High-speed 3D Particle Tracking using Tomographic Holographic Reconstruction

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ABSTRACT

This paper demonstrates ultra-high-speed tomographic digital in-line holographic velocimetry measurements of the three-component, three-dimensional velocities and trajectories of micron-sized particles in a supersonic underexpanded jet flow. In high-speed digital in-line holography the depth resolution of the measurement is severely restricted by the limited resolution of the high-speed digital recording array, which leads to significant elongation of the reconstructed particles in the depth direction. It is shown that by applying tomographic digital holography this limitation is relaxed and that accurate reconstruction of the three-dimensional particle intensities can be obtained without the depth-of-field restriction. In the first part, the paper discusses guidelines and limitation as they apply to ultra-high-speed holography at frame rates of up to 1,000,000fps and provides a detailed description and implementation of the high-speed tomographic holographic velocimetry technique. In the second part of this paper, the method is applied to an axisymmetric particle-laden underexpanded jet flow to obtain the time-resolved three-component particle velocities and to asses the measurement uncertainty.

1. INTRODUCTION

Understanding particle-fluid interaction in particle-laden high-speed flows is relevant to many applications such as particle impactors [4], needle-free-dug delivery [13] and cold-gas-dynamic spray processes [12]. In these processes, micron-sized particles (1–50µm) are typically accelerated within a gas flow to high particle velocities (300–1200 m/s) so that they become suitable for particle delivery or coating purposes. The gas flow in these processes is unsteady and turbulent and often supersonic and impinges onto a a surface. The optimal application of these high-speed multi-phase processes benefits from a detailed understanding of the complex fluid-particle interaction, which requires well-resolved and accurate experimental data in both time and space. However, such data is still difficult to obtain due to the large dimensional gap between the size of the very small particles (O(10^{-6})m) and their very large velocities (O(10^3)m/s). The need to resolve the three-dimensional displacement associated with these particles over a length scale comparable to their size requires purpose designed high-speed imaging and particle reconstruction and tracking techniques such as the one presented in this paper.

1.1 High-Speed Imaging

The quantitative understanding of the underlying flow physics associated with high-speed particle laden flows requires high-speed imaging and tracking to obtain the particle position and velocity at a series of time steps. It is therefore important to capture the particle motion in both time and space at the highest possible resolution and with good contrast. This is to fulfill the temporal and spatial Nyquist criteria, to maximise the signal-to-noise ratio (SNR) at the imaging sensor and to minimise motion blur [16]. For high-speed imaging the optimal temporal sampling or frame rate f can be estimated as

\[ f = n \cdot u / D \]  

(1)

where D and u are typical length and velocity scales and n is the required number of snapshots to fulfill the sampling criterion (i.e. \( n \geq 2 \)). For example a velocity of 100m/s corresponds to 100µm/µs, which means that if imaging at the micron-size level Mfps frame rates are required to achieve optimal temporal resolution. Additionally, the exposure time of each frame should be short enough to avoid motion blur, but at the same should be long enough to obtain an adequate signal level. Defining \( \varepsilon = \tau u / d \) as the displacement of a particle of diameter d during the exposure time \( \tau \), motion blur will be minimised if \( \varepsilon \leq 1 \), which leads to a minimum exposure or illumination time of

\[ \tau \leq d / u \]  

(2)

It must be noted that decreasing the exposure time will result in a linear decrease in signal level, which in turn must be compensated by an increased illumination level to maintain adequate SNR at short exposure times. Equation 1 and 2 are illustrated in Figure 1, which shows the maximum required frame rate and minimum exposure time for a range of particle diameters and particle velocities. As can be seen, already for relatively large particles and moderate velocities (\( d = 100\mu m, u_p = 200m/s \)), exposure times of less than 0.5µs and frame rates in excess of 200,000fps are required. The specifications of selected standard CCD, high-speed CMOS and ultra-high-speed CCD cameras and summarised in Table 1, where it can be seen that recording rates at Mfps and ns exposure times are achievable with todays technology. However, the increase in temporal resolution is always offset by a decrease in spatial resolution,
which creates serious problems when imaging micron-sized particles that are at the resolution limit of the imaging array. By using appropriate magnification optics this lack of spatial resolution can be overcome and the spatial Nyquist criterion can be fulfilled; This is that the object or particle must be sampled by at least 2 pixels on the CCD/CMOS array. This leads to a minimum optical magnification of the high-speed imaging system of

\[ M \geq 2\Delta x/d \quad (3) \]

where \( \Delta x \) is the spatial resolution of the recording array and \( M \) the magnification factor. As seen from Figure 2, the minimum particle or object size for a given high-speed imaging array decreased inversely with optical magnification and for moderate particles sizes (e.g. \( d = 100\mu m \)) and MPFs recording (i.e. Shimadzu HPV-1) magnifications \( M > 3 \) are already required. Large magnifications result in higher spatial sampling, but also result in a reduced field of view and lower SNR. Therefore the image magnification is limited by the optimal object sampling and available illumination intensity [16]. For classical imaging techniques, high magnification levels also mean strongly reduced depth of focus, which renders classical three-dimensional volumetric imaging at high magnifications levels (i.e. \( M > 1 \) impractical.

One popular mean of recording three-dimensional particle location and velocity is via digital holography. Holography is not limited by the depth-of-focus of the imaging system and readily provides three-dimensional particle position. Furthermore, by combining high-speed digital holography with three-dimensional particle tracking or cross-correlation techniques it is possible to extract the three-component particle velocity. However, the ability of in-line holograph to accurately resolve the depth location of the particles is limited by the particle size and the resolution and size of the recording media. Particularly for high-speed applications the limited sensor resolution again severely restricts the depth resolution of such measurements as will be discussed in this paper. To improve the

![Figure 1: Required frame rate (a) and exposure time (b) for high-speed imaging of micron-sized particles (\( D = 2 \)).](image)

![Figure 2: Spatial resolution limit of selected high-speed imaging arrays.](image)

**Table 1**: Specifications of selected high-resolution cameras, high-speed cameras and ultra-high-speed cameras.

<table>
<thead>
<tr>
<th>Camera</th>
<th>Chip size (px×px)</th>
<th>Resolution (μm/px)</th>
<th>Exposure time (μs)</th>
<th>Frame rate (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperx IGV-B482</td>
<td>4920×3280</td>
<td>5.5</td>
<td>4.0</td>
<td>3.1</td>
</tr>
<tr>
<td>pco.dimax</td>
<td>2016×2016</td>
<td>11.0</td>
<td>1.5</td>
<td>1,279</td>
</tr>
<tr>
<td>Photon Fastcam SA-X</td>
<td>1024×1024</td>
<td>20.0</td>
<td>1.0</td>
<td>12,500</td>
</tr>
<tr>
<td>Shimadzu HPV-1</td>
<td>312×260</td>
<td>66.3</td>
<td>0.25</td>
<td>1,000,000</td>
</tr>
</tbody>
</table>
depth-resolution of the measurement, a combination of magnified digital in-line holography [9] and tomographic holographic particle image velocimetry (TDHPIV) [15] is used. TDHPIV overcomes the holographic depth-of-field problem by combining information from two or more instantaneous holographic recordings at different viewing angles in order to reconstructed the three-dimensional particle distribution without the depth-of-field constraint. The technique is applied to an axisymmetric particle-laded underexpanded jet flow to measure the time-resolved, three-dimensional position and velocity of micron-sized particles.

2. MULTI-CAMERA DIGITAL IN-LINE HOLOGRAPHY

2.1 Depth Resolution in digital in-line Holography

The greatest impediment to the use of digital holography is the limited depth resolution normal to the hologram plane. As a consequence it is generally difficult to accurately estimate the depth location of an individual particle in digital particle holography [8]. This limited depth encoding manifests as an ellipsoid or cigar like reconstruction of what should instead be a spherical particle. The hologram normal elongation depends on the effective aperture angle and in practical cases can be several orders of magnitude larger than the true particle diameter [18]. The effective resolution or elongation of a reconstructed particle \( \tau \) in the hologram normal direction can be derived from the intensity spread around a reconstructed particle due to the diffraction through the hologram and the defocusing about the particles true location [10] and is expressed as

\[
\tau = \frac{\lambda}{\Theta^2},
\]

(4)

where \( \lambda \) is the wavelength and \( \Theta \) the aperture half angle. In in-line holography the scattered light (assuming Mie scattering) is predominantly contained in the forward scattering lobe [6], and as such can limit the effective half angular aperture of the hologram to

\[
\Theta_p = \frac{\lambda}{d},
\]

(5)

where \( d \) is the particle diameter. In digital holography the limited spatial resolution of the recording media \( \Delta x \) results in an under-sampling of the interference fringe spacing, which reduces the effective aperture angle to [8]

\[
\Theta_r = \tan^{-1}\left(\frac{\lambda}{2\Delta x}\right).
\]

(6)

In digital high-speed holography this resolution limit is of particular concern due to the very large pixel size of the digital high-speed recording array as discussed in the previous section. The influence of the limited spatial resolution can be mitigated through appropriate magnification of the hologram plane such as done in microscopic holography [14, 9]. By placing a lens or microscope objective in front of the camera, the interference fringes are magnified and the effective resolution of the hologram increased. The reduced field-of-view associated with this magnification however limits the upper bandwidth of the recorded hologram and the angular aperture half angle may become limited by the hologram size

\[
\Theta_s = \tan^{-1}\left(\frac{N\Delta x}{2MZ}\right),
\]

(7)

where \( N \) is the number of pixels across the digital recording array and \( Z \) is the distance from the object plane to the hologram plane.

In in-line holography the ability to accurately resolve the depth location of a given particle may be limited by any one of these effects depending on the particle size, the pixel resolution and size of the digital recording array as shown in Figure 3. For ultra-high-speed applications the limited sensor resolution and large sensor size severely restrict the depth resolution of such measurements. In addition, the motion of the particle during the exposure (i.e. motion blur) causes a ‘smearing’ of the recorded diffraction pattern, which also reduce the effective aperture angle and hence the depth resolution of the reconstructed hologram. Typically this effect becomes notable if \( \varepsilon \geq 1/10 \), but remains acceptable as long as \( \varepsilon \leq 1/2 \) (see [17] for more details) effectively reducing the minimum exposure time in Eq. 2 by half.

Figure 3: Depth-of-field in the limit of particle diffraction (\( \tau_p \)), sensor resolution (\( \tau_r \)) and sensor size (\( \tau_s \)); \( N = 312, \Delta x = 66.3\mu m, Z = 40, \lambda = 532nm, d = 45\mu m \).
2.2 Principle of Tomographic Holography

Tomographic digital holographic particle image velocimetry (TDHPIV) overcomes the holographic depth-of-field problem by combining information from two or more instantaneous holographic recordings of a particle seeded flow at different viewing angles as explained in detail in [2, 3, 15]. The setup of TDHPIV is similar to that of digital in-line holography. The in-line laser beam for each holographic recording is aligned normal to the hologram plane of each camera and arranged such that each beam intersects the measurement volume. By exploiting the fact that the particle elongation in each hologram extends in a different direction an approximation of the actual particle can be determined from the volume intersection of each ellipsoid as shown in Figure 4a. The mapping between the global coordinate system located in the measurement volume \( \hat{X} \) and the local coordinate system of each camera \( \hat{x}_k \) (Fig. 4b) is given in the following form:

\[
\hat{x}_k = f_k(\hat{X}),
\]

where \( f_k \) is the mapping function. The digital intensity distributions \( I_H(x, y, 0) \) produced by the interference between the object and reference wave are simultaneously recorded, such that intensity and phase information associated with each particle is simultaneously recorded in a digital hologram from each viewing angle. The complex object amplitude field \( U(\hat{x}_k) \) associated with the 3D particle distribution as seen by each hologram \( k \) is then reconstructed relative to the local hologram coordinates using the Rayleigh-Sommerfeld diffraction formula [5],

\[
U(\hat{x}_k) = \frac{1}{i\lambda} \int I_H(x, y, 0) \exp(i\hat{k}r_{01}) \cos(\hat{n}, r_{01}) \, dx \, dy,
\]

where \( \hat{k} = 2\pi/\lambda \), the wavenumber, \( r_{01} \) is the distance from a point of the sensor \( (x, y, 0) \) to a point in the reconstruction plane \( (\hat{x}_k) \) and \( \hat{n} \) is the outward unit normal vector of the diffraction surface. If the dimensions of the hologram are small compared to the distance \( r_{01} \) one can assume \( \cos(\hat{n}, r_{01}) \approx 1 \), [8]. Equation (9) is evaluated as the convolution between \( I_H(x, y, 0) \) and the diffraction kernel

\[
h(\hat{x}_k; x, y) = \frac{\exp(i\hat{k}r_{01})}{i\lambda r_{01}},
\]

such that the complex object amplitude at any point in the reconstruction is given by

\[
U(\hat{x}_k) = \int I_H(x, y, 0) h(\hat{x}_k; x, y) \, dx \, dy.
\]

This convolution is efficiently performed in Fourier space such that the complex amplitude field is evaluated as

\[
U(\hat{x}_k) = \mathcal{F}^{-1} \left[ \mathcal{F} [I_H(x, y, 0)] \mathcal{F} [h(\hat{x}_k; x, y)] \right].
\]

In order to minimize the influence of the virtual image associated with the in-line holographic reconstruction, the reconstruction is performed using an iterative filter proposed in [11]. After reconstruction, the intensity field associated with each hologram is given by \( I_k(\hat{x}_k) \), where the intensity is given by the product of the complex object amplitude field and its complex conjugate,

\[
I_k(\hat{x}_k) = U(\hat{x}_k) U^*(\hat{x}_k).
\]

Once reconstructed, the intensity field associated with each hologram is represented in terms of its location in the global coordinates by applying the mapping function \( f_k \)

\[
I_k(\hat{x}_k) = I_k(f_k(\hat{X})).
\]

In practice this mapping is performed by interpolation of the reconstructed intensity field of each hologram onto a common discretised grid in the global domain. In order to remove the background intensity associated with each reconstruction the mapped volumes

\[\text{Figure 4: (left) Schematic of intersection of elongated particle reconstructions from multiple holograms; (right) Schematic of the global and local coordinate system}\]
are thresholded such that the remaining non-zero intensities closely correspond to the true particle locations. The tomographic reconstruction of the $K$ holograms is then performed by taking the product of the $K$ mapped intensity fields as given by

$$I(\vec{X}) = \prod_{k=1}^{K} I_k(\vec{f}_k(\vec{X})), \quad (15)$$

such that only the overlapping regions of non-zero reconstructed intensities remain.

The influence of the number of holograms and their angular separation on the particle shape, intensity and reconstruction artefacts due to overlapping ellipsoids (ghost particles) has been investigated using synthetic simulations [2]. For moderate particle densities of $10^{12}$ particles per m$^3$, the results show a diminishing improvement when more than three holograms are used with optimal angles of 45 to 90 between each hologram. For the present investigation only two ultra-high-speed cameras are available and the measurements are therefore limited to relatively sparsely seeded volumes (i.e. $< 10^{12}$ particle per m$^3$).

3. EXPERIMENTAL SETUP

3.1 Supersonic Jet Facility

The particle-laden high-speed gas flow is produced via a converging nozzle of diameter $D = 2$ mm, which allows the controlled injection of micron-sized particles into the gas flow (see [7, 3] for more details). The gas flow through the nozzle and hence the particle exit velocity is adjusted by changing the nozzle pressure ratio NPR defined as the ratio between the static pressure upstream of the nozzle and the ambient pressure. For NPR > 1.9 the flow in the nozzle becomes choked and the nozzle reaches its maximum exit velocity of approximately 313 m/s. For higher NPR the flow becomes underexpanded, yet the exit velocity remains constant. The solid particles (Vestosint 1301) with a mean diameter of $d = 110 \mu$m and a specific density of $\rho = 1.06$ g/cm$^2$ are suspended in the airflow co-axially via a designated seeding system and the nozzle is operated at NPR = 2.

3.2 Tomographic Holography Setup

The optical setup for the magnified tomographic in-line digital holography system is shown in Figure 5. The illumination is provided by a 200mW continuous single mode diode laser with a wavelength of 532nm. The laser beam is passed through a set of neutral density filters (ND) and expanded and collimated by a series of spherical lenses ($L_1, L_2$) with a final diameter of 25mm. The laser beam is subsequently divided into two beams by a 50:50 beam splitter (BS) and directed via two mirrors ($M_2, M_3$) through the measurement volume and onto the high-speed recording array.

The two ultra-high-speed cameras are of the type Shimadzu HPV-1 (see Table 1) and are capable of recoding 102 consecutive frames at a frame rate of up to 1,000,000fps. The digital array has size of $312 \times 260$ pixel$^2$, spatial resolution $\Delta x = 66.3 \mu$m and a 10bit intensity dynamic range. In order to magnify the hologram plane, each camera is equipped with a 105mm focal length lens (Micro Nikkor) mounted on a extendable bellow. The cameras are located with an angular separation of 90 degrees and mounted on three-axis micrometer traverses to accurately position the field-of-view and to adjust the distance between the object and hologram plane. The magnification for each camera system is determined by imaging a calibration grid (1mm $\times$ 1mm spacing) located in the focal plane and is $M = 2.97$ and $M = 2.95$ for the first and second camera system, respectively. At this magnification the corresponding effective spatial resolution is 22.32$\mu$m and 22.48$\mu$m, which implies a minimum resolvable particle diameter of approximately 45$\mu$m.

A summary of the acquisition parameters for the current high-speed tomographic holography experiment is given in Table 2 together with the required parameters calculated by Equations (1)–(3) for a 50$\mu$m and 110$\mu$m particle.

![Figure 5: Schematic of the optical setup used for high-speed tomographic holography.](image-url)
Table 2: Required and actual acquisition parameters of the high-speed particle tracking experiment; \((u = 200\text{m/s}, D = 2\text{mm}, n = 4)\).

<table>
<thead>
<tr>
<th>(d) (µm)</th>
<th>(f) (Hz)</th>
<th>(\tau_p/d)</th>
<th>(M)</th>
<th>(\tau_p/d)</th>
<th>(\tau_d/d)</th>
<th>(\tau_s/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>&gt;400,000</td>
<td>&lt;0.125</td>
<td>&lt;0.5</td>
<td>&gt;2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>&gt;400,000</td>
<td>&lt;0.275</td>
<td>&lt;0.5</td>
<td>&gt;1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>current ((d = 110\mu m))</td>
<td>500,000</td>
<td>0.25</td>
<td>0.45</td>
<td>\approx 3.0</td>
<td>206.8</td>
<td>34.3</td>
</tr>
</tbody>
</table>

Figure 6: Holographic imaging of 110µm particles suspended in an underexpanded jet at NPR=2 recorded at 500,000fps and 250ns exposure time: (a) recorded hologram; (b) hologram after background subtraction; (c) reconstruction of the hologram at the jet centre axis. Flow is from top to bottom.

3.3 Camera Calibration

The camera mapping function required for the mapping of the reconstructed intensity volumes from the local coordinates from each camera \(k\) to the global cameras is obtained by solving the following linear transformation

\[ x_{ik} = (a_{ik,j}X_j - O_{ik}) \cdot M, \]

where \(a_{ik,j}\) is the transformation tensor for camera \(k\), \(O_{ik}\) the distance of the local coordinates to the origin of the global coordinate system and \(i,j = 1,2,3\). The individual coefficient \(a_{ik,j}\) are determined by imaging a calibrate target and using geometric relations between the local and global coordinates (see [3, 15] for a more detailed description). The accuracy of the geometric calibration is approximately 0.1% of the target grid spacing, which at the current magnification corresponds to approximately 1µm. When taking into account the uncertainty in the translation of the calibration target (±2.5µm) the overall calibration uncertainty is ±3.5µm or 0.16 pixel.

4. RESULTS

4.1 3D Particle Reconstruction

The estimated depth resolution of the reconstructed holograms is dominated by the angular aperture of the forward scattering lobe of the particles and is approximately \(\tau_p/d = 206.8\), which is one to two orders of magnitude larger than the size- and resolution-limited depth of field (see Tab. 2). An example of the recorded hologram and reconstruction is shown in Figure 6. Prior to the holographic reconstruction the recorded holograms are pre-processed using a background subtraction and bicubic interpolation to a common image resolution of 22µm/pixel. The objects intensity distribution is reconstructed in the depth direction with a resolution of 20µm over a domain of 4 nozzle diameter (8mm), symmetrically around the jet centre axis. Subsequently, the reconstructed intensity distributions are mapped to the global coordinate system using the mapping function and volume intersection weighted interpolation [1].

The volumes are then thresholded to remove the background intensity and multiplied to obtain the final three-dimensional intensity distribution. An example of the reconstructed, mapped and multiplied intensity fields at varying thresholds levels is shown in Figure 6 for a sub-volume of \(2 \times 1 \times 2\text{mm}^3\). As can be seen the reconstruction noise is progressively suppressed as the threshold level is increases. In the present example, four individual particles are reconstructed in the centre of the sub-volume. For the current configuration with only two cameras the particles become slightly elongated in the \(x - z\)-plane due to the limited number of cameras and their arrangement. As a consequence, and the particle volume is over-estimated by approximately 27% (see for more details [15]).

4.2 3D Particle Tracking

The complete reconstruction of the supersonic under expanded particle-laded jet at NPR=2 is shown in Figure 8a. A total of four particles are reconstructed at successive time steps recorded at 500,000fps with an exposure time of 250ns over a period of 200µs (i.e. 102 frames). The reconstructed particles are represented by a constant threshold corresponding to 2500 counts (12bit dynamic range) and colour-coded by the respective time step.

The particle velocity is determined via cross-correlation of the reconstructed particle intensitis at successive time-steps and the particle trajectory is formed using a nearest neighbour tracking scheme. The axial particle velocity along its trajectory is plotted in Figure 7b.
for particle 2 and 4 identified in Figure 8a. The particles exit the jet nozzle with a velocity of approximately $100 - 110\text{m/s}$, which is significantly lower than the gas velocity of $313\text{m/s}$. Due to the relatively large particle size their response time is large ($> 1\text{ms}$) and the particles are not able to accelerate to the gas velocity before leaving the nozzle. After exiting the nozzle the particles begin to decelerate until an axial distance of $1.5D$ before they start accelerating again. The transversal particle velocities are plotted in Figure ??b and are one to two order of magnitude lower than the axial velocity. In the $x-z-$ plane the particle show some oscillating behaviour, which is potentially caused by a tumbling motion of the non-spherical particles or fluctuations due to the unsteady and turbulent nature of the velocity field.

4.3 Measurement Uncertainty

It can be shown from geometric considerations that the error in the centroid location of the reconstructed particles $\varepsilon_c$ is proportional to the calibration error $\delta_k$ in the local $x_1$ and $x_2$ direction and is given as

$$\varepsilon_c = \sqrt{2}\delta_k$$

(17)

where it is assumed that $\delta_{1k} = \delta_{2k} = \delta_k$ for both cameras. The calibration error in the $\delta_k$ can be neglected as it is typically much smaller than the depth-of-field. Recalling that $\delta_k = 0.16\text{pixel}$, the particle centroid error becomes approximately $0.23\text{pix} (5.2\text{µm})$ or $0.05$ particle diameter. In comparison, alternative methods of determining the centroid of the elongated particle including ellipse or complex amplitude fitting of the reconstructed in-line holograms have an accuracy of approximately $1 - 2$ particle diameter (see [8] for more details). The error in the measured particle velocity is proportional to twice the particle centroid error divided by the time separation (note, the cross-correlation error of $\approx 0.05 - 0.1$ pixel is neglected).

$$\varepsilon_u = \frac{2\varepsilon_c}{\Delta t} = \frac{4}{\sqrt{2}} \frac{\delta_k}{\Delta t}$$

(18)

For a frame rate of $500,000\text{fps}$ this error becomes $\varepsilon_u \approx 5\text{m/s}$ or approximately $4.5\%$ of the particle velocity at the nozzle exit (see error bars in Fig. 8)b.
Figure 8: (a) Reconstruction of micron-sized particles in a supersonic underexpanded jet (NPR = 2) recorded at 500,000fps. Particle trajectories are colour-coded by time after the start of the recording; (b) Measured particle velocities along the trajectory $|x|/D$: (top) streamwise velocity; (bottom) traversal velocities. Errors bars are ±5 m/s.

Since the depth position of the particles is now known the individual in-line holograms can be reconstructed at the respective depth location and the nearest neighbour tracking scheme can be repeated on a 2D frame without the inaccuracies introduced by the tomographic reconstruction. This provides an alternative measure of the axial particle velocity, but does not allow for the recovery of the transversal velocities. The axial particle velocity along the 2D projection of the particle trajectory is also plotted in Figure 8b. Both, the 3D and 2D projection measurement agree reasonably well with the difference between the two approaches being within the estimated error range.

5. CONCLUDING REMARKS

This paper has presented three-dimensional ultra-high-speed particle tracking of micron-sized particles in a supersonic under expanded gas flow. The necessary requirements and limitations to perform high-speed imaging at up to 1,000,000fps were discussed and guidelines for the appropriate choice of frame rate, exposure time and spatial resolution were derived. While modern digital high-speed recording arrays are now capable of recording at Mfps, it is the limited spatial resolution and field of view of these arrays that are the creates impediment for these measurements. The limited spatial resolution can be overcome by using appropriate magnification optics, however large magnification severely restrict the depth-of-focus rendering conventional imaging at high magnification nearly impractical.

Therefore we have introduced a magnified digital tomographic holography system to measure three-dimensional particle location and particle velocity. By combining high-speed holograms simultaneously recorded from multiple direction, it is possible to reconstruct the three-dimensional particle intensity distribution without the limitation in depth resolution typically encountered in digital in-line holography. Experimental demonstration of this approach was presented by measuring 110µm particles suspended in a supersonic jet at a frame rate of 500,000fps and 250ns exposure time. The three-dimensional intensity distribution associated with the particles was reconstructed at 102 consecutive time steps (204µs) and the particle velocities and trajectories were determined via a nearest neighbour tracking scheme. The uncertainty associate the with three-dimensional reconstruction of the particle position is dominated by the accuracy of the camera calibration and amounts to approximately 5% of the particle diameter. This precision is one order of magnitude better than conventional methods of determining the particle centroid via ellipse or complex amplitude fitting, which usually have a precision of 1 – 2 particle diameters. The resulting precision of the particle velocity measurement is approximately 5m/s or 4.5% of the particle velocity.

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REFERENCES


