PIV study of vortex breakdown in low- and high-swirl flames in a model combustor

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ABSTRACT

The present work is devoted to a comparative study of low- and high-swirl flows in a model lean combustor. Lean methane-air mixture with equivalence ratio of 0.6 burned at atmospheric pressure. A high-repetition stereoscopic PIV system was used for the velocity measurements both in the reacting and non-reacting flows. During analysis of the velocity data sets, the emphasis was put on application of statistical tools to build a reduced model of flows as stochastic dynamical systems. Proper orthogonal decomposition and dynamic mode decomposition routines were used in the study. It was observed that despite the combustion significantly affected the time-averaged characteristics of the strongly swirling flow, the dynamics of both flows were associated with a global helical instability mode, which corresponded to a strong precession of the vortex core. In contrast, velocity fluctuations above the nozzle exit of the studied low-swirl flows were associated with local instability modes. In particular, this result indicates that the low-swirl flame should be more sensitive to external flow perturbations for combustion control.

INTRODUCTION

Flow swirling can is used for a reliable stabilisation of jet-flames at rather adverse conditions for combustion, e.g., ultra lean mixtures. In particular, lean combustion regimes are known to be efficient to achieve a low level of NOx and CO emission from gas-turbine combustors [1]. However, lean flames are rather sensitive to instabilities, resulting in thermo-acoustic resonance and blow-off phenomena. Significant efforts have been made to avoid these adverse effects. For example, periodical perturbations of the mixture flow rate can be used as an active way to control the combustion process. However, as many studies have shown, the response of the swirling flames to periodic forcing appears to be strongly non-linear due to a number of mechanisms (e.g., [2]): flame stretching and extinction, local variations of equivalence ratio and temperature, variations of boundary conditions. In particular, for a swirling jet flow, nonlinear dynamics of the flow itself have a significant impact on the flame kinematics. Complementary method to reduce NOx and CO emission from gas-turbine combustors is to use a low-swirl injector [3]. In contrast to commonly used high-swirl flow configurations, in which central recirculation zone (RZ) provides stabilisation of the flame, no RZ is present in case of low swirl and less pollutants are emitted. At the same time, low-swirl flames manifest similar stability characteristics, as in case of high swirl. Thus, experimental data on flow dynamics in swirl combustors is necessary for deeper understanding the flame behaviour and for designing more efficient combustion devices. Evidently, particle image velocimetry (PIV) is an essential tool to retrieve information about role of fluid dynamics in stabilization of swirling flames. For example, [4] have shown that precessing vortex core (PVC) promoted stability of strongly swirling flame via a high turbulent mixing rate and enlarged flame surface.

It is recognized that helical waves dominate (e.g., [5], [6]) shear layer of a jet under sufficient swirl intensity. A further increase in the swirl rate leads to a breakdown of the swirling jets’ vortex core ([7], [8], [9]). Remarkable that flow structure of strongly swirling jets with bubble-type vortex breakdown (VB) and precession of the vortex core manifests some common features, even for rather different nozzle geometries. Based on PIV measurements and numerical simulations, the authors of [10], [11] reported that precession of the vortex core in a strongly swirling jet is the result of global helical instability mode $|m| = 1$ growth in an absolutely unstable jet flow with an initially axisymmetric RZ, whereas the RZ appears due to centrifugally unstable swirling jet column under a sudden expansion. According to [11], two co-rotating spirals were present in the outer and inner mixing layers of the strongly swirling jet after development of the bubble-type VB. Based on phase-averaging in a turbulent strongly swirling jet Cala et al. [12] determined three-dimensional shape of coherent structures in the flow as a pair of secondary helical vortices induced by the PVC. One secondary vortex was located inside the vortex core (corresponded to spiral shape of the main vortex), while the other one was induced in the outer mixing layer. Legrand et al. [13] used correlation coefficients of proper orthogonal decomposition (POD, [14]) modes with PIV snapshots to reconstruct a phase portrait of quasi-periodic PVC and
coherent helical vortices in the outer and inner mixing layer of turbulent swirling jet. In [15] the first two modes of POD of PIV data were used for a low-order reconstruction of the coherent structures associated with PVC in a strongly swirling jet. The authors of [15] demonstrated remarkable coincidence of the low-order reconstruction with a shape of the absolutely unstable mode of the flow. POD used in [4] to study impact of PVC on swirling flame dynamics in a model gas turbine combustor by means of time-resolved POD and OH* planar laser-induced fluorescence. Based on PIV data for fuel-rich lifted swirling flame, Alekseenko et al. [16] demonstrated that outer helical vortex can be replaced by periodic ring-like vortices, when the external forcing of the flow rate is applied with amplitude about intensity of the reverse flow. Consequently, combustion efficiency can be increased by entraining more ambient air upstream the flame base. In the recent paper [17], the initiation of VB and appearance of PVC in a swirling water jet was studied by gradually increasing swirl rate. The obtained time-resolved PIV data revealed that with increase of the swirl the RZ appeared intermittently in the instantaneous velocity fields, and it could not be detected in the time-averaged velocity distribution. Further increase of the swirl rate resulted in permanent presence of VB; and then, above a critical swirl rate, the swirling jet flow became absolutely unstable to a global helical mode, corresponding to the precession of the flow.

The present work is devoted to a comparative study of low- and high-swirl lean flames in a model combustor chamber. A high-repetition stereoscopic PIV system was used to measure ensembles of the instantaneous velocity fields during few seconds. A linear algebra tools were applied to retrieve information about global self-sustaining oscillations in the flows, viz., POD and dynamic mode decomposition (DMD, [18]) routines were used in the study.

**EXPERIMENTAL SETUP AND DATA PROCESSING**

The measurements were carried out in a combustion rig consisted of a transparent combustor chamber, swirling nozzle, flow seeding device, premixing pipe and section for the air and methane flow rate control. The model chamber represented a cylinder (with 120 mm height and inner diameter of 77 mm) made of fused silica and placed on a flat pedestal with a contraction nozzle inside (see Figure 1a). A vane swirler inside the nozzle (exit diameter \(d = 15\) mm) could be changed to vary the swirl rate. By using two swirlers with different inclination angle of blades, the swirl rate \(S\) (e.g., see [7] for the definition) was 0.41 and 1.0. The Reynolds number \(Re_{air}\) (based on \(d\) and on the mean flowrate and viscosity of the air) was fixed as 5 000. The equivalence ratio \(\Phi\) of the methane-air mixture was 0.6. Both for the cases of strong and weak swirl (\(S = 1.0 \) and 0.41, respectively), V-shape inverted flames were formed. In the former case, apex of the inverted cone was located inside the nozzle, above the central body of the swirler. In the latter case, the lifted inverted flame was significantly above the nozzle exit, and its surface undergone significant turbulent fluctuations. In order to provide PIV measurements, the flows were seeded by TiO\(_2\) particles with the average diameter of one micrometer. The seeding device instantly introduced solid particles to the air flow by using a stirring rod. This was done to achieve a steady combustion regime before the high-repetition PIV measurements. After the particles were introduced to the flow, PIV images were acquired before the fused silica walls of the chamber became non-transparent. The PIV system composed of a double-pulsed Nd:YLF Pegasus PIV laser and a pair of PCO 1200HS CMOS cameras was running at approximately 770 Hz frequency. A laser sheet, formed by the system of lenses, passed thru the central (horizontal) plane of the flows and had a thickness of 0.8 mm in the measurement section. The cameras were equipped with narrow-bandwidth optical filters admitting the light from the laser and suppressing the radiation of the flame. The system was operated by a computer with "ActualFlow" software. Background was removed from each captured 10 bit PIV images, and they were processed by an iterative cross-correlation algorithm with a continuous image shifting and deformation [19]. The size of a final interrogation area was 32×32 pixels. 75% overlap rate between the interrogation areas was used to increase robustness of the validation procedures (e.g., 5×5 moving average filter) applied between iterations of the cross-correlation algorithm. The calculated instantaneous velocity vectors were validated using a signal-to-noise criterion for cross-correlation maxima and with an adaptive median filter proposed by [20]. Stereo calibration was performed using the multi-level calibration target and a 3rd-order polynomial transform. In addition, to minimise the stereo calibration error, an iterative correction procedure of possible misalignment of the laser sheet and target plane was applied [21]. For each flow case, 900 instantaneous three-component velocity fields were estimated.
For analysis of the spatial structure of coherent vortex structures in the flows, POD and DMD were applied to the ensembles of the instantaneous velocity fields. Both methods provide representation of each velocity field as a finite series of products of spatial functions with time-dependent coefficients. POD for an ensemble $V_1^N = \{ u^1, u^2, u^3, \ldots, u^N \}$ of $N$ instantaneous velocity fields $u^i$ (snapshots) provides:

$$u^i(x) = \sum_{n=1}^{N} a_n^i \varphi_n(x),$$

where $\varphi_n$ are the orthonormal basis functions (viz., $\langle \varphi_n \varphi_m \rangle = \delta_{nm}$, where $\delta_{nm}$ is the Kronecker delta, and $\langle \rangle$ is the spatial averaging over the domain considered) with "temporal" (correlation) coefficients $a_n^i$. The snapshot POD [22] is based on determining optimal orthonormal basis functions $\varphi_n$ that are the most correlated to the velocity fields. The number of snapshots $N$ should be large enough for fulfillment of ergodic hypothesis. The variational problem for the optimal basis is solved as a solution of a Fredholm integral equation of the second kind, where the kernel of the integral operator is a cross-correlation function between the instantaneous velocity fields.

The POD modes are defined as

$$\varphi_n(x) = \sum_{n=1}^{N} a_n^i u^i(x)$$

and sorted according to the corresponding eigenvalues $\lambda_n$. Thus, sums of the correlation coefficients $\sum (a_n^i)^2 = \lambda_n$ are subsequently maximized for $n = 0, n = 1$ and so on. The least-squares problem for POD

$$\frac{1}{N} \sum_{m=1}^{N} \langle u^m u^m \rangle \alpha_m = \lambda \alpha_n,$$

can be solved via singular value decomposition of $N \times M$ matrix $V_1^N$: $V_1^N = U \Lambda W^H$, where columns of matrix $U$ correspond to the POD modes [18], and $A$ is the diagonal matrix with elements equal to square root of $\lambda_n$.

DMD can be interpreted as decomposition of subsequent velocity fields in $V_1^N$ into a number of spatial basis functions $\varphi_i(x)$ with amplitudes $b_n(t)$ oscillating as complex Fourier harmonic $b_n(t) = \exp(i \omega_n t)$:

$$u(x, t) = \sum_{n=1}^{N} b_n(t) \varphi_n(x) = \sum_{n=1}^{N} e^{i \omega_n t} \varphi_n(x),$$

where $\omega_n$ are the complex values, with $\text{Re} \omega_n$ representing the growth rate of harmonic with frequency of $\text{Im} \omega_n$. If the time step $\Delta t$ between the snapshots is sufficiently small, the problem is solved as spectral decomposition of matrix $B$ defined as:

$$u^{i+1} = Bu^i = e^{i \Delta t} u^i,$$

This can be performed by using a modified Arnoldi method of Schmid [18]. Instead of finding $B$, that connects two reduced ensembles of the snapshots as $V_2^N = BV_1^{N-1}$, the last snapshot $u^N$ is represented as the linear combination of the previous ones:

$$u^N = b_1 u^1 + b_2 u^2 + \ldots + b_{N-1} u^{N-1} + r = V_1^{N-1} b + r.$$
Consequently, one can obtain matrix $S$: $V_2^N = V_1^{N-1}S + \mathbf{r}e^{N-1}$ (where $\mathbf{e}^{N-1}$ is the unit vector $[0,0,0,\ldots,1]$), with eigenvalues and eigenvectors of $S$ approximating those of $B$. The last-squares problem

$$\min_{\mathbf{u}} \left\| u^N(x) - \sum_{n=1}^{N-1} b_n u^n(x) \right\|_2^2$$

can be solved via QR decomposition $V_1^{N-1} = QR \Rightarrow S = R^{-1}Q^H V_2^N$ [23]. Alternatively the low-dimensional system matrix $\hat{S} = U^H B U$ can be obtained via singular decomposition of $V_1^{N-1} = U \Lambda W^H \Rightarrow \hat{S} = U \Lambda^2 W^{-1}$ [18]. Eigenvectors and eigenvalues of $\hat{S}$ also approximate those of $B$, while the spectral decomposition of $\hat{S}$ is more robust in comparison to decomposition of $S$. Dynamic modes $\Phi_i = [\phi_1, \phi_2, \phi_3, \ldots, \phi_{N-1}]$ are obtained as $\phi_i = U y_i$, and growth rates and frequencies are found from $\omega_i = \frac{\ln(\mu_i)}{\Delta t}$. Since the stationary turbulent flows are investigated in the present study, the obtained DMD modes correspond to $\Re(\omega_n) \approx 0$. Thus, the spectra of the DMD modes are defined as $L_2$ norm of $\phi_n$ (projected on ten the most coherent modes of the POD basis) versus frequency $\Im(\omega_n)$. The spectra are displayed for $\Im(\omega_n) > 0$ only, because they are symmetric relatively to zero since the decomposed data are real.

**RESULTS**

Figure 2 shows the time-averaged velocity field and squared intensity of the axial velocity fluctuations in the non-reacting flows. For the case of strong swirl, viz., for $S = 1.0$, a central RZ (visualized by a red heavy line) is present around the jet axis. For the case of weak swirl, expansion of the jet core causes stagnation of the axial flow around the jet axis, but the mean axial velocity remains positive. In both cases of swirl, an outer RZ is present between the expanding jet and corner of the cylindrical chamber. As the axial component of turbulent kinetic energy demonstrates, velocity fluctuations in the case $S = 1.0$ are significantly greater than in the case of $S = 0.41$. For the latter swirl rate, the velocity fluctuations arise both in the outer shear layer of jet flow and in the inner shear layer due to the axial velocity stagnation.

Figure 3 shows DMD and POD spectra for these flows. A pronounced peak for 223 Hz is present in the DMD spectrum in the case of $S = 1.0$. At the same time, the first two POD modes contain almost 17% of turbulent kinetic energy in the measurement domain, when the third POD mode represents only about 3% of the energy. The Lissajous figure for this case of swirl clearly shows that the first two POD modes are statistically correlated. Moreover, variation in time of their correlation coefficients (see the example for the first mode in Figure 3) was associated with amplitude-modulated harmonics, shifted on phase of $\pi/2$. As can be seen from Figure 4, the spatial distributions of the real and imaginary parts of the DMD mode for 223 Hz remarkably coincide with the first and second POD modes, respectively (by taking into account phase shift between results of the decompositions). Thus, both POD and DMD extracted the flow dynamics associated with limit cycle oscillations due to a global instability mode, corresponding to the precession of the vortex core (see Figure 12a for the low-order reconstruction of the coherent structures corresponding to the PVC).
For the case of \( S = 0.41 \), no dedicated modes can be distinguished except of 9 Hz (not observed in an open configuration of the swirling jet flow). The POD spectrum corresponds to a gradual decrease of the energy of the modes. The spatial distributions of the first four POD modes are shown in Figure 5. The first three modes appear to be quite similar with respect to rotation around the jet axis, and sufficient asymmetry with respect to \( z \) axis appears only in the fourth mode for the region above stagnation of the axial velocity. The second mode is associated primarily with coherent fluctuations of the axial velocity in the stagnation region. Analysis of the correlation coefficient for this mode support conclusion of [17] on intermittent formation of central RZ in a weakly swirling jet flow (without pronounced breakdown of the vortex core). The mode also reveals weak coherence of these near-axis oscillations with long-wave fluctuations in the corner RZ. Depicted in Figure 6 spatial distributions of the real and imaginary part of the DMD mode for 9 Hz have similar distributions as the first and third POD modes, respectively. These modes are the most likely associated with low-frequency variations of jet's opening angle and intensity of flow in the corner RZ. Thus, the modes demonstrate that corner RZ and expanding swirling jet interacted in an unsteady manner. Noteworthy, comparatively good axial symmetry of the modes indicate that this interaction was not associated with precession of the jet (for example, procession of reattachment point of weakly swirling jet after a sudden expansion was reported in [24] and [25]).

The distributions of the time-averaged velocity and squared axial velocity fluctuations for the lean methane/air flames (see direct images in Figure 1b and c) are plotted in Figure 7. A tiny zone of negative axial velocity can be detected for the weakly swirling jet flow with combustion; however, there are no substantial changes in distributions of the data between the reacting and non-reacting flows. For the case of strong swirl, combustion changed the flow pattern dramatically. The annular jet flow took shape of a cone, and the expansion rate of the jet became significantly greater. Thus, the central RZ represents a cone, and the reverse flow inside is less intensive and weakly turbulent. The combustion also resulted in more intensive flow in the corner RZ (in contrast to the flame effect on the flow with the
weak swirl). Additionally, one can observe that the strong velocity fluctuations took place only in the region of the annular jet.

![Figure 5](image1.png)  
**Figure 5** POD modes in a weakly swirling non-reacting flow.

![Figure 6](image2.png)  
**Figure 6** DMD mode in a weakly swirling non-reacting flow. The shown DMD mode for $\text{Im}\omega \approx 9$ Hz is normalized to allow comparison with POD modes.

![Figure 7](image3.png)  
**Figure 7** Time-averaged distributions of velocity (top) and squared axial velocity fluctuations (bottom) in a swirling methane/air flame. Left: strong swirl; right: weak swirl. Red heavy line visualizes central recirculation zone.

Despite these significant changes in the time-averaged flow structure, the POD and DMD spectrum for the strongly swirling flame (Figure 8) show similarity of the flow dynamics to the case of strongly swirling flow without combustion. A strong peak for 257 Hz is present in the DMD spectrum, and amplitudes of the first two POD modes are clearly discriminative with respect to the rest ones. Despite values of the correlation coefficients of the first two POD modes do not form a clear circle in the Lissajous plot, their evolutions in time represent harmonic oscillations with $\pi/2$ shift (but owing to the combustion, their amplitudes became more modulated). As in the case of the non-reacting strongly swirling flow, the spatial distributions of dominant DMD and POD modes are very similar.

For the case of the weak swirl, POD spectrum shows one mode with clearly distinguishable amplitude in terms of the velocity fluctuations (the eigenvalues for the cases with combustion do not correspond to turbulent kinetic energy, since the density is not constant). The first POD mode, presented in Figure 10 (the second POD mode is also quite similar), show that fluctuations of the axial velocity near the stagnation region are more intensive and in overall are correlated with variations of opening angle of the jet flow. Consequently, the combustion process intensified intermittent
appearance of the central RZ in the weakly swirling flow, and a tiny zone of negative axial velocity is present at the jet axis in the time-averaged velocity field. The third POD mode represents significant radial and azimuthal fluctuations in the region of the axial velocity stagnation and apparently corresponds to local development of convective helical instabilities due to the shear around the stagnation region.

**Figure 8** DMD and POD spectra (top), and POD correlation coefficients (bottom) in a swirling methane/air flame.

**Figure 9** POD (top) and DMD (bottom) modes in a strongly swirling methane/air flame. The shown DMD modes for \( \text{Im} \omega_i = 257 \text{ Hz} \) are normalized to allow comparison with POD modes.

**Figure 10** POD modes in a weakly swirling methane/air flame.
The plotted in Figure 11 distributions of the real and imaginary parts of the DMD mode for 9 Hz, show that oscillations at this frequency took place mainly for $r/d > 0.5$, and corresponded to variations of the reverse flow intensity of the corner RZ. As in the case of non-reacting weakly swirling flow, no significant asymmetry can be detected in this mode.

Figure 12 compares phase-averaged spatial distributions of the velocity and iso-surface of positive "Q" criterion [26], obtained from the low-order reconstruction based on the first two POD modes: $U + (2\lambda_1)^{1/2}\sin(\theta)\phi_1 + (2\lambda_2)^{1/2}\cos(\theta)\phi_2$, where $\theta$ is the azimuthal angle. Obviously, in terms of the phase jitter effect this way of phase averaging is less precise than the procedure of Legrand et al. [27]. Nevertheless, the iso-surfaces provide information about 3D shape of coherent structures in the flow and about region of their phase-averaged location. In both cases, two helical vortices are detected. One vortex is located in the outer mixing layer between the jet flow and corner RZ, while the second one is in the inner mixing layer between the jet and the reverse flow (central RZ). Similar phase-averaged flow structure was reported in [16] for the unconfined jet flow produced by the same type of nozzle (without pedestal). The helical vortex in the inner layer corresponds to the PVC, winding around $z$ axis. For the case with combustion the inner and outer helical vortices are located on opposite sites of V-shaped flame surface and the eccentricity of their trajectory during the precessing motion of the flow is significantly greater than in the non-reacting case. Significant deflection of PVC from the axis for the lean flame in a model combustor chamber was also demonstrated in [4].

CONCLUSIONS

A high-repetition stereoscopic PIV system was used to measure sets of subsequent velocity fields in strongly- and weakly swirling lean methane/air flame in a model combustor chamber. Non-reacting flows were also studied, when the gas flow was terminated. The PIV data were post-processed by POD and DMD routines to retrieve information about the flow dynamics and characteristics of the most powerful coherent structures. It is observed, that despite the combustion significantly affected time-averaged characteristics of the strongly swirling flow, dynamics of both non-reacting and reacting flows were associated with a global helical instability mode, which corresponds to a precession of the swirling flow. Both POD and DMD analysis reveal a pair the most powerful modes (for the DMD analysis this couple corresponds to the modes with $\text{Im}\omega = \pm f_p$, where $f_p$ is the frequency of the precession), and the pairs of the modes are similar, taking into account phase shift between the results of the decompositions. From analysis of the instantaneous velocity fields (not shown in the paper) and results of the decompositions (distributions of POD modes and evolution of their correlation coefficients in time) for the weakly swirling flows, it is suggested that no stable central RZ was present, and a small regions of reverse flows appeared intermittently. Thus, no RZ core be detected in
the time-averaged velocity distributions. However, a region of flow deceleration took place around the jet axis after the expansion of the weakly swirling jets. Thus, the inner shear layer was formed around this region, where significant velocity perturbations took place. It is concluded that these fluctuations corresponds to development of local convective instability and formation of helical eddies. Despite global low-frequency oscillations in the weakly swirling flows were likely induced by a corner RZ, they are sufficiently weak in comparison to the velocity fluctuations induced by the PVC in the strongly swirling flows. Consequently, the absence of strong precession in the low-swirl flame is in agreement with scenario of VB by [17] and suggests that such flames should be more sensitive to an external active control.

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