:

1st calibration point:

\[ U = U_\infty, \phi = \phi_o, c = c_o, E = E_o, \ell_o = E_o/U_\infty \]

2nd calibration point:

\[ U = U_\infty, \phi = \phi_o, c = c_n, E = E_n, \ell_n = E_n/U_\infty \]

Using eq. (A.9) we find:

\[ \ell_n = \ell_o + r_o \frac{(c_n - c_o)}{c_n} \quad \text{or} \quad r_o = \frac{(\ell_n - \ell_o)}{(c_n - c_o)} \]  \hspace{1cm} (A.11)

and using eq. (A.5):

\[ E_n = E_o + m_o \frac{c_n - c_o}{c_n} \quad \text{or} \quad m_o = \frac{(E_n - E_o)}{(c_n - c_o)} \]  \hspace{1cm} (A.12)
From this it also follows:

\[ m_0 = U_\infty r_0 \]  \hspace{1cm} (A.13)

We conclude that \( k_0, l_0, m_0 \) and \( r_0 \) are all known. So the complete expression for the hot-wire anemometer output becomes:

\[
E = E_0 + l_0 \Delta U + k_0 \Delta \phi + m_0 \Delta c + k_0 l_0 \Delta U \Delta \phi + \\
+ r_0 \Delta U \Delta c + k_0 m_0 \Delta \phi \Delta c + k_0 r_0 \Delta U \Delta \phi \Delta c
\]  \hspace{1cm} (A.14)

We now put:

\[ \Delta U = U - U_\infty \]

For small \( \Delta \phi \) we may write:

\[ V = UD \phi \]

Also we recall \( E_0 = l_0 U_\infty \).

Substituting this into (A.14) and using (A.13) we find after some manipulation:

\[
E = (l_0 + r_0 \Delta c)(U + k_0 V)
\]  \hspace{1cm} (A.15)

Special cases.

a. normal wire.

For a normal wire \( k_0 = 0 \). We then have:

\[
E = (l_0 + r_0 \Delta c)U
\]  \hspace{1cm} (A.16)

For a turbulent flow we put:

\[
E = \bar{E} + \epsilon, \hspace{0.5cm} \text{and} \hspace{0.5cm} U = \bar{U} + u
\]

After taking the average we find:

\[
\bar{E} = (l_0 + r_0 \Delta c)\bar{U}
\]  \hspace{1cm} (A.17)

Subtracting (A.17) from (A.16):

\[
e = (l_0 + r_0 \Delta c)u
\]
or:
\[
\bar{e}^2 = (\ell_o + r_o \Delta c)^2 \frac{u^2}{\bar{v}} \tag{A.18}
\]

(A.17) and (A.18) are the correction formulas for normal hot-wires with calibration drift.

b. Identical slant wires.

Again separating the mean and fluctuating part we find from (A.15):
\[
\bar{E} = (\ell_o + r_o \Delta c) (\bar{U} + k_o \bar{V})
\]
\[
\bar{e}^2 = (\ell_o + r_o \Delta c)^2 \left( \frac{u^2}{\bar{u}} + 2k_o \bar{u} v + k_o^2 \frac{v^2}{\bar{v}} \right)
\]

It can be seen easily that the turbulent normal and shear stresses can be determined from two identical wires with the same \(\ell_o, r_o\), but \(k_o\) of opposite sign (i.e. \(\phi_o\) of opposite sign).

\[
e_1 = (\ell_o + r_o \Delta c)(u + k_o v)
\]
\[
e_2 = (\ell_o + r_o \Delta c)(u - k_o v)
\]

Then:
\[
\frac{(e_1 + e_2)^2}{(e_1 - e_2)^2} = 2(\ell_o + r_o \Delta c)^2 \frac{u^2}{\bar{v}}
\]
\[
\frac{(e_1 - e_2)^2}{e_1^2 - e_2^2} = 2k_o^2 (\ell_o + r_o \Delta c)^2 \frac{v^2}{\bar{u} \bar{v}} \tag{A.19}
\]

C. Non-identical wires.

We now consider the case of the present shear stress measurements: two traverses with different \(\ell_o, m_o\), perhaps different \(U_{\infty}\), and \(k_o\) of opposite sign. We then have:
\[
\bar{E}_1 = (\ell_{o_1} + r_{o_1} \Delta c_1) (\bar{U}_{1} + k_o \bar{V}_1) \tag{A.20}
\]
\[ E_2 = (l_{o2} + r_{o2} \Delta c_2)(u_{2} - k_o \bar{v}_2) \]

\[ e_1^2 = (l_{o1} + r_{o1} \Delta c_1)^2 (u_{1} + k_o \bar{v}_1^2 + 2k_o \bar{u} \bar{v}_1) \]

\[ e_2^2 = (l_{o2} + r_{o2} \Delta c_2)^2 (u_{2} + k_o \bar{v}_2^2 - 2k_o \bar{u} \bar{v}_2) \]  \( \text{(A.20)} \)

If we now suppose that:

\[ \bar{U}_1/U_{\infty 2} = \bar{U}_2/U_{\infty 2}, \quad \bar{V}_1/U_{\infty 1} = v_{1}/U_{\infty 1}, \quad u_{1}/U_{\infty 1}^2 = u_{2}/U_{\infty 2}^2, \]

\[ v_{1}/U_{\infty 1}^2 = v_{2}/U_{\infty 2}^2, \quad u_{1}/U_{\infty 1}^2 = u_{2}/U_{\infty 2}^2, \]

which is correct for not too large a variation of \( U_{\infty} \), we find from (A.20):

\[ \bar{U}/U_{\infty} = E_1(2U_{\infty} (l_{o1} + r_{o1} \Delta c_1)) + E_2/(2U_{\infty} (l_{o2} + r_{o2} \Delta c_2)) \]  \( \text{(A.21)} \)

\[ \bar{u}v/U_{\infty}^2 = \frac{1}{4k_o} (e_1^2/(U_{\infty 1}^2 (l_{o1} + r_{o1} \Delta c_1)^2) - e_2^2/(U_{\infty 2}^2 (l_{o2} + r_{o2} \Delta c_2)^2)) \]  \( \text{(A.22)} \)

For the present measurements \( l_o \) was about .2 V sec/m, while \( r_o \Delta c \) was about .5% of this. From the yaw calibration the value for \( k_o \) of \( k_o = 1.07 \) was found with an uncertainty due to the scatter in the experimental points of about 3%. 
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<th>b /θ∞</th>
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1) from velocity profile plot

2) from δ∞, θ and exponential profile

Table 1. Compilation of slant wire data.
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1) from velocity profile plot
2) from δ*, θ and exponential profile

Table 2. Compilation of normal wire data.
Fig. 1. Definition of $U_\infty$ and $b$

Fig. 2. Experimental set-up
Fig. 7. Non-dimensional comparison of mean velocity profile parameters: normal and slant wires.
Fig. 8. Mean velocity profile at x=1000 mm, at different z-stations.

Fig. 9. Longitudinal turbulence profile at x=1000 mm, at different z-stations.
Fig. 10. Non-dimensional velocity profiles; normal wire.

Fig. 11. Non-dimensional velocity profiles; normal wire.
Fig. 12. Non-dimensional velocity profiles; normal wire.

Fig. 13. Non-dimensional velocity profiles; normal wire.
Fig. 14. Non-dimensional velocity profiles; normal wire.

Fig. 15. Non-dimensional velocity profiles; normal wire.
Fig. 16. Non-dimensional velocity profiles; normal wire.

Fig. 17. Non-dimensional velocity profiles; normal wire.
Fig. 18. Non-dimensional velocity profiles; slant wire.

Fig. 19. Non-dimensional velocity profiles; slant wire.
Fig. 20. Non-dimensional velocity profiles; slant wire.

Fig. 21. Non-dimensional velocity profiles; slant wire.
Fig. 22. Non-dimensional velocity profiles; slant wire.
Fig. 23. The influence of the correction for calibration drift.
From two traverses with slant wire at x = 1600 mm.
First traverse: total calibration drift of 0.52%.
Second traverse: total calibration drift of 0.26%.
Fig. 24. Non-dimensional shear stress profiles.

Fig. 25. Non-dimensional shear stress profiles.
Fig. 26. Non-dimensional shear stress profiles.

Fig. 27. Non-dimensional shear stress profiles.
Fig. 28. Non-dimensional shear stress profiles.
Fig. 29. The development of various ratios of turbulence and mean velocity scales.
Fig. 30. The development of the turbulence velocity scale.
Fig. 31. Non-dimensional turbulence intensity profiles.

Fig. 32. Non-dimensional turbulence intensity profiles.
Fig. 33. Non-dimensional turbulence intensity profiles.

Fig. 34. Non-dimensional turbulence intensity profiles.
Fig. 35. Non-dimensional turbulence intensity profiles.

Fig. 36. Non-dimensional turbulence intensity profiles.
Fig. 37. Non-dimensional turbulence intensity profiles.

Fig. 39. Non-dimensional turbulence intensity profiles.
Fig. A1. Definition of angle between hot-wire and flow direction

Fig. A2. Definitions of $\phi_o$ and $E\phi_o$, from the calibration curves of a slant wire in two positions.