Preferred location of droplet collisions in turbulent flows

Vincent E. Perrin and Harm J. J. Jonker

Delft University of Technology, 2600 AA Delft, The Netherlands

(Received 16 November 2012; revised manuscript received 26 July 2013; published 6 March 2014)

This study investigates the local flow characteristics near droplet-droplet collisions by means of direct numerical simulation of isotropic cloudlike turbulence. The key finding is that, generally, droplets do not collide where they preferentially concentrate. Preferential concentration is found to happen as expected in regions of low enstrophy (vorticity magnitude), but collisions tend to take place in regions with significantly higher dissipation rates (up to a factor of 2.5 for Stokes unity droplets). Investigation of the droplet history reveals that collisions are consistently preceded by dissipative events. Based on the droplet history data, the following physical picture of a collision can be constructed: Enstrophy makes droplets preferentially concentrate in quiescent flow regions, thereby increasing the droplet velocity coherence, i.e., decreasing relative velocities between droplets. Strongly clustered droplets thus have a low collision probability, until a dissipative event accelerates the droplets towards each other. We study the relation between the local dissipation rate and the local collision kernel and vary the averaging scale to relate the results to the globally averaged collision and dissipation rates. It is noted that, unlike enstrophy, there is a positive correlation between the dissipation rate and collision efficiency that extends from the largest to the smallest scales of the flow.

DOI: [10.1103/PhysRevE.89.033005](https://doi.org/10.1103/PhysRevE.89.033005)

PACS number(s): 47.55.Kf, 92.60.hk, 47.27.Jv

I. INTRODUCTION

In cloud physics, droplets and the way they collide are at the very base of the formation of clouds and the initiation of warm rain. This stage in which droplet growth is dominated by collision and coalescence occurs after the condensation stage, in which condensation is the leading process. The third and last stage of rain formation is the sedimentating stage in which gravity plays a crucial role (see, e.g., Ref. [1]).

The idea about in-cloud droplet dynamics is that turbulence may significantly increase the collision probability and therefore accelerate rain formation [1–3]. Turbulent fluctuations in a flow lead to continuously varying drag forces on a droplet, which leads to large variations in droplet velocities and more collisions [4]. Turbulence also tends to cluster droplets at the smallest scale in regions of low enstrophy, a process called preferential concentration [5].

Previous studies [6–8] show that the collision efficiency can be expressed as a function of the mean dissipation rate $\langle\epsilon\rangle$, to the extent that increasing the mean dissipation rate increases the collision efficiency. Yet, for high Reynolds number flows it is well known that the spatial and temporal distribution of dissipation rates is far from uniform, with large regions of low dissipation interspersed by localized bursts of very high dissipation rates [9]. This particular characteristic of turbulence, referred to as intermittency, makes one wonder whether the mean value $\langle\epsilon\rangle$ is sufficiently able to represent the collision process or, phrased differently, how large should an averaging length l be for ϵ_l to be a meaningful proxy for collision efficiency.

To further explore this issue, let us consider the following gedanken experiment. Consider three domains with the same mean dissipation rate $\langle\epsilon\rangle$ and the same number of droplets, but with different spatial arrangements of ϵ (see Fig. 1). We divide domain 2 into four subdomains and domain 3 in nine subdomains and vary the intermittency by setting some subdomains to zero dissipation while increasing ϵ in others. Pursuing the notion that the mean collision rate in

a subvolume depends on the mean dissipation rate of that subvolume, one concludes that no collisions take place in the (white) subdomains with zero dissipation rate. So to maintain the mean collision rate averaged over the entire volume, the subdomains of domains 2 and 3 must locally produce collision rates that are 4 and 9 times higher, respectively. This can only hold if the collision rate in a subdomain depends *linearly* on the value of ϵ in that subdomain. However, even if this linear dependence were true for large enough subdomains, one might further refine the subdomains until one reaches a scale where a high dissipation rate might be associated with ejection of the droplets, making them cluster in more quiescent regions. In other words, at some (small) scale it is not unthinkable that a local high dissipation rate suppresses rather than enhances the local droplet collision rate.

The above cartoon of intermittency is obviously much too simplistic, but it underlines the importance of the nonuniformity of the dissipation field for collisions. After all, the flow field of a cumulus cloud is far from uniform, in particular near the cloud edges [10–12]. It also underlines the importance of understanding not only *where* droplets collide on average in a turbulent flow, but *also* the circumstances preceding a collision. The aim of this study is therefore to gain a better physical understanding of the processes surrounding droplet collisions in turbulent flows. To this end, we study isotropic turbulence with direct numerical simulation (DNS), tracking droplets and collisions in a Lagrangian framework. To get a better understanding of collisions, we investigate the role of the local flow field conditions of dissipation and enstrophy in this process. Conditional sampling allows us to find which flow conditions favor collisions and to investigate whether the positive correlation between dissipation and collisions still holds at small scales. We will also look at mean droplet trajectories just before a collision to investigate the flow conditions a droplet has traveled through. The effect of both the Stokes number and the Reynolds number on the collision preferences will be studied as well.

FIG. 1. Three domains with the same mean dissipation rate $\langle \epsilon \rangle$ with different spatial distributions of ϵ . As a simplistic representation of intermittency, the white subdomains have zero dissipation rate.

Furthermore, the role of nonuniformity is addressed by determining the relation between the local dissipation rate and the local collision kernel and by comparing the results with their mean counterparts. Finally, we make an estimate of the relevant scales involved in droplet dynamics and the scales involved in collision dynamics.

II. BACKGROUND

A. Droplet dynamics

Every droplet in a turbulent flow is to some extent influenced by turbulence. The full equations of motion have been described by Maxey and Riley [13]. Many terms in these equations can be neglected when considering cloud droplets since the density of cloud droplets ρ_p is high compared to the density of air ρ_f and since the radius r of droplets is small compared to the Kolmogorov scale η of the flow. These assumptions reduce the equations to

$$\frac{dv_i(t)}{dt} = \frac{u_i[x_i(t), t] - v_i(t)}{\tau_p}, \quad (1)$$

$$\frac{dx_i(t)}{dt} = v_i(t), \quad (2)$$

where $v_i(t)$ is the droplet velocity, $x_i(t)$ the droplet position, and $u_i[x_i(t), t]$ the local flow field at the droplet position. Gravity is omitted in this paper since it adds complexity to the problem in a delicate way. The combined effect of turbulence and gravity is not merely an addition of separate phenomena (see Ref. [14]). Under the assumption of Stokes drag, $\tau_p = 2\rho_p r^2 / (9\rho_f \nu)$ is the droplet relaxation time with ν the kinematic viscosity of the carrier fluid. The interaction between the flow and the droplet can be described with the use of τ_p by the Stokes number

$$St = \frac{\tau_p}{\tau_\eta}, \quad (3)$$

where $\tau_\eta = (\nu/\epsilon)^{1/2}$ is the Kolmogorov time scale. In the limit of $St \rightarrow 0$, droplets follow the flow, and in the limit of $St \rightarrow \infty$ droplets are not influenced by the flow. For $St \approx 1$, droplets resonate with the flow and cluster in regions of low enstrophy and become preferentially concentrated [15].

B. Collision statistics

The average number of collisions $\dot{N}_{12}$ per unit volume and unit time between two groups of droplets with radii r_1 and r_2

is given by

$$\dot{N}_{12} = N_1 N_2 \Gamma_{12}, \quad (4)$$

where N_1 and N_2 are number concentrations of the two different groups and Γ_{12} is the collision kernel. The collision kernel can be expressed as follows [16]:

$$\Gamma_{12} = 2\pi R^2 \langle |w_r| \rangle g(R), \quad (5)$$

where $R = r_1 + r_2$, $|w_r|$ is the magnitude of the radial relative velocity, and $g(R)$ is the radial distribution function (RDF) describing spatial nonuniformity of the droplet concentration. A value of $g(R) = 1$ indicates a uniform droplet concentration, whereas higher values are indicative of clustering. Equation (5) shows that the chance of colliding increases proportionally to the relative velocity between the droplets and proportionally to the droplet RDF.

C. Flow field statistics

Both the local dissipation rate ϵ and the local enstrophy $\Omega = \nu|\omega|^2$, where $|\omega|^2$ is the vorticity magnitude, turn out to play an important role in the spatial distribution of the droplets. Luo *et al.* [17] showed that the instantaneous spatial distribution of inertial droplets correlates well with the Laplacian of pressure $\nabla^2 p$. This was also shown for very light particles [18]. For isotropic turbulence a direct relation can be established between the enstrophy, the dissipation rate, and the Laplacian of pressure [9]:

$$\Omega - \epsilon = \nabla^2 P, \quad (6)$$

where P is a rescaled pressure $P = (2\nu/\rho_f)p$. The dissipation has been computed using its formal definition $\epsilon = 2\nu S_{ij} S_{ij}$, where $S_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$ is the symmetric part of the deformation tensor of the flow. Since we are focusing on local values of the dissipation rate, it is important to precisely specify the employed definition of dissipation rate. For example, using $\tilde{\epsilon} = \nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}$ (termed pseudodissipation by, e.g., Pope [19]) will yield the same volume averaged values but can differ locally. A useful dimensionless number in isotropic turbulence is the Taylor-based Reynolds number Re_λ , which can be calculated as follows:

$$Re_\lambda = \frac{u'\lambda}{\nu}, \quad \lambda = \left(\frac{15\nu u'^2}{\epsilon} \right)^{1/2}, \quad (7)$$

where λ is the Taylor microscale.

III. NUMERICAL SETUP

In order to explicitly simulate the turbulence, we use a direct simulation code [20] to solve the incompressible Navier-Stokes equations on a uniform staggered grid:

$$\frac{\partial u_i}{\partial x_j} = 0, \quad (8)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2}, \quad (9)$$

where u_i are the three velocity components, p is the pressure field, ν is the kinematic viscosity, and ρ_f is the fluid density.

TABLE I. Overview of the simulations. The R simulations study the impact of the Reynolds number, the S simulations have been performed to investigate the Stokes number effect, and the E simulations explore the effect of the mean dissipation rate. Each simulation is shown together with the dimensions of the domain L , the number of grid points N_x , the Taylor-based Reynolds number Re_λ , the radius of the droplets r , the Stokes number St , and the number of droplets N_p . Note that simulations R1 and S3 are identical and R3 and E2 are also identical and that all simulations are monodisperse.

Run	L (m)	N_x	Re_λ	$\langle \epsilon \rangle$ (m^2/s^3)	u' (m/s)	r (μm)	St	$N_p/10^6$
R1	0.2	160^3	110	4.5×10^{-2}	0.15	30	0.64	0.5
R2	0.4	320^3	190	4.7×10^{-2}	0.20	30	0.64	2
R3	0.6	512^3	230	3.9×10^{-2}	0.22	30	0.64	15
S1	0.2	160^3	110	4.5×10^{-2}	0.15	10	0.07	3
S2	0.2	160^3	110	4.5×10^{-2}	0.15	20	0.29	1
S3	0.2	160^3	110	4.5×10^{-2}	0.15	30	0.64	0.5
S4	0.2	160^3	110	4.5×10^{-2}	0.15	40	1.14	0.4
S5	0.2	160^3	110	4.5×10^{-2}	0.15	50	1.78	0.4
S6	0.2	160^3	110	4.5×10^{-2}	0.15	60	2.57	0.4
E1	0.6	512^3	199	8.5×10^{-3}	0.14	30	0.29	15
E2	0.6	512^3	268	3.9×10^{-2}	0.20	30	0.64	15
E3	0.6	512^3	292	1.0×10^{-1}	0.31	30	0.91	15

The Navier-Stokes equations are discretized by the finite-volume method, with second-order central differences in space and Adams-Bashforth differences in time. We use a triple-periodic computational domain. Time stepping is restricted by the Courant-Friedrich-Lewy criterion using a Courant number of 0.25. The code also makes use of the MPI communication protocol as it is parallelized by domain decomposition in two dimensions, making the code highly scalable and fit for modern supercomputers.

Since a turbulent system is inherently dissipative, energy is injected at the lowest wave number. To this end, we employ a forcing scheme similar to that used by Woittiez *et al.* [14], using a nudging time scale $\tau_{\text{forc}} = 0.5\sqrt{\nu/\epsilon_t}$ [21] to add kinetic energy to the largest scales. This energy has been set to $(0.25\epsilon_t L)^{2/3}$; ϵ_t denotes the target mean dissipation rate of the simulation, which is the mean dissipation rate we aim for (it is not necessarily exactly equal to the mean dissipation rate of the actual simulation), and L denotes the physical size of the computational domain. The use of DNS limits the range of scales that can be resolved. As a result, the domain size and the Reynolds number are limited and several orders of magnitude lower than in real convective clouds.

Typical energy and dissipation spectra of such a simulation (in this case R3; see Table I for more details) are shown in Fig. 2. It can be seen that the DNS properly resolves the flow down to the smallest scales. In addition we performed several resolution dependence tests to ascertain that flow and droplet features were both properly resolved (see the Appendix).

The equations of motion of the droplets (1) and (2) are solved using a second-order Runge-Kutta scheme. The velocity of the flow field at the droplet position is computed using trilinear interpolation.

The collision routine checks the number of collisions and computes the collision kernel both *dynamically* using Eq. (4) and *kinematically* using Eq. (5). The algorithm of Chen

FIG. 2. Energy and dissipation spectra for simulation R3.

et al. [22] is used to detect collisions, which uses cell indexing and linked lists to check only droplet pairs that could collide within one time step. The cost of this algorithm is $O(\frac{27N_p^2}{2N_xN_yN_z})$, where N_x , N_y , and N_z define the size of the computation domain in the x , y , and z directions, respectively, and N_p is the number of droplets present in the computational domain. To ensure that all collisions are detected, the maximum travel distance of the droplets is restricted to half a grid distance by using a dynamically adaptive time step for the droplets.

Since the radial distribution function in Eq. (5) is defined at the contact of two droplets and is therefore theoretically determined only by droplets that are exactly a distance R apart, a greater number of samples is acquired by considering all droplet pairs that are separated by a distance of $R \pm \delta$. The same value as that used by Wang *et al.* [23] has been used (i.e., $\delta = R/100$).

Three different sets of simulations have been performed to investigate the effect of the Reynolds number (marked as R runs in Table I), the effect of the Stokes number (marked as S runs), and the similarities between local and mean collision kernels (marked as E runs). All simulations (except the E marked runs in Table I) have been performed using a target dissipation rate of $\epsilon_t = 0.05 \text{ m}^2/\text{s}^3$. The runs E1, E2, and E3 have been run with ϵ_t set equal to 0.01, 0.05, and 0.1 m^2/s^3 , respectively. The spatial resolution limit of the simulations $k_{\text{max}}\langle\eta\rangle$, where $k_{\text{max}} = N_x/2$ and $\langle\eta\rangle = (\nu^3/\langle\epsilon\rangle)^{1/4}$, has a value in between 1.2 and 2.1 for all simulations. More on resolution effects can be found in the Appendix. In all simulations, droplets are released after 2.0 s of simulation time and the collision routine starts after 5.0 s.

IV. RESULTS

This section investigates the flow field characteristics favorable to droplets ($St \simeq 1$) and collisions. The results are obtained from the R3 simulation (see Table I). Figure 3 shows probability density functions (PDFs) of the dissipation rate ϵ ,

FIG. 3. (Color) The PDFs of (a) ϵ , (b) Ω , and (c) $\nabla^2 P$, sampled over the flow field (black line), sampled over the locations of the droplets and sampled over the locations of the collisions. The insets show the tails of the distributions. (d) shows the mean of (a) and (b).

the enstrophy Ω , and the Laplacian of the (rescaled) pressure $\nabla^2 P$. Black lines represent the flow field, red lines the field when conditionally sampled on positions of the droplets, and blue lines the field conditionally sampled on locations of the collisions. Comparing the PDFs of the dissipation and the enstrophy of the flow field, it can be observed that enstrophy has a broader PDF and is therefore more intermittent, which has been reported previously in literature for low Reynolds numbers; see, e.g., Ref. [24].

Figure 3 shows that droplets concentrate in regions characterized by a lower Ω value than the flow. This is clearly consistent with the theory of preferential concentration [15]: Enstrophy *swings out* the droplets. Dissipation does not seem to influence the spatial position of the droplets. Droplets also tend to cluster where $\nabla^2 P \simeq 0$, which has previously also been found by Luo *et al.* [17].

Figure 4 shows the joint PDF of the dissipation and the vorticity of the flow field [Fig. 4(a)], of the field field conditionally sampled on the droplet positions [Fig. 4(b)], and of the field field conditionally sampled on the collision locations [Fig. 4(c)] of simulation R3. The joint PDF of the flow field agrees very well with results previously found by Yeung *et al.* [25], both performed at a Reynolds number $Re_\lambda \approx 230$.

It is tempting to presume that collisions occur where the droplet concentration is highest, implying that the statistics conditioned on collision positions would yield similar results as the statistics conditioned on droplet positions, but Figs. 3 and 4 prove otherwise. Both figures show that collisions occur at places where the dissipation rate is significantly *higher* than where droplets reside (up to a factor of 2.5).

Also the enstrophy at collision locations is higher than at droplet locations. Both effects can also be seen in the PDF of $\nabla^2 P$ [Fig. 3(c)], which for collisions is strongly skewed towards negative values. Figure 3(d) concisely summarizes the main message as it shows the averages conditioned on droplet locations and collision locations, respectively, in comparison to the mean flow properties. The latter obeys $\langle\epsilon\rangle = \langle\Omega\rangle$, which implies that the point $(\langle\epsilon\rangle, \langle\Omega\rangle)$ must be located right on the black line that represents $\nabla^2 P = 0$. The graph shows that, on average, droplets reside in regions of low enstrophy and average dissipation, whereas collisions favor regions with appreciable dissipation rates and with enstrophy values that are comparable to or slightly higher than the flow average value. The figure leads to the conclusion that collisions do not tend to occur where most of the droplets are located. This makes sense to the extent that preferential concentration favors droplet clustering, thereby increasing the velocity coherence of droplets within a cluster. This increase in coherence implies a decrease in the relative velocity between droplets and reduces the collision probability. Recalling Eq. (5), it can be concluded that the gain via the increased radial distribution function is outweighed by the loss in relative velocity.

To get a better view on the flow conditions surrounding droplet collisions, we have sampled the local flow field around every collision so as to obtain radial profiles of dissipation, enstrophy, and $\nabla^2 P$. The result is shown in Fig. 5. A local maximum can be observed in the dissipation, whereas a minimum can be observed in the enstrophy as well as in the pressure Laplacian. Note that the local enstrophy is maximum near $r/\eta \approx 15$, where $\eta = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov microscale. Apart from a spatial analysis, it is also interesting to conduct a

FIG. 4. Joint PDFs of the dissipation and the enstrophy sampled on (a) the flow field, (b) the droplets positions, and (c) the collision locations. Both the x axis and the y axis are on a logarithmic scale. The isolines are also scaled logarithmically and correspond to the values $10^{-0.3}$, 10^{-1} , 10^{-2} , 10^{-3} , 10^{-4} , and 10^{-5} , respectively. The black dot indicates the mean of the corresponding PDF.

temporal analysis. For each collision, a mean droplet trajectory is determined for 100 time steps before and after a collision. Figure 6 shows the mean droplet trajectory prior to a collision in terms of dissipation rate, enstrophy, and absolute velocity when averaged over all collisions, where $t = 0$ represents the actual collision (dashed line). It is interesting to note that prior to a collision the enstrophy profile is still *increasing*. From this, one can infer that collisions are generally not the result of droplets that are swung out of a vortex, which excludes vortices as being the primary source of relative velocities and initiators of collisions. However, a significant dissipation peak can be found prior to a collision, which emphasizes the important role of dissipative events for the collision process. The time scale involved is of the order of τ_η , which is of the same order as the droplet relaxation time τ_p . Absolute velocity is only marginally increased.

Based on these results, the following physical picture emerges of the collision process in turbulent flows for droplets

with $St \simeq 1$. Droplets preferentially cluster under the influence of enstrophy; however, they do not collide yet since the increase in velocity coherence yields a decrease in relative velocity. Once clustered, dissipative events are found to precede collisions because they appear vital for decorrelating the droplet velocities. A possible source for these dissipative events could be the interaction between vortex filaments [26]. More research is needed to investigate this further.

At first sight, this physical picture seems contradictory to studies performed by Falkovich *et al.* [27] and more recently by Falkovich and Pumir [28] on the so-called sling effect. Droplets are swung out of a vortex, which results in an increase of the relative velocity of the droplets and a significant increase of the collision kernel as computed by Saffman and Turner [29]. The difference in both views and how they can be reconciled will be discussed in Sec. IV B.

It should be noted that this physical picture of the collision process is reconstructed for a monodisperse droplet

FIG. 5. Radial profile of the sampled flow field around collisions. Shown are the dissipation rate (solid line), the enstrophy (dashed line), and the Laplacian of the pressure (dotted line).

FIG. 6. Conditionally sampled dissipation rate (solid line), enstrophy (dashed line), and velocity magnitude (dash-dotted line) of droplets before and after they collide. The x axis shows the time prior and after a collision.

FIG. 7. (Color) Effect of the Reynolds number on the preferred droplet positions (red dots) and collision locations (blue dots). The larger the dot, the higher the Reynolds number. The axes have been scaled with the mean flow dissipation and enstrophy. Simulation details can be found in Table I.

distribution. In real clouds, this is by far not the case [30]. According to Celani *et al.* [31], turbulent velocity fluctuations cause a large spread of the droplet size distribution. It is very interesting to investigate to what extent the physical picture sketched for monodisperse droplet collisions is valid for a polydisperse droplet size distribution. Collisions in such a distribution depend less on the decorrelation of the droplet velocities by a dissipative event since significantly less coherence is found between droplets of different size [14].

A. Reynolds number and Stokes number effects

This section examines the effects of the Reynolds number and the Stokes number on the preferred droplet and collision locations. The first three simulations have been performed with varying Reynolds numbers (R1–R3; see Table I). From the data we construct a figure comparable to Fig. 3(d) in order to show the impact of the Reynolds number (see Fig. 7). Interestingly, the droplet preferred location ($0.6\langle\Omega_{flow}\rangle$ and $\langle\epsilon_{flow}\rangle$) is not influenced by the Reynolds number. The collisions tend to occur at even larger dissipation rates and enstrophy when increasing the Reynolds number.

It is interesting to note that for high Reynolds numbers, Nelkin [24] analytically shows that the anomalous scaling exponents of dissipation and enstrophy are equal. Recent studies by Schumacher *et al.* [25,26] suggest that for high Reynolds number, not only do dissipation and enstrophy scale in the same manner, but extreme events in both would also tend to spatially occur together. The effect this spatial correlation in extreme events would have on mean droplets and collision statistics would be interesting to study, but is presently computationally very demanding.

A similar plot can be made to show the impact of the Stokes number. In simulations S1–S6, droplets with a radius between

FIG. 8. (Color) Effect of the Stokes number on the preferred droplet positions (red dots) and collision locations (blue dots). The larger the dot, the higher the Stokes number. The axes have been scaled with the mean flow dissipation and enstrophy. Simulation details can be found in Table I.

10 μm ($St = 0.07$) and 60 μm ($St = 2.45$) have been used (see Table I). Results are presented in Fig. 8. In the limit cases of $St \rightarrow 0$ and $St \rightarrow \infty$, both droplet and collision preferred locations are statistically identical to the flow field mean values (black dot). Droplets with zero Stokes number follow the streamlines and will not preferentially concentrate. Stokes infinity droplets are not influenced by the flow field at all and will also not preferentially concentrate. The radial distribution function for a simulation with $\epsilon_t = 0.05 \text{ m}^2/\text{s}^3$ peaks at a droplet radius of 30 μm [14] since they become resonant with the small-scale vortices. The higher this resonance, the easier the droplets can reach quiescent regions of the flow (i.e., with low enstrophy). This can be seen in Fig. 8, where the red dot corresponding to 30- μm droplets has the lowest enstrophy value. Droplets with a larger or smaller radius are all located closer to the flow mean (black dot). For the collisions, we see that the blue dots follows an elliptical path, with the 20- μm droplets at the extremity.

The reason why heavy droplets prefer a lower enstrophy is not yet entirely clear. This could be due to the sling effect [28].

B. Local versus mean collision kernel

In this section we return to the thought experiment formulated in the Introduction, where the issue of intermittency was addressed. The question was whether a mean collision kernel can be properly represented by the mean dissipation rate or whether a more local approach is needed. We concluded that a mean approach might work only if the collision kernel is locally a linear function of the dissipation rate; otherwise the nonuniformity of the dissipation field has to be taken into account.

By measuring the local dissipation rate at each collision, it is possible to determine the number of collisions between ϵ

FIG. 9. Relation between the local dissipation rate and the local collision kernel Γ_ϵ . The local collision kernel is related to the mean collision kernel (indicated as dots with corresponding line style) via Eq. (10).

and $\epsilon + d\epsilon$. This enables one to link the collision statistics to ϵ and hence to calculate within a single simulation the collision efficiency Γ_ϵ as a function of the (local) dissipation rate ϵ . One can regard Γ_ϵ as a decomposition of the volume mean collision efficiency $\langle \Gamma \rangle$ since they are related via

$$\langle \Gamma \rangle = \int_0^\infty \Gamma_\epsilon \mathcal{P}(\epsilon) d\epsilon, \quad (10)$$

where $\mathcal{P}(\epsilon)$ represents the PDF of ϵ of the flow field [see, e.g., the black line in Fig. 3(a)]. We have conducted such an analysis for three different target dissipation rates ϵ_t , leading to three different values of $\langle \epsilon \rangle = \int \epsilon \mathcal{P}(\epsilon) d\epsilon$; see runs E1–E3 in Table I. The results have been plotted in Fig. 9. In the same plot, the mean values of the collision kernel and the mean dissipation rate are indicated as dots. It is interesting to note that all three local collision kernels are monotonically rising functions of ϵ . However, one also notes that the dependence of the local collision kernels on the local values of ϵ is essentially *nonlinear*. We can therefore conclude that intermittency and the nonuniformity of the flow field make it nontrivial to derive a general relationship between the mean collision kernel and the mean dissipation rate if no reference to the averaging length scale is specified.

To better understand the influence of the averaging size, we repeat the analysis where we conditionally sample enstrophy and dissipation rate based on the occurrence of droplets and collisions; however, now, instead of using the local values of Ω and ϵ , we first determine coarse-grained values Ω_l and ϵ_l when a collision is detected. This analysis provides conditional averages $\langle \Omega_l | c \rangle$ and $\langle \epsilon_l | c \rangle$ for collisions and $\langle \Omega_l | d \rangle$ and $\langle \epsilon_l | d \rangle$ for droplet locations. Here l denotes the linear size of the averaging volume, which we vary from $l = \Delta$ (grid scale) to $l = L$ (domain size) by factors of 2. For $l = \Delta$, one exactly retrieves the results of the previous section based on the local values, whereas for $l = L$ the conditionally sampled averages

FIG. 10. (Color) Conditionally sampled statistics of coarse-grained values of dissipation rate and enstrophy. The sampling is based on droplet positions (red lines) or collision locations (blue lines). The averaging length scale varies from grid size $l = \Delta$ to domain size $l = L = 512\Delta$.

should get very close to the flow-field averages (there is still a small difference due to temporal correlations that make $\langle \epsilon_l | c \rangle \gtrsim \langle \epsilon \rangle$).

Figure 10 is obtained from simulation R3 and shows the *coarse-grained* dissipation rate and enstrophy when conditionally sampled on the droplet positions (red lines) and collision locations (blue lines) for different length scales l . As expected, one observes for larger averaging volumes a gradual convergence to the mean flow statistics. It is interesting to note that the differences between ϵ and Ω disappear at $l/\Delta \geq 16$, a scale that perhaps could be associated with the value of the Taylor scale [see Eq. (7)]. Apparently at this scale the average of the pressure Laplacian effectively vanishes, implying that enstrophy and dissipation are interchangeable from this scale onward. However, at smaller scales it is important to retain the distinction between ϵ and Ω . Indeed, one notices a consistent positive correlation between collisions and ϵ_l all the way to the smallest scale of the flow. However, this is not the case for Ω_l , which, after a maximum, exhibits a reduced correlation at smaller scales. This finding is consistent with Fig. 5, which revealed that the enstrophy field increases further away from the collision location; so if one increases the averaging area, one includes the distant enstrophy contributions in the average.

Recalling the discussion on the sling effect in Sec. IV, this interchangeability of dissipation and enstrophy at larger scales could explain the different collision mechanisms as presented by Falkovich and Pumir [28] and in the current work. Both mechanisms attribute an increase in relative velocities either to vorticity or to dissipation, but as can be concluded from Fig. 10, both are interchangeable for somewhat larger scales. Falkovich and Pumir [28] treat the droplet concentration as a continuum and approximate the radial relative velocity as $\eta St^{1/2}/\tau$. These assumptions break down at the smallest scales of the flow and do not allow one to capture the small-scale

phenomena involved in collision dynamics. The present work can therefore be regarded as a more elaborate local explanation for the increase in collision chances by turbulence.

Finally, Fig. 10 provides an estimate of the relevant scales involved in preferential concentration and collisions. The effect of preferential concentration becomes small at a scale $l/\Delta \gtrsim 16$. The collision-based statistics, however, exhibit a scale dependence up to much larger scales, i.e., almost as large as domain size.

V. CONCLUSION

Our key finding is that droplets statistically do not collide where they preferentially concentrate. Droplets preferentially cluster in regions of low enstrophy ($\approx 0.6\langle\Omega\rangle$) and dissipation ($\approx\langle\epsilon\rangle$), whereas collisions favor regions with appreciable dissipation rates (up to $\approx 2.5\langle\epsilon\rangle$) and with enstrophy values that are comparable to or slightly higher than the flow average value. The higher dissipation values serve to enhance the collision rate by decorrelating the motion of nearby droplets. By studying the history data of droplets, one can observe a distinct dissipation peak preceding a collision. In particular at small scales it is important to make a distinction between enstrophy and dissipation as they both play a different role. Enstrophy causes droplets to cluster, but the resulting preferential concentration decreases the relative velocities between droplets, making them less likely to collide. A dissipative event provides them with the necessary acceleration towards each other.

The Reynolds number does not have a large effect on the dissipation and enstrophy levels at which droplets reside in the flow. Collisions, however, tend to occur in regions of higher dissipation and enstrophy for higher Reynolds numbers. The Stokes number exerts a large influence on the collision conditioned flow statistics. For very low and very high Stokes numbers, these statistics resemble that of the mean flow, but for moderate Stokes numbers the differences are pronounced.

We have studied the relation between the local dissipation rate and the local collision kernel. A consistent positive correlation exists between both, but the relation is nonlinear. To understand the relation between local and global collision kernels and their relation with the local or global dissipation rates, we determined coarse-grained values of enstrophy and dissipation. Unlike enstrophy, a consistent positive correlation between the dissipation and collision rate was found from the largest to the smallest scales of the flow.

ACKNOWLEDGMENTS

This work is part of the research program of the Foundation for Fundamental Research on Matter, which is part of the Netherlands Organisation for Scientific Research (NWO). This work was sponsored by the Stichting Nationale Computerfaciliteiten for the use of supercomputer facilities, with financial support from NWO.

APPENDIX: RESOLUTION DEPENDENCE

In this appendix, a resolution dependence study is performed to investigate the numerical accuracy and therefore

FIG. 11. (Color) Comparison of the dissipation (solid line) and enstrophy (dashed line) conditionally sampled on the droplets positions (red lines) and conditionally sampled on the collision locations (blue lines) as a function of the grid size. The x axis shows the spatial resolution limit $k_{\max}\langle\eta\rangle$. The value of 1 is shown by the dotted black line. The solid black line shows value of $k_{\max}\langle\eta\rangle$ of simulation R3, which is used for most of the statistics. The gray area indicates the range of the values of $k_{\max}\langle\eta\rangle$ used in this paper. For comparison, the crosses are results obtained from a pseudospectral code.

the trustworthiness of our results. This is especially important since the code makes use of second-order numerical schemes. The simulation setup is very comparable to simulations R1 and S3, however on a small domain of $L = 0.1$ m in which 10^5 droplets with a radius of $30\text{ }\mu\text{m}$ are released. A wide range of grid sizes is considered (i.e., 16^3 , 32^3 , 64^3 , 80^3 , 160^3 , and 320^3). All the results in the paper are obtained with a resolution equivalent to 80^3 , which is equal to that used by Woittiez *et al.* [14]. The simulation time has been set to 100 large eddy turnover times. Note that a doubling of the resolution corresponds to a increase in computational cost by a factor of 16. Figure 11 shows our key finding, namely, that droplets statistically do not collide where they preferentially concentrate, as a function of the grid size N_x ($=N_y = N_z$). Although the effect enstrophy has on droplets and collisions is very resolution sensitive, only a small resolution effect can be observed for grid sizes larger than 80^3 . The effect of dissipation is much less resolution dependent.

For comparison, results obtained using a pseudospectral code have been added to Fig. 11 as crosses. This pseudospectral code solves the spectral Navier-Stokes equation using pseudospectral methods. Time integration is performed using a third-order Adams-Bashforth scheme. Both advection and diffusion are treated explicitly and the 3/2 rule is used to deal with the aliasing errors (see, e.g., Ref. [32]). Judging from the figure, it appears that the resolution of 80^3 on a domain of $L = 0.1$ m used in the paper is sufficient to capture the essential features of colliding droplets.

In simulation R3 (which has been extensively analyzed), more than 99% of all droplets reside in regions of the flow where $\epsilon/\langle\epsilon_{flow}\rangle < 10$ (see Fig. 3). More than 97% of all

collisions occur where $\epsilon/\langle\epsilon_{\text{flow}}\rangle < 10$. These are regions of the flow that are well captured by the current resolution (see, e.g., the works of Watanabe and Gotoh [33] and Schumacher *et al.* [34], showing that the core of the PDF of the dissipation rate is well resolved for low resolutions).

In this paper we are mainly interested in mean quantities. However, if one is interested in individual (extreme) events, higher resolutions (e.g., 160^3 or 320^3 with $L = 0.1$ m) and/or higher-order methods are recommended (see Refs. [25,33–36]).

-
- [1] R. A. Shaw, *Annu. Rev. Fluid Mech.* **35**, 183 (2003).
 - [2] P. Jonas, *Atmos. Res.* **40**, 283 (1996).
 - [3] W. W. Grabowski and L.-P. Wang, *Annu. Rev. Fluid Mech.* **45**, 293 (2013).
 - [4] M. B. Pinsky and A. P. Khain, *Q. J. R. Meteorol. Soc.* **123**, 1517 (1997).
 - [5] K. D. Squires and J. K. Eaton, *Phys. Fluids A* **3**, 1169 (1991).
 - [6] O. Ayala, B. Rosa, L.-P. Wang, and W. W. Grabowski, *New J. Phys.* **10**, 075015 (2008).
 - [7] C. N. Franklin, P. A. Vaillancourt, M. K. Yau, and P. Bartello, *J. Atmos. Sci.* **62**, 2451 (2005).
 - [8] A. Seifert, L. Nuijens, and B. Stevens, *Q. J. R. Meteorol. Soc.* **136**, 1753 (2010).
 - [9] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, 1995).
 - [10] S. M. A. Rodts, P. G. Duynkerke, and H. J. J. Jonker, *J. Atmos. Sci.* **60**, 1895 (2003).
 - [11] H. Siebert, K. Lehmann, and M. Wendisch, *J. Atmos. Sci.* **63**, 1451 (2006).
 - [12] T. Heus and H. J. J. Jonker, *J. Atmos. Sci.* **65**, 1003 (2008).
 - [13] M. R. Maxey and J. J. Riley, *Phys. Fluids* **26**, 883 (1983).
 - [14] E. J. P. Woittiez, H. J. J. Jonker, and L. M. Portela, *J. Atmos. Sci.* **66**, 1926 (2009).
 - [15] L. P. Wang and M. R. Maxey, *J. Fluid Mech.* **256**, 27 (1993).
 - [16] S. Sundaram and L. R. Collins, *J. Fluid Mech.* **335**, 75 (1997).
 - [17] K. Luo, J. Fan, and K. Cen, *Phys. Rev. E* **75**, 046309 (2007).
 - [18] P. Davidson, *Turbulence: An Introduction for Scientists and Engineers* (Oxford University Press, New York, 2004).
 - [19] S. Pope, *Turbulent Flows* (Cambridge University Press, Cambridge, 2000), p. 132.
 - [20] M. van Reeuwijk, H. J. J. Jonker, and K. Hanjalić, *Phys. Rev. E* **77**, 036311 (2008).
 - [21] M. Overholt and S. Pope, *Comput. Fluids* **27**, 11 (1998).
 - [22] M. Chen, K. Kontomaris, and J. B. McLaughlin, *Int. J. Multiphase Flow* **24**, 1079 (1998).
 - [23] L. P. Wang, A. S. Wexler, and Y. Zhou, *J. Fluid Mech.* **415**, 117 (2000).
 - [24] M. Nelkin, *Phys. Fluids* **11**, 2202 (1999).
 - [25] P. K. Yeung, D. A. Donzis, and K. R. Sreenivasan, *J. Fluid Mech.* **700**, 5 (2012).
 - [26] J. Schumacher, B. Eckhardt, and C. R. Doering, *Phys. Lett. A* **374**, 861 (2010).
 - [27] G. Falkovich, A. Fouxon, and M. G. Stepanov, *Nature (London)* **419**, 151 (2002).
 - [28] G. Falkovich and A. Pumir, *J. Atmos. Sci.* **64**, 4497 (2007).
 - [29] P. G. Saffman and J. S. Turner, *J. Fluid Mech.* **1**, 16 (1956).
 - [30] J.-L. Brenguier and L. Chaumat, *J. Atmos. Sci.* **58**, 628 (2001).
 - [31] A. Celani, G. Falkovich, A. Mazzino, and A. Seminara, *Europhys. Lett.* **70**, 775 (2005).
 - [32] C. Canuto, M. Y. Hussaini, A. Quarteroni, and T. A. Zang, *Spectral Methods: Fundamentals in Single Domains* (Springer, Berlin, 2006).
 - [33] T. Watanabe and T. Gotoh, *J. Fluid Mech.* **590**, 117 (2007).
 - [34] J. Schumacher, K. R. Sreenivasan, and V. Yakhot, *New J. Phys.* **9**, 89 (2007).
 - [35] D. A. Donzis, P. K. Yeung, and K. R. Sreenivasan, *Phys. Fluids* **20**, 045108 (2008).
 - [36] D. A. Donzis and K. R. Sreenivasan, *J. Fluid Mech.* **647**, 13 (2010).