Modeling end-stop nonlinearity of Tuned Mass Dampers in Offshore Wind Turbines

Jelte van Til

Report no : EM 2016.032
Coaches : dr.ir. S. N. Voormeeren, prof.dr.ir. W. Lacarbonara, dr.ir. F. Aljani
Professor : prof.dr.ir. F. van Keulen
Specialization : Engineering Mechanics
Type of report : Master Thesis
Date : 18 July 2016
Modeling end-stop nonlinearity of Tuned Mass Dampers in Offshore Wind Turbines

by

Jelte van Til

in partial fulfillment of the requirements for the degree of

Master of Science

in Mechanical Engineering

at Delft University of Technology,
to be defended publicly on 18 July 2016.

Student number: 1376608

Committee: Prof.dr.ir. F. van Keulen TU Delft (Chairman)
Dr.ir. F. Alijani TU Delft
Prof.dr.ir. W. Lacarbonara Sapienza - Università di Roma
Dr.ir. S. N. Voormeeren Siemens Wind Power
Dr.ir. A. Tsouvalas TU Delft

Faculty of Mechanical, Maritime and Materials Engineering
Department of Precision and Microsystems Engineering
Specialization: Engineering Mechanics

An electronic version of this thesis is available at: http://repository.tudelft.nl.

Cover photo: www.siemens.com/press/IM2015100044WPEN
Abstract

Offshore wind energy is one of the main leaders in sustainable energy. For water depths up to around 40 m, the monopile support structure is preferred, mainly because of its simplicity. The resulting slender structure including the tower above sea level is however poorly damped in the first bending modes. As a result, expensive support structures are needed to limit the fatigue damage caused by the cyclic hydrodynamic loadings from the sea. The situation can be improved by installing a Tuned Mass Damper (TMD), which consists of a heavy body, a spring mechanism and an optional damper. The first bending mode can be optimally damped by installing the TMD high in the structure, where the response amplitude is large. In order to calculate the fatigue damage in the wind turbine for many different scenarios, fast simulation tools are of interest. Since the displacements and velocities of the wind turbine are relatively small, the dynamics can be linearized without significant loss of accuracy. A TMD however may accompany nonlinearities that cannot easily be discarded. In this work, the nonlinear behavior of end-stops is studied, which can be designed to prevent the TMD from colliding with the (internal) structure. Frequency domain methods for nonlinear dynamics were investigated as faster alternative to conventional nonlinear time-integration.

Frequency domain based solution techniques for nonlinear dynamics can be classified into those containing just a single or (possibly) multiple harmonics in the solution and/or external load. The hydrodynamic loads in the problem are derived from a wave frequency spectrum. At the same time, this spectrum is concentrated around a peak period. Therefore both a single and a multiple harmonic solution technique are considered. Sequential path-following is chosen as single harmonic technique, as it is a simple and robust method widely applied in nonlinear dynamical system analysis, while harmonic balancing based on alternating frequency / time (AFT-HB) is chosen as the algorithm for multiple harmonics. The AFT-HB algorithm has the main advantage of easy control over the number of chosen harmonics in the solution. The method has been applied successfully in past studies to several mechanical systems under single harmonic external loads, and also to external loads containing a few harmonic components. The application scope of the AFT-HB method is extended in this work to a mechanical system with multiple degrees of freedom containing an extremely stiff end-stop nonlinearity, under spatially distributed external loads containing in the order of $10^2$ harmonics. A nonlinear Newmark time-integration based algorithm is applied in this work for numerical verification of the results obtained by the frequency domain methods.

All solution techniques discussed in the previous paragraph are applied to one and two degree of freedom systems, in order to obtain a step-wise verification, and to gain insight in nonlinear end-stop behavior. Two extreme cases of external loads are considered: single harmonic, and a spectrum with uniform distribution, and random phase correlation
between the load harmonics. The AFT-HB algorithm is found to accurately reproduce the time-integration results, for all considered cases. Reasonable consistency is found for the sequential path-following applied to single harmonic external load cases. The results for single harmonic loads are further validated with an existing renowned pseudo-arclength path-following tool. Finally, both techniques are applied to a reduced finite element model of a wind turbine, with a TMD and an end-stop. The AFT-HB method shows promising potential in reproducing the internal bending moments, under multi-harmonic loading, while this is found far from the case for sequential path-following after comparison with time-integration results. From the latter it is also shown that an external load with multiple harmonics is needed to represent hydrodynamic loading from sea. Gains in computational speed in the order of a factor 10 with respect to time-integration are accomplished by the AFT-HB method, depending strongly on the severity of the external loading. The fatigue damage equivalent loads (DEL) are not accurately reproduced by either method, and it is left as a recommendation to improve the harmonic balancing method or its implementation in calculating DEL results.

The last contribution of this work is a non-monolithic frequency domain based approach to solving mechanical systems with multiple degrees of freedom. In monolithic approaches, the dynamics of the full model containing nonlinearity is solved at each iteration step. In non-monolithic algorithms, local iterations are performed on linear and nonlinear parts of the system separately, while global iterations solve for the combined system, leading to significant increase in computational speed in existing time-integration techniques. In this work, a frequency domain alternative is proposed for single external loading, based on sequential path-following. From a two degree of freedom system with end-stop nonlinearity, the pre-computed numerical response from a standalone base-excitation problem for the degree of freedom associated to the end-stop nonlinearity is used to compute the response of the coupled system afterwards. The results matched those obtained by monolithic sequential path-following, while a factor 8 in computational speed is gained.
Acknowledgements

During this thesis project I felt very privileged. I was given the chance to crystallize and investigate a very challenging problem in my own way, leading to my most significant academic accomplishment so far. This accomplishment was not possible without the help of several people. I would like to start by thanking everybody who has had a supervising/coaching role in my project.

Sven Voormeeren has consistently supported me throughout this entire project, both academic and practical. As this project made several turns, he challenged me to keep an optimistic, independent and confident stance. In this way I kept learning, took a firm grip on the project, and exploited my own potential and the opportunities that came along our path. Walter Lacarbonara has helped me during different phases of the project, and I would like to thank him for his help and for feeding positive energy into the project. The discussions we had over skype were of good help, and he has also shown great enthusiasm for my work, and my ideas - this really boosted my confidence. Also, I would like to thank Biagio Carboni for his suggestions during the skype conversations, and for his extensive support during my stay in Rome. Farbod Alijani has always provided instant support and ideas whenever it was needed. Together with Walter he challenged me to explore harmonic balancing as a method for my research, which ultimately led to the most important results of my work. Fred van Keulen was the formal supervisor of this work, and he has helped me during the progress meetings. Despite not being specialized in nonlinear dynamics, he has given very useful feedback, including the suggestion to focus on end-stop nonlinearity, which is key to this work.

Further, I would like to thank everybody who has ever taught me something in a way that made me passionate about a subject. Not only have you inspired me to learn more, but also to explain what I have learned to others, which is something I truly enjoy. My academic learning path has been fueled by inspiring lectures and books. When I had just finished my BSc in applied physics, I was looking for something different. I decided to see if mechanical engineering was something for me. When I had listened to a couple of lectures given by Paul van Woerkom from the BSc course “Dynamica 2”, I became truly passionate about the subject, decided to follow the corresponding specialization in the MSc and the result is what I am writing here.

I would also like to thank my fellow students, and the other employees at Siemens Wind Power in The Hague for making me feel at home, and for helping me spontaneously whenever it was needed. I was pleased to interact with you in several settings, for example at the football table - my apologies to the department for the fatigue damage that I have caused to several components of the table, as a result of my enthusiastic playing style.

I would like to thank my close friends for their support. You guys make me smile! Minke en Margreet, jullie zijn geweldig - bedankt voor wie jullie zijn! Papa en mama,
ik ben trots op jullie. Onze tijd vroeger is gemonet door verschillende landen, culturen, kleuren en talen - door alles heen bleven jullie trouw aan het gezin, en aan de Heer - en bovenal blijft Hij trouw aan ons!
Contents

Abstract i
Acknowledgements iii
List of Figures viii
List of Tables xiii
Nomenclature xv

1 Introduction 1
1.1 Research context . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1
1.2 Tuned Mass Dampers in Offshore Wind Turbines . . . . . . . . . . . . . . 3
1.2.1 Examples . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3
1.2.2 Nonlinearities . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5
1.3 Load calculation procedures . . . . . . . . . . . . . . . . . . . . . . . . . . 6
1.3.1 Motivation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 6
1.3.2 Methods for nonlinear structural dynamics . . . . . . . . . . . . . . 7
1.4 Thesis objective and outline . . . . . . . . . . . . . . . . . . . . . . . . . . 8

I Theory & Modeling 9

2 Time and frequency domain fatigue load calculations 11
2.1 Wave load model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 11
2.2 Time-domain load calculations . . . . . . . . . . . . . . . . . . . . . . . . . 13
2.2.1 Load timeseries . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 13
2.2.2 Internal loads from time simulation . . . . . . . . . . . . . . . . . . . . 13
2.2.3 Rainflow Counting . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14
2.2.4 Calculating fatigue loads . . . . . . . . . . . . . . . . . . . . . . . . . . 14
2.3 Frequency domain load calculations . . . . . . . . . . . . . . . . . . . . . . 14
2.3.1 Structural transfer functions . . . . . . . . . . . . . . . . . . . . . . . . 15
2.3.2 Structural response spectra . . . . . . . . . . . . . . . . . . . . . . . . 16
2.3.3 Dirlik’s method for calculating fatigue loads . . . . . . . . . . . . . . 17
2.3.4 Nonlinear structural models . . . . . . . . . . . . . . . . . . . . . . . . 17
2.4 Summary and outlook . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 19
3 Modeling of structures with Tuned Mass Dampers 21
  3.1 Linear 2DoF damped structure ........................................... 21
    3.1.1 Tuning the TMD parameters ........................................... 22
  3.2 Nonlinearities ............................................................. 24
    3.2.1 External loads .......................................................... 24
    3.2.2 Internal forces: geometric and material nonlinearities ............. 24
    3.2.3 Internal forces: end-stop nonlinearity ................................ 24
  3.3 Extension to mDoF systems ................................................. 26
    3.3.1 Dynamic Substructuring ............................................... 26
    3.3.2 Application to monopile + TMD ...................................... 28
  3.4 Model order reduction .................................................... 29
  3.5 Summary and outlook ..................................................... 31

4 Solution strategies for nonlinear structural dynamics 33
  4.1 Newton-Raphson method for nonlinear systems .......................... 34
  4.2 Time domain methods (monolithic) ...................................... 34
    4.2.1 Time-integration (single / multi-harmonic) ......................... 35
    4.2.2 Pseudo-arclength path-following (single / multi-harmonic) ........ 35
  4.3 Frequency domain based methods (monolithic) .......................... 35
    4.3.1 Sequential path-following (single harmonic) ........................ 36
    4.3.2 Perturbation Techniques (single / multi-harmonic) .................. 37
    4.3.3 Harmonic Balancing (single / multi-harmonic) ....................... 38
  4.4 Non-monolithic methods .................................................. 43
    4.4.1 Time-integration (single / multi-harmonic) ........................ 43
    4.4.2 Sequential path-following (single harmonic) ....................... 43
  4.5 Conclusions and outlook ................................................ 45

II Case studies & Results 47

5 End-stop behavior for one and two DoF systems 49
  5.1 Duffing Oscillator ....................................................... 50
  5.2 1DoF oscillator + fractional-order spring ............................ 51
  5.3 1DoF oscillator with fractional-order end-stop ........................ 52
    5.3.1 Single harmonic external loading .................................... 53
    5.3.2 Multi harmonic external loading .................................... 54
  5.4 2DoF damped structure with fractional-order end-stop .................. 59
    5.4.1 Single harmonic external forcing, monolithic approach .......... 59
    5.4.2 Single harmonic external forcing, non-monolithic approach ...... 60
    5.4.3 Multi harmonic external forcing, monolithic approach .......... 62
  5.5 Including hysteresis in the end-stop model ............................ 63
  5.6 Summary and outlook .................................................... 67

6 Case study on an offshore wind turbine model including a TMD 69
  6.1 Wind turbine and external loads ....................................... 69
    6.1.1 Offshore Wind Turbine model ....................................... 69
    6.1.2 Sea states ............................................................ 70
  6.2 Results for displacement response ..................................... 71
6.3 Results for internal bending moments ........................................... 71
  6.3.1 Mild sea state: $H_s = 1m, T_p = 4.2s$ .................................. 72
  6.3.2 Moderate sea state, $H_s = 4m, T_p = 8.3s$ .......................... 72
6.4 Results for fatigue damage .......................................................... 73
6.5 Computational speed ................................................................. 75
6.6 Discussion of the results .............................................................. 75

7 Conclusions & Recommendations 77
  7.1 Conclusions ................................................................. 77
  7.2 Recommendations ............................................................ 78

A Newmark Time integration method 81
  A.1 Constant average acceleration: $\beta = \frac{1}{4}, \alpha = \frac{1}{2}$ .................. 82
  A.2 Application to structural dynamics ........................................... 82

Bibliography 85
List of Figures

1.1 Foundations for offshore wind turbines [12]. From left to right: monopile, tripile, jacket, gravity-based. .................................................. 2
1.2 Location of the natural frequency $\omega_0$ of the monopile structure (red line), between the sea spectrum (blue line) and the 3P band [16]. ................. 2
1.3 Fore-Aft (left) and Side-Side modes (right) [15]. When in operation, the wind turbine structure is most vulnerable to fatigue loads when wind and wave are misaligned, causing cyclic wave loads in the Side-Side mode. .... 3
1.4 Schematic of a TMD and its position on the wind turbine structure. In order for the TMD to optimally attack the first bending mode shown in Fig. 1.3, the TMD must attached to a relatively high position in the structure. A practical and safe location is at a high point within the tower housing. ... 4
1.5 Examples of Tuned Mass Damper systems [15]. .......................... 4
1.6 Nonlinear and linearized model. .................................................... 5
2.1 Wave load JONSWAP spectrum for $H_s = 1m$, $T_p = 4s$. ................. 12
2.2 Obtaining the linear structural response spectra $S_{R,m}^m(\omega)$ for an output DoF $m$ of interest, corresponding to the output node (marked red). $S_{R,m}^m(\omega)$ is obtained by summing the squared load transfer functions $|H_{m1}(\omega)|^2$ from the different input DoF to the output DoF $m$, multiplied by the respective wave load spectra $S_{f,z,k}(\omega)$. Although node $m$ may also be contain input DoFs to an output DoF at the same node, this has been omitted for clarity purposes. The red and blue vertical lines in the spectra mark the resonance frequency of the structure $\omega_0$ and the reciprocal of the peak period of the wave spectrum $\omega_p = 2\pi/T_p$. .................................................. 18
2.3 Dirlics method for obtaining fatigue load results. .......................... 18
3.1 Linear 2DoF damped structure. .................................................... 21
3.2 Frequency response of the linear 2DoF damped structure, with $\frac{m_d}{m_s} = 0.01$. The response of the stand-alone structure has been added as a reference. .... 22
3.3 Influence of $\zeta_d$ on the structure response. $\tilde{m} = 0.01$, $k_d = \tilde{k}_d$. The response of the stand-alone structure has been added as a reference. .................. 23
3.4 Splitting a structure up into two substructures, where the chosen boundary DoF $u_b$ are copied to be included in the DoF of both substructures. The unknown interface forces/moments $g^{(s)}$ working between the substructures are treated as external forces on the substructures, indicated with arrows. ... 26
4.1 Difference between linear and nonlinear solution scheme. The linear scheme takes one computation step in order to find equilibrium $\hat{q}$, while the nonlinear scheme needs to iterate three times, where the tangent stiffness $\partial f / \partial q$ needs to be computed at each iteration step $^{(k)}$. 34

4.2 Harmonic Balancing using the Alternating Frequency / Time method: 3 cm
1. Construct a timeseries from state vector $\hat{q}^{(m)}$ through the Inverse Discrete Fourier Transform. 2. Compute the nonlinear force, as well as its derivative, as timeseries. 3. Obtain the harmonic coefficients of the nonlinear force, as well as the Jacobian, through the Discrete Fourier Transform. 4. Compute the residual in the harmonic equation of motion. 5. Compute and apply the update of the state vector. 6. Check whether convergence is reached. If not, proceed to step 7 and repeat steps 1-5. 7. Update the iteration counter $m$. 41

4.3 Original (coupled) system, and base-excitation problem. 44

5.1 Frequency response for the Duffing oscillator, under single harmonic external loading, with $k_2 = k$. The response is scaled to the linear static response $u_{0,lin}$. The pseudo-arclength path-following algorithm [21] was borrowed and applied with permission. 50

5.2 Time-integration results of the steady-state response for the Duffing oscillator, $\omega = 1.25\omega_0$. 51

5.3 Frequency response of the fractional order oscillator, under single harmonic external forcing. The response is scaled to the linear static response $u_{0,lin}$. The pseudo-arclength path-following algorithm [21] was borrowed and applied with permission. 52

5.4 Frequency components resulting from the Harmonic Balance method, for the oscillator with the fractional order nonlinearity. The superharmonic observed in 5.3 at $\omega = 0.5\omega_0$ is revealed, with relatively high contributions from the higher harmonic components. 53

5.5 1DoF oscillator with end-stops, indicated by the vertical red barriers. 53

5.6 Frequency response of the oscillator with the end-stop nonlinearity, under single harmonic external forcing. The response is scaled to the linear static response $u_{0,lin}$. The pseudo-arclength path-following algorithm [21] was borrowed and applied with permission. 55

5.7 Time-integration results of the steady-state response for the oscillator with nonlinear end-stop, $\omega = 1.05\omega_0$. 55

5.8 Frequency components resulting from the Harmonic Balance method, for the oscillator with the end-stop nonlinearity, $\omega = 1.05\omega_0$. 56

5.9 Frequency response of the oscillator with the end-stop nonlinearity, under single harmonic external forcing, for different relative end-stop stiffnesses. The response is scaled to the end-stop free distance $b_{free}$. PSA = Pseudo-Arclength. The Harmonic Balancing method was carried out using 7 harmonics. The pseudo-arclength path-following algorithm [21] was borrowed and applied with permission. 56

5.10 Time-series sample of the multi-harmonic forcing as as defined in Eq. 5.8. The period of the time-simulation is bordered by the red vertical lines. 57

5.11 Time-response of the oscillator with the end-stop nonlinearity, under multi harmonic external forcing. 57
5.12 Frequency response of the oscillator with end-stop, under multi-harmonic forcing as defined in Eq. 5.8, obtained by windowing and calculating the FFT of a displacement response timeseries. The result is normalized to the static linear response $u_{0,\text{lin}}$. .......................................................... 58

5.13 Frequency response of the oscillator with the end-stop nonlinearity, under multi harmonic external forcing, averaged over 134 different realizations of random phase in the external load (Eq. 5.8). .................. 58

5.14 2DoF damped structure with end-stops limiting the motion of the damper mass $m_d$. The end-stop mechanism, drawn in red, is connected rigidly to the structure mass $m_s$. .......................................................... 59

5.15 Frequency response of the 2DoF damped structure with the end-stop nonlinearity, under multi harmonic external forcing. The linear response (with no end-stop) is shown as reference. The sequential path-following method is in this case called Forward Newton-Raphson. ...................... 60

5.16 Time response of the 2DoF structure + TMD with the end-stop nonlinearity, under single harmonic external forcing, $\Omega = \omega_s$. .................. 61

5.17 Time response of the 2DoF structure + TMD with the end-stop nonlinearity, under single harmonic external forcing, $\Omega = 1.03\omega_s$. .................. 61

5.18 Displacement transfer map, as a function of base excitation force level and forcing frequency. .......................................................... 63

5.19 Frequency response of the structure mass for the 2DoF structure with TMD under single harmonic external loading, obtained with both the monolithic and non-monolithic sequential path-following method. .................. 64

5.20 Frequency response of the 2DoF damped structure with the end-stop nonlinearity, under multi harmonic external forcing. .................. 64

5.21 Time response of the oscillator with the end-stop nonlinearity, under multi harmonic external forcing. .......................................................... 65

5.22 The effect of including hysteresis in the endstop force on the frequency response of the structure mass. The response is constructed by time-simulations where the structure mass is harmonically forced. .................. 65

5.23 Time domain simulation results for the relative displacement between structure and damper, $u_{d}-u_s$, with $e_R = 0.10$. Both figures represent the same timeseries, with different plot ranges. .................. 66

5.24 Time-integration results for the relative displacement between structure and damper, $u_{d}-u_s$, with $e_R = 1.00$. Both figures represent the same timeseries, with different plot ranges. .................. 66

6.1 Schematic of the FEM model for the offshore wind turbine system consisting of the foundation and the tower, with the RNA and the TMD. ............... 70

6.2 Time-response of the relative displacement between structure and TMD resulting from two realizations of external force timeseries representing different sea-states. Results have been plotted versus dimensionless time $t = \omega_0 t$. ........... 71

6.3 Results for the normalized internal bending moments at interface and mudline positions, with sea state $H_s = 1m, T_p = 4.2s$, obtained by different methods. .................. 73
6.4 Results for the normalized internal bending moments at interface and mudline positions, with sea state $H_s = 4m, T_p = 8.3s$, obtained by different methods.
List of Tables

2.1 Definition of parameters used in Eq. 2.15. ............................................. 15
2.2 Definition of parameters used in Eq. 2.28. ............................................. 17

4.1 Overview of solution strategies for nonlinear structural dynamics, with references to the sections in this chapter. Combinations marked bold/italic have been applied in this work, and a star (*) indicates that additions to the existing methods have been proposed and applied. TD = Time Domain, FD = Frequency Domain. ......................................................... 33

5.1 Coefficients of the polynomial giving $\sum_{r=1}^{7} c_r u^r$ as an approximation of $\text{sign}(u)k_2|u|^2$, obtained with the MATLAB function 'polyfit'. ............... 51
5.2 Results for simulation time needed to obtain the frequency response of the 2DoF structure with TMD, under single harmonic external loading, using sequential path-following. ......................................................... 62
5.3 Results for maximum penetration depth, for different hysteresis levels in the end-stop force ............................................................... 66

6.1 Normalized Damage Equivalent Loads (DEL) for the internal bending moment relative to the situation with no TMD, obtained with different solution techniques. The time-integration results also serve as reference, and are shown in italic. In particular, the relative increases in DEL due to the end-stop, are shown in bold. TI = time-integration, HB = harmonic balancing, SPF = sequential path-following. ..................................................... 75
6.2 Computational times (minutes) need to obtain the fatigue damage results for the internal bending moments in Table 6.1, and the spectra in Figs. 6.3 - 6.4. .............................................................. 75
# Nomenclature

**Latin symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>cosine components of solution in harmonic balancing</td>
<td>m</td>
</tr>
<tr>
<td>$B$</td>
<td>boolean matrix</td>
<td>-</td>
</tr>
<tr>
<td>$b_{free}$</td>
<td>free displacement before end-stop</td>
<td>m</td>
</tr>
<tr>
<td>$b$</td>
<td>sine components of solution in harmonic balancing</td>
<td>m</td>
</tr>
<tr>
<td>$C$</td>
<td>damping matrix</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$c$</td>
<td>cosine components of nonlinear force in harmonic balancing</td>
<td>N</td>
</tr>
<tr>
<td>$C_M$</td>
<td>inertia coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$\ddot{c}$</td>
<td>end-stop damping constant</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of monopile foundation</td>
<td>m</td>
</tr>
<tr>
<td>$d$</td>
<td>sine components of nonlinear force in harmonic balancing</td>
<td>N</td>
</tr>
<tr>
<td>$D_s$</td>
<td>damage done to structure</td>
<td>-</td>
</tr>
<tr>
<td>$d_w$</td>
<td>water depth</td>
<td>m</td>
</tr>
<tr>
<td>$e$</td>
<td>end-stop restitution factor</td>
<td>-</td>
</tr>
<tr>
<td>$F$</td>
<td>external force</td>
<td>N</td>
</tr>
<tr>
<td>$f^{NL}$</td>
<td>nonlinear internal force</td>
<td>N</td>
</tr>
<tr>
<td>$g$</td>
<td>interface force between substructures</td>
<td>N</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>$g_{DC/cos/sin}$</td>
<td>equation of motion components in harmonic balancing</td>
<td>N</td>
</tr>
<tr>
<td>$H$</td>
<td>structural transfer function</td>
<td>m/N, rad/N, m/nM, rad/Nm</td>
</tr>
<tr>
<td>$H_s$</td>
<td>significant wave height</td>
<td>m</td>
</tr>
<tr>
<td>$J$</td>
<td>Jacobian matrix</td>
<td>N/m</td>
</tr>
<tr>
<td>$K$</td>
<td>stiffness matrix</td>
<td>N/m</td>
</tr>
<tr>
<td>$K$</td>
<td>number of harmonics in harmonic balancing solution</td>
<td>-</td>
</tr>
<tr>
<td>$k_2$</td>
<td>stiffness of nonlinear internal force</td>
<td>N/m</td>
</tr>
<tr>
<td>$k_{wave}$</td>
<td>wave number</td>
<td>-</td>
</tr>
<tr>
<td>$K_{el}$</td>
<td>element stiffness matrix</td>
<td>N/m</td>
</tr>
<tr>
<td>$\tilde{k}_d$</td>
<td>optimal TMD stiffness</td>
<td>N/m</td>
</tr>
<tr>
<td>$L$</td>
<td>localization matrix</td>
<td>-</td>
</tr>
<tr>
<td>$L$</td>
<td>load range</td>
<td>N, Nm</td>
</tr>
</tbody>
</table>

---

*Note: The table represents the nomenclature used in the document.*
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>mass matrix</td>
</tr>
<tr>
<td>$m$</td>
<td>Wohler slope</td>
</tr>
<tr>
<td>$\dot{m}_d$</td>
<td>relative damper mass</td>
</tr>
<tr>
<td>$N_a$</td>
<td>allowed number of stress cycles</td>
</tr>
<tr>
<td>$N_c$</td>
<td>number of stress cycles</td>
</tr>
<tr>
<td>$p$</td>
<td>harmonic components of external force in harmonic balancing</td>
</tr>
<tr>
<td>$q$</td>
<td>solution variable in Newton-Raphson iteration scheme</td>
</tr>
<tr>
<td>$r$</td>
<td>residual in iterative scheme</td>
</tr>
<tr>
<td>$R$</td>
<td>modal reduction matrix</td>
</tr>
<tr>
<td>$s$</td>
<td>complex external force amplitude</td>
</tr>
<tr>
<td>$S$</td>
<td>spectral density function</td>
</tr>
<tr>
<td>$T_{life}$</td>
<td>lifetime of structure</td>
</tr>
<tr>
<td>$T_{sim}$</td>
<td>simulation time</td>
</tr>
<tr>
<td>$\tilde{t}$</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>$T_p$</td>
<td>peak period</td>
</tr>
<tr>
<td>$u$</td>
<td>displacement</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity</td>
</tr>
<tr>
<td>$w$</td>
<td>solution components in direct harmonic balancing</td>
</tr>
<tr>
<td>$x$</td>
<td>dimensional mode shape</td>
</tr>
<tr>
<td>$Y$</td>
<td>receptance matrix</td>
</tr>
<tr>
<td>$Z$</td>
<td>dynamic stiffness matrix</td>
</tr>
<tr>
<td>$z$</td>
<td>complex displacement amplitude</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>vertical position</td>
</tr>
</tbody>
</table>

**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\alpha}$</td>
<td>numerical softening in Newton-Raphson update</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>order of nonlinear polynomial force</td>
</tr>
<tr>
<td>$\beta$</td>
<td>end-stop stiffness</td>
</tr>
<tr>
<td>$\delta$</td>
<td>end-stop penetration depth</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>convergence criterion</td>
</tr>
<tr>
<td>$\eta$</td>
<td>modal participation factor</td>
</tr>
<tr>
<td>$\dot{\eta}$</td>
<td>intensity of internal vibration modes in Guyan reduction</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>eigenfrequency matrix</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>peak enhancement factor</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength of surrounding fluid</td>
</tr>
<tr>
<td>$\mu$</td>
<td>modal mass</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>angular forcing frequency</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>linear natural frequency</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency</td>
</tr>
</tbody>
</table>
\( \bar{\omega}_d \) optimal TMD angular frequency \( \text{rad/s} \)
\( \tilde{\omega} \) dimensionless frequency -
\( \phi \) phase in external force \( \text{rad} \)
\( \Psi \) cosine/sine part of external force in harmonic balancing \( N \)
\( \rho_w \) water density \( \text{kg/m}^3 \)
\( \sigma \) spectral bandwidth \( \text{rad/s} \)
\( \theta_z \) rotation about z-axis \( \text{rad} \)
\( \bar{\zeta}_d \) optimal TMD damping ratio \( \text{Ns/m} \)

**Subscripts**
- \( b \) boundary
- \( d \) (tuned mass) damper
- \( ES \) end-stop
- \( i \) internal
- \( s \) (offshore wind turbine) structure
- \( (s) \) mode number
- \( w \) water

**Superscripts**
- \( (d) \) damper substructure
- \( el \) element
- \( (m) \) monopile substructure
- \( NL \) nonlinear
- \( (s) \) substructure index

**Abbreviations**
- AFT alternating frequency-time
- CB Craig-Bampton
- DEL damage equivalent load
- DoF degree(s) of freedom
- EoM equation(s) of motion
- FLS fatigue limit state
- HB harmonic balancing
- mDoF multi-degree-of-freedom
- NR Newton-Raphson
- PF path-following
- PSA pseudo-arclength
- sDoF single-degree-of-freedom
- TMD tuned mass damper
- ULS ultimate limit state
Chapter 1

Introduction

1.1 Research context

Offshore wind energy

In a world with a growing energy demand, climate change and a decline in resources from fossil fuels, there is a strong need for renewable energy. Wind energy is one of the main leaders in renewable energy, making up for over 40% of the newly installed capacity in 2014 [28], and since 1997 there has been an annual growth of no less than 10% in cumulative installed wind capacity. Offshore locations provide better wind resources than most onshore sites, with higher mean wind speeds, less turbulence and less wind shear. Furthermore, the spatial availability is much greater. These offshore systems are however more expensive than onshore systems because of the support structures that must withstand additional wave loading and the grid connections must be installed under water. A 40% cost reduction has been agreed in industry by 2020 with respect to 2012 in order for offshore wind energy to become economically viable [27].

Monopile foundation structures

Several types of foundation structures exist for offshore wind turbines, and the choice depends heavily on the water depth (Fig. 1.1). Monopile foundations lead the offshore wind energy market, with a total installed share of 75% in 2013 [1]. In particular for water depths up to 40 m as studied in this work, monopile structures are almost always the preferred choice. This is mainly because the structure is the most simple of all, as it consists of the fewest components / materials. This not only makes monopiles the most affordable, and least complex to install, but also demands the easiest supply chain.

Lack of damping

The main problem with monopiles is that they are poorly damped in the first bending mode. Because of this, they are usually designed such that the corresponding natural frequency lies safely outside of the dominant frequency spectra associated with two external loads: the sea wave spectrum, and the periodic wind-induced loading through the blades (as shown in Fig. 1.2). The latter is characterized by two bands: one corresponding to the rotational frequency of the blades (1P), and the other to 3 times (3P) this frequency (since...
1. INTRODUCTION

Figure 1.1: Foundations for offshore wind turbines [12]. From left to right: monopile, tripile, jacket, gravity-based.

there are 3 blades). The 1P and 3P frequencies are bands rather than single frequencies, since the rotational speed varies with the wind speed.

Figure 1.2: Location of the natural frequency $\omega_0$ of the monopile structure (red line), between the sea spectrum (blue line) and the 3P band [16].

From Fig. 1.2 it can be noted that the natural frequency of the monopile structure $\omega_0$ is caught inside the tail of the wave load spectrum. Moreover, this spectrum belongs to a specific sea state, as will be explained in Chapter 2. For other sea states, the peak frequency (also called $1/T_p$ with $T_p$ the peak period) may vary towards values close to $\omega_0$.

The main sources of damping are aerodynamic damping, structural damping and damping due to the soil, in which the foundation is embedded. The monopile structure is most vulnerable to fatigue loads in situations where the aerodynamic damping is low, leaving the structure only with structural and soil damping. There are two main situations where this is the case. The most obvious is idling: when the blades are very slowly rotating, which is when the wind speed is outside of the operational range. The second is when the wind and wave directions are misaligned. The first bending modes of the structure are in fact two, since it can bend in two directions in the horizontal plane. The convention is to define the two modes in the direction normal to the rotor plane (Fore-Aft) and perpendicular to the blade plane (Side-Side) - see Fig. 1.3. Thus, the Side-Side mode of the structure is poorly damped when the turbine is in operation.

**Damper Systems**

Decreasing the amplitude of vibration in the first bending mode (and hence, resulting fatigue loads) would make the construction cheaper and more sustainable, as less material would be needed to reach the same design lifetime. For this purpose one must find a way to increase the total damping of the structure. This can be done by increasing the structural damping in the monopile construction itself, or by attaching a damper system tuned to specifically attack the modes of interest. Increasing the structural damping of
1.2 Tuned Mass Dampers in Offshore Wind Turbines

Most structural damper systems are designed for onshore, high-rise buildings, while the application of these systems in offshore wind turbines is in a more explorative/experimental phase, and aims to reduce loads induced mainly by sea waves, which are different in nature than wind- or seismic loadings, for which onshore damping systems are designed. Several passive damper systems exist, to be divided into three main categories: systems containing only stiffness (via a spring mechanism), those containing only a damping mechanism and systems having both - the latter are called Tuned Mass Dampers (TMD). In offshore wind turbines, only TMD’s are considered, because they are the only types of damper systems that allow for tuning of natural frequency, as well as damping, as explained further in Chapter 3. The schematic of a TMD is shown in Fig. 1.4, as well as its typical position on the structure. A few examples of TMD’s are shown in the following.

This section starts with few examples of TMD’s in Sec. 1.2.1, followed by an explanation of how these devices introduce nonlinearities in the combined system in Sec. 1.2.2, as modeling these nonlinearities is the main focus of this work.

1.2.1 Examples

Tuned Pendulum Damper. Tuned Pendulum Dampers are generally employed in high-rise buildings, to counteract seismic and wind loadings. Although there is some damping present in the hinge connection, and the connections between the body and the cables, the damping level is generally low and not controlled. Damping can be added to an ordinary Pendulum Damper, for example through pistons with viscous fluids, connected to the countermoving mass. These tuned pendulum dampers are used in high-rise buildings to effectively reduce seismic loads. Eg Taipei101, see Fig. 1.5b.
1. INTRODUCTION

**Tuned Mass Damper**

![Schematic of a TMD and its position on the wind turbine structure.](image)

Figure 1.4: Schematic of a TMD and its position on the wind turbine structure. In order for the TMD to optimally attack the first bending mode shown in Fig. 1.3, the TMD must attached to a relatively high position in the structure. A practical and safe location is at a high point within the tower housing.

(a) Tuned Pendulum Damper as in Taipei101. Pendulum dampers consist of a solid body of high density material (e.g., steel), suspended somewhere inside the structure (usually in the top) via cables. The eigenfrequency of oscillation of the pendulum can be determined beforehand by choosing the cable length and the mass.

(b) Tuned Liquid Column Damper. In a TLCD, the stiffness of the system is caused by a height difference of the fluid within two columns. Damping is obtained by friction within the moving fluid, dissipating as heat.

![Examples of Tuned Mass Damper systems](image)

Figure 1.5: Examples of Tuned Mass Damper systems [15].

**Tuned Liquid Column Damper.** Often the Tuned Liquid Column Damper is categorized under TLD’s (Tuned Liquid Dampers). However, the dynamics are very similar to that of a TMD, if the fluid is considered incompressible, and side-effects / turbulence are neglected. The main deficit is the damping tunability, as the damping is provided mainly by friction in the water motion. The damping could be tuned further by substituting water with a different viscous fluid. G. Stewart and M. Lackner [34] set up a limited D. O. F. model of the monopile, and showed the potential to reduce fatigue loads from wind/wave excitation up to 55%. However, validation is needed with a more detailed FEM model, including an appropriate soil model.
1.2. Nonlinearities

Nonlinearities are present in all physical systems. In many cases, however, the nonlinear effect ends up being relatively small within the operating range of the system. In these cases, the nonlinear behavior can be linearized. This is illustrated in Fig. 1.6 for a nonlinear relationship between the externally applied load and the resulting deflection of a one-dimensional mechanical system, yielding a linear model.

Introducing a TMD in a monopile structure complicates the modeling of the structural dynamic behavior in the monopile. The monopile is a slender structure, fixed to the ground, and undergoes relatively small deformations due to the external wind/wave loads. Therefore the monopile’s dynamics can often be linearized, whilst maintaining sufficient accuracy in the load calculations. A TMD on the other hand may exhibit several nonlinearities that can not easily be discarded. This is mainly because the TMD is designed to exhibit a stroke that is typically large compared to its size, in order to effectively dampen the structure to which it is attached. Aside from a large displacement, a damper system can exhibit various other behaviors such as hysteresis, internal dissipation of energy as heat and end-stop behavior. These behaviors introduce nonlinearities in the total model, depending on the type of TMD.

Geometric vs. material nonlinearities

Geometric nonlinearities cause nonlinear load-displacement behaviour due to the change in the geometrical configuration of the system with respect to the reference position. In a pendulum damper, for instance, the linearization of stiffness force/moment due to gravity could no longer be valid for large enough angular displacements. Material nonlinearities are nonlinear stress-strain relationships that are inherent to the materials or constructions. In hysteresis-based TMD’s the stress-strain relationship is designed to be non-holonomic in order to generate dissipative forces/moments.

End-stop nonlinearity

As a TMD experiences large displacements, an end-stop effect can occur, either as a direct result from the configuration, or built in purposefully. An example of an end-stop resulting from the configuration is a Tuned Liquid Column Damper, where the fluid end can reach the bottom corners of the container upon large oscillations. In this case, turbulent effects may occur, by which the phase advantage of the TMD motion is lost and its performance may be severely degraded. For any kind of Tuned Mass Damper with a solid body, suitable
end-stops should be implemented on purpose in order to prevent the TMD from colliding with the structure walls. In this work, particular focus is given to this specific end-stop nonlinearity, for a few reasons:

- Once the end-stop is hit, the dynamical behavior of the TMD can be changed significantly. Firstly because the phase difference between TMD and structure motion is disturbed. Secondly, because the nonlinearity does not work over the entire range of the displacement, in which case the associated load-displacement relationship could be linearized.

- End stop mechanisms exist in various types of Tuned Mass Dampers, either intentionally built for safety reasons, or automatically present. Furthermore, the end-stops of TMD’s are a specific example of impact dynamics in general, which applies to any situation where collision between structures occurs. Therefore, a far larger class of problems than TMD’s in offshore wind turbines can benefit from a good calculation method for end-stop behavior.

Load calculation procedures for Offshore Wind Turbine structures can be affected by nonlinearities such as end-stops, as described in the following section.

1.3 Load calculation procedures

1.3.1 Motivation

Every offshore wind farm operates under a unique combination of circumstances: wind/water current and wave levels/directions, water depth and soil properties. These circumstances can be collected as a set of wind farm data, and are passed as inputs to the load calculation procedure that predicts the design driving loads upon the structure in its initial design. The initial design is updated in order to meet the lifetime requirements that are placed upon the offshore wind turbine. The loads calculation procedure is repeated for the updated design, and this process continues until the lifetime requirements are met.

It becomes clear that the load calculations have to be carried out iteratively for each specific project. Consequently, it is an important challenge to optimize the load calculation procedure to be fast, whilst maintaining accuracy in the load results.

Lifetime load scenarios: ULS and FLS

Typically there are two types of loading upon the monopile structure that are considered in structural design procedures, namely ultimate loading and cyclic loading. The ultimate loads that the structure undergoes are described by the Ultimate Limit State (ULS), and the cyclic loading is described by the Fatigue Limit State (FLS).

**Ultimate Limit State (ULS)** To prevent an irrestorable deformation of the structure, all ultimate loads provided by the set of ULS strains (bending, torsion, shear or stretching/compression) must be kept under their respective yield strengths, in any direction. In this way, the structure is deformed elastically, apart from rarely severe load cases for which the structure may suffer yielding, without collapsing. Further ULS criteria are buckling of the structure and soil stability.
Fatigue Limit State (FLS)  When a structure undergoes periodic loading and unloading, microscopic cracks can be formed. Over a longer period of time, more cracks are formed and they grow, causing damage to the structure known as fatigue. Fatigue can ultimately lead to failure of the structure.

Relevance of fatigue loads

The fatigue loads form the main contributor to the lifetime of the structure, since these loads are cyclic, and cause damage over a longer time. This is contrary to incidental ultimate loads, which are not supposed to cause any significant damage to the structure, since it is designed such that the structure only deforms elastically under ultimate wind/wave loading. More importantly, monopile designs are currently generally driven by FLS, rather than ULS.

1.3.2 Methods for nonlinear structural dynamics

The internal loads in an offshore wind turbine structure are derived from the cyclic stresses resulting from deformations within the structure due to the external wind and wave loads. In order to compute these deformations, a finite element structure is built in a mathematical/scientific-programming tool. In this work, MATLAB was used for this purpose. The next step is to define an external loading that interacts with the structure, causing deformation, and stress. Computing the deformations based on the external loads / initial conditions can be accomplished through time and frequency domain methods, depending on whether a linear or nonlinear structural dynamics model is used, and on the type of the external loading.

Time vs. frequency domain

In time-domain methods or time integration schemes, the initial conditions of the wind turbine structure must be defined, after which the states (displacements, velocities, accelerations) for each subsequent timestep can be obtained via a time-marching algorithm. While a representative finite-element structure in combination with a robust time domain simulation method can deliver accurate load results, these simulations can be computationally expensive, especially when a nonlinear model of the structural response is needed. The previous section explained why nonlinearities in Tunded Mass Dampers could change the entire structural dynamics modeling of a monopile system.

In frequency-domain based methods to obtain the response of a nonlinear system, the solution is expressed in harmonic components, with respective weights. If the structural dynamics model as well as the external loads are linear, a transfer function could be obtained, relating the input load to the output response as a function of frequency. Hereby the response of the system could be obtained in one single matrix-vector multiplication, which is the fastest possible method. For nonlinear models, a range of methods exist, as explained further in Chapter 4.

Single vs. multi-harmonic external loads

In time-domain simulations, the external load is also given as function of time, and may therefore contain many frequency components. The majority of frequency domain methods for computing the response of a nonlinear system, however, are based on a single harmonic
loading. In fact, no application of current frequency domain methods has been found in literature to handle a problem similar to that of this work, namely a multi-degree-of-freedom (mDoF) structure with a highly nonlinear component, subject to an external load containing many harmonic components.

1.4 Thesis objective and outline

This introduction started by motivating the attempt of making monopiles in the offshore wind energy cheaper, by introducing a Tuned Mass Damper (TMD) in Sec. 1.2.1. TMD’s can potentially decrease the fatigue loads and hence, the required amount of material. These damper systems however introduce complexity to the modeling of the structural dynamics of monopile systems, as explained in Sec. 1.2.2. In particular, end-stop nonlinearity can have a significant impact on the fatigue loads, and should be modeled and computed correctly. In order to handle this nonlinearity, time and frequency domain methods exist, as introduced in Sec. 1.3.2. Although time domain methods are capable of simulating the system in question, these methods require much computational effort. While several frequency domain methods for nonlinear dynamics exist, these techniques are limited/unexplored when it comes to modeling systems with multiple degrees of freedom, and under external loads with several harmonic components (Sec. 1.3.2). Therefore, the research goal of this work is formulated as follows:

“Investigate frequency domain methods to model / describe end-stop nonlinearity of Tuned Mass Dampers in offshore monopile structures under multi-frequency hydrodynamic loads, and determine whether these methods are capable of fast and accurate modeling of fatigue loads, relative to time domain methods.”

In chapter 2, an overview is given of load calculation procedures, time and frequency based. The mathematical modeling of structures with damper systems is explained in chapter 3. Chapter 4 gives an overview of different methods to obtain the response of systems containing nonlinearities, in particular that of end-stop nonlinearities in Tuned Mass Dampers. In chapter 5, the modeling framework as laid out in Chapters 3 and 4 is applied to one and two DoF systems, under external harmonic loads. In Chapter 6, results are presented for the offshore monopile structure as a case study. The work is completed by a set of conclusions and recommendations in Chapter 7.
Part I

Theory & Modeling
Chapter 2

Time and frequency domain fatigue load calculations

Fatigue loads due to cyclic loading on an offshore monopile structure are calculated using the internal loads in the structure, derived from the structural deformations resulting from a given external load. In the time-domain method, the external loads and resulting structural deformations are represented by timeseries, in contrast to the frequency domain method, where the responses are obtained by multiplying load input and structural transfer functions. As a result, the post-processing step in calculating the fatigue loads is different in both methods. In the time domain, this method is called Rainflow Counting, while in the frequency domain Dirlik’s method is applied. This chapter is started by presenting the wave load model governing both time and frequency domain based load calculation procedures (Sec. 2.1). In Sections 2.2 and 2.3 respectively, time domain and frequency domain based methods for load calculations are explained.

For nonlinear structural models, fatigue loads are computed through the time domain method. The frequency domain method, as used nowadays, is incompatible with structural models containing nonlinearities, as explained in Sec. 2.3.4. The main focus of this work is to find a way to incorporate nonlinear structural models in nowadays frequency domain tools for calculating fatigue loads.

2.1 Wave load model

The first step in the analysis is to obtain the height-distributed wave load spectrum. The wave load spectrum is based on a sea state, which is in the case of the current study described by just two parameters: a significant wave height $H_s$ and a peak period $T_p$. The energy distribution based on these inputs for different wave frequencies in the North sea is commonly described with the JONSWAP spectrum [3]. This distribution is, with $f = \omega / 2\pi$ and $f_p = 1/T_p$:

$$S_{JW}(\omega) = 0.3125 C(\gamma) H_s^2 \left(\frac{\omega_p}{\omega}\right)^{-5} \exp\left[\frac{5}{4} \left(\frac{\omega_p}{\omega}\right)^{-4}\right] \gamma^r,$$

$$r = \exp(\alpha),$$

where $C$ and $\alpha$ are defined as follows:
\[ C(\gamma) = 1 - 0.278 \ln(\gamma), \quad (2.3) \]
\[ \alpha = -\frac{(\omega - \omega_p)^2}{2\sigma^2 \omega_p^2}, \quad (2.4) \]
\[ \sigma = \begin{cases} 0.07 & \omega \leq \omega_p \\ 0.09 & \omega > \omega_p \end{cases}. \quad (2.5) \]

Here \( \gamma \) is a peak enhancement factor. An example spectrum is shown in Fig. 2.1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{spectrum.png}
\caption{Wave load JONSWAP spectrum for \( H_s = 1m, T_p = 4s \).}
\end{figure}

From this spectrum, the distribution of velocities and accelerations can be computed, via the dispersion relation, to obtain the wave numbers \( k_{\text{wave}} \) and water depth \( d_w \):

\[ \omega^2 = g k_w \tanh(k_{\text{wave}} d_w). \quad (2.6) \]

The velocity and acceleration spectra are computed as follows, with vertical distance \( z \) beneath the sea surface:

\[ S_V(\omega, z) = \nu S_{JV}(\omega), \quad (2.7) \]
\[ S_A(\omega, z) = \nu \omega^2 S_{JV}(\omega), \quad (2.8) \]
\[ \nu = \frac{\omega^2 \cosh^2(k_{\text{wave}} z)}{\sinh^2(k_{\text{wave}} d_w)}. \quad (2.9) \]

Given the distribution of velocities and accelerations present in the wave loads, the external loads on the structure can be computed. Morison’s equation is the leading model in current offshore engineering practice [37]. The exact form of the Morison equation depends on the geometry of the structure, which is in the case of the current study a relatively thin structure with \( \pi D/\lambda < 0.5 \) where \( D \) is the diameter of the structure, and \( \lambda \) is the (main) wavelength of the surrounding fluid. The resulting equation consists of a drag and an inertial part:
\[ dF = f_{\text{drag}} + f_{\text{inertia}}, \]
\[ = \frac{1}{2} C_D \rho_w D |v_w - v_s| (v_w - v_s) + \frac{\pi}{4} C_M \rho_w D^2 (\dot{v}_w - \dot{v}_s). \]

Here, \( \rho_w \) is the water density, \( C_M \) is the inertia coefficient and \( C_D \) the drag coefficient. \( v_w \) is the water particle velocity and \( v_s \) is the velocity of the structure. In this model, no fluid structure interaction is assumed: the loads are directly applied to the structure, independent from its response. This assumption is valid when the velocities and accelerations of the structure (\( v_s \) and \( \dot{v}_s \)) are small compared to those of the surrounding fluid (\( v_w \) and \( \dot{v}_w \)), which has been shown to be the case in previous studies [2]. In this case the following approximation can be made for the relative fluid velocity/acceleration: \( v_w - v_s \approx v_w \), \( \dot{v}_w - \dot{v}_s \approx \dot{v}_w \). The total load \( F \) on the structure can be found by integrating \( dF \) over the height of the part of the structure where wave loading is present.

### 2.2 Time-domain load calculations

Calculating fatigue loads due to wave loading in the time domain consists of four main steps. Firstly the height-distributed wave load timeseries are computed via the Morison equation. The next steps are obtaining an internal force-series from time integration of the equation of motion, performing a rainflow count and summing the Damage Equivalent Loads (DEL - see Eq. 2.15) resulting from different load ranges. These steps are explained in this section.

#### 2.2.1 Load timeseries

Morison’s equation (Eq. 2.11) is calculated in the time domain via the JONSWAP spectrum and the dispersion relation, as explained in the previous section, to ultimately give load time series that can be applied to structure nodes. In order to do so, random phase was given when summing the contribution of the different harmonics (uniform distribution over the frequencies), rendering unique and unbiased realizations of possible waves and emulating their actual behavior. The generation of these wave load vectors based on was done using the in-house tool SWAG (SIEMENS Wave Generator [24]), and the sea state parameters \( H_s \) and \( T_p \) were passed as inputs.

#### 2.2.2 Internal loads from time simulation

The displacement at each node can be obtained by passing the distributed load through a (nonlinear) time integration method. These displacements are then converted into internal loads, using an element stiffness matrix that contains the node. The internal force for the nodes neighboring element \( n \) is given by:

\[ \begin{bmatrix} g_{n-1} \\ g_n \end{bmatrix} = K_{n}^{el} \begin{bmatrix} u_{n-1} \\ u_{n} \end{bmatrix}. \]

Here, \( K_{n}^{el} \) is a 12 by 12 stiffness matrix relating the displacements of both neighboring nodes, each containing 3 displacements and 3 rotations in global coordinates.
2.2.3 Rainflow Counting

Fatigue in the structure is caused by the cyclic deformation, where the total fatigue damage results from the number of cycles, and the amplitude of these cycles. In order to obtain a total damage value, the deformation time series is then divided into a specified number of load ranges $L_i$, based on the maximum deformation present in the time series. The number of cycles per load range $n_i$ is then obtained via the Rainflow Counting algorithm. The standard available tool in MATLAB was used for this purpose. The reader is referred to Endo & Matsuishi [23] for an explanation on this algorithm.

2.2.4 Calculating fatigue loads

The total damage done to a structure due to cyclic loading can be expressed by a dimensionless number $D_s$:

$$D_s = \sum \frac{n_{c,i}}{N_{S_i}}. \quad (2.13)$$

This relationship is known as the Palmgren-Miner rule [7]. Here, $n_i$ is the number of cycles for a given stress range, where $i = 1, 2, ..$ comprises all stress ranges present in the deformation time series, and $N_{S_i}$ is the number of allowed cycles for the respective stress range until failure, which depends on the material properties of the structure and the type of connections within the structure (e.g., welds between plates). $N_{S_i}$ is usually derived from an SN-curve, relating stress ranges to allowed number of cycles [16]. The value of $D_s$ must be smaller than 1 in order to prevent failure of the material. The number of allowed cycles is related to the stress $S$ or $\sigma$ as:

$$N_a = N_{S_i} \left(\frac{\sigma_c}{\sigma_e}\right)^m. \quad (2.14)$$

Here $\sigma_e$ is the amplitude of the stress value, constant for all cycles, $\sigma_c$ is the endurance limit, $m$ is a measure for the sensitivity of the allowable number of cycles to the stress range and $N$ is the prescribed number of cycles. Hence, if the applied load $L_i$ is smaller than the endurance limit $L_c$ for a corresponding number of maximum allowable cycles $N_{S_i}$, the reference number of cycles $N_a$ may be greater than $N_{S_i}$, and vice-versa.

After obtaining the number of cycles for different load ranges from rainflow count (described in the previous subsection), the equivalent fatigue load from a given timeseries can be calculated by combining Eqs. 2.13 and 2.14:

$$\text{DEL} = \left(\sum \frac{n_i T_{hf} L_i^m}{N_a T_{sim}^m} \right)^{1/m}. \quad (2.15)$$

An overview of the parameters is given in Table 2.1. Here DEL is the damage equivalent load value of a constant amplitude cyclic load suffered by the structure during its lifetime.

2.3 Frequency domain load calculations

In this section the frequency domain based method to calculate fatigue loads is explained. Firstly, methods are presented on how to obtain the linear structural frequency response of the offshore wind turbine structure. In what follows, the linearized Morison equation
Table 2.1: Definition of parameters used in Eq. 2.15.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_i )</td>
<td>counted number of cycles</td>
<td>(-)</td>
</tr>
<tr>
<td>( N_n )</td>
<td>specified number of cycles</td>
<td>(-)</td>
</tr>
<tr>
<td>( T_{life} )</td>
<td>lifetime</td>
<td>(s)</td>
</tr>
<tr>
<td>( T_{sim} )</td>
<td>simulation time</td>
<td>(s)</td>
</tr>
<tr>
<td>( L_i )</td>
<td>load range</td>
<td>(N), (Nm)</td>
</tr>
<tr>
<td>( m )</td>
<td>Wöhler slope</td>
<td>(-)</td>
</tr>
<tr>
<td>DEL</td>
<td>damage equivalent load</td>
<td>(N), (Nm)</td>
</tr>
</tbody>
</table>

for the frequency domain is presented, and finally, the Dirlik method for obtaining fatigue loads in the frequency domain is presented.

### 2.3.1 Structural transfer functions

Structural transfer functions can be obtained for linear mechanical models. This subsection briefly presents how to obtain the response of a linear mDoF system. For linear systems containing many degrees of freedom, direct inversion is less attractive, and techniques may be applied in order to speed up the computation of the transfer function, usually sacrificing accuracy as assumptions are made with respect to the system of interest, light damping for example. The most common technique is called modal synthesis, as will be explained briefly.

The starting point for obtaining the frequency response of a linear system under a forced harmonic excitation is the following equation, with mass, damping and stiffness matrices \( M, C \) and \( K \):

\[
M\ddot{u} + Cu + Ku = se^{j\Omega t}.
\]  (2.16)

Here, the external force has amplitude \( s \) and frequency \( \Omega \). The steady-state response of the structure can be assumed harmonic, conform the external force:

\[
\dot{u} = j\Omega u,
\]  (2.17)

\[
\ddot{u} = -\Omega^2 u.
\]  (2.18)

Equation 2.20 can then be rewritten as (omitting the time dependency for readability):

\[
\left( -\Omega^2 M + j\Omega C + K \right) z = s.
\]  (2.20)

In the following subsections, different methods are explained that can render the transfer function.

**Direct inversion**

In the direct inversion, the transfer function of frequency \( \omega \) is found directly by computing:

\[
Y(\omega) = (M\ddot{u} + Cu + K)^{-1}.
\]  (2.21)

Here, \( Y \) is the receptance matrix such that \( z = Y(\omega)s \). This is the most computationally expensive way to compute the response of the system, and the most accurate, since it does not make any further implicit assumptions about the system. An individual structural
transfer function $Y_{mk}(\omega)$ giving the transfer from input $k$ to output $m$ is obtained by simply looking up the row entry $m$ and the column entry $k$ from $Y$.

**Modal synthesis**

The modal synthesis technique relies on the assumption that the eigenmodes of the system are only lightly affected by the damping. The solution to Eq. 2.20 can be expressed as the sum of the eigenmodes $x_s$ with $s = 1..n$ in the system with respective generalized coordinates $\eta_s$:

$$z = \sum_{s=1}^{n} \eta_s x_s. \quad (2.22)$$

When Eq. 2.27 is substituted in 2.20, the following result can be obtained:

$$\eta_s = \frac{x_s^T s}{\mu_s (\omega_s^2 - \omega^2 + 2j\zeta_s \omega_s \omega)}. \quad (2.23)$$

Here, $\zeta_s$ is the damping ratio corresponding to mode $s$, and $\mu_s$ the modal masses:

$$\mu_s = x_s^T M x_s, \quad (2.24)$$

$$\zeta_s = \frac{x_s^T C x_s}{2\omega_s \mu_s}. \quad (2.25)$$

The derivation of Eq. 2.23 can be found in [31] and is based on the orthogonality of eigenmodes and the assumption that the damping is light. Using this result, the expression for the response becomes:

$$z = \sum_{s=1}^{n} \frac{1}{\omega_s^2 - \omega^2 + 2j\zeta_s \omega_s \omega} x_s x_s^T \mu_s. \quad (2.26)$$

The structural transfer from load input degree of freedom $k$ to output degree of freedom $m$ becomes:

$$\frac{z_m}{s_k}(\omega) = H_{mk}(\omega) = \sum_{s=1}^{n} \frac{1}{\omega_s^2 - \omega^2 + 2j\zeta_s \omega_s \omega} x_m x_k \mu_s. \quad (2.27)$$

In combination with the load spectra, one obtain the structural response spectra, as explained in the following subsection.

**2.3.2 Structural response spectra**

Morison’s equation can be employed in the frequency domain, after being linearized, since the drag term (Eq. 2.11) is proportional to the velocity squared. In order to do so, the linearization method suggested by Borgmann [8] was used, by using the first term in a Fourier series expansion. The resulting equation, using the velocity and acceleration spectra from Eqs. 2.8,2.9:
2.3. FREQUENCY DOMAIN LOAD CALCULATIONS

\[ S_f(\omega, z) = \frac{8c_{\text{mor}}^2 \sigma^2}{\pi} S_V(\omega, z) + k_{\text{mor}}^2 S_A(\omega, z), \] (2.28)

\[ c_{\text{mor}} = \frac{1}{2} C_D \rho D, \] (2.29)

\[ k_{\text{mor}} = C_M \rho \frac{\pi}{4} D^2, \] (2.30)

\[ \sigma^2 = \int_0^\infty S_{JW}(\omega) d\omega. \] (2.31)

A summary of the parameters and functions is provided in Table 2.2.

| \(S_V\) | velocity spectrum (m\(^2\)/s) |
| \(S_A\) | acceleration spectrum (m\(^2\)/s\(^3\)) |
| \(S_{JW}\) | JONSWAP spectrum (m\(^2\)) |
| \(\rho\) | water density (kg/m\(^3\)) |
| \(\omega\) | angular frequency = \(2\pi f\) (rad/s) |
| \(z\) | depth (m) |
| \(D\) | monopile diameter (m) |
| \(C_M\) | inertia coefficient (-) |
| \(C_D\) | drag coefficient (-) |

The load spectrum \(S_f(z, \omega)\) is discretized for \(z_k\), for \(k = 1, 2, \ldots, K\). The internal loads at a DoF of interest \(m\) can be computed by summing the transfers from all load input DoF \(z_1, z_2, \ldots, z_K\) to the output DoF \(m\):

\[ S_m^m(\omega) = \sum_{k=1}^{K} |H_{mk}(\omega)|^2 S_{f,z_k}(\omega). \] (2.32)

In Fig. 2.2 it is shown how the input wave loads are combined with the structural transfer functions leading to the structural response spectra, for an arbitrary output DoF \(m\). A typical DoF of interest is the bending moment at mudline.

2.3.3 Dirlik’s method for calculating fatigue loads

Once the structural response spectra are found for the output DoF of interest, the fatigue loads can be calculated through Dirlik’s method, which is the frequency-domain equivalent of rainflow counting in time domain analysis. For a complete description of the model, the reader is referred to Ziegler et al. [4]. The method computes the damage equivalent loads through an expected number of cycles and load ranges, based on a probability density function, in turn derived from the power spectral density of the internal loads. A flowchart of the process is shown in Fig. 2.3.

2.3.4 Nonlinear structural models

In the previous section it was shown how the internal load spectra can be computed from summing transferfunctions (see Eq. 2.32 and Fig. 2.2). When the structural model is nonlinear, the structural transfer functions can not readily be computed for in nonlinear...
Wave Load Spectra

Structural Response Spectrum at DoF $m$

$$S^m_R(\omega) = \sum_{k=1}^{K} |H_{mk}(\omega)|^2 S_{f,z_k}(\omega)$$

Structural Load Transfer Functions to DoF $m$

Figure 2.2: Obtaining the linear structural response spectra $S^m_R(\omega)$ for an output DoF $m$ of interest, corresponding to the output node (marked red). $S^m_R(\omega)$ is obtained by summing the squared load transfer functions $|H_{mk}(\omega)|^2$ from the different input DoF to the output DoF $m$, multiplied by the respective wave load spectra $S_{f,z_k}(\omega)$. Although node $m$ may also be contain input DoFs to an output DoF at the same node, this has been omitted for clarity purposes. The red and blue vertical lines in the spectra mark the resonance frequency of the structure $\omega_0$ and the reciprocal of the peak period of the wave spectrum $\omega_p = 2\pi/T_p$.

Figure 2.3: Dirlik's method for obtaining fatigue load results.

structural models, as in for instance Eq. 2.21. Moreover, once the individual transferfunctions are obtained through an alternative method, they cannot simply be summed to give the total response as in Eq. 2.32, as the principle of linear superposition no longer holds.

The above forms the motivation for Chapter 4, in which methods are discussed and presented to obtain the frequency response of nonlinear structural models, in order to proceed with Dirlik’s method to calculate fatigue values in the frequency domain.
2.4 Summary and outlook

In this chapter, time and frequency domain methods were presented in Sections 2.2 and 2.3, as used in this work to calculate fatigue load results. The time domain method allows to include nonlinear structural models, while the current frequency domain tools are based solely on linear models, giving rise to the request of an alternative way to come up with frequency responses. In Chapters 3 and 4 respectively, a modeling framework of the structural dynamics of monopile structures with Tuned Mass Dampers is presented, followed by a proposal of methods to obtain the nonlinear frequency response of this combined system.
Chapter 3

Modeling of structures with Tuned Mass Dampers

The most basic representation of a monopile with a TMD consists of two connected mass-spring-damper systems, under external loads, as explained Section 3.1. In Section 3.2, possible nonlinearities in damper systems are discussed, in particular end-stop nonlinearity in Section 3.2.3. In Section 3.3 the full equations of motion of the TMD and monopile are presented, as well as their coupling, through a technique called dynamic substructuring. Finally, a basic method is explained that allows for reducing the full DoF models to fewer DoF using a selection of vibration modes in Section 3.4.

3.1 Linear 2DoF damped structure

The linear 2DoF system consisting of a main structure and damper components, denoted by subscripts $s$ and $d$ respectively, is shown in Fig. 3.1. When the external loads are assumed to apply to the structure only, the equations of motion are as follows:

\[
\begin{bmatrix}
    m_d & 0 \\
    0 & m_s
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_d \\
    \ddot{u}_s
\end{bmatrix}
+ \begin{bmatrix}
    c_d & -c_d \\
    -c_d & c_s + c_d
\end{bmatrix}
\begin{bmatrix}
    \dot{u}_d \\
    \dot{u}_s
\end{bmatrix}
+ \begin{bmatrix}
    k_d & -k_d \\
    -k_d & k_s + k_d
\end{bmatrix}
\begin{bmatrix}
    u_d \\
    u_s
\end{bmatrix}
= \begin{bmatrix}
    F
\end{bmatrix}
\]  

(3.1)

The frequency response of this system is shown in Fig. 3.2, together with the response of a stand-alone structure body. The TMD is designed to move out of phase by (almost) 90 degrees, such that a maximum amount of kinetic energy from the TMD can be used to suppress the structure motion. The spring ($k_d$) serves to define the natural frequency
of the TMD, leading to the phase difference with the structure, and to keep the TMD in place. The damper reduces the motion of the structure while dissipating some of the energy as heat.

![Diagram](image)

**Figure 3.2:** Frequency response of the linear 2DoF damped structure, with $\frac{m_d}{m_s} = 0.01$. The response of the stand-alone structure has been added as a reference.

### 3.1.1 Tuning the TMD parameters

Tuning a TMD implies choosing $c_d$ and $k_d$, given $m_d$. Usually, the damper mass $m_d$ is specified beforehand, and typically ranges from 0.01 - 0.1 times $m_s$ [10]. From here, the theoretical optimal natural frequency of the attached damper, $\bar{\omega}_d$, can be derived. Since the damping components $c_s, c_d$ change the eigensolution of the system, finding the optimal values of $k_d$ and $c_d$ is far from straightforward. For a derivation of this optimum condition for the system of Eq. 3.1, a curve-fitting procedure must be carried out and the curious reader is referred to, for instance, Connor [10]. One could simplify the problem in this study by noting that the structural damping of the monopile is relatively low, and therefore setting it to zero. This approach yields a well-tuned system for engineering purposes, especially considering the practical limitations in implementing a theoretically optimal tuned system.

**Stiffness Tuning**

In the case of $\zeta_s = 0$, the optimal frequency is given by (see Connor [10] for a derivation):

$$\bar{\omega}_d = \sqrt{\frac{1 - 0.5\hat{m}}{1 + \hat{m}}} \omega_s,$$

$$\hat{m} = \sqrt{\frac{m_d}{m_s}},$$

after which the stiffness of the TMD can be obtained:

$$\bar{k}_d = \bar{\omega}_d^2 m_d.$$
3.1. LINEAR 2DOF DAMPED STRUCTURE

In a Tuned Liquid Column Damper (Section 1.2.1), the stiffness can be varied by changing the water level. In a pendulum TMD, the stiffness is tuned by varying the length of the cable / rod attaching the TMD to the structure.

Damping tuning

In [10] the optimal value for $\zeta_d$ is also derived, under the assumption $\zeta_s=0$. The result is

$$\bar{\zeta}_d = \sqrt{\frac{\bar{m}(3 - \sqrt{0.5\bar{m}})}{8(1 + \bar{m})(1 - 0.5\bar{m})}}. \quad (3.5)$$

In a Tuned Liquid Column Damper, $\bar{\zeta}_d$ is determined by the viscosity of the damper fluid, and in a Pendulum Damper it depends on the viscosity / dimensions of the fluid in the dashpots connecting the TMD to the structure (or possibly, a fluid in which the pendulum mass sits), limiting the horizontal motion. Fig. 3.3 shows how the value of $\zeta_d$ influences the frequency response of the structure. The effect of adding a TMD causes the single resonance from the structure $\zeta_d$ to split up in two resonances around $\omega = \omega_s$, as can be seen clearly for the case $\zeta_d = 0$. Also, an antiresonance appears in between both peaks. At this frequency the dynamic forces/moments that the structure receives from its own stiffness and damping $k_s, c_s$ exactly balance those of the TMD $k_d, c_d$, resulting in standstill. Upon increasing $\zeta_d$, more energy dissipates through $c_d$, and the resonance peaks lower as a result. The damping in the TMD also disturbs the force balance that made the antiresonance possible, making it less present. The peaks are also brought closer upon increasing $\zeta_d$ to the point of being merged to indistinguishable, for $\zeta_d > \bar{\zeta}_d$.

Throughout this work, $\zeta_d$ was chosen as over-damped: $\zeta_d \leq \zeta_d \leq 2\bar{\zeta}_d$. The drawback is that the response around resonance increases with respect to $\zeta_d$, as the motion of the TMD becomes more constrained in the velocity, hindering its performance. The advantage is that the performance becomes less sensitive to the design implementation inaccuracies, in the area $0 < \omega \leq \omega_d$, which may fall in a significant portion of the wave load spectrum (Fig. 1.2).

![Figure 3.3: Influence of $\zeta_d$ on the structure response. $\bar{m} = 0.01$, $k_d = \bar{k}_d$. The response of the stand-alone structure has been added as a reference.](image-url)
3.2 Nonlinearities

In any mechanical structure nonlinearities may exist, and this may have an influence on the description given in the previous section on linear structures with TMDs. Nonlinearities can be present in the external forces acting on a system, or the internal forces. After briefly going over the possible nonlinearities in the monopile with TMD system, this section continues by elaborating on the main nonlinearity studied in this work, that of an end-stop mechanism.

3.2.1 External loads

When nonlinearities are present in the external loads, the expression for the external forces $F$ in Eq. 3.1 becomes dependent not only explicitly on time, but also on the states, such that

$$F = F(t, \ddot{u}(t), \dot{u}(t), u(t)).$$ (3.6)

The external loads on a monopile structure are wave, wind and gravity loads. This work only takes into account the wave loads, since they are the main fatigue causing load, as explained in the Introduction. In Section 2.1 it was explained that this interaction can be neglected, since the monopile velocities are low compared to that of the surrounding fluid.

Obviously, the external loads can also interact with a damper system. This is not taken into account in Eq. 3.1. An example would be the drag force caused by the surrounding air of a moving damper body and/or connection parts. However, a TMD is designed to move at velocities comparable to that of the monopile, for which the interaction with the surrounding load has been left out for the same reason, despite the fact that water has a density three orders of magnitudes higher than air. Hereby it is motivated to consider all loads upon the monopile (only wave loads are chosen) as linear.

3.2.2 Internal forces: geometric and material nonlinearities

Nonlinearities are typically divided into geometric or material nonlinearities, as explained in the Introduction (Chapter 1). Since the displacements of the monopile are relatively small compared to its size, no nonlinearities in the internal loads of the monopile are considered. In the Introduction was also motivated why focusing on end-stop nonlinearity, a nonlinear force accompanied by the TMD. A model is presented for this phenomenon, in the next subsection.

3.2.3 Internal forces: end-stop nonlinearity

The load magnitude of impact between the TMD and this end-stop obviously depends on the material properties of both parts, and their geometries. Several studies have been carried out in order to derive models for the impact forces between structural components, upon for instance earthquake-induced pounding ([18], [5]). Based on these works, Jankowski presented a nonlinear viscoelastic model ([17], [17]) for the pounding of structures, where the result can be described by the following equations relating the nonlinear force $f^{NL}$ opposing a structure upon impact with another structure, with the penetration depth $\delta$:

$$f^{NL}.$$
3.2. NONLINEARITIES

\[
\begin{align*}
    f^{NL} &= \frac{3}{2} \bar{\beta} \delta^3 + \bar{c} \dot{\delta} \quad \delta > 0, \dot{\delta} > 0 \quad \text{(approach)} \\
    f^{NL} &= \frac{3}{2} \bar{\beta} \delta \quad \delta > 0, \dot{\delta} \leq 0 \quad \text{(restitution)} \\
    f^{NL} &= 0 \quad \delta \leq 0
\end{align*}
\]  

(3.7)

Here, \( \bar{\beta} \) describes the elastic stiffness and \( \bar{c} \) the dissipative damping term, modeled as a linear dashpot-damper during the approach period of collision. The fractional-order dependency is a result of contact mechanics modeling between spherical bodies. An extensive overview of the contact forces between two bodies with different shape combinations can be found in [29]. The derivations of these forces are based on Hertz compressive effects, which rely on the following assumptions:

- The surfaces in contact are perfectly smooth.
- The elastic limits of the materials are not exceeded.
- The materials are homogeneous.
- There are no frictional forces within the contact area.

The \( \frac{3}{2} \) - power dependency on \( \delta \) holds for the following configurations of colliding bodies: sphere-sphere, sphere-plane, and cylinder-cylinder (with axes misaligned). When the colliding shapes are arbitrary, it is conventional to use the sphere-sphere approximation. The specific mentioned cases happen to cover all the examples treated in this work and will be used. The damping term \( \bar{c} \) is expressed as:

\[
\bar{c} = 2\bar{\zeta} \sqrt{\bar{\beta} \delta m_1 m_2 / (m_1 + m_2)}.
\]

(3.8)

Here, \( \bar{\zeta} \) is the damping ratio and \( m_1 \) and \( m_2 \) are the masses of the respective colliding bodies. In general, for a structure \( m_1 \) to which the end-stop is rigidly connected, and a TMD \( m_2 \), the structure is much heavier: \( m_1 >> m_2 \). In this case, Eq. 3.9 reduces to:

\[
\bar{c} \approx 2\bar{\zeta} \sqrt{\bar{\beta} \delta m_2}.
\]

(3.9)

The damping ratio \( \bar{\zeta} \) is usually related to a restitution factor \( e \) (0 \leq e \leq 1) relating the prior-impact and post-impact (final) relative velocities, \( \dot{\delta}_0 \) and \( \dot{\delta}_f \) respectively:

\[
e = \frac{|\dot{\delta}_f|}{\dot{\delta}_0} \quad \text{(3.10)}
\]

The relationship, as derived by Jankowski [17], reads:

\[
\bar{\zeta} = \frac{9\sqrt{5}}{2} \cdot \frac{1 - e^2}{e(e(6\pi - 16) + 16)}.
\]

(3.11)

Jankowski’s model has been tested through comparisons between numerical studies of structure impacts, and have shown very good agreements with experimental results. This is the motivation for using this end-stop model throughout the rest of this work.
3.3 Extension to mDoF systems

In the previous sections, the basic two DoF structure with TMD system was described, as applied to this work. Real-life monopile structures and TMDs consist of many more DoF. Since the main focus of this work is on the nonlinear behavior of the TMD, and since this is a relatively small part of the total structure, it would be of interest to study the TMD equations of motion separately. For this purpose the Dynamic Substructuring (DS [19]) method is presented in the first part of this section. The second motivation for DS is that there exist methods to reduce the size of a linear system of equations, making DS even more powerful, as shown in the second part of this section.

3.3.1 Dynamic Substructuring

In Fig. 3.4 it is shown how an arbitrary structure can be divided into two substructures. In general, the total structure is divided into \( n \) substructures, separated by carefully chosen interface nodes, with corresponding DoF \( u_b \). \( u_b^{(s)} \) are the DoF belonging to the interface nodes separating substructure \( s \) from the other substructures. The interior DoF of each substructure is denoted by \( u_i \):

\[
\begin{align*}
&u_1 \quad u_b \\
&\text{substructure 1} \\
&\downarrow \\
&u_b^{(1)} \quad g^{(1)} \\
&\downarrow \\
&u_b \quad u_b^{(2)} \\
&\text{substructure 2} \\
&\downarrow \\
&u^{(1)} \quad u_i \quad u^{(2)} \\
&\text{original structure} \\
\end{align*}
\]

Figure 3.4: Splitting a structure up into two substructures, where the chosen boundary DoF \( u_b \) are copied to be included in the DoF of both substructures. The unknown interface forces/moments \( g^{(s)} \) working between the substructures are treated as external forces on the substructures, indicated with arrows.

The partitioned, but general EoM for each substructure reads, with internal forces \( f \), external forces \( F \) and interface forces \( g \):

\[
\begin{bmatrix}
\dot{f}_i^{(s)}(u^{(s)}, \dot{u}^{(s)}, \ddot{u}^{(s)}) \\
\dot{f}_b^{(s)}(u^{(s)}, \dot{u}^{(s)}, \ddot{u}^{(s)})
\end{bmatrix} = \begin{bmatrix}
\dot{F}_i^{(s)}(u^{(s)}, \dot{u}^{(s)}, \ddot{u}^{(s)}) \\
\dot{F}_b^{(s)}(u^{(s)}, \dot{u}^{(s)}, \ddot{u}^{(s)})
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix} + \begin{bmatrix}
g^{(s)}(u^{(s)}, \dot{u}^{(s)}, \ddot{u}^{(s)})
\end{bmatrix}. \tag{3.13}
\]

From here on, the state-dependencies are omitted for readability. The internal forces/moments can be separated into a linear (inertial, stiffness and damping) and a nonlinear part (denoted by \( f^{NL} \)), giving:
3.3. EXTENSION TO MDOF SYSTEMS

\[
\begin{bmatrix}
M^{(s)} & M^{(s)}_b \\
M^{(s)}_b & M^{(s)}_{bb}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}^{(s)}_i \\
\ddot{u}^{(s)}_b
\end{bmatrix}
+ \begin{bmatrix}
C^{(s)}_i & C^{(s)}_b \\
C^{(s)}_b & C^{(s)}_{bb}
\end{bmatrix}
\begin{bmatrix}
\dot{u}^{(s)}_i \\
\dot{u}^{(s)}_b
\end{bmatrix}
+ \begin{bmatrix}
K^{(s)}_i & K^{(s)}_b \\
K^{(s)}_b & K^{(s)}_{bb}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}^{(s)}_i \\
\ddot{u}^{(s)}_b
\end{bmatrix}
+ 
\begin{bmatrix}
\dot{f}^{NL(s)}_i \\
\dot{f}^{NL(s)}_b
\end{bmatrix}
= 
\begin{bmatrix}
F^{(s)}_i \\
F^{(s)}_b
\end{bmatrix}
+ 
\begin{bmatrix}
g^{(s)}
\end{bmatrix},
\] 

(3.14)

If one stacks the DoF in the system with \( S \) substructures as

\[
u^T = [u^{(1)^T}_i u^{(1)^T}_b u^{(2)^T}_i u^{(2)^T}_b \ldots u^{(S)^T}_i u^{(S)^T}_b],
\] 

(3.15)

the unique DoF can be identified through a Boolean matrix with a pair \([1, -1]\) at the locations of the two overlapping interface DoF from both substrucures, expressing the compatibility condition as follows:

\[
Bu = 0.
\] 

(3.16)

The nullspace of \( B \) contains all vectors that become zero upon multiplying with the rows of \( B \), or in other words - which lose their identity upon ‘B-transformation’. These vectors gathered as columns are called the localization matrix \( L \), which can be used to express \( u \) in terms of the generalized, unique DoF \( q \):

\[
L = \text{null}(B) \leftrightarrow BL = 0,
\] 

(3.17)

\[
u = Lq.
\] 

(3.18)

The system matrices and forces of the system can be assembled similarly to 3.15:

\[
f^{NL} = [f^{NL(1)}_i f^{NL(2)}_i \ldots f^{NL(S)}_i]^T, f^{NL(s)} = \begin{bmatrix} f^{NL(s)}_i \\ f^{NL(s)}_b \end{bmatrix},
\] 

(3.19)

\[
F = [F^{(1)}_i F^{(2)}_i \ldots F^{(S)}_i]^T, F^{(s)} = \begin{bmatrix} F^{(s)}_i \\ F^{(s)}_b \end{bmatrix},
\] 

(3.20)

\[
\dot{g} = [\dot{g}^{(1)}_i \dot{g}^{(2)}_i \ldots \dot{g}^{(S)}_i]^T, \dot{g}^{(s)} = \begin{bmatrix} \dot{g}^{(s)}_i \\ \dot{g}^{(s)}_b \end{bmatrix},
\] 

(3.21)

\[
M = \begin{bmatrix}
M^{(1)} & 0 & 0 & 0 & 0 \\
0 & M^{(2)} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & M^{(S)}
\end{bmatrix}, M^{(s)} = \begin{bmatrix}
M^{(s)}_i & M^{(s)}_b \\
M^{(s)}_b & M^{(s)}_{bb}
\end{bmatrix},
\] 

(3.22)

\[
C = \begin{bmatrix}
C^{(1)} & 0 & 0 & 0 \\
0 & C^{(2)} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & C^{(S)}
\end{bmatrix}, C^{(s)} = \begin{bmatrix}
C^{(s)}_i & C^{(s)}_b \\
C^{(s)}_b & C^{(s)}_{bb}
\end{bmatrix},
\] 

(3.23)
3. MODELING OF STRUCTURES WITH TUNED MASS DAMPERS

\[
K = \begin{bmatrix}
K^{(1)} & 0 & 0 & 0 \\
0 & K^{(2)} & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & K^{(S)}
\end{bmatrix},
K^{(s)} = \begin{bmatrix}
K_{ii}^{(s)} & K_{ib}^{(s)} \\
K_{ib}^{(s)T} & K_{bb}^{(s)}
\end{bmatrix}.
\tag{3.24}
\]

Then, the equation of motion can be written as

\[
ML\dddot{q} + CL\dot{q} + KLq + f^\text{NL} = F + \hat{g}.
\tag{3.25}
\]

Left-multiplication by \(L^T\) yields

\[
\tilde{M}\dddot{q} + \tilde{C}\dot{q} + \tilde{K}q + \tilde{f}^\text{NL}(\dot{q}, \ddot{q}, q) = \tilde{F},
\tag{3.26}
\]

with:

\[
\tilde{M} = L^TML,
\tag{3.27}
\]
\[
\tilde{C} = L^TCL,
\tag{3.28}
\]
\[
\tilde{K} = L^TKL,
\tag{3.29}
\]
\[
\tilde{F} = L^TF,
\tag{3.30}
\]
\[
\tilde{f}^\text{NL} = L^Tf^\text{NL}.
\tag{3.31}
\]

The term \(L^Tg = 0\), since the entries of \(L\) are rows containing [1,1] pairs, corresponding to the interface forces that are equal and opposite - this is referred to as the force equilibrium condition.

3.3.2 Application to monopile + TMD

The equations of motion become, when cast in the form of Eq. 3.14, denoting the monopile with \(^{(m)}\) and the TMD with \(^{(d)}\):

\[
\begin{bmatrix}
M^{(m)}_{ii} & 0 \\
0 & M_{bb}^{(m)}
\end{bmatrix}\begin{bmatrix}
\dot{u}^{(m)}_i \\
\dot{u}^{(m)}_b
\end{bmatrix} + \begin{bmatrix}
C^{(m)}_{ii} & C^{(m)}_{ib} \\
C^{(m)T}_{ib} & C^{(m)T}_{bb}
\end{bmatrix}\begin{bmatrix}
\ddot{u}^{(m)}_i \\
\ddot{u}^{(m)}_b
\end{bmatrix} + \begin{bmatrix}
K^{(m)}_{ii} & K^{(m)}_{ib} \\
K^{(m)T}_{ib} & K^{(m)T}_{bb}
\end{bmatrix}\begin{bmatrix}
\dot{u}^{(m)}_i \\
\dot{u}^{(m)}_b
\end{bmatrix} = \begin{bmatrix}
F^{(m)}_i \\
F^{(m)}_b + g^{(m)}
\end{bmatrix},
\tag{3.33}
\]

\[
\begin{bmatrix}
M^{(d)}_{ii} & 0 \\
0 & M_{bb}^{(d)}
\end{bmatrix}\begin{bmatrix}
\dot{u}^{(d)}_i \\
\dot{u}^{(d)}_b
\end{bmatrix} + \begin{bmatrix}
C^{(d)}_{ii} & C^{(d)}_{ib} \\
C^{(d)T}_{ib} & C^{(d)T}_{bb}
\end{bmatrix}\begin{bmatrix}
\ddot{u}^{(d)}_i \\
\ddot{u}^{(d)}_b
\end{bmatrix} + \begin{bmatrix}
K^{(d)}_{ii} & K^{(d)}_{ib} \\
K^{(d)T}_{ib} & K^{(d)T}_{bb}
\end{bmatrix}\begin{bmatrix}
\dot{u}^{(d)}_i \\
\dot{u}^{(d)}_b
\end{bmatrix} + \begin{bmatrix}
f_{NL}^{(d)}(\ddot{u}^{(d)}_i, \dot{u}^{(d)}_i) \\
f_{NL}^{(d)}(\ddot{u}^{(d)}_b, \dot{u}^{(d)}_b)
\end{bmatrix} = \begin{bmatrix}
0 \\
g^{(d)}
\end{bmatrix}.
\tag{3.34}
\]

Here, the damping and elastic forces on the monopile consist of a structural part, and an interaction with the soil (the latter keeping the structure in place):
3.4 Model order reduction

The external forces $F^{(s)}$ are not considered a function of the states, as explained in Section 3.2.1. The external loads on the monopile consists of wave loads (as explained in Chapter 2), and work on the internal DoF of the monopile (the TMD is located above sea level):

$$F^{(m)} = F_{\text{wave}}(t).$$

(3.37)

The only nonlinearity considered in this work is the endstop force in the TMD, described by Eq. 3.7, which depends on the velocity and displacement of the TMD body and works on the TMD body itself (not on the interface with the monopile structure):

$$f_{NL}^{(d)} = f_{NL}^{(d)}(\dot{u}(d), u(d)).$$

Now that the equations of motion of both the wind turbine and the TMD have been organized with knowledge of Dynamic Substructuring, one can apply model order reduction to the linear wind turbine model to reduce the size of the combined system, as explained in the following section.

3.4 Model order reduction

In the previous section, the Dynamic Substructuring technique was presented as a way to partition the equations of motion of a structure into smaller subsets, and it was applied to yield a set of equations corresponding to the monopile and one set belonging to the TMD. Since the equations of motion of the monopile are linear, one could describe its behavior with fewer DoF by selecting a number of its linear internal vibration modes. In this work, the Craig-Bampton reduction technique is used for this purpose [32].

Craig-Bampton method

The idea of the Craig-Bampton method is that a solution to a dynamic problem can always be decomposed into static and a dynamic contributions. Taking Eq. 3.33 without inertia, damping and external forces, gives

$$\begin{bmatrix} K_{ii}^{(m)} & K_{ib}^{(m)} \\ K_{ib}^{(m)T} & K_{bb}^{(m)} \end{bmatrix} \begin{bmatrix} u_i^{(m)} \\ u_b^{(m)} \end{bmatrix} = \begin{bmatrix} 0 \\ g^{(m)} \end{bmatrix}.$$  

(3.39)

From the first row, the boundary DoF can be condensed upon the internal DoF, thus giving the static part of the solution:

$$u_i^{(m)} = - (K_{ii}^{(m)})^{-1} K_{ib}^{(m)} u_b^{(m)}.$$  

(3.40)
If the boundary DoF were clamped, the eigensolution of the internal DoF reads, as the dynamic contribution:

$$K_{ii}^{(m)} u_{i}^{(m)} = \omega^{2} M_{ii}^{(m)} u_{i}^{(m)}. \quad (3.41)$$

If a number of eigensolutions $N_{m}$ is selected (for instance, 10% of the eigenfrequencies, starting from lowest) and placed in a matrix $\Gamma_{N_{m}}^{(m)}$, then

$$\Gamma_{N_{m}}^{(m)T} K_{ii}^{(m)} \Gamma_{N_{m}}^{(m)} = \text{diag}\left(\left(\omega_{1}^{(m)}\right)^{2}, \left(\omega_{2}^{(m)}\right)^{2}, \ldots, \left(\omega_{N_{m}}^{(m)}\right)^{2}\right), \quad (3.42)$$

$$\Gamma_{N_{m}}^{(m)T} M_{i}^{(m)} \Gamma_{N_{m}}^{(m)} = I. \quad (3.43)$$

Then, a reduction matrix can be constructed:

$$R^{(m)} = \begin{bmatrix} - \left(K_{ii}^{(m)}\right)^{-1} K_{ib}^{(m)} & I_{N_{m}}^{(m)} \end{bmatrix}. \quad (3.44)$$

Now the DoF in the monopile substructure can be represented by the number of DoF on its boundary with the TMD substructure, plus the number of selected internal vibration modes $N_{m}$:

$$u^{(m)} = \begin{bmatrix} u_{i}^{(m)} \\ u_{b}^{(m)} \end{bmatrix},$$

$$= R^{(m)} \begin{bmatrix} u_{i}^{(m)} \\ \eta^{(m)} \end{bmatrix},$$

$$= R^{(m)} \hat{u}^{(m)}. \quad (3.45)$$

Here, $\eta^{(m)}$ is a measure of the intensity of the internal vibration modes of the substructure. Then, the Equation of Motion can be rewritten as:

$$\hat{M}^{(m)} \ddot{\hat{u}}^{(m)} + \hat{C}^{(m)} \dot{\hat{u}}^{(m)} + \hat{K}^{(m)} \hat{u}^{(m)} = \hat{F}, \quad (3.46)$$

with:

$$\hat{M}^{(m)} = R^{(m)T} M^{(m)} R^{(m)},$$

$$\hat{C}^{(m)} = R^{(m)T} C^{(m)} R^{(m)},$$

$$\hat{K}^{(m)} = R^{(m)T} K^{(m)} R^{(m)},$$

$$\hat{F}^{(m)} = R^{(m)T} F^{(m)} R^{(m)}. \quad (3.47)$$

This method has been found to work well for the problem of interest - accurate frequency response functions and internal loads results were found when for instance 5% of the lowest modes were taken into account, significantly speeding up the analysis, as shown in Chapter 6.
3.5 Summary and outlook

In this chapter, the basic behavior of a structure with a Tuned Mass Damper was explained, for a 2DoF linear system (Sec. 3.1). In what followed, possible nonlinearities were introduced in Sec. 3.2, and a model was presented for end-stop nonlinearities in Sec. 3.2.3. In Sec. 3.3, the full equations of motion of the offshore wind turbine system including TMD were presented, where Dynamic Substructuring was used as a powerful tool that allows focus on the dynamics of the OWT system and TMD, separately. Finally, the Craig-Bampton method was shown in Sec. 3.4 as a way to reduce the size of the linear part of the model, covering the entire OWT structure. In Chapter 4, methods will be presented to obtain the nonlinear response of the combined structural system.
Chapter 4

Solution strategies for nonlinear structural dynamics

Structural models that contain nonlinear dynamics can be solved in several ways, mainly to be divided in time domain methods and frequency domain based methods, as explained in the introduction of this work (Chapter 1). Some classes of solution techniques have time domain as well as frequency domain sub-methods. Furthermore, the solution procedure for computing the total system of equations can be monolithic (by solving for all DoF at the same time) or non-monolithic: solving for separate domains of the structure, and solving the coupled system on the interface. Lastly, the external load has been divided into single and multi-harmonic. An overview is given in Table 4.1. In this overview it is shown which sub-methods have been studied and/or applied in this work. In particular, an extension of the Harmonic Balancing method is proposed, in order to include many DoF and harmonic components in the external load. This method will be applied in Chapters 5 and 6. Also, a non-monolithic variant of the sequential path-following method has been proposed, and applied in Chapter 5.

This chapter starts by presenting the Newton-Raphson iteration scheme in Sec. 4.1, which is the algorithm at the heart of every method applied in this work to solve for nonlinear structural dynamics. In Sections 4.2 and 4.3, monolithic time and frequency domain methods are presented, respectively. Finally, non-monolithic time and frequency domain methods will be treated in Sec. 4.4.

Table 4.1: Overview of solution strategies for nonlinear structural dynamics, with references to the sections in this chapter. Combinations marked bold/italic have been applied in this work, and a star (⋆) indicates that additions to the existing methods have been proposed and applied. TD = Time Domain, FD = Frequency Domain.

<table>
<thead>
<tr>
<th>Solution Procedure</th>
<th>External Forcing →</th>
<th>Single harmonic</th>
<th>Multi harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time-integration</td>
<td>TD 4.2.1</td>
<td>FD 4.2.1</td>
</tr>
<tr>
<td>Monolithic</td>
<td>Path-Following</td>
<td>4.2.2</td>
<td>4.2.2</td>
</tr>
<tr>
<td></td>
<td>Perturbation Analysis</td>
<td>4.3.2</td>
<td>4.3.2</td>
</tr>
<tr>
<td></td>
<td>Harmonic Balancing</td>
<td>4.3.3</td>
<td>4.3.3 ⋆</td>
</tr>
<tr>
<td>Non-Monolithic</td>
<td>Time-integration</td>
<td>4.4.1</td>
<td>4.4.1</td>
</tr>
<tr>
<td></td>
<td>Path-Following</td>
<td>4.4.2 ⋆</td>
<td>4.4.2 ⋆</td>
</tr>
</tbody>
</table>
4.1 Newton-Raphson method for nonlinear systems

Figure 4.1 illustrates the difference between a linear and a nonlinear system, for a one-dimensional problem. The variable \( q \) satisfies some equation at \( q = \hat{q} \):

\[
f(\hat{q}) = 0.
\]

For an initial guess \( q^{(0)} \) (often based on equal to the solution at the previous solution timestep), Eq. 4.1 will most likely not be satisfied, and the result is a residual error \( r^{(0)} = f(q^{(0)}) \), giving \( r^{(k)} \) for each subsequent guess. The procedure is continued until \( |r^{(k)}| < \epsilon \), where \( \epsilon \) is a user-specified tolerance. The most efficient way to iterate is by using the information of the derivative of the residual (when known). This is called the tangent stiffness:

\[
J^{(k)} = \frac{\partial f}{\partial q} \bigg|_{q^{(k)}} = \frac{\partial r}{\partial q} \bigg|_{q^{(k)}}.
\]

(4.2)

In the Newton-Rapson method [20], the update is computed as

\[
\Delta q^{(k)} = -(J_n^{(k)})^{-1} r^{(k)},
\]

(4.3)

such that \( q^{(k+1)} = q^{(k)} + \Delta q^{(k)} \). In case of a linear scheme, \( J \) does not depend on \( q \), and therefore convergence is always found within one step. For a non-linear simulation, many iterations may be needed, depending on the smoothness and relative magnitude of the nonlinearity.

4.2 Time domain methods (monolithic)

In time domain methods, the response of the system (displacement / velocity / acceleration) is found by solving the governing ordinary differential equations (ODEs) as a function of time. These ODEs are discretized, after which the solution at each subsequent time step is found through a time marching scheme.
4.3 Frequency domain based methods (monolithic)

4.2.1 Time-integration (single / multi-harmonic)

In so-called time-integration methods, the solution is given directly by the resulting states from the time marching, as time series. In order to obtain a frequency response, for instance the Fast Fourier Transform could be applied, or steady state amplitudes could be taken, if the forcing is single-harmonic.

While there exist many types of solvers to achieve time marching, this work uses the Newmark scheme introduced in 1959 by N.M. Newmark [32]. The Newmark method has proven to be versatile, as it has been implemented in many commercial computer programs [9]. Furthermore, this scheme can be made unconditionally stable, and simulations at even relatively large time steps can lead to accurate results in the response. In this work, the Newmark scheme is used to govern the time domain based approach. The method is very general, and therefore briefly explained in the Appendix A.

4.2.2 Pseudo-arclength path-following (single / multi-harmonic)

In pseudo-arclength path-following [20], short timesimulations govern the search of a the frequency response. The method searches stable periodic solutions, and walks through the spectrum to find steady state solutions. These stable periodic solutions are also referred to as ‘fixed points’ on the Poincaré Map [20], which connects the solutions of instantaneous states after each period, starting from an initial state.

An additional benefit is that that it allows to find unstable branches of the solution (more than one solution per frequency). However the number of DoF in the system, as well as the number of spectral components in the external forcing, are limited. Furthermore, the method has difficulty converging for steep nonlinearities, as was found after testing it for the end-stop problem in this work. Therefore the method was not further investigated.

However, an existing algorithm for this method developed by Lacarbonara and Carboni at the Sapienza University of Rome, is borrowed and applied in this work, as a verification tool for sDoF systems, under single harmonic loading. This tool has been used in for instance [36] and [22], where interesting results obtained with the algorithm were shown.

4.3 Frequency domain based methods (monolithic)

In frequency domain based methods, the response solution of the system is expressed as a summation of harmonic components, with respective amplitudes and phases. As with time domain algorithms, frequency domain methods can be used to obtain the response of linear and nonlinear structural models, with corresponding sub-methods. Following with the example of the previous subsection, the response of the structure is sought in terms of frequency $\omega$ rather than time $q = q(\omega)$. When the linear transfer function $H(\omega)$ of the structure, and the spectrum of the forcing $F(\omega)$ (for instance, Fig. 1.2) are known, the solution can simply be computed as:

$$q(\omega) = (H(\omega))^{-1}F(\omega).$$  \hspace{1cm} (4.4)

It is clear that for a linear equation of motion, the frequency domain approach is the fastest possible. When the EoM is nonlinear, the transfer function $H(\omega)$ (or the external forcing $F(\omega)$) could also depend on the state: $H = H(\omega, q(\omega))$ (or $F(\omega, q(\omega))$), in which case an alternative procedure must be carried out. Several different methods for this purpose are discussed in this section.
4.3.1 Sequential path-following (single harmonic)

The most obvious frequency domain method for nonlinear dynamics is sequential path-following [20]. This is the frequency domain equivalent of the Newton-Raphson scheme illustrated in the previous subsection. In this method, the frequency response of a system can be obtained for a predefined set of frequencies (not necessarily equally spaced), via an iterative scheme. For each frequency $\omega_i$ of interest, the displacement solution $u$ to the following problem is sought for:

$$M\ddot{u} + Cu + Ku + f^{NL}(u) = F. \quad (4.5)$$

When a single harmonic solution is assumed, the solution can be written in the following form:

$$u = q_0 e^{j\omega t}, \quad (4.6)$$
$$\dot{u} = j\omega q_0 e^{j\omega t} = j\omega u, \quad (4.7)$$
$$\ddot{u} = -\omega^2 q_0 e^{j\omega t} = -\omega^2 u. \quad (4.8)$$

If a single harmonic external forcing is applied at frequency $\Omega$, the external force can be written as $F = F_0 e^{j\Omega t}$, and Eq. 4.5 can be rewritten as:

$$(-\Omega^2 M + j\Omega C + K) q_0 e^{j\Omega t} + f^{NL}(q_0 e^{j\Omega t}) = F_0 e^{j\Omega t}, \quad (4.9)$$

since the system will only be able to respond in the forcing frequency in order to satisfy the E. o. M. The nonlinear force $f^{NL}$ can be approximated by the first term in a Taylor series, after which the equation of motion is rewritten in the variables $q_0$, after division by $e^{j\Omega t}$:

$$f^{NL}(q_0 e^{j\Omega t}) \approx f^{NL}(q_0) e^{j\Omega t}, \quad (4.10)$$

$$(-\Omega^2 M + j\Omega C + K) q_0 + f^{NL}(q_0) = F_0. \quad (4.11)$$

The solution of the steady-state response amplitude $q_0$ can be found through an iterative process. In the Newton-Raphson scheme, the derivative of the E. o. M. with respect to the solution variable is used to iterate towards solution. The process is started by re-writing Eq. 4.11 into a residual form:

$$r(q_0) := (-\Omega^2 M + j\Omega C + K) q_0 + f^{NL}(q_0) - F_0 = 0. \quad (4.12)$$

At iteration $k$, the residual expression $r^{(k)}$ becomes:

$$r^{(k)} := (-\Omega^2 M + j\Omega C + K) q_0^{(k)} + f^{NL}(q_0^{(k)}) - F_0. \quad (4.13)$$

The Jacobian of the residual $r(q_0^{(k)})$ becomes:

$$J(q_0^{(k)}) = r_{q_0}(q_0^{(k)}) = -\Omega^2 M + j\Omega C + K + f^{NL}_{q_0}(q_0^{(k)}). \quad (4.14)$$

The Jacobian $J(q_0^{(k)})$ is used to compute the solution at iteration $k + 1$:
4.3. FREQUENCY DOMAIN BASED METHODS (MONOLITHIC)

\[ q_{0}^{(k+1)} = q_{0}^{(k)} - \left[ J^{(k)} \right]^{-1} r^{(k)}. \]  

(4.15)

This Newton-Raphson sequence is repeated until convergence is found, such that \( r^{(k)} < \epsilon \) specified by a tolerance set by the user. In this work, a small number \( \epsilon_{0} \left( 10^{-6} - 10^{-4} \right) \) times the internal forces \( (\Omega^2 M + j\Omega C + K) q^{(k)} \) is used, such that:

\[ \epsilon = \epsilon_{0} \left( \Omega^2 M + j\Omega C + K \right) q^{(k)}. \]  

(4.16)

Remarks

- The approximation made in the nonlinear term 4.10 can only be accurate when the nonlinear force contribution is small compared to the linear part, when the nonlinearity works over the entire range of motion \( q_{0} \). In this work, an end-stop nonlinearity is considered, where a relatively high nonlinear force may occur, starting from a threshold displacement. Often one may be interested in obtaining the total displacement \( q \) in a collision problem (rather than the impact depth itself). If the free displacement before impact is relatively large compared to the impact depth, the accuracy criterion is reversed, because a large nonlinear force leads to a small impact depth, and hence, the relative error over the total displacement remains small.

- The solution \( q_{0} \), by definition, contains only one harmonic. Therefore, the solution can only be accurate if the contribution from harmonics other than the forcing frequency is relatively small.

- The speed of convergence depends completely on the conditioning of the Jacobian matrix \( J(q_{0}) \). When the system matrices \( M, C \) and \( K \) are well conditioned, the convergence speed will depend on the smoothness of the first derivative of the nonlinear force term: \( f_{NL_{q_{0}}}^{(k)}(q_{0}) \). When the nonlinear force-displacement curve is very steep (as with the end-stop problem in this work), cycling in the solution may start to occur. In this work it was found that the algorithm could be stabalized by introducing a relaxation parameter \( \bar{\alpha} \), modifying the iterative update step:

\[ q_{0}^{(k+1)} = q_{0}^{(k)} - \bar{\alpha} \left[ J^{(k)} \right]^{-1} r^{(k)}, \]  

(4.17)

where \( 0 < \bar{\alpha} \leq 1 \).

4.3.2 Perturbation Techniques (single / multi-harmonic)

Another option is to obtain a closed form solution by treating the nonlinear force as a perturbation on the linear system (Nayfeh [25]). The problem is that the solutions to systems with very simple nonlinearities (far simpler than the end-stop considered in this work) can be mathematically complex, and based on assumptions to further simplify the problems. Furthermore, the external forcing considered in this work are wave load spectra that consists of hundreds of frequency components, while the external forcings considered in existing perturbation techniques go as far as a few components, where the frequencies are related to one another in a particular way (for instance, closely spaced). Therefore, this class of methods has not been further investigated in this work.
4.3.3 Harmonic Balancing (single / multi-harmonic)

The starting point of the harmonic balancing method [33] is to express the solution to the equation of motion as a set of harmonics \( k = 1, 2, \ldots, K \), with corresponding amplitudes \( a_k \) and \( b_k \):

\[
\begin{align*}
\mathbf{u}_K(t) &= a_0 + \sum_{k=1}^{K} \left[ a_k \cos(\omega_k t) + b_k \sin(\omega_k t) \right], \\
\dot{\mathbf{u}}_K(t) &= \sum_{k=1}^{K} \left[ \omega_k (-a_k \sin(\omega_k t) + b_k \cos(\omega_k t)) \right], \\
\ddot{\mathbf{u}}_K(t) &= \sum_{k=1}^{K} \left[ \omega_k^2 (-a_k \cos(\omega_k t) - b_k \sin(\omega_k t)) \right].
\end{align*}
\] (4.18)

Usually, the higher harmonics are constructed as integer multiples of the first harmonic \( \omega_1 \), so that \( \omega_k = k \omega_1 \). The external forcing may consist of a single frequency (or a few, as found in all applications in literature), but in case of for instance a sea wave loading, a whole spectrum of frequencies \( \omega_s \) with corresponding phases \( \phi_s \) is contained in the external forcing. Hereby the following problem is defined:

\[
M \ddot{\mathbf{u}} + C \dot{\mathbf{u}} + \mathbf{Ku} + \mathbf{f}^{NL}(\mathbf{u}) = \mathbf{F} = \sum_{s=1}^{S} \mathbf{p}_s \sin (k \omega_s t + \phi_s). \tag{4.21}
\]

The phases \( \phi_s \) can be transferred to the amplitude by using a trigonometric identity, doubling the number of harmonic terms in \( \mathbf{F} \):

\[
\mathbf{p}_s \sin (k \omega_s t + \phi_s) = \mathbf{p}_s \cos (k \omega_s t) \sin \phi_s + \mathbf{p}_s \sin (k \omega_s t) \cos \phi_s. \tag{4.22}
\]

The end goal is to obtain \( 2NK + N \) equations: \( NK \) for the cosine contributions, another \( NK \) for the sines, and \( N \) for the DC terms (\( a_0 \)), with \( 2NK + N \) unknowns: \( a_0, a_k \) and \( b_0 \). The nonlinear force \( \mathbf{f}_{NL} \) can also be expressed as a sum of cosines/sines:

\[
\mathbf{f}^{NL}_K(t) = c_0 + \sum_{k=1}^{K} [c_k \cos(\omega_k t) + d_k \sin(\omega_k t)]. \tag{4.23}
\]

Hereby, another \( 2NK + N \) unknowns are introduced. The reverse relationship lies in the Fourier Transform [26]:

\[
\begin{align*}
c_0 &= \frac{1}{2\pi} \int_0^{2\pi} \left( \mathbf{f}^{NL}_K(t) \right) \, dt, \\
c_k &= \frac{1}{\pi} \int_0^{2\pi} \left( \mathbf{f}^{NL}_K(t) \cos(\omega_k t) \right) \, dt, \\
d_k &= \frac{1}{\pi} \int_0^{2\pi} \left( \mathbf{f}^{NL}_K(t) \sin(\omega_k t) \right) \, dt.
\end{align*}
\] (4.24 - 4.26)

Eqns. 4.18 - 4.23 can be combined to give the harmonic equation of motion, for the DC components and the parts corresponding to \( \cos(\omega_k t) \) and \( \sin(\omega_k t) \), respectively:
4.3. FREQUENCY DOMAIN BASED METHODS (MONOLITHIC)

\[ g_{DC} = K a_0 + c_0 = 0, \]  
\[ g_{k,\cos} = -\left(\omega_k^2 M - K\right) a_k + \omega_k C b_k + c_k - \Psi_{k,\sin} = 0, \]  
\[ g_{k,\sin} = -\omega_k C a_k - \left(\omega_k^2 M - K\right) b_k + d_k - \Psi_{k,\cos} = 0. \]

with \( \Psi_{k,\cos} = p_k \cos \phi_k \) and \( \Psi_{k,\sin} = p_k \sin \phi_k \). The extra unknowns \( c_0, c_k \) and \( d_k \) can be dealt with in different ways. In what follows, a classification is made for the different methods found in literature. After explaining and motivating the Alternating Frequency / Time approach, this method is applied to the nonlinear end-stop problem as formulated in Chapter 3.

**Classical method**

One could attempt to express \( c_0, c_k \) and \( d_k \) as a function of \( a_0, a_k \) and \( b_k \), dissolving the extra \( 2NK + N \) unknowns and allowing for different solution branches - in this work, this is referred to as the Direct Harmonic Balancing method. This is especially interesting when one would like to trace unstable solution branches. This could be done in two ways:

**Analytic approach.** In this approach, the first step is to find a power series as an approximation of \( F_{NL}(t) \):

\[ f_{NL}^p(q) \approx \sum_{d=1}^{D} c_d q^d = \sum_{d=1}^{D} c_d \left( a_0 + \sum_{k=1}^{K} [a_k \cos(\omega_k t) + b_k \sin(\omega_k t)] \right)^d, \]  

where \( D \) is the number of powers (including zero, DC) included in the approximation. The next step is to use multinomial expansion, resulting in a summation of multiplication of individual powers:

\[ (w_1 + w_2 + w_3 + \ldots + w_{2K+1})^d = \sum_{s_1 + s_2 + \ldots + s_r = d} \prod_{1 \leq t \leq r} w_t^{s_t}, \]

\[ w_1, w_2, w_3, \ldots, w_{2K+1} = a_0, a_1 \cos(\omega_1 t), b_1 \sin(\omega_1 t), \ldots, a_K \cos(\omega_K t), b_K \sin(\omega_K t). \]

One proceeds with trigonometric identities to pull apart the multiplications between sines, cosines and powers thereof, from the resulting expression. Finally, one could group these sines and cosines corresponding to the different frequencies \( k \), and the equation can be solved in one step. It has been found in this work that at least 20 terms are needed in order to find any match between \( f_{NL} \) and \( f_{NL}^p \) (see Chapter 5). In this case, expression 4.23 must be expanded to the power of 20. If the forcing is represented by for instance 30 harmonic components (as would be around the minimum to compute fatigue loads accurately), the expression for \( f_{NL}(t) \) becomes very lengthy. For \( d \geq 3 \) the expansion by hand runs into pages of derivation, therefore it would be another option to solve computationally. The CPU memory however runs full for \( d \geq 3 \). Even though a cluster of CPUs might be able to reach a power of 10 within a reasonable time, this would still not be
sufficient if any system parameter would change, as in for instance an optimization process. Also, the frequency response as a result from a power series approximation (found through the Alternating Frequency / Time approach, described later in this chapter) has not been found to accurately represent end-stop behavior, even with 20 powers, as shown in Chapter 5.

Integration method. Alternatively, one could construct Eqn. 4.18 using symbolic variables for $a_0$, $a_k$, and $b_k$, and insert in Eqns. 4.24 - 4.26. The first problem with this approach is that a solution is not always guaranteed. Especially for the end-stop problem under study, MATLAB nor MAPLE were not able to find a solution, even when just a few harmonics were included in the solution. For power series approximations, closed form solutions were only obtained for low powers (around 5). The other disadvantage is that solving systems with many symbolic variables has been found to be computationally expensive.

Alternating Frequency / Time method

The AFT Harmonic Balance (AFT-HB) method has been used in literature in several classes of problems, having the main advantage of easy controllability of the number of harmonics in the solution. This is particularly interesting for the problem studied in this work, since hydrodynamic loads with a relatively large number of harmonics are of interest. In [6] the AFT-HB technique was applied to one and two DoF systems, with bilinear hysteretic nonlinearity, under single harmonic loading. In [38] the same method was applied to a bearing-rotor problem with a similar end-stop nonlinearity as the one treated in this work, where frequency responses were successfully obtained using the AFT-HB algorithm. In [33] the method was applied to a similar problem, containing piece-wise nonlinearities. While all of the mentioned authors studied the response of the respective nonlinear systems, in [11] the response was studied of dry-damped one and two DoF systems under loads containing a few harmonic components, after which the results showed good agreement with results obtained by time-integration. It remains unclear however how the AFT-HB algorithms performs in solving for a system with multiple DoF (such as a wind turbine), under loading with many harmonic components ($\sim 10^2$), with a non-linearity as severe as an end-stop.

The above review forms the motivation for further investigating the AFT-HB method in this work. After the method is explained in the current section, it will be applied to several mechanical systems representing stand-alone TMD systems (Chapter 5), or structures such as wind turbines, with a TMD (Chapters 5 and 6), all containing end-stop nonlinearity.

The AFT-HB method [33],[14] uses the Discrete Fourier Transform and its inverse in order to obtain the harmonic nonlinear force coefficients $\tilde{f}^{NL}$ (comprised of $c_0$, $c_k$, and $d_k$) numerically, based on an initial estimate of the state vector $\tilde{q}$ (comprised of $a_0$, $a_k$, and $b_k$). Inserting $\tilde{q}$ and $\tilde{f}^{NL}$ in the harmonic equations of motion (Eq. 4.27 -4.29) results in a residual $r$. Therefore the Newton-Raphson method is employed to iterate towards a solution. The complete schematic of this method is shown in Fig. 4.2.

The process is started by an initial estimation of the response, with $a_0^{(0)}$, $a_k^{(0)}$, and $b_k^{(0)}$. From this estimate, a discrete timeseries of the response can be constructed numerically, by applying the Inverse Discrete Fourier Transform (IDFT):
4.3. FREQUENCY DOMAIN BASED METHODS (MONOLITHIC)

\[ q(m) \]

\[ f_{NL}[n] \]

\[ \tilde{f}_{NL} \]

\[ r^{(m)} \]

\[ \Delta \tilde{q}^{(m)} = \tilde{q}^{(m)} + \Delta \tilde{q}^{(m)} \]

\[ \text{time domain} \]

\[ \text{END} \]

\[ \text{START} \]

\[ m = 0 \]

\[ \tilde{q}^{(m)} \rightarrow \text{IDFT} \]

\[ q[n] \]

\[ \partial f_{NL}^{N}[n] / \partial q \]

\[ J \]

\[ \Delta \tilde{q}^{(m)} = -[J]^{-1} r^{(m)} \]

\[ \tilde{q}^{(m+1)} = \tilde{q}^{(m)} + \Delta \tilde{q}^{(m)} \]

\[ m \rightarrow m + 1 \]

\[ | r^{(k)} | < \epsilon ? \]

\[ \text{no} \]

\[ \text{yes} \]

\[ \text{no} \]

\[ \text{yes} \]

**Figure 4.2:** Harmonic Balancing using the Alternating Frequency / Time method:

1. Construct a timeseries from state vector \( \tilde{q}^{(m)} \) through the Inverse Discrete Fourier Transform.
2. Compute the nonlinear force, as well as its derivative, as timeseries.
3. Obtain the harmonic coefficients of the nonlinear force, as well as the Jacobian, through the Discrete Fourier Transform.
4. Compute the residual in the harmonic equation of motion.
5. Compute and apply the update of the state vector.
6. Check whether convergence is reached. If not, proceed to step 7 and repeat steps 1-5.
7. Update the iteration counter \( m \).

\[
q[n] = a_0 + \sum_{k=1}^{K} \left[ a_k \cos \left( \frac{2\pi nk}{M} \right) + b_k \sin \left( \frac{2\pi nk}{M} \right) \right].
\]  

(4.33)

From here, a discrete timeseries of the nonlinear force can be computed:

\[
f_{NL}[n] = f_{NL}^{NL} (q[n]).
\]

(4.34)

The integrals in Eqns. 4.24 - 4.26 are then carried out numerically, using the Discrete Fourier Transform (DFT):

\[
c_0 = \frac{1}{M} \sum_{n=0}^{M-1} f_{NL}^{NL}[n],
\]

(4.35)

\[
c_k = \frac{2}{M} \sum_{n=0}^{M-1} \left[ f_{NL}^{NL}[n] \cos \left( \frac{2\pi nk}{M} \right) \right],
\]

(4.36)

\[
d_k = \frac{2}{M} \sum_{n=0}^{M-1} \left[ f_{NL}^{NL}[n] \sin \left( \frac{2\pi nk}{M} \right) \right].
\]

(4.37)

Inserting the estimated response and the resulting nonlinear spectral coefficients in the EoM (Eqns. 4.27-4.29), will result in a residual:
4. SOLUTION STRATEGIES FOR NONLINEAR STRUCTURAL DYNAMICS

\[ r_{DC} = g_{DC}(a_0, c_0), \]
\[ r_{k,\text{cos}} = g_{k,\text{cos}}(a_k, c_k), \]
\[ r_{k,\text{sin}} = g_{k,\text{sin}}(b_k, d_k). \]

The total residual \( r \), state coefficients \( \tilde{q} \) and nonlinear force coefficients \( \tilde{f}^{NL} \) vectors are composed as follows:

\[
\begin{bmatrix}
    g_{DC} \\
g_{1,\text{cos}} \\
g_{1,\text{sin}} \\
g_{2,\text{cos}} \\
g_{2,\text{sin}} \\
g_{K,\text{cos}} \\
g_{K,\text{sin}}
\end{bmatrix}
\quad ; \quad (4.41)
\]

\[
\begin{bmatrix}
a_0 \\
a_1 \\
b_1 \\
a_2 \\
b_2 \\
a_K \\
b_K
\end{bmatrix}
\quad ; \quad (4.42)
\]

\[
\begin{bmatrix}
c_0 \\
c_1 \\
d_1 \\
c_2 \\
d_2 \\
c_K \\
d_K
\end{bmatrix}
\quad ; \quad (4.43)
\]

A Newton-Raphson scheme, similar to that in sequential path-following (previous section) can be employed, to iterate towards solution \( \tilde{q} \). The updates are found by inverting the Jacobian of the residual equation with respect to the states:

\[
\tilde{q}^{(m+1)} = \tilde{q}^{(m)} - [J^{(m)}]^{-1} r^{(m)},
\]

with

\[
J = \frac{\partial r}{\partial \tilde{q}} = \frac{\partial g}{\partial \tilde{q}} = \begin{bmatrix}
K & 0 & 0 & 0 & 0 \\
0 & \Phi_1 & 0 & 0 & 0 \\
0 & 0 & \Phi_2 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \Phi_K
\end{bmatrix} + \frac{\partial \mu}{\partial \tilde{q}},
\]

\[
\Phi_k = \begin{bmatrix}
-(\omega_k^2 M - K) & \omega_k C \\
-\omega_k C & (\omega_k^2 M - K)
\end{bmatrix}.
\]

The terms in \( J \) due to the nonlinear force, \( \frac{\partial f^{NL}}{\partial \tilde{q}} \), are found by taking the Jacobian of Eqns. 4.35-4.37 with respect to \( \tilde{q} \):

\[
\frac{\partial c_0}{\partial \tilde{q}} = \frac{1}{M} \sum_{n=0}^{M-1} \frac{\partial f^{NL}_K[n]}{\partial \tilde{q}}, \]
\[
\frac{\partial c_k}{\partial \tilde{q}} = \frac{2}{M} \sum_{n=0}^{M-1} \frac{\partial f^{NL}_K[n]}{\partial \tilde{q}} \cos \left( \frac{2\pi nk}{M} \right), \]
\[
\frac{\partial d_k}{\partial \tilde{q}} = \frac{2}{M} \sum_{n=0}^{M-1} \frac{\partial f^{NL}_K[n]}{\partial \tilde{q}} \sin \left( \frac{2\pi nk}{M} \right), \]
\[
\frac{\partial f^{NL}_K[n]}{\partial \tilde{q}} = \frac{\partial f^{NL}_K[n]}{\partial q[n]} \frac{\partial q[n]}{\partial \tilde{q}}.
\]
4.4. NON-MONOLITHIC METHODS

where, from Eqn. 4.33:

\[
\frac{\partial q[n]}{\partial \tilde{q}_{i,j}} = \begin{cases} 
1 & i = 1 \\
\cos\left(\frac{2\pi nj}{M}\right) & i = 2, 4, 6, ..., 2K \\
\sin\left(\frac{2\pi nj}{M}\right) & i = 3, 5, 7, ..., 2K + 1
\end{cases}
\]  

(4.51)

and \(\partial f_{NL}^K[n]/\partial q[n]\) is the discrete form of \(\partial f_{NL}^K(t)/\partial q(t)\) (known from the analytic model of the nonlinear force), to be obtained numerically.

The main advantage of the AFT method is that any expression can be used for the nonlinear force \(f_{NL}(q)\). Furthermore, no symbolic variables are needed, and in terms of computational effort, the method seems to be the most promising. Furthermore, no lengthy derivations have to be made, where potential errors may be hard to identify. The downside is that it may take many iterations until convergence is reached. Furthermore, a Jacobian is needed in the iterative process which may be expensive to construct for each iterative step. Lastly, there may exist situations with no stable branches of the solution, in which case no solution is found. However, instabilities are usually associated to a single harmonic external excitation. In the problem of interest with the TMD under multi-frequency forcing from the sea, the system will not likely be given the chance to exhibit unstable motion associated with a single forcing frequency.

Given the above reasoning, the AFT method seems to be the most promising for the problem of interest, and is therefore applied to the end-stop problem studied in this work.

4.4 Non-monolithic methods

Often the nonlinear behavior of the total structure is due to only part of the structure, as with this work, a nonlinear TMD attached to a linearly modeled structure.

4.4.1 Time-integration (single / multi-harmonic)

Previous work of v. d. Valk [35] has shown that solving such systems in the time domain in a non-monolithic way (solving for the linear and nonlinear parts with respective time-marching algorithms, and solving at the interface), can significantly increase computation speed when the number of DoF associated to the nonlinear forces is relatively small compared to the entire structure. This method has not been applied to the current work, since the number of DoF in the total system is not so large that the computational gain is expected to be significant, especially with the created overhead, as a nested global vs. local (on substructure level) iteration framework is needed.

4.4.2 Sequential path-following (single harmonic)

In this work, a non-monolithic approach is presented for structures with nonlinear components, linear coupling forces between the (linear and nonlinear) components, a single harmonic external loading. The method starts by considering a base-excitation problem of the nonlinear part of the structure, as shown in Fig. 4.3, with a forced base-excitation amplitude \(\tilde{q}_b\) and forcing frequency \(\omega\).
The EoM for this system reads, with $Z_d = -\omega^2 M_d + j\omega C_d + K_d$:

$$Z_d \ddot{q}_d + f_{NL}^d(\ddot{q}_b, \ddot{q}_d) = f_b(\ddot{q}_b), \quad (4.52)$$

$$f_b(\dddot{q}_b) = (j\omega C_d + K_d) \ddot{q}_b. \quad (4.53)$$

where $\ddot{q}_d$ and $\dddot{q}_b$ are the amplitudes of the harmonic responses of the damper and interface DoF, respectively. Hence, $\ddot{q}_d$ can be solved by applying the Newton-Raphson scheme to Eq. 4.52 by choosing $\ddot{q}_b$. By repeating this process for a range of $\ddot{q}_b$ and applying the force defined in Eq. 4.53 to the standalone nonlinear system, a response map can be obtained that gives the force-steady state displacement relationship:

$$Z_{NL,d,k} = \tilde{f}_{b,l} \tilde{q}_{d,k}. \quad (4.54)$$

Hereby the assembled EoM is reduced to a quasi-static problem:

$$\begin{bmatrix} Z_{d,k}^{NL}(\ddot{q}_b) & Z_{ds} \\ Z_{sd} & Z_{ss} \end{bmatrix} \begin{bmatrix} \ddot{q}_d \\ \ddot{q}_s \end{bmatrix} = \begin{bmatrix} \tilde{F}_d \\ \tilde{F}_s \end{bmatrix}, \quad (4.55)$$

where $Z_{sd} = Z_{ds} = -j\omega C_d - K_d$ and $\tilde{F}_d$ and $\tilde{F}_s$ denote the complex amplitudes of the external forcing on the nonlinear and linear structures, respectively. Similar as in the Guyan reduction [32], the problem can be further reduced by condensing the linear part of the solution onto the nonlinear part. Rewriting the linear equation gives:

$$\ddot{q}_s = Z_{ss}^{-1} \left( \tilde{F}_s - Z_{sd} \ddot{q}_d \right). \quad (4.56)$$

Inserting 4.56 in the first line of Eq. 4.55 and rearranging, gives:

$$\dot{Z}_{d,k}^{NL}(\ddot{q}_d) \ddot{q}_d = \ddot{f}_d, \quad (4.57)$$

with:
\[
Z_{d}^{NL}(\ddot{q}_d) = Z_{d}^{NL}(\ddot{q}_d) - Z_{ds}Z_{ss}^{-1}Z_{sd},
\]
(4.58)
\[
\hat{f}_d = \hat{F}_d - Z_{ds}Z_{ss}^{-1}\hat{F}_s,
\]
(4.59)

where the dependency \( \hat{Z}_{NL}^{d}(\dddot{q}_b \in \dddot{q}_s) \) has been changed freely to \( \hat{Z}_{NL}^{d}(\dddot{q}_d) \), given the direct relationship 4.56. A Newton-Raphson scheme can be employed to find the solution of \( \dddot{q}_d \), and the response of the structure \( \dddot{q}_s \) can be computed in one step, with Eq. 4.56. Once \( Z_d^{NL} \) is obtained, it can always be used to compute the total response of the (arbitrary!) structure to which it is connected, as long as the nonlinear system component does not change \( (M_d, K_d, C_d, f_{NL}(\dddot{q}_i, \dddot{q}_d)) \). As most of the computational cost is most likely spent in the iterations needed to obtain \( \hat{Z}_d^{NL}(\dddot{q}_d) \), a relatively small cost is expected to remain for computing the response of the coupled system afterwards. This can be of benefit in situations where many different simulations are required (such as in wind industry, mechatronics, etc.) to represent different scenarios, or when optimization is carried out for the linear part of the structure. In Sec. 5.4.2 this method is applied to the two DoF structure shown in Fig. 4.3, including end-stop nonlinearity.

4.5 Conclusions and outlook

In this work, several methods were discussed that can be used to obtain the dynamic response of nonlinear structural models. Four classes of solution techniques were covered, containing time and/or frequency domain based sub-methods. The Newmark time-integration method was selected as the time domain analysis tool for all possible situations. For single harmonic external forcing, harmonic balancing (using Alternating Frequency / Time - AFT) was selected, as well as sequential path-following. For the latter, a new technique has been presented in Sec. 4.4 to solve for systems in a non-monolithic way. For multi harmonic external loading, the harmonic balancing (with AFT) method has been selected as well, including a proposal to include far more harmonics in the external forcing than in conventional applications in Sec. 4.3.3.

In Chapter 5, the modeling frameworks as defined in Chapters 3 and 4 will be applied to single and simple mDoF systems. In Chapter 6 these methods are applied to the monopile-based offshore wind turbine system with a one-dimensional model of a Tuned Mass Damper, in combination with the time and frequency fatigue load calculation procedures as described in Chapter 2.
Part II

Case studies & Results
Chapter 5

End-stop behavior for one and two DoF systems

In this chapter, the performance of the techniques for nonlinear dynamics presented in Chapter 4 is studied and evaluated, by applying them to several illustrative one DoF and simple two DoF systems. The one DoF system could be seen as a standalone TMD. As an introductory example, the Duffing oscillator is presented in Sec. 5.1, after which the example is repeated in Sec. 5.2 with a fractional-order spring instead of a cubic spring. Sec. 5.3 builds further on the fractional-order spring, however placing a threshold in the displacement; this is an endstop mechanism for a stand-alone damper system, as introduced in Sec. 3.2.3.

After treating the description of one DoF systems, a two DoF damped structure with an end-stop is presented in Sec. 5.4. The model is simplified by leaving out the hysteresis term in the end-stop force. After studying the behavior of the structure under a single harmonic excitation at constant amplitude, the response is studied under multi-frequency excitations, as a means of representing wave loads. Finally, the end-stop model is complicated by inclusion of a hysteresis term, in Sec. 5.5, given by the material model for end stops as presented in Sec. 3.2.3. As a last step, the results from the different methods are discussed in Sec. 5.6. The performance of the methods is discussed in terms of accuracy, computational cost, ability to give insight in the physical behavior and the required assumptions.

Two frequency domain methods are investigated in this chapter: the harmonic balancing method based on Alternating Frequency/Time (AFT-HB) from Sec. 4.3.3 and sequential path-following from Sec. 4.4.2. The leading verification tool for all results obtained in this chapter is nonlinear time-simulation (Sec. 4.2.1). The results for sDoF systems under single harmonic loading have been further verified with a renowned pseudo-arclength path-following tool [21] (introduced in Sec. 4.2.2), as was mentioned in Sec. 4.2.2.

In the results shown in this chapter, the linear resonance frequency will be referred to as $\omega_0$. In case of a 2DoF structure, $\omega_0$ is the linear resonance frequency corresponding to the standalone structure. Furthermore, timeseries will be plotted against dimensionless time $\tilde{t} = \omega_0 t$. 

49
5.1 Duffing Oscillator

The duffing oscillator has been extensively studied in literature, for instance [30]. The equation of motion is that of a linear oscillator, with an additional cubic spring with stiffness $k_2$:

$$m\ddot{u} + c\dot{u} + ku + k_2u^3 = F \sin \Omega t.$$  \hspace{1cm} (5.1)

In Fig. 5.1 the frequency response is shown, with $k_2 = k$. For the harmonic balancing method, the number of included harmonics is $K = 7$. The total stiffness resulting from $k$ and $k_2$ increases with the oscillation amplitude, due to the displacement dependency of the elastic forces in the nonlinear term. An increase in stiffness in turn results in increased oscillation frequency - this explains why the response peak lies on a frequency higher than the linear resonance frequency $\omega_0$, namely around $\omega/\omega_0 \approx 1.3$. The latter described effect is referred to as hardening. In the example shown in Fig. 5.1, the nonlinearity is so severe that the resonance peak is tilted to the point of introducing three different solutions for $1.3 \leq \omega \leq 1.35$, as revealed by the solution obtained with the pseudo-arclength path-following method. All other methods rely on iteration schemes that yield one solution per frequency, hence the different solution branches are not obtained. Good agreement is shown between time-integration, harmonic balancing and pseudo-arclength path-following. This is in contrast to the sequential path-following, which overestimates the nonlinear term, as higher order harmonics are not taken into account. Similar observations can be made in the time-response, which is shown in Fig. 5.2.

![Figure 5.1: Frequency response for the Duffing oscillator, under single harmonic external loading, with $k_2 = k$. The response is scaled to the linear static response $u_{0,lin}$. The pseudo-arclength path-following algorithm [21] was borrowed and applied with permission.](image)
5.2 1DoF oscillator + fractional-order spring

The 1DoF nonlinear oscillator with a fractional-order spring is governed by the following equation:

\[ m\ddot{u} + c\dot{u} + ku + \text{sign}(u)k_2\vert u \vert^\alpha = F \sin \Omega t. \]  

Here, \( \alpha \) is chosen as \( 3/2 \). In Fig. 5.3 the frequency response is shown. Similar observations can be made as with the Duffing oscillator from Sec. 5.1 with regard to hardening behavior. Furthermore, the natural frequency of the system is shifted past \( \omega_0 \), suggesting that the nonlinear force adds to the linear stiffness \( k \). This can be understood intuitively, since a fractional order power cannot be expressed directly as a higher order power. The linear term in a Taylor expansion around equilibrium (\( u=0 \)) would give:

\[
\frac{d}{du}(\text{sign}(u)\vert u \vert^\alpha) = \frac{d}{du}((2\tilde{w}_s(u) - 1)\vert u \vert^\alpha),
\]

\[
= 2\tilde{D}_d(u)\vert u \vert^\alpha + \alpha (2\tilde{w}_s(u) - 1)\text{sign}(u)\vert u \vert^{\alpha-1},
\]

where \( \tilde{w}_s \) and \( \tilde{D}_d \) denote the step and delta Dirac functions respectively. Since \( \tilde{D}_d(u) \) and \( u \) are infinite respectively and zero at \( u = 0 \), the resulting contribution is not easily defined mathematically. Alternatively, one could carry out a numerical polynomial fitting procedure. When using the ‘polyfit’ function in MATLAB on the region \(-1 \leq u \leq 1\) this gives, up to 5 coefficients of \( c_r u^r \):

<table>
<thead>
<tr>
<th>( r )</th>
<th>( c_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4146</td>
</tr>
<tr>
<td>3</td>
<td>1.5186</td>
</tr>
<tr>
<td>5</td>
<td>-1.6905</td>
</tr>
<tr>
<td>7</td>
<td>0.7676</td>
</tr>
</tbody>
</table>

**Table 5.1:** Coefficients of the polynomial giving \( \sum_{r=1}^{7} c_r u^r \) as an approximation of \( \text{sign}(u)k_2\vert u \vert^{3/2} \), obtained with the MATLAB function 'polyfit'.

**Figure 5.2:** Time-integration results of the steady-state response for the Duffing oscillator, \( \omega = 1.25\omega_0 \).
The linear contribution $c_1$ suggests that the stiffness increases by a factor $1.4$ due to the nonlinear term, which would cause the natural frequency to increase by a factor $\sqrt{1.4} \approx 1.2$. It must be noted that these parameters are very sensitive to chosen range of $u$ for fitting, and for the chosen number of coefficients $c_r$, and that this fitting example serves merely as an illustration.

Lastly, a superharmonic can be observed at $\omega \approx 0.5\omega_0$ in Fig. 5.3, suggesting that a higher harmonic coincides with the natural frequency of the system. This phenomenon is revealed by all methods except for sequential path-following, which takes only one harmonic in the solution. In Fig. 5.4 this observation is confirmed, where the harmonic components resulting from the harmonic balance method are shown. The dominance of the response at $p = 3$ suggests that the linear eigenfrequency of the system has shifted to around $1.5\omega_0$ (3 times the frequency at which the superharmonic was observed).

![Figure 5.3: Frequency response of the fractional order oscillator, under single harmonic external forcing. The response is scaled to the linear static response $u_{0,lin}$. The pseudo-arclength path-following algorithm [21] was borrowed and applied with permission.](image)

### 5.3 1DoF oscillator with fractional-order end-stop

In Sections 5.1 and 5.2 the performance of the different techniques discussed in Chapter 4 was shown for single harmonic loading. In this section, the sDoF system is of the oscillator with the end-stop is treated. Both single and multi frequency external loadings are considered, since they result in very different system behavior, and since the loading scenario from the sea is caught between these two extremes.

A setup of this system is shown in Fig. 5.5. For the end-stop nonlinearity, the model as described in Eq. 3.7 is applied:
5.3.1 Single harmonic external loading

For a single harmonic loading, the external load becomes

\[ F(t) = \hat{F} \sin \Omega t. \]  

(5.7)
In Fig. 5.6 the frequency response is shown. The end-stop is hit in the region $0.93 \leq \omega/\omega_0 \leq 1.05$, where the response is saturated: the relative amplitude reaches a limit of around $u/u_{0,lin} = 4.1$. Again, agreement is shown between all methods except for the sequential path following method. Although all methods start to saturate the response at $\omega/\omega_0 = 0.93$, the sequential path-following overestimates the RMS amplitude response as $\Omega$ is increased. A time-response reveals that the higher harmonics cause the response to become triangular, decreasing the RMS value of the amplitude, as shown in Fig. 5.7. In Fig. 5.8 the results for the frequency components from the Harmonic Balance method are shown, revealing higher harmonics at $\omega = k\Omega$ with $k = 3, 5, 7...$. Although the higher harmonics are present, these start at almost 2 orders of magnitude less than that for $k = 1$, indicating that the amount energy contained in the higher harmonics is relatively small.

Furthermore, the pseudon-arclength path following displays an extreme hardening branch for $1.05 \leq \omega/\omega_0 \leq 1.12$. The sequential path-following method is able to detect part of this hardening region. It must be noted that the results for sequential path-following have been obtained by forward sweeping. With a backward sweep, this hardening branch is not detected. In order to investigate this hardening further, the dependence of the frequency response on the end-stop stiffness is portrayed in Fig. 5.9. For $k_2/k = 10$ and 50, the hardening branch is analogous to the Duffing and fractional order nonlinear oscillators treated in Sections 5.1 and 5.2. As the relative end-stop stiffness is increased between $10 \leq k_2/k \leq 5 \cdot 10^2$, the response in the end-stop region $0.93 \leq \omega/\omega_0 \leq 1.04$ becomes more suppressed, to the point of no longer increasing, but decreasing for $k_2/k = 5 \cdot 10^2$, as was also revealed in the RMS values from time series results (Fig. 5.7). The phenomenon can be explained as follows: the amplitude response can be seen as a multiplication of a static response and a dynamic amplification factor. The dynamic amplification factor depends on the damping $c$, while the static response amplitude depends on the stiffness, composed of $k$ and $k_2$. As $\Omega$ is increased, the stiffness of the system increases as well due to the hardening effect, as described in Sec. 5.1. As a result, the total response amplitude relative to the linear case decreases for increasing $\omega$.

The harmonic balance method again shows a good agreement with the pseudon-arclength path-following method, in tracing the stable parts ($0.93 \leq \omega/\omega_0 \leq 1.04$) of the nonlinear response.

### 5.3.2 Multi harmonic external loading

One could start to complicate the analysis so far in this chapter by increasing the harmonic components to just two, or a few more. An advantage of such an approach is that very specific nonlinear behavior could be studied, for instance that of higher harmonics from (part of) the external load coinciding with the resonance frequency of the structure. However, in Sec. 5.3 it was shown that the first higher harmonic in the system is at $\omega = 3\Omega$. If one were to excite the structure at $\Omega = \frac{1}{3}\omega_{res}$, the load level would need to be very high in order for the end-stop to be hit in the first place. Secondly, an accurate model for a sea wave spectrum starts with no less than 50 harmonic components, with random phase relationship. The approach also changes inherently when moving from a single to a multi-harmonic sea-state representative loading in that one tries to mimic a situation with no steady-state response. With this reasoning it was chosen to study directly the other extreme, namely a uniformly distributed external load spectrum with closely spaced frequency components, up to a cutoff frequency around $2\omega_0$. 
A multi-harmonic force vector with uniform distribution can be constructed by summing a number of $n_F$ individual sinusoids:

$$F(t) = \frac{\hat{F}}{n_F} \sum_{i=1}^{n_F} \sin(\Omega_i t + \phi_i).$$

(5.8)
Figure 5.8: Frequency components resulting from the Harmonic Balance method, for the oscillator with the end-stop nonlinearity, $\omega = 1.05\omega_0$.

Figure 5.9: Frequency response of the oscillator with the end-stop nonlinearity, under single harmonic external forcing, for different relative end-stop stiffnesses. The response is scaled to the end-stop free distance $b_{free}$. PSA = Pseudo-Arclength. The Harmonic Balancing method was carried out using 7 harmonics. The pseudo-arclength path-following algorithm [21] was borrowed and applied with permission.

$\hat{F}$ is the amplitude, which is constant so that all frequencies in the given spectrum carry the same force level. $\phi_i$ is a random phase, and there is no correlation between the phases from the different sinusoids. In Fig. 5.10 an example of this forcing vector is shown. Other uniformity could be chosen as an alternative, such as a uniform power spectrum. However, since this extreme case of uniform distribution serves mainly as illustration, the different possibilities for distributions were not extensively treated, and it was chosen to proceed
with the intuitive uniform force distribution defined in Eq. 5.8.

![Figure 5.10](image)

**Figure 5.10**: Time-series sample of the multi-harmonic forcing as as defined in Eq. 5.8. The period of the time-simulation is bordered by the red vertical lines.

After the force vector from Eq. 5.8 is passed to the nonlinear system (Eqn 5.6), a time response can be obtained, as shown in Fig. 5.11. The frequency response has been obtained through the FFT, after applying a Hamming window. The result is shown in Fig. 5.12.

![Figure 5.11](image)

**Figure 5.11**: Time-response of the oscillator with the end-stop nonlinearity, under multi harmonic external forcing.

The apparent noisiness of the frequency response is due to the stochastic nature of the phases in the external forcing, causing a biased response due to the nonlinearity. When the process of generating a force vector (Eq. 5.8), time-integrating Eq. 5.6 and hence obtaining different frequency responses through the FFT is repeated many times and averaged, a smoother response is obtained. The result is shown in Fig. 5.13. The response has also been obtained with the Harmonic Balancing method, as discussed in Sec. 4.3.3, by applying the forcing defined in 5.8 and assuming the response in the same harmonic
components. The response obtained with harmonic balancing shows good agreement with time-simulations.

Figure 5.12: Frequency response of the oscillator with end-stop, under multi-harmonic forcing as as defined in Eq. 5.8, obtained by windowing and calculating the FFT of a displacement response timeseries. The result is normalized to the static linear response $u_{0,\text{lin}}$.

Figure 5.13: Frequency response of the oscillator with the end-stop nonlinearity, under multi harmonic external forcing, averaged over 134 different realizations of random phase in the external load (Eq. 5.8).

In contrast to the response under single harmonic loading (Fig. 5.6), no threshold behavior of the end-stop is apparent in the averaged frequency response that can be seen in Fig. 5.13. Due to the random phase relationship between the harmonic components in the external forcing, the values of $\omega$ at which end-stop is hit carry a probability distribution rather than a sharp transition. A smooth lowering of the amplitude, as well as a slight hardening can be observed in Fig. 5.13. Just as with the response amplitude reduction, this hardening is of smooth nature as opposed to the hardening branches corresponding to the single harmonic external loading, as were observed in Fig. 5.6.
5.4 2DoF damped structure with fractional-order end-stop

The 2DoF damped structure is the most basic representation of any structure, damped by a TMD. This system is shown in Fig. 5.14, where the end-stop mechanism is also sketched. The system consists of a rigid body representing the damped structure, connected to ground via a linear spring and dashpot set. The structure is subject to an external force $F$, which is generally time-dependent. The damper is in turn connected to the structure, via a linear spring and a dashpot, and its free motion relative to the main structure is limited by the end-stop to $b_{\text{free}}$. The equations of motion for this system are defined as follows, with structure and damper denoted with subscripts $s$ and $d$, respectively:

$$
\begin{bmatrix}
    m_s & 0 \\
    0 & m_d 
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_s \\
    \ddot{u}_d 
\end{bmatrix} +
\begin{bmatrix}
    c_s - c_d & -c_d \\
    -c_d & c_s + c_d 
\end{bmatrix}
\begin{bmatrix}
    \dot{u}_s \\
    \dot{u}_d 
\end{bmatrix} +
\begin{bmatrix}
    k_s & -k_d \\
    -k_d & k_s + k_d 
\end{bmatrix}
\begin{bmatrix}
    u_s \\
    u_d 
\end{bmatrix} +
\begin{bmatrix}
    f_{\text{ES}}(u_s, u_d) \\
    0 
\end{bmatrix} =
\begin{bmatrix}
    0 \\
    F(t) 
\end{bmatrix},
$$

(5.9)

$$
f_{\text{ES}}(u_s, u_d) = \begin{cases}
    k_2 (|u_d - u_s| - b_{\text{free}})^{\alpha} \cdot \text{sign}(u_d - u_s) & |u_d - u_s| - b_{\text{free}} \geq 0 \\
    0 & |u_d - u_s| - b_{\text{free}} < 0
\end{cases}.
$$

(5.10)

As with the 1DoF oscillator in Sec. 5.3, the response of the system is studied for both single- and multi-frequency external loadings in Secs. 5.4.1 and 5.4.3. Furthermore, the non-monolithic approach as formulated in Sec. 4.4.2 is applied to this 2DoF system for single harmonic loading in Sec. 5.4.2, hereby proposing a faster way to compute the total response of the system.

![Figure 5.14: 2DoF damped structure with end-stops limiting the motion of the damper mass $m_d$. The end-stop mechanism, drawn in red, is connected rigidly to the structure mass $m_s$.](image)

5.4.1 Single harmonic external forcing, monolithic approach

For a single harmonic external load, Eq. 5.7 has been used. In Fig. 5.15 the frequency response is shown, both for the structure displacement response, as well as the relative TMD displacement, and the response of the stand-alone structure.

The response of the main mass approaches that of the situation without a damper, or even worse, since the parasite damper mass $m_d$ is added to $m_s$, the latter also being responsible for the lowering of the natural frequency of the system. Just as with the 1DoF oscillator with the end-stop, the response curves show a threshold behavior; they deviate from the linear case for $||u_d - u_s|| \geq b_{\text{free}}$. As with the 1DoF examples, the sequential path-following (of forward Newton-Raphson) overshoots the displacement with respect to
the results from time domain simulations, although the values for \( \omega \) at which the curves start deviating from the linear reference curve seem to coincide for both methods.

Around resonance (\( \omega = \omega_0 \)), the time-integration method reveals a sharp increase in the relative TMD response. Time simulations are shown in Fig. 5.16, and reveal an asymmetric response from the time-simulations (the DC component is \( u = -0.4b_{\text{free}} \)). In an attempt to trace this behavior with the harmonic balancing method, the number of harmonic components was increased to around 50 (including DC terms), but the method was not able to yield this particular asymmetric branch of the solution.

Around \( \omega/\omega_0 = 1.01 \) the amplitude response of the structure drops to values slightly below the linear curve. Especially the time-integration method shows erratic behavior in the relative TMD displacement. In Fig. 5.17 the time response of the relative TMD displacement is shown. The behavior seems a ‘beating’ effect, where the difference between \( \omega_0 \) and \( \Omega \) appears as a dominant frequency component in the displacement response. This frequency \( \Omega - \omega_0 \) is far lower than the forcing \( \Omega \), and is hence not caught by the harmonic balancing method (as the lowest frequency in the response is \( \Omega \)), nor the sequential path-following method.

### 5.4.2 Single harmonic external forcing, non-monolithic approach

The method as proposed in Sec. 4.4.2 has been applied to obtain the frequency response of the 2DoF damped structure with end-stop nonlinearity, using sequential path-following.
5.4. 2DOF DAMPED STRUCTURE WITH FRACTIONAL-ORDER END-STOP

![Figure 5.16](image1.png)

**Figure 5.16:** Time response of the 2DoF structure + TMD with the end-stop nonlinearity, under single harmonic external forcing, $\Omega = \omega_s$.

![Figure 5.17](image2.png)

**Figure 5.17:** Time response of the 2DoF structure + TMD with the end-stop nonlinearity, under single harmonic external forcing, $\Omega = 1.03\omega_s$.

When setting up the base-excitation problem sketched in Fig. 4.3, a map can be obtained giving the nonlinear displacement transfer $z^{NL}$ from the base-excitation force $f_b$ the TMD displacement $u_d$, as a function of the forcing frequency $\omega$ and the forcing amplitude $\tilde{f}_b$. The base excitation force is defined as:

$$\tilde{f}_b(\tilde{u}_b) = (j\omega c_d + k_d) \tilde{u}_b.$$

Hence, by selecting a suitable range for $\tilde{u}_b$, the range for $\tilde{f}_b$ follows directly. A resulting map is shown in Fig. 5.18. As $\tilde{f}_b / \max(\tilde{f}_b)$ is increased, the TMD starts to hit the end-stop, where part of the response is limited by the end-stop. For $\omega / \omega_0 > 1$, saturation occurs, as was also portrayed in Fig. 5.6. The response can be obtained as explained in Sec. 4.4.2, after partitioning the system of equations as follows:
The resulting frequency response is shown in Fig. 5.19. Apart from a small deviation observed around $\omega/\omega_0=0.96$, the non-monolithic approach matches the monolithic sequential path-following result. The small deviation in the result is due to the resolution of the sample points on the map, for which cubic interpolation was used. Upon choosing a finer grid, these deviations were found to decrease significantly. The simulation times needed for both methods are listed in Table 5.2.

### Table 5.2: Results for simulation time needed to obtain the frequency response of the 2DoF structure with TMD, under single harmonic external loading, using sequential path-following.

<table>
<thead>
<tr>
<th>Method</th>
<th>Simulation Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monolithic</td>
<td>48 s</td>
</tr>
<tr>
<td>Non-monolithic</td>
<td>6 s</td>
</tr>
</tbody>
</table>

Hence, a factor 8 in computational speed is gained by using the non-monolithic sequential path-following method as proposed in this work. The gain is made in the iterations needed when searching on the steep nonlinear end-stop force-displacement relationship curve, which is taken care of when creating the response map, beforehand. If this cost is taken into account, the non-monolithic approach would be slower than the monolithic approach. However, if the same TMD with end-stop model is used for more than one simulation (in offshore wind turbine load calculations, thousands of cases may be applicable), one can obviously benefit from the non-monolithic approach.

#### 5.4.3 Multi harmonic external forcing, monolithic approach

In this final part of this chapter, the 2DoF structure with TMD is studied with the structure subject to multi-harmonic external loading, as defined in Eq. 5.8. In Fig. 5.20 the frequency response is shown for both the structure and the relative TMD displacement, after averaging around 150 realizations of the random phase in the external force $\phi_i$ in Eq. 5.8, as was done in Sec. 5.3.2. The main purpose of this section is to investigate the performance of the harmonic balance method, which shows good agreement with time-integration. As with the 1DoF oscillator in Sec. 5.3.2, the frequency response is of completely different nature than that under single harmonic loading (section 5.4.3). Instead of a sharp transition to a degradation of the performance of the TMD, a much milder effect is portrayed. In the region $0.9 \leq \omega/\omega_0 \leq 1.2$ the response of the TMD mass is slightly perturbed with respect to the linear case, causing a relative increase of the structure response in the order of 20%.

In Fig. 5.21 the time response is shown, for an arbitrary realization of $\phi_i$. Again, good agreement is shown between the harmonic balance method and time-integration.
5.5 Including hysteresis in the end-stop model

For the examples treated so far in this chapter has been assumed that the end-stop model is purely elastic - there is no dissipative term depending on velocity. In Sec. 3.2.3 it was explained how the dissipative term can be defined through a restitution coefficient $e_R$, which is defined as the return velocity divided by the approach velocity. For purely elastic collisions, $e_R = 1$. In reality, $e_R$ is always between 0 and 1. For a perfect inelastic collision, $e_R = 0$ and the damping ratio in the end-stop force term would be infinite, which would lead to singularity upon numerical simulation.

Fig. 5.22 shows that the effect of including hysteresis in the end-stop force is relatively small. The threshold values for the degradation of the response are unchanged, since they

\[ \tilde{F}_i / \max(\tilde{F}_i) \]

\[ \tilde{f}_b / \max(\tilde{f}_b) \]

\[ \omega / \omega_0 \]
64 5. END-STOP BEHAVIOR FOR ONE AND TWO DOF SYSTEMS

Figure 5.19: Frequency response of the structure mass for the 2DoF structure with TMD under single harmonic external loading, obtained with both the monolithic and non-monolithic sequential path-following method.

Figure 5.20: Frequency response of the 2DoF damped structure with the end-stop nonlinearity, under multi harmonic external forcing.

depend on relative displacement only. The hysteresis seems to introduce a slight hardening, and a further degradation past resonance. In Figs. 5.24 and 5.23 the time domain
5.5. INCLUDING HYSTERESIS IN THE END-STOP MODEL

Figure 5.21: Time response of the oscillator with the end-stop nonlinearity, under multi harmonic external forcing.

Figure 5.22: The effect of including hysteresis in the endstop force on the frequency response of the structure mass. The response is constructed by time-simulations where the structure mass is harmonically forced.

Simulation results are shown for \( \omega/\omega_s = 0.995 \) (around resonance), and for \( \omega/\omega_s = 1.004 \) (the beating effect as described in Sec. 5.4 - more specifically, Figs. 5.15 and 5.17), for two different hysteresis levels.

The general displayed time domain behavior with high hysteresis (\( e_R = 0.10 \)) is similar to the case without hysteresis (\( e_R = 1.00 \)), although the penetration depth into the endstop is significantly reduced (more than a factor 5), as shown in Tab. 5.3.

The gained accuracy in especially the frequency responses is not expected to outweigh the increase in complexity in the model by introducing hysteresis, given the observations made in this section. Therefore it was decided not to further investigate in the end-stop model including the hysteresis term.
Figure 5.23: Time domain simulation results for the relative displacement between structure and damper, \( u_d-u_s \), with \( e_R = 0.10 \). Both figures represent the same timeseries, with different plot ranges.

Figure 5.24: Time-integration results for the relative displacement between structure and damper, \( u_d-u_s \), with \( e_R = 1.00 \). Both figures represent the same timeseries, with different plot ranges.

Table 5.3: Results for maximum penetration depth, for different hysteresis levels in the end-stop force

<table>
<thead>
<tr>
<th>( e_R )</th>
<th>( \frac{u_d-u_s}{b_{free}_{\text{max}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.052</td>
</tr>
<tr>
<td>0.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>
5.6 Summary and outlook

In this chapter, the frequency domain tools as presented in Chapter 4 have been applied to simple mechanical systems. After treating sDoF systems with several types of nonlinearities in Sections 5.1-5.3, insight was gained in the nonlinear behavior of a 2DoF system as the basic representation for an offshore structure with a TMD. The observations made in the current chapter can be summarized into the following points:

• The frequency response of a structure with an end-stop nonlinearity has been found very dependent on the type of external loading. Two extreme scenarios were considered: single harmonic loading (Sections 5.1 - 5.3.1 and 5.4.1), and multi-harmonic loading, uniformly distributed in frequency (Sections 5.3.2 and 5.4.3). The end-stop was found to saturate the response of both sDoF and mDoF systems in very pronounced fashion under single harmonic loading, and degrading the performance of the TMD severely for the mDoF system in Sec. 5.4.1. For multi-harmonic loading with random phase relationship, this degradation was found to be much milder and less binary, due to the stochastic nature of the external forcing.

• The harmonic balancing method as introduced in Sec. 4.3.3 shows consistency with the reference time-integration method for all different problems treated in this chapter, including the multi-harmonic external loading. The results have been further verified by comparison with a state of the art existing frequency domain method for nonlinear dynamical systems, namely a pseudo-arclength path-following tool ([21]).

• A non-monolithic sequential path-following method for single harmonic loading was introduced in Sec. 4.4.2 and applied in Sec. 5.4.2 to the 2DoF structure with a TMD and a nonlinear end-stop. The method was able reproduce the response obtained by the monolithic method for the total system, with a factor 8 gain in computational speed.

• The hysteresis term in the end-stop model was not found to influence the frequency response of the mDoF system significantly, as found in Sec. 5.5, and was omitted for the other examples treated in this chapter.

In the next and final chapter of this work (Chapter 6) the sequential path-following and harmonic balancing method are applied to a more complex system, representing the offshore monopile structure with a TMD, with the ultimate goal to obtain fatigue damage values. Time-integration results are also obtained, and are mainly used as a reference. Special focus will be given on the harmonic balancing method, as this method has been shown effective in this chapter in delivering the response simple mechanical systems under multi frequency external loads, which is needed in order to properly represent a sea state.
Chapter 6

Case study on an offshore wind turbine model including a TMD

The main goal of this chapter is to investigate the potential of frequency domain based methods to compute the response of the monopile structure with a nonlinear Tuned Mass Damper under a multi frequency external wave load governed by a sea-state. To this end, the theory and modeling in this work are applied: calculating fatigue loads from external sea forces (Chapter 2), modeling of monopile systems with TMD’s (Chapter 3) and solution techniques for nonlinear dynamics (Chapter 4). The insights gained in Chapter 5 on sDoF and simple mDoF systems are used to explain the behavior in the more complex system treated in the current chapter.

After the FEM model of the structure and the sea states are briefly presented in Sec. 6.1, the time domain behavior is illustrated in Sec. 6.2 and results for internal load spectra are presented in Sec. 6.3, obtained by time-domain as well as frequency domain methods. Throughout this chapter, the time-domain based results are used a reference, since they are the current state-of-the-art tool to predict the structural response under a multi frequency loading such as from a sea spectrum. In Sec. 6.4 the fatigue damage results are computed. Lastly, the computational efforts for all different methods are compared in Sec. 6.5. The chapter finalizes with a brief discussion in Sec. 6.6.

6.1 Wind turbine and external loads

In this work a finite element beam model developed by SIEMENS Wind Power is used to represent the multi-MW offshore wind turbine system consisting of the tower with monopile foundation. After this model is briefly presented in Sec. 6.1.1, Sec. 6.1.2 introduces a few sea states that are used in this chapter to represent mild and more severe loading scenarios respectively.

6.1.1 Offshore Wind Turbine model

FEM model

The structure consisting of the tower and monopile foundation was modeled with about 50 bi-nodal Timoshenko beam elements [13]. Each node has 6 DoF: 3 translations and rotations. Part of the bottom set of nodes was suspended in the soil, which was mod-
eled with linear springs and dampers, perpendicular to the monopile’s cylinder centerline, equally distributed around the surface. The bottom node was fixed in z-translation and z-rotation. A measurement-based modal damping was assigned to the lowest 6 modes of the structure, to represent the structural damping. The Rotor Nacelle Assembly (RNA) was modeled as a point mass. A schematic of the total system is shown in Fig. 6.1.

![Schematic of the total system](image)

**Figure 6.1:** Schematic of the FEM model for the offshore wind turbine system consisting of the foundation and the tower, with the RNA and the TMD.

### Reduced model

The FEM model as presented in the previous section was reduced using the Craig-Bampton reduction method as presented in Sec. 3.4. Hereby the original FEM model consisting of around 300 DoF was reduced to 7 DoF in total, with 5 internal DoF representing the lowest 5 eigenmodes, and 2 DoF for the x and y coordinate at the TMD attachment location.

#### 6.1.2 Sea states

The external loads of the sea at a certain instant in time is characterized by a sea load spectrum, carrying a significant wave height $H_s$ and a peak period $T_p$, as was explained in Sec. 2.1. The occurrences of the possible combinations of $H_s$ and $T_p$ are typically collected in scatter diagrams. From these diagrams it becomes apparent that small waves (low $H_s$) have shorter periods (low $T_p$), and rules of thumb can be adopted to obtain combinations with high occurrences. The following rule of thumb is used in this work:

$$T_p = 13 \sqrt{H_s/g},$$  \hspace{1cm} (6.1)

where $g$ is the gravity constant = 9.81 $m/s^2$. Two relatively often occurring sea states were selected in this work: a mild sea state with $H_s = 1m$, $T_p = 4.2s$, and a moderate sea state with $H_s = 4m$, $T_p = 8.3s$. 

6.2 Results for displacement response

Timeseries of the relative TMD displacement, as obtained by time-integration, are shown in Fig. 6.2. Since the harmonic spacings and cut-off frequencies for the time and frequency domain methods are not exactly the same, the results obtained by harmonic balancing cannot be plotted for comparison. For $H_s = 1\text{m}$, the endstop is barely hit, while a significant perturbation in the response can be observed for $H_s = 4\text{m}$. The linear response is picked up again between $315 < \omega_0 t < 320$. The time domain characteristic is very similar to the behavior of the sDoF system under multi frequency harmonic loading with uniform distribution, as was shown in Fig. 5.11.

$$H_s = 1\text{ m}, \ T_p = 4.2 \text{ s}$$

$$H_s = 4\text{ m}, \ T_p = 8.3 \text{ s}$$

Figure 6.2: Time-response of the relative displacement between structure and TMD resulting from two realizations of external force timeseries representing different sea-states. Results have been plotted versus dimensionless time $\tilde{t} = \omega_0 t$.

6.3 Results for internal bending moments

After the displacement responses were obtained with the different methods for nonlinear dynamics, the internal bending moments were computed using Eq. 2.2.2, which are in turn needed to compute the damage equivalent loads, presented in Sec. 6.4. This section presents the results for the internal bending moment spectra, obtained by applying the different solution techniques for nonlinear dynamics to the OWT model with TMD as presented in Sec. 6.1 - hereby displaying the influence of the end-stop in the TMD.

For time-integration and harmonic balancing, the solution was found for 90 different realizations of the multi frequency external force with random phases, similar to Sections 5.3.2 and 5.4.3. For the time-simulations, the FFT was taken from the time response, after which the resulting spectra were averaged over the 90 seeds. Furthermore, a 10-sample moving average filter was applied to smooth the results. For sequential path-following, a
Lastly, the total external load in the harmonic balance method had to be rescaled in order to match the same load level as in the time-integration approach. The difference in mean amplitude in the external load between both methods was caused by a difference in cutoff frequencies for the generating load spectrum, as well as different frequency spacing. In the time-integration method, a fine frequency spacing was used as input spectrum generating the load timeseries. A coarser spacing was chosen in the harmonic balance method, in order to gain computational speed, and to interpolate the results afterwards. With the same purpose, a lower cutoff frequency of the input load was used in the harmonic balance method. The rest of the response spectrum was replaced by the results from the linear model (with no end-stop). Little difference was found between this augmented approach and taking a higher cutoff frequency to start with.

In order to correct for these differences, setting the RMS values of the resulting time series of the external force was the starting point. Without knowledge of a more correct approach (left as recommendation), the forces were rescaled by a single factor, to match the peaks of the harmonic balancing and the time-integration results (Figs. 6.3 and 6.4). Similar to the discussion in Sec. 5.3.2, the possibility must be noted that it could be more sensible to investigate the energy content of both load spectra. A further recommendation for future work would be to further optimize the choice of frequencies in the forcing in the harmonic balance methods, as this was found to have a significant impact on the computational time.

6.3.1 Mild sea state: $H_s = 1m, T_p = 4.2s$

In Fig. 6.3 the results for the internal bending moment spectra are shown, at interface and mudline positions respectively. The responses are zoomed in to the region where most deviation occurs from the situation without the end-stop. The time-integration results point out that the inclusion of an end-stop hardly changes the response, for both positions. While the harmonic balancing method is consistent with the time-integration results, the sequential path-following method predicts severe degradation of the response for $0.93 < \omega/\omega_0 < 1.07$, similarly as was observed in Sec. 5.4.1. Replacing the sea state load spectrum with a frequency sweep containing single harmonic loading clearly does not lead to representative end-stop behavior.

6.3.2 Moderate sea state, $H_s = 4m, T_p = 8.3s$

In Fig. 6.4 the internal bending moment spectra are shown again at both interface and mudline positions, for a moderate sea state. The time-integration and harmonic balancing results show that the end-stop clearly increases the internal loads around $\omega/\omega_0 \approx 0.98$, in the same gradual manner as was displayed for the structure displacement in the 2DoF structure under uniformly distributed spectral external loading (Fig 5.13). The response at mudline actually contains another peak below $\omega/\omega_0 = 0.85$, corresponding to the peak period $T_p = 8.3s$. Since the mudline position corresponds to a relatively low amplitude region of the first bending mode, the resonance from the sea spectrum (which occurs at a lower frequency) is relatively more present in the resulting structural response spectrum.

The sequential path-following once again over-estimates the response resulting from the nonlinear end-stop. The harmonic balancing method is very consistent with the time-integration results, apart from a slight over-estimation around $\omega/\omega_0 = 0.95$. While it
6.4 Results for fatigue damage

In this section the results for fatigue damage values are presented, as well as the computational effort needed. For the time integration method, the procedure in Sec. 2.2 was applied, including rainflow counting (Sec. 2.2.3). For the frequency domain techniques for nonlinear dynamics (sequential path-following and harmonic balancing) the procedure in Sec. 2.3 was applied. The linearized Morison equation (Eq. 2.31) was applied, after which the internal loads were computed via Eq. 2.32. The linear structural transfer functions were replaced by the responses obtained with the respective techniques for nonlinear dynamics. Finally, Dirlik’s method from Sec. 2.3.3 was applied to obtain fatigue damage values.
The fatigue level results are shown in Table 6.1. The TMD decreases the fatigue load values to around 40% with respect to the situation with no TMD for all cases, with the exception of the mudline position under $H_s = 4m$, where 65% of the damage is still done. For this combination, the response due to the peak in the external load spectrum at $1/T_p \approx 0.12$ Hz (to which the TMD is not tuned) is significant with respect to the response around resonance.

The time integration method predicts hardly any increase in damage due to the end-stop for $H_s = 1m$, and a more significant (27%) increase for $H_s = 4m$. The sequential path-following technique results in fatigue damage that is worse than the situation with no TMD at all (values greater than 1), for both sea states, and both positions - interface and mudline, hereby clearly overestimating the influence of the end-stop, as was also observed in the internal bending moment spectra is Sec. 6.3. The harmonic balancing method on the other hand underestimates the fatigue damage added by the end-stop. In the final columns for both sea states in Table 6.1, the relative differences in the results including the end-stop with time-integration are shown. For $H_s = 4.2m$ at interface level, the relative under-estimation of the harmonic balance method with respect to time-integration is as
high as 14%. A possible reason for this lack of correspondence could be that widening the frequency spacing in the external load spectrum from the harmonic balancing method with regard to the time-integration method (see explanation in the introduction of Sec. 6.3) leads to a decrease in number of extrema in the internal bending moment spectra in the harmonic balance results, which have a relatively high contribution to the total damage, due to the Wohler slope in Eq. 2.15.

Table 6.1: Normalized Damage Equivalent Loads (DEL) for the internal bending moment relative to the situation with no TMD, obtained with different solution techniques. The time-integration results also serve as reference, and are shown in italic. In particular, the relative increases in DEL due to the end-stop, are shown in bold. TI = time-integration, HB = harmonic balancing, SPF = sequential path-following.

<table>
<thead>
<tr>
<th>Method</th>
<th>Sea state → TMD Configuration →</th>
<th>Configuration →</th>
<th>$H_s = 1m, T_p = 4.2$ s incr. $\Delta$ end-st.</th>
<th>$H_s = 4m, T_p = 8.3$ s incr. $\Delta$ end-st.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI</td>
<td>Interface (1 = No TMD)</td>
<td>linear</td>
<td>0.391 0.402 2.9%</td>
<td>0.444 0.562 27%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>end-stop</td>
<td>0.389 0.390 0.4%</td>
<td>0.442 0.486 10% -14%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>w.r.t. w.r.t.</td>
<td>-3% -3%</td>
<td>10% -14%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TI</td>
<td>0.388 1.02 160% +150%</td>
<td>0.443 1.06 140% +89%</td>
</tr>
<tr>
<td>HB</td>
<td></td>
<td>linear</td>
<td>0.391 0.392 0.4%</td>
<td>0.651 0.687 5.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>end-stop</td>
<td>0.389 0.402 2.4%</td>
<td>0.653 0.674 3.3% -2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>w.r.t. w.r.t.</td>
<td>-2% -2%</td>
<td>3% -2%</td>
</tr>
<tr>
<td>SPF</td>
<td></td>
<td>linear</td>
<td>0.391 1.03 160% +160%</td>
<td>0.654 1.05 61% +53%</td>
</tr>
</tbody>
</table>

6.5 Computational speed

The computational times needed to obtain the results from Sections 6.3 and 6.4 are given in Table 6.2. Both frequency domain methods show a significant decrease in computational effort needed compared to the time-integration method. For the time-integration, there is little variance between the different simulation cases, in contrast to the frequency domain methods, where many iterations are needed when the TMD body comes into the end-stop nonlinear region. For time-domain computations, the extra number of iterations is relatively small, also in view of the amount of overhead in this method.

Table 6.2: Computational times (minutes) need to obtain the fatigue damage results for the internal bending moments in Table 6.1, and the spectra in Figs. 6.3 - 6.4.

<table>
<thead>
<tr>
<th>Method</th>
<th>Sea state → TMD Configuration →</th>
<th>$H_s = 1m, T_p = 4.2$ s Linear</th>
<th>$H_s = 1m, T_p = 4.2$ s With end-stop</th>
<th>$H_s = 4m, T_p = 8.3$ s Linear</th>
<th>$H_s = 4m, T_p = 8.3$ s With end-stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-integration</td>
<td></td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Harmonic Balancing</td>
<td></td>
<td>0.30</td>
<td>0.52</td>
<td>0.30</td>
<td>4.8</td>
</tr>
<tr>
<td>Sequential path-following</td>
<td></td>
<td>0.17</td>
<td>1.4</td>
<td>0.17</td>
<td>3</td>
</tr>
</tbody>
</table>

6.6 Discussion of the results

In this final chapter, the techniques for solving nonlinear dynamics (presented in Chapter 4) were applied to a reduced version of a full FEM model of a wind turbine structure including an sDoF TMD with an end-stop nonlinearity. Time-integration results served as the reference for the chosen frequency-domain based results: sequential path-following and harmonic balancing.
After the FEM model and the modeled sea states as external loads were explained in Sections 6.1.1 and 6.1.2 respectively, the results for bending moment spectra were presented in Sec. 6.3, obtained with the different solution techniques. The harmonic balancing method showed good consistency with the time-integration results, while the sequential path-following severely overestimates the response. It became clear that a sequential sweep of single harmonic external forcings can not represent a sea state spectrum, when applied to the end-stop nonlinearity.

The results for fatigue damage were presented in Sec. 6.4. As expected from the bending moment spectra results shown in Sec. 6.3, the fatigue damage was severely overestimated by the sequential path-following method. The harmonic balancing method underestimates the fatigue damage, in contrast to the consistency in the results for internal bending moment spectra from Sec. 6.3. In the previous section, the influence of the different frequency spacing in the load spectra was given as a possible explanation for the inconsistency in the DEL results.

A few recommendations can be given for future work, with regards to the harmonic balance method:

- The choice of the external load spectrum could be further optimized, by choosing for instance non-uniform spacing (with coarser spacing where less variance in the results occurs, or where the response is linear).

- The scaling of the input load spectrum must be verified / obtained analytically, instead of manual tuning.

In the following and final chapter of this work, the main conclusions and recommendations are given from this entire work, mainly based on the results obtained in Chapters 5 and 6.
The research aim of this work was defined in Sec. 1.4 from the Introduction:

“Investigate frequency domain methods to model / describe end-stop nonlinearity of Tuned Mass Dampers in offshore monopile structures under multi-frequency hydrodynamic loads, and determine whether these methods are capable of fast and accurate modeling of fatigue loads, relative to time domain methods.”

Current frequency domain load calculation procedures are based on linear transfer functions, as explained in Chapter 2, which motivates the search for a nonlinear frequency domain solution technique to handle the end-stop nonlinearity accompanied by the TMD. In Chapter 3 the modeling of TMDs was explained in more detail, as well as the chosen model for the end-stop. In the same chapter, the modeling approach for the combined nonlinear system was presented, using dynamic substructuring. After a study and comparison of different solution strategies for nonlinear dynamics, the alternating frequency/time harmonic balancing method (AFT-HB) was implemented in this work, as well a sequential path-following method. A time-integration (Newmark) method was also implemented, and was used to produce reference simulation data. Furthermore, a non-monolithic solution strategy was proposed in Sec. 4.4 in combination with the sequential path-following, that can be used to solve for multi-DoF systems containing nonlinearities, under a single harmonic loading.

In Chapter 5 the harmonic balancing method, as well as sequential path-following were applied to sDoF and simple mDoF systems, containing end-stop nonlinearity. This way, not only was the performance of the different solution techniques evaluated, but also thorough description of the nonlinear end-stop behavior was given. Finally, the same techniques were applied to a finite element model of a wind turbine system with a TMD, under spatially distributed hydrodynamic loading with many frequency components. In what follows, conclusions will be given from this investigation, as well as recommendations for future work.

7.1 Conclusions

From the case studies and results presented in Chapters 5 and 6, two main conclusions can be drawn:
• It can be concluded that frequency domain based methods for solving the dynamics of wind turbines with TMDs that carry end-stop nonlinearity must include multiple frequency components in the external load as well as the solution. Although hydrodynamic loads are characterized by a peak period, the dynamics of a system with end-stop nonlinearity cannot be accurately be reproduced by a collection of results from external loads with single harmonic components. A collection of single harmonic external loads results in frequency responses with distinct linear and nonlinear parts, separated by sharp transitions at combinations of frequency and magnitude of the external load resulting in collision with the end-stop. Multiple frequency external loads on the other hand result in more smooth responses, with no distinct linear and nonlinear parts.

• Harmonic balancing is a promising frequency domain alternative to time-integration, for solving nonlinear dynamics of wind turbines with TMDs that contain end-stop nonlinearity. The harmonic balancing method has been able to accurately portray the frequency response of the structure, under a hydrodynamic external load containing in the order of $10^2$ frequency components. Furthermore, a significant amount of computational speed was gained with respect to time-integration. The time and frequency domain behavior obtained with harmonic balancing has been verified with time-integration for both single and multi harmonic external load cases, different sDoF and mDoF systems, and different types of nonlinearities. The results for single harmonic external loads were further verified with a renowned pseudo-arclength path-following technique [21].

Lastly, a non-monolithic frequency domain technique has been proposed in Sec. 4.4.2 for nonlinear multi-DoF systems under single harmonic loads and implemented in Sec. 5.4.2. From the implementation results it can be concluded that the proposed method can successfully reproduce the results from monolithic sequential path-following applied to end-stop nonlinearity, and a significant gain in computational speed is accomplished.

7.2 Recommendations

As described in the conclusions in the previous section, the main research goal of this work has been addressed: to find a frequency domain alternative to time-integration for modeling the dynamics of an offshore wind turbine with a TMD containing end-stop nonlinearity. Even though this goal has been accomplished, further improvement is possible, and some issues remain unclear, motivating a few generic recommendations for future research. Furthermore, this work had a particular focus on the harmonic balancing method, as it was found a promising tool to solve the dynamics of the system of interest. A set of recommendations are given for this method, separately.

General

• More complex TMD systems. Throughout this entire work, an sDoF model with a linear spring and damper has been used as the most simple model of a Tuned Mass Damper. As a side-work, a pendulum TMD with finite pendulum length and angular displacements was implemented in the time-integration, which yielded similar end-stop behavior as with the sDoF model. It would be of interest to investigate the
performance of the frequency domain methods (in particular harmonic balancing) to this more realistic/specific TMD model.

- **Closed-form solution.** All solution techniques for nonlinear dynamics treated in this work, based on the time as well as the frequency domain, make use of Newton-Raphson iterations. For the frequency domain methods, hundreds of iterations were required to deal with the end-stop, especially when the iteration schemes had to be softened in order to reach convergence. With a closed form solution, no iterations would be needed. In Chapter 4, perturbation techniques were introduced, as a way to obtain solutions of closed form. Although this approach is limited to systems with only a few DoF, this work has shown that end-stop behavior of such simple systems can be very similar to (reduced versions of) full FEM models. Furthermore, the end-stop behavior would probably have to be modeled with a more continuous function. Although this work briefly explored and discarded polynomial and arctangent fits, further research could be done to obtain a more complete and thorough overview of candidate replacing functions.

- **Non-monolithic approach for multi-harmonic loads.** The non-monolithic approach as proposed in this work (Sec. 4.4.2) could be further investigated, in order to be extended to nonlinear mechanical systems under loads with many frequency components. An attempt was done by setting up a base-excitation problem under a multi-frequency load with uniform distribution, and solving by means of time-integration. However, the resulting phases were very noisy, hindering the solution of the combined system afterwards. Furthermore, the definition of the non-monolithic problem would need to change, as this was also based on single harmonic loading.

**Harmonic Balancing**

- **Re-scaling of the external load vector.** As discussed in Sec. 6.3, the external load level in the harmonic balancing method has to be rescaled in order to represent the same load series from time-integration, when the governing spectra carry different cutoff frequencies and spacings. The suggested approach was to match the RMS values - however, there may exist alternative approaches, and a more thorough research would need to be done with regards to, for instance, the energy content of the load timeseries.

- **Improvement of Damage Equivalent Load results.** The results for internal bending moment spectra as obtained with the harmonic balancing technique corresponded well with those obtained through time-integration. The Damage Equivalent Loads however were under-estimated by the Harmonic Balancing method, as found in Sec. 6.4. While the suggested cause lies in the difference in frequency spacing of the load spectra used in both methods, this mismatch must be thoroughly investigated in order to start improving the harmonic balancing method or its implementation.

- **Non-uniform frequency spacing.** The computational speed of the harmonic balancing relies heavily on the size of its system matrix, which is governed both by the number of DoF of the mechanical system and the number of frequency components. While both were kept at a minimum for the case study in Chapter 6, the number of
frequencies could be further reduced by a non-uniform frequency spacing in the forcing, for relatively smooth regions, or regions of frequencies far from the significant nonlinear behavior.

- **Sub-excitation frequencies.** Frequency components lower than the excitation frequency could be used in the implemented harmonic balancing tool to include for effects such as ‘beating’, as was observed in the 2DoF system in Fig. 5.17, revealed by the pseudo-arclength path-following.

- **Verification for mDoF systems.** The results from the harmonic balancing method applied do the sDoF oscillator under single frequency external loading were thoroughly verified, using time-integration as well as Pseudo-arclength path-following methods. The latter revealed unstable behavior that was not identified by either of the former methods. The results for the 2DoF model under single frequency forcing could also be examined with the reference pseudo-arclength path-following tool [21].
Appendix A

Newmark Time integration method

In this work, the Newmark time-integration method was applied to obtain the time domain results from Chapters 5 and 6, which helped in understanding end-stop behavior, and served as a reference tool for the results obtained with the frequency domain methods studied and applied throughout the course of this work. The Newmark method for solving the states of a system relies on a Taylor series expansion around the previous timestep $t_{n-1}$, to give the states at the current timestep $t_n$:

$$\dot{q}_n = \dot{q}_{n-1} + \int_{t_{n-1}}^{t_n} \ddot{q}(\tau) d\tau,$$
$$q_n = q_{n-1} + h\ddot{q}_{n-1} + \int_{t_{n-1}}^{t_n} (t_n - \tau) \dddot{q}(\tau) d\tau.$$  \hfill (A.1)

The integral in Eqs. A.1 can be approximated by using numerical quadrature. The accelerations in the integrands are in turn expressed as a Taylor expansion for the time interval $\tau = [t_{n-1}, t_n]$:

$$\ddot{q}_{n-1} = \ddot{q}(\tau) + q^{(3)}(\tau) (t_{n-1} - \tau) + q^{(4)}(\tau) \frac{(t_{n-1} - \tau)^2}{2} + ..., \hfill (A.2)$$
$$\ddot{q}_n = \ddot{q}(\tau) + q^{(3)}(\tau) (t_n - \tau) + q^{(4)}(\tau) \frac{(t_n - \tau)^2}{2} + ...$$

The Taylor series expressions can be used in the quadrature expression that uses weights $\gamma$ and $2\beta$ assigned to the accelerations at $t_{n-1}$ and $t_n$:

$$\ddot{q}(\tau) = (1 - \gamma) \ddot{q}_{n-1} + \gamma \ddot{q}_n + q^{(3)}(\tau) (\tau - h\gamma - t_{n-1}) + O(h^2 q^{(4)}),$$
$$\ddot{q}(\tau) = (1 - 2\beta) \ddot{q}_{n-1} + 2\beta \ddot{q}_n + q^{(3)}(\tau) (\tau - h2\beta - t_{n-1}) + O(h^2 q^{(4)}). \hfill (A.3)$$

Neglecting higher order (>2) derivatives gives, after substituting Eqs. A.3 in Eqs. A.1:

$$\dot{q} = \dot{q}_{n-1} + (1 - \gamma)h\ddot{q}_n + \gamma h\dddot{q}_n = \dot{q}_n + \gamma h\dddot{q}_n,$$
$$q_n = q_{n-1} + h\ddot{q}_{n-1} + \left(\frac{1}{2} - \beta\right)h^2 \dddot{q}_{n-1} + \beta h^2 \dddot{q}_n = \dddot{q}_n + \beta h^2 \dddot{q}_n.$$  \hfill (A.4)
In equation A.4, \( \ddot{q}_n \) are the predictors, i.e., the influence of the previous state at \( t-1 \) on the current state \( q_n \) at \( t \).

### A.1 Constant average acceleration: \( \beta = \frac{1}{4}, \alpha = \frac{1}{2} \)

Several choices exist for combinations of \( \beta \) and \( \gamma \), each leading to different stability and accuracy characteristics. By choosing \( \beta = \frac{1}{4} \) and \( \alpha = \frac{1}{2} \), average constant acceleration is assumed. This scheme renders unconditional stability, has zero amplitude error and a periodicity error of \( \Delta T = \frac{\omega^2 h^2}{12} \). In this work, the combination of \( \omega \) and \( h \) was chosen that gave a relative periodicity error of \( 5 \cdot 10^{-5} \). Due to the stability and its low relative error, the Constant Average Acceleration scheme has been implemented throughout this study.

### A.2 Application to structural dynamics

The Newmark scheme can be applied to structural dynamics, both to a linear system of equations as well as a nonlinear dynamic system. In the linear case, the equation of motion reads:

\[
M \ddot{q} + C \dot{q} + Kq = F. \tag{A.5}
\]

By introducing the following so-called stepping matrix:

\[
S = (M + \gamma h C + \beta h^2 K), \tag{A.6}
\]
the accelerations in Eq. A.7 at timestep \( t_n \) can be expressed in terms of the velocities and the displacement at previous timesteps, using Eq. A.4:

\[
S \ddot{q}_n = F - C \dot{\ddot{q}} - K \ddot{q}. \tag{A.7}
\]

Since \( S \) is constant, it can be inverted once for all to explicitly deliver the accelerations at each subsequent timestep.

For nonlinear dynamical systems, the starting point is again the equation of motion, which needs to be expressed differently in order to contain the nonlinearities:

\[
M(q)\ddot{q} + p(q, \dot{q}) = F. \tag{A.8}
\]

This equation of motion is rewritten to express a residual, which should be zero at each timestep:

\[
r_n = M(q_n)\ddot{q}_n + p(q_n, \dot{q}_n) - F_n = 0. \tag{A.9}
\]

An iterative procedure is applied at each timestep until this residual drops below a user-defined tolerance \( \bar{\epsilon} \) (in this work this tolerance is set as a fraction of the internal forces - in this work the following tolerance was chosen: \( \bar{\epsilon} = 10^{-6} \ p(q, \dot{q}) \)). A first-order Taylor approximation is used around the residual at iteration \( k \) in order to compute the residual at the next iteration \( k+1 \), at the timestep \( n \):

\[
r(q_n^{k+1}) = r(q_n^k) + r'(q_n^k)(q_n^{k+1} - q_n^k). \tag{A.10}
\]
The derivative of the residual to the displacement becomes the nonlinear stepping matrix at iteration $k$:

$$r'(q^k_n) = \left[ \frac{\partial r}{\partial q} \right]_{q^k_n} = S(q^k_n).$$  \hspace{1cm} (A.11)

This concludes the summary of the basic Newmark time-integration scheme as was applied throughout this work.
Bibliography


