The influence of the deorbit-burn manoeuvre on the footprint of a re-entry capsule

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Title: The influence of the deorbit-burn manoeuvre on the footprint of a re-entry capsule

Author(s): E. Mooij

Abstract: The implementation of the deorbit-burn manoeuvre in the entry simulation software START is presented. Two models are derived. The first model assumes an impulsive $\Delta V$, in which the (inertial) velocity change is vectorially added to the velocity of the spacecraft. The second model takes the finite time of the manoeuvre into account and the manoeuvre is a thrusted one. The two models are evaluated and the differences in flight time, entry conditions and entry path are shown to be substantial. Finally, the impulsive model is used to do a sensitivity analysis for the Assured Crew Return Vehicle (ACRV). The influence of initial height, magnitude and direction of the burn on the footprint of the vehicle is shown and discussed.

Keyword(s): deorbit burn, re-entry, footprint, START
<table>
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<th>Issue No.</th>
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<th>Pages Changed/ Added/ Deleted</th>
<th>Topics Introduced</th>
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</tbody>
</table>
Table of contents.

Notations. .......................................................... iv

1. Introduction. .................................................... 1

2. The flight-dynamics software START. .......................... 2
   2.1. START Version 2.0. ........................................ 2
   2.2. The equations of translational motion. .................. 5

3. The deorbit-burn manoeuvre. ................................... 9
   3.1. Introduction. ............................................. 9
   3.2. Reference frames and frame transformations. .......... 9
   3.3. Transformations between different sets of state variables. 12
       3.3.1. Cartesian and spherical position and velocity. 12
       3.3.2. Orbital and spherical position and velocity. 16
   3.4. The transformation from the O-frame to the I-frame. 22
   3.5. The impulsive \( \Delta V \)-manoeuvre. .................. 26
   3.6. The \( \Delta V \) manoeuvre with finite thrust. .......... 27
   3.7. The crossrange. ....................................... 31

4. START Version 2.1. ........................................... 33
   4.1. Implementation of the derived models. ................. 33
   4.2. Evaluation. ............................................. 33
       4.2.1. Introduction. ...................................... 33
       4.2.2. The exo-atmospheric phase. ....................... 36
       4.2.3. Combined results. ................................ 39

5. Deorbit-burn analysis. ........................................ 62
   5.1. Introduction. ........................................... 62
   5.2. The ACRV database. ..................................... 63
   5.3. Sensitivity analysis. .................................. 66
       5.3.1. List of simulations. ............................... 66
       5.3.2. Results and discussion. ......................... 67

6. Conclusions and recommendations. ............................. 78

References. ....................................................... 81

Appendix A - Input file of ACRV.
## Notations.

### Roman

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$a$</td>
<td>semi-major axis</td>
<td>m</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag force coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift force coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$C_S$</td>
<td>side force coefficient</td>
<td>-</td>
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<td>$d$</td>
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<td>N</td>
</tr>
<tr>
<td>$e$</td>
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<td>-</td>
</tr>
<tr>
<td>$e$</td>
<td>ellipticity</td>
<td>-</td>
</tr>
<tr>
<td>$E$</td>
<td>eccentric anomaly</td>
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<td>$g_0$</td>
<td>reference gravitational acceleration at surface</td>
<td>m/s²</td>
</tr>
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<td>acceleration due to gravity in radial direction</td>
<td>m/s²</td>
</tr>
<tr>
<td>$g_b$</td>
<td>acceleration due to gravity in meridional direction</td>
<td>m/s²</td>
</tr>
<tr>
<td>$g_{\text{load}}$</td>
<td>occurring dimensionless deceleration (deceleration/$g_0$)</td>
<td>-</td>
</tr>
<tr>
<td>$h$</td>
<td>height (geometric altitude)</td>
<td>m</td>
</tr>
<tr>
<td>$H$</td>
<td>angular momentum per unit mass</td>
<td>m²/s²</td>
</tr>
<tr>
<td>$i$</td>
<td>inclination</td>
<td>rad</td>
</tr>
<tr>
<td>$L$</td>
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<td>N</td>
</tr>
<tr>
<td>$m$</td>
<td>mass</td>
<td>kg</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
<td>-</td>
</tr>
<tr>
<td>$M$</td>
<td>mean anomaly</td>
<td>rad</td>
</tr>
<tr>
<td>$p$</td>
<td>semi-latus rectum</td>
<td>m</td>
</tr>
<tr>
<td>$r$</td>
<td>distance to the Centre of Mass of the central body</td>
<td>m</td>
</tr>
<tr>
<td>$R_e$</td>
<td>radius at equator</td>
<td>m</td>
</tr>
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<td>$S$</td>
<td>side force</td>
<td>N</td>
</tr>
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<tr>
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<td>time</td>
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<td>$y$</td>
<td>cartesian y-position</td>
<td>m</td>
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### Greek

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<th>Description</th>
<th>Unit</th>
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<tr>
<td>$\alpha$</td>
<td>angle of attack</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta$</td>
<td>angle of sideslip</td>
<td>rad</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>flight-path angle</td>
<td>rad</td>
</tr>
<tr>
<td>$\Delta V$</td>
<td>change in velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$\delta$</td>
<td>planetocentric latitude</td>
<td>rad</td>
</tr>
<tr>
<td>$\varepsilon_T$</td>
<td>in-plane angle of thrust vector</td>
<td>rad</td>
</tr>
<tr>
<td>$\varepsilon_{\Delta V}$</td>
<td>in-plane angle of $\Delta V$ vector</td>
<td>rad</td>
</tr>
</tbody>
</table>
\begin{align*}
\eta & \quad \text{auxiliary variable} \\
\theta & \quad \text{true anomaly} \\
\mu & \quad \text{gravitation parameter} \quad \text{m}^3/\text{e}^2 \\
\xi & \quad \text{auxiliary variable} \\
\sigma & \quad \text{bank angle} \quad \text{rad} \\
\tau & \quad \text{planetocentric longitude} \quad \text{rad} \\
\tau & \quad \text{time of pericentre passage} \quad \text{s} \\
\chi & \quad \text{heading} \quad \text{rad} \\
\psi_T & \quad \text{out-of-plane angle of thrust vector} \quad \text{rad} \\
\psi_{\Delta V} & \quad \text{out-of-plane angle of } \Delta V \text{ vector} \quad \text{rad} \\
\omega_{cb} & \quad \text{rotational rate of the central body} \quad \text{rad/s} \\
\omega & \quad \text{argument of pericentre} \quad \text{rad} \\
\Omega & \quad \text{longitude of the ascending node} \quad \text{rad} \\
\end{align*}

\textit{Indices}

\begin{align*}
I & \quad \text{inertial planetocentric frame} \\
R & \quad \text{rotating planetocentric frame} \\
B & \quad \text{body frame} \\
A & \quad \text{aerodynamic frame} \\
T & \quad \text{trajectory frame} \\
V & \quad \text{vertical frame} \\
O & \quad \text{orbital frame} \\
\end{align*}
1. Introduction.

Manoeuvres are quite common in space missions. When a spacecraft is flying in a particular orbit, it does not necessarily mean that it will do that for the rest of its remaining lifetime. A communications satellite, for instance, can be launched into a Geostationary Transfer Orbit (GTO) for the sake of minimizing the launch cost. Then, when it has reached the apogee\(^1\) of this orbit, a thrust manoeuvre brings the satellite into a geostationary orbit (GEO - Geostationary Earth Orbit). Or, if we consider a spacecraft on an interplanetary mission, once it has reached the vicinity of the target planet, it must perform a manoeuvre to be able to orbit that planet.

When we look at a manned spacecraft orbiting the Earth, e.g., the Space Shuttle, then we can assume that after performing its mission, the vehicle will return to the surface of the Earth. This means, that the current orbit should be left, which can be accomplished by changing the vehicle’s velocity and/or direction of motion. The related manoeuvre is the so-called deorbit-burn manoeuvre, which is the main topic of this report.

The Simulation Tool for Atmospheric Re-entry Trajectories (START) is an open loop, six-degrees-of-freedom simulation tool, with which entry and descent analyses can be performed for several central bodies, e.g., the Earth and Titan. Within the scope of the transfer of START to ESTEC, the current version of the software has to be extended with a capability of simulating a (n impulsive) deorbit-burn manoeuvre (Paris, 1992). To demonstrate the updated version of START, a sensitivity analysis has to be performed of the deorbit-burn manoeuvre of the Assured Crew Return Vehicle (ACRV), a so-called rescue or return vehicle for the International Space Station ‘Freedom’.

Chapter 2 introduces the flight-dynamics software START Version 2.0. A brief description of the capabilities and the implemented models is given. The Chapter concludes with a statement of the equations of motion.

In Chapter 3, the theory of the deorbit-burn manoeuvre is discussed. Two alternative implementations will pass review, i.e., the impulsive $\Delta V$-manoeuvre and the finite-thrust manoeuvre. This Chapter includes some transformations between related sets of state variables (spherical and cartesian components, and classical orbital elements).

The implementation of the deorbit-burn manoeuvre in the START software is briefly discussed in Chapter 4. To assess the validity of the impulsive-$\Delta V$ model, it is compared with the finite-thrust model. The differences between the two models are evaluated.

Chapter 5, finally, presents the sensitivity analysis of the ACRV. A description of the input data is given as well.

---

\(^1\) The apogee and perigee of an elliptical orbit are the points closest to and furthest away from the Earth. The general names of these points are the apocentre and pericentre, respectively. Since the deorbit-burn model, which is going to be discussed in this report, is in principal not restricted to Earth orbits we will use the names ‘apocentre’ and ‘pericentre’ from now on.
2. The flight-dynamics software START.

2.1. START Version 2.0.

START can be defined to be a six-degrees-of-freedom (6-dof) open-loop re-entry trajectory simulation tool. Open-loop means here, that there are no guidance and control capabilities. The equations of motion, which can be divided into equations of translational motion (position and velocity) and equations of rotational motion (attitude and angular motion), are based on the following state variables: \( r, \tau \) and \( \delta \) for the position; \( V, \gamma \) and \( \chi \) for the velocity; \( \rho, \varphi \) and \( \tau \) (the rotational rate of the body w.r.t. the inertial planetocentric frame) for the angular motion; and \( \alpha \) (angle of attack), \( \beta \) (angle of sideslip) and \( \sigma \) (bank angle) for the attitude of the re-entry vehicle w.r.t. the oncoming flow. As an alternative for the attitude, the so-called quaternions have been implemented.

As is indicated in Fig. 2.1, the actual flight dynamics code, the core of the program, is embedded in a User Interface (UI). It provides the user with a friendly tool to edit all the input data necessary for the trajectory analysis and to start the simulation itself. In other words, all possible actions can only be activated by means of this UI.

The input data can be divided into four blocks, in Fig. 2.1 indicated with trapezoids. The first block is related with the re-entry vehicle. A vehicle can be described as a number of mass elements, each with its own mass, Centre of Mass (CoM) and inertia tensor. This way of entering the vehicle enables a user to 'build' a re-entry vehicle on basis of fundamental geometrical shapes with readily available inertia tensors. The global inertia tensor will be computed during the simulation. Besides, this concept of mass elements can also be used for configuration changes by just deleting one (or more) of the mass elements.

A major part of the vehicle data consists of the aerodynamic database. Each of the (six) force and moment coefficients can be written as a Taylor series, as a function of a number of independent variables, for instance:

\[
CF = CF_0 + CF_{X_1} \cdot X_1 + CF_{X_2} \cdot X_2 + \ldots + CF_{X_1,X_2} \cdot X_1 \cdot X_2 + \ldots
\]

The variables \( X_i \) are called derivation variables (e.g. \( \alpha, \beta \), one of the three angular rates); the order of derivation can be 0, 1 or 2. Besides, each of the coefficients \( CF_\cdot \) can be entered as a table (max. 10x10), as a function of 0, 1 or 2 table variables (e.g. \( \alpha, \beta, q_{dyn}, M \)). This way of entering aerodynamic data is very flexible. And, by storing the data in a separate file, the aerodynamic data can be updated after each configuration change, by simply reading in an aerodynamic database file. The aerodynamic forces and moments consist of the drag, side and lift forces \( D, S, \) and \( L \), and the rolling, pitching and yawing moments \( L, M \) and \( N \), respectively.
USER INTERFACE

INITIAL STATE PARACHUTE OPERATIONS CONFIGURATION CHANGES

MISSION

MASS LOCATION & M MASS
INERTIA TENSOR
REFERENCE GEOMETRY
AERODYNAMICS
PARACHUTE SYSTEMS
EXTERNAL FORCES AND MOMENTS

MISSION EVENTS

PARACHUTE DEPLOYMENT
PARACHUTE REEFING
PARACHUTE RELEASE
CONFIGURATION CHANGES

FLIGHT DYNAMICS

EQUATIONS OF MOTION
EXTERNAL FORCES AND MOMENTS

ACCELERATIONS

POSITION
SPEED
ATTITUDE
ANGULAR VELOCITIES

INTEGRATION OF MOTION

INTEGRATION METHOD
OUTPUT FILEING RATE
TOLERANCES
STOP CRITERIA

INITIALIZATION
START SIMULATION
3 DOF / 6 DOF
ATTITUDE VARIABLES

SIMULATION

STOP CRITERIA
FULFILLED

YES

STOP SIMULATION
RETURN TO MAIN MENU

NO
As a parachute model, a three-dimensional, 6-dof model has been implemented. Both the parachute and the re-entry vehicle are thought to be rigid bodies (no mass points), connected by a rigid bar. This rigid-bar connection prevents the two bodies to rotate w.r.t. each other. However, the two bodies can spin w.r.t. each other, along this hypothetical bar, because a (non-ideal) swivel has been implemented in the connection between parachute and re-entry vehicle. Non-ideal means here, that the swivel causes a friction moment that counteracts the spinning motion. The air in and under the parachute is taken into account as added mass. The shape of the canopy is defined to be a hemisphere. The model may also include reefing (i.e. step-wise opening) of the parachute. Three parachute systems can be defined.

To create additional external moments, a simplified spin-vane model has been implemented. Each vane is considered to be a rectangular flat plate. The lift and drag contribution for one vane is computed, based on flat-plate theory. These forces result in a rolling moment that drives the re-entry vehicle. The contribution of all the vanes is taken into account by multiplying the computed rolling moment by the number of vanes. This implies, that the flow direction for each vane is assumed to be identical, so no transverse winds can be studied, for instance.

The second block deals with the environment. In the first place, a central body can be selected. This can either be the Earth, the Moon, Mars or Titan. Depending on the central body is the choice of the gravity model (central field plus harmonics up to $J_2$ and that of the atmospheric model. For the Earth, there are three available models: US Standard atmosphere 1962 (US62), US76 and a simple tabulated model based on US76 (up to 90 km) and CIRA86 (above 90 km). The Martian atmosphere is also tabulated and based on Viking-1 data. For the Titan atmosphere, the tabulated minimum, nominal and maximum Lellouch-Hunten model of October 1987 are used.

A major part of the environmental data consists of the wind database. Presently, the wind model may exist of a steady-state wind and horizontal wind gusts. The steady-state wind can be defined as two components, either a zonal and a meridional component, or a modulus and direction component. The components can be entered in tabular form, being a function of two independent variables at the most (i.e. the atmospheric pressure, the height and the latitude, with a maximum of 40 entries per variable). It is also possible to define a zonal component only, being a (simple) function of the planetary rotation. In addition, the so-called Flasar wind model has been implemented for Titan. This is an engineering model, defining a zonal steady-state wind as a function of altitude (atmospheric pressure) and latitude.

A total of 10 horizontal wind gusts can be specified. Main parameters are the initial altitude, the thickness, the maximum velocity and the direction of each gust. Two shapes have been predefined.

The mission block enables the user to define the mission of the re-entry vehicle. Data correspond to the initial state of the vehicle, parachute deployment, reefing and release and (other) configuration changes.

The fourth block, simulation, contains data for executing the actual simulation, i.e. the choice of the integrator (fourth-order fixed-step Runge-Kutta, seventh-order variable-step Runge-Kutta-
Fehlberg, variable-order variable-step Adams-Bashforth, Adams-Moulton, second-, third- and fourth-order, fixed-step Adams-Bashforth), the maximum integration step size, the output filing rate for both the quicklook data (information for user during the simulation) and the plot data, the tolerances of the state variables (used by the variable step size integrators) and the stop criteria (e.g. \( t, h, M \)).

Input data can be stored in a data file and retrieved when necessary. The aerodynamic data, the parachute systems and the wind model can be stored and retrieved separately.

The actual simulation can be performed in two modes, namely 3 and 6 dof. In the first mode, the re-entry vehicle is considered to be a mass point and only translations can be analyzed (little CPU-time is used). In the second mode, the vehicle is thought of as a rigid body and both translations and rotations can be examined. For the attitude of the body, both aerodynamic angles (\( \alpha, \beta, \sigma \)) and quaternions can be selected.

Each time step, the equations of motion are integrated to give position and speed (3 dof) and attitude and angular velocities (6 dof). Mission events can be executed whenever a predefined flight condition occurs. Computations proceed until a stop criterion is met. In that case, control is returned to the UI.

More (technical) details about START, including two applications (mission analyses of the ESA Huygens Probe), can be found in Mooij (1991a, 1991b and 1992, respectively).

### 2.2. The equations of translational motion.

The equations of translational motion in START are based on spherical components for both the position (distance \( r \), planetocentric longitude \( \tau \) and planetocentric latitude \( \delta \)) and velocity (groundspeed \( V \), flight-path angle \( \gamma \) and heading \( \chi \)) as defined in Fig. 2.2. For the case that we do not consider the influence of wind, the following equations can be derived (Mooij, 1991a):

The dynamic equations:

\[
\hat{V} = \frac{F_V}{m} + \omega_c^2 r \cos \delta (\sin \gamma \cos \delta - \cos \gamma \sin \delta \cos \chi)
\]  

(2.2.1a)

with

\[
F_V = -D - T (\cos \alpha \cos \beta \cos \psi \tau \cos \epsilon \tau + \sin \beta \sin \psi \tau \cos \epsilon \tau - \sin \alpha \cos \beta \sin \epsilon \tau) + \\
- mg_s \sin \gamma - mg_b \cos \gamma \cos \chi
\]

(2.2.1b)
Fig. 2.2 - Definition of the six flight parameters, the position \((r, \tau, \delta)\) and velocity \((V, \gamma, \chi)\).

\[
V_\gamma = \frac{F_x}{m} + 2 \omega_{cb} V \cos \delta \sin \chi + \frac{V^2}{r} \cos \gamma + \\
+ \omega_{cb}^2 \cos \delta (\cos \delta \cos \gamma + \sin \gamma \sin \delta \cos \chi)
\]  

(2.2.2a)

with

\[
F_x = -(S + T \cos \alpha \sin\beta \cos \psi \cos \epsilon_T - T \cos \beta \sin \psi \cos \epsilon_T - T \sin \alpha \sin \beta \sin \epsilon_T) \sin \sigma + \\
+ (L + T \sin \alpha \cos \psi \cos \epsilon_T + T \cos \alpha \sin \epsilon_T) \cos \sigma + \\
- mg \cos \gamma + mg_s \sin \gamma \cos \chi
\]  

(2.2.2b)

\[
V \cos \gamma \chi = \frac{F_x}{m} + 2 \omega_{cb} V (\sin \delta \cos \gamma - \cos \delta \sin \gamma \cos \chi) + \\
+ \frac{V^2}{r} \cos^2 \gamma \tan \delta \sin \chi + \omega_{cb}^2 \cos \delta \sin \delta \sin \chi
\]  

(2.2.3a)

with
\[ F_x = -(S + T \cos \alpha \sin \beta \cos \psi_T \cos \epsilon_T - T \cos \beta \sin \psi_T \cos \epsilon_T - T \sin \alpha \sin \beta \sin \epsilon_T) \cos \sigma + \\
- (L + T \sin \alpha \cos \psi_T \cos \epsilon_T + T \cos \alpha \sin \epsilon_T) \sin \sigma + \\
+ mg_0 \sin \chi \]

(2.2.3b)

As can be seen in Eq. (2.2.3a), there is a singularity in the set of dynamic equations, namely for \( \gamma = \pm 90^\circ \).

The kinematic equations:

\[ \dot{r} = V \sin \gamma \]  
(2.2.4)

\[ \dot{\psi} = \frac{V \sin \chi \cos \gamma}{g_0 \cos \delta} \]  
(2.2.5)

\[ \dot{\delta} = \frac{V \cos \chi \cos \gamma}{\epsilon} \]  
(2.2.6)

Also in Eq. (2.2.5) there is a singularity, namely for \( \delta = \pm 90^\circ \), which corresponds with a location at the North Pole or the South Pole.

In the above equations, the following notations are used:

- \( V, \gamma, \chi \) : state variables for velocity (m/s, rad, rad)
- \( r, \tau, \delta \) : state variables for position (m, rad, rad)
- \( D, S, L \) : aerodynamic drag, side and lift force (N)
- \( \alpha, \beta, \sigma \) : angles of attack and sideslip, and bank angle (rad)
- \( T \) : thrust force (N)
- \( \epsilon_T, \psi_T \) : in-plane and out-of-plane angles of thrust vector (rad)
- \( m \) : mass of vehicle (kg)
- \( g_r, g_\delta \) : acceleration due to gravity in radial and meridional direction (m/s²)
- \( \omega_{cb} \) : rotational rate of the central body (rad/s)
- \( c, s \) : short-hand notation for 'cosine' and 'sine', respectively

The aerodynamic angles \( \alpha, \beta \) and \( \sigma \), and the thrust angles \( \epsilon_T \) and \( \psi_T \) are defined in Figs. 2.3 and 2.4.
Fig. 2.3 - Definition of the aerodynamic angles $\alpha$, $\beta$ and $\sigma$.

Fig. 2.4 - Definition of the thrust angles $\varepsilon_T$ and $\psi_T$. 
3. The deorbit-burn manoeuvre.

3.1. Introduction.

The purpose of the deorbit-burn manoeuvre is to change the spacecraft's velocity in magnitude and/or direction by operating the propulsion system. When we look at ways how to implement the deorbit-burn manoeuvre, we arrive in principal at two different ways. The first way assumes that the manoeuvre is instantaneous. The boost of the propulsion system is an impulsive shot, and all fuel, which is needed for the manoeuvre, is used in zero time. This way is usually used for preliminary mission design (see, for instance, Jensen et al. (1962) and Battin (1987)) and gives reasonably good results, especially when the duration of the manoeuvre is short.

However, when the thrust level is low, the duration of the complete manoeuvre becomes comparably longer and the application of the impulsive-shot concept is doubtful. If we look, for instance, at the deorbit burn of the space plane Hermes, initially the manoeuvre lasted 20 minutes, due to the very low thrust level (Raillon et al., 1992). Therefore, the second way focuses on a finite-thrust manoeuvre, meaning that the propulsion system will thrust until the $\Delta V$ has been reached.

Linked with the departure from orbit are the conditions at the entry of the atmospheric boundary. A small difference between the actual and computed entry conditions can result in completely different (and maybe unacceptable) trajectories through the atmosphere. Besides, when the recovery area at the Earth's surface is specified, the spacecraft must deorbit at a particular position in orbit. Both cases are examples of two-point boundary value problems, in which guidance plays an important role. However, this aspect will not be studied in this report. Only the case of the free-fall, unguided trajectory, starting at an arbitrary point in orbit with a typical $\Delta V$ change will be studied here. Bluntly spoken, we will restrict to the application of a deorbit-burn manoeuvre and after that we will see where the spacecraft arrives.

The layout of this Chapter is as follows. In Section 3.2, we will give the reference frames which are needed for the derivation of the deorbit-burn model. Besides, the transformations between these frames are given. Section 3.3 introduces the different forms of initial conditions. When a spacecraft is revolving around a planet, position and velocity are usually specified with orbital elements or a cartesian state vector (w.r.t. an inertial frame). Since the equations of translational motion of START are based on spherical components w.r.t. a rotating frame, several transformations are necessary. These will also be briefly discussed. In Section 3.4, the transformation from the orbital frame to the inertial planetocentric frame is derived. This transformation is used in the following two Sections (3.5 and 3.6), where the two deorbit-burn models, i.e., the impulsive $\Delta V$ and the finite-thrust manoeuvre, are presented.

3.2. Reference frames and frame transformations.

In this Section, we define the relevant reference frames and the interrelation between each one of them. By means of simple matrix operations, vectors can be transformed from one frame to
another.

_Inertial planetocentric reference frame, index I._

The origin of the inertial reference frame is located at the Centre of Mass (CoM) of the central body\(^2\). The \(OX_YZ\) plane coincides with the equatorial plane of the central body. The \(Z_I\)-axis is pointing north and the reference meridian which determines the direction of the \(X_I\)-axis, is defined by the zero-longitude meridian at zero time. The \(Y_I\)-axis completes the right-handed system.

_Orbital reference frame, index O._

The origin of this inertial frame is located at the CoM of the re-entry vehicle at epoch \((t = 0)\). The \(X_O\)-axis is aligned with the inertial velocity. The \(Z_O\)-axis is aligned with the angular momentum vector of the vehicle (perpendicular to the orbital plane) and the \(Y_O\)-axis is the right-handed supplement.

_Rotating planetocentric reference frame, index R._

This frame is fixed to the central body and is coincident with the inertial planetocentric frame at zero time. The \(Z_R\)-axis is pointing north, the \(X_R\)-axis intersects the equator at zero longitude and the \(Y_R\)-axis completes the right-handed system.

The origin of the following reference frames is located at the CoM of the re-entry vehicle.

_Vertical reference frame, index V._

The \(Z_V\)-axis is pointing towards the CoM of the central body (along the radial component of the gravity force). The \(X_V\)-axis lies in a meridian plane, perpendicular to \(Z_V\) and points to the northern hemisphere. The \(Y_V\)-axis is the right-handed supplement.

_Trajectory reference frame, index T._

The axes of the \(T\)-frame are defined as follows:

\[ X_T : \text{ along the velocity vector.} \]
\( Z_T \) : in vertical plane, pointing downwards.
\( Y_T \) : completes right-handed system.

**Propulsion reference frame, index \( P \).**

For the propulsion frame, only the \( X_P \)-axis is of importance. This axis is collinear with the (resulting) thrust force, and is positive in thrust direction. The orientation of the (right-handed) propulsion frame w.r.t. the body frame is given by the azimuth and elevation angles of the thrust.

The frame transformations used in this report are based on unit axis rotations. The unit rotation matrices are (\( \alpha \) is here used as an arbitrary angle):

\[
C_1(\alpha) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{bmatrix}
\]

\[
C_2(\alpha) = \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\]

\[
C_3(\alpha) = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Any rotation from frame \( i \) to frame \( j \) can always be decomposed into a number of sequential unit axis rotations. This implies that the resulting transformation matrix can be formed as the product of (a combination of) the matrices \( C_1 \), \( C_2 \) and \( C_3 \).

Throughout this paper several transformation matrices are used. A matrix written as \( C_{ij} \) means that the matrix defines the transformation from the \( j \)-frame to the \( i \)-frame. In (Mooij, 1991), the related transformation matrices have been derived. They will be stated below in the introduced short-hand notation.

\[
C_{iR} = C_3(-\omega_{cb} \hat{t})
\]

\[
C_{R,V} = C_3(-\gamma) C_2(\frac{\pi}{2} + \delta)
\]
\[ C_{V,T} = C_3(-\gamma) C_2(-\gamma) \]  
(3.2.6)

\[ C_{T,A} = C_1(\sigma) \]  
(3.2.7)

\[ C_{A,B} = C_2(\beta) C_2(-\alpha) \]  
(3.2.8)

Each of the matrices \( C_1, C_2 \) and \( C_3 \) is orthonormal. Also the product of orthonormal matrices is orthonormal again. The inverse of this kind of matrix is simply its transpose. So with the above notation we can write:

\[ C_{ij}^{-1} = C_{ij}^T = C_{ji} \]  
(3.2.9)

3.3. Transformations between different sets of state variables.

In orbital mechanics, the position and velocity of a satellite or a spacecraft orbiting the Earth, is usually given by either orbital or cartesian components. Both ways of defining the state of the vehicle are confined to inertial space, which means that position and velocity are expressed with respect to the inertial planetocentric frame (index \( I \)). The equations of motion, which are implemented in START, define the state of the vehicle with respect to the rotating planetocentric frame (index \( R \)); besides, the position and velocity are expressed with spherical parameters.

By definition of the several reference frames, the \( I \)- and \( R \)-frame are coincident for \( t = 0 \). This means, that if the initial state of the spacecraft is given in orbital or cartesian parameters (\( I \)-frame), the position vector can directly be transformed to spherical parameters (\( R \)-frame). The velocity vector, however, needs to be compensated for the Earth's rotation. In the next few subsections, the transformation from cartesian and orbital to spherical parameters is presented.

3.3.1. Cartesian and spherical position and velocity.

Using cartesian components, the position and velocity are defined by the following variables (Fig. 3.1):

Position: \( x, y, z \)

Velocity: \( \dot{x}, \dot{y}, \dot{z} \) (in the \( R \)-frame, these components are usually indicated by \( u, v \) and \( w \))

The relation between the cartesian position in the \( I \)- and the \( R \)-frame is given by:

\[ X_R = C_{R,I} X_I \]  
(3.3.1)

Together with Eq. (3.2.4), this results in:
Fig. 3.1 - Definition of the cartesian position and velocity components.

\[ x_R = \cos(\omega_{cb\,f}) \, x_I + \sin(\omega_{cb\,f}) \, y_I \]  
(3.3.2a)

\[ y_R = -\sin(\omega_{cb\,f}) \, x_I + \cos(\omega_{cb\,f}) \, y_I \]  
(3.3.2b)

\[ z_R = z_I \]  
(3.3.2c)

For the velocity, we proceed in the same way; however, first we need to compensate for the Earth's rotation. Starting with the inertial velocity \( V_I \), we get

\[ V_R = C_{R,I} \, (V_I - \omega_{cb} \times r_I) \]  
(3.3.3)

or

\[
\begin{pmatrix}
\dot{x}_R \\
\dot{y}_R \\
\dot{z}_R
\end{pmatrix}
= \begin{bmatrix}
\cos(\omega_{cb\,f}) & \sin(\omega_{cb\,f}) & 0 \\
-\sin(\omega_{cb\,f}) & \cos(\omega_{cb\,f}) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{z}_I
\end{pmatrix}
- \begin{pmatrix} 0 \\ 0 \end{pmatrix} \times \begin{pmatrix}
x_I \\
y_I
\end{pmatrix}
\]  
(3.3.4)

The relation between spherical and cartesian position in the \( R \)-frame is given by the following well-known equations (see also Fig. 2.2):

\[ r = \sqrt{x_R^2 + y_R^2 + z_R^2} \]  
(3.3.5a)

\[ \tau = \arctan \left( \frac{y_R}{x_R} \right) \]  
(3.3.5b)
\[ \delta = \arcsin \left( \frac{z_R}{\sqrt{x_R^2 + y_R^2 + z_R^2}} \right) \]  

(3.3.5c)

The relation between the spherical and cartesian velocity is not that straightforward. Let us start with the cartesian velocity with respect to the R-frame, \( \mathbf{V}_R = (\dot{x}_R, \dot{y}_R, \dot{z}_R)^T \). The modulus of the velocity vector, which is by definition aligned with the \( X \)-axis of the trajectory frame (index \( T \)), can directly be computed from the three cartesian components,

\[ V = \sqrt{\dot{x}_R^2 + \dot{y}_R^2 + \dot{z}_R^2} \]  

(3.3.6)

For the two direction angles, \( \gamma \) and \( \chi \), we need to compute the cartesian velocity in the vertical frame (index \( V \)), \( \mathbf{V}_V = (v_x, v_y, v_z)^T \), see also Fig. 3.2:

Fig. 3.2 - For the computation of the cartesian velocity in the \( R \)-frame, \((u, v, w)^T\), the spherical velocity \( (V; \gamma, \chi) \) is first transformed to a cartesian velocity in the \( V \)-frame, 
\((v_x, v_y, v_z)^T\).
Fig. 3.3 - Transformation of the spherical velocity components to cartesian components in the V-frame.

\[ V_V = C_{V,R} V_R \]  
(3.3.7)

where

\[ C_{V,R} = C_2(-\delta - \frac{\pi}{2}) C_3(t) = \begin{bmatrix} -c(t) s(t) & -c(t) d(t) & c(t) \\ -s(t) c(t) & c(t) c(t) & 0 \\ -s(t) d(t) & -c(t) d(t) & -s(t) \end{bmatrix} \]

The vectorial summation of \( v_x \) and \( v_y \) is equal to the projection of \( V \) (the modulus) in the local horizontal plane, which gives us (see Fig. 3.3):

\[ V \cos \gamma = \sqrt{v_x^2 + v_y^2} \]  
(3.3.8)

As can be seen in the same figure, we find for the heading \( \chi \):

\[ \tan \chi = \frac{v_y}{v_x} \Rightarrow \chi = \arctan \left( \frac{v_y}{v_x} \right) \]  
(3.3.9)

The projection of \( V \) in the local vertical plane (Fig. 3.3) gives

\[ V \sin \gamma = -v_z \]  
(3.3.10)
Eq. (3.4.6) divided by Eq. (3.4.4) yields

\[ \tan \gamma = -\frac{v_z}{\sqrt{v_x^2 + v_y^2}} \Rightarrow \gamma = -\arctan \left( \frac{v_z}{\sqrt{v_x^2 + v_y^2}} \right) \]  \hspace{1cm} (3.3.11)

Nota bene: since \( \gamma \) is restricted to values between -90° and +90°, we can suffice with Eq. (3.3.10), since the arcsine function is defined for the same interval. In that case we have:

\[ \gamma = -\arcsin \left( \frac{v_z}{V} \right) \]  \hspace{1cm} (3.3.12)

3.3.2. Orbital and spherical position and velocity.

In case of an initial state defined by the classical orbital elements, we will restrict ourselves to elliptical orbits. The six parameters, defining the position and velocity of a spacecraft in an elliptical orbit are (see also Figs. 3.4 and 3.5):

![Diagram of spacecraft orbit](image)

**Fig. 3.4** - Definition of semi-major axis \( a \) and the eccentricity \( e \). The spacecraft is moving at a distance \( r \); the true anomaly is indicated by \( \theta \).

- \( e \): the eccentricity (0 \( \leq \) \( e \) < 1)
- \( a \): the semi-major axis (\( a > R_e \))
- \( i \): the inclination (0° \( \leq \) \( i \) \( \leq \) 180°)

- \( \omega \): argument of pericentre (0° \( \leq \) \( \omega \) < 360°)
- \( \Omega \): the longitude of the ascending node (0° \( \leq \) \( \Omega \) < 360°)
- \( M \): mean anomaly (0° \( \leq \) \( M \) < 360°)
Fig. 3.5 - Definition of the three orbital parameters $\Omega$, $\omega$ and $i$. The spacecraft is moving at a distance $r$ with a velocity $v_i$ w.r.t. to the inertial planetocentric frame (index $I$). The true anomaly is indicated by $\theta$.

The orbit of a spacecraft in the orbital plane is a conic section, with the central body (in this report the Earth) at one focus. The shape is defined by the eccentricity, $e$, and the size by the semi-major axis, $a$. The orientation of the conic in the orbital plane is specified by the argument of pericentre, $\omega$, which is measured from the line of intersection of the orbital plane with the $XY$-plane of the inertial frame, here the equatorial plane of the Earth towards the pericentre. This intersection line is also called the line of nodes. It intersects the orbit at two points, the ascending node (the $z$-coordinate of the spacecraft changes from negative to positive) and the descending node. The position of the ascending node in the $XY$-plane is specified by the angle $\Omega$, which is called the longitude of the ascending node. It is measured positively from the $X_f$-axis eastward. The angle $i$ between the $XY$-plane and the orbital plane is called the inclination. It is considered to be the (smallest) angle between the positive $Z_f$-axis and the orbital angular momentum vector. To determine the position of the spacecraft in its orbit at a given time, we need a sixth parameter, the time of pericentre passage $\tau$. However, often $\tau$ is replaced by the mean anomaly at the epoch $t_0$, $M_0$, defined as

$$M_0 = n(t_0 - \tau)$$  \hspace{1cm} (3.3.12)

with

$$n = \sqrt{\frac{\mu}{a^3}}$$  \hspace{1cm} (3.3.13)

The mean anomaly $M$ is then

$$M = M_0 + n(t - t_0) = n(t - \tau)$$  \hspace{1cm} (3.3.14)
which is, in fact, the product of a mean angular velocity and the time, which has passed since the last pericentre passage.

To transform the orbital (l-frame) to spherical (R-frame) parameters, we use the results of the above discussion. We first transform from orbital to cartesian components, and then from cartesian to spherical as has been discussed above.

The cartesian position w.r.t. to the l-frame can be written as (Wakker, 1989):

\[
x_l = l_1 \xi + l_2 \eta \\
y_l = m_1 \xi + m_2 \eta \\
z_l = n_1 \xi + n_2 \eta
\]

with

\[
l_1 = \cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i
\]

\[
m_1 = \cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i
\]

\[
n_1 = \sin \omega \sin i
\]

\[
l_2 = -\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos i
\]

\[
m_2 = -\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos i
\]

\[
n_2 = \cos \omega \sin i
\]

The two variables \( \xi \) and \( \eta \) are defined as\(^3\)

\[
\xi = r \cos \theta \\
\eta = r \sin \theta
\]

In these equations, \( r \) and \( \theta \) are the radial distance and the true anomaly, respectively. They can be computed from the orbital parameters with (Fig. 3.6):

\[
r = a(1 - e \cos E)
\]

\(^3\) \( \xi \) and \( \eta \) are the coordinates of a cartesian frame, with the \( \xi \eta \) plane coinciding with the orbital plane. The origin is the CoM of the Earth, the \( \xi \)-axis is positive pointing towards the perigee and the \( \eta \)-axis is positive in the direction of the velocity in the perigee.
\[ \theta = 2 \arctan \left( \frac{1+e}{\sqrt{1-e^2}} \right) \tan \frac{E}{2} \]  

(3.3.18)

The eccentric anomaly \( E \) is defined in Fig. 3.6. Kepler's equation relates the position, through the angle \( E \), to time elapsed after pericentre passage \( \tau \):

\[ E - e \sin E = \sqrt{\frac{\mu}{a^3}} (t-\tau) = M \]  

(3.3.19)

To get a numerical value for \( E \), the above equation needs to be solved with a numerical iteration method, for instance with the method of Newton-Rhapson. The numerical scheme is given by

\[ E_{k+1} = E_k - \frac{f(E_k, M)}{\frac{d}{dE} f(E_k, M)}|_{E=E_k} = E_k - \frac{E_k - e \sin E_k - M}{1 - e \cos E_k} \]  

(3.3.20)

For the cartesian velocity w.r.t. the \( i \)-frame, Wakker (1989) gives

\[ \dot{x}_I = \frac{\mu}{H} [-\ell_1 \sin \theta + \ell_2 (e + \cos \theta)] \]  

(3.3.21a)

\[ \dot{y}_I = \frac{\mu}{H} [-m_1 \sin \theta + m_2 (e + \cos \theta)] \]  

(3.3.21b)
\[
\dot{z}_l = \frac{\mu}{H} \left[ -n_1 \sin \theta + n_2 (e + \cos \theta) \right] \\
(3.3.21c)
\]

In these equations, \( \mu \) is the gravitation parameter and \( H \) the angular momentum per unit mass. \( H \) can be written as

\[
H = \sqrt{\rho \mu} \\
(3.3.22)
\]

with the semi-latus rectum \( p \) as (see also Fig. 3.6):

\[
p = a(1 - e^2) \\
(3.3.23)
\]

The above obtained cartesian position and velocity are now transformed from the inertial to the rotating planetocentric frame with Eqs. 3.3.2 and 3.3.4. Finally, Eqs. 3.3.5-6, 3.3.9 and 3.3.11 will give us the spherical components in the \( R \)-frame.

For the inverse transformation, we approach in a similar manner (Cornelisse et al., 1979). In this case, we will start with the inertial cartesian position and velocity. To arrive at this point, the several transformations as discussed in the previous Subsections, can be used.

\[
r = \sqrt{x_i^2 + y_i^2 + z_i^2} \\
(3.3.24)
\]

\[
V_i^2 = \dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2 \\
(3.3.25)
\]

\[
a = \frac{\mu r}{2 \mu - r V_i^2} \\
(3.3.26)
\]

\[
esinE = \frac{1}{\sqrt{\mu a}} (x_i \dot{x}_i + y_i \dot{y}_i + z_i \dot{z}_i) \\
(3.3.27a)
\]

\[
ecosE = 1 - \frac{r}{a} \\
(3.3.27b)
\]

From Eqs. (3.3.27), both \( e \) and \( E \) can be derived. When both right-hand sides of these equations are zero, it follows that \( e = 0 \). In that case, \( E \) is not defined. When \( e \) is not equal to zero, \( E \) follows from

\[
E = \arctan \left( \frac{esinE}{ecosE} \right) \\
(3.3.28)
\]

where the right-hand sides of Eqs. (3.3.27) have to be substituted in Eq. (3.3.28), of course.
When $E = 0$, Eq. (3.3.27b) has to be used to compute $e$. Similarly, in case $E = 90^\circ$, Eq. (3.3.27a) gives $e$.

The following two parameters, i.e., $M$ and $\theta$, are only defined when $e \neq 0$.

$$M = E - esinE$$  \hfill (3.3.29)

$$\theta = 2 \arctan \left( \frac{1 + e}{\sqrt{1-e^2}} \tan \left( \frac{E}{2} \right) \right)$$  \hfill (3.3.30)

$$H = \sqrt{\mu a(1-e^2)}$$  \hfill (3.3.31)

$$i = \arccos \left( \frac{x_i \hat{y}_i - y_i \hat{x}_i}{H} \right), \quad 0 \leq i \leq 180^\circ$$  \hfill (3.3.32)

When $i = 0$, the remaining parameters, $\Omega$ and $\omega$, are not defined. Similarly, when $e = 0$, $\omega$ is not defined.

$$\sin \Omega = \frac{y_i \hat{z}_i - z_i \hat{y}_i}{H \sin i}$$  \hfill (3.3.33a)

$$\cos \Omega = \frac{x_i \hat{z}_i - z_i \hat{x}_i}{H \sin i}$$  \hfill (3.3.33b)

$$\Omega = \arctan \left( \frac{\sin \Omega}{\cos \Omega} \right)$$  \hfill (3.3.33c)

$$\sin(\omega + \theta) = \frac{z_i}{r \sin i}$$  \hfill (3.3.34a)

$$\cos(\omega + \theta) = \frac{y_i}{r} \sin \Omega + \frac{x_i}{r} \cos \Omega$$  \hfill (3.3.34b)

$$\omega = \arctan \left( \frac{\sin(\omega + \theta)}{\cos(\omega + \theta)} \right) - \theta$$  \hfill (3.3.34c)
3.4. The transformation from the \( O \)-frame to the \( I \)-frame.

In the next two Sections, the deorbit-burn models are derived. We will see that we need the transformation matrix from the orbital frame (index \( O \)) to the inertial planetocentric frame (index \( I \)). The related transformation will be derived in this Section. Since the transformation matrix is dependent on the type of orbit, i.e., elliptical versus circular, inclined or not inclined, or a combination of both, we will look at each of the possible orbits and derive the matrix accordingly.

*Inclined, elliptical orbit; \( e \neq 0 \) and \( i \neq 0 \)*

The general case concerns an elliptical orbit with inclination. In Fig. 3.7, the geometry of the orbit is depicted. From this Figure, we can derive

![Fig. 3.7 - Geometry of an inclined, elliptical orbit.](image)

\[
C_{I,O} = C_3\left(-\Omega - \frac{\pi}{2}\right) C_2(i) C_3(\gamma - \omega - \theta) \tag{3.4.1}
\]

In this equation, \( \gamma \) is the angle between the local horizontal plane (perpendicular to \( I \)) and the (inertial) velocity, the flight-path angle for \( V \), \( \gamma \) is given by

\[
\gamma = \arcsin \left( \frac{\mu \sin^2 \theta}{V_i H} \right) \tag{3.4.2}
\]

with

\[
H = \sqrt{\mu a(1 - e^2)} \tag{3.4.3}
\]
Using the basic frame transformations of Section 3.2 and taking into account that

\[ \sin \left( -\Omega - \frac{\pi}{2} \right) = -\cos \Omega \]
\[ \cos \left( -\Omega - \frac{\pi}{2} \right) = -\sin \Omega \]

it follows that, using the shorthand notation 's' for sine and 'c' for cosine:

\[ \mathbf{C}_{iO} = \begin{bmatrix} -s\Omega c/(\omega + \theta - \gamma) & s\Omega s/(\omega + \theta - \gamma) & s\Omega s/i \\ c\Omega c/(\omega + \theta - \gamma) & -c\Omega s/(\omega + \theta - \gamma) & -\Omega s/i \\ s\Omega c/(\omega + \theta - \gamma) & s\Omega s/(\omega + \theta - \gamma) & c/i \end{bmatrix} \]

(3.4.4)

**Inclined, circular orbit; e = 0 and i ≠ 0**

As we saw in Eq. (3.4.1), we need four angles to compute \( \mathbf{C}_{iO} \), i.e., \( \Omega \), \( i \), \( \omega \) and \( \theta \). When \( e = 0 \) (circular orbit), \( \omega \) and \( \theta \) are not defined, because the pericentre is not defined. Therefore, we introduce the angle \( \theta^* \). From Fig. 3.8, it follows that

\[ \theta^* = \arcsin \left( \frac{\cos \delta \sin \tau^*}{\cos i} \right) \]

(3.4.5)

with

Fig. 3.8 - Geometry of an inclined, circular orbit.
\[ \tau^* = \bar{\tau} - \Omega = \omega_{cb} t + \tau - \Omega \]

Nota bene: \( \tau \) is the planetocentric longitude and not the time of pericentre passage; \( \bar{\tau} \) is the celestial longitude.

In Eq. (3.4.5), we applied standard spherical trigonometry to the spherical triangle, as shown in Fig. 3.8. The particular law, which has been used, is known as the second cosine law.

Since the arcsine function gives only angles in the range from \(-90^\circ\) to \(90^\circ\), and \( \theta^* \) is in principle defined from 0 to \(360^\circ\), we need to correct the outcome of Eq. (3.4.5) accordingly.

\[ \tau^* \in \left[ \frac{-\pi}{2}, 0 \right] : \theta^* = \theta^* + 2\pi \]
\[ \tau^* \in \left( \frac{\pi}{2}, \frac{3\pi}{2} \right] : \theta^* = \pi - \theta^* \]

The transformation matrix can now be written as

\[ C_{t,0} = C_3(-\Omega - \frac{\pi}{2}) \quad C_2(i) \quad C_3(-\theta^*) \]  

\[ (3.4.6) \]

or fully written out

\[
C_{t,0} = \begin{bmatrix}
-s\Omega \cos \theta^* & -c\Omega \sin \theta^* & s\Omega \sin i \\
c\Omega \cos \theta^* & -s\Omega \sin \theta^* & -c\Omega \cos i \\
\sin \theta^* & -\sin \theta^* & c \end{bmatrix} 
\]  

\[ (3.4.7) \]

Non-inclined, elliptical orbit; \( e \neq 0 \) and \( i = 0 \)

When the orbital plane coincides with the equatorial plane, the ascending and descending node cannot be determined. As a result, \( \Omega \) and \( \omega \) are not defined. Looking at Fig. 3.9, we find for \( C_{t,0} \):

\[ C_{t,0} = C_3(\gamma - \frac{\bar{\tau}}{2} - \frac{\pi}{2}) = \begin{bmatrix}
-sin(\bar{\tau} - \gamma) & -cos(\bar{\tau} - \gamma) & 0 \\
\cos(\bar{\tau} - \gamma) & -\sin(\bar{\tau} - \gamma) & 0 \\
0 & 0 & 1 
\end{bmatrix} \]  

\[ (3.4.8) \]
Non-inclined, circular orbit; \( e = 0 \) and \( i = 0 \)

For a non-inclined, circular orbit, both \( \Omega \), \( \omega \) and \( \theta \) are not defined. However, the transformation matrix is very simple and can easily be derived. Based on Fig. 3.10, we find:

\[
C_{10} = C_3(-\tilde{\tau} - \frac{\pi}{2}) = \begin{bmatrix}
-sin\tilde{\tau} & -cos\tilde{\tau} & 0 \\
-cos\tilde{\tau} & sin\tilde{\tau} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(3.4.9)

Nota bene: when we consider the transformation matrix at epoch \( t = 0 \), we can replace \( \tilde{\tau} \) with \( \tau \), because

\[
\tilde{\tau}_0 = \tau_0 + \omega_{cb} t_0 \quad (= 0) = \tau_0
\]
3.5. The impulsive $\Delta V$-manoeuvre.

The idea behind the impulsive $\Delta V$-manoeuvre is, that the (inertial) $\Delta V$-vector is vectorially added to the initial velocity of the spacecraft. $\Delta V$ is given as a modulus and two direction angles, relative to the inertial velocity of the vehicle (Fig. 3.11).

The ranges of the two angles are:

\begin{align*}
0^\circ & \leq \varepsilon_{\Delta V} < 360^\circ \\
-90^\circ & \leq \psi_{\Delta V} \leq 90^\circ
\end{align*}

The velocity of the spacecraft is in principle given in spherical components w.r.t. the $R$-frame, although the initial velocity can be given in orbital elements and cartesian components (w.r.t. the $R$-frame) as well. For reasons of simplicity, we assume that the initial velocity has already been transformed to spherical components.

In order to add $\Delta V$ to the velocity of the spacecraft, the latter has to be transformed to the $I$-frame, as discussed in Section 3.3.1. In short, we transform the spherical components ($R$-frame) first to cartesian components ($R$-frame), compensate for the Earth's rotation and transform the resulting velocity vector to the $I$-frame. The result is the velocity vector $V_{I,I}$, the velocity w.r.t. inertial space expressed in components of the $I$-frame.

Next, we define an inertial frame (the orbital frame), with the $X$-axis collinear with $V_{I,I}$, so:

\[ V_{I,O} = (V_I,0,0)^T \]  

(3.5.1)

with

\[ V_I = \|V_{I,I}\| \]
We will now transform $\Delta V$ to the $O$-frame. Based on Fig. 3.11, we find

$$
\Delta V_O = \begin{bmatrix}
    \cos \Delta \psi \cos \Delta \phi \Delta V \\
    \sin \Delta \phi \Delta V \\
    \sin \Delta \psi \Delta V
\end{bmatrix}
$$

(3.5.2)

so the total velocity vector in the $O$-frame becomes

$$
V_{t,tot} = V_{t,0} + \Delta V_O
$$

(3.5.3)

Nota bene: the index '0' stands for $t = 0$ (initial velocity).

$V_{t,tot}$ is now to be transformed to the $I$-frame to get cartesian components in the $I$-frame,

$$
V_{i,tot} = C_{i,O} V_{i,tot}
$$

(3.5.4)

with $C_{i,O}$ as discussed in the previous Section. Next, we transform to cartesian components in the $R$-frame and finally from to spherical components. The result is that we have added $\Delta V$ to the initial velocity, and we have the resulting velocity available in state variables so that the integration of the equations of motion can begin.

### 3.6. The $\Delta V$ manoeuvre with finite thrust.

Also in this case, the inertial $\Delta V$-vector is given as a modulus and two (relative) direction angles (Fig. 3.11). The $\Delta V$ is now achieved by a thrusted manoeuvre. The direction of the thrust vector $T$ is computed on basis of the (fixed) orientation in inertial space\(^4\). So when the spacecraft is orbiting the central body, the thrust direction is constantly adjusted, because the orientation of the body frame (in which the thrust is defined) is continuously changing w.r.t. the inertial frame.

As a stop criterion for thrusting, the achieved (total) $\Delta V$ in thrust direction is taken. This implies, that we assume a drag-free deorbit-bum manoeuvre, i.e., an exo-atmospheric manoeuvre.

First, we transform $\Delta V$ to the $I$-frame with

$$
\Delta V_I = C_{i,O} \Delta V_O
$$

(3.6.1)

$\Delta V_O$ is given by Eq. (3.5.2) and $C_{i,O}$ is the transformation matrix from the orbital to the inertial planetocentric frame, extensively described in Section 3.4.

\(^4\) The proposed, simple guidance law is just one possibility out of many. The goal of this report was not to develop efficient guidance algorithms, so that is why we use a very simple scheme. An even simpler guidance law would have been a constant thrust direction, based on the initial orientation of the $\Delta V$-vector.
The orientation of $\Delta V_l$, as given by Eq. (3.6.1), is now frozen in inertial space. Based on this orientation, we define the orientation of the thrust vector. To guide the spacecraft in the direction of the required $\Delta V_l$, we align $T$ (w.r.t. body frame) with $\Delta V_l$ (w.r.t. inertial frame). Therefore, we need to transform $\Delta V_l$ to the body frame:

$$\Delta V_B = C_{B,l} \Delta V_l$$  \hspace{1cm} (3.6.2)

with

$$C_{B,l} = C_{B,T} C_{T,V} C_{V,l}$$

For 3-dof simulation, $C_{B,T} = C_2(\alpha) C_3(-\beta) C_1(-\sigma)$ is constant. In exo-atmospheric flight, $\alpha = 180^\circ$ and $\beta = \sigma = 0^\circ$, so

$$C_{B,T} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$  \hspace{1cm} (3.6.3)

$C_{T,V}$ and $C_{V,l}$ are given by the inverse matrices of Eqs. (3.2.4-6).

Normalizing $\Delta V_B$ results in

$$n = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \frac{\Delta V_B}{\|\Delta V_B\|} = \frac{1}{\sqrt{\Delta V_x^2 + \Delta V_y^2 + \Delta V_z^2}} \begin{bmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{bmatrix}_B$$  \hspace{1cm} (3.6.4)

The thrust vector is defined by its magnitude $T$ and two direction angles ($\epsilon_T$ and $\psi_T$). In the case of the thrusted $\Delta V$-manoeuvre, $T$ is considered to be user input. $T$ is aligned with $\Delta V_B$, so based on $n$, we can compute $\epsilon_T$ and $\psi_T$.

The relation between the body frame and the propulsion frame is given by:

$$C_{B,P} = C_2(\epsilon_T) C_3(-\psi_T) = \begin{bmatrix} \cos \epsilon_T \cos \psi_T & -\cos \epsilon_T \sin \psi_T & -\sin \epsilon_T \\ \sin \epsilon_T \sin \psi_T & \cos \epsilon_T \cos \psi_T & -\sin \psi_T \\ \sin \epsilon_T \cos \psi_T & \cos \epsilon_T \sin \psi_T & \cos \psi_T \end{bmatrix}$$  \hspace{1cm} (3.6.5)

With this result, the normalized thrust components in the body frame can be written. These components must be equal to the corresponding components of $n$, since we want a thrust vector with the same orientation as $n$. So:
\[
\frac{T_x}{T} = \cos\psi \cos \theta = n_x
\] (3.6.6a)

\[
\frac{T_y}{T} = \sin \psi = n_y
\] (3.6.6b)

\[
\frac{T_z}{T} = \sin \theta \cos \psi = n_z
\] (3.6.6c)

From the above equations, we can extract \(\theta\) and \(\psi\) with

\[
\theta = \arctan \left( \frac{n_z}{n_x} \right), \quad -180^\circ \leq \theta < 180^\circ
\] (3.6.7)

\[
\psi = \arcsin(n_y), \quad -90^\circ \leq \psi \leq 90^\circ
\] (3.6.8)

Due to the definition of \(\theta\) \(\left(0^\circ \leq \theta < 360^\circ\right)\) and \(\psi\), the outcome of the above equations has to be shifted, i.e.,

\[
\theta := -\theta \text{ for } \theta \leq 0^\circ
\]

and

\[
\theta := \pi - \theta \text{ for } \theta > 0^\circ
\]

\[
\psi := -\psi
\]

The stop criterion for the thrusted manoeuvre is, of course, based on the acquired \(\Delta V\) (in thrust direction). It will be obvious, that when the fuel mass is not sufficient, the \(\Delta V\)-criterion will not be met. Assuming there is enough propellant, we will proceed in the following manner.

On-board a spacecraft the accelerometers measure the accelerations resulting from velocity changes. However, the acceleration due to gravity cannot be measured, because accelerometers are sensitive to surface forces, such as drag and thrust, and not to volume forces, like gravity. The velocity change resulting from integrating the equations of motion, however, includes the effect of gravity, so we must either compensate for this or find an alternative.

Due to assumptions made for this study, there is a simple alternative. The atmospheric boundary is assumed to be at 120 km altitude, and since the deorbit-burn manoeuvre has finished long before the atmosphere is reached, the only surface force during the manoeuvre is the thrust force (the drag-free phase is called the exo-atmospheric phase). Integrating the acceleration due to the (constant) thrust would therefore meet the requirements. It is, however, possible to make use of Tsiolkovsky's Equation, which gives us the \(\Delta V\) for a given fuel mass (Cornelisse et al., 1979):
\[ \Delta V = g_0 I_{sp} \ln \frac{m_0}{m} \quad (3.6.9) \]

with

- \( g_0 \) = acceleration due to gravity at sea level (\( g_0 = 9.798 \text{ m/s}^2 \))
- \( I_{sp} \) = specific impulse (s)
- \( m_0 \) = initial mass of spacecraft (kg)
- \( m \) = mass of spacecraft, after acquiring \( \Delta V \) (kg); this mass is equal to the dry mass of the spacecraft \( m_{\text{dry}} \) plus possible fuel mass, which is left.

When we assume, that for the deorbit-burn manoeuvre just as much fuel as needed, has been stored, we can compute the fuel mass \( m_f \) with Tsiolkovsky’s Equation and thrust until \( m_f \) is zero. Based on Eq. (3.6.9), we find

\[ m_f = m_0 \left( \frac{\Delta V}{g_0 I_{sp}} - 1 \right) \quad (3.6.10) \]

since \( m = m_0 - m_f \).

On the other hand, when the total fuel mass is specified, we can use Eq. (3.6.10) to come to the total burn time of the engine. Since the burn-out mass is not known, we have to iterate to compute the actual Tsiolkovsky mass:

\[ m_{f_{\text{total}}} = 0 \quad (3.6.11a) \]

\[ m_f |_k = (m_{\text{dry}} + m_{f_{\text{total}}} |_k) \left( \frac{\Delta V}{g_0 I_{sp}} - 1 \right) \quad (3.6.11b) \]

\[ m_{f_{\text{total}}} |_{k+1} = m_{f_{\text{total}}} |_k - m_f |_k \quad (3.6.11c) \]

The iteration has converged when

\[ m_{f_{\text{total}}} - \varepsilon \leq m_f |_{k+1} + m_{f_{\text{total}}} |_{k+1} \leq m_{f_{\text{total}}} + \varepsilon, \text{ with } \varepsilon = 10^{-10} \quad (3.6.12) \]

It may be obvious that when the computed \( m_f \) is larger than the available \( m_p \), the latter value is taken. The burn time follows with

\[ m_f = \frac{T}{g_0 I_{sp}} \quad (3.6.13) \]
\[ t_{\text{burn}} = \frac{m_f}{m_i} \quad (3.6.14) \]

### 3.7. The crossrange.

The crossrange of a spacecraft is here defined as the deviation perpendicular to the original groundtrack, i.e., the groundtrack which would have been flown without steering of the spacecraft. In our case, we take the orbit around the Earth as the nominal orbit, and study the effect of the deorbit-burn maneuver on the crossrange at the final altitude. The geometry of the crossrange concept has been depicted in Fig. 3.12.

![Diagram of crossrange](image)

**Fig. 3.12 - Definition of the crossrange.**

The final position in the deorbited trajectory is given by \( \tau_2 \) and \( \delta_2 \). The original trajectory is known, so for the given \( \tau_1 = \tau_2 \) we can compute \( \delta_1 \) and hence \( \Delta \delta \). \( \alpha_1 \) follows from the tangent in position 1, i.e.,

\[ \alpha_1 = \frac{\pi}{2} - \arctan \left( \frac{\Delta \delta}{\delta_1} \right) \quad (3.7.1) \]

With the sine law applied to the spherical triangle, we find for the crossrange

\[ \sin CR = \sin \Delta \delta \sin \alpha_1 \quad (3.7.2) \]

or

\[ CR = \arcsin(\sin \Delta \delta \sin \alpha_1), \quad -90^\circ \leq CR \leq 90^\circ \quad (3.7.3) \]

\( CR \) will in our case have only positive values smaller than 90°, so Eq. (3.7.3) can directly be used. For the crossrange in km (at the Earth’s surface), we use
\[ CR \text{ (km)} = CR \text{ (rad)} \cdot R_e \] (3.7.4)

The above equation for the crossrange is an approximation, because we assumed here a spherical Earth, while in reality the Earth has an elliptical shape. Since the maximum latitude of the groundtrack will be about 30°, which means that the corresponding Earth radius is about 5 km smaller than the equatorial radius, and the variation of this radius for the small crossranges appearing in this report is small, the introduced error will also be small.

Nota bene: the above derivation can only be applied to the sine-shaped groundtracks of 'normal' elliptical and circular orbits, of which the orbits discussed in this report are an example.

4.1. Implementation of the derived models.

The models, as have been discussed in the previous Chapter, have been implemented in START Version 2.0. It concerns the impulsive deorbit-burn manoeuvre and the finite thrust manoeuvre. The crossrange computation is not implemented in the START software, but was used off-line.

The basic layout of the implementation is as follows. Based on the initial conditions, either spherical or cartesian components w.r.t. the $R$-frame, the initial state vector is determined (spherical components w.r.t. $R$-frame). Depending on the fact, whether the deorbit-burn manoeuvre has been selected, some initial computations are performed. These include: the transformation matrix from the orbital to the inertial planeto-centric frame, $C_{iO}$; the transformation of the $\Delta V$-vector to the $I$-frame; the addition of $\Delta V$ to the initial velocity, in case of the impulsive deorbit burn; and the computation of the needed fuel mass and the burn time of the engine, in case of the finite-thrust manoeuvre.

During the simulation, the orientation of the (constant) thrust vector is derived and the motion due to the external forces is computed. Mass properties are updated and the finite-thrust manoeuvre lasts for the complete burn time of the engine, or until there is no fuel mass left. From that moment on, thrust-free motion is being simulated until a stop criterion is reached.

*User-Interface extensions*

The UI extensions are related to two of the four input-data blocks, i.e., the vehicle and the mission block. Part of the vehicle data is the propulsion system; for the deorbit-burn engine the constant specific impulse and thrust in vacuum can be edited. W.r.t. the mission data, the entry of the initial state has been extended with the possibility to edit cartesian position and velocity, and to enter the classical orbital elements. Finally, propulsion operations have been added to the menu structure: the deorbit-burn manoeuvre can be selected, and a choice can be made out of three models, i.e., the impulsive $\Delta V$, and two finite-thrust manoeuvres ($m_f$ computed according to Tsiolkovsky's Equation or $m_f$ specified by the user).

4.2. Evaluation.

4.2.1. Introduction.

The first step of the evaluation of the newly achieved software was a thorough debugging to see whether no physically impossible results were obtained. The second step deals with a comparison between the impulsive $\Delta V$-manoeuvre and the finite-thrust manoeuvre. The outcome of that analysis will be presented in this Section.
The vehicle configuration and mission is based on the ACRV, which is used for the sensitivity analysis in the next Chapter. For all related data, the reader is referred to Section 5.2\(^5\).

For each of the two manoeuvres, we vary the deorbit-burn parameters, i.e., the magnitude and the direction of the \(\Delta V\)-vector. Each of the parameters is varied in three steps: \(\Delta V\) ranges from 150 to 250 m/s, \(\varepsilon_{\Delta V}\) from 180° down to 140° and \(\psi_{\Delta V}\) from 0 to 60°. To limit the total number of simulations, the initial altitude is assumed to be constant for each of the simulations \((h_0 = 280 \text{ km})\). This results in 27 simulations per implementation, so a total of 54 simulations is considered. The list with 27 simulations, which have been selected for both the impulsive and the finite-thrust model, can be found in Table 4.1.

<table>
<thead>
<tr>
<th>Run</th>
<th>(h_0) (km)</th>
<th>(\Delta V) (m/s)</th>
<th>(\varepsilon_{\Delta V}) (°)</th>
<th>(\psi_{\Delta V}) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>280</td>
<td>150</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>280</td>
<td>150</td>
<td>180</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>280</td>
<td>150</td>
<td>180</td>
<td>60</td>
</tr>
<tr>
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<td>150</td>
<td>160</td>
<td>0</td>
</tr>
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<td>280</td>
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<td>30</td>
</tr>
<tr>
<td>6</td>
<td>280</td>
<td>150</td>
<td>160</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>280</td>
<td>150</td>
<td>140</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>280</td>
<td>150</td>
<td>140</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>280</td>
<td>150</td>
<td>140</td>
<td>60</td>
</tr>
<tr>
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<td>280</td>
<td>200</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>280</td>
<td>200</td>
<td>180</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>280</td>
<td>200</td>
<td>180</td>
<td>60</td>
</tr>
<tr>
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<td>280</td>
<td>200</td>
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<td>0</td>
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<td>200</td>
<td>160</td>
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<td>200</td>
<td>160</td>
<td>60</td>
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<tr>
<td>16</td>
<td>280</td>
<td>200</td>
<td>140</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>280</td>
<td>200</td>
<td>140</td>
<td>30</td>
</tr>
<tr>
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<td>280</td>
<td>200</td>
<td>140</td>
<td>60</td>
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<td>250</td>
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<tr>
<td>20</td>
<td>280</td>
<td>250</td>
<td>180</td>
<td>30</td>
</tr>
<tr>
<td>21</td>
<td>280</td>
<td>250</td>
<td>180</td>
<td>60</td>
</tr>
</tbody>
</table>

\(^5\) The reason for not introducing the database here can be explained as follows. The main goal of this report is the derivation of an impulsive deorbit-burn model and the execution of a sensitivity analysis for the ACRV. The finite-thrust manoeuvre has been added to assess the validity of the model. For the reader, who is basically interested in the sensitivity analysis, we have put the outcome of this analysis, including the data with which the results were obtained, in one chapter.
To synchronize with the sensitivity analysis, the same (constant) attitude of the vehicle will be defined. This attitude is a(n) (trimmed) angle of attack of 165°, an angle of sideslip of 0° and a bank angle of 45°. The angle of attack is chosen such, that for the simulations of the sensitivity analysis, the maximum allowable g-load will not exceed 4 g. We will see later on, that for this evaluation this mission requirement is not met. However, since it is not our goal to analyze a mission here, we will not pay attention to this problem. The initial position and velocity are the same for all simulations. We consider a circular, 28.5°-inclined orbit; the vehicle is located at zero-longitude and zero-latitude, corresponding to the ascending node.

For all thrusted manoeuvres, we assume a constant thrust and specific impulse. The values are based on data used for similar studies within European industry (Paris, 1993a), and are partially back-upped by literature (Sackheim et al., 1980). The related values are:

\[
T = 1600 \text{ N}
\]
\[
l_{sp} = 300 \text{ s}
\]

With these values, the deorbit-burn time easily follows with Eqs. (3.6.13) and (3.6.14), if we assume that the fuel mass is just sufficient to perform the manoeuvre, as given by Eq. (3.6.10). The total fuel mass and corresponding burn times are listed in Table 4.2 (\(m_f = 0.544 \text{ kg/s}\)):

<table>
<thead>
<tr>
<th>(\Delta V) (m/s)</th>
<th>(m_f) (kg)</th>
<th>(t_{burn}) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>340.3</td>
<td>625.2</td>
</tr>
<tr>
<td>200</td>
<td>457.7</td>
<td>840.9</td>
</tr>
<tr>
<td>250</td>
<td>577.0</td>
<td>1060.1</td>
</tr>
</tbody>
</table>

Table 4.2 - Fuel consumption and burn times for three \(\Delta V\)-values.
The parameters, which we will study, are the same as for the sensitivity analysis in Section 4.3, and are the following:

- $t_e$: time of entry (s)
- $V_e$: entry velocity (m/s)
- $\gamma_e$: entry flight-path angle (°)
- $\Delta t$: entry duration (s)
- $g$-load<sub>max</sub>: maximum occurring $g$-load (-)
- $t_f$: final time (s), i.e., $t_f = t_e + \Delta t$
- $\tau_f$: final longitude (°)
- $\delta_f$: final latitude (°)
- $CR$: crossrange at final position (km)

The numerical results of the simulations can be found in Table 4.3, at the end of this Section. The relative differences between the thrusted and the impulsive manoeuvre are shown in Table 4.4. These values are based on the results obtained with the impulsive manoeuvre (i.e., the 100%-values).

The discussion of the results is divided into two parts. Subsection 4.2.2 presents the exo-atmospheric phase for one simulation, and Subsection 4.2.3 gives the combined results of all simulations.

### 4.2.2. The exo-atmospheric phase.

We will begin the evaluation by focusing on one simulation for both implementations to get a bit of a feeling about what is happening. We have selected, quite arbitrarily, simulation #9, with deorbit-burn values of $\Delta V = 150$ m/s, $\epsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$. In Fig. 4.1, the height $h$ as a function of flight time $t$ is depicted. The differences between the two curves are obvious. The trajectory of the thrusted manoeuvre is smoothly changing, compared with the abrupt change at $t = 0$ for the impulsive manoeuvre. We see that the vectorial addition of $\Delta V$ has a change in the tangent at $t = 0$ as a result, meaning that the spacecraft is immediately moving downwards. Looking at the thrusted manoeuvre, the tangent is zero at $t = 0$ and is only slowly becoming more negative (denoting a more downward motion) according as the deorbit-burn engine is operating longer.

The two curves intersect at $t = 950$ s, which results in a shorter flight time for the thrusted flight (although the difference is very small). We will see later, that this simulation is the only one with a longer flight time for the impulsive deorbit burn. As an example, we have included the two related curves for simulation #27 ($\Delta V = 250$ m/s, $\epsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$) in Fig. 4.2, where the difference in $t$ is some 200 s (the second 'thrusted' curve is obtained with another guidance scheme; we will come back to this later). The dissimilarity can be explained as follows.

---

6 The entry point is considered to be the point at which the atmospheric boundary is reached. We assume this to be at 120 km altitude.
When the velocity in flight direction is changed, the (orbital) velocity is too small to maintain the current orbit. As a result, the spacecraft moves down (towards the Earth) thereby increasing the velocity. The flight-path angle \( \gamma \) becomes negative (starting at zero for a circular orbit) and the spacecraft is accelerating even more, due to the influence of gravity. There is now a continuous interaction between \( \gamma \) and \( V \). It may be obvious that a discrete change in \( V \) brings on a more rapid change in \( \gamma \) than a gradual change of \( \Delta V \), as in the thrusted manoeuvre.

A \( \Delta V \) which is not collinear with the flight direction, but, for instance, in the orbital plane, has a direct effect on \( \gamma \) (and thus on the direction of motion)\(^7\). For the impulsive manoeuvre, this manifests itself in a discrete change in \( \gamma \). Again, the rate of change of \( \gamma \) for the thrusted manoeuvre is much smoother. However, there is another aspect which is important. The thrust direction is based on the inertial direction of \( \Delta V \). As we can see in Fig. 4.3, the orientation of \( T \), defined here by \( \varepsilon_T \) only, is continuously changing due to the rotation of the body frame. In fact, \( \varepsilon_T \) is getting smaller, which results in a greater effect on \( \gamma \). It is now the combination of magnitude and direction of \( \Delta V \) which determine the downward motion. Apparently is the effect of the relatively large value of \( \varepsilon_{\Delta V} (140^\circ) \) dominating over the effect of the relatively small value of \( \Delta V (150 \text{ m/s}) \) for the thrusted manoeuvre (Fig. 4.1). In Fig. 4.2, \( \Delta V \) is 250 m/s, which is the dominating effect, so the trajectory of the impulsive manoeuvre is steeper right from the beginning.

Fig. 4.4 shows the height-velocity profile. The two curves start in the same point. The instantaneous change in velocity can be seen clearly in the 'impulsive' curve. As we discussed before, after the deorbit-burn manoeuvre the velocity is increasing again. The rate of change is only slightly increasing, as the gravitational force is more or less constant. During the thrusted manoeuvre, the velocity is gradually decreasing. However, the thrust level is relatively low (1600 N) compared to the mass \( (m_{\text{burn-out}} = 6500 \text{ kg}) \), so the decelerating effect is less than 0.25 m/s\(^2\). When \( \gamma \) is becoming more negative, the influence of gravity is getting stronger and finally dominating, which results in an acceleration of the spacecraft (but not as much as it would have been without thrust). The end of the thrusted manoeuvre is at \( V = 7335 \text{ m/s} (h = 240 \text{ km}) \), which is far below the height at which the vehicle was still decelerating \( (h = 275 \text{ km}) \). Because \( T \) had to compensate for gravity as well, the change in velocity is not as high as for the impulsive manoeuvre, partly the cause of a higher end velocity.

Nota bene: when the deorbit-burn engine stops thrusting, the spacecraft has reached a certain state of total energy. This state remains constant, because there are no energy-dissipating forces, such as the drag force, acting on the vehicle. This means that the change in potential energy is directly converted into kinetic energy. Since this is also valid for the impulsive manoeuvre, we see two parallel lines for the height-velocity profile (giving the relation between potential and kinetic energy). It must be stressed that, when we look at the velocity as a function of flight time, the lines are not parallel. In the thrusted case, the velocity increases more in time, due to a different \( \gamma \), as can be seen in Fig. 4.5.

The straight line from \( \gamma = 0^\circ \) to \( \gamma = -0.4^\circ \) for the impulsive manoeuvre represents the instant-

\(^7\) Here, we only study \( \Delta V \)-vectors which are directed horizontally or downwards \( (\varepsilon_{\Delta V} \leq 180^\circ) \).
aneous change in direction and magnitude of V. Due to this change, the spacecraft is being 'pulled down' by gravity, so γ keeps on increasing\textsuperscript{8}. The curve is flattening, and had we moving in the atmosphere, we might have called this effect 'rebound'. However, due to the high velocity (and therefore the large distance covered on the Earth's surface), the local horizontal plane rotates, while the orientation of the velocity vector is more or less fixed in space. As a result, γ is getting smaller. The γ-profile is therefore a combination of the physical 'pulling' and the relative rotation of the local horizontal plane (both effects are counteracting).

A similar effect can be partly seen in the curve for the thrusted manoeuvre. As we discussed before, the fixed orientation of the thrust vector in inertial space, results in a more negative γ. When the thrust has ended, the additional 'pulling' disappears, and the profile is as would have been the case under the influence of gravity only. Due to the higher velocity, the relative motion of the local horizontal plane has a stronger effect on γ, so γ\textsubscript{e} is smaller.

In Fig. 4.6, the γ-profiles of simulation #27 are depicted. These profiles represent the more general case of a longer flight time for the thrusted manoeuvre (see Fig. 4.2). Similar effects, as discussed above, can be discerned. Due to the longer thrust time, the 'rebound' part is much smaller. Again, two curves are shown for the thrusted manoeuvre. These represent two guidance schemes. The 'thrusted-#1' curve has been obtained with the 'normal guidance of a fixed orientation of T'in inertial space. The second scheme assumes a constant orientation of T\textsubscript{w.r.t.} the spacecraft, based on the inertial orientation of ΔV. Because T is rotating as well, in that case, the velocity is decreasing to a greater extent (nota bene: the lower velocity results in an increase of γ, which compensates the difference in thrust angle for guidance scheme #1). Due to the lower velocity, the flight time (#2 compared to #1) is higher (Fig. 4.2). V\textsubscript{e} comes much closer to the one after the impulsive ΔV, but on the other hand, the relative rotation is also smaller, so γ\textsubscript{e} differs. Since the entry profile is very sensitive to the initial conditions at the entry interface, it is impossible to predict which of the two guidance schemes would give results close to the impulsive-shot theory. It needs a lot more study, including other guidance schemes, to come to conclusions. Since this is beyond the scope of the present study, we will restrict now to guidance scheme #1, knowing that the conclusions concerning the results have to be drawn with reservations.

To continue with our discussion on the exo-atmospheric phase of simulation #9, we have arrived at the heading as function of the flight time, as indicated in Fig. 4.7. Three curves are shown, which start all in the same point at t = 0. For the impulsive manoeuvre, the discrete change of V shows as a jump in the χ-profile as well. Since the variation of χ in time is much more gradual for the thrusted manoeuvre, we see the corresponding curve only slowly moving away from the curve of the nominal orbit. The two curves intersect the curve of the nominal orbit at t = 1100 s, because after the deorbit burn, the spacecraft is moving at a continuously decreasing altitude. This means that the vehicle is revolving the Earth at a higher rate, and since the orbit is inclined, χ increases at a higher rate (nota bene: χ gives the orientation of the relative velocity vector w.r.t. the local north).

\textsuperscript{8} We use the term increasing, because the angle between the local horizon and the velocity vector is getting larger, although in numerical terms γ = -1.4°, for instance, is smaller than γ = -1.2°.
The corresponding groundtracks are given in Fig. 4.8. It may be obvious, that a smaller heading results in a more northern motion, which shows as a shift in the groundtrack. The largest difference can be found for the impulsive manoeuvre, because the heading is most northerly directed. This Figure concludes the discussion of the exo-atmospheric phase.

4.2.3. Combined results.

Basis of the discussion will be the results in Tables 4.3 and 4.4. As can be seen in Fig. 4.9, the flight time till entry has a regular pattern for both the impulsive and the thrusted manoeuvre. The 27 simulations can be divided into three groups of nine for every value of ΔV (these groups contain simulation numbers 1-9, 10-18 and 19-27). Per group, we see three smaller sets of three, indicating a variation in $\psi_{\Delta V}$ only. With increasing $\psi_{\Delta V}$, the flight time increases substantially for the impulsive manoeuvre. This is due to the fact that the flight path is much shallower (see also Fig. 4.11). Decreasing values for $\epsilon_{\Delta V}$ (180° down to 140°) will decrease the flight time, because the path will be more directed towards the CoM of the Earth. Higher values for $\Delta V$, finally, will make the path steeper as well, as we already discussed in the previous Subsection.

The differences between the two implementations were covered there as well. The relative differences vary between -3.2% and 86.2% (!). We will restrict to one last remark, that, although at first sight the pattern of the entry times for the thrusted manoeuvre resembles the pattern for the impulsive one, there are some differences as well. In the first place, the relative differences are not as large, because of the gradual change of $V$, and in the second place, every third subset seems to have shifted up a bit. Apparently, the optimum $\epsilon_{\Delta V}$ to minimize the entry time lies within the interval of [180°, 140°], while for the impulsive manoeuvre the optimum value has not been reached yet.

Fig. 4.10 shows the entry velocity for each of the simulations. Again, we see for the impulsive manoeuvre a regular pattern, but for the thrusted manoeuvre this pattern looses its shape for larger values of $\Delta V$. A larger $\Delta V$ means a comparably longer thrust time, and therefore a more gradual change in velocity (and a vaguer pattern as a result). The differences between the two manoeuvres look substantial, but because of the high velocities, the relative differences are quite small (less than 2%).

These differences are much larger (up to 22.5%) for the entry angle (Table 4.4), although at first sight the results compare quite well (Fig. 4.11). We will see later, that differences in entry conditions can have a large effect on the entry trajectory. For smaller values of $\epsilon_{\Delta V}$ the differences between the two implementations get smaller as well. We discussed in the previous Subsection, that because of the applied guidance scheme, $\gamma$ is more negative than it would be in free fall. For longer thrust times, the effect on $\gamma$ will be larger and so will be the difference with the impulsive manoeuvre.

The entry duration (Fig. 4.12) is directly dependent on the entry conditions ($V_e$ and $\gamma_e$). The results between the two implementations compare quite well, although in some cases the difference is over 20%. That the entry trajectory is sensitive to the entry conditions can be seen for
simulation #9. Here, the entry lasts 200 s longer, although there is only a small difference in entry conditions.

The differences in case of the maximum $g_{load}$ show a more irregular pattern (Fig. 4.13). There is a direct relation between the entry conditions and the maximum $g_{load}$ (Mooij, 1991b). As we saw before, the two implementations result in quite substantial differences in these entry conditions. The results are therefore not surprising. In most cases, the impulsive manoeuvre gives a higher $g_{load}$, which would result in an over-design of the mission, when using this method.

To conclude the discussion on the entry conditions, we give the relation between $V_e$, $\gamma_e$, the entry duration and the maximum $g_{load}$ in Figs. 4.14-4.20. In general terms, the results obtained with the finite-thrust implementation give a more irregular pattern. A distinct relation can be found for the maximum $g_{load}$ as a function of the entry duration (Fig. 4.16). There is an obvious optimum, namely for an entry duration of about 575 s, the maximum $g_{load}$ reaches a minimum value of 3.15 g. A relation is also clear for the maximum $g_{load}$ as a function of the entry conditions (Figs. 4.17 and 4.18), but only for the impulsive manoeuvre. This is evident when we look at the three-dimensional plots of Figs. 4.19 and 4.20.

In Fig. 4.21, the height as a function of the flight time is shown for the two simulations (#1 and #9). Since the whole trajectory is shown, we can see how important the correct entry conditions are for the entry phase. The two simulations are designated the 'no-skip' entry, meaning that the entry path is steep and the entry duration short, and the 'skip entry' with a shallower path. For the latter type of entry, at lower altitudes the dynamic pressure (and thus also the aerodynamic drag) is getting so high that the descent rate, i.e., the radial component of the velocity vector, is decreasing substantially, showing as a flattening of the curve. Nota bene: with 'skip' we do not mean that the vehicle is skipping in the atmosphere and ascending again; we use this name to indicate the different type of entries.

The no-skip entry does not show much of a difference between the two implementations, although there is a shift in time for the thrusted manoeuvre due to the finite-thrust time. A similar shift can be seen for the skip entry, but now the shift is not constant. During the burn, the difference is increasing, starting from zero. The two curves intersect and remain quite close, even in the first part of entry. For the thrusted manoeuvre, the entry angle is slightly smaller and the velocity is somewhat higher than for the impulsive manoeuvre. The combination of the two parameters is finally the cause of the increasing difference. The skip is even more emphatically visible in the case of the thrusted manoeuvre.

The $g_{load}$ is shown in relation to the height in Fig. 4.22. The two types of entry for the thrusted manoeuvre are shown. The curves indicate, that for the steeper entry, the $g_{load}$ is increasing sooner (at higher altitudes), but as a result the maximum value is smaller, as has already been explained. The small irregularity at the end of the curves should be studied in more detail to give a full explanation. It is assumed that it is related with the initiation of a spiralling motion of the ACRV (due to banking), as we will see later in the groundtracks. Fig. 4.23 gives the corresponding velocity-height profiles.
Figs. 4.24-4.27 show the groundtracks of both the no-skip and skip entry, for the two implementations. First, Fig. 4.24 shows the groundtrack for the no-skip entry. The differences are not very large, because the deorbit burn was no out-of-plane manoeuvre. Due to the longer flight time, the groundtrack for the thrusted manoeuvre extends over a longer distance. At the end of the curves, especially the impulsive groundtrack, we see the initiation of the spiralling motion of the ACRV. Due to the bank angle of $45^\circ$, the lift force is tilted, and the vehicle is going to fly a circular trajectory, whilst at the same time moving downwards. The skip entry gives the second set of groundtracks (Fig. 4.25). Since the burn is an out-of-plane manoeuvre, we see a difference occurring right from $t = 0$. The graph does not give surprising results, compared with the results which we have already seen. The same applies for the third and fourth set of groundtracks in Figs. 4.26 and 4.27, because they give the same groundtracks as before, but in a different combination. They have been added for the sake of completeness.

Figs. 4.28-4.30 give the footprints for the three $\Delta V$-values. Per Figure, the two implementations have been put together. We see of course a shift in the footprint for the thrusted manoeuvre, because of the longer flight time (and the same initial position). What is more surprising, however, is that the same order of simulations (labelled in the graphs) does not give the same order in the results. In Fig. 4.28, for example, the first three results are the ones of simulations #7, #4 and #1 (impulsive manoeuvre) compared with #4, #1 and #5 (thrusted manoeuvre). Besides, in Fig. 4.30, the results of the thrusted manoeuvre are much more condensed than the ones of the impulsive manoeuvre.

Of course, this result does not come like a bolt from the blue. We already extensively discussed the influence of the entry conditions on the remaining flight. What is more critical, is that the two different implementations are the cause of these different entry conditions. The influence on the crossrange is significant, although in principle the results of the two implementations cannot be compared directly. If the initial positions for the two implementations could be shifted such, that in both cases the ACRV would arrive at the top of the sinusoid ($\tau_f = 84.14^\circ$), the results are not as bad as it seems.

We have done four additional simulations for #9 (out of plane) and #25 (in plane), in order to achieve this. The results are given in Table 4.5. The conclusion, which can be drawn from the results, is that it is actually a rather big difference. The two crossranges differ about 37 (27.7%) and 12 km (15.7%). It must be noted, however, that the choice of guidance scheme could have a large impact on the results, so at least further study with different guidance schemes is required to come to a final conclusion.

In Figs. 4.31 and 4.32, the groundtracks of the above mentioned simulations are shown. The discontinuity is due to the definition of the longitude ($0^\circ \leq \tau < 360^\circ$). Especially in Fig. 4.32 the large difference in covered distance can be seen.
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Table 4.3 - Simulation results for the evaluation of the Impulsive (I) and the finite-thrust (FT) do not disturb manoeuvre.
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Table 4.4 - Relative differences of the finite-thrust deorbit-burn manoeuvre w.r.t. the impulsive manoeuvre.
### Table 4.5 - Crossrange comparisons for similar final longitude.

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<th>$\delta_0$ (°)</th>
<th>$\delta_{\text{max}}$ (°)</th>
<th>CR (km)</th>
<th>$\Delta CR$ (%)</th>
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### Fig. 4.1 - Height as a function of flight time for the exo-atmospheric phase. The two simulations considered are the impulsive (solid line) and thrusted (dashed line) implementation for $\Delta V = 150$ m/s, $\epsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$. 
Fig. 4.2 - Height as a function of flight time for the exo-atmospheric phase. The two simulations considered are the impulsive (solid line) and thrusted (dashed lines) implementation for $\Delta V = 250 \text{ m/s}$, $\varepsilon_{AV} = 140^\circ$ and $\psi_{AV} = 60^\circ$. The thrusted-#1 curve represents guidance based on the fixed inertial orientation of the thrust force, whereas 'thrusted #2' indicates a constant thrust direction based on the initial orientation of the $\Delta V$-vector.

Fig. 4.3 - The orientation of the thrust vector is based on the inertial orientation of $\Delta V$, here indicated by $\varepsilon_{AV}$. When the spacecraft is orbiting around the Earth, the orientation of the body frame is continuously changing (for the sake of convenience, we assume that the velocity vector indicates the body frame; for exo-atmospheric flight, the negative $X_P$-axis is aligned with the velocity w.r.t. the $R$-frame. Simplifying the orbit to a (constant) circular orbit, we see that the thrust angle is decreasing. This results in a stronger effect on the flight-path angle, which is getting more negative.
Fig. 4.4 - Height as a function of velocity for the exo-atmospheric phase. The two simulations considered are the impulsive (solid line) and thrusted (dashed line) implementation for $\Delta V = 150$ m/s, $\epsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$.

Fig. 4.5 - Flight-path angle as a function of velocity for the exo-atmospheric phase. The two simulations considered are the impulsive (solid line) and thrusted (dashed line) implementation for $\Delta V = 150$ m/s, $\epsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$. 
Fig. 4.6 - Flight-path angle as a function of velocity for the exo-atmospheric phase. The two simulations considered are the impulsive (solid line) and thrusted (dashed lines) implementation for $\Delta V = 250$ m/s, $\varepsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$. The thrusted-$\#1$ curve represents guidance based on the fixed inertial orientation of the thrust force, whereas thrusted $\#2$ indicates a constant thrust direction based on the initial orientation of the $\Delta V$-vector.

Fig. 4.7 - Heading as a function of flight time for the exo-atmospheric phase. The two simulations, the impulsive (long dashes) and thrusted (short dashes) implementation for $\Delta V = 150$ m/s, $\varepsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$, are compared with the nominal orbit (solid line).
Fig. 4.8 - The groundtrack for the exo-atmospheric phase. The two simulations considered are the impulsive (solid line) and thrusted (dashed line) implementation for $\Delta V = 150$ m/s, $\epsilon_{\Delta V} = 140^\circ$ and $\psi_{\Delta V} = 60^\circ$.

Fig. 4.9 - The entry time for each of the simulations. The solid line represents the impulsive manoeuvre, whereas the dashed line is related to the finite-thrust manoeuvre. The line connections do not imply a physical relation.
Fig. 4.10 - The entry velocity for each of the simulations. The solid line represents the impulsive manoeuvre, whereas the dashed line is related to the finite-thrust manoeuvre. The line connections do not imply a physical relation.

Fig. 4.11 - The entry angle for each of the simulations. The solid line represents the impulsive manoeuvre, whereas the dashed line is related to the finite-thrust manoeuvre. The line connections do not imply a physical relation.
Fig. 4.12 - The entry duration for each of the simulations. The solid line represents the impulsive manoeuvre, whereas the dashed line is related to the finite-thrust manoeuvre. The line connections do not imply a physical relation.

Fig. 4.13 - The maximum $g_{load}$ during entry for each of the simulations. The solid line represents the impulsive manoeuvre, whereas the dashed line is related to the finite-thrust manoeuvre. The line connections do not imply a physical relation.
Fig. 4.14 - The entry duration versus the entry velocity. Squares show the results of the impulsive manoeuvre and pluses of the finite-thrust manoeuvre.

Fig. 4.15 - The entry duration as a function of the entry angle. Squares show the results of the impulsive manoeuvre and pluses of the finite-thrust manoeuvre.
Fig. 4.16 - The maximum $g_{load}$ versus the entry duration. Squares show the results of the impulsive manoeuvre and pluses of the finite-thrust manoeuvre.

Fig. 4.17 - The maximum $g_{load}$ as a function of the entry velocity. Squares show the results of the impulsive manoeuvre and pluses of the finite-thrust manoeuvre.
Fig. 4.18 - The maximum $g_{load}$ as a function of the entry angle. Squares show the results of the impulsive manoeuvre and pluses of the finite-thrust manoeuvre.

Fig. 4.19 - The maximum $g_{load}$ as a function of the entry velocity and the entry angle for the impulsive deorbit-burn manoeuvre.
max. \textit{g\_load} (-)

7350 7400 7450

entry velocity (m/s)

-2.5 entry angle (deg)

-1

-2

4.5

4

3.5

Fig. 4.20 - The maximum \textit{g\_load} as a function of the entry velocity and the entry angle for the finite-thrust deorbit-burn manoeuvre.

Fig. 4.21 - The height as a function of flight time for a skip entry (right two lines: the solid line is the impulsive manoeuvre and the dashed line the finite thrust one) and a no-skip entry (left two lines).
Fig. 4.22 - The $g_{load}$-height profile for the no-skip (solid line) and skip (dashed line) entry. Results obtained with the finite-thrust implementation.

Fig. 4.23 - The velocity-height profile for the no-skip (solid line) and skip (dashed line) entry. Results obtained with the finite-thrust implementation.
Fig. 4.24 - The ground track for the no-skip entry. The two curves represent the impulsive (solid) and thrusted (dashed) implementation.

Fig. 4.25 - The ground track for the skip entry. The two curves represent the impulsive (solid) and thrusted (dashed) implementation.
Fig. 4.26 - The groundtrack for the no-skip (solid line) and the skip (dashed line) entry. The results are obtained with the impulsive implementation.

Fig. 4.27 - The groundtrack for the no-skip (solid line) and the skip (dashed line) entry. The results are obtained with the thrusted implementation.
Fig. 4.28 - The footprint for the deorbit-burn manoeuvre with a $\Delta V$ of 150 m/s. Results are obtained with the impulsive-$\Delta V$ implementation (top) and the finite-thrust implementation (bottom). The numbers in the graphs represent the simulation numbers.
Fig. 4.29 - The footprint for the deorbit-burn manoeuvre with a $\Delta V$ of 200 m/s. Results are obtained with the impulsive-$\Delta V$ implementation (top) and the finite-thrust implementation (bottom). The numbers in the graphs represent the simulation numbers.
Fig. 4.30 - The footprint for the deorbit-burn manoeuvre with a $\Delta V$ of 250 m/s. Results are obtained with the impulsive-$\Delta V$ implementation (top) and the finite-thrust implementation (bottom). The numbers in the graphs represent the simulation numbers.
Fig. 4.31 - Crossrange comparison for simulation configuration #9. The final point is at the top of the sinusoid. The groundtracks for the nominal orbit (solid line), the impulsive (dashed line) and thrusted (small dashes) manoeuvre are shown.

Fig. 4.32 - Crossrange comparison for simulation configuration #25. The final point is at the top of the sinusoid. The groundtracks for the nominal orbit (solid line), the impulsive (dashed line) and thrusted (small dashes) manoeuvre are shown.
5. Deorbit-burn analysis.

5.1. Introduction.

The international space community (NASA, ESA, NASDA) is preparing for the development of a permanently manned space station, the International Space Station 'Freedom'. For this station, NASA has recognised that the crew should always have a possibility to return to Earth. This might be the case when one of the astronauts becomes ill, but also when a hazardous situation occurs like an unavoidable collision with big fragments of space debris. These kinds of return missions can be performed by the so-called Assured Crew Return Vehicle, abbreviated to ACRV.

ESA recognised that the design and development of the ACRV could be part of a cooperation between the Agency and its American counterpart, NASA. Succeeding Phase-A studies showed the following main results (Nérault, 1993):

- a Viking or Apollo type capsule could satisfy the requirements;
- due to launcher diameter constraints the ACRV has to be made up of two modules (Fig. 5.1). A resource module holding the propulsion subsystem would be separated just before re-entry and would then burn up. The other, the re-entry capsule would carry the crew and provides for the necessary life-support equipment and the landing system;

Fig. 5.1 - The Pase-A configuration of the ACRV; the resource module is shown on the left (Nérault, 1993).
• the landing system exists of parachutes (similar to Apollo), retro-rockets and shock-absorbing devices to reduce the impact shock;

• the overall mass of the ACRV would be in the order of 8.6 tonnes at the time of separation from the Space Station; this includes 1.9 t for the resource module.

An activity, which fits well within the framework of the shadow engineering performed at ESTEC, is the sensitivity analysis of the deorbit-burn manoeuvre on the footprint of the ACRV. This sensitivity analysis will be described in this Chapter. The analysis takes an impulsive $\Delta V$ into account, as has been requested by ESTEC (Paris, 1993a). Section 5.2 describes the ACRV database, which has been used for all the simulations. In Section 5.3, the actual sensitivity analysis is introduced. A list of all simulations, which have been executed, is presented. Then the outcome of the sensitivity analysis is shown, in the form of graphs and tables. A discussion on the results concludes this Section.

5.2. The ACRV database.

As has been mentioned in the introduction, the ACRV consists basically of two elements, i.e., the resource module and the actual re-entry capsule. The resource module is of no use after the deorbit-burn manoeuvre and it is assumed that this module is jettisoned briefly after the burn (Paris, 1993a). Since we will consider an impulsive $\Delta V$ and the trajectory from burn to re-entry is a free-fall trajectory outside the atmosphere, only the re-entry capsule is important to us\(^9\).

With respect to the mass properties of the ACRV re-entry capsule, we can suffice with the total mass of the vehicle, since only three-degrees-of-freedom simulations will be executed. This mass is\(^10\)

\[ m = 6500 \text{ kg} \]

A major part of the database consists of the aerodynamic data. The ACRV is an axisymmetric body, and because we do not consider a slipping flight, we only need to have aerodynamic coefficients for lift and drag. These coefficients are based on the aerodynamic database of the Apollo, as described by Mosely et al. (1967), and are given in Table 5.1 and 5.2. Both coefficients are a function of the Mach number $M$ and the angle of attack $\alpha$.

---

\(^9\) When the deorbit-burn manoeuvre is considered to be a finite-thrust manoeuvre, the thrust force must accelerate (or decelerate) the total mass of the vehicle. In that case, the resource module must be taken into account and can only be jettisoned when the deorbit burn has been concluded (not to mention the fact, that the propulsion system is integrated in the resource module!). However, as we explained in the previous Chapter, we considered an identical mass for both the impulsive $\Delta V$ and the finite-thrust manoeuvre, in order to study the effect of different implementations only. For accurate mission analyses of the ACRV, this approach is not valid, of course.

\(^10\) The database, which is presented in this Section, has been specified by ESTEC for this particular analysis and might differ for other analyses.
The reference surface, needed to compute the actual aerodynamic forces is based on a capsule diameter of 4.4 m, so

\[ S_{ref} = \frac{\pi d^2}{4} = 15.2053 \text{ m}^2 \]

The extrapolation method, which is used to compute coefficients outside the table range, is the one using the boundary value.
Environmental parameters concern the choice of the central body (Earth), the atmosphere model (US Standard Atmosphere 1976, with the atmospheric boundary at 120 km) and the gravity model (central field, including $J_2$ harmonic, with $J_2 = 1.082627 \cdot 10^{-3}$). Besides, there is no wind.

The initial state vector is given by classical orbital elements for position and velocity, and the attitude in the form of aerodynamic angles\textsuperscript{11}. The orbit from which the ACRV departs is a circular one ($e = 0$), with an inclination of 28.5°. The semi-major axis $a$, or in this case the radius, of the orbit is not the same for all simulations, since we will study three departure heights: $h_0 = 280$ km, 350 km and 450 km. The Earth's radius, as defined in START, is $R_e = 6378.139$ km, so we get for the three values of $a$:

\[
\begin{align*}
  a_1 &= 6658.139 \text{ km} \\
  a_2 &= 6728.139 \text{ km} \\
  a_3 &= 6828.139 \text{ km}
\end{align*}
\]

The remaining three (initial) orbital elements, i.e., $\omega$, $\Omega$ and $M$, are identical to zero for all simulations.

The attitude (aerodynamic angles) of the vehicle has (have) been defined as follows. The angle of sideslip and the bank angle were specified, i.e.,

\[
\begin{align*}
  \beta &= 0^\circ \text{ (no slip)} \\
  \sigma &= 45^\circ \text{ (deviation to the North)}
\end{align*}
\]

The angle of attack has to be chosen such, that the maximum $g_{\text{load}}$ does not exceed 4 g. A value of $\alpha = 165^\circ$, the so-called trimmed angle-of-attack, gave peak values of about 3.5 g for all simulations, as we will see later, and was considered to be acceptable.

The deorbit-burn specifications are variable, and will be introduced in the next Section.

Throughout the simulations, we use the same numerical integration method, namely the Runge-Kutta-Fehlberg with variable step size. In all cases, the maximum step size and the local error conditions were the same, i.e.,

\[
\begin{align*}
  dt_{\text{max}} &= 1 \text{ s} \\
  dr &= 0.1 \text{ m} \\
  d(t, \delta) &= 5.73 \cdot 10^{-3} \degree \\
  dV &= 0.1 \text{ m/s} \\
  d(\gamma, \chi) &= 5.73 \cdot 10^{-3} \degree
\end{align*}
\]

\textsuperscript{11} Despite the fact that we do only 3-dof simulations, we have to specify a (constant!) attitude of the vehicle w.r.t. the oncoming flow. This attitude is only taken into account for a flight in the atmosphere; for exo-atmospheric flight the equilibrium attitude is maintained: $\alpha = 180^\circ$ (so-called 'zero-angle-of-attack'), $\beta = \sigma = 0^\circ$. 
The stop criterion will be the final height: \( h_f = 5 \text{ km} \). As a safety measure for a possible skip entry, a maximum flight time of 5000 s has been added as a stop criterion.

The complete input set, which was used for the sensitivity analysis, is given in Appendix A.

### 5.3. Sensitivity analysis.

#### 5.3.1. List of simulations.

The simulations, which have to be part of the sensitivity analysis, are given by ESTEC (Paris, 1993b). However, since the orientation of the \( \Delta V \)-vector was specified differently compared with the definition used in this report, the direction angles had to be adjusted. The resulting values for the deorbit-burn manoeuvres are given in Table 5.3.

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Table 5.3 - List of simulations for the sensitivity analysis.
5.3.2. Results and discussion.

The parameters, which we will study in this sensitivity analysis, are the same as we studied during the evaluation of the two deorbit-burn implementations. For the sake of convenience, they will be restated below:

\[ t_e \] : time of entry (s)
\[ V_e \] : entry velocity (m/s)
\[ \gamma_e \] : entry flight-path angle (°)
\[ \Delta t \] : entry duration (s)
\[ g\text{-load}_{\text{max}} \] : maximum occurring g-load (-)
\[ t_f \] : final time (s), i.e., \( t_f = t_e + \Delta t \)
\[ \tau_f \] : final longitude (°)
\[ \delta_f \] : final latitude (°)
\[ CR \] : crossrange at final position (km)

The numerical results of the simulations can be found in Table 5.4 at the end of this Section.

We will start the discussion with looking at the entry variables (at \( h_0 = 120 \text{ km} \)). In Fig. 5.2, the entry angle is show as a function of entry time. We see three curves, indicating the three initial heights of \( h_0 = 280, 350 \) and 450 km. The flight starting at a higher altitude takes a longer time, of course. Besides, when \( \Delta V \) is larger, the capsule will follow a steeper path, as we saw in Section 4.2.2. But, as we see in Fig. 5.3, the entry velocity is smaller for larger values of \( \Delta V \). This is due to the fact that the ACRV starts with a lower velocity after the burn, and despite the steeper path it will not reach a higher velocity because the trajectory is too short for that. Starting at a higher altitude, results in a higher \( V_e \), of course, because of the longer flight time (and therefore longer acceleration due to gravity). This behaviour can also be seen in Figs. 5.4 and 5.6.

To explain the differences in \( \gamma_e \) is a bit trickier. On one hand, the flight-path angle increases (more negative!) when \( \Delta V \) is larger - the lower the velocity is compared to the nominal velocity for that height, the steeper the path will be. On the other hand, the vehicle will follow a shallower path when the flight lasts longer, due to the rotation of the local horizontal plane (l.h.p.). It is this combination, which determines \( \gamma \) at \( h_0 = 120 \text{ km} \). To illustrate this, we have included the results of four additional simulations (two \( \Delta V \)-values, \( \Delta V 125 \) and 250 m/s for two initial heights, \( h_0 = 280 \) and 450 km). The \( \Delta V \)-angles are for all simulations the same, i.e., \( \varepsilon_{\Delta V} = 180^\circ \) and \( \psi_{\Delta V} = 0^\circ \).

Figs. 5.4 and 5.5 show the height as a function of velocity and flight-path angle (\( \Delta V = 125 \text{ m/s} \)). As we discussed before, \( V_e \) is larger when the initial height is larger (compare with Fig. 5.6: larger \( \Delta V \)-values give lower entry velocities). Starting at the higher altitude, the 'rebound' due to the rotation of the l.h.p. shows clearly. For the lower initial height, the rebounding effect is there, but before the flight could turn into a shallower one, the atmospheric boundary is reached. This results in an intersection of the two curves and a corresponding lower \( \gamma_e \) for \( h_0 = 450 \text{ km} \). Fig. 5.7, however, gives the opposite result for \( \gamma_e \) - Due to the lower velocity throughout the trajectory, the path is steeper and the rotation of the l.h.p. is smaller. The flight
to the entry height is not long enough to get a shallower path.

Nota bene: the changes in $\gamma$ are quite large, especially before and after the minimum value. This means that a small difference in flight time due to different values for $\Delta V$ and $h_D$ can have a large effect on $\gamma$. This shows in all the related Figures as a larger spreading of the results for $h_D = 450$ km.

In the previous Chapter, we already saw the sensitivity of the entry trajectory to the entry parameters $V_e$ and $\gamma_e$. Since the relative change of $\gamma_e$ is much larger than the change in $V_e$\textsuperscript{12}, we will focus on $\gamma_e$. In Fig. 5.8, the entry duration is depicted as a function of the entry angle. Shallower entries correspond with comparably longer flight times, as is clearly shown.

For higher initial orbits, the entries are steeper (shorter entry duration). However, the entry velocities are larger, and, in combination with larger values for $\gamma$, the vehicle has a tendency to skip slightly in the atmosphere, which increases the flight time again. This effect results, for instance, in a larger entry duration for $h_D = 450$ km and $\Delta V = 125$ m/s than one might expect on basis of the variation of the results. The difference in entry angle ($= 1^\circ$) results in differences of some 175 s in entry duration.

One might expect, that a shorter entry duration will result in larger decelerations and thus higher maximum $g_{loads}$. Looking at Fig. 5.9 and Table 5.4, however, we see that the opposite is true. Steeper entries (with higher velocities) result in a higher dynamic pressure at higher altitudes. As a result, the deceleration due to drag is more effective at these higher altitudes, but as an absolute value still small. Due to this stronger decrease in velocity, the maximum dynamic pressure (and thus also the maximum $g_{load}$) will be lower. For all simulations, we see that the maximum $g_{load}$ stays well below the constraint on 4 g. This indicates, that the ACRV could fly with a higher trimmed angle of attack than it is doing now ($\alpha = 165^\circ$). This would increase the flight time in the atmosphere, and that would have a positive effect on the crossrange (especially for a banking flight), as we will see later.

The three remaining Figures, i.e., Figs. 5.10-5.12, show the groundtracks of the deorbited flights compared with the groundtrack of the nominal orbit (this is the orbit in which the ACRV would revolve the Earth, when it would not have deorbited). Besides, the position at the final height of $h_f = 5$ km are indicated, thus giving the footprint of the ACRV.

It must be noted, that crossrange comparisons are in principal not possible, because the final positions are not the same. Due to differences in longitude, the final latitude changes, which has an influence on the flight dynamics\textsuperscript{13}. A good comparison can therefore only be made when the final longitude will be the same for all simulations (which means different initial

\textsuperscript{12} The entry angle ranges from -1.30$^\circ$ down to -2.29$^\circ$, which means a variation of 43%. The entry velocity varies from 7431.5 m/s to 7498.1 m/s, which is only a 0.9%-variation.

\textsuperscript{13} This influence is a result of the rotation of the Earth. The velocity relative to the Earth (and related to this, the groundtrack relative to the Earth) can be divided into an inertial component and a tangential velocity due to the planetary rotational rate, and the latter component is a function of both latitude and height, see also Fig. 5.13).
positions in each case). As we can see, the maximum crossrange appears at the top of the sinusoid, so the corresponding longitude should be the reference value. Since it was not directly possible for this study, the current simulations have been done for the same initial position.

The crossrange varies from 137 to 277 km, but is not only the result of different deorbit-burn manoeuvres. The ACRV has a constant bank angle of 45°, which tilts the lift force out of plane so the vehicle is deviating from its nominal path. When we study the groundtracks somewhat closer, we see that the groundtrack is strongly curved near the final position. This is due to the banking. The velocity at the final height is relatively low ($V_f = 110$ m/s), and the banking has a relative strong effect on the trajectory so that the vehicle starts spiralling. It may be obvious that this has a large effect on the crossrange. In order to see the influence of the deorbit burn manoeuvre only, it is therefore advised to simulate with zero bank-angle.

The value of $\Delta V$ and the initial height have a major influence on the flight time and therefore the final position. A higher $\Delta V$ shortens the total flight (see Table 5.4), which means a final latitude closer to the top of the sinusoid. Starting at higher altitudes, however, increase the flight times substantially, so a longer trajectory is flown. The curves verify these statements.

<table>
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<tr>
<th>Nr</th>
<th>$t_0$ (s)</th>
<th>$V_e$ (m/s)</th>
<th>$\gamma_e$ (°)</th>
<th>$\Delta t$ (s)</th>
<th>$g$-load$_{max}$ (-)</th>
<th>$t_f$ (s)</th>
<th>$\gamma_f$ (°)</th>
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<th>CR (km)</th>
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Table 5.4 - Results of the sensitivity analysis.
Fig. 5.2 - The entry angle as a function of the entry time. Three curves can be distinguished, indicating the three initial heights 280, 350 and 450 km, from left to right. Increasing $\Delta V$ shows from top to bottom.

Fig. 5.3 - The entry velocity versus the entry time. The three curves give the results for the three initial heights 280, 350 and 450 km, from right to left. Increasing $\Delta V$ shows from top to bottom.
Fig. 5.4 - The height as a function of the velocity for the two initial heights of $h_0 = 280$ (solid line) and 450 km (dashed line). Deorbit-burn values are $\Delta V = 125$ m/s, $\varepsilon_{\Delta V} = 180^\circ$ and $\psi_{\Delta V} = 0^\circ$.

Fig. 5.5 - The height versus the flight-path angle for the two initial heights of $h_0 = 280$ (solid line) and 450 km (dashed line). Deorbit-burn values are $\Delta V = 125$ m/s, $\varepsilon_{\Delta V} = 180^\circ$ and $\psi_{\Delta V} = 0^\circ$. 
Fig. 5.6 - The height as a function of the velocity for the two initial altitudes of $h_0 = 280$ (solid line) and 450 km (dashed line). Deorbit-burn values are $\Delta V = 250$ m/s, $\epsilon_{\Delta V} = 180^\circ$ and $\psi_{\Delta V} = 0^\circ$.

Fig. 5.7 - The height versus the flight-path angle for the two initial altitudes of $h_0 = 280$ (solid line) and 450 km (dashed line). Deorbit-burn values are $\Delta V = 250$ m/s, $\epsilon_{\Delta V} = 180^\circ$ and $\psi_{\Delta V} = 0^\circ$. 
Fig. 5.8 - The entry duration as a function of the entry angle. Three curves can be distinguished, indicating the three initial heights 280, 350 and 450 km, from right to left. Increasing ΔV shows from top to bottom.

Fig. 5.9 - The maximum g_load as a function of the entry angle. Three curves can be distinguished, indicating the three initial heights 280, 350 and 450 km, from right to left. Increasing ΔV shows from top to bottom.
Fig. 5.10 - The groundtrack of the trajectories after the deorbit burn, for an initial height of $h_0 = 280$ km (top). The solid line indicates the nominal orbit. The deorbited groundtracks read from bottom to top for increasing $\Delta V$-values. In the lower graph, the footprint after the deorbit burn at $h_f = 5$ km is shown. The labels indicate the $\Delta V$-values in m/s.
**Fig. 5.8** - The entry duration as a function of the entry angle. Three curves can be distinguished, indicating the three initial heights 280, 350 and 450 km, from right to left. Increasing $\Delta V$ shows from top to bottom.

**Fig. 5.9** - The maximum $g_{load}$ as a function of the entry angle. Three curves can be distinguished, indicating the three initial heights 280, 350 and 450 km, from right to left. Increasing $\Delta V$ shows from top to bottom.
Fig. 5.10 - The groundtrack of the trajectories after the deorbit burn, for an initial height of $h_0 = 280$ km (top). The solid line indicates the nominal orbit. The dotted groundtracks read from bottom to top for increasing $\Delta V$-values. In the lower graph, the footprint after the deorbit burn at $h_f = 5$ km is shown. The labels indicate the $\Delta V$-values in m/s.
Fig. 5.11 - The groundtrack of the trajectories after the deorbit burn, for an initial height of $h_0 = 350$ km (top). The solid line indicates the nominal orbit. The deorbited groundtracks read from bottom to top for increasing $\Delta V$-values. In the lower graph, the footprint after the deorbit burn at $h_f = 5$ km is shown. The labels indicate the $\Delta V$-values in m/s.
Fig. 5.12 - The groundtrack of the trajectories after the deorbit burn, for an initial height of $h_0 = 450$ km (top). The solid line indicates the nominal orbit. The deorbited groundtracks read from bottom to top for increasing $\Delta V$-values. In the lower graph, the footprint after the deorbit burn at $h_f = 5$ km is shown. The labels indicate the $\Delta V$-values in m/s.
Fig. 5.13 - The tangential velocity due to the rotation of the Earth, as a function of latitude and height. The five curves indicate increasing heights, starting at $h = 100$ km (solid line) and ending with $h = 500$ km (dash-dot line); $\Delta h = 100$ km.
6. Conclusions and recommendations.

The Assured Crew Return Vehicle (ACRV) is designed to be a means for the astronauts of the Space Station 'Freedom' to return safely to the Earth at any time. After leaving the Space Station, the ACRV will leave the orbit of the Station by means of a deorbit-burn manoeuvre, and move towards the atmosphere. The choice of magnitude and direction of the burn has an impact on both the entry conditions and the crossrange capabilities of the vehicle.

This report presents the implementation of the deorbit-burn manoeuvre in the Simulation Tool for Atmospheric Re-entry Trajectories (START). Two models have been derived, i.e., the impulsive and finite-thrust deorbit burn. To match with different ways of entering the initial conditions (spherical and cartesian components for the position and velocity, and the classical orbital elements), the relation between the different sets of state variables has been discussed.

The two derived and implemented models were evaluated, to indicate possible differences in the results. It became obvious that the two different implementations resulted in large shifts in flight time, differences in entry conditions (entry angle and velocity) and maximum occurring $g_{load}$. The chosen guidance scheme (constant inertial thrust direction) appeared to have a significant influence on the entry conditions, and therefore the whole entry phase. The footprints for the two implementations showed large differences, partly because the same initial position for all simulations was chosen. Additional simulations to arrive at the same final longitude, still showed a rather large difference in crossrange (the maximum difference was almost 30%).

A final conclusion about the comparison of the two models is not completely possible without further study. It became obvious, however, that the impulsive $\Delta V$ gives a good indication about what is happening, although one should keep in mind that large differences with a 'real-life' thrusted manoeuvre can occur. The efficiency of the thrusted manoeuvre is, on the other hand, depending on a wisely selected guidance scheme.

Finally, START was used to do a sensitivity analysis for the ACRV. After presenting the database of the ACRV, 18 simulations were executed, taking three initial heights (280, 350 and 450 km) and 6 $\Delta V$-magnitudes (125 to 250 m/s) with all different directions for the $\Delta V$-vector into account. The ACRV flew with a trimmed angle of attack of $\alpha = 165^\circ$ to satisfy a maximum-$g_{load}$ constraint of 4 g, and a constant bank angle of $\sigma = 45^\circ$.

The simulations showed crossranges varying from 137 to 277 km. These were not the maximum crossranges, because for all simulations the same initial position was taken so that the final latitude was not the maximum one. The influence of the several deorbit burns on entry time and duration were quite large ($\Delta t_e = 400$ s (27% of the minimum value) and $\Delta \Delta t = 173$ s (30%) for $h_0 = 450$ km). Differences in entry conditions ($7431.5$ m/s $\leq V_e \leq 7498.1$ m/s and $-2.29^\circ \leq \gamma_e \leq -1.30^\circ$) resulted in a 10%-variation of the maximum $g_{load}$, but the values were well within the maximum allowable value of 4 g (maximum occurring value: $g_{load} = 3.47$ g).

The deorbit-burn manoeuvre can have a large influence on the entry conditions, with an effect on the entry duration and the maximum occurring $g_{load}$. This has its influence on the crossrange of the vehicle, and the well-being of (possibly sick) astronauts on-board, so careful
mission design is essential. Before firm conclusions regarding the sensitivity analysis can be made, more simulations have to be executed. One should consider to increase the thrust level of the deorbit-burn engine of the ACRV, because especially for higher $\Delta V$-values, the burn time becomes very high, and the burn will only be completed some 10-20 km before entry.

During the study, a number of practical extensions or improvements arose. Some major recommendations will be listed below.

**General**

- The deorbit-burn manoeuvre could be considered as a two-point boundary value problem. Based on a landing point at the Earth's surface, the time and location in orbit should be determined, thereby taking the entry conditions into account. Another possibility might be the fast-landing approach: what will be the conditions for the deorbit burn, when the spacecraft has to leave orbit as soon as possible and to land on one ore more specified landing places, without violating pre-specified constraints.

- Subjected to specified boundary conditions and constraints, the deorbit burn could be optimized, e.g., to minimize fuel mass or flight time.

- W.r.t. the implementation in START, the deorbit-burn manoeuvre could be generalized as a $\Delta V$-manoeuvre, which can occur at any time during the mission, depending on a pre-defined flight condition. At the moment, this condition is zero flight-time, or, in other words, the very beginning of the mission.

**Finite-thrust model**

- The finite-thrust model has been implemented with constant thrust and a simple guidance scheme. Other guidance schemes (which might include variable thrust), taking for instance the entry conditions into account, should be implemented. A possible guidance scheme could follow from the optimization of the burn.

**Evaluation**

- So far, not much literature on real-life deorbit-burn manoeuvres could be found. An additional literature study should be performed, to increase insight. The question of what are acceptable burn times of the deorbit-burn engine should be answered.

- The evaluation of the two implemented models is not yet complete. A further activity should concentrate on other guidance schemes, take different initial orbits into account, and should also focus on the influence of the deorbit-burn manoeuvre only, i.e., assuming no bank.

- Crossrange comparisons should be done for a specified location, e.g., the maximum lati-
tude of the unperturbed, nominal orbit. In that case, a realistic indication of the differences is obtained.

**Sensitivity analysis**

- The sensitivity analysis should be done again, but now with the finite-thrust model. Besides, the final point of the capsule should be specified as height and longitude, so that the maximum crossrange can be derived.

- The trimmed angle of attack of $\alpha = 165^\circ$ left a margin of 0.53 g (= 13% of the maximum value of 4 g). Additional simulations with higher $\alpha$ could be done to maximize the crossrange even more.
References.

1) Battin, R.H.;
   An introduction to the mathematics and methods of astrodynamics;
   AIAA Education Series, second printing;

   Rocket propulsion and spaceflight dynamics;

   Design guide to orbital flight;
   Chapter IX: Satellite recovery (pp. 437-534);

4) Mooij, E.;
   Development of START, a six degrees of freedom Simulation Tool for Atmospheric Re-
   entry Trajectories;
   ESTEC Working Paper EWP 1633;
   ESTEC, Noordwijk, 1991a.

5) Mooij, E.;
   ESA Huygens probe entry trajectory analysis;
   ESTEC Working Paper EWP 1634;
   ESTEC, Noordwijk, 1991b.

6) Mooij, E.;
   START User Manual, Version 1.0;
   Final thesis Delft University of Technology, Faculty of Aerospace Engineering;
   Delft, 1991c.

7) Mooij, E.;
   ESA Huygens probe entry and descent analysis;
   ESTEC Working Paper EWP 1679;

8) Mooij, E.;
   The motion of a vehicle in a planetary atmosphere: the six-degrees-of-freedom equations
   of motion;
   Note 10064;
   Delft University of Technology, Faculty of Aerospace Engineering, To Be Published 1993.

9) Mosely Jr., W.C., Moore Jr., R.H. and Hughes, J.E.;
   Stability characteristics of the Apollo command module;
   NASA TN D-3890;

10) Nérault, M.;  
The Assured Crew Return Vehicle (ACRV);  
Reaching for the skies, no. 8, June 1993, pp. 6-7;  

11) Paris, D.A.P.;  
Statement of work, DRAFT 9, Issue 02;  
Fax dd. December 11, 1992;  

12) Paris, D.A.P. (ESTEC, WKA);  
Private communication, 1993a.

13) Paris, D.A.P.;  
Input data for deorbitation simulations;  
Fax dd. May 24, 1993;  
ESTEC WKA, Noordwijk, The Netherlands, 1993b.

14) Raillon, E., Parnis, P. and Devaux, N.;  
Flying qualities of the Hermes space plane and the shape definition process;  

Performance trends in spacecraft auxiliary propulsion systems;  

16) Wakker, K.F.;  
Beweging van ruimtevoertuigen (Dutch text);  
Lecture notes Ω7b;  
Delft University of Technology, Faculty of Aerospace Engineering, 1989.
Appendix A - Input file of ACRV

July 1993

Ir. E. Mooij

TU Delft
Delft University of Technology

Faculty of Aerospace Engineering
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** MASS PROPERTIES **

Number of mass elements

| 1 |

************> Data for mass element # 1

Mass at epoch (kg)

| 0.65000000000D+04 |

X-coordinate CoM (m)

| 0.00000000000D+00 |

Y-coordinate CoM (m)

| 0.00000000000D+0 |

Z-coordinate CoM (m)

| 0.00000000000D+0 |

Ixx (kg m²)

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Iyy (kg m²)

| 0.00000000000D+0 |

Izz (kg m²)

| 0.00000000000D+0 |

Ixz (kg m²)

| 0.00000000000D+0 |

Iyz (kg m²)

| 0.00000000000D+0 |

***************

** REF. GEOMETRY **

Ref. length d (m) for aerod. coeff.

| 0.44000000000D+01 |

Ref. area S (m²) for aerod. coeff.

| 0.15205300000D+02 |

Ref. length Lref (m) for Re-number

| 0.44000000000D+01 |

***************

** AERODYNAMICS **

Re-entry vehicle is axisymmetric body

| 1 |

Interpolation method

| 2 |

***************

** COEFFICIENT CD **

Number of coefficient components

| 1 |

************> Data for coefficient component # 1

Order of derivation

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Number of table variables

| 2 |

Table variable #1

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Table variable #2

| 1 = alpha |

Number of entries for table variable #2

| 5 |

| Entry 1 | 0.14000000000D+03 |
| Entry 2 | 0.15000000000D+03 |
| Entry 3 | 0.16000000000D+03 |
| Entry 4 | 0.17000000000D+03 |
| Entry 5 | 0.18000000000D+03 |

Function values for coefficient component

| C(  1,  1) | 0.70000000000D+00 |
| C(  2,  1) | 0.80000000000D+00 |
| C(  3,  1) | 1.00000000000D+01 |
| C(  4,  1) | 1.00000000000D+0 |
| C(  5,  1) | 1.10000000000D+0 |
| C(  6,  1) | 1.05000000000D+0 |
| C(  7,  1) | 1.00000000000D+0 |
| C(  8,  1) | 0.95000000000D+0 |
| C(  9,  1) | 0.95000000000D+0 |
| C(10,  1) | 0.90000000000D+0 |
| C(  1,  2) | 0.80000000000D+0 |
********** Data for coefficient component #1

Order of derivation
0

Number of table variables
2

Table variable #1
5 = Mach

Number of entries for table variable #1
10

Table

<table>
<thead>
<tr>
<th>Entry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.0000000000e+00</td>
</tr>
<tr>
<td>2</td>
<td>9.0000000000e+00</td>
</tr>
<tr>
<td>3</td>
<td>1.1000000000e+01</td>
</tr>
<tr>
<td>4</td>
<td>1.2000000000e+01</td>
</tr>
<tr>
<td>5</td>
<td>1.3000000000e+01</td>
</tr>
<tr>
<td>6</td>
<td>1.4000000000e+01</td>
</tr>
<tr>
<td>7</td>
<td>1.5000000000e+01</td>
</tr>
<tr>
<td>8</td>
<td>1.6000000000e+01</td>
</tr>
<tr>
<td>9</td>
<td>3.0000000000e+01</td>
</tr>
<tr>
<td>10</td>
<td>3.4000000000e+01</td>
</tr>
</tbody>
</table>

Table variable #2
1 = alpha

Number of entries for table variable #2
5

Table

<table>
<thead>
<tr>
<th>Entry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4000000000e+03</td>
</tr>
<tr>
<td>2</td>
<td>1.5000000000e+03</td>
</tr>
<tr>
<td>3</td>
<td>1.6000000000e+03</td>
</tr>
<tr>
<td>4</td>
<td>1.7000000000e+03</td>
</tr>
<tr>
<td>5</td>
<td>1.8000000000e+03</td>
</tr>
</tbody>
</table>

Function values for coefficient component

C( 1, 1) 5.2000000000e+00
C( 2, 1) 5.4000000000e+00
C( 3, 1) 6.4000000000e+00
C( 4, 1) 6.4000000000e+00
C( 5, 1) 6.4000000000e+00
C( 6, 1) 6.4000000000e+00
C( 7, 1) 5.6000000000e+00
C( 8, 1) 5.2000000000e+00
C( 9, 1) 5.2000000000e+00
C(10, 1) 5.2000000000e+00
C( 1, 2) 4.0000000000e+00
C( 2, 2) 4.4000000000e+00

**************
** COEFFICIENT CL **
**************

Number of coefficient components 1
<table>
<thead>
<tr>
<th>C(3, 2)</th>
<th>0.56000000000E+00</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(4, 2)</td>
<td>0.56000000000E+00</td>
</tr>
<tr>
<td>C(5, 2)</td>
<td>0.60000000000E+00</td>
</tr>
<tr>
<td>C(6, 2)</td>
<td>0.58000000000E+00</td>
</tr>
<tr>
<td>C(7, 2)</td>
<td>0.56000000000E+00</td>
</tr>
<tr>
<td>C(8, 2)</td>
<td>0.52000000000E+00</td>
</tr>
<tr>
<td>C(9, 2)</td>
<td>0.52000000000E+00</td>
</tr>
<tr>
<td>C(10, 2)</td>
<td>0.50000000000E+00</td>
</tr>
<tr>
<td>C(1, 3)</td>
<td>0.32000000000E+00</td>
</tr>
<tr>
<td>C(2, 3)</td>
<td>0.34000000000E+00</td>
</tr>
<tr>
<td>C(3, 3)</td>
<td>0.40000000000E+00</td>
</tr>
<tr>
<td>C(4, 3)</td>
<td>0.40000000000E+00</td>
</tr>
<tr>
<td>C(5, 3)</td>
<td>0.44000000000E+00</td>
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<tr>
<td>C(6, 3)</td>
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<td>C(7, 3)</td>
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</tr>
<tr>
<td>C(8, 3)</td>
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<tr>
<td>C(9, 3)</td>
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<tr>
<td>C(10, 3)</td>
<td>0.40000000000E+00</td>
</tr>
<tr>
<td>C(1, 4)</td>
<td>0.18000000000E+00</td>
</tr>
<tr>
<td>C(2, 4)</td>
<td>0.20000000000E+00</td>
</tr>
<tr>
<td>C(3, 4)</td>
<td>0.22000000000E+00</td>
</tr>
<tr>
<td>C(4, 4)</td>
<td>0.22000000000E+00</td>
</tr>
<tr>
<td>C(5, 4)</td>
<td>0.24000000000E+00</td>
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<tr>
<td>C(6, 4)</td>
<td>0.20000000000E+00</td>
</tr>
<tr>
<td>C(7, 4)</td>
<td>0.22000000000E+00</td>
</tr>
<tr>
<td>C(8, 4)</td>
<td>0.22000000000E+00</td>
</tr>
<tr>
<td>C(9, 4)</td>
<td>0.22000000000E+00</td>
</tr>
<tr>
<td>C(10, 4)</td>
<td>0.22000000000E+00</td>
</tr>
<tr>
<td>C(1, 5)</td>
<td>0.00000000000E+00</td>
</tr>
<tr>
<td>C(2, 5)</td>
<td>0.00000000000E+00</td>
</tr>
<tr>
<td>C(3, 5)</td>
<td>0.00000000000E+00</td>
</tr>
<tr>
<td>C(4, 5)</td>
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<tr>
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<tr>
<td>C(8, 5)</td>
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</tr>
<tr>
<td>C(9, 5)</td>
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</tr>
<tr>
<td>C(10, 5)</td>
<td>0.00000000000E+00</td>
</tr>
</tbody>
</table>

** PARACHUTE SYSTEMS **

- Number of parachute systems: 0

** EXTERNAL FORCES/MOMENTS **

** Configuration #0 **

- Spin vanes selected: N

** PROPULSION SYSTEM **

- Deorbit-burn engine present: N

** ENVIRONMENT **

- Central body: 1 = Earth
- Gravitational model: 2 = J2 included
- Atmosphere model: 2 = US76

** PERTURBATIONS **

- Percentage of density deviation: 0.0000000000E+00

** WIND MODEL **
No wind model defined

***********************
** INITIAL STATE **
***********************

Choice of initial position and velocity 3 = orb. elem.

************ Classical orbital elements

eccentricity (-) 0.0000000000D+00
semi-major axis a (km) 0.6658139000D+04
inclination i (deg) 0.2850000000D+02

argument of pericentre omega (deg) 0.0000000000D+00
longitude of ascending node OMEGA (deg) 0.0000000000D+00
mean anomaly M (deg) 0.0000000000D+00

************ Attitude

Angle of attack alpha (deg) 0.1650000000D+03
Angle of sideslip beta (deg) 0.0000000000D+00
Bank angle sigma (deg) 0.4500000000D+02

************ Angular rates

Angular roll rate p (deg/s) 0.0000000000D+00
Angular pitch rate q (deg/s) 0.0000000000D+00
Angular yaw rate r (deg/s) 0.0000000000D+00

************ Configuration changes

Number of configuration changes 0

************ PROPULSION OPERATIONS

Deorbit-burn manoeuvre selected Y

Deorbit-burn implementation 1 = impulsive dV
Delta V (m/s) 0.1250000000D+03
Elevation angle (deg) 0.1620000000D+03
Azimuth angle (deg) 0.4800000000D+02

************ Integration

Integration method 2 = V.S. 7(8) RKF
The maximum integration step size (s) 0.1000000000D+01
The quicklook data filing rate (s) 0.1000000000D+02
The plot data filing rate (s) 0.5000000000D+01

************ Tolerances

Tolerance for velocity (m/s) 0.1000000000D+00
Tolerance for flightpath angle (deg) 0.5729577951D-02
Tolerance for heading (deg) 0.5729577951D-02
Tolerance for position (m) 0.1000000000D+00
Tolerance for position angles (deg) 0.5729577951D-02
Tolerance for angular rates (deg/s) 0.5729577951D-04
Tolerance for attitude (deg) 0.5729577951D-02

************ STOP CRITERIA

Stop by flight time Y
<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elapsed flight time in sec</td>
<td>5.00000000000000E+04</td>
</tr>
<tr>
<td>Stop by height</td>
<td>Y</td>
</tr>
<tr>
<td>Height in meter</td>
<td>5.00000000000000E+04</td>
</tr>
<tr>
<td>Stop by velocity</td>
<td>N</td>
</tr>
<tr>
<td>Stop by Mach number</td>
<td>N</td>
</tr>
<tr>
<td>Stop by dynamic pressure</td>
<td>N</td>
</tr>
</tbody>
</table>