REFLECTION AND TRANSMISSION OF ACOUSTIC WAVES BY THE PERIODIC INTERFACE BETWEEN A SOLID AND A FLUID

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The redistribution of acoustic energy incident on a spatially periodic interface between a solid and a fluid is investigated theoretically. The main tools in the analysis are the elastodynamic Green-type integral relations. One of these relations is a vectorial integral equation from which the elastodynamic field quantities can be determined. The solid-medium part of it is identical to the case of the interface between two solids investigated in a previous paper, the fluid-medium part, however, needs reformulation. Numerical results are presented for the sinusoidal interface between granite and sea water. The computations have been carried out for four different heights of the periodic profile (the plane interface included), a single frequency of operation and the three types of excitation.

A peaked behaviour of the reflection and transmission factor occurs at angles of incidence where an elastodynamic spectral mode changes from propagating to evanescent and vice versa. An additional anomaly occurs in cases where the horizontal wave number of one of the spectral orders coincides with the horizontal wave number of a Scholte wave along the corresponding plane interface. The latter phenomenon is the more pronounced, the shallower the corrugation of the interface is.

1. Introduction

In this paper we investigate the reflection and transmission of acoustic waves by the spatially periodic interface between a solid and a fluid. The fluid is taken to be ideal, which leaves only its compressibility to be taken into account. Although in essence the theory developed in [1], where the corresponding problem for the spatially periodic boundary between two solids is investigated, remains valid, the fluid part of the problem needs reformulation. A new set of Green-type integral relations, adapted to the present problem, is derived. In these integral relations, the Green state in the solid is identical to the one considered in [1] but in the fluid another Green state is introduced.

Numerical results are presented for an application of seismological interest, viz. the sinusoidal interface between granite and sea water. The computations have been carried out for four different heights of the periodic profile (the plane interface included) and a single frequency of operation. Three cases of incidence are investigated, viz. P-wave incidence in granite, SV-wave incidence in granite and P-wave incidence in sea water.

2. Green type integral relations pertaining to the solid/fluid interface

In [1], we have investigated the properties of a spatially periodic interface separating two different *solids*. In this paper, we will analyse the reflection and transmission properties of the interface between a solid (medium I) and a fluid (medium II). Then a reformulation of our problem is necessary. We take the fluid to be perfect in the sense that internal friction is neglected, which leaves only its compressibility to be taken into account (see [2, p. 78]).

As a consequence, the shear part of the stress tensor vanishes and the stress tensor reduces to the diagonal form

$$\tau_{\alpha\beta}^{\text{II}} = -p\delta_{\alpha\beta}, \quad (2.1)$$

where p denotes the scalar pressure. Further, the constitutive relation pertaining to the fluid is written as

$$p = -\lambda^{\text{II}} \partial_\beta u_\beta^{\text{II}}. \quad (2.2)$$

In fluid dynamics, it is usual to write the constitutive relation as $p = -K \partial_\beta u_\beta^{\text{II}}$, where K denotes the modulus of compression.

Substituting (2.1) in the equation of motion (2.6) of [1], we obtain

$$-\partial_\alpha p + \rho^{\text{II}} \omega^2 u_\alpha^{\text{II}} = 0. \quad (2.3)$$

When we perform the operator ∂_α on (2.3) and use (2.2) we arrive at

$$\partial_\alpha \partial_\alpha p + (\rho^{\text{II}} \omega^2 / \lambda^{\text{II}}) p = 0, \quad (2.4)$$

which is nothing but the homogeneous two-dimensional, scalar Helmholtz equation. Eq. (2.4) admits plane wave solutions that are of the P-type (compressional waves). Following the same systematic approach as in [1], we introduce plane P-waves of general spectral order n in the fluid; their corresponding quantities are listed in Table 1.

As in [1], we take the incident field to be an arbitrary superposition of waves of all spectral orders, in the fluid as well as in the solid. Again, the scattered field can, in the uniform regions within a single cell, be expanded in terms of outgoing spectral orders. In the solid (medium I), the relevant expansions are given by (2.15) and (2.20) of [1].

In the fluid (medium II), we characterize the incident field by the particle displacement $u_\alpha^{\text{inc,II}}$ and the pressure p^{inc} . Using the normalized quantities of Table 1 as constituents, the modal structure of $u_\alpha^{\text{inc,II}}$ and p^{inc} is written as

$$u_\alpha^{\text{inc,II}} = \sum_{n=-\infty}^{\infty} \text{AP}^{\text{II}}(k_n) \text{UP}_\alpha^{+, \text{II}}(k_n), \quad (2.5)$$

Table 1

Quantities associated with the normalized plane P-wave of spectral order n ($n = 0, \pm 1, \pm 2, \dots$) pertaining to a fluid

Quantity	P-wave (fluid)
wave number	$kP = \omega/cP$
wave speed	$cP = [\lambda/\rho]^{1/2}$
wave vector	$\mathbf{kP}^\pm(k_n) = kP_1(k_n)\mathbf{i}_1 \pm kP_2(k_n)\mathbf{i}_2$
horizontal wave number	$kP_1(k_n) = k_n = k^{\text{inc}} + 2\pi n/\mathcal{D}$
vertical wave number	$kP_2(k_n) = (kP^2 - k_n^2)^{1/2}$ with $\text{Re}(kP_2) \geq 0$ and $\text{Im}(kP_2) \geq 0$
vertical wave impedance	$ZP(k_n) = \rho cP kP_2(k_n)/kP$
normalized vertical intensity*	$IP(k_n) = \frac{1}{2}\omega^2 ZP(k_n)$
normalized particle displacement	$\text{UP}_\alpha^\pm(k_n) = [kP_\alpha^\pm(k_n)/kP] \exp(i\mathbf{kP}^\pm(k_n) \cdot \mathbf{x})$
normalized pressure	$P^\pm(k_n) = -i\lambda kP \exp(i\mathbf{kP}^\pm(k_n) \cdot \mathbf{x})$

* For propagating spectral orders only

and

$$p^{inc} = \sum_{n=-\infty}^{\infty} AP^{\text{II}}(k_n)P^{+, \text{II}}(k_n). \quad (2.6)$$

The particle displacement and the pressure of the scattered field in the fluid are introduced as

$$u_{\alpha}^{sc, \text{II}} = u_{\alpha}^{\text{II}} - u_{\alpha}^{inc, \text{II}}, \quad (2.7)$$

and

$$p^{sc} = p - p^{inc}. \quad (2.8)$$

In the domain $-\infty < x_2 < x_{2, \min}$, the particle displacement $u_{\alpha}^{sc, \text{II}}$ and the pressure p^{sc} are represented by

$$u_{\alpha}^{sc, \text{II}} = \sum_{n=-\infty}^{\infty} BP^{\text{II}}(k_n)UP_{\alpha}^{-, \text{II}}(k_n), \quad (2.9)$$

and

$$p^{sc} = \sum_{n=-\infty}^{\infty} BP^{\text{II}}(k_n)P^{-, \text{II}}(k_n), \quad \text{when } -\infty < x_2 < x_{2, \min}. \quad (2.10)$$

The boundary conditions at the solid/fluid interface require the normal component of the particle displacement and the normal component of the traction to be continuous across the interface, while the tangential components of the traction vanish on the interface. Hence

$$u_{\beta}^{\text{I}} n_{\beta} = u_{\beta}^{\text{II}} n_{\beta} = n_{\beta} \partial_{\beta} p / (\lambda^{\text{II}} k P^{\text{II}^2}), \quad (2.11)$$

and

$$\tau_{\alpha\beta}^{\text{I}} n_{\beta} = \tau_{\alpha\beta}^{\text{II}} n_{\beta} = -p n_{\alpha}, \quad \text{on } \Lambda, \quad (2.12)$$

where we have used the relations (2.1) and (2.3).

To derive Green-type integral relations in the fluid, we apply the two-dimensional form of the Green theorem to the domain D_2 (Fig. 1) and the field quantity p^{sc} , followed by an application to the domain D_1

Fig. 1. Domains of interest pertaining to the integral relations in case of the solid/fluid interface.

and the field quantity p^{inc} (cf. [3]). By using (2.8), the procedure leads to

$$-\int_L [G(x'; x) n_\beta(x) \partial_\beta p(x) - p(x) n_\beta(x) \partial_\beta G(x'; x)] ds(x) + p^{\text{inc}}(x') = \{1, \frac{1}{2}, 0\} p(x'),$$

when $x' \in \{D_2, L, D_1\}$, (2.13)

where the scalar Green function G is subject to the same conditions as regards quasi-periodicity as the elastodynamic Green tensors in the solid and is given by

$$G(x'; x) = \frac{1}{2i(\lambda^{\text{II}} k P^{\text{II}})^2 \mathcal{D}} \sum_{n=-\infty}^{\infty} \frac{P^{+, \text{II}}(x', k_n) P^{-, \text{II}}(x, -k_n)}{k P_2^{\text{II}}(k_n)}, \quad \text{when } -\infty < x_2 < x'_2, \quad (2.14)$$

and

$$G(x'; x) = \frac{1}{2i(\lambda^{\text{II}} k P^{\text{II}})^2 \mathcal{D}} \sum_{n=-\infty}^{\infty} \frac{P^{-, \text{II}}(x', k_n) P^{+, \text{II}}(x, -k_n)}{k P_2^{\text{II}}(k_n)}, \quad \text{when } x'_2 < x_2 < \infty. \quad (2.15)$$

The function G satisfies, in a unit cell, the inhomogeneous Helmholtz equation

$$\partial_\alpha \partial_\alpha G(x'; x) + k P^{\text{II}2} G(x'; x) = -\delta(x - x'). \quad (2.16)$$

In medium I (solid) the following integral relation holds (cf. eq. (6.5) of [1]):

$$\int_L [u_{\gamma\alpha}^{\text{G}, \text{I}}(x'; x) \tau_{\alpha\beta}^{\text{I}}(x) - u_\alpha^{\text{I}}(x) \tau_{\gamma\alpha\beta}^{\text{G}, \text{I}}(x'; x)] n_\beta(x) ds(x) + u_\gamma^{\text{inc}, \text{I}}(x') = \{0, \frac{1}{2}, 1\} u_\gamma^{\text{I}}(x'),$$

when $x' \in \{D_2, L, D_1\}$. (2.17)

Using the boundary conditions (2.11) and (2.12), (2.17) and (2.13) can be rewritten as

$$-\int_L [u_{\gamma\alpha}^{\text{G}, \text{I}}(x'; x) n_\alpha(x) p(x) + u_\alpha^{\text{I}}(x) \tau_{\gamma\alpha\beta}^{\text{G}, \text{I}}(x'; x) n_\beta(x)] ds(x) + u_\gamma^{\text{inc}, \text{I}}(x') = \{0, \frac{1}{2}, 1\} u_\gamma^{\text{I}}(x'),$$

when $x' \in \{D_2, L, D_1\}$, (2.18)

and

$$-\int_L [G(x'; x) (\lambda^{\text{II}} k P^{\text{II}2}) u_\beta^{\text{I}}(x) n_\beta(x) - p(x) n_\beta(x) \partial_\beta G(x'; x)] ds(x) + p^{\text{inc}}(x') = \{1, \frac{1}{2}, 0\} p(x'),$$

when $x' \in \{D_2, L, D_1\}$. (2.19)

In view of the arbitrariness of the extent of D_1 and D_2 within a unit cell in D^{I} and D^{II} , respectively, (2.18) and (2.19) hold in an entire unit cell. When x' is chosen below L , (2.18) yields an integral representation of the scattered field in the solid. In particular, by choosing $x'_2 > x_{2, \text{max}}$ and substituting the relevant Green state in the solid, we arrive at

$$u_\gamma^{\text{sc}, \text{I}} = \sum_{n=-\infty}^{\infty} \left[\frac{i}{2\omega \mathcal{D} Z P^{\text{I}}(k_n)} \int_L [U P_\alpha^{-, \text{I}}(-k_n) n_\alpha p + u_\alpha^{\text{I}} T P_{\alpha\beta}^{-, \text{I}}(-k_n) n_\beta] ds \right] U P_\gamma^{+, \text{I}}(k_n) \\ + \sum_{n=-\infty}^{\infty} \left[\frac{i}{2\omega \mathcal{D} Z S^{\text{I}}(k_n)} \int_L [U S_\alpha^{-, \text{I}}(-k_n) n_\alpha p + u_\alpha^{\text{I}} T S_{\alpha\beta}^{-, \text{I}}(-k_n) n_\beta] ds \right] U S_\gamma^{+, \text{I}}(k_n),$$

when $x_{2, \text{max}} < x'_2 < \infty$. (2.20)

In a similar way, by choosing $x'_2 < x_{2,\min}$ we obtain from (2.19), using the expansion (2.15),

$$p^{sc} = \sum_{n=-\infty}^{\infty} \left[\frac{i}{2(\lambda^{II} k P^{II})^2 k P_2^{II}(k_n) \mathcal{D}} \int_L [\lambda^{II} k P^{II2} u_{\beta}^I n_{\beta} - i p n_{\beta} k P_{\beta}^{+,II}(-k_n)] P^{+,II}(-k_n) ds \right] P^{-,II}(k_n),$$

when $-\infty < x'_2 < x_{2,\min}$. (2.21)

Comparing (2.20) with the relevant modal expansion in the solid and (2.21) with the modal expansion (2.10), it follows that in the case of the solid/fluid interface, we have the following representations for the complex amplitudes of the scattered field:

$$\begin{aligned} BP^I(k_n) &= \frac{i}{2\omega \mathcal{D} Z P^I(k_n)} \int_L [U P_{\alpha}^{-,I}(-k_n) n_{\alpha} p + u_{\alpha}^I T P_{\alpha\beta}^{-,I}(-k_n) n_{\beta}] ds, \\ BS^I(k_n) &= \frac{i}{2\omega \mathcal{D} Z S^I(k_n)} \int_L [U S_{\alpha}^{-,I}(-k_n) n_{\alpha} p + u_{\alpha}^I T S_{\alpha\beta}^{-,I}(-k_n) n_{\beta}] ds, \end{aligned} \quad (2.22)$$

and

$$BP^{II}(k_n) = \frac{i}{2(\lambda^{II} k P^{II})^2 k P_2^{II}(k_n) \mathcal{D}} \int_L [\lambda^{II} k P^{II2} u_{\beta}^I n_{\beta} - i p n_{\beta} k P_{\beta}^{+,II}(-k_n)] P^{+,II}(-k_n) ds. \quad (2.23)$$

From these representations it follows that $BP^I(k_n)$, $BS^I(k_n)$ and $BP^{II}(k_n)$ can be calculated as soon as the components of the particle displacement u_{α}^I and the pressure p on L are known. The latter quantities follow from (2.18) and (2.19) by taking $x' \in L$. As a result we have,

$$\begin{aligned} \frac{1}{2} u_{\gamma}^I(x') + \oint_L [u_{\gamma\alpha}^{G,I}(x'; x) n_{\alpha}(x) p(x) + u_{\alpha}^I(x) \tau_{\gamma\alpha\beta}^{G,I}(x'; x) n_{\beta}(x)] ds(x) &= u_{\gamma}^{inc,I}(x'), \\ \frac{1}{2} p(x') + \oint_L [G(x'; x) (\lambda^{II} k P^{II2}) u_{\beta}^I(x) n_{\beta}(x) - p(x) n_{\beta}(x) \partial_{\beta} G(x'; x)] ds(x) &= p^{inc}(x'), \quad \text{when } x' \in L, \end{aligned} \quad (2.24)$$

where $\oint$ denotes the Cauchy principal value of the relevant integral. Eq. (2.24) represents a system of three coupled integral equations with the scalar pressure and the two components of the particle displacement on L (limiting values in the solid) as unknowns. For the numerical solution of this system and the numerical evaluation of the representations (2.22) and (2.23), we refer to the numerical techniques discussed in [1]. The results for a sinusoidal solid/fluid interface will be discussed in the next section.

3. Numerical results pertaining to the sinusoidal granite/sea water interface

Our computer program for determining the reflection and transmission properties of spatially periodic solid/fluid interfaces, is quite general. As an example, we have considered in more detail the sinusoidal interface between granite and sea water. The results for the plane interface ($h/\mathcal{D} = 0$) for this particular geological application were discussed by Ergin [4] and by Ewing, Jardetzky and Press [5, pp. 81–83]. As medium parameters, we have taken those by Ergin [4]. It is noticed, that the parameters pertaining to granite here differ from those used in [6], where the granite/slate interface is analysed and where the parameters have been borrowed from a different source. This is conceivable, since the parameters obviously apply to a totally different geological site. The values of the parameters are shown in Table 2.

Fig. 2. Sinusoidal interface between granite and sea water, incident SV-wave in granite and indication of some reflected and transmitted P- and SV-waves of spectral order n (the waves for $n = -1$ are shown).

Table 2

Medium parameters pertaining to granite and sea water

Material	Poisson ratio ν	P-wave velocity c_P (m/s)	SV-wave velocity c_S (m/s)	Mass density ρ (kg/m ³)
granite	0.24	5550	3268	2.9×10^3
sea water	0.5	1500	—	1.03×10^3

Along the plane interface between a solid and a fluid, a surface wave is always possible (cf. [7; 8; 5, pp. 105–107; 9, Chapter 14]). Following the suggestion of Cagniard [9, p. 245], we shall call this type of surface wave a Scholte wave, since Scholte was the first to recognize its general existence. With the aid of the formula given by Ewing, Jardetzky and Press [5, p. 106, eq. (3.111)], we have calculated the, non-dispersive, Scholte-wave velocity for the granite/sea water interface to be $c^{\text{SCH}} = 1496$ m/s.

Since there exist no SV-waves in medium II, those elements in the power scattering matrix introduced in [6], that refer to the propagating SV-waves in medium II are to be deleted. In a corresponding manner, the reciprocity relations and the power relations are to be modified.

Following the same strategy as in [6] we have chosen the frequency of operation such that the value 9 is assigned to the normalized wave number of the smallest wave velocity ($c_P^{\text{II}} = 1500$ m/s). The other normalized wave numbers then follow from the velocity ratios. The relevant normalized wave numbers and their corresponding normalized wave lengths are listed in Table 3.

For the normalized Scholte-wave number and corresponding normalized wave length, we have obtained the values $k^{\text{SCH}} \mathcal{D} = 9.02$ and $\lambda^{\text{SCH}} / \mathcal{D} = 0.697$, respectively.

Table 3

Normalized wave numbers and normalized wave lengths pertaining to the granite/sea water interface, as used in the computer program

Material	$kP\mathcal{D}$	$kS\mathcal{D}$	$\lambda P/\mathcal{D}$	$\lambda S/\mathcal{D}$
granite (medium I)	2.43	4.13	2.59	1.52
sea water (medium II)	9	—	0.70	—

The computations have been performed for the values $h/\mathcal{D} = 0, 0.1, 0.3$ and 0.5 . The results for $h/\mathcal{D} = 0.1, 0.3$ and 0.5 were obtained with the aid of our computer program, $kP^I\mathcal{D}$, $kS^I\mathcal{D}$, $kP^{II}\mathcal{D}$ and ρ^{II}/ρ^I being the input parameters. The results for $h/\mathcal{D} = 0$ were obtained by directly applying the boundary conditions pertaining to the plane interface, to the spectral combinations of order zero, cP^{II}/cP^I , cP^I/cS^I and ρ^{II}/ρ^I serving as input parameters.

The results pertaining to the granite/sea water interface are presented in the following figures:

Figs. 3 and 4 for an incident P-wave in granite,

Figs. 5 and 6 for an incident SV-wave in granite,

Figs. 7 and 8 for an incident P-wave in sea water.

They show, for $h/\mathcal{D} = 0, 0.1, 0.3$ and 0.5 , the values of the elements of $[II]_0$ as a function of the angle of incidence (varying between 0° and 90°). A few prominent properties will now be referred to.

From Figs. 3 and 4, that present the results for an incident P-wave in granite, we observe that the redistribution of incident energy is such that the reflected SV-wave of spectral order zero collects the main part of the incident energy in the range 30° – 85° of the angle of incidence. However, at greater depths of the profile this becomes less pronounced. Horizontal phase matching would occur in the following ranges.

For an incident P-wave

$$\begin{aligned}
 m > 0: \quad k_m\mathcal{D} &= k^{\text{SCH}}\mathcal{D} \quad \text{if } c^{\text{SCH}}/cP \leq \mathcal{D}/(m\lambda P) \leq c^{\text{SCH}}/(cP - c^{\text{SCH}}), \\
 m < 0: \quad k_m\mathcal{D} &= -k^{\text{SCH}}\mathcal{D} \quad \text{if } c^{\text{SCH}}/(cP + c^{\text{SCH}}) \leq \mathcal{D}/(|m|\lambda P) \leq c^{\text{SCH}}/cP,
 \end{aligned}
 \tag{3.1}$$

Fig. 3. Range of propagation of the different spectral orders for the reflected and transmitted waves and their corresponding angles of emergence as a function of θP_0^I for a P-wave incident in granite on a granite/sea water interface.

Fig. 4. Acoustic reflectances and transmittances, in the vertical direction, of the zero spectral orders as a function of θP_0^I for a sinusoidal interface between granite (medium I) and sea water (medium II) for a P-wave incident in granite.

For an incident SV-wave

$$\begin{aligned} m > 0: \quad k_m \mathcal{D} &= k^{\text{SCH}} \mathcal{D} \quad \text{if } c^{\text{SCH}}/cS \leq \mathcal{D}/(m\lambda S) \leq c^{\text{SCH}}/cS - c^{\text{SCH}}, \\ m < 0: \quad k_m \mathcal{D} &= -k^{\text{SCH}} \mathcal{D} \quad \text{if } c^{\text{SCH}}/(cS + c^{\text{SCH}}) \leq \mathcal{D}/(|m|\lambda S) \leq c^{\text{SCH}}/cS. \end{aligned} \quad (3.2)$$

However, for the P-wave type of incidence and the chosen frequency of operation, none of the inequalities is ever satisfied. This implies that there exists no positive real angle of incidence for which horizontal phase matching occurs.

When the reflected SV-wave of spectral order -1 becomes propagating (which occurs at the angle of incidence $\theta P_0^I = 62.4^\circ$), we have noticeable variations in the curves representing $\Pi S P_0^{I,I}$ for $h/D = 0.1, 0.3$ and in the curve representing $\Pi P P_0^{I,I}$ for $h/D = 0.5$.

Fig. 5. Range of propagation of the different spectral orders for the reflected and transmitted waves and their corresponding angles of emergence as a function of θS_0^I for an SV-wave incident in granite on a granite/sea water interface.

From Figs. 5 and 6, that present the results for an incident SV-wave in granite, we notice that for the plane interface ($h/\mathcal{D} = 0$) exactly at the critical angle of incidence $\theta S_0^I = 36^\circ$, when the reflected P-wave of spectral order zero becomes evanescent, all incident energy is reflected in the SV-wave. This phenomenon disappears with increasing depths of the profile.

Checking the inequalities (3.2) for an incident SV-wave, we conclude that horizontal phase matching occurs for $m = 1$ ($k_1\mathcal{D} = k^{SCH}\mathcal{D}$) at $\theta S_0^I = 41.5^\circ$ and for $m = -2$ ($k_{-2}\mathcal{D} = -k^{SCH}\mathcal{D}$) at $\theta S_0^I = 59.1^\circ$. For these values of the angle of incidence, we observe rapid variations in the curves representing $\Pi SS_0^{I,I}$ and $\Pi PS_0^{II,I}$ for $h/\mathcal{D} = 0.1$ and this phenomenon is most pronounced at the angle of incidence $\theta S_0^I = 41.5^\circ$, where $k_1\mathcal{D} = k^{SCH}\mathcal{D}$. For increasing depth, this phenomenon disappears. The phenomenon is also absent in the curves representing ΠPS_0^I . This follows from the obvious fact that we have no propagating P-waves of spectral order zero in the range of angles of incidence for which horizontal phase matching can occur. Also, no horizontal phase matching occurs for the reflected P-wave of order -1 in its domain of propagation.

From Figs. 7 and 8, that present the results for an incident P-wave in sea water, we again observe that for the plane interface ($h/\mathcal{D} = 0$) exactly at critical incidence $\theta P_0^{II} = 15.7^\circ$, total reflection of the incident P-wave occurs. Further, for the angles of incidence $\theta P_0^{II} = 17.7^\circ$ and $\theta P_0^{II} = 23.2^\circ$ we have the horizontal phase matchings $k_1\mathcal{D} = k^{SCH}\mathcal{D}$ and $k_{-2}\mathcal{D} = -k^{SCH}\mathcal{D}$, respectively, and again rapid variations are observed in the curves representing $\Pi PP_0^{II,II}$ and $\Pi SP_0^{I,II}$ for $h/\mathcal{D} = 0.1$.

In the case of a plane interface, the incident P-wave is totally reflected in the range 27.3° – 90° of the angle of incidence. However, for greater depths of the profile, this phenomenon disappears and most of the incident energy is redistributed over the propagating waves of higher spectral order. Finally, it is remarked, that it is difficult to follow the details in the curves representing $\Pi SP_0^{I,II}$ (Fig. 8). This is due to the crowding of data in a small range of values of the angle of incidence. The reciprocal curves for $\Pi PS_0^{II,I}$ (Fig. 6) show the same details, but on an extended horizontal scale.

Fig. 6. Acoustic reflectances and transmittances, in the vertical direction, of the zero spectral orders as a function of $\theta_{S_0}^I$ for a sinusoidal interface between granite (medium I) and sea water (medium II) for an SV-wave incident in granite.

4. Conclusions

From the results in the preceding section, we conclude that in all cases considered, most of the energy goes into the propagating reflected P- and SV-waves. This can be explained by the large acoustic contrast between the two media. Another consequence of this large contrast is that the influence of the roughness of the interface is considerable, especially in the cases of an incident SV-wave in granite and an incident P-wave in sea water. Further, we notice that the influence of horizontal phase matching between an evanescent spectral order and the (Scholte) surface wave is in the curves with $h/D = 0.1$ stronger than in the corresponding case of the copper/flint glass interface (cf. [6]). This, too, is due to the greater contrast in elastic parameters.

Fig. 7. Range of propagation of the different spectral orders for the reflected and transmitted waves and their corresponding angles of emergence as a function of θP_0^{II} for a P-wave incident in sea water on a granite/sea water interface.

As a consequence of the reciprocity relations (2.10) of [6] the following relationships exist:

$\Pi SP_0^{I,I}$ in Fig. 4 corresponds to $\Pi PS_0^{I,I}$ in Fig. 6,

$\Pi PP_0^{II,I}$ in Fig. 4 corresponds to $\Pi PP_0^{I,II}$ in Fig. 8,

$\Pi PS_0^{II,I}$ in Fig. 6 corresponds to $\Pi SP_0^{I,II}$ in Fig. 8.

The reciprocity relations have served as a first check on the accuracy of the computations. As a second check, we have verified the power relations (2.11), (2.12) and (2.13) of [6] for each type of incident field. For some details of the applied numerical techniques, we refer to the remarks made in [1] and [6]. We shall conclude this section with an indication of the computing times needed by our program to obtain the results for a single angle of incidence. They are given in Table 4, for various depths of the profile.

Comparing the computing times in Table 4 to those of the solid/solid interface [6], we observe a reduction for each depth of profile. This is due to the fact that less linear equations are needed, since the Green state in medium II (the fluid) is a scalar one.

Table 4

Number of spline subintervals (K), number of linear equations (M), number of terms in the numerical evaluation of the Green state ($2N+1$), and computing time per angle of incidence (T), pertaining to a single interface problem for various values of $h/\mathcal{D}$ (for the values of $kP\mathcal{D}$ and $kS\mathcal{D}$ see Table 3)

$h/\mathcal{D}$	K	M	$2N+1$	T
0.1	10	30	21	21 s
0.3	13	39	21	34 s
0.5	15	45	21	46 s

Fig. 8. Acoustic reflectances and transmittances, in the vertical direction, of the zero spectral orders as a function of $\theta_{P_0}^{II}$ for a sinusoidal interface between granite (medium I) and sea water (medium II) for P-wave incident in sea water.

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