Multi-path sound transfer from resiliently mounted shipboard machinery

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Experimental methods for analyzing and improving noise control

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PREFACE

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REFERENCES

DANKBETUIGING

CURRICULUM VITAE
SYMBOLS

General remarks:
Complex quantities are indicated by underlining. In cases where no confusion is expected this has been omitted.

Sound quantities and electrical quantities marked with a single prime belong to a reciprocal experiment, i.e. an experiment in which sound source location and receiver location have been interchanged.

The symbol definitions are tabulated below. Symbols that are used only in passing are not included. The meaning of subscripts is given for only a few cases.

Roman capitals:
A  equivalent absorption area
B  bending stiffness
C  constant; correction factor
D  outer diameter of circular cylindrical pipe
D_o inner diameter of circular cylindrical pipe
E  sound energy in shallow cavity; modulus of elasticity
F  force
G_{x,x} one-sided power spectral density function of x(t)
G_{x,y} one-sided cross-spectral density function of x(t) and y(t)
J  principal moment of inertia of rigid body
K  dimensionless geometry parameter of pipe wall
L_a acceleration level (analogously for other quantities)
M  torque, force-moment
N  integer
P  injected sound power; energy flow
Q_c ratio of radiation resistance of acoustical point source in shallow cavity and that in the free field
R  radius; distance from origin in cylindrical coordinate system
R_{rad} radiation resistance of acoustical point source
S  area; transfer function of ship structure for force- or torque excitation at a resilient mounting location and with a receiver at large distance
T_{60} \quad \text{reverberation time}

T_a \quad \text{acceleration transmissibility}

T_{p_j, a_j} \quad \text{blocked mounting transfer function of resilient mounting, representing the ratio between blocked force in j-th direction on ship side and acceleration in i-th direction on machine side}

U \quad \text{volume acceleration}

V \quad \text{volume; volume velocity}

Y \quad \text{mechanical admittance}

Z \quad \text{mechanical impedance}

Z_t \quad \text{blocked transfer impedance of resilient mounting}

Z_{j, i} \quad \text{blocked transfer impedance of resilient mounting, representing the ratio between blocked force at j-th "network port" and velocity at i-th "network port"}

\text{Lower-case Roman letters:}

a \quad \text{acceleration}

c \quad \text{propagation speed of sound}

d \quad \text{pipe wall thickness}

e \quad \text{open circuit voltage of reciprocal electrodynamic transducer}

f \quad \text{frequency}

h \quad \text{height}

i \quad \text{integer; driving current through reciprocal electrodynamic transducer}

j \quad \text{integer}

k \quad \text{integer; wave number}

m \quad \text{mass}

m_e \quad \text{mass of excitation block in resilient mounting test rig}

m_s \quad \text{mass of terminating block in resilient mounting test rig}

n \quad \text{integer}

p \quad \text{sound pressure}

r \quad \text{unspecified receiver quantity; cylindrical coordinate}

s \quad \text{resilient mounting stiffness; standard deviation}

t \quad \text{time}

u \quad \text{axial pipe wall displacement}

v \quad \text{velocity; circumferential pipe wall displacement (only in Ch. 5)}

v_m \quad \text{velocity on machine side of resilient mounting}

w \quad \text{radial pipe wall displacement}
x, y, z \quad \text{coordinates in Cartesian system}
\dot{x}, \dot{y}, \dot{z} \quad \text{translational velocities in Cartesian coordinate system}
\ddot{x}, \ddot{y}, \ddot{z} \quad \text{translational accelerations in Cartesian coordinate system}
x(t), y(t) \quad \text{signals in time domain}

\textbf{Greek capitals:}
\Gamma \quad \text{correlation coefficient}
\Delta \quad \text{accelerometer spacing for structureborne energy flow measurements}
\Omega \quad \text{normalized frequency}

\textbf{Lower-case Greek letters:}
\dot{\alpha}, \dot{\beta}, \dot{\gamma} \quad \text{rotational velocities in Cartesian coordinate system}
\ddot{\alpha}, \ddot{\beta}, \ddot{\gamma} \quad \text{rotational accelerations in Cartesian coordinate system}
\gamma_{x,y} \quad \text{coherence function of } x(t) \text{ and } y(t)
\delta \quad \text{ratio between transverse- and main axis sensitivity of accelerometers}
\eta \quad \text{loss factor}
\theta \quad \text{phase angle of cross-spectral density function; angular coordinate}
\lambda \quad \text{sound wavelength}
\nu \quad \text{Poisson's ratio}
\rho \quad \text{density}
\sigma \quad \text{radiation efficiency}
\phi \quad \text{phase angle}
\chi \quad \text{noise to signal ratio}
\omega \quad \text{radian frequency}
1. GENERAL INTRODUCTION

The installation of shipboard machinery on resilient elements is a well-known and, generally speaking, quite effective noise reduction measure. However, methods for systematic investigation of the factors which limit the effectiveness of this measure in practice, are largely incomplete. This concerns methods for determining the remaining vibrational energy flow through the supporting resilient mountings and also that along so-called flanking paths.

This thesis presents new contributions to the methodology for analyzing the multi-path sound transfer in resilient mounting systems on board ships and for improving such systems in an economical way.

1.1 Machinery noise isolation in ships

Noise problems in ships may arise from quite a number of sound sources. Examples of very powerful sound sources are cavitating propellers, large machinery such as diesel engines, turbines and gearing and various types of pumps. The term sound is not confined to aerial vibrations, but covers generally the vibrations in gases, liquids and solids within the range of audible frequencies, i.e. between 20 Hz and 15 kHz. Dependent on the medium a distinction is made between airborne sound, structureborne sound and liquidborne- or fluidborne sound.

Efforts on noise reduction on board ships have the following backgrounds:
- High noise levels on bridge wings caused by the exhaust system may lead to dangerous situations when navigation sound signals become masked.
- Within the ship the noise may interfere with comfort and communication, whereas in engine rooms it also presents a hazard with respect to hearing damage.
- For naval vessels, fishery vessels and seismic research vessels low underwater sound levels are of vital importance. Underwater sound radiation degrades the sonar performance on a ship and moreover in naval vessels it increases the vulnerability. Acoustic mines may be actuated and more generally the probability is increased of being detected or targeted by means of the enemy's passive sonar systems.
In Fig. 1.1 the typical frequency ranges are indicated which correspond with the various motives for noise control in ships.

In the last few decades preventive shipboard noise control has become a specialists' branch of technical acoustics. This has resulted in a large amount of technical reports and articles on sound source properties, sound propagation through the ship structure and methods for sound transfer reduction.

Because of the multitude of sound transfer paths in ships, noise isolation is most effective in regions close to the sound source and close to the receiver. The present thesis is concerned with noise control near machinery, more in particular with the effectiveness of resilient mounting systems.
The reduction of machinery sound transfer is of interest both for comfort in inboard spaces (e.g. cabins) and for the control of the radiated underwater sound.

Within the ship, machinery noise is especially important for accommodation close to the engine room and at sufficient distance from the other very powerful sound source, the propeller(s). For the radiation of underwater sound, machinery noise is predominant at low operational speeds in absence of propeller cavitation. In these cases the most powerful machinery such as diesel engines, turbines and gearing are usually responsible for the excess of noise requirements, with emphasis on the frequency range between 20 Hz and 2 kHz.

The installation of shipboard machinery on resilient elements is a well-known and, generally speaking, quite effective noise reduction measure. This measure is meant to reduce the sound energy flow into the ship structure and to reduce in this way the sound transfer to distant receiver locations. For noise reduction in the engine room itself also acoustic enclosure of the machinery is needed because of the direct sound transfer through the air. The extent to which the sound transfer to distant receiver locations is reduced depends, however, not only on the remaining transfer through the support mountings but also on the relative strength of sound transfer via flanking paths. These are formed by the air around machinery and by mechanical links such as pipes, propeller shaft etc., although flexible isolators may also be inserted in pipe and shafting paths. The sound transfer from resiliently mounted shipboard machinery is therefore fundamentally of a multi-path nature.

The acoustical aspects of resilient mounting systems in general and on board ships in particular, have been studied quite extensively during the last thirty years. However, most studies concentrate on the sound energy flow through the support mountings underneath the machinery. Only a few analyze either the relative importance of flanking transfer paths or the multi-path system effectiveness, see e.g. References 1.2 - 1.6.
The state of the art with respect to the understanding of shipboard resilient mounting systems may be indicated quite appropriately with the following comment by Heckl /1.7/ as reviewer at the closure of the International Symposium on Shipboard Acoustics 1976:

"The second problem with respect to ship quieting methods is the use of proper mounting devices, especially resilient mounts. This subject has been covered quite thoroughly during this symposium. It was somewhat surprising and possibly disappointing to hear that the insertion loss* by elastic engine mounts of main propulsion machinery is only 10-15 dB, but since Helmut Müller and I measured similar values many years ago I am convinced that these values are correct. It is still not quite understood why the values are so low. Airborne sound transmission may be important at high frequencies, but certainly not everything can be blamed on this effect. It is likely that resilient mounts often are by-passed by shafts and other connections".

Although the quotation refers to results for propulsion diesel engines, Steenhoek /1.6/ states that the situation for auxiliary diesel engines is quite similar. This situation is very unsatisfactory because in most cases a large noise reduction is necessary to fulfil noise requirements. If this large noise reduction is not achieved then extra measures become necessary either in way of the accommodation or in way of the engine.

![Figure 1.2](image)

*Schematic representation of resilient mounting systems:
- a. single stage system
- b. double stage system with or without acoustic enclosure.

- Reduction of sound pressure levels in cabins due to the insertion of resilient mountings.
Improvement of the mounting system may be attempted, for example, by special seating design /1.8/ or by changing from a more conventional single stage to a double stage mounting system with or without acoustic enclosure (Fig. 1.2).

However, such additional measures are very costly. It is therefore of great economical interest to start by determining which factors are limiting the effectiveness of the conventional single stage mounting systems and what measures are needed to improve such systems. For such an analysis it is necessary to quantify both the sound transfer through the support mountings underneath an engine and that via the flanking paths.

Complete mathematical analysis of a multi-path system may prove too complex in most cases. On the other hand experimental evaluation requires measuring methods which can be applied under the very restrictive conditions on board ships. Usually when ships are available for acoustic investigations, this is only for a short period and with few possibilities to remove parts of machinery installations for acoustic path analysis.

Reliable and practicable analysis methods for shipboard use, especially for a number of flanking paths, are lacking. As a result many design studies use the knowledge and analysis methods for only a very limited number of aspects and there are good reasons for the suspicion that in some so-called "sophisticated" isolation systems the importance of flanking paths is overlooked. However, as will be discussed later the paths analyzed traditionally, i.e. through the support mountings and through the air, deserve also further study.

In view of this situation it is the general objective of this thesis to contribute to the methodology for analysing the noise reduction properties of multi-path shipboard mounting systems and for improving these systems in the most economical way.
1.2 General approach to the analysis of multi-path sound transfer in resilient mounting systems

To understand the sound transfer from resiliently mounted machinery, requires that a number of paths should be analyzed. This is illustrated in Figs. 1.3 and 1.4 with the main paths from a resiliently mounted propulsion diesel engine to the adjacent ship structure.

Note:

The diesel engine has been chosen as illustration because of the great interest for many ships which are built nowadays in the Netherlands.

Several of the analysis methods developed in this thesis have been tested with respect to the transfer of diesel engine noise. However, they are applicable to other type of machinery as well.

In the present thesis the path types no. 1-3 are studied, i.e. the path via the resilient support mountings, the path via the surrounding air and the path via pipes.

Path type no. 4, i.e. via the propeller shaft, and path type no. 5, i.e. via the resiliently mounted exhaust system, are not studied in detail. In principle they can be analyzed in the same way as path no. 1 via the support mountings.
Figure 1.4:
Simplified block diagram of the main sound transfer paths between a resiliently mounted propulsion diesel engine and adjacent ship structure. Paths no. 1, 4 and 5 are called "structureborne sound paths" and path no. 2 is called "airborne sound path". Path no. 3 is possibly a strongly coupled "liquidborne" and "structureborne" sound path. For the paths no. 3 and 4 there exist a number of system variants. In case of diesel driven generators, path no. 4 is replaced by electricity cables.

Not shown in Fig. 1.4 and neglected in this thesis are the direct liquidborne path to the water through cooling water pipes and the transfer paths to decks through the gas inside exhaust systems. The latter form no real problem when well-designed silencers are fitted. The liquidborne path via cooling water can form an important transfer path for the underwater sound radiation of pump noise, but it is not believed to be important for the underwater sound radiation of diesel engine noise.
For the path analysis problem two different approaches can be distinguished. Within the scope of this introduction only their main characteristics are discussed. The usual approach is to analyse each path individually.

A transfer path system is broken down into a number of sub-systems from which the sound transfer and coupling properties are to be determined either by calculations or by experiments. A great advantage of this approach is its flexibility. The choice of sub-systems may be adapted to the most promising analysis methods. Moreover, the analysis results have the appropriate format for investigating the influence of system modifications in an economic manner. This is especially important when designing new ships. However, the unsatisfactory state of the art with respect to the path analysis problem is due to practical difficulties in determining the acoustical properties of certain sub-systems with sufficient detail and accuracy. A number of such problems have been discussed by Ten Wolde /1.1/ and some of them will be discussed in subsequent chapters of this thesis.

In view of such limitations Ten Wolde /1.1/ has studied an alternative, purely experimental approach in which the transfer paths are no longer investigated individually. Some further investigations on the assumptions and on the applicability of this approach have been reported by Verheij /1.9/.

Figure 1.5:
Principle of method for flanking sound transfer path detection by artificial excitation of the engine. In the actual experiments not the engine is excited as in case b, but a deck for example (case c). With this reciprocal excitation the effect is studied of e.g. uncoupling a pipe (case d). This pipe forms a predominant flanking path if $a_2 > a_1'$. 

![Diagram](image-url)
The concept of this approach is illustrated in Fig 1.5. For the path analysis, measurements are performed with the engine stopped. In principle the machinery noise is simulated by excitation upon the outside of the engine with a number of incoherent exciters. It is assumed that for the frequency range where the engine no longer vibrates as a rigid body, the internal excitation may be modelled sufficiently accurately by this artificial excitation. With sufficiently accurate modelling is meant that the relative distribution of sound energy flow along the various transfer paths is approximately the same as for the case of the engine in operation. Only by using this artificial excitation does it become practicable to interrupt sound transfer paths, e.g. by removing a flexible pipe coupling. If then for constant excitation the sound transfer to a distant position decreases, this forms a quantitative indication for the relative importance of the interrupted path. An essential step simplifying these experiments is to perform the transfer measurements reciprocally, see Fig. 1.5c-d. The great advantage of this approach might be therefore that predominant transfer paths can be detected with relatively simple experiments. Moreover, quantitative insight is obtained as to what extent a certain resilient mounting system can be improved without modifying the paths through the mountings and through the air.

However, within the scope of this introduction some important practical and fundamental limitations have to be mentioned. When mounting systems are to be investigated on board ships that are in service, a systematic uncoupling of connections between engine and ship, preferably repeated in different sequences, may require more time than available. Moreover, sometimes the slight risk of damage by such actions is unacceptable. A more fundamental limitation is formed by the type of information which is obtained. A predominant flanking path is probably found, but although this is essential it is not sufficient for economical system improvements. In addition insight is needed in which way individual paths can be improved. This makes additional analysis necessary according to the more usual methods.

It is because of the greater flexibility in obtaining and using results of sub-system analysis that this thesis concentrates on investigating a number of transfer paths individually.
1.3 Outline of the study

The following study is divided into part A and part B. Part A is formed by the Chapters 2 through 5, in which analysis methods are developed for respectively the sound transfer via resilient mountings, that via shallow air cavities below engines and that along pipes. Part B is formed by Chapters 6 and 7. In Chapter 6 noise reduction properties of a shipboard mounting system are analyzed using methods developed in Part A. In Chapter 7 a simple method is developed for aiding the design of improved structureborne sound isolation on the ship's side of flexible isolators.

All methods which have been studied have in common that they are mainly experimental and that they can be applied on board ships without disturbing seriously normal ship programs. In addition most of them can be applied in the laboratory when investigating new designs, e.g. with the aid of scale models; see note below.

Finally a brief discussion on the remaining transfer paths which have not been studied in this thesis is included in the evaluation in Chapter 8.

Note:
On the use of scale models:
Scale models are used in this thesis for testing the validity of path investigation methods as being developed in Chapter 2, 3, 4 and 7. For such type of investigations it is sufficient if a scale model section behaves as a typical ship structure, or more specifically, if it shows representative densities of resonant vibration modes in the frequency bands of interest. Therefore, a representative distribution of mass and stiffness is important, whereas a correct modelling of damping is less critical.

The ship-like scale model structures in this thesis were designed on basis of the following model law:
- if the model is geometrically similar to the full scale system with all linear dimensions reduced by a factor s
- if the frequency is scaled up with this factor s
- if the model consists of the same elastic solids and fluids as the original at corresponding positions and if the elasticity properties are frequency independent
then the wavelengths of all wavetypes are scaled down at the same rate as the linear dimensions and the model can vibrate geometrically similarly to the original system; see e.g. Rayleigh /1.10/ and Schoch and Feher /1.11/. Similarity relations for acoustic properties of ship structures and also the role of damping in scale model experiments have been discussed by Ten Wolde /1.1, 1.12/. For a more general discussion on scale modelling and its engineering applications, see e.g. Langhaar /1.13/ and Baker et al. /1.14/.
Part A:

EXPERIMENTAL METHODS FOR QUANTIFYING TRANSFER PATHS
2. MULTI-DIRECTIONAL STRUCTUREBORNE SOUND TRANSFER THROUGH RESILIENT MOUNTINGS

For the experimental analysis of the structureborne sound transfer through resilient mountings underneath engines and through the adjacent ship structure a new method is developed which takes into account 6 degrees of freedom for motions and forces at the interfaces on both sides of a mounting. Key elements are relatively simple techniques for measuring multi-directional sound transfer properties of mountings in a laboratory test rig and reciprocity techniques for measuring ship transfer functions.

2.1 Introduction

The complexity of the sound transfer problem for the resilient mounting path follows from the inherent multi-directional nature of machinery motions. This leads to the necessity of determining the sound transfer functions of the mounting with 6 degrees of freedom and of determining the sound transfer through the ship structure for each one of the excitation components.

Most of the literature has, in general, considered only vertical translational vibration, see e.g. 2 recent review articles respectively by Klyukin /2.1/ and Snowdon /2.2/. However, in transport vehicles and ships the properties of the structureborne sound sources, isolators and seating structures can cause other excitation components than the vertical one to dominate the sound transfer. This has been recognized for a long time by a number of authors, see e.g. Janssen /2.3,2.4/, Darby /1.5/, Sykes /2.5/, Ten Wolde /1.1/, Popkov /2.6/, Damberg et al./2.7/ and Sainsbury /2.8, 1.8/.

As far as is known to the author, the only rather complete multi-directional analysis of a mounting system has been attempted by Sainsbury /1.8/. He presents an analysis of a seating structure which was composed of multi-layer-sandwich beams with V-shaped rubber blocks in between. The analysis of the sound transfer through these rubber blocks included the excitation by rotational components. The method which was used, however, needs sophisticated measurement and calculation facilities. Moreover, as
will be argued later, this method may not be generally applicable for the types of mounting which are normally used on board ships. There is a need therefore to develop a simpler and practical method as described in this thesis.

General theoretical analysis:
To formulate the proposed analysis in general terms a usual method of approach is followed by splitting the transfer system into at least 3 parts; viz. source, mountings and the remaining path through the ship structure to a receiver location, see Fig. 2.1.

It is assumed that the analysis is limited to the frequency range in which the mounting end flanges can be considered rigid. Then the engine velocity (or displacement or acceleration) on top of a mounting and the force upon the seating in one direction are fully characterized by point quantities $v_m$ and $F_s$. These may be assigned to the centres of the interfaces with a mounting. For a multi-directional analysis accurate estimates are needed for 3 forces and 3 torques upon the seating and for 6 complementary ship transfer functions at each mounting location.

As will be discussed later even for the uni-directional case estimating $F_s$ may be rather complicated. However, a large impedance mismatch at the mounting seating interface is quite representative of practical conditions and is supposed to occur throughout in this thesis. For that case it will be shown in 2.2.1 that the force upon the seating for a given velocity on top of the mounting is determined by the so-called blocked transfer impedance of the mounting ($Z_{t}$). This will be shown to be solely a mounting property.

- mechanical impedance is defined as force divided by velocity in complex notation
For the case of a single mounting and a single vibration direction the response "r" at a receiver location is obtained from multiplying the source velocity \( v_m \) by the mounting transfer impedance \( Z_m \) and by the ship transfer function \( S \), according to (see Fig. 2.1)

\[
\begin{align*}
    r &= \begin{bmatrix} r \\ Fs \\ F_s \end{bmatrix} \cdot \begin{bmatrix} Fs \\ F_s \\ v_m \end{bmatrix} \\
    &= v_m = S \cdot Z_m \cdot v_m.
\end{align*}
\] (2.1a, 2.1b)

The quantity "r" remains unspecified at this stage because many choices are possible.

Eq. (2.1) is valid for harmonic time dependence and \( r, F_s \) and \( v_m \) represent complex phasors. Any other time dependence can be related to harmonic processes by means of Fourier analysis.

For the case of \( n \) identical mountings and \( 6 \) vibration directions at each mounting location, Eq. (2.1) is to be replaced by

\[
\begin{align*}
    r &= \sum_{k=1}^{n} \left\{ \sum_{j=1}^{6} S_{jk} \cdot \sum_{i=1}^{6} Z_{ji} \cdot v_{m,ik} \right\}.
\end{align*}
\] (2.2)

Mounting transfer functions:
The sound transfer through a resilient mounting for which the motions of the flange on the seating side are blocked is determined by \( 36 \) transfer impedances at maximum. The frequency dependence of these transfer functions may be fairly complex in the frequency range of interest (say 20 - 2000 Hz) because resonances of several wavetypes will occur in the mounting. The only accurate way to determine these transfer functions for mountings of representative complexity with respect to shape and composition is, in the opinion of the author, by experiment.

Other approaches have been discussed by Sainsbury /1.8/. For the analysis of a large V-shaped rubber block he has presented a technique which combines mathematical modelling and measurements. The mathematical model of the block consisted of a number of coupled mass-spring systems. Its properties were matched by an iterative procedure until its parameters were
in agreement with data from multi-directional point receptance* measurements. These measurements were made on top of the rubber block with the "terminals" on the seating side blocked. However, disadvantages of this technique are difficulties of mathematical modelling for mountings of general complexity and the complicated measuring techniques which are needed and which become even more complicated if static preloading of the mounting is required.

An alternative approach, suggested by Sainsbury /1.8/, is to use Finite Element Method calculations. However, besides probably high modelling costs for mountings of practical type, inaccuracies are to be expected from uncertainties on material properties and on the influence of static preloads. For these reasons purely experimental determination as is described and tested in sections 2.3 and 2.4 will appear simpler, cheaper and in most cases more reliable and accurate.

**Ship transfer functions:**

For determining the six transfer functions $S_{i,j} = \frac{r_i}{F_{j}}$ for each mounting location, Ten Wolde /1.1,2.9/ has shown that reciprocity measuring methods provide a simple and, under shipboard conditions, most practicable solution. These methods can be applied in scale models also when investigating a new ship and/or mounting system design. In this thesis ship transfer functions from the mounting path have been determined in this reciprocal way for the scale model experiments of chapter 3 and for the shipboard experiments of chapter 6.

However, when the sound transfer is to be analyzed during the design stage of a new ship the costs of building a representative scale model may be prohibitive. In that case the use of theoretical models for the sound transfer through ship structures might form the only alternative. For existing ships such models are not competitive with experimental results with respect to reliability and accuracy.

*receptance is defined as displacement divided by force in complex notation*
Until now two types of theoretical models have been proposed for the frequency range which is of interest for diesel engine noise reduction. One is the so-called Statistical Energy Analysis (SEA) method, see e.g. Jensen /2.10/, Kihlman, Plunt /2.11, 2.12/. The other method, as proposed by Nilsson /2.13/, uses a more deterministic coupled plate element model and is potentially more accurate than SEA at frequencies below a few hundred Hz, see also Ref. 2.14. Both types of mathematical models need as input data the power which is injected into the structure by the sound sources.

If the above theoretical methods are applied to resilient mounting systems, the power injection for a certain frequency bandwidth (e.g. 1/3-octaves) is to be summed for all force and torque components which excite the seating. In cases that the sound transfer for the excitation in different directions and/or at different mounting locations may be considered as uncorrelated, the total power injected within a bandwidth Δf is found in analogy with Eq. (2.2) from

\[
P_{Δf} = \sum_{k=1}^{n} \sum_{j=1}^{6} \frac{Δf}{Δs, jk} \left( \sum_{i=1}^{6} \frac{P^2_{m, ik}}{s, ijk} \right) = \sum_{k=1}^{n} \sum_{j=1}^{6} \frac{Δf}{Δs, jk} \left( \sum_{i=1}^{6} \left| \frac{Y_{m, ik}}{s, ijk} \right|^2 \cdot v^2 \right) \quad \text{(2.3a, 2.3b)}
\]

In this equation, \( P^2 \) and \( v^2 \) denote the time mean square values of band-filtered force and velocity. The factors \( \Re \frac{Y_{s, jk}}{Δf} \) denote the frequency averaged values of the real parts of the multi-directional point admittances* of the seating structures. Again because of the complexity of the structures generally speaking, these quantities can only be obtained accurately from measurements. In Chapter 7 a simple measuring method is described for this purpose. During the design stage of a new ship such data can be obtained from representative seating structures in existing ships or from relatively limited scale model sections.

Note:

The admissibility of neglecting the interaction of different mountings as in Eq.(2.3), is studied in 3.3. and is also discussed briefly by Petersson /2.15/.

* mechanical admittance is defined as velocity divided by force in complex notation
2.2 Analysis scheme for the mounting path
The generalized analysis expressed in Eqs. (2.2) and (2.3) contains functions and quantities which need to be examined in greater detail. Subsequently the characterization of mounting transfer functions and the measurement of ship transfer functions are discussed.

2.2.1 Mounting transfer functions
The proposed characterization of resilient mountings by means of a matrix of blocked transfer impedances originates from linear mechanical network theory; see for fundamentals of this theory with respect to vibration problems e.g. Molloy /2.16/, Sykes /2.5/ and Snowdon /2.17/. For the proposed description to be true certain assumptions must be made and conditions fulfilled. It is necessary therefore to consider whether these can be fulfilled in a typical mounting system.

Mounting construction:
It is assumed that the mounting behaves as a linear element and that the contact areas with machine and seating vibrate as rigid planes in the frequency range of interest. In practice the mounting may be composed of a rubber element and steel end plates (flanges). The degree of undistorted plane vibrational motion of these contact areas is not only determined by the mounting flange properties but also by the adjacent structures. As an example it is noted that for large rubber mountings the first "beam" resonance of the relatively thin steel end flanges may occur at frequencies well below 1 kHz when the mounting is considered in isolation. However, for such mountings the adjacent seating and engine (-raft) structures are usually much stiffer. As long as in these adjacent structures the bending wavelengths are much larger than the dimensions of the mounting, the forces and motions on each side of the mounting may be characterized with 6 network ports. In the following discussion of the analysis scheme this situation is assumed and for measurements of mounting properties such conditions should be fulfilled.

Single mounting, single vibration direction:
First the case of uni-directional vibration is considered, see Fig. 2.2. The mounting is modelled by a two-port network with ports no. 1 and no. 2. Port no. 3 is the input port of the seating. For harmonic time dependence
and using complex notation, two impedance equations define the relations between the forces and velocities on both sides of the mountings as follows:

\[ F_1 = Z_{1,1} v_1 + Z_{1,2} v_2, \tag{2.4} \]
\[ F_2 = Z_{2,1} v_1 + Z_{2,2} v_2. \tag{2.5} \]

This may be written as a matrix equation

\[ F = Z v \tag{2.6} \]

The impedance matrix consists of the complex elements \( Z_{1,1}, Z_{1,2}, Z_{2,1} \) and \( Z_{2,2} \), where

\[
\begin{align*}
Z_{1,1} &= \frac{F_1}{v_1} \bigg|_{v_2=0} ; \\
Z_{2,2} &= \frac{F_2}{v_2} \bigg|_{v_1=0} ; \\
Z_{1,2} &= \frac{F_1}{v_2} \bigg|_{v_1=0} ; \\
Z_{2,1} &= \frac{F_2}{v_1} \bigg|_{v_2=0}.
\end{align*}
\tag{2.7}
\]

with

- \( Z_{1,1} \): driving-point impedance at port no. 1 when port no. 2 is blocked,
- \( Z_{2,2} \): driving-point impedance at port no. 2 when port no. 1 is blocked,
- \( Z_{1,2} \): blocked transfer impedance for excitation at port no. 2,
- \( Z_{2,1} \): blocked transfer impedance for excitation at port no. 1.
Because the reciprocity principle is valid for a linear mounting

\[ Z_{1,2} = Z_{2,1} \]  
(2.8)

At low frequencies when the inertia forces in the mounting may be neglected the two-port network reduces to a bilateral element with impedance \( Z \).

For typical rubber mountings with low internal damping

\[ Z = s(1+j\eta)/j\omega, \]  
(2.9)

with nearly frequency independent stiffness \( s \) and loss factor \( \eta \); \( \omega \) denotes the radian frequency.

For coupling to the seating port the force is given by

\[ F_3 = Z_3 v_3 \]  
(2.10)

with \( Z_3 \) the driving-point impedance of the seating.

Forces and velocities at ports no. 2 and no. 3 are related by:

\[ F_3 = F_2, \]  
(2.11)

\[ v_3 = -v_2. \]  
(2.12)

The signs of Eqs. (2.11) and (2.12) follow from the sign convention adopted in Fig. 2.2. From Eqs. (2.5), (2.11) and (2.12) it follows that

\[ \frac{F_3}{v_1} = \frac{Z_{2,1}}{1 + Z_{2,2}/Z_3}. \]  
(2.13)

Eq. (2.13) shows that in the general case \( F_3 \) is determined both by the mounting properties and the seating impedance \( Z_3 \).

For measuring \( Z_3 \) the mounting has to be removed, but on board ships which are in service this is mostly impossible. However, this measurement is unnecessary if \( |Z_3| >> |Z_{2,2}| \). In that case it follows from Eq. (2.13) that

\[ F_3 \sim Z_{2,1} v_1. \]  
(2.14)
Thus, if a large impedance mismatch occurs at the mounting seating interface, $F_3$ is no longer dependent on the seating impedance, but is solely determined by the blocked transfer impedance of the mounting. Then $F_3$ is equal to the so-called blocked force on the seating side of the mounting.

**Single mounting, 6 vibration directions:**

For the generalized case with 6 degrees of freedom on each mounting side, the mounting may be characterized with a linear twelve-port. The translational velocities parallel to the axes of a Cartesian coordinate system are denoted by $\dot{x}$, $\dot{y}$, $\dot{z}$. The rotational velocities about these axes are denoted by $\dot{\alpha}$, $\dot{\beta}$, $\dot{\gamma}$. The forces and torques are denoted by $F_x$, $F_y$, $F_z$ and $M_x$, $M_y$, $M_z$ respectively. The way in which these quantities are assigned to the input ports no. 1-6 and to the output ports no. 7-12 which correspond with the seating ports no. 13-18, is indicated in Fig. 2.3.

The 12 impedance equations are of the form

$$F_{m,x} = -Z_{1,1} \dot{x}_m + Z_{1,2} \dot{y}_m - \ldots - Z_{1,12} \dot{y}_s.$$  \hspace{1cm} (2.15)
They can be represented by the same matrix equation as in Eq. (2.6). Now the impedance matrix contains 144 elements \( Z_{i,j} \) and again because of reciprocity \( Z_{i,j} = Z_{j,i} \). The 12 x 12 matrix \( Z \) may be divided in 4 sub-matrices \( A, B, C \) and \( D \) as follows:

\[
\begin{bmatrix}
Z_{1,1} & \ldots & Z_{1,6} & Z_{1,7} & \ldots & Z_{1,12} \\
\vdots & & \ddots & \vdots & & \vdots \\
Z_{6,1} & \ldots & Z_{6,6} & Z_{6,7} & \ldots & Z_{6,12} \\
Z_{7,1} & \ldots & Z_{7,6} & Z_{7,7} & \ldots & Z_{7,12} \\
\vdots & & \ddots & \vdots & & \vdots \\
Z_{12,1} & \ldots & Z_{12,6} & Z_{12,7} & \ldots & Z_{12,12}
\end{bmatrix}
\]

(2.16)

The sub-matrices \( A \) and \( B \) contain the driving-point impedances on respectively the machine and the seating side of the mounting. The sub-matrices \( C \) and \( D \) are blocked transfer impedance matrices, with \( C \) the matrix relating "blocked" forces on the seating with velocities of the engine.

In the same way as above for the uni-directional case, Ten Wolde /1.1/ has shown that when each of the driving point impedances of sub-matrix \( B \) is much smaller than the corresponding driving point impedances of the seating, the forces and torques on the seating are no longer dependent on the seating properties. Thus, if \( |Z_{13,13}| > |Z_{7,7}|, |Z_{13,14}| > |Z_{7,8}| \), etc., the forces and torques on the seating follow from the matrix equation
In brief notation Eq. (2.17) can be written as:

$$F_s = C \cdot V_m.$$  \hspace{1cm} (2.18)

**Multi-mounting system:**

For the multi-mounting case the total sound transfer can be calculated by linear superposition of the single mounting contributions, as in Eq. (2.2). The conditions under which neglect of phase relationships is allowed will be investigated later.

**Large impedance mismatch:**

It is difficult to prove in general whether the requirement of large impedance mismatch at the mounting seating interface is met. In principle this would require measurement of complete driving-point admittance matrices of mounting and seating. Such measurements are very complex /1.8/ and in many cases impracticable under shipboard conditions. A practical, but not absolute, indication of a large impedance mismatch is a large ratio of corresponding velocity components on top and underneath the mounting over the whole frequency range of interest, say when $\Delta L_{v1} > 10$ dB.

**Symmetry properties of resilient mountings:**

The blocked transfer impedance matrix $C$ of Eqs. (2.17) and (2.18) contains 36 elements. If there is some degree of symmetry a number of them are equal or vanish. This is worked out for common cases of symmetry and applied on typical mounting shapes. In the discussion below it is supposed that the machine/mounting and mounting/seating interfaces are perpendicular to the $z$-direction. The following cases of symmetry are considered.

A. Mirror plane perpendicular to $z$-axis; see Fig 2.5

B. Two mirror planes; one perpendicular to the $x$-axis and the other to the $y$-axis; see Fig. 2.4
Figure 2.4:
Examples of symmetrical mountings (See Fig. 2.5 for Case A).

Figure 2.5:
Equivalent "direct" and "reciprocity" measurement of \( Z_{8,4} \).
The cross section of the fancy mounting shape has a symmetry type A.

Figure 2.6:
Equivalence of \( Z_{4,8} \) and \( Z_{10,2} \) visualized for the same mounting shape as in Fig. 2.5.
C. as B, with identical shape with respect to both mirror planes; see Fig. 2.4
D. Combination A + B, e.g. rectangular block mounting; see Fig. 2.4
E. Combination A + C; see Fig. 2.4

Case A. Mirror plane perpendicular to z-axis: Reciprocity of the mounting implies $Z_{i,j} = Z_{j,i}$. From this property it follows that the blocked transfer impedance matrix $C$ of Eqs. (2.17) and (2.18) becomes skew when the mounting is symmetrical with respect to a plane perpendicular to the z-axis. This is visualized in the Figures 2.5 and 2.6 with the aid of a fancy mounting cross section. Because of reciprocity $Z_{8,4} = Z_{4,8}$. The measurement of $Z_{8,4}$ is shown on the left hand side of Fig. 2.5, that of $Z_{4,8}$ on the right hand side.

$$Z_{8,4} = \frac{F_{s,y}}{s} \frac{M_{m,x}}{s} \text{ other 11 velocities blocked, i.e.} = \frac{M_{m,x}}{s} \text{ other 11 velocities blocked, i.e.} = Z_{4,8} \quad (2.19)$$

Because of the symmetry $Z_{8,4} = -Z_{10,2}$. This follows from Eq. (2.19) and from comparing Fig. 2.6 with the right hand side of Fig. 2.5. The same can be proven for all elements of $C$ which lie symmetrically with respect to the diagonal in the same way as $Z_{8,4}$ and $Z_{10,2}$. The number of different transfer impedances is therefore reduced from 36 to 21.

Case B. Mirror planes perpendicular to x-axis and y-axis: In this case a number of transfer impedances vanish. There remain 12 different non-zero elements.

$$C = \begin{bmatrix}
Z_{7,1} & 0 & 0 & 0 & Z_{7,5} & 0 \\
0 & Z_{8,2} & 0 & Z_{8,4} & 0 & 0 \\
0 & 0 & Z_{9,3} & 0 & 0 & Z_{9,6} \\
0 & Z_{10,2} & 0 & Z_{10,4} & 0 & 0 \\
Z_{11,1} & 0 & 0 & 0 & Z_{11,5} & 0 \\
0 & 0 & Z_{12,3} & 0 & 0 & Z_{12,6}
\end{bmatrix} \quad (2.20)$$

- a matrix is called "skew" if $a_{i,j} = -a_{j,i}$ and at least one $a_{i,i} \neq 0$
Case C. Equal shape with respect to symmetry planes of case B: The matrix of Eq. (2.20) has now 8 different non-zero elements, because $Z_{7,1} = Z_{8,2}$; $Z_{7,5} = Z_{8,4}$; $Z_{10,2} = Z_{11,1}$; $Z_{10,4} = Z_{11,5}$.

Case D. Rectangular block (combination A + B): The matrix of Eq. (2.20) has now 9 different non-zero elements, because it is skew.

Case E. Square block or circular cylinder (combination A + C): In this case the matrix of Eq. (2.20) has 6 or 5 different non-zero elements as follows: 4 "diagonal elements" $Z_{7,1} = Z_{8,2}$; $Z_{9,3}$; $Z_{10,4} = Z_{11,5}$; $Z_{12,6}$ and moreover $Z_{7,5} = -Z_{8,4} = -Z_{11,1} = Z_{10,2}$; $Z_{9,6} = Z_{12,3}$ ($= 0$ for circular cylinder).

2.2.2 Reciprocal measurement of ship transfer functions

For determining the ship transfer functions reciprocal measurements are paramount. In reciprocal measurements the locations of sound source and receiver have been interchanged. A number of reciprocity relations for ship transfer functions are discussed by Ten Wolde /1.1, 2.9/ and in Appendix A of this thesis. When the transfer functions for point force and torque excitation of the seating are measured reciprocally, six velocity (or acceleration) components have to be measured on the seating at the mounting location.

Such reciprocity experiments offer 2 distinct advantages. These are as follows:

a. The difficulties of proper multi-directional excitation in the ship with known forces and torques are avoided. In the reciprocity experiment six velocity (or acceleration) components are derived from the output signals of normal accelerometers /2.9/.

b. There are no space problems of locating large exciters at the mounting locations. It is sufficient to remove the mounting and to substitute accelerometers.

In most cases even removal of the mounting is not necessary, because the motions at the centre of the contact area can be deduced with a suitable configuration of accelerometers adjacent to the mounting or on the bottom side of the seating top plate (see e.g. Chapter 6). It is assumed then that the mounting and the engine form a negligible dynamical load.
Direct and reciprocal measurement of ship transfer function. On the right hand side the mounting is still in position. When it presents no significant dynamical load on the seating, then \( (a'_{s1} + a'_{s2})/2 = a'_{s} \).

for the seating. An illustration is presented in Fig. 2.7, where

\[
\frac{a_d}{F_s} = \frac{a'_{s}}{F'_d} = \frac{(a'_{s1} + a'_{s2})/2}{F'_d}.
\]

On the left hand side the "direct" measurement is shown and in the middle the equivalent reciprocal measurement. In the reciprocal experiment on the right hand side the seating acceleration \( a'_{s} \) is estimated from the accelerations \( a'_{s1} \) and \( a'_{s2} \) on both sides of a resilient mounting which has been left in position.

2.3 Method for measuring multi-directional mounting properties

The blocked transfer impedances of a mounting are defined by

\[
Z_{j,i} = \frac{F_{s,j}}{v_{m,i}} \left| \begin{array}{c} \text{other } \| \text{velocities zero} \end{array} \right.
\]

The conditions which are to be fulfilled in an arrangement for the measurement of \( Z_{j,i} \) follow from the j-th impedance equation for the mounting, see Eqs. (2.6), (2.15) and (2.16). The requirement of zero velocities is sufficiently met if \( |Z_{j,i} v_{m,i}| \) is much larger than the modulus of the sum of all other terms.
In the actual test arrangement the measurements are carried out with normal uni-axial accelerometers. For source strength measurements on engines this is the case also usually. Because the relations which have been discussed in sections 2.1 and 2.2 remain valid if the velocity components are replaced by the corresponding accelerations, it is convenient to define transfer functions and source levels in terms of accelerations rather than velocity. Therefore in the subsequent discussion blocked mounting transfer functions are considered of the type

\[ T_{j,i} = \frac{F_{s,j}}{a_{m,i}} \text{ other } \mu \text{ accelerations blocked, i.e. zero} \]  \hspace{1cm} (2.23)

For the sake of clarity the identification by subscripts is expanded as illustrated by the following examples

\[ T_{7,5} = T_{F} \overset{\text{def}}{=} \frac{F_{s,x}}{E_{m}} ; \quad T_{11,1} = T_{M} \overset{\text{def}}{=} \frac{M_{s,y}}{E_{m}} \]  \hspace{1cm} (2.24)

General measuring principle:
The principle of the test arrangement is shown in Fig. 2.8, see also Verheij /2.18/. The test mounting is secured between 2 blocks which vibrate as rigid bodies in the frequency range of interest. Underneath the lower block and on top of the upper block auxiliary resilient mountings are used respectively for supporting and for applying a static preload.

The upper block, which excites the mounting, has a dual function:
- its rigidity is used for clamping the upper metal flange of the resilient mounting in the same way as under a heavy engine (-raft), so that the flange vibrates without deformation
- its mass and rotational inertias should be sufficiently large such that the 6 natural frequencies of the mass mounting combination, when installed on a rigid foundation, are well below the frequency range of the measurements. In that case it is possible to create 6 situations for each of which 1 of the acceleration components on the machine side of the mounting is very large and the others are suppressed. This can be achieved with simple exciter configurations.
The lower block has in analogy also a dual function:
- its rigidity is used now for a realistic boundary condition on the seating side of the mounting
- its mass $m_s$ and rotational inertias should be so large that it forms a high impedance termination for all excitation components of the mounting. Therefore the 6 natural frequencies of this mounting and mass combination, when installed on a rigid foundation, should be well below the frequency range of the measurements (see subsequent discussion). The forces and torques exerted by the mounting on this block are then approximately equal to the blocked forces and torques. These can be derived from accelerations of the lower block.
The role of the auxiliary mountings is discussed later.

Vertical translation:
First it is assumed that the auxiliary mounting(s) underneath $m_s$ have a stiffness which is negligibly small compared to that of the test mounting. For harmonic excitation and using complex notation the ratio $\bar{z}_s/\bar{F}_m$ is a complex quantity which is known as (acceleration-)transmissibility ($T_a$). It will be shown that with an appropriate choice of $m_s$ the blocked mounting transfer function can be obtained over a large frequency range from...
\[ T_{F_z,\ddot{z}} = m_s T_{a} = m_s \frac{\ddot{z}}{m}. \]  
\[ (2.25) \]

For a mounting which may be considered as a massless spring with frequency independent stiffness \( s = s(1 + j\eta) \) the modulus and the phase angle of \( T_{a} \) are given respectively by

\[
|T_{a}| = \frac{(1 + \eta)^{\frac{1}{2}}}{\sqrt{(1 - \Omega^2)^2 + \eta^2}},
\]
\[ (2.26) \]

and

\[
-\pi < \phi_{T_{a}} = \arctan \left[ \frac{-\eta \Omega^2}{1 - \Omega^2 + \eta^2} \right] < 0,
\]
\[ (2.27) \]

where \( \Omega^2 = \omega^2/\omega_o^2 \) and \( \omega_o^2 = s/m_s \); see e.g. Snowdon /2.19/. As an example Fig. 2.9 shows 20 lg|\( T_a \)| for such an idealized mounting having a loss factor \( \eta = 0.1 \). The blocked transfer function for such a mounting may be written as

\[
\frac{T_{F_z,\ddot{z}}}{F_{s,z}}\defeq \frac{\ddot{z}}{m} = \frac{-s}{m} = \frac{-(1 + j\eta)}{\Omega^2},
\]
\[ (2.28) \]

thus with modulus

\[
|\frac{T_{F_z,\ddot{z}}}{F_{s,z}}| = \frac{m_s}{\Omega^2},
\]
\[ (2.29) \]

and with phase angle

\[
\pi < \phi_{T_{F_z,\ddot{z}}} = \arctan \eta < 0.
\]
\[ (2.30) \]

From Eqs. (2.26), (2.27), (2.29) and (2.30) it follows that

\[
20 \lg \left| \frac{m_s T_{a}}{T_{F_z,\ddot{z}}} \right| = 20 \lg \frac{\Omega}{\sqrt{(1-\eta^2)^2 + \eta^2}} \leq 1 \text{ dB for } f \geq 3f_o,
\]
\[ (2.31) \]

and for \( \eta \leq 0.1 \) (typical for shipboard mountings)

\[
\phi_{T_{a}} - \phi_{T_{F_z,\ddot{z}}} < 1^\circ \text{ for } f \geq 3f_o.
\]
\[ (2.32) \]

The conclusion from Eqs. (2.31) and (2.32) is that for \( f \geq 3f_o \) the approximation in Eq. (2.25) is accurate. For \( f < 3f_o \) \( T_{F_z,\ddot{z}} \) can be obtained from the following equation:

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For actual mountings the behaviour may deviate considerably from Fig. 2.9 because of frequency dependence of the material properties, but especially because of the stiffening effect due to internal resonances. An appropriate choice of \( m_s \) is such that \(|X^*|<<1\) remains valid at frequencies where this stiffening occurs. Then the impedance mismatch condition for measuring \( T_{Fz}z^2 \) correctly, is still fulfilled and Eq. (2.25) may still be used for \( f \geq 3f_o \).

In the actual test arrangement the support mountings underneath \( m_s \) may complicate the situation. As long as their total stiffness is much smaller than that of the test mounting they do not influence the accuracy of Eq. (2.25) for \( f \geq 3f_o \). However, usually auxiliary mountings with small stiffness in unloaded condition show a much increased stiffness when subjected to the nominal preload for the test mounting. Therefore for measurements under high static loads the total stiffness of support mountings may be expected to be of the same order of magnitude as for the test mounting. Fortunately, this does not influence very much the frequency range for which Eq. (2.25) is accurate. For instance, for the case of a
support mounting identical with the test mounting, in analogy with Eqs. (2.31) and (2.32),

\[
20 \log \frac{\frac{\ddot{z}_S}{m_S}}{\frac{\ddot{z}_m}{m_m}} = 20 \log \frac{\Omega^2}{(2-\eta^2)^2 + 4\eta^2} \leq 1.2 \text{ dB}
\]

for \( f \geq 4f_o \)

and for \( \eta = 0,1 \)

\[
\phi_{\ddot{z}_S}/\ddot{z}_m - \phi_{\ddot{z}_m}/\ddot{z}_m < 1^0 \text{ for } f > 4f_o .
\]

Use of the same measurement principle for small size unloaded mountings is reported by Heckl /1.4/.

**Horizontal translation:**

The same procedure as for the vertical translation may be used for horizontal translational excitation. To suppress rotation of the excitation block in the frequency range of the measurements, the transverse excitation should occur along a line through the mass centre of the rigid body formed by the excitation block and the upper mounting flange. Moreover, the mass and the rotational inertia of the excitation block should be so large that the natural frequencies of the "rocking-modes" are sufficiently low.

In theory, uncoupling of the transverse translational mode and a rotational mode may be obtained by applying the static preload via an auxiliary mounting on top of \( m_e \) which is identical with the test mounting and symmetrically placed with respect to \( m_e \). However, in practice this is impossible when there is just one test sample available and if it is possible the symmetry is not always ideal. A validity test procedure for checking the influence of rotational excitation will be discussed therefore in section 2.4.

**Rotational excitation and measurement of blocked torques:**

A similar measurement procedure as described above is also applied to the measurement of the rotational components. The problems here are the
suppression of the translational excitation on top of the mountings and the measurement of blocked torques beneath them.

The measurement of $T_{m,s,ys}$ is now considered; see Fig. 2.10a. The excitation of $\ddot{x}_m$ is realized with the aid of a lever. The height of the exciter position is adjusted to minimize $\ddot{x}_m$. This is obtained when the translation in the contact plane due to the rotation of the excitation block about a line through the mass centre is cancelled by the rigid body translation, which is equal to the translation of the mass centre. At low frequencies again the most favourable condition is obtained when rotational and translational modes are uncoupled by using an auxiliary mounting on top which is identical with the test mounting.

For deriving the blocked torque $M_{s,y}$ it should be noted that the total external moment on the terminating block is determined by the torque $M_{s,y}$ as well as by the force-moment due to the transverse force $F_{s,x}$; see Fig. 2.10b. The torque $M_{s,y}$ is found from

$$M_{s,y}(t) = J_{s,ys} \dot{\omega}_{s,y}(t) - m_s \ddot{x}_s(t) h/2,$$

(2.36)
Figure 2.11: Reciprocity principle: $T_{M_{y},x}^{\alpha} = T_{F_{x},x}^{\alpha}$ for the mounting inverted.

where $J_{s,y}$: principal moment of inertia of terminating block about y-axis;

$h/2$: distance between mass centre and plane of excitation.

Sometimes an elegant simplification is practicable, by using a terminating block the mass centre of which lies in the contact plane with the mounting.

For some type of transfer functions the measurement of blocked torques can be either avoided or checked on its accuracy by performing an equivalent reciprocal measurement. The reciprocity relations follow from the symmetry of the impedance matrix in Eq. (2.16). For example $T_{M_{y},x}^{\alpha}$ can be determined from $T_{F_{x},x}^{\alpha}$, for the mounting turned upside down; see Fig. 2.11. In case of symmetry type A, D or E of Figs. 2.4 and 2.5 this rotation of the mounting is, of course, not necessary.

Test facilities:

Test rigs for resilient mountings and flexible shaft couplings are shown in respectively Figs. 2.12 and 2.13. The mounting test rig in Fig. 2.12 has a working height of 1.6 m and a width of 1 m. The maximum compression preload is 600 kN which is applied via a hydraulic actuator. With pneumatic auxiliaries, shear loads and torsional loads can also be applied. The test rig for shaft couplings in Fig. 2.13 has a working height of 1.5 m; a width of 2.4 m. The torsional preload capacity is up to 20 kNm with the pneumatic auxiliaries shown in Fig. 2.13.
Figure 2.12:
Test rig for measuring acoustic properties of preloaded resilient mountings.

Figure 2.13:
Test rig for measuring acoustic properties of preloaded flexible shaft couplings.
In Fig. 2.12 a measuring configuration is shown for excitation of a mounting with rotation about a horizontal axis. For measuring a blocked torque exerted by the test mounting on the lower block, translation of its mass centre and rotation of the block are measured simultaneously. The signals are processed according to Eq. (2.36) with the aid of a special purpose electronic device. Rotational accelerations and translational acceleration of inaccessible locations are measured by subtracting or adding the signals of sensitivity matched uni-directional accelerometers which are located at adjacent locations.

**Liquid filled flexible bellows and hoses:**

The multi-directional structureborne sound transfer properties of flexible bellows and hoses can be characterized under certain conditions with the same measuring principle as for mountings. However, such tests are more complicated when the test components must be filled with a liquid and pressurized in the same way as under operational conditions. An extensive discussion on the complex acoustical behaviour of liquid filled isolators is beyond the scope of the present discussion. Only two test arrangement aspects are mentioned here.

It appeared in practice that, using the same test arrangement as for mountings, liquidborne sound excitation of the terminating block may determine the blocked forces in axial direction completely. However, such test rig boundary conditions with a blocked liquid column on both sides are not representative of field situations. A way to avoid this liquidborne excitation in axial direction, is the application of an annular terminating block and an auxiliary bellows in a way as in Fig. 2.14.

In principle, such a test arrangement can be used also for measuring blocked forces in transverse directions and for measuring blocked torques. However, because of the flexibility of the bellows the sound propagation speed in the liquid column is much lower than in a rigid-walled pipe. Therefore, initiation of non-axially symmetric wave propagation (see 5.3) occurs probably within the measurement frequency range. This might lead to significant net forces in transverse directions.

A practical way of proving the predominance of structureborne sound transfer via the test bellows to $m_s$, is by choosing the auxiliary bellows
in Fig. 2.14 identical with the test bellows and by measuring accelerations on both bellows. If the accelerations on the auxiliary bellows are much lower than on the test bellows, this indicates predominance of structureborne sound transfer to $m_s$. If there is no significant attenuation across $m_s$, the probable cause is liquidborne flanking sound transfer.

2.4 Experimental aspects of measuring mounting transfer functions

Some results are presented in Fig. 2.15 as an illustration of the feasibility of the measuring technique of section 2.3. Results are shown for the test mounting of Fig. 2.12. This is a rectangular rubber block (0.19 x 0.19 x 0.10 (height) m$^3$) with steel end plates. Its dynamic compression stiffness is $2.2 \times 10^6$ Nm$^{-1}$. 

**Figure 2.14:** Measuring arrangement for liquid filled bellows and hoses.

**Figure 2.15:** Transfer functions and validity test results for rectangular rubber block of Fig. 2.12.
Data are shown on $T_{M_{y,x}}$ and $T_{F_{x,y}B}$ for measurements in a prototype test rig without static preload (except for the load presented by the excitation blocks). These transfer functions should be equal because of reciprocity and symmetry; see Fig. 2.11. For the measurements the vibration exciter was driven with a white noise spectrum. For the signal processing a 1/3-octave analyzer was used. In a separate investigation it was checked that the power spectral density function of the excitation accelerations was sufficiently flat within the 1/3-octave bandwidths, so that the results may be considered as good approximations of $|T|^2/\Delta f$, i.e. frequency averages of the squared modulus of the transfer functions.

The transfer function $T_{M_{y,x}}$ has been measured using elimination of the force moment $F_{s,x}^{(h/2)}$ according to Eq. (2.36). The approximate equality with $T_{F_{x,y}B}$ in Fig. 2.15 indicates the feasibility of this elimination procedure.

Independent validity checks are also shown. $V_{F_{x,y}B}$ and $V_{M_{y,x}}$ represent respectively calculation results for $F_{s,x}/B$ as caused by still existing $x_m$ for the $B$ excitation mode and for $M_{s,y}/x_m$ as caused by $B_m$ for the $x_m$—excitation mode; see further discussion below. It appears that for $f < 200$ Hz $M_{s,y}/x_m$ is easier to measure reliably than $F_{s,x}/B$. Of course, this depends on the mounting type as well as on the test rig.

The level differences in Fig. 2.15 between the dotted line representing $T_{F_{x,y}B}(h/2)$ and the line representing $T_{M_{y,x}}$ is a measure for the ratio between the blocked torque $M_{s,y}$ and the force-moment $F_{s,x}(h/2)$, which had to be eliminated for measuring $M_{s,y}$. The results at higher frequencies show that force-moments which were 5 times (14 dB) larger than the blocked torques have been eliminated without leading to large inaccuracies.

A more general discussion is now presented on aspects which are to be considered when judging the reliability of the measured results.
Airborne sound transfer:
Resonance in the coupled system of the vibration exciter, push rod and (as in the case of Figs. 2.10 and 2.12) the lever, may lead to intense sound radiation which causes unwanted excitation of the terminating block. The best way to check whether this has influenced measured results is by interrupting the mounting path, leaving the airborne sound field unaltered. This can be attained by hanging the excitation block in elastic supports close to the test mounting. If, instead, the exciters are disconnected from the excitation block, the sound field changes in a complex way. This can not be characterized accurately by the change of the sound pressure at a single microphone location and therefore sometimes leads to erroneous conclusions.

Separation of excitation components:
A critical part of the validity test procedure is the determination of contributions due to unwanted, but not completely avoidable, excitation components. When measuring $T_{M_y,\beta}$ and $T_{F_x,\beta}$ for example, it is desirable to know whether the results have been influenced due to excitation by $\ddot{x}_m'$. On the other hand, when measuring $T_{F_x,\ddot{x}}$, knowledge is needed as to whether the results have been influenced by $\ddot{\beta}_m$. In this test procedure it is difficult to avoid arguing in a circle. For example, for testing the validity of $T_{F_x,\ddot{x}}$, one needs $T_{F_x,\ddot{\beta}}$ and for testing $T_{F_x,\ddot{\beta}}$, one needs $T_{F_x,\ddot{x}}$ for the calculation of unwanted contributions. This does not present problems as long as the calculated unwanted contributions are much smaller than the measured values for both transfer functions. If this is not the case then it is necessary to carefully analyse "worst case conditions" and possibly reject one or both results. However, as in the example of Fig. 2.15, sometimes an extra independent indication on the reliability of a transfer function may be obtained from an equivalent reciprocal measurement.

Measurement of blocked torques:
Measuring a blocked torque with the aid of a terminating block as in Figs. 2.10 and 2.12 requires elimination of the force-moment $F_{s,x} (h/2)$ using signal processing according to Eq. (2.36). As has been shown in Fig. 2.15 force-moments which were 5 times larger than the blocked torques have been eliminated without resulting in large inaccuracies. The following brief
error analysis indicates that for significantly larger ratios of force-moment and torque this procedure probably becomes inaccurate. The critical part of Eq. (2.36) may be written as

\[ q = \bar{\beta} - C \bar{x} , \]  

(2.37)

with \( C = \frac{m_s h}{2J_s y} \). The relative error in \( q \) may be written as a function of the relative error in \( \bar{\beta}, C \) and \( \bar{x} \) as follows

\[
\frac{\Delta q}{q} = \frac{\Delta \bar{\beta}}{C \bar{x}} + \frac{1}{1 - \frac{\bar{\beta}}{\bar{x}}} \left( \frac{\Delta C}{C} + \frac{1}{1 - \frac{\bar{\beta}}{\bar{x}}} \frac{\Delta \bar{x}}{\bar{x}} \right). \tag{2.38}
\]

The factor \( (1 - \frac{\bar{\beta}}{C \bar{x}})^{-1} \) determines the relative error in \( q \) caused by a given relative error in \( C \) and \( \bar{x} \) and for values of \( \frac{\bar{\beta}}{C \bar{x}} \) close to 1 ("blow-up" region), also with good approximation that caused by a relative error in \( \bar{\beta} \).

Note:
For the measurement of \( M_{s,x} \), the Eqs. (2.37) and (2.38) are to be modified replacing \( \bar{\beta} \) and \( \bar{x} \) by \( \bar{\alpha} \) and \( -\bar{y} \) respectively.

In the example of Fig. 2.15 the maximum level difference between force-moment and torque is some 14 dB. This corresponds with

\[
\frac{F_{s,x}}{F_{s,y}} \frac{(h/2)}{M_{s,y}} = \frac{C \bar{x}}{q} \approx 10^{14/20} \approx 5 , \tag{2.39}
\]

or with Eq. (2.37)

\[
\frac{\bar{\beta}}{C \bar{x}} = 1 \pm 0.2 . \tag{2.40}
\]

For the ratio as in Eq. (2.40) the relative errors in \( \bar{\beta}, C \) and \( \bar{x} \) are thus magnified with a factor 5. Therefore the relative error in \( q \) and thus in the measured blocked torque, will not exceed some 10 percent (1 dB) as long as the relative errors of \( \bar{\beta}, C \) and \( \bar{x} \) are of the order of 1 percent.

Apparently this has been realized with the electronic devices and the accelerometers (type B & K 4368) which have been used for producing the
measured results of Fig. 2.15. However, for a number of reasons relative errors in $\hat{\beta}$, $C$ and $\hat{x}$ which are significantly less than 1 percent cannot be guaranteed in general. Therefore in test arrangements which indicate higher ratios of force-moments and torques than say 5, the terminating block should be modified for assuring reliable torque measurements.

2.5 Discussion of some practical aspects of mounting path analysis
A brief discussion is presented on some practical aspects which should be considered for the application of the mounting path analysis procedure.

Validity of the analysis method at higher frequencies:
Some of the fundamental assumptions of the analysis method may become more or less invalid at higher frequencies, viz. the undistorted plane vibrational motion of the mounting flanges and the large impedance mismatch at the mounting/seating interface.

For the distortion of the mounting flanges two causes can be distinguished.
Firstly, the bending wavelengths in the adjacent structures can become too small compared with the dimensions of the mounting. A similar analysis procedure can probably be used in that case, but now with some averaging of source levels and sound transfer functions over the contact areas. The reciprocal measurement of transfer functions makes such averaging procedures especially simple.

A second cause of local deformations is an incompletely rigid connection between mounting and adjacent structures over the whole interface area. Especially for large size mountings with relatively thin flanges a perfect "clamping" contact seems difficult to realize if no special milling (or filling) procedure is prescribed. As a result resonances in the end plates may occur above a few hundred Hz which may lead to a considerable increase of sound transfer. This phenomenon has been observed by the present author in some mounting test rig measurements and is also reported by Sainsbury /1.8/, p.379.

Another problem may be formed by the requirement of a large impedance mismatch at the mounting/seating interface. At higher frequencies this requirement may be violated when a mounting with a relatively heavy flange
is installed on a relatively lightweight seating structure. In many cases this problem can be solved very practically by considering the mounting flange as an integral part of the seating structure. In that case the ship transfer functions are to be determined for excitation on the flange or on a dummy flange. Then in the mounting test rig the measurements should also be adjusted.

**Measurement of resilient mounting properties:**
The structural components of the mounting test rig, i.e. the blocks and the auxiliary mountings, should be adjusted both to the test mounting type and to the frequency range of interest. For low frequency measurements the auxiliary mountings should be soft and the masses $m_e$ and $m_s$ sufficiently large. For high frequency measurements the size of the blocks should be rather small to preserve their behaviour as rigid bodies. These somewhat contradictory requirements for measurements over a large frequency range, say $30 - 2000$ Hz, necessitate at least two sets of excitation and measuring blocks. However, in many cases the measurements at low frequencies, say $f < 80$ Hz, are not necessary because reliable estimates can be made from extrapolating observed "pure spring" behaviour, for e.g. $80 - 160$ Hz, to lower frequencies. In that case priority can be given to reliable "high" frequency measurements and one set of test rig blocks might be sufficient.

2.6 **Summary**

1. A new experimental procedure has been developed for determining the sound transfer through the resilient mountings underneath shipboard machinery. The method is based on linear mechanical network theory and takes into account six degrees of freedom for the motions on top of a mounting and for the excitation of the seating underneath. The sound transfer system is split conceptually into 3 sub-systems, viz. the source, the mountings and the remaining path through the ship structure to a receiver location.

2. The resilient mountings are characterized by transfer functions for a "blocked" terminal on the seating side, in analogy with the well-known concept of "blocked" transfer impedances. A method and laboratory facilities have been described for measuring these transfer functions for independent excitation with 6 orthogonal translations and rotations.
Measurement examples illustrate the feasibility of this measuring method. An important feature of the test rig is that the mountings can be investigated under static pre-loads which are representative of shipboard situations. Variants of the method for investigating flexible propeller shaft couplings and liquid filled flexible pipe couplings have also been discussed.

3. For the measurement of transfer functions through the ship structure previously published reciprocity techniques are to be used. In these methods a distant sound source is used (e.g. on a deck or in the water) and accelerations are measured at the resilient mounting locations. In this way it becomes practicable to include transfer functions for in-plane force excitation and for torque excitation on the seating. Moreover, in many cases there is no need to remove the resilient mountings when these reciprocity techniques are applied. This has the advantage that measurements on board ships which are in service can be performed with a minimum disturbance of normal ship operations.
3. EXPERIMENTS ON THE ACCURACY OF MOUNTING PATH ANALYSIS

Because of the multi-directional vibrations the complete analysis of the resilient mounting path for a multi-mounting system requires the measurement of an enormous amount of data. In this chapter it is investigated in scale model experiments how the accuracy of the analysis is affected when simplified procedures are applied.

3.1 Introduction

In Chapter 6 a case study is presented on quantifying sound transfer paths for a large resiliently mounted shipboard diesel engine. This concerns an engine which is installed on 18 resilient mountings. The analysis of the mounting path according to the exact scheme of Eq. (2.2) requires an enormous amount of measured data to be collected on board, i.e. $18 \times 6$ source level spectra and $18 \times 6$ ship transfer functions, all data containing amplitude and phase information. Usually the time and effort needed for such a complete analysis are prohibitive. It is obvious therefore to consider simplified procedures. Because this has consequences for the accuracy, it seems useful to distinguish between 2 objectives for such a mounting path analysis. Firstly the analysis can be part of an investigation comparing several transfer paths over a wide frequency range. In that case it might be acceptable to adopt a simplified statistically oriented but perhaps somewhat less accurate procedure. Secondly the analysis can be directed on the identification of the cause of noise requirement violation at one or a few discrete frequencies. This might ask for a deterministic analysis.

It is crucial to investigate the accuracy of the analysis procedure for the path complete from source to receiver and the influence of simplifications. To do this, experiments were carried through on a scale model system which is, as will appear in Chapter 6, representative of a resiliently mounted propulsion diesel engine in a car-ferry. In this scale model the situation is created that the sound transfer from the "engine" to the "accommodation" is determined completely by the resilient mounting path. The structureborne sound levels on an accommodation deck are compared with calculations according to Eq. (2.2) and according to simplified versions.
3.2 Description of scale model
The objective of the scale model investigations is testing path quantifying methods. Therefore it suffices to build a typical shiplike structure rather than to model a particular ship accurately.

Ship section:
The steel test section is shown in Fig. 3.1 and its principal dimensions are given in Figs. 3.2 and 3.3. The section forms a periodic structure over a depth (i.e. ship length) of 13 frame spaces. Dimensions have been derived from corresponding sections in car-ferries and in a fleet tanker in which the propulsion diesel engines have been mounted resiliently. The scaling factor is 1:8.
The section consists of a double bottom structure, a hull on one side, a between deck and three accommodation decks. The decks have been turned outside for experimental convenience. The bottom section consists of a tank top and plate-like stiffeners in athwartship and in fore-and-aft direction. The shell plating of the bottom was left out for practical reasons. The hull is stiffened with frames and web frames and the decks are stiffened in a similar way. Extra damping was introduced in the bottom and in the lower deck with the aid of wooden beams, which in turn were resiliently mounted on the wooden main support. Numbers of measurement locations are given in Fig. 3.3.

**Sound source:**
To simulate the complex excitation of the mounting path by an engine, a model structure has been used which corresponds with an engine raft for a large medium speed propulsion diesel engine. In Fig. 3.4 this structure is
Figure 3.4:
Resiliently mounted engine raft with 9 electrodynamic exciters (loudspeakers).

Figure 3.5:
Cross section of engine raft and of airborne sound insulation measures.
Figure 3.6:
Test arrangement for measuring $T_{m_y}^B$ of the scale model mounting:
$$m_e = 0.16 \text{ kg (aluminium)}; \quad m_s = 0.44 \text{ kg (steel)}.$$ For measuring other transfer functions the same blocks have been used except for $T_{p_z}^B$ at $f < 1 \text{ kHz}$, where $m_s = 6 \text{ kg}$.

shown connected to the ship section by means of 16 rubber mountings and with on top 9 electrodynamic exciters for exciting the raft in three directions. This arrangement has been used for the investigations in 3.4.

To create a situation in which the structureborne sound transfer through the mountings was predominant, it was necessary to reduce the airborne sound transfer. The noise reduction measures which were taken to achieve this are shown in Figs. 3.4 and 3.5. However, it appeared necessary to remove in addition most of the raft bottom plate. The air cavities inside the raft were filled with sound absorbing foam plastic.

Resilient mountings:
Although not essential for the experiments in the present chapter, the mountings have been modelled after the mountings in the car-ferry system which is analyzed in Chapter 6. They were composed of rectangular rubber blocks vulcanized to steel end plates. The rubber dimensions are in mm: $54 \times 25 \times 8.5$. The rubber hardness is 45° Shore A. The dimensions of the steel end plates are in mm: $54 \times 42 \times 1.5$. The end plates were connected with six bolts both to the seating and to the raft; see Fig. 3.9. In Fig. 3.6
the test arrangement is shown for measuring the scale model mounting transfer functions.

3.3 Investigations on the role of correlation

General theoretical considerations:
The analysis of a mounting system according to the procedure of Eq. (2.2) or its variants provides a time mean square value of receiver response \( r \) which may be written as

\[
<r^2> = <(\sum_i \sum_j \sum_k r_{i,j,k})^2>
\]

(3.1)

where \(<.>\) denotes time averaging.

Energy summation of the individual components, thus neglecting phase relationships, is justified if

\[
<(\sum_i \sum_j \sum_k r_{i,j,k})^2 > = \sum_i \sum_j \sum_k <r_{i,j,k}^2>
\]

(3.2)

For any analysis procedure it is most important to avoid large underestimates of the relative path strength.

The simplest case to consider is the summation of two components \( r_1 \) and \( r_2 \) with harmonic time dependence. Let \( \hat{r}_1 \) and \( \hat{r}_2 \) be the amplitudes and \( \phi \) the phase difference. Then the total response is given by the following equation:

\[
<r^2> = r_1^2 + r_2^2 + 2<r_1 r_2> = \frac{\hat{r}_1^2 + \hat{r}_2^2}{2} + \hat{r}_1 \hat{r}_2 \cos \phi .
\]

(3.3)

It is seen from this equation that the maximum underestimate of the level of \( r \) due to neglecting correlation is 3 dB, namely for the case that \( \hat{r}_1 = \hat{r}_2 \) and \( \phi = 0 \).

For \( n \) components Eq. (3.3) is to be replaced by

\[
<r^2> = \frac{1}{2} \sum_{u=1}^{n} \hat{r}_u^2 + \sum_{v=1}^{n-1} \sum_{w=v+1}^{n} \hat{r}_v \hat{r}_w \cos \phi_{v,w} .
\]

(3.4)

The maximum possible underestimate is in this case 10 lg \( n \) dB. However, in general it may be expected that the probability for such a maximum underestimate decreases with an increase of \( n \). Positive and negative errors will tend to cancel each other at least partially.
The influence of a neglect of phase relations on the accuracy of the mounting path analysis will be negligible as long as the second term on the right hand side of Eq. (3.4) is much smaller than the first term. For investigating whether this will be the case it is useful to distinguish between the correlation between the various excitation components (i.e. source vibrations) and the "correlation" for the ship transfer functions. The role of the latter can be illustrated with the situation that at all mounting locations the source vibrations have equal phase and amplitude. Then still the second term on the right hand side of Eq. (3.4) can be much smaller than the first term for an appropriate distribution of $\phi_{v,w}$, i.e. phase angles of the transfer functions. It will be discussed later on in this section (see Experiment 2) that correlation coefficients form an appropriate measure for characterizing both the sound source and the transfer system.

In general it may be expected that factors which tend to diminish the influence of correlation in diesel engine noise transfer are:
- increase of the number of mountings and vibration directions which contribute to the total sound transfer
- increase of the analysis bandwidth, so that the total response level at a receiver location is composed from contributions at several discrete frequencies
- increase of frequency so that more resonant vibration modes in the engine and in the ship structure are excited, which tends to randomize phase relationships both for the accelerations on top of the mountings and for the ship transfer functions
- increase of the number of partially incoherent primary sources, especially the number of cylinders.

Some of these factors were investigated in the following experiments.

Experiment 1: Influence of neglecting phase relationships:
In the first experiment the engine raft was installed on 8 mountings, each 2 frame spaces apart. The sound transfer from the engine raft to the middle accommodation deck was measured as well as determined indirectly with 2 variants of Eq. (2.2). One variant is exact. The other is a simplified version in which the phase relationships are neglected.
The frequency range of the analysis covered the 1/3-octave bands with centre frequencies 80 - 1600 Hz, thus corresponding with the full scale low frequency range 10 - 200 Hz. For this frequency range the sound transfer is completely dominated by translations in the compression direction of the resilient mountings: see also 3.4. Therefore the analysis could be limited to 8 source accelerations $\ddot{z}_m$, 1 mounting transfer function, i.e. $T_{Fz}\ddot{z}$, and 8 ships transfer functions. For the response at the receiver location on the deck the open circuit voltage of a reciprocal electrodynamic loudspeaker was chosen. The cone of this loudspeaker was connected mechanically to the deck plate. The ship section transfer functions $[e/F_z]$ were measured reciprocally as $[\ddot{z}'/\omega i']$, see Appendix A. Therefore the variant of Eq. (2.2), which was used may be written for the case of harmonic time dependence as

$$<e^2> = \left| T_{Fz}\ddot{z} \right|^2 \cdot \left( \sum_{k=1}^{8} \frac{\ddot{z}'_k}{\omega i'} \cdot \ddot{z}_m,k \right)^2,$$

or, when the phase relationships between the transfer functions and between source accelerations are neglected as

$$<e^2> = \sum_{k=1}^{8} \frac{\ddot{z}'^2_k}{\omega^2<i'^2>} \cdot \left| T_{Fz}\ddot{z} \right|^2 \cdot \ddot{z}_m,k^2.$$

For the actual experiments broadband excitation was used and signal processing analogous to Eqs. (3.5) and (3.6) was performed in the frequency domain, using Discrete Fourier Transform methods on a digital computer system.

For exciting the raft in vertical direction three exciters were used, which were driven from a single signal generator with a periodic chirp at repetition frequencies of respectively 40 Hz and 4 Hz. The spectrum with line distance $\Delta f = 40$ Hz was chosen to simulate the spectrum of a medium speed four stroke diesel engine running at 600 rev./min. The frequency spacing between the harmonic components of the cylinder firing frequency which usually dominate the excitation spectrum of such a diesel engine is $\Delta f = 5$ Hz. Due to the scaling factor 1:8 this corresponds with $\Delta f = 40$ Hz in the scale model investigations. The scale model spectrum with $\Delta f = 4$ Hz is not representative for any diesel engine, but it was chosen to illustrate the decreasing influence of correlation when the number of excitation frequencies per bandwidth increase.

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Figure 3.7:
Sound transfer from engine raft on 8 resilient mountings to transducer on accommodation deck. The calculations have been performed analogous to respectively Eq. (3.5) and Eq. (3.6).

Results: In Fig. 3.7a the 1/3-octave spectra of "e" are shown for the experiments with Δf = 40 Hz. It is seen that calculated results for which the phase relationships were taken into account agree quite well with the directly measured results. The mean level difference between measurement and calculation for the 12 1/3-octave bands is -0.8 dB and the standard deviation for the level differences is 2.3 dB. When neglecting the phase relationships the mean level difference is -0.5 dB and standard deviation is 6.3 dB. This is significantly larger than when taking the phase relationships into account. Large overestimates (11 dB at 500 Hz) and underestimates (8 dB at 160 Hz and 800 Hz) are seen.

In Fig. 3.7b the results are shown for the similar measurements with Δf = 4 Hz. The mean level difference between measurement and "exact" calculation is -0.9 dB and the standard deviation for the level difference 2.8 dB. For the calculation without phase relationships the mean level...
The difference is -1.4 dB and the standard deviation 3.5 dB, which is significantly smaller than for Δf = 40 Hz and more or less the same as for the "exact" calculation. This result confirms that increasing the number of frequency components reduces the influence of correlation.

The results of the "exact" calculations for both measurements are an indication of the accuracy which can be obtained with the path analysis method under the most favourable conditions.

**Experiment 2: Measurement of correlation coefficients:**

Correlation coefficients for pairs of source vibrations as well as for pairs of ship transfer functions may give an appropriate indication of the potential importance of phase relationships. For instance, if they are small for source accelerations or for ship transfer functions at adjacent mounting locations, this may also be expected to be the case for more distant locations.

Measured results of such correlation coefficients are presented below. With respect to source vibrations some correlation coefficients for a diesel engine are presented by Popkov /2.6/. However, the application of the same concept on the transfer system is new as far as is known to the author.

The correlation coefficient for two signals \( u_1(t) \) and \( u_2(t) \) is defined as

\[
\Gamma \stackrel{\text{def}}{=} \frac{\langle u_1 u_2 \rangle}{\left( \langle u_1^2 \rangle \cdot \langle u_2^2 \rangle \right)^{\frac{1}{2}}} .
\]

(3.7)

The signals \( u_1 \) and \( u_2 \) may represent either accelerations on an engine or response quantities at some distant receiver location due to in-phase excitation at two mounting locations or in two directions at the same mounting location. It is easy to prove that if \( |\Gamma| \ll 1 \) either for source accelerations or for transfer functions, then neglecting the phase relationships between them cannot lead to significant underestimates of the sound transfer.

The correlation coefficients can be obtained by signal processing in the frequency domain. For two source accelerations \( a_1 \) and \( a_2 \) and a filter
bandwidth $\Delta f$ is easy to derive that

$$
\Gamma_{\Delta f} = \frac{\int_{f_1}^{f_1+\Delta f} \text{Re} \ G_{a_1, a_2} \, df}{\left( \int_{f_1}^{f_1+\Delta f} G_{a_1, a_1} \, df \right)^{\frac{1}{2}} \left( \int_{f_1}^{f_1+\Delta f} G_{a_2, a_2} \, df \right)^{\frac{1}{2}}},
$$

(3.8)

where $G_{a_1, a_1}$ and $G_{a_1, a_2}$ denote respectively one sided power and cross-power spectral density functions.

For ship transfer functions correlation coefficients can be determined most easily with reciprocal measurements. As an example the measurement is discussed of the correlation coefficient for in-phase excitation by point forces $F_1$ and $F_2$ at two mounting locations on a seating and for sound transfer to a receiver location on a deck. As a response on the deck the open circuit voltage "$e"$ of a reciprocal electrodynamic transducer is considered as in Experiment 1. To get an unambiguously defined quantity in this study the correlation coefficient $\Gamma_{\Delta f}$ for transfer functions is defined for excitation forces (torques) with a flat power spectral density function within the analysis bandwidth $\Delta f$. If $e_1$ is the deck response due to $F_1$ and $e_2$ that due to $F_2$ then the correlation coefficient may be written in analogy with Eq. (3.8) as

$$
\Gamma_{\Delta f} = \frac{\int_{f_1}^{f_1+\Delta f} \text{Re} \ G_{e_1, e_2} \, df}{\left( \int_{f_1}^{f_1+\Delta f} G_{e_1, e_1} \, df \right)^{\frac{1}{2}} \left( \int_{f_1}^{f_1+\Delta f} G_{e_2, e_2} \, df \right)^{\frac{1}{2}}},
$$

(3.9)

The cross-spectral density function $G_{e_1, e_2}$ cannot be measured in a single experiment. But $\Gamma_{\Delta f}$ is easily measured reciprocally. In the reciprocal measurement, the reciprocal transducer on the deck is driven with a current $i'$ and at the seating locations the accelerations $a'_1$ and $a'_2$ are measured in
the same directions as the forces $F_1$ and $F_2$ (see Appendix A). Using the
reciprocity relations of the type $[e/F] = [a'/\omega i']$ it can be derived (see
Appendix B) that

$$\Gamma_{\Delta f} = \frac{\int_{f_1}^{f_1+\Delta f} \frac{\text{Re} \left\{ G_{a_1', a_1'} \right\}}{\omega^2 G_{i', i'}} \, df}{\left( \int_{f_1}^{f_1+\Delta f} \frac{G_{a_1', a_1'}}{\omega^2 G_{i', i'}} \, df \right)^{\frac{1}{2}} \left( \int_{f_1}^{f_1+\Delta f} \frac{G_{a_2', a_2'}}{\omega^2 G_{i', i'}} \, df \right)^{\frac{1}{2}}}.$$  (3.10)

For other physical quantities than "e" similar formulae can be derived for measuring $\Gamma_{\Delta f}$ reciprocally.

**Correlation coefficients for transfer functions:**

Measurements for determining $\Gamma_{\Delta f}$ according to Eq. (3.10) have been
performed on the scale model section. The middle accommodation deck was
excited with a loudspeaker in the same way as in the foregoing experiment.
Accelerations have been measured at pairs of seating locations as follows
(see Fig. 3.3): 1 frame space apart (1,2); 2 frame spaces apart (8,10); 3
frame spaces apart (1,4); at the same frame number but on different
seatings (1,8); at the same mounting location but for excitation in normal
and transverse direction (n,t). The loudspeaker was again driven with a
periodic broadband chirp with repetition frequency 40 Hz to simulate diesel
engine noise. The correlation coefficients have been determined for the
1/3-octave bands with centre frequencies 0.4-10 kHz, corresponding with
50-1250 Hz on full scale.

**Results:** In the Figs. 3.8a and 3.8b the results are presented. For the
location pair (1,2), i.e. the adjacent locations, the values of $\Gamma_{\Delta f}$ are
close to +1 for $f < 2$ kHz ($f < 125$ Hz on full scale). Also for other pairs
of locations values of $|\Gamma_{\Delta f}| > 0.5$ occur at these low frequencies. At
higher frequencies for all pairs the values of $|\Gamma_{\Delta f}|$ decrease and are on
the average < 0.5. Therefore it may be expected that at these higher
frequencies phase relationships can play hardly a role in the accuracy of
the mounting path analysis even for perfectly correlated excitations at all
positions, whereas at lower frequencies they can.
Figure 3.8a,b:
Correlation coefficients of transfer function pairs for excitation with periodic signal (bf = 40 Hz):
(1,2) 1 frame distance apart; normal direction
(8,10) 2 frame distances apart; normal direction
(1,4) 3 frame distances apart; normal direction

(1,8) at same frame but on different seatings; normal direction
(n,t) at same location but for normal and transverse direction

Figure 3.8c,d:
Correlation coefficients for acceleration pairs on the feet of a medium speed diesel engine:
(12,14) 1 cylinder distance apart; normal direction
(12,15) 2 cylinder distances apart; normal direction
(12,16) same cylinder but on both sides of engine; normal direction
(n,t) same location but for normal and transverse direction
Correlation coefficients for accelerations on a diesel engine:
For the medium speed propulsion diesel engine of the case study in Chapter 6 correlation coefficients have been measured for pairs of accelerations on the raft. A set of locations was chosen similar to that on the seatings in the scale model section. The full scale frequency range 50-800 Hz corresponds with a large part of the frequency range of the foregoing scale model measurements.

Results: In the Figs. 3.8c and 3.8d the results are presented. The position pair (12,14) for adjacent cylinders shows significantly larger values for $|\Gamma_{\Delta f}|$ than the pair (12,15) for locations two cylinder spaces apart.

Note:
The decrease of $|\Gamma_{\Delta f}|$ on the diesel engine with increased spacing of the accelerometers is not solely a result of frequency averaging, but also of the imperfect coherence between engine foot accelerations at different cylinder positions; see 6.3.4.

From the combination of these correlation data for a source and for a representative ship section the conclusion becomes reasonable that the phase relationships may be neglected when analyzing the sound transfer for a resiliently mounted medium speed diesel engine, as in Chapter 6. This neglect will not affect the accuracy of the analysis appreciable for frequencies above, say, 80 Hz. At lower frequencies a more refined analysis might also be of interest as appeared from Experiment 1.

In general a practical indication for the preferability of an "exact" analysis might be that the analysis concerns a sound source which causes troubles at one particular running speed and for which small but deliberate changes of speed cause large sound level fluctuations at receiver locations.

3.4 Scale model test of a considerably simplified analysis procedure
For the complete analysis of the mounting transfer path in a multi-mounting system with multi-directional vibrations it would be attractive if not only the correlations between the various components could be neglected but the number of shipboard transfer functions measurements could also be considerably limited. The influence of such a simplification has been investigated with the scale model system.
Measurements and calculations are discussed for the engine raft installed on 16 mountings and excited by 9 exciters in 3 directions; see Fig. 3.4. All sixteen mountings were installed at similar locations both with respect to seating structure and raft structure, namely against stiffeners in the seating and in the raft. The exciters were driven from a single signal generator. The signal which simulated the engine excitation was again a periodic chirp with repetition frequency of 40 Hz. The frequency range of the analysis covers the 1/3-octave bands with centre frequencies 0.4-8 kHz, corresponding with 50-1000 Hz on full scale. The objective of the test was to compare the measured acceleration at a receiver location on the middle deck with a calculation according to a simplified variant of Eq. (2.2).

Analysis procedure:
For the calculation the following simplifications have been introduced:

Source accelerations: These have been determined using 1/3-octave band filters and for only 8 of the 16 mounting locations, 4 on each side of the raft. The time mean squared values averaged over the measurement positions are indicated with \( \overline{a^2_{m,i}} \Delta f \) for the directions \( i = 1-6 \).

Mounting transfer functions: The mounting has a symmetry type D (see Chapter 2). This implies that the matrix of blocked transfer functions contains 12 non-zero elements, 9 of which are different because the matrix is skew. In analogy with Eq. (2.20) the matrix may be written as

\[
\begin{bmatrix}
  T_{Fx},\ddot{x} & 0 & 0 & 0 & T_{Fx},\ddot{y} & 0 \\
  0 & T_{Fy},\ddot{y} & 0 & T_{Fy},\ddot{z} & 0 & 0 \\
  0 & 0 & T_{Fz},\ddot{z} & 0 & 0 & T_{Fz},\ddot{y} \\
  0 & T_{Mx},\dddot{y} = -T_{Fx},\dddot{y} & 0 & T_{Mx},\dddot{z} & 0 & 0 \\
  T_{My},\dddot{y} = -T_{Fx},\dddot{y} & 0 & 0 & T_{My},\dddot{z} & 0 & 0 \\
  0 & 0 & T_{Mz},\dddot{z} = -T_{Fz},\dddot{y} & 0 & 0 & T_{Mz},\dddot{y} \\
\end{bmatrix}. \quad (3.11)
\]
Local Cartesian coordinate system at mounting locations.

See Fig. 3.9 for local coordinate system.

For 10 of the 12 mounting transfer functions of Eq. (3.11) the 1/3-octave averages of the squared moduli have been determined. These are indicated below with $|T_{i,j}|^2_{\Delta f}$. The 2 transfer functions $T_{F_{x,y}}$ and $T_{M_{z,x}}$ have been neglected a priori for practical reasons, because it was intuitively felt that these "secondary wave type" transfer modes will contribute much less than the compression and torsion modes.

Ship section transfer functions: For 3 adjacent mounting locations (no. 1-3 in Fig. 3.3) a complete set of 6 transfer functions have been determined.

For each of the 6 excitation directions $j = 1-6$ the 1/3-octave band averaged squared modulus $|a$_{\text{deck}}$/F_{s,j}|^2_{\Delta f}$ has been averaged over the 3 locations. These averages are indicated with $|S_{j}|^2_{\Delta f}$. The transfer functions have been measured reciprocally, i.e. with an exciter on the deck (see Appendix A). This exciter was driven with a white noise signal and the force and acceleration signals have been analyzed with a 1/3-octave analyzer.

Correction factor: The number of mountings which contribute to the sound transfer is 16. If there were no systematic differences between the transfer functions for the 16 mounting locations it would be sufficient to
multiply the above mentioned averaged results for positions no. 1-3 by 16, to obtain the total sound transfer. However, there may occur frequency dependent systematic differences for the sound transfer from locations along the same seating or between locations on the 2 opposite seatings. Such effects have been investigated experimentally and as a result a correction factor C has been derived, which had to be used for deriving the total sound transfer function from the average over adjacent positions no. 1-3.

The total sound transfer is calculated by summing the individual contributions for the 10 mounting transfer functions. For each mounting transfer function the total contribution for all 16 mountings has been calculated according to

$$<a^2_{1,j}(\text{deck})>_{\Delta f} = C \sum |s_{j}|^2 \Delta f |T_{-1,j}|^2 \Delta f <a^2_{m,i}>_{\Delta f} \quad (3.12)$$

The values of 10 lg C which have been used are +12 dB for f < 1 kHz, +11 dB at 1 kHz and + 10 dB for f > 1 kHz.

The procedure of Eq. (3.12) is justified as follows:
- The ship transfer functions $|a(\text{deck})/F_{s,j}|^2 \Delta f$ show a very small scatter for the locations nr. 1-3. This is true for all 6 degrees of freedom.

Note:
At f < 2 kHz this small scatter is due to the white noise excitation for these transfer function measurements. Results from the experiments in 3.3 show that if periodic excitation with $\Delta f = 40$ Hz had been used, a much larger scatter would have been found at these low frequencies.

- Additional measurements of transfer functions $\left| a(\text{deck})/F_{s,z} \right|^2 \Delta f$ for positions no. 5 and 7 on the seating adjacent to the hull and for positions no. 8, 10, 12 and 14 on the more distant seating show that there are no systematic differences for mounting locations along the same seating. Therefore $\left| a(\text{deck})/F_{s,z} \right|^2 \Delta f$ averaged over all mounting locations on the seating adjacent to the hull is approximately equal to $\left| S_{p} \right|^2 \Delta f$ averaged over positions no. 1-3. On the basis of structural similarity at all mounting locations the same is postulated also for the other type transfer functions (see Chapter 7 for justification).
Figure 3.10:
Translational acceleration levels averaged over 8 mounting locations on the raft.

Figure 3.11:
Rotational acceleration levels averaged over 8 mounting locations on the raft.

Figure 3.12:
Ship section transfer functions for force excitation averaged over mounting locations nos. 1-3.

Figure 3.13:
Ship section transfer functions for torque excitation averaged over mounting locations nos 1-3.
To determine the propagation loss in athwartship direction, transfer functions have been compared for opposite locations on both seatings. The difference between the value of $10 \lg \left| \frac{a(\text{deck})}{F_{s,z}} \right|^2$ averaged over positions no. 1, 3, 5 and 7 and that averaged over positions no. 8, 10, 12 and 14 is 0 dB for $f < 1$ kHz, 3 dB for $f = 1$ kHz and $5 \pm 1$ dB for $f > 1$ kHz (where $f_c$ denotes the centre frequency of the 1/3-octave bands).

On basis of this limited set of sound transfer functions $\left| \frac{a(\text{deck})}{F_{s,z}} \right|^2$, it has been assumed that for $f_c < 1$ kHz all 16 mountings contribute equally to the sound transfer. This has been expressed with a correction $10 \lg C = 10 \lg 16 = +12$ dB. For the higher frequencies the combined effect of the number of mountings and the propagation loss in athwartship direction leads to corrections of +11 dB for $f_c = 1$ kHz and +10 dB for $f_c > 1$ kHz.

Results:
Source levels: The Figs. 3.10 and 3.11 show the source strength levels. For the measurements, accelerometers type B & K 8303 have been used. Some low frequency values which were influenced by background noise have been deleted. It is seen that the translation for the z-(normal) direction, the y-(athwartship) direction and at higher frequencies also for the x-(fore-and-aft) direction are of the same order of magnitude. For the rotations this is true for all three components. This is consistent with measurements on diesel engines.

Ships transfer functions: Fig. 3.12 and Fig. 3.13 show the ship section transfer functions. For the three force components the transfer functions are not very different. For torque excitations they show a large scatter. Especially at higher frequencies the sound transfer for a torque about the y-(athwartship) direction is much higher than for a torque about the x-(fore-and-aft) direction. This is caused by local bending properties of the seating top plate.

For the reciprocal measurement the force on the deck has been measured with the force transducer of a miniature impedance head type Wilcoxon Z11. However, a check on the accuracy of $\left| \frac{a(\text{deck})}{F_{s,z}} \right|$ by measuring the same transfer function directly, i.e. for excitation on the seating revealed...
Figure 3.14:
Scale model mounting transfer functions $T_{F_z}$, $T_{F_x}$, $T_{F_y}$, $T_{M_1}$, $T_{M_2}$, and $T_{M_3}$.

Figure 3.15:
Scale model mounting transfer functions $T_{M_1}$, $T_{M_2}$, $T_{M_3}$, $T_{M_4}$, $T_{M_5}$, and $T_{M_6}$.

Figure 3.16:
Scale model mounting transfer functions $T_{M_1}$, $T_{M_2}$, $T_{M_3}$, $T_{M_4}$, $T_{M_5}$, $T_{M_6}$, and $T_{M_7}$. Values which have been influenced by unwanted excitation are marked with an asterix.
systematic differences. For this direct measurement the force transducer of a B & K type 8001 impedance head was used. Investigations revealed that this discrepancy was caused, at least partially, by the fact that the deck plate was excited not solely by a point force, but probably also by torques. The reciprocity principle provides simple methods for checking the purity of point force excitation (see Appendix C). It appeared that the directly measured transfer function was correct. From the difference between the directly and reciprocally measured transfer functions a correction factor was derived which was applied also on the other types of reciprocally measured transfer functions.

This experience forms not an argument against reciprocal measurements, but rather a warning for problems which may arise by choosing transfer functions which necessitate excitation with a point force. In the experiments of 3.3, Chapter 4 and Chapter 6 these problems have been avoided by using other types of transfer functions.

Mounting transfer functions: Figs. 3.14 through 3.16 show the measurement results for 8 mounting transfer functions. For \( T_{M,y,x} \) validity tests indicated that some values were unreliable due to the influence of an unwanted excitation component. These values have been marked in Fig. 3.16. However, no effort was made to improve these measurements because, as will appear, this transfer mode and \( T_{F,x,y} \) did not play an important role.

Instead of \( T_{F,x,y} \) and \( T_{F,y,z} \) the equivalent spectra of \( T_{M,y,x} \) and \( T_{M,x,y} \) have been used for the calculations.

Calculated results for the sound transfer: Figs. 3.17 through 3.19 show the results of the sound transfer calculations. The sum of the contributions of all mounting transfer modes together and the individual contributions are compared with the calculated result for the normal translation mode \( T_{F,y,z} \). It is seen that this transfer mode predominates at most frequencies. Only at 2-4 kHz some other transfer modes become equally important.
Figure 3.17:
Calculated contribution to $L_a$ (deck) by various mounting transfer modes minus the contribution by $T_{F_x,\dot{\gamma}}$.

Figure 3.18:
As in Fig. 3.17.

Figure 3.19:
As in Fig. 3.17.

Figure 3.20:
Level differences between calculated and measured deck accelerations.
Comparison between measured and calculated sound transfer: Fig. 3.20 shows the level differences between calculated and measured deck accelerations. Both octave and 1/3-octave results are given. The differences at low frequencies are of the same order of magnitude as in 3.3, where neglect of correlation was the only simplification. The mean value of level differences between calculated and measured deck accelerations is for 1/3-octave bands -0.15 dB, the standard deviation is 5.4 dB. For octave bands these values are 0.8 dB and 3.5 dB respectively.

The answer on the question whether such inaccuracies can be accepted in practice depends on the objective of the analysis. They are fully acceptable if the objective is to find out whether there are flanking paths which contribute much more to the sound transfer than the mounting path; e.g. in the case study in Chapter 6. However, if it is known a priori that the mounting path is predominant and if the objective of the analysis is to determine which modifications are needed to improve the noise reduction, a more exact analysis may be necessary. The feasibility of such a procedure has been shown in 3.3. However, in that situation it may be advantageous also to start with a simplified procedure to select the potential predominant transfer modes.

For an "exact" analysis of the investigated scale model system 96 source accelerations and 96 ship transfer functions would have been needed, phase relationships included. Whereas for the preceding analysis 48 source accelerations and 24 transfer functions have been used, neglecting the phase information. Taking into account the small scatter in source levels at the 8 raft locations, half the number of source accelerations would have been sufficient.

In view of the rather small inaccuracies for octave band results and the considerable saving of time needed for shipboard measurements, the simplified procedure could be adopted in many practical situations.

3.5 Discussion on some practical aspects

When the simplified analysis procedure is adopted, important aspects are the selection of the mounting locations for which the limited number of transfer functions are determined and the way in which these transfer
functions are measured. In the example of 3.4 all resilient mountings had been installed at locations of similar construction. In many shipboard multi-mounting systems there will be different types of mounting locations, e.g., directly above stiffeners and between them. The minimum set of ship transfer functions needed for a simplified analysis should contain information of such different types of locations and preferably on mounting locations on both sides of an engine.

With respect to the measurement of ship transfer functions in 3.4 the 1/3-octave band values were determined using white noise instead of a periodical engine-like excitation. Such a procedure has some useful effects, namely of averaging the sound transfer for excitation at various running speeds of an engine and of reducing the scatter of sound transfer from locations of similar construction. It seems to be more or less equivalent with averaging over a much larger number of mounting locations for excitation with a limited number of spectral components.

3.6 Summary

1. Various aspects concerning the accuracy of the analysis procedure for the mounting path have been investigated with a scale model of a ship section and of a resiliently mounted sound source which represents a large diesel engine. Situations were created in which the sound transfer from the engine to the ship accommodation was solely determined by the mounting path. The directly measured sound transfer has been compared with calculations according to several variants of the analysis procedure.

2. Neglect of phase relationships: For the analysis of multi-mounting systems with multi-directional vibrations a question of practical interest is whether phase relationships may be neglected in the analysis. Especially for machinery with a very stable speed and with a spectrum dominated by discrete frequency components this possibly might lead to significant inaccuracies. This matter has been investigated for a noise spectrum which is typical for a medium speed diesel engine running at 600 rpm.
For a frequency range which corresponds to 10-200 Hz on full scale the measured sound transfer has been compared with results from the analysis procedure. Both "exact" calculations and calculations in which the phase relationships had been neglected have been compared. It appears that especially at $f < 100$ Hz (on full scale) the neglect of phase relationships can lead to rather large errors. The "exact" calculation agrees quite well with the measured result.

At frequencies above 80 Hz (on full scale) the influence of neglecting phase information will not lead to significant errors for diesel engine noise. This has been made probable by presenting correlation coefficients for source vibrations on a medium speed diesel engine and also for transfer functions in the scale model ship section.

Application of the correlation coefficient concept on transfer functions seems new. It provides valuable information which is as relevant as that for source vibrations. To obtain this information in a practical way, reciprocal measurements are indispensable.

3. Use of a limited set of ship transfer functions: For a complete analysis of a multi-mounting system with multi-directional vibrations the number of transfer functions which are to be determined on board can be very large. Therefore it is of practical interest to investigate the accuracy of an analysis procedure which uses much less transfer data.

In the scale model system a considerably simplified analysis procedure has been tested. Not only the phase relationships were neglected, but only 24 of the 96 ship transfer functions were used. The analysis has been made for 10 different transfer modes of the resilient mounting. For the calculations $1/3$-octave data have been used for source accelerations, mounting transfer functions and ship transfer functions. For source accelerations and ship transfer functions averages over several mounting locations have been used.

The calculated octave band levels of the acceleration on a deck differ 6 dB at maximum from the measured levels. The mean octave band level difference between calculation and measurement is 0.8 dB and the standard deviation is 3.5 dB, in a frequency range covering 5 octave bands. This
inaccuracy is fully acceptable when the objective of the analysis is to investigate whether there are strong flanking paths.

In the discussion practical suggestions have been made for the selection of mounting locations and for transfer function measurements.
4. AIRBORNE SOUND TRANSFER THROUGH SHALLOW CAVITIES BELOW RESILIENTLY MOUNTED MACHINERY

Two experimental methods are described for determining the airborne sound transfer through reverberant cavities below resiliently mounted machinery, in cases where these cavities are inaccessible for loudspeakers as substitution sound sources. One of the methods is validated by scale model tests. The theoretical analysis leads also to some improvements of previously published mathematical models for sound transfer through shallow or narrow cavities.

4.1 Introduction

One of the potential flanking sound transfer paths for resiliently mounted machinery is through the surrounding air. Heckl /1.4,1.5/ has presented some illustrative calculations which make predominance of airborne sound transfer plausible especially at higher frequencies, say \( f > 500 \) Hz. However, as will appear in Chapter 6 this may also occur at much lower frequencies. In Ref. 1.4 Heckl compares the sound energy flow\(^*\) through resilient mountings underneath a diesel engine with that through the air around the engine. In Ref. 1.5 the same is done for airborne sound transfer through a cavity below the engine.

For the analysis of practical situations a choice has to be made between calculating or measuring the contribution of airborne sound transfer.

For existing ships measurements might be preferred as being most accurate. In general such experiments are quite simple because loudspeakers can be used as substitution sources in the engine room. However, in some cases there is a shallow cavity below an engine which is inaccessible for loudspeakers of sufficient strength; see the example in Fig. 4.1. The present chapter deals with solving the experimental problems for such cases. In the case study of Chapter 6 also airborne sound transfer via the engine room is considered.

For shallow reverberant cavities relatively simple measuring methods are developed in 4.2, which make use of reciprocal measurements.

\* rate of energy transport in a certain direction, also called "power flow"
One of the proposed experimental analysis methods has been tested with a scale model. These tests are described in 4.3. They have been performed in the frequency range in which the width of the cavity is equal to or larger than half the wavelength of sound in air, whereas its height is smaller than half the wavelength. In this frequency range the method appears accurate within a few dB.

In 4.4 some variants of the proposed methods are presented, especially for shipboard application. In addition, an illustration is given of how one of the methods can simplify field investigations on flanking sound transfer in buildings.

For ships in the design stage it may happen that no appropriate scale model is available for measurements. In that case a calculation of the airborne sound transfer might be performed along the lines of mathematical methods which have been mentioned already in Chapter 2; see Ref. 2.10-2.13. As input data for these calculations the amount of sound energy flow into the ship structure adjacent to the engine is needed. However, the energy flow from a reverberant shallow cavity with a two-dimensional sound wavefield into an adjacent panel has not been analyzed correctly in previous
Figure 4.2:
Direct and reciprocal transfer functions. a: Fraction of $e_2^2$ caused by $p_1^2$ when the engine is in operation. b: Reciprocal transfer functions measured when the engine has been stopped. For method A $(p_1^2/\omega^2i_2^2)$ is needed and for method B $(a_1^2/\omega^2i_2^2)$.

literature. The theoretical analysis in 4.2 (see also Verheij /4.1/) and a recent article by Schroter and Fahy /4.2/ imply that in previous articles on narrow or shallow cavities by Price and Crocker /4.3/, Heckl /1.5/ and Verheij /4.4/ fundamental aspects have been neglected. This subject is discussed in 4.5.

In 4.6 some of the practical aspects are discussed which are of interest for an experimental or theoretical analysis of the sound transfer through shallow cavities. Moreover, some possible consequences of the theoretical insights for effective sound transfer reduction from such cavities are briefly considered.

4.2 Experimental methods for quantifying the sound transfer in-situ

Outline of 2 methods:
For the derivation of the measuring methods the example in Fig. 4.2 is used (see 4.4 for variants). The sound transfer to a receiver location on an accommodation deck is considered. As a response quantity the open circuit voltage $e_2$ of a reciprocal electrodynamic transducer is chosen. This quantity can be measured when the ship is in service. The problem is how to
determine the fraction of $e_2$ which is caused by the excitation due to airborne sound excitation in the reverberant cavity below the engine. This problem is solved using reciprocal transfer function measurements. In these reciprocal measurements the deck transducer is driven with a current $i_2$ and in the cavity either sound pressures $p_j$ (Method A) are measured or accelerations $a_j$ on tank top (Method B). A flat bar above the symbols $p_j^2$ and $a_j^2$ in Fig. 4.2 indicates averaging over the cavity space and over the tank top surface respectively. The desired transfer function can be found from Eq. (4.1) or (4.2). The derivation of these equations will follow later.

Method A:

$$\frac{e_2^2}{p_1^2} = \frac{17,6 \pi^2 V}{\rho^2 c Q_c T_{60}} \cdot \frac{p_j^2}{\omega^2 i_2^2},$$  \hspace{1cm} (4.1)

Method B:

$$\frac{e_2^2}{p_1^2} = \frac{S_{o(2d)}}{\pi Q_c} \lambda^2 \cdot \frac{a_j^2}{\omega^2 i_2^2},$$  \hspace{1cm} (4.2)

where

- $c$ propagation speed of sound in air (ms$^{-1}$)
- $Q_c$ ratio between space averaged radiation resistance of acoustical point source in shallow cavity and the radiation resistance in the free field
- $S$ area of cavity, i.e. length $\times$ width (m$^2$)
- $T_{60}$ reverberation time of cavity (s)
- $V$ volume of cavity (m$^3$)
- $\lambda$ sound wavelength in air (m)
- $S_{o(2d)}$ in-situ radiation efficiency of tank top for distant mechanical excitation (2-dimensional wave field)
- $\omega$ radian frequency (s$^{-1}$)

The squared measuring quantities denote time mean values, as in the remainder of this chapter.

**Relationship between Methods A and B:**

The following discussion is limited to the single frequency case. However, for practical application 1/3-octave or octave band filtering will be used. Consequences for the signal processing in those cases are discussed in 4.3.
Method B, i.e. Eq. (4.2), follows from Method A, i.e. Eq. (4.1), using the expression in Eq. (4.3c) for sound radiation by the tank top into the cavity during the reciprocal experiment, which may be derived from textbooks on acoustics as follows:

\[ o_0(2d) = \frac{P'_1}{SpC V_1^{1/2}} = \frac{\eta_1 \omega E'_1}{SpC a_1^{1/2} \omega^{-2}} = \frac{4,4\pi}{T_60} \frac{P'_1^{1/2} V}{SpC a_1^{1/2} \omega^{-2}}, \quad (4.3a - 4.3c) \]

where

- \( E'_1 \): sound energy in cavity in reciprocal experiment (J)
- \( P'_1 \): radiated acoustical power by tank top into cavity (W)
- \( \eta_1 \): loss factor of reverberant sound field in cavity (= 2,2/\( fT_60 \)).

**Derivation of Method A:**

For the derivation of Eq. (4.1), i.e. Method A, the following procedure is used:

1. Suppose the engine stopped and imagine an acoustical point source placed in the cavity as a substitution source.
2. Calculate which volume acceleration \( U_1 \) this substitution source should have, averaged over all possible excitation positions in the cavity, to generate the same \( p_1^{2/2} \) as the engine.
3. Measure the sound transfer to the accommodation for this substitution source reciprocally and average over a number of assumed source positions in the cavity.

These steps are presented in a formula as follows,

\[ \frac{e_2^2}{p_1^2} = \frac{e_2^2}{U_2^2} : \frac{p_1^1}{U_1^2} = \frac{p_1^{1/2}}{\omega^2 i_2^{1/2}} : \frac{p_1^2}{U_1^2}. \quad (4.4a; 4.4b) \]

Ensemble averaging over excitation positions of the assumed substitution source is indicated in Eq. (4.4) with a flat bar above transfer functions. The equivalence of Eqs. (4.4a) and (4.4b) follows from the reciprocity of the transfer system, see Appendix A, but now in a distributive formulation instead of point-to-point.
The second factor of Eqs. (4.4a) and (4.4b), i.e. the ratio of $p_1^2$ and $U_1^2$ averaged over all excitation positions of the acoustical point source, can be derived from an ensemble average of the energy flow balance for the reverberant sound field which is generated by the substitution source, as follows:

$$
\overline{P_\text{rad}} = \overline{P_\text{diss}} .
$$

(4.5)

The ensemble average over substitution source locations for the radiated sound power can be written as

$$
\overline{P_\text{rad}} = U_1^2 \omega -\overline{R(2d)_{\text{rad}}} = U_1^2 \overline{Q_c \rho / 4\pi c} ,
$$

(4.6)

where $\overline{R(2d)_{\text{rad}}}$ denotes the space averaged radiation resistance for an acoustical point source in a shallow cavity. The right-hand side of Eq. (4.6) follows from

$$
\overline{R(2d)_{\text{rad}}} = \overline{Q_c R_{\text{rad}}} = \overline{Q_c \omega^2 \rho / 4\pi c} ,
$$

(4.7)

where $R_{\text{rad}}$ denotes the radiation resistance of an acoustical point source in the free field, which is well known from textbooks on acoustics. The ensemble average of the dissipated sound power can be written as

$$
\overline{P_\text{diss}} = \eta_1 \omega E_1 = 4.4\pi \overline{p_1^2} V/(\rho c^2 T_{60}) ,
$$

(4.8)

where the double line above $p_1^2$ denotes space averaging for each substitution source location as well as ensemble averaging over substitution source locations.

From Eqs. (4.5), (4.6) and (4.8) it follows that

$$
\begin{bmatrix}
\overline{p_1^2} \\
\overline{U_1^2}
\end{bmatrix} = \begin{bmatrix}
\rho^2 c Q \overline{T_{60}} \\
17.6 \pi^2 V
\end{bmatrix} .
$$

(4.9)

Combination of Eqs. (4.4) and (4.9) leads to Eq. (4.1).
Differences between Methods A and B:

When method A is used, i.e. the reciprocity measurement with microphones in the cavity, an estimate of \( T_{60} \) and \( Q_c \) is needed. Determination of \( Q_c \) is discussed below. The reverberation time can be measured in-situ. However, in many cases measurements in a scale model cavity will be simpler.

When method B is used an estimate of \( \sigma^{(2d)}/Q_c \) is needed. This quantity can be obtained from measurements in a (scale model) shallow cavity. But in 4.6.1 it is argued that accurate estimates can be obtained from

\[
\frac{\sigma^{(2d)}}{Q_c} \approx \frac{\sigma}{2} \quad \text{for } 4h < \lambda,
\]

where \( h \) denotes the height of the cavity and \( \sigma \) the radiation efficiency of tank top for radiation into a half space. This latter quantity can be estimated from calculations, see e.g. Maidanik /4.5/, or from simple scale model measurements. A further discussion on practical aspects is presented in 4.6.3.

**Determination of \( Q_c \):**

Both theoretical and experimental methods of estimating \( Q_c \) are considered. Theoretical estimate: For the case of a large reverberant cavity between two perfectly reflecting parallel boundaries at distance \( h \) the space averaged value of \( Q_c \) is found from

\[
Q_c \equiv \frac{R^{(2d)}}{R_{\text{rad}}} = \frac{\lambda}{2h} \quad \text{for } 2h < \lambda.
\]

This result can be derived by using a similar mathematical model as applied by Lyon /4.6/ for calculating \( R_{\text{rad}} \) of a large reverberant room with a 3-dimensional sound field. Instead of the asymptotic expression for the modal density in a large room, used by Lyon, now the asymptotic expression \( dN/df = \omega S/c^2 \) for a reverberant space with a 2-dimensional sound field is to be used.

Another derivation of the result in Eq. (4.10) is found if it is assumed in analogy to many other cases in acoustics, that the space averaged value of the radiation resistance for a point source in a large finite cavity is equal to the radiation resistance in an infinitely large cavity. This
latter quantity can be found via the radiated sound power into such a cavity. This sound power is equal to $p^2(r) \cdot \frac{2\pi rh}{pc}$, where $p(r)$ denotes the sound pressure caused by a point source at distance $r \gg h$. An expression for $p(r)$ in a flat layer of infinite extent can be derived from theory, see e.g. Brekhovskikh /4.7/.

Note:
Waterhouse /4.8/ has obtained a result for a point source amidst two parallel walls, which is slightly different from that in Eq. (4.10). However, that result is inconsistent with his analysis which, when worked out correctly, provides the same result as in Eq. (4.10).

**Experimental estimate:** For actual cavities below engines the length and width are usually not in the whole frequency range of interest much larger than $\lambda$. It may be expected that at relatively low frequencies the open cavity boundaries will lower $Q_c$ compared to the asymptotic value given in Eq. (4.10). A simple experimental method for determining $Q_c$ uses a scale model cavity in a scale model reverberant room; see Fig. 4.3. The reverberant room is excited by a loudspeaker successively at a number of locations. The value $\bar{Q}_c$ for the full scale cavity is found at corresponding scale model frequencies from

$$\bar{Q}_c = \frac{\bar{p}_c^2}{\bar{p}_r^2}$$

where $p_c$ and $p_r$ denote the sound pressure in the cavity and in the reverberant room respectively. The flat bar above the transfer function denotes again ensemble averaging, in this case over loudspeaker locations in the reverberant room.
For the derivation of Eq. (4.11) the following assumptions are considered:

Sit. 1: Acoustical point source with volume acceleration $U_r$ in reverberant room.

Sit. 2: Acoustical point source with volume acceleration $U_c$ in reverberant cavity.

In analogy with Eqs. (4.5)-(4.8) the following energy flow balances are valid for ensemble averaging over point source locations:

\[
\text{Sit. 1: } U^2_r = \frac{4, 4 \pi V p^2_{1r}}{\rho c^2 T_{60,r}}, \tag{4.12}
\]

\[
\text{Sit. 2: } U^2_c = \frac{4, 4 \pi V p^2_{2r}}{\rho c^2 T_{60,r}}, \tag{4.13}
\]

(energy dissipation in the cavity itself is neglected).

From Eqs. (4.12) and (4.13) follows:

\[
\overline{Q_c} = \begin{bmatrix} \overline{U^2_c} \\ \overline{p^2_{2r}} \\ \overline{p^2_{1r}} \\ \overline{U^2_r} \end{bmatrix}, \tag{4.14}
\]

A distributive variant of a well known reciprocity relation (see Appendix A) which uses spatial averaging can be written as

\[
\begin{bmatrix} \overline{p^2_{2r}} \\ \overline{U^2_c} \end{bmatrix} = \begin{bmatrix} \overline{p^2_{1r}} \\ \overline{U^2_r} \end{bmatrix}. \tag{4.15}
\]

Eq. (4.11) follows from Eqs. (4.14) and (4.15). When the size of the reverberant room is chosen in such a way that a large number of room resonances is excited within the analysis bandwidth, one loudspeaker location may suffice for the measurement according to Eq. (4.11).

Note:

A conceptually analogous method for determining the ratio of point admittances of two coupled plates is described by Cremer et al. /4.9/, p.441. For an air cavity with closed boundaries $\overline{Q_c}$ might be determined by coupling the cavity to a reverberant thin plate.
4.3 Validity tests in a scale model

Measuring arrangement:

The accuracy of Method A according to the variant of Eq. (4.1) has been tested in the scale model section which has been described already in Chapter 3. A schematic presentation of the test arrangement is given in Fig. 4.4. A shallow cavity has been modelled between tank top and a chipboard panel which was supported by soft rubber mountings. The dimensions of the model cavity are as follows: length 0.86 m, width 0.26 m, and height 0.01 m and 0.02 m respectively. With the height of 0.02 m this cavity is an accurate model at 1:8 scale of the cavity below the shipboard propulsion diesel engine of the case study in Chapter 6.

For the direct and reciprocal transfer function measurements a loudspeaker was mounted in the middle of the accommodation deck. For the direct experiment the sound in the cavity was generated via holes in the tank top. Two driving units of horn loudspeakers had been connected to the tank top with short flexible tubes. Experiments have been performed with 2 and 4 excitation openings in the tank top. The reason for preferring this type of cavity excitation in these scale model experiments was that earlier measurements with a sound radiating "raft bottom" (stiffened plate)
presented severe experimental problems, such as flanking sound transfer and insufficient signal to noise ratio at higher frequencies.

The frequency range of the measurements covered the 1/3-octave bands with centre frequencies 0.4-10 kHz, corresponding with 50-1250 Hz on full scale.

Both for the direct and the reciprocal measurements a periodical chirp signal has been used with a repetition frequency of 40 Hz. Similarly to the experiments in Chapter 3 this type of signal was chosen to simulate the spectrum of a four stroke medium speed diesel engine running at 600 rev./min. An additional advantage of a periodical test signal, which has been exploited here and also in Chapter 3, is the suppression of the destructive influence of structural reverberation on ordinary coherence functions which are determined by using finite discrete Fourier Transforms. Such coherence functions have been used in the experiments to suppress the influence of background and electronic instrumentation noise. It is shown by Verhulst and Verheij /4.10/ that the strong multiple delays in highly reverberant systems cause a bias error in ordinary coherence function estimates, but that this is avoided when appropriate periodical test signals are used.

The sound pressures in the cavity have been averaged over 5 microphone locations both in the direct and the reciprocal experiment.

**Measurement of \( T_{60} \):**

Reverberation times of the cavity have been determined for octave bandwidths. For these measurements the cavity was excited with a stationary white noise signal which was interrupted suddenly. For each microphone location at which \( T_{60} \) was determined, 25 decay signals have been stored in a digital computer system. With Fourier Transform techniques the complex analytic signal was calculated. Its modulus, the so-called envelope, represents the momentary amplitude of the decay signal; see e.g. Meyer and Guicking /4.11/. This envelope has been averaged over the 25 decay signals and after logarithmic transformation \( T_{60} \) has been derived from the early decay time curve over some 10 or 20 dB, using linear regression. The reverberation time measurements have been performed in-situ, i.e. in the cavity on the ship section, as well as in a similar cavity between the
Figure 4.5:
Measuring results for $10 \log Q_c$ in 1/3-octave bands. a, b and c: Averaged over 5 cavity positions. d: Averages over 5 and over 13 cavity positions.
In Figs. 4.5a and 4.5b also asymptotic values for a large cavity are shown.

chipboard panel and the laboratory floor for sound radiation into a large semi-anechoic room. The values of $T_{60}$ (averaged over 4 microphone locations) differed for these two environments and for the two different cavity heights $h = 0.01$ m and $h = 0.02$ m by not more than 25 percent (1 dB) from the values which have been used for the calculations.

Measurement of $\overline{Q_c}$:
For determining $Q_c$ according to Eq. (4.11) a scale model reverberant room has been used ($V = 1.13$ m$^3$). The cavity was modelled between a chipboard panel and the concrete laboratory floor, see Fig. 4.3. The same periodic test signal was used as for the transfer measurements, i.e. $\Delta f = 40$ Hz. In addition one measurement has been performed with $\Delta f = 5.5$ Hz. Only one loudspeaker location was used for exciting the reverberant room. The sound pressure has been averaged over 13 microphone locations in the reverberant room and over sets of respectively 5 and 13 locations in the cavity. The set of 5 microphone locations was identical to the set of 5 positions used for the reciprocal transfer measurements on the ship section.
Figure 4.6:
Ratio between directly and reciprocally measured sound transfer, presented according to Eq. (4.16).

(2,1): h = 0.01 m, excitation of cavity through 2 openings in tank top
(4,1): h = 0.01 m, " " " " 4 " " " "
(4,2): h = 0.02 m, " " " " 4 " " " "

Measured results:
The results for \( Q_c \) for \( \Delta f = 40 \) Hz and for averaging over 5 microphone locations in the cavity are shown in Fig. 4.5a for \( h = 0.01 \) m and in Fig. 4.5b for \( h = 0.02 \) m. For the signal analysis a 1/3-octave analyzer was used. It is seen that at low frequencies \( Q_c \ll \lambda/2h \). In Fig. 4.5c results for \( \Delta f = 40 \) Hz and \( \Delta f = 5.5 \) Hz are compared and in Fig. 4.5d for averaging over 5 and 13 microphone locations. For frequencies above 1 kHz there is nearly no influence of increasing the number of frequency components or the number of microphone locations, i.e. assumed substitution source locations. At lower frequencies slight differences are seen.

The accuracy of Method A according to Eq. (4.1) can be seen from Fig. 4.6. A logarithmic value \( \Delta L \) is presented for the ratio of the directly and the reciprocally measured transfer functions, with

\[
\Delta L = 10 \log \left( \frac{e_2'^2}{p_1} : \frac{17.6 \pi^2 V}{\rho c T_{60} Q_c} \cdot \frac{p_1'^2}{\omega^2 i^2} \right) . \tag{4.16}
\]

For Fig. 4.6a the 1/3-octave band values of the measuring quantities \( e_2'^2, \), \( p_1'^2, \), \( p_1, \), \( i_2'^2, \), and \( \omega^2 i^2 \) have been used. For \( \omega \) the centre frequency of the 1/3-octave bands was used. For \( T_{60} \) the octave band values were used for the calculations.
Fig. 4.6b shows the results for octave bands which have been obtained after averaging the 1/3-octave band values of respectively $e_2^2/\Delta f/p_1^2/\Delta f$, $p_1^2/\omega^2 i_2^2$, and $G_c$.

Because Eq. (4.1) is, strictly speaking, only valid for discrete frequencies it has been checked whether the results of Fig. 4.6 were sensitive to changes in the power spectral density functions of respectively $p_1$ and $i_2$ within the analysis bandwidths. To obtain this, different signal processing procedures have been applied. The direct transfer function $e_2^2/p_1^2$ has been determined both from 1/3-octave results for $e_2^2$ and $p_1^2$ and from the frequency average of the power spectral density ratio $G_{e_2^2,e_2^2}/p_1^2,p_1^2$. The reciprocal transfer function $p_1^2/\omega^2 i_2^2$ has been determined in three ways, namely by frequency averaging $G_{p_1^2,p_1^2}/G_{i_2^2,i_2^2}$ (equivalent with $G_{i_2^2,i_2^2}$, flat), $G_{p_1^2,p_1^2}/\omega G_{i_2^2,i_2^2}$ (equivalent with slope $G_{i_2^2,i_2^2}$: $-3$ dB/oct.) and $G_{p_1^2,p_1^2}/\omega^2 G_{i_2^2,i_2^2}$ (equivalent with slope $G_{i_2^2,i_2^2}$: $-6$ dB/oct.) and by dividing these averages by respectively $\omega_c^2, \omega_c$ and 1, where $\omega_c = 2\pi \times$ centre frequency of 1/3-octave bands. The results for these variations differed not more than 1 dB.

The results in Fig. 4.6 show that there are no significant systematic errors in the method, not even at low frequencies where the width of the cavity is of the same order of magnitude as $\lambda/2$. The size of the random errors is at least partially determined by the small number of excitation sources in the "direct" experiment. The standard deviation of $p_1^2$ was much larger than in preliminary measurements where the cavity was excited with a stiffened plate simulating the raft bottom. No further investigations have been performed on statistical aspects of the accuracy. If the objective of the analysis is to determine whether a cavity forms an important flanking transfer path for the resilient mountings, the inaccuracy of a few dB as in Fig. 4.6b is fully acceptable. Consequently the method is used in the case study of Chapter 6 for a similar cavity on board.

It is interesting to discuss the reverberation properties of the cavity and the sound absorption at the open boundaries.
Figure 4.7:
Relative amplification of sound radiation from two loudspeaker tubes by cavity resonances
C: arbitrary constant
p_{j}: sound pressure in cavity
p_{f, f}^{2}: sound pressure at 0.1 m distance from tube openings when radiating into large semi-anechoic room.

In Fig. 4.7 the reverberation characteristic of the cavity is illustrated. The quantity which is presented is 10 lg(p_{j}^{2}/p_{f, f}^{2}) where p_{j}^{2} denotes the space averaged value in the cavity and p_{f, f}^{2} the averaged value at 10 cm distance from the flexible loudspeaker tubes when radiating into a large semi-anechoic room. As is seen the amplification due to cavity resonances rises sharply at approximately 500 Hz.

In Fig. 4.8 the results of the reverberation time measurements are given in two different presentations. One line represents the ratio of the apparent absorbing area A_{(2d)} at the open boundaries and the actual open area A_{o} (=h \times \text{circumference}). It has been calculated as follows (see e.g. Ref. 4.12 and 4.13):

\[ A_{(2d)} = \frac{6\pi V \ln 10}{c T_{60}} \quad (4.17) \]
The "reflectivity" of the open boundaries at low frequencies is seen from 
\[ A^{(2d)} / A_o \ll 1 \]. The other line represents the mean absorption coefficient at 
the open boundaries calculated according to Eyring [4.14], using 
\[ \alpha^{(2d)} = 1 - e^{-A^{(2d)} / A_o} \] (4.18)

4.4 Variants of the methods

Shipboard applications:

There are a number of variants of Methods A and B dependent on the type of 
receiver location and measuring quantity. In Eqs. (4.19)-(4.21) variants of 
Eq. (4.1) are given. Similar variants can be derived for Eq. (4.2). See 
Appendix A for reciprocity relations.

\[ \frac{a^2}{p_1^2} = \frac{17.6 \pi^2 V}{\rho^2 c T_{60} \overline{Q_c}} \cdot \frac{p_1^2}{F_2^2} \quad (4.19) \]

\[ \frac{p_2}{p_1} = \frac{17.6 \pi^2 V}{\rho^2 c T_{60} \overline{Q_c}} \cdot \frac{p_1}{U_2} \quad (4.20) \]

where 
\[ a \] acceleration on deck
\[ p_2 \] sound pressure in cabin or water
\[ F_2 \] point force on deck
\[ U_2 \] volume acceleration of acoustical point source in cabin or in the 
water.

For a cabin where during the reciprocal measurement a diffuse sound field 
is generated with a loudspeaker, Eq. (4.21) can be used instead of

* see "Addition during printing" at the end of this chapter.
Figure 4.9:
Application of Method B according to Eq. (4.22) for determining flanking sound transfer in buildings reciprocally.

Eq. (4.20). This follows from Eq. (4.9) with $Q_{\text{cabin}} = 1$.

$$\frac{p_2^2}{p_1^2} = \frac{V_{c,60,\text{cabin}}}{Q_{c,60,\text{cabin}}} \cdot \frac{p_1^2}{p_2^2}.$$  \hspace{1cm} (4.21)

The application of Eq. (4.19) requires a transfer function measurement with pure point force excitation on the deck. In practice to attain this may be difficult, because this reciprocal transfer function measurement requires a sufficiently strong (thus large) exciter and preferably an excitation location with large point admittance. This makes it difficult to avoid contamination of the excitation by unwanted torques, for instance due to constraining the rotation at the exciter location (see also 3.4, p. 67). Simple reciprocity methods for checking the purity of point force excitation are discussed in Appendix C.

**Application of Method B in building acoustics:**

Another type of application for method B can be found in field investigations on flanking sound transfer. In Fig. 4.9 three adjacent rooms in a building are shown. The potentially annoying sound source is located in room $j = 1$. The airborne sound transfer to rooms $j = 2$ and $j = 3$ occurs via walls indicated with subscript $i$. The sound transfer via wall $i$ to room $j$ can be found using Method B, i.e. Eqs. (4.2), (4.9) and a variant analogous to Eq. (4.20), as follows:
For the "direct" sound transfer to room $j = 2$ via wall $i = 2$, for airborne sound excitation is needed. For all other cases $\sigma$ for mechanical excitation is needed. The radiation efficiencies can be obtained either from laboratory measurements or from theoretical estimates.

### 4.5 Discussion of previous literature

Some workers have endeavoured to develop simple mathematical models on the sound transfer via reverberant cavities, notably Heckl /1.5/ (followed by Verheij /4.4/) on the sound transfer through cavities below engines with open boundaries and Price and Crocker /4.3/ on sound transfer through double panels. It is of interest to compare these mathematical models with the analysis in this thesis.

**Paper by Heckl:**

The objective of Heckl's calculation /1.5/ is to illustrate with a simplified model that the sound transfer through resilient mountings is rather easily bypassed by flanking airborne sound transfer. Part of the calculation is repeated here as far as it serves the discussion in this section and that in 4.6.

The calculation in Ref. 1.5 concerns a flat shallow cavity similar to that considered in the earlier part of this chapter. Instead of the terminology and symbols of Ref. 1.5 those of the above treatment are used.

Eqs. (4.23)-(4.25) below have been quoted from Ref. 1.5:

\[
\frac{p_{i,j}^2}{p_i^2} = \frac{S \cdot \sigma \cdot \lambda^2}{\pi} \cdot \frac{\rho^2 \cdot c \cdot T_{60,j}}{17.6 \cdot \pi^2 \cdot V_j} \cdot \frac{a_{i,j}^2}{p_j^2}.
\] (4.22)

\[
P_o = \rho c \cdot S \cdot \sigma \cdot v_o^2,
\] (4.23)

\[
\frac{p_o^2}{p_1^2} = \frac{4 \rho c \cdot P_o}{A},
\] (4.24)
where

\[ P_o \] airborne sound power radiated from engine (raft) bottom into the reverberant cavity (W)

\[ v_o \] velocity of engine (raft) bottom (ms\(^{-1}\))

\[ \sigma_o \] radiation efficiency of the engine (raft) bottom

\[ A \] absorption area in cavity (m\(^2\))

\[ P_{1a} \] sound power radiated from the cavity into the tank top (W)

\[ Z_{p_\infty} \] mechanical point impedance of homogenous unstiffened infinite plate with the same properties as the tank top panels (kg s\(^{-1}\)). \( Z_{p_\infty} \) is assumed to be equal to the inverse of the real part of the point admittance averaged over the tank top surface.

Substitution of Eqs. (4.23) and (4.24) into Eq. (4.25) leads to

\[
\frac{P_{1a}}{P_1} = \frac{\rho c S_o \lambda^2}{\pi v_o^2 Z_{p_\infty}} \quad \text{(4.26)}
\]

However, for a more accurate analysis the 2-dimensionality of the sound wave field in the cavity is to be taken into account as has been shown in 4.2. Then Eqs. (4.23)-(4.26) are replaced by

\[
P_o = \rho c S_o (2d) \frac{v_o^2}{v_o^2} \quad \text{(4.27)}
\]

\[
\frac{P_{1a}}{P_1} = \frac{\rho c P_o}{A(2d)} \quad \text{(4.28)}
\]

\[
\frac{P_{1a}}{P_1} = \frac{S_o (2d) \lambda^2}{\pi Z_{p_\infty} Q_C} \quad \text{(4.29)}
\]

\[
\frac{P_{1a}}{P_1} = \frac{\rho c^2 S_o (2d) \sigma (2d) \lambda^2}{Z_{p_\infty} A(2d) \Omega_C} \quad \text{(4.30)}
\]
The main differences with the previous results in Eqs. (4.23)-(4.26) are the modified radiation efficiencies and the factor \( Q_c \) in Eqs. (4.29) and (4.30). From observations by Verheij /4.4/ and from theoretical analysis by Schroter and Fahy /4.2/ it is known that \( Q \) can be significantly larger for radiation into a shallow cavity than for radiation into a half space.

Eq. (4.25) is valid for the coupling of a large reverberant space with a reverberant thin plate. It can be derived in various manners. In Ref. 4.15 Heckl has used a reciprocity method. Assumptions are that on the plate \( \bar{R}_{rad} = \frac{1}{\bar{P}} \) and that in the reverberant space \( \bar{R}_{rad} = \bar{R}_{rad}^{(f,f)} \).

Another derivation method uses Statistical Energy Analysis (SEA) /4.16/. Assumptions which are equivalent with those for \( \bar{R}_{p} \) and \( \bar{R}_{rad} \) in the foregoing method are now that the modal densities for the plate and for the reverberant space are equal to the high frequency asymptotic values, i.e.

For the plate:
\[
\frac{\Delta N}{\Delta f} = \frac{S}{3.6 \, c_L d} \tag{4.31}
\]

For the air space:
\[
\frac{\Delta N}{\Delta f} = \frac{\omega^2 \, S}{\pi c^3} \tag{4.32}
\]

where
- \( c_L \) propagation speed of quasi-longitudinal waves in plate (m/s)
- \( d \) plate thickness (m)

The shallow cavity result of Eq. (4.29) can be derived in a similar way as Eq. (4.25). However, now for the reciprocity method analogous to Ref. 4.15 \( R_{rad}^{(f,f)} = \frac{1}{c_L} R_{rad}^{(f,f)} \) can be introduced, thus without assuming that \( Q_c \) is equal to the high frequency asymptotic value \( 2h/\lambda \). As has been discussed in 4.2 the true \( Q_c \) can be estimated experimentally.

For the derivation using SEA the high frequency asymptotic value for the modal density in a shallow cavity can be used, i.e.
\[
\frac{\Delta N}{\Delta f} = \frac{S}{c^2} \tag{4.33}
\]
In that case the result of Eq. (4.29) is found with $Q_c = 2h/\lambda$. A theoretical correction of Eq. (4.33) for the low frequency case is difficult to derive for cavities with open boundaries.

If for the analysis of sound transfer through a shallow cavity Eq. (4.26) or an equivalent procedure is used instead of Eq. (4.30), this might lead to a considerable underestimate of the sound transfer, dependent on the increase of the radiation efficiency of the raft in the cavity, i.e. $\sigma_o^{(2d)}$ compared to $\sigma_o$. For the use of Eq. (4.25) such an underestimate is less serious because the increase of $\sigma_1$ is largely compensated by the increase of $Q_c$. See for further discussion 4.6.2.

**Paper by Verheij:**

In the paper by Verheij /4.4/ an experimental method was derived for determining the sound transfer through a shallow cavity below an engine. For the derivation of the method Eq. (4.25) was used, quoted from Heckl's paper /1.5/. The resulting method corresponds with Method B (see Eq. (4.2) in this chapter. Indeed the radiation efficiency of the tank top was modified in Ref. 4.4 by taking $\sigma_o^{(2d)}$ instead of the value for radiation into a half space. However, the factor $Q_c$ was missing.

For the experimental verification in Ref. 4.4 the same ship section and model cavity had been used as in 4.3. According to the measured results for $Q_c$ in Fig. (4.5) of the present study the erroneous theory in Ref. 4.4 should have produced a significant discrepancy between the results for the reciprocal method and the direct measurements at low frequencies. Unfortunately there was misleadingly a good agreement between measurements and calculations in Ref. 4.4. New investigations revealed that reciprocally measured transfer functions of the type $(a_1^2/F_2^2)$, i.e. for the point force excitation on the accommodation deck, were seriously in error especially at low frequencies for the same reason as discussed in 3.4 (p. 67). In the direct measurements there were also some inaccuracies, due to flanking sound transfer from the structure which was used to excite the engine raft. All these error sources have been eliminated in the experiments which have been reported in 4.3.
Article by Price and Crocker:
The objective of the article by Price and Crocker /4.3/ is to analyse the airborne sound transfer through double panels using SEA. The double panel which is analyzed forms a partition wall between two reverberant rooms. It is composed of two parallel homogenous panels with a narrow cavity in between. Both "resonant" and "non-resonant" sound transfer is considered. The difference with the cavity below an engine in the foregoing discussion is that the wall cavity is closed at its boundaries.

A part of the analysis in Ref. 4.3 for the "resonant" sound transfer is of interest for the present discussion. This concerns the coupling of the narrow air cavity with the adjacent panels. For calculating the ratio of coupling loss factors for the energy flow from panel to cavity and back, in Ref. 4.3 the values of Eqs. (4.31) and (4.33) are used for the modal densities. In terms of the foregoing analysis this type of calculation would produce the asymptotic value $\lambda/2h$ for $\overline{Q}_c$ in Eqs. (4.29) and (4.30). For deriving the coupling loss factor for energy flow from panel to cavity, Eq. (22) in Ref. 4.3 uses for panel radiation resistances twice the value for radiation into a half space, taking into account in this way the mirror effects at the closed cavity boundaries. These boundaries are adjacent to those panel parts which dominate the low frequency sound radiation. However, the typical narrow cavity effects on the radiation efficiency are neglected. In Ref. 4.3, $h = 7,1 \text{ cm}$. With this small dimension it might be expected that $\sigma^{(2d)}$ will be very much greater than $\sigma$ (half space), particularly at low frequencies, say below 500 Hz. It follows from this that an underestimate of the resonant sound transfer would result when using the theory in Ref. 4.3. However, comparison in Ref. 4.3 with measurements does not reveal such a discrepancy. The reason for this is not understood at the time being.

4.6 Discussion of some practical aspects
Some aspects of measuring procedures, simplified calculations and effectiveness of noise reduction measures in a shallow cavity are worthy of further comment.
4.6.1 Measuring procedures

If the reciprocity methods are applied for e.g. diesel engines with a variable running speed it is advisable to perform the reciprocal measurements with a test signal which averages over all relevant operational engine spectra. This means that a white noise signal or a broadband periodic signal with a small spectrum line distance (e.g. \( \Delta f = 1 \) Hz) is to be preferred above a periodic signal which is typical for one particular running speed (e.g. \( \Delta f = 5 \) Hz, corresponding with 600 rev./min). In the latter case there is a risk, especially at lower frequencies, to underestimate the worst case sound transfer.

4.6.2 Estimating the radiation efficiency in a shallow cavity

Application of Method B according to Eq. (4.2) or variants requires a reciprocal transfer function measurement and moreover estimates of \( \sigma^{(2d)} \) and \( Q_c \). Although \( \sigma^{(2d)} \) and \( Q_c \) can be determined from (scale model) measurements, the method would be simplified if, instead, only a (theoretical) estimate of \( \sigma_1 \) for radiation into a half space is needed. For this purpose a hypothesis has been investigated to some extent, which states that

\[
\frac{\sigma^{(2d)}}{Q_c} \approx \frac{\sigma_1}{2} \text{ for } 4h < \lambda ,
\]

where \( \sigma_1 \) denotes the radiation efficiency for coupling to a half space. This hypothesis is based on the supposition that the radiation resistance of a panel changes with approximately the same ratio as the radiation resistance of an acoustical point source located at the panel surface. This latter quantity is twice the free field value when the panel surface is coupled to a half space (see e.g. Ref. 4.8) and \( Q_c \) times the free field value when the panel is coupled to a shallow cavity (see 4.2). The approximate validity range, i.e. \( 4h < \lambda \), is assumed because at higher frequencies the asymptotic value of \( Q_c \) according to Eq. (4.10) becomes smaller than 2. This would imply according to Eq. (4.34) that \( \sigma^{(2d)} \) becomes smaller than \( \sigma_1 \), which is not reasonable to assume, at least not in a frequency averaged sense.
The hypothesis of Eq. (4.34) has been tested both by using some published data and by a single experiment. As is shown below, the results of both tests are in favour of the hypothesis.

Published data: A limited investigation has been performed on basis of available graphical data. A more extensive numerical study with the available theory was not possible within the scope of the present study. Schroter and Fahy /4.2/ present graphical results for radiation efficiencies \( \sigma^{(2d)} \) of some panel modes of a baffled square panel which is simply supported at its boundaries. The radiation takes place into an infinite large shallow cavity between parallel reflecting planes. At the panel surface in such a cavity \( Q_c \) has the asymptotic value \( \lambda/2h \) (see 4.2). The radiation efficiencies \( \sigma \) for the same panel modes when radiating into a half space are presented graphically by Wallace /4.17/. Table 4.1 gives a logarithmic presentation of the ratios \( \sigma^{(2d)}/\sigma \) and \( Q_c/2 = \lambda/4h \). The calculated results are valid for a single frequency which is equal to \( 1/9 \)th of the critical frequency* for the radiating panel. Because of the presentation of the results for \( \sigma^{(2d)} \) in Ref. 4.2, the cavity height and thus the value for \( Q_c \) is not the same for all modes of Table 4.1.

For this limited investigation the agreement with the hypothesis of Eq. (4.34) is quite encouraging. Only for the (3,1) mode is a discrepancy of 3 dB found.

Experiment: An experimental test of the validity of Eq. (4.34) has been performed for a 1:8 scale model tank top plate which had been mounted flush in the floor of a chipboard scale model reverberant room (\( V = 1.13 \text{ m}^3 \)). The sound radiation into the reverberant room has been compared for the cases with and without a shallow cavity above the radiating plate, whereas the velocity levels on the plate were the same.

---

* the critical frequency is the frequency at which the sound wavelength in air equals the wavelength of a free bending wave in the panel
Table 4.1: Values of $10 \log (Q_c/2)$ and of $10 \log (\sigma^{(2d)}/\sigma)$ taken from Refs. 4.2 and 4.17 for a square panel at $f = f_{\text{crit}}/9$.

<table>
<thead>
<tr>
<th></th>
<th>$10 \log (Q_c/2)$ dB</th>
<th>$10 \log (\sigma^{(2d)}/\sigma)$ dB</th>
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<tr>
<td>&quot;monopole modes&quot;</td>
<td></td>
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<tr>
<td>1,1</td>
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For the two situations with and without cavity the sound pressures in the reverberant room have been measured at 10 microphone locations. On the tank top accelerations have been measured with a lightweight accelerometer at 8 randomly chosen locations. For the frequency analysis a third-octave analyzer has been used.

For testing the validity of Eq. (4.34) the ratio $\sigma^{(2d)}/\sigma$ has been derived from the space averaged mean square sound pressures $p_1^2$ and $p_2^2$ in the reverberant room for respectively the situations with and without cavity.
Figure 4.10:
Part of the scale model test arrangement for measuring the radiation efficiency of a "tank top" for coupling respectively to a shallow cavity and a reverberant room

a: cavity underneath wooden panel
b: steel bottom section with 6 vibration exciters
c: wooden panel covers only 70 percent of tank top area.
Part of the measuring arrangement is shown in Fig. 4.10. Only the floor of the reverberant room is shown. In the experiments a chipboard box was put over this floor, serving as a reverberant room. The steel plate which simulates the tank top has the same dimensions as the tank top part between the engine seatings of the scale model in 4.3. It extends over a ship length of 11 frame spaces (see also Fig. 3.3). Underneath it is stiffened by two plates in fore-and-aft direction and by 12 plates in athwartship direction. These stiffeners can be seen in Fig. 4.10b. The mechanical excitation of the model bottom structure occurred on the stiffeners in athwartship direction with the aid of six small electro-dynamic exciters; see Fig. 4.10b. The driving signal was a periodic broadband chirp with repetition frequency 40 Hz. To introduce energy dissipation which is representative of a ship structure, the bottom section was mounted on a rubber layer; see Fig. 4.10a. The cavity was formed below a wooden panel whose area was 70 percent of the tank top area; see Figs. 4.10a and 4.10c. The cavity had exactly the same dimensions as that in the experiments of 4.3 for \( h = 0.02 \text{ m} \).

Because the cavity area is only 70 percent of the radiating plate area

\[
\frac{P_{\text{rad},1}}{P_{\text{rad},2}} = \frac{p_1^2}{p_2^2} = \frac{0.7 \sigma^{(2d)}}{\sigma} + 0.3 \sigma \quad (4.35a; \quad 4.35b)
\]

From (4.35b) it follows that

\[
\frac{\sigma^{(2d)}}{\sigma} = \frac{(p_1^2/p_2^2) - 0.3}{0.7} \quad (4.36)
\]

This ratio has been determined for octave bands by calculating the octave band sound pressure levels for the case of a flat 1/3-octave level spectrum for the tank top velocity. In Fig. 4.11 the ratio \( \sigma^{(2d)}/\sigma \) is compared with \( \bar{Q}_c/2 \) using

\[
\Delta L = 10 \lg \frac{\sigma^{(2d)}}{\sigma \bar{Q}_c} \quad (4.37)
\]

For \( \bar{Q}_c \) the octave band averages have been used for the measured results in Fig. 4.5d, i.e. for 13 positions in the cavity (\( h = 0.02 \text{ m} \)).
Figure 4.11:
Test results according to Eq. (4.37) for the hypothesis that the radiation efficiency of the tank top changes due to coupling with a shallow cavity with approximately the same ratio as $R_{\text{rad}}$ of an acoustical point source at the tank top surface; see also Eq. (4.34).

The maximum value of $|\Delta L|$ in Fig. 4.11 is 1 dB for $4h < \lambda$. Therefore this experimental result supports the hypothesis of Eq. (4.34) also. For $f_{\text{model}} = 8$ kHz, i.e. $\lambda = 2h$, the result in Fig. 4.11 shows that $\sigma^{(2d)}/Q_c$ differs only 1 dB from $\sigma$. It therefore seems probable that a simplification of Method B which replaces the measurements of $\sigma^{(2d)}$ and $Q_c$ by measuring or calculating $\sigma$ for radiation into a half space, keeps the method sufficiently accurate for the comparison with contributions of other transfer paths.

4.6.3 Choice of Method A or B

Both measurement methods A and B require the same source strength measurement, i.e. of $p_{1}^2$. The differences between the methods concern the reciprocal measurements, i.e. $p_{1}^2$ versus $a_{1}^2$ and the measurement of cavity properties, i.e. $Q_c$ and $T_{60}$ versus $\sigma_{1}^{(2d)}/Q_c$.

Using the hypothesis of Eq. (4.34) the ratio $\sigma_{1}^{(2d)}/Q_c$ can be estimated from $\sigma_{1}/2$ for radiation into a half space. For many structures in practice $\sigma_{1}$ can be estimated reliably from literature on sound radiation by weakly damped stiffened thin plates. Therefore, if it is practicable to measure $a_{1}^2$, Method B might require less measuring effort than Method A. However, the measurement of $a_{1}^2$ inside the cavity is usually hampered by its inaccessibility. Sometimes this problem can be circumvented by measuring $a_{1}^2$ on the bottom side of the tank top panels as in the case study of Chapter 6. In some other cases $a_{1}^2$ may be estimated reliably from accelerations on the bottom plates adjacent to the engine, if these have the same dimensions as the tank top plates underneath the engine (see Chapter 7).

If no reliable or easy estimate of $a_{1}^2$ or $\sigma_{1}$ is possible Method A should be used.
4.6.4 Consequences of the foregoing analysis with respect to the effectiveness of noise reduction measures

In cases where a shallow cavity below the engine cavity forms a predominant flanking path, noise reduction measures might be considered. Due to operational conditions the possibilities are rather limited. Filling of the cavity with sound absorbing material, for example, is impracticable because in a short time this material would be saturated with oil. A few other possibilities are discussed briefly, showing the consequences of the foregoing analysis. The measures which are considered are respectively reduction of sound radiation from the engine (raft) bottom, increase of the cavity height and application of a damping layer or a sound insulating structure on the tank top.

Reducing the sound radiation by the engine (raft) bottom:

This reduction can be achieved by application of a damping layer (as suggested by Heckl /1.5/ or by a sound insulating structure underneath the engine (raft) bottom, (see Chapter 6). However, it is questionable whether such measures can be effective. Eq. (4.11) shows that the squared sound pressure in the cavity is already $Q_c$ times larger than that in the engine room even in complete absence of sound radiation by the engine (raft) bottom. In 6.4.2 some shipboard measurements are reported. Indeed, these indicate that due to this flanking sound transfer into a cavity below the engine, reduction of sound radiation of an engine raft was ineffective at $f < 500$ Hz.

Increasing the cavity height:

The influence of increasing $h$ can be studied using the results of Eqs. (4.30), (4.34) and (4.11). The direct sound transfer from the engine (raft) bottom to tank top is described by Eq. (4.30). According to the hypothesis of Eq. (4.34) the ratio $\sigma_{(2d)} / Q_c$ is nearly unaffected. The value of $Q_c$ is approximately inversely proportional to $h$ (compare Fig. 4.5a with Fig. 4.5b). Therefore this will also hold for $\sigma_{(2d)}$. Moreover, the sound absorbing area $A_{(2d)}$ and thus $p_{(2d)}^2$ is proportional with $h$. Hence it may be expected that doubling the cavity height decreases the direct sound transfer with some 5 or 6 dB.
The flanking sound transfer from the engine room into the cavity is proportional with $Q_c$ according to Eq. (4.11) also. Therefore doubling of $h$ will decrease the flanking sound transfer with some 2 or 3 dB (see Fig. 4.5a and 4.5b).

Consequently, dependent on the ratio between direct and flanking sound transfer, the sound transfer reduction due to doubling the cavity height will vary from 2-6 dB. However, in practice the possibilities for deliberately designing such an increased height are rather limited, especially in case of propulsion engines.

Measures on the tank top:
The effectiveness of noise reduction measures on the tank top can be evaluated using Method B according to Eq. (4.2) or variants.

The top layer of an insulating structure can be designed to have lower values for $\sigma_{r_{2d}}$ than the bare tank top and lower acceleration levels $a^{22}_{1}$ in the reciprocity experiment. These properties lead to a reduction of sound transfer via tank top. However, if the height of this insulating structure is not small compared to the cavity height, this reduction is partially cancelled by increase of the sound pressure levels in the cavity which are caused both by the direct and flanking sound transfer (see foregoing discussion). A practical disadvantage is that protection measures against oil penetration are to be carried out very carefully, otherwise the isolation is lost by short circuits.

A better solution is the application of constrained layer damping. This has the advantage of a small layer thickness. Moreover, protection of such a layer against oil penetration is rather simply realized without spoiling the damping properties. The effectiveness of the measure can be evaluated again with Method B. If the layer reduces $a^{12}_{1}$ considerably without increasing $\sigma_{r_{2d}}$, the sound transfer is effectively reduced by this type of measure. Full scale and model experiments are reported in 6.4. These confirm the effectiveness of this measure in practice.
4.7 Summary

1. Two experimental procedures, referred to as Method A and B have been developed in 4.2 for quantifying the sound transfer through flat shallow reverberant air cavities below shipboard engines. These methods, which use reciprocal transfer function measurements, are convenient in cases when "direct" measurements with substitution sources in the cavity are impossible due to lack of space. For Method A sound pressures in the cavity are to be measured in the reciprocal experiment, whereas for Method B accelerations of the cavity bottom are needed.

2. Differences in sound transfer properties of cavities with a 2-dimensional and a 3-dimensional sound field have been discussed extensively. Several aspects are elucidated by the introduction of a factor \( Q_c \). This factor represents the space averaged ratio of the radiation resistance of an acoustical point source in a 2-dimensional sound field and in the free field respectively. Theoretical and experimental methods for estimating \( Q_c \) have been described in 4.2. The relation to and differences with previous literature on sound transfer through narrow cavities have been discussed in 4.5.

3. Scale model experiments in 4.3 indicate that the proposed measuring methods are accurate within a few dB provided the length and the width of the cavity are larger than half the sound wavelength in air and provided frequency averaging is applied over octave bandwidths.

4. Variants of the methods are discussed in 4.4. Moreover, it is demonstrated there that one of the methods can be applied in building acoustics for field investigations on flanking sound transfer.

5. Practical guidance for measurements and for simplified calculations are given in 4.6.1 - 4.6.3. In 4.6.2 a very simple method is suggested for estimating the radiation efficiency of a plate coupled to a flat shallow cavity from the value for radiation into a half space. In many cases it will not be necessary to measure radiation efficiencies of the tank top plate for radiation into a shallow cavity, when Method B is used.
6. Noise reduction measures in the cavity should be applied preferably on the "receiver side" (e.g. tank top). Measures on the "source side", e.g. damping or insulating the engine (raft) bottom, are probably ineffective, especially at low frequencies. The reason for this follows from the theory in this chapter which states in Eq. (4.11) that the squared sound pressure in the cavity is already $\bar{Q}_c$ times larger than that in the engine room, even in complete absence of sound radiation by the engine (raft) bottom.

**Addition during printing**

*Application of Methods A and B for air path in engine room:*
The reciprocal methods for path assessment may also be applied on the airborne sound transfer through the engine room. For that application in all equations derived for the cavity, $\bar{Q}_c$ has to replaced by 1. Because direct experiments with loudspeakers can be performed, this application of reciprocity seems not obvious. However, in many practical shipboard situations direct experiments are hampered because of strong background noise at distant receiver locations. With the above reciprocal methods these practical difficulties may be circumvented.
5. STRUCTUREBORNE SOUND TRANSFER ALONG PIPES

New experimental methods are investigated for quantifying the sound transfer from resiliently mounted machinery via pipe systems. Direct determination of the sound transfer is attempted by measuring the structureborne energy flow. Indirect determination of pipe path contribution is attempted with 2 substitution source methods, one on basis of energy flow measurements, the other on basis of radial pipe wall accelerations. Laboratory experiments show the feasibility of the methods.

5.1 Introduction

For resiliently mounted machinery potentially important flanking sound transfer paths are formed by pipe systems, e.g. cooling water and lubrication oil pipes. It is usual to insert in these pipes flexible bellows or hoses which should reduce the structureborne sound transfer according to the same principle as for mountings underneath the machinery. Therefore, it would seem obvious to assess the structureborne sound transfer via flexible pipe isolators according to the same experimental method as in Chapter 2 for the support mountings. However, for many existing shipboard systems this method is expected to be inaccurate. The probable reason for a failure of this method is that for poorly isolated pipes the impedance mismatch between the flexible isolator and the pipe on the ship side of the isolator, is insufficiently large. Then the flexible isolator cannot be characterized with "blocked" transfer functions. Situations where this probably occurs, are recognized in many shipboard systems just by visual inspection. Experimental indications of such a situation are the occurrence of small or even negative differences between structureborne sound levels on the machine side and the ship side of the isolator.

Other practical methods for quantitative path assessment are not available. Looking for possibilities to develop such a method, it should be considered whether a mathematical or an experimental approach is most promising. In the opinion of the author an experimental method has to be developed. The reason is that even if a mathematical method should exist or could be developed to describe the complex sound transfer mechanisms in liquid filled pipe systems of representative complexity, it would be impracticable to determine the excitation functions.
In the present chapter experimental methods for pipe path assessment are investigated. The pipe systems which are considered have circular cylindrical symmetry for the straight sections, but they may contain elbow sections and fixing clamps at arbitrary positions. The pipe wall material is homogeneous and isotropic.

In section 5.2 the principles of 3 path quantifying methods are discussed.

Section 5.3 presents some discussion of theoretical aspects of propagation and measurement of the structureborne energy flow through a pipe wall.

Section 5.4 describes laboratory experiments in which the path quantifying methods were tested.

In section 5.5 the experiments are evaluated.

5.2 Experimental methods for quantifying the structureborne sound transfer along pipes

In the following study direct determination of the sound transfer along a pipe is attempted by measuring structureborne energy flow. Such a path strength measure can be used for ranking the contributions of different pipes. Moreover, it can be used for comparison with the sound transfer through mountings and via the air. For the latter purpose the energy flow via the mountings has to be estimated with Eq. (2.3) and that via the air paths with Eqs. (4.25) and (4.29).

Note:

An alternative of using the procedure of Eq. (2.3) for the mounting path might be the in-situ measurement of energy flow through the mountings. This might be done with a multi-directional variant of a cross-spectral density method which has been proposed recently by Pinnington and White /5.1/. For this method the complex multi-directional mounting transfer functions are needed as defined in Chapter 2 of this thesis. However, test results in Ref. 5.1 for the uni-directional case indicate that further study is needed with respect to sensitivity for errors.

Indirect determination of the sound transfer along a pipe is attempted with 2 substitution source methods. For one method the sound transfer is
characterized, as above, by the structureborne energy flow through a
certain cross section, whereas for the other method radial accelerations
are averaged over a certain pipe length. To determine the path strength
for a certain pipe, these quantities are to be measured both when the
machine is running and when the machine has been stopped. In the latter
case the pipe is excited by substitution sources at locations on the
ship side of the flexible isolator. If necessary, the transfer functions
for these substitution sources to a distant receiver location can be
measured reciprocally, in the same way as in foregoing chapters.

5.2.1 Energy flow measurements at low frequencies
The transport of sound energy from shipboard machinery to the adjacent
ship structure via liquid filled pipes, is a very complex physical
phenomenon. For determining the structureborne energy flow only measure­
ments of kinematical quantities are practicable, i.e. of accelerations,
velocities, displacements and/or strains. At each position the vibrations
on a thin-walled pipe can be separated into radial, tangential and axial
motions. However, before such vibration quantities can be exploited for
energy flow measurements, a thorough understanding is needed of the
various wave types which exist in the pipes. Only on the basis of such
an understanding, can methods be formulated for measuring the sound energy
in those wave types which contribute to the energy transport.

For the general case of an analysis over a wide frequency range the above
suggested approach seems practically impossible due to the complexity of
pipe wall vibrations. However, at relatively low frequencies, say below
1 kHz, for many practical shipboard pipe systems the situation might be
more favourable for energy flow measurements. It will be shown later, that
below a certain frequency the structureborne energy transport is dominated
by 3 propagating wave types. Two of these wave types are axially symmetric,
i.e. one which corresponds to a quasi-longitudinal wave in a thin plate
and another which is equivalent to a torsional wave in a rod. The 3rd
wave type is equivalent to the bending wave in a beam; see section 5.3.

For the measurement of the energy flow in the above 3 wave types, relatively
simple methods are described in Appendix D. In principle the total struc­
tureborne energy flow can be estimated from
\[ P = P_L + P_T + P_{B1} + P_{B2}, \]  
(5.1)

where the subscripts L, T and B denote respectively the 3 wave types, with for bending waves 2 terms, because of 2 allowed polarizations.

5.2.2 Substitution source methods

Besides the need of comparing the relative importance of different pipes and of comparing pipe paths with mounting- and airborne paths, also of special interest there is the relative contribution of one particular pipe to the total sound transfer. For this purpose 2 substitution sources methods are investigated in this chapter. They show some analogy with the principles used in 4.2.

The general principles are illustrated in Fig. 5.1, where sound transfer to a receiver location on a deck is considered. In the following discussion time averaged measuring quantities are used for a filter bandwidth \( \Delta f \). As a response quantity on the deck, the acceleration \( a_{d1, \Delta f} \) is chosen. This quantity can be measured when the machinery is in operation. The problem is how to determine the fraction \( a_{d1, \Delta f} \) of this deck response, which originates from the sound transfer along a particular pipe. This problem is solved by using substitution sources.

The first method, which will be called Method A, uses energy flow measurements. For the case in Fig. 5.1a, i.e. when the machinery is in operation \( P_{1, \Delta f} \) is determined according to Eq. (5.1). For the case in Fig. 5.1b, when the machinery has been stopped, the pipe is excited with a substitution source on the ship side of a pipe isolator. For this substitution source excitation let the energy flow measured according to Eq. (5.1) be equal to \( P_{2, \Delta f} \), and the deck response equal to \( a_{2, \Delta f} \). Then, under the reasonable assumption that in case of substitution source excitation the pipe path is predominant, \( a_{d1, \Delta f} \) is found from:

\[
\text{Method A: } a_{d1, \Delta f} = \frac{a_{d2, \Delta f}}{P_{2, \Delta f}} \cdot P_{1, \Delta f} \tag{5.2}
\]

The hypothetical assumption behind Method A is that, averaged over e.g. 1/3-octave or octave bands, the fraction of the energy flow in the pipe wall that is injected into the ship, is rather independent of the type of
Figure 5.1:
Substitution source methods A and B for the assessment of sound transfer along pipes; see Eqs. (5.2), (5.3), (5.5) and (5.6).

excitation (i.e. by machine or substitution source) and thus of the way in which the energy flow is distributed over the wave-types. Expectedly most of the energy flow is injected into the ship structure and not dissipated in the pipe system.

The second method, which will be called Method B, is based on the same hypothetical assumption as Method A, but now instead of energy flow, the radial accelerations \( a_{p}^{2} \) averaged over a certain pipe length are used as path strength descriptor. In analogy with Eq. (5.2), \( a_{d1}^{2} \) can now be found from:

\[
\text{Method B: } a_{d1}^{2} = \frac{a_{d2}^{2}}{a_{p2}^{2}} a_{p1}^{2} \Delta f
\]

The reason for investigating 2 different methods is that they are, at least partially, complementary.

At low frequencies the radial accelerations of the pipe wall are probably dominated by, in the frequency domain, relatively widely spaced "beam-bending" resonances. The radial accelerations may show a large scatter and it is uncertain whether they are then proportional with the total energy flow through the pipe. For example, energy flow in non-resonant...
longitudinal waves might be as important as in bending waves, but might not be properly reflected in the radial pipe vibrations. In those cases Method A might be appropriate.

Above a certain cut-off frequency pipe wall vibration modes of circumferential order \( n = 2 \) are initiated; see 5.3. Then the simplifying assumptions for measuring the energy flow according to Eq. (5.1) are no longer valid. However, above this cut-off frequency the density of resonant vibration modes increases and because of wave type conversion at discontinuities (bends, supports) it is expected that averaged over 1/3-octave or octave bands the energy distribution over the various shell vibration modes becomes rather independent from the type of excitation. The total energy flow is then expectedly proportional with the radial accelerations averaged over a certain pipe length. These accelerations will show a rather small scatter. For those cases Method B might be applicable.

In the foregoing discussion the exciter on the ship side of the flexible pipe isolator is called the substitution source. A slightly different way of looking at the methods proposed is to consider the artificially excited pipe itself as the substitution source and thus the pipe excited by the machinery as the original source.

A more detailed discussion on the use of substitution sources is presented in Chapter 7. Here it suffices to mention that in general such methods become less accurate when the number of resonant vibration modes in the transfer system between source and receiver location becomes small. In that case a precise modelling of the machinery excitation by the substitution source(s) becomes increasingly important. A practical way to reduce statistical scatter in calculations according to Eqs. (5.2) and (5.3), might be averaging over locations or excitation directions of the substitution exciters as well as over several representative receiver locations.

A practical problem may arise when the substitution source is of insufficient strength to exceed the background noise at a distant receiver location in the ship accommodation or in the water. In many cases this problem can be solved by an additional reciprocal measurement of a transfer function from receiver location to substitution exciter location. This is illustrated
in Figs. 5.1b and 5.1c for the case that the pipe is excited with a force source of strength $F^2_{0,\Delta f}$. The transfer function to the deck location can be measured reciprocally according to

$$\begin{bmatrix} a^2_{d2,\Delta f} \\ F^2_{0,\Delta f} \end{bmatrix} = \begin{bmatrix} a^2_{0,\Delta f} \\ F^2_{d,\Delta f} \end{bmatrix}.$$  (5.4)

Now the unknown terms in Eqs. (5.2) and (5.3) can be found from

$$\begin{bmatrix} a^2_{d2,\Delta f} \\ P^2_{2,\Delta f} \end{bmatrix} = \begin{bmatrix} a^2_{0,\Delta f} \\ P^2_{d,\Delta f} \end{bmatrix} : \begin{bmatrix} F^2_{2,\Delta f} \\ F^2_{0,\Delta f} \end{bmatrix}.$$  (5.5)

and

$$\begin{bmatrix} a^2_{d2,\Delta f} \\ a^2_{p2,\Delta f} \end{bmatrix} = \begin{bmatrix} a^2_{0,\Delta f} \\ P^2_{d,\Delta f} \end{bmatrix} : \begin{bmatrix} a^2_{b2,\Delta f} \\ F^2_{0,\Delta f} \end{bmatrix}.$$  (5.6)

Variants of these methods with other types of excitation and receiver quantities can be derived in the same way as in 4.4.

5.3 Theoretical considerations with respect to the measurement of structure-borne energy flow

The objective of the following discussion is to provide qualitative insight in the sound wave mechanism in pipes and in the possibilities of energy flow measurements. Moreover, measurement techniques and measurement error sources are discussed.

Sound propagation along circular cylindrical pipes has been the subject of a large amount of published literature. The present discussion has no pretension of adding something new to the theoretical understanding of the wave mechanisms. However, no previous authors have considered the application of energy flow measurements on pipes for path assessment according to Eq. (5.1) and according to the substitution source method of Eq. (5.2) on basis of Eq. (5.1). Therefore, it will be necessary to determine for which frequency range Eq. (5.1) is valid, and what type of measurement complications may occur within this frequency range. Moreover, basic mechanisms of wave type conversion in practical systems will be discussed, showing that no wave types may be neglected a priori.
5.3.1 Some fundamentals of pipe wall vibrations

Semi-infinite straight pipes in vacuo:

An extensive summary of the literature on sound wave propagation along thin-walled pipes in vacuo has been presented by Leissa /5.2/.

The shell geometry and the coordinate system which are used in the following discussion, are shown in Fig. 5.2. At each location the pipe wall vibrations in a cylindrical coordinate system may be separated into the orthogonal displacements \( u, v \) and \( w \) for respectively axial, circumferential and radial direction.

In the present discussion a mode is defined as an allowed displacement variation with respect to length and circumference, which can exist at a certain frequency in an infinitely long shell. A mode itself is independent of all other modes. In general, it will contain all three displacement components \( u, v \) and \( w \).
To find the modes which can exist on a pipe the equations of motion are to be derived and to be solved for the displacements u, v and w. The solutions which are found depend on the approximations that are used for deriving the equations of motions. Leissa /5.2/ summarizes amongst others the results of so-called zero-order approximations, which neglect shear deformation and rotational inertia in the pipe wall.

The characteristic equation is found by substituting travelling wave solutions with harmonic time dependence in the equations of motion and by expanding the determinant of the coefficients in the remaining algebraic equations. The wave solutions are divided in different classes according to the number of nodal lines of the modes around the circumference. Axially symmetric modes are of the order \( n = 0 \). For modes of the order \( n = 1 \), each of the displacements \( u, v \) and \( w \) has 2 nodal lines on opposite sides of the pipe. Not all nodal lines for \( u, v \) and \( w \) coincide (see e.g. Frymoyer /5.3/, Fig. 2.12.2). For \( n = 2 \) there are 4 nodal lines; see Fig. 5.3 for displacement pattern of the cross section.

Zero-order theories provide for any particular value of \( n \) and for each propagation direction, 4 solutions of the characteristic equation, i.e. 4 axial wave numbers at any particular frequency. If these solutions are substituted in the equations of motion, amplitude ratios \( u/w, v/w \) and \( w/v \) are found.

At each frequency, real, imaginary and complex wave numbers are found. Real wave numbers correspond with vibration modes which transport energy along the pipe. Imaginary wave numbers correspond with nearfields which decay exponentially. According to Frymoyer /5.3/ and Fuller /5.4/, complex axial wave numbers occur in such combinations that they correspond with standing waves which decay also exponentially. Because the modes with real wave numbers are important for the energy transport, these are to be separated for energy flow measurements. However, as will be discussed later, the occurrence of the other types might hamper good separation.

*higher order equation in wave number and frequency.
**Modes with real wave numbers:**

**n = 0:** There are 2 types of axially symmetric modes that commence at zero frequency. This means that they have real wave numbers at all frequencies.

One mode is a torsional mode which corresponds with a torsional wave in a circular cylindrical rod. The circumferential motion is uncoupled from the axial motion and from the radial motion that are both equal to zero. The propagation speed (phase-speed) of this torsional wave is independent of frequency and of geometry. It is given by

\[ c_T = \sqrt{\frac{G}{\rho_p}} \], \hspace{1cm} (5.7)

where \( G \) denotes the shear modulus and \( \rho_p \) the density of the pipe wall material.

For the other axially symmetric mode the circumferential motion is zero, but the axial motion is coupled to the radial motion by the Poisson effect. At frequencies well below the so-called ring frequency \(|\tilde{u}/\tilde{G}| \gg 1\); (see e.g. Fuller /5.4/, Fig. 3). Then the mode corresponds with a quasi-longitudinal wave in a thin plate. The low frequency propagation speed is given by

\[ c_L = \sqrt{\frac{E}{\rho_p (1 - \nu^2)}} \], \hspace{1cm} (5.8)

where \( \nu \) denotes the Poisson constant.

The ring frequency is defined as the frequency at which the wavelength of a quasi-longitudinal plate wave equals the pipe circumference. It can be found from

\[ f_{\text{ring}} = \frac{1}{2\pi R} \sqrt{\frac{E}{\rho_p (1 - \nu^2)}} \]. \hspace{1cm} (5.9)

Figure 5.4 presents the ring frequency for steel pipes with

\( 0.01 \text{ m} \leq R \leq 0.1 \text{ m} \), i.e. for pipe diameters representative of most shipboard pipes. For these cases the ring frequency exceeds 8 kHz.

In many publications on pipe wall vibrations the frequency is normalized upon this ring frequency and denoted by \( \tilde{f} \). Therefore this parameter will also be used sometimes in the following discussion.
Figure 5.4:
Ring frequencies according to Eq. (5.9) for steel pipes.

Figure 5.5:
Normalized lower cut-off frequency of $n = 2$ circumferential mode for steel pipes, according to Eq. (5.13).
There is 1 first order mode which starts at zero frequency. All orthogonal displacements are coupled. At frequencies far below the ring frequency this mode corresponds with a bending wave in a slender beam. For \( f << f_{\text{ring}} \) the amplitude ratios are given by

\[
\left| \frac{\hat{v}}{\hat{w}} \right| = 1; \quad \left| \frac{\hat{u}}{\hat{w}} \right| = R \left( \frac{3 \hat{\omega}}{2 \hat{\kappa}} \right) = k_B R. \tag{5.10a; 5.10b}
\]

The bending wave number \( k_B \) can be found from

\[
k_B = \sqrt{\frac{4 m' \omega^2}{B}}, \tag{5.11}
\]

where \( m' \) denotes mass per unit length and \( B \) the bending stiffness. For thin-walled pipes with \( d/R \leq 0.1 \), \( k_B \) may be approximated within 0.5 percent by

\[
k_B = \sqrt{\frac{2 \rho \omega^2}{\pi d^2}} \tag{5.12}
\]

which shows that \( k_B \) is largely independent of the wall thickness.

Calculated results have been presented by Leissa /5.2/, Fig. 2.4 and by Fuller /5.4/, Fig. 3 for \( d/R = 0.05 \). This ratio is of the order of magnitude representative of many shipboard pipe systems. The calculated results indicate that the equivalence of the first order mode with a bending wave on a beam is valid up to \( \Omega = 0.2 \) for representative pipes, i.e. up to frequencies when \( \lambda_B \sim 6D \). This corresponds with the frequency range in which rotational inertia and shear can be neglected when considering bending wave propagation in a slender beam; see e.g. Cremer et al. /4.9/. In Table 5.1 it is shown that for representative pipes the lowest cut-off frequency of \( n = 2 \) modes is well below \( \Omega = 0.2 \). Because energy flow measurements are only considered for frequencies below the cut-off of \( n = 2 \) modes, it may be concluded that the low frequency approximation of the first order mode by a beam bending mode is correct for such measurements.

Another first order mode starts at \( \Omega > 1 \) and is not of interest for the present discussion.

\[ n = 2: \] "Zero-order" theories predict 2 second order modes that commence at non-zero frequency. Expressions for the cut-off frequencies are presented
by Leissa /5.2/, Table 2.1. He presents also numerical results, for d/R = 0,05 and d/R = 0,002; see /5.2/, Table 2.2. As has been said, the ratio d/R = 0,05 is of the order of magnitude representative of many shipboard pipes. For this ratio Flügge and Donnel-Mushtari theories predict values for the lower n = 2 cut-off frequency which are some 35 percent higher than results from many other theories and which are also inconsistent with an observation reported in 5.4. The predictions by Love-Timoshenko theory are both for d/R = 0,05 and for d/R = 0,002 consistent with many other theories and also rather well with the measurement result in 5.4. Therefore it seems reasonable to use this theory for estimating n = 2 cut-off frequencies for shipboard pipes (without liquid!).

The n = 2 cut-off frequencies can be found from

\[ \frac{\Omega^2}{c^2} = \frac{1}{2} \left\{ 5 \left( 1 + 4 K \right) - (25 + 56 K)^{1/2} \right\} \]

(5.13)

where

\[ K = \frac{d^2}{12 R^2} \]

In Fig. 5.5 the lower cut-off frequency for n = 2 modes on steel pipes is presented for 0 < d/R < 0,1. Table 5.1 presents some numerical results. The dimensions R = 0,08 m and d = 0,005 m are close to those of the test pipe in 5.4.

Table 5.1: Lower cut-off frequency of n = 2 circumferential modes (steel:
\( E = 2,1 \times 10^{11} \text{ Nm}\text{ }^{-2}; \rho_p = 7,8 \times 10^3 \text{ kgm}^{-3}; \nu = 0,33 \)).

<table>
<thead>
<tr>
<th>R(m)</th>
<th>d(m)</th>
<th>d/R</th>
<th>( f_{\text{ring}} \text{(Hz)} )</th>
<th>( \Omega_{c2} )</th>
<th>( f_{c2} \text{(Hz)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,08</td>
<td>0,005</td>
<td>0,062</td>
<td>( 1,09 \times 10^6 )</td>
<td>0,048</td>
<td>528</td>
</tr>
<tr>
<td>0,045</td>
<td>0,003</td>
<td>0,067</td>
<td>( 1,93 \times 10^6 )</td>
<td>0,052</td>
<td>997</td>
</tr>
<tr>
<td>0,02</td>
<td>0,0025</td>
<td>0,125</td>
<td>( 4,35 \times 10^6 )</td>
<td>0,097</td>
<td>4212</td>
</tr>
</tbody>
</table>

Wavelengths: In Fig. 5.6 wavelengths in steel pipes are presented for the above discussed wave types. For the frequency range below 1 kHz the bending
wavelengths on representative pipes are much smaller than the longitudinal and torsional wavelengths. For the \( n = 2 \) mode the corresponding axial wavelength drops very fast after initiation of this mode and becomes of the same order of magnitude as bending wavelengths. The result for the \( n = 2 \) mode in Fig. 5.6 \((R = 0.08 \text{ m and } d = 0.005 \text{ m})\), has been obtained from interpolation of graphical data presented by Goudriaan /5.5/ (see also Leissa /5.2/, Figs. 2.4 and 2.5) and has no high accuracy.

**Liquid filled pipes**

For the transfer of machinery noise via pipes, especially pipes filled with water or oil are of interest. A description of the sound transfer along such pipes is more complex than in the above case of pipes in vacuo, because of the interaction between the elastic pipe wall and the liquid. The low frequency sound propagation along semi-infinite straight pipes filled with liquid will be discussed now, along the same lines as above for pipes in-vacuo.

For the extreme case of a pipe with rigid wall the sound propagation is confined within the liquid. At low frequencies, when the wavelength of
sound in the liquid is at least 8 times greater than the diameter of the pipe, the sound propagates in plane waves. The phase speed is frequency independent (water: \( c_f \approx 1500 \text{ ms}^{-1} \)). At higher frequencies the sound pressure becomes dependent on radial and angular coordinates. Above a certain frequency higher order duct modes occur. The lowest cut-off frequency is of a non-axially symmetric mode and is given by (see Morse and Ingard /5.6/):

\[
f_c = \frac{0.586}{D_0} c_f, \tag{5.14}
\]

where \( D_0 \) denotes the inner diameter of the pipe.

For the case of a flexible pipe wall the vibrations in the liquid and in the pipe wall are coupled. The coupled system modes are discussed only insofar they are important for low frequency energy flow measurements on the pipe wall, i.e. for circumferential order \( n \leq 2 \).

\( n = 0 \): In contrast to the "in-vacuo" case there are now 3 axially symmetric modes that start at zero frequency.

The first mode is a torsional mode. The vibrations in the pipe wall are uncoupled from the liquid (viscosity effects are neglected) and therefore this mode is equal to that in the in-vacuo pipe.

The second mode is close to the in-vacuo quasi-longitudinal shell mode. The radial wall motion due to the Poisson effect is responsible for the coupling to the liquid. At \( \Omega < 0.5 \) the phase speed is accurately approximated by Eq. (5.9); see Fuller and Fahy /5.7/, Figs. 3 and 5.

The third mode is close to the plane duct wave. However, because of wall flexibility, the phase velocity is dependent on wall thickness and frequency. White and Sawley /5.8/ give the following expression:

\[
c_f' = c_f \left\{ \frac{\Omega^2 - 1}{\Omega^2 - (1 + \frac{\rho_f c_f^2 D_0}{dE})} \right\}^{1/2}, \tag{5.15}
\]

where \( \rho_f \) denotes the density of the liquid and \( c_f \) the plane wave speed in a
rigid-walled pipe. This results, for example, for a waterfilled pipe with \( R = 0.08 \text{ m} \) and \( d = 0.005 \text{ m} \) in \( c' = 0.87c_f \approx 1300 \text{ ms}^{-1} \). For \( \Omega \ll 1 \), Eq. (5.15) can be approximated by the frequency independent expression:

\[
c_f' = \frac{c_f}{(1 + \frac{\rho_f c_f^2 D}{(d E)})^{1/2}}.
\] (5.16)

Note:

Although the wall flexibility also lowers the cut-off frequencies of higher order duct modes, for representative shipboard pipes they still remain far above the frequency range of interest for energy flow measurements. Within flexible bellows this may be different; see 2.3.

\( n = 1 \): There is 1 first order mode which starts at zero frequency. At \( \Omega \ll 1 \) it corresponds again to the beam-bending wave, but now the phase speed is lowered due to the mass loading effect of the water. For \( d/R \ll 1 \), the mass per unit length may be approximated by

\[
m' = 2\pi R_d p + \pi R^2 \rho_f.
\] (5.17)

This results, for example, for a water filled steel pipe with \( R = 0.08 \text{ m} \) and \( d = 0.005 \text{ m} \) in doubling of \( m' \) compared with the in-vacuo case. According to Eq. (5.11), that results in a reduction of the wavelength with a factor \( \sqrt{2} \approx 1.2 \).

\( n = 2 \): A formula for the lower \( n = 2 \) cut-off frequencies of liquid filled pipes can be obtained from Bentley and Firth /5.9/ as follows:

\[
\Omega_{c2}^2 = \frac{36K}{5 + \frac{\rho_f D}{(dE)}}.
\] (5.18)

For the water-filled steel pipes with dimensions as in Table 5.1 the cut-off frequencies according to Eq. (5.18) are respectively 390 Hz, 758 Hz and 3880 Hz. For the case without liquid, i.e. \( \rho_f \) equals zero, Eq. (5.18) provides results equal to those in Table 5.1.

Energy distribution over pipe wall and liquid: The energy distribution in \( n = 0 \) and \( n = 1 \) circumferential modes has been studied recently by Fuller
and Fahy /5.7/. Although Ref. /5.7/ contains only very limited graphical data, interesting conclusions can be drawn for the present discussion.

In the case of structureborne sound excitation the energy transport occurs at low frequencies by the $n = 0$ torsional and quasi-longitudinal modes and by the $n = 1$ "beam-bending" mode. The results in Fig. 9 of Ref. /5.7/ indicate that in that case at least for $\Omega < 0.2$ and $d/R > 0.05$ the energy flow in the pipe wall is much greater than that in the liquid.

In the case of a sound source in the liquid, the energy is transported in the $n = 0$ duct mode. Now the energy flow in the liquid is by far predominant for $\Omega < 0.2$ and $d/R > 0.05$.

**Note:**
The above indicated limits $\Omega < 0.2$ and $d/R > 0.05$ have been derived from the graphical data in Ref. /5.7/. They form no strict limits, but indicate only an established validity range.

**Finite pipes with bends, flanges etc.:**
The above results are valid for semi-infinite pipes. However, actual pipe systems are comprised of straight sections and bends, generally in a non-planar configuration. They contain also cross-sectional changes like flanges and are clamped to the ship structure at several locations. Some consequences for energy flow distribution and energy flow measurements are now discussed.

**Wave reflections and wave type conversion:** At pipe wall discontinuities wave reflection occurs. Because at low frequencies the beam-bending wavelengths are much smaller than for quasi-longitudinal and torsional waves, beam-type resonances, might be expected to dominate the pipe wall vibrations at low frequencies. However, because of strong coupling between the various wave types, this does not imply that only bending waves should be considered for energy flow measurements. This can be elucidated qualitatively with the simple planar pipe configuration in Fig. 5.7. Two straight sections no. 1 and no. 2 are connected by a $90^\circ$ elbow. Let section no. 1 be excited by a radial point force in arbitrary angular direction. The excitation generates in section no. 1 a beam-bending wave field which may be separated into
Figure 5.7: Planar pipe configuration comprised of 2 straight sections and a 90° elbow.

bending waves in the plane of the pipe assembly, denoted by $B_{11}$, and perpendicular to this plane, denoted by $B_{12}$. These wavefields generate waves in section no. 2.

The bending wave field $B_{11}$ generates in section no. 2 a bending wave field $B_{21}$ and a longitudinal wave field $L_2$. The secondary waves are reflected at the free end and return energy to section no. 1 via $B_{11}$ and a longitudinal wave field $L_1$.

The bending wave field $B_{12}$ generates in section no. 2 a bending wave field $B_{22}$ and a torsional wave field $T_2$. In analogy with the above case these wave-types return energy to section no. 1 via $B_{12}$ and a torsional wave field $T_1$. In a non-planar configuration all wave types are coupled.

Perhaps the coupling mechanism is even more complex than outlined above, but the purpose was to illustrate wave type conversion. Due to this wave type conversion "non-resonant" longitudinal or torsional vibrations on a particular pipe section may become very important for the energy transport due to the coupling with bending-type resonances in other pipe sections or in the adjacent ship structure. Therefore, a priori neglect of longitudinal and torsional waves, might lead to serious errors in energy flow estimates.

Coupling of "shell waves" and "duct wave": For straight semi-infinite pipes it was seen above, that at low frequencies the energy flow is predominantly either in the pipe wall or in the liquid, dependent on the type of excitation.
For pipe assemblies with elbows there occurs in these elbows a strong coupling between the plane wave in the liquid and the beam-bending vibrations. This has been shown theoretically and experimentally by Davidson et al. /5.10, 5.11/; see also El-Raheb /5.12/ and Firth /5.13/. Because of the above discussed wave conversion mechanisms there will be also an important indirect coupling of the duct wave with other pipe wall vibrations. After commencement of the \( n = 2 \) circumferential shell-modes, also the coupling between this wave type and the plane duct wave is strong; see Firth /5.13/.

Note:
This coupling to \( n = 2 \) shell vibrations is neglected in the theoretical analysis by El-Raheb /5.12/, who presents a numerical example with only beam-type vibrations up to 4 kHz, whereas for the pipe under investigation the \( n = 2 \) cut-off frequency is at approximately 400 Hz.

On basis of the above discussed coupling mechanisms it might be expected that, generally speaking, the distribution of energy flow over pipe wall and liquid is not heavily dependent on the type of excitation. At low frequencies this would imply that energy flow measurements taken from the pipe wall provide a reliable measure for the total sound transfer, giving a fraction of the total energy flow which is rather independent of the type of excitation.

Pipe wall motions near discontinuities: At low frequencies the energy flow through the pipe wall is governed by a small number of propagating wave types. However, near pipe wall discontinuities the pipe wall vibrations can also be determined by exponentially near-fields and by exponentially decaying standing waves. This might hamper in practice the separation of energy flow components according to Eq. (5.1).

Investigations concerning this problem are only known for beam-bending waves. A measuring method for separating travelling waves and near-fields has been proposed by Pavić /5.14, 5.15/; see also Verheij /5.16/, (Appendix D of this thesis). A theoretical study of the errors in bending wave energy flow measurements due to neglect of near-fields has been presented by Noiseux /5.17/ and according to the same principle, but in more detail, by
Verheij /5.18/. A general error analysis for wave fields on pipes is not known to the author and is not undertaken in this thesis. Experiments should reveal whether a more detailed analysis is worthwhile.

On basis of the foregoing discussion it can be concluded that for \( \frac{f}{c_2} < f \), the structureborne energy flow is correctly described by Eq. (5.1) and that it forms a representative measure of the total sound transfer along a pipe. In the following paragraphs the measurement of the various components of Eq. (5.1) is discussed.

5.3.2 The measurement of structureborne energy flow
In 5.4 measurements are reported of the energy flow components of Eq. (5.1). For these measurements use has been made of methods based on a finite-difference principle and on signal processing in the frequency domain, i.e. of so-called cross-spectral density methods.

Finite-difference methods:
The basic operation for measuring energy flow is the multiplication of a force and a velocity or of a torque and an angular velocity. For the wave types of present interest the following energy flow components need to be determined:

- **longitudinal wave**: \((\text{axial force}) \times (\text{axial velocity})\)
- **torsional wave**: \((\text{torque about pipe axis}) \times (\text{angular velocity})\)
- **bending wave**: \((\text{transverse shear force}) \times (\text{transverse velocity}) + (\text{bending torque}) \times (\text{angular velocity})\).

For the determination of velocities there are appropriate transducers. However, special transducer configurations are needed for separating the desired components.

For the measurement of the shell stresses there are no appropriate transducers and therefore the dynamical quantities are derived from kinematical quantities, using simple elasticity relations. By this procedure the dynamical quantities can be measured also with vibration transducers. However, because the elasticity relations contain spatial derivatives of the kinematical quantities (see Appendix D, Eqs. (3.1), (4.1) and (4.2)), approximations are needed when use is made from point measurements. For wave types in beams and on plates, methods based on finite-difference approximations
have been described by Pavić /5.14, 5.15, 5.19, 5.20/. These are used in this thesis.

**Cross-spectral density methods:**

In the present study time averaged values of the energy flow components in Eq. (5.1) are considered, which means that correlation between the various components is neglected.

With respect to the signal processing a choice has been made for processing in the frequency domain, using cross-spectral density methods as formulated by Verheij /5.16/; see Appendix D. The energy flow components are derived from the cross-spectral density of accelerations at 2 closely spaced locations, according to the following equations (see Appendix D, Eqs. (3.7), (4.3) and (4.4)):

\[
\frac{\partial P_B}{\partial f} = \frac{2(Bm')^{1/2} \text{Im} G_{a_{B2},a_{B1}}}{\Delta \omega^2}, \quad (5.19)
\]

\[
\frac{\partial P_L}{\partial f} = \frac{\text{SE} \text{Im} G_{a_{L2},a_{L1}}}{\Delta \omega^3}, \quad (5.20)
\]

\[
\frac{\partial P_T}{\partial f} = \frac{T \text{Im} G_{\alpha_{T2},\alpha_{T1}}}{\Delta \omega^3}, \quad (5.21)
\]

where

- \( a_{B1}, a_{B2} \) transverse accelerations caused by bending waves at closely spaced locations no. 1 and no. 2 (ms\(^{-2}\))
- \( a_L \) axial acceleration caused by quasi-longitudinal wave (ms\(^{-2}\))
- \( G_{a_2,a_1} \) one-sided cross-spectral density of \( a_2 \) and \( a_1 \) (m\(^2\)s\(^{-3}\))
- \( T_p \) torsional stiffness of pipe; /4.9/, p. 92 (Nm\(^2\))
- \( \alpha_T \) angular acceleration about pipe axis (s\(^{-2}\))
- \( \Delta \) distance between locations no. 1 and no. 2 (m)

\( P \) is positive if the net flow direction is from location no. 1 to no. 2.

In the measurements of 5.4 the accelerometer spacing is different for the various wave types. The energy flow components have been integrated over 1/3-octave bands.
In a reply to Ref. 5.16, Pavic /5.21/ has defended a certain superiority of time domain processing over frequency processing, suggesting that Ref. 5.16 would imply the opposite conclusion. However, nowhere in Ref. 5.16 general superiority of frequency domain processing was claimed, neither implicitly nor explicitly. The main purpose was to formulate methods that use uni-axial accelerometers and that can be implemented on the widely available 2-channel Fourier analyzers. Signal processing in the time domain requires the development of high quality special purpose instrumentation.

Within the scope of methodological studies, as in this thesis, the use of available Fourier analyzers is especially attractive because the concentration has to be on methodological problems rather than on developing instrumentation. In 5.3.3 and 5.4 it will appear that the type of information which comes available by frequency domain processing is essential in the assessment of various error sources. However, if certain applications of energy flow measurements have been established, the availability of portable instrumentation based on signal processing in the time domain, may be very attractive for field applications.

Separation of wave types:
The point vibrations u, v and w on the pipe wall are a superposition of motions of all wave types and can therefore not be used for substitution in Eqs. (5.19) - (5.21). The wave types of interest are separated by using at each cross section no. 1 and no. 2 a pair of sensitivity- and phase matched accelerometers of which the output signal are either added or subtracted. Fig. 1 of Appendix D shows the accelerometer configurations for longitudinal and torsional waves. In Fig. 5.8 the configuration for bending waves is shown and also that for detecting the initiation of the n = 2 circumferential wave. These accelerometer configurations have been used for the energy flow measurements of 5.4.

5.3.3 Error sources in energy flow measurements
Several types of error sources have been discussed in the literature on sound intensity measurements in air, where cross-spectral density methods have been applied in a way analogous to Eqs. (5.19) - (5.21); see e.g. Refs. 5.22 - 5.27. For structureborne energy flow measurements Pavic /5.14/ presents some discussion on error sources. With respect to measurements on
pipes Verheij and Van Ruiten /5.25/ have made a detailed analysis of error sources, and they have described some practical methods for testing whether actual measurement results are sensitive for certain types of errors. A summary of results and test methods will be presented here.

The errors are divided in systematic or bias errors and random errors. Sources of bias errors are:

- the approximations inherent to finite-difference methods
- instrument phase mismatch
- coherent noise signals on both instrument channels. These can originate from imperfect separation of wave types due to transverse sensitivity of accelerometers, or due to asymmetry in pipe wall geometry, or due to neglect of near-fields etc. They can also originate from electronic cross-talk between the 2 instrument channels. On board ships adjacent machinery can lead to coherent noise.

Sources of random errors are:

- the use of a finite number of finite sample records for cross-spectral density estimates
- any type of coherent or incoherent noise
- slow random fluctuations in the source strength of the machinery

Finite-difference errors:

Bias errors due to finite-difference approximations are treated by several authors /5.14, 5.22, 5.23/. For the case of one-dimensional wave propagation, as on pipes at low frequencies, there is a very simple correction procedure as follows:
\[
\frac{\partial P}{\partial f}_{\text{true}} = \frac{k \Delta}{\sin k \Delta} \cdot \frac{\partial P}{\partial f}_{\text{measured}},
\]

(5.22)

where \(\frac{\partial P}{\partial f}_{\text{measured}}\) denote the quantities obtained from Eqs. (5.19) - (5.21). The correction procedure of Eq. (5.22) has been applied on all measurement results in 5.4.

**Phase mismatch errors:**

Bias errors due to instrument phase mismatch are treated also by several authors; see e.g. Ref. 5.24. The relative error due to a phase mismatch \(\Delta \theta \ll 1\) can be approximated by

\[
\varepsilon_{\Delta \theta}(f) = \frac{\frac{\partial P}{\partial f}_{\text{measured,} \Delta \theta \neq 0}}{\frac{\partial P}{\partial f}_{\text{measured,} \Delta \theta = 0}} - 1 \approx \Delta \theta \cot \theta_{12},
\]

(5.23, 5.24)

with \(\theta_{12} = \theta_1 - \theta_2\), where \(\theta_1\) and \(\theta_2\) denote respectively the phase spectra of the unbiased accelerations \(a_1\) and \(a_2\).

From Eq. (5.24) it is seen that the error "explodes" for \(\theta_{12}\) close to 0 and \(\pi\). For the special case of wave propagation solely in positive direction, \(\theta_{12}\) equals \(k \Delta\). This implies, for constant \(\Delta \theta\), increase of the relative error with decreasing frequency. For the general case of wave propagation in both directions large relative errors may also occur at higher frequencies, namely when \(\theta_{12} \ll k \Delta\).

A simple method to check whether measured results are sensitive for phase mismatch errors has been proposed by the author /5.27/. Deliberate phase errors are introduced in the cross-spectral density function by multiplying it with \(e^{j\Delta \theta}\), with \(\Delta \theta\) independent of frequency.

**Bias errors due to coherent noise:**

Verheij and Van Ruiten /5.27/ have studied the relative error for noise signals in the 2 instrument channels that are mutually coherent but incoherent with the uncorrupted acceleration signals. The "worst case" relative error is given by

\[
|\varepsilon|_{\text{max}} = \frac{\chi}{\sin \theta_{12}}
\]

(5.25)

where \(\chi\) denotes the signal ratio. This is the ratio between the power
spectral density of the noise signal and the power spectral density of the uncorrupted acceleration. It is assumed that $\chi$ is approximately the same at the closely spaced cross sections of the measurements.

The result of Eq. (5.25) is for $\theta_{12} \ll 1$ equal to that in Eq. (5.24) for phase mismatch. Therefore the conditions under which the measurements are sensitive for correlated noise are the same as for phase mismatch and also the same sensitivity test can be used, namely by multiplying the cross-spectral density function with $e^{+j\chi}$. For example, for coherent noise due to the transverse sensitivity of accelerometers $\chi$ can be estimated from

$$\chi_{\Delta f} = \delta^2 \frac{\Delta f_{\text{transverse}}}{\Delta f_{\text{normal}}}$$

(5.26)

where $\delta$ denotes the ratio between transverse- and main axis sensitivity.

**Random errors:**

The random error of energy flow measurements according to Eqs. (5.19) - (5.21) is determined by the random error in the estimate of the imaginary part of the cross-spectral density. The random error, being defined as the standard deviation, can be estimated from $n$ sample records as follows (see Ref. 5.27)

$$s(\text{Im } G_{a_2, a_1}) = \frac{1}{n} \left( \frac{1}{2\sin^2 \theta} \frac{1}{\gamma_{a_2, a_1}} \right)^{1/2} \cdot \text{Im } G_{a_2, a_1},$$

(5.27)

where $\{\cdot\}$ denotes a smoothed estimate; $\gamma_{a_2, a_1}$ denotes the squared coherence function for actually measured acceleration spectra (see 6.3.4 for definition). The random errors in the energy flow components of Eq. (5.1) are found by substitution of Eqs. (5.19) - (5.21) into Eq. (5.27), i.e. by

$$s(\frac{B}{\omega}) = \frac{2(Bm')^{1/2}}{\omega^\Delta},$$

(5.28)

It is seen from Eq. (5.27) that the minimum value for the random error equals $(1/n)^{1/2} \cdot \text{Im } G_{a_2, a_1}$ for the case of perfect coherence, i.e. for $\gamma_{a_2, a_1} = 1$. If $\gamma_{a_2, a_1} \ll 1$, the random error increases strongly when $\theta_{12}$ is close 0 or $\pi$, i.e. in the same situations where the measurements are sensitive for phase mismatch.
The derivation of Eq. (5.27) has been based on an article by Burros /5.28/.
The result differs slightly from an expression presented by Seybert /5.26/.
Seybert assumes that the covariance of $G_{2,1}$ and $E_{12}$ is zero, whereas
Burros does not use such an approximation. In practice the difference is small.

For the measurements in 5.4, the random error in the total energy flow
according to Eq. (5.1) is of interest and for filter bandwidths of
1/3-octaves. The variance of the total energy flow is estimated from:

$$
\sigma^2 \{P_{\Delta f}\} = \sum_{i=1}^{4} \sum_{j=1}^{N} \sigma^2_{i,j} \{P_i(f_j)\}, \quad (5.29)
$$

where $i = 1-4$ denote the 4 energy flow components of Eq. (5.1) and where
$j = 1-N$ denote the Fourier spectrum lines within $\Delta f$.

5.4 Laboratory experiments
In this section laboratory experiments are discussed which show the
feasibility of structureborne energy flow measurements and the validity
of the substitution source methods for pipe path assessment.

5.4.1 Test arrangement
Water tank and pump: The test arrangement was a cooling water pump circuit
connected with the ship-like hull of a 360 m³ steel water tank. Figure 5.9
shows a sketch of the water tank. Figure 5.10 shows the electric driven
cooling water pump installed via a subbase and 4 rubber mountings upon a
seating against the tank hull. During the measurements the pump had been
hooded by a wooden box to attenuate airborne sound transfer from the pump
to the hull.

Pipe system: The pressure side of the pipe system has a straight section
over a length of approximately 7 m along the tank hull. At a distance of
approximately 2 m from the pump the pipe has been connected with the tank
hull; see Figs. 5.10, 5.11 and 5.13. A second support is at approximately
7 m distance from the pump, against a building pillar. The outer diameter of
the steel pipe varies between 160 + 0,5 mm, the wall thickness is
Figure 5.9:
Steel water tank, dimensions: 12 m(length) × 6 m(width) × 5 m(depth).
Accelerometer positions no. 1-4 have been used as receiver locations for testing the validity of the substitution sources methods. The positions no. 2-4 are located on the hull 2.5 m below the water surface level.

4.9 + 0.1 mm. The flexible bellows close to the pump on the pressure side, was mechanically short-circuited to increase the sound transfer via the pipe at frequencies above 500 Hz.

Sound sources: Measurements have been performed for the following types of excitation:
1. Cavitating pump (1472 rev./min; water volume flow: 0.05 m³s⁻¹).
2. Electrodynamic exciters against the pump. Two exciters, type Derritron VP4, had been mounted against the pump housing; see Fig. 5.12. They were driven simultaneously from a single signal generator with periodical chirps, to simulate line spectrum sources.
3. Electrodynamic exciters against the pipe. Two exciters, type B & K 4810, were mounted against the pipe as substitution sources for respectively excitation by pump cavitation and artificial excitation on the pump housing.
Figure 5.10:
Resiliently mounted water pump with acoustic enclosure removed. The arrow points towards the single connection between the pipe system and the water tank hull.

Figure 5.11:
Schematic representation of pipe system, with locations of supports, substitution sources and measurements on the pipe wall.
Figure 5.12:
Electrodynamie exciters mounted against pump house (pump inside enclosure).

Figure 5.13:
Substitution exciters on pipe between pump and connection with hull (arrow points towards this connection).
The substitution sources are seen in Fig. 5.13. They excited the pipe in 2 perpendicular radial directions. Also these exciters were driven simultaneously from a single signal generator. The location of the substitution sources was between the pump and the connection with the hull, at some 0.4 m distance from the hull support.

**Measurements on the pipe:** The cross section at which the energy flow was measured, was between the pipe support on the tank and that on the building pillar, at a distance of some 2.5 m from each; see Fig. 5.11. Over a length of 1.6 m around this cross section, also radial accelerations have been measured for testing the validity of substitution source Method B according to Eq. (5.3). These radial accelerations have been measured in 2 perpendicular directions, with 3 accelerometers for each direction.

For all radial, tangential and axial measurements on the pipe wall, accelerometers type B & K 4368 have been used.

### 5.4.2 Energy flow measurements

**Instrumentation:**

**Signal processing:** The on-line data acquisition and the signal processing for the energy flow measurements have been performed with a 2-channel digital analysis system, type Time Data TDA 33/L. The frequency range of the measurements was 40 Hz – 500 Hz.

**Accelerometers:** With a special test procedure on a vibration exciter (type Derritron VP 4), accelerometers were selected having a ratio between transverse- and main sensitivity smaller than or equal to 0.01 for frequencies below 1 kHz.

**Phase matching of instrument channels:**

For phase matching the phase properties of the accelerometers were checked and the phase mismatch between the instrument channels (preamplifiers, anti-aliasing filters) was eliminated with a special correction procedure.

**Accelerometers:** Accelerometers were selected with a phase mismatch smaller than 0.11 degree. To measure this, pairs of accelerometers were mounted on opposite sides of a specially fabricated stud on top of a vibration exciter.
The arrangement was such that the main axes of the accelerometers coincided with the symmetry axes of stud and exciter, so that the accelerometers did undergo the same acceleration in the direction of their main axes. The 2 transducer signals were subsequently added and subtracted with a special purpose electronic device. For the "accepted" accelerometers the level difference between the sum and difference signals, i.e. the "common mode rejection" was at least 60 dB for 50 Hz - 1000 Hz, whereas the electronic device itself has a "common mode rejection" of at least 85 dB. The measured minimum "common mode rejection" of 60 dB was limited due to the transverse vibrations of the exciter and the transverse sensitivity of the accelerometers. It implies a transducer phase mismatch error smaller than 0,11 degree.

Instrument channels: For the elimination of phase mismatch between the channels without transducers, an identical electric signal (white noise) was inserted in both channels, replacing the accelerometer signals. The phase angle of the measured cross-spectral density function for these calibration signals have been used for correcting the subsequent measurements. Calibrations were performed for each different pair of pre-amplifier settings. To suppress statistical errors, a moving average technique was used, so that the "effective averaging" occurred over 500 sample records. The remaining phase mismatch errors are smaller than 0,04 degree. Therefore, for the channels with accelerometers included, the phase mismatch errors remain smaller than 0,15 degree.

Measurement procedures:
For all energy flow measurements, cross-spectral density functions have been estimated from 100 sample records.

The spacing \( \Delta \) between the accelerometer pairs was 0,2 m for bending wave measurements and 1,25 m for longitudinal and torsional waves.

Test conditions:
The excitation conditions and the frequency resolution of the Discrete Fourier Analysis (DFA) are given in Table 5.2.
Table 5.2: Test conditions:

<table>
<thead>
<tr>
<th>Code</th>
<th>Pipe</th>
<th>Excitation</th>
<th>Frequency resolution for DFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>water-filled</td>
<td>cavitating pump</td>
<td>2 Hz</td>
</tr>
<tr>
<td>II</td>
<td>water-filled</td>
<td>exciters on pump; line spectrum: Δf = 2 Hz</td>
<td>2 Hz</td>
</tr>
<tr>
<td>III</td>
<td>water-filled</td>
<td>exciters on pump; line spectrum: Δf = 8 Hz</td>
<td>8 Hz</td>
</tr>
<tr>
<td>IV</td>
<td>empty</td>
<td>exciters on pump; line spectrum: Δf = 2 Hz</td>
<td>2 Hz</td>
</tr>
</tbody>
</table>

Test conditions I and II were chosen to compare the measurement accuracy for a stochastic and a periodical sound source. However, the spectrum line distance Δf = 2 Hz is too small to be a representative spectrum of a resiliently mounted diesel engine. Therefore, condition III was included, with the spectrum line distance Δf = 8 Hz being representative of the spectrum of a 4-stroke diesel engine running at 960 rev./min. Test condition IV, with the pipe empty, was included to investigate whether a liquid path forms a complication for the substitution source method A which is discussed later.

Test results:

Cut-off frequencies of n = 2 circumferential wave: The cut-off frequency is approximately 515 Hz on the empty pipe and approximately 380 Hz on the water-filled pipe. This could be determined very accurately from summing the radial accelerations for opposite pipe locations according to Fig. 5.8. In the narrow band spectrum a very steep level increase was observed at the above mentioned frequencies. Also the squared coherence function for acceleration sums at 2 adjacent cross sections increases steeply at those frequencies, from \( \gamma^2 \ll 1 \) to \( \gamma^2 = 1 \).

Note:

The cut-off frequency calculated according to Eqs. (5.9) and (5.13) for the empty pipe is 553 Hz, when for the calculation the same material data is used as in Table 5.1. For the water-filled pipe the calculated result according to Eq. (5.18) is 41 Hz. These results exceed the observed values less than 10 percent.

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Energy flow results: In Fig. 5.14 the energy flow components $P_L$, $P_T$, and $P_B = P_{B1} + P_{B2}$ are shown for the 4 test conditions of Table 5.2. The measured spectra have been summed over 1/3-octave bands. It is seen that the distribution of the energy flow over the wave types varies with the excitation. Generally speaking, all 3 wave types are important. At the very low frequencies the longitudinal wave component is predominant in most cases. A circle indicates that the energy flow vector of the corresponding wave type is pointing towards the pump. This return of energy flow may be explained by wave type conversion at downstream discontinuities.
Figure 5.15:
Measured total energy flow levels of 3 wave types for test conditions I – IV of Table 5.2. The broken lines are calculated results for artificially induced phase mismatch errors: $\Delta \theta = \pm 0.2^\circ$.

Error analysis:
Sensitivity to phase mismatch: In Fig. 5.15 the total energy flow according to Eq. (5.1) is presented, together with calculated results for deliberate phase mismatch errors $\Delta \theta = \pm 0.2$ degree. The direction of the total energy flow is always outgoing from the pump. It is seen that the influence of a small phase mismatch is, generally speaking, very moderate. This is due to the fact that the largest relative errors usually occur for wave types and frequency components that are not predominant in the total energy flow within the 1/3-octave filter bandwidth. Only for the cavitating pump (Fig. 5.15a) the low frequency results are very sensitive to phase mismatch errors, indicating very reactive wave fields.
It is interesting to notice the $\pm 1.5$ dB errors in Fig. 5.15d at $f_c = 160$ Hz. From Fig. 14d it can be seen that at this frequency the predominant energy flow components for torsion and bending are of opposite sign and cancel each other. The much weaker energy flow in the longitudinal wave field becomes predominant. This longitudinal wave field appears to be very reactive.

**Bias errors due to coherent noise:** One cause of coherent noise is the transverse sensitivity of accelerometers. Because of the equivalence of Eqs. (5.25) and (5.24) for $\theta_{12} \ll 1$, Eqs. (5.26) can be used to calculate the level difference between accelerations transverse to the main axis and that in the main axis direction, which would cause a noise to signal ratio $\chi$ equivalent with a phase mismatch $\Delta \theta = 0.2$ degree ($0.0035$ radians). Because $\delta = 0.01$, it follows from Eq. (5.26) that the transverse vibration should be at least $15$ dB stronger than those in the main axis direction.

In Fig. 5.16 the acceleration levels of the various wave types are shown for test conditions I and II. Acceleration spectra of the respective energy flow measurements have been used. All results have been averaged over the 2 acceleration pairs used for the cross-spectral density measurements. For bending waves and for $n = 2$ circumferential waves additional averaging was performed over 2 perpendicular directions. The initiation of the $n = 2$ circumferential wave at approximately $380$ Hz is clearly seen in Fig. 5.16.

For the transverse sensitivity error analysis only the error in the total energy flow is of interest, in the same way as in Fig. 5.15 for the phase mismatch error. Therefore, to interpret Fig. 5.16 with respect to the influence of transverse sensitivity, the following procedure has to be followed:

- for a particular test condition and frequency band the predominant wave type is found in Fig. 5.14;
- it is checked in Fig. 5.16 whether the accelerations corresponding to this wave type are considerably exceeded by those of other wave types.

For example, in Fig. 5.15a it appears that the total energy flow for excitation by the cavitating pump is very sensitive to phase mismatch errors at $50$ Hz. From Fig. 5.14a, it is seen that at this frequency the energy flow in longitudinal waves is by far predominant. From Fig. 5.16a it is seen that the strongest "transverse" vibration level, i.e. that for bending, is $2$ dB.
lower than the longitudinal acceleration level. This corresponds with a noise to signal ratio \( \chi = 6 \times 10^{-5} \), which is equivalent with a phase mismatch \( \Delta \theta = 0.004 \) degree. This leads to negligible bias errors.

With the above derived criterion of an allowed level difference of 15 dB between "transverse" accelerations and predominant wave type accelerations, it is easily found from the Figs. 5.14 and 5.16 that in all cases transverse sensitivity errors are much smaller than bias errors due to \( \Delta \theta = \pm 0.2 \) degree.

Other causes of coherent noise have not been investigated in detail. Because the asymmetries in pipe cross sections are smaller than 0.6 percent of the pipe diameter, it is improbable that imperfect separation of wave types is an important error source. Also bias errors caused by bending wave near-fields are expectedly low. The shortest distance between measurement...
Figure 5.17:
Measured total energy flow levels of 3 wave types for test conditions I - IV of Table 5.2. The broken lines represent calculated results of $P_{\Delta f} \pm 2 \tilde{s}_{P_{\Delta f}}$, where $\tilde{s}_{P_{\Delta f}}$ denotes the random error in $P_{\Delta f}$.

cross sections and pipe wall discontinuities was some 2.5 m or, as can be seen from Fig. 5.6, greater than $\lambda_B/2$. In such a situation both the theoretical analysis in Ref. 5.18 and the results of previous experiments in a somewhat different test arrangement for the same pipe, indicate that the errors due to near fields will remain smaller than some 2 dB. In those previous experiments the power injection for point force excitation was compared with the total bending wave energy flow $P_{B1} + P_{B2}$, measured according to Eq. (5.19).

Random errors: In Fig. 5.17 the total energy flow according to Eq. (5.1) is presented, together with calculated results of $P_{\Delta f} \pm 2 \tilde{s}_{P_{\Delta f}}$ with the standard deviation $\tilde{s}_{P_{\Delta f}}$ calculated according to Eqs. (5.27) - (5.29).
It is seen that the errors remain moderate due to the averaging over \( n = 100 \) sample records. The maximum errors occur under the same conditions as those for phase mismatch. For periodic excitation they remain very small. For the cavitating pump they are somewhat larger, especially at low frequency. For this stochastic excitation the squared coherence function for the accelerations at adjacent positions is for many frequencies considerably smaller than unity. Recent shipboard measurements on cooling water pipes of a diesel engine showed squared coherence function values close to unity for all harmonics of the cylinder firing frequencies. Therefore, the random errors in Fig. 5.17b-5.17d are expectedly representative of measurements for "line spectrum" sound sources.

### 5.4.3 Substitution Source Methods

To test the validity of the substitution methods, acceleration levels on the tank hull calculated according to Eqs. (5.2) and (5.3) have been compared with measured acceleration levels. The 4 test conditions of Table 5.2 were used. It was checked that for all test conditions the sound transfer from pump to water tank hull via the pipe connection was much stronger than via other transfer paths. The accelerations on the hull were measured at positions no. 1-4; see Fig. 5.9. The substitution sources were driven with the same spectra as the exciters on the pump. For substituting the pump cavitation, a broadband periodical chirp with spectrum line distance \( \Delta f = 2 \) Hz was used, equal to that used for simulating excitation for test condition II of Table 5.2. In the laboratory tests it was not necessary to use the reciprocity techniques according to Eqs. (5.5) and (5.6) for measuring the sound transfer from the substitution sources.

#### Method A according to Eq. (5.1):

In addition to the energy flow measurements of Fig. 5.14, similar measurements were made for the substitution source excitations. Fig. 5.18 shows the measured energy flow components for substitution excitation with \( \Delta f = 2 \) Hz on the water-filled pipe. Comparison of these results with those for test conditions I and II in Figs. 5.14a and 5.14b shows that the energy flow in bending waves has become relatively more important and that in longitudinal waves much less important. This is probably caused by the fact that only substitution source excitation in radial directions occurred. Still at 50, 63 and 315 Hz the longitudinal waves are predominant.
Figure 5.18:
Energy flow levels of different wave types on water-filled pipe for excitation by substitution sources (line spectrum: \(\Delta f = 2\ Hz\)).

Figure 5.19:
Level difference between the indirectly and directly determined accelerations at hull position 1.

a. Test condition IV of Table 5.2; Method A.

b. Test condition IV of Table 5.2; Method B.

In Fig. 5.19a the level difference is shown between the acceleration at hull position no. 1, calculated according to Eq. (5.2) and that measured for test condition IV of Table 5.2. Both 1/3-octave band and octave band results are shown, according to

\[
\Delta L = 10 \ \log \ \frac{a^2_{\Delta f} \text{(calculated)}}{a^2_{\Delta f} \text{(measured)}}
\]  

(5.30)

The 1/3-octave bands values of \(\Delta L\) vary between -12 dB and +10 dB and the octave band values between -1 dB and +6 dB. It is the general trend, also in the other test results, that the 1/3-octave values of \(\Delta L\) show a large
Level difference between the indirectly and directly determined acceleration on the water tank hull for test conditions I, II and III of Table 5.2.

- a. Method A; hull position no. 1.
- b. Method A; 4 hull positions, see Eq. (5.31).
- c. Method B; hull position no. 1.
- d. Method B; 4 hull positions, see Eq. (5.31).

scatter around 0 dB, whereas for octave band results the scatter remains moderate. In the following figures only octave band values of $\Delta L$ are presented.

In Fig. 5.20a, $\Delta L$ for hull position no. 1 is shown for the test conditions I-III of Table 5.2. The maximum deviations from 0 dB are -5 dB and +6 dB. In Fig. 5.20b, results are presented for the same test conditions, but now averaged over hull positions no. 1-4, according to

$$\Delta L = 10 \log \left[ 4 \sum_{i=1}^{4} \frac{a_i^2 \Delta f_{\text{calculated}}}{a_i^2 \Delta f_{\text{measured}}} \right]. \quad (5.31)$$

The results in Fig. 5.20b differ not significantly from those in Fig. 5.20a for the single receiver location.
Figure 5.21:
Scatter of ratios of indirectly and directly determined accelerations at 4 hull positions, calculated according to Eq. (5.32) for test conditions I, II and III of Table 5.2.

In Fig. 5.21a, a measure is shown of the scatter of $a_i^2/Af_{measured}$ (measured) for the 4 receiver positions on the hull. This quantity $\Delta E$ is defined as

$$\Delta E = 10 \log \left(1 + \frac{s_{x_i}}{\bar{x}_i}\right)$$

where $x_i$ denotes the ratio $a_i^2/Af_{measured}$; $s_{x_i}$ denotes the standard deviation of the 4 "$x_i$'s" and $\bar{x}_i$ denotes the averaged value.

It is seen in Fig. 5.21a, that the largest scatter of $x_i$ occurs for excitation with $Af = 8$ Hz (test condition III).

Comparison of the results for test conditions II and IV in Figs. 5.19a and 5.20a shows no systematic influence of the liquidborne sound path on the accuracy of Method A.

Method B according to Eq. (5.2):
In Fig. 5.19b, the level difference is shown between the acceleration at hull position no. 1 calculated according to Eq. (5.3) and that measured for test condition IV of Table 5.2. The frequency range of the measurements covered now 13 $1/3$-octave bands, i.e. $50$ Hz $< f < 800$ Hz. Again, both $1/3$-octave band and octave band results of $\Delta L$ are shown. For $1/3$-octave bands, $\Delta L$ varies between $-5$ dB and $+8$ dB and for octave bands, between $-1$ dB and $+3$ dB.

In Fig. 5.20c, octave band values of $\Delta L$ for hull position no. 1 are shown for the test conditions I-III of Table 5.2. The values vary between $-3$ dB and
+7 dB. In Fig. 5.20d, results for the same conditions are presented, but now averaged over the 4 hull positions, according to Eq. (5.31). Again, the values of $\Delta L$ vary between -3 dB and +7 dB.

In Fig. 5.21b, $\Delta E$ is shown for the results of Method B. Again the largest scatter of $x_i$ occurs for excitation with $\Delta f = 8$ Hz.

Accuracy of the substitution source methods:
The results in Figs. 5.19 and 5.20 show that both substitution source methods provide path assessment results with a maximum inaccuracy of some +5 dB. To obtain this, it appeared necessary to convert the calculated 1/3-octave band responses to octave band responses. Additional averaging over 4 receiver locations, seemingly, did not lead to significantly more accurate results. However, especially for test condition III, i.e. for excitation with a line spectrum $\Delta f = 8$ Hz, the scatter in results for the 4 receiver locations was such that much less accurate results than those in Figs. 5.20a and 5.20c could have been presented for another receiver location. Therefore, averaging over the 4 receiver locations decreases the confidence interval and thus increases the reliability of the path assessment.

For the test conditions I, II and IV, Method A and Method B are equivalent with respect to the accuracy of the results. For test condition III with the diesel engine-like excitation ($\Delta f = 8$ Hz), there is slight evidence from Fig. 5.20 that at frequencies below 500 Hz Method A based on energy flow measurements is somewhat more accurate than Method B based on radial pipe wall accelerations.

Future investigations should reveal whether the observed inaccuracies of some +5 dB are typical also on other pipe systems. If this should be the case, then the substitution source methods are sufficiently good to find on board those pipe connections which form an unwanted limitation of the sound reduction effectiveness of resilient mounting systems.

5.5 Evaluation of the experiments
The sound path assessment methods as investigated in the present chapter have not been explored previously. Therefore the promising results of the foregoing section for a single test facility and a single pipe configuration
cannot be conclusive with respect to the general applicability of the methods on board ships. In the present section it will be discussed to what extent the system investigated is representative of shipboard pipe systems and what type of investigations are needed to obtain further evidence of the validity of the path assessment methods.

**Energy flow measurements:**

Measurement of structureborne flow according to Eq. (5.1) appeared feasible on the laboratory pipe circuit. The upper limit of the frequency range is determined by \( f_{c2} \), i.e. the cut-off frequency of \( n = 2 \) circumferential waves. For the water-filled test pipe this frequency is at 380 Hz.

However, for many interesting shipboard pipe systems this 2nd order mode will occur at higher frequencies, so that energy flow measurements can be performed over a wider frequency range than in the above tests. Notably cooling water and lubrication oil pipes of auxiliary diesel engines have usually a smaller diameter and a greater wall thickness/diameter ratio than the above test pipe, leading to \( f_{c2} \) at approximately 800 Hz.

With respect to the potential errors caused by exponentially decaying near fields and exponentially decaying standing waves, the pipe system investigated is not fully representative of typical shipboard systems. On the test pipe the minimum distance between, on the one hand the measurement cross sections and on the other hand pipe clamps and bends, is 2.5 m. This is much more than possible in many shipboard situations, where this minimum distance is typically 1 m. Preliminary investigations on the test pipe of 5.4, indicate that at a distance of 1 m of a pipe clamp the influence of near fields etc. on the energy flow measurements is very small for 50 Hz \( \leq f \leq 400 \) Hz. However, near bends and other pipe wall discontinuities stronger near fields may occur than near usually "non-rigid" clamps. Therefore this error source needs further experimental investigation.

A very important observation in 5.4.2 is that at low frequencies where the bending wavelengths are much shorter than the longitudinal- and torsional wavelengths, the latter wave types may transport much more energy than the bending waves; see e.g. Fig. 5.14c. Therefore, reliable determination of energy flow on shipboard pipe systems will always require measurements for all wave types.
Substitution source methods:

On basis of the limited experience of the tests in 5.4 there is no indication of significant difference in accuracy between Method A and Method B. Therefore, because of the simplicity of measurements and because of the wider frequency range of application, Method B would seem more attractive. However, the advantage of Method A is that the energy flow strengths which come available can also be used for direct comparison with the energy flow through resilient support mountings and air paths.

Further tests on other pipe systems should reveal whether there are cases where one of the methods is fundamentally more accurate than the other.

With respect to the substitution sources several aspects are worthy to be discussed in more detail. In the tests of 5.4, simultaneous excitation on the pipe in 2 perpendicular radial directions was used. The simultaneousness is not essential. Similar results may be expected for summation over subsequent excitation for the 2 directions. Because the distribution of the energy flow over the various wave types appeared rather different for excitation by the pump or upon the pump house and by the substitution sources, perhaps somewhat better results would have been obtained if also substitution source excitation in the axial direction (e.g. against a flange) had been used. Generally speaking, it may be expected that when the number of resonant modes in the pipe system and in the transmitting ship structure is small, the details of the excitation become more important for the sound transfer. Also the number of connections between pipe and ship structure may be of interest. In the test of 5.4 only 1 connection was used. In actual shipboard systems there are more connections, making perhaps the sound transfer from pipe to ship less sensitive to details of the excitation. In fact the test arrangement forms a rather extreme case because only a small fraction of the energy flow is injected into the tank structure. The remaining part is flowing into the building supports and to the auxiliary water basin (see Fig. 5.11). The reasonably accurate results even for this extreme situation make the methods proposed very promising for actual shipboard systems.

Another important aspect concerns the substitution source spectrum. It is interesting to know whether it is necessary to simulate periodical engine
Figure 5.22:
Level difference between the indirectly and directly determined accelerations on the water tank hull; 4 hull positions, see Eq. (5.31).

Line no. 1: Excitation on pump: $\Delta f = 8$ Hz
Substitution excitation on pump: $\Delta f = 8$ Hz

Line no. 2: Excitation on pump: $\Delta f = 8$ Hz
Substitution excitation on pump: $\Delta f = 2$ Hz

excitation or whether continuous spectra can be used, e.g. generated using experimentally simple transient excitation techniques. To investigate this, in Fig. 5.22 different values of $\Delta L$ according to Eq. (5.31) are presented for test condition III of Table 5.2, i.e. diesel engine-like excitation with $\Delta f = 8$ Hz. One line represents test results for which $a_{11}^2 \Delta f$ (calculated) has been obtained from substitution source excitation with $\Delta f = 8$ Hz, thus the same results as in Figs. 5.20b and 5.20d. The other line represents test results for $a_{11}^2 \Delta f$ (calculated) obtained from substitution source excitation with $\Delta f = 2$ Hz. As is seen from Fig. 5.22 the latter procedure does not lead to less accurate results. Therefore, substitution sources with continuous spectra seem acceptable.

5.6 Summary
1. New methods have been investigated for quantifying the sound transfer from resiliently mounted machinery via pipe systems. Direct determination of the sound transfer is attempted by measuring structureborne energy flow. This quantity for pipe systems can directly be compared with energy flows via resilient mountings underneath machinery and that via air paths. Indirect determination of pipe path contribution is attempted with 2 substitution source methods, one on basis of energy flow measurements on the pipe systems, the other on basis of radial pipe wall accelerations.
2. In the theoretical section 5.3, some fundamentals of pipe wall vibrations have been discussed and moreover the techniques for measuring energy flow in longitudinal-, torsional- and beam-bending waves. The theoretical discussion explains that energy flow measurements are simple only below the initiation of $n = 2$ circumferential shell waves. In that frequency range the energy flow is determined by the 3 above mentioned wave types.

3. Laboratory tests have been performed on a pipe system at a cooling water pump, which was connected to the ship-like hull of a steel water tank. The test results show the feasibility of accurate energy flow measurements. Moreover, it has been shown that none of the 3 above mentioned wave types may be excluded a priori, if the total structureborne energy flow has to be determined. For several test conditions the low frequency energy flow was dominated by longitudinal waves.

To test the 2 substitution source methods, indirectly obtained results, i.e. by using the substitution source methods, have been compared with directly measured responses on the water tank hull. Both methods appear accurate within some 5 dB, provided both the indirectly and the directly determined hull responses are converted to octave band levels. Averaging over several receiver locations on the hull increases the reliability of these path assessment methods.

4. The evaluation in 5.5 presents suggestions for additional research and also some practical guidance for field measurements.
Part B:

NOISE REDUCTION PROPERTIES OF RESILIENT MOUNTING SYSTEMS
6. CASE STUDY ON SOUND TRANSFER FROM A RESILIENTLY MOUNTED PROPULSION DIESEL ENGINE

The sound transfer from a resiliently mounted medium-speed propulsion diesel engine on board a sea-going passenger and car-ferry to the accommodation has been analyzed. For frequencies below 500 Hz the contributions of the resilient mounting path and the airborne paths are much below the total sound transfer. This indicates predominant flanking sound transfer through the remaining paths, i.e. flexible shaft coupling, pipes, exhaust system, etc. On the basis of both shipboard and scale model measurements, factors which are important for the sound transfer through the resilient mountings and through the air are discussed for the system investigated. Estimates are given for the upper limit of the insertion loss for similar single stage mounting systems without acoustic enclosure.

6.1 Introduction

In certain types of sea-going passenger and car-ferries, medium speed propulsion diesel engines are installed at some 35 m distance from the propellers. When the propellers are not excessively noisy this implies that the noise in the passenger accommodation above the main engine room is dominated by noise from these propulsion engines. To reduce this noise to acceptable levels it appeared in past designs necessary to apply floating floors in the accommodation and resilient mountings underneath the main engines, see Buiten et al. /6.1/.

To attain a good noise reduction over a broad frequency range, until now it is judged to be necessary to use a resilient mounting system with its natural frequencies below 10 Hz. Because the engine speeds are in the range 350-600 rev./min, such a mounting system has to be designed very carefully to avoid "mass-spring-system" resonances due to excitation by engine or propellers. Prevention of low frequency vibrational problems in such a mounting system is discussed by Steenhoek /6.2/.
The noise reduction by such mounting systems has been investigated by Buiten. His analysis method and some results have been reported by Steenhoek /1.6/. For two rather similar mounting systems in different ships the reduction of noise levels in the accommodation was found to be some 10-15 dB for the octave bands with centre frequencies 63 Hz - 1 kHz. This was less than expected for these systems with soft rubber mountings and heavy seating structures. Because there was no evidence that other sound sources such as the main reduction gears or the propellers had become predominant, it was supposed that flanking transfer from the main diesel engines was the limiting factor for the multi-path system insertion loss.

In 1978 the author got the opportunity to investigate the sound transfer from a resiliently mounted propulsion diesel engine on board a new passenger and car-ferry /6.3/.

The main engine installation and the mounting systems on this ship were rather similar to those in the earlier ships analyzed by Buiten. The primary objective of the measurements was to investigate for a single engine to what extent the sound transfer through the resilient mountings was exceeded by flanking transfer paths. Although the path analysis methods of Chapters 2 and 4 had not been developed in detail at that time, appropriate measurements were made to apply these methods afterwards. In this way it could be calculated which fractions of total sound transfer were determined by respectively the sound transfer through the resilient mountings and through the air below the engine. The contribution of the airborne sound transfer from the engine directly to the hull was also determined. A detailed examination of sound transfer along pipes was not included in the project.

In section 6.2 the measurement procedures are briefly discussed and only the main results are presented. These concern a comparison between the measured total sound transfer and the calculated contributions for some of the transfer paths.

In section 6.3 it is discussed in more detail which procedures have been followed for the measurements and calculations.
Section 6.4 is directed on a more detailed study of some noise reduction properties of the system investigated. Practical possibilities are discussed for improving the noise reduction effectiveness.

6.2 Measurements and multi-path analysis results

General measurement procedures:
The sound source under investigation was a 4 MW medium-speed propulsion diesel engine on board a twin propeller passenger and car-ferry. Measurements have been performed both at sea during a normal service trip and at the yard before the ship came into service.

During the measurements at sea the 2 outer of the 4 main engines were in service. They were running deliberately at different speeds to facilitate source identification. The engine closest to the receiver location was running at 512 rev./min, the more distant engine at 480 rev./min. The receiver location, which has been used for determining the sound transfer to the accommodation, was on the hull at approximately 10 m above the tank top; see Fig. 6.1. To characterize the total sound transfer to this location, the open circuit voltage of a reciprocal electrodynamic vibration exciter (type Derritron VP4) was used. This transducer had been mounted against a web frame. Its response at sea was completely determined by the nearest main engine for the frequency range of the path analysis, i.e., the 63 Hz-1 kHz octave bands. The way in which this has been determined is discussed later.
For path analysis purposes, source strength levels have also been measured at sea. For the resilient mounting path, accelerations have been measured on the engine raft for various translational and rotational directions. For the airborne paths, sound pressures have been measured at positions in the cavity below the engine (dimensions: $7.5 \times 2 \times 0.15$ m$^3$) and at positions in the space between engine and hull.

To determine transfer functions, measurements have been performed at the dockyard. During these measurements no engines were in operation. For the path through the mountings and the path through the cavity below the engine, reciprocal transfer functions of the type $\left(\frac{a'^2}{\omega^2 i^2}\right)$ have been measured, analogous to the experiments in 3.3 and 4.3. For these measurements the reciprocal transducer upon the web frame was used as exciter (driving current: $i'$). Because the double bottom was accessible the accelerations $a'$ could be measured at appropriate positions directly underneath resilient mountings and on the underside of the tank top plates below the air cavity. For the direct airborne path to the hull, the sound transfer has been measured using 4 loudspeakers in the space between engine and hull as a substitution sound source.
Using these shipboard measurement data and in addition some data obtained from the similar scale model system of 3.4, the sound transfer for the mounting path and the two airborne paths has been calculated.

The measured and calculated structureborne sound spectra for the receiver location on the hull are shown in Fig. 6.2 as follows:

a. measured at sea
b. calculated for the hypothetical situation of a rigidly mounted engine. This result has been obtained by Buiten /6.4/ using the same analysis method as described in Ref. 1.6.
c. calculated for the mounting path
d. calculated for the airborne sound transfer directly from the engine to the adjacent hull
e. calculated for the path through the cavity below the engine. In this cavity noise reduction measures had been applied, viz., constrained layer damping on the tank top (visco-elastic layer: 4mm; steel top plates: 3 mm) and an insulating structure against the raft bottom (5 mm steel plates upon 50 mm oil resistant closed cells rubber).

The most important conclusion from Fig. 6.2 is that at \( f \leq 250 \text{ Hz} \) the multi-path system insertion loss is by no means determined by the resilient mountings and the airborne paths. However, for higher frequencies these paths also become important. At the low frequencies the differences between the total response and the calculated contributions for mounting and airborne paths are so large that this conclusion is not affected by some uncertainties in the calculation procedures. A more detailed discussion follows in 6.4.
6.3 Discussion of analysis procedures

In the foregoing section measurement results for the total sound transfer and calculation results for the sound transfer via individual paths have been presented. In this section a more detailed discussion is presented of the procedures which have been followed to obtain these results and of the estimated accuracy of the results.

6.3.1 Resilient mounting path

In principle, the multi-directional sound transfer through the resilient mountings was analyzed according to the same scheme as used for the scale model experiments of 3.4. The sound transfer path has been split into 3 sub-systems, viz. the engine raft representing the sound source, the resilient mountings and the ship structure (see Fig. 2.1). The same type of simplifications have been used as in 3.4, i.e. phase relationships have been neglected and source levels and ship transfer functions have been measured for a very limited number of mounting locations.

However, the analysis is not just a repetition of the scale model investigation in 3.4. Due to contractual limitations only 2 transfer functions of the resilient mounting could be measured in contrast to the 10 transfer functions for the scale model mounting in 3.4. These full scale transfer functions are \( T_{F_y,y} \) and \( T_{F_z,z} \), i.e. for "shear" in athwartship direction and for compression (the same notation as in Chapters 2 and 3 is used). Therefore, with the complementary source strength data and ship transfer functions, the sound transfer of only these 2 transfer modes could be calculated. Nevertheless, these results will appear later in this paragraph to be representative of the total sound transfer through the mountings.

Shipboard measurement data:

Source levels: The acceleration \( \ddot{z}_m \) (translation in normal direction) has been measured at 4 engine raft positions for mounting locations close to raft stiffeners. The scatter of \( \ddot{z}_m^2 \) for these positions was very small (\( \leq 3 \text{ dB} \) for 1/3-octave band levels). At 1 of these locations source levels have been measured for 4 degrees of freedom, viz. \( \ddot{y}_m, \ddot{z}_m, \ddot{\alpha}_m \) and \( \ddot{\beta}_m \) (same coordinate system as in Figs. 2.3 and 3.9).
Ship transfer functions: Ratios \( \frac{e^2}{F^2_{s,z}} \) for excitation on the seatings have been measured (reciprocally) for 4 mounting locations close to seating stiffeners, 2 on each side of the engine, and moreover for 2 mounting locations between stiffeners. In this way the propagation attenuation in fore-and-aft and in athwartship direction was investigated in the same way as in 3.4. For 1 mounting location close to a stiffener and 1 mounting location between stiffeners, ship transfer functions have been measured for 4 excitation components, viz. \( F_{s,y}, F_{s,z}, M_{s,x} \) and \( M_{s,y} \). The differences in sound transfer for these 2 types of locations were to be taken into account because 6 of the 18 mountings had been located between stiffeners.

Calculations:
Using 1/3-octave band data for source strengths and transfer functions, contributions to the hull transfer response "e" have been calculated for the 2 mounting transfer functions \( T_{F_{y,y}} \) and \( T_{F_{z,z}} \). For this calculation Eq. (6.1) has been used in analogy with Eq. (3.12). The final results have been converted into octave band levels.

\[
e_{F_{y,y}}^{2} = C_{y} S_{F_{y}}^{2} T_{F_{y,y}}^{2} y_{m}^{2} \tag{6.1a}
\]

\[
e_{F_{z,z}}^{2} = C_{z} S_{F_{z}}^{2} T_{F_{z,z}}^{2} z_{m}^{2} \tag{6.1b}
\]

For the ship transfer functions \( S_{F}^{2} \), viz. \( e^2/F_{s,y}^2 \) and \( e^2/F_{s,z}^2 \), data have been used averaged for two mounting locations close to seating stiffeners on the seating adjacent to the hull.

\( C_{y} \) and \( C_{z} \) are correction factors which take into account the total number of mountings, the influence of propagation attenuation in athwartship direction and especially at higher frequencies the increased source levels and sound transfer for mounting locations amidst stiffeners. This increase is due to more or less "local" bending wave resonances in the seating and raft plates against the mountings. For the determination of \( C_{y} \) and \( C_{z} \) the same procedure was followed as described in 3.4.
Table 6.1 shows the values of $C$ which have been used for the calculations and also those which would have been valid if all 18 mountings would have been located at stiff locations on seatings and raft structure. Especially at higher frequencies there is a considerable increase of sound transfer for the mountings located between the stiffeners.

Table 6.1: Estimated values of $10 \ lg C$.

<table>
<thead>
<tr>
<th>$f_c$ (oct):</th>
<th>63</th>
<th>125</th>
<th>250</th>
<th>500</th>
<th>1k</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual system</td>
<td>$C_y$</td>
<td>15</td>
<td>11</td>
<td>16</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>$C_z$</td>
<td>13</td>
<td>14</td>
<td>14</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>all mounting at stiff locations</td>
<td>$C_y$</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>$C_z$</td>
<td>12</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

Comparison of ship and scale model systems for a limited number of excitation modes:

Owing to the limited data that could be obtained on board the ship and of the mounting, it was necessary to rely on additional data obtained from the 1:8 scale model used previously in 3.4. For this scale model it has been shown that the sound transfer due to the compression of the model mountings was most predominant at all frequencies except for the 1/3-octave bands with centre frequencies 315 and 400 Hz (full scale frequencies); see Figs. 3.17-3.19. For these frequency bands the sound transfer for "shear" in athwartship direction and for a few other transfer modes became of equal importance. Therefore, if the full and model scale mountings are similar and if the relative levels of the various source accelerations, together with the ship transfer functions are similar, then it would be reasonable to assume that also for the ship the sum of the sound transfer for compression and athwartship "shear" will be representative of the total sound transfer through the mountings, with perhaps a slight underestimate at frequencies around 315 Hz.

To demonstrate this, similarity between ship and model systems was investigated for a limited number of transfer modes, namely $T_{Fz,z}$ (compression), $T_{Fy,y}$ (athwartship "shear"), $T_{My,x}$ and $T_{Mx,y}$ (bending).
A straightforward manner of showing the predominance of sound transfer by compression plus athwartship "shear" would consist of the 3 following steps:

- Prove the similarity of the full scale and model mounting by comparing the transfer functions that were measured for both, i.e. $T_{F,\ddot{z}}$ and $T_{F,\ddot{y}}$.
- Derive from this comparison the "translation" factor for estimating full scale mounting transfer functions that were not measured, on basis of the scale model results.
- Use these estimated mounting transfer data in combination with sound source levels and ship transfer functions measured on board and calculate the sound transfer for each excitation mode with variants of Eq. (6.1). The calculation results might be represented in the same way as in section 3.4, Figs. 3.17-3.19.

In principle the procedure as outlined above has been followed below. However, the presentation is somewhat different from that in 3.4, to provide some additional information on the degree of similarity of full and model scale sound source and of the sound transfer in full and model scale ship.

**Similarity of shipboard and scale model mounting path for compression versus "shear":**

Similar acoustical behaviour of the full scale and scale model mountings might be expected on basis of geometric similarity and approximately equal rubber hardness; see 1.3. Such a supposition is sustained to a large extent by the following comparison of the two measured sound transfer functions $T_{F,\ddot{y}}$ and $T_{F,\ddot{z}}$ of the full scale mounting with those of the scale model mounting.

The full scale mounting transfer functions have been measured in the test rig of Fig. 2.12. The test arrangement is shown in Fig. 6.3. The block shape rubber mounting is a type Metalastik 17/1359 with rubber dimensions (in m); $0.4 \times 0.19 \times 0.085$ (dynamical compressional stiffness: $8.4 \times 10^6$ N m$^{-1}$). Because the mountings on board are inclined at an angle of 45° the test rig measurements have been performed under the representative deformation caused by simultaneous compressional and shear preloads (respectively $5 \times 10^4$ N and $8 \times 10^3$ N).
Figure 6.3:
Resilient mounting in sound transfer test rig subjected to simultaneous compression and shear preload.

Figure 6.4:
Sound transfer functions of resilient mounting under nominal preload.
The measurement results for $T_{F_y}$, $\ddot{y}$ and $T_{F_z}$, $\ddot{z}$ are shown in Fig. 6.4. For $f < 100$ Hz the measured values for $T_{F_z}$, $\ddot{z}$ have been corrected according to Eq. (2.33) because the mass of the terminating block ($m_s = 176$ kg) was too small for an unbiased measurement. It is seen that around 200 Hz the "shear" transfer function exceeds that for compression due to a rather undamped resonance at 200 Hz. In Fig. 6.5 the level differences of $T_{F_y}$, $\ddot{y}$ and $T_{F_z}$, $\ddot{z}$ are presented both for the full scale and the scale model mounting. A great similarity is seen at low and high frequencies. However, around 250 Hz the stiffening effect of the "shear" resonance is much more pronounced on full scale. This is probably due to differences in rubber compositions and due to frequency dependence of the loss factors of the rubbers.

Note:

For the scale model results in Fig. 6.5 the frequency scale has been "translated" to full scale, i.e. divided by a factor 8. This holds also for Figs. 6.6-6.10.
Comparison of ship transfer functions: Sound levels in accommodation due to $10 \log \left( \frac{S_F^2}{S_{F_0}^2} \right)$ for mounting location that due to $T_y$, $T_z$ close to seating stiffeners. Figure 6.7:

In Fig. 6.6 and Fig. 6.7 the level differences of respectively the source accelerations $\ddot{y}_m$ and $\ddot{z}_m$ and the ship transfer functions $S_F^y$ and $S_F^z$ are presented both for the shipboard system and the scale model system at corresponding frequencies. The differences for both systems are on the average smaller than 5 dB for mounting locations close to stiffeners. For the actual shipboard system with mounting locations also between stiffeners, additional differences with the scale model follow from Table 6.1.

In Fig. 6.8 the level difference of the calculated hull transducer response "$e" for both mounting transfer modes is compared with a corresponding ratio for the deck response "$a" of the scale model. The latter ratio has been derived from Fig. 3.17. Although there are considerable differences between both systems when looking at details, the similitude is that at low frequencies and high frequencies the sound transfer due to compression of the mountings predominates very strongly, whereas in a rather small frequency range around 250 Hz the "shear" transfer becomes important. If the analysis of the full scale system was limited to compression, the sound transfer for the 250 Hz octave band would have been underestimated with at least 9 dB.
Figure 6.9:
Level differences for source accelerations $\ddot{x}_m$ and $\ddot{y}_m$ close to raft stiffener and normalized according to Eq. (6.2).

Similarity for compression versus "bending":
To compare the relative importance of the sound transfer for mountings in compression and bending for the shipboard system and in the scale model of 3.4, both the source strengths and the ship transfer functions need to be considered. The similarity of the full scale and model scale mountings with respect to sound transfer caused by "bending" has been assumed without measuring $T_{M_x,\dot{u}}$ and $T_{M_y,\dot{v}}$ for the full scale mountings, but in analogy with the above mentioned similarity for compression and shear.

To compare the relative strength of rotational source acceleration components, the normalized level difference defined by

$$\Delta L = 10 \log \frac{\ddot{\beta}_m^2}{\ddot{x}_m^2 k_{B,r}^2}, \quad (6.2)$$

is shown in Fig. 6.9 both for scale model and the shipboard engine raft, where $k_{B,r}$ denotes the bending wave number of raft plate supported by the mountings. The results for both systems should be equal at corresponding frequencies for an accurately scaled model raft connected to an accurately scaled engine. However, although the thickness of the scale model raft plate to which the mountings had been fitted was scaled correctly, this was not the case for the scale model raft structure as a whole and moreover there was no scale model engine connected to it in the experiments of 3.4. This leads to differences in $\Delta L$, as defined according to Eq. (6.2), for ship and scale model respectively. The differences in Fig. 6.9 show that compared to the normal translational acceleration $\ddot{x}_m$ the rotation $\ddot{\beta}_m$ of the shipboard system is greater than in the scale model at low frequencies.
To compare the relative importance of ship transfer functions the normalized level difference given by

\[ \Delta L = 10 \log_{10} \frac{S_M}{S_F} \left( \frac{k_B^2 S}{k_B^2 s} \right) \]  \hspace{1cm} (6.3)

is shown in Fig. 6.10 both for the scale model section and the car-ferry, where \( k_B, s \) denotes the bending wave number of the seating top plate. Again the results of Eq. (6.3) should be invariant for the scale at corresponding frequencies in case of a correctly modelled ship. However, although the top plate of the scale model seating was scaled correctly, a lot of dissimilarities exist between the scale model seating and bottom structure and those in the ship. The results shown in Fig. 6.10 indicate that in 3 of the 5 octave bands the scale model structure of 3.4 is relatively more sensitive for torque excitation than the "ideal" scale model of the car-ferry. If similarity of the mountings is assumed, the combination of the data from Figs. 6.9 and 6.10 and a correction factor ratio \( C_y/C_z \) in analogy with \( C_y/C_z \), leads to the conclusion: it is only for the octave band with centre frequency 63 Hz that the sound transfer due to \( T_{M_y} / C \) is relatively stronger in the shipboard system than in the scale model. This increase in comparison with the sound transfer due to \( T_{F_z} / C \) is some 15 dB. However, in the scale model the sound transfer due to \( T_{F_z} / C \) exceeded that due to \( T_{M_y} / C \) by some 30-40 dB (see Fig. 3.18) for the corresponding octave band. Therefore, despite the increase of the relative importance of the sound transfer due to \( T_{M_y} / C \) with some 15 dB for the shipboard system, the sound transfer due to \( T_{F_z} / C \) may still be expected to be predominate at 63 Hz. The same holds for the other octave bands.

A similar analysis has been made for \( T_{M_x} / C \), leading to the same conclusion.

**Conclusion with respect to the accuracy of mounting path analysis:**

On the basis of the above similarity of the ship and the scale model system it is concluded that for the ship the sum of the sound transfer by \( T_{F_z} / C \) and that by \( T_{F_y} / C \) is representative of the total sound transfer, \( T_{F_z} / C \) being predominant in the 63, 125, 500 and 1000 Hz octave bands and \( T_{F_y} / C \)
in the 250 Hz octave band. There is a slight uncertainty that by neglecting other transfer modes of the mounting some underestimate might have been made for the octave band with centre frequency 250 Hz.

The possible inaccuracies of the above calculation may be expected to be of the same order of magnitude as those in Fig. 3.20, i.e. some 5 dB at low frequencies \( f_c \leq 250 \) Hz and some 3 dB at higher frequencies. Uncertainties of this order of magnitude do not affect the conclusions given at the end of 6.2 concerning the relative importance of the mounting path.

6.3.2 Path through the cavity below the engine

The sound transfer through the below engine cavity has been determined using Method B of Chapter 4 according to Eq. (4.2) which is repeated below.

\[
\frac{e_2^2}{P_1^2} = \frac{S_{\alpha}^{(2d)}}{Q_c} \cdot \frac{\lambda^2}{a_1} \cdot \frac{i_2^2}{\omega^2}
\]

Note:

Perhaps the use of Method A according to Eq. (4.1) would have been simpler.

However, this method had not yet been discovered at the time of the measurements.
For the reciprocal transfer function measurement the acceleration $a'$ on tank top has been measured on the underside of the tank top plates at 2 positions between the stiffeners. The difference in 1/3-octave band levels for $a'^2$ at these 2 positions was on the average smaller than 2 dB.

The radiation efficiency $\sigma^{(2d)}_1$ of the tank top has been measured in the scale model of 4.3. This was a measurement for a tank top without constrained layer damping, whereas in the ship the tank top had been damped. However, with an additional scale model experiment (see 6.4.2) it has been shown that the radiation efficiency of the shipboard tank top is not affected by the constrained layer damping. The value of $Q_c$ has been taken from the scale model result in Fig. 4.5d (result for 13 measuring positions), because the scale model cavity in the experiments of 4.3 forms an accurate model of the shipboard cavity.

Because the similarity of scale model and shipboard system it is assumed that the uncertainty in the analysis result in Fig. 6.2 for the sound transfer through the cavity is not larger than a few dB, as was the case for the scale model results in Fig. 4.6b.

6.3.3 Path through the air between the engine and the hull

The sound transfer for the airborne path to the hull has been estimated by a measurement with loudspeakers as substitution sources for the engine. Over the length of the diesel engine 4 loudspeakers have been placed close to the engine in the space between the engine and the hull. Inaccuracies of the octave band values for the transfer function $e^2/p^2$ introduced by the substitution source principle and by averaging $p^2$ over only 3 microphone locations in the engine room, are expected to be less than 3 dB.

6.3.4 Sound source identification for measurements at sea

To investigate whether the open circuit voltage "e" of the hull transducer during the trip at sea was solely determined by the nearest main diesel engine several checks have been made.
Analysis of the power spectral density function $G_{e,e}(f)$ showed clearly that the harmonics of the cylinder firing frequency of the main diesel engine(s) are predominant. Significant contribution of other sound sources (propellers, main reduction gear etc.) was not observed.

An independent indication that the predominant source is on tank top follows from an investigation on the attenuation of acceleration levels along the hull. The differences between the acceleration levels on the shell plating in the main engine room and those on the shell plating close to the receiver location are approximately equal for excitation by the loudspeakers in the engine room and that by the running engine(s).

A clear indication that the contribution of the nearest engine is much stronger than that from the more distant engine follows from some investigations using coherence functions. Simultaneous measurements have been performed of $e(t)$ and $a_1(t)$ and of $e(t)$ and $a_2(t)$, where $e(t)$ denotes the response of the hull transducer, $a_1(t)$ the acceleration on a foot of the nearest engine and $a_2(t)$ the acceleration at an identical foot position on the more distant engine.

The ordinary (squared) coherence function, between $e(t)$ and $a(t)$ is defined by

$$\gamma^2_{e,a}(f) = \frac{|G_{e,a}(f)|^2}{G_{e,e}(f) G_{a,a}(f)},$$  \hspace{1cm} (6.4)

where the "G's" are the one-sided power and cross-spectral density functions; see e.g. Bendat and Piersol /6,5/. The value of this coherence function, i.e.

$$0 \leq \gamma^2_{e,a}(f) \leq 1,$$  \hspace{1cm} (6.5)

is a measure for the degree of linear dependence between $e(t)$ and $a(t)$. If there is a perfectly linear relationship between $e(t)$ and $a(t)$ the value of $\gamma^2$ equals unity. If there is no linear relationship at all the value of $\gamma^2$ equals zero.
The fractions of $G_{e,e}$ which are linearly related to respectively $a_1$ and $a_2$ are called coherent output spectra. These are respectively $\gamma_{e,a_1}^2(f) \times G_{e,e}(f)$ and $\gamma_{e,a_2}^2 \times G_{e,e}(f)$. The octave band levels of these coherent output spectra are shown in Fig. 6.11. It is seen that the octave band levels of the fraction of $G_{e,e}$ which is coherent with $a_2$ on the more distant engine are significantly lower than those coherent with $a_1$. An essential condition for obtaining such differences is a low degree of linear dependence between $a_1$ and $a_2$. This was effectively realized by choosing different running speeds, i.e. 512 rev./min and 480 rev./min respectively. The large difference in Fig. 6.11 between the coherent output spectra is an indication of the athwartship attenuation along the ship bottom.

Note:
It has been checked that the level differences between the coherent output spectra are not a bias error due to use of finite records for the Discrete Fourier Transforms and caused by a larger time delay between $a_2(t)$ and $e(t)$ than between $a_1(t)$ and $e(t)$.

Another important aspect of sound source identification follows from a comparison in Fig. 6.11 between the octave band levels of the measured total output for "e" and the output levels coherent with $a_1$. The latter are much lower than the former. Above, it has been argued already that the measured total output is determined by the diesel engine noise. These two
facts imply that the fraction of "e" which is coherent with \( a_1 \), is only a small fraction of the diesel engine originated output "e". The physical explanation is that the coherence between the large number of sources in the 8-cylinder line engine is only partial due to statistical fluctuations in combustion processes, etc. Such a multiple sound source cannot be represented properly by a single acceleration response at one of the engine feet. This conclusion was confirmed by looking at the values of the coherence function \( \gamma^2_{a_1,a_3} (f) \) for two accelerations on the same engine two cylinder distances apart. These were typically smaller than 0.6 for \( 100 \text{ Hz} \leq f \leq 1 \text{ kHz} \). Results as in Fig. 6.11 illustrate that the use of ordinary coherence functions for e.g. separating diesel engine originated noise from propeller originated noise can lead to very large errors.

6.4 Discussion of some sound transfer properties of the multi-path system investigated

An estimate of the reduction of noise in the accommodation of the car-ferry due to application of the mounting system has been presented already in Fig. 6.2. The octave band values of this so-called insertion loss vary from 11-17 dB. This is of the same order of magnitude as on the ships previously investigated which were referred to in 6.1. It appeared that for \( f < 500 \text{ Hz} \) the sound transfer from the engine is dominated by one or more flanking paths other than the surrounding air. Therefore an improvement of the insertion loss values requires identification of these flanking paths and reduction of their influence. Investigations on identifying these paths are underway. At \( f \geq 500 \text{ Hz} \) the mounting path and the airborne path to the hull are also to be improved to increase the multi-path system insertion loss.

Whether an improved system is of any practical interest with relation to future ships of similar type is not considered in this present work. Instead the discussion concentrates on more general aspects of acoustical system properties, particularly with respect to the mounting and airborne paths. The strong and weak points in these transfer paths are discussed. Moreover it is indicated for which changes the sound transfer along these paths is most sensitive. Finally the maximum insertion loss which can be attained for an improved single stage mounting system for a similar engine is estimated.
6.4.1 *Resilient mounting path*

On basis of the foregoing results an assessment can be made of what the insertion loss values might be if only the mounting path is considered. Further which system modifications may improve or impair the present noise reduction qualities. Aspects which will be considered are respectively the influence of the admittances of engine raft and seating structures, the mounting properties and the influence of structural details on the relative importance of other vibration components than $\tilde{z}_m$. Moreover the insertion loss values of the present system and an improved variant are compared with data from literature on insertion losses of single and double stage mounting systems.

**Insertion loss values for the mounting path:**

The difference between the noise levels in the accommodation for the case of the rigidly mounted engine and those which are caused by the sound transfer solely through the resilient mountings is called the insertion loss (IL) for the mounting path. This insertion loss can be derived from Fig. 6.2. Its values are presented in Fig. 6.12 together with estimated values for the modified system with all 18 mountings at stiff engine raft and seating locations (see also Table 6.1).

IL at low frequencies, i.e. $f_c \leq 250$ Hz: The low frequency values of the insertion loss are approximately 50 dB and it is of practical interest to discuss under which conditions such values can be attained.
a. rigidly mounted:

\[
\text{engine: } Y_m \quad \xrightarrow{s,1} \quad \text{seating: } Y_s
\]

b. resiliently mounted:

\[
\text{engine: } Y_m \quad \xrightarrow{\text{mounting: } Y_1 = j\omega/s} \quad \text{seating: } Y_s
\]

Figure 6.13:
Block diagram of simplified model for the calculation of the insertion loss of a single stage mounting system.

To make some estimates a well-known very simplified calculation model is used; see e.g. Ungar and Dietrich /6.6/. The situation of uni-directional normal translation is considered and the effect is determined of inserting a single resilient mounting between engine and seating. The 2 situations, i.e. the rigidly and the resiliently mounted cases, are schematically represented in Fig. 6.13.

Considering the case of harmonic excitation and using complex notation, the point admittance of the engine is denoted by \( Y_m \) and that of the seating by \( Y_s \). The mounting is considered to be massless and is represented by a bilateral element with an admittance \( Y_1 \) corresponding with that of a spring with frequency independent stiffness \( s \), i.e. \( Y_1 = j\omega/s \).

Note:
Strictly speaking Fig. 6.13 is not representative of a resiliently mounted main engine if, as in the case of the car-ferry, besides resilient mountings an engine raft is also inserted, which is meant to increase the low frequency torsional stiffness of the engine after dynamically uncoupling from the ship. The point admittance of the raft at the mounting location will usually differ from that of the engine.

Linearity of the transfer system implies that for the hypothetical case of Fig. 6.13 the reduction of the sound pressure in the accommodation by inserting the resilient mounting is equal to the reduction of the force \( F_s \) exerted on the seating. The insertion loss can be found from (see Ref. 6.6)

\[
IL \overset{\text{def}}{=} 20 \log \left| \frac{F_s,1}{F_s,2} \right| = 20 \log \left| 1 + \frac{Y_1}{Y_m + Y_s} \right|. \tag{6.6}
\]
It is tentatively supposed that in a certain low frequency range both \( Y_m \) and \( Y_s \) show a spring-like behaviour and are of approximately equal magnitude, i.e. \( Y = Y_m = Y_s \). In practical cases this concerns frequencies well above the range where the engine plus raft vibrate as a rigid body and well below the range where local resonances occur in the raft and seating plates connected to the resilient mountings. For that case Eq. (6.6) can be replaced by

\[
\text{IL} = 20 \log \left| \frac{Y_j}{2Y} \right|, \quad \text{for } |Y_j| \gg |Y|
\]  

(6.7)

To attain, as in the case of the shipboard system, an insertion loss of 49 dB at 125 Hz with a resilient mounting stiffness: \( s = 8.4 \times 10^6 \text{ Nm}^{-1} \), an admittance value of

\[
|Y| = \frac{2\pi \times 125}{8.4 \times 10^6 \times 2 \times 10^{49/20}} = 1.7 \times 10^{-7} \text{ s kg}^{-1}
\]  

(6.8)

is needed, as can be derived from Eq. (6.7). An analysis of some measured data using a method which is discussed in Chapter 7, makes it probable that such admittance values have been realized on the very stiff car-ferry seatings and on the similarly constructed raft, both structures having top plate thicknesses of 0.045 m. Typical values for less stiff seating structures are of the order \( |Y| = 10^{-5} \text{ s kg}^{-1} \), see e.g. Burgtorf et al. /6.7/. For a mounting system with this larger value for \( |Y_m| \) and \( |Y_s| \), Eq. (6.6) predicts an insertion loss of only 13 dB at 125 Hz. It is of great practical interest to take notice of this large variation in possible insertion losses for single stage mounting systems.

Nelson and Burroughs /6.8/ present an insertion loss spectrum for typical single stage mounting systems with low natural frequencies, i.e. below 10 Hz. This result is shown in Fig. 6.12 also. It concerns an insertion loss estimate on a theoretical basis. Further details of the system properties are not given in Ref. 6.8. However, from the large discrepancy at low frequencies with our results for the car-ferry system it is obvious that this refers to a system with much larger engine and seating admittances. The car-ferry results prove that such a system can be improved considerably at low frequencies by diminishing the engine and seating admittances. Diminishing the engine admittance has the effect of lower

* see "Addition during printing" at the end of this chapter.
source levels on top of the resilient mountings and therefore lower forces exerted by the mountings on the seating. Diminishing the seating admittance reduces the energy flow into the ship structure due to the excitation by the resilient mountings.

IL at high frequencies, i.e. \( f > 500 \) Hz: At higher frequencies the insertion losses of the car-ferry system in Fig. 6.12 show a sudden drop of some 25 dB, even for the hypothetical case with all mountings at stiff locations. This decreased IL can be explained, at least partially, by the stiffening of the resilient mountings due to internal standing waves (see Fig. 6.4). Locating all mountings at stiff positions would increase IL considerably, especially at 500 Hz.

Double stage mounting system: In Ref. 6.8 also an estimate is presented for the insertion loss of a double stage mounting system. This result is shown in Fig. 6.12 as well. At the lower frequencies the estimated gain compared with the single stage mounting system of Ref. 6.8 is some 12 dB. To attain such a gain an intermediate structure (see Fig. 1.2) is needed whose mass is a considerable fraction of the engine mass. The low frequency results in Fig. 6.12 for the single stage car-ferry system suggest that an equal or larger gain can be obtained by spending some of the necessary extra weight for improving the impedance properties of the engine (raft) and the seating structures.

At \( f > 500 \) Hz the estimated insertion loss of the double stage mounting system is considerably larger than the values found for the car-ferry mounting system. A very practical way to reduce the high frequency sound transfer through the mounting path for a diesel engine as on the car-ferry might be the insertion of stiff resilient mountings between the engine and the raft. In this mounting system variant the engine raft acts at high frequencies as intermediate structure of a double stage mounting system, whereas there is no increase of weight as in the case of a low frequency double stage mounting system. The stiffness of the extra mountings should be chosen in such a way that the low frequency vibrational properties of the engine installation are unaffected to preserve the "stiffness increasing" function of the raft, whereas at \( f > 500 \) Hz there is a significant extra acoustical gain due to the transition from single to
double stage mounting system. At the frequency where this transition occurs, say 80 Hz, there would be some decrease of insertion loss, but still some 35-40 dB insertion loss would be retained.

**Sound transfer properties of the resilient mountings:**

An acoustical property of the block shape rubber mountings which has an unfavourable influence on the system investigated, is their significant stiffening at \( f \geq 500 \text{ Hz} \) due to internal standing waves, see Fig. 6.4. There are other mounting shapes, e.g. conical ones, which show a less pronounced stiffening. However, such mountings are as stiff in transverse directions as in compressional direction. With such mounting types it appears impossible in practice to design a mounting system for a medium-speed propulsion engine which both excludes resonances at low frequency excitation components \( (f < 10 \text{ Hz}) \) and behaves properly under seaway motions. However, for other types of engines sometimes such mounting shapes might be preferred, for acoustical reasons, instead of block shape mountings.

**Source accelerations and seating transfer properties:**

In the Figs. 6.6-6.10, source accelerations \( \ddot{y}_m, \ddot{z}_m, \ddot{\beta}_m \) and transfer functions \( S_{Fy}, S_{Fz}, S_{M} \) have been compared.

\[
\begin{align*}
\text{The 2 translational source accelerations } & \ddot{z}_m \text{ and } \ddot{y}_m \text{ are of the same order of magnitude; see Fig. 6.6. This is typically of diesel engines.} \\
\text{The ship transfer functions for excitation by } & F_{s,z} \text{ and } F_{s,y} \text{ are also of the same order of magnitude; see Fig. 6.7. However, this is a favourable characteristic of the particular type of seatings used in the car-ferry. In contrast, especially underneath auxiliary diesel engines sometimes lightweight seating structures are built which show for transverse excitation } & F_{s,x} \text{ and/or } F_{s,y} \text{ and for torque excitation } M_{s,x} \text{ and/or } M_{s,y} \text{ a sharp increase of sound transfer, especially in the frequency range } 100 \text{ Hz-1 kHz. In such cases the vibration modes of these seating structures have, at their resonance frequencies, high admittances especially for the above mentioned excitation components. Typical examples are column-shape seatings and so-called cantilever seatings. Especially for resiliently mounted machinery with a broadband excitation spectrum such seating designs should be avoided.}
\end{align*}
\]
Although for the mounting transfer path in the system investigated the contribution of rotational components seems negligible, it is interesting to discuss in more detail under which circumstances they become significant.

The relative strength of raft accelerations \( \ddot{x} \) and \( \ddot{y} \) is shown in Fig. 6.9. Except at 50 Hz the ratio of \( \ddot{y}^2 \) and \( \ddot{x}^2 \) is of the same order of magnitude as \( k_B^2 \) for the adjacent raft plate. It is worthwhile to notice that for a slender beam of infinite length the ratio of the real parts of the point admittances for respectively torque excitation \( M_y \) and lateral point force excitation \( F_z \) is equal to \( k_B^2 \), see Cremer et al. /4.9/. In Chapter 7 of the present thesis it is explained that on a structure with many resonant vibration modes within a bandwidth \( \Delta f \) the ratio between \( \ddot{y}^2 / \Delta f \) and \( \ddot{x}^2 / \Delta f \) is given by

\[
\frac{\ddot{y}^2}{\Delta f} \approx \frac{\text{Re} \left( \frac{Y_{M_y}}{Y_{F_z}} \right)}{\Delta f},
\]

(6.9)

where

- \( \ddot{y}^2 / \Delta f \) and \( \ddot{x}^2 / \Delta f \) denote time mean squared band filtered accelerations measured simultaneously at a certain location and generated by a distant sound source
- \( \text{Re} \left( \frac{Y_{M_y}}{Y_{F_z}} \right) / \Delta f \) denote frequency averages of real parts of point admittances for respectively point force excitation \( F_z \) and torque excitation \( M_y \) at the location where \( \ddot{x} \) and \( \ddot{y} \) are measured.

If Eq. (6.9) is valid for the engine raft structure then the results in Fig. 6.9 imply that \( \left( \text{Re} \frac{Y_{M_y}^\Delta f}{Y_{F_z}^\Delta f} \right) \) on the full scale engine raft top plate is approximately equal to the corresponding ratio on an infinite beam with the same thickness. Because \( k_B^2 = \omega / d \) (\( d \) denotes thickness of top plate) the value of \( 10 \log \frac{\ddot{y}^2}{\ddot{x}^2} \) might be expected to increase with approximately 3 dB when the frequency is doubled and such an increase of 3 dB might also be expected when the top plate thickness is halved.
A similar behaviour might be expected for the relative strength of the ship transfer functions $S_{M_y}$ and $S_{F_z}$. The latter hypothesis is indeed confirmed by unpublished measurement results for seatings with thinner top plates.

A practical consequence of this type of system properties can be illustrated by considering some hypothetical modifications of the scale model of 3.4. In Fig. 3.18 it is shown that at 4 and 8 kHz the sound transfer for the mounting transfer function $T_{M_y}^{\gamma}$ is respectively 8 dB and 3 dB less than the total sound transfer which was dominated by the transfer mode $T_{F_z}^{\gamma}$. Now the hypothetical situation is supposed that the thickness of the model raft and seating top plates is 2 mm instead of 6 mm and that the resilient mountings are turned over an angle of 90 degrees in such a way that $T_{M_y}^{\gamma}$ is replaced by $T_{M_x}^{\gamma}$ (see Fig. 3.15). In that case the sound transfer for the new "bending" mode $T_{M_y}^{\gamma}$ might be expected to increase by some 14 dB at 4 kHz and some 13 dB at 8 kHz, becoming in this way much more important than the sound transfer for $T_{F_z}^{\gamma}$. For the same reasons it might be expected that in the similar car-ferry system a decrease of plate thicknesses and a change of mounting orientation would impair the mounting path insertion loss, especially at $f > 250$ Hz.

The foregoing discussion illustrates clearly that when a shipboard resilient mounting system is to be designed for high insertion loss values, a thorough analysis of other vibration directions than the normal translation is indispensable.

6.4.2 Airborne sound transfer paths

Next the influence of airborne sound transfer on the multi-path system insertion loss of the car-ferry system is examined in detail. Moreover the effectiveness of some possible noise reduction measures in the cavity below the engine and on the engine room boundaries is considered and evaluated.
The difference between the noise levels in the accommodation for the case of the rigidly mounted engine and those which are caused by the sound transfer solely through the cavity below the engine may be called the insertion loss limit due to the cavity path. This limit can be derived from Fig. 6.2. Its values are presented in Fig. 6.14 (line d). In this figure also estimated values are presented for a cavity without the noise reduction measures which have been applied on board the car-ferry, i.e. without the insulating structure against the engine raft bottom and without constrained layer damping on the tank top (line b). The derivation of these results is discussed later.

In a similar way as for the cavity path the insertion loss limit due to the airborne path to the hull can be defined. This quantity follows also from Fig. 6.2 and it is also presented in Fig. 6.14 (line a). From comparison with line b it is seen that even if there would had been no other structural flanking paths, the noise reduction measures in the cavity would have been ineffective because of the predominance of the airborne sound transfer to the hull.
Noise reduction measures in the cavity:
The noise reduction of the insulating structure on the raft bottom (i.e. 5 mm steel plates upon 50 mm closed cells rubber) is equal to the difference between line (d) and line (c) in Fig. 6.14. It is seen that this measure is totally ineffective at $f_c < 500$ Hz.

To arrive at this conclusion a comparison has been made with the sound transfer in a cavity without noise reduction measures, below an identical engine on board a similar passenger and car-ferry. The only further difference between the 2 shipboard cavities is the height, viz. $0.15$ m for the cavity with noise reduction measures and $0.30$ m for the cavity without noise reduction measures.

In Fig. 6.15 for both ships level differences are shown between the mean squared sound pressure in the cavity and that in the space between engine and hull. Both quantities have been averaged over 3 microphone locations. It is seen that at $f_c < 500$ Hz the sound pressure levels in both cavities exceed to an equal extent those around the engine. Because the insulating structure is expected to become effective already at lower frequencies this result forms an indication of flanking sound transfer from the engine room into the cavity; see discussion in 4.6.3. At $f_c \geq 500$ Hz the reduction of the sound pressure levels in the cavity due to the insulating structure seems to be some 7-10 dB. These values follow from adding 3 dB to the differences in Fig. 6.15, taking into account the ratio of the cavity heights in agreement with 4.6.3. This estimated reduction of 7-10 dB has been subtracted from the results of line d in Fig. 6.14 to arrive at line c.
The reduction of the sound transfer due to constrained layer damping on tank top is estimated to be at least 10 dB independent of frequency. This estimate has been used to derive line b from line c in Fig. 6.14. To arrive at this conclusion the right hand side of Eq. (4.2) has been used. It has been investigated to what extent \( \sigma_{1}^{(2d)} \) of the tank top plates changes due to constrained layer damping and also estimation has been made of the reduction of the tank top accelerations \( a_{1}' \).

The influence of the damping layer on \( \sigma_{1}^{(2d)} \) has been investigated in a scale model (see note below). It appeared that the radiation efficiency remained unaffected by typical constrained layer damping.

The influence of the shipboard damping layer on the acceleration levels on the tank top underneath the engine has been estimated from comparison with the accelerations on the undamped tank top plate between engine and hull (see Fig. 6.1). For the reciprocal measurements the acceleration levels on the damped plates underneath the engine were as a function of frequency on the average some 15 dB lower than those on the tank top plates between engine and hull. This means that, after correction for some 5 dB propagation attenuation in athwartship direction across the outer seating, the acceleration reduction due to constrained layer damping is at least some 10 dB in the reciprocity experiment. When this result is combined with the conclusion of an unaffected radiation efficiency, it follows from Eq. (4.2) that the sound transfer through the cavity is reduced with at least some 10 dB by the damping layer.

Note:
The scale model experiments on the influence of damping on \( \sigma_{1}^{(2d)} \) have been performed with the test arrangement of Fig. 4.10: see discussion in 4.6.2.

For constructing the scale model ship bottom with the damped tank top, a "sandwich" plate (type Bondal N) has been used, composed of a 1.5 mm thick steel plate as tank top, a 0.3 mm thick visco-elastic core and a 0.5 mm thick steel top plate. The similarity of the dissipation properties of the scale model damping layer and the full scale damping appears from the observation that the scale model damping reduces the tank top accelerations also with some 10 dB for the whole frequency range. For the untreated tank top which was used for comparison, the loss factors were equal to typical shipboard values, i.e. decreasing from \( 10^{-2} \) at 500 Hz to \( 3.5 \times 10^{-3} \) at 8 kHz (corresponding full scale frequency range: 63 Hz - 1 kHz).
Noise reduction measures in the path to the hull:

In 6.4.1 it has been stated that the insertion loss for the resilient mounting path can be improved to some 25 dB at \( f \geq 500 \) Hz when all mountings are located against stiffeners (see Fig. 6.12). However, from Fig. 6.14, lines a and b, it is seen that such an insertion loss can be obtained only if the airborne sound transfer is also reduced. A reduction of 10 dB is needed for the airborne path to the hull compared to the present shipboard situation and moreover the damping layer in the cavity should be maintained. To attain a reduction of 10 dB for the path via the engine room it is necessary that at least 90 percent of the area of the engine room boundaries, i.e. of hull, tank top, deckhead and tween-deck, is covered with an effective insulating structure or damping layer. For the car-ferry under investigation this would mean for the whole engine room an area of 300-400 \( m^2 \).

Relative importance of cavity path and direct path to hull:

The conclusion that in the shipboard system investigated the noise reduction measures in the cavity are useless because of the predominance of the direct airborne path to the hull, even for the case that the present predominant flanking paths would have been suppressed, may not be generalized. Experience teaches that in cases where the cavity height is a few cm only, sound pressure levels in the cavity are found which exceed by 15-20 dB those in the rest of the engine room over a rather broad frequency range. In such situations it depends on the ratio between the cavity area and the area of the uncovered engine room boundaries whether the sound transfer through the cavity may become more important than the airborne sound transfer to the engine room boundaries. The cavity may form an important flanking sound transfer path for the resilient mountings even at low frequencies.

6.4.3 Upper limit of multi-path insertion loss of improved single stage mounting system

For the shipboard system investigated no improved multi-path system insertion loss can be attained unless the predominant flanking paths are identified and technical solutions for improving them are developed. However, it seems reasonable to expect that this will appear to be feasible
in general. Therefore it makes sense to summarize the results of the foregoing multi-path system analysis and to estimate the maximum insertion loss for two improved variants of the present car-ferry system. These are:

a. present predominant flanking paths improved
   - all resilient mountings at stiff raft and seating locations
   - without noise reduction measures in cavity
   - without extra sound insulating structures against hull, deckhead etc.

b. present predominant flanking paths improved
   - all resilient mountings at stiff raft and seating locations
   - constrained layer damping on complete tank top area and on tween deck
   - sound insulation against remaining engine room boundaries, which reduce the airborne sound energy flow into these structures with at least 12 dB at $f_c \geq 500$ Hz.

The estimated maximum insertion losses for these variants are shown in Fig. 6.16 together with the insertion losses for the present shipboard system. The maximum insertion losses are $> 30$ dB for $f_c \leq 250$ Hz and some 25 dB for $f_c \geq 500$ Hz.

As far as is known to the author such estimates on basis of shipboard measurements taking into account both the sound transfer via resilient mountings and via the air have not been published previously. This type of data is especially useful when the well-known alternative of a double stage mounting system with acoustic enclosure is considered. It should always be
remembered that the latter solution is especially meant for increasing the insertion loss limits for the mounting path and the flanking airborne paths. For the other flanking paths the same technological restrictions exist as in the case of a single stage mounting system. Therefore the choice of a double stage mounting system with enclosure can be justified only after a thorough analysis has proved that the mounting and/or airborne paths are predominant for an optimized single stage alternative. The discussion of the present section and the resulting estimates of insertion losses in Fig. 6.16 may illustrate the potentially high quality performance of single stage mounting systems and stress the high priority of flanking path treatment.

6.5 Summary

Sound transfer properties of mounting system investigated:

1. For the resilient mounting system of a medium-speed propulsion diesel engine on a car-ferry the multi-path insertion loss appears to be some 12-17 dB for octave bands with centre frequencies 63 Hz < f_c < 1 kHz. For f_c < 500 Hz the sound transfer through the mountings and the airborne paths is much below the total sound transfer. This implies the predominance of flanking paths such as piping, propeller shaft coupling etc. For f_c ≥ 500 Hz the calculated contribution for the mounting path is nearly equal to the total sound transfer.

Analysis methods:

2. For the analysis of the mounting path a simplified procedure has been applied in the same way as in the scale model experiments of 3.4. As has been found in 3.4 (Fig. 3.20) such a simplified approach may lead to uncertainties in the calculated results at low frequencies of some 5 dB. However, as can be seen from Fig. 6.2 the main conclusions for the shipboard system with respect to the importance of flanking paths are not affected by uncertainties of that order of magnitude.

3. For the analysis of the path through the air cavity below the engine Method B of Chapter 4 appeared practicable. Moreover, with some additional scale model experiments it appeared feasible to evaluate the effectiveness of the shipboard constrained layer damping on tank top for reducing the contribution of the cavity path.
General aspects of mounting path:

4. The analysis results show that if solely the sound transfer through the mountings is considered, insertion loss values of 50 dB can be obtained for $63 \, \text{Hz} \leq f_c \leq 250 \, \text{Hz}$. These values are significantly larger than estimates found in the literature. It is indicated that very low admittances both on the engine(raft) and the seatings are needed to attain such large insertion losses.

5. At $f_c \geq 500 \, \text{Hz}$ the insertion losses for the shipboard mounting path are some 15 dB. It is indicated that values of some 25 dB can be attained by locating all resilient mountings at stiff raft and seating positions. A characteristic property of the shipboard system investigated is that the sound transfer through the mountings is predominated by the normal translation at these higher frequencies. Critical analysis will show which structural modifications may change this situation. As an example it is calculated that the insertion loss of the mountings is reduced at high frequencies by increased sound transfer for rotation when the plate thicknesses of the engine raft and seating structures are decreased and when at the same time the resilient mountings are orientated in a different way. This result underlines the necessity of analyzing multi-directional vibrations when systems with high insertion losses are required.

General aspects of airborne paths:

6. The analysis results for the airborne paths show that both the direct path to the hull and the path via the cavity may form important flanking paths for the resilient mountings. The airborne path to the hull contributes much more to the noise in the accommodation than the mounting path for the octave bands with $f_c \leq 250 \, \text{Hz}$. For the cavity path this would have been the case also if no noise reduction measures had been applied. At higher frequencies the airborne path to the hull would become predominant if all mountings were positioned at stiff locations.

7. From comparison with a sister ship it appears that the insulating structure against the engine raft bottom is quite ineffective for reducing the airborne sound transfer through the cavity, especially for $f_c \leq 250 \, \text{Hz}$. Probably this is caused by flanking sound transfer into the cavity.
8. From combining shipboard and scale model measurements it appears that the shipboard constrained layer damping on tank top has diminished the sound transfer via the cavity path by some 10 dB over the whole frequency range of the investigations.

9. The relative importance of each airborne path depends on factors as the ratio between sound pressures in the cavity and engine room, the ratio between cavity area and area of engine room boundaries and on the noise reduction measures which have been applied. Especially in the case of a cavity height of a few cm the cavity may become a very important flanking path.

**Improved multi-path insertion loss for single stage mounting systems:**

10. The insertion loss of single stage resilient mounting systems for medium speed propulsion diesel engines as fitted in several of today's seagoing passenger and car-ferries is no more than some 10-15 dB for the octave bands with centre frequencies 63 Hz-1 kHz. It has however been shown that by a careful analysis of each part of the multi-path sound transfer mechanism, as outlined in this thesis, their relative importance may be determined. On that basis the most appropriate cost effective design can be adopted to increase the multi-path insertion loss. The upper limit is equal to or greater than 30 dB for \( f_C \leq 250 \) Hz and is some 25 dB for \( f_C \geq 500 \) Hz. In principle, double stage mounting systems with an acoustic enclosure can provide larger insertion loss values, especially at higher frequencies. However, the problem of identifying and reducing flanking sound transfer via the shaft and via pipes remains the same as in the case of a single stage mounting system.
Admittance of car-ferry seating:
The estimated insertion loss values for the mounting path of some 50 dB at low frequencies and the admittance estimation in Eq. (6.8) of some $1.7 \times 10^{-7} \text{ skg}^{-1}$ are respectively exceptionally high and exceptionally low compared to data known from literature. Burgtorf et al. /6.7/ mention $10^{-6} \text{ skg}^{-1}$ for a seating on board a tanker. Therefore, it seems useful to expand the foregoing chapter with some additional explanation.

Although the results of Fig. 6.2 and therefore the estimation of IL (mounting path) has some inaccuracy and although the use of Eq. (6.7) for estimating $|Y_s|$ and $|Y_{\text{m}}|$ is somewhat questionable for several reasons, viz. multi-directionality of low-frequency sound transfer in the rigidly mounted case, insertion of engine raft and tentative assumption on spring-like character of the admittance, it would be surprising if this would lead to an estimation error of 1 order of magnitude. A factor 10 in admittance values corresponds with 20 dB difference in insertion loss. However, the key argument for claiming the small admittance values for the car-ferry seating is not the estimation in Eq. (6.8), but the interpretation of shipboard measurements according to a method developed in the next chapter; see Eq. (7.2). On board the car-ferry measurements have been performed similar to those reported in Chapter 7 for the scale model ship section described in Chapter 3. During excitation on the web frame (see Fig. 6.1) accelerations have been measured on the undamped tank top plates between engine and hull and also on the adjacent seating top plate. On the tank top 2 positions near the centre of panels were used and on the seating 2 positions close to stiffeners and 1 position at the centre of a top-plate field. Because the dimensions of the tank top plates are in m: $1.1 \times 0.7 \times 0.012$, the fundamental "local" resonance occurs at approximately 85 Hz. Therefore at $f \geq 100$ Hz, averaged over the 2 tank top positions one may expect

$$\Delta f \approx Y_{\text{m}} \approx (2,3 \rho_p c_L d^2)^{-1} \approx 7,5 \times 10^{-5} \text{ skg}^{-1}.$$ Using Eq. (7.2),

$$\Delta f \approx \text{Re } Y_{FZ} \text{ at seating positions may be estimated from the ratio of squared accelerations on tank top and on the seating.}$$
Estimations for 1/3-octave bands near 125 Hz are shown in the Table below. Also estimations of $|Y_{FZ}|$ are shown. These estimations have been made on basis of measurements on the scale model section of Figs. 7.3 and 7.4. The level difference

$$10 \lg \left( \frac{|Y_{FZ}|}{\text{Re} \ Y_{FZ}} \right)$$

at position 4 (between stiffeners) is some 5 dB for 1/3-octave bands with centre frequencies 800-1250 Hz. On basis of reasonable similarity this level difference is also assumed for the car-ferry seating at the corresponding full scale frequencies 100-160 Hz. For the position above a seating stiffener (position no. 9 in Fig. 3.3) the corresponding "level" difference between modulus and real part is 2,5 dB. Also this level difference is assumed to be valid for the car-ferry.

Table 6.2: Estimations of 1/3-octave band values of $\text{Re} \ Y_{FZ}$ and $|Y_{FZ}|$

for the car-ferry seating.

<table>
<thead>
<tr>
<th>f_c</th>
<th>100</th>
<th>125</th>
<th>160</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>above Re $Y_{FZ}$</td>
<td>1</td>
<td>0.8</td>
<td>1.25</td>
<td>$x 10^{-7}$ skg$^{-1}$</td>
</tr>
<tr>
<td>$</td>
<td>Y_{FZ}</td>
<td>$</td>
<td>1.8</td>
<td>1.4</td>
</tr>
<tr>
<td>between Re $Y_{FZ}$</td>
<td>5</td>
<td>1.8</td>
<td>2.5</td>
<td>$x 10^{-7}$ skg$^{-1}$</td>
</tr>
<tr>
<td>$</td>
<td>Y_{FZ}</td>
<td>$</td>
<td>15.8</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Literature on measured admittances usually presents the modulus $|Y_{FZ}|$. It is seen in Table 6.2 that for the stiff locations on the car-ferry seating the moduli of the admittances are some $2 \times 10^{-7}$ skg$^{-1}$. As has been said above, this value is much smaller than data published for shipboard seatings. Indeed, the car-ferry seating is very stiff due to the 45 mm thick horizontal and inclined top plates (see Fig. 6.1). On the other hand sometimes literature data on $|Y_{FZ}|$ might be overestimations due to impedance head dimensions.
On heavy and stiff structures such data may be determined by local elasticity when the excitation area of the impedance head is too small /6.9, 6.10/. The "contact stiffness" at the boundary of a semi-infinite solid with a normal load uniformly distributed over a circular area with diameter $D$ is given by (see /6.11/):

$$s = \frac{3\pi^2}{32} \frac{DE}{(1-\nu^2)} , \quad (6.10)$$

where $E$ denotes modulus of elasticity and $\nu$ the Poisson ratio. For steel, where $\nu = 0.33$ the following approximation is valid:

$$s \approx ED \quad (6.11)$$

In the opinion of the author the admittance modulus relevant for structures underneath representative resilient mountings is that of an "area" admittance. The excitation area of the force transducer should be so large that the "local" elasticity is not determining $|Y_{Pz}|$ when this is not the case for excitation by the mounting. Therefore, measuring admittance moduli on heavy and stiff structures requires an impedance head with a sufficiently large contact area against the structure or, alternatively, a force transducer and a symmetrically placed pair of accelerometers at some distance. For example, the measurement with an impedance head of an "area" admittance of some $2 \times 10^{-7}$ skg$^{-1}$ at 160 Hz on a steel structure requires a "contact stiffness" admittance smaller than, say, $7 \times 10^{-8}$ skg$^{-1}$, which implies $D > 0.066$ m. At higher frequencies even larger diameters are required. Measuring Re $Y_{Pz}$ presents no severe problems because the "contact stiffness" dissipates no energy. The only complication might be an increased sensitivity for phase mismatch errors due to increased "reactivity" of the admittance.
7. SIMPLE EXPERIMENTAL METHOD FOR AIDING THE DESIGN OF IMPROVED MULTI-DIRECTIONAL VIBRATION ISOLATION

The structureborne sound transfer through resilient isolators into a ship can be reduced by diminishing the real parts of the point admittances of the ship structure at isolator locations. A simple engineering method is described and tested for estimating frequency bandwidth averages of the real parts of point admittances of 6 degrees of freedom of shipboard structures.

7.1 Introduction
Effective isolation of structureborne sound with resilient elements requires large impedance mismatches both on source and receiver side of these elements. If this condition is met on the receiver side, the resilient elements act upon the receiving structure as structureborne sound sources with low internal impedance, i.e. as "force sources". In that case the power injection by the isolators into a ship can be described with Eq. (2.3) for the general, i.e. multi-directional, case. Diminishing the real parts of point admittances "Re Y" of the ship structure at isolator locations would reduce the sound transfer.

With respect to the reduction of the sound transfer from resiliently mounted shipboard machinery, the collection of multi-directional admittance data might be useful, for example, for comparing the acoustic quality of different seating types and for designing good pipe systems. The following may illustrate this.

Seatings: For comparison of the acoustic quality of different seating types the "Re Y's" might be a more practical measure than ship transfer functions to a distant receiver location. The reason is that the latter quantities depend on the admittance characteristics of the seating as well as on the sound transfer system between seating and receiver location. The latter dependence complicates a comparison of seatings located in different parts of a ship or in different ships.

Pipes: In cases where it is necessary to avoid that the sound transfer along pipes exceeds that via the resilient support mountings, the "Re Y's"
of the pipe system on the ship side of a flexible bellows or hose should not exceed certain values. This may require a careful design of pipe supports, because without special measures pipes may be very susceptible to excitation in any direction. The admissible upper limits of the "Re Y's" can be estimated from Eq. (2.3) using the requirement that the power injection into the pipe is smaller than that into the seating. For this calculation source accelerations and blocked transfer functions of the support mountings and bellows are needed as well as the admittance characteristics of the seating. For designing appropriate pipe seatings, scale models might be used.

However, until now the above mentioned applications have been hampered because of the difficulty of multi-directional admittance measurements. A detailed discussion of measurement techniques is presented by Sainsbury /1.8/. In contrast to the relatively simple admittance measurement for point force excitation in normal direction, the measurement for in-plane forces and for torques is quite complicated. Moreover, on board ships at many locations of interest the space is lacking for the appropriate measuring equipment. There is a need therefore to develop a simple estimation method as described in this chapter; see also Verheij /7.1/.

7.2 Method for estimating frequency bandwidth averages of real parts of point admittances of six degrees of freedom

The quantity Re $Y_{Fz}$ for point force excitation in normal direction may be determined from the measurement of force $F_z$ and of acceleration $\ddot{z}$ and by using some special signal processing. With the advent of 2-channel Fourier analyzers, signal processing in the frequency domain has become widely available. For measurement of Re $Y_{Fz}$ a cross-spectral method for measuring energy flow, as proposed by Fahy and Pierri /7.2/, can be used as follows:

$$\text{Re } Y_{Fz} = \frac{\partial P_{FZ} / \partial f}{\partial G_{Fz,Fz}^z} = \frac{\partial G_{Fz,Fz}^zz}{\partial G_{Fz,Fz}^z} ,$$  

(7.1a,7.1b)

where $P_{Fz}$ denotes the energy flow (or "power injection") into the structure and the $G$'s the power- and cross-spectral density functions.
For the measurement of \( \text{Re} \ Y_{F_1} \) for other excitation components \( F_i \), the engineering method developed in this chapter uses normal point force \( F_z \) excitation as a substitution source, because for this excitation \( \text{Re} \ Y \) can be measured relatively simply.

**Formulation of the method and its theoretical basis:**
The proposed engineering method is as follows. A structure is excited with a broadband sound source which causes a large number of vibration modes to resonate within a frequency bandwidth \( \Delta f \); see e.g. the left-hand side in Fig. 7.1. The mean squared band-filtered accelerations are measured at 2 positions, labeled respectively as no. 1 and no. 2. These positions are close together but at sufficiently large distance from the sound source. At position no. 1 the squared band-filtered translational acceleration in normal direction is considered, i.e. \( z_1^2 \), and at position no. 2 the squared acceleration \( a_{2i}^2 \) for an arbitrary direction, called here \( i \)-direction. In addition at position no. 1 the frequency bandwidth average of \( \text{Re} \ Y_{F_1} \) is measured, i.e. \( \text{Re} \ Y_{F_1 z}^{\Delta f} \). Then an estimate of \( \text{Re} \ Y_{2i z}^{\Delta f} \) for the excitation force or torque which corresponds with \( a_{2i}^2 \) follows from:

\[
\text{Re} \ Y_{2i z}^{\Delta f} \approx \frac{a_{2i}^2}{z_1^2} \frac{\Delta f}{\Delta f} \cdot \text{Re} \ Y_{F_1 z}^{\Delta f}
\]

with for the accelerations time mean squared values.

**Note:**
In this chapter no special notation is used for time averaging. All squared quantities denote time mean squared quantities.

In words Eq. (7.2) implies that the ratio of the frequency bandwidth averages of the real parts of point admittances at adjacent positions, or at the same position but for different directions, can be found from the ratio of the corresponding accelerations at these positions in case of excitation by a distant sound source. According to Eq. (7.2) the difficult measurement of an admittance \( Y_{F_2 i} \) for in-plane force excitation or for torque excitation, may therefore be replaced by the much simpler measurement of an in-plane acceleration or rotational acceleration \( a_{2i}^2 \) and by an admittance measurement for excitation with a point force in normal direction.
Figure 7.1:
Shipboard situations for illustrating the underlying principles of the estimation method. The reciprocity principle states that the transfer functions for excitation on the deck (on the left) are equivalent with those for excitation in the engine room (on the right).
A substitution source principle states that the deck accelerations for subsequent excitation at positions no. 1 and no. 2 (on the right) are approximately equal provided the power injection is kept equal.

For deriving Eq. (7.2) the assumed shipboard situations of Fig. 7.1 are used.

In the situation on the left-hand side a deck is excited by a point force $F_d$, which has a flat power spectral density function. Position no. 2, for which estimates are needed of multi-directional admittances $\text{Re} \, Y_{21}^{\Delta f}$, is on top of a seating in the engine room. The auxiliary position no. 1 is on a bottom plate close to the seating. Strictly speaking positions no. 1 and no. 2 may coincide, but in practice the use of an auxiliary position on a rather thin plate may be advantageous from an experimental viewpoint, e.g. in the case that at position no. 2 a resilient mounting is installed. In that case accelerations $a_{21}$ on the receiver side of the isolator are to be determined with suitable accelerometer configurations around (see Fig. 2.7) or underneath the isolator.

On the right-hand side of Fig. 7.1 the situation is shown in which the structure is excited in the engine room, subsequently with $F_{1z}$ and $F_{21}$. Both excitation forces have a flat spectral density function. Now it is
hypothetically stated that the squared acceleration \( a_d^2 \) at the deck position is approximately equal for both excitations, provided the injected power is equal at positions no. 1 and no. 2, i.e.

\[
\left[ a_d^2, \Delta f / P_{F_{1z}}, \Delta f \right] \sim \left[ a_d^2, \Delta f / P_{F_{2i}}, \Delta f \right].
\]  

(7.3)

For this equation to be valid it is necessary that for each excitation \( F_{1z}, \Delta f \) and \( F_{2i}, \Delta f \) the power is fed into the same set of vibration modes of the transfer system. Substitution in Eq. (7.3) of

\[
P_{F_{1i}, \Delta f} = P_{i, \Delta f} Re \ Y_{F_{1i}}
\]

leads to

\[
\left[ a_d^2, \Delta f / P_{F_{1z}}, \Delta f \right] / Re \ Y_{F_{1z}} \sim \left[ a_d^2, \Delta f / P_{F_{2i}}, \Delta f \right] / Re \ Y_{F_{2i}}.
\]  

(7.5)

Then Eq. (7.2) follows from Eq. (7.5) when the transfer functions between brackets are replaced by those for the equivalent reciprocal experiments on the left-hand side of Fig. 7.1, viz. \( \left[ a_d^2, \Delta f / P_{F_{1z}}, \Delta f \right] \) and \( \left[ a_d^2, \Delta f / P_{F_{2i}}, \Delta f \right] \). The reciprocity relations are exact for broadband signals under the condition that the power spectral density functions of \( F_{1z} \), \( F_{2i} \) and \( F_d \) have the same frequency dependence; see Appendix A.

With similar arguments Eq. (7.2) may be derived also for other situations than those in Fig. 7.1. For instance, an underwater sound source could have been used instead of an exciter on a deck.

The proposed method of Eq. (7.2) is based upon the hypothesis that a response at a distant receiver location due to excitation in the proximity of position no. 2, is largely invariant for the excitation manner or position as long as the power injection into the structure is kept equal. The formulation "...largely invariant..." already indicates that certain conditions should be met and that the nature of this substitution source method is statistical and not exact. Studies on the acoustics of reverberant structures and spaces, either separate or coupled, have produced many results which are equivalent to or based upon the above mentioned hypothesis. As an example the substitution source method in section 4.2 of this thesis can be mentioned.
Practical requirements:
For practical applications it would be most convenient if there are no special requirements with respect to the environment of the distant sound source which is used for determining \( a_{z_2, \Delta f}^2 / \Delta f \). Both a reverberant environment (e.g. exciter on deck or hull) and a semi- or non-reverberant environment (e.g. underwater sound source in harbour) should be admissible. Moreover, the positions no. 1 and no. 2 should not be located necessarily on a highly reverberant sub-system, e.g. at the centre of large plate fields. On the contrary, many locations of interest are on seatings which have been composed of elements with a very low modal density in the frequency range of interest.

Therefore, for practical reasons preferably the only general conditions for obtaining acceptable accuracy should be that the complete ship section between distant source and accelerometer locations contains sufficient resonant vibration modes in the frequency bands of the measurements and that the responses \( z_{1, \Delta f}^2 \) and \( a_{z_2, \Delta f}^2 \) are governed by the same set of modes. A very practical method to check whether these conditions are fulfilled is to look at the scatter of \( a_{z_2, \Delta f}^2 / \Delta f^2 \) for different distant excitation locations and/or excitation directions.

7.3 Some theoretical considerations with respect to the statistical nature of the method
The purpose of the following theoretical analysis is to elucidate that a sufficiently large number of resonant vibration modes in the transfer system is required for the proposed method to be accurate. It should be explained that in practical applications, generally speaking, only qualitative indication on statistical accuracy can be obtained from the scatter of \( a_{z_2, \Delta f}^2 / \Delta f^2 \) for excitation at different distant sound source locations.

7.3.1 Infinite plate without stiffeners
First a thin homogeneous plate without stiffeners is investigated to determine under which conditions, in analogy with Eq. (7.2),
where \( \ddot{a} \) denotes a rotational acceleration about an arbitrary in-plane axis and \( Y_M \) the corresponding torque admittance. Both quantities belong to the same position as \( \ddot{z} \) and \( Y_F \), which position is chosen as the origin of a Cartesian coordinate system; see Fig. 7.2. Quantities which belong to this origin are indicated with subscript "1", e.g. \( \ddot{u}_1; \ddot{z}_1; M_1 = M_x; F_1 = F_z \).

Let the plate be excited with a stochastic force \( F_2 = F_{2z} \) at a position no. 2 with cylinder coordinates \( (r = R, \theta) \). The distance \( R \) be much larger than the bending wavelength in the plate, i.e. \( k_B R >> 1 \). The power spectral density function \( G_{F_2, F_2} \) be independent of frequency.

**Single frequency, single source:**

For an infinitesimal bandwidth \( \Delta f \) the left-hand side of Eq. (7.6) may be replaced by

\[
\lim_{{\Delta f \to 0}} \frac{\ddot{u}_1^2}{\ddot{u}_1^2} = \frac{G_u}{G_{u_1, \ddot{u}_1}}, \quad (7.7)
\]
and the right-hand side by

\[
\lim_{\Delta f \to 0} \frac{\text{Re} \ Y_{M_1}}{\Delta f} \cdot \frac{\text{Re} \ Y_{M_1}'}{\Delta f} = \frac{\text{Re} \ Y_{F_1}'}{\text{Re} \ Y_{F_1}}. \tag{7.8}
\]

Because of symmetry the ratio in Eq. (7.8) is independent of the orientation of the x-axis. For a thin plate in vacuum it is known that

\[
\frac{\text{Re} \ Y_{M_1}}{\text{Re} \ Y_{F_1}} = \frac{k_B^2}{2}, \tag{7.9}
\]

see Cremer, Heckl, Ungar /4.9/, p. 279. Now it is easy to prove that for averaging over excitation from different directions

\[
\frac{1}{2\pi} \int_0^{2\pi} G_{\alpha_1', \alpha_1}(\theta) \frac{G_{\alpha_1', \alpha_1}(\theta)}{G_{\alpha_1', \alpha_1}(\theta)} d\theta = \frac{k_B^2}{2}, \tag{7.10}
\]

and thus, because of Eq. (7.9), that

\[
<< \frac{G_{\alpha_1', \alpha_1}}{G_{\alpha_1', \alpha_1}} >> = \frac{\text{Re} \ Y_{M_1}}{\text{Re} \ Y_{F_1}}, \tag{7.11}
\]

where <<...>> denotes averaging over \(\theta\).

The proof of Eq. (7.10) is as follows. For excitation at large distance \(R\) the incident wave may be considered as a straight-crested free wave in the region near the origin. Then for a certain direction \(\theta\) the spectral density of the rotational acceleration \(\ddot{\alpha}_1\), can be written as

\[
G_{\ddot{\alpha}_1, \ddot{\alpha}_1}(\theta) = G_{\ddot{\alpha}_1, \ddot{\alpha}_1}(\theta) \cdot \cos^2 \theta = k_B^2 G_{\ddot{\alpha}_1, \ddot{\alpha}_1}(\theta) \cos^2 \theta, \tag{7.12}
\]

where \(G_{\ddot{\alpha}_1, \ddot{\alpha}_1}(\theta)\) denotes the power spectral density of the rotational acceleration in the plane through the excitation position and the z-axis. Because \(G_{\ddot{\alpha}_1, \ddot{\alpha}_1}(\theta)\) is independent of \(\theta\), Eq. (7.10) is proved by substitution of the right-hand side of Eq. (7.12) for \(G_{\ddot{\alpha}_1, \ddot{\alpha}_1}(\theta)\).
Note:

For a fluid loaded plate Eq. (7.10) is also valid, with \( k_B \) the wavenumber for the fluid loaded plate. Eq. (7.9) is also valid at frequencies far below the critical frequency, when the radiated power is much less than the power injected into the plate. To prove this, e.g. work by Crighton /7.3/, Eqs. (22) and (25) and by Heckl /7.4/, Eq. (28) can be used.

Single frequency, diffuse field:

Another assumed situation is that the stochastic field in the region near the origin is diffuse, i.e. isotropic (the same wave intensity is incident from all \( \theta \)) and homogeneous (the expected values for \( G_{\alpha,\alpha} \) and \( G_{z,z} \) are the same for all positions near the origin because waves incident from different \( \theta \) are uncorrelated). Such a situation can be created by a uniform distribution of uncorrelated point force sources of equal strength, located on a circle with large radius \( R \). The ratio \( G_{\alpha,\alpha} / G_{z,z} \) may now be written as

\[
\frac{G_{\alpha,\alpha}}{G_{z,z}} = \frac{2\pi}{\int_0^{2\pi} G_{\alpha,\alpha}(\theta) d\theta} \frac{\int_0^{2\pi} G_{z,z}(\theta) d\theta}{2\pi}.
\]

(7.13)

Because \( G_{z,z}(\theta) \) is independent of \( \theta \), substitution of the right hand side of Eq. (7.12) into Eq. (7.13) and use of Eq. (7.9) leads to

\[
\frac{G_{\alpha,\alpha}}{G_{z,z}} = \frac{\text{Re } Y_{M_1}}{\text{Re } Y_{F_1}}.
\]

(7.14)

Therefore it can be concluded that for an infinite plate the ratio \( \text{Re } Y_{M_1} / \text{Re } Y_{F_1} \) can be determined from \( G_{\alpha,\alpha} / G_{z,z} \), either under diffuse field conditions or with single sound sources by averaging over \( \theta \) with uniform weighting in the interval \( (0,2\pi) \).

Note:

The result of Eq. (7.11) can also be derived without knowledge of the result in Eq. (7.9). The principle of a derivation which uses energy flow balances and the reciprocity principle, has been described by Smith /7.5/ for a similar purpose.
Moderate bandwidth, single source:

The ratio $\frac{\tilde{\alpha}_1^2, \Delta f}{\tilde{\alpha}_1, \Delta f}$ for excitation with a single source $F_2$, but averaged over source positions uniformly distributed over a circle, is written as

$$\frac{\tilde{\alpha}_1^2, \Delta f}{\tilde{\alpha}_1, \Delta f} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\tilde{\alpha}_1^2, \Delta f(\theta)}{\tilde{\alpha}_1, \Delta f(\theta)} d\theta = \frac{\int_{f_1}^{f_2} \frac{k_B}{2} G_{\tilde{z}_1, \tilde{z}_1} df}{\int_{f_1}^{f_2} G_{\tilde{z}_1, \tilde{z}_1} df}. \quad (7.15)$$

The power spectral density of response $\tilde{z}_1$ is related to that of the excitation force $F_2$ by

$$G_{\tilde{z}_1, \tilde{z}_1} = \frac{k_B^3}{32\pi \rho m''} G_{F_2, F_2}, \quad (7.16)$$

where $m''$ denotes plate mass per unit area; see Cremer et al./4.9/, p. 264, Eqs. (63) and (65a). With the assumption $G_{F_2, F_2}$ independent of frequency and with $k_B = (\omega^2 m''/B')^{1/4}$, substitution of Eq. (7.16) into Eq. (7.15) leads to

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{\tilde{\alpha}_1^2, \Delta f(\theta)}{\tilde{\alpha}_1, \Delta f(\theta)} d\theta = \frac{1}{2} \int_{f_1}^{f_2} \frac{k_B^3 df}{f_1} = \frac{5\pi}{7} \sqrt{\frac{m''}{B'}} \frac{\left(\frac{f_2}{2} - \frac{f_1}{2}\right)^{7/2}}{\left(\frac{f_2}{2} - \frac{f_1}{2}\right)^{7/2}}, \quad (7.17)$$

where $B'$ denotes plate bending stiffness per unit width.

Because $\text{Re} Y_F$ is independent of frequency and using Eq. (7.9), the right-hand side of Eq. (7.6) can be written as
For moderate bandwidth $\Delta f$ the values of Eqs. (7.17) and (7.18) are approximately equal. For example for 1/3-octave bands $1,14 f_1$ versus $1,13 f_1$ is found and for octave bands $1,58 f_1$ versus $1,50 f_1$.

Moderate bandwidth, diffuse field:

For diffuse field conditions as defined above, the ratio $\tilde{a}_1, \Delta f / \tilde{z}_1, \Delta f$ may be written as

\[
\tilde{a}_1, \Delta f / \tilde{z}_1, \Delta f = \frac{1}{\Delta f} \int_{f_1}^{f_2} \frac{G_{\alpha_1, \bar{z}_1} (\theta) \sqrt{m''} \Delta f}{2 \sqrt{B r}} \, df = \pi \sqrt{\frac{m''}{B r}} \frac{f_2 - f_1}{2}. \quad (7.18)
\]

This is the same result as in Eq. (7.15) for averaging over single sources in different directions.

The foregoing analysis shows that for determining the ratio $(\text{Re } Y_{\Delta f}/\text{Re } Y_{\Delta f})$ from $(\tilde{a}_1, \Delta f / \tilde{z}_1, \Delta f)$, averaging over distant sound source locations is equivalent with measuring under diffuse field conditions. In the next section it is shown that this equivalence is not generally valid for finite systems.

7.3.2 Finite plate without stiffeners

Next the conditions for the validity of Eq. (7.6) are investigated for a finite plate without stiffeners and with arbitrary shape and boundary conditions. Again the quantities of Eq. (7.6) belong to a position which is
indicated with subscript "1". Sound source positions which are used for determining \( \ddot{u}_1, \Delta f/\ddot{z}_1, \Delta f \) are again indicated with subscript "2".

**Single frequency, single source:**

For an infinitesimal bandwidth \( \Delta f \) the left-hand side of Eq. (7.6) can be replaced by the ratio of power spectral densities, as in Eq. (7.7). The relation is investigated between \( \text{Re } Y_{M1x}/\text{Re } Y_{F1z} \) and

\[
\left< \frac{G_{\alpha_{1}}}{G_{\alpha_{1}'}} \right> \triangleq \frac{1}{S} \int_S \left| \frac{G_{\alpha_{1}}}{G_{\alpha_{1}'}} \right| dx_2 dy_2,
\]

where \( S \) denotes the plate area and \( \left< \ldots \right> \) averaging over all possible excitations \( F_2 \equiv F_z(x_2,y_2) \) on the plate.

The power injection into a weakly damped plate for arbitrary excitation can be related to the spatially averaged spectral density of \( \ddot{z}_2 \) according to

\[
\frac{\partial P}{\partial f} = \frac{n_m''}{\omega} \int_S G_{z_2,\ddot{z}_2} \, dx_2 dy_2,
\]

where \( n \) denotes the "apparent" loss factor which represents the rate of vibrational energy loss in the plate material, at the plate boundaries and by sound radiation, see Cremer et al./4.9/, p.297, Eq. (110). Let for subsequent excitation at position no. 1 with a point force \( F_1 \equiv F_z(x_1,y_1) \) and a torque \( M_1 \equiv M_x(x_1,y_1) \) the plate acceleration be denoted by respectively \( \ddot{z}_2 \) and \( \ddot{z}_2' \). Then with Eqs. (7.1a) and (7.21) the ratio of the real parts of the admittances can be written

\[
\frac{\text{Re } Y_{M1}}{\text{Re } Y_{F1}} = \frac{\partial P/\partial f}{\partial P/\partial f} = \frac{\int_S G_{z_2,\ddot{z}_2} \, dx_2 dy_2}{\int_S G_{z_2,\ddot{z}_2} \, dx_2 dy_2}.
\]
Because of reciprocity (see Appendix A), Eq. (7.22) may be replaced by

$$
\frac{\text{Re } Y_{M_1}}{\text{Re } Y_{F_1}} = \frac{\iint S \frac{G_{\alpha_1, \beta_1}}{G_{F_2, F_2}} \text{d}x_2 \text{d}y_2}{\iint S \frac{G_{\alpha_1, \beta_1}}{G_{F_2, F_2}} \text{d}x_2 \text{d}y_2}.
$$

(7.23)

There is no loss of generality by assuming $G_{F_2, F_2}$ independent of the excitation position, so that Eq. (7.23) is equivalent with

$$
\frac{\text{Re } Y_{M_1}}{\text{Re } Y_{F_1}} = \frac{\langle \langle G_{\alpha_1, \beta_1} \rangle \rangle}{\langle \langle G_{\alpha_1, \beta_1} \rangle \rangle}.
$$

(7.24)

From comparing Eqs. (7.20) and (7.24) it appears that $\text{Re } Y_{M_1}/\text{Re } Y_{F_1}$ can only be estimated from averaging $G_{\alpha_1, \beta_1}/G_{\alpha_1, \beta_1}$ over many source positions $(x_2, y_2)$ provided

$$
\frac{\langle \langle G_{\alpha_1, \beta_1} \rangle \rangle}{\langle \langle G_{\alpha_1, \beta_1} \rangle \rangle} \approx \frac{\langle \langle G_{\alpha_1, \beta_1} \rangle \rangle}{\langle \langle G_{\alpha_1, \beta_1} \rangle \rangle}.
$$

(7.25)

The approximate equality in Eq. (7.25) is only valid when the scatter of the integrand in Eq. (7.20) is small. This can be seen from writing

$$
\langle \langle \frac{G_{\alpha_1, \beta_1}}{G_{\alpha_1, \beta_1}} \rangle \rangle = A,
$$

(7.26)

and

$$
\frac{G_{\alpha_1, \beta_1}}{G_{\alpha_1, \beta_1}} = c(x_2, y_2)A.
$$

(7.27)
Substitution of Eq. (7.27) into Eq. (7.24) leads to

\[ \frac{\text{Re } Y_{M_1}}{\text{Re } Y_{F_1}} = A \cdot \frac{\int_S c(x_2, y_2) \cdot G_{z_1, z_1} \, dx_2 \, dy_2}{\int_S G_{z_1, z_1} \, dx_2 \, dy_2} \]  

(7.28)

If \( c(x_2, y_2) \) differs not too much from unity (except perhaps for a small region near \((x_1, y_1)\)), which then may be excluded from all integrals in Eqs. (7.20) through (7.28), then

\[ \frac{\text{Re } Y_{M_1}}{\text{Re } Y_{F_1}} \approx A = \ll\ll \frac{G_{z_1, z_1}}{G_{z_1, z_1}} \gg \gg . \]  

(7.29)

The fundamental difference with the foregoing analysis of the infinite plate is that averaging over a large number of different sources positions \((x_2, y_2)\) leads not in all cases to a reliable estimate of \( \text{Re } Y_{M_1}/\text{Re } Y_{F_1} \). The small scatter which is required for \( G_{z_1, z_1}/G_{z_1, z_1} \) is equivalent with requiring approximate diffuse field conditions for each excitation position \((x_2, y_2)\). This is only obtained when a sufficiently large number of plate vibration modes resonates.

**Moderate bandwidth:**

For the case of moderate bandwidth \( \Delta f \) the same requirement can be derived as above, but in this case the requirement will be met at lower frequencies because of the larger number of resonant vibration modes which is excited simultaneously.

**7.3.3 Ship structure**

A similar analysis as above seems very difficult for a ship section of representative complexity. However, an extension of the foregoing analysis might be the analysis of a finite flat plate (or several coupled plates) with stiffeners in two perpendicular directions. Energy dissipation might occur at the plate boundaries (representing the energy losses in the rest of the ship) and by sound radiation from the plate into the surrounding medium.
At high frequencies, where the plate bending wavelengths are smaller than the length and width dimensions of the individual panels, a stiffened plate can be considered as a system of coupled reverberant panels. Then for each distant exciter location, either in the ship or in the water, a more or less diffuse field is generated on each of the individual panels.

In that case it might be expected that for a position near the centre of such a panel and for moderate bandwidth \( \Delta f \)

\[
<< \frac{\tilde{w}^2}{\Delta w} >> \approx \left( \frac{\text{Re } Y_M}{\text{Re } Y_F} \right) \text{ infinite plate without stiffeners} = \frac{k_B^2}{2}. \tag{7.30}
\]

Indeed this has been observed by the author both on a tank top panel of the scale model section described in 3.2 and on a hull panel of the laboratory water tank described in 5.4 and 7.4.2.

At low frequencies where the plate bending wavelengths are much larger than the dimensions of the individual panels, the stiffened plate behaves as an orthotropic plate.

**Orthotropic plate:**

For the low frequency case of orthotropic plate behaviour it might be derived along the same lines as above, that the validity of the approximation in Eq. (7.6) for averaging over successive excitation at different source locations requires a sufficiently small scatter of \((\tilde{w}^2 / \Delta w)\) for these different excitations. Again this is equivalent with requiring a "diffuse" field condition and therefore the excitation of a sufficiently large number of resonant vibration modes. However, in this case the requirement refers to orthotropic plate modes rather than to vibration modes which dominate locally on the subpanels.

**Other wave types than bending waves:**

The above theoretical discussion was limited to acceleration responses \( \ddot{z} \) and \( \ddot{u} \) that correspond to bending wave fields in a plate. However, the question is whether Eq. (7.3) is valid for arbitrary pairs of responses and excitations. There are good reasons for expecting this to be the case for
the rather complex structures such as ships. Assume, for example, the extreme case that for a hull plate the in-plane acceleration $\ddot{y}^2_{\Delta f}$ and the corresponding $\text{Re} \ Y^2_{y \Delta f}$ replace the above $\ddot{z}^2_{\Delta f}$ and $\text{Re} \ Y^2_{z \Delta f}$. Then both $\ddot{y}^2_{\Delta f}$ and $\ddot{z}^2_{\Delta f}$ on the hull in case of distant excitation are expected to be governed by the same resonant transfer system modes, because the vibration modes of complex structures with edges etc., include all wave types due to the coupling at structural discontinuities. This has been explained qualitatively in Chapter 5 with the aid of a simple pipe configuration. For the same reason the energy transport through the ship structure to distant locations for excitation by respectively $F_y$ and $F_z$ may be expected to be governed by the same set of transfer system modes.

Note:
The fact that for the transfer system modes all wave types are coupled, provides a probable explanation of observed equality of in-plane and normal velocities on a ship hull plate (see Kihlman, Plunt /7,6/) at low frequencies where the in-plane wavelengths are much larger than typical structural dimensions.

Seatings:
The foregoing analysis should not lead to the conclusion that the position for which the admittances are to be estimated, should be located upon a plate field. That this is by no means the case can be elucidated with a further analysis of the finite unstiffened plate model of paragraph 7.2.1. Assume that a shiplike engine seating is mounted upon a plate. Let it be composed of steel parts in which no locally dominant vibration modes resonate in the frequency range of interest. Its mass, stiffness and dimensions be such that $\text{Re} \ Y^x_{M \Delta f}$ and $\text{Re} \ Y^z_{F \Delta f}$ for excitation upon the seating are much smaller than for excitation upon the unloaded plate. When the mass of the seating is much smaller than the plate mass, Eq. (7.21) may be used to determine the power injection for arbitrary excitation upon the seating. In that case it follows, in the same way as for the unloaded plate, that

$$\frac{\text{Re} \ Y^x_{M \Delta f}}{\text{Re} \ Y^z_{F \Delta f}} \ll \frac{\Delta f}{\omega} \gg F_2,$$

(7.31)

provided a sufficiently large number of plate modes resonates for excitation at each source location $(x_2, y_2)$.  

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Accuracy:

Quantitative statistical criteria for the accuracy of the proposed method can only be derived for simple structures for which the number of resonant vibration modes and the response statistics can be predicted rather accurately; see e.g. Lyon /4.6/ for single reverberant panels. In the frequency range which is of interest for machinery noise reduction, this is not possible for representative ship structures. Only tests on structures of representative composition and dimensions can provide quantitative indications. The same is true for application in other than ship structures. Some experimental tests on ship-like structures are described in the next section.

7.4 Validity tests

The accuracy of Eq. (7.2) has been tested on model scale in the ship section which has been described already in Chapter 2, and on full scale on the ship-like hull of the steel water tank which has been described already in Chapter 5. In both tests the real part of the point admittance at positions with low admittance was calculated from the measured $\text{Re } Y_p^{\Delta f}$ at an adjacent position on a plate field and from "reciprocally" measured acceleration level differences. This calculated value has been compared with a measured admittance. To facilitate the tests, also for the low admittance positions the admittance for point force excitation in normal direction was chosen. The full scale and model tests cover a corresponding frequency range, namely 50 Hz - 1 kHz on full scale. For the analysis 1/3-octave bandwidths were chosen.

7.4.1 Scale model ship section

Measurement procedure:

The measurement positions and exciter locations on the scale model section are indicated in the Figures 7.3 and 7.4. The calculation procedure according to Eq. (7.2) has been used to estimate $\text{Re } Y_p^{\Delta f}$ and $\text{Re } Y_p^{\Delta f}$, i.e. for positions no. 2 and no. 4 on top of the engine seatings. The auxiliary positions were on tank top plates, viz. position no. 1 close to no. 2 and position no. 3 close to no. 4. The thickness of the seating top plates is 6 mm, whereas on tank top the plate thickness is 1,25 mm. The positions no. 2 and no. 4 are at the centre of seating top plates, i.e. at locations between seating stiffeners.
Figure 7.3:
Measurement positions and sound source locations on scale model ship section (cross sectional view). Positions no. 1-4 were used for measuring Re $Y_R$; locations A-D were used for distant excitation.

Figure 7.4:
As in Figure 7.3; top view on tank top.
For the measurement of the acceleration ratios \((z_1^2/\Delta f, z_2^2/\Delta f)\) 3 source locations have been used, viz. A, B and C at distances corresponding to respectively 23 m, 7 m and 5 m on full scale. In a similar way source locations A, B and D have been used for determining \((z_3^2/\Delta f, z_4^2/\Delta f)\). The distant sound source was an electrodynamic exciter driven with a pink noise signal.

The measurements of \(\text{Re } Y_{Z_F}\) have been performed with the cross-spectral method of Eq. (7.1). At the thin plate positions no. 1 and no. 3 an impedance head type B & K 8001 has been used (see 7.5 for discussion on error sources). At the positions no. 2 and no. 4 a force transducer type B & K 8200 was mounted against the seating and a 3 grams accelerometer underneath. For the excitation a mini-shaker type B & K 4810 was used. This was driven with single block pulse signals. The choice of short duration transients was made to avoid bias errors in the measurement of the cross-spectral density function \(C_{Z_F,Z}\), which would be caused by structural reverberation when using stationary stochastic excitation. All measurement results were averaged over 50 transients.

Note:

The alternative method of using a prescribed periodical signal, which has been suggested by Verhulst and Verheij /A.10/, appeared not appropriate because of insufficient frequency resolution on the available digital analysis system.

Results:
The acceleration level differences are shown in the Figs. 7.5 and 7.6. The 1/3-octave band levels of \(\text{Re } Y_{Z_F}\) are presented in the Figs. 7.7 and 7.8.

To obtain the calculated results for positions no. 2 and no. 4, the ratios of the squared accelerations have been averaged over the 3 distant source locations which have been mentioned above. In the Figs. 7.7 and 7.8 the theoretical value of \(\text{Re } Y_{Z_F}\) is also shown for an unstiffened infinite steel plate with the same thickness (1.25 mm) as at positions no. 1 and no. 3; see Cremer et al./4.9/. At the higher frequencies this result is approximately equal to the measured values on tank top.
Figure 7.5:
Acceleration level difference
$10 \lg \left( \frac{H_1^2}{H_2^2} \right)$ for sound source locations A, B and C in scale model.

Figure 7.6:
Acceleration level difference
$10 \lg \left( \frac{H_3^2}{H_4^2} \right)$ for sound source locations A, B and D in scale model.

Figure 7.7:
Measured and calculated values for $10 \lg \left( \frac{Re Y_{p2}}{1 \text{ s/kg}^{-1}} \right)$ at positions no. 1 and 2 in scale model.

Figure 7.8:
As in Fig. 7.7 for positions no. 3 and 4.
The frequency averaged level difference between the estimated and measured values of $\Re \left( \frac{Y_1 - \Delta f}{2z} \right)$ in Fig. 7.7 is +1.3 dB. The standard deviation of the level differences in the 14 1/3-octave bands is 1.6 dB. For $\Re \left( \frac{Y_1 - \Delta f}{4z} \right)$ in Fig. 7.8 these quantities are respectively 1.6 dB and 2.2 dB.

7.4.2 Steel hull of laboratory water tank

The measurements on the steel hull of the laboratory water tank are to some extent a repetition of the foregoing experiments, but now on full scale. However, also some new aspects were investigated.

Firstly, the size of the tank structure is relatively smaller than that of the ship structure represented by the scale model; see Fig. 7.9. The area of hull and deck plates is approximately 300 m², whereas the corresponding full scale dimension for the scale model section equals approximately 950 m² (480 m² for the tank top and the bottom stiffeners alone). This implies that the number of resonant structural vibration modes in the tank hull is smaller than in the scale model for corresponding frequency bands.

Another new aspect is that the sound sources for the "reciprocal" measurement of acceleration ratios were positioned both on the reverberant tank structure and in the water. For frequencies above 400 Hz, the water volume of 360 m³ behaves as a reverberant chamber with a reasonably diffuse sound field for excitation with 1/3-octave band filtered noise. However, at frequencies below 200 Hz there are no "room" resonances at all.

Measurement procedure:
The measurement positions and exciter locations are indicated in Fig. 7.9. The low admittance position no. 2 was on top of a web frame. The auxiliary position no. 1 was at the centre of a rectangular flat hull panel (dimensions in m: 3 x 0.6 x 0.008). The distance between the 2 positions is 1.5 m.

The hull was excited at 4 distant locations A, B, C and D, respectively at a distance of approximately 7 m, 10 m, 10 m and 10 m from position no. 2. For excitation in the water 4 randomly chosen positions were used. Both the vibration exciter and the underwater sound source were driven with pink noise.
Figure 7.9:
Steel water tank: dimensions: 12 m (length) × 6 m (width) × 5 m (depth).
Measurement positions no. 1 and no. 2. Sound source locations A, B, C and D, with B, C and D located on the hull 2.5 m below the water surface level.

The admittances were measured with a force transducer type B & K 8200 mounted against the hull and with 2 accelerometers symmetrically positioned with respect of the force transducer. The output signal of these accelerometers were summed instantaneously. For the excitation transients were again used, in this case generated by hammer blows. Two hammers with different elasticity were used to adapt the pulse shape to the frequency ranges of interest. Only for f < 90 Hz periodically swept sines were used with a repetition frequency 5 Hz (Δf = 0.2 Hz), because the results for hammer blows showed relatively large random errors. Because of the large reverberation time in the hull, the determination of the cross-spectral density function required the use of the largest available record times for the Fourier Analysis, to capture the transients as completely as possible. Table 7.1 shows the reverberation times measured for octave band filtering and the record times used for the discrete Fourier transformation. All measurements were averaged over 25 hammer blows. Above 80 Hz the squared coherence functions for the transient force and acceleration signals were almost perfectly equal to 1.
The acceleration level differences are shown respectively in Fig. 7.10 for excitation on the tank structure and in Fig. 7.11 for excitation in the water. For excitation in the water the scatter in Fig. 7.11 becomes very small at $f > 200 \text{ Hz}$, i.e. above the lowest natural frequency of reverberant room modes. At lower frequencies the scatter is of the same order of magnitude as for excitation on the structure.

The $1/3$-octave levels of $\mathbb{Re} \frac{Y_f^\Delta f}{Z}$ are presented in the Figs. 7.12-7.14. The calculated result in Fig. 7.12 is obtained by using \( \left( \frac{Z_2^2}{Z_1^2}, \frac{\Delta f}{\Delta f_1} \right) \) averaged over the 4 exciter locations on the structure, that in Fig. 7.13 for averaging over the 4 underwater sound source locations and that in Fig. 7.14 for averaging over all 8 source locations. In the Figs. 7.12-7.14 also $\mathbb{Re} \frac{Y_f^\Delta f}{Z}$ is shown for an infinite unstiffened plate with the same thickness as the hull panel and also water-loaded on one side.
Figure 7.12:
Measured and calculated values for $10 \log \left( \frac{\Delta f}{\text{Re} Y_p} / \text{s kg}^{-1} \right)$ at positions no. 1 and 2 on water tank. Calculated result for distant exciters on hull.

Figure 7.13:
As in Fig. 7.12. Calculated result for under water sound sources.

Figure 7.14:
As in Fig. 7.12. Calculated result for all 8 distant exciter locations.

Figure 7.15:
Scatter of the 8 acceleration ratios in Figs. 7.10 and 7.11, calculated according to Eq. (5.32).
see Heckl /7.4/. The measured result for position no. 1 is on the average quite close to this value, even at low frequencies where the number of "panel resonances" is very small.

The frequency averaged level difference between the estimated and measured values of $\text{Re} \frac{Y_{z}}{Y_{z}}$ is for the Figs. 7.12-7.14 respectively +1 dB, +0.6 dB and +1 dB. The standard deviation of the level differences in the 13 1/3-octave bands is for the Figs. 7.12-7.14 respectively 2,8; 2,2 and 1,8 dB.

In Fig. 7.15 for each 1/3-octave band the scatter is shown for the 8 acceleration ratios from Figs. 7.10 and 7.11. For the calculation of this scatter the procedure of Eq. (5.32) has been used. From a comparison with Fig. 7.14 it appears that at frequencies where the largest discrepancies between measured and calculated $\text{Re} \frac{Y_{z}}{Y_{z}}$ occurs, there is no exceptional scatter in acceleration level differences.

7.4.3 Discussion of error sources

Three types of error sources are distinguished, viz. errors inherent to the method, measurement errors in $\text{Re} \frac{Y_{z}}{Y_{z}}$ and violation of the "substitution source hypothesis" in Eq. (7.3) due to an improper choice of auxiliary positions.

Errors inherent to the method:

With respect to the errors which are inherent to the method a distinction can be made between statistical and systematic errors. The statistical errors are those that can be reduced by averaging the acceleration ratios over more distant-sound-source locations. The systematic errors are those that remain after averaging over a large number of source locations in the case that insufficient vibration modes resonate in the transfer system or modes that do not couple all wave types. In cases of suspicion, special attention should be given to the choice of distant exciter locations and directions for measuring the ratios of the squared accelerations.

The results of the foregoing scale model and the full scale tests illustrate clearly the beneficial effect of averaging over several source locations. After this averaging procedure the largest discrepancies, i.e.
some 4 to 5 dB, between the measured and the indirectly estimated values remained predominantly at low frequencies, namely for the water tank at 63 Hz and 100 Hz (see Fig. 7.14) and for the scale model at 630 Hz, which corresponds with 80 Hz on full scale (see Figs. 7.7 and 7.8). However, on average the discrepancies between measurements and calculations remain within a few dB even at low frequencies. Additional experiments are needed for determining the frequency range of validity for smaller size structures.

The question may arise whether the foregoing tests are sufficiently general, because only normal point force excitation was considered (however, notice that in the scale model tests the bottom structure was also excited in an in-plane direction due to the inclined seating top plates). In favour of a more general validity are observations of the type reported in Chapter 3. The ratios of the ship transfer functions of 6 degrees of freedom of Figs. 3.12 and 3.13 show a small scatter for different seating locations of similar structural composition. These observations would find a good explanation on the basis of Eqs. (7.2) and (7.3).

For the time being the conclusion seems to be justified that the proposed estimation method is sufficiently accurate in typical ship structures for frequencies above 50 Hz, provided the total area of shell, bottom and deck plating is at least some 250 m² and for averaging the acceleration ratios over at least 4 distant sound source locations. Smaller size structures are still to be investigated.

Admittance measurements:
For the measurement of $\text{Re} \frac{Y}{\Delta f}$ with the cross-spectral density method of Eq. (7.1), the following potential error sources are discussed:
- phase mismatch in instrument channels, especially detrimental at frequencies where the admittance is very "reactive"
- bias errors in the cross-spectral density function due to large ratios between the reverberation times of the measurement object and the sample record times of the digital analysis
- the dynamical loading of the measurement object, firstly by the mass added between force transducer and measurement object, which causes the
measured force to differ from the force exerted on the object and secondly by the rotational inertia of the exciter system.

Phase mismatch errors: These have been eliminated by a special calibration procedure, as in Chapter 5. The remaining statistical errors are smaller than 0.04 degree. In tests of the sensitivity for phase errors it appeared that all 1/3-octave band results of $\text{Re} \ Y^f_{Fz} \Delta f$ were insensitive for deliberately inserted shifts in the phase angle of the cross-spectral density of $\pm 0.2$ degree. Therefore it may be concluded that errors due to phase mismatch are negligible.

Influence of structural reverberation: For measurements on the water tank the ratio $T_{60}/T_{\text{rec, max}}$ appeared to be approximately equal to 4 (see Table 7.1). Tests showed that for halving the record times, thus for ratios $T_{60}/T_{\text{rec}} = 8$, the measured results for $\text{Re} \ Y^f_{Fz} \Delta f$ hardly changed, whereas for larger ratios they did. Therefore, no significant bias errors due to strong reverberation occur in the results presented.

Improper loading of measurement object: The most difficult problem appeared the realization of a "torque-free" point force excitation at the auxiliary positions no. 1 and no. 3 on the scale model tank top. Reciprocity checks, as described in Appendix C, were helpful for improving the measurement arrangement. A satisfactory solution was obtained by pressing the impedance head against the plate via a small stud with a spherical head. The difference between the force on the object and the measured force, which is caused by the inertia of the mass between force transducer and object, was cancelled by special signal processing on $F_z$ and $z$ in the frequency domain.

Improper choice of auxiliary positions:
The approximate equality in Eq. (7.3), i.e. the "substitution source hypothesis", may be expected to be violated if the auxiliary position and the position of the unknown admittance are too far apart, e.g. spatially separated by a strongly reflecting barrier. For example, for the scale model section (see Figs. 7.3 and 7.4) the choice of tank top position no. 1 as an auxiliary position for determining $\text{Re} \ Y^f_{4z} \Delta f$ would have been improper, especially at scale model frequencies above 1 kHz. At these higher
frequencies Eq. (7.3) is invalid due to a significant propagation attenuation in athwartship direction. For instance, between the ratios \( \frac{\frac{2}{2}, \Delta f}{\frac{2}{2}, \Delta f} \) measured for the source locations A and B in opposite directions, systematic level differences of approximately 10 dB would have been found for \( f \geq 1 \text{ kHz} \). In practice, therefore, such systematic differences in acceleration ratios for excitation sources in opposite directions may form an indication of an improperly chosen auxiliary position.

Another situation where erroneous auxiliary positions might be chosen is when damping treatments have been applied locally. For example, in the scale model section of Fig. 7.3 the tank top position no. 3 would have been an improperly chosen auxiliary position for seating position no. 4, when the tank top plates between the seatings would have been damped in the same way as in the passenger and car-ferry of Chapter 6 (see Fig. 6.1). In that case Eq. (7.3) is invalid, because a considerable fraction of the structureborne sound energy injected at position no. 3 is dissipated in the damping layer. For the energy injected at position no. 4 the locally dissipated fraction will be smaller because of the flanking sound transfer via other bottom elements and because of the direct coupling to other than bending wave types.

7.5 Summary

1. A simple experimental method has been proposed for estimating the frequency bandwidth averages of real parts of mechanical point admittances of 6 degrees of freedom. These real parts of the point admittances govern the energy flow into the ship structure at resilient isolator locations when the impedance mismatches between isolator and receiving structure are sufficiently large. Multi-directional measurement of these quantities is very difficult and on board ships in many cases impracticable.

The proposed method uses relatively simple measurements at the position for which the admittance has to be estimated and at an adjacent auxiliary position. At this auxiliary position the frequency averaged value of the real part of the admittance for point force excitation in normal
direction is determined. In addition, for excitation with a distant sound source the acceleration ratio is determined for appropriate components at the position with the unknown admittance and at the auxiliary position. From this ratio and from the measured admittance at the auxiliary position the real part of the unknown admittance is derived. The validity of the method has shown to be possible on basis of a substitution source hypothesis and using reciprocity relations for transfer functions.

2. The proposed estimation method is of a statistical nature. In a theoretical discussion it has been proved that to reduce statistical errors, it will be necessary, in general, to average the acceleration ratio for the two adjacent positions over excitation at several distant locations. Systematic errors may be expected to occur when the number of resonant vibration modes in the transfer system between the distant source locations and the positions at which the accelerations are measured, becomes too small and/or when 2 accelerations correspond to fundamentally different sets of modes. The latter situation is improbable in ship structures due to the coupling of all wave types at discontinuities.

3. The accuracy of the proposed estimation method has been tested on a scale model of a ship section and on a full scale ship-like hull of a water tank. Both types of experiments covered a corresponding frequency range, namely 50 Hz-1 kHz on full scale. For the analysis 1/3-octave bandwidths were chosen.

The conclusion from the experiments is that the proposed method may provide estimations which are accurate within a few dB in typical ship structures for frequencies above 50 Hz, provided the total area of shell, bottom and deck plating is at least some 250 m² and provided the acceleration ratios are averaged over at least 4 distant sound source locations. Additional tests on structures of smaller size are still to be done.

4. The proposed estimation method can be applied on board for quickly collecting multi-directional admittance data of engine seatings at resilient mounting locations and of pipes on the ship side of flexible isolators. Moreover, it can be used in the laboratory for designing seatings and pipe supports with prescribed multi-directional admittances.
An important practical aspect of the proposed method, especially for shipboard applications, is that by introducing an auxiliary position for point admittance measurements, the estimation of admittances becomes possible at locations where resilient isolators cannot be removed for practical reasons.

Multi-directional admittance data of the ship structure are complementary to the multi-directional transfer functions of resilient isolators which can be obtained with the measuring methods of Chapter 2.
EVALUATION, SUMMARY, SAMENVATTING
8. EVALUATION

A brief evaluation of the foregoing study is presented and an indication is given of the type of work still to be done.

8.1 Methods for sound transfer path assessment

One of the objectives of the foregoing study was the development of reliable and practicable methods for the shipboard analysis of the multi-path sound transfer from resiliently mounted machinery. There is a need for such an analysis, because a cost-effective improvement of resilient mountings is only possible on the basis of a quantitative knowledge of the sound transfer via the various transfer paths that are involved. The present evaluation is limited to the main sound transfer paths from resiliently mounted diesel engines (see Figs. 1.3 and 1.4).

Paths investigated:

Resilient support mountings: For this widely investigated transfer path a fundamental step forward has been made by the development of a relatively simple method which takes the multi-directionality correctly into account in combination with the static preload effects on the sound transfer properties of mountings. Although there are practical and fundamental limitations with respect to the validity of the method, as has been discussed in Chapter 2, for the majority of practical situations a more reliable analysis can be made than before. The method is also directly applicable for the resilient mountings that support the exhaust system (path no. 5 in Figs. 1.3 and 1.4) provided there is a sufficiently large impedance mismatch between mountings and casing seatings. According to the author the method described is sufficiently far developed for an accurate analysis of situations where a large impedance mismatch occurs at mounting-seating interfaces. For conditions of small impedance mismatch an accurate multi-directional analysis will prove completely impracticable for most shipboard situations.

Airborne paths: For shallow reverberant air cavities below machinery the indirect methods of Chapter 4 have solved the problem of path assessment sufficiently accurately. In principle the transfer path through the engine room to hull or deckhead can be investigated with loudspeakers as substitution sources.
However, in practice the sound response at a receiver location may be masked by background noise. For that case, variants of the methods of Chapter 4 have been proposed for the airborne path through the engine room. In the opinion of the author the proposed methods will solve the practical problems of experimental airborne path assessment with sufficient accuracy for the majority of shipboard situations.

Pipes: The newly investigated path assessment methods of Chapter 5 for pipes appear promising. However, because of the limited experience at the present stage, additional investigations are needed with respect to the range of pipe systems for which the methods can be applied and with respect to the frequency range of validity. Of special concern for further investigations are pipes with a length shorter than a half beam-bending wavelength. Such a short length might form a complication both for accurate energy flow measurements and for the substitution source method that is based on radial pipe wall accelerations. However, a more detailed variant of this latter method, which uses also substitution source excitation and vibration measurements for longitudinal and torsional directions might possibly be successful.

Remaining paths:
Of the transfer paths that have not been investigated in the present thesis, only path no. 4 of Fig. 1.4 is discussed here, i.e. the sound transfer via the flexible propeller shaft coupling of a propulsion diesel engine or that via the electricity cables of diesel driven generators.

Flexible shaft couplings: The path assessment method of Chapter 2 for resilient mountings is not straightforwardly applicable for flexible shaft couplings. There are differences both with respect to the measurement of source levels and with respect to the measurement of sound transfer properties of couplings.

The multi-directional vibration levels on the machine side of the flexible coupling need to be measured on a rotating shaft or flange. In principle this seems feasible and the development of special instrumentation is underway. Acceleration measurements on the adjacent engine bearing may possibly serve as an usable approximation.
The measurement of the shaft coupling properties is to be performed in a test rig similar to that in Fig. 2.13. For small size couplings of representative designs such measurements have been performed successfully. However, for large size couplings (0.5 m < diameter ≤ 2 m) serious problems may arise. Such couplings are not produced in great numbers and only in exceptional cases a specimen will be available for laboratory tests. However, even if a specimen is available there remain fundamental and practical problems. The modelling of the coupling by a linear 12-port network, as in Fig. 2.3, seems questionable for a large part of the frequency range of interest, because of the large size of the coupling flanges. These cease to vibrate as rigid bodies at relatively low frequencies. Moreover, it will be very difficult both in the laboratory and on board to measure at positions on the ship side of the coupling in way of the large impedance mismatches. These problems deserve further study. Methods that can be applied, if only shipboard measurements can be performed, are still to be developed. In special cases it is certainly worthwhile to consider the method of Fig. 1.5.

Electricity cables: The sound transfer properties of electricity cables can be determined in the same way as for resilient mountings. However, for shipboard path assessment the method of Chapter 2 seems not applicable for the same reason as discussed for pipes. In many practical cases the cables are connected to the ship structure via relatively low-impedance supports. For path assessment purposes it seems worthwhile in those cases, to investigate a substitution source method similar to Method B of Chapter 5, i.e. using lateral vibrations on the cables as a path strength measure.

8.2 Mounting system improvements

The multi-path analysis of shipboard resilient mounting systems provides the information about factors limiting the acoustical effectiveness. Moreover, it can be derived to what extent the system may be improved without going, for example, from a single stage to a double stage mounting system. Chapter 6 of the present thesis provides a practical example of such an analysis. In Chapter 6 and 7 some directions are indicated which are to be followed to improve noise reduction in the resilient mounting path and in several flanking paths. However, for some of the flanking paths it is not yet clear to what extent the noise reduction can be improved.
This concerns in particular the transfer paths via flexible shaft couplings and via liquid filled pipes.

Flexible shaft couplings:
Several small size couplings of similar design as representative of propulsion system couplings have been investigated acoustically in the test rig of Fig. 2.13. Using scale model rules the measured results have been translated to estimate the acoustical properties of large size couplings. From this procedure it became very clear that in many cases currently used flexible shaft couplings will be much stiffer "acoustically", than the support mountings underneath an engine. Therefore such couplings may form a very important flanking transfer path. For critical situations the coupling design has to be considered in the very early design stage of a ship. For example, if it would appear necessary to apply a double stage coupling composed of 2 flexible couplings connected by an intermediate shaft, then extra length for the propulsion system would be required. Generally speaking, it would be of great technological value to re-design the present type of single stage couplings for improved acoustical behaviour, maintaining at the same time their mechanical properties. The measurement techniques of Chapter 2 may be used as one of the design tools.

Liquid filled pipes:
Several representative types of liquid filled flexible bellows have been investigated in a test rig according to Fig. 2.14. From these measurements it may be concluded that in many cases the pipe seatings on the ship side of flexible pipe isolators have to be improved considerably to avoid strong flanking sound transfer via pipes. A practical method for evaluating improved seatings has been discussed in Chapter 7. However, even if the proper multi-directional impedance mismatch between flexible bellows and pipe system has been realised, it is uncertain whether this is sufficient. At the present stage it is not known when the structureborne sound transfer via the flexible bellows is exceeded by liquidborne sound transfer. It is also unknown how this depends on the position of the bellows with respect to the sound source and how this depends on the type of excitation. These problems deserve further study.
9. SUMMARY

The sound transfer from resiliently mounted shipboard machinery to the ship structure is fundamentally of a multi-path nature. It occurs simultaneously via the resilient mountings, via the surrounding air and via mechanical links such as pipes, propeller shaft etc.

At the present stage it is usually unknown which factors limit the effectiveness of a resilient mounting system as a noise reduction measure. This hampers a cost-effective improvement. Complete theoretical analysis of a multi-path system is too complex. On the other hand experimental evaluation requires measuring methods which can be applied under the very restrictive conditions on board ships. For most sound transfer paths such methods are lacking.

In the Chapters 2-5 of this thesis new experimental methods have been developed and tested for quantifying the sound transfer respectively via the resilient mountings underneath machinery, via shallow air cavities below machinery and via pipes. All these methods can be applied on board without disturbing seriously normal ship programs.

The Chapters 6 and 7 are concerned with a case study of the multi-path noise reduction properties in a representative shipboard mounting system and with the development of a simple experimental method for aiding the design of improved multi-directional structureborne sound isolation.

In Chapter 1 an overview is given of the knowledge with respect to the effectiveness of resilient mounting systems on board ships. Different approaches for the in-situ analysis of multi-path mounting systems are compared and an outline of the thesis is presented.

In Chapter 2 a method is described for the experimental analysis of the multi-directional structureborne sound transfer through the resilient mountings and through the ship structure. Basic elements are a newly developed technique for measuring multi-directional sound transfer properties of mountings in a laboratory test rig and previously published reciprocity techniques for measuring ship transfer functions.
The feasibility of the measurements on resilient mountings is illustrated with some test results.

In *Chapter 3*, the mounting path analysis procedure is investigated in a scale model for the complete path from a diesel engine-like vibration source, via resilient mountings and ship structure, to an accommodation deck. Because of the multi-directional vibrations the complete analysis for a multi-mounting system requires the measurement of an enormous amount of data. Investigations were carried through to what extent the accuracy of the analysis is affected when simplified procedures are applied.

*Chapter 4* describes two experimental methods for determining the airborne sound transfer through shallow reverberant cavities below resiliently mounted machinery, in cases where these cavities are inaccessible for loudspeakers as substitution sources. Basic elements are the introduction of hypothetical acoustic point sources in a cavity and reciprocal transfer function measurements for such point sources. One of the methods is tested and validated by scale model experiments. The theoretical analysis leads also to improvements in theoretical models for sound transfer through shallow or narrow cavities published previously.

In *Chapter 5*, experimental methods are investigated for the assessment of structureborne sound transfer along pipes. Laboratory tests show that direct determination of the sound transfer using energy flow measurements is feasible at frequencies below the initiation of 2nd order circumferential waves. Two substitution source methods for indirect determination of the sound transfer appear also feasible. One of the methods uses energy flow measurements on the pipe, whereas the other method uses squared radial accelerations averaged over a certain pipe length. The latter method is also usable at frequencies above the cut-off frequency of 2nd order circumferential waves. Of great practical interest is the use of reciprocal measurement of the sound transfer from the substitution sources, when the signal to noise ratio for direct measurements is low.

The sound transfer from a resiliently mounted medium-speed propulsion diesel engine to the accommodation is analysed in *Chapter 6*. It concerns a mounting system representative of several modern passenger and car-ferries.
The multi-path system insertion loss is some 12-17 dB for octave bands with centre frequencies 63 Hz - 1 kHz, which is typical for similar systems in other ships too. For octave bands with centre frequencies up to 250 Hz the contributions of the resilient mounting path and the airborne paths appear to be much smaller than the total sound transfer. On the basis of both shipboard and scale model measurements, system parameters which are important for the sound transfer through the resilient mountings and through the air are discussed for the system investigated. Estimates are given for the upper limit of the insertion loss for similar single stage mounting systems without acoustic enclosure. Compared to the present situation an improvement can be obtained of maximally some 20 dB for the octave bands with centre frequencies 63-250 Hz and of some 10 dB for the 500 Hz and 1 kHz octave bands.

In Chapter 7, a simple experimental method is described and tested for estimating frequency bandwidth averages of real parts of point admittances for each of 6 degrees of freedom. Again, use is made of a substitution source principle and of reciprocity relations for transfer functions. The method is very useful for collecting multi-directional admittance data at resilient isolator locations on board ships. Moreover, it is of great practical use as a tool for designing seating structures, taking into account the multi-directionality of machinery vibrations and the multi-directional sound transfer properties of flexible isolators.

Finally, in Chapter 8, an attempt is made to evaluate to what extent problems of the experimental analysis of multi-path resilient mounting systems has been solved in the present thesis and what type of work has still to be done. Moreover, some factors are indicated that may form either a practical or a fundamental limitation for mounting system improvement.
10. SAMENVATTING

Bepaalde typen machines en werktuigen aan boord van schepen worden op ver­­rende draagelementen geïnstalleerd om de geluidoverdracht naar de scheeps­­constructie te verminderen. Momenteel is het doorgaans niet bekend door wel­­ke factoren de effectiviteit van deze lawaaibestrijdingsmaatregel wordt be­­perkt. Daardoor is ook niet bekend hoe tegen acceptabele kosten deze effec­­tiviteit eventueel zou kunnen worden verhoogd.

Om de geluidoverdracht vanaf een verend opgestelde machine naar de scheeps­­constructie voldoende goed te kunnen analyseren, is het noodzakelijk om in rekening te brengen dat deze tegelijkertijd plaats vindt langs vele pa­­den. Behalve geluidoverdracht via de verende draagelementen, is er zogenaamde flankerende overdracht via de lucht in de machinekamer en via andere mecha­­nische verbindingen met het schip, zoals bijvoorbeeld via pijpen, of via een schroefas. Omdat de geluidopwekking en de geluidoverdracht voor ver­­schillende paden bijzonder gecompliceerd is, is een gedetailleerde theore­­tische behandeling veelal niet uitvoerbaar. Aan de andere kant zijn er voor een experimentele analyse van bestaande systemen ook belangrijke beperkingen. Tijdens normaal scheepsbedrijf is de mogelijkheid tot experimenteren zeer gering en ook tijdens verblijf in havens of op een werf is er meestal slechts korte tijd beschikbaar en is er weinig mogelijkheid om systemen te demonteren voor het aanbrengen van meetapparatuur. Voor bijna alle geluid­­overdrachtpaden ontbreken tot nu toe methoden die ook onder deze beperken­­de omstandigheden toch tot een betrouwbare schatting van de geluidoverdracht leiden. Met de in dit proefschrift beschreven methoden wordt getracht deze leemte voor een aantal belangrijke overdrachtpaden te vullen.

In de hoofdstukken 2-5 zijn methoden ontwikkeld en getoetst, voor het kwant­­ificeren van de geluidoverdracht, respectievelijk via de draagveren, via platte galmende luchtvolumes onder machines en via pijpen. Al deze methoden kunnen worden toegepast zonder ernstige storing van het normale scheepsbe­­drijf.

Hoofdstuk 6 bevat een analyse van de geluidreducerende eigenschappen van een representatieve verende opstelling en in hoofdstuk 7 wordt een eenvoudige ex­­perimentele methode beschreven om de geluidisolerende kwaliteit van construc­­ties aan de scheepszijde van flexibele isolatoren te beoordelen.
Na een algemene inleiding in hoofdstuk 1 wordt in hoofdstuk 2 een methode beschreven voor de experimentele analyse van de geluidoverdracht van het pad door de draagveren en daarna door de scheepsconstructie. Aan de machinekant van elke veer worden de constructiegeluidstrillingen ontbonden in 3 translatie- en 3 rotatiebewegingen. Aan de scheepskant van elke veer wordt de aanstoting beschreven met 3 krachten en 3 koppels. De methode is gebaseerd op nieuw ontwikkelde technieken om de geluidoverdracht eigenschappen van draagveren te meten in een laboratorium testbank en op reeds eerder ontwikkelde methoden voor reciproque meting van de geluidoverdracht via de scheepsconstructie. De bruikbaarheid van de varenmeetmethode wordt geïllustreerd aan de hand van enkele meetresultaten.

In hoofdstuk 3 worden een aantal experimenten in een schaalmodel beschreven, waarin de analyseprocedure voor het varenpad nader is onderzocht. Hierbij werd een compleet overdrachtsysteem beschouwd vanaf een machine-achtige geluidbron via draagveren en via de scheepsconstructie naar een ontvangpositie op een accommodatiepak. Vanwege de noodzaak, om in het algemene geval, de trillingen en krachten in 6 componenten te ontbinden, vereist de analyse van een varenopstelling met vele draagveren een enorme hoeveelheid meetgegevens. Onderzocht is in hoeverre de nauwkeurigheid van de varenpad-analyse wordt aangetast, wanneer vereenvoudigde procedures worden toegepast.

In hoofdstuk 4, worden twee experimentele methoden beschreven om de luchtgeluidoverdracht te bepalen via platte ruimten onder machines waarin geen luidsprekers als vervangingsbronnen kunnen worden geplaatst. De methoden zijn gebaseerd op een rekenmodel waarin denkbeeldige vervangingsbronnen dezelfde hoeveelheid geluid in de spouw opwekken als de machine. Voor deze denkbeeldige vervangingsbronnen wordt de geluidoverdracht naar een ontvangpositie op grote afstand bepaald met behulp van reciproque overdrachtmetingen. Een van de methoden is getoetst in een schaalmodel en geldig bevonden. De theoretische analyse in hoofdstuk 4 leidt ook tot een verbetering van eerder gepubliceerde theoretische modellen voor de geluidoverdracht in platte galmende ruimten.

In hoofdstuk 5 worden experimentele methoden beschreven en getoetst om de geluidoverdracht langs pijpen te kwantificeren. Laboratoriummetingen tonen aan dat een directe bepaling van de geluidoverdracht met behulp van energiestroommetingen uitvoerbaar is in het frequentiegebied beneden de afsnijfrequentie voor zogenaamde "2e-orde-golven".
Het betreft hier energiestroommetingen in longitudinale golven, torsiegolven en buiggolven. Indirecte bepaling van de geluidoverdracht met behulp van twee vervangingsbron-methoden blijkt eveneens uitvoerbaar. Bij de ene methode wordt ook gebruik gemaakt van energiestroommetingen. Bij de andere methode wordt gebruik gemaakt van gekwadrateerde radiale versnellingen van de pijpwand, gemiddeld over een zekere pijplengte. De laatste methode is ook bruikbaar bij frequenties boven de afsnijfrequentie van "2e-orde-golven". Van groot praktisch belang is de mogelijkheid om de geluidoverdracht vanaf de vervangingsbron(nen) naar een ontvangpositie reciprook te meten voor gevallen waarin er hoge stoorniveaus optreden op de ontvangpositie. Dit zal in de praktijk vaak voorkomen.

In hoofdstuk 6 wordt de geluidoverdracht vanaf een veren opgestelde hoofdmotor naar de accommodatie geanalyseerd. Het betreft hier een representatieve verende opstelling van een midden-snellelopende dieselmotor aan boord van een zeevaartvaartuig. Een soortgelijke verende opstelling is ook in een aantal andere veerschepen toegepast. De geluidvermindering in de accommodatie ten gevolge van het aanbrengen van de verende opstelling is ca. 12-17 dB voor de octaafbanden met middenfrequenties 63 Hz-1 kHz. Deze waarde is ook typerend voor vergelijkbare systemen in andere schepen. Het blijkt dat voor octaafbanden met middenfrequenties lager dan 500 Hz geluidoverdracht via de draagveren en via de lucht rondom de motor ver onder de totale geluidoverdracht ligt. Er treedt dus in sterke mate flankerende overdracht op via pijpen, via de schroefas en/of via de uitlaatgassenleiding. Op basis van meetingen aan boord en van metingen in een schaalmodel, wordt een analyse gegeven van de factoren die betrekking hebben op de mate van geluidoverdracht via de draagveren en via de lucht. Er wordt een schatting gemaakt van de bovengrens voor de geluidoverdrachtvermindering die kan worden gerealiseerd met een enkelvoudige verende opstelling zonder omkasting. In vergelijking tot de onderzochte situatie kan een verbetering worden bereikt van maximaal ca. 20 dB voor de octaafbanden met middenfrequenties 63-250 Hz en van ca. 10 dB voor de 500 Hz en 1 kHz octaafbanden.

In hoofdstuk 7 wordt een eenvoudige experimentele methode ontwikkeld en getoetst, om de over frequentiebanden gemiddelde waarden van reële delen van mechanische puntadmittanties te schatten voor 6 vrijheidsgraden van beweging.
Het betreft hier constructie-eigenschappen die bepalend zijn voor de energiestroom vanuit een flexibele isolator naar de aangrenzende scheepscstructie. Opnieuw wordt gebruik gemaakt van een vervangingsbronprincipe en van reciprociteitsbetrekkingen voor overdrachtsfuncties. Met de methode kan aan boord op snelle wijze een groot aantal admittanties van representatieve veer- en pijpfundaties worden bepaald, voor 6 vrijheidsgraden. Bovendien vormt de methode een bijzonder praktisch hulpmiddel bij het ontwerpen van fundaties die in alle richtingen goed zijn aangepast bij geluidbronsterkten en bij geluidoverdrachteigenschappen van flexibele isolatoren.

In hoofdstuk 8 tenslotte, wordt nagegaan in hoeverre problemen met betrekking tot de experimentele analyse van de diverse geluidoverdrachtpaden bij verend opgestelde machines zijn opgelost met resultaten in dit proefschrift. Er wordt aangegeven in welke richtingen het onderzoek moet worden voortgezet. Verder worden enkele factoren besproken die een praktische of een fundamentele beperking kunnen vormen voor een verdere akoestische verbetering van verende opstellingen.
APPENDICES
APPENDIX A: RECIPROCITY RELATIONS FOR TRANSFER FUNCTIONS

The applications of the reciprocity principle in this thesis concern the equality of point-to-point transfer functions when sound source and receiver location are interchanged. An extensive discussion about the validity of reciprocity is beyond the scope of this Appendix. Treatments can be found in literature on electrical circuits, on electro-acoustics, on sound and vibration and on dynamical analogies; see e.g. Rayleigh /1.10/, Bose and Stevens /A.1/, Cremer, Heckl, Ungar /4.9/, Ten Wolde /1.1, A.2/ (with many references), Belousov and Rimskii-Korsakov /A.3/ (with many references) and Olson /A.4/. Only a brief statement of some principles is presented here. The main purpose is to present a number of reciprocity relations the use of which may be advantageous when measuring transfer functions in electrical, mechanical or acoustical systems and in combinations of them.

**Electrical circuits:**

The simplest representation of a network composed of linear, passive electrical circuit elements is a 2-port network as given in Fig. A.1. At the 2 ports the conjugate* variable pairs \((e_1, i_1)\) and \((e_2, i_2)\). The sign convention is chosen in such a way that a positive energy flow is directed inside. This holds also for all other systems that are discussed later in this Appendix. Harmonic signals are assumed and \(e_1, i_1, e_2, i_2\) denote the complex phasors. Consider 2 different sets of voltages and currents, i.e. \((e_1', i_1', e_2', i_2')\) and \((e_1, i_1, e_2, i_2)\), resulting respectively from subsequent excitation at port no. 1 and port no. 2. Then the network is said to be reciprocal if

\[
e_{11}'i_1 + e_{22}'i_2 = e_{11}i_1' + e_{22}i_2'.
\]  

(A-1)

* conjugate means that the time averaged product equals the mean energy flow.

Figure A.1:

Electrical 2-port network.
From this general reciprocity relationship 3 special reciprocity relations follow, corresponding to 3 different pairs of external connections, i.e.

\[ \frac{e_2}{i_1} \bigg|_{i_2=0} = \frac{e_1'}{i_2'} \bigg|_{i_1'=0} , \quad \text{(A-2)} \]

\[ \frac{i_2}{e_1} \bigg|_{e_2=0} = \frac{i_1'}{e_2'} \bigg|_{e_1'=0} , \quad \text{(A-3)} \]

\[ \frac{e_2}{e_1} \bigg|_{i_2=0} = -\frac{i_1'}{i_2'} e_1'=0 \quad \text{(A-4)} \]

Another relation which is equivalent to Eq. (A-4) is found by interchanging the subscripts 1 and 2 in Eq. (A-4). For the present discussion it is of interest that the above transfer functions can be measured "directly", i.e. with excitation at port no. 1, as well as "reciprocally", i.e. with excitation at port no. 2.

If the circuit has "n" network ports the reciprocity relation of Eq. (A-1) is replaced by

\[ \sum_{j=1}^{n} e_j' i_j = \sum_{j=1}^{n} e_i j' j . \quad \text{(A-5)} \]

The total number of special reciprocity relations for transfer function between all possible pairs of ports, analogous to Eqs. (A-2) - (A-4), equals

\[ N = 3 \sum_{i=1}^{n} (i-1) . \quad \text{(A-6)} \]

For the equivalent "direct" and "reciprocal" measurements the external conditions at all ports follow from Eq. (A-5).

The above reciprocity relations are valid when the system is composed of bilateral elements, i.e. resistances, inductances and capacitances. It may also contain transformers.
Gyrators are non-bilateral elements that can make a circuit no longer reciprocal. A gyrator may be represented by a "anti-reciprocal" 2-port. For an anti-reciprocal 2-port Eq. (A-1) is replaced by

\[ e_1' i_1 - e_2' i_2 = e_1 i_1 - e_2 i_2 . \]  

(A-7)

Then the Eqs. (A-2) - (A-4) are replaced by

\[
\begin{align*}
\frac{e_2}{i_1} & \bigg|_{i_2=0} = - \frac{e_1'}{i_2'} \bigg|_{i_1'=0} , \\
\frac{i_2}{e_1} & \bigg|_{e_2=0} = - \frac{i_1'}{e_2'} \bigg|_{e_1'=0} , \\
\frac{e_2}{e_1} & \bigg|_{i_2=0} = \frac{i_1'}{i_2'} \bigg|_{e_1'=0} .
\end{align*}
\]  

(A-8)  

(A-9)  

(A-10)

However, it is of practical interest for the following discussion to notice that for an anti-reciprocal 2-port still the transfer functions can be measured "directly" as well as "reciprocally". The same remains true if an anti-reciprocal 2-port is connected in series with reciprocal 2-ports. Then the resulting 2-port is anti-reciprocal according to the Eqs. (A-7) - (A-10).

The foregoing discussion was restricted to signals with harmonic time dependence. However, using Fourier analysis special reciprocity relations similar to those in Eqs. (A-2) - (A-4) can also be derived for stationary stochastic signals and for transient signals. An example of a reciprocity relation for power spectral density functions of stationary stochastic signals is as follows:

\[ \frac{G_{e_2,e_2}}{G_{i_1,i_1}} \bigg|_{i_2=0} = \frac{G_{e_1',e_1'}}{G_{i_2,i_2}} \bigg|_{i_1'=0} . \]  

(A-11)

An example of a reciprocity relation for mean squared bandfiltered signals is as follows:

\[ \frac{\langle e_2^2, \Delta f \rangle}{\langle i_1^2, \Delta f \rangle} \bigg|_{i_2=0} = \frac{\langle e_1'^2, \Delta f \rangle}{\langle i'_2, \Delta f \rangle} \bigg|_{i_1'=0} . \]  

(A-12)
For Eq. (A-12) to be valid the power spectral density functions of the driving currents \( i_1 \) and \( i_2 \) must be similar within the filter bandwidth \( \Delta f \).

**Mechanical systems:**

Passive, linear mechanical systems that vibrate about a configuration of stable equilibrium and that do not contain non-bilateral elements such as gyrators, are reciprocal. For a mechanical system which can be modelled as \( n \)-port the general reciprocity relation can be written, in analogy with Eq. (A-5), as

\[
\sum_{j=1}^{n} F'_j v_j = \sum_{j=1}^{n} F_j v'_j.
\]  
(A-13)

For the case of a 2-port the special reciprocity relation analogous to Eq. (A-3) is

\[
\frac{v_2}{F_1} \bigg|_{F_2=0} = \frac{v'_1}{F'_2} \bigg|_{F'_1=0}.
\]  
(A-14)

However, in many situations it is of practical interest to consider the point forces and point velocities of six degrees of freedom. Then the system between a single source and a single receiver location is represented by a 12-port (see for example Fig. 2-3). With \( F_j \) and \( v_j \) being generalized expressions for the orthogonal forces, couples, translational and rotational velocities it follows from Eq. (A-13) that Eq. (A-14) is replaced by

\[
\frac{v_2}{F_1} \bigg|_{\text{other } 11 F_j \text{ zero}} = \frac{v'_1}{F'_2} \bigg|_{\text{other } 11 F'_j \text{ zero}}.
\]  
(A-15)

The analogy with Eq. (A-2) is

\[
\frac{F_2}{v_1} \bigg|_{\text{other } 11 v_j \text{ zero}} = \frac{F'_1}{v'_2} \bigg|_{\text{other } 11 v'_j \text{ zero}}.
\]  
(A-16)

see the examples in Eq. (2.19) and in Fig. 2.11.
Unto now conjugate pairs of variables have been considered. Although this is useful for heuristic purposes, to state that it is necessary, as is sometimes done (see e.g. /A.5/), imposes unnecessary and impractical restrictions. The special reciprocity relations analogous to Eqs. (A-2) - (A-4) and to Eqs. (A-11) and (A-12) remain valid for time integrated and time differentiated signals. This is particularly useful with respect to structureborne sound measurements, because accelerometers are more widely used than velocity transducers. Therefore it may be advantageous to replace in the special reciprocity relations velocities by accelerations; see the example in Fig. A.2a.

**Acoustical systems:**

Passive, linear acoustical systems which vibrate about a configuration of stable equilibrium are reciprocal. For a 2-port transfer system the general reciprocity relation analogous to Eq. (A-1) is

\[
 p_1'V_1 + p_2'V_1 = p_1V_1' + p_2V_2' ,
\]

(A-17)

where \( V \) denotes volume velocity. A well-known special reciprocity relation equivalent with Eq. (A-2) is

\[
 \frac{p_2}{V_1} \bigg|_{V_2=0} = \frac{p_1'}{V_2'} \bigg|_{V_1'=0} .
\]

(A-18)

In Eq. (A-18) the volume velocities may be replaced by volume accelerations; see the example in Fig. A.2b. Within the scope of the present discussion only measurements with omnidirectional point sources and point receivers are considered, but under certain conditions reciprocity relations can be extended to directive sources and receivers.

**Heterogenous systems:**

**Mechanical-Acoustical systems:** These systems are reciprocal under the same conditions as above for the separate systems. The general reciprocity relation for a n-port transfer system analogous to Eq. (A-5) is

\[
 \sum_{i=1}^{m} \sum_{j=n-m+1}^{n} (F_i'v_i + p_j'v_j) = \sum_{i=1}^{m} \sum_{j=n-m+1}^{n} (F_i'v_i + p_j'v_j) .
\]

(A-19)
Figure A.2:
Examples of equivalent transfer function measurements on basis of reciprocity.

Of particular interest is the case of 6 mechanical ports corresponding to 6 degrees of freedom at a single structural location and of 1 acoustical port. For this case the special reciprocity relations analogous to Eq. (A-4) are

\[
\frac{p_2}{F_1j} \bigg|_{y_2=0} = -\frac{V'_{1j}}{V_2'} \bigg| \begin{array}{l} \text{all } 6 \ F_1' \\ \text{zero} \end{array} \quad \text{other } 5 \ F_1 \text{ zero} \quad (A-20)
\]
Again the velocities and volume velocity in Eq. (A-20) can be replaced by the corresponding accelerations; see Fig. A.2c.

Reciprocity relations for the variables at 2 mechanical ports or at 2 acoustical ports of the heterogenous system are the same as in the above discussed homogenous systems.

**Electrical-Mechanical and Electrical-Mechanical-Acoustical systems:**

These are classes of heterogenous systems of great practical interest because they include reversible transducer types in isolation or in combination with mechanical and/or acoustical systems. Whether the transfer systems are reciprocal or anti-reciprocal depends on the types of transducer that are included. Moving coil transducers are anti-reciprocal /A.6/ because they use a permanent magnetic field both in the "source mode" and in the "receiver mode". If a heterogenous system includes a single anti-reciprocal transducer in series connection with the other systems, then the system is anti-reciprocal. If 2-anti-reciprocal transducers are included in series with the other subsystems (see below), then the system is reciprocal.

Some of the combinations for practical use are as follows:

1. The transfer system is between the electrical terminals of 2 transducers that are connected by an intermediate mechanical or mechanical-acoustical system. Figure A.2d shows an example with 2 moving coil transducers. For the resulting 2-port the special reciprocity relations in Eqs. (A-2) - (A-4) are valid. Such a configuration can be used to test the reciprocity of certain mechanical (-acoustical) systems.

2. The transfer system is between the electrical terminals of an electrical-mechanical transducer and a mechanical transfer system. If an anti-reciprocal transducer is included the general reciprocity relation for 6 mechanical ports at a single location in

\[
e'_{11} - \sum_{j=1}^{n} F'_{2j} v'_{2j} = e'_{11} - \sum_{j=1}^{n} F_{2j} v_{2j}.
\]  

(A-21)
Anti-reciprocity relations analogous to Eq. (A-10) are of the following type:

\[
\frac{v_{2j}}{i_1} \begin{cases} 
0 & \text{all } F_2 \\
F_2 \end{cases} = \frac{e'_1}{F_2} \begin{cases} 
1' = 0 & \text{zero} \\
0 & \text{other} F'_2 
\end{cases} \tag{A-22}
\]

If the velocity is replaced by the corresponding acceleration (see Fig. A.2e), the left hand side of Eq. (A-22) is replaced by \(a_{2j}/(j\omega i_1)\).

3. The transfer system is between the electrical terminals of an electrical-mechanical transducer and a single acoustical port. The general reciprocity relation in case of an anti-reciprocal transducer is

\[
e'_1 i_1 - p'_2 v_2 = e'_1 i_1 - p_2 v_2 
\tag{A-23}
\]

In analogy with Eq. (A-8) it follows that

\[
\frac{p_2}{i_1} \begin{cases} 
v_2 = 0 & \text{zero} \\
v_2 \end{cases} = -\frac{e'_1}{v'_2} \begin{cases} 
i'_1 = 0 & \text{zero} \\
i'_1 \end{cases} \tag{A-24}
\]

If the volume velocity is replaced by the volume acceleration (see Fig. A.2f), the left hand side of Eq. (A-24) is replaced by \(p_2/(j\omega i_1)\).
APPENDIX B: RECIPROCAL MEASUREMENT OF CORRELATION COEFFICIENTS FOR TRANSFER FUNCTIONS

The equality of the right-hand sides of Eqs. (3.9) and (3.10) is proved as follows. Let the complex transfer functions for the direct and reciprocal experiments be defined as

\[ S_1 = \frac{G_{F_1,F_1},e_1}{G_{F_1,F_1}} \quad f > 0 , \quad (B-1) \]
\[ S_2 = \frac{G_{F_2,F_2},e_2}{G_{F_2,F_2}} \quad f > 0 , \quad (B-2) \]
\[ S_1' = \frac{G_{i',i'},a'_1}{G_{i',i'}} \quad f > 0 , \quad (B-3) \]
\[ S_2' = \frac{G_{i',i'},a'_2}{G_{i',i'}} \quad f > 0 , \quad (B-4) \]

where the G's denote the one-sided power and cross-spectral density functions. According to Appendix A the following reciprocity relations can be derived:

\[ S_1 = S_1'/j\omega , \quad (B-5) \]
\[ S_2 = S_2'/j\omega , \quad (B-6) \]
\[ \frac{G_{e_1,e_1}}{G_{F_1,F_1}} = \frac{G_{a'_1,a'_2}}{\omega^2 G_{i',i'}} \quad , \quad (B-7) \]
\[ \frac{G_{e_2,e_2}}{G_{F_2,F_2}} = \frac{G_{a'_2,a'_2}}{\omega^2 G_{i',i'}} \quad (B-8) \]

From Eqs. (B-5) and (B-6) it follows that

\[ \text{Re} \left( S_1^* S_2 \right) = \frac{1}{\omega^2} \text{Re} \left( S_1'^* S_2' \right) \quad , \quad (B-9) \]

where the asterix means complex conjugate.
If for the direct experiment the point force excitations at the seating positions no. 1 and no. 2 are identical, i.e.

\[ F_1(t) = F_2(t), \quad (B-10) \]

then

\[ \text{Re} \left\{ S_1^* S_2 \right\} = \frac{\text{Re}\{G_{F_1}^*, e_1 \cdot GF_{F_1, e_2}\}}{G_{F_1, F_1}^2} = \frac{\text{Re} \{G_{e_1, e_2}\}}{G_{F_1, F_1}}. \quad (B-11) \]

In the same way it may be proved that

\[ \text{Re} \left\{ S_1^* S_1' \right\} = \frac{\text{Re} \{G_{a_1, a_2}'\}}{G_{i', i}}. \quad (B-12) \]

With Eqs. (B-7) - (B-12) it follows that the correlation coefficient defined according to Eq. (3.9) is equal to

\[
\Gamma_{\Delta f} = \frac{\int_{f_1}^{f_1 + \Delta f} \frac{\text{Re} \{G_{e_1, e_2}\}}{G_{F_1, F_1}} \, df}{\left( \int_{f_1}^{f_1 + \Delta f} \frac{\text{Re} \{G_{e_1, e_1}\}}{G_{F_1, F_1}} \, df \right)^{\frac{1}{2}} \left( \int_{f_1}^{f_1 + \Delta f} \frac{\text{Re} \{G_{e_2, e_2}\}}{G_{F_1, F_1}} \, df \right)^{\frac{1}{2}}} =
\]

\[
= \frac{\int_{f_1}^{f_1 + \Delta f} \frac{\text{Re} \{G_{a_1, a_2}'\}}{\omega^2 G_{i', i'}} \, df}{\left( \int_{f_1}^{f_1 + \Delta f} \frac{\text{Re} \{G_{a_1, a_1}'\}}{\omega^2 G_{i', i'}} \, df \right)^{\frac{1}{2}} \left( \int_{f_1}^{f_1 + \Delta f} \frac{\text{Re} \{G_{a_2, a_2}'\}}{\omega^2 G_{i', i'}} \, df \right)^{\frac{1}{2}}}.
\quad (B-13) \]
APPENDIX C: METHODS FOR VERIFYING THE PURITY OF POINT FORCE EXCITATION

Measurements of transfer functions, mechanical admittances or power injection for point force excitation may be subjected to systematic errors if other excitation forces and torques than the desired are generated. The unwanted excitation may originate from "misbehaviour" of the exciter, but in many cases the inertial loading on the excited structure is responsible when it affects the vibrations in other than the excitation direction. Reciprocity checks may reveal such errors. In many cases the test arrangement of Fig. A2-e is very practical for such a check. Violation of the reciprocity relation

\[
\frac{2}{j\omega_1} \begin{cases} \text{all } 6 \ F_2 \text{'s} = - \frac{e'_1}{F_{2z}} \text{ if } i'_1 = 0; \text{ other } \ F_2 \text{'s zero} \\
\text{otherwise} \end{cases}
\]

(C-1)

indicates excitation errors. The (anti-)reciprocity of the transducer may be checked in a system similar to that in Fig. A2-d.
The following Appendix has been published as "Letter to the Editor" in Journal of Sound and Vibration, 70, (1980), 133 - 138. The permission from Academic Press Inc. (London) Ltd. for using a photographic copy in this thesis is gratefully acknowledged.
CROSS SPECTRAL DENSITY METHODS FOR MEASURING
STRUCTURE BORNE POWER FLOW ON BEAMS AND PIPES

1. INTRODUCTION

Measuring acoustic intensity, structure borne wave intensity and mechanical power flow is of interest for purposes such as determining acoustic source strengths, identifying sound sources and transmission paths, studying the character of sound fields and testing conceptual approaches like Statistical Energy Analysis. With respect to the measurement methods the advent of digital equipment has stimulated the use of signal processing in the frequency domain. So-called cross spectral density methods have been reported for various applications; see, e.g., references [1-6]. The reliability of the cross spectral density method as a signal processing method is very good owing to the elegant switching techniques for the elimination of instrument phase mismatch; see reference [2].

Recently the author has been considering the feasibility of power flow measurements for pipes in the low frequency range where only bending, longitudinal and torsional waves occur. In this letter cross spectral density methods are formulated for measuring the one-dimensional power flow for these three wave types. For bending waves the method given by Pavić in references [5, 6] is used, but with modifications in respect to the use by Pavić of signal processing in the time domain and vibration transducers other than accelerometers.

2. EQUIVALENT SIGNAL OPERATIONS

The mathematical basis for the cross spectrum methods can be found in the theory of Fourier transforms. If one works out the equivalence of signal processing in the time domain and in the frequency domain, the definitions of certain operations and functions, like cross spectral density, depend of course on the nature of the signals: e.g., stationary random signals or deterministic transients. This subject, however, is sufficiently covered in textbooks and articles. Therefore in this letter only the evident relations are put forward for sinusoidal signals, and on the basis of similarity those for broadband signals, in this way conveying the principles.

The following short notation is used:

\[ \langle xy \rangle_t \triangleq \langle x(t)y(t) \rangle_n \quad \langle x \rangle_t \triangleq \langle x(t) \rangle_n \]

\[ \langle x \rangle_0 \triangleq \langle x(t) \int y(\tau) \, d\tau \rangle_n \]  

(2.1, 2.2)

e.t., where \( \langle \cdots \rangle_t \) means time averaging. For sinusoidal signals the notation

\[ x(t) = \bar{X} e^{j\omega t}, \quad y(t) = \bar{Y} e^{j\omega t} \]  

(2.3)

is used and for broadband signals the one-sided cross spectral density

\[ G(x, y, f) = G^*(y, x, f) \quad \text{for } f \geq 0 \]  

(2.4)

is postulated where \( ^* \) means complex conjugate.

In Table 1 the equivalent operations in the time domain and frequency domain are presented. One observes from the table that the time average product of two time-varying signals may be obtained from their cross spectral densities. Whether one needs the real or the imaginary part of the cross spectral density depends on the number of integrations.
which are required for the signals in the time domain. The equivalence of the operations on
the second line of this table has been used in the cross spectral density methods to which
reference has been made in the introduction, as follows.

For mechanical power flow due to a point force measured with force transducer and
accelerometer [4],
\[ \langle P_m \rangle_t = \langle Fv \rangle_t = \int_0^\infty \frac{\text{Im} G(F, a, f)}{\omega} df, \] (2.5)

where \( v = \int a \) (i.e., \( v(t) = \int_0^t a(\tau) d\tau \)).

For acoustic intensity near a radiating surface, measured with a microphone and an
accelerometer [3],
\[ \langle I_a \rangle_t = \langle pv \rangle_t = \int_0^\infty \frac{\text{Im} G(p, a, f)}{\omega} df, \] (2.6)

where \( a \) is the acceleration of the radiating surface.

For acoustic intensity, measured by the two microphones method [1, 2],
\[ \langle I_a \rangle_t = \langle pv \rangle_t = \frac{1}{\rho \Delta} \left( p_2 \int p_1 \right) = \frac{1}{\rho \Delta} \int_0^\infty \frac{\text{Im} G(p_2, p_1, f)}{\omega} df, \] (2.7)

where \( v \) is the particle velocity, \( \rho \) is the density of air and \( \Delta \) is the distance between the
microphones. The approximations of \( \langle I_a \rangle_t \) in equation (2.7) both in the time domain
and the frequency domain follow from the well known pressure-sum pressure-difference
principle; see, e.g., references [7, 8, 2]. In the next sections the measurement of structure
borne power flow is discussed. Where in the acoustic case the two microphones method eliminates the measurement of a kinematical quantity (i.e., particle velocity), in the structure borne case the measurement of dynamical quantities (i.e., forces and moments) is to be eliminated. It will be shown for the case of one-dimensional wave propagation on beams and pipes that, by using the relations on the fourth line of Table 1, the power flow can be formulated in terms of cross spectral densities of accelerations.

3. BENDING WAVES ON A BEAM

A method for measuring the intensity in bending waves has been discussed by Pavić [5, 6]. For the one-dimensional case of a beam and with “pure” bending waves, two terms contribute to the power flow: i.e., a force term and a moment term. The power flow may be written in terms of the lateral displacement as

$$\langle P_B \rangle_t = B \left( \delta^2 \eta / \delta x^2 \eta / \delta t - \delta^2 \eta / \delta x^2 \delta \eta / \delta x \partial t \right)_n.$$  

where $B$ is the bending stiffness of the beam and $\eta$ is the lateral displacement. In terms of the lateral acceleration one obtains

$$\langle P_B \rangle_t = B \left( \int \int \delta^3 a / \delta x^3 \right) a - \int \int \delta^2 a / \delta x^2 \int \delta a / \delta x \right)_t.$$  

For the special case without “near fields”, the time averages of the force and moment terms are equal. This situation occurs sufficiently far (see reference [9]) from the excitation position and discontinuities. Then the power flow may be written as

$$\langle P_B \rangle_t = -2B \left( \int \int \delta^2 a / \delta x^2 \int \delta a / \delta x \right)_t.$$  

Using the relation on the fourth line of Table 1, one obtains, in the frequency domain,

$$\langle P_B \rangle_t = 2B \int_\omega^\infty \frac{\text{Im} G(\delta^2 a / \delta x^2, \delta a / \delta x, f)}{\omega^3} df.$$  

For the situation without “near fields”, the relation between the Fourier components of $\delta^2 a / \delta x^2$ and $a$ is

$$F_{\delta^2 a / \delta x^2} = -k_B^2 F_a,$$  

where $k_B$ is the bending wave-number.

The power flow can now be obtained from the cross spectral density of two closely spaced accelerations. An acceleration-sum acceleration-difference principle follows from

$$F_a = (F_{a1} + F_{a2})/2, \quad F_{\delta a / \delta x} = (F_{a2} - F_{a1})/\Delta,$$  

where $\Delta$ is the distance between the two accelerometers. Then, as in the acoustic case, one may prove, by using equations (3.3)-(3.6), that

$$\langle P_B \rangle_t = \frac{2(\omega m'/B)^{1/2}}{\Delta} \int_0^\infty \text{Im} \frac{G(a_2, a_1, f)}{\omega^2} df,$$  

if one substitutes $\omega (m'/B)^{1/2}$ for $k_B^2$, where $m'$ is the mass per unit of length. For this special case Pavić has formulated a similar cross spectral density method, but in terms of lateral velocity. For the measurements he unnecessarily used integrators in combination with accelerometers.

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For the general case with "near fields" the power flow on a beam can be measured with a linear array of four accelerometers at equal spacing $A$. Substitution of finite difference approximations for the spatial derivatives of the lateral acceleration in equation (3.1a) leads to (compare this with Pavić’s equation (5.3.8) in reference [5] and equation (11) in reference [6]).

$$\langle P_B \rangle_t = (B/A^3) \left[ 4 \int a_3 \int a_2 - \int a_4 \int a_2 - \int a_3 \int a_1 \right]_t. \quad (3.8)$$

The equivalent expression in the frequency domain follows from the relation on the fourth line of Table 1 and equation (2.4):

$$\langle P_B \rangle_t = \frac{B}{A^3} \int \int \frac{\text{Im} \ G(a_2, a_3, f)}{\omega^3} \text{df} - \int \int \frac{\text{Im} \ G(a_2, a_4, f)}{\omega^3} \text{df} - \int \int \frac{\text{Im} \ G(a_1, a_3, f)}{\omega^3} \text{df}. \quad (3.9)$$

(Note: to agree with reference [6], the accelerometer number decreases with increasing x co-ordinate.)

The disadvantage of signal processing in the frequency domain is that it is more time consuming. This disadvantage, however, becomes less relevant as faster processors become available. A great advantage is, besides the flexibility with respect to frequency resolution, that no careful development and/or testing of electronics is necessary. This is especially attractive if one is still in the stage of a feasibility study on the application of power flow methods for path identification under realistic circumstances, as was the case in the author’s studies. In that case one wants to concentrate on methodological problems rather than on instrumentation.

4. LONGITUDINAL AND TORSIONAL WAVES

For measuring power flow in longitudinal and torsional waves on beam-like structures, cross spectral density methods can be formulated which are analogous to the acoustic two microphones case. For two closely spaced accelerometer positions the power flow can again be approximated by using an acceleration-sum acceleration-difference method. For longitudinal waves one obtains

$$\langle P_L \rangle_t = \langle F \partial \xi/\partial t \rangle_t = -SE \langle \partial \xi/\partial x \partial \xi/\partial t \rangle_t = -SE \left[ \int \int \partial a/\partial x \int a \right]_t, \quad (4.1)$$

where $\xi$ is the displacement in the propagation (x-)direction, $S$ is the cross-section area, and $E$ is the modulus of elasticity. For torsional waves on beams or pipes with rotationally symmetric cross sections (pure transverse waves) one obtains (see reference [10])

$$\langle P_T \rangle_t = \langle M \partial \chi/\partial t \rangle_t = -T \langle \partial \chi/\partial x \partial \chi/\partial t \rangle_t = -T \left[ \int \int \partial \alpha/\partial x \int \alpha \right]_t, \quad (4.2)$$

where $\chi$ and $\alpha$ are, respectively, the angular displacement and angular acceleration and $T$ is the torsional stiffness. The sum and difference approximations follow from $a = (a_1 + a_2)/2$, $\partial a/\partial x = (a_2 - a_1)/\Delta$, $\alpha = (\alpha_1 + \alpha_2)/2$, and $\partial \alpha/\partial x = (\alpha_2 - \alpha_1)/\Delta$, with $\Delta$ the separation distance for the closely spaced accelerometer positions. In analogy with the acoustic case one gets in the frequency domain the following cross spectral formulations:

$$\langle P_L \rangle_t = -\frac{SE}{\Delta} \int \int \frac{\text{Im} \ G(a_1, a_2, f)}{\omega^3} \text{df}, \quad \langle P_T \rangle_t = -\frac{T}{\Delta} \int \int \frac{\text{Im} \ G(a_1, a_2, f)}{\omega^3} \text{df}. \quad (4.3, 4.4)$$
For the proper measurement of \( a \) and \( \alpha \), respectively, one may use, at either of the locations 1 and 2, two common accelerometers as in Figure 1.

![Figure 1. Accelerometer configurations for measuring longitudinal and angular accelerations.](image)

The simplest procedure now is to realize the addition of the accelerometer signals in the time domain and carry out the rest of the signal processing in the frequency domain. Adding devices, with an adjustable gain factor in one of the channels for transducer matching, are cheap and accurate. Then equations (4.3) and (4.4) are to be replaced by

\[
\langle P_L \rangle_t = \frac{SE}{4A} \int_0^\infty \text{Im} \left( \frac{a_1''', a_2'''', f}{\omega^3} \right) df,
\]

\[
\langle P_T \rangle_t = -\frac{T}{2AD^2} \int_0^\infty \text{Im} \left( \frac{a_1'''', a_2'''', f}{\omega^3} \right) df,
\]

where \( a''' \) and \( \alpha''' \) stand for \( a' + a'' \) in the situations of Figure 1. If one performs all signal processing in the frequency domain one has to replace the cross spectral density functions in equations (4.3) and (4.4) by a sum of four cross spectral densities, but this is unnecessarily cumbersome.

5. CONCLUSION

For measuring power flow along beams or pipes cross spectral density methods have been formulated for bending, longitudinal and torsional waves. The transducers which are needed are normal accelerometers.

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Bij de voltooiing van deze studie erken ik met dankbaarheid dat God mij er de werkracht voor gaf en bovendien gezondheid aan mijn vrouw en kinderen. Daarnaast betuig ik gaarne mijn dank aan degenen die hebben meegewerkt aan de totstandkoming van dit proefschrift.

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- De experimenten zijn voor een belangrijk deel uitgevoerd door F. Goos (hfdst. 6), A. Huisman (hfdst. 3.3, 4, 5 en 7) en H.M.A. Karreman (hfdst. 2 en 3.4).

- De meetopstellingen zijn verzorgd en deels ontworpen door A.W. Witvliet.

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9 december 1943 geboren te 's-Gravenhage
1956 - 1962 's-Gravenhaags Christelijk Gymnasium; diploma Gymnasium-β
1962 - 1963 natuurkundig assistent, Natuurkundig Laboratorium van Philips, Eindhoven
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1969 - heden Technisch Physische Dienst TNO-TH, Delft; werkzaam bij de afdeling Scheepsakoestiek onder leiding van ir. J.H. Janssen
1. Het sterk experimentele karakter van het scheepsakoestisch onderzoek in Nederland hangt samen met de geografische omstandigheid van geringe afstand tussen onderzoekinstelling enerzijds en werven en havens anderzijds.

2. Het toepassen van een dubbelverende opstelling bij scheepsmotoren is een dure vergissing wanneer de geluidreductie in flankerende overdrachtpaden, zoals pijpen en schroefas, niet zeer hoog is (hoofdstukken 6 en 8 van dit proefschrift).

3. De theoretische beschrijving van de geluidisolatie van spouwwanden met behulp van Statistische Energie Analyse\(^1\) houdt tot nu toe geen rekening met de in hoofdstuk 4 van dit proefschrift beschreven invloed van de spouw-diepte op de stralingsweerstand van de spouwbladen. De discrepantie tussen metingen en berekeningen\(^2\) is zonder voldoende theoretische of experimentele grond toegeschreven aan "niet-resonante stijfheids-koppeling" door de lucht tussen de spouwbladen\(^3\).


4. Bij de constructiegeluidoverdracht in complexe systemen (bijv. schepen en gebouwen) zijn alle typen elastische golven sterk gekoppeld. Bij de experimentele bepaling van energiestromen dienen derhalve niet een of meer golftypen a priori te worden verwaarloosd, ook niet wanneer de bijbehorende golflengten veel groter zijn dan de afmetingen van de constructie (hoofdstuk 5 van dit proefschrift).

5. Constructiegeluidisolatoren voor scheepstoepassing moeten isoleren in alle trillingsrichtingen. Vele van de in de handel zijnde isolatoren zijn ontworpen voor het isoleren van één-dimensionale trillingen en vertonen ongewenst hoge stijfheden voor de andere richtingen.
6. Voor de suggestie dat er fundamenteel een aanzienlijke discrepantie zou zijn tussen dissipatie-verliesfactoren verkregen uit nagalmmetingen en uit energiestroommetingen, bestaat geen goede theoretische of experimentele grond.

7. De bijdrage van resonerende trilvormen aan de geluidsuitstraling door een scheepshuid in water ten gevolge van puntkrachtaanstoting bij frequenties ver beneden de grensfrequentie, wordt door een aantal auteurs sterk onderschat.
2. T. ten Wolde, Proefschrift, Delft, 1973

8. De benaming "alternatief" voor de homoeopatische geneeswijze is niet passend omdat daarmee de complementaire en de additieve functie van deze geneeswijze wordt genegeerd. Een vooruitstrevend volksgezondheidsbeleid zou het kader moeten scheppen waarbinnen constructief onderzoek mogelijk is om vast te stellen op welke gebieden deze complementaire en additieve functies op grotere schaal kunnen worden benut dan nu het geval is.


10. Eén van de toetsingscriteria voor deugdelijkheid van een herziene structuur van voortgezet onderwijs zou moeten zijn, dat voor de laatste onderwijsfase geen niveauverlagingen noodzakelijk blijken. Een goede experimentele toetsing kan derhalve op zijn vroegst pas twaalf jaar na de gedeeltelijke invoering voltooid zijn.

11. Met moed, beleid en/of angst verdedigt een promovendus zich tot en met de laatste stelling.
