STEREOPHOTOGRAPHY OF

OCEAN WAVES

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1. Introduction

The Delft University of Technology and the Ministry of Public Works in the Netherlands jointly developed a system to measure the instantaneous three-dimensional sea surface by means of stereophotography from helicopters. The objective of this report is to describe this system.

The system was developed as one of the elements in a study of the directional characteristics of ocean waves. This study was primarily aimed at estimating the two-dimensional spectrum of waves which gives the distribution of wave energy over wavelengths and directions. The system and procedures which are described in this report are based on the requirements to measure this spectrum.

The arrangements of the chapters is as follows. In chapter 2 some basic elements of the system are described. A description of the operational system and procedures is given in chapters 3 and 4 respectively. The procedures for the analysis of the photographs are described in chapter 5.

Information on the system has also been published elsewhere. In Holthuijsen et al. (1974) an integrated, but less detailed, review is presented and in v.d. Vliet (1972, 1974) a detailed description of the electronic components is given. Some preliminary results with emphasis on wind wave properties have been reported in Holthuijsen (1978).

2. Basic elements of the system

Stereophotogrammetry is a well-established technique in the survey of land surfaces. In oceanography it can be used to monitor the sea surface but this requires some adaptations from the conventional methods.

Some relevant aspects of stereophotogrammetry will be outlined briefly based on Thompson (1966) and some basic system requirements will be indicated.

2.1. Principles of stereophotogrammetry

To reduce the system to its most essential structure, it is assumed that the photographs are made looking downwards, that is, the camera axes are truly vertical during the exposure of the film and there is no tilt whatsoever. The image in the photograph will be a projection of the photographed terrain, and the distance between points on the ground at equal elevation can be determined fairly simple if the scale of photography is known. This scale can be expressed in terms of f and h as in equation (1) where s is the scale, defined as the ratio of corresponding linear lengths in the terrain and in the photograph, f is the focal length of the lens and h is the altitude of photography above the horizontal terrain level ABC (fig. 1).

$$s = f/h \quad (1)$$

If the values of f and h are known the positions of A, B and C can be determined relative to the camera from the images a , b and c . But if the terrain is hilly or mountainous, two photographs are needed to determine the geometry of that terrain.

To describe the latter situation two photographs are illustrated in fig. 2. Consider them to be taken truly vertically by two identical cameras from the same altitude.

Fig. 1. Vertical photography of plane ABC

Fig. 2. Stereo photo pair (after Thompson, 1966)

The elevation above a horizontal plane of reference is h and the distance between the cameras is $00' = b$. An x-axis parallel to $00'$ and through the geometric centres of the pictures n and n' has been adopted in each picture. The elevation of a point A in the terrain is Δh . Its image in the lefthand picture is a and in the righthand picture it is a' . The projections of a and a' on the x-axes are a_1 and a'_1 , which are the images of A_1 . The values of the abscissas na_1 and $n'a'_1$ are different and by definition this difference is the parallax of A for the two pictures. Its value is given in the following equation where p is the parallax.

$$p = na_1 - n'a'_1 \quad (2)$$

It can be shown by similar triangles that

$$h = h - \frac{bf}{p} \quad (3)$$

This equation is the basis of the photogrammetric analysis to determine the elevation of points in the photographed terrain.

2.2. System requirements

The concept of parallax is central to the analysis. Note that the derivation of equation (3) is based on three assumptions (a) no tilt of the photographs, (b) equal camera elevation and (c) equal focal lengths. These conditions will not be met exactly in practice and allowance for deviations must be made in the analysis of pictures taken during actual field operations. These corrections are described in standard text books such as Thompson (1966), Schwedfsky (1963) or Finsterwalder and Hofman (1968).

It appears that the corrections can be carried out most readily when the deviations from the ideal situation are small. In that case conventional instruments and procedures can be used

to analyze the pictures. Values related to the position and the orientation of the pictures which reflect directly on the operational system are listed in table 1.

- altitude difference	< 10%
- overlap of pictures in x-direction	> 80%
- overlap of pictures in y-direction	50 - 70%
- tilt	< 3°
- difference in orientation	< 15°

Table 1. Limitations on camera position and orientation.

For most analysis procedures the stereoscopic effects are optimal if the overlap of the pictures is about 60% in the x-direction. This is illustrated in fig. 3. The format of the picture in x-direction is l'_x , corresponding to a distance l_x in the terrain.

Fig. 3. Stereophotographic overlap

By considering similar triangles in fig. 3 and by equating b to $0.4 l_x$ as required for a 60% overlap in x-direction, the following relationship can be found.

$$\frac{h}{b} = \frac{f}{0.4 l'_x} \quad (4)$$

For a given set of cameras with f and l'_x fixed, it appears that the ratio h/b is fixed. This leaves h or b to be chosen for the actual photographic operations.

The altitude for a photographic mission is chosen on the basis of two considerations. They concern the error in the measurements and the size of the area to be photographed. More information on this subject is given in paragraph 4, where the operational procedures are described and in Appendix I.

In conventional geodetic survey the stereophotographs are usually obtained by a camera looking vertically downward from an airplane which is flying directly over the terrain of interest. The pictures are taken in sequence such that consecutive pictures cover overlapping areas on the ground. In these areas of overlap, the stereo effects are used to determine the geometry of the terrain. Note that the two camera stations needed to produce the stereo effects are two different positions of one camera and that the time interval between the pictures is irrelevant for fixed (non-moving) terrains.

The sea surface changes rapidly and if the conventional technique were used, serious distortions in the stereo effects would occur due to the change in surface geometry between two exposures. To limit these distortions to an acceptable level, the time interval between the exposures should be at most a few ms (see below). It is not possible with conventional aircraft to position the camera at the two stations within this time interval and consequently two cameras are needed which take

the pictures more or less simultaneously.

In the literature on this subject several investigators have indicated a maximum value for this interval, without, however, going into detail as to how they arrived at the value. Cote et al. (1960) stated a desired value of less than 10 ms and Cruset (1952) recommended an interval of less than 5 ms. One exception to the conclusion that two synchronized cameras are needed is provided by Howard (1969)* who estimated that a 40 ms interval would still produce satisfactory results. From this assumption he proceeded to implement a system consisting of one high speed camera (movie camera) in a fast flying airplane. Howard (1969) did obtain stereo effects and the pictures were analyzed to produce wave number spectra.

An effort will be made here to estimate the maximum permissible time interval but it is difficult to state the problem quantitatively and only an order of magnitude will be established. The measurements in the stereo photographs are based on parallax effects in the images, that is, on horizontal displacements in the pictures. These should solely be the result of the three-dimensional geometry of the surface. But horizontal displacements are also caused by the horizontal or vertical movements of the surface between the two exposures. In particular the horizontal movement was considered to be important since its speed is much greater than that of the vertical movement. It was estimated by photogrammetric experts that a horizontal displacement in the surface of a decimeter, say, would result in an error of a few centimeter in determining the surface elevation from a few hundred meters altitude. This error seems to be acceptable. To

* The title of one of the references (Dubovskoy and Perkis, 1956) in Krylov et al. (1968) suggests that this reference provides another exception.

correctly establish from this spatial limitation the requirement for the time interval is very complicated, if alone for the random nature of the sea surface. The problem is reduced appreciably if it is assumed that the phase speed of the significant wave can be used to transform the spatial limitations into a time interval. The photographic system has been designed to be used in situations where this phase speed would not exceed 10 m/s (significant period less than 7 s). It follows that in the most unfavourable conditions the above mentioned limit of one decimeter, and consequently the error of a few centimeter, is reached in 10 ms. For good measure the acceptable limit was set at 5 ms. These values are consistent with those mentioned by Cote et al. (1960) and Cruset (1952).

The value of 5 ms seems to be unnessecerily small when compared with the exposure time of the film which is 5 to 10 ms. But the effect of the finite exposure time is to "blur" the image which in turn "blurs" the parallax. The differences in timing, however, produces false parallax which can cause larger errors.

Using two cameras simultaneously rather than one camera in sequence is the single most important difference between the systems as used in conventional geodetic survey and as used in the present system.

Other basic requirements on the system are standard for aerial photographic survey and will not be discussed here.

3. Operational system

The operational system consisted primarily of two vertically downward looking cameras each mounted in a helicopter. The cameras were triggered by a radio signal to ensure synchronization.

3.1. Cameras

The central elements in the operational system are obviously the cameras. The task to select and synchronize the cameras was given to the Central Electronics Service Department of the Delft University of Technology. This task was performed in close cooperation with the Survey Department of the Ministry of Public Works which eventually provided the selected cameras and radio equipment.

In the operations for normal geodetic survey, cameras are used which have been specially designed for high-quality aerial photography. The performance of these cameras is usually adequate and for this reason an investigation was started to establish whether this type of camera could be synchronized to the specified degree.

The cameras usually require very limited manual operation. They can be triggered electronically, that is, the command pulse to start the sequence in the camera to open and close the shutter can be given by an electrical signal. In the cameras investigated this sequence consisted of moving mechanical parts in the camera. It was found that the time between the triggering of the camera and the actual opening of the shutter varied randomly from exposure to exposure. In this report this interval between triggering and shutter opening will be called camera delay. It consists of an average delay and a random variation which for the purposes of this report is adequately described by the standard deviation of the

camera delay. When two of these cameras are triggered perfectly, simultaneously, the film in the two cameras would still be exposed at different times due to the random differences in camera delays.

When activating the cameras simultaneously one could conceivably anticipate the moment of exposure and compensate electronically. This approach was seriously considered for those cameras with a rotary disk shutter where such an anticipation can be made. But major mechanical and electronic adaptations in the cameras would be needed and the proprietors (commercial firms) did not wish to have the cameras adapted to such an extent. Instead a less demanding approach was chosen. If the standard deviation of the camera delay could be reduced to an acceptable level (1 or 2 ms, say) only the differences in the average camera delays of the two cameras would need compensation. This has been done by providing each camera with an electric unit to generate an additional delay for each camera such that the total average delays for both cameras were equal within acceptable narrow limits. The variation of the difference in the random camera delays was approximately equal to the sum of the individual variations. The basic idea is illustrated in fig. 4 where the time diagram of the sequence is schematically indicated. The first trigger for each camera in the present system could be produced by a radio signal.

Fig. 4. Time diagram of triggering

If the exposure in the two cameras is to be initiated within a time lapses of 5 ms, it follows that the standard deviation of the camera delay should be at most a few ms and even then the 5 ms limit would sometimes be exceeded. Laboratory investigations of two types of conventional aerial survey cameras showed that the variation of the camera delay was too large and that reducing it could only be achieved by major alterations in the cameras.

Two other types of cameras were also considered. The first type was Hasselblad 500 EL, which is a fine camera but which has not been designed as a survey camera. The geometric stability and the lensdistortions are not up to the usual standards for survey photography. However, it was found that the standard deviation of the camera delay could be reduced to approximately 0.3 ms with only minor alterations in the camera. In view of this it was decided to use these cameras and to improve on or correct for metric shortcomings. The second type of camera is the UMK camera of Jenoptik (Jena, DDR) which came available one year after the implementation of the Hasselblads. The metric qualities of this camera are good and the standard deviation of the camera delay was found to be on the order of 1.5 ms. This camera too has not been designed for aerial survey photography. The focal length f and the image format in x-direction l'_x are 5.0 cm and 5.0 cm for the Hasselblad cameras and 10.0 cm and 17.0 cm for the UMK cameras^{*}.

For both types of cameras control units were built. These units were built in larger units which were totally self-contained for the photographic operations. That is, they carried such peripheral equipment as radio receiver, suction pump (for flattening the film during exposure) and power supply. Each unit provided support for one of the cameras but a second camera could also be plugged in. It could be triggered simultaneously with the other cameras. The function of this additional camera will be explained later when the problem of

^{*} The nominal formatsize is 1.0 cm larger but the edges of the pictures were not used due to reduction of photographic quality at the edges.

determining the scale of photography will be discussed. For a detailed description reference is made to v.d. Vliet (1972, 1974).

The equipment was tested extensively both in the laboratory and during testflights (5 flights in all, each with approximately 60 exposures). During the first flight with the Hasselblads, the cameras were mounted outside the aircraft. This mounting proved to be unsatisfactory because the camera delays grew to unacceptable values during the flight. The delays could be measured during the flights through the use of the flash gun contact which closed at the initiation of the exposure. Upon checking the cameras in the laboratory it was found that the camera delays were sensitive to changes in temperature and humidity. To avoid this problem, the cameras were mounted inside the aircraft and sealed from the outside atmosphere by a window pane of optical glass. The Hasselblad cameras have performed perfectly ever since. The same type of mounting was used for the UMK cameras. These cameras, however, failed to perform from time to time due to failures of electronical or mechanical parts inside the cameras. These failures could not always be traced or repaired during the photographic missions and some missions have had to be abandoned. It should perhaps be restated in this context that the cameras have not been specifically designed for survey photography from aircraft and that they have been subject to conditions which are probably far outside the manufacturer's specifications (e.g. excessive vibrations, see paragraph 3.3 on camera mounting).

3.2. Aircraft

Other major components in the system are the camera carriers and from the outset it was decided to use aircraft for this purpose. Blimps, which have been used by other investigators (e.g. Horikawa and Sasaki, 1972) were not seriously considered because of availability and operational problems. In consultation with experts several types of aircraft were checked and both military and commercial institutions were visited. It does not seem relevant

to relate the findings here and it may suffice to state that the Royal Netherlands Air Force (RNAF) was able and willing to provide assistance far superior to alternative options available.

The aircraft chosen were helicopters which were operated by the Search and Rescue team of Soesterberg airbase (now at Leeuwarden air base). Two of these helicopters were assigned to the study for a total of 120 flying hours over a period of three years. The primary peace-time tasks of this team is to perform search and rescue operations at sea. A second peace-time task is to carry out photographic missions. Obviously this was a very fortunate combination of capabilities and technical facilities.

The helicopters were of the type Alouette III with a transportation capacity of six persons. These helicopters, as used by the Search and Rescue team did not carry sophisticated equipment to aid in navigation and only magnetic compass and speedometer were used. This seemed to be adequate since the requirements for the accuracy of navigation were not very severe because the primary purpose of the study referred to in the introduction was to study the directional characteristics of the waves without necessarily relating these to the position of observation. The latter was a secondary objective at best. For the purpose of the study it was considered acceptable to know the position with an accuracy consistent with the variations of characteristic parameters of the wave field. The orientation of the helicopters was needed to determine the orientation of the pictures relative to the wind direction. An average deviation of 5° and a standard deviation of 5° in the helicopter orientation seemed to be adequate for this purpose.

3.3. Camera mounting

The helicopters were accommodated with drop doors. These were open during the photographic missions and the cameras were mounted

over these doors on a wooden plank. As noted before, a window pane of optical glass sealed the cameras from the outside air. It was found during the test flights that the floor of the helicopter tilted approximately 7° (head down) at a forward speed of about 70 knots which was a typical speed for all operations. For convenience in the analysis of the pictures this tilt was removed from the cameras by tilting the mountings. The cameras were secured with bolts and nuts to the wooden plank which in turn was bolted down to the helicopter floor. A consequence of this construction was that the motion of the helicopter, including vibrations were transmitted directly to the cameras. A gyro-stabilized and vibration-damping construction was considered but the cost was prohibitive.

In addition to the two downward looking cameras (one in each helicopter), a third camera was introduced. This camera was used for two purposes both related to estimating the distance between the helicopters: maintaining the helicopter formation and determining the scale of photography.

In conventional aerial survey the scale of photography is usually determined from known linear distances in the photographs between points in the terrain. The equivalent situation at sea would be to have a ship or other structure with known dimensions in the area of photography. This is very well possible and in SWOP (Cote et al., 1960) a ship was used which towed a buoy. The distance between the ship and the buoy was continuously monitored and provided the required linear distance in the pictures. A major drawback of this approach is that the area of operation is restricted to the immediate vicinity of such structures. To avoid this restriction another method was adopted here which is based on determining the distance between the cameras (or the helicopters) photographically. The third camera (also a Hasselblad 500 EL) was used for this. It was mounted in one of the helicopters with the other helicopter in the field of view through an open window. The image of that helicopter was projected on the film and since the

geometry of that helicopter was known the distance could be computed. The accuracy of this method was estimated by checking the method with the helicopters on the ground and it was found to be better than 3%.

This third camera was also used as a range-finder during the flight to maintain the relative position of the helicopters. A number of calibration lines was super-imposed on the viewer of the camera which helped an observer to estimate the distance between the helicopter. Differences in altitude could also be observed through this camera.

3.4. Flight performance

The performance of the helicopters has been evaluated through an extensive test flight over sea and it was found that all of the requirements listed in table 1 could be met within acceptable limits. The windspeed during the flight was approximately 10 m/s and the atmospheric conditions were rather turbulent. The altitude varied from 200 to 500 m. The observed values of camera position and orientation are listed in table 2.

	average	standard deviation
- altitude difference	10.6%	1.6% (of altitude)
- overlap of pictures in x-direction	79%	6.5%
- overlap of pictures in y-direction	60%	9.5%
- tilt	1.9°	2.1°
- difference in orientation	5.6°	4.7°

Table 2. Observation of camera position and orientation (see also table 1).

The numbers in this table are based on approximately thirty observations except for the difference in orientation which was based on ten observations.

The average deviation and standard deviation of the helicopter orientation relative to true North was 1.5° and 3.3° respectively. These values are based on ten observations over a fixed platform at sea with known orientation.

4. Operational procedures

The operational procedures are of two kinds: those related to the preparations for the photographic mission and those related to the actual flight. As regards the preparations it may suffice to indicate that the logistics for the helicopters in the study referred to in the introduction were mostly arranged through military channels. Details on these arrangements, including transportation, housing, licences etc. do not seem to be relevant here. The preparation of the photographic equipment was limited to setting the instrumentation on stand-by with a film type depending on anticipated weather conditions. During the actual flight attention was concentrated on the second kind of procedures which will be discussed next.

The total crew in the two helicopters consisted usually of seven people but in later operations, when the UMK cameras failed from time to time, an extra crew member was introduced. Before and after each flight the crew was briefed and debriefed, the pattern for a typical flight being of the following nature.

During one flight observations were to be carried out at a number of locations. The large scale procedure was to fly from the airport (not necessarily the home base of the helicopters but the airport where the different groups assembled) to a pre-arranged area of observation and to hop from one point of observation to the other. During the legs of the flight to and from and between the points of observation the usual flight procedures in air traffic were observed.

At a point of observation the procedures were concentrated on obtaining a sequence of non-overlapping stereo-photographs . In the study referred to in the introduction this sequence consisted of approximately ten photo pairs. Such an action to obtain the sequence was called a sortie. The pilots found it convenient to agree on the following procedure: one helicopter (the "leader") would keep

to a steady course at a specified altitude, direction and speed. The other (the "follower") would take its position relative to the leader such that the two helicopters were flying side by side at a specified distance. Finding this position was based to a large extent on the information from the viewer of the third camera which acted as a range finder. The choice of the altitude for a given set of cameras depends on several considerations related to the wave field. These considerations and the results thereof are discussed in Appendix I.

When the required helicopter formation was achieved, the photographer started the photographic sequence by operating the radio transmitter. Careful timing located the pictures within a few hundred meter of the specified position. But many times the operational procedures were frustrated by camera failures, atmospheric disturbances, clouds, rain etc. and the sortie had to be relocated, flown again or abandoned.

5. Photogrammetric analysis

To quantify the information of the stereo photo pairs, equation (3) was evaluated for each point of interest of the sea surface. In this equation the values of b (the camera distance) and f (the focal length of the lens) were known, h (the altitude) could be chosen arbitrarily for elevations relative to an arbitrary level of reference and p (the parallax) had to be determined from the pictures.

Determining the parallax of a point requires the identification of two points, one in the lefthand picture and one in the righthand picture. These points are called homologous points. In conventional stereophotographic survey the identification of the homologous points is relatively easy through visual inspection of the pictures separately (land terrain). If homologous points are not so easy to identify or if the number of points of interest is great, some degree of automation is called for. The identification is then basically carried out by a correlation technique.

Two processes can be used to carry out the correlation. The first technique will be indicated only briefly because it was not used in this study. It is a fully automated process in the sense that human interference is very limited. It is based on fairly recent developments in the analysis of stereo pictures and it does not seem to be fully operational in the sense that it can be used successfully in a large variety of terrain types. The pictures are scanned to digitize the optical density of the film and the position of a large number of homologous points is calculated through a correlation procedure (e.g. Crawley, 1975, Brnjac et al., 1976). Various modes of operation are possible such as finding the position of contourlines in the terrain at a specified elevation or finding the elevation of points on a regular grid. This process is a very promising alternative to the second process which will be described below. It was seriously considered for use in the study and a

commercial firm was contracted. However, during a test phase of the analysis procedure it was found that the firm could not fulfill the contract because the equipment would not be available.

The second process is conventional in stereophotogrammetry. It is based on the human interpretation of the stereo photo pair which in a sense, may also be considered as a correlation technique. The human interpretation seems to be much more sophisticated than that of the computer since a human being uses additional information in the perception of depth (color, size, texture etc.).

When each of the two pictures of a stereo pair is presented to each of the eyes the brain interprets these two pictures as one image of a three-dimensional space. The analysis of the pictures is based on observations in this fictitious space with the aid of fairly complicated stereoscopic viewing devices. On each picture of the pair a small dot is projected. When the pictures are observed through the viewing device the two dots seem to create one "floating" point in the fictitious space created by the stereo photo pair. A human operator can move the floating point by varying the positions of the dots in the pictures. He is thus able to identify the floating point with a point of interest in the terrain. Having made the identification, the positions of the dots in the pictures determine the parallax, and the coordinates of the point in the three-dimensional space are recorded.

A cartesian system of x, y and z coordinates was defined in the three-dimensional space created in the stereoscopic viewing device. The x- and y-axes defined the horizontal plane and the z-axis was pointing upward. The operator was asked to determine the sea surface elevation on a square grid in an area as large as possible. The mode of operation was a profiling method: the sea surface was scanned along the gridlines parallel to the y-axis (which pointed into the flight direction). While the x- and y-position was controlled automatically, the operator controlled the vertical position of the floating point which followed the sea surface

as close as possible. Every time the horizontal position of the floating point crossed a gridpoint the three coordinates of that point were recorded on tape. In some instances the surface could not be perceived with any accuracy (loss of stereoscopy due to photoquality, sun-glitter etc.) and the concerned gridpoints were either labelled or skipped.

The analysis of each picture resulted in a set of observations of the elevation of the sea surface relative to an arbitrary frame of reference. This set is given symbolically in equation (5).

$$\eta^{\mathbf{x}} = \eta^{\mathbf{x}}(x_i, y_j) \quad (5)$$

where $\eta^{\mathbf{x}}$ is the surface elevation, x_i and y_j are the coordinates of the gridpoints when $x_i = (i-1)\Delta x$ and $y_j = (j-1)\Delta y$ and Δx and Δy are the mesh size. The general idea is illustrated in fig. 5.

Fig. 5. View of area of analysis in stereo photo pair

The information from the analysis just described was not in a format suitable for the spectral analysis needed to obtain the two-dimensional spectrum (see Appendix II) The surface information was adapted so as to meet the specifications listed below. These specifications follow from the method which

was used to Fourier transform the data (based on a FFT method, Singleton, 1960).

- a the surface elevation should have the two-dimensional linear trend removed
- b the area of surface information should be a square of specified size
- c at the positions where the observations of the surface elevation is missing or not realistic, zero's are to be inserted.

Before removing the two-dimensional linear trend or clipping or extending the area of surface information to the requested shape and size spikes and gaps in the photogrammetric results were identified. That is, locations in the grid where the surface data failed to pass a test were rejected from the analysis. This test was essentially one-dimensional in nature but it was applied in two directions: the x- and y-direction.

Consider one row of data points (e.g. y-value constant) in an analyzed area. Usually this series of data points had a small but significant linear trend and may contain spikes or gaps (see fig. 6).

Fig. 6. Spikes and gaps in data sequence

The spikes and gaps were identified with the following procedure. For every point in a sequence of data points the difference in value with the following point (larger x-value) was checked. If larger than a threshold value a spike or gap is detected and the next acceptable data point is searched for. This next acceptable data point is the first point which falls within a critical wedge formed by two lines with specified slopes starting from the last accepted data point (see fig. 7).

Fig. 7. Critical wedge in error detection

Subsequently the two-dimensional linear trend is removed by a least squares technique. The results of the photogrammetric analysis can be expressed as in equation (6) where the terms $ax_i + by_i + c$ represent the two-dimensional linear trend.

$$\eta^*(x_i, y_j) = \eta(x_i, y_j) + ax_i + by_j + c \quad (6)$$

The function to be minimized is given in equation (7)

$$V = \sum_{i=1}^m \sum_{j=1}^n \{\eta(x_i, y_j)\}^2 = \sum_{i=1}^m \sum_{j=1}^n \{\eta(x_i, y_j) - ax_i - by_j - c\}^2 \quad (7)$$

where m and n are the maximum number of data points in x- respectively y-direction.

The minimization results in the following matrix equation (8).

$$\begin{array}{c}
 \left| \begin{array}{ccc}
 \sum_{i=1}^m \sum_{j=1}^n x_i^2 & \sum_{i=1}^m \sum_{j=1}^n x_i y_j & \sum_{i=1}^m \sum_{j=1}^n x_i \\
 \sum_{i=1}^m \sum_{j=1}^n x_i y_j & \sum_{i=1}^m \sum_{j=1}^n y_j^2 & \sum_{i=1}^m \sum_{j=1}^n y_j \\
 \sum_{i=1}^m \sum_{j=1}^n x_i & \sum_{i=1}^m \sum_{j=1}^n y_j & \sum_{i=1}^m \sum_{j=1}^n 1
 \end{array} \right. \times \begin{array}{c} a \\ b \\ c \end{array} = \begin{array}{c} \sum_{i=1}^m \sum_{j=1}^n \eta^{**}(x_i, y_j) x_i \\ \sum_{i=1}^m \sum_{j=1}^n \eta^{**}(x_i, y_j) y_j \\ \sum_{i=1}^m \sum_{j=1}^n \eta^{**}(x_i, y_j) \end{array} \quad (8)
 \end{array}$$

$$\begin{array}{c}
 \left| \begin{array}{ccc}
 \frac{m n (n-1) 2n-1}{6} & \frac{n m (n-1) (m-1)}{4} & \frac{m n (n-1)}{2} \\
 \frac{m n (n-1) (m-1)}{4} & \frac{n m (m-1) (2m-1)}{6} & \frac{n m (m-1)}{2} \\
 \frac{m n (n-1)}{2} & \frac{n m (m-1)}{2} & \frac{nm}{1}
 \end{array} \right. \times \begin{array}{c} a \\ b \\ c \end{array} = \begin{array}{c} \sum_{i=1}^m \sum_{j=1}^n \eta^{**}(x_i, y_j) x_i \\ \sum_{i=1}^m \sum_{j=1}^n \eta^{**}(x_i, y_j) y_j \\ \sum_{i=1}^m \sum_{j=1}^n \eta^{**}(x_i, y_j) \end{array} \quad (9)
 \end{array}$$

The data points labelled "spike" or "gap" are excluded from the summation terms in the matrix. It may be pointed out that if no data are excluded from the summation, the terms in the matrix can be replaced by analytical expressions for the sum. This simplified matrix is given in equation (9). It is identical to the one given in Cote et al. (1960).

Finally the areas taken from a series of pictures taken during one sortie are clipped or extended (by adding zeros) to a square of a common size. An illustration of the results from one stereo photo pair is given in fig. 8.

After the analysis the photogrammetric results were available for calculating the two-dimensional wave number spectrum. Such calculations have been carried out and preliminary results have been published elsewhere (Holthuijsen, 1978).

The system can also be used to measure waves in situations different from those assumed in this report such as in the surfzone, in a harbour or near breakwaters. The arguments used in this report would probably be very similar. As a matter of fact a number of stereo pictures of breaking waves have been made over the breakwaters of Rotterdam harbour (with the Hasselblad cameras) and even the spray from these breaking waves could be observed in three dimensions in these pictures. The system has also been successfully used to measure waves generated in a hydraulic laboratory.

Fig. 8. Contourline plot of the seafloor.
Area size = $170 \times 170 \text{ m}^2$, significant wave
height = 0.90m, contourline interval
= 0.20 m.

List of symbols

A	area of integration
a	constant
b	distance between cameras
b	constant
c	constant
E	variance (energy) density
f	focal length
f_m	peak frequency, frequency of peak of frequency spectrum
g	gravitational acceleration
H	modulus squared of Fourier transform of η
H_s	significant wave height, average of highest 1/3 of zero-crossing waves
$\hat{H}_s$	dimensionless significant wave height
h	altitude of photography
$h_{\max}$	maximum altitude of photography
$h_{\min}$	minimum altitude of photography
$\hat{h}$	dimensionless altitude of photography
i	integer
j	integer
$\vec{k}$	wave number vector
k_m	magnitude of wave number at maximum of $E(\vec{k})$
L_x	dimension of area of analysis
L_y	dimension of area of analysis
l_x	linear distance
l'_x	image format
m	maximum number of data points
m_o	variance of waves
n	constant
n	maximum number of data points
p	parallax

q	constant
$\vec{R}$	boundary of area of integration
r	resolution bandwidth
s	scale
s	constant
t	time
U	wind speed
V	sum of squared differences
$\vec{x}$	horizontal place vector
x	horizontal coordinate
x_i	coordinate of gridpoint
y	horizontal coordinate
y_j	coordinate of gridpoint
z	vertical coordinate
Δ	increment
η	sea surface elevation with zero mean
η^{ref}	sea surface elevation relative to arbitrary plane of reference
σ	standard deviation
ρ	specific density of water

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Appendix I

Altitude of photography

The altitude of a photographic mission is chosen on the basis of two considerations. The first consideration concerns the error in the measurements (noise), the second concerns the size of the area to be photographed.

The maximum altitude of photography is related to noise considerations. For conventional geodetic survey the expression for the measurement error is given in equation (1) where σ is the standard deviation of the error.

$$\sigma \approx 10^{-4} h \quad (1)$$

For stereophotography of the sea surface, where high grade pictures are not expected considering the type of "terrain", photogrammetric experts anticipated a standard deviation three times as large, equation (2).

$$\sigma \approx 3 \cdot 10^{-4} h \quad (2)$$

A widely used parameter to quantify the relative importance of noise is the noise-signal ratio. In this case the appropriate entities are the variance of the measurement error, σ^2 , and the variance of the waves, m_o . Accepting a maximum noise level of 10% of the wave variance, it follows that the maximum altitude of photography (denoted by $h_{\max}$) is given by equation (3)

$$h_{\max} \approx 1000 m_o^{\frac{1}{2}} \quad (3)$$

Relating the maximum altitude of photography to the significant wave height H_s by equating m_o to $1/16 H_s^2$ results in equation (4).

$$h_{\max} \approx 250 H_s \quad (4)$$

The minimum altitude of photography depends on the size of the area to be analyzed which is directly related to the resolution in the two-dimensional wavenumber spectrum. The relationship between resolution and area dimensions are given in equations (5) and (6).

$$r_x = 1/L_x \quad (5)$$

$$r_y = 1/L_y \quad (6)$$

where r_x and r_y are the resolution bandwidths and L_x and L_y are the sides of the area. For convenience of the subsequent analysis the values of L_x and L_y are taken equal.

It was required in the study referred to in the introduction that the resolution be at most some small fraction ($1/n$, say) of k_m , the wavenumber at the peak of the spectrum. This requirement is formulated in equation (7)

$$r_x = r_y = k_m/n \quad (7)$$

It follows from equation (5), (6) and (7) that L_x and L_y are larger than n/k_m as stated in equation (8).

$$L_x = L_y > n/k_m \quad (8)$$

The minimum altitude of photography, $h_{\min}$, can be determined from this resolution requirement and from the geometry of fig. 3 of paragraph 2.2. if 60% overlap in the x-direction is required, the following expression (9) for $h_{\min}$ is found.

$$h_{\min} = \frac{n}{k_m} \frac{f}{0.6 l'_x} \quad (9)$$

The limitations of the altitude of photography expressed in equations (4) and (9) can be combined into equation (10)

$$\frac{n}{k_m} \frac{f}{0.6 l'_x} < h < 250 H_s \quad (10)$$

Before determining the altitude of photography in an actual photographic operation with a given camera system, it is apparently necessary to estimate k_m and H_s .

If the wave field to be photographed is being generated by the local wind it is possible to replace k_m by the windspeed U as a relevant parameter in equation (10). For operational procedures this is often more convenient. The replacement is based on the fact that for these sea states the dimensionless wavenumber $\tilde{k}_m (= k_m U^2/g)$ and the dimensionless significant waveheight $\tilde{H}_s (= g H_s/U^2)$ are related as in equation (11), e.g. Hasselman et al. (1976).

$$\tilde{k}_m = s \tilde{H}_s^q \quad (11)$$

where s and q are constants. Using dimensionless representations for h and H_s , equation (10) can be written as equation (12).

$$\frac{n}{s} \frac{f}{0.6 l'_x} \tilde{H}_s^q < \tilde{h} < 250 \tilde{H}_s \quad (12)$$

To estimate the values of s and q , the JONSWAP relationships (Hasselmann et al., 1973) can be used and $s = 1.57 \times 10^{-2}$ and $q = -1.32$ are found. With these values of the constants substituted in equation (12) the functional form of the limitations of the altitude of photography can be plotted as in fig. 1.

Fig. 1 Limitations on altitude of photography

The condition that $\tilde{h}_{min}$ be less than $\tilde{h}_{max}$ is met. for young sea states (with the above values of s and q and $\tilde{H}_s < 0.05$, say) if for the Hasselblad system $n < 6$ and for the UMK system $n < 10$.

The difference between $\tilde{h}_{min}$ and $\tilde{h}_{max}$ will usually leave some leeway for the final choice of h . It can be used to make fine adjustments depending on photographic conditions during the field operations.

Appendix II

The wavenumber spectrum

The two-dimensional wave number spectrum $E(\vec{k})$ distributing the wave variance over wave number space is defined by the following equations where E is the variance density, $\vec{k}$ is the wave number vector, H is the modulus squared of the spatial Fourier transform of η the sea surface elevation with zero mean, A is the area of integration and $\vec{R}$ the spatial boundary of integration, $\vec{x}$ is the horizontal coordinate.

$$E(\vec{k}) = \lim_{A \rightarrow \infty} \left\langle \frac{H(\vec{k})}{A} \right\rangle$$

where

$$H(k) = \left| \iint_{\vec{R}} \eta(\vec{x}) e^{-i 2 \pi \vec{k} \cdot \vec{x}} d\vec{x} \right|^2$$

and

$$A = \iint_{\vec{R}} d\vec{x}$$

and $\langle \rangle$ denotes ensemble average. The observations available from one stereo photo pair are considered to be a realization from the ensemble on which the estimation of $E(\vec{k})$ can be based. In the text of the report the term energy density is used for variance density, the difference is a constant factor ρg where ρ is the specific density of water and g is the acceleration due to gravity.

Standard procedures to estimate variance density spectra are described in textbooks such as e.g. Bendat and Piersol (1971).