Gait initiation for a biped robot

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MASTER OF SCIENCE THESIS

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In the last decade great progress has been made in the field of biped walking robots. Developing and studying biped robots is done for several reasons, such as understanding how humans walk, creating assistive robots to fulfill household tasks or for commercial reasons.

At the Delft Biorobotics Laboratory (DBL) various biped robots are built and studied to get a better understanding of how humans walk. The goal of DBL is to develop a humanoid robot with low energy consumption, high versatility and the ability to deal with large disturbances during active dynamic walking.

In 2007 research started on 3D stability in limit cycle walking. For this purpose a humanoid biped robot named Flame was built. At the start of this master thesis, the biped robot Flame was able to walk. However the problem was that the biped was not able to start walking from a standstill. The initial velocity had to be given by hand to reach a stable walking cycle. With automatic stable gait initiation the biped robot Flame would be able to function more autonomously and thereby provide better test circumstances for different stability analyses. This is mainly due to constant initial conditions for every walking cycle.

The goal of this study is to develop a gait initiation procedure where, after the gait initiation (at the first heelstrike), the biped robot is within the basin of attraction and will thus perform a stable walking motion.

In this study the dynamic behavior of the biped robot during falling is modeled with two different inverted pendulum models. One inverted pendulum model is used to simulate the forward behavior and one inverted pendulum model to simulate the sideways behavior. This simulation showed promising results. After a falling motion from a standstill, the heelstrike state approximates the average heelstrike state. The average heelstrike state is found from steps during a stable walking motion of Flame.

A performance measure is defined to analyze the performance of the gait initiation procedure in the biped robot Flame. There is some variation in the different gait initiation trials, but Flame shows that a stable walking motion can be reached from a standstill by using a falling trajectory. After conducting several tests the biped robot Flame is able to automatically start walking from a standstill.
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Nomenclature

Symbols
\( \alpha \) Angle between the lower leg and the ankle spring
\( \phi \) Angle y-direction (pitch)
\( \sigma \) Standard deviation
\( k_a \) Spring stiffness
\( \theta \) Angle x-direction (roll)
\( I \) Moment of inertia
\( L \) Length
\( M \) Moment
\( m \) Mass
\( P \) Transformation matrix (PCA)
\( T \) Data in the transformed coordinate system (PCA)

Subscripts
\( a \) Ankle
\( d \) Damping
\( g \) Gravity
\( h \) Hip
\( k \) Knee
\( l \) Left
\( r \) Right
\( s \) Spring
Chapter 1

Introduction

The last decade there has been much progress in the field of biped walking robots. This research has been done for several reasons, such as understanding how humans walk, building a robot to help in the household or for commercial reasons.

![Examples of biped research.](image)

**Figure 1-1**: Examples of biped research.

Figure 1-1 shows some of these research topics. A robot developed by Honda (Asimo, Figure 1-1(a)) is able to serve people in a canteen. Some researchers compare the human walking
motion with the walking motions of a biped robot (Figures 1-1(b) and 1-1(c)). The models of those biped robots are very complex (Figure 1-1(d)) and with the analysis of biped robots better prostheses are built (Figures 1-1(e) and 1-1(f)).

Mainly there are two categories in which this research can be divided, trajectory controlled biped robots and limit cycle walkers.

**Trajectory controlled biped robots**

The trajectory controlled approach aims at full local controllability of the humanoid robot. This approach originates from the conventional control of industrial robot manipulators. The idea is to stabilize a robot’s motion along a desired trajectory. A problem is that industrial robots are mounted to the ground, for a biped this means that the stance foot makes full contact to the ground. The most well-known approach to ensure full contact is the “Zero Moment Point” [24] approach (ZMP), which aims to keep the “Zero Moment Point” (center of pressure) at all times inside the foot polygon away from the foot edges. The big advantage is that the walking motion of a trajectory controlled biped is impressive as the robots are highly versatile\(^1\). For example, most ZMP walkers are able to start from a standstill and change their walking speed.

The main drawback is that the energy consumption is high, because of accurate tracking of the trajectories which are not necessarily close to the natural dynamics of the robot. Different research groups all over the world are working on these trajectory controlled biped robots [1], [5], [20]. Figure 1-2 shows a few of the trajectory controlled biped robots.

![Figure 1-2: Trajectory based controlled robots](image)

(a) HRP-3 [1]  
(b) JOHNNIE [14]  
(c) Wabian2 [18]  
(d) ASIMO [20]

**Limit cycle walkers**

The Limit Cycle approach emphasizes the passive dynamics of the legs, and generally avoids the use of high-gain control. This group of humanoid robots will be called Limit Cycle

\(^1\) Versatile: capable of moving freely in all directions
Walkers [6]. Limit cycle walking is based on the concept of Passive Dynamic Walking [16]. Passive Dynamic Walkers can perform a stable walking motion down a slope, without any actuation or control. Limit cycle walkers are able to perform a stable walking motion, based on passive dynamic walking, on a flat surface with actuation and control. The essence of limit cycle walking is that it is possible to obtain stable periodic walking without locally stabilizing the walking motion at every instant during gait [8]. The 3D limit cycle walker should be designed in such a way that it is almost stable without control, close to the natural dynamics of the limit cycle walker. To make the limit cycle walker more stable (reject large disturbances) and more versatile some control is needed. Figure 1-3 shows biped robots that are able to walk with this second approach.

At the Delft Biorobotics Laboratory (DBL) various biped robots are studied to get a better understanding of how humans walk. Over the last decade, several robots have been built for various analyses [10], [9]. The goal of DBL is to develop a humanoid robot with low energy consumption, high versatility and the ability to deal with large disturbances.

In 2007 research started on 3D stability in limit cycle walking. For this purpose a humanoid biped robot named Flame (Figure 1-3(e)) was built. The biped robot Flame is designed for walking, where it is not locally stable but the whole walking motion is stable. This is the difference with the trajectory controlled biped robots and the design of Flame makes it challenging to develop a stable gait initiation procedure.

At the start of the current study, the biped robot Flame was able to walk, the problem was that the biped was not able to start walking from a standstill. The initial velocity had to be
given by hand to reach a stable walking cycle. With automatic stable gait initiation the biped robot Flame would be able to function more autonomously and thereby provide better test circumstances for different stability analyses. This is mainly due to constant initial conditions for every walking cycle. Therefore the goal of this study is to design a stable gait initiation procedure for the biped robot Flame.

1-2 Method and outline

To be able to understand what is happening inside the biped robot Flame that keeps it in a stable walking motion, a description of Flame is given in Chapter 2. This chapter shows the mechanical setup, the control system architecture and the limit cycle walking approach that leads to a stable walking motion. To get a better understanding of the behavior of the biped robot Flame during the gait initiation a simulation is done in (Chapter 3). This simulation consists of two inverted pendulum models. One inverted pendulum model to model the behavior in the sagittal plane\(^2\) and one inverted pendulum model to model the behavior in the coronal plane\(^3\). With the results from the simulation a gait initiation procedure is derived. Flame has more degrees of freedom than the simulation, therefore the gait initiation procedure is modified to work on the biped robot Flame. Chapter 4 describes the gait initiation procedure, and also describes a performance measure to analyze the results. Finally Chapter 5 gives the overall conclusion of the developed gait initiation procedure during this research.

\(^2\)The sagittal plane divides the biped robot into a left and right portion

\(^3\)The coronal plane divides the biped robot into a back and front portion
Chapter 2

Description of the biped robot Flame

As mentioned in Chapter 1 it is useful to build biped robots to increase the understanding of the dynamic principles of human walking. The Delft Biorobotics Lab has recently built a limit cycle walker called Flame. Flame is built to demonstrate the potential of limit cycle walking in 3D.

This chapter describes the biped Flame to give more insight in the control of this biped robot. Later, this knowledge is used to simulate the biped robot as two separate inverted pendulum models (Chapter 3) and to develop a gait initiation procedure (Chapter 4) for the biped robot Flame.

To understand more about the control of Flame, this chapter first discusses the overall design layout with the requirements for limit cycle walking Section 2-1. The mechanical realization of the biped robot Flame is showed in Section 2-2 and the control system architecture is showed in Section 2-3. Limit cycle walking is then described in Section 2-4 and in Section 2-5 data of twenty stable walking steps is shown.

2-1 Overall design layout

Flame is designed as a limit cycle walker, which is able to walk straight at varying speed, with the ability of lateral foot placement for stabilizing in 3D. This means that the actuators should be able to supply the limit cycle walker with sufficient controllability. However, to obtain the concept of limit cycle walking, the actuators need to leave the natural dynamics of the walker intact. Also by minimizing the mass at the end of the legs, less torque is needed to accelerate the leg. Therefore with minimal energy the swing-leg motion can be fast, which increases the disturbance rejection [25].

Flame has nine internal degrees of freedom to be able to walk in a straight line in three dimensions. Figure 2-1(a) shows that the biped has joints in the hip (φ_h) and knee (φ_k) that can make the leg segments swing forward and backward in both legs. This degree of freedom is also in the ankle (φ_a), but here is an extra joint (θ_a) perpendicular to that joint, making the
foot able to roll sideways. There is one more degree of freedom which adds the possibility to spread the legs ($\theta_h$). This joint is actuated by a spindle, which is parallel to the spine, which pushes a part up and down. There are two bars connected to this part and both the hips, so by moving the part up, the bars will pull the sides of the hip up and the legs well spread out. Due to this design the sideway angle between the upper body and the leg is always half the angle between both legs, therefore the upperbody will always be in the middle of the two legs.

Figure 2-1: This 3D bipedal walker has nine internal degrees of freedom, out of which seven are actuated (gray). The letters $l$ and $r$ indicate the left and right leg. The direction of rotation is positive as indicated by the arrows (Figure 2-1(a)). The design and realization of the prototype Flame is shown in (Figure 2-1(b)), adopted from [6].

The actual design and realization of the biped robot Flame is shown in Figure 2-1(b). Flame weighs 15 kg, is 1.2 m tall and has a leg length of 0.6 m. The Maxon motors used for the actuation for all the joints have a power of 90W each. The gear reduction and the range of motion are different for all the joints.

2-2 Mechanical realization

This section covers the most interesting mechanical features that lead to the desired degrees of freedom, the low actuator impedance and proper natural dynamics as mentioned in Section 2-1.
2-2 Mechanical realization

2-2-1 Series elastic actuation

Six degrees of freedom are actuated by series elastic actuation [19] which allows torque control in the joints. The series elastic actuator leaves the natural dynamics of the system intact. The actuated joints are driven by a geared electric DC motor that is connected to the joint via a cable that holds an elastic element. This cable is on both sides of the joints therefore the motor is able to rotate the joint in both directions (e.g. Figure 2-2). There is one incremental encoder mounted on the actuator and one incremental encoder on the joint. With these two signals the length of the elastic element can be calculated. By controlling the length of this element, the actuation system allows the application of force/torque control as shown by the schematic diagram in Figure 2-3.

Figure 2-2: Example of a joint that is actuated by series elastic actuation. Design of the hip pitch joint in the prototype Flame. The hip pitch motor is placed in the upper leg orthogonal to the joint (Zoom of Figure 2-4)

Figure 2-3: Schematic diagram of the concept of series elastic actuation. An electric motor drives a joint through an elastic/compliant element. By measuring the elongation of this element, the torque that the actuator delivers to the joint can be controlled, adopted from [6].

To give an example of the series elastic actuation the hip pitch is shown. The mechanical implementation of the series elastic actuation is done through a steel cable drive that holds
two tension springs (Figures 2-2 and 2-4 upper right corner). The use of cables offers the possibility to place the motor at various positions and orientations relative to the joint axis. In Flame the motor is placed in the upper leg orthogonal to the hip pitch joint. A 30000 counts per revolution incremental encoder (Scancon) is used at the joint side for accurate position and velocity estimation. By measuring the difference in orientation of the encoder on the motor and joint side, the extension of the spring is determined. The torque exerted on the joint can be calculated from this measurement.

Figure 2-4: Design of the hip pitch joint (above) and foot plus ankle (below) in the prototype Flame. The hip pitch motor is placed in the upper leg orthogonal to the joint. The ankle pitch motor is placed in the upper body and connects to the joint through a Bowden cable construction, adopted from [6].

Figure 2-5(a) shows the measured dynamics of a freely swinging fully passive robot leg. The dotted line in Figure 2-5(b) shows how directly connecting a geared electric motor severely
hampers these natural dynamics. In contrast, the dashed line in Figure 2-5(b) shows how a series elastic actuator with (zero) torque controller leaves the original dynamics intact.

![Graph showing hip pitch joint angle during passive swinging](image)

**(a)** Original (passive) dynamics  
**(b)** With geared electric motor connected (dotted) and series elastic actuation (dashed)

**Figure 2-5:** Measured hip pitch joint angle during passive swinging of a leg (Figure 2-5(a)), “swinging” with a geared electric motor directly connected (Figure 2-5(b) dotted) and with a series elastic actuator connected (Figure 2-5(b) dashed). Using an elastic actuator leaves the original dynamics of the leg intact, in contrast to the geared electric motor, adopted from [6].

There is one more advantage of the series elastic actuation. When impacts occur in the robot (which happens frequently when walking), the series elasticity protects the motor and especially the gears in the gearbox from being damaged. A disadvantage of the series elasticity is that it forms a limitation on the position control bandwidth that can be obtained on the joints. This is not a problem for limit cycle walking as it typically does not depend on the application of stiff position control, but it could be a problem for the gait initiation. During a stable walking motion some joints are position controlled and during walking these controllers are sufficient. But during walking the velocity of all the joints is higher than the velocity of the joints when standing or starting. During the rest of this thesis, the error between the reference signal and the actual position was acceptable and caused no problems.

### 2-2-2 Foot and ankle design

As can be seen at the bottom of Figure 2-4 the ankle and foot design is another special design feature of Flame. The toe of the foot is actively controlled in the downward direction through the motor and, in the counter direction, passively by a return spring. Besides this ankle pitch actuation design, the foot design incorporates the ability to implement variable stiffness around the passive ankle roll joint through tension springs with various attachment points. This passive ankle roll joint is not actively controlled, so the biped robot is not actively controlled in the sideways direction. Flame is controlled in the sideways direction by using active lateral foot placement [6] when walking. This underactuation in the sideways direction in the ankle could be a problem when starting from a standstill. During the gait initiation the biped robot has to stand on one leg. If the forward velocity is still zero, when standing on one leg, the active lateral foot placement will not be sufficient to avoid the biped from falling. Active lateral foot placement can only be used during a walking motion. Therefore the biped robot should have forward velocity when standing on one leg.
2-3 Control system architecture

This section describes the control hardware and the lower and higher level control structure of limit cycle walker Flame.

2-3-1 Hardware architecture

Table 2-1 shows all the control hardware components of Flame. All the sensor data acquisition (encoders, inertial sensor, switches) is done by several boards on a central PC104 stack in a parallel fashion, through custom made electronics. The low power electronics and the motor/amplifiers combinations are powered separately by lithium polymer battery packs that can deliver a robot operating time of approximately 30 minutes.

<table>
<thead>
<tr>
<th>Computer</th>
<th>Diamond Systems Athena 400 MHz CPU board plus Data Acquisition</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/O cards</td>
<td>Diamond -MM-16-AT (16-bit analog I/O)</td>
</tr>
<tr>
<td></td>
<td>Mesa Electronics 4I36 (2x) (counters)</td>
</tr>
<tr>
<td>Inertial sensor</td>
<td>Xsens MTi</td>
</tr>
<tr>
<td>Joint encoders</td>
<td>Scancon 2rRMHF 7500 pulses/rev (8x)</td>
</tr>
<tr>
<td>Motor encoders</td>
<td>Agilent 500 pulses/ref (7x)</td>
</tr>
<tr>
<td>Foot sensor</td>
<td>Microswitches (4x)</td>
</tr>
<tr>
<td>Motors</td>
<td>Maxon RE35 (7x)</td>
</tr>
<tr>
<td>Amplifiers</td>
<td>AMC Z12A8 6 A RMS (7x)</td>
</tr>
<tr>
<td>Electronics battery</td>
<td>Kokam 3 cell 4 Ah LiPo</td>
</tr>
<tr>
<td>Motor battery</td>
<td>Kokam 8 cell 26.4 Ah LiPo</td>
</tr>
</tbody>
</table>

2-3-2 Typical walking controller

This section shows the structure of a typical walking controller. The controller of Flame has a low level torque controller and a higher level state machine.

Local torque control

The local torque controller is the same in all joints with series elastic actuation. The torque is measured as explained in Section 2-2-1. This actual torque \( \tau_{\text{act}} \) is subtracted from the current desired torque \( \tau_{\text{des}} \) and fed back through a proportional-derivative controller to give a motor torque signal. Every local torque controller incorporates a unit feedforward component from the desired joint torque to the motor torque signal, based on a simple, static model of the motor. Figure 2-6 shows a scheme of a local torque controller. The system constant \( K \) consists of the gear ratio, pulley ratio and the motor constant.
Higher level state machine

The desired torques $\tau_{\text{des}}$ are calculated by a higher level controller. This higher level controller is an event based state machine. This state machine has four states in the walking motion that alternate in the following order:

1. double stance, left leg is leading leg
2. single stance, left leg is stance leg
3. double stance, left leg is trailing leg
4. single stance, left leg is swing leg

Events trigger a transition from one state to another. The transition from “single stance” to “double stance” is triggered by the foot switch which is beneath the foot. The event that triggers the transition from “double stance” to “single stance” is that the torque in the pitch ankle of the trailing leg gets below a threshold value (near to zero). During a stable walking motion these four states succeed each other.

2-3-3 Software implementation

The controller in Flame is built in C++. This section shows an overview of the implemented state machine.

The state machines for the biped robot Flame are divided into “Walking”, “Standing” etc. (Figure 2-7). Based on “rules”, transitions are made between one and another state.

The sample frequency is 1000 Hz. A state machine calculates new input values for the controllers, the controllers calculate new input values for the actuators and this signal is sent to the actuators in each time step.

To understand how the C++ program is programmed an overview of the state machine is given in this section. Figure 2-7 shows an overview of the total state machine of Flame. There are four main states, namely “Exercise”, “Standing”, “Start Walking” and “Walking”. 

Figure 2-6: Local torque controller that incorporates a unit feedforward component from the desired joint torque to the motor torque signal, based on a simple, static model of the motor. The system constant $K$ consists of the gear ratio, pulley ratio and the motor constant.
With different transition conditions other states can be reached. The overall state machine of Flame shows that an operator is able to decide which state will be entered, the standing or the exercise state. After both these states the operator can make another decision of the next state by pressing a button.

**Figure 2-7:** Overview of the state machines. All the triggers for switching between the states are shown in this diagram. In the “Exercise” state the biped robot is initialized, in the “Standing” state the robot is standing still, in the “Walking” state the robot is walking (Figure 2-8) and the “StartWalking” state is the core of the research in this thesis (yellow).

**Figure 2-8:** Walking state. All the triggers for switching between the states are shown in this diagram.

Figure 2-8 shows the walking state, there are two scenarios when a transition to the “Walking” state is made. The first one is at the end of the “Exercise” state when “toggle2” is in the upright position. The second scenario is at the end of the “StartWalking” state. This state is the core of the research in this thesis.
The “Walking” state always starts with the “GetReady” state. Here the left leg is set as the stance leg as an initiation.

2-4 Limit cycle walking

As explained in the introduction, the biped robot Flame is a limit cycle walker. This section describes the theory (Section 2-4-1) of limit cycles. A model used for limit cycle walking is described (Section 2-4-2). This model is then used to analyze different stability measures. In Section 4-4 one of these methods is used to measure whether the gait initiation leads to a stable walking motion or not.

2-4-1 Limit cycle analysis

To explain what a limit cycle actually is, this section gives some theory and a few examples.

Theory

A limit cycle is an isolated closed trajectory [22]. Isolated means that the neighboring trajectories are not closed; they spiral either toward or away from the limit cycle (Figure 2-9).

![Figure 2-9: Different limit cycle behavior](image)

(a) Stable Limit Cycle   (b) Unstable Limit Cycle   (c) Half-stable Limit Cycle

Three limit cycles are distinguished: stable, unstable and half stable (Figure 2-9). A limit cycle is stable if the all neighboring trajectories approach the limit cycle. These stable systems oscillate even in the absence of external periodic forcing. An external force may be applied, but this force should be time independent to match the definition of a limit cycle. If such a system is perturbed slightly, it always returns to the cycle. A limit cycle is unstable if all the neighboring trajectories go away from the limit cycle and, in exceptional cases, a limit cycle is half-stable if there are neighboring trajectories that approach the limit cycle and there exist neighboring trajectories that go away from the limit cycle.

Due to the fact that a limit cycle is an isolated trajectory, it is a nonlinear phenomenon. A linear system can have closed orbits, but they will not be isolated. If a linear system has a closed orbit the amplitude is set entirely by the initial conditions, any disturbance to the amplitude will persist forever.
Examples of limit cycle behavior

There are countless examples of limit cycles. The beating of the heart, periodic firing of a pacemaker neuron, daily rhythms in human body temperature and hormone secretion are examples of limit cycles where the limit cycle is really desired. But there are more examples where the limit cycle is disliked: chemical reactions that oscillate spontaneously or dangerous self-excited vibrations in bridges and airplane wings. The following sections show two examples of limit cycles.

Van der Pol oscillator

The van der Pol oscillator is one example of a limit cycle that played a central role in the development of the nonlinear dynamics. Balthasar van der Pol, a Dutch electrical engineer and physicist, found stable oscillations in electrical circuits employing vacuum tubes. If a sinusoidal E.M.F. (of the form $E_0 \sin \omega t$) is present in such a system, it was found that this system was only capable of oscillating with discrete frequencies.

The van der Pol oscillator (given by the van der Pol equation (Equation 2-1)) has a long history of being used in both the physical and biological sciences. For instance, in biology, Fitzhugh and Nagumo extended the equation in a planar field as a model for action potentials of neurons. The equation has also been utilized in seismology to model the two plates in a geological fault.

The van der Pol equation is given by:

$$\ddot{x} + \mu (x^2 - 1) \dot{x} + x = 0 \tag{2-1}$$

where $\mu \geq 0$ is a parameter. Equation 2-1 looks like a simple harmonic oscillator, but with a nonlinear damping term $\mu (x^2 - 1) \dot{x}$. This term causes large amplitude oscillations to decay, but it pumps them back up if they become too small.

Figure 2-10 represents the solution of the van der Pol equation (Equation 2-1) for $\mu = 1.5$, starting from $(x(t), \dot{x}(t)) = (0.5, 0)$ at $t = 0$. To visualize the limit cycle the solution to van der Pol equation is shown in a phase plane (Figure 2-10(a)), it is a plot of two state variables against each other and often used for non-linear systems.

The van der Pol oscillator is a clear example of a stable limit cycle without any external periodic forcing.

Pendulum

Another example is given by a simple pendulum with damping. The equation of motion is as follows:

$$m \ddot{\theta} = -b \dot{\theta} - \frac{mg}{l} \sin \theta,$$

1. There is a spontaneous polarization in the sinus node, that triggers the contraction of the heart. No impulse from the brain is needed. If the sinus node does not function, or the impulse generated in the sinus node is blocked before it travels down the electrical conduction system, a group of cells further down the heart will become the heart’s pacemaker.

2. E.M.F.: Electromotive Force
where is $m$ the point mass of the pendulum, $\theta$ is the angle of the pendulum (as shown in Figure 2-11), $b$ is the damping, $g$ is gravity and $l$ is the length of the pendulum. This pendulum will oscillate around the middle ($\theta = 0$) and damp out over time as shown in Figure 2-12. The initial condition of the pendulum is set at an angle $\theta = -0.4$ rad ($\approx -23^\circ$) and a speed of zero ($\dot{\theta} = 0$). The phase plane figure (Figure 2-12(a)) shows that the maximum speed of the angle $\dot{\theta}$ is decreasing over time and will eventually stop in the middle ($\theta = 0$ and $\dot{\theta} = 0$).

Now, a force is applied when the pendulum reaches its maximum position on the left hand site ($\dot{\theta} = 0$) until the pendulum moves with a certain speed ($\dot{\theta} = 0.5$). This force compensates for the energy losses due to the damping and with the right timing the pendulum will show a limit cycle behavior.

Figure 2-13(a) shows that the state space of the pendulum will become a connected line. This means that the pendulum will continue oscillating. This is a limit cycle because the external force is not applied with a pre-defined period but depends totally on a certain state of the system. A stable limit cycle is an isolated closed trajectory that oscillates even in the absence of external periodic forcing. The external force will evolve to a periodic force due to the properties of the system and the external force is time independent. The robot Flame has this same property, the forces applied to the robot are depending on the state of the robot (Section 2-3-2).
Figure 2-12: Pendulum with damping. The horizontal axes represents the angle $\theta$ of the pendulum with respect to the vertical line, the vertical axes represents the speed of the angle $\dot{\theta}$ (Figure 2-12(a)). Figure 2-12(b) shows the angle of the pendulum with respect to time. Both figures show the same behavior of the pendulum where the amplitude is of the angle is damped.

Figure 2-13: Pendulum with damping and force. The horizontal axes represents the angle $\theta$ of the pendulum with respect to the vertical line, the vertical axes represents the speed of the angle $\dot{\theta}$ (Figure 2-13(a)). The force is applied when the pendulum is at its maximum condition to the left, with the right force a limit cycle behavior is reached. Figure 2-13(b) shows the angle of the pendulum with respect to time.
2-4 Limit cycle walking

2-4-2 Limit cycle analysis of walking

The theory for a limit cycle is applicable in biped robots. This theory can easily be seen in a passive walking robot and the Simplest Walking Model. Daan Hobbelen [6, 8] gave a definition for “Limit Cycle Walking” as follows:

*Limit Cycle Walking is a nominally periodic sequence of steps that is stable as a whole but not locally stable at every instant in time.*

A closed trajectory in state space is the intended walking motion. This is a trajectory where the biped robot is not stable during a step, but the multiple steps will repeat themselves (a nominal trajectory) and thus be stable. This type of stability is called ‘cyclic stability’ or ‘orbital stability’ [22] and can be compared to the definition of a limit cycle as given in Section 2-4-1.

**The Simplest Walking Model**

To give an example of limit cycle walking the Simplest Walking Model [3] is used. The Simplest Walking Model is a model of a biped robot that walks down a slope without any actuation. This model will also be used to analyze stability. The Simplest Walking Model is a simplified version of the two-dimensional straight-legged walker with more general mass distribution [16].

![Diagram of the Simplest Walking Model](image)

*Figure 2-14: The simplest walking model; A typical passive walking step. The new stance leg (lighter line) has just made contact with the ramp in the upper left picture. The swing leg (heavier line) swings until the next heelstrike (bottom right picture). The top-center picture gives a description of the variables and parameters that are used. $\theta$ is the angle of the stance leg with respect to the slope normal. $\phi$ is the angle between the stance leg and the swing leg. $M$ is the hip mass, and $m$ is the foot mass. $l$ is the leg length, $\gamma$ is the ramp slope, and $g$ is the acceleration due to gravity, adopted from [3].*

A drawing of the simplest walking model is shown in Figure 2-14. It has two rigid legs connected by a frictionless hinge at the hip. The only mass is at the hip and the feet. The hip mass $M$ is much larger than the foot mass $m$ ($M \gg m$) so that the motion of a swinging foot does not affect the motion of the hip. This linked mechanism moves on a rigid ramp of slope $\gamma$. When a foot hits the ground (ramp surface) at heelstrike, it has a plastic (no-slip,
no-bounce) collision and its velocity jumps to zero. That foot remains on the ground, acting like a hinge, until the swinging foot reaches heelstrike. During walking, only one foot is in contact with the ground at any time; double support occurs instantaneously. This does not hold for the biped robot Flame during walking.

There is one nonphysical assumption that the swing foot can briefly pass through the ramp surface when the stance leg is near vertical. This concession is made to avoid the inevitable scuffing problems of straight legged walkers.

2-4-3 Stability analysis

Different analyses can be done to show whether a limit cycle walker is stable or not. In further research for the gait initiation these analyses could be used to find a place in the state space where the variation of the nominal behavior is maximal. This could be the point where the limit cycle is easy to enter and thus the best end of the gait initiation. The following sections describe two methods for this stability analysis. First Section 2-4-3 iteratively calculates the Basin of Attraction (BoA), for the simplest walking model. Then Section 2-4-3 describes a method to approximate the Basin of Attraction.

Basin of attraction

To analyze the stability of the walking motion, the Limit Cycle Walker is observed on a step-to-step basis. One complete step is considered as a function or “mapping” from the walker’s state \( v_n \) at a definite point, within the motion of a step, to this exact same point the next step \( v_{n+1} \). Here \( v \) is a vector with all the state variables (\( \phi, \dot{\phi}, \theta \) and \( \dot{\theta} \)) and \( n \) is a step counter. The mapping is generally called a Poincaré map in nonlinear dynamics [22]. For walking, this definite point could be the moment just after heel strike and the mapping is called the “stride function” \( S \) [16]:

\[
v_{n+1} = S(v_n)
\]  

(2-2)

This stride function \( S \) contains all the dynamics of the dynamic walker. There will be a periodic motion if the mapping of the walker’s state gives exactly the same state one step later. This specific state \( v^* \) is called the “fixed point” of the function \( S \):

\[
v^* = S(v^*)
\]

The entire collection of initial conditions that lead to this fixed point is the basin of attraction (BoA). This complete BoA can be found by using the cell mapping method [11]. With this cell mapping method the general behavior of the stride function (Equation 2-2) can be studied.

For the simplest walking model, a step will start at heelstrike. At heelstrike \( \phi = 2\theta \), this reduces the Poincaré section to a 2D area. Figure 2-15(a) shows the Poincaré section for the simplest walking model at a slope \( \gamma = 0.004 \) [rad]. To analyze the BoA Figure 2-15(b) zooms in on this small area (point A in Figure 2-15(a)). This BoA is obtained with the application of the cell mapping method with a discretization of about 200 x 250 points [21].
To explain the behavior of the walker in the different areas the vertical line, $\theta = 0.2 \ [rad]$ is used. If started at the initial condition ($\theta = 0.2 \ [rad], \dot{\theta} = -0.23 \ [rad]$) the walking model will fall forwards. The swing leg is allowed to pass through the ground (as explained in the simplest walking model), but will not rise above the floor level anymore. Going up this vertical line, and getting closer to the basin of attraction, the behavior changes. The walker will not fall the first step after the initial conditions, but the walker will fall forward after some steps. The closer to the basin of attraction the more steps the walker will take before falling.

Figure 2-16 shows several points towards the fixed point. The first initial condition is just inside the basin of attraction and after nine steps the walker is close to the fixed point, which will be asymptotically approached. If the vertical line ($\theta = 0.2$) is followed even more the area’s (A, B and F) will be crossed several times, until $\theta + \dot{\theta} = \gamma$. Above this boundary the input energy is not enough to overcome the “dead point” (comparable with an inverted pendulum) and the walker will fall backwards.

**Multi-way principal component analysis**

Above a method to calculate the basin of attraction is shown, but this is only possible for a simple model (only two states) and the way it is described, only the BoA is known at one instant during a step. It is very hard to analyze the BoA of a real walking robot due to the very high dimensionality and the non-linear behavior. To analyze the performance of the gait initiation another method is shown here.

Another way to do a stability analysis is shown by Karssen [12]. He describes a way to detect upcoming falls of a limit cycle walker with a Multi-way Principal Component Analysis (MPCA) detection method [12]. Karssen describes a way to model the nominal behavior by
letting the robot walk for a number of steps. This nominal behavior is inside, and smaller than, the Basin of Attraction (Section 2-4-3), because the robot is walking for numerous steps. In this thesis the method for monitoring a robot could be used to find a place in the state space where the variation of the nominal behavior is maximal. This could be the point where the limit cycle is easy to enter and thus the best end of the gait initiation.

The monitoring of walking bipeds is possible with MPCA. The MPCA method was introduced to monitor a batch process [17]. The process can be compared with the walking motion of a biped, because it is a non-linear cyclic process and has a high number of state variables with different units.

This method could be implemented on the real walking robot Flame and a nominal trajectory with a variation could be found. This is an approximation of the BoA during a whole step. This tube can now be analyzed and a place in state space could be found were the variation is maximum. This maximum variation could define a point where the limit cycle is entered the most easiest, because once the robot is within the BoA, the robot will stay in this BoA and perform a stable walking motion.

**Modeling of nominal behavior with MPCA**

The following six steps describe the way to determine the nominal trajectory and the nominal variation.

1) *Obtain “good” cycles:* First of all a set of “good” cycles (Figure 2-17(a)) is obtained by letting the robot walk for a number of steps, on a floor with small irregularities, without falling. With “good” cycles, cycles are meant where the system is running for a number of cycles in an environment with small disturbances.

2) *Determine nominal trajectory:* To estimate the nominal trajectory, the time average of the trajectories of the “good” cycles are taken. Before taking the average, the time is normalized...
(a) “Good” walking cycles  (b) Planes perpendicular  (c) A tube of the combined planes to the nominal trajectory

**Figure 2-17:** A set of 50 non-linear trajectories in a 3-dimensional state space. The dashed lines indicate that the trajectories make a jump in state space (Figure 2-17(a)). 10 hyperplanes slice the trajectories perpendicular to the nominal trajectory. These planes are the cross-sections that are used for analyzing the variation of the “good” trajectories around the nominal trajectory (Figure 2-17(b)). With the points in the planes the average trajectory and the boundary are estimated. The boundary in a plane is an ellipse. The ellipses of all the planes combined result in a tube. Note that the shown tube is created with more planes than are shown (Figure 2-17(c)), adopted from[12]

with the cycle time, to prevent problems with differences in cycle duration.

3) **Scale state variables:** The state variables can be measured using a variety of units. To be independent of the units used for the state variables, the data of the “good” cycles and the nominal trajectory are scaled. Each state variable is scaled with the distance between the minimal and maximal value of the state variables in the set of “good” cycles.

4) **Determine cross-sections:** Cross-sections are taken to look at how the “good” trajectories are spread around the nominal trajectory. These cross-sections are hyperplanes in state space that are perpendicular to - and spaced equally over - the nominal trajectory (Figure 2-17(b)). The intersections, between the trajectories and the hyperplanes, are determined by taking a linear interpolation between the two sample points of each trajectory closest to the hyperplane. The number of planes used is as large as possible, but limited by computational power.

**Figure 2-18:** A random set of points with linear correlation between the variables. With PCA the coordinate system is rotated so that linear correlation axis lines up with a coordinate axis, adopted from [12].
5) **Remove correlation:** Figure 2.18 shows that the linear correlation can be removed by a Principal Component Analysis (PCA) \[3\]. In a PCA the coordinate system is redefined in such a way that coordinate axes are in line with the correlation axes. The new coordinate axes are called principal components and are a rotated version of the original coordinate axes. In matrix notation the rotation of the coordinate system can be written as:

\[ T_k = P_k (X_k - \bar{X}_k) \]

in which \( X_k \) is the data in the original coordinate system, \( \bar{X}_k \) is the average point of the data, \( T_k \) is the data in the new coordinate system and \( P_k \) the transformation matrix. Note that the PCA is done for each plane separately and that the index \( k \) refers to plane \( k \).

6) **Determine standard deviation:** To form a tube (Figure 2.17(c)) around the nominal trajectory in state space the 95% confidence interval for each plane is used. The confidence interval of the multivariate normal distribution is an ellipsoid with its axes on the principal component axes. The relative size of the ellipsoid is determined by the standard deviation of the points in each principal component.

---

\[3\] PCA involves the calculation of the eigenvalue decomposition of a singular value decomposition of a data matrix.
2-5 Internal joint angles and torques during twenty steps of the biped robot Flame

This section showed stable walking data of twenty walking steps of the biped robot Flame. The stable walking data will be used for the MPCA method.

![Graphs showing joint angles and torques](image)

**Figure 2-19:** Internal joint angles and torques during twenty steps of the biped robot Flame. The solid and dashed lines give the information of the stance leg, the dotted and dash-dotted lines for the swing leg. The blue lines are where the left leg is the stance leg and the red lines are where the right leg is the stance leg.

The computer in the biped robot Flame is able to store all the data of the different sensors. In total there are 115 variables which are recorded every thousandth of a second. The heel sensors at both the feet are used to define a new step. At heelstrike a new step starts. Figure 2-19 shows twenty steps of a stable walking motion of the biped robot Flame. There is a slide variance in all the steps. This data (and more stable walking steps) is later used for the PCA in Section 4-4.
This chapter shows some of the design aspects of the robot Flame to understand more about the control of this robot. As can be seen from this design, the robot is especially made for limit cycle walking. This special design is very suitable for walking but could be a problem in gait initiation. The two most important problems are, bad position control (due to the series elastic actuation) and underactuated in the sideways direction in the ankle (which can be a problem when standing on one leg).

The biped robot Flame is a limit cycle walker and this chapter also describes the limit cycle walking approach. A limit cycle is an isolated closed trajectory, for a stable limit cycle this means that the neighboring trajectories are not closed, but they spiral towards the limit cycle. In a limit cycle walker this means that the walker itself will correct for small disturbances around the fixed point. A limit cycle walker is not necessarily stable at every instant in time, but has a nominally periodic sequence of steps that is stable as a whole.

The basin of attraction can be calculated for a simple walking model. The shape of all the initial conditions that lead to a stable walking motion is roughly a small pointy boomerang. A disadvantage of this method is that the basin of attraction is only calculated at one instant in a step, so the basin of attraction is unknown for the whole step. The shape of the basin of attraction could be different at other places in the step. Another disadvantage is that the basin of attraction is hard to calculate for a more complex robot. The more degrees of freedom a robot has the more calculation time is needed. The grid needed to calculate the basin of attraction is very small so with more degrees of freedom the calculation time will increase. The last disadvantage is that a model of the robot is needed to simulate the behavior of the robot, because the BoA can not be measured on the real robot. To find the boundaries of the BoA, the model mismatch should be very small.

To overcome the disadvantages of the calculation method for the basin of attraction a different method is explained to calculate the nominal trajectory with a nominal variation. The downside is that with the Multi-way principal component analysis (MPCA) the variation around the fixed point becomes even smaller than the actual basin of attraction. Which means it will be even harder to reach a state close to the nominal trajectory and within the nominal variation. The positive side of the MPCA is that the maximal variation around the nominal trajectory can be easily calculated, which could be used as the best start of the limit cycle.
Chapter 3

Modeling and simulation

The previous chapter showed that the biped robot Flame is able to perform a walking motion. The biped robot is launched by hand and to perform a stable walking motion the state at heelstrike must be appropriate. This chapter investigates whether a falling motion from an upright position will also lead to a state at heelstrike that is appropriate for a stable walking motion. A simulation of the biped robot is made to investigate the motions during falling.

To describe the movements in the sagittal (Section 3-1) and in the coronal plane (Section 3-2), two different inverted pendulum models are used. Section 3-3 compares the falling motion of these two models with walking data from the biped Flame. The conclusion (Section 3-4) of this chapter shows whether the falling motion of the pendulum models is a way to reach an appropriate state at heelstrike for a walking motion.

3-1 Sagittal movement behavior

To model the behavior of the biped robot in the sagittal plane an inverted pendulum model is used. The ankle design (Figure 2-4 lower right) shows that there are two ankle springs that pull the lower leg of the biped Flame forward and there is one actuated spring that is able to pull the lower leg backwards. The actuated spring is neglected in the model because the force in this spring will be controlled to zero in order to fall forward. The two other ankle springs are incorporated in the model (Figure 3-1). These springs exert a force in the forward direction and ensure that the robot falls forward from the initial upright position.

There are three moments on the inverted pendulum; the moment of the springs in the ankle $M_s$, the moment of gravity $M_g$ and the moment of the damping in the ankle $M_d$.

The force of the springs $F_s = 2k_a \Delta L_s \sin(\alpha)$ generate a moment:

$$M_s = h_1 F_s$$
Figure 3-1: A schematic drawing of the foot and lower leg, where two ankle springs pull the lower leg forward. Here \( \alpha \) is the angle between the lower leg and the spring, \( \phi_a \) is the angle of the lower leg, \( k_a \) is the spring stiffness and \( h \) is the distance between the ankle and the point on the lower where the springs are attached.

around the pivot point. Where \( \Delta L_s \) is the elongation of the spring and \( \alpha \) is the angle between the lower leg and the ankle spring. For the gravity force we have

\[
F_g = mg \sin(\phi) \quad (3-1)
\]
\[
M_g = mgl \sin(\phi) \quad (3-2)
\]

where \( m \) is the point mass, \( g = 9.81 \, m/s^2 \) and \( l \) the distance from the center of mass to the rotation point in the ankle.

The equation of motion of the inverted pendulum model is:

\[
I \ddot{\theta} = M_a + M_g + M_d \quad (3-3)
\]

where \( I = mL^2 \) is the moments of inertia.

The simulink model (Figure 3-2) is evaluated for different values of the spring stiffness \( k \) to show that the dynamics of the model are not significant depending on the spring stiffness but on the gravity of the center of mass (CoM). The maximum angle of the pendulum is 0.26 rad, this is the average ankle angle \( \phi_a \) at heelstrike.
Figure 3-2: Simulink model of the inverted pendulum model with ankle springs in the sagittal plane.
Figure 3-3 shows the model behavior for different values of the spring stiffness $k$. And by analyzing the moments on the pendulum (Figure 3-4) it is seen that the motion of the pendulum is more depending on the gravity than on the ankle spring moment.

This Section showed a one degree of freedom model for the sagittal movements of a biped robot. In Section 3-3 this inverted pendulum model is verified with data from the biped robot Flame.

3-2 Coronal movement behavior

Another inverted pendulum model is used for the simulation in the coronal direction. A predefined initial angle is used to analyze the inverted pendulum model behavior. From this initial angle of the pendulum falls sideways. Later an initiation procedure is needed to get the biped robot Flame to this initial angle.

Figure 3-5 shows the initial and final positions of the model. The angle of inverted pendulum $\theta_p$ is depending on the roll angle of the upperbody $\theta_{ub}$.

$$\theta_p = \beta - \theta_{ub}$$
where $\beta = \tan^{-1}\left(\frac{B/2}{h_{CoM}}\right)$ is the fixed angle between the pendulum and the upperbody.

The initial angle of the pendulum is the average maximum angle of the upperbody during walking of the biped robot Flame. From this initial angle the pendulum falls to the right and reaches the final angle, where the right heel strikes the ground.

![Initial and Final Positions](image)

**Figure 3-5:** The initial and final positions of the simulation in the coronal plane. The dotted red line shows how a pendulum can be modeled. Here $\beta$ is the fixed angle between the pendulum $\theta_p$ and the upperbody $\theta_{ub}$.

The inverted pendulum model in the coronal plane only considers the gravity force and models the foot on the ground as a pivot point. Figure 3-6 shows the angle of the upperbody ($\theta_p$) during the simulation.

![Simulation of Roll Angle](image)

**Figure 3-6:** Simulation of the roll angle of the upperbody while falling.
3-3 Verification of the models with data from Flame

To verify whether the heelstrike state of the inverted pendulum model simulations is corresponding to the heelstrike state of the biped robot Flame, data from the biped robot Flame is used. The question remains: is the falling motion of the models sufficient enough to reach a stable walking motion?

3-3-1 Sagittal movement behavior

Figure 3-7 shows the ankle angle $\phi_a$ during ten walking steps (where the left leg is the stance leg) and the angle modeled by the inverted pendulum (Section 3-1). Angle $\phi_a$ defines the position of the center of mass, because the knee angle $\phi_k$ will be zero. The modeled ankle angle and the angle of the biped robot Flame at heelstrike are similar as can be concluded from this figure.

![Figure 3-7: Ankle angle of Flame and the model in the sagittal plane. The blue rounds in Figure 3-7(b) show the angles at heelstrike and the black x shows the angle of the inverted pendulum model.](image)

3-3-2 Coronal movement behavior

Figure 3-8 shows the roll angle during ten walking steps (where the right leg is the stance leg) and the upperbody angle modeled by the inverted pendulum (Section 3-2). The modeled upperbody angle and the roll angle of the biped robot shows similar behavior during the last phase of a step. And, as can be seen in Figure 3-8(b) the angle of the modeled upperbody corresponds to the roll angle of the biped robot at heelstrike.
3-4 Conclusion

This chapter describes two inverted pendulum models that are used to simulate the behavior of the biped robot Flame. The falling motion of the inverted pendulum models, used in both the sagittal and coronal plane, show promising results in reaching an acceptable state at heelstrike. From these results a falling controller can be formed, that should be able to let the robot fall into a stable walking motion, where both legs are on the ground as the initial position.

Figure 3-8: Roll of the upperbody of Flame during ten walking steps and the inverted pendulum model in the coronal plane. Showing the roll angle at heelstrike of the model and the biped robot are similar. The blue rounds in Figure 3-8(b) show the angles at heelstrike and the black crosses show the angle of the upperbody modeled by the inverted pendulum model. An initiation sequence is needed to reach the predefined initial angle.
Chapter 4

Gait initiation controller

The previous chapter showed that a falling motion from a standstill of the biped robot Flame, modeled by two inverted pendulums, can lead to a stable walking motion. This chapter describes a procedure that initiates the falling behavior of the biped robot. The aim is to reach a heelstrike state, that lies within the basin of attraction, from a standing position.

Simulation of the two inverted pendulum models (see Section 3-1 and Section 3-2) gave initial parameters of a time driven gait initiation trajectory. But the inverted pendulum simulations had a different falling duration time, the falling motion in the sagittal plane has a longer duration than the falling motion in the coronal plane. Therefore the heelstrike moments of the two inverted pendulum models are synchronized, and the synchronized inverted pendulum simulations give a better insight of the gait initiation (Section 4-1). A time driven gait initiation trajectory is then adjusted for the biped robot Flame (Section 4-2). Three different measures of the performance are discussed in Section 4-3. Finally the result of the gait initiation procedure is discussed.

4-1 Controller in simulation

To define a gait initiation procedure for the biped robot Flame, the data of the simulation of the two inverted pendulum models (Section 3-1 and Section 3-2) is used as a basis. The simulation of the two inverted pendulum models shows that it takes more time to reach the heelstrike state in the sagittal plane than in the coronal plane. The heelstrike state is the state of the biped robot where the heel of the swing leg hits the ground. This could be during a stable walking motion or after the gait initiation procedure. From the upright position in the sagittal plane ($t_1$) it takes 0.76 s to reach the heelstrike state and it takes 0.41 s to reach the heelstrike state from the maximum roll of the upperbody in the coronal plane. These two models should reach the heelstrike state at the same time, therefore the falling forward motion should be initiated 0.76 − 0.41 = 0.35 s before the falling motion in the coronal plane. This extra time is used to gain forward speed and to reach the maximum roll. To reach the maximum roll of the upperbody, the right leg should be “elongated”, this is done by changing...
the pitch angle of the right ankle $\phi_{a,r}$ ($t_2$). After the weight is transferred to the left ($t_3$), the right leg can be swung forward and the biped robot will fall to the right and forward until heelstrike ($t_5$).

Figure 4-1 shows a Gantt chart of the gait initiation for the biped robot simulation. The times defined in the Gantt chart are also shown in Figure 4-2, where snapshots of the roll, the ankle angle and the interleg angle of the two synchronized inverted pendulum models are shown. To show the gait initiation behavior more clearly Figure 4-3 shows the two synchronized inverted pendulum models at different times during the gait initiation. The behavior in the sagittal plane is a falling motion forward and in the coronal plane the biped robot first moves to the left and then falls to the right.

The gait initiation procedure can not directly be used on the biped robot Flame, because Flame has more degrees of freedom. The gait initiation procedure used in simulation is modified and described in the next section.
Figure 4-2: The top row of figures shows the two inverted pendulum models (dashed magenta line), the left leg (blue line) and the right leg (red line) at the heelstrike state. The lower left figure shows the angular roll of the upperbody during the gait initiation procedure, where it behaves like an inverted pendulum from $t_3 \rightarrow t_5$. The lower right figure shows a falling behavior of the inverted pendulum from $t_1 \rightarrow t_5$ and that the right leg is swung forward from $t_3 \rightarrow t_4$. 
Figure 4-3: Phase in the simulation of the gait initiation. The left column of figures show rear views of the simulated robot and the right column of figures show side views of the simulated robot. The magenta dashed line shows the inverted pendulum, the blue line shows the left leg and the red line shows the right leg. The simulation starts from an upright initial position (Figure 4-3(a)). Then the weight is transferred to the left (Figure 4-3(b)), whereafter the right leg can be swung forward (Figure 4-3(c)). The last phase is waiting for the heelstrike (Figure 4-3(d)).
4-2 Gait initiation controller for the biped Flame

The gait initiation procedure used in the simulation is modified for the biped robot Flame. The biped robot starts with falling forward by changing the ankle angle $\phi_a$ ($t_1$). The left ankle angle is actively changed until $t_3$, thereafter the robot will keep on falling forward because the controller of the left ankle is not very stiff. If the left ankle angle $\phi_{a,l} < -0.03$ rad, the weight will be shifted to the left leg ($t_2$). The weight is shifted to the left leg by “elongating” the right leg, that is done by changing the right ankle angle $\phi_{a,r}$. To lift the right leg from the ground it is important to perform three actions at the same time ($t_4$). The right foot has to be parallel to the ground, the right knee has to be bent and the right upperleg should be swung forward (by the use of the right hip). By performing these three actions at the same time the foot can be lifted from the ground. If the right foot is lifted from the ground the right knee can be straighten again ($t_7$) and the right leg can be swung forward until $\phi_{h,r} = -0.4$ rad ($t_8$). Also the right ankle angle $\phi_{a,r}$ should be ready for heelstrike ($t_9$), therefore this angle is changed to the normal angle before heelstrike. A switch is now made to the normal walking controller and at heelstrike ($t_9$) the biped robot will perform a normal walking motion. Figure 4-4 shows a Gantt chart of the developed gait initiation controller.

![Gantt chart of gait initiation](image)

**Figure 4-4:** Gait initiation procedure. The biped robot starts by falling forward ($t_1 \rightarrow t_3$), then weight is shifted to the left ($t_2 \rightarrow t_4$). If the weight is transfered the left foot ($t_4$) the right leg is swung forward by bending the knee, holding the foot parallel to the ground and by swinging the upperleg. Before heelstrike ($t_9$) the knee is stretched again and the ankle is put in the heelstrike position.

The heelstrike state after the gait initiation procedure should be within the basin of attraction to achieve a stable walking motion. To analyze the performance, and to find the right values of the times ($t_1$ to $t_9$), of the gait initiation procedure a measure is needed. Section 4-4 describes the result of this controller on the biped robot Flame.

4-3 Performance measures

Three common methods to measure the performance of a stable walking motion are considered in this section. These three measures should be adapted to measure the performance of the gait initiation procedure.

The first performance measure is calculating the distance from the heelstrike state to the average walking heelstrike state. Due to the shape of the basin of attraction (Figure 2-15),
this distance can only be compared if the direction in phase space is corresponding to the average walking heelstrike state. This angle is not always corresponding, that makes this measure unreliable. Therefore the measure will not be used to analyze the performance.

The second performance measure is the gait sensitivity norm \([7]\). The gait sensitivity norm is a measure for disturbance rejection. This measure uses all the steps after the disturbance to calculate the difference in a chosen gait indicator. For example a gait indicator could be the step time. The step time of all the steps after gait initiation are compared to the average walking step time. This gait sensitivity cannot be used for the biped robot Flame due to practical reasons such as the ground is not even (therefore there are other disturbances during walking).

The third measure is a principal component analysis (PCA). By using principal components the linear correlation between variables is removed \([12]\).

First 44 heelstrike states are analyzed to obtain a nominal heelstrike state with a nominal variation around this heelstrike state. These 44 heelstrike states are obtained from a stable walking motion of the biped robot Flame. The fixed point of the limit cycle can not be obtained in the physical system, therefore the nominal heelstrike state will be taken as the average of the 44 heelstrike states.

The data of the heelstrike state is scaled to be independent of the different units for different state variables. The variables are scaled between the minimal and the maximal value of the state variable. The linear correlation is removed by a Principal Component Analysis (PCA). The principal components are a rotated version of the original coordinate axes, where the new coordinate system is defined in such a way that the axes are in line with the correlation axes. A normal distribution is assumed and the standard deviation in each direction is defined separately.

To define the performance of the gait initiation the error with the nominal heelstrike state can be calculated and scaled with the standard deviation. This scaling is done to allow a bigger error if there is a lot of variation. Finally the mean squared error of the scaled error can be calculated. This mean squared error is the error between the normal walking motion state at heelstrike and the state at heelstrike after the gait initiation procedure. This measure will be used to quantify the performance of the gait initiation procedure in the next section.

### 4.4 Results

The state at heelstrike \(\mathbf{x}\) is completely defined by the following variables,

\[
\mathbf{x} = \begin{bmatrix}
\phi_{a,l} \\
\dot{\phi}_{a,l} \\
\phi_{h,l} \\
\dot{\phi}_{h,l} \\
\phi_{h,r}
\end{bmatrix}
\]

where \(\phi\) and \(\dot{\phi}\) denote the pitch angle and pitch angular velocity, \(a\) stands for the ankle, \(h\) for hip, \(l\) for left and \(r\) for right. The ankle of the right leg is not important as this is the ankle
that hits the ground. And because the right leg hits the ground, the speed of the right leg \( (\dot{\phi}_{h,r}) \) will immediately change during impact. Therefore only the right hip ankle \( \phi_{h,r} \) of the right leg is used to describe the state at heelstrike.

Figure 4-5 shows a gait initiation procedure (blue line) and the normal walking motion (dashed black line) for these states variables. The gait initiation takes about 1.3 s before heelstrike of the right foot and all the state variables start from zero. During the whole initiation procedure the biped robot Flame is falling forward as the left ankle angle \( \phi_{a,l} \) is decreasing (Figure 4-5(a)). The angular velocity and the angle of the left ankle at heelstrike is comparable with the average heelstrike. At heelstrike the angle of the left hip \( \phi_{h,l} \) is smaller and the angular velocity is bigger than the average. The right hip angle \( \phi_{h,r} \) is very close to the average angle at heelstrike. Figure 4-5 shows the whole motion of the first step, but to reach a stable limit cycle motion the heelstrike states of the gait initiation procedure and of the stable walking motion should be compared. From Figure 4-5 it is concluded that the heelstrike state of after the gait initiation procedure is comparable to the average heelstrike state. But to calculate the performance a Principal Component Analysis (PCA) is done at the heelstrike state.
Figure 4-5: Comparison of the gait initiation procedure and the average walking motion in time. The dashed black line shows the average walking motion and the yellow band represents the standard deviation. The average step duration is shorter than the gait initiation procedure, to compare the state at heelstrike the average walking motion start later. The blue line show a gait initiation procedure. Heelstrike occurs after about 1.2 s.
4-4 Results

4-4-1 PCA example

To describe the Principal Component Analysis (PCA) in detail only the first two variables are used (Figure 4-6). This figure shows two variables of x at heelstrike for nine \(^1\) “normal” walking steps. These nine heelstrike states are recorded during a stable walking motion.

These variables are scaled and mean centered before the linear correlation is removed by the PCA. To make all the variables independent of their units the variables are scaled.

\[
\tilde{x}_j = \frac{x_j}{c_j}
\]  \hspace{1cm} (4-1)

where \(j\) is the number of state variables and \(c\) is a scaling factor. The first two variables mean-centered and scaled are shown in Figure 4-7.

The linear correlation between the different variables is then removed by the Principal Component Analysis (PCA). Where:

\[
T = P(X - \bar{X})
\]  \hspace{1cm} (4-2)

Where \(T\) is the uncorrelated data and \(P\) the transformation matrix. Figure 4-8 shows the two principal component of the first two variables. The standard deviation is calculated:

\[
\sigma_j = \left( \frac{1}{I-1} \sum_{i=1}^{I} (T_{ij})^2 \right)^{\frac{1}{2}}
\]  \hspace{1cm} (4-3)

where \(I\) is the number of walking steps used. An ellipse is plotted to show two times the standard deviation, which encircles all the heelstrike states.

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\(^1\)Nine steps during a stable walking motion are used here as an example. The principal component analysis used to calculate the performance of the gait initiation procedure uses 44 stable walking step.

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mean-centered scaled values of first two states

Figure 4-7: First two scaled mean-centered variables at heelstrike.

Principal components plus an ellipse of 2 x standard deviation

Figure 4-8: Two principal components of the nine normal walking steps with an ellipse of two times the standard deviation (dashed black line).
Calculation of scaled root mean squared error for a given point

After calculating the PCA for the normal walking the error at the heelstrike state of the gait initiation at the average walking heelstrike state can be computed. To show the calculation only two variables are used here:

\[
x^* = \begin{bmatrix} \phi_{a,l} \\ \dot{\phi}_{a,l} \end{bmatrix}
\]  

This heelstrike state after the gait initiation procedure \( x^* \) is scaled:

\[
\hat{x}_j = \frac{x_j}{c_j}
\]  

The error \( e \) (between the gait initiation procedure heelstrike state \( \hat{x} \) and the nominal state \( \bar{X} \)) is given by:

\[
e = \hat{x} - \bar{X}
\]  

The projected error \( \tilde{e} \) on the principal components is:

\[
\tilde{e} = Pe
\]  

This error is scaled with the standard deviation \( \sigma_j \)

\[
\tilde{e}_j = \frac{\tilde{e}_j}{\sigma_j}
\]  

The root mean squared error \( e_{\text{rms}} \) of scaled vector \( \tilde{e} \) is:

\[
e_{\text{rms}} = \sqrt{\frac{1}{J} \sum_{j=1}^{J} \tilde{e}_j^2}
\]  

Figure 4-9 gives an example for two variables. Here the root mean squared error between the heelstrike state after gait initiation and normal stable walking is \( e_{\text{rms}} = 1.7 \).
4-4-2 PCA of the gait initiation procedure

The previous section showed how the principal component analysis works. In this section the PCA is applied to check the performance of the gait initiation procedure.

The “normal” walking heelstrike state is calculated by using 44 heelstrikes during a stable walking motion. Figure 4-11 and Figure 4-12 show the data of these “normal” walking heelstrike states and eight successful and five unsuccessful heelstrike states after a gait initiation trials. The (scaled) root mean squared errors of these thirteen heelstrike states are shown in Figure 4-10. The smaller this error the closer the heelstrike state after the gait initiation is to the average heelstrike state during stable walking. The stable gait initiations show smaller errors than the unsuccessful gait initiation, therefore this error can be measure to check the performance of the gait initiation. The unsuccessful gait initiation are a result of mechanical problems, such as the pretension of the springs in the series elastic actuated joints, of the biped robot Flame.

![Root mean squared error between the gait initiation and the average walking heelstrike state. The error is scaled with the standard deviation. The green bars show the RMS of the successful gait initiations and the red bars show the unsuccessful gait initiations.](image)

**Figure 4-10**: Root mean squared error between the gait initiation and the average walking heelstrike state. The error is scaled with the standard deviation. The green bars show the RMS of the successful gait initiations and the red bars show the unsuccessful gait initiations.

For all the different principal components three times the standard deviation is calculated and an ellipse is plotted in Figure 4-12. As can be seen from this figure almost all the successful gait initiations are inside this ellipse and the unsuccessful gait initiations are outside this ellipse.

Figure 4-11 shows a boxplot of the PC’s separately. The data of the thirteen gait initiations is also shown.
Figure 4-11: Principal component boxplot of the stable heelstrike states. The green circles represent eight successful gait initiation procedures and the red squares represent five unsuccessful gait initiation procedures.
Figure 4-12: The five principal components (PC). The blue crosses are 44 stable walking heel-strike states, the green circles are eight successful gait initiation procedures and the red squares represent five unsuccessful gait initiation procedures. The black dashed ellipse corresponds to three times the standard deviation. Notice the difference in the axis scaling, principal component 1 has the largest variation and principal component 5 the smallest. The error between the successful gait initiation procedures and the "normal" walking heelstrike states is apparently small enough, to lead to a stable walking motion.
Figure 4-13 and Figure 4-14 show the unscaled variables of 44 stable walking heelstrike states and thirteen gait initiation procedures. These figures also show that the gait initiation procedures lead to a heelstrike state that is comparable with the “normal” walking heelstrike state.

**Figure 4-13**: Boxplot of the variables of the stable heelstrike states. The green circles represent eight successful gait initiation procedures and the red squares represent five unsuccessful gait initiation procedures. The angle variables $\phi_{a,l}$, $\phi_{h,l}$ and $\phi_{h,r}$ show a small variation and the angular velocity variables $\dot{\phi}_{a,l}$ and $\dot{\phi}_{h,l}$ show a larger variation.
**Figure 4-14:** Unscaled variables used to analyze the performance of the gait initiation procedure. The blue crosses are 44 stable walking heelstrike states, the green circles are eight successful gait initiation procedures and the red squares represent five unsuccessful gait initiation procedures. Notice the difference in the axis scaling. The error between the successful gait initiation procedures and the “normal” walking heelstrike states is apparently small enough, to lead to a stable walking motion.
4-5 Conclusion

This chapter shows that the two inverted pendulum models can be combined and give an insight of the gait initiation controller for the biped robot Flame. The falling dynamics of the biped robot Flame are preserved and therefore at heelstrike the biped robot should be within the basin of attraction. The simulated gait initiation controller is adjusted for (and implemented on) the biped robot Flame, because it has more degrees of freedom. To analyze the performance of the gait initiation procedure three measures are evaluated for usability, of which only the MPCA seems suitable. The implemented gait initiation procedure shows acceptable results. The principal component analysis is used to analyze the scaled root mean squared error between the heelstrike states after the gait initiation procedure and the heelstrike state of stable walking motions. From this PCA it is concluded that the performance of the defined gait initiation procedure is acceptable and that the gait initiation procedure leads to a stable walking motion.
At the start of this current study, the biped robot Flame was able to walk, the problem was that the biped robot was not able to start walking from a standstill. The goal of this study was to design a stable gait initiation procedure for the biped robot Flame.

5-1 Conclusions

- The position controller of the series elastic actuator is sufficient enough to control the biped robot Flame during standing and during the gait initiation developed in this thesis.

- The gait initiation procedure developed in the thesis deals with the underactuation in the sideway direction of the ankle by letting the robot fall in a forwards direction first and then move the weight to one leg.

- Simulation of the biped robot Flame by two inverted pendulum models shows that a falling motion form an upright standing position can lead to a heelstrike state that is within a cloud of normal stable walking heelstrike states.

- Based on the simulation a timed based gait initiation procedure is developed that controls a falling motion of the biped robot Flame from a standstill position to a stable limit cycle walking motion.

- Principal component analysis can be used as a reliable measure for the performance of a gait initiation procedure. The smaller the scaled root mean squared error the closer the heelstrike state after gait initiation is to the average walking heelstrike state.

- The trajectories of the biped robot Flame during the gait initiation still differ, even after applying identical gait initiation procedures.
5-2 Recommendations

The falling procedure described in this thesis performs a stable gait initiation, but there is some variation even after applying identical gait initiation procedures. To investigate stability of the biped robot Flame during walking in further research the same initial heelstrike state is desirable, so that initial heelstrike state is not disturbing the walking motion. To minimize the variation of the heelstrike state after the gait initiation procedure the following recommendations are given:

- The distance between the feet on the ground (before gait initiation) influences the falling behavior of the biped robot Flame. Therefore to get the same upright position for every experiment the feet of the biped robot Flame can be placed within a frame. Then the distance between the feet in the sagittal and in the coronal plane will be the same for every experiment, which will lead to a smaller variation of the heelstrike after gait initiation.

- The gait initiation procedure could be more dependent on the state of the robot. The initiation upright position is not exactly the same every experiment and the gait initiation procedure should compensate for this.

- With a complete simulation of the biped robot Flame a fixed point of the walking motion can be found. The fixed point is approximated in this research with the average of 44 stable walking heelstrike states. A stable walking motion of the biped robot Flame is more likely to be reached as the heelstrike state after gait initiation is closer to the fixed point.

- The springs used in the series elastic actuators show plastic deformation after a while. This causes a change in the behavior of the robot, which makes it hard to compare different gait initiation procedures. To overcome this problem different springs could be used.
Bibliography


