Spatial variability of the wave bottom boundary layer over movable rippled beds

S. Rodríguez-Abudo, D. L. Foster, and M. Henriquez

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[1] Observations of the spatially dependent velocity field over movable bed forms subjected to slightly skewed and asymmetric regular wave forcing were collected. The dynamics between the ripple elements is dominated by coherent vortices, characterized by the swirling strength, and evidenced in the temporal and spectral characterization. Within the boundary layer, spectral energy in the second harmonic (3f0) is amplified at the ripple slopes and is consistent with the location of the expected strongest pressure gradients. First-moment and second-moment velocity statistics were used to address the spatial variability of the intra-ripple hydrodynamics. Estimates of displacement and momentum thicknesses (δv and δmom) are smaller than suggested by the higher harmonics, but consistently highlight areas of adverse and favorable pressure gradients. Shear stress and roughness estimates were inferred by fitting a logarithmic model to first-moment and second-moment statistics of the velocity field. The maximum Shields parameter was observed to peak at the stoss slope of asymmetric ripples during the strongest and shorter half-wave period (onshore). First-moment roughness estimates are similar in magnitude to bed load parameterizations provided by Li et al. (1997), and about a factor of 3 larger than second-moment estimates. Assessment of the vertical transfer of horizontal momentum derived using a Reynolds decomposition suggests that stresses inferred from the logarithmic law using first-moment velocity statistics appropriately reproduce the mean momentum transfer for the longer and weaker offshore half-wave period.


1. Introduction

[2] In the coastal ocean, the initiation of sediment transport occurs in the wave bottom boundary layer (WBBL), as sediment particles mobilize when a critical threshold in the shear stress and/or horizontal pressure gradient is exceeded [Shields, 1936; Sleath, 1999]. On a flat sediment bed, shear stress acts via skin friction, the name collectively given to viscous stresses acting on sediment grains plus pressure drag imparted by each individual grain [Fredsoe and Deigaard, 1992; Maddux et al., 2003]. On a rippled bed, the large roughness elements enhance vortex formation and ejection [Bagno, 1946; Van der Werf et al., 2007; Nichols and Foster, 2007], further complicating momentum transfer in the WBBL. Due to the turbulent nature of the flow, the unsteadiness of the free stream, and the spatial nonuniformity imposed by the rippled bed, the shear stress is often parameterized with empirical and quasi-empirical formulations (for a review see Nielsen [1992] and Fredsoe and Deigaard [1992]). Parameterizations are often based on bulk flow estimates and not generally derived from direct estimates of the bottom boundary layer dynamics. Furthermore, the formulations lack a detailed spatial characterization of the relative contributions of the individual mechanisms.

[3] Early formulations of shear stress and bed roughness were largely based on field observations of flow velocity outside the WBBL [Drake and Cacchione, 1986; Drake et al., 1992]. The WBBL is characterized by a reduction in velocity magnitude and a phase lead near the sea bed [Trowbridge and Agrawal, 1995; Foster et al., 2000]. Most detailed field scale observations of fluid-sediment interactions in the coastal ocean have primarily been limited to one-dimensional profiles mainly outside the WBBL [Osborne and Vincent, 1996; Traykovski et al., 1999; Foster et al., 2000; Crawford and Hay, 2001; Doucette, 2002; Smyth et al., 2002; Thorne and Hanes, 2002; Chang and Hanes, 2004]. Recent two-dimensional (2-D) field observations of the flow field in the lower water column obtained by Doron et al. [2000] and Nimmo Smith et al. [2005]
provided significant insight on the anisotropic nature of the boundary layer and its role in wave energy dissipation, yet they lacked information very close to the bed (O(1) cm).

[4] After the early efforts of Darwin [1883], Ayton [1910], and Bagnold [1946], most detailed observations of the WBBL over rippled beds have been obtained in oscillatory flow tunnels. Over fixed rippled beds, Jonsson and Carlsson [1976] observed boundary layer thicknesses and estimated friction factors, while Ahmed and Saito [2001] reported flow separation from 2-D velocity observations. Over movable sediment beds, the occurrence, growth, and evolution of sand ripples has been shown to be dependent on the initial conditions and inconsistent with an equilibrium evolution of sand ripples. [5] In a wave flume investigation, with vertical orbital motions and a fixed rippled bed, Fredsøe et al. [1999] showed that even in combined wave-current flows, coherent vortices dominate the near bed dynamics, resulting in an increase of one order of magnitude in the apparent roughness as opposed to currents alone. Similarly, but at a slightly larger scale, Davies and Thorne [2005] identified the primary sediment entraining mechanism under monochromatic waves as a lee vortex that ejects during flow reversal; while Nichols and Foster [2007] and Nichols and Foster [2009] found correlation between these events, ripple geometry, and irregular wave forcing conditions. Field observations of Trawkowski et al. [1999] suggest that vortex shedding may be responsible for a net offshore transport of suspended sediment, while Smyth et al. [2002] showed it is the dominant process suspending and distributing sediments in environments dominated by irregular and cross ripples.

Table 1. Summary of Experimental Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave height, ( H )</td>
<td>5 cm</td>
</tr>
<tr>
<td>Wave period, ( T )</td>
<td>2 s</td>
</tr>
<tr>
<td>Wavelength, ( \lambda )</td>
<td>3.3 m</td>
</tr>
<tr>
<td>Horizontal velocity amplitude, ( u_0 )</td>
<td>11.8 cm s⁻¹</td>
</tr>
<tr>
<td>Horizontal velocity amplitude onshore, ( u_{0,cm} )</td>
<td>13.2 cm s⁻¹</td>
</tr>
<tr>
<td>Horizontal velocity amplitude offshore, ( u_{0,off} )</td>
<td>10.3 cm s⁻¹</td>
</tr>
<tr>
<td>Wave orbital semiexcursion, ( A_0 = u_0T/2π )</td>
<td>3.8 cm</td>
</tr>
<tr>
<td>Water depth, ( h )</td>
<td>31 cm</td>
</tr>
<tr>
<td>Wave asymmetry, ( As )</td>
<td>0.58</td>
</tr>
<tr>
<td>Wave skewness, ( S_{k} )</td>
<td>0.12</td>
</tr>
<tr>
<td>Kinematic viscosity of water, ( \nu )</td>
<td>1.004 × 10⁻⁶ m² s⁻¹</td>
</tr>
<tr>
<td>Sediment median grain diameter, ( d_{50} )</td>
<td>0.54 mm</td>
</tr>
<tr>
<td>Sediment specific gravity, ( \rho_s/\rho )</td>
<td>1.2</td>
</tr>
<tr>
<td>Sediment settling velocity, ( w_0 )</td>
<td>1.8 cm s⁻¹</td>
</tr>
<tr>
<td>Ripple height, ( \eta_b )</td>
<td>0.8–1 cm</td>
</tr>
<tr>
<td>Ripple wavelength, ( \lambda_b )</td>
<td>4.5–7 cm</td>
</tr>
<tr>
<td>Migration rate, ( c_b )</td>
<td>–0.005 cm s⁻¹</td>
</tr>
<tr>
<td>Reynolds number, ( Re = u_0^2T/2\nu )</td>
<td>4400</td>
</tr>
<tr>
<td>Particle Reynolds number, ( Re_p = w_0d_{50}/\nu )</td>
<td>9.7</td>
</tr>
<tr>
<td>Stokes number, ( Sk = \pi d_{50}^3/9\eta_bT )</td>
<td>0.06</td>
</tr>
<tr>
<td>Wave friction factor, ( f_{2,5} = \exp \left[5.213(2.5d_{50}/A_0)^{0.394} - 5.977\right] )</td>
<td>0.039</td>
</tr>
<tr>
<td>Grain roughness Shields parameter, ( \theta_{2,5} = f_{2,5}u_0/2(s^2 - 1)gd_{50} )</td>
<td>0.26</td>
</tr>
<tr>
<td>Sleuth parameter, ( S = 2u_{0,k}^2(s^3 - 1)/gT )</td>
<td>0.19</td>
</tr>
<tr>
<td>Kolmogorov length scale, ( l_k \sim \eta_b Re^{-3/4} )</td>
<td>O (0.01) mm</td>
</tr>
<tr>
<td>Kolmogorov velocity scale, ( u_k \sim u_0Re^{-1/4} )</td>
<td>O (1) cm s⁻¹</td>
</tr>
<tr>
<td>Kolmogorov time scale, ( t_k \sim \eta_k/u_k )</td>
<td>O (0.001) s</td>
</tr>
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2. Observations

2.1. Facility

[7] The observational data for this study was collected in a small-scale (1:15) wave flume at the Fluid Mechanics Laboratory at Delft University of Technology, the Netherlands. The flume is 42 m long, 0.8 m wide, and 1 m high. A 1:20 rigid slope starting 32 m from the wave generator was covered with sediment extending to a few meters from the wave generation area. The wave generator is capable of producing regular waves, 5 cm in height, and 2 s in wave period as observed 29 m from the wave generator in 31 cm water depth. Additional wave characteristics are summarized in Table 1. The active reflection compensation system reduced the effect of reflected waves (its mechanisms are
described in Van den Boomgaard [2003]). The coordinate system of this study defines $x$ positive offshore, $z$ positive upward, and $y$ positive out-of-the-paper.

[s] Correct physical modeling of the nearshore sediment transport in a laboratory setting requires significant scaling of the sediment particles. In order to preserve the Froude number, particle Reynolds number, Shields parameter and relative settling velocity for these wave characteristics, a sediment specific gravity ($s$) of 1.2, and median grain diameter ($d_{50}$) of 0.54 mm was necessary [Henriquez et al., 2008]. These sediment characteristics produced a grain roughness Shields parameter of 0.26, a particle Reynolds number of 9.7, and ripples of approximately 1 cm in height and 5 cm in wavelength, for the prescribed wave conditions (Table 1). Self similar field conditions would include quartz sediment grains with $d_{50} = 0.22$ mm and $s = 2.65$, and waves 50 cm in height, 6 s in wave period in 3 m of water depth.

[s] A Shields parameter of 0.26 would result in a local morphology squarely within the ripple regime and consistent with the observed bed forms. However, for the case of this scaled sediment, the horizontal pressure gradient may also contribute to the sediment dynamics. Bed dilation and subsequent plug flow conditions occur when the ratio of wave-induced pressure gradients to the immerse weight of the grains, the Sleath parameter, exceeds 0.29 according to the oscillatory laboratory studies by Sleath [1999], or 0.1 according to field observations by Foster et al. [2006]. The wave forcing conditions in this effort yielded a Sleath parameter of 0.19 (Table 1), which suggests pressure gradients may contribute to the sediment dynamics. Ancillary support for the possible influence of the pressure gradient in these conditions was provided by visual observations of a persistently thick ($\sim$10 grain diameters) mobile layer in this dynamic rippled bed. However, Henriquez et al. [2008] conclude that the mobility of the sediment bed may be larger than expected due to the scaling effects associated with a mismatch in the Sleath parameter.

2.2. Velocity Measurements

[10] A Dantec particle image velocimetry (PIV) system resolved the 2-D velocity field and ripple profile (Figure 1). The free-stream velocity was measured with an acoustic Doppler velocimeter (ADV), time-synchronized with the PIV. The PIV system consisted of a 120 mJ ND:YAG laser vertically located 27 cm above the bed, illuminating a vertical ($x$-$z$) slice of the water column. In contrast to previous efforts [Nichols and Foster, 2007], the laser was operated in low power mode, to reduce bed reflections (Figure 2a). A 1 megapixel camera (1016 pixels × 1008 pixels) located outside the wave flume obtained image pairs over a 11 cm × 11 cm sampling window at 12 Hz for 60 s bursts, leading to a total of 720 velocity fields (30 waves in each burst). The time between image pair members was selected as 10 ms to prevent particles from moving more than one third of a correlation window (21 pixels, 2.3 mm). Seeding material included sediment particles, microbubbles, and organic matter.

[11] Velocity vectors were calculated using, MatPIV 1.6.1, developed by Sveen [2004]. Correlations were calculated with two passes with interrogation windows of size 64 pixels in the horizontal and 32 pixels in the vertical, with 50% overlap. Because velocity vectors result from the average particle displacement within each correlation window, rectangular windows were employed to decrease the effect of vertical velocity gradients on the Reynolds stress and other estimated quantities, while increasing their vertical resolution. This approach was justified by the magnitude difference between the horizontal and vertical velocity components ($u$ and $w$, respectively), with particles traveling about 10 times the distance in $x$ than in $z$. The spatial resolution of the resulting velocity field is 3.5 mm in the
horizontal and 1.7 mm in the vertical. Particle displacements were calculated at the subpixel level using a three-point Gaussian estimator [Raffel et al., 2007].

[12] Noise resulting from unrealistic velocity estimates associated with the PIV technique was reduced with a three standard deviation filter. The total number of spurious vectors detected corresponded to less than 3.5% of the observations, for both $u$ and $w$. After detection, outliers were replaced with the local phase average defined as

$$
\tilde{q}(\tilde{t}) = \frac{1}{M} \sum_{m=0}^{M} q(t + mT),
$$

(1)

where $\tilde{q}$ represents any phase-averaged flow quantity, $\tilde{t}$ is the phase, $M = 30$ is the total number of waves, and $T = 2$ s is the wave period.

[13] The free-stream velocity vectors were validated with point measurements provided by the ADV following Nichols and Foster [2007]. The uncertainty level of the horizontal velocity was computed following Cowen et al. [2003] and Nichols and Foster [2007], with a bootstrap analysis at the 95% significance level [Efron and Tibshirani, 1993]. Taking the ADV time series as the true value, and the filtered PIV time series at $x = 5.23$ cm and $z = 9.24$ cm, resulted in an uncertainty of $\pm 0.72$ cm s$^{-1}$ ($\pm 6\%$ the horizontal velocity amplitude, $u_0$). Please note that this estimate also includes any uncertainty related to the ADV measurements.

[14] Closer to the bed, velocity gradients, bed reflections, and sediment suspensions may influence PIV displacement calculations. The effects of bed reflections and velocity gradients were reduced by lowering the power of the laser (Figure 2a), and using rectangular interrogation windows, respectively. Their potential influence on the PIV velocity estimates is depicted by the ratio of the highest to second highest correlation peak in each interrogation window. Generally, correlation peaks used to calculate displacements were influenced by their proximity to the bed (Figures 2b–2d). However, on average, they still met the 1.2 threshold proposed by Keane and Adrian [1992], and previously used by van der A et al. [2011]. Nevertheless, care must be taken when interpreting the results of the first cell above the bed estimate, where strongest reflections and sharpest velocity gradients occur.

[15] The size and number of particles in an interrogation window is strongly related to the accuracy of the final velocity measurement [Dong et al., 1992; Huang et al., 1999]. Near the bed, the 0.54 mm sediment particles corresponded roughly to a $5 \times 5$ pixel trace size, with an average of 20 particles per interrogation window, including other seeding material. However, coherent sediment plumes
(further discussed later in the paper) resulted in a variability of \( \pm 10 \) particles near the crests of the ripples. The trace size and number of particles allow for a conservative uncertainty estimate of 0.3 pixels, based on previous efforts by Bertuccioli et al. [1999] and Nimmo Smith et al. [2005]. With an expected displacement of 12 pixels during peak flows, the corresponding uncertainty is 2.5\%. For the case of the Reynolds stress estimate, the uncertainty improves with the square root of the total number of points averaged (i.e., \( \sqrt{720} \)), but increases by one order of magnitude for differential quantities, such as \( \mu_k/u_c^2 \) [Nimmo Smith et al., 2005].

[16] The settling velocity of the bed sediment was estimated to be 1.8 cm s\(^{-1}\), following Cheng [2009]. The corresponding wave-based Stokes number was 0.06 (Table 1) and, therefore, particles were assumed to follow the wave-induced motions closely (\( S_{tk} \ll 1 \)). However, the relatively large size of the bed sediment particles prevented resolution of the smallest scales of turbulence (\( l_k \sim 0.01 \) mm, \( u_k \sim 0(1) \) cm s\(^{-1}\), \( k \sim 0(0.001) \) s, Table 1. In this investigation, we will focus the analysis on integral flow quantities, rather than the turbulence microscales.

[17] As the PIV laser sheet illuminated both bed sediment suspended in the water column and other smaller particles with substantially less settling potential (sediment fragments, organic matter, etc.), the effect of sediment settling velocity on the PIV measurements was expected to be minimal and, therefore, no settling was subtracted from the vertical velocity signal. The same argument yields to the assumption of minimal settling-induced turbulence, and therefore negligible trajectory bias [Maxey, 1987].

[18] Similar efforts that have successfully achieved near-bed PIV velocity measurements with strong presence of loose sediments and/or spatial nonuniformity in particle concentration, include those of Ahmed and Sato [2001], Nichols and Foster [2007], Van der Werf et al. [2007], and van der A et al. [2011].

### 2.3. Bed Elevation

[19] Previous optical and acoustic studies have approximated the bed position to generally within 1 cm [Crawford and Hay, 2001; Doucette and O’Donoghue, 2006a; Van der Werf et al., 2007; Nichols and Foster, 2007, 2009]. However, they either rely on sonar scanning times that are long compared to the wave period [Doucette and O’Donoghue, 2006a], laser point measurements during inactive flow conditions [Van der Werf et al., 2007], or time averages of image intensity over entire data realizations [Nichols and Foster, 2007, 2009]. In the latter, bed reflections associated with the PIV laser sheet presented serious difficulties within the lowest 1 cm from the bed. In this study, we will follow the method of Nichols and Foster [2007], replacing the time-averaged image with the phase average defined with equation (1). Image phase averaging is a valid approach for determining bed elevations in this data set, as the total migration of the ripples is considerably slow (0.005 cm s\(^{-1}\)) compared to the duration of the realization (60 s). Bed elevation estimates from phase-averaged pixel images were contaminated by recurrent coherent sediment plumes obscuring the bed (Figure 3b). In this effort, the effect of the coherent plumes was eliminated by using the phase-maximum pixel intensity (\( I_{\text{max}}(x,z,t) \), Figure 3a) as the bed elevation baseline. The phase-maximum pixel intensity is defined with

\[
I_{\text{max}}(x,z,t) = \max(I(x,z,t + mT), \forall m = 0, 1, 2, ..., M - 1),
\]

(2)

where \( I \) is the intensity value in each pixel and \( M = 30 \) is the total number of waves. A bed position baseline, \( z_{\text{base}}(x,t) \), is taken as the lowest-elevation point with the maximum pixel intensity in the phase-maximum pixel image (Figure 3a). The actual bed position, \( z_b(x,t) \), is assumed to be the baseline estimate plus a constant given by

\[
C(t) = \langle z_b(x,t) - z_{\text{base}}(x,t) \rangle,
\]

(3)

where the angle brackets represent spatial average in the \( x \) direction and \( z_b \) is the centroid of the phase-averaged pixel image as suggested by Nichols and Foster [2007]. The final bed height is given by

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**Figure 3.** (a) Phase-maximum and (b) phase-averaged pixel image at \( \bar{t} = 235^\circ \) showing: \( z_{\text{base}} \) (red), \( z_c \) (blue), and \( z_b \) (green). (c) Vertical profiles of phase-maximum (red) and phase-averaged (blue) pixel intensity at \( x = 4.32 \) cm with diamonds corresponding to the values in Figures 3a and 3b as shown by the black dashed line. Images have been zoomed into the near-bed region.
Finally, a running average of size 30 pixels (3.2 mm) was applied to $z_b$.

The variability of $z_b$ over $\tilde{t}$ fluctuated between 2 mm at the trough and 6 mm at the onshore slope of the rightmost ripple. This approach is different from that used by Nichols and Foster [2007] by a root-mean-square difference of $\sim$2 mm, computed by comparing the centroid of the mean intensity image over the 720 samples to the mean bed elevation found by averaging the 24 phases. As $z_b$ is resolved at 1 pixel resolution, velocity measurements corresponding to interrogation windows at or below $z_b$ were excluded (masked) from the analysis.

2.4. Wave Characteristics

Figure 4a shows that skewed and asymmetric waves, characterized by longer offshore-directed half-wave periods (positive horizontal velocity with lower peak magnitudes) and shorter onshore-directed half-wave periods (negative horizontal velocity with higher peak magnitudes), were present at the instrument location. The wave skewness ($Sk = 0.12$) and asymmetry ($As = 0.58$) were estimated using the free-stream ($x = 5.23$ cm, $z = 9.24$ cm) velocity skewness and temporal asymmetry, respectively, following Gonzalez-Rodriguez and Madsen [2007]. Based on the work by Elfrink et al. [2006], these wave characteristics will be present in field conditions with a constant bed slope of 1:20 and $Hh/C_0 \geq 10$.

Positively asymmetric waves have a forward leaning face and a gentle rear, which induces highest fluid accelerations in the onshore direction. The implications include higher bed shear stresses, and consequently higher sediment pick up rates, during the onshore half-wave period [Nielsen, 2006; Gonzalez-Rodriguez and Madsen, 2007]. Wave skewness results from the evolution of the wave field.

Figure 4. (a) Free-stream horizontal (black) and vertical (red) velocity for one wave period at $x = 5.23$ cm and $z = 9.24$ cm. (b) Onshore-averaged flow and (c) offshore-averaged flow (red vectors) over the corresponding half-wave averaged swirling strength ($\lambda_{ci}$), as shown by the gray colormap. (d) Standard deviation velocity field (red vectors) over the rms $\lambda_{ci}$ (gray colormap). (e) Mean velocity field (red vectors) over the mean image intensity computed over the 60 s PIV realization. Scale vectors represent 10 cm s$^{-1}$. Velocity data under $z_b$ (green solid line) have been masked. Onshore flow is directed to the left. Blue dots on Figure 4e provide the horizontal location for the data presented in Figure 5.
along the tank and is consistent with shoaling of ocean waves [Elgar and Guza, 1985]. Narrow crests and wide troughs result in longer excursion times in the offshore direction, and the potential for offshore sediment transport [Hassan and Ribberink, 2005]. We can anticipate that these nonlinearities in the wave field will affect the boundary layer dynamics [van der A et al., 2011] and bed form evolution [Crawford and Hay, 2001].

3. First-Moment and Second-Moment Velocity Statistics

[23] Approximations of various hydrodynamic- and sediment transport-related characteristics are evaluated with both the first and second moments of the regular monochromatic wave observations. The first moment of the velocity field is defined with the time-average of the velocity field over half-wave periods, which were detected by finding the zero-crossings in the horizontal free-stream velocity. Half-wave periods in each horizontal flow direction (onshore and offshore) were averaged over an entire realization, resulting in a mean velocity field for each flow direction, consisting of \( \bar{u}(x,z) \) and \( \bar{w}(x,z) \) in the onshore (negative) direction and \( u_{off}(x,z) \) and \( w_{off}(x,z) \) in the offshore (positive) direction (Figures 4b and 4c).

[24] The second moment of the velocity field is defined with the square root of the variance (i.e., the standard deviation) of the horizontal and vertical velocity components, \( \sigma_{u}(x,z) \) and \( \sigma_{w}(x,z) \) (Figure 4d). Standard deviation velocity estimates are common in the literature [Cox et al., 1994; Trowbridge and Lentz, 1998; Shaw and Trowbridge, 2001] as they allow for simple assessment of the magnitude of the wave field.

[25] The slightly skewed and asymmetric wave forcing and the presence of a rippled boundary induce a complex flow pattern, characterized by shedding vortices and a “jet regime” [Gimenez-Curto and Corniero Lera, 1996] above the ripple crests. While this paper will focus the analysis on boundary layer characteristics and flow quantities derived from first-moment and second-moment velocity statistics, we understand that a brief temporal and spectral description of the flow within the WBBL is in order, before furthering the quasi-steady analysis.

4. Temporal Variability

[26] Inflection points in the vertical profiles of the phase-averaged horizontal velocity near the central trough of the ripples (\( x = 4.5 \) and \( 5.9 \) cm, Figures 5a and 5c, respectively) provide evidence of boundary layer separation during phases of flow reversal (roughly 0° and 180°). The separated boundary induces a phase lag that is evident at elevations roughly between 4 and 5 cm, and a phase lead confined to elevations below 4 cm. Phase separation will be further discussed in section 5.

[27] Sharp transitions in the velocity profiles near the ripple trough are more evident during peak flows above lee slopes (Figures 5a and 5c), that is, peak offshore (\( f = 120° \)) at \( x = 4.5 \) cm and peak onshore (\( f = 270° \)) at \( x = 5.9 \) cm. Although we cannot suggest a physical reason for these slope discontinuities other than experimental noise, their occurrence suggests a dependence on boundary geometry given perhaps by the adverse pressure gradients at the lee slopes. Clear signatures of velocity overshoot, typical in oscillating boundary layers and explained with the velocity defect acting as a “dampened wave” propagating from the boundary [Nielsen, 1992], are observed during peak offshore flow, at the rightmost ripple crest (\( x = 8 \) cm, Figure 5c). Velocity profiles during peak flows at this location also suggest a slipping boundary.

[28] This temporal and vertical complexity in the nearbed velocity may be a result of periodic “jets” associated with the vortical structures generated by the large-scale boundary irregularities, as previously suggested by Sleath [1987] and Gimenez-Curto and Corniero Lera [1996]. Following Zhou et al. [1999] and Nichols and Foster [2007], coherent flow structures were identified with the swirling strength criterion (\( \lambda_{w} \)) computed from phase-averaged horizontal and vertical velocities. Half-wave averages of \( \lambda_{w} \) (Figures 4b and 4c) are consistent with swirling motions generated at the ripple crests and prevailing over the lee slope of the ripples. The strongest signatures are present for the offshore half-wave period, and may be explained by the temporal asymmetry and velocity skewness characterizing the wave field. The rms swirling strength (Figure 4d) show two distinct layers separated at roughly \( z = 4 \) cm. Consistent with qualitative observations of the phase-averaged pixel intensity provided in Rodriguez-Abudo [2011], and previous observations by Van der Werf et al. [2007], the weak swirling signatures above \( z = 4 \) cm, may be related to vortices generated by adjacent ripple crests, which have been shed into the water column and advected into the flow with a slight loss of strength.

5. Spectral Variability

[29] Stacks of horizontal velocity spectra for three different horizontal locations (Figures 5b, 5d, and 5f) clearly show adequate resolution throughout the water column of the incident wave band and the next harmonic. Consistent with the ratio of the highest to second highest PIV correlation peaks (Figure 2), the noise floor increases generally from right to left in the image, and from top to bottom.

[30] To further explore the space-dependent spectral decay/enhancement of the horizontal velocity, the power spectral density and corresponding phase separation at the incident wave band \( f_{0} \) and two higher harmonics (2\( f_{0} \) and 3\( f_{0} \)) with respect to the free stream are presented in Figure 6. The energy at the incident band shows an amplitude decay of roughly 80% in the immediate proximity to the bed (Figure 6a). In a laboratory study concerning the spectral decay of surface gravity waves, Doering and Baryla [2002] observed an amplitude decay of roughly 50% just outside the boundary layer under Stokes waves over a rippled sediment bed.

[31] The local phase separation was computed from cross-spectral analysis against the spatially averaged free-stream velocity. Space- and time-dependent horizontal velocity, \( u(x,z,t) \), was compared against the corresponding free-stream velocity, \( u_{\infty}(x,t) \). Consistent with Figure 5e, phase leads are present roughly \( \eta_{\phi} \) above the crest. Since waves propagate from right to left, there is a slight phase lead at the offshore side of the image, and a phase lag at the onshore side. Generally, the boundary layer leads the free
stream by 0–25°. These values show variability over the ripple, but are greater than previous observations of the WBBL over flat beds, which showed roughly 10° phase lead [Trowbridge and Agrawal, 1995; Foster et al., 2000]. Note that the near-bed phase lead at the central trough (x = 5.1 cm) is not as high as at the other troughs (x = 0.5 cm and x = 10.2 cm), likely due to stronger adverse pressure gradients present in this relatively deeper ripple trough. Higher in the water column, a phase lag of 10° is observed above the main ripple trough, showing consistency with Figures 5a and 5c. This lag is suggestive of vortices being shed from the ripples during flow reversals and is not inconsistent with the rms swirling strength presented in Figure 4d.

At the first harmonic (2f₀), the spectral energy amplitude is lower than at the incident band (f₀) and decreases near the bed (95% decay). A strong phase lead of roughly 100° is observed at the main ripple trough (Figure 6e), and may be suggestive of the role of horizontal pressure gradients in this relatively deeper trough. Free-stream spectral energy at the second harmonic (3f₀) is about 5% of the energy contained in the first harmonic (2f₀). However, a statistically significant increase of almost two orders of magnitude is observed with proximity to the bed (order 1 and 100 given by the colors cyan and yellow, respectively, in Figure 6c and top colorbar). The strong amplification of spectral energy in the near-bed region suggests that not only the nonlinearities imposed by the skewed wave field play a role within the boundary layer [van der A et al., 2011], but also that these are highly amplified due to presence of a rippled boundary. This is consistent with numerical [Ning et al., 2012] and experimental [Beji and Battjes, 1993] studies showing increasing amplitude in the higher harmonics of a wave field at the lee side of large-scale bars in the nearshore.

Slight phase leads (30°) at the second harmonic (3f₀) are visible at the left and right ripple troughs (Figure 6f). Contrastingly, at the main ripple trough, a strong phase separation (±180°) with respect to the free stream is observed at the ripple slopes. As a matter of fact, in their study over flat sand beds, van der A et al. [2011] saw the strongest phase separation occurring at the second harmonic. They also found that almost perfectly sinusoidal flows, like the one in the present study, are more susceptible to strong
phase separations at the second harmonic than are highly acceleration skewed flows. For the present study, we expect the asymmetry in the rippled bed, provided by the deeper ripple trough, to further increase the phase separation due to the adverse and favorable pressure gradients imposed by each ripple slopes. Once again, these findings suggest that nonlinearities in the wave field and ripple geometry may play an important role in the characterization of the near-bed turbulent flow and boundary layer thickness. In general, Figure 6 suggests that above \( z / C_2^5 \lesssim 5 \text{ cm} \), the spectral characteristics are uniform, and the boundary layer is confined to \( \sim 2 \text{ cm} \) at the troughs, and \( \sim 1 \text{ cm} \) at the crests.

6. Boundary Layer Thickness

[34] The thickness of the boundary layer was also characterized for first-moment and second-moment statistics with an integral approach for the displacement and momentum thicknesses. The displacement thickness, \( \delta^* \), is the distance the bed will be upwardly displaced in a frictionless flow so as to maintain the same mass flux [Schlichting, 1979]:

\[
\delta^*(x) = \int_{z_0}^{\infty} \left( 1 - \frac{u(x,z)}{u_\infty(x)} \right) dz. \tag{5}
\]

[35] Figure 7 shows \( \delta^* \) for first-moment (onshore and offshore) and second-moment (standard deviation) flow statistics. Consistent with laminar boundary layers, \( \delta^* \) becomes thicker in response to the adverse pressure gradient present in the ripple trough. The increased thickness is also consistent with the expanded logarithmic layers shown in Figure 8. Moreover, the smaller boundary layer thicknesses at the ripple crests are expected due to flow acceleration and are particularly evident in Figure 7e. Flow amplification at the ripple crest may result in enhanced shear [Nielsen, 1986], and induce a slip boundary. The relationship between the shear stress and the bottom boundary will be further discussed later in the paper.

[36] The momentum thickness, \( \delta_{mom} \), represents the distance the bed would be displaced upward in an inviscid flow in order to conserve the same momentum deficit:

\[
\delta_{mom}(x) = \int_{z_0}^{\infty} \frac{u(x,z)}{u_\infty(x)} \left( 1 - \frac{u(x,z)}{u_\infty(x)} \right) dz, \tag{6}
\]

[37] Figure 7 shows that \( \delta_{mom} \) and \( \delta^* \) behave similarly, although the magnitudes of \( \delta_{mom} \) are slightly lower than \( \delta^* \), especially above ripple troughs. The ratio of \( \delta^* \) to \( \delta_{mom} \) provides information regarding flow separation in turbulent flows [White, 1994]. Large ratios imply a momentum loss.
Figure 7. Viscous shear stress (color scale) in the nearbed region (right column) and over the entire imaged area (left column) as calculated with $u_{on}$ ($x, z$) (a, b), $u_{off}$ ($x, z$) (c, d), and $u_{std}$ ($x, z$) (e, f), with corresponding bed elevation (gray), for a single realization. Data under $z_b$ have been masked. The displacement thickness, $\delta$, and momentum thickness, $\delta_{mom}$, are indicated by the black dashed and solid lines, respectively. Black arrows indicate the direction of the flow.

Figure 8. Horizontal velocity profiles (open circles) plotted on a logarithmic scale at every second horizontal location for (top) onshore-directed flow, (center) offshore-directed flow, and (bottom) standard deviation. The least-squares fit to the horizontal velocity profiles is plotted with a black solid line. The mean bed elevation is shown in gray.
neat the boundary in response to adverse pressure gradients
inducing flow separation. This is evident in Figures 7a and
7b, as flow constricts up the ripples’ stoss slopes and crests
maintaining a low \( \delta / \theta_{\text{nom}} \) ratio, and then diverges at the
lee slopes and troughs with higher \( \delta / \theta_{\text{nom}} \) ratios. This is not as
evident for the standard deviation estimates, although \( \delta / \theta_{\text{nom}} \) remains high at the center ripple trough.

[38] As shown in Table 2, the integral approach for esti-
mating boundary layer thickness using \( \delta \) and \( \theta_{\text{nom}} \) yields results that are almost four times smaller than boundary
layer thicknesses suggested by the decay of the higher har-
monics (O(1–2) cm), although their relationship may be
more complex. It should be noted that the integral approach
becomes problematic when velocity overshoots or stream-
lines narrow above the ripple crests, resulting in \( u > u_{\infty} \),
and negative values of \( \delta \), as seen in Figure 7b for \( x < 1 \) cm,
and Figure 7c at the rightmost ripple crest (\( x = 8 \) cm).

7. Viscous Stress

[39] Also shown in Figure 7 is the viscous shear stress
\( -\mu \frac{d\bar{u}}{dz} \), where \( \mu \) is the dynamic viscosity of water). In turbu-
lent flows, the viscous shear should be several orders of
magnitude less than the total shear, however, it provides a
qualitative notion of the flow behavior. Viscous shear is
larger on the stoss side of the ripples for both onshore
directed and offshore directed flows. This is also consistent
with flow acceleration as it constricts up the ripple face.
Conversely, the flow decelerates on the lee side of the rip-
ple, forming a wake-like area that remains present until it is
affected by the presence of the next ripple [Nelson and
Smith, 1989; Wiberg and Nelson, 1992; Li, 1994; Coleman
et al., 2008]. This suggests the presence of a recirculating
area after the ripple crest, in a manner analogous to a back-
ward facing step [Nichols and Foster, 2007], and consistent
with the average swirling strength signal (Figures 4b and
4c).

[40] The magnitude of the viscous shear is higher during
the offshore directed flow. This is opposite to the onshore
directed bed form migration, suggesting that higher order
terms may be responsible for ripple mobilization, such as
skewness of the near-bed orbital velocity [Crawford and
Hay, 2001], and vortex formation and ejection [Traykovski
et al., 1999]. During the strong onshore directed flow (Fig-
ure 7a), high shear is generated at the stoss side of the rip-
ple (\( x = 3–4.5 \) cm), where the pressure gradient is
favorable, and then shed into the water column forming a
wake-like region, with little to no viscous shear at the
lee side. This is less evident for the offshore shear (Figure 7a),
where viscous shear appears to follow the ge-
ometry of the bed more closely, probably due to the milder
free-stream flow and longer excursion times. The longer
excursion times allow for increased boundary layer devel-
opment. In the standard deviation case, the viscous shear
goes to zero at heights below 4.5 cm regardless of location
along the ripple, and the sharpest gradients occur at the off-
shore ripple slopes and crests, suggesting a strong depend-
ence on the dynamics present during the onshore directed
flow. Each of the three cases show qualitatively that the
region where velocity gradients are considerable is compa-
rable to boundary layer thickness estimates using the higher
harmonics (O(1–2) cm).

8. Shear Velocity and Roughness Estimates

[41] Horizontal velocity profiles across the rippled bed
for the onshore, offshore, and standard deviation flows
are presented in Figure 8. Although the shape of the veloc-
ity profiles is affected by vortex shedding, and the logarith-
mic law may not be valid in this complex environment,
Figure 8 suggests that the first and second moments of the
horizontal velocity generally decay logarithmically.

[42] In his study concerning atmospheric boundary
layers over a urban canopy, Jackson [1981] showed that
velocity profiles above large roughness elements follow a log-
arithmic law of the form

\[
u = \frac{u_{\infty}}{\kappa} \ln \left( \frac{z - d}{z_0} \right), \tag{7}
\]

where \( u_{\infty} \) is the shear velocity, \( \kappa = 0.4 \) is the Von Karman
coefficient, \( z_0 \) is the hypothetical vertical position where
\( u = 0 \), and \( d \) is the displacement height. Jackson [1981]
showed that \( d \) represents the average level at which the
mean drag appears to act on a rough boundary. In the case
of a rippled boundary, \( d \) may be associated with the mani-
festation of the flow blockage introduced by the ripple
crest, and therefore dependent on the ripple size and shape.

[43] Iterative methods are commonly used to find \( d \) in
studies where the lowest velocity measurement is taken just
above the tallest roughness element [Cox et al., 1996]. The
spatial resolution in this effort allows us to obtain velocity
observations between the roughness elements, and there-
fore examine the spatially variable displacement height.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Onshore</th>
<th>Offshore</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>[cm]</td>
<td>0.57 ± 0.22</td>
<td>0.30 ± 0.21</td>
<td>0.28 ± 0.16</td>
</tr>
<tr>
<td>( \theta_{\text{nom}} )</td>
<td>[cm]</td>
<td>0.40 ± 0.14</td>
<td>0.21 ± 0.13</td>
<td>0.22 ± 0.12</td>
</tr>
<tr>
<td>( \Delta x )</td>
<td>[cm]</td>
<td>0.42 ± 0.17</td>
<td>0.44 ± 0.20</td>
<td>0.38 ± 0.15</td>
</tr>
<tr>
<td>( d )</td>
<td>[cm]</td>
<td>2.34 ± 0.39</td>
<td>2.31 ± 0.42</td>
<td>2.35 ± 0.38</td>
</tr>
<tr>
<td>( u_{\infty} )</td>
<td>[cm s(^{-1})]</td>
<td>( -1.42 ± 0.43 )</td>
<td>( 1.23 ± 0.40 )</td>
<td>( 1.04 ± 0.29 )</td>
</tr>
<tr>
<td>( \theta_{\text{nom}} )</td>
<td>[cm]</td>
<td>0.21 ± 0.13</td>
<td>0.17 ± 0.11</td>
<td>0.11 ± 0.06</td>
</tr>
<tr>
<td>( z_0 )</td>
<td>[cm]</td>
<td>( 0.23 ± 0.31 \leq 0.38 )</td>
<td>( 0.20 &lt; 0.27 ≤ 0.33 )</td>
<td>( 0.07 &lt; 0.11 ≤ 0.15 )</td>
</tr>
<tr>
<td>( R )</td>
<td>[cm]</td>
<td>( 7.0 ≤ 9.2 ≤ 11.3 )</td>
<td>( 5.9 ≤ 8.0 ≤ 9.9 )</td>
<td>( 2.2 ≤ 3.2 ≤ 4.4 )</td>
</tr>
<tr>
<td>( \Delta u_{\text{rms}} )</td>
<td>[cm]</td>
<td>2.76 ± 0.29</td>
<td>2.75 ± 0.29</td>
<td>2.73 ± 0.29</td>
</tr>
<tr>
<td>( \min \left( \Delta u_{\text{rms}} \right) )</td>
<td>[cm]</td>
<td>2.27 ± 0.08</td>
<td>2.25 ± 0.09</td>
<td>2.23 ± 0.08</td>
</tr>
<tr>
<td>( \max \left( \Delta u_{\text{rms}} \right) )</td>
<td>[cm]</td>
<td>3.25 ± 0.13</td>
<td>3.25 ± 0.12</td>
<td>3.23 ± 0.13</td>
</tr>
</tbody>
</table>

3500
with \( d_* = z_{\text{base}} - \Delta z \), where we define \( \Delta z \) as the adjustment height for logarithmic layers that lie between the ripple roughness elements (Figure 9). On a rippled bed, one would expect that (1) at the ripple crests the bed form-induced effects on the velocity profiles are minimal and \( d_* \approx z_{\text{base}} \) and (2) at the ripple trough the bed form-induced effects on the velocity profiles are considerable and consequently \( \Delta z \) reaches its maximum. Consistent with visual observations, this formulation also allows for a slip boundary condition to exist at \( z_{\text{base}} \).

Substituting for \( d_* \) in equation (7), yields

\[
\frac{u(x, z)}{u_*} = \frac{\Delta z / \kappa}{z_0(x)} \ln \frac{z - z_{\text{base}}(x)}{z_0(x)}.
\]

for both, first-moment and second-moment statistics. Following Cox et al. [1996], Fredsøe et al. [1999], and others, least-squares regression to the observations (Figure 8) allows for estimates of \( u_* \), \( z_{\text{base}} \), and \( z_0 \). The lower limits of the regression analysis were located at the first cells above \( z_{\text{base}} \), where sediment concentration is assumed to be a maximum by noting that pixels do not saturate below this point. The upper limits of the regression analysis were located at the cell immediately below the velocity overshoot or at the lowest elevation, where \( u(x) \geq 0.99 u_{\infty}(x) \). Degrees of freedom (DOF = \( n - 2 \), with \( n \) being the number of points in the regression), for each regression fit are shown in Figure 10a. The adjustment height \( \Delta z \), Figure 10c) was found iteratively by increasing \( \Delta z \) by 0.1 mm until the percent difference of the square of the correlation coefficient of the regression (\( r^2 \), Figure 10b) between consecutive \( \Delta z \) increments was less than or equal to 0.1%. The initial \( \Delta z \) used at every horizontal location was 0.5 mm.

\( \Delta z \) is correlated to the bed elevation (see Figure 10), increasing at the ripple troughs and decreasing at the crests. \( \Delta z \) supports the significance of the bed form-induced pressure gradients and shedding vortices present in the WBBL, while also suggesting a possible significant role of additional spatial gradients.

[46] Jackson [1981] showed that \( d_* \) is, for a rough boundary, the average level at which the mean drag appears to act. The present analysis results in \( d_* \) values located slightly above the ripple trough (min \( z_{\text{base}} \), Table 2). The mean drag appears to act just above the main ripple trough, with standard deviations that are similar in magnitude to half a ripple height (\( \eta/2 \)), and suggests a relaxation of the no-slip boundary condition at \( z_{\text{base}} \) that is not inconsistent with visual observations of significant sediment movement at the ripple crests and slopes. These results suggest that the mean drag on this artificial sediment bed acts closer to the base of the ripple rather than at the expected centroid.

[47] In steady flow, the nondimensional number governing the ratio between the disturbing and stabilizing forces was suggested by Shields [1936],

\[
\theta_u = \frac{u^2}{(s - 1)g d_0}
\]

where \( u_* \) is obtained from the least-squares fit performed in equation (8) and \( g \) is the gravitational acceleration. The critical Shields parameter for incipient motion, \( \theta_{u_{\text{c}}} \), is taken from the modified Shields diagram [Madsen and Grant, 1976] as 0.04. For \( \theta_{u_{\text{c}}} > 0.8 \), the bed is typically considered to have sheet flow conditions and flattens. If we allow \( u_* \) to vary spatially, then \( \theta_{u_{\text{c}}} \) is also spatially variable. Figure 10d shows \( \theta_{u_{\text{c}}}(x) \) for onshore, offshore, and standard deviation flows, with vertical bars indicating the standard error of \( \theta_{u_{\text{c}}} \) with 90% confidence. The values of \( \theta_{u_{\text{c}}} \) are consistent with a bed within the ripple regime.

[48] In general, the values of \( \theta_{u_{\text{c}}} \) are comparable with the grain roughness Shields parameter defined in Table 1, where the friction factor is given by Swart [1974], following Nielsen [1992], Crawford and Hay [2001], and Nichols and Foster [2007], among others. For irregular waves, the velocity scale used in the \( \theta_{u_{\text{c}}} \) expression is usually taken as the significant wave orbital velocity \( u_{\text{rms}} = 2u_{\text{rms}} \) [Maier and Hay, 2009], which for this effort would yield a value of 0.46, only exceeded by the onshore directed shear stress at the stoss slope of the leftmost ripple crest (\( x = 3.5 \) cm and \( x = 5 \) cm). High stress on the stoss slope of the ripple crest is consistent with the direction of bed form migration and is also consistent with the asymmetry of the ripples. Given that the center trough is deeper than the other two, the leftmost ripple may act as a forward facing step during the higher onshore flows.

[49] The overall statistics of \( \theta_{u_{\text{c}}} \) for the four realizations (Table 2) show significant variability of \( \theta_{u_{\text{c}}} \) values, as expected due to the highly variable bed elevation. While the \( \theta_{u_{\text{c}}} \) values tended to increase on the stoss side of the steep ripple, no significant correlation between bed elevation and \( \theta_{u_{\text{c}}} \) was found. The magnitude of \( \theta_{u_{\text{c}}} \) obtained from standard deviation velocity profiles is generally smaller than the other two estimates, with standard error significantly lower. This is not surprising given that the standard deviation velocity profiles do not quantify the directional characteristics of the flow field (including the coherent vortices).
Both first-moment (onshore and offshore) estimates of $z_0$ peak at the leftmost ripple trough, specifically at the stoss slope during onshore directed flow, and lee slope during offshore directed flow. Higher order effects induced by asymmetry of the flow and ripples, such as coherent motions and flow acceleration, may contribute to this amplification of $z_0$. The offshore estimate peaks at $x = 4.9$ cm, consistent with the location of the strongest offshore average of the swirling strength (Figure 4c). Similar to $\theta_{u*}$, second-moment estimates of $z_0$ and their standard error, are consistently smaller than the first-moment estimates. Additionally, overall statistics (Table 2) show that the range of $z_0$ values from second-moment velocity estimates is half the range of the values derived with the first-moment velocity estimates.

In turbulent boundary layers over rough surfaces, we can regard $z_0$ as a measure of roughness [Kundu and Cohen, 2004]. It is common in the WBBL literature to parameterize the bottom roughness with the equivalent Nikuradse roughness ($r$) defined as $r = 30z_0$ (Table 2). First-moment $r$ estimates are similar for both flow directions and comparable in magnitude to roughness estimates computed with $\theta_{2.5}$ based on the bed load expression for combined waves-currents flow given in Li et al. [1997] ($r_L = 522d_50(\theta_{2.5} - \theta_1)_{75} = 9.1$ cm). As with $z_0$, second-moment estimates are about a factor of 3 lower and in

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**Figure 10.** (a–c) Regression parameters including degrees of freedom (DOF), correlation coefficient squared ($r^2$), and adjustment height ($\Delta z$). (d) Shields parameter and (e) $z_0$ at each horizontal position as calculated from the least-squares regression fit over onshore, offshore, and standard deviation velocity profiles for a single realization. Vertical bars indicate the standard error with 90% confidence. The dashed line in Figure 10e indicates one ripple height. (f) $z_{\text{base}}$ used as the lower limit of the velocity profiles used in the regression fits.
better agreement with the empirical formulations for bed load roughness given by Grant and Madsen [1982] and Nielsen [1992], that is, \( r_{GM} = 160(s + 0.5)d_{50}\theta_e \left( \sqrt{\frac{\theta_{2.5}}{\theta_e} - 0.7} \right)^2 = 2.0 \text{ cm} \) and \( r_N = 170d_{50}\sqrt{\theta_{2.5}} - \theta_e = 4.3 \text{ cm} \). Please note that we have purposely excluded the ripple-induced roughness, since our analysis is concerned with the intraripple flow. Although a formal assessment of \( r \) based on theoretical grounds is still required, these results suggest that, in general, the log-layer approach may be able to capture the processes involved in some of the available parameterizations for bed roughness.


[52] The fundamental assumption in deriving a logarithmic model for the horizontal velocity distribution relies on a no-slip boundary condition at the wall, resulting in a wall shear stress defined by \( \tau_0 \equiv \mu \frac{u}{\varepsilon} \). Following Tennekes and Lumley [1972], near a smooth wall, there is a small inner layer where molecular diffusion dictates the shape of the velocity profile, and therefore, \( u = f(u, \nu, z) \). Far away from the wall, but still within the boundary layer, the velocity deficit is primarily due to the turbulent transfer of momentum imposed by the no-slip boundary condition at the wall. It is, therefore, expected that in this outer layer, \( u = f(u_e, \delta, z) \). The small overlapping layer where the solutions for both, the inner and outer layers hold, was shown, from purely dimensional arguments by Millikan [1938], to follow a logarithmic law. The common logarithmic law for hydrodynamically rough surfaces results from requiring a boundary condition, where \( u(z_0) = 0 \).

[53] The logarithmic model, while not explicitly requiring steady flows, zero pressure gradients, or a particular stress distribution, is only valid for a small layer, where \( u \neq f(\nu, \delta) \). While independence of \( \nu \) is expected in this hydrodynamically rough environment (Figure 7), the region where the logarithmic regression was performed represents a considerable portion of the boundary layer, roughly 50% when taking \( \delta \) as the estimate provided by the higher harmonics (O(2) cm). Despite the proximity of the logarithmic layer to \( \delta \), and visual observations of a slipping boundary, fitting a logarithmic model to the data yields statistically sound inferences (Figure 10).

[54] We assess the logarithmic layer approach with independent estimates of the momentum transfer within the same layer of thickness \( \delta_{log} \). Similar to Mellor [2002], let \( \tilde{\tau}(x, z) \equiv \tilde{\tau}_{xz}(x, z, t) \) (Figures 11a–11c), where the overbar (\( \tilde{\cdot} \)) represents time average and \( \tau_{xz} \) is the vertical transfer of horizontal momentum, which is approximated by performing a Reynolds decomposition on the velocity and pressure fields, and subsequently phase-averaging the equations of motion. Velocity and pressure are decomposed into

![Figure 11](image-url)

Figure 11. (a) Onshore, (b) offshore, and (c) rms estimates of the time-independent vertical transfer of horizontal momentum as depicted by \( \sqrt{\tau_{xz}/\rho} \). Black solid lines represent \( \delta_{log} \). (d) Onshore, (e) offshore, and (f) rms estimates of \( u_e \) (black dotted lines, equation (15), with corresponding variation within \( \delta_{log} \), as computed from the standard deviation (gray shaded area). Values of \( u_e \), (equation (8)) and their corresponding error are presented in red (Figures 11d and 11e) and blue (Figure 11f). Empirical estimates of the shear velocity given by \( \sqrt{f_{2.5}/2 u_0} \) (Table 1) are shown by the black dashed line.
mean (current), phase (wave), and fluctuating (turbulent) components defined by

\[ u_t = \bar{u} + \tilde{u} + u'_t \]  
\[ p = \bar{p} + \tilde{p} + p' \]  

where overbar (') represents time average, over tilde (') denotes phase average with zero mean, and prime (') represents departure from the latter. Time and phase averages are calculated following Nielsen [1992] with

\[ \bar{u}_i(t) = \frac{1}{N} \sum_{n=1}^{N} u_i(t_n) \]  
\[ \tilde{u}_i(t) = \frac{1}{M} \sum_{m=0}^{M} u_i(t + mT) - \bar{u}_i \]  

where \( N = 720 \) is the total number of samples, \( M = 30 \) is the total number of waves, and \( T = 2 \) s is the wave period.

[55] Substituting equation (10a)–(10b) in the Navier-Stokes equations for incompressible flow in Cartesian tensor notation, and taking the phase average yields

\[ \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}'_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} \]  

where it has been assumed that the mean quantities are negligible (Figure 4c). In Cartesian coordinates, the terms that represent the vertical transfer of horizontal momentum [Nielsen, 1992] are given by

\[ \tau_{wz}(x, z, t) = \mu \frac{d\tilde{u}}{dz} - \rho \tilde{w} - \rho \tilde{w}' \]  

where the first term represents the viscous stress, the second term is the “wave” stress, and the third term is the Reynolds stress. In contrast to Mellor’s expression, which was originally developed for rigid lid oscillatory flows and flat beds (\( \nu = 0 \)), equation (14) includes the effects of both, bed form-induced acceleration/deceleration and bed form-induced coherent motions, given by the highly correlated terms \( \tilde{u}\tilde{w} \). The corresponding shear velocity is defined as

\[ u'_t(x) = \rho \log \int_{z_0}^{\infty} \tau_m(x, z) dz \]  

[56] This approach for computing \( u'_t \) provides for a direct comparison with values of \( u'_t \) inferred from the log-layer fits to the rms velocity signal (or \( u_{30} \) with zero mean, Figure 11f). Please note that while there appears to be a signal at the upper left corner of the rms \( \tau_m \) (Figure 11c), it does not affect the \( u'_t \) estimates, since the integration only includes elevations below \( \theta_{log} \). Additionally, comparisons with half-wave averages require a modulation of \( \pi (2\sqrt{2}) \) since the rms of a half-wave period is mathematically different from the average (Figures 11d and 11e). It is also worth noting that the positive definition of \( \tau_m \) does not allow for partitioning of \( u_t \) between the viscous, wave, and turbulent components.

[57] Shear velocity comparisons generally show consistency between the two methods (rms deviations of 0.53, 0.35, and 0.57 cm s\(^{-1}\) for the onshore, offshore, and rms cases, respectively). Both estimates are considerably close to empirical formulations for \( u_t \) given by the wave friction factor \( f_{2z} \), which only takes into account the grain roughness (2.5\( d_{50} \)).

[58] Shear velocities inferred from the vertical transfer of horizontal momentum (\( u_t \)) are consistently weaker at the ripple crests. This is expected since the flow above the crests is nearly horizontal, and consequently \( \tilde{u}\tilde{w} \) is a minimum, and suggests that the small-scale turbulence is the main mechanism responsible for the vertical transfer of momentum above the ripple crests. At the stoss and lee slopes, \( u_t \) increases due to flow acceleration and formation of coherent structures, respectively.

[59] While the offshore half-wave period is slightly longer and weaker than the onshore, Figures 4b and 4c suggest that the bed form-induced flow structures are slightly more coherent and organized during the offshore-directed flow. This may influence the estimates of \( \tilde{u}\tilde{w} \), and consequently may account for the agreement between \( u_t \) and \( u'_t \) at the stoss slope during the offshore directed flow. The estimates of \( u_t \) and \( u'_t \) from rms velocity statistics (Figure 11f) are of similar magnitude, but can disagree by as much as a factor of 2. Their spatial mismatch is expected, as the rms velocity signal is not able to capture the nearbed directional-dependence associated with the coherent structures.

[60] The results above suggest that (1) Small-scale turbulence, as accounted for with the Reynolds decomposition, is not the only mechanism responsible for transferring momentum within the logarithmic layer, and therefore contributions from \( \tilde{u}\tilde{w} \) must be included for rippled beds; (2) Spatial correlation between \( u_t \) and \( u'_t \) for the longer and weaker offshore half-wave period suggest that the flow may be more similar to a steady flow environment, where flow acceleration effects are less significant. This is in contrast to the lack of spatial correlation during the shorter duration high-acceleration onshore half-wave period; and (3) While intraripple measurements of the WBBL may not account for the entire drag, they provide for estimates that are similar in magnitude to the grain roughness-induced skin friction, however, spatial variability can be more than a factor of 2.

10. Summary and Concluding Remarks

[61] New detailed observations of the 2-D velocity field over a mobile rippled bed subjected to slightly skewed and asymmetric wave forcing conditions are presented in this paper. The intraripple spatial characterizations of the hydrodynamics within the WBBL are examined in the time and frequency domains, and with first-moment and second-moment velocity statistics.

[62] Spectral analysis of the horizontal velocity field obtained from PIV showed that spectral energy vertically decays at the zeroth and first harmonics (\( f_0 \) and 2\( f_0 \)), but is significantly intensified at the second harmonic (3\( f_0 \)) near the ripple slopes, where vortices occur. Cross-spectral analysis showed a phase separation at the zeroth harmonic that
is mostly consistent with vortex signatures, provided by the swirling strength. Stronger phase separation at the second harmonic is not inconsistent with previous flat bed efforts, yet its higher magnitude is attributed to bed form-induced pressure gradients.

[63] First-moment (onshore and offshore directed flow) and second-moment (standard deviation flow) statistics were used to infer spatially variable hydrodynamic quantities. Integral (displacement and momentum thickness) and differential (viscous shear stress) flow characteristics show sensitivity at illustrating the impact of a rippled boundary in the surrounding flow field. The estimates of boundary layer thickness show signatures of adverse pressure gradients and wakes in the troughs and lee side of the ripples, and shrunken boundary layers at the ripple crests. The integral approaches, while useful for characterizing flow separation, provide estimates of boundary layer thickness that are much smaller than suggested by the higher harmonics and vertical velocity gradients.

[64] Fitting a logarithmic model to first-moment and second-moment velocity profiles between ripple elements provides estimates of shear stress, mean drag elevation, and bed roughness. Shear stresses characterized by the Shields parameter locate the bed within the ripple regime, and suggest stress amplification at the stoss slope of asymmetric ripples during the strongest flow direction (onshore). The mean drag, while not formally quantified in this effort, appears to act slightly above the ripple trough and not at the expected centroid of the ripple, based on the interpretation of d. by Jackson [1981].

[65] An assessment of z0 estimates indicates an increased bed roughness for first-moment velocity statistics that is well modeled by the empirical expression given in Li et al. [1997]. The second-moment statistics provide roughness estimates that are roughly a factor of 3 lower, and in better agreement with formulations by Grant and Madsen [1982] and Nielsen [1992]. Our results imply that the logarithmic layer approach and empirical bed load parameterizations may be able to properly capture the quasi-steady intraripple flow characteristics at most locations along the rippled bed, but that the logarithmic model localized to a single position does not accurately represent the dynamics over the entire ripple.


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