Nonlocal Damage Models for Concrete

Implementation and investigation of the nonlocal damage models to study concrete damage under dynamic loading

MSc Thesis

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Preface

This thesis is the result of the graduation project which is a part of the curriculum of the Structural Engineering track of the MSc in Civil Engineering at Delft University of Technology. The research is carried out in cooperation with TNO, Rijswijk.

I would like to thank my daily supervisor, Ir. Y. S. Khoe for the valuable guidance and useful advice. I would like to thank Prof.dr.ir. L.J.Sluys and Dr.ir. J. Weerheijm for their continuous support and guidance during my graduation period. I would also like to express gratitude towards Dr.ir. M. Ruess for the feedback during the completion stages of the project.

During the months spent at TNO, Rijswijk I have had the opportunity to work in a dynamic and friendly environment. I would like to thank my colleagues at TNO for the welcoming environment and support. Special thanks goes to Mark Tyler-Street for his support during the implementation of the nonlocal models in LS-DYNA.

Finally I would like to thank my friends and family for the support and encouragement.

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R. S. Maravalalu Suresh
Abstract

An understanding of response of concrete structures to impact loading is required to make safe designs for dynamic loading conditions. The Explosions, Ballistics and Protection department of TNO is interested in understanding the response of concrete to impact loads, i.e. the damage patterns and residual strength. Numerical simulations conducted to study this response require very small elements to capture the shockwave propagation and the subsequent response.

Most of the material models produce results which are mesh dependent. To resolve this issue an enrichment to the current models by means of explicit introduction of a length scale is proposed in the nonlocal damage models. The original nonlocal model produces incorrect description of the fracture process. The nonlocal model is implemented in LS-DYNA to investigate the incorrect fracture process by means of certain benchmark tests. The use of the constant length scale and the isotropic weight function is found to be the cause of the drawbacks of the nonlocal model.

Giry et al (2011) proposed a stress based nonlocal model which addresses these drawbacks of the nonlocal model. This model is also implemented in LS-DYNA to study its features and limitations. The behavior leading to the resolution of the drawbacks of the nonlocal model are studied using the benchmark tests. The stress based nonlocal model implemented resolves the issues and produces a physically relevant fracture process.

The behavior under dynamic load setting is analyzed as application in dynamic setting is the final goal. Using a model with parameters presented for quasi static conditions certain limitations are observed in the dynamic load setting. These limitations are further exposed when the model is validated with experimental results. The model cannot capture the varying fracture processes with different loading rates as seen in experiments but still produces mesh independent results.
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1. INTRODUCTION

Concrete structures like high rise structures, bridges, tunnels and protective structures need to withstand large stresses and strains through impact loadings. To design structures to resist impact loadings, it is important to understand the response of concrete to impact loading. The behavior of concrete under impact loading, i.e. damage patterns and residual strength is of interest to the Explosions, Ballistics and Protection section of TNO. Numerical simulations are used to study the response to dynamic loadings when the experimental approach is not feasible. This thesis investigates methods to further improve the numerical analysis of concrete under dynamic loads.

The failure of concrete is characterized by the progressive damage due to the occurrence of localized deformations which lead to material degradation causing failure. The continuum damage mechanics approach is the most direct and commonly used approach to study the softening behavior. This approach generally uses concrete models that follow local constitutive laws, i.e. the stress state at a point is dependent on the deformations at that point. Concrete should be analyzed using these local models when the element size is above a certain length scale defined by the material structure. As the length scale defines the zone of influence of a material point, local models which cannot be applied to elements smaller than the length scale. Additionally, the width of localization obtained using the local models is governed by the size of the mesh chosen leading to solutions dependent on the element size.

In high dynamic applications very small elements are required to describe localized events and to accurately resolve the shock loading. These elements are generally smaller than the material length scale of concrete. The local models generally provide incorrect description of the damage distribution which is generally mesh dependent. This issue must be addressed in order to study the localized events. Certain regularization methods, such as fracture energy...
approaches and nonlocal methods are available in literature to obtain the mesh independent results.

The nonlocal model is a regularization approach based on the interactions between the micro-cracks within the damaged zone. It acts as an enrichment in the continuum to capture the real processes more adequately than the local models. The original nonlocal model, Pijaudier-Cabot and Bažant (1987) provides mesh independent solutions but has certain drawbacks which lead to incorrect description of the damage process. Recently, Giry et al.(2011) provided an improved stress based nonlocal model, which addresses the drawbacks of the original nonlocal model. The stress based nonlocal model has further physical backing with the consideration of the evolving interaction domains.

This thesis aims at understanding the nonlocal models and its behavior leading to the regularized results when simulating concrete damage under dynamic loads. The following research objectives are defined to reach the goal of this thesis:

- A study of the features and limitations of the nonlocal models.
- The behavior in dynamic loading conditions compared to quasi-static loading.
- Experimental validation of the stress based nonlocal model.

To understand the advantages of the stress based nonlocal model over nonlocal model and its limitations the study of its features and limitations is required. The comparison of the behavior of the nonlocal models in dynamic conditions to quasi static conditions is necessary, as most nonlocal models are primarily presented for quasi static loading conditions. Finally, the capability of the stress based nonlocal model to reproduce fracture processes under dynamic loading is investigated by validation with experimental results.

The implementation is carried out using LS-DYNA, a commercially available finite element package which is well suited for simulating high dynamic conditions. The software provides a means to implement user subroutines which can be modified to implement the necessary material models and the regularizations.

The report is structured as follows. Chapter 2 describes the physical properties of concrete and the progressive damage under tensile loading. Additionally a brief description of the rate effects observed for dynamic loading of concrete is presented. Chapter 3 presents the numerical modeling of damage of concrete and the mesh sensitivity is shown using a benchmark test. The original nonlocal model is presented with benchmark tests to show the mesh independency and its drawbacks. Chapter 4 presents the stress Based Nonlocal model and benchmark tests conducted to compare it with the nonlocal model. The experimental validation is presented in Chapter 5. An appendix is attached with the document describing the implementation of the isotropic damage model and the nonlocal damage models in LS-DYNA subroutines.
2. PHYSICAL PROPERTIES

Concrete is a composite material with heterogeneous internal structure. This heterogeneity can be observed on different scales. Coarse aggregates and mortar can be seen at mesoscale and at the microscale mortar is visible, which is made up of sand and cement paste. The cement paste has grains in the range of 0.5-50 µm and sand grains are in the range of 0.1-4 mm. The coarse aggregates are 4-32 mm large. The heterogeneous structure at the different scales of the material is shown in Fig 2.1. The sizes of the constituents of the lower scales are usually significantly smaller than the structural dimensions and can be homogenized. For example, cement paste is considered as homogeneous at mesoscale and the mortar and coarse aggregates are seen as homogenous materials at macroscale.

In addition to the geometry of the structure, the global mechanical response of concrete is determined by its constituents, their distribution, properties and respective interactions. For example, the bond strength at the interfaces are low compared to the tensile strength of the mortar matrix or the coarse aggregates and hence have high influence on the strength as shown by Appa Rao (2004).

Failure of concrete occurs through formation of cracks. Crack formation can be categorized as tensile cracks (mode 1), in plane shear (mode 2), out of plane shear (mode 3) or a combination of modes (mixed mode). Crack formation in concrete originates at the mesoscale and alters the interaction of the constituents, leading to a reduction in strength beyond the peak strength. This behavior is often referred to as strain softening (see figure 2.2).
Figure 2.2. Characteristic load displacement curve of a softening material in tension (Sluys (1992))

A representative curve showing the strain softening behavior for a displacement controlled tensile loading scenario for concrete in tension is shown in Fig 2.2. Initially the behavior is elastic showing linear behavior up to about 80% of the tensile strength. At this point cracks arise at the aggregate-mortar interfaces, leading to nonlinear behavior as it approaches the peak strength. Concurrently cracks in the mortar also develop. The peak strength is the point when the interface and mortar cracks start to interact and microcracks develop. The development, growth and coalescence of microcracks leads to a local reduction of effective cross sectional area available of transmitting the force, i.e. material degradation. This degradation is responsible for the steep drop in the descending branch of the curve. As the displacement increases the density of microcracks increases. The growth and localization of microcracks forms macrocracks.

Progressive damage, the process of gradual loss of strength occurs at three scales starting from the microscale and moving up to the macroscale. The zone of accumulated microcracks is the fracture process zone (FPZ) which has a finite width as shown by Bazant and Pijaudier-Cabot (1989).

At different strain rates, the material response of concrete changes. While the stiffness of concrete only shows small variations with changing strain rates, the strength and fracture energy vary significantly.

The increase of tensile strength from various experiments is represented as a dynamic intensity factor (DIF), which is defined as the ratio of the dynamic strength to the static strength. Fig 2.3 shows the dynamic intensity factors of various strain rates ranging from static to very fast dynamic cases from various experiments. It is seen that with increasing strain rates there is an increase of strength. A moderate effect is seen ranging from static to intermediate strain rates (up to 1/s). Beyond moderate strain rates dynamic effects significantly increase the strength of concrete.

The global response of any material should be distinguished as material and structural response. At higher rates of loading the inertial response can be seen at structural level (macroscale) and material level (meso and microscale). The inertial response carries a part of the load causing the material to obtain higher resistance. The inertia at structural level influences the average stress distribution. At meso and micro level the inertia affects the local
stress distribution and induces limitations in crack initiation and propagation, which affect the fracture process within the material. The rate of deformation in the fracture process zone is higher with higher loading rates. This will also contribute to the inertia in the zone of fracture which influences the fracture process and hence the strength and fracture energy.

The quality of concrete, aggregate size and moisture conditions also influence the dynamic tensile strength of concrete (Belaoura and Brara (2003)). Strain rate effects in concrete are controlled by parameters such as the water/cement ratio, porosity aggregate size. Moisture content is indicated as the most influential factor regarding rate effects in concrete. The moisture content in the pores lead to a viscous effect, which leads to increased strength at higher loading rates. This effect is known as the Stefan effect. Hence with an increase in the moisture content we will observe a higher strain rate effect.

The strain rate effect for the intermediate loading rates, up to 1 /s, is caused mainly by the moisture content and the pore structure of the material. The inertia effects are quite low in this range of strain rates. The moisture content within the capillary pores causes the rate effects in the moderate regime, Weerheijm et al (2009).

Higher strain rates show a much steeper increase in the strength. This increase is accompanied by a relatively ductile softening behavior. The width of the fracture process zone increases considerably (factors 5, 3 and 8 for dry, normal and wet concrete respectively). The increase in the amount of microcracking is relatively small compared to the increase in the fracture process zone. The strain rate effects can be attributed to the moisture content in the pores, the additional microcracking and most importantly the inertia response of the fracture process.

Figure 2.3: Experimental data of the Dynamic Increase Factor (DIF=$f_{tdyn}/f_{t,stat}$) (Weerheijm and Vegt (2013))
zone. A summary of the dominant mechanisms for the different rate effects is shown in Table 2.1, (Weerheijm and Vegt (2013)).

<table>
<thead>
<tr>
<th>Rate Effect</th>
<th>Dominant mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma &lt; 50 \text{ GPa/s}$</td>
<td>• Moisture &amp; pore structure</td>
</tr>
<tr>
<td>$\sigma &gt; 50 \text{ GPa/s}$</td>
<td>• Moisture &amp; pore structure</td>
</tr>
<tr>
<td></td>
<td>• Micro inertia (crack initiation &amp; propagation)</td>
</tr>
<tr>
<td></td>
<td>• Structural inertia pre-peak softening zone</td>
</tr>
</tbody>
</table>

Table 2.1: Dominant mechanisms influencing the rate effects
3. NUMERICAL MODELLING OF CONCRETE

Many physical phenomena in engineering can be described using partial differential equations which can be solved using analytical methods. Solving these complex problems by means of the analytical methods is either very cumbersome or impossible. The finite element method is an approximate numerical approach that is used to solve the partial differential equations (PDE). The physical problem usually involves a structure which is subject to certain loads. The structure is discretized into a set of finite material volumes (elements), each calculating the material response locally which together give the global response of the system. A mathematical model describes the constitutive behavior of a material within each element. With the mathematical model and the boundary conditions a solution scheme can be used to solve the complete problem. A simple representation of the finite element framework is shown in Fig 3.1.

![Finite Element Analysis Process Diagram](image)

**Figure 3.1:** The finite element analysis process (Bathe (1982))

Various methods exist to solve the system of PDEs. A commercial finite element software package, LS-Dyna is used for this thesis. This package primarily uses the explicit time integrations to solve the system of PDEs. The explicit method uses current state of the system...
(e.g. displacements of nodes) at time \( t \), to calculate the state of the variable at time \( t + \Delta t \).
The stability of this numerical integration scheme is limited by the time step size, \( \Delta t \). The critical time step size is defined by the Courant Stability Limit and is equal to the time taken for a wave to propagate through the smallest element of the discretization.

The material model is the theoretical framework which helps describe the physical process of a material. In this study it deals with the deformations, strains, stresses and the variables describing the nonlinear behavior of the material. The material model can be described at the various scales of the material. The scale at which the material is mathematically described depends on the scope of the study. The microscale models are chosen if the physio-chemical mechanisms between the concrete constituents need to be studied. The mesoscale models are chosen if the mechanisms linking the aggregates and mortar are in focus. The macroscale models are chosen when global response of the concrete structure is important. The focus of this study being the global response of concrete to mechanical loads the material model to describe concrete is placed at macroscale.

The fracture of concrete can be modeled by the discrete or the continuous approach (shown in Fig 3.2). The discrete approach presents the macrocrack as a discontinuity in the geometry. The cracks are placed along the edges of the elements of the structure, and the opening of the crack is represented by the separation of the elements which depends on the traction law. This method needs prior knowledge of the possible crack path to correctly position the element boundaries. The alternative would be to have a remeshing of the model to have edges along the possible crack path. To represent the crack openings along the edges of elements interface/cohesive elements of zero thickness are used, which are not trivial to implement. The remeshing techniques need the stiffness and mass matrix to be reconstructed throughout the analysis time. This model leads high computational costs when large structures are analyzed for fracture.

The continuous approach or crack band model, considers the crack to be part of the continuous homogenous system. The crack is now modeled as a band of distributed microcracks, a band of localized strains which are smeared out over the elements. The macrocrack forms

![Figure 3.2: Concrete Fracture, (A) discrete approach (discrete crack) and (B) Continuous approach (shaded crack band)](image_url)
along the path of completely damaged points in the process zone, Fig 3.3. The use of the crack band approach is justified by Bazant (1983) and its features are shown by Chabole (1988). The elements record the distributed microcracks, deterioration of the material beyond peak strength using a “damage variable”.

![Crack defined by the completely damaged zone (Torrenti et al (2010))](image)

This model estimates the level of damage during the analysis unlike the crack opening provided by the fracture mechanics approach. Compared to the discrete approaches this method is simpler to implement in a finite element framework and computationally not as heavy. As the smeared crack approach is used the elements are homogenous and are placed at macroscale. The additional benefits of a clear definition of the state of damage and the description of the mechanical behavior of the damaged material makes this the preferred method to model failure in this study.

The continuous approach requires the formulation of a material model where the damage can be recorded using the stress deformation relations. This is done using the post peak softening behavior of the material. The softening behavior leads to redistribution of the stresses which occurs in a partially damaged region termed fracture process zone. Damage evolution in this zone determines the direction of crack propagation. The “damage variable” thus represents the reduction in stiffness and can be either single scalar parameter Jirasek (2008), for simple isotropic cases as used in this study or a vector damage variable for more complex failure models.

### 3.1. Local isotropic damage model

To study the strain softening behavior in concrete a simple isotropic damage model is used. In this macroscopic model the damage is controlled by a single scalar parameter, $\omega$. It is the macroscopic representation of deterioration of strength, due to the formation, growth and coalescence of the defects at the microscale and mesoscale. This parameter $\omega$ ranges from zero to one, where zero is elastic behavior with no damage and one is complete failure. The scalar parameter is the ratio of the local effective damaged cross sectional area to the area prior to damage, eq (3.1).

$$\omega = 1 - \frac{\bar{A}}{\bar{A}} = \frac{A_d}{\bar{A}}$$

(3.1)
\( \bar{A} \) represents the local effective cross sectional area, \( A_d \) the damaged local effective cross sectional area and \( A \) the total area.

The isotropic damage model works under the assumption that the stiffness degradation is isotropic. With the assumption that the Poisson’s ratio is not affected by damage a single variable can be used to record the damage. A degraded stiffness is now defined as given by eq (3.2), where \( D_e \) is the undamaged elastic stiffness tensor.

\[
D = (1 - \omega)D_e
\]

(3.2)

Applying the Hooke’s law to obtain the effective stresses now we have the eq (3.3)

\[
\sigma = D : \varepsilon = (1 - \omega)D_e : \varepsilon
\]

(3.3)

The evolution of the damage parameter is dependent on the equivalent strain in the material.

\[
\omega = G(\varepsilon_{eq})
\]

(3.4)

The equivalent strain is a single parameter which can characterize the strain state of the material. The equivalent strain can be evaluated by various methods. The Mazar’s definition, eq(3.5) and the Modified von-Mises strain, eq(3.6) formulations have been implemented.

\[
\varepsilon_{eq} = \sqrt{\sum_{i=1}^{2} \langle \varepsilon_i \rangle^2}
\]

(3.5)

with \( \varepsilon_i \), the principal strains, \( \langle \varepsilon_i \rangle = \varepsilon_i \) if \( \varepsilon_i > 0 \) and \( \langle \varepsilon_i \rangle = 0 \) if \( \varepsilon_i < 0 \).

\[
\varepsilon_{eq} = \frac{k - 1}{2k(1 - 2\nu)} I_1 + \frac{1}{2k} \sqrt{\frac{(k - 1)^2}{(1 - 2\nu)^2} I_1^2 + \frac{12k}{(1 + \nu)^2} I_2}
\]

(3.6)

with \( k \), the ratio of compressive to tensile strength and \( I_1, I_2 \) are strain tensor invariants.

The function \( G \) affects the shape of the stress-strain plot. This function should be monotonically increasing as the damage in any material cannot reduce. The damage being a function of equivalent strain, cannot decrease. Therefore a history parameter is used to record the maximum peak strain value representing the damage value. \( \kappa(t) \) is the history parameter which records the maximum value of equivalent strain.

\[
\kappa(t) = \max_{\tau \leq t} \varepsilon(\tau)
\]

(3.7)

Here \( t \) can signify not just time but any monotonically increasing parameter controlling the loading process.

The nonlinearity before the peak strength in concrete is not considered in this simple model. Instead the material is considered to have complete elastic behavior up to the peak strength,
after which the damage initiates to initiate the descending branch. The function $g$ controls this descending branch. The softening is represented by the descending branch using a linear or exponential softening law, given by the functions (3.8) and (3.9) respectively. Representative plots of the same are shown in Fig 3.4.

$$\omega(\kappa) = \frac{\kappa_c}{\kappa} \left( \frac{\kappa - \kappa_i}{\kappa_c - \kappa_i} \right)$$

(3.8)

$$\omega(\kappa) = 1 - \frac{\kappa_i}{\kappa} \left[ 1 - \alpha - \alpha e^{-\beta(\kappa - \kappa_i)} \right]$$

(3.9)

$\omega(\kappa)$, is the damage, $\kappa$ is the history parameter recording the peak equivalent strain. $\kappa_i$ is the peak strain at which the damage initiates. For the linear law, $\kappa_c$ is the ultimate strain value at which there is failure. For the exponential law, $\alpha$ controls the residual stress value when damage reaches its maximum value for exponential softening, where $\alpha = 1$ gives no residual stress and $\alpha = 0$ gives residual stress equal to the tensile strength. $\beta$ controls the steepness of the descending branch of the stress strain curve, higher the value steeper is the curve.

Figure 3.4. (A) Linear softening, (B) Exponential softening

The exponential softening law is commonly used as it closely represents the softening behavior of concrete as shown by Reindhart et al (1986) and Haidar et al (2005). Hence, it is used for all the numerical analysis carried out in this thesis. The formation and accumulation of microcracks are represented in the first part and the formation and growth of the macrocrack in the second part.

We introduce a loading function $f(\varepsilon, \kappa) = \varepsilon - \kappa$, to define the loading/unloading conditions in the Kuhn-Tucker form,

$$f \leq 0, \quad \dot{\kappa} \geq 0, \quad \kappa f = 0$$

(3.10)

The first condition states that $\kappa$ cannot be smaller than $\varepsilon$. The second condition states $\kappa$ cannot decrease. The third condition states $\kappa$ can increase only if it is equal to $\varepsilon$, the equivalent strain. With all these equations combined a representative force displacement relation for tensile failure scenario is shown in Fig 3.5.
Within the finite element framework used to study strain softening, the descending branch of the load displacement curve leads to ill-posedness of the underlying PDEs. From a mathematical point of view, these issues can be associated with the loss of ellipticity for static cases and hyperbolicity for dynamic cases of the governing differential equations. This leads to the loss of a unique solution with dependence on the given data, ill-posed boundary value problem (Chapter 7, Jirasek (2009)). This instability is accompanied by the localization of the strains to the damaged elements. The strains localize and damage is limited to the initially damaged elements. This leads to incorrect description of the FPZ width as it is dependent on the chosen mesh size (Bazant (1976)). This in turn leads to mesh dependent force displacement profiles in the post peak regime. For decreasing mesh sizes the descending branch of the force displacement plots become steeper, i.e. reduction in dissipated fracture energy. This leads to solutions with a vanishing energy dissipation for smaller mesh sizes. A simple case is set up to study the localization of strains to demonstrate mesh sensitivity and its effects.

### 3.1.1. Mesh sensitivity with local damage models

A cantilever beam is considered, Fig 3.6 for the study with a slight reduction of damage threshold in the element at the center of the beam (shaded zone) to initiate damage. At the free end a tensile load is applied by displacement control. The cross sectional area $A = 10 \text{ mm}^2$, length $L = 100 \text{ mm}$, Young’s modulus $E = 30 \text{ GPa}$, Poisson’s ratio $\nu = 0.0$ were used for the analysis. The threshold value for damage initiation was chosen as $\kappa_i = 1.45\times10^{-4}$ in the weak zone as opposed to $1.5\times10^{-4}$ in the rest of the beam. The exponential damage evolution law (eq 3.9) is used with parameters $\alpha = 0.95$ and $\beta = 100$. The beam is discretized with varying mesh sizes along the length.

A relatively low number of elements (21,51,81) are used to show the results, as the higher number of elements will lead to convergence issues. These issues are due to the descending
post peaks for higher number of elements as seen in Fig 3.7A. When a large number of elements is used very small time step size needs to be used obtain converging results.

![Load vs Displacement plot and Strain profile along the length](image)

Figure 3.7: Local damage model : (A) Load vs Displacement plot, (B) Strain profile along the length

When the stress in the predefined weak element exceeds the tensile strength, damage initiates. Consequently the stresses reduce due to the stiffness degradation. The rest of the bar will unload elastically as the strength threshold is not exceeded. Hence, the softening zone is dictated by the weak element size. This can be seen in the Fig 3.7B where the strain localizes to a single element at the midpoint of the bar, the weak element. The localized strain in the weak element increases for smaller elements since the deformation is localized to a smaller width. The larger value of localized strains leads to quicker damage evolution and hence, a varying force displacement plot, Fig 3.7A. The dissipated fracture energy reduces for a decrease in the mesh size. This mesh dependency is not physically desirable as it shows the same bar damaging under the same loading conditions by dissipating different fracture energies with different fracture widths.

There is a need to enrich the continuum description in order to prevent the ill posedness of the equations and avoid strain localization to a single element. This enrichment is achieved by introducing a length parameter to regularize the localization process by introducing damage dispersion, which serves as the localization limiter (Chapter 7, Jirasek (2009)).

The fracture energy approach and the nonlocal model both aim at introducing damage diffusion by means of a length parameter to overcome the localization problem. The fracture energy approach aims to properly reproduce the energy dissipation, $G_F$, in the softening band, $L_s$. This is achieved by adjusting the energy dissipated in the softening band (area under stress strain curve after subtraction of pre peak dissipation) $g_F = G_F / L_s$. This method removes the sensitivity to the mesh size and provides objective global solutions, but the strain and displacement fields still show some mesh dependency. The fracture process zone can be affected by mesh induced direction bias.

The nonlocal model on the other hand, introduces the length scale at the local element level by replacing the local variables with a nonlocal counterpart (obtained by weighted average).
This influences the global solution to be mesh independent. The strains and deformation in the continuum also show converging results irrespective of mesh size. The fracture process zone does not show any mesh-induced directional bias. Hence, the nonlocal model is the preferred regularization method although the computational costs may be higher.

### 3.2. Non-Local damage model

The assumption that the material response at a certain point is not only dependent on the stress state at that point but also depends on the state of stresses in the surrounding region is the underlying principle of the nonlocal models. This is because the damage zone consists of microcracks and a certain level of interactions exist between them (Bazant, Z.P., 1994). This nonlocal behavior is employed in continuum damage mechanics using an internal length scale which accounts for the interactions of the microcracks. The nonlocal damage model aims to provide mesh independent solutions, i.e. the width of the strain localization and the fracture energy are mesh independent.

The nonlocal model works by replacing certain local material properties with their weighted average over a certain domain to describe the state of the material. The models were initially implemented in the integral form by Pijaudier-Cabot and Bazant (1987). Differential formulations of the nonlocal behavior were implemented by Peerlings et al. (1996) as the gradient enhanced damage model. This was done by using the local isotropic damage model where the equivalent strain was replaced with the nonlocal equivalent strain used to calculate the damage. It is also possible to use the nonlocal counterparts of other parameters (Chapter 7, Jirasek (2009)), namely the damage energy release rate, the damage parameter, the specific fracturing strain. It was found that the nonlocal equivalent strain provides the most stable set of results (Chapter 8, Jirasek (2009)).

The nonlocal model serves primarily as a localization limiter, (Bazant, Z.P., 2002)) and helps account for the effects of heterogeneity in a homogenized model. On a macroscopic level the stress averaged over a certain area is dependent on the average strain value and not the strain at the center point which can be different. The use of the nonlocal parameter which is the average of the strains, helps to account for the effects of heterogeneity. In other words the growth of a crack is dependent on the overall energy release from the surrounding finite volume, i.e. the average deformation of the volume. The use of the nonlocal models results in distributed cracking which is observed physically.

From the experimental results obtained for mode I failure analysis a certain characteristic length, \( l_{ch} \) has been defined which is a measure of the brittleness of the material. The material behaves less brittle with higher \( l_{ch} \). The \( l_{ch} \) increases with the aggregate size, thus the brittleness reduces with increase in heterogeneity. As the brittleness of a material reduces, failure is delayed and hence more fracture energy is dissipated. The internal length scale used in the nonlocal model is related to the \( l_{ch} \). Physically, the internal length scale is a function of (amongst others) the maximum aggregate size used in concrete, Bazant and Pijaudier-Cabot (1989). Hence, the internal length scale of the nonlocal model can be related to the brittleness of the material.
The non-local averaging aims at replacing variables by their non-local counterparts which are obtained by weighted average over a defined spatial neighborhood of the point where the variable is to be determined. The spatial neighborhood, termed as the domain of influence or the interaction domain is defined using the internal length scale. If \( f(x) \) is a local field in a domain \( V \) then the corresponding non-local field is \( \tilde{f}(x) \).

\[
\tilde{f}(x) = \int_{V} \alpha(x, \xi) f(\xi) \, d\xi
\]

The \( \alpha(x, \xi) \) term is the given weight function, which prescribes the contribution of surrounding elements to the nonlocal average. The weight function depends on the distance, \( r \) between the source point \( \xi \), and the target point \( x \). The weight function is a monotonically decreasing function as \( r \) increases as the source points closest to the target point have highest influence compared to ones farther away. When applied to softening materials, it is required that the nonlocal operator should not alter a uniform field, which requires the weight function to satisfy the normalizing condition \( \int_{V} \alpha(x, \xi) \, d\xi = 1 \).

In an isotropic and homogeneous medium, the weight function depends on the distance, \( r \) between the source point \( \xi \), and the target point \( x \). The function can be rewritten as \( \alpha(x, \xi) = \alpha_{o}(\|x - \xi\|) \). In the vicinity of a boundary the body the averaging is performed only on the part of the interaction domain that lies within the body. To satisfy the normalizing condition the weight function is defined as

\[
\alpha(x, \xi) = \frac{\alpha_{o}(\|x - \xi\|)}{\int_{V} \alpha_{o}(\|x - \zeta\|) \, d\zeta}
\]

(3.12)

It has been shown that the averaging of the equivalent strains produce the most stable solutions when using the nonlocal integral damage models. Hence the average equivalent strain can be written as

\[
\bar{\varepsilon}_{eq}(x) = \frac{\int_{V} \alpha_{o}(\|x - \xi\|) \varepsilon_{eq}(\xi) \, d\xi}{\int_{V} \alpha_{o}(\|x - \zeta\|) \, d\zeta}
\]

(3.13)

In the finite element formulation the integration points are considered as the source and target points. To calculate the weighted average of the equivalent strains at a target integration point \( x \), the integration points within the interaction domain are considered as source points. Written in equation forms the nonlocal strains at an integration point are obtained by eq (3.14) and (3.15) where \( x_{k} \) is the target point, \( x_{i} \) is the source point and \( w_{i} \) is the weight of the integration point.

\[
\bar{\varepsilon}_{eq} = \sum_{i=1}^{n_{gp}} \alpha_{ki} \bar{\varepsilon}_{eq,i}
\]

(3.14)
\[
\alpha_{ki} = \frac{\alpha_o(\|x_k - x_i\|)w_i}{\sum_{j=1}^{n_{gp}} \alpha_o(\|x_k - x_j\|)w_j}
\] (3.15)

The nonlocal models replace the local parameter with its nonlocal counterpart. Hence, the local equivalent strain must be replaced with the nonlocal equivalent strain. This nonlocal equivalent must be evaluated using the local strains from the same cycle of analysis. This cannot be done when implementing the nonlocal model with the LS-DYNA user subroutine framework. The modifications that can be done to the solution process using the user subroutine has certain limits. The replacement of equivalent strains is outside this limit and hence cannot be implemented, (Appendix C). Hence, to calculate the nonlocal equivalent strain the local equivalent strains from the previous load cycle are used. This obviously would introduce an error in the obtained result but can be reduced. To reduce the error a very small time step size is chosen compared to the critical time step size of the discretized model. The stress state between two consecutive load cycles being very small, will reduce the error marginally.

Using the equivalent nonlocal strain, \(\bar{\varepsilon}_{eq}\), the damage parameter is evaluated. In the stress-strain relations the damage parameter is evaluated from the nonlocal equivalent strain but the strain used in eq (3.3) is kept local, \(\varepsilon\). This is done to keep the elastic behavior of the materials local. The commonly used weight functions are the Bell shaped function, eq (3.16) and the Gauss distribution function (3.17)

\[
\alpha_o(r) = \left(1 - \frac{\|x_k - x_i\|^2}{R^2}\right)^2 \quad 0 \leq \|x_k - x_i\| \leq R
\] (3.16)

\[
\alpha_o(r) = \exp\left(-\frac{4\|x_k - x_i\|^2}{R^2}\right) \quad 0 \leq \|x_k - x_i\| \leq R
\] (3.17)

![Figure 3.8: Weight Functions (A) Comparison of Bell function and Gaussian function for 1-D cases, (B) Isotropic weight function](image)

For this study the Gaussian distribution function is used. As seen from the figure the Gaussian distribution is thinner Fig 3.8A. This leads to the surrounding points having lower weights...
than bell function and hence a lesser spread of damage. In other word the Gaussian distribution will lead to a slightly more brittle response which has a very small effect on the solution. The Gaussian distribution is used for all the numerical analysis conducted in this thesis. Irrespective of choice of the weight function, the current nonlocal model uses an isotropic function for the nonlocal models Fig 3.8B.

**Iterative scheme of the nonlocal model implemented**

To clearly understand the flow of the analysis an iterative procedure is provided of the nonlocal damage model

0. Before the computations are started LS-DYNA computes the stable time step size required for the explicit integration scheme, $\Delta t_{\text{stable}}$.

1. At the first cycle of analysis, the interaction area and weights of this interaction area are calculated which are constant throughout the analysis.

2. Compute the strain increment, $\Delta \varepsilon_{j+1}$ from the displacement increment $\Delta a_{j+1}$ which in turn are obtained from the acceleration increment from the equation $M \ddot{a}_j = f_{\text{ext}}^j - f_{\text{int}}^j$.

3. Update the total strain, $\varepsilon_{j+1} = \varepsilon_j + \Delta \varepsilon_{j+1}$

4. Compute the equivalent strain, $\bar{\varepsilon}_{j+1} = F(\varepsilon_{j+1})$

5. Compute nonlocal equivalent strain in case of nonlocal models. $\bar{\varepsilon}_{j+1} = G(\bar{\varepsilon}_j)$. Here the equivalent strains of the previous cycle are used.

6. Evaluate the damage loading function, $f(\bar{\varepsilon}, \kappa) = \bar{\varepsilon}(\varepsilon) - \kappa$

7. If $f \geq 0$, $\kappa_{j+1} = \bar{\varepsilon}_{j+1}$ else $\kappa_{j+1} = \kappa_j$

8. Update the damage variable, $\omega_{j+1} = \omega(\kappa_{j+1})$

9. Compute new stresses, $\sigma_{j+1} = \left(1 - \omega_{j+1}\right)D^e \varepsilon_{j+1}$

10. Compute internal forces, $f_{\text{int}}^{j+1} = \sum_{n_{\text{ele}}} Z_c^{T} \sum_{i=1}^{n_{\text{int}}} w_i \det J_l B_{i,j+1}^{T} \sigma_{i,j+1}$

11. Use internal forces calculated to obtain the displacement increment $\Delta a$, using the equation used in explicit integration scheme.
3.2.1. One dimensional uniaxial tension test (Static)

The uniaxial bar under tension, Fig 3.6 is analyzed again. Now the nonlocal damage model is applied to the model. A fictitious length scale of 6 mm is used which is greater than the largest mesh size to ensure the spread of damage to a wider zone which is consistent with different mesh sizes. The bar has been discretized using 51, 81, 101 and 201 elements.

![Figure 3.9: Nonlocal damage model (A) Load vs Displacement plot, (B) Damage profile along the length (C) Eq Strain at disp = 0.04mm and (D) Eq Strain at disp = 0.075mm](image)

As expected the nonlocal model shows negligible mesh sensitivity. It can be seen from the force displacement plot, Fig 3.9A that the post peak behavior of the plots are nearly identical for all mesh sizes. They converge as the element size decreases, as the gradient of the strains are better captured with higher number of elements. Mesh independent results are also obtained in the damage profiles Fig 3.9B. It is also observed that the strain localization width is constant for the different mesh sizes, Fig 3.9C and Fig 3.9D. The slight variations observed is due to the different number of elements within the interaction domain. This ensures that the fracture energy is the same irrespective of the mesh size. Hence we can state that this model is mesh independent and has many advantages in comparison with the mesh dependent local models.
The mesh independent behavior is introduced by the use of the nonlocal equivalent strains instead of the local equivalent strains. The weighted average of the local strains over the interaction domain always leads to a difference between the local and nonlocal parameters. This difference leads to a delay in strain accumulation when using the nonlocal models which is larger with for larger length scales. This delay introduces a delay in damage initiation and hence a higher peak force, strength of the material is obtained when using the nonlocal model.

The nonlocal model fails to form a macrocrack and does not show complete failure, i.e. the damage at the midpoint does not reach one Fig 3.9B. This leads to a spurious additional strength of the material when damage is high. Hence, the nonlocal model fails to represent open macrocracks with localized strains.

3.2.2. One dimensional uniaxial tension test (Dynamic)

A simple case is set up to compare the behavior of the nonlocal and stress based nonlocal model in a dynamic setup. A cantilever beam is considered, Fig 3.10 for the study which is fixed at the left end and a load is applied at the right end. The load produces a stress pulse shown in fig 2.1 which should cause damage at the fixed end upon reflection. The cross sectional area $A = 10 \text{ mm}^2$, length $L = 100 \text{ mm}$, Young’s modulus $E = 30000 \text{ MPa}$, Poisson’s ratio $\nu = 0.0$ and density of 2500 kg/m$^3$ were used for the analysis. The threshold value for damage initiation of $\kappa_t = 1.5e^{-4}$ is chosen. The exponential damage evolution law (eq 3.9) is used with parameters $\alpha = 0.99$ and $\beta = 2000$ and the equivalent strain is evaluated by Mazar’s definition (eq 3.5). The beam is discretized using 100, 150 and 200 elements and a length scale of 8 mm is used.

Figure 3.10: Uniaxial Bar fixed at the left end

Figure 3.11: Original Nonlocal model (A) Strain (B) Damage profile along the bar length at $t = 6e^{-5}$s
The location of damage is predefined due to the loading pattern. The nonlocal model spreads the damage over a certain region and shows mesh independent behavior. The peak damage as expected is at the fixed end of the bar and we see convergence of the damage profiles for the different meshes, Fig 3.11B. We also observe a convergence of the equivalent strain profiles, Fig 3.11A.

As observed for the static case the nonlocal model cannot show complete failure. A macrocrack is expected at the fixed end reducing it to free boundary. But with the nonlocal model a macrocrack is not formed. This leads to an incorrect description of the damage process which leads to unrestricted spread of damage.

To capture the effect of the loading rate on the fracture process, specially the effect on the peak damage values and the spread of damage an additional analysis using the discretization with 100 elements is used. The loading rates are produced by changing the slope of the loading F(t). The current slope represents a loading rate of 8.33 /s and by manipulation the rates are increased to 16.66 /s and 33.32 /s but on reflection the loading rate at the fixed end is twice the input rate.

![Figure 3.12: (A) Damage profiles and (B) Stress time plots of the FPZ for the different loading rates at t = 6e-5s](image)

The damage profiles recorded for the different strain rates are plotted in Fig 3.12A. The width of the damage is independent of the loading rate as the width is dependent on the input length scale. With higher rates, quicker development of the damage zone is seen resulting in a higher value of peak damage. The stress time plot at the fixed end, Fig 3.12B shows an increase in strength for higher loading rates. For higher rates, larger number of elements within the interaction domain have a small value of strain within the interaction domain. Hence, the nonlocal strains calculated have larger delay in strain accumulation, and delay in damage initiation leading to the apparent increase of strength. This apparent increase in strength with increasing loading rates reaches a maximum value when a rectangular pulse is used.

This delay observed is similar to the quasi static case. The increase observed when using the nonlocal model for the dynamic loading cases shows a high variation with different loading rates. This is mainly due to the variations of the stress levels within the material for the given
loading. In quasi static cases the rate loading of loading being extremely low the stress levels throughout the material are almost constant and the apparent increase observed may not vary much.

The incorrect damage initiation at crack tip and inadequate descriptions of interactions in the vicinity of boundaries are other drawbacks of the nonlocal model. In an attempt to understand these limitations of the nonlocal model these drawbacks are investigated using certain benchmark tests.

3.2.3. Compact tension test

Damage initiation at the crack tip was the first problem to be analyzed when using the nonlocal models. This can be observed in the problem of an elastic plate loaded in tension and weakened by a sharp crack perpendicular to the applied loads. It has been shown that nonlocal models erroneously show that the point of the maximum stress is located away from the crack tip, and this point moves further for a larger length scales Simone et al. (2004). This leads to the damage initiation to be away from the crack tip instead of initiating at the crack tip.

To study the erroneous damage initiation we use the compact test specimen shown in Fig 3.13 as used by Simone et al. (2004). The notch is located at halfway in the specimen (EF), and has a length equal to half the width of the specimen. The tensile load is applied from both sides by means of incremental displacement control. The section has a height $4h = 2$ mm with a notch $h = 0.5$ mm long. The Young’s modulus $E = 1$ GPa, Poisson’s ratio $v = 0.0$ were used for the analysis. The threshold value for damage initiation was chosen as $\kappa_i = 3e^{-4}$ and the equivalent strain was obtained using the Mazar’s definition (eq 3.5). The exponential damage evolution law (eq 3.9) is used with parameters $\alpha = 0.99$ and $\beta = 1000$. The structure is discretized using 20 $\mu$m square elements and a length scale of 0.2mm has been used for the analysis.

Figure 3.13:Compact tension test specimen
The contour plots for equivalent strains have been plotted at the point of damage initiation to locate the point of damage initiation, Fig 3.14A. The peak nonlocal strains have shifted inwards from the notch and this shift increases for larger length scales, Fig 3.14C. The shift of the peak strains leads to damage initiation within the specimen and not at the notch. When calculating the nonlocal equivalent strains at the crack tip the interaction domain considers the elements along the predefined crack. This reduces the weighted average at the crack tip as the elements along the crack have a low stress state and lower equivalent strains. As we move inwards the isotropic interaction domain contains fewer elements along the crack. Hence, the calculated nonlocal equivalent strains will be higher than the crack tip as observed at damage initiation.

As the crack is propagating along FG the damage is expected to evolve with initiation of damage away from expected point. With very fine meshes, this behavior will lead to jumps in the load displacement plots. The load displacement plot, Fig 3.14D does not show any jumps. Either the mesh is not fine enough or the explicit integration scheme does not show results.
with jumps in the load displacement plots. The explicit integration scheme uses small time steps to calculate the new state unlike the implicit integration scheme which uses larger time steps. These small time step sizes may lead to gradual evolution of damage and cannot capture the jumps in a explicit finite element setting.

The peak strength includes a spurious increase in strength. The nonlocal model induces a delay of damage initiation due to the elements with low stress states along the crack in the interaction domain. This will lead to a delay in the equivalent strain accumulation and hence, an increase in strength.

The analysis is carried out up to complete failure Fig 3.14B. It can be seen that the damage has now also spread over the predefined notch. This is not physical behavior as damage is not expected to develop along the predefined notch. When the nonlocal equivalent strains are evaluated for the elements over the crack they consider the interaction domain which includes the crack tip which damages during the analysis and have very high strains. Hence, higher nonlocal equivalent strains are calculated for these elements which at some point induce damage. This damage also spreads in the vertical direction as the damage grows horizontally. The elements surrounding the damaged zone consider the high strains of the damaged elements in the constant isotropic interaction domain creating a spurious spread of damage. These interactions, which depend on the distance between two points considers the damage zone with high strains leading to the damage diffusion phenomenon.

3.2.4. Spalling test

The description of the interaction domains near the boundaries of the models is another problem with the nonlocal model. The isotropic interaction domains work well in the infinite continuum but when approaching boundaries the domains are truncated and lead to problems. This problem also exists with the interaction across boundaries (macrocracks in completely damaged elements) created during the analysis. The attraction of damage by the boundaries is demonstrated by Krayani et al (2009).

\[
\begin{align*}
\text{t < } l_c & \quad v = v_c \\
\text{t > } l_c & \quad v = 0
\end{align*}
\]

Figure 3.15: Split Hopkinson Bar test

In order to study the results in the vicinity of boundaries a simple one dimensional dynamic tension test, spalling of the Split Hopkinson Bar is considered. A rectangular compressive pulse as shown in Fig 3.15 is introduced into the bar. Upon reflection the pulse at the end of the bar, changes to a tensile pulse that adds onto the incoming compressive pulse. The loading is such that the compressive pulse has a magnitude greater than the tensile strength of the bar and the damage initiates as soon the tensile pulse length is larger than the compressive pulse
length after reflection. This leads to damage initiation at a length equal to half the length of the compressive pulse introduced. The length of the compressive pulse is chosen such that the damage initiates close the free boundary to study the damage evolution.

The analysis is carried out using a bar with length, \( l = 250 \text{ mm} \) discretized using 250 elements along the length. A modulus of elasticity, \( E = 1 \text{ Mpa} \) is chosen with Poisson’s ratio \( v = 0.0 \) and density of \( 1 \text{ kg/m}^3 \). The threshold value for damage initiation was chosen as \( \kappa_1 = 1e-6 \) and the equivalent strain was obtained using the Mazar’s definition (eq 3.5). The exponential damage evolution law (eq 3.9) is used with parameters \( \alpha = 1 \) and \( \beta = 2e + 6 \). A length scale of 30 mm is used for the analysis. The compressive pulse is introduced by means of velocity boundary condition, \( v_0 = 2.5 \text{ mm/s} \) at the left end of the bar for a time \( t_0 = l_0/c \), where \( l_0 \) is the length of the pulse and \( c \) is the wave speed in the material. To study the damage evolution near the boundary we consider a small pulse \( l_0 = 30 \text{ mm} \) such the damage initiates 15 mm from the boundary.

![Figure 3.15: (A) Nonlocal strain evolution and (B) Damage evolution](image)

Even though the damage initiates away from the boundary and the material is brittle we observe that the damage has spread to the boundary. When the domain is truncated by the boundary the denominator of the eq (2.13) reduces leading to higher amplitude of interactions. This leads to high nonlocal strain values evaluated closer to the boundary which can be seen in Fig 3.16A. This effect combined with the other drawbacks of the nonlocal model shows the spurious damage evolutions, Fig 3.16B. The drawbacks are incapability of the nonlocal damage model to show compete failure and the isotropic weight function considering the damaged region to calculate the nonlocal strains.

### 3.3. Conclusions

Strain softening behavior of concrete can be captured using the exponential softening law used in the local constitutive law. These local constitutive laws show results which are dependent on the mesh discretization due to the ill-posedness of the governing equations. The nonlocal model is a regularization method which avoids the ill-posedness of the equations using the localization limiter, which also offers physical reliability to the results. The
localization limiter, the length scale, introduces the nonlocal nature of the material as it is related to the characteristic size of the material, aggregate size for concrete.

The damage calculated using the nonlocal equivalent strains is obtained by the weighted average of the local equivalent strains. The weighted average introduces a delay in the nonlocal strain accumulation resulting in delayed damage initiation and an apparent strength increase in the material. The increase is almost constant for quasi static loading conditions but increases with increasing loading rates in dynamic conditions. This is due to the variation of stress states in the interaction domain in the dynamic loading conditions. The loading rates only influence the development of the damage zone but not its size which is controlled by the length scale.

The nonlocal model comes with certain drawbacks, mainly due to the generic description of the interactions in the material considered. The nonlocal model cannot capture complete failure due to the inability to define the macrocracks. The erroneous damage initiation at the crack tip leads to a spurious strength increase as the strain accumulation is delayed. Damage diffusion and the attraction of damage by the boundaries are other issues which can provide unreliable results. The isotropic interaction domains used are the root cause of the drawback of the model. These issues with the nonlocal model may turn out to be more severe when dynamic loading conditions are considered. Hence, a better interaction domain description with more relation to the material behavior needs to be implemented to obtain physically relevant results.
4. **STRESS BASED NONLOCAL MODEL**

The original nonlocal model shows certain drawbacks which need to be resolved to study the damage profiles in concrete:

1. Incorrect damage initiation in the crack tip problem.
2. Failure to represent open macrocracks as boundaries.
3. Inadequate treatment of interactions in the vicinity of boundaries.
4. Unrestricted growth of damage over time.

A number of proposals are available to create a modified nonlocal model which can address the shortcomings of the original nonlocal model. Desmorat et al. (2007) presented a nonlocal model wherein the interaction depends on the wave propagation time rather than the distance between points. This introduces an evolving interaction length effect. As damage initiates and evolves, the wave speed reduces, reducing the interaction length which eventually shows local behavior as damage approaches maximum value. The evaluation of the wave propagation time takes considerable amount of time.

The most common problem with the nonlocal model is in the vicinity of boundaries. Generally the averaging is performed only on the part of the domain that is within the boundary, for which a modification is made to the weight function. Borino et al. (2003) introduces a change in the weight function using a dirac-delta function when approaching the boundaries. But these solutions still show problems with the error in damage initiation point and is incapable of introducing a macrocrack due to damage.

Bazant et al. (2010) proposed a boundary layer approach to overcome the boundary errors. This was done by introducing a layer of elements with the local damage model and the elements in the bulk of the material are treated with the nonlocal model. As only one layer of the elements is being treated as local a mesh dependency is introduced when the element size changes. Also, this method does not consider the new boundaries (macrocracks) formed in the structure.

Krayani et al. (2009) proposed modifying the domain of interactions in the vicinity of boundaries. The motivation behind the work was that the stress redistribution due to interactions should decrease in the vicinity of boundaries in the direction normal to these boundaries. The nonlocality in the normal direction of the boundary for a point located on the boundary is nonexistent.

It was shown by Piaudier-Cabot, G. et al.(2010) that nonlocality corresponds to stress redistribution due to presence of defects with the help of the microcrack interactions. They demonstrated that the nonlocality disappears in the direction normal to the boundary but is preserved in the other directions. This model also accounted for the boundary effects with the new surfaces created during the analysis. The interactions between neighboring points depends on the state of damage is the underlying concept. The model works by remapping the
neighbors to new distance which increases with increase in damage. As the damage at a point increases the neighbors are remapped and eventually they are so far that the point with complete damage behaves locally. This gives anisotropic interaction domains which work well around boundaries and evolve over time. When implementing the model in the finite element framework it is quite cumbersome to keep track of the remapping of the neighbors.

Giry et al.(2011) proposed a nonlocal damage model with a dynamic interaction domain. The interaction domain is dependent on the magnitude and directions of the principal stresses. In this method the amount of interactions “s” can distribute to its surroundings is under consideration for computations rather than interactions “x” can receive. Since the interaction domain is now anisotropic the width can vary from 0 to $L_c$, the length scale (a material property). This method may be computationally time consuming due to the recalculation of the interaction domain at every cycle.

In a study regarding the microcrack interactions, Bazant et al. (1994) shows that the interactions depend on the orientation of the stresses around the penny shaped cracks. Based on this a nonlocal method with an additional term depending on the principal stress directions was added to the original model. The interactions may be more or less based on the principal stress directions. This leads to an anisotropic interaction domain depending on the local stress state.

Pijaudier-Cabot, G. et al.(2010) and Giry et al.(2011) propose models with evolving anisotropic interaction domains which best describe the interactions throughout the continuum of the model. The models arise from the concept that the interactions are dependent on the state of damage at a point. In the former model the damage parameter is directly used to define the interaction domains, while with latter model it is used indirectly in the form of the principal stresses. The latter model also works well for 2D cases and also accounts the orientation of the interactions as shown by Bazant et al. (1994). With respect to implementation the latter is easier as the length scale is the one that evolves unlike the former model wherein the remapping needs to be tracked.

**Stress Based Nonlocal damage model**

The stress based nonlocal model, Giry et al.(2011) defines the nonlocality using the source point, $s$ distributions with an influence domain. The target point, $x$ receives influence from certain neighboring source points as it is within the influence domains. The interaction domain now consists of the source points which have the target point within their respective influence domains. The influence domain evolves over time, depending on the stress state at the point and hence is anisotropic. The anisotropic influence domain is an ellipse which is oriented along the principal stress direction. The axes of the ellipse are now dependent on the principal stress magnitude. This leads to the evolving anisotropic interaction domains. The nonlocality now depends completely on the stress field. This easily helps the description of interaction domains near free boundaries and developing fracture zones.

The evolution of influence is controlled by a scalar parameter, $\rho_i$ in each principal stress direction. The product of this scalar quantity and the characteristic length $l_c$ gives the internal
length of the of the model. The scalar parameter, $\rho_t$ ranges from 0 for an unloaded material where the material shows local behavior to 1 when approaching tensile strength when the material shows full nonlocality. The ellipse is centered at $s$ and the major axis is oriented along the first principal stress direction. The length of the major and minor axes is given by $\rho_1 l_c$ and $\rho_2 l_c$ respectively, where $\rho_t = |\sigma(s)/f_t|$. In this equation $\sigma(s)$ is the principal stress and $f_t$ is the tensile strength of the material. The influence domain can vary from a point, an ellipse to complete circle. These influence domains of the source points lead to interaction domains of target points to vary from single points to complete circles. The eq (4.1) defines the ellipse representing the ellipse over which weights are distributes by a point $s(x_1, y_1)$. The orientation of the ellipse is given by the angle $\theta$ to the horizontal, the major axis $a = \rho_1 l_c$ and the minor axis $b = \rho_2 l_c$.

$$\frac{(x-x_1)^2}{a^2} + \frac{(y-y_1)^2}{b^2} = 1$$

$$\tan \theta = \frac{\sigma_x - \sigma_y - \sqrt{(\sigma_x - \sigma_y)^2 + (2\sigma_{xy})^2}}{2 \sigma_{xy}}$$

The domain of influence is scaled according to the ratio of the current principal stress with respect to the tensile strength. The characteristic length $l_c$ defines the maximum size of the interaction domain. Hence $\rho_t l_c$ has an upper limit when approaching the tensile strength of the material, leading to a maximum value of $\rho_t$ to be 1 under loading directions where the principal stresses are greater than the tensile strength. The factor $\rho$ introduces an anisotropic influence domain, Fig 4.1A which consequently has an anisotropic weight function, Fig 4.1B.

![Influence domain](image)

**Figure 4.1 : Influence domains (A) Comparison between local, nonlocal and stress based nonlocal model (B) anisotropic weight function**

The weight function takes the form (eq 4.3) with $l_m$ equal to the distance from the center of the ellipse to the edge along the direction of the $(s - x)$ vector. To evaluate $l_m$ easily we can use the polar coordinate form the ellipse, as we already know the angle $\varphi$ of the vector to the horizontal (4.4).
Unlike the original nonlocal model the new interaction domains are calculated at every step of the analysis leading to a longer computational time. The new interaction domain calculation has two parts. Firstly the calculation of the influence domains of each integration point and secondly the definition of the interaction domain from these influence domains. The new influence domain calculation is done using the stress state at the points from the previous step. The procedure of the calculation of damage and stresses remains the same with the major change being in the calculation of the nonlocal equivalent strains. To prevent complications in the evaluation of the ellipse a minimum value for the axes of the ellipse is defined. The model switches to a local model when the length scale is smaller than the element size. Hence, \( \rho_l l_c = \text{element length} \) (1D cases) and \( \rho_l l_c = \sqrt{\text{area of element}} \) (2D cases) are set as minimum length scales.

**Iterative scheme of the stress based nonlocal model implemented**

0. Before the computations are started LS-DYNA computes the stable time step size required for the explicit integration scheme, \( \Delta t_{\text{stable}} \).

1. At the first cycle of analysis, the largest possible interaction area is calculated which are constant throughout the analysis.

2. Compute the strain increment, \( \Delta \varepsilon_{j+1} \) from the displacement increment \( \Delta a_{j+1} \) which are obtained from the acceleration increment from the equation, \( M \ddot{a}_j = f_j^\text{ext} - f_j^\text{int} \).

3. Update the total strain, \( \varepsilon_{j+1} = \varepsilon_j + \Delta \varepsilon_{j+1} \)

4. Compute the equivalent strain, \( \bar{\varepsilon}_{j+1} = F(\varepsilon_{j+1}) \)

5. Computation of the nonlocal equivalent strain:

   5.1. Calculate the principal stresses and their directions, \( \sigma_{j+1}^{(1)}, \sigma_{j+1}^{(2)}, \theta \).

   5.2. Calculate the parameters, \( \rho_1 \) and \( \rho_2 \) using \( \rho = \frac{\sigma_s}{f_t} \).

   5.3. Calculate the new interaction domain, ellipse with major axis \( a = \rho_1 l_c \) and the minor axis \( b = \rho_2 l_c \) rotated by angle \( \theta \).

   5.4. Calculate the weights distributed by a point in this ellipse.

   5.5. Using these weights distributed the nonlocal strain is computed \( \bar{\varepsilon}(\varepsilon) \).

6. Evaluate the damage loading function, \( f(\bar{\varepsilon}, \kappa) = \bar{\varepsilon}(\varepsilon) - \kappa \)

7. If \( f \geq 0 \), \( \kappa_{j+1} = \bar{\varepsilon}_{j+1} \) else \( \kappa_{j+1} = \kappa_j \)
8. Update the damage variable, \( \omega_{j+1} = \omega(\kappa_{j+1}) \)

9. Compute new stresses, \( \sigma_{j+1} = (1 - \omega_{j+1})D^e e_{j+1} \)

10. Compute internal forces, \( f_{j+1}^{\text{int}} = \sum_{e=1}^{n_{ele}} Z_e^T \sum_{i=1}^{n_{int}} w_i \text{det} J_i B_{i,j+1}^T \sigma_{i,j+1} \)

11. Use internal forces calculated to obtain the displacement increment \( \Delta u \), using the equation used in explicit integration scheme.

4.1. One-dimensional uniaxial tension test (Static)

The uniaxial bar under tension presented in section 3.2.1 is analyzed again, now using the stress based nonlocal model. The bar has been discretized using 51, 81, 101 and 201 elements. With the stress based nonlocal damage model, the damage spreads over a region but the strain localizes to the weak element at complete failure. The element where the strain localizes is damaged completely signifying a macro crack.

Figure 4.2: Nonlocal damage model (A) Load vs Displacement plot, (B) Damage profile along the length (C) Eq Strain at disp = 0.04mm and (D) Eq Strain at disp = 0.075mm

The load displacement plots show some mesh sensitivity when approaching failure, Fig 4.2A. With increase in damage the influence domains reduce in size, which reduces the interaction.
domain, transforming the model to a local damage model. We observe mesh sensitivity when the model transforms to a local damage model. Having the largest elements the largest mesh size transforms to the local model first. Hence, we see failure first in the mesh with 51 elements which is accompanied by loss of convergence in the solution, Fig 4.3A. This does not have much influence on the damage zone width which almost converge, Fig 4.3B. The minor variation in the damage profiles is similar to the nonlocal model which is due to the different number of elements within the interaction domain of the different meshes at the same load cycle.

This leads to a single element damaging completely to show failure, i.e. damage reaches a value of 1 at midpoint of the bar, Fig 4.2B. The failure of the midpoint of the bar is also seen in the load displacement plot. Mesh independent strain profiles are observed when the damage is not very high, Fig 4.2C and localize to a single element when approaching failure Fig 4.2D. This is due to the evolution of the interaction domains as discussed.

**Comparison of models**

![Load vs Displacement plot](image1)
![Eq Strain](image2)
![Damage profile](image3)
![Zoomed in Damage profile](image4)

Figure 4.3: Comparison of models (a) Load vs Displacement plot, (b) Eq Strain and (c) Damage profile and (D) Zoomed in Damage profile along the length using mesh with 81 elements

The load displacement plots, strain profiles and the damage profiles for the different models are plotted using the mesh with 81 elements. The load displacement plots, Fig 4.3A show the
nonlocal and the stress based nonlocal model having a higher peak compared to the local damage model due to the delay of strain accumulation. Also the nonlocal and stress based nonlocal model have a similar profile at lower levels of damage. For low levels of damage the shrinking of the interaction domain is not high and will induce damage diffusion close to that of the nonlocal model. From the strain profiles, Fig 4.4B the stress based nonlocal and local model show strain localization over the same width unlike the nonlocal model. The localization of the strain profiles corresponds to the value of damage reaching 1, Fig 4.3C and zoomed in plots, Fig 4.3D depicting complete failure unlike the nonlocal model.

The size of the elements in the expected zone of damage should be regulated to obtain reliable results. The global strength of the structure depends on the element size chosen and the interaction length scale obtained from the experiments. Hence, a clear definition of acceptable range of element sizes in the expected zone of failure needs to be provided where the strains may localize and form a macrocrack.

4.2. One-dimensional Uniaxial tension test (Dynamic)

The dynamic uniaxial tension carried out using the cantilever bar in section 3.2.2 is carried out again using the stress based nonlocal model. The beam is discretized using 100, 150 and 200 elements and a length scale of 8 mm is used.

![Figure 4.4: Stress based nonlocal models (A) Strain profile (B) Damage profile along bar length at t = 4.75e-5s](image)

To observe the behavior of the stress based nonlocal model at lower levels of damage the strain and damage plots at t = 4.75e-6 are recorded. The strain profiles, Fig 4.4A and the damage plots, Fig 4.4B show good convergence. As the damage has not reached a high value, the interaction domains span a number of elements leading to nonlocal behavior and hence show convergence. The number of elements covered within the interaction domain may vary for the different meshes at the same time of analysis which leads to the minor variations in the strain profiles. This in turn results in the variations in the damage profiles.

With the growth of damage the interaction domain shrinks causing the model to behave as local damage model. This occurs at the element attached to the fixed end where the strains
STRESS BASED NONLOCAL MODEL

localize within this element on the growth of damage leading to strain profiles which do not converge, Fig 4.5A. The damage profile, Fig 4.5B shows minor mesh dependency but the spread of damage and the peak value at the fixed end are consistent.

Figure 4.5: Stress based nonlocal model (A) Strain profile (B) Damage profile along bar length at $t = 6e^{-5}s$

To observe the effects of the loading rate on the damage profile, its peak and the spread, the analysis presented in 3.2.2 with different loading rates is repeated with using the stress based nonlocal model.

Figure 4.6: (A) Stress deformation plots of the FPZ and (B) Damage profiles for the different loading rates at $t = 6e^{-5}s$

The damage profiles recorded for the different strain rates is plotted in Fig 4.6A. The loading rates do not affect the width of the damage zone as seen in the nonlocal model. The evolution of the damage zone is faster with higher loading rates, Fig 4.6B. An increase of the material strength observed is due to the delay of nonlocal strain accumulation as seen with the nonlocal model. The variation of the increase in the strength is due to averaging of strains, similar to the nonlocal model, as this delay occurs near the tensile strength when the interaction domains are equal to the ones used in the nonlocal model.
Unlike the original nonlocal model the stress based nonlocal model captures the formation of a macrocrack at the fixed end resulting in the formation of a free end. The wave propagating now reflects at the free end as a compressive wave. This results in a lower stress state in the bar and no additional damage evolution is expected.

The nonlocal and the stress based nonlocal models should behave in a similar manner when the material is not approaching complete failure. To test this the same analysis was carried out with a reduced post peak slope of the softening model i.e. $\beta = 1000$ keeping all the other parameters the same. As the model already agrees on the mesh dependency we shall take the mesh with 100 elements for this analysis.

The strain profiles, Fig 4.7A obtained from the two models are almost the same with minor differences. The damage profiles, Fig 4.7B in agreement with the strain profiles and show a small change in the spread and the slope of the damage profile. This clearly shows that the nonlocal and stress based nonlocal model behave in an identical manner when the damage does not approach failure.

4.3. Compact tension test

The compact tension test analyzed using the nonlocal model has shown some deficiencies. The analysis from section 3.2.3 is carried out using the stress-based nonlocal model to check if the issues have been resolved.

To investigate the damage initiation at the crack tip the contour plots of the equivalent strains are plotted when damage initiates, Fig 4.8A. The peak equivalent strains are located at the crack tip and hence damage also initiates at the crack tip. The point of damage initiation was also confirmed to be at the crack tip irrespective of the length scale chosen, Fig 4.8B. To better understand the model behavior leading to the correct location of damage initiation the interaction domains in the proximity of the crack are to be understood. The interaction domains are dependent on the influence domains which are derived from the principal stress vectors when damage initiates. Hence the principal stress vectors, the influence domains and the interaction domains at three points along EG, along the crack (a), at the crack tip (b) and...
away from the crack tip (c) are considered as shown in Fig 4.9A. The principal stress vectors are plotted in Fig 4.9B, the derived influence domains in 4.9C and the interaction domains in 4.9D.

Figure 4.8: (A) Nonlocal strain contour plot and (B) Nonlocal strain along FG for different length scales at damage initiation

Figure 4.9: (A) Location of the points, (B) First principal stress vectors at damage initiation, (C) Influence domains near the points and (D) Interaction domains at the points
At point (a) the stresses are quite low as this area is shielded by the crack. The points in this region have no influence on the surrounding elements due to the low stress state and leads to local behavior of the point (a). In the vicinity of the point (c) the principal stress vectors are almost vertical and hence the influence domains are thin ellipses which are aligned in this direction. The ellipses are thin as the second principal vector is very small compared to the tensile strength. The interaction domain at (c) is an ellipse oriented along the vector as shown.

The points in the vicinity of point (b) at the crack tip have principal stresses which vary in orientation and magnitude. These variations in the stress state cause influence domains of these points to be disturbed as seen in Fig 4.9C. These influence domains combine to form a complex interaction domain. This interaction domain includes most points around it except the points along the crack tip on the left side. On the right side the boundary is almost vertical due to the principal stress orientation. When the nonlocal strain is evaluated at the crack tip the highest value is obtained at the crack tip.

The damage once initiated at the crack tip now spreads out to the vicinity in the shape of the interaction domain. This spread occurs up to the point when a macrocrack starts forming, i.e. damage value at crack tip is equal to 1. Then the damage propagates along the line FG, representing the growth of the macrocrack along this line.

![Figure 4.10: (A) Damage contour plot at complete failure and (B) Load-displacement plots](image)

From the damage plot, Fig 4.10A at complete failure we see that the damage is limited to the right side of the crack tip as the interaction domains are defined better with relation to the stress state. Unlike the nonlocal model we do not see a spurious spread of damage on either side of the crack. With progressive damage, the area of influence reduces, making the damaged elements behaving locally. This will avoid interactions across the damaged zone and produces a thin zone of failure which is closer to reality. The model now shows a lesser global strength, Fig 4.10B. The damage initiates at a lower load compared to the nonlocal model as
the interaction domains at the crack tip do not consider the elements with low stress states which introduced the delay of damage initiation and the higher peak load.

4.4. Spalling test

The spalling test carried out on a simplified Split Hopkinson Bar test, section 3.2.4 is analyzed again using the stress based nonlocal model. The nonlocal model showed an incorrect failure pattern with damage diffusion across the crack and damage attraction by the boundaries. With the stress based nonlocal model it is expected to observe spalling, the complete damage at a single point without the attraction of damage to the boundaries.

![Figure 4.11: (A) Nonlocal strain evolution and (B) Damage evolution](image)

The evolution of the nonlocal equivalent strains and damage profile are shown in Fig 4.11. From the damage plots, Fig 4.11A we can observe that the maximum damage within the bar away from the free end and approximately at 15 mm from the free end which is half the length of the input pulse. This shows a clear location of failure, i.e. spalling of the specimen modeled. The strain profile is in agreement showing the localization at the expected location. There seems to be a small shift of the peaks of the profiles to the right, which is due to the reflection of waves at the domain which is damaging.

The width of the influence domains being dependent on the stress state of the material leads to the correct description of the interaction domains. Close to the boundary the reflected wave which is tensile adds to the compressive wave giving a zero stress state up to 15 mm of the bar. The interactions start as the tensile wave grows larger than the compressive wave. The influence of these elements is maximum when the tensile wave is activated. This wave initiates damage as the input pulse is larger than the tensile strength. With the quick damage evolution the influence of these elements to the neighboring elements reduces. This helps to avoid the spreading of the damage to the boundary. Most importantly this helps to avoid interactions across the damage zone with high strains avoiding spurious damage diffusion.
4.5. Conclusions

The description of the influence domains, dependent on the principal stresses helps to capture the anisotropic interaction domains which evolve during the analysis. The issues of the nonlocal model are resolved using these evolving interaction domains. Macrocrack formation is captured with the strain localization to a thin region. The stress based nonlocal model is capable of locating the correct damage initiation and the propagation. The better description of the interaction domains near boundaries with the formation of the macrocrack helps resolve the spurious damage diffusion, damage attraction by the boundaries and unrestricted damage growth in time.

The model transforms to a local damage model with the shrinking interaction domains to form the macrocrack, localized strains. Similar to the local damage model the final results are dependent on the number of elements within the interaction domain. The dissipated fracture energy increases with decreasing element sizes. Hence, a smoother transition of the model to local model needs to be facilitated. In spite of this problem the model does show objective results for the fracture process widths.

The stress based nonlocal model shows an increase in the strength of the material for increasing rates when simulating failure under dynamic loading conditions. This increase is due to the delay of damage initiation as observed with the nonlocal model as the stress based nonlocal model also shows full interaction in the proximity of tensile strength of the material. With higher loading rates the development of the damage zone and formation of macrocrack is faster but the width remains constant. The quicker development of the damage zone leads to faster shrinking of the interaction domains which may lead to a relatively brittle response.

The stress based nonlocal model clearly distributes damage within the interaction domain defined by the stress state of the material. Hence, the results will be independent of the mesh orientation and may replicate physical fracture process zones with the right parameters.
5. EXPERIMENTAL VALIDATION

The stress based nonlocal model is now used to study the dynamic failure of concrete. The results are compared with the results from the Split Hopkinson Bar test for moderate strain rates and the Modified Split Hopkinson Bar test for high strain rates (Vegt (2013)). The experiments are aimed at capturing the rate sensitive behavior of concrete for the two domains of strain rate effects. The tests involves the axial impact of loading of a notched concrete specimen. The influence of the loading rate on the tensile strength, young’s modulus and the fracture energy is the focus of the experiments. Using the stress based nonlocal model, it is attempted to capture the same behavior of failure in concrete.

<table>
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<th>( \varepsilon ) [-]</th>
<th>St.dev. [-]</th>
<th>( f_t ) [MPa]</th>
<th>( f_{t}/f_{\text{stat}} ) [-]</th>
<th>St.dev. [MPa]</th>
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<td>( 1.0 \cdot 10^{-8} )</td>
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<td>0.32</td>
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<td>4.40</td>
<td>10.47</td>
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Table 5.1: Tensile strength increase from experiments (Vegt (2013))

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<th>( G_f ) [N/m]</th>
<th>( G_f/G_f_{\text{stat}} ) [-]</th>
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<td>5.6</td>
<td>241.8</td>
</tr>
</tbody>
</table>

Table 5.2: Fracture energy obtained from experiments (Vegt (2013))

![Figure 5.1: Stress Displacement plots from experiments of the static test, Split Hopkinson Bar test and the Modified Split Hopkinson Bar test (Vegt (2013))](image-url)
In Table 5.1 the experimental results for the rate-dependent tensile strengths are listed, and Table 5.2 shows the fracture energies. As discussed earlier, the rate-dependency of concrete in tension is divided into two regimes, the moderate strain rate (4 GPa/s-50 GPa/s) and the high strain rate (>50 GPa/s) regime. The Split Hopkinson Bar Test is conducted for the moderate strain rate regime and the Modified Split Hopkinson Bar Test for the high strain rate regime. Representative stress displacement plots of the static, moderate and high strain rates are shown in Fig 5.1.

The rate effects in the moderate rate regime is largely influenced by the moisture in the pore system. A moderate increase in tensile strength is observed but the fracture energy shows a small deviation. The strength increase is attributed to the Stefan effect due to the moisture in the pores. The quick damage accumulation of concrete with the inertial effects of the damaged zone leads to a similar dissipated fracture energy.

In high rate regime a steep increase in the tensile strength and a steeper increase in the fracture energy is seen. The inertia at microscale and mesoscale is associated to the rate effects in high rate regime. For static case we are aware that the microcracks develop at around 80% of the tensile strength. The inertia due to the high rate of loading will delay this process and along with the moisture in the pores causes the steep increase in the observed tensile strength. The concrete at high strain rates behaves less brittle compared to the lower strain rates. The combined effects of moisture content and the inertial forces due to the softening of the fracture process zone is responsible for this behavior.

5.1. Split Hopkinson Bar test

The concrete is tested for the moderate rates, (4 GPa/s-50 GPa/s) using the gravity driven Split Hopkinson Bar test (SHB) apparatus shown in Fig 5.2. The apparatus consists of two long cylindrical aluminum bars (74 mm diameter) between which the notched concrete specimen (100 mm long) is glued. The aluminum bars are long enough to avoid reflections at the ends which may affect the results. A drop weight which slides along the lower bar to hit the anvil at the base, generating a tensile pulse that propagates upward through the lower aluminum bar. This tensile pulse causes failure in the specimen at the notch. The notch (4mm wide x 2mm deep) is placed to ensure that the damage always occurs at the same location and to avoid multiple fracture zones.

The tensile pulse transmitted from the failure zone is measured in the upper aluminum. This tensile pulse is synchronized to obtain the exact transmitted wave at the notch, the damage region. The fracture zone deformation is measured by subtracting the elastic deformations of the concrete specimen (measured at SG2) from the elongation of the concrete specimen, measured using the linear variable differential transducers Fig 5.2. Using the reproduced stress pulse and the deformation the stress displacement plot is obtained using which the fracture energy is evaluated. A detailed explanation of the SHB used for these experiments located at Delft university of technology is available in Kormeling (1986) and Weerheijm (1992).
The SHB experiment is simulated in LS Dyna using the stress based nonlocal model. The concrete is modeled with a Static Young’s Modulus, $E_{\text{static}} = 35.7$ GPa and density, $\rho = 2400$ kg/m$^3$ and Poisson’s ratio, $\nu = 0.2$. The static tensile strength is $f_{t,\text{static}} = 3.3$ MPa, which occurs at a peak strain, $\varepsilon_{o,\text{static}} = 9.24 \cdot 10^{-5}$. These are properties at very low loading rates and need to be altered as suggested by CEB 1988 for higher strain rates. The young’s modulus and the peak strain values change with the loading rate but the Poisson’s ratio is not affected.

To obtain the dynamic mechanical properties the equations 5.1 and 5.2 are used.

$$\frac{E_{\text{dyn}}}{E_{\text{static}}} = \left(\frac{\dot{\varepsilon}_{\text{dyn}}}{\dot{\varepsilon}_{\text{stat}}}\right)^{0.016} \tag{5.1}$$

$$\frac{\varepsilon_{o,dyn}}{\varepsilon_{o,static}} = \left(\frac{\dot{\varepsilon}_{\text{dyn}}}{\dot{\varepsilon}_{\text{stat}}}\right)^{0.02} \tag{5.2}$$

The $\dot{\varepsilon}_{\text{dyn}}$ represents the dynamic loading rate = 1.1 /s for the Split Hopkinson Bar test, and the $\dot{\varepsilon}_{\text{stat}}$ represents the static loading rate = $2.7 \cdot 10^{-6}$ /s. Hence the Dynamic Young’s Modulus, $E_{\text{dyn}} = 43.9$ GPa and the peak strain, $\varepsilon_{o,dyn} = 1.2 \cdot 10^{-4}$. The exponential damage evolution law (eq 3.5) is used with parameters $\alpha = 0.95$ and $\beta = 5000$. An internal length scale of 34 mm is used, which is approximately 3.5 times the maximum aggregate size. These values are in accordance with Haidar et al (2005) who presents experimentally relevant parameters for the exponential softening and the internal length scale.

A simplified two dimensional plane stress model, Fig 5.3 is used to reduce computational cost. It is assumed that the stress pulse measured at SG1 is completely transmitted into the concrete specimen. This helps model the lower aluminum bar into a smaller bar using the
properties of the concrete specimen to ensure complete propagation. A function \( F(t) \) which describes the incoming tensile wave measured is fitted to the strain measured at SG1.

The stress transmitted is recorded as the average stress pulse across notch cross section which represents the strength of the fracture zone. The upper bar is modeled as a system of two bars used to kill the wave after transmission without any reflections at the fixed end. The first part is modeled with the properties of concrete to ensure complete transmission and the second part is modeled using an imaginary material which ensures complete transmission from but with a very slow wave speed, which is done using acoustic impedance. The deformation is measured by capturing the change in length of the fracture process zone. The complete numerical setup is modeled using plane stress single integration point quad elements.

Before the actual analysis, the influence of the nonlocal approach used and the influence of the discretization used on the results are determined. The concrete specimen has been discretized using three meshes of different mesh sizes, shown in Fig 5.4. To compare the results provided by the nonlocal model and the stress based nonlocal model the mesh B is chosen.

![Figure 5.4: The three different meshes used for the analysis.](image)

Damage initiating away from the notch and failure to form a macrocrack leads to unrestricted spread of damage resulting in the unrealistic damage profile, Fig 5.5A. The stress based nonlocal model presents a better damage profile, Fig 5.5B which shows a macrocrack across the notch as expected. As discussed earlier the stress based nonlocal model is the better approach to study damage in concrete.
Figure 5.6: Stress deformation plots with the different meshes

Figure 5.7: Split Hopkinson Bar test using different meshes: Contour plots of (A) Eq Strain and (B) Damage
The stress deformation plots for the three meshes are plotted in Fig 5.6. A convergence in the plots is seen with the different meshes. The contour plots of the damage at rupture is shown in Fig 5.7A. The fracture process zone widths obtained from the different meshes are almost equal and show a similar pattern. The contour plots of the equivalent strains are shown, Fig 5.7B. The approximate value of eq strain when the damage equals one, i.e. formation of macrocrack is calculated using eq (3.9), $\varepsilon_{f_{\text{int}}}$ = $7e - 4$. It can be observed that the macrocrack is across the cross section and also has a similar profile. The results obtained are mesh independent with minor variations originating from the number of elements within the evolving interaction domains during the analysis.

We expect the meshes to show some mesh sensitivity when approaching failure as seen in the quasi static cases. This is not seen here as the macrocrack is formed by localization of the strains to a thin zone across the notch and not a line of elements.

Figure 5.8: Stress deformation plots : Comparison with the experimental data

As the mesh sensitivity of the results is now alleviated, the focus is on the experimental validation of the model. The stress displacement plots of the experimental and the numerical analysis are compared in Fig 5.8. We can see that the elastic behavior of the concrete up to the peak strength almost agrees with the experimental result. The peak strength obtained from the numerical results is 5.34 MPa which is slightly higher compared to the input 5.24 MPa which is equal to the experimentally obtained value. The small increase is due to the delay of the damage introduced due to the weighted averaging.

The focus is on the post peak curve, i.e. softening behavior. The results are mesh independent but do not match the experimental results. The dynamic loading leads to quick damage propagation and evolutions which result in reduction of the principal stresses and hence quick shrinking of interaction domains. This will result in a relatively brittle response. Additionally, the use of rate independent material model and its combination with the stress based nonlocal model leads to the observed brittle response. In reality this damage is delayed by the additional resistance provided by the moisture in the microcracks and the minor inertia effects which cannot be captured using simple isotropic damage models.
5.2. Modified Split Hopkinson Bar test

For very high strain rates (>50 GPa/s) the Modified Split Hopkinson Bar setup (MHSB) is used. The MSHB is based on the principle of spalling, damage caused by the tensile wave, which is generated by the reflected compressive wave at the free end. The setup uses a single steel bar (74 mm diameter) which is glued to a concrete specimen (300 mm long) on one end Fig 5.9. The concrete bar is notched at 65 mm from the free end. At the free end of the steel bar an explosive charge is placed which introduces the compressive pulse which is transmitted into the concrete specimen. The setup is placed such that the wave propagation is not affected by the supports.

![Experimental setup of the Modified Split Hopkinson Bar](image)

Figure 5.9: Experimental setup of the Modified Split Hopkinson Bar

To ensure no damage occurs due to the input compressive wave, the introduced pulse is smaller than the static compressive strength of concrete. This compressive pulse on reflection at the free end changes to a tensile pulse. The tensile pulse causes damage at the notch. The input pulse and transmitted pulse are measured in the concrete bar at the same location R2. A different method, which does not manipulate the results like the Linear variable differential transformer is used to measure the deformation. The deformation is now measured by considering the deformations between two points 5mm away from the notch on each side. The transmitted pulse recorded at R9 is reproduced to obtain the transmitted pulse from the damaged notch. Using this and the deformation the stress displacement plot and fracture energy are obtained.

The numerical setup used for the MSHB is shown in Fig 5.10. The concrete specimen is the main focus and no measurements are required in the steel bar. To transmit the compressive wave into the concrete specimen a 100 mm bar with the concrete properties is used. The load $F(t)$ which describes the incident pulse is factored by 2 to account for the pulse splitting into the compressive wave moving to the right and the tensile wave to the left. This tensile wave is eliminated by using a bar with very low wave speed by using acoustic impedance which is fixed at the left end. Similar to the SHB we obtain the transmitted pulse by using the average of the stress time plots at the notch cross section. The deformation is measured by obtaining the deformation time plots between two points as shown in Fig 5.9.

The material properties of concrete used for the analysis are as stated in the previous section except the Young’s modulus and the peak strain as they depend on the loading rate. The loading rate $= 46.5$ /s for the MSHB. Using equations 5.1 and 5.2 the Dynamic Young’s Modulus, $E_{dyn} = 46.61 \text{ GPa}$ and the peak strain, $\varepsilon_{o,dyn} = 1.289 \cdot 10^{-4}$. The input tensile strength is 6 MPa which is almost half the experimental value. To obtain the same strength in the material the peak strain value is tuned to obtain the experimental strength. The new peak
strain value is $\varepsilon_{o,dyn} = 2.486 \cdot 10^{-4}$ which should provide the same strength as observed in the experiments, 11.59 MPa.

Figure 5.10: Numerical setup of the Modified Split Hopkinson Bar

Figure 5.11: Modified Split Hopkinson bar specimen (A) Eq strain profile (B) Damage profile

Figure 5.12: Stress deformation plot of the numerical and experimental results.

Mesh sensitivity analysis is not performed, since the SHB study in the previous section has shown mesh independent results. The Fig 5.11 shows the equivalent strain profile and the damage profile. The equivalent strain profile a curved macrocrack across the notch and
EXPERIMENTAL VALIDATION

Additionally shows small macrocracks originating from the edges away from the notch, Fig 5.11A. The damage profile shows a corresponding macrocracks with a distributed damage zone, Fig 5.11B. The macrocrack at the notch shows a curved path which is different from the experimental results wherein the crack is almost perpendicular to the specimen length across the cross section. The damage initiates at the notch and spreads in the direction of the principal stress orientation. The damage having spread at an angle to the notch will now spread in the curved path on the left of the notch. This is because the wave with magnitude larger than the concrete strength has already travelled ahead within the delay of damage initiation.

The response of the concrete under high loading rate is compared with the experimental results using the stress displacement plots, Fig 5.12. The stress is obtained using the average of the stresses in the completely damaged elements and the deformation of the fracture zone is considered. The numerical analysis shows a peak strength of 12.4 MPa compared to the input value equal to the experimental value of 11.59 MPa. This is due the weighted averaging leading to a delay of damage initiation. The difference is larger compared to the SHB as the loading rate is higher. This is similar to the behavior observed in the one dimensional dynamic tension test, section 4.3.

The tuning of the peak strain helps in obtaining the peak strength of the material as the increase cannot be obtained in the rate independent model used. The numerical result does not capture the pre peak nonlinearity as the material model is linear up to damage initiation. The post peak curve is also not captured. The rate independent material model cannot capture the effects of the various mechanisms introducing a delay in damage evolution. The dynamic loading induces a quick shrinking of the interaction domains and faster damage evolution. Hence, we observe quicker damage evolution and a relative brittle response from the numerical results which is a result of the combination the rate independent material model and the stress based nonlocal model.

5.3. Conclusions

The stress based nonlocal model produces mesh independent results but cannot capture the true dynamic behavior of concrete. Concrete failure in dynamic cases shows varying fracture process zone widths with different loading rates. The current model produces the same width irrespective of the loading rates as the length scale is primarily defined for quasi static conditions.

The rate independent constitutive law chosen cannot capture the rate effects of concrete, i.e. the increase in strength and the slow evolutions of damage. Hence, the results obtained show a brittle response compared to the experimental results. The isotropic damage model considers homogenous behavior at all the scales. This assumption is not valid as additional mechanisms at smaller scales are activated at high loading rates in concrete. These additional mechanisms are material responses and should be included in the material model.

The influence domain depends on the principal stresses and has a direct relation with the damage. As the isotropic damage models are used the damage evolutions are quick. This
results in interaction domains shrinking quickly. The quick shrinking of the interaction domains leads to relatively brittle response. This leads to a quick loss of strength, formation of the macrocrack and the failure of the specimen.

The brittle response of the numerical results compared to the experimental results is due to the combination of the rate independent material models and quick shrinking interaction domains but a clear separation cannot be made on the dominant effect.
6. CONCLUSIONS

Concrete having a complicated internal structure cannot be considered to behave locally when the mesh is discretized with elements smaller than the material length scale. With the use of the local constitutive model, a severe mesh sensitivity of results is observed. To obtain mesh independent results, the length scale is explicitly introduced using the nonlocal models. The nonlocal model is implemented and investigated and found to have certain drawbacks. To resolve the issues of the nonlocal model an improved stress based nonlocal model is implemented. A study of the features and limitations of the nonlocal models in quasi static and dynamic loading conditions is carried out. Finally, the effectiveness of the stress based nonlocal model to study the damage of concrete under dynamic loading is investigated.

The original nonlocal model is implemented to alleviate the influence of the mesh discretization on the results. The benchmark uniaxial tension test shows converging results for the various meshes used. The nonlocal model comes with certain limitations mainly due to the generic description of the interaction domains which lead to unrealistic fracture processes. Firstly, the incorrect location of damage initiation leads to spurious increase in strength. Secondly, the inability to form macrocracks leads to damage diffusion and additional fracture energy dissipated. Thirdly, the inaccurate description of interactions near boundaries leads to damage attraction towards the boundaries.

The original nonlocal model introduces a delay in damage initiation compared to the local damage model which leads to a strength increase of the material. This increase is almost constant in quasi-static cases but increases with increasing loading rates in the dynamic conditions. With increasing loading rates the width of the damage zone is not affected but initiation and evolution of the damage within this zone is faster.

The stress based nonlocal model presented by Giry et al.(2011) effectively solves the issues of the original nonlocal model by dynamically orienting and scaling the interaction domains proportional to the magnitude and orientation of the principal stresses. The physically relevant description of the interaction domains leads to better description of the fracture process. The damage initiates at the correct location giving better predictions of the strength. Macrocracks are formed representing fracture. The damage diffusion and damage attraction is also avoided by the new description of the interaction domains. While the model alleviates a significant portion of mesh dependency, the partial local nature of the model introduces a degree of mesh dependency, especially in fully fractured zones. This leads to the width of the macrocracks to be mesh dependent and some variation in the dissipated fracture energy.

An apparent strength increase is observed due to this delay resulting in additional strength is observed similar to the original nonlocal model in quasi static and dynamic loading conditions. The width of the damage zone is not affected due to the loading rate. For increasing loading rates the damage initiates earlier and evolves faster to form a macrocrack.
The stress based nonlocal model provides mesh independent results for the SHB test and provides a more realistic damage profile compared to the nonlocal model. The dynamic Young’s modulus and the peak strain are tuned to tensile strength of concrete obtained from the experiments as the isotropic damage model is rate independent and cannot capture the increase in strength. The delay of damage initiation shows some increase of strength beyond the input value which is higher for the MSHB compared to SHB as it has a higher rate of loading.

The stress based nonlocal model cannot capture the dynamic behavior of concrete. The constitutive law used being rate independent fails to capture the rate effects which shows a delay in damage evolutions. The length scale used defined for the quasi static conditions does not help in capturing the rate effects. The quick evolution of damage at these loading rates results in the shrinking of the interaction domains. The combination of these results in brittle responses compared to the experiment. To capture the rate effects the additional mechanisms contributing to the material response are to be involved in the constitutive law used.

**Recommendations**

The goal of implementing a regularization method to produce mesh independent results is fulfilled with small limitations. The following recommendations are given which could be addressed in future research.

- The minor mesh sensitivity observed can be eliminated by providing a smoother transition of the stress based nonlocal model to a local model. This can be achieved using a very fine mesh to rectify this issue, which may be computationally heavy or by using remeshing techniques in the vicinity of the expected macrocrack. A lower bound of element sizes should be defined to avoid using infinitesimally small elements to obtain the smooth transition.
- The distribution of damage being dependent on the stress state may help in capturing crack branching and formation of curved cracks which need to be investigated.
- The current definition of the evolving interaction domain depends on the tensile strength of the material. The effect of using the tensile strength in mode 2 and mixed mode failure need investigation.
- A rate dependent material model which captures the rate effects and implicitly introduces a length scale. To avoid relatively ductile behavior and spurious fracture process zones due to over estimation of the length scale when using the rate dependent material model, investigation of the length scales implicitly introduced to obtain a correct combination of length scales is necessary (Desmorat et al (2010)).
- The length scale distributes damage but shows constant fracture process zone widths. To introduce a varying fracture process width in dynamic cases an additional length scale should be introduced to capture the inertial effects at the microscale level as suggested by Askes et al (2007). To capture the moisture effects a viscoelastic material model can be used along with the additional enrichment, Combe (2013).
REFERENCES


REFERENCES


Belaoura M., Brara A. (2003), Tensile behaviour of concrete at high strain rates. ‘International Conference of Earthquake Engineering to Mark 40 Years from Catastrophic 1963 Skopje Earthquake and Successful City Reconstruction (SE 40 EEE) ’, 26-29 August 2003, Skopje-Ohrid, Macedonian.


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Appendix A – Dynamic Analysis

The Dynamic analysis carried out in LS Dyna follows the explicit integration scheme. The equations of dynamic equilibrium are solved by direct time integration schemes, which are classified as implicit and explicit time integration schemes.

Implicit time integration methods rely upon the information at time step \((n + 1)\) and the present time step \(n\). The displacement \(u(t + \Delta t)\) is calculated by considering the equilibrium equation at time \((t + \Delta t)\). These methods require the solution of a set of equations at each time step which requires the inversion of the effective stiffness matrix of the form

Explicit integration methods rely only on the information at time step \(n\) and earlier steps. Generally the displacement \(u(t + \Delta t)\) is determined by considering equilibrium at time \(t\).

LS Dyna primarily works with an explicit integration scheme, a modified central difference method. The modified central difference method used in LS DYNA is presented.

**Modified version used by LS Dyna**

Velocity is given by:

\[
\frac{\dot{u}_{n+\frac{1}{2}}}{\Delta t} = \frac{1}{\Delta t} \left( u_{n+1} - u_n \right)
\]

Equation of motion for nonlinear cases:

\[M \ddot{u}_n = P_n - F_{n}^{int} - C \dot{u}_n\]

Giving the acceleration update:

\[\ddot{u}_n = M^{-1} \left( P_n - F_{n}^{int} - C \dot{u}_n \right)\]

The velocity update:

\[\dot{u}_{n+\frac{1}{2}} = \dot{u}_{n-\frac{1}{2}} + (\Delta t_n) \ddot{u}_n\]

The displacement update:

\[u_{n+1} = u_n + (\Delta t_{n+\frac{1}{2}}) \dot{u}_{n+\frac{1}{2}}\]

To start the procedure for the first time step

\[u_0 = \bar{u}_0 + u_{stat}\]

\[\bar{u}_0 = \frac{1}{2} \left( \dot{u}_{-\frac{1}{2}} + \dot{u}_{\frac{1}{2}} \right)\]

\[\dot{u}_0 = M^{-1} \left( P_0 + P_{stat} - F_0^{int} (u_{stat}) - C \dot{u}_0 \right)\]
\[ \dot{u}_{\frac{1}{2}} = \dot{u}_0 + \frac{1}{2}(\Delta t_0)\ddot{u}_0 \]

\[ P_{stat} - F_{int}^{u_{stat}}(u_{stat}) = 0 \]

The critical time step does not have a predefined formula but is computed by LS DYNA. It is calculated as the time taken for a wave to travel through the smallest element in finite element model. Control over this time step size is possible, as by default it is 0.9 times the actual stable size calculated.
Appendix B - Material models:

For the proposed work a simple softening material model with linear or exponential softening is necessary. Since LS-DYNA does not provide a simple softening model to which an implemented nonlocal model can be added, a user implemented material is required. Hence, the user materials umat49 (Plane stress) and umat50 (Plane strain) material models inclusive of the linear and exponential softening laws. The equivalent strain evaluation is carried out using the Mazar’s and the modified von mises criterion which have also been implemented within the material models.

The Plane stress model is an approximation which helps in analyzing a 3-dimensional case using 2-dimensions. This can be done as it is assumed that the third principal stress is zero. A thin plate subjected to in-plane loading, the state of stress and deformation within the plate is called plane stress. In this case only two dimensions (in the plane of plate) are required for the analysis. The constitutive law for the user material model umat_49 reads

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix} = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 0.5(1 - \nu) \end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{xy}
\end{bmatrix}
\]

The Plane strain model is another approximation which helps in analyzing a 3-dimensional case using 2-dimensions. This can be done by assuming that the third principal strain is zero. If a long body is subjected to transverse loading and its cross section and loading do not vary significantly in the longitudinal direction, a small thickness in the loaded area can be treated as subjected to plane strain. The constitutive law for the user material model umat_50 reads

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{xy}
\end{bmatrix} = \frac{E}{1 + \nu(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & \nu & 0 \\ \nu & 1 - \nu & \nu & 0 \\ \nu & \nu & 1 - \nu & 0 \\ 0 & 0 & 0 & 0.5 - \nu \end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{xy}
\end{bmatrix}
\]

The material models in LS DYNA receives strain increments at every cycle of the analysis. These strains are accumulated to obtain the total strain within each element at every cycle which are then used to evaluate the local equivalent strains. The local strains are stored for the evaluation of the nonlocal strains.

The damage is then evaluated using the nonlocal strains obtained by the weighted average of the local strains obtained from the previous cycle of the analysis. In principle the nonlocal strains need to be evaluated using the local strains of the same stress state, i.e. the same cycle of the analysis. Due to limitations of modifications that can be made to the user subroutines it is not possible (appendix C).

Using the total strains and the damage of the element the stresses are evaluated which are used to obtain the new strain increments for the next cycle of the analysis. With regard to the stress based nonlocal model a further calculation is necessary to obtain the parameters which define the influence domain of each element. These parameters are the ratios of the principal
stresses to the tensile strength of the material and the principal stress orientations. They are stored along with the total strains, the local and nonlocal equivalent strains and the damage.

The keycard for the implemented material model reads with default values

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<th>RO</th>
<th>MT</th>
<th>LMC</th>
<th>NHV</th>
<th>IORTHO</th>
<th>IBULK</th>
<th>IG</th>
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<td>-</td>
<td>49 or 50</td>
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<th>ITERM</th>
<th>IHYPER</th>
<th>IEOS</th>
<th>LMCA</th>
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<th>-</th>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
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<th>P10</th>
<th>P11</th>
<th>P12</th>
<th>P13</th>
<th>P14</th>
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</tbody>
</table>

MID | Material identification  
RO | Mass density  
MT | User material type, 49 for plane stress and 50 for plain strain  
LMC | Number of material constants to be input  
NHV | Number of history variables to be stored  
IORTHO |  
ibulk | Address of bulk modulus  
ig | Address of shear modulus  
ivect | Vector flag turned off = 0  
ifail | Failure flag turned off = 0  
itherm | Temperature flag turned off = 0  
ihyper | Deformation gradient turned off = 0  
ieos | Equation of state flag turned off = 0  
lmca | Length of additional material constant array  
P1 | Modulus of elasticity  
P2 | Poisson’s ratio  
P3 | Bulk modulus  
P4 | Shear modulus  
P5 | Peak strain  
P6 | Ultimate strain (used only for linear softening)  
P7 | Softening flag (1 = linear, 2 = exponential)
P8 | Nonlocal flag (1 = off, 2 = on)
P9 | Internal length scale
P10 | Equivalent strain flag (1 = mazars, 2 = modified von mises)
P11 | $\alpha$ of exponential softening
P12 | $\beta$ of exponential softening
P13 | Smallest element size
P14 | Ratio of compressive to tensile strength

Remarks:
1. The model has been implemented for quad elements with 1 point integration.
2. Convergence issues of the solution can be solved by reducing the time step size.

The user material models umat49 and umat50 have been implemented in the dyn21.f FORTRAN file provided by LS Dyna. This file contains the part of the LS Dyna code which can be manipulated to implement the user material models and the solution process. The material model is implemented in the “subroutines umat49” and “subroutine umat50.”
Appendix C – Nonlocal models (Original nonlocal model and Stress based nonlocal model)

The nonlocal models are implemented by modifying the subroutines which call the material models, “subroutine urmathn” and the subroutine which allows solution control, “subroutine uctrl1.” The original nonlocal model requires evaluation of the interaction domain once as it does not evolve. The stress based nonlocal model evolves during the analysis and constant evaluation is necessary.

The user implemented nonlocal models can be broken down into 3 different processes. The first process being the evaluation of the interaction domains at the first cycle which serves different purposes in the original nonlocal and stress based nonlocal model. The second is the accumulation of the strains from the various material models. And finally the evaluation of the nonlocal equivalent strains using the weighted average over the interaction domain.

The first one is where the interaction domains, circles with the length scale being the radius is evaluated. This process is done in the first cycle of analysis for every element and the surrounding elements within the interaction domain are stored. For the original nonlocal model the interaction domain remains constant throughout the analysis and hence the weights are also calculated for the weighted averaging carried out later.

The stress based nonlocal model uses evolving interaction domains which are evaluated at every cycle of the analysis. Since these evolving interaction domains have a maximum size equal to the circular interaction domains of the original nonlocal model, the interaction domain data is stored termed maximum interaction domain. For the subsequent interaction domain evaluations, only the elements within the maximum interaction domain which influence the target element are chosen.

The material model addresses the evaluation of the local equivalent strains. To calculate the damage at a certain stress state, cycle of analysis the equivalent strains at the same cycle are required. Hence, the nonlocal equivalent strains need to be evaluated from the local equivalent strains of the same cycle and used to calculate damage. This requires an additional intermediate step within the subroutine processing the element data. All the elements need to be processed up to the evaluation of the local equivalent strains. At this point the intermediate step should evaluate the nonlocal equivalent strains from the local equivalent strains. With these nonlocal strains the damage parameter of each element needs to evaluated after which the stresses are evaluated.

LS-DYNA has a certain limit on the manipulation of the subroutines which alters the flow of the program. This limitation does not allow the introduction of an intermediate cycle to evaluate the nonlocal equivalent strains. Hence, the nonlocal strains can only be obtained by the weighted average of the local equivalent strains of the previous cycle. If the change in the stress state of the elements is large between two consecutive cycles it will lead to spurious results. Hence, it is important to introduce a time step size which does not result in a big difference in the stress states between consecutive cycles.
The time step size chosen by LS-DYNA are based on the time taken for a wave to travel through the smallest element. The loading rate is very low in quasi static loading cases, leading to small variations in the stress states between consecutive cycles. Using time steps equal to 0.6 times the critical time step size should be sufficient. For Dynamic loading cases the loading rate being high will result in a considerable change in the stress state between consecutive analysis cycles when critical time step size is considered. Care needs to be taken that the variation between two cycles is quite small. This can be done by reducing the time step size. After trial and error it was found that time step sizes smaller than 0.1 times the critical time step size give minimal variations and hence suitable for the analysis.

The third process is the evaluation of the nonlocal equivalent strains by weighted average of the local equivalent strains within the interaction domain. The original nonlocal model considers a constant influence domain of the source points. This leads to constant interaction domains of the target points. Hence, the weights calculated with respect to the target point and the source points will be the same, due to the isotropic weight function used. Therefore along with the maximum interaction zone evaluated at the first cycle the respective weights for the source points within the interaction domain are obtained and stored which will be used for every nonlocal equivalent strain evaluation.

The stress based nonlocal model requires evaluation of the influence domains at every cycle of the analysis for each element based on the parameters obtained from the principal stresses of the respective element. These influence domains have an upper limit on the size which are equal to the maximum interaction domains evaluated at the first cycle of analysis. This data of points within the maximum interaction domain is used to evaluate the points which lie within the current influence domain. The weights of influence distributed are also evaluated and stored using the equation (4.3). Each target point will receive distributions from itself and surrounding points. A search of all the source point distributing influence to the target point in focus, the weight of influence and the source point’s local equivalent strain are used to form the new interaction domains of each target point. The nonlocal equivalent strains are evaluated from this data.

From the described process the same problem will be analyzed faster using the original nonlocal model compared to the stress based nonlocal model. The compact tension test was analyzed up to a loading point using the local damage model, the original and the stress based nonlocal model. The results of the computational time are reported in table C.1.

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAT_003 +MAT_NONLOCAL</td>
<td>521</td>
</tr>
<tr>
<td>umat49/50 + nonlocal</td>
<td>62</td>
</tr>
<tr>
<td>Umat49/50 +Stress based nonlocal</td>
<td>259</td>
</tr>
</tbody>
</table>

Table C.1: Computational time using different methods to analyze compact tension test

The implemented nonlocal model is faster than the stress based nonlocal model. This is because the interaction domains and respective weights are evaluated at the start and used
repeatedly. The stress based nonlocal model needs to evaluate the interaction domains at every cycle of the analysis leading to a longer analysis time. But the implemented stress based nonlocal model is faster than the inbuilt nonlocal model of LS-DYNA.