Stopping criteria for Krylov methods and finite-element approximation of variational problems

Mario Arioli* and Daniel Loghin
Rutherford Appleton Laboratory
Chilton, Didcot, Oxfordshire, OX11 0QX, UK
e-mail: m.arioli@rl.ac.uk
web page: http://www.numerical.rl.ac.uk/people/marioli/marioli.html

ABSTRACT

We combine linear algebra techniques with finite element techniques to obtain a reliable stopping criterion for Krylov method based algorithms. The Conjugate Gradient method has for a long time been successfully used in the solution of the symmetric and positive definite systems obtained from the finite-element approximation of self-adjoint elliptic partial differential equations. Taking into account recent results [5,6,7] which make it possible to approximate the energy norm of the error during the conjugate gradient iterative process, in [1] we introduce a stopping criterion based on an energy norm and a dual space norm linked to the continuous problem. Moreover, we show that the use of efficient preconditioners does not require us to change the energy norm used by the stopping criterion.

In [3], we extend the previous results on stopping criteria to the case of nonsymmetric positive-definite problems. We show that the residual measured in the norm induced by the symmetric part of the inverse of the system matrix is relevant to measuring convergence in a finite element context. We then provide alternative ways of calculating or estimating this quantity.

Finally, we extend the results of [1] to the Block Conjugate Gradient (BCG) algorithm [2,4]. In particular, we show that the simple rule proposed in [1] for computing the stopping criterion can be easily extended to the BCG algorithm with a cheap cost proportional to the square of the block size.

REFERENCES


