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2pPA8. Computation of three-dimensional, pulsed, nonlinear acoustic wavefields from medical phased array transducers in very large domains

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For the optimization and development of medical ultrasound transducers and imaging modalities, the Iterative Nonlinear Contrast Source (INCS) method has been developed. This numerical method predicts the nonlinear acoustic pressure field, generated by a pulsed, plane source with an arbitrary aperture, and propagating in a three-dimensional tissue-like medium that extends over a very large domain of interest. The INCS method obtains the acoustic pressure from the nonlinear acoustic wave equation by treating the nonlinear term as a contrast source. The full nonlinear wave field is then found by iteratively solving the linearized wave problem using a Green's function method. By employing the Filtered Convolution method discussed in a companion paper, accurate field predictions are obtained at a discretization approaching two points per wavelength or period of the highest frequency of interest. In this paper, very large-scale, nonlinear field profiles are presented for transducers with cylindrical as well as phased array geometries, excited with a pulsed waveform having a center frequency of 1-2 MHz. Comparison with results obtained from models of reduced complexity shows that in all cases the INCS method accurately predicts the nonlinear field. [Work supported by STW and NCF.]

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FIGURE 1. Overview of the paper

In this paper, we present a novel method for the computation of the nonlinear acoustic pressure field excited by a medical diagnostic phased array transducer. The method is referred to as the Iterative Nonlinear Contrast Source (INCS) method. First we discuss the application for which the model has been designed and the model requirements resulting from the application. We briefly review a number of existing approaches for the modeling of nonlinear acoustic fields. Next, we discuss the INCS method, starting with the nonlinear wave equation and following with the solution approach and the efficient discretization and evaluation of the occurring fields. Subsequently, we present results generated by the method, and we compare the results with those of other, existing methods. The paper closes with some conclusions.

Application

- In medical echography, image quality is improved by selectively imaging the 2nd harmonic frequency component that is caused by nonlinear propagation (THI – Tissue Harmonic Imaging)

Images courtesy
of Siemens Acuson

- Employment of the 3rd to the 5th harmonic components (SuperHarmonic Imaging) is likely to further enhance this improvement (Bouakaz and De Jong 2003)

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FIGURE 2. The application for which the method has been designed.

In the recent decade, employment of the nonlinear characteristics of acoustic propagation has resulted in a significant improvement of the image quality of diagnostic echography systems.¹ The imaging modality called Tissue Harmonic Imaging (THI) is based on the selective imaging of the second harmonic frequency component that results from the nonlinear distortion of the pressure field. In the two images of the same heart shown in Fig. 2, obtained with the traditional Fundamental Imaging (FI) method (left) and with THI (right), THI exhibits a clearer delineation of the borders of the ventricles and an increased signal to noise ratio. Recently, a study has shown that selective imaging of the third to the fifth harmonic frequency components, an approach that is referred to as SuperHarmonic Imaging (SHI), is bound to yield an even higher image quality.² However, due to the limited bandwidth of modern day phased array transducers, SHI would require a new generation of transducers that exhibits a wideband or a dual band frequency sensitivity. The accurate prediction of the nonlinear pressure field excited by such a transducer is an essential ingredient in its design process and in its optimization for SHI or, eventually, THI.

Requirements

- To accurately predict the nonlinear acoustic field from diagnostic medical phased array transducers, we need to account for
 - Weak nonlinear distortion
 - In a three-dimensional, large-scale domain
 - Pulsed excitation
 - Medium: water (lossless, homogeneous) or human soft tissue (lossy, inhomogeneous)

- Scale
 $f_0 = 1 \text{ MHz}$, $c_0 = 1500 \text{ m/s}$
 5th harmonic $\rightarrow f_{\max} = 6 f_0$
- Dimensions
 Spatial: $400 \times 400 \times 80$ wavelengths
 Temporal: ~ 500 periods
- Grid size at 2 ppw
 Total = 102×10^9 pts
 Ac. pressure in dp $\rightarrow$ **763 GB of data**

FIGURE 3. Model requirements as dictated by the application.

The accurate prediction of the nonlinear field from a phased array transducer by a numerical evaluation method requires the method to account for weak nonlinear distortion, as the field strength is limited because of the diagnostic purpose of the application. The field needs to be obtained in a three-dimensional, large-scale domain in the order of 100 wavelengths in each dimension, where the wavelength is related to the highest frequency of interest in the problem. Because of the pulsed excitation that is often employed, the method needs to compute the field in a time frame that may have a length in the order of 100 periods of the maximum frequency of interest. The medium under study is that of human soft tissue, which exhibits acoustic loss and inhomogeneity. However, most often water is employed as a sufficient approximation of tissue, which can be considered lossless and homogeneous. In this paper, we will limit ourselves to studying nonlinear acoustic fields in water.

To give an idea of the complexity of the computational domain, we will compute the number of points that are needed to describe the acoustic field in a typical spatiotemporal domain of $0.1 \text{ m} \times 0.1 \text{ m} \times 0.02 \text{ m}$, consisting of water ($c_0 = 1500 \text{ m/s}$). A center frequency of the excitation pulse that is typical for medical diagnostic ultrasound is $f_0 = 1 \text{ MHz}$. Further suppose that we are interested in the nonlinear acoustic field up to the maximum frequency $f_{\max} = 6 f_0$. At this maximum frequency, the spatial domain is about $400 \times 400 \times 80$ wavelengths. The time frame needed for the pulse to traverse the domain from the source to an extreme corner is about 500 periods at $f_{\max}$. When the entire spatiotemporal domain is sampled at a minimal discretization of two points per wavelength, as prescribed by the Nyquist-Shannon theorem for band limited signals, and the pressure in the sample points is described as a real variable of 8 bytes, then a total memory size of 763 GB results. This clearly shows the order of magnitude that the problem poses in describing the nonlinear acoustic field in a spatiotemporal domain.

Existing approaches

- Forward wave methods (plane to plane marching)
 - Parabolic approximation: KZK equation (Lee and Hamilton 1995, Bouakaz et al. 1999, Khokhlova et al. 2002, Yang and Cleveland 2006)
 - + Fast and accurate for quasi-plane beams
 - Not appropriate for large angles off-axis, for strongly focused beams or for inhomogeneity
 - Phenomenological approach (Christopher and Parker 1991, Tavakkoli et al. 1998, Zemp et al. 2003)
 - + No parabolic approximation in the diffraction term
 - Stepwise quasi-plane nonlinear distortion

FIGURE 4. Existing approaches for the modeling of the nonlinear propagation of the acoustic pressure field: Forward-wave methods.

In the recent decades, several groups of researchers have invested their effort in developing a numerical method that meets the challenge of accurately predicting the nonlinear ultrasound field. Most currently used 3D nonlinear acoustic models assume forward-wave propagation and are based on an evolution equation that, starting from a certain source plane, marches the solution from plane to plane in the main direction of propagation. Within this group of models, a first line of research is based on the Kuznetsov-Zabolotskaya-Khokhlov (KZK) equation.³⁻⁶ The KZK equation is a nonlinear wave equation derived from the basic nonlinear acoustic equations under a parabolic approximation,⁷ and it accounts for the nonlinear terms up to the second order in the field quantities. Because of its parabolic approximation it is valid for beams with quasi-plane wavefields. For this reason, the region of validity is generally taken not too far off the main transducer axis and not too close to the source. A second line of forward-wave models uses a phenomenological approach and splits each marching step into separate operations that account for the effects of diffraction, absorption and nonlinearity.^{8,9} The diffraction step is incorporated without using a paraxial approximation.

For the nonlinear propagation step, all of the above references use a plane-wave solution in the time domain or in the frequency domain, thus assuming that the main nonlinear distortion is in the direction of the transducer axis. Although at each diffraction step the nonlinear distortion is spreaded again, this may not be accurate for wavefields that are strongly focused or that propagate at an angle significantly different from the transducer axis. A correction method has been proposed by Christopher and Parker,⁸ and Fox et al. implemented a KZK method with a steered propagation axis.¹⁰ However, when studying wide-angle phenomena like grating lobes or propagation through heterogeneous media, we need to account for nonlinear distortion in all directions. A method that handles nonlinear propagation in all forward directions was proposed by Varslot et al.,¹¹ but this method is limited by its quasilinear approximation.

Existing approaches

- Full-wave methods (Sparrow and Raspet 1991, Ginter and Liebler 2002, Hallaj and Cleveland 1995, Wojcik et al. 1998)
 - + Diffraction and nonlinear distortion free of directionality
 - Large computational grid because the step size in all dimensions is coupled to smallest significant wavelength
- Current method: Iterative Nonlinear Contrast Source (INCS) Method
 - Full-wave method, therefore no assumed directionality
 - Discretization is as coarse as possible, at the Nyquist limit (2 PPW)

FIGURE 5. Existing approaches for the modeling of the nonlinear propagation of the acoustic pressure field: Full-wave methods and the INCS method.

Apart from the forward-wave models, a number of models has been developed for full-wave nonlinear propagation of acoustic fields. Because of the computational effort involved, these models have been limited to 2D cartesian or cylindrical implementations.¹²⁻¹⁵ None of these models has an assumed directionality in the wave propagation or in the nonlinear distortion. However, a problem of full-wave models is the large number of points per wavelength and per period (referred to as PPW) in space and time that is needed for an accurate computation. The Nyquist-Shannon sampling theorem for band limited signals prescribes a minimum of 2 PPW at the highest frequency of interest. Finite Difference and Finite Element methods need a much higher number of PPW, and this makes these methods particularly unfavorable for application to large-scale, three-dimensional problems. The best effort in this respect has been made by Wojcik et al.,¹⁵ who present a pseudospectral method that handles the spatial differentiation in the k -space domain and includes propagation in inhomogeneous media. It needs 4 PPW in the spatial dimensions, and with a Courant number of 0.2 it employs at least 20 PPW in the temporal dimension.

The Iterative Nonlinear Contrast Source (INCS) method presented in this paper can be classified as a full-wave method, and therefore it does not suffer from an assumption on the directionality of the wavefield as made with all described forward-wave methods. The method is able to compute the weak to moderate nonlinear acoustic field at a discretization of 2 PPW at the highest frequency of interest.

INCS Method

- The Iterative Nonlinear Contrast Source method is based on the Westervelt equation:

$$c_0^{-2} \partial_t^2 p - \nabla^2 p = S + \frac{\beta}{\rho_0 c_0^4} \partial_t^2 p^2$$

- The source term S may excite a velocity or pressure jump at $z=0$:
 $S = \rho_0 \partial_t [\Delta V \delta(z)]$
 $S = -\partial_z [\Delta P \delta(z)]$

- The nonlinear term is interpreted as a *contrast source* $S^N(p)$

$$c_0^{-2} \partial_t^2 p - \nabla^2 p = S + S^N(p) \quad S^N(p) = \frac{\beta}{\rho_0 c_0^4} \partial_t^2 p^2$$

- The acoustic pressure may obtained as a convolution of the source with the free space Green's function:

$$p = G *_{\mathbf{x},t} [S + S^N(p)] \quad G(\mathbf{x}, t) = \frac{\delta(t - \|\mathbf{x}\| / c_0)}{4\pi \|\mathbf{x}\|}$$

FIGURE 6. The INCS Method: Nonlinear wave equation, contrast source formulation and Green's function.

The INCS method is based on the Westervelt equation,^{7,16} which is a nonlinear wave equation that has been derived under a second-order approximation from the basic conservation laws of the acoustic field. In the form of the Westervelt equation as employed here we have neglected the attenuation and we have included a primary source term S . In this paper, the source term is used to describe the situations of a normal velocity jump of magnitude $\Delta V(x,y,t)$ or a pressure jump of magnitude $\Delta P(x,y,t)$ across the plane $z = 0$.

We interpret the nonlinear distortion of the acoustic field as a correction to the linear field solution. Because of this, we write the nonlinear term on the right-hand side of the Westervelt equation and consider it as a nonlinear contrast source $S^N(p)$. Formally, the pressure may be solved from the wave equation as the spatiotemporal convolution of the primary and contrast source $[S + S^N(p)]$ with the free space Green's function. However, the appearance of the pressure in the nonlinear contrast source on the right-hand side results in an implicit solution rather than an explicit solution.

INCS Method

$$p = G *_{\mathbf{x},t} [S + S^N(p)]$$

- p and $S^N(p)$ are estimated through the iterative scheme

$$p^{(0)} = G *_{\mathbf{x},t} S^{(0)}, \quad S^{(0)} = S,$$

$$p^{(j)} = p^{(0)} + G *_{\mathbf{x},t} S^{(j)}, \quad S^{(j)} = S^N(p^{(j-1)}), \quad j \geq 1.$$

- Convolution of the Green's function and the primary or contrast source:

$$G *_{\mathbf{x},t} S^{(j)} = \int_{\mathcal{D}_S^{(j)}} \int_{T_S^{(j)}} G(\mathbf{x} - \mathbf{x}', t - t') S^{(j)}(\mathbf{x}', t') dt' d\mathbf{x}'$$

FIGURE 7. The INCS Method: Iterative solution scheme and primary and contrast source domains.

If the nonlinear field distortion is weak, then in zero order we may approximate the acoustic field with the linear field solution $p^{(0)}$ obtained by the spatiotemporal convolution of the primary source with the Green's function over the spatial and temporal domains of the primary source. With this, we then obtain a first approximation of the nonlinear contrast source as $S^N(p^{(0)})$. This enables us to compute an estimate $p^{(1)}$ of the nonlinear acoustic pressure field. Repeating these steps, the contrast source formulation enables the computation of a successive estimate $p^{(j)}$ of p , and it results in the iterative scheme as shown in Fig. 7.

For the linear field solution $p^{(0)}$ that is computed as a function of the primary source S , the spatial domain over which the convolution is performed is equal to the support of the primary source, i.e. the plane $z = 0$. For the nonlinear field corrections computed to obtain $p^{(j)}$, $j \geq 1$, the spatial domain of the convolution is in principle the support of $S^{(j)}$, which is equal to the support of $p^{(j-1)}$. The latter support spans the entire space where an acoustic pressure is observed. In practice however, the domain in which the contrast sources contribute most significantly to the nonlinear pressure in a certain point of observation is limited to the domain between the primary source and the point of observation.

INCS Method

- Discretization of the convolution integral with a left Riemann sum yields

$$\left[G *_{x,t} S^{(j)} \right]_{\mathbf{k},l} \approx \Delta x^3 \Delta t \sum_{\mathbf{m}} \sum_n G_{\mathbf{k}-\mathbf{m},l-n} S_{\mathbf{m},n}^{(j)}$$

- Efficient discretization by spatiotemporal filtering of the Green's function and the sources before sampling

$$G(\mathbf{x}, t) \rightarrow G_K^\Omega(\mathbf{x}, t)$$

$$S^{(j)}(\mathbf{x}, t) \rightarrow S_K^{\Omega,(j)}(\mathbf{x}, t)$$

- Evaluation with Fast Fourier Transforms:

$$\left[G *_{x,t} S^{(j)} \right]_{\mathbf{k},l} \approx \Delta x^3 \Delta t \text{FFT}^{-1} \left\{ \text{FFT}[G_{\mathbf{k},l}] \text{FFT}[S_{\mathbf{k},l}^{(j)}] \right\}$$

FIGURE 8. The INCS method: discretization and numerical evaluation

The convolution integral over the Green's function and the primary or contrast source may be discretized to yield a convolution sum. The discretization in the directions of integration is determined by the number of points per wavelength or period (PPW) related to the maximum spatial and temporal frequency of interest. To minimize PPW, the Green's function, and the primary and contrast source are spatiotemporally filtered and windowed. This approach enables a discretization down to 2 PPW and is described in a companion paper.¹⁷ The resulting discrete convolution sum may be efficiently evaluated with a Fast Fourier Transform (FFT) method.¹⁸

The INCS method has been implemented in Fortran for a distributed multi-core computer system, enabling the evaluation of large-scale problems. The implementation allows for the field computation in parallelogram-shaped domains, thus enabling the computation of steered beams in the x - and y -dimensions, and allowing for a co-moving time frame in the t -dimension. For problems involving weak to moderate nonlinear distortion, an accurate solution is already obtained with a small number of iterations. We have employed the program to evaluate the nonlinear pressure field for a number of cases that show the characteristics of the method, as will be discussed next.

Results – cylindrical transducer

- Unfocused cylindrical transducer, $R = 5\text{ mm}$, $f_0 = 1\text{ MHz}$, $P_0 = 500\text{ kPa}$
- INCS method is run with 2 PPW at $f_{\max} = 4\text{ MHz}$, iteration $j = 4$
- Comparison with three other methods:
 - Finite Difference Westervelt method (Hallaj&Cleveland, 1999)
 - Angular Spectrum NLP method (Zemp et al. 2003)
 - KZK method (Lee&Hamilton, 1995)

FIGURE 9. Numerical prediction of the nonlinear field of an unfocused cylindrical piston. Comparison of the axial and lateral profiles generated by the INCS method with those generated by three other numerical methods.

As a first case, we compute the nonlinear acoustic pressure field from a cylindrical piston transducer and we compare the results of the INCS method with those of three existing numerical methods. The first method is a Finite Difference Time Domain approximation of the Westervelt equation for cylindrical geometries (FD-WV).^{14,19} The second method is the angular spectrum nonlinear propagation (AS-NLP) model developed by Zemp et al.⁹ The third model we employ for comparison is a numerical model for cylindrical geometries based on the KZK equation.³ Because of the parabolic approximation of the KZK equation, the KZK model should result in sufficiently accurate answers in a region not too close to the source and not too far away from the transducer axis.²⁰ The transducer has a 5 mm radius and it generates a pressure jump perpendicular to the source plane. The signature of the source signal is a harmonic signal modulated with a Gaussian envelope,

$$\Delta P = 2P_0 \exp\left[-(2t/T_w)^2\right] \sin(2\pi f_0 t).$$

Here, the center frequency $f_0 = 1\text{ MHz}$, the pulse width $T_w = 3/f_0$ and the source pressure amplitude $P_0 = 500\text{ kPa}$. To compute the nonlinear pressure field with sufficient accuracy up to the third harmonic frequency component, the INCS method employs a discretization of 2 PPW at a maximum frequency of 4 MHz, and we employ the results for iteration $j = 4$. The plots in Fig. 9 show the spectral profiles of the fundamental (F0), the second harmonic (2H) and the third harmonic (3H) frequency components at the transducer axis and in the radial direction at an axial distance $z = 20\text{ mm}$. From these figures we observe for all three profiles a good agreement between the results of the INCS method, the FD-WV method and the AS-NLP method. Within the region of validity of the parabolic approximation, the results from the KZK method coincide well with those of the other models. This shows that, despite the different approaches, the results are similar, and for the purpose of this paper, this gives us confidence in the accuracy of the INCS method.

Results – point source

- Point source at $|\mathbf{x}|=0$, $f_0=1$ MHz, $P_0=500$ kPa at $|\mathbf{x}|=1$ mm
- The INCS method does not exhibit any directionality in the linear field nor in the nonlinear field correction

FIGURE 10. Numerical prediction of the nonlinear acoustic field of a point source. On the left, the field profile is shown. On the right, the variation in the linear field solution $p^{(0)}$ and in the nonlinear field correction $p^{(3)}$ are shown on the half-circle depicted in the field profile.

In the second case, we demonstrate the lack of dependency on the direction of the wavefield by obtaining the nonlinear pressure field of a point source acting as a monopole. The point source is located at the origin and it is excited with an identical pulse as in the previous case, with a source amplitude such that the amplitude of the pressure field is 500 kPa at a distance of 1 mm from the source. We compute the nonlinear pressure field in a domain of interest of size 8.6 mm $\times$ 8.6 mm $\times$ 8.6 mm, centered at the origin. We employ the INCS method with a discretization of 2 PPW at a maximum frequency of 5 MHz. We study the linear field solution from iteration $j = 0$ and the nonlinear field correction from iteration $j = 3$ on a half-circle with the source as the center and a radius of 4 mm. We observe that on this half-circle the relative variation of the linear field solution as well as the nonlinear field correction is very small, within ± 0.05 dB from the average value. This shows the directional independence of the INCS method in both the linear and the nonlinear field solution.

Results – phased array transducer

- 48-element phased array transducer, element $W \times H = .21 \times 12$ mm, pitch = .5 mm, $f_c = 1$ MHz, $P_0 = 500$ kPa, focus at $z = 57$ mm, elev. focusing

FIGURE 11. Numerical prediction of the nonlinear pressure field of a phased array transducer. Depicted are the field profiles of the fundamental, second harmonic and third harmonic frequency components in the plane $y=0$.

In the third case, we compute the nonlinear field excited by a phased array transducer (48 elements of size $W \times H = 0.21 \text{ mm} \times 12 \text{ mm}$ and a pitch of 0.5 mm) focused at $(x, z) = (0, 57)$ mm including elevation focusing at $z = 57$ mm. The transducer excites a pressure jump in the medium of the same form as used in the first case with a pressure amplitude $P_0 = 250$ kPa. The INCS method is employed with a discretization of 2 PPW at the maximum frequency of 4 MHz and we use the nonlinear field estimate for iteration $j = 4$. This should yield an accurate prediction of the nonlinear field up to the third harmonic. In Fig. 11, the spectral profiles of the fundamental, second harmonic and third harmonic components are shown in the plane $y = 0$. We observe the well-known characteristics of the nonlinear field components: a reduced near field and side lobe level and a narrowed main beam. The maximum overall pressure level at the focus is 414 kPa. This case shows that the INCS method is very well able to deal with an unsteered beam from a phased array transducer.

Results – phased array transducer

- 48-element phased array transducer, element $W \times H = .21 \times 12$ mm, pitch = .5 mm, $f_c = 1$ MHz, $P_0 = 250$ kPa, focus at $z = 57$ mm, elev. focusing
- Comparison with AS-NLP method

FIGURE 12. Numerical prediction of the nonlinear pressure field of a phased array transducer. Comparison of the axial and lateral profiles generated by the INCS method with those generated by the AS-NLP method.

The results of the INCS method for the case presented in Fig. 11 are compared with results of the AS-NLP method for the same case. Figure 12 shows the spectral profiles of the fundamental, second harmonic and third harmonic frequency components on the axis of the transducer and in the lateral direction at $z = 57$ mm. We observe good agreement for all profiles, which confirms the observations from Fig. 9.

Results – phased array transducer

- 48-element phased array transducer, element $W \times H = .42 \times 12$ mm, pitch = **1.0** mm, $f_c = 1$ MHz, $P_0 = 250$ kPa, focus at $(x, z) = (40, 40)$ mm, elev. focusing

FIGURE 13. Numerical prediction of the nonlinear pressure field of a phased array transducer exhibiting a steered beam and a grating lobe. Depicted are the field profiles of the fundamental, second harmonic and third harmonic frequency components in the plane $y=0$.

As a fourth case we compute the nonlinear pressure field for a phased array transducer that has a doubled element width and pitch as compared to the previous case. In view of this, we expect to generate a grating lobe. With this configuration, we illustrate the ability of the INCS method to predict the nonlinear field for wide-angle phenomena. The transducer is focused at $(x, z) = (40, 40)$ mm with elevation focusing at $z = 40$ mm, and it is excited with the same pulse as in the previous case. The fundamental, second harmonic and third harmonic field profiles are obtained in two simulations where we employ two parallelogram-shaped domains of size $60 \text{ mm} \times 18 \text{ mm} \times 50 \text{ mm}$, one of which making an angle of 45° and capturing the main beam, and the other making an angle of -45° and capturing the grating lobe. We employ the INCS method with a discretization of 2 PPW at a maximum frequency of 4.5 MHz, and we use iteration $j = 4$. We observe a considerably reduced level of the second harmonic and third harmonic frequency components at the grating lobes as compared to the respective levels at the focus. The maximum pressure level at the focus is now 536 kPa. This case shows that the INCS method is very well able to deal with a strongly steered beam from a phased array transducer.

Conclusions

- The INCS method accurately predicts the nonlinear acoustic pressure field excited by medical diagnostic phased array transducers.
- Diffraction and nonlinearity are free of assumed directionality.
- Because of its coarse discretization, the INCS method is capable of handling a three-dimensional, large-scale domain.

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FIGURE 14. Conclusions and Acknowledgements.

In this paper we have presented the Iterative Nonlinear Contrast Source (INCS) method for the accurate prediction of acoustic pressure fields with weak to moderate nonlinear distortion, and we have presented a number of results obtained with this method. The first case showed that for fields near the transducer axis, the INCS method predicts the nonlinear pressure field from medical diagnostic phased array transducer with equal accuracy as three other methods. The second case showed that, with the INCS method, diffraction as well as nonlinear distortion are free of assumed directionality of the wavefield. The third and fourth cases showed that the INCS method is capable of predicting the nonlinear field components at driving frequencies and in a domain with a size and direction that is typical for the problems occurring in the design of medical phased array transducers.

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