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Lateral and Directional Control and the Measurement
of Aerodynamic Coefficients in Steady Asymmetric
Flight and in Flight on Asymmetric Power

by

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Abstract.

In this paper desirable control characteristics in steady asymmetric flight are discussed. These characteristics are control position stability and control force stability in steady turns on ailerons and rudder only and sideslip stability as to aileron and rudder control positions and control forces in steady, straight sideslips. Applying these concepts to steady asymmetric flight conditions, control position and control force stability parameters have been introduced for steady turns on ailerons and rudder only, as well as sideslip stability parameters for steady, straight sideslips. Flight tests have been presented to determine these control stability parameters.

Furthermore, flight tests have been described to measure aileron and rudder effectiveness coefficients and a method has been given to calculate from the results of tests in steady flight the stability derivatives with respect to sideslip and rate of yaw. The reduction of data has been illustrated with an example.

In an appendix the relation between control position stability and dynamic stability has been shown.

Attention has also been given to the control characteristics both in steady flight on asymmetric power and during the recovery upon failure of an engine. The basic considerations governing these flight conditions have been briefly dealt with. Tests on the control in flight on asymmetric power based on these considerations have been presented.

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1. Introduction.

In the study of flying qualities first consideration is usually given to the behaviour of the aircraft in different steady flight conditions.

The control characteristics in steady symmetric flight have been studied already very early by Glauert and others. In the Netherlands a report on this subject has been published in 1929 (Ref. 1), in which a theory as well as flight test methods for the determination of the control characteristics have been presented.

Flight tests on the control in steady asymmetric flight have been carried out since 1927. A report on this subject appeared in 1940 (Ref. 2). Since then some improvements have been made on the flight test methods.

This paper presents a review of the theory of control in steady asymmetric flight as well as a description of the relevant flight test methods.

Concerning the theory and flight test methods on the control characteristics in flight on asymmetric power, not only steady flight has to be considered, but also the recovery of control after a sudden engine failure. Both aspects of the problem have been treated in this paper.

2. Symbols.

a	cable length, viz. Fig. 16.
a	coefficient of denominator of frequency response function.
b	wing span.
b	coefficient of numerator of frequency response function.
$\bar{c}$	mean aerodynamic chord
j	$\sqrt{-1}$
ℓ	arm of parachute drag with respect to c.g. of aircraft, viz. Fig. 16.
ℓ	distance from the thrust line to the plane of symmetry of the aircraft.
ℓ_v	distance from c.g. of aircraft to c.p. of vertical tail.
m	mass
m	positive integer.
n	positive integer
n	engine r.p.m.
p	rolling angular velocity.
q	dynamic pressure.
r	yawing angular velocity.
t	time.
$x_{c.g.}$	x-coordinate of aircraft c.g.
A	} coefficients of characteristics equation.
B	
C	
D	
E	
F_a	aileron control force.
F_r	rudder control force.
N	yawing moment.
P	measured force, viz. Fig. 16.
S	wing area.
T	drag of parachute, viz. Fig. 16.
ΔT	change in thrust due to engine failure.
V	airspeed.
V_c	calibrated airspeed
V_{mc}	minimum control speed
	index g: on or near the ground
	index a: in free air.

C_L	lift coefficient
C_Y	side force coefficient
C_{ℓ}	rolling moment coefficient
C_{ℓ_e}	rolling moment coefficient due to engine failure
C_n	yawing moment coefficient
C_{n_e}	yawing moment coefficient due to engine failure
β	angle of sideslip
δ_a	total aileron deflection
δ_r	rudder deflection
δ_{t_r}	rudder trim tab deflection
θ_i	input signal of dynamic system
θ_o	output signal of dynamic system
λ	root of characteristic equation
μ_b	relative density of aircraft ($\frac{m}{\rho S b}$)
ρ	density
ϕ	angle of bank
ψ	aircraft heading
ω	angular frequency

β , δ_a and δ_r used as subscripts indicate partial differentiation, r and p used as a subscript indicate partial differentiation with respect to $\frac{rb}{2V}$ and $\frac{pb}{2V}$.

The stability axis system has been used, the origin of which coincides with the centre of gravity of the aircraft. The positive X-axis points forward in the direction of the undisturbed flow in the plane of symmetry. The positive Y-axis points to the right, perpendicular to the plane of symmetry. The positive Z-axis points downwards in the plane of symmetry, perpendicular to the X-axis, viz. Fig. 1a.

3. Desirable control characteristics in steady flight.

3.1. Introduction.

The control characteristics of an aircraft with its many degrees of freedom present a complicated problem, as they are governed by a large number of parameters, some of which are interconnected in an intricate way. The study of this problem has been simplified a great deal by first considering stability and control in steady flight.

The relations between the parameters governing these flight conditions and the behaviour of the aircraft upon small deviations from steady flight have been studied and correlated with the flying characteristics, considered desirable from the pilot's point of view. This approach has been combined with the study of the static stability characteristics of an aircraft. Reports on this subject have been published amongst others by Glauert as early as 1918.

The control characteristics of an aircraft can roughly be divided into two groups; the first of which covers those characteristics bearing on the effectiveness of the controls and the control power, whereas the second one relates to the control positions, and control forces and the subsequent motion of the aircraft. In the following the second group of control characteristics will be analysed.

In order to keep the pilot's task as simple as possible and to increase the safety of flight, there should be logical and straightforward relations between the control positions, control forces and the motion of the aircraft.

From the pilot's point of view, the desirable control characteristics of an aircraft in steady flight are primarily its "stability" as regards control positions and control forces. This control position and control force stability has to be considered for the principal controls (longitudinal, lateral and directional). It describes the behaviour of the aircraft upon transition from one steady flight condition I to another neighbouring steady flight condition II, when one of the controls is deflected, whilst the others are held fixed.

When considering certain steady flight conditions, an aircraft can be termed to be stable in condition I as regards the position of one of its controls, if the position of this control in the neighbouring steady flight condition II, when viewed from the position it had in the steady condition I, has changed in the same

direction as that of the initial control movement, necessary for achieving the transition from condition I to condition II, the other controls being held fixed. In Fig. 2 the relevant control positions have been indicated.

Likewise an aircraft can be termed to be stable as regards the control forces of one of its controls in the steady flight condition I, if the increase in stick force in the neighbouring steady flight condition II compared with the force in flight condition I, has the same direction as the initial change in stick force, necessary for achieving the transition from condition I to condition II, the other controls being held fixed.

The use of these definitions can be illustrated, for instance, by considering the longitudinal control positions. When, starting from a condition of steady flight (I) an initial forward movement of the control column is made, whilst all other controls are held fixed, the speed of the aircraft increases. Then a new steady flight condition (II) is obtained, in which the airspeed is slightly higher than in condition I. The aircraft is stable as to the elevator control position, when the control position required to maintain condition II is also forward relative to the position in condition I.

Although it is desirable that the aircraft possesses both control position and control force stability, cases occur in which an aircraft in a certain steady condition of flight is stable as regards the control position whilst being unstable as to the control force of one of its controls and vice versa.

3.2. Asymmetric flight.

The definitions of control position and control force stability will now be applied to asymmetric flight.

The control characteristics in steady asymmetric flight are best indicated, if only one of the two controls - ailerons or rudder - is displaced, whilst the other is held fixed. Generally the aircraft then describes a steady slipping or skidding turn. This motion is the most elementary steady flight condition of the asymmetric flight.

When applying the notion of control position stability to asymmetric flight, both control position stability as to the ailerons and control position stability as to the rudder have to be considered. This holds also true for the control force stability.

3.3. Aileron control position stability.

It will be clear that, in a transition from a certain steady flight condition to another flight condition with a different angle of bank, the initial aileron control deflection has to be in the direction of the required change of the angle of bank.

Now the case will be considered in which, starting from steady symmetric flight, an initial aileron control deflection is applied, rudder control being held fixed. If the aileron control deflection is to the right, the aircraft thereupon banks to the right. According to the definition the aircraft is control position stable as to the ailerons, if an aileron control deflection to the right is required to maintain steady flight with right bank. It will be shown later on, that the aircraft then yaws to the right and in normal cases ($C_{n\beta} > 0$) has a positive angle of sideslip. (For positive directions of angular displacements and velocities, viz. Fig. 1a, b, c, d).

From the foregoing it follows, that aileron control position stability can be thus defined:

an aircraft is control position stable as to the ailerons, if after a transition - by means of the ailerons only - from a steady flight condition I to a neighbouring steady flight condition II in which the aircraft has a larger angle of bank (in a positive sense), the aileron control in the steady condition II has been deflected to the right relative to the position it had in condition I.

As an aileron control deflection to the right causes a negative aileron deflection (right aileron up) and an angle of bank to the right is called positive, the mathematical formulation of aileron control position stability is:

$$\frac{d\delta_a}{d\phi} < 0$$

viz. Fig. 3

3.4. Rudder control position stability.

As the aircraft itself possesses no neutral heading which can be used as a reference, the angle of yaw is unsuitable as a yardstick of the change in the aircraft's motion caused by the control deflection, as is the angle of bank. Resort has to be taken, therefore, to the rate of yaw.

In order to initiate a transition from one steady flight

condition to another flight condition with a larger rate of yaw (in a positive sense), the right rudder pedal has to be moved forward.

Starting from a steady, symmetric flight, the aircraft will initially turn to the right, if the right rudder pedal is moved forward, whilst the aileron control is held fixed. According to the definition of control position stability, the aircraft is control position stable as to the rudder, if the rudder pedal has to be kept forward of the original position in order to maintain a steady (slipping) turn to the right.

Rudder control position stability can, therefore, be thus defined:

an aircraft is control position stable as to the rudder, is after a transition - by means of the rudder only - from a steady flight condition I to a neighbouring steady flight condition II in which the aircraft has a larger rate of yaw (in a positive sense), the right rudder pedal in the steady condition II has been moved forward relative to the position in condition I.

As a forward movement of the right rudder pedal causes a negative rudder deflection and a rate of yaw to the right is called positive, the mathematical formulation of rudder control position stability is:

$$\frac{d\delta_r}{d \frac{rb}{2V}} < 0$$

viz. Fig. 4

These differential quotients, expressing the change of either the aileron or the rudder control surface deflection with the change of angle of bank or dimensionless rate of yaw, will be called control position stability parameters.

Aileron and rudder control force stability can be defined in a similar way as the control position stability.

3.5. Aileron control force stability.

An aircraft is control force stable as to the ailerons if, after a transition - by means of the ailerons only - from a steady flight condition I to a neighbouring flight condition II in which the aircraft has a larger angle of bank (in a positive sense), the change in control force in steady condition II relative to the one

in condition I is directed to the right.

The mathematical formulation is:

$$\frac{dF_a}{d\phi} < 0$$

viz. Fig. 5

an aileron control force to the right being negative and an angle of bank to the right being positive.

3.6. Rudder control force stability.

An aircraft is control force stable as to the rudder if, after a transition - by means of the rudder only - from a steady flight condition I to a neighbouring steady flight condition II in which the aircraft has a larger rate of yaw (in a positive sense), the force on the right rudder pedal has been increased.

In formula:

$$\frac{dF_r}{d\frac{r_b}{2V}} < 0$$

viz. Fig. 6

a pedal force on the right rudder pedal force being negative and a rate of yaw to the right being positive.

3.7. Sideslip stability.

Another desirable control characteristic will be discussed now, pertaining to the control positions and control forces in steady, straight sideslips.

When a straight sideslip is initiated from steady symmetric flight, whilst the aircraft maintains a constant heading, the aileron control has to be deflected to the right and - neglecting the adverse aileron yawing moment - the rudder has to be deflected to the left (left rudder pedal moved forward). The aircraft then banks and sideslips to the right (positive angle of bank and angle of sideslip).

It is desirable that, in order to maintain a steady, straight sideslip to the right, the aileron control remains deflected to the right and the rudder remains deflected to the left. If this is the case, the aircraft will be called sideslip stable.

In as far as this characteristic pertains to the aileron control deflection, it will be referred to as sideslip stability as to aileron control position, and in as far as it pertains to the rudder control deflection, it will be referred to as sideslip

stability as to rudder control position.

From the above, the following definition of sideslip stability can be inferred:

an aircraft possesses sideslip stability as to the control positions if, after the transition from a steady, straight sideslip to a neighbouring steady, straight sideslip with a larger angle of sideslip (in a positive sense), or a larger angle of bank, the aileron control has been deflected to the right and the left rudder pedal has been moved forward, relative to the positions in the original flight condition.

Mathematically, sideslip stability can be thus expressed:

sideslip stability as to aileron control position:

$$\frac{d\delta_a}{d\beta} < 0 \quad \text{viz. Fig. 7}$$

sideslip stability as to rudder control position:

$$\frac{d\delta_r}{d\beta} > 0 \quad \text{viz. Fig. 8}$$

These differential quotients $\frac{d\delta_a}{d\beta}$ and $\frac{d\delta_r}{d\beta}$ will be called the sideslip stability parameters.

Similar desirable control characteristics exist as regards the control forces in steady, straight sideslips. The definition of sideslip stability as to the control forces can, therefore, be formulated as follows:

an aircraft is sideslip stable as to the control forces, if, after the transition from a steady, straight sideslip to a neighbouring steady, straight sideslip with a larger angle of sideslip (in a positive sense), or a larger angle of bank, the change in aileron control force is directed to the right and the change in control force on the left rudder pedal is directed forward.

The relevant mathematical expressions are:

sideslip stability as to aileron control force:

$$\frac{dF_a}{d\beta} < 0 \quad \text{viz. Fig. 9}$$

sideslip stability as to rudder control force:

$$\frac{dF_r}{d\beta} > 0 \quad \text{viz. Fig. 10}$$

For positive directions of control deflections and control forces, viz. Fig. 1d.

3.8. Rudder control at large angles of sideslip; rudder lock.

In connection with the desirable characteristic of sideslip stability as to rudder control force mention must be made of an instability which can occur at large angles of sideslip. Due to the airflow over the fin and rudder breaking away ("fin stall") the rudder floating angle may increase considerably, thereby reducing the rudder control force, sometimes even reversing sign and attaining a large value in the opposite sense. The relation between the rudder control force and the sideslip angle then has the character, illustrated in Fig. 11. In some cases the rudder control force can after reversal - increase beyond the maximum value which can be exerted by the pilot, in which case the aircraft gets out of control and eventually must be abandoned. It is clear that this phenomenon of "rudder lock" is, therefore, an unacceptable characteristic which should be prevented by either increasing the sideslip angle at which fin stall occurs or by reducing the maximum sideslip angle obtainable in steady flight.

3.9. Steady coordinated turns.

In asymmetric flight, besides the turns on one control and straight sideslips, several other categories of steady flight conditions exist, e.g. coordinated turns and flat turns. Of these, coordinated turns are the most important.

In analogy with the above, there could also be indicated a desirable direction of the change in control positions and control forces after the transition from one steady asymmetric flight condition into another in which the angle of sideslip remains equal to zero, i.e. in coordinated turns.

This would lead to the point, that after the transition from one steady coordinated turn to another with a larger angle of bank and a larger rate of yaw (in a positive sense), the aileron control has to be deflected to the right and the right rudder pedal has to be moved forward relative to the positions they had in the original steady flight condition.

With a normal aircraft (negative C_{n_r} and positive C_{l_r}) this condition will always be satisfied as regards the rudder control, whereas it never will be as regards the ailerons. In steady

coordinated turns the aileron control has, after the initial displacement, to be reversed beyond the original position. This constitutes an objectionable control characteristic as becomes apparent especially in the initial stages of pilot training.

This characteristic is inherent to the configuration of conventional aircraft as will be shown later on. It is caused primarily by the additional requirement of zero sideslip in coordinated turns. This makes these flight conditions into more complicated ones, in which it is hard to expect all control requirements to be satisfied.

The latter remark could be also applied to flat turns.

4. Relations between control position and sideslip stability and aerodynamic coefficients of the aircraft.

4.1. Introduction.

In the preceding Section 3 the desirable sign of the changes of control positions and control forces with the changes of the components of the aircraft's motion in a series of steady asymmetric flight conditions have been shown.

The relations between the control surface deflections and the components of the aircraft's motion in steady flight - i.e. the control position stability and sideslip stability parameters as defined in Section 3 - will now be expressed as functions of the aerodynamic coefficients of the aircraft. Thereby, the way in which the control characteristics, as seen from the pilot's point of view, are linked up with the aerodynamic characteristics of the aircraft, will be demonstrated.

In Section 6 the results thus obtained will be used to deduce various aerodynamic coefficients from the results of measurements of control positions and the aircraft's motion in flight.

4.2. Equations of equilibrium.

Using the stability axis system, the non-dimensional equations of equilibrium for an arbitrary horizontal or nearly horizontal steady, asymmetric flight condition can be written as follows:

$$C_{L_{\beta}} \cdot \text{tg } \phi + C_{Y_{\beta}} \cdot \beta - 4 \mu_b \cdot \frac{rb}{2V} + C_{Y_{\delta_r}} \cdot \delta_r = 0 \quad (1)$$

$$C_{l_{\beta}} \cdot \beta + C_{l_r} \cdot \frac{rb}{2V} + C_{l_{\delta_a}} \cdot \delta_a = 0 \quad (2)$$

$$C_{n_{\beta}} \cdot \beta + C_{n_r} \cdot \frac{rb}{2V} + C_{n_{\delta_a}} \cdot \delta_a + C_{n_{\delta_r}} \cdot \delta_r = 0 \quad (3)$$

Less important aerodynamic effects and coefficients have been omitted, and in this Section 4 the - usually small - aileron yawing moment $C_{n_{\delta_a}} \cdot \delta_a$ will be neglected. For small deviations

from symmetric flight, $\text{tg } \phi$ will be replaced by ϕ (measured in radians).

4.3. Steady turns on ailerons only.

In order to analyse the control position stability in asymmetric flight, the steady motion which the aircraft describes when only the ailerons have been deflected will be considered. To this end, the variation of the dimensionless rate of yaw and the angle of sideslip with the angle of bank will be deduced. Also the variation of the aileron angle with angle of bank, expressed in the differential quotient $\frac{d\delta_a}{d\phi}$ and already discussed in Section 3 will be calculated.

The first quantity to be deduced from the equations of equilibrium is the quotient $\frac{d \frac{rb}{2V}}{d\phi}$. Substituting $\delta_r = 0$ in Eqs. (1), (2) and (3) yields:

$$\frac{d \frac{rb}{2V}}{d\phi} = \frac{C_L}{4 \mu_b + \frac{C_{Y\beta} \cdot C_{n_r}}{C_{n_\beta}}} \quad (4)$$

In all practical cases $\frac{C_{Y\beta} \cdot C_{n_r}}{C_{n_\beta}}$ is negligible with respect to $4 \mu_b$. In the example discussed later on in this paper for instance, $\frac{C_{Y\beta} \cdot C_{n_r}}{C_{n_\beta}}$ amounts to 0,68, whereas $4 \mu_b$ at sea level is equal to 27,0.

This means, that for positive values of the lift coefficient:

$$\frac{d \frac{rb}{2V}}{d\phi} > 0$$

The general conclusion is, therefore, that in steady turns on ailerons only with a positive angle of bank (right wing down), the aircraft yaws to the right.

Now the quantity $\frac{d\beta}{d\phi}$ will be considered. From the Eqs. (1), (2) and (3) it follows:

$$\frac{d\beta}{d\phi} = - \frac{C_{n_r}}{C_{n_\beta}} \cdot \frac{C_L}{4 \mu_b + \frac{C_{Y\beta} \cdot C_{n_r}}{C_{n_\beta}}} \quad (5)$$

It can be seen, that for normal configurations where

$4 \mu_b \gg \frac{C_{Y\beta} \cdot C_{n_r}}{C_{n\beta}}$ and $C_{n_r} < 0$, and for positive values of the lift coefficient the sign of the angle of sideslip in a turn to the right depends on the static directional stability $C_{n\beta}$. With a positive value of $C_{n\beta}$ - as is normal - the aircraft will have a positive angle of sideslip in a turn on ailerons only to the right.

Finally the aileron control position stability parameter $\frac{d\delta_a}{d\phi}$ will be calculated. The Eqs. (1), (2) and (3) yield:

$$\frac{d\delta_a}{d\phi} = \frac{1}{C_{\ell_{\delta_a}}} \cdot \frac{C_{\ell_{\beta}} \cdot C_{n_r} - C_{\ell_r} \cdot C_{n\beta}}{C_{n\beta}} \cdot \frac{C_L}{4 \mu_b + \frac{C_{Y\beta} \cdot C_{n_r}}{C_{n\beta}}} \quad (6)$$

For normal configurations, $C_{\ell_{\delta_a}}$ is negative and $4 \mu_b \gg \frac{C_{Y\beta} \cdot C_{n_r}}{C_{n\beta}}$,

so the sign of $\frac{d\delta_a}{d\phi}$ depends on the signs of the static directional stability $C_{n\beta}$ and the quantity $C_{\ell_{\beta}} \cdot C_{n_r} - C_{\ell_r} \cdot C_{n\beta}$. The latter

combination of stability derivatives also determines whether or not the aircraft is spirally stable (coefficient E of the dynamic stability quartic, viz. Appendix 1).

It has been shown in Section 3, that in order to obtain satisfactory control characteristics, the condition

$$\frac{d\delta_a}{d\phi} < 0$$

has to be satisfied. From Eq. (6) it is evident, that this will be the case if both $C_{n\beta}$ and $C_{\ell_{\beta}} \cdot C_{n_r} - C_{\ell_r} \cdot C_{n\beta}$ have the same sign.

For normal aircraft $C_{n\beta}$ is positive and if the aircraft is spirally stable, $C_{\ell_{\beta}} \cdot C_{n_r} - C_{\ell_r} \cdot C_{n\beta}$ is positive as well.

4.4. Steady turns on rudder only.

Before calculating the control position stability parameter $\frac{d\delta_r}{d\frac{rb}{2V}}$, the character of the steady turns on rudder only will be investigated, by studying the variation of angle of bank and sideslip angle with rate of yaw.

If the - usually small-- rudder side force $C_{Y\beta} \cdot \delta_r$ is neglected in Eq. (1), the variation of angle of bank with rate of yaw is expressed by:

$$\frac{d\phi}{d\frac{rb}{2V}} = \frac{4\mu_b + \frac{C_{Y\beta} \cdot C_{\ell_r}}{C_{\ell\beta}}}{C_L} \quad (7)$$

as follows from the Eqs. (1), (2) and (3) by equating δ_a to zero.

In all normal cases the quantity

$\frac{C_{Y\beta} \cdot C_{\ell_r}}{C_{\ell\beta}}$, being of the same order of magnitude as $\frac{C_{Y\beta} \cdot C_{n_r}}{C_{n\beta}}$, is small

if compared with $4\mu_b$. At positive values of the lift coefficient, therefore:

$$\frac{d\phi}{d\frac{rb}{2V}} > 0$$

The general conclusion can be, that in a turn on rudder only to the right (positive rate of yaw) the aircraft also banks to the right.

The relation between angle of sideslip and rate of yaw follows at once from Eq. (2):

$$\frac{d\beta}{d\frac{rb}{2V}} = - \frac{C_{\ell_r}}{C_{\ell\beta}} \quad (8)$$

As the normal sign of C_{ℓ_r} is positive, the aircraft will sideslip to the right in a right hand turn on rudder only, if it has positive effective dihedral ($C_{\ell\beta} < 0$).

Solving the Eqs. (1), (2) and (3) for the rudder control position stability parameter $\frac{d\delta_r}{d\frac{rb}{2V}}$ as introduced in Section 3, yields:

$$\frac{d\delta_r}{d\frac{rb}{2V}} = \frac{-1}{C_{n\delta_r}} \cdot \frac{C_{l\beta} \cdot C_{n_r} - C_{l_r} \cdot C_{n\beta}}{C_{l\beta}} \quad (9)$$

For aircraft of normal configuration, $C_{n\delta_r}$ is negative.

The sign of $\frac{d\delta_r}{d\frac{rb}{2V}}$ depends, therefore, on the signs of $C_{l\beta}$ and the quantity $C_{l\beta} \cdot C_{n_r} - C_{l_r} \cdot C_{n\beta}$ which has already been encountered in the discussion of the turns on ailerons only. The desirable sign of $\frac{d\delta_r}{d\frac{rb}{2V}}$ is:

$$\frac{d\delta_r}{d\frac{rb}{2V}} < 0$$

as has been shown in Section 3. It can be seen from Eq. (9), that this condition will be satisfied, if $C_{l\beta}$ and $C_{l\beta} \cdot C_{n_r} - C_{l_r} \cdot C_{n\beta}$ have opposite signs. Normally $C_{l\beta}$ is negative, and if the aircraft is spirally stable, $C_{l\beta} \cdot C_{n_r} - C_{l_r} \cdot C_{n\beta}$ is positive.

4.5. Steady, straight sideslips.

In steady, straight sideslips both controls have been deflected in order to maintain rectilinear flight.

The sideslip stability parameters $\frac{d\delta_a}{d\beta}$ and $\frac{d\delta_r}{d\beta}$ can be deduced from the equations of equilibrium Eqs. (2) and (3) by substituting: $\frac{rb}{2V} = 0$.

The results are:

aileron sideslip stability parameter:

$$\frac{d\delta_a}{d\beta} = - \frac{C_{l\beta}}{C_{l\delta_a}} \quad (10)$$

rudder sideslip stability parameter:

$$\frac{d\delta_r}{d\beta} = - \frac{C_{n\beta}}{C_{n\delta_r}} \quad (11)$$

As the control effectiveness coefficients $C_{\ell_{\delta_a}}$ and $C_{n_{\delta_r}}$ are negative in all normal cases, the signs of the parameters $\frac{d\delta_a}{d\beta}$ and $\frac{d\delta_r}{d\beta}$ only depend on the sign of $C_{\ell_{\beta}}$ and $C_{n_{\beta}}$ respectively.

It has been shown in Section 3, that the desirable signs of the sideslip stability parameters are:

$$\frac{d\delta_a}{d\beta} < 0$$

and

$$\frac{d\delta_r}{d\beta} > 0$$

From Eqs. (10) and (11) it now follows, that these conditions will be met, if $C_{\ell_{\beta}} < 0$ and $C_{n_{\beta}} > 0$. The aircraft then possesses positive effective dihedral and positive static directional stability.

4.6. Steady coordinated turns.

In Section 3 it has been mentioned, that in steady coordinated turns, after the initial deflection, necessary to bank the aircraft, the aileron control has to be reversed beyond the original position. This means, that in a steady turn to the right, the aileron control has been deflected to the left, or:

$$\frac{d\delta_a}{d \frac{rb}{2V}} > 0$$

By substituting $\beta = 0$ in Eq. (2), the following expression can be obtained:

$$\frac{d\delta_a}{d \frac{rb}{2V}} = - \frac{C_{\ell_r}}{C_{\ell_{\delta_a}}} \quad (12)$$

From this equation it will be clear, that $\frac{d\delta_a}{d \frac{rb}{2V}}$ always will be positive, because in normal cases $C_{\ell_{\delta_a}}$ is negative and C_{ℓ_r} is positive. The objectionable aileron control characteristic in steady coordinated turns is, therefore, unavoidable.

4.7. Conclusions.

From the equations derived in the above, representing the relations between the control position and sideslip stability parameters and the aerodynamic coefficients of the aircraft, some important conclusions can be drawn:

1) From Eq. (10) it follows:

the aircraft possesses positive effective dihedral ($C_{\ell_\beta} < 0$), if it is sideslip stable as to the aileron control position ($\frac{d\delta_a}{d\beta} < 0$).

2) From Eq. (11) it follows:

the aircraft possesses positive static directional stability ($C_{n_\beta} > 0$), if it is sideslip stable as to the rudder control position ($\frac{d\delta_r}{d\beta} > 0$).

3) From Eq. (6) it follows:

the aircraft is spirally stable with controls fixed ($C_{\ell_\beta} \cdot C_{n_r} - C_{\ell_r} \cdot C_{n_\beta} > 0$), if it is:

- a) control position stable in turns on ailerons only ($\frac{d\delta_a}{d\phi} < 0$) and
- b) sideslip stable as to the rudder control position ($\frac{d\delta_r}{d\beta} > 0$ and $C_{n_\beta} > 0$, viz. conclusion 2).

4) From Eq. (9) it follows:

the aircraft is spirally stable with controls fixed, if it is:

- a) control position stable in turns on rudder only ($\frac{d\delta_r}{d\frac{r_b}{2V}} < 0$) and
- b) sideslip stable as to the aileron control position ($\frac{d\delta_a}{d\beta} < 0$ and $C_{\ell_\beta} < 0$, viz. conclusion 1).

In Appendix 1 the relationship between the control characteristics in steady asymmetric flight and the dynamic lateral stability has been further developed.

5. Flight tests for determination of the control position and sideslip stability parameters and control effectiveness coefficients.

5.1. Introduction.

The values of the control position, control force, and sideslip stability parameters, discussed in the foregoing Sections 3 and 4, together with the stability derivatives of an aircraft can be determined from flight test measurements. The results of these flight tests will be presented as trim curves, which are defined as curves representing control surface deflections and control forces as functions of the components of the aircraft's motion in a series of steady flight conditions. The slopes of these trim curves at the points representing symmetric flight are equal to the values of the relevant stability parameters.

In Section 6 use will be made of the measured control position and sideslip stability parameters, in order to calculate the values of the stability derivatives. It will be shown that for this purpose the values of the control effectiveness coefficients, to be mentioned later on, have to be known. The flight tests to be discussed will, therefore, include those tests necessary to determine these coefficients.

It is obvious, that if the stability derivatives are to be calculated for a certain aircraft configuration, all necessary flight tests have to be made in this configuration.

High engine power usually reduces the static directional stability. It also adversely affects the effective dihedral, particularly for conditions with wing flaps deflected. The flight tests should include, therefore, especially those configurations in which high engine power and large flap deflections are combined¹⁾.

1) For instance the configurations PA, L and WO, as defined in Ref. 3 and given in Appendix 2.

5.2. Determination of trim curves.

5.2.1. Steady straight sideslips.

a) Pilot technique.

The aircraft is brought into the required configuration and trimmed carefully for symmetric flight at the prescribed trim speed. The aircraft is then banked and equilibrium is obtained in a series of straight sideslips at the given trim speed, with different increasing angles of bank, both to the left and to the right.

Records of aileron and rudder angles and control forces are made, together with angle of bank and sideslip angle, at each equilibrium condition. The tests should be continued until the allowable or full rudder or aileron deflection has been reached, or until the control forces have become excessive. The pilot should note any tendency towards rudder force reversal.

Unless a swivelling pitot-static tube is used, a correction must be applied to the IAS readings at large angles of sideslip. Ref. 4 gives a simple method of obtaining this correction, which is as follows.

The aircraft is placed in a straight sideslip with fairly high sideslip angle. Then the rudder is rapidly centred and the increase in IAS reading is noted as the sideslip angle decreases. When making the test run for the trim curve, the IAS is varied according to the correction thus found.

b) Reduction of data.

Aileron and rudder angles and control forces, and angle of bank are plotted against sideslip angle. The slopes of these curves at the points for zero sideslip are determined. As mentioned in the introduction to this Section, the values of $\frac{d\delta_a}{d\beta}$, $\frac{d\delta_r}{d\beta}$, $\frac{dF_a}{d\beta}$ and $\frac{dF_r}{d\beta}$ are the sideslip stability parameters as to the control positions and control forces, used in Section 3.

The foregoing is illustrated with an example of measurements in steady asymmetric flight, made by the N.L.L. on a North American AT-6 "Harvard IIB" advanced trainer, viz. Ref. 5. The leading particulars of this aircraft have been summarized in Tabel 1.

Table 1

North American AT-6 "Harvard IIB", advanced trainer.

Weight	$W = 2500 \text{ kg}$
Wing area	$S = 23,6 \text{ m}^2$
Wing span	$b = 12,8 \text{ m}$
Vertical tail length	$l_v = 5,6 \text{ m}$
True air speed	$V = 280 \text{ km/h}$
Lift coefficient during measurements	$C_L = 0,31$

No aileron or rudder control forces were measured in this specific aircraft. It has, however, been chosen as an example because the data available on this aircraft being in other respects fairly complete and the aircraft type and its qualities are well known in many countries.

For the "Harvard" aircraft, mentioned above, Fig. 12 a, b and c show the aileron and rudder control and angle of bank against angle of sideslip in steady straight sideslips. The slopes of the curves at $\beta = 0$ are:

$$\frac{d\delta_a}{d\beta} = -1,30; \quad \frac{d\delta_r}{d\beta} = 1,12 \quad \text{and} \quad \frac{d\phi}{d\beta} = 2,03.$$

These values will be used later on in this paper (Section 6); they indicate that the aircraft is sideslip stable as to the aileron as well as to the rudder control position in the configuration tested.

5.2.2. Steady turns on ailerons only.

a) Pilot technique.

The aircraft is carefully trimmed at the given speed in the required configuration. Then, with the rudder being held fixed, the aircraft is slowly banked using ailerons only. A steady flight is obtained in which the aircraft yaws and slips (turn needle deflected, ball not centred). In this condition aileron angle and control force, angle of bank, sideslip angle and rate of yaw are recorded. The angle of bank is increased and a new steady condition is reached, in which the same items are recorded, etc. The elevator may be used to maintain the trim speed. The measurements are made both in turns to the right and to the left.

b) Reduction of data.

Aileron angle and control force are plotted versus angle of bank, and sideslip angle and angle of bank are plotted versus dimensionless rate of yaw $\frac{rb}{2V}$. The slopes of the trim curves at the point for straight flight represent the control position and control force stability parameters $\frac{d\delta_a}{d\phi}$ and $\frac{dF_a}{d\phi}$ respectively, whereas the plots of β and ϕ as functions of $\frac{rb}{2V}$ will be used in Section 6 in the calculation of the stability derivatives.

Fig. 13a shows δ_a as a function of ϕ and Fig. 13b and c give β and ϕ v.s. $\frac{rb}{2V}$ for the "Harvard" aircraft in the same configuration as used in Fig. 12. It appears that:

$$\frac{d\delta_a}{d\phi} = 0; \quad \frac{d\beta}{d\frac{rb}{2V}} = 0,94 \quad \text{and} \quad \frac{d\phi}{d\frac{rb}{2V}} = 99$$

The results $\frac{d\delta_a}{d\phi} = 0$ indicates that in the configuration tested the aircraft possesses neutral control position stability in turns on ailerons only.

5.2.3. Steady turns on rudder only.

a) Pilot technique.

The aircraft is carefully trimmed at the given speed in the required configuration. Some rudder is applied slowly to yaw and bank the aircraft, the ailerons being held fixed. A steady flight is obtained in which the aircraft yaws and slips as in the aileron-only turns. Rudder angle and control force, angle of sideslip and rate of yaw are recorded. Then some more rudder is applied slowly and a new steady flight is obtained in which the same items are recorded, etc. The elevator may be used to maintain the trim speed. The measurements are made both in turns to the right and to the left.

b) Reduction of data.

Rudder angle, rudder control force and sideslip angle are plotted as functions of the dimensionless rate of yaw $\frac{rb}{2V}$. The slopes of the trim curves at the point for straight flight represent the control position and control force stability parameters

$$\frac{d\delta_r}{d\frac{rb}{2V}} \quad \text{and} \quad \frac{dF_r}{d\frac{rb}{2V}} \quad \text{respectively.}$$

Fig. 14a and b give the values of δ_r and β v.s. $\frac{rb}{2V}$ for the "Harvard" aircraft, used as an example. For $\frac{rb}{2V} = 0$ it appears that:

$$\frac{d\delta_r}{d\frac{rb}{2V}} = -0,30 \quad \text{and} \quad \frac{d\beta}{d\frac{rb}{2V}} = 0,69$$

The negative value of the control position stability parameter $\frac{d\delta_r}{d\frac{rb}{2V}}$ indicates that in the configuration tested the aircraft is control position stable in steady turns on rudder only.

5.3. Determination of rudder effectiveness and rudder side force coefficients.

5.3.1. General.

Two methods can be applied to measure in flight the rudder effectiveness and side force coefficients $C_{n_{\delta_r}}$ and $C_{Y_{\delta_r}}$. The first one is based on the measurement of rudder angles in steady, straight sideslips at two c.g. positions. The second one makes use of a wing parachute to produce a yawing moment of known magnitude.

5.3.2. Measurements at different c.g. positions.

a) Introduction.

In steady straight sideslips the equations (1), (2) and (3) read:

$$C_L \cdot \text{tg } \phi + C_{Y_{\beta}} \cdot \beta + C_{Y_{\delta_r}} \cdot \delta_r = 0 \quad (13)$$

$$C_{l_{\beta}} \cdot \beta + C_{l_{\delta_a}} \cdot \delta_a = 0 \quad (14)$$

$$C_{n_{\beta}} \cdot \beta + C_{n_{\delta_a}} \cdot \delta_a + C_{n_{\delta_r}} \cdot \delta_r = 0 \quad (15)$$

In the following, the angle of bank is assumed to be small and the rudder side force $C_{Y_{\delta_r}} \cdot \delta_r$ in Eq. (13) will be neglected.

Then the equations can be written:

$$C_L \cdot \frac{d\phi}{d\beta} + C_{Y_\beta} = 0 \quad (16)$$

$$C_{L_\beta} + C_{L_{\delta_a}} \cdot \frac{d\delta_a}{d\beta} = 0 \quad (17)$$

$$C_{n_\beta} + C_{n_{\delta_a}} \cdot \frac{d\delta_a}{d\beta} + C_{n_{\delta_r}} \cdot \frac{d\delta_r}{d\beta} = 0 \quad (18)$$

ϕ in radians

If the c.g. is shifted a distance $\Delta x_{c.g.}$ (positive if backwards), the only aerodynamic coefficients to change are C_{n_β} and $C_{n_{\delta_r}}$. As $C_{n_{\delta_r}}$ is produced by a sideforce acting on the vertical tail, and the tail length is large in comparison with the c.g. displacement, $C_{n_{\delta_r}}$ may also be assumed to be independent of c.g. position.

The change in C_{n_β} can be calculated from Fig. 15. It is easily seen that:

$$C_{n_{\beta_2}} = C_{n_{\beta_1}} + C_{Y_\beta} \cdot \Delta \frac{x_{c.g.}}{b} \quad (19)$$

Or:

$$\Delta C_{n_\beta} = C_{Y_\beta} \cdot \Delta \frac{x_{c.g.}}{b} \quad (20)$$

If in steady straight sideslips at two or more different c.g. positions, aileron and rudder angles and angle of bank are measured as functions of angle of sideslip, it follows from Eqs. (16) and (17), that the slopes of the curves $\frac{d\phi}{d\beta}$ and $\frac{d\delta_a}{d\beta}$ are independent of longitudinal c.g. position.

C_{Y_β} can be calculated from Eq. (16):

$$C_{Y_\beta} = - C_L \cdot \frac{d\phi}{d\beta} \quad (21)$$

Writing Eq. (18) for both c.g. positions:

$$C_{n_{\beta_2}} + C_{n_{\delta_a}} \cdot \frac{d\delta_a}{d\beta} + C_{n_{\delta_r}} \left(\frac{d\delta_r}{d\beta} \right)_2 = 0 \quad (22)$$

$$C_{n_{\beta_1}} + C_{n_{\delta_a}} \cdot \frac{d\delta_a}{d\beta} + C_{n_{\delta_r}} \left(\frac{d\delta_r}{d\beta} \right)_1 = 0 \quad (23)$$

results in:

$$\Delta C_{n_{\beta}} + C_{n_{\delta_r}} \cdot \Delta \left(\frac{d\delta_r}{d\beta} \right) = 0 \quad (24)$$

Or, using Eqs. (20) and (21):

$$C_{n_{\delta_r}} = C_L \cdot \frac{d\phi}{d\beta} \cdot \frac{\Delta \frac{x_{c.g.}}{b}}{\Delta \left(\frac{d\delta_r}{d\beta} \right)} \quad (25)$$

According to Eq. (25) the rudder effectiveness coefficient can be calculated from measurements in steady, straight sideslips at two or more c.g. positions.

If the distance of the centre of pressure on the vertical tail behind the c.g. of the aircraft (vertical tail length) can be estimated, the rudder sideforce coefficient can be calculated from $C_{n_{\delta_r}}$ by putting

$$C_{Y_{\delta_r}} = - C_{n_{\delta_r}} \cdot \frac{b}{\ell_v} \quad (26)$$

in which ℓ_v is the vertical tail length, viz. Fig. 17.

b) Pilot technique.

As described under 5.2. The tests are performed for at least two different c.g. positions, covering a c.g. range as large as possible.

c) Reduction of data.

Rudder angle and angle of bank are plotted as functions of sideslip angle for the different c.g. positions and the slopes of the curves for $\beta = 0$ are noted. From the given weight and IAS the lift coefficient C_L is calculated. Then the rudder sideslip stability parameter $\frac{d\delta_r}{d\beta}$ is plotted against c.g. position and the quotient

$$\frac{\Delta \frac{x_{c.g.}}{b}}{\Delta \left(\frac{d\delta_r}{d\beta} \right)}$$

is determined. $C_{n_{\delta_r}}$ and $C_{Y_{\delta_r}}$ are calculated, using Eqs. (25) and (26).

5.3.3. Measurements with wing parachute.

a) Introduction.

In the second method for determining the rudder effectiveness and side force coefficients $C_{n_{\delta_r}}$ and $C_{Y_{\delta_r}}$, a yawing moment of known magnitude is obtained by means of a wing parachute. Fig. 16 shows the general test arrangement.

It must be possible to stream the parachute during flight, to prevent a take-off being made with a trailing parachute. Also the pilot must be able to jettison the parachute in case of emergency.

The yawing moment due to the parachute:

$$\Delta N = T.l \quad (27)$$

can be expressed in terms of a , $x_{c.g.}$ and P , all of which can be measured, together with the rudder angle. In order to increase the accuracy of the tests, measurements should be made at several values of the cable length a .

The condition for equilibrium at zero sideslip is, viz. Fig. 17:

$$\Delta C_n + C_{n_{\delta_r}} \cdot \Delta \delta_r = 0 \quad (28)$$

wherein

$$\Delta C_n = \frac{\Delta N}{\frac{1}{2} \rho V^2 S b} \quad (29)$$

From Eq. (28) the rudder effectiveness parameter $C_{n_{\delta_r}}$ is found:

$$C_{n_{\delta_r}} = - \frac{\Delta C_n}{\Delta \delta_r} \quad (30)$$

The rudder sideforce coefficient follows from $C_{n_{\delta_r}}$ according to Eq. (26).

b) Pilot technique.

The aircraft is placed in a series of steady straight sideslips at the given speed in the required configuration. In each steady flight rudder angle, angle of sideslip and angle of bank are recorded together with the force P indicated in Fig. 16. Sideslips to the right and to the left are made. The test is repeated for at least one more value of cable length a .

c) Reduction of data.

The calculation of $C_{n_{\delta_r}}$ and $C_{Y_{\delta_r}}$ is shown by means of an example, for which the same North American "Harvard IIB" as in paragraph 5.2 has been chosen.

From the measured quantities, yawing moment coefficients C_n are calculated. These coefficients and the measured rudder angles are plotted versus sideslip angle or angle of bank for the different values of cable length, as shown in Fig. 18 a and b. A new plot is made, viz. Fig. 18c, of yawing moment coefficient versus rudder angle for zero sideslip. The slope of the latter curve is substituted in Eq. (30) to calculate $C_{n_{\delta_r}}$. The results in this specific case is:

$$C_{n_{\delta_r}} = - 0,037$$

Using Eq. (26), with a vertical tail length $l_v = 5,6$ m and wing span $b = 12,8$ m, $C_{Y_{\delta_r}}$ reads:

$$\begin{aligned} C_{Y_{\delta_r}} &= 0,037 \cdot \frac{12,8}{5,6} \\ &= 0,085 \end{aligned}$$

Strictly speaking it would be sufficient to measure yawing moment and rudder angle in two symmetric flights only, but then there would be no possibility of fairing the curves.

5.4. Determination of aileron effectiveness and aileron yawing moment coefficients.

5.4.1. General.

Rolling and yawing moments produced by aileron deflection can be determined by means of measurements in steady, straight sideslips at different c.g. positions. C.g. shifts along the lateral and top axis can be used.

As the possible displacement in height of the c.g. usually is rather limited, the accuracy of the results will in most cases be insufficient. For this reason the method using vertical c.g. shifts will not be discussed here.

The second method, entailing the measurement of aileron deflections at c.g. positions outside the plane of symmetry has been described below.

5.4.2. Measurements with wing tip weight.

a) Introduction.

If the aircraft is fitted with tip tanks, fuel in these tanks may be used to obtain the required lateral c.g. shift. Otherwise a heavy weight is installed in the wing tip. In the latter case, however, flutter calculations must be performed beforehand.

The fuel or the weight produces a rolling moment, which has to be balanced in flight by an appropriate aileron deflection. So, measuring aileron angles yields aileron power data if the rolling moment is known.

The condition for equilibrium at zero sideslip is, viz. Fig. 19:

$$\Delta C_l + C_{l_{\delta_a}} \cdot \Delta \delta_a = 0 \quad (30)$$

Or:

$$C_{l_{\delta_a}} = - \frac{\Delta C_l}{\Delta \delta_a} \quad (31)$$

The aileron deflection induces a dimensionless yawing moment $C_{n_{\delta_a}} \cdot \Delta \delta_a$, which in its turn is counteracted by a rudder deflection $\Delta \delta_r$:

$$C_{n_{\delta_a}} \cdot \Delta \delta_a + C_{n_{\delta_r}} \cdot \Delta \delta_r = 0 \quad (32)$$

From this it follows:

$$C_{n_{\delta_a}} = - C_{n_{\delta_r}} \cdot \frac{\Delta \delta_r}{\Delta \delta_a} \quad (33)$$

If the rudder effectiveness coefficient $C_{n_{\delta_r}}$ has been determined according to paragraph 5.3, the aileron yawing moment coefficient $C_{n_{\delta_a}}$ can be calculated from Eq. (33).

b) Pilot technique.

For each value of wing tip weight, a series of steady, straight sideslips is performed at the given speed in the required configuration. In each steady condition, aileron angle, rudder angle, sideslip angle and angle of bank are recorded. Sideslips to the right and to the left are made. The test has to be made for at least two laterally different c.g. positions.

c) Reduction of data.

The determination of aileron effectiveness and yawing moment coefficients is demonstrated with an example, using the measurements on a North American "Harvard IIB" mentioned earlier in this paper.

Plots are made of aileron and rudder angles versus sideslip angle or angle of bank, viz. Fig. 20 a and b. From the known applied rolling moments and the given airspeed, rolling moment coefficients can be calculated. Then a new plot is made, viz. Fig. 20 c, showing rolling moment coefficient against aileron angle at zero sideslip. The slope of the latter curve is substituted in Eq. (31) to give the aileron effectiveness coefficient $C_{\ell_{\delta_a}}$. In this case the result is:

$$C_{\ell_{\delta_a}} = - 0,068$$

From Fig. 20 a and b rudder angle is plotted versus aileron angle, also for symmetric flight, as shown in Fig. 20 d. The slope of this curve, together with the rudder effectiveness coefficient $C_{n_{\delta_r}}$ yields the aileron yawing moment coefficient, viz. Eq. (33). The result is:

$$\begin{aligned} C_{n_{\delta_a}} &= 0,037.0,113 \\ &= 0,0042 \end{aligned}$$

As is the case for the determination of rudder power, measurements in symmetric flight only would be sufficient, but they would give no possibility of fairing curves.

6. Calculation of stability derivatives from results of flight tests.

6.1. General.

The relevant stability derivatives are derived from the slopes of the trim curves, determined in the preceding Section 5, using the known values of the forces and moments produced by the control surface deflections.

The sideslip derivatives are calculated from the measurements in steady straight sideslips, the yawing velocity derivatives follow from the measurements in steady turns on ailerons and rudder only.

In the following the calculation of the stability derivatives is illustrated with the results of the measurements on the North American "Harvard IIB" given in Section 5. For convenience the quantities obtained thus far have been collected in table II.

Table II.

$C_{n_{\delta_r}} = - 0,037$	$C_{l_{\delta_a}} = - 0,068$
$C_{Y_{\delta_r}} = 0,085$	$C_{n_{\delta_a}} = 0,0042$
<u>Steady straight sideslips.</u>	
$\frac{d\delta_a}{d\beta} = - 1,30$	$\frac{d\delta_r}{d\beta} = 1,12 \quad \frac{d\phi}{d\beta} = 2,03$
<u>Steady turns on ailerons only.</u>	
$\frac{d\delta_a}{d\phi} = 0$	$\frac{d\beta}{d \frac{rb}{2V}} = 0,94 \quad \frac{d\phi}{d \frac{rb}{2V}} = 99$
<u>Steady turns on rudder only.</u>	
$\frac{d\delta_r}{d \frac{rb}{2V}} = - 0,30$	$\frac{d\beta}{d \frac{rb}{2V}} = 0,69$

6.2. Steady straight sideslips.

It follows from the equations of equilibrium (1), (2) and (3) for $\frac{rb}{2V} = 0$ and small angles of bank:

$$C_{Y_\beta} = - C_L \cdot \frac{d\phi}{d\beta} - C_{Y_{\delta_r}} \cdot \frac{d\delta_r}{d\beta} \quad (34)$$

$$C_{l_\beta} = - C_{l_{\delta_a}} \cdot \frac{d\delta_a}{d\beta} \quad (35)$$

$$C_{n_\beta} = - C_{n_{\delta_a}} \cdot \frac{d\delta_a}{d\beta} - C_{n_{\delta_r}} \cdot \frac{d\delta_r}{d\beta} \quad (36)$$

Substituting the values of table II in the above equations results in:

$$C_{Y_\beta} = - 0,31 \cdot 2,03 - 0,085 \cdot 1,12 = - 0,73$$

$$C_{l_\beta} = - 0,068 \cdot 1,30 = - 0,088$$

$$C_{n_\beta} = 0,0042 \cdot 1,30 + 0,037 \cdot 1,12 = 0,047$$

6.3. Steady turns on ailerons only.

From Eq. (3) follows for $\delta_r = 0$:

$$C_{n_r} = - C_{n_\beta} \cdot \frac{d\beta}{d \frac{rb}{2V}} - C_{n_{\delta_a}} \cdot \frac{d\delta_a}{d \frac{rb}{2V}} \quad (37)$$

The static directional stability C_{n_β} obtained in paragraph 6.2 and the values of $C_{n_{\delta_a}}$, $\frac{d\beta}{d \frac{rb}{2V}}$ and $\frac{d\delta_a}{d \frac{rb}{2V}}$ given in table II are used to calculate the damping in yaw coefficient C_{n_r} for the "Harvard IIB". The result is:

$$C_{n_r} = - 0,047 \cdot 0,94 = - 0,044.$$

6.4. Steady turns on rudder only.

Substituting $\delta_a = 0$ in the equation of equilibrium (2) leads to:

$$C_{l_r} = - C_{l_\beta} \cdot \frac{d\beta}{d \frac{rb}{2V}} \quad (38)$$

The coefficient of rolling moment due to yawing for the "Harvard IIB" is found, using the effective dihedral calculated in paragraph 6.2 and the value of $\frac{d\beta}{d \frac{rb}{2V}}$ given in table II:

$$C_{l_r} = 0,088 \cdot 0,69 = 0,061.$$

Finally in table III all the aerodynamic coefficients and derivatives determined by means of flight tests have been summarized.

Table III

Stability derivatives and control effectiveness coefficients, measured in steady asymmetric flight.

North American AT-6 "Harvard IIB"

Configuration: Power-on Clean.

$C_L = 0,31$ c.g. at $0,304 \bar{c}$

$$C_{Y\beta} = - 0,73$$

$$C_{l\beta} = - 0,088$$

$$C_{n\beta} = 0,047$$

$$C_{Y\delta_r} = 0,085$$

$$C_{n\delta_r} = - 0,037$$

$$C_{l_r} = 0,061$$

$$C_{n_r} = - 0,044$$

$$C_{l\delta_a} = - 0,068$$

$$C_{n\delta_a} = 0,0042$$

7. Lateral and directional control in flight on asymmetric power

7.1. Introduction.

When discussing lateral and directional control characteristics in flight on asymmetric power two aspects have to be considered, firstly the control effectiveness in restoring a steady flight condition upon the sudden failure of an engine and secondly the characteristics in steady flight with one or more engines inoperative. In paragraphs 7.2 and 7.3 respectively the basic considerations for these two aspects will be discussed; in Section 8 the flight tests for the determination of the principal parameters will be described.

7.2. Sudden failure of an engine.

7.2.1. General considerations.

For an analysis of the behaviour of an aircraft upon sudden failure of an engine the basic equilibrium equations (1), (2) and (3), given in paragraph 4.2 should be extended to read:

$$C_{L_p} \cdot \phi - 4\mu_b \cdot \frac{rb}{2V} + C_{Y_\beta} \cdot \beta + C_{Y_{\delta_r}} \cdot \delta_r = 0 \quad (39)$$

$$C_{\ell_p} \cdot \frac{pb}{2V} + C_{\ell_r} \cdot \frac{rb}{2V} + C_{\ell_\beta} \cdot \beta + C_{\ell_{\delta_a}} \cdot \delta_a + C_{\ell_e} = 0 \quad (40)$$

$$C_{n_p} \cdot \frac{pb}{2V} + C_{n_r} \cdot \frac{rb}{2V} + C_{n_\beta} \cdot \beta + C_{n_{\delta_a}} \cdot \delta_a + C_{n_{\delta_r}} \cdot \delta_r + C_{n_e} = 0 \quad (41)$$

In these equations C_{ℓ_e} and C_{n_e} respectively are the coefficients of the rolling and yawing moments due to engine failure.

The yawing moment is a direct result of the loss in thrust from the failing engine. For a first approximation the coefficient C_{n_e} can be calculated from:

$$C_{n_e} = \frac{\Delta T \cdot \ell}{q \cdot S \cdot b} \quad (42)$$

wherein:

ΔT = change in thrust from the failing engine, including the drag of the windmilling or feathered propeller or the idling jet turbine;
 ℓ = distance from the thrust line to the plane of symmetry of the aircraft (positive to the left).

The principal contributions to the rolling moment coefficient C_{ℓ_e} are for a propeller-driven aircraft the loss in lift due to the loss of slipstream and the change in engine torque, the former being the most important factor. These effects may be either complementary

or counteracting, depending on the position of the failing engine and the sense of rotation of the propeller. For a jet-propelled aircraft only the negligibly small engine torque contributes to C_{ℓ_e} which is, therefore, much smaller than for a propeller-driven aircraft.

Considering engine failure on the starboard side C_{ℓ_e} and C_{n_e} will be positive. As a result of these moments the aircraft obtains a positive rate of yaw (nose turns to the right) and a positive rate of roll (right wing down). The aircraft then sideslips towards the live engine ($\beta < 0$) and banks towards the dead engine ($\phi > 0$).

The induced rolling moments due to sideslip ($C_{\ell_\beta} < 0$) and due to rate of roll ($C_{\ell_r} > 0$) both tend to increase the angle of bank, which for some aircraft types rapidly assumes a large value. In order to pick up the dead-engined wing a positive aileron deflection ($\delta_a > 0$) is required. This aileron deflection does, however, induce an adverse yawing moment, which for most aircraft types cannot be neglected in this case and which contributes to increase the rate of yaw ($C_{n_{\delta_a}} > 0$).

For a conventional aircraft the yawing moments due to p , r and β counteract the motion of the aircraft upon engine failure. It depends mainly on the magnitude of C_{n_r} and C_{n_β} at which rate the angle of sideslip and the rate of yaw will develop after an engine failure. Should these attain large values then fin stalling and rudder lock (see paragraph 3.8) may occur. In order to prevent this from happening the yawing motion of the aircraft upon engine failure should be counteracted as quickly as possible by an appropriate rudder deflection. The ailerons should, in the first instance, not be deflected as these contribute adversely to the rate of yaw. Only after the rate of yaw and the corresponding sideslipping motion have been checked and the danger of fin stalling and rudder lock is over, the ailerons should be deflected in order to reduce the angle of bank which has meanwhile developed.

When flying at large angles of attack e.g. during take-off or final approach - there is also the danger of wing tip stalling due to the rate of roll and the aileron deflection, which both increase the angle of attack on the dead-engined wing. In this respect it is also important first to counteract the yawing moment

by rudder deflection (as this also reduces the induced rate of roll) before a large aileron deflection is given in order to bring the aircraft back to a wings-level attitude.

7.2.2. Minimum control speed.

Further considering the rudder deflection required to check the yawing motion resulting from an engine failure it should be noted that the maximum obtainable yawing moment coefficient due to rudder deflection ($C_{n_{\delta_r}} \cdot \delta_r$)_{max} is virtually independent of the airspeed,

neglecting compressibility and deformation effects. The yawing moment coefficient due to engine failure C_{n_e} varies, however, with the airspeed, according to Eq. (42). For a propeller-driven aircraft the thrust - and with it ΔT - can, in a first approximation, be taken to vary inversely with the airspeed V ; C_{n_e} is, therefore, inversely proportional to V^3 . For a jet-propelled aircraft the thrust is, in the first instance, independent of V and in this case C_{n_e} varies inversely with V^2 , according to Eq. (42).

As illustrated in Fig. 21 there is, therefore, a minimum speed below which it is impossible to regain and thereafter maintain control in steady, straight flight after an engine failure. This speed is usually referred to as the minimum control speed, V_{mc} . The maximum obtainable rudder deflection can be the limiting factor in determining the minimum control speed, but in some cases the rudder control force will become excessive before the rudder has been fully deflected. The minimum control speed is, of course, dependent on the power setting and the aircraft configuration. For a specific aircraft the minimum control speed will, therefore, have different values, according to the flight condition. In the following paragraph the principal flight conditions will be considered in more detail.

Take off.

During take-off both the low speed and the full power used, contribute to give a large value of C_{n_e} , according to Eq. (42). Furthermore due to the danger of losing height or hitting the ground due to the angle of bank assuming an excessive value, engine failure during take-off is one of the most dangerous and difficult situations (viz. Ref. 6). The minimum control speed in

the take-off configuration, often denoted as V_{mcg} , "minimum control speed on or near the ground" may, in exceptional cases, be determined by the aircraft attaining a dangerous attitude during the recovery even before the rudder control forces become excessive or the rudder reaches its stops; this is, of course, a most undesirable characteristic.

It should be noted that as long as the aircraft is still on the runway cross-wind during take-off also influences the value of V_{mcg} ; cross wind coming from the dead engine side tending to increase the swing, thereby unfavourable affecting V_{mcg} . Furthermore, if the engine failure occurs transition from level to climbing flight the induced gyroscopic moments due to pitching, also significantly influence the yawing moment due to engine failure. If the propeller rotates in a clockwise direction - viewed from the rear - the port outer engine is the critical engine in this condition.

For the Douglas DC-5, to be used as an example later on in this paper the gyroscopic moment for a rate of pitch of $10^\circ/\text{sec}$ amounts to as much as 20% of the yawing moment due to engine failure.

Climb at low airspeed.

The most critical condition for engine failure in climbing flight is the initial climb during take-off, when the engines are still developing take-off power and the aircraft is in the take-off configuration except for the undercarriage being retracted. The minimum control speed for this condition is usually referred to as the "minimum control speed in free air", V_{mca} .

Approach on asymmetric power.

A situation similar to that resulting from the sudden failure of an engine can arise when during flight on asymmetric power the live engines are suddenly opened up. The most critical condition is that during the final approach of an aircraft with all engines on one side inoperative. The control problems are usually less severe than during take-off or initial climb, as the propellers of the inoperative engine have been feathered and the live engines can be opened up gradually. The minimum control speed should also be determined for this condition.

7.2.3. Engine failure during cruising flight.

In order to make sure that the directional stability of the aircraft is sufficiently large to cope with the yawing moment due to engine failure during cruising flight without danger of fin stalling or rudder lock occurring, it is usually required that it shall be possible to regain control in case of engine failure during cruising flight by use of the ailerons only; the rudder being either kept free or centralised. During recovery the aileron control force should not become excessive.

During a turn failure of an inside engine causes a tightening of the turn. It shall be possible to counteract this effect by moderate control deflections.

7.3. Steady, straight flight on asymmetric power.

7.3.1. General considerations.

For the steady flight condition after engine failure the equations (39), (40) and (41) reduce to:

$$C_{L\beta} \cdot \beta + C_{Y\beta} \cdot \beta + C_{Y\delta_r} \cdot \delta_r = 0 \quad (43)$$

$$C_{l\beta} \cdot \beta + C_{l\delta_a} \cdot \delta_a + C_{l_e} = 0 \quad (44)$$

$$C_{n\beta} \cdot \beta + C_{n\delta_a} \cdot \delta_a + C_{n\delta_r} \cdot \delta_r + C_{n_e} = 0 \quad (45)$$

From Eqs. (43), (44) and (45) it appears that the aileron and rudder deflection during steady, straight flight on asymmetric power can be determined from:

$$\delta_a = - \frac{C_{l\beta}}{C_{l\delta_a}} \cdot \beta - \frac{C_{l_e}}{C_{l\delta_a}} \quad (46)$$

$$\delta_r = - \frac{C_{n\beta}}{C_{n\delta_r}} \cdot \beta - \frac{C_{n_e}}{C_{n\delta_r}} + \frac{C_{n\delta_a}}{C_{n\delta_r}} \left(\frac{C_{l\beta}}{C_{l\delta_a}} \beta + \frac{C_{l_e}}{C_{l\delta_a}} \right) \quad (47)$$

For conventional aircraft the last term of Eq. (47) usually is small in relation to the other two terms. For our purpose it can, therefore, be neglected and Eq. (47) reduces to:

$$\delta_r = - \frac{C_{n\beta}}{C_{n\delta_r}} \cdot \beta - \frac{C_{n_e}}{C_{n\delta_r}} \quad (48)$$

It is usually required, that during steady, straight flight on asymmetric power it shall be possible to reduce all control forces to zero by means of the trimming devices. On the basis of this requirement a minimum control speed can be determined below which it is impossible to trim out all control forces. It appears that the required control deflection and control forces during flight on asymmetric power are greatly influenced by the angle of bank and angle of sideslip or the aileron and rudder control coordination. In this respect a number of characteristic conditions can be indicated; these conditions will be described in more detail in the following paragraphs.

7.3.2. Flight with wings level. ($\phi = 0$).

In this condition it appears from Eq. (43) that the aircraft is sideslipping, the side force resulting from this sideslip being required to counteract the side force on the vertical tail due to rudder deflection. As both the yawing moment due to engine failure (C_{n_e}) and that due to the sideslip ($C_{n\beta} \cdot \beta$) are in the same direction a fairly large rudder deflection will be required (viz. Eq. 47). Considering an engine failure on the starboard wing both β and δ_r will be positive; $C_{n\delta_r}$ being negative. This condition is illustrated in Fig. 22. The required angle of sideslip β usually is fairly small ($\phi < 5^\circ$).

7.3.3. Flight with zero sideslip. ($\beta = 0$)

From Eq. (47) it appears that in this condition the required rudder deflection is smaller than in flight with wings level. Eq. (43) indicates that the aircraft should be banked (live engine down) in order to counteract the side force on the vertical tail due to rudder deflection by the lateral component of the aircraft weight. The required angle of bank is small usually and within 3° .

This condition gives the most favourable aircraft performance and should, therefore, be recommended for flight on asymmetric power, especially under those conditions where the performance is critical (take-off, initial climb). The required angle of bank should be determined from flight tests and mentioned in the pilots instructions.

7.3.4. Flight with rudder centralised.

For the condition in which the rudder is kept centralised it appears from Eq. (47), that

$$\beta = - \frac{C_{n_e}}{C_{n_\beta}} \quad (49)$$

Considering an engine failure on the starboard side, β will be negative; the aircraft sideslips towards the live engine. A large angle of sideslip will be required as the yawing moment due to sideslip should fully compensate the yawing moment due to engine failure. Eq. (43) indicates that the live engine should be banked down to balance the side force due to sideslip. The aileron deflection is, according to Eq. (46), larger than in the cases considered under 7.3.2. and 7.3.3, because the aileron rolling moment should not only compensate the rolling moment due to asymmetric power but also that induced by the large angle of sideslip. In this condition rudder lock can occur, due to the large value of the sideslip angle β .

7.3.5. Flight with rudder free.

If the rudder has a tendency to float with the wind (aerodynamically underbalanced rudder) there is a yawing moment due to rudder deflection which supplements the yawing moment due to asymmetric thrust. The angle of sideslip required to compensate the yawing moment, is, therefore, even larger than in flight with rudder centralised (7.3.4); the possibility of rudder lock occurring is largest in this condition. The angle of bank required to compensate for the sideforce due to sideslip will, however, not necessarily be larger than in condition 7.3.4, as the side force due to rudder deflection also has a compensating effect, viz. Fig. 23.

If the rudder floats against the wind (aerodynamically overbalanced rudder) the reverse holds true. Then β is smaller and ϕ is larger than in condition 7.3.4.

8. Flight tests on asymmetric power.

8.1. Introduction.

The flight tests on asymmetric power can also be subdivided into two categories. The first group comprises the tests carried out to determine the aircraft behaviour after a sudden failure of an engine and the possibility to manoeuvre the aircraft on asymmetric power. The second group consists of measurements in a series of steady flight conditions on asymmetric power. In the following paragraph the tests for determining the minimum control speeds and other items discussed in Section 7 will be described. The aircraft configurations, the trim speeds used and the limits of allowable control forces should be in accordance with the relevant requirements.

8.2. Sudden failure of an engine and manoeuvring on asymmetric power.

8.2.1. Determination of the minimum control speeds

V_{mca} and V_{mcg} .

a) Pilot technique.

The aircraft is brought into the required take-off configuration and trimmed carefully for symmetric flight at the prescribed speed. All engines shall be operating at maximum take-off power. Sudden failure of the critical engine shall be simulated by closing its throttle as rapidly as possible. The pilot must be able to regain and thereafter maintain control in a steady, straight flight without reducing the thrust of the live engines or operating propeller or trim controls.

During recovery records of control angles and forces are made together with the airspeed, height, angle of bank, sideslip angle and heading of the aircraft. The test should be repeated at progressively lower speeds untill during recovery the maximum allowable control deflection has been reached, the control forces have become excessive, a dangerous attitude has been attained or the speed is equal to the stalling speed. The tests for determining V_{mca} can also be limited by the attainment of the minimum speed in the steady flight, see paragraph 8.3.1.

b) Reduction of data.

For each speed a time history of the aircraft behaviour shall be plotted, i.e. control angles and forces, angle of bank,

sideslip angle, aircraft heading, height and airspeed shall be plotted versus time. The minimum speed and the limiting factor shall be indicated.

The foregoing is illustrated with an example of measurements made by the N.L.L. on a Douglas DC 5 transport aircraft, viz. Ref. 7. Fig. 24 shows rudder and aileron angle, angle of bank, heading, calibrated airspeed and r.p.m. of the port engine versus time. In this case the initial speed was lower than the stalling speed in the glide.

8.2.2. Cruising flight.

a) Pilot technique.

The aircraft is trimmed at a prescribed cruising speed in the required configuration. All engines are operating at maximum continuous power. In this condition a failure of the critical engine should be simulated. No action should be taken until the aircraft attains a dangerous attitude. Then with the rudder left free, the pilot should apply aileron control only and restore the aircraft to a wings level steady straight flight condition at the initial airspeed. The test should be repeated with the rudder held fixed in the central position. The pilot should note any tendency towards rudder force reversal or fin stalling.

During and after recovery a continuous record should be made of control deflections and forces, angle of bank, angle of sideslip, airspeed and height.

b) Reduction of data.

Time histories shall be given of the dynamical tests, showing the control angles and forces, angle of bank, angle of sideslip, airspeed and height versus time.

Typical examples for these curves are shown in Fig. 25 for the Douglas DC-5 aircraft, used as an example. In this case the rudder was held fixed in the central position. An engine failure on the port wing was simulated.

8.2.3. Turns.

a) Pilot technique.

The aircraft is brought into the configuration for cruising flight and trimmed at the speed for maximum range. In this condition banked turns with an angle of bank up to 30° should be made in both directions. When a steady condition is reached the throttle of the

critical engine on the inside wing should be closed rapidly. The pilot should be able to restore the steady flight condition with zero angle of bank.

During recovery records should be made of the control angles and forces, angle of bank and rate of yaw.

b) Reduction of data.

Plots of time histories should be made, showing the control angles and forces, angle of bank and rate of yaw.

This is illustrated again by tests made on the Douglas DC-5 aircraft. Fig. 26 shows some of the measured quantities. An engine failure on the port wing was simulated in a turn to the left.

8.2.4. Manoeuvring on asymmetric power.

a) Control during approach.

The aircraft is brought into the required final approach configuration and trimmed for gliding flight. All engines on one side are made inoperative (only the outboard feathered) and the live engines are idling. In this condition the pilot should open up the live engines untill they develop maximum take-off power. He should determine whether it is possible to counteract the swing and roll without ^{attaining} a dangerous attitude or encountering dangerous flight characteristics. The pilot should note the limiting factor.

b) Turns on asymmetric power.

The aircraft is trimmed for cruising flight and the aircraft brought into the required configuration. All engines on one side are made inoperative, while the others are developing maximum continuous power. In this condition the pilot shall make banked turns up to 30° angle of bank both with and against the inoperative power units. During these manoeuvres excessive control forces should be noted and the pilot shall report on any difficulties encountered.

8.3. Steady flight on asymmetric power.

8.3.1. Determination of minimum control speeds.

a) Pilot technique.

The aircraft is brought in the same configuration as for determining the minimum control speed in free air and trimmed for symmetrical flight. The throttle of the critical engine should be closed and the propeller allowed to windmill. By means of the

lateral and directional controls the aircraft is brought in a steady, straight flight at a sufficiently high speed with either wings level or zero sideslip (angle of bank within 5°). In this steady condition records should be taken of control angles and forces, angle of bank, angle of sideslip and the airspeed. The speed should then be reduced in convenient steps, records being taken at each step untill the allowable or full rudder deflection is reached, the control forces become too high or the speed is equal to the stalling speed in this condition.

These tests can be combined with those mentioned in Section 8.2.1.

In order to obtain more information on the flying qualities, the above-mentioned tests should be repeated in the configuration for cruising flight, with the live engines developing maximum continuous power. These tests should be made with one, two or more engines on one side windmilling untill full asymmetric power is reached.

b) Reduction of data.

The measured quantities should be plotted versus calibrated airspeed. There should be one plot for each configuration tested. The minimum speed obtained and the limiting factor shall be noted.

As an example measurements are given for the Douglas DC-5, taken from Ref. 7. Fig. 27 shows the rudder pedal force versus calibrated airspeed for two cases: either the port or the starboard engine being inoperative. As can be seen there is some difference between the effects from the two engines. Fig. 28 shows the rudder deflection in a percentage of the maximum deflections for the same flight conditions.

8.3.2. Determination of the minimum trim speed.

a) Pilot technique.

The tests mentioned in Section 8.3.1 should be run again, but this time the trim controls should be used to reduce the control forces to zero at various equilibrium speeds. Records should be taken of the control and trim tab deflections, angle of bank and sideslip and after full trim tab deflection has been reached also of the control forces.

b) Reduction of data.

The measured quantities should be plotted versus calibrated airspeed. There should be one plot for each configuration tested. The speed at which one of the trim tabs is fully deflected should

be noted.

Fig. 29 shows the rudder trim tab deflection as a function of the calibrated airspeed for the Douglas DC-5 used throughout this Section 8 as an example for tests on asymmetric power. In the configuration undercarriage retracted, flaps closed the trim tab is fully deflected at a speed of 150 km/h.

9. Summary.

In the foregoing Sections an outline has been given of a theory control in steady asymmetric flight, using the concepts of control position and control force stability and sideslip stability.

The following conclusions can be drawn concerning the sideslip stability and the control position stability in steady asymmetric flight:

- 1) If the aircraft possesses sideslip stability as to aileron control position, it possesses positive effective dihedral.
- 2) If the aircraft possesses sideslip stability as to rudder control position, it possesses static directional stability.
- 3) If the aircraft possesses:
 - a. aileron control position stability in steady turns on ailerons only, and
 - b. sideslip stability as to rudder control position, it is spirally stable with controls fixed.
- 4) If the aircraft possesses:
 - a. rudder control position stability in steady turns on rudder only, and
 - b. sideslip stability as to aileron control position, it is spirally stable with controls fixed.

Based on the theory, flight tests have been described for the determination of the relevant control position, control force and sideslip stability parameters.

Flight tests have also been given to measure the forces and moments produced by aileron and rudder deflections.

A method has been indicated to calculate from the results of the flight tests the stability derivatives with respect to sideslip angle and rate of yaw.

The problem of control in flight on asymmetric power has been divided into two parts, firstly to regain control immediately after an engine failure and secondly to maintain control in steady flight on asymmetric power. Minimum control speeds and some characteristics in steady flight has been discussed.

Flight tests for the determination of the minimum control speeds and the manoeuvrability on asymmetric power have been described.

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Appendix 1.

Relation between control position stability and dynamic stability.

In order to investigate the relation between control position stability and dynamic stability, a control surface deflection varying sinusoidally with time is considered. The angular frequency of the oscillation be ω .

After the motion due to the initial disturbance has died out (supposing a stable flight condition) the response of the aircraft to the control surface oscillation will also be sinusoidally, with the same angular frequency ω .

The amplitude and phase relationship between the output (a component of the aircraft's motion) and the input (control surface deflection) of the oscillating system is given by the frequency response function. This is a frequency dependent complex number, the modulus of which indicates the amplitude ratio and the argument of which represents the phase relationship between output and input.

If the behaviour of a dynamical system can be described by linear equations, its frequency response function will consist of a numerator and a denominator, both of which are polynomials in the imaginary variable $j\omega$.

Be θ_i the input of the system - e.g. the deflection of one of the control surfaces of an aircraft - and θ_o the output - e.g. one of the components of the aircraft's motion.

Then the frequency response function can be written as:

$$\left[\frac{\theta_o}{\theta_i} \right] = \frac{b_m (j\omega)^m + b_{m-1}(j\omega)^{m-1} + \dots + b_1 (j\omega) + b_0}{a_n (j\omega)^n + a_{n-1}(j\omega)^{n-1} + \dots + a_1 (j\omega) + a_0} \quad (50)$$

It is known from the theory of control, that the coefficients in the denominator are equal to the coefficients of the characteristic equation of the system:

$$a_n \lambda^n + a_{n-1} \lambda^{n-1} + \dots + a_1 \lambda + a_0 = 0 \quad (51)$$

whereas the coefficients in the numerator depend on which input and which output signal of the system are considered.

Now the angular frequency of the input signal is gradually reduced to zero. It can be seen, that in the latter case a series

of steady conditions with different but constant values of the input and output signals is obtained. As follows from Eq. (50), the output is proportional to the input and:

$$\left[\frac{\theta_o}{\theta_i} \right]_{\omega=0} = \frac{b_o}{a_o} \quad (52)$$

In the case of an aircraft responding to a control surface oscillation, the input signal is the deflection of one of the control surfaces, e.g. the rudder, measured from the position in steady, symmetrical flight:

$$\theta_i = \delta_r \quad (53)$$

whereas the output is one of the components of motion of the aircraft, e.g. the dimensionless rate of yaw:

$$\theta_o = \frac{rb}{2V} \quad (54)$$

It can be seen, that in this case the ratio of the input to the output signal at zero frequency is identical with the parameters for control position stability in steady turns on rudder only, as introduced in Section 3:

$$\left[\frac{\theta_i}{\theta_o} \right]_{\omega=0} = \frac{d\delta_r}{d \frac{rb}{2V}} = \frac{a_o}{b_o} \quad (55)$$

The characteristic equation of the disturbed lateral motion of an aircraft is usually written as:

$$A \lambda^4 + B \lambda^3 + C \lambda^2 + D \lambda + E = 0 \quad (56)$$

wherein:

$$E = \frac{1}{2} C_L (C_{\ell_\beta} \cdot C_{n_r} - C_{n_\beta} \cdot C_{\ell_r}) \quad (57)$$

From Eqs. (51) and (56) it follows:

$$E = a_o \quad (58)$$

Substitution of Eq. (58) in Eq. (55) results in:

$$\frac{d\delta_r}{d \frac{rb}{2V}} = \frac{E}{b_o} \quad (59)$$

It is thus proved, that the control position parameter is proportional to the constant term of the characteristic equation of the aircraft's disturbed motion.

Whether a certain flight condition of the aircraft is laterally stable or not depends solely on the sign and the magnitude of the coefficients A, B, C, D and E of the characteristic equation. One of the conditions for stability is:

$$E > 0 \quad (60)$$

or, for asymmetric motion and for positive values of the lift coefficient:

$$C_{\ell_\beta} \cdot C_{n_r} - C_{n_\beta} \cdot C_{\ell_r} > 0 \quad (61)$$

When E has changed sign and is negative, the aircraft is laterally dynamically unstable (spiral instability) and, at the same time, it is control position unstable as to the rudder in steady turns on rudder only, as follows from Eq. (59).

If a deflection of the ailerons instead of the rudder is considered, the foregoing holds equally true, the only difference being in the value of the coefficient b_0 in the numerator of the frequency response function. When E changes sign, the aircraft also becomes unstable as to aileron position in steady turns on ailerons only.

Appendix 2.

Definition of aircraft configurations.

The aircraft configuration PA, L and WO are defined as follows, viz. Ref. 3.

Configuration PA. Power Approach. Power for level flight at $1,20 V_{S_L}$, flaps down and undercarriage down.

Configuration L. Landing. Power off, flaps down and undercarriage down.

Configuration WO. Wave-off. Take-off power, flaps and undercarriage down.

V_{S_L} is the indicated stalling speed, corrected for instrument and position error, in the landing configuration.

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carriage and flaps up.

Fig. 1a. Stability axis system and angular velocities.

Fig.1b. Definition of sideslip angle β .

Fig.1c. Definition of angle of bank ϕ .

Fig.1d. Definition of positive control surface deflections and control forces.

Fig. 2. Illustration of control position stability.

Fig. 3. Control position stability as to ailerons in steady turns on ailerons only.

Fig. 4. Control position stability as to rudder in steady turns on rudder only.

Fig. 5. Control force stability as to ailerons in steady turns on ailerons only.

Fig. 6. Control force stability as to rudder in steady turns on rudder only.

Fig. 7. Sideslip stability as to aileron position.

Fig. 8. Sideslip stability as to rudder position.

Fig. 9.

Sideslip stability as to aileron control force .

Fig. 10.

Sideslip stability as to rudder control force .

Fig.11 . Reversal of rudder pedal force.

Fig.12 . Measurements in steady straight sideslips, North American "Harvard II B",
configuration: Power-on Clean
 $C_L = 0.31$, c.g. at $0.304 \bar{c}$.

Fig. 13. Measurements in steady turns on ailerons only, North American "Harvard II B", configuration: Power-on Clean $C_L = 0,31$, c.g. at $0,304 \bar{c}$.

Fig. 14. Measurements in steady turns on rudder only.
 North American "Harvard II B" configuration:
 Power-on Clean, $C_L = 0.31$ c.g. at 0.304 c

Fig. 15. Change in static directional stability $C_{n\beta}$ due to c.g. displacement.

Fig.16. Measurement of rudder effectiveness with wing parachute.

Fig.17. Equilibrium of yawing moments with wing parachute at zero sideslip.

Fig. 18. Measurement of rudder effectiveness with wing parachute, North American "Harvard II B", configuration: Power-on Clean $C_L = 0,31$, c.g. at $0,304 \bar{c}$.

Fig. 19. Measurement of aileron effectiveness with wing tip weight;
equilibrium of rolling moments at zero sideslip.

a) $\Delta\delta_a - \beta$

b) $\Delta\delta_r - \beta$

c) $C_L - \delta_a$, zero sideslip.

d) $\delta_r - \delta_a$, zero sideslip.

Fig. 20. Measurement of aileron effectiveness with wing tip weight, North American "Harvard II B", configuration Power-on Clean.

$C_L = 0,31$; c.g. at $0,304 \bar{c}$.

Fig. 21. Minimum control speed.

Fig.22.

Equilibrium of yawing moments for flight on asymmetric power with wings level.

Fig.23. Equilibrium of yawing moments for flight on asymmetric power with rudder free.

Fig.24. Measurements after simulated failure of the port engine, Douglas DC-5, configuration: Cruising power, undercarriage and flaps up.

Fig.25.

Measurements after simulated failure of the port engine, Douglas DC-5, configuration: Power-on Clean, undercarriage down, flaps up, rudder centralised.

Fig. 26. Measurements after simulated failure of the port engine in a turn to the left, Douglas DC-5, cruising power, undercarriage and flaps up, $V_c = 265$ km/h.

Fig. 27. Measurements of the rudder pedal force in steady straight flight on asymmetric power with wings level, Douglas DC-5, configuration: Power-on Clean, undercarriage and flaps up.

Fig. 28. Measurements of rudder deflection in steady, straight flight on asymmetric power, wings level. Douglas DC-5, configuration: Power - on Clean, undercarriage and flaps up.

Fig.29. Measurements of trim tab deflection for zero pedal force in steady, straight flight on asymmetric power, wings level.
Douglas DC-5, configuration: Power - on Clean, under - carriage and flaps up.