Validity of wind assumptions for wave predictions in the case of lakes

DEWANDRE Cedric

Rapport de recherche présenté en vue de l’obtention du diplôme de maîtrise en Génie Civil.

Seconde partie.

Directeurs de recherche : Pr. H. Verhagen, Technische Universiteit Delft,
Pr. C. Marche, Ecole Polytechnique Montréal.

December 2005
Introduction

It is important for dams and dikes planning to take the generation of waves within their reservoir into account. However, the theories used to determine the characteristics of those wind-generated waves often rely on conditions applicable in open sea. This paper will try to investigate their applicability to conditions present in reservoirs and lakes.

The main tool in this research will be the study of the shape of the wind speed profile which could be sensibly different in each case. Indeed, theories often use as unique wind data –apart from the duration- the wind speed at the standard height of ten meters above water level, making the assumption of a standard wind field implying a logarithmic wind profile. Yet the effective wave-generating wind is the one at water level, thus depending upon the shape of its profile.

Here, we will be studying the wind profile likely to be present in the case of lakes and reservoirs. From there on, it will be possible to assert whether the hypothesis of the representativeness of a ten-meter high wind is well-founded.
State of the art

Two kinds of wave prediction methods can be distinguished. First of all, the method of the characteristic wave produces a significant wave height and a significant period. Secondly, the method of the standard spectrum describes the water state as the distribution of frequencies according to their energies. The later thus carries more information than the former.

Both these methods rely on the assumption of a homogenous stationary wind known as the standard wind field. This standard wind field is characterized as infinitely wide. Moreover, the wind speed is assumed to be nil at $t<0$ and to suddenly start blowing at $t=0$. Because of its stationary character, it blows at the same constant speed $U$ starting at $t=0$.

The method of the characteristic wave puts the wind characteristics (the speed $U$ and wind duration $t$) and the fetch $F$ in relation with the significant wave height $H_s$ and period $T_s$. Amongst others, Brettschneider (1973) as well as Sverdrup and Munk (1946) have proposed such relations.

In most publications a time-average wind speed is used, measured at a constant height $z$ above the mean water level. This height varies between 7.5 and 20 meters, but the large majority of authors chose a height $z=10$ m and thus rely on a definition of the wind speed $U_{10}$. Other authors prefer to use the friction velocity $u_*$, which can be set in relation with $U_{10}$ according to the following expression:

$$u_*^2 = C_{10} U_{10}^2$$

where $C_{10}$ is the drag coefficient. Furthermore, a logarithmic profile of the wind is being commonly accepted, defining the wind speed at any other height $z$ as:

$$U_z = U_{10} \left[ 1 + \frac{C_{10}^{1/2}}{\kappa} \ln \left( \frac{z}{10} \right) \right]$$

or in another form

$$U_z = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right)$$

with $\kappa$ being the von Kármán constant; $z_0$ is the surface roughness length which characterizes the roughness of the surface, or in a more physical definition, is the height above surface at which mean wind extrapolates to zero.

However, according to Holthuijsen (1980), the actual wind speed can sensibly differ from this postulate as we will see later on, which can jeopardize the prediction of waves.

The advantage of methods such as the characteristic wave is that the wind field is very schematized, narrowing down the number of parameters characterizing it. This gives
it an attractive simplicity and ease of use. However, this goes at the expense of its accuracy and range of applications.

**Procedure**

In this paper, we will try to establish the type of wind profile that should be expected on lakes, in order to determine whether the commonly used formulas for calculation wave generation are applicable to this particular case or not. The whole question is to verify the validity of the commonly made assumption of taking the speed $U$ at a standard height of 10 meters as representative enough for wave generation.

This will be done in the following manner. We will be taking the most significant differences between a lake model and an ocean model, or at least the hypotheses involved with it, and assessing their effect on the wind field.

Lakes, and in particular dam reservoirs, are often likely to be situated in hilly regions. This means that most these lakes would probably be characterized by a strong spatial variation of the wind field. Furthermore, the influence of the surrounding topography on the wind field grows inversely proportional to the lake’s size. This study will therefore be based on a model of a hills-surrounded lake. A certain amount of studies has been achieved about the air flow over either hills or dunes, and in particular the evolution of the wind speed profile around them. The hypothesis of a lake within a hilly area allows us to make use of these results with nonetheless the needed precautions.

Another aspect by which a lake differentiates itself from the open sea is the influence of the shoreline. In open sea the land-sea discontinuity is to be totally neglected. However, coming to lakes, in some cases the wind sheltering along steep densely vegetated sections of the shoreline is expected to produce inhomogeneities of the wind field across the water surface (Rubbert, Königter, 2004). This influence is to take more importance in the case of small fetches.

Furthermore, according to Young (1999), one of the primary assumptions adopted in all fetch limited studies is that the wind speed is constant over the fetch. Dobson et al (1989) recognized that, even in the most stable of meteorological situations, such conditions do not occur. (…discontinuity in the surface roughness, as a result, an internal marine boundary layer begins to grow within the thicker terrestrial boundary layer.)

Moreover, wind speed profiles can strongly be influenced by thermal exchange at the water-air boundary. Indeed, in unstable conditions higher velocities are expected closer to the ground than in stable conditions; the stability depending on the temperature difference between the air and the water (Young, 1999). A neutral boundary layer will however be assumed during the rest of the study.

Through a combination of these aspects – hilly surroundings, and the influence of the nearby water-land discontinuity – we will attempt to check the validity of the ten meter high used value of the wind speed. Accordingly, we will then come up with a more realistic image of the wind profile or at least establish guidelines for the prediction of a profile in such cases. First of all a qualitative analysis will be undertaken, followed by a quantitative study. In the quantitative study, it would ideally be interesting to investigate the relative importance of each velocity deficit source. The aim is first and foremost to get
an understandable image of the problematic and all of its aspects to take into account, in a qualitative way before/instead of falling directly into too technical terms.

In a later part, its effects on wave generation will be explored, ideally examining the two usual types of wave prediction methods: the characteristic wave and the energy spectrum.
Chapter 1

The influence of the topography

It seems obvious that the presence of hills is likely to disturb significantly the airflow, and that the downstream region would benefit from a sheltering effect. The logarithmic profile of the incident wind is to be inevitably affected. Results about this matter have been borrowed from various sources and from different applications.

Flow Separation

For airflow over low hills, reasonable understanding can be provided by linear theory. Nevertheless, with the steepness of hills grows the likeliness of observing flow separation. Separation grows with the roughness of the terrain as well. Separation is defined as the case where air flow streamline takes off from the surface and creating a bubble behind it with possible recirculation. This matter is however less understood (Finnigan, 1988).

Separation naturally has a drastic effect on the wind profile as this region is controlled by turbulence. The whole question is the importance of this region. The presence and size of a separation bubble naturally depends on the hills topography, more particularly its steepness and curvature, as well as its roughness. The influence of separation on the wind profile above a lake that would be situated behind the hill is questionable. Indeed, since the separation occurs right behind the hill, more specifically on its downward slope, it would take a quite mountainous region for the air flow above the lake to be in this separation region. The lake shouldn’t be too large either, although a separation effect localized on a fraction of the lake would have a fetch-reducing effect. The presence of the separation bubble and its influence downstream is very case-specific. There is however sufficient data from a variety of hills in order to establish relationships between these cases and the case to assess, in such a way to have a reasonable idea of the separation bubble that is likely to be present in this specific case. Yet, it can be said that there are critical slope values of 10 to 20° at which separation generally appear.

If present, its effects would be expected to be significant, as it can provoke recirculation thus winds in the opposite direction on the lake or part of it.
Figure 1: Idealized flow over a hill and the separation and turbulence created in the region behind it. Here different stability conditions are defined by the values of the Froude number \( Fr = U/(NL) \), where \( U \) is the wind speed and \( L \) is the length scale of the hill (from Stull, 1988, p.602, fig. 14.4).

Finnigan (1988), however, came up with a striking conclusion comparing different gathered data: the separation has a high sensitivity to surface roughness, even on very abrupt hills where one would expect topographical effects to totally dominate.

**Influence of the wake**

At a greater distance from the hill, whether or not a region of true separation exists behind a hill, a pronounced wake region is always established. This region is characterized by a reduced mean velocity and increased turbulence and is remarkably persistent. It is essentially the study of this region, as well the near wake as the far wake, that will probably give us the most significant information of what the wind profile is likely to look like in the present case.

The effect of the wake is clearly observable in Finnigan’s experiments as can be seen on the following figures:
The interesting section to us is the one behind the hill. The following figure illustrates a comparison of the wind profile at a distance upflow from the hill, at the downflow foot of the hill and some distance downflow of the hill. The first profile should be following a logarithmic profile. The wake effect on at the foot of the hill is clearly visible, with a significant speed reduction near the ground, and a logical acceleration above the height of the hill’s top. Some distance further down, the profile tends back to a logarithmic profile but the wake is still visible as speed reduction is generalized along the elevation.
Figure 4: Comparison of three velocity profiles measured by Finnigan in his experiments for $x=-1.8$ (blue), 0.6 (red) and 1.8 (black) from figure 2. They correspond to upstream the obstacle, at the downward toe of the obstacle and some distance downstream the obstacle. The dotted line represents an idealized logarithmic profile.

Behind 2-D ridges, complete recovery of the mean velocity profile does not occur before traveling about $60h$ downwind (Arya and Shipman 1981). Behind 3-D hills, recovery is usually more rapid although axial vortices can be very persistent (Finnigan, 1988).

It is pretty straight forward that the wake triggered by the obstacle will provoke a velocity deficit in this region. The velocity deficit will be most important just downstream the obstacle, its effect decreasing in intensity along the wake.

For a free wake model, so without boundary layer, Townsend (1976) suggests that the rate of spread of the wake and the decay rate for the velocity perturbation should follow a power law. If $x_0$ is the effective origin for the far wake, then the wake thickness is found to increase at a rate proportional to $(x-x_0)^a$ while the centerline velocity deficit decays as $(x-x_0)^b$ (Taylor, 1988). Values for $a$ and $b$ are given for both two and three-dimensional wake flows, and are equal to $\frac{1}{2}$ and $-\frac{1}{2}$ respectively for the former.

Concerning surface-mounted obstacles, Taylor (1983) introduces a linearized two-dimensional planetary boundary layer wake model. He further distinguishes two regions in the wake. The model applies to the wake far downstream of the obstacle in the region $EF'CB$ of figure 5, and will consider its decay subject to the boundary-layer approximations of neglecting horizontal diffusion and pressure-gradient perturbations. This far wake can generally be thought of as $x > 10h$ for two-dimensional obstacles and
perhaps $x > 5h$ in some three-dimensional cases, with $h$ being the height of the obstacle (Taylor, 1988). At the other end, for what concerns velocity perturbations, a distance of $30h$ is a reasonable estimate, given the perturbations are usually less than 5% at that distance.

Taylor (1988), assumes an initial wake profile just downstream the obstacle of the form:

$$ u = -\alpha U_0 e^{-\frac{kz}{h}} - \delta \frac{dU_0}{dz} $$

where $U_0(z)$ is the upstream undisturbed velocity profile, $\alpha$ is a scaling constant and $\delta = \alpha(1 - e^{-kz})/k$ is the streamline displacement. The initial depth of the wake is characterized by $k^{-1}$. According to Taylor (1983), $\alpha$ should be equal to one. Assuming a logarithmic incoming speed profile with $U_{10} = 10$ m/s, an initial wake depth of the order of the obstacle height of 10 meters and $\alpha = 1$, we would come up with the following profile right downstream the obstacle:
Figure 6: Comparison of the logarithmic velocity profile (blue), velocity deficit at the downstream hill toe (red) and the resulting velocity profile at the downstream hill toe (green).

From this figure we can see the considerable deficit in velocity caused by the obstacle, particularly near the ground and decreasing with the elevation. However, this only shows the profile at the downstream foot of the obstacle and not its evolution down the wake.

Taylor and Richards’ (1979) analysis about the profile in the far wake eventually come to a solution for the velocity perturbation of the form:

\[ u = x^{-f}(\eta) \]  

(5)

where \( f \) defines a non-dimensional stream function \( \Psi = x^a f(\eta) \), with \( \eta = z/x^a \). As seen above, \( a \) and \( b \) are here the power law coefficients for the wake thickness and the velocity deficit respectively. Much research has been carried out with different results for the values for \( a \) and \( b \), but it can be generally agreed that the later would \( \approx -1 \) while the former \( \approx 0.5 \) or \( \approx 1 \).

In the case of the near-wake, results tend to be very obstacle specific. No real generally applicable analysis can be set. The presence of flow separation complicates this matter even more. Although non-separating flow still exhibit velocity deficit and turbulence, the wake characteristics are critically dependent on the flow separation, and by corollary on the hill slope.

As a rough guide, Taylor (1988) suggests we should expect separation to occur for essentially two-dimensional hills or ridges when the maximum slopes are of order 0.3.
There will furthermore be a certain dependence on surface roughness and on the detailed shape of the hill.

![Figure 7: a) Hill profiles studied by Arya et al. (1987). b) The recirculating cavity in the lee of hill 3 (Taylor).](image)

**Influence of the hill’s surface**

Not only is it the hill that influences the flow profile, but also what covers it. In fact, hills are often forested. This not only gives different roughness properties, modifying the atmospheric boundary layer above the forest top, but principally affects considerably the flow within this canopy. As our study is mainly aimed at the profile close to the ground, at low height, the presence of a forest canopy cannot be omitted.

Ross and Vosper (2005) have carried out numerical simulations using a canopy model representing the forest through additional drag being exerted on the flow within the canopy. This model has turned out to give results in close agreement with previous research within its application range.

The model offers a mixing-length turbulence closure scheme, altered to include a canopy at the lower boundary. The additional drag on the air flow within the canopy is modeled through an additional term in the momentum equation:

\[
\rho \frac{DU}{Dt} = -\nabla p + \nabla \cdot \tau - \rho CaU|U| \tag{6}
\]

In the absence of a canopy a large roughness length is used to represent the forest. The canopy however gives much better results which is a considerable improvement.

Furthermore, an analytical model developed by Finnigan and Brunet (1995) comes up with an analytic expression for the velocity profile along hill both in and above the canopy. Nevertheless, the model requires that \(H/L \ll 1\) where \(H\) is the hill height and \(L\) its quarter wavelength, which makes it applicable to small hills only. In a more down to earth
explanation, once the separation region extends above the canopy the analytical solution is no longer valid, which also means the canopy must be sufficiently deep.

![Figure 8: Wind speed profile comparison for wind tunnel measurements and numerical solutions (Ross and Vosper, 2005).](image)

From figure 8, we can see the canopy model reproduces relatively well the experimental data. Furthermore, the flow modification due to the forest canopy is undeniable compared with a more classical profile as seen above the canopy top or in figure 2 above. As in our case hills are often covered with forests, leading to a canopy height of the order of tens of meters, its effect cannot be denied in the case of low hills. With higher hills, the influence of the canopy would be less felt in the wake due to greater wake and flow separation as these would be taking over and the effect of the canopy could thus be neglected anyway.

Furthermore, for the case of a canopy over flat terrain, a simplified version of the theory of Lalic and Mihailovic (2002) makes it quite straightforward to describe the wind field in and above the canopy. In fact, in the case of our study, the model can be simplified without much loss of precision to a constant wind speed within the canopy and a logarithmic profile above it. The two would be linearly linked in a transition area around the canopy top. An alternative way of dealing with the problem could indeed be to consider a non-vegetated hill with a canopy in its wake prior to the lake.
Figure 9: Typical wind speed profiles obtained by Lalic and Mihailovic (2002).
Chapter 2

The influence of the shoreline

The logarithmic speed profile is valid for conditions found in open sea. More specifically, the model is valid for a constant surface roughness $z_0$ and for a constant shearing stress

$$\tau = \rho_u u_*^2$$

Moreover, the model also assumes neutral thermal conditions.

The shoreline is accompanied by a change in roughness from the rough land to the relatively smooth water surface. The shear stress also encounters a variation. To what extent are thus those two hypotheses valid in the area near the shoreline? The same goes for neutral thermal stratification, where the flow regime is affected by the land sea discontinuity.

Change in roughness

The change in roughness affects the wind profile progressively. The region that is affected is known as the internal boundary layer. Above the internal boundary layer, the flow field is characteristic of the upstream conditions. Very near the ground, an inner or ‘equilibrium’ layer exists where the wind profile has completely adjusted to the local conditions. Within the internal boundary layer itself, the velocity gradually changes from the logarithmic form of the downstream roughness to that of the upstream one (Garratt, 1990).

Many laws have been proposed for the evolution of the internal boundary layer thickness. We will use Taylor and Lee’s proposal (1984) which represents the thickness $\delta_i$ as

$$\delta_i = 0.75z_0 \left( \frac{x}{z_0} \right)^{0.8}$$

where $x$ is the fetch length. Amongst other proposals, according to Taylor and Lee (1984) the fractional change in wind speed $\Delta R = [U(x) - U_u] / U_u$ (where the subscript $u$ refers to quantities at a point upwind of $x$) is given by

$$\Delta R = \begin{cases} 
0 & z \geq \delta_i \\
\frac{\ln (z/z_0) \ln (\delta_i/z_{0w})}{\ln (z/z_{0w}) \ln (\delta_i/z_0)} - 1 & z < \delta_i 
\end{cases}$$

14
Here, $U_u$ is taken as the wind speed at the shoreline and $z_{0u}$ as the roughness length for the upwind land surface. This formulation accounts for the progressive effect in elevation on the wind profile due to the roughness change. The roughness length is calculated from the Charnock relation:

$$z_0 = c_h \frac{u_*^2}{g}$$  \hspace{1cm} (10)

with $c_h$ being the Charnock parameter assumed constant, we will take it equal to 0.0185 (Wu, 1982) although a range of values have been reported in the literature.

However, Taylor and Lee assume that both the upwind boundary layer is logarithmic and that the developing internal boundary layer is logarithmic for $z < \delta$. This isn’t the best way to check for the validity for a logarithmic profile, but lack of better, we will use this model for non-logarithmic profiles. This abuse can be accepted, as the purpose of our study is not to end up with precise results, but is mainly qualitative.

This formulation leads to the following results in the case of an initial $U_{10}=10$ m/s. It can be clearly seen the logarithmic profile is being progressively altered and evolves from one logarithmic profile and tends to another. The results look satisfactory in general. However, it seems Taylor and Lee’s model is not applicable to very low elevations. In fact, at elevations lower than 1 or 2 meters, the results become senseless. As a result of which we are inclined to conclude this model has a certain application range that hadn’t been specified before.

![Graph showing velocity and elevation relationship](image-url)
Figure 10: Evolution of the wind profile down the fetch following a roughness change at the shoreline. Profiles are shown for fetches of: 10, 30, 50, 100, 500, 1000 and 5000 meters respectively.

Furthermore, the deduced wind speed evolution at one specific height matches the behaviour of previously published results (Young, 1999, …). It can be noticed that at lower height, wind speed increases faster, is this due to the limited application range of Taylor and Lee’s model? Anyhow, from 7 meters onwards, velocities follow similar evolutions. These values somewhat vary with the case studied and their roughness lengths.

In the case that a logarithmic profile is assumed within the internal boundary layer, the part of the fetch that needs particular attention is the one for which the internal boundary layer is beneath the ten meter height. Once the internal boundary layer is thicker than ten meters,

**Constancy of shear stress**

As expected, it is near the shore that this hypothesis is most put at stake. At a certain distance offshore, the hypothesis can be accepted, at least for a certain elevation.

The next figure shows Taylor’s (1970) results when simulating the case of a lake in the great lakes region, including step changes in surface roughness as well as in surface temperature. In this case, it is not before 1 kilometer offshore that it can be said the shear constancy is valid for the first ten meters in elevation. However, the shear stress profile at 1 kilometer offshore still tells us a wind value measured at a ten meter height would yield an erroneous prediction; since a logarithmic profile still cannot be assumed at the height of ten meters.
Again it is for smaller lakes, where the influence of the shoreline is larger, that this hypothesis is least respected and leads to larger errors.

Figure 12: Shear stress profile over a lake according to Taylor (1970). Upstream roughness length $z_0=1$ cm, downstream $z_1=0.1$ cm. Step change in surface temperature, $RL^*_0=-2.5 \times 10^{-3}$ (Taylor, 1970).

**Thermal stability**

The origin of wind in the case of lakes lies in the temperature differences between the air above the land and above the water surface. In fact, during daytime or in the summertime, the air flows from the warmer land towards the colder water surface, forming what is known as the sea or lake breeze. During the night or winter, the opposite happens, known as cold air advection. Any hypothesis assuming neutral thermal stability should then be revised in the case of lakes, as the source of the wind is precisely the change in temperature. Incongruously, as seen above, the assumption of a logarithmic wind profile lies on the hypothesis of neutral thermal conditions…

**Constancy of wind speed along the fetch**

According to Young (1999), one of the primary assumptions adopted in most fetch limited studies is that the wind speed is constant over the fetch. Dobson et al (1989) recognized that, even in the most stable of meteorological situations, such conditions do not occur.

As seen above, the explanation comes from the existence at the shoreline of a discontinuity in the surface roughness, from the aerodynamically “rough” land surface to the relatively smooth water surface. As a result, an internal marine boundary layer begins to grow within the thicker terrestrial boundary layer. Hence, the wind speed measured at a
constant reference height will gradually increase moving down the fetch. This is illustrated by the data of Smith and MacPherson (1987), as presented by Dobson et al (1989).

Figure 13: A comparison between measured aircraft winds at 50 m elevation, $U_{50}$, as measured by Smith and MacPherson (1987) with the prediction of Taylor and Lee (1984).

This data, although measured at a 50 meters elevation, shows the same tendency as our results at lower height derived from Taylor and Lee in figure 11.

Rubbert and Kongeter (2004) state, in their research on wind-induced currents, that according to Ottesen Hansen (1979), the gradual increase of the shear at the water surface may be compared to the shear stress distribution behind a backward facing step. It takes a distance of about $\alpha=6$ times the height of the step until the separated flow re-attaches to the surface. From that point, the shear stress increases exponentially towards its maximum value, $\tau_{w0}$, across a distance of $\beta H_{w}$, where $\beta=6$ (Bradshaw and Wong, 1972; Le et al. 1997). Values of $\tau_{w0}$ are calculated from

$$\tau_{w0} = c_{02} \rho_a U_{02}^2$$  \hspace{1cm} (11)

Where $\rho_a$ denotes the density of the atmosphere, $U_{02}$ represents the wind speed at 2 m above the water surface and $c_{02}$ is the drag coefficient.
The relation between wind speed and shear stress $\tau$ is known to be as follows:

$$\tau = \rho_a U_{10}^2 = \rho_a C_d U_{10}^2$$  \hspace{1cm} (12)

where $\rho_a$ is the density of air, $U_{10}$ the wind speed measured at a reference height of 10 m and $C_d$ is the drag coefficient. The relation between the shear stress and the wind speed is however not that straightforward. Indeed, the drag coefficient varies itself according to the following expression:

$$C_d = u^*_r / U_{10}^2 = \frac{\kappa^2}{\ln^2 \left( \frac{10}{z_0} \right)}$$  \hspace{1cm} (13)

Experimental data show however a large scatter. Garratt (1977) suggested that in the velocity range $4 < U < 21$ m/s, the drag coefficient can be approximated by a power law relation:

$$C_{10} \approx 0.51 \cdot 10^{-3} U_{10}^{0.46}$$  \hspace{1cm} (14)

or by a linear form:

$$C_{10} \approx (0.75 + 0.067 U_{10}) \cdot 10^{-3}$$  \hspace{1cm} (15)
Chapter 3

Results

a) Flat terrain

In this section, we will try to use the different theories explained in the sections above to model the effect of all those factors together on the wind profile.

We will first consider a flat terrain, covered by a forest, adding the hill afterwards. In this first case, three effects will be involved: the canopy effect (Lalic and Mihailovic), the sheltering effect (Ottensen Hansen) and the roughness change effect (Taylor and Lee).

The first two could be considered somewhat similar. In fact the effect a canopy has is to drastically reduce wind speed within it, past the shoreline this profile has to recover a more classical logarithmic profile. The length it takes for the profile to recover its characteristics near the water level can be considered similar to the sheltering effects recovery described by Ottensen and Hansen.

In simple terms, this effect can be mainly cut down to a reduced effective fetch length on the lake. Following Ottensen and Hansen’s theory, the effective fetch would thus be simply reduced of $12^*H_w$ with $H_w$ being the height of the shelter (for the first $6^*H_w$ stress is nil, for the following $6^*H_w$ the stress is not fully effective). A forest with trees of 20 meters would thus provide a fetch reduction of nearly 250 meters. In the case of smaller fetches this effect is not negligible.

Concerning the roughness change effect, it should be much more influential than the shelter effect described above, given the roughness length is switching from $z=0.5m$ to $z=10^{-3}m$. However, the law used to describe the effect on the wind profile is the one by Taylor and Lee (1984) stated above. This theory gives unsatisfactory results at low heights, and would yield a wind profile evolution of similar tendency to that of figure 10.

This means the model used here gives reasonable results when there is substantial hill influence, when the roughness change effect does not stand alone.

b) Hilly terrain

In the second scenario, we will try to model the profile affected by the additional presence of a hill in front of the lake. Naturally, a lot depends on the geometry and roughness of the hill, in particular concerning flow separation. We will be taking the example of hills of characteristics 10 and 100 meters in height. We make the hypothesis of the lake starting at the downflow toe of the hill.
In order to model the hill, the modified wind profile at the downstream foot of the hill has been calculated according to Taylor (1988). Furthermore, the wake has been modeled using Townsend’s decay law (1976). Townsend was preferred, although it applies to a free wake model, because of Taylor and Richard’s model (1979) being too complicated to carry out in the scope of this qualitative study.

The fact of using a model applying to a free wake model shouldn’t be too much of a hazard for the validity of the results, given the results were checked to fit usual surface mounted obstacles behaviour. More specifically, the model was applied in the far wake region, considered to be located between \( x=10h \) and \( x=30h \), perturbations being less than 5% at the farthest boundary (more secure than Arya and Shipman’s 60h (1981) or Taylor’s 100h (1988) perturbations range). Before \( x=10h \), in the near wake region, it was assumed there would be no change in profile compared to the foot of the hill. This may not be the best approach, but it is the best guess to be made given the obstacle-specific character of this region and is even conservative regarding recirculation involving inverse circulation in this region (see Figure 7).

Furthermore, concerning the speed deficit decay exponent \( b \) as explained at pages 8 and 10, a value of -1 has been judged incompatible with the postulate of having 5% perturbations at a distance of 30h. It was preferred to choose values applicable to the 2-D free wake case, checking for the postulate to be fulfilled, as it is conservative compared to other general observations.

Regarding the change in roughness due to the shoreline, Taylor and Lee is used again. A roughness length of 0.07 meters for the upwind land surface is assumed, whereas a roughness length growing from 0.019 \( 10^{-3} \) to 0.291 \( 10^{-3} \) meters is assumed on the lake surface (typical sea roughness length values near the shore are inferior to 4 \( 10^{-4} \) meters).

### i) 10 meters high hill

Figure 15 presents the results obtained in the case of a 10 meter high hill. A value of 0.56 has been used for the speed deficit decay exponent \( b \). In the beginning, the speed deficit is much more important near the water surface. At higher altitude, the profile is still unchanged, the effect of the change in roughness having not yet reached that elevation. The progressive modification of the profile due to the change in roughness can be clearly identified. The effect of the roughness is then an increase in wind speed. Although it slowly tends to one, it can be noticed there is never a point where a perfect logarithmic profile is obtained, except if going a certain distance further down the fetch. In fact, the profile at the 10 kilometer fetch becomes very similar to a logarithmic profile, when looking at the lowest 10 meters that retain our attention. This logarithmic profile is however only valid for the 10 kilometer fetch, as it still evolves down the fetch.

Furthermore, we can deduct from these results that the wake’s effect is most felt for smaller fetches, the roughness change effect taking over for longer fetches. This can clearly be seen at around 25 meter heights where the wake-influenced profile tends to a logarithmic profile as the roughness change’s effect hasn’t reached that height yet.

However, as seen above, there exists a fetch-reducing sheltering effect due to the presence of trees near the shore. In this case, wind profiles only should be taken into account starting at a fetch of 250 meters onwards (for 20 meter high trees).

Concerning the overall shape of profile to be found over the lake, for a same value of the wind speed at the reference height of 10 meters, lower wind speeds will be
underestimated while wind speeds above will be overestimated when using a logarithmic profile law. The main cause, in this case, is the roughness change at the shoreline discontinuity. However, yielded profiles at a certain distance down the fetch remain close to logarithmic profiles, for the lowest 10 meters.

Figure 15: Results for wind profile evolution for a 10 meter high hill, for fetches of: 200 (g), 500 (y), 1000 (r), 1500 (y), 2000 (b), 3000 (b), 4000(b), 5000 (r) and 10000 (r) meters respectively. The first red dotted line represents the original logarithmic profile while the blue dotted line represents the supposed logarithmic profile on the lake at a fetch of 10000 meters, the two other red dotted lines represent the supposed profiles at 2000 and 5000 meters (b=0.56).

ii) 100 meters high hill

The results for a 100 meter high hill are presented in figure 16. The wake-effect due to the hill is now generalized, and it becomes so important that the roughness change is much less identifiable. At distances larger than three kilometers (30h), only the roughness change effect should be left over in the profile. From the profiles at 5 and 10 kilometers, we see the roughness change has much less influence than in the previous case.
Figure 16: Results for wind profile evolution for a 100 meter high hill, for fetches of: 1010, 1100 (g), 1500 (y), 2000 (b), 3000 (b), 4000(b), 5000 (r) and 10000 (r) meters respectively. The green dotted line represents the original logarithmic profile while the blue dotted line represents the supposed logarithmic profile on the lake at a fetch of 10000 meters, the red dotted lines represent the supposed profiles at 1010, 2000 and 5000 meters (b=0.4).

The fact that the wake effect is much more important in this case, yields much more uniform profiles. The wake-affected region is also much larger than previously.

The badly defined near-wake which extends up to 1000 meters from the shore incorporates the first part of the roughness-change effect on the profile, as can be seen when comparing to figure 15. Therefore that area cannot be considered as taking much part in the generation of waves in this case. It can also be noticed the shelter effect due to the vegetation on the shore isn’t much of application anymore since it is now entirely located in this badly defined near-wake region.

Next, in the far-wake region, the wake perturbation will decay until about 3000 meters from the shore, the remaining profile perturbation being caused by the roughness change. At this point however, its effect is quite restrained and a logarithmic profile is reached except for heights lower than eight meters where a deviation persists. This deviation, however light, could be due either simply the logarithmic profile underestimating the actual profile, or to the flaws revealed in Taylor and Lee’s model (1984) regarding lower heights. According to figure 10, this questionable validity occurs for lower heights than is the case here, but we cannot at this stage be sure of anything.
The profile at the 10 kilometer fetch is representative of the profiles met at larger fetches, at the scale of lakes.

Like above, we can conclude from the yielded profile that a logarithmic profile underestimates the wind speed for lower heights and overestimates it at larger heights. It is however much more pronounced than in the previous case. This tendency can be understood through the respective effects: the wake having a generalized effect, and the change in roughness having its largest effect at lower heights.

However, we should be careful at results for lower heights, Taylor and Lee’s (1984) validity being questionable for such heights as seen in figure 10; even though the tendency is not so visible in figures 15 and 16.

Nevertheless, the kind of profile yielded at 5 or 10 kilometers fetches is acceptable. Taylor (1970) gives the following profile evolution for an example of a lake breeze, which unlike our case is blowing onshore; thermal effects are also included in his profile:

![Figure 17: Velocity profiles for lake breeze with $z_0=0.1\text{ cm}$, $z_1=10\text{cm}$ and $RL^*_j=-10^2$ (Taylor, 1970). The roughness change effect is first dominant, thermal effects taking over afterwards.](image)

Besides the fact that the effects are the inverse of our case, the tendencies match our own, once the wake-effect has faded away, and setting aside thermal effects. This gives some additional credit to our results.

Out of the topic, it can be seen how thermal variations can cause the wind to deviate away from its thermally stable logarithmic profile.

Anyway, it takes a long time for the wind to reach an ‘equilibrium’ profile, a constant profile that would be valid over the rest of the fetch. It is clear that if a constant
speed were to be assumed over the lake for wave predictions, either the evolution of the speed from the shore should be included in the calculations, or an ‘effective’ fetch reduction should simply be considered.

As a whole, a hill has the opposite effect on the wind profile as that of a change in roughness. The former provokes a speed deficit decaying along the fetch while the later causes a speed increase. However, depending on the hill’s height, their respective relative effects are dominant in distinct regions of the fetch.

c) Discussion

In the case of a relatively flat hill, the wake’s effect loses importance and the canopy’s effect takes over, at least for smaller fetches. Finnigan and Brunet (1995)’s analytical model can be then be used (see pages 11 and 12) to obtain a better prediction of the yielded wind profile. For reminder, their model requires that $H/L << 1$ where $H$ is the hill height and $L$ its quarter wavelength.

For cases where no hills are present, Lalic and Mihailovic’s (2002) model can also be used. However, as in the above paragraph, the effects on the profile are then only tangible at relatively small fetches.

At the other extreme, there comes a point to which the topography becomes so overwhelming that a closer study of the actual flow patterns is required.

Indeed, another way of looking at the problem is working from the source of the wind, at what induces the wind. This could have the advantage of getting a more precise idea of the wind flow and profile in a given topography. However, this complicates once more the initial problem, making it harder to get to acceptable solutions, as well as being additionally case-specific.

The case studied in this paper is a combination of air flowing over a non-homogenous flat terrain, with air flow over a hilly terrain. The source of the wind can be identified in each case to characterize the resulting flow.

In the first case, differences in terrain features, such as variation in roughness length, are reflected into horizontal variations of surface temperatures and turbulent fluxes. These cause growth of internal boundary layers and circulations (Finardi et al, 1997). The best example for this is the periodical land-sea/lake breeze driven by different thermal properties of the land and sea or lake.

In the second case, we may consider the case of a mountainous area. Mountain/valley breezes originate from the differential warming of the mountain sides that give rise to horizontal temperature and pressure gradients. They determine the direction and profile of the wind in the valley, in our case host of a lake, as shown in the following figure (Finardi et al, 1997):
Figure 18: Dynamics of wind flow in valleys (Finardi et al., 1997).

This shows how case-specific a wind profile can sometimes be, and that a general applicable theory will then not be suitable for the determination of the yielded wind profile.

Furthermore, even though we put thermal effects on the side during our study, these can often play a large role, besides the fact they are often at the origin of the creation of wind as seen above.

For instance, it can be seen for the case of cold air-advection in figure 17 that the roughness length change is accountable for the profile change up to 100 meters from the shoreline. Beyond that point, it is starting to be dominated by thermal effects which have the opposite effect on the shear stress and thus on the wind profile. The relative significance of those thermal effects is dependent upon the importance of the roughness change.
Chapter 5

Effect on wave prediction

a) Evolution down the fetch

The prediction laws that will be used for the comparison are the one by Smith 1991 as well as the Shore Protection Manual formula. Indeed, it hasn’t been proven that any prediction law was overruling the other ones. These are laws commonly being used for predicting waves in lakes and reservoirs. According to Smith (1991), the significant wave height $H_s$ and period are defined as follows:

\[
H_s = 0.0015g^{-0.5} F^{0.5} (U \cos \Phi)
\]

(16)

\[
T = 0.385 g^{-0.72} F^{0.28} (U \cos \Phi)^{0.44}
\]

(17)

where $\Phi$ represents the angle between the wind direction and the fetch direction. The Shore Protection Manual states the following formulae:

\[
g \frac{H_{m0}}{U_A^{-2}} = 1.6 \cdot 10^{-3} \left( \frac{g F}{U_A^{-2}} \right)^{1/2}
\]

(18)

\[
g \frac{T_m}{U_A} = 2.857 \cdot 10^{-1} \left( \frac{g F}{U_A^{-2}} \right)^{1/3}
\]

(19)

where $H_{m0}$ and $T_m$ are respectively the spectral wave height and the peak spectral period. These are deep-water equations, but the effect is expected to be the same for shallow-water equations. The above computed wind profile evolution from the shoreline down the fetch yields the following wave predictions along the fetch, using the shore protection manual prediction. The last case of the previous chapter has been used: a 100 meters high hill.
Figure 19: Evolution of the wave height (blue) and period (red) down the fetch based on the wind profiles using Smith’s (1991) prediction law. The dotted lines represent the same evolution if we use the wind speed at a 5 meters height instead of a 10 meters height.

The jump at a distance of 1000 meters offshore coincides with the point where the air flow reattaches to the water surface, at the limit between the near and far-wake. The values are however overestimated, as the use of these formulae assumes a constant wind speed over its fetch, which is clearly not the case in the near-wake. Besides, these formulae do not account for the depth change in the lake as they apply to deep water.

We can also notice the fact of using the wind speed at a height of 5 meters has some influence but not as important as what could’ve been expected.

As said before, the hill-effect on the wave generation can simply be modeled by a fetch reduction of a distance of $10h$, apart from the lesser velocity deficit in the far-wake beyond that point. In fact, wind profiles before that distance are badly defined and are not likely to yield a significant contribution to wave generation.

The comparison between this modeled evolution and the classical way of determining the wave height evolution down a fetch for a given wind is shown in figure 20. The classical wave prediction for a given fetch, also used for the design of dikes and dams, assumes one constant and unique wind value over the entire fetch (the wind speed at fetch 10 kms was used).
Figure 20: Comparison between classical prediction method for wave heights and methods applied to modified wind profiles. Smith’s method (1991): original evolution (blue), corrected evolution (dotted blue), classical approach (red). Shore Protection Manual’s formulae: original evolution (black), corrected evolution (dotted black), classical approach (yellow).

The results show the effect on the wave generation can be modeled as a $10h$ translation which remains valid for most of the fetch. In this case, the actual effects on wave characteristics are a decrease of 29% to 5% for fetches of 2 to 10 kilometers for the wave height and a smaller 22% to 3.5% decrease for the period. The period evolution shows the same pattern. The correction made to the wave height evolution is also valid for the period, even though the period jump at 1000 meters is not as important, as it is the period of the most significant waves.

Figure 20 should be close to reflecting reality in a relatively sensible way, as it can be noticed that the wind speed doesn’t vary all too much from 1000 meters onwards. This would make the hypothesis of a constant speed for every fetch more or less acceptable.
Another issue is the actual shape of the wind profile. From figures above, wind profiles remain close to the logarithmic law. This is especially for larger fetches, at larger distances from the shore. Profiles can however differ sensibly from the logarithmic law, but looking at our results, it remains a relatively good guess. Now the manner in which our results have been obtained may not be the most realistic ones. Finally, types of profiles like the ones obtained by Taylor in figure 17 may raise some more questions about this matter.

**b) Case study**

The case to which the first part of this project is applied (“Experimental validation of a dike overflow model”) is the Moncouche dike at the Kenogami reservoir in Quebec (Canada). It is an important reservoir with a 900 hm³ capacity, and the dike that was studied faces a fetch of more than 25 kilometers. During the investigations, the waves that have been modeled are typically produced at wind speeds of 50 to 70 km/h, which represent a recurrence rate of one in five years.
Even though the surrounding terrain looks quite hilly, the topography in the axis of the fetch can mainly be considered as a plain. In fact, the fetch is situated in the length of a valley, with plains at its extremity. The topography is thus not expected to have a significant influence on the wind profile evolution.
If we still try to apply our model (thus using the roughness change as expressed by Taylor and Lee) to this pure roughness change case, the following results would be obtained for the influence on wave generation:

![Figure 24: Comparison between classical prediction method for wave heights and methods applied to modified wind profiles. Case without a hill. Smith's method(1991): modeled evolution (blue), classical approach (red). Shore Protection Manual's formulae: modeled evolution (black), classical approach (yellow).](image)

This is the case of a larger roughness change than above; with an initial rough land with a roughness length value of 0.5 m. It can be observed there is an overestimation of the local wave height in the order of up to 3 cm, but the predictions match again after a certain distance. However here again, a constant wind speed is assumed on the fore lying fetch, which means the modeled evolution is a somewhat overestimated itself. It can however be asserted without much risks that with a fetch of 25 kilometers, the roughness change won’t have much influence on the eventual wave generation.

In the case of the Netherlands however, 25 kms-long fetches aren’t very common. Hence the roughness change will play a more important role and might have a reduction effect on the final wave height.

This case is comparable to the kind of topography in the Netherlands. Of course, looking at the 3-D situation, we don’t know the effects the hills on the sides of the reservoir would have on the air flow. There could either be a tunnel effect with acceleration in this valley, or lower speeds due to the friction on the flanks of the hills on either side of the fetch. This situation would thus probably require a case-specific study if the exact influence of the wind irregularities on wave generation was to be investigated for the design of the Moncouche dike.
Conclusion

This research has attempted to put in evidence flaws of the classical wave prediction methods and their hypotheses in the case of lakes. This has been mainly done by investigating two effects specific to the case: the wake effect due to the presence of hills and the roughness change effect at the shoreline. Both these aspects affect the wind profile, which in turn affects the generated wave characteristics.

The main conclusion is that classical wave prediction methods rely on hypotheses that may be valid in the open sea, but can be jeopardized in other situations. This should encourage the taking into account of these deviations from the idealized case. It should also uphold specific case studies when hypotheses are clearly not fulfilled, when such action is pertinent.

It has been seen the effects on wave predictions could be summarized to fetch reductions. In fact, the most controversial hypothesis is the assumption of a constant speed over the whole fetch. This effect’s relative influence on a calculated design wave naturally depends on the length of the fetch. Depending on the topography, it is foremost for fetches smaller than 10 kilometers that the overestimation of classical methods is felt. The smaller the fetch, and the rougher the topography, the more this deduction is to be made. Hence the relative importance of the classical prediction methods’ overshoot is case-sensitive. Due to the specificities of each and every case, these are the limits of the general conclusions to be made. Proposals have been made in the first part of this report for other research paths.

Others aspects have been left aside like thermal variations, whose influence on wave generation remains a subject to be investigated further although the Monin-Obukhov theory helps a big deal. Indeed, thermal variations are often the origin of wind, whereas thermal stability is assumed for wind-wave generation predictions.


Robbe, R., “Modélisation des Débordements de Batillage et de leurs Conséquences éventuelles sur la Stabilité des Digues”, 2004


Rubbert, S., Königter, J., “Measurements and three-dimensional simulations of flow in a shallow reservoir subject to small-scale wind field inhomogeneities induced by sheltering”, July 2004, Aquatic Sciences, 67 (104-121).


function wakeTaylor

% profil au pied du hill selon taylor

pas=0.01;
z=[pas:pas:50];
n=length(z);
U10=10;%hyp
kapa=0.4;
C10=(0.75+0.067*U10)*10^-3;
g=9.81;

U0(1)=0;
for i=2:n
    U0(i)=U10*(1+C10^(1/2)/kapa*log(z(i)/10));
    dU0dz(i)=U10*C10^(1/2)/kapa/z(i);
end

%%%%%%%%%%%%%%%%%%%%%
h=100;%m
%%%%%%%%%%%%%%%%%%%%%

k=h^-1;%hyp
alpha=1;%hyp
u(1)=0;
for i=1:n
    delta=alpha*(1-exp(-k*z(i)))/k;
    u(i)=alpha*U0(i)*exp(-k*z(i))-delta*dU0dz(i);
end

figure(1)
plot(U0,z)
hold
plot(u,z,'r')
plot(U0-u,z,'g')
hold

xlabel('Velocity u [m/s]')
ylabel('Elevation z [m/s]')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Taylor Lee

x=[1 10 50 101 200 300 400 500 600 700 800 900 1001 1010 1100 1300
   1500 2000 2500 3000 3500 4000 4500 5000 6000 7000 8000 9000 10000];
n=length(x);

a=0.0185;%wind wave devplt over estuary
%Dobson: a=ch=0.01
U=10;%U10
k=0.4;%von karman

for i=1:n
\[ u_{\text{star}}(i) = ((0.8 + 0.065 * U) * 10^{-3} * U^2)^{1/2}; \] % formule uniquement valable pour \( U = U_{10} \)

\[ z_0(i) = 10 / (\exp(U * k / u_{\text{star}}(i))) ; \] % \( a * u_{\text{star}}(i)^2 / 9.81; = 0.04; \) % charnock

\[ \delta_i(i) = 0.75 * z_0(i) * (x(i) / z_0(i))^0.8; \]

%%
for \( i = \) 1: \( n \)
    \( U = 9; \)
    \( U_{\text{test}} = 10; \)
    while abs(U - U_{\text{test}}) > 0.01
        \( U = U_{\text{test}}; \)
        \( u_{\text{star}}(i) = ((0.8 + 0.065 * U) * 10^{-3} * U^2)^{1/2}; \) % formule uniquement valable pour \( U = U_{10} \)
        \( z_0(i) = 10 / (\exp(U * k / u_{\text{star}}(i))) ; \) % \( a * u_{\text{star}}(i)^2 / 9.81; = 0.04; \) % charnock
        \( \delta_i(i) = 0.75 * z_0(i) * (x(i) / z_0(i))^0.8; \) % boundary layer thickness
        for \( j = 1: \) \( n_{z} \)
            \( U_{\text{log}}(j) = U_{0}(j); \) %
        end
    end
end

%%% puissance de la degradation du deficit
if \( h = 10 \) | \( h = 0 \)
    \( \text{fact} = 0.56; \% 0.5; \)
elseif \( h = 100 \)
    \( \text{fact} = 0.4; \)
end

%%% for \( i = 1: \) \( n \)
    \( U = 9; \)
    \( U_{\text{test}} = 10; \)
    while abs(U - U_{\text{test}}) > 0.01
        \( U = U_{\text{test}}; \)
        \( u_{\text{star}}(i) = ((0.8 + 0.065 * U) * 10^{-3} * U^2)^{1/2}; \) % formule uniquement valable pour \( U = U_{10} \)
        \( z_0(i) = 10 / (\exp(U * k / u_{\text{star}}(i))) ; \) % \( a * u_{\text{star}}(i)^2 / 9.81; = 0.04; \) % charnock
        \( \delta_i(i) = 0.75 * z_0(i) * (x(i) / z_0(i))^0.8; \) % boundary layer thickness
        for \( j = 1: \) \( n_{z} \)
            \( U_{\text{log}}(j) = U_{0}(j); \) %
        end
    end
if \( x(i) < 10 * h \)
    \( U_{\text{test}} = \delta_i(i, 10 / \text{pas}) + U_{\text{log}}(10 / \text{pas}) - u(10 / \text{pas}); \)
else
    \( U_{\text{test}} = \delta_i(i, 10 / \text{pas}) + U_{\text{log}}(10 / \text{pas}) - u(10 / \text{pas}) / ((x(i) - 10 * h)^{\text{fact}}); \)
end
if \( i <= n \) \( \% = 24 \)
if \( x(i) < 10 * h \)
    \( U_{10}(i) = U_{\text{log}}(10 / \text{pas}) + \delta_i(i, 10 / \text{pas}) - u(10 / \text{pas}); \)
    \( U_{3}(i) = U_{\text{log}}(3 / \text{pas}) + \delta_i(i, 3 / \text{pas}) - u(3 / \text{pas}); \)
    \( U_{15}(i) = U_{\text{log}}(15 / \text{pas}) + \delta_i(i, 15 / \text{pas}) - u(15 / \text{pas}); \)
    \( U_{5}(i) = U_{\text{log}}(5 / \text{pas}) + \delta_i(i, 5 / \text{pas}) - u(5 / \text{pas}); \)
else

38
\[
U_{10}(i) = U_{\log}(10/\text{pas}) + \delta_R(i, 10/\text{pas}) - u(10/\text{pas})/((x(i) - 10*h)^\text{fact});
\]
\[
U_{3}(i) = U_{\log}(3/\text{pas}) + \delta_R(i, 3/\text{pas}) - u(3/\text{pas})/((x(i) - 10*h)^\text{fact});
\]
\[
U_{15}(i) = U_{\log}(15/\text{pas}) + \delta_R(i, 15/\text{pas}) - u(15/\text{pas})/((x(i) - 10*h)^\text{fact});
\]
\[
U_{5}(i) = U_{\log}(5/\text{pas}) + \delta_R(i, 5/\text{pas}) - u(5/\text{pas})/((x(i) - 10*h)^\text{fact});
\]
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{if } x(i) < 10*h
\]
\[
\begin{align*}
U_x(i, :) &= U_{\log} + \delta_R(i, :) - u; \\
\text{else} \\
U_x(i, :) &= U_{\log} + \delta_R(i, :) - u/((x(i) - 10*h)^\text{fact});
\end{align*}
\]
\[
\text{end}
\]
\[
z_0
\]
\[
\text{for } j = 1:nz
\]
\[
\begin{align*}
U_{\text{lac}}(j) &= u_{\text{star}}(n)/k*\log(z(j)/z_0(n)); \\
U_{\text{lac}5}(j) &= u_{\text{star}}(n-1)/k*\log(z(j)/z_0(n-1)); \\
U_{\text{lac}2}(j) &= u_{\text{star}}(18)/k*\log(z(j)/z_0(18)); \\
U_{\text{lac}01}(j) &= u_{\text{star}}(14)/k*\log(z(j)/z_0(14));
\end{align*}
\]
\[
\text{end}
\]
\[
\text{figure(2)}
\]
\[
\text{if } h = 10 | h = 0
\]
\[
\begin{align*}
\text{plot}(U_0, z, 'r:') \\
\text{hold} \\
\text{plot}(U_x(5,:), z, 'g') \% x = 200 \\
\text{plot}(U_x(8,:), z, 'y') \% x = 500 \\
\text{plot}(U_x(13,:), z, 'r') \% x = 1001 \\
\text{plot}(U_x(15,:), z, 'g') \% x = 1100 \\
\text{plot}(U_x(17,:), z, 'y') \% x = 1500 \\
\text{plot}(U_x(18,:), z) \% x = 2000 \\
\text{plot}(U_x(20,:), z) \% x = 3000 \\
\text{plot}(U_x(22,:), z) \% x = 4000 \\
\text{plot}(U_x(24,:), z, 'r') \% x = 5000 \\
\text{plot}(U_x(29,:), z, 'r') \% x = 10000 \\
\text{plot}(U_{\text{lac}}, z, ':') \\
\text{plot}(U_{\text{lac}5}, z, 'r:') \\
\text{plot}(U_{\text{lac}2}, z, 'r:') \\
\text{plot}(U_{\text{lac}01}, z, 'r:') \\
\text{hold} \\
\text{elseif } h = 100
\end{align*}
\]
\[
\begin{align*}
\text{plot}(U_0, z, 'r:') \\
\text{hold} \\
\text{plot}(U_x(1,:), z) \% x = 1 \\
\text{plot}(U_x(2,:), z, 'r') \% x = 10 \\
\text{plot}(U_x(3,:), z, 'k') \% x = 50
\end{align*}
\]

39
% Altered wave generation

d=20;
nxx=length(xx);
for i=1:nxx
    F(i)=xx(i);
    Hs10(i)=0.0015*g^(-0.5)*F(i)^0.5*U10(i);
    T10(i)=0.385*g^(-0.72)*F(i)^0.28*U10(i)^0.44;
    UA=U10(i);
    Hm0(i)=UA^2/g*1.6*10^-3*(g*F(i)/UA^2)^(1/2);
    Tm(i)=UA/g^2.857*10^-1*(g*F(i)/UA^2)^(1/3);
    if F(i)<=10*h
        Hs10cor(i)=0;
        Hm0cor(i)=0;
        Tmcor(i)=0;
    end
end

figure(2)

xlabel('Fetch [m]')
ylabel('Velocity [m/s]')
title('Velocity evolution down the fetch for heights of 3m (red), 5m (black), 10m (blue) and 15m (green)')
\[ H_{s10\text{cor}}(i) = 0.0015 g^{-0.5} (F(i) - 10h)^{0.5} U_{10}(i); \]
\[ H_{m0\text{cor}}(i) = U_{A}^2/g \times 1.6 \times 10^{-3} (g(F(i) - 10h)/U_{A}^2)^{1/2}; \]
\[ T_{m\text{cor}}(i) = U_{A}/g \times 2.857 \times 10^{-1} (g(F(i) - 10h)/U_{A}^2)^{1/3}; \]
\[ H_{s10\text{b}}(i) = 0.0015 g^{-0.5} F(i)^{0.5} U_{10}(29); \]
\[ H_{m0\text{b}}(i) = U_{10}(29)^2/g \times 1.6 \times 10^{-3} (gF(i)/U_{10}(29)^2)^{1/2}; \]
\[ T_{mb}(i) = U_{10}(29)/g \times 2.857 \times 10^{-1} (gF(i)/U_{10}(29)^2)^{1/3}; \]
\[ H_{s5}(i) = 0.0015 g^{-0.5} F(i)^{0.5} U_{5}(i); \]
\[ T_{5}(i) = 0.385 g^{-0.72} F(i)^{0.28} U_{5}(i)^{0.44}; \]
\[ H_{s\text{hal}}(i) = U_{A}^2/g \times 0.283 \tanh(0.53(g*d*U_{A}^2)^{(3/4)}) \tanh((0.00565(g*F(i)/U_{A}^2)^{1/2})/(\tanh(0.53(g*d/UA^2)^{(3/4)})); \]
\[ H_{brett}(i) = U_{A}^2/g \times 0.283 \tanh(0.0125(gF(i)/U_{A}^2)^{0.42}); \]
\[ U_{A} = u_{\text{star}}(i); \]
\[ H_{m0\text{2}}(i) = U_{A}^2/g \times 4.13 \times 10^{-2} (gF(i)/U_{A}^2)^{1/2}; \]
\end{verbatim}

```matlab
else
Hs10cor(i)=0.0015*g^(-0.5)*(F(i)-10*h)^{(0.5)}*U10(i);
Hm0cor(i)=U^2/g*1.6*10^-3*(g*(F(i)-10*h)/U^2)^{(1/2)};
Tmcor(i)=U/g*2.857*10^-1*(g*(F(i)-10*h)/U^2)^{(1/3)};
end
Hs10b(i)=0.0015*g^(-0.5)*F(i)^{(0.5)}*U10(29);
Hm0b(i)=U10(29)^2/g*1.6*10^-3*(g*F(i)/U10(29)^2)^{(1/2)};
Tmb(i)=U10(29)/g*2.857*10^-1*(g*F(i)/U10(29)^2)^{(1/3)};
Hs5(i)=0.0015*g^(-0.5)*F(i)^{(0.5)}*U5(i);
T5(i)=0.385*g^(-0.72)*F(i)^{(0.28)}*U5(i)^{(0.44)};

figure(4)
plot(xx,Hs10)
hold
plot(xx,Hs5,':')
plot(xx,T10/10,'r')
plot(xx,T5/10,'r:');
xlabel('Fetch [m]')
ylabel('Hs [m], T/10 [s]')
hold
figure(5)
plot(xx,Hs10)
hold
plot(xx,Hs10b,'r')
plot(xx,Hs10cor,:')
plot(xx,Hm0,'k')
plot(xx,Hm0b,'y')
plot(xx,Hm0cor,'k:')
hold
xlabel('Fetch [m]')
ylabel('Hs [m]')
grid on
axis([0 10000 0 0.53])
figure(6)
plot(xx,T10)
hold
plot(xx,Hs10b,'r')
plot(xx,Hs10cor,:')
plot(xx,Tm,'k')
plot(xx,Tmb,'y')
plot(xx,Tmcor,'k:')
hold
xlabel('Fetch [m]')
ylabel('T [s]')
grid on
```