A Hydromechanic-Electrokinetic Model for CO₂ Sequestration in Geological Formations

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ABSTRACT

In this contribution, a finite element model for simulating coupled hydromechanic and electrokinetic flow in a multiphase domain is outlined. The model describes CO₂ flow in a deformed, unsaturated geological formation and its associated streaming potential flow. The governing field equations are derived based on the averaging theory and solved numerically based a mixed discretization scheme. The standard Galerkin finite element method is utilized to discretize the deformation and the diffusive dominant field equations, and the extended finite element method, together with the level-set method, is utilized to discretize the advective dominant field equations. The level-set method is employed to trace the CO₂ plume front, and the extended finite element method is employed to model the high gradient in the saturation field front. The mixed discretization scheme leads to a highly convergent system, giving a stable and effectively mesh-independent model. The capability of the model is evaluated by verification and numerical examples. The numerical analysis shows that the streaming potential peak moves with the saturation front, and hence, measuring the streaming potential can be utilized for monitoring CO₂ flow remotely.

INTRODUCTION

CO₂ sequestration in geological formations gives rise to a variety of strongly coupled physical, chemical, thermal and mechanical processes, including hydromechanical deformation and electrokinetic flow. This paper focuses on modeling these two processes and phenomena.

Multiphase fluid flow due to injection of CO₂ in an unsaturated reservoir is accompanied by continuous redistribution of pore pressures and effective stresses, causing local and regional deformations and probably major uplifting or subsidence. It is also accompanied by electrokinetic flow. Electrokinetic flow is a natural process occurring due to the movement of ions in a porous medium electric double layer under the action of fluid flow. The driving hydromechanical force, due to pressure

gradient, and the restraining force, due to electrical resistance, give rise to various electrokinetic effects, including the streaming potential, known also as the self-potential (SP). Since the electrical conductivity of CO₂ is lower than that for the formation brine, it can be detected by measuring the self-potential. Based on this, SP can be used as a monitoring technique, which is necessary to ensure that the geological sequestration is both safe and effective. The self-potential has been extensively utilized and modelled for geothermal exploration (Ishido et al. 2010), groundwater flow (Bolève et al. 2007) and oil reservoir (Saunders et al. 2008). However, it seems that so far no work has been introduced on the numerical modeling of the self-potential associated with CO₂ flow in porous media.

Despite that CO₂ sequestration in geological formations is relatively new, an enormous amount of theoretical and experimental work has already been conducted. In spite of the versatilities of the available numerical tools, they mostly require parallel computing with tens of computer processors and days of CPU time. This makes the development of numerical tools for CO₂ sequestration difficult and rather expensive. To tackle this issue, in this work, a computationally efficient numerical model based on a mixed discretization scheme is introduced. The standard Galerkin finite element method (SG) is utilized to discretize the deformation and the diffusive dominant field equations, and the extended finite element method (XFEM), together with the level-set method (LS), is utilized to discretize the advective dominant field equations. The level-set method is employed to trace and locate the CO₂ plume front, and the extended finite element method is employed to model the associated high gradient in the saturation field front. Details of this model is given in Talebian et al. 2013.

GOVERNING FIELD EQUATIONS

We adopt the pressure-saturation formulation, namely $P_g - S_w$, because it gives a concise set of equations.

Equilibrium field equation

The equilibrium equation can be described as

$$\text{div} \left[\mathbf{D}_e (\hat{\mathbf{L}} \mathbf{u}) - \mathbf{m}^T \alpha (P_g - P_c S_w) \right] + \rho_{eff} \mathbf{g} = 0 \quad (1)$$

where $\mathbf{D}_e$ is the stiffness tensor of the solid phase, $\mathbf{u}$ is its displacement vector, $\hat{\mathbf{L}}$ is the displacement-strain operator, p_g is CO₂ pressure, p_c is the capillary pressure, S_w is water saturation, α is Biot's constant, $\mathbf{g}$ is the gravity acceleration, and the effective density is defined as

$$\rho_{eff} = (1 - \phi) \rho_s + \phi S_w \rho_w + \phi S_g \rho_g \quad (2)$$

Mass balance field equations

The mass balance field equation can be expressed as

$$d_1 \frac{\partial P_g}{\partial t} + d_2 \frac{\partial S_w}{\partial t} + d_3 \mathbf{m}^T \frac{\partial \boldsymbol{\varepsilon}}{\partial t} + \nabla \cdot (-\mathbf{c}_1 \nabla P_g - \mathbf{c}_2 \nabla S_w - \mathbf{c}_3 \nabla V + \mathbf{G}_1) = Q_w \quad (3)$$

$$d_4 \frac{\partial P_g}{\partial t} + d_5 \frac{\partial S_w}{\partial t} + d_6 \mathbf{m}^T \frac{\partial \boldsymbol{\varepsilon}}{\partial t} + \nabla \cdot (-\mathbf{c}_4 \nabla P_g + \mathbf{G}_2) = Q_g \quad (4)$$

where $\boldsymbol{\varepsilon}$ is the total strain, $\mathbf{m}^T = [1, 1, 1, 0, 0, 0]^T$, Q_w and Q_g are source terms, and $d_1 \dots d_6$, $\mathbf{c}_1 \dots \mathbf{c}_4$, and $\mathbf{G}_1 \dots \mathbf{G}_2$ are coefficients described in Table 1.

Electric current density balance field equations

The SP field equation can be expressed as

$$\nabla \cdot (-\mathbf{c}_5 \nabla P_g - \mathbf{c}_6 \nabla S_w - c_7 \nabla V + \mathbf{G}_3) = 0 \quad (5)$$

where $\mathbf{c}_5 \dots \mathbf{c}_7$ and $\mathbf{G}_3$ are coefficients described in Table 1.

Table 1. Model parameters

$\mathbf{C}$	d	$\mathbf{G}$
$\mathbf{c}_1 = \mathbf{k} \frac{k_{rw}}{\mu_w}$	$d_1 = \frac{\alpha - \phi}{K_s} S_w + \frac{\phi S_w}{K_w}$	$\mathbf{G}_1 = \mathbf{k} \frac{k_{rw}}{\mu_w} \rho_w \mathbf{g}$
$\mathbf{c}_2 = -\frac{dP_c}{dS_w} \mathbf{k} \frac{k_{rw}}{\mu_w}$	$d_2 = -\frac{\alpha - \phi}{K_s} P_c S_w + \phi - \frac{\alpha - \phi}{K_s} S_w^2 \frac{dP_c}{dS_w} - \frac{\phi S_w}{K_w} \frac{dP_c}{dS_w}$	$\mathbf{G}_2 = \mathbf{k} \frac{k_{rg}}{\mu_g} \rho_g \mathbf{g}$
$\mathbf{c}_3 = \mathbf{C} C_r \sigma_e \sigma_r$	$d_3 = \alpha S_w$	$\mathbf{G}_3 = \rho_w \mathbf{g} \mathbf{C} C_r \sigma_e \sigma_r$
$\mathbf{c}_4 = \mathbf{k} \frac{k_{rg}}{\mu_g}$	$d_4 = \frac{\alpha - \phi}{K_s} - S_w \frac{\alpha - \phi}{K_s} + \frac{\phi(1 - S_w)}{K_g}$	-
$\mathbf{c}_5 = \mathbf{C} C_r \sigma_e \sigma_r$	$d_5 = -\frac{\alpha - \phi}{K_s} P_c + \frac{\alpha - \phi}{K_s} S_w P_c - \phi - \frac{\alpha - \phi}{K_s} S_w \frac{dP_c}{dS_w} + \frac{\alpha - \phi}{K_s} S_w^2 \frac{dP_c}{dS_w}$	-
$\mathbf{c}_6 = -\mathbf{C} C_r \sigma_e \sigma_r \frac{dP_c}{dS_w}$	$d_6 = \alpha(1 - S_w)$	-
$c_7 = \sigma_e \sigma_r$	$d_7 = 0$	-

where ϕ is the porosity, K_s , K_w and K_g are the bulk modulus of the solid, water and CO₂ respectively, $\mathbf{k}$ is the intrinsic permeability tensor, μ_π and $k_{r\pi}$ are the dynamic viscosity and the relative permeability of π -phase respectively, σ_e is electric conductivity, $\mathbf{C}$ is the electrokinetic (voltage) coupling coefficient, C_r and σ_r are relative values ranging from 0 to 1.

MIXED DISCRETIZATION SCHEME

Eqs. 1-5 involves the motion of an immiscible CO₂ plume under the combined action of solid deformation and viscous, capillary and gravity forces. At the front of the plume, there is a relatively high gradient in the saturation field. Capturing and modeling this front can lead to spurious oscillations. Using standard finite element discretization schemes requires fine or adaptive meshes, and probably CPU time of the order of several days or weeks to conduct an analysis at a regional level. To tackle this problem, here, the governing equations are solved using a mixed discretization scheme. The standard Galerkin finite element method (SG) and the extended finite

element method (XFEM), coupled to the level-set method (LS), are utilized. Coupling between LS and XFEM is essential for effectively capturing and modeling the CO₂ plume front. Physically; for a typical CO₂ sequestration problem, capturing the front of the CO₂ plume at the exact location is not very important; but numerically, it is vital because it leads to a locally conservative discrete system, making the scheme stable and convergent. For the equilibrium field equation and the diffusive dominant equations, the standard Galerkin method suffices. A complete treatment of this model is given in Talebian et al. (2013).

NUMERICAL EXAMPLE

In Talebian et al. 2013, several numerical examples have been presented to examine the capability of the model to simulate CO₂ sequestration in geological formations. Here we present a numerical example addressing the electrokinetic flow.

Front tracking of electrokinetic potential

Electrokinetic measurement has proven to be an efficient technique for monitoring fluid motion in underground formations in response to pumping or injection of fluids or contaminants. It seems that no computational work on monitoring CO₂ movement in underground formations has yet been published. Hence, in order to compare, here, we simulate a numerical example obtained from petroleum engineering, particularly that of Saunders et al. (2008). They simulated water encroachment during oil production towards a well and the resulting electrokinetic potential response.

Based on Saunders et al. (2008) example, a 2D reservoir domain, consisting of six layers, is modelled, Figure 1. The reservoir is bounded by conductive, low permeability layers, representing reservoir seals. The seal layers are located between two high permeability sandstone layers. At the top of the geometry, a highly resistive weathered layer exists. At the left boundary of the reservoir, a water aquifer exists. As oil is pumped, the water in the aquifer expands and moves into the reservoir, displacing oil. The material and other simulation parameters are given in Table 2.

A large domain (2000 m x 2000 m) is simulated to allow for setting zero potentials at the boundaries, used as a reference for the electrokinetic measurements. No electrical potential flux is applied on the domain surface. The capillary pressure and the gravitational forces are neglected.

Water and oil relative permeability are calculated using Brook and Corey relationships. The electrical conductivity of the bulk formation is calculated following Glover et al. (2000), as

$$\sigma_e = \phi^m \left[\sigma_g + (S_w)^n (\sigma_w - \sigma_g) \right] \quad (6)$$

where n is Archie's saturation exponent (here 2), m is the cementation exponent (here 1.8), σ_g and σ_w are the gas and water phase conductivities, respectively.

Figure 2 shows the streaming potential versus horizontal distance, along a section in the centre of the reservoir, at different time intervals. Salinity of $C_f=0.01$ mol/L is assumed. Figure 3 shows the water saturation front at the same time intervals. Comparing Figures 2 and 3, reveals that the electric potential peaks follow the water front. Therefore, even though this example is for analysing water-oil front motion, it

can be deduced that the electrokinetic potential technique can be utilized for monitoring CO₂ motion in underground formations.

Table 2- geometry, material and physical parameters

Reservoir thickness, $w_l = 100\text{m}$	Production rate, $q = 8.1 \times 10^{-4} \text{ kg/m.s}$
Lateral dimensions of model, $L = 2000\text{m}$	Initial pressure, $P_i = 10 \text{ M Pa}$
Reservoir permeability, $k_l = 1.0 \times 10^{-13} \text{ m}^2$	Residual water saturation, $S_{wr} = 0.2$
Confining rock permeability, $k_2 = 1.0 \times 10^{-15} \text{ m}^2$	Residual oil saturation, $S_{nr} = 0.2$
Upper layer permeability, $k_3 = 3.3 \times 10^{-13} \text{ m}^2$	Pore size distribution index, $\lambda = 2$
Reservoir porosity, $\phi_l = 0.25$	Upper layer conductivity, $\sigma_{r1} = 3 \times 10^{-5} \text{ (m.}\Omega)^{-1}$
Confining rock Porosity, $\phi_2 = 0.01$	Sandstone conductivity, $\sigma_{r2} = 0.0097 \text{ (m.}\Omega)^{-1}$
Upper layer porosity, $\phi_3 = 0.3$	Confining rock conductivity $\sigma_{r3} = 0.0135 \text{ (m.}\Omega)^{-1}$
Brine viscosity, $\mu_w = 1.0 \times 10^{-3} \text{ Pa s}$	Oil conductivity, $\sigma_{nw} = 1 \times 10^{-5} \text{ (m.}\Omega)^{-1}$
Oil viscosity, $\mu_o = 1.00 \times 10^{-3} \text{ Pa s}$	Oil density, $\rho_o = 900 \text{ kg/m}^3$

Figure 1: Model geometry

Figure 2: Streaming potential at different time intervals

Figure 3: Water saturation profile at different time intervals

CONCLUSION

The numerical example has shown that the proposed mixed SG-XFEM-LS numerical scheme is effectively capable of simulating coupled phenomena and processes involved in multiphase and electrokinetic flow in unsaturated porous media.

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