COMBINED AEROSTRUCTURAL WING AND HIGH-LIFT SYSTEM OPTIMIZATION

THESIS REPORT

by

Koen T.H. van den Kieboom

in partial fulfillment of the requirements for the degree of

Master of Science
in Aerospace Engineering

at the Delft University of Technology,
to be defended publicly on Monday August 22, 2016 at 09:00 AM.

Supervisor: Dr. A. Elham
Thesis committee: Prof. dr. ir. L. L. M. Veldhuis
Dr. ir. H. G. Visser

Thesis registration number: 085#16#MT#FPP
Student number: 4004299

Cover image: Credits: Graham, https://www.flickr.com/photos/gsairpics/

An electronic version of this thesis is available at http://repository.tudelft.nl/.

Department of Flight Performance and Propulsion
Faculty of Aerospace Engineering • Delft University of Technology
The undersigned hereby certify that they have read and recommend to the Faculty of Flight Performance and Propulsion for acceptance a thesis entitled “Combined Aerostructural Wing and High-Lift System Optimization” by Koen Theodorus Hubertus van den Kieboom in partial fulfillment of the requirements for the degree of Master of Science.

Dated: August 22, 2016

Head of department: Prof. dr. ir. L. L. M. Veldhuis

Supervisor: Dr. A. Elham

Reader: Dr. ir. H. G. Visser
This thesis has been written as final part of my Master program at the chair “Flight Performance and Propulsion” of the faculty of Aerospace Engineering at the TU Delft. The research was conducted in an effort to include the design of high-lift devices at the start of the wing optimization process.

I would like to thank my supervisor dr. Ali Elham for his support and guidance throughout the past year. He has truly been an inspiration to me and has helped me to achieve more that I could have predicted at the start of my thesis. I am especially glad for the time he took to discuss exciting ideas and challenges, and for his help during the publication of this research. I would also like to thank Prof. dr. ir. Leo Veldhuis for chairing the exam committee.

Furthermore I would like to thank my friend Jesper for his support throughout my studies and my fellow students of “Kamerijke 1” for the fun times and advice. I would also like to thank all of my past and present housemates from the “Hagedis” for taking over cooking and cleaning over the last couple of months. Lastly, I want to thank my girlfriend Lotte and my family for their love and support.

Koen T.H. van den Kieboom
Delft, August 2016
This thesis has been performed in an effort to develop an analysis and optimization framework which combines the high-lift wing design with the cruise wing design. By combining both designs at an early stage, crucial trade-offs between the two designs can be made resulting in more optimum wing designs, instead of limiting the high-lift design space by cruise performance requirements.

The work in this thesis expands on an existing aerostructural analysis and optimization tool which is able to accurately compute aerodynamic loads and structural deformation using minimal computational effort. This analysis tool makes use of a quasi-three-dimensional methodology in which a three-dimensional inviscid analysis is coupled with two-dimensional viscous analyses performed at several spanwise sections. The three-dimensional analysis is performed using a vortex lattice method and the two-dimensional analysis by MSES, a viscous/inviscid Euler solver. Structural analysis is performed using FEMWET, which is able to compute structural deformation and stresses as well as provide an accurate wing weight estimation. The primary structural weight is computed by modeling the wing box internal structure as equivalent panels, of which the thicknesses are defined as design variables. The secondary weights are computed as a function of wing surface area. The aerostructural system is solved using the Newton method for iteration and the total derivative of any function of interest with respect to the design variables is computed through a combined use of Automatic Differentiation, chain rule of differentiation and the coupled-adjoint method, enabling gradient based optimization.

While the existing aerostructural analysis tool has been validated to accurately predict aerodynamic loads and aeroelastic effects in cruise condition, it is unable to analyze the performance of high-lift devices and perform high-lift system optimization. In order to enable the tool to perform analysis and optimization of both the cruise and high-lift wing design, modifications to this tool are therefore required. High-lift devices are included in the vortex-lattice method by shifting the vortex points aft to simulate chord extension and rotating the points to simulate flap deflection. Also a cosine spacing of the vortex points in both spanwise and chordwise direction in order to obtain a good resolution near the high-lift device edges.

When optimization of high-lift devices is considered, it is key to determine the maximum attainable lift due to deployment of the high-lift devices. Two methods of predicting $C_{L_{\text{max}}}$ are investigated: The physics-based $\alpha$ method and the semi-empirical Pressure Difference Rule. While the $\alpha$ correction method has shown good results in previous researches, it is highly dependent on the convergence capability of the two-dimensional viscous analysis. In the case of the Pressure Difference Rule, two-dimensional analysis may be performed using an inviscid panel method which is both robust and fast.

The Pressure Difference Rule is implemented in the existing Q3D aerodynamic analysis by replacing the viscous two-dimensional analysis with an inviscid 2D panel method, developed by the author. This panel method is based on a linear strength vortex model and is able to compute the derivatives of the outputs with respect to the inputs. The output of the panel method is the pressure difference between the suction peak and trailing-edge. This analysis is performed at several spanwise sections in order to establish a pressure difference distribution. Hereafter, an allowable pressure distribution is established based on chord Reynolds number and free stream Mach number. It is then determined for which angle of attack the pressure difference of one of the sections is equal to the allowable pressure difference at which maximum lift is expected. The respective $C_{L_{\text{max}}}$ is then determined by performing the VLM analysis at the specific angle of attack.

In order to include aeroelastic effects in both the implementation of the $\alpha$ correction method and the Pressure Difference Rule, the aerodynamic analyses are coupled to the structural solver FEMWET, enabling for the first time maximum lift prediction while taking into account wing elasticity. While coupling of the $\alpha$ method requires only replacing the induced angle of attack with the viscous correction angle as state variable, the Pressure Difference Rule is coupled using the Kreisselmeier-Steinhauser aggregation function. This function aggregates the condition for maximum lift of each separate section to a single governing equation.
Both systems are then solved using the Newton method for iteration.

In order to include the specific weight of the high-lift devices, a key aspect in high-lift device design optimization, the reference wing weight estimation was extended using the method of Torenbeek for computing the secondary weights. In this method, the secondary wing weight is computed from the separate weights of the fixed leading and trailing edge structure, high-lift devices, ailerons and spoilers. Aircraft performance is measured in terms of both cruise and airfield performance. Cruise performance is measured by fuel weight based on the fuel weight fraction method of Roskam in which the fuel use during cruise is obtained using the Bréguet range equation. Airfield performance is computed based on the method of Raymer which takes into account regulations as listed in FAR 25.

Validation of the implemented aerodynamic tools revealed that although the $\alpha$ correction method was able to accurately predict lift and drag up to stall, it failed to predict wing maximum lift as MSES proves to be highly unstable at high angles of attack. The Pressure Difference Rule on the other hand was able to accurately predict maximum lift with minimal computational effort, and is therefore used in the research as maximum lift prediction method. Furthermore, the computed sensitivities of the landing distance with respect to several design variables showed good resemblance. It can therefore be concluded that the developed method is suitable for performing combined aerostructural wing and high-lift system optimization.

As test case the aerostructural optimization of a Fokker 100 class wing is considered. Eight load cases are analyzed to compute structural failure performance, cruise performance and airfield performance. The optimization objective was fuel weight and constraints were put on structural failure, aileron effectiveness and take-off and landing distance. Two optimization schemes were examined, where one resembled a conventional optimization scheme, where the high-lift devices are bound to an optimized cruise wing design. The second optimization resembled the proposed combined optimization, including high-lift design considerations at the start of the optimization. The combined aerostructural optimization resulted in a fuel weight reduction of 9.65% while maintaining airfield performance of the reference aircraft, compared to a fuel weight reduction of 9.20% for the sequential optimization.

While the developed analysis and optimization framework enables the aerostructural analysis and optimization of an elastic wing, improvements can be made by replacing the semi-empirical Pressure Difference Rule by the physics-based $\alpha$ correction method for which MSES needs to be replaced by a more robust 2D viscous solver. Furthermore, more complex high-lift devices should be included to enable the analysis and optimization of a wide range of wing designs.
# Contents

<table>
<thead>
<tr>
<th>List of Figures</th>
<th>xi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomenclature</td>
<td>xiii</td>
</tr>
</tbody>
</table>

## 1 Introduction
1.1 Background ................................................. 1
1.2 Research goal .............................................. 1
1.3 Thesis outline ............................................. 2

## 2 Theoretical Background
2.1 Mission requirements ........................................... 3
2.2 High-lift devices ............................................ 5
  2.2.1 The need for high-lift devices ......................... 5
  2.2.2 High-lift device design process ....................... 6
  2.2.3 Flow physics of high-lift configurations ............. 8
  2.2.4 Design topology of high-lift-devices .................. 10
2.3 Aerostructural analysis and optimization framework ............ 11
  2.3.1 Q3D aerodynamic analysis .............................. 11
  2.3.2 FEMWET .................................................. 14
  2.3.3 Aerostructural coupling ................................ 16
  2.3.4 Coupled-adjoint method ............................... 17
2.4 Maximum lift prediction methods ................................ 17
  2.4.1 $\Gamma$ methods ....................................... 18
  2.4.2 $\alpha$ methods ....................................... 18
  2.4.3 Semi-emperical methods ............................... 19
2.5 Optimization algorithm ...................................... 21
  2.5.1 SNOPT ................................................... 21

## 3 Methodology
3.1 Two methods for the prediction of $C_{L_{\text{max}}}$ ............ 23
  3.1.1 $\alpha$ Method ........................................ 23
  3.1.2 Pressure Difference Rule .............................. 23
3.2 High-lift aerodynamic analysis .................................. 26
  3.2.1 High-lift vortex lattice method ....................... 26
  3.2.2 MSES - Multi-element airfoil design and analysis software .. 26
  3.2.3 Multi-element linear strength vortex panel method ........ 30
3.3 Aerostructural coupling ...................................... 31
  3.3.1 Systems of equations ................................... 31
  3.3.2 Sensitivity analysis ................................... 32
3.4 Airfoil shape parameterization .................................. 33
3.5 Wing weight estimation ...................................... 33
3.6 Aircraft performance model ................................... 35
  3.6.1 Cruise .................................................. 35
  3.6.2 Take-off ................................................ 36
  3.6.3 Landing .................................................. 37
3.7 Verification and validation .................................... 37
  3.7.1 VLM grid convergence study ......................... 37
  3.7.2 2D panel method verification ......................... 39
  3.7.3 Pressure Difference Rule ............................. 40
  3.7.4 $\alpha$ Method ........................................ 41
  3.7.5 Wing weight ............................................ 41
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Typical mission profile</td>
<td>3</td>
</tr>
<tr>
<td>2.2</td>
<td>Take-off procedure</td>
<td>4</td>
</tr>
<tr>
<td>2.3</td>
<td>Landing procedure</td>
<td>4</td>
</tr>
<tr>
<td>2.4</td>
<td>High lift device effects</td>
<td>6</td>
</tr>
<tr>
<td>2.5</td>
<td>Various types of high-lift devices</td>
<td>6</td>
</tr>
<tr>
<td>2.6</td>
<td>Airbus high lift design process in 1993 (1993)</td>
<td>7</td>
</tr>
<tr>
<td>2.7</td>
<td>Boeing 777 high lift design considerations (1995)</td>
<td>7</td>
</tr>
<tr>
<td>2.8</td>
<td>Airbus A380 high lift device design process (2002)</td>
<td>7</td>
</tr>
<tr>
<td>2.9</td>
<td>Trend in trailing edge high lift device complexity</td>
<td>8</td>
</tr>
<tr>
<td>2.10</td>
<td>Mutual interaction effect</td>
<td>9</td>
</tr>
<tr>
<td>2.11</td>
<td>Flow on high lift airfoil</td>
<td>10</td>
</tr>
<tr>
<td>2.12</td>
<td>High lift device topology definitions</td>
<td>10</td>
</tr>
<tr>
<td>2.13</td>
<td>Slot geometry effect</td>
<td>10</td>
</tr>
<tr>
<td>2.14</td>
<td>Flap cove design</td>
<td>11</td>
</tr>
<tr>
<td>2.15</td>
<td>Forces and angles of a 2D wing section</td>
<td>12</td>
</tr>
<tr>
<td>2.16</td>
<td>Vortex ring model</td>
<td>12</td>
</tr>
<tr>
<td>2.17</td>
<td>Forces and angles of wing sections due to wing deformation</td>
<td>14</td>
</tr>
<tr>
<td>2.18</td>
<td>Aerostructural mesh</td>
<td>15</td>
</tr>
<tr>
<td>2.19</td>
<td>Wing box element positions</td>
<td>15</td>
</tr>
<tr>
<td>2.20</td>
<td>Wing box modeling</td>
<td>16</td>
</tr>
<tr>
<td>2.21</td>
<td>$\alpha$ method routine</td>
<td>19</td>
</tr>
<tr>
<td>2.22</td>
<td>Critical pressure difference for given chord Reynolds number</td>
<td>20</td>
</tr>
<tr>
<td>2.23</td>
<td>PDR implementation</td>
<td>20</td>
</tr>
<tr>
<td>2.24</td>
<td>PDR validation for flap and slat deflection</td>
<td>20</td>
</tr>
<tr>
<td>3.1</td>
<td>RAE Wing planform</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>Q3D PDR procedure</td>
<td>25</td>
</tr>
<tr>
<td>3.3</td>
<td>Comparison of aerostructural High-lift aerostructural mesh</td>
<td>25</td>
</tr>
<tr>
<td>3.4</td>
<td>MSES automatic mesh generation</td>
<td>26</td>
</tr>
<tr>
<td>3.5</td>
<td>GA (W)-2 airfoil lift curve</td>
<td>26</td>
</tr>
<tr>
<td>3.6</td>
<td>MSES modification results for GA (W)-2 airfoil</td>
<td>27</td>
</tr>
<tr>
<td>3.7</td>
<td>Turbulence development</td>
<td>28</td>
</tr>
<tr>
<td>3.8</td>
<td>Effect of n-factor on 2D friction drag and pressure coefficient distribution</td>
<td>29</td>
</tr>
<tr>
<td>3.9</td>
<td>Linear strength vortex panel method nomenclature</td>
<td>30</td>
</tr>
<tr>
<td>3.10</td>
<td>High-lift aerostructural grid deformation</td>
<td>30</td>
</tr>
<tr>
<td>3.11</td>
<td>Airfoil shape perturbation using Chebyshev modes</td>
<td>32</td>
</tr>
<tr>
<td>3.12</td>
<td>Secondary wing weights</td>
<td>33</td>
</tr>
<tr>
<td>3.13</td>
<td>VLM grid convergence study</td>
<td>34</td>
</tr>
<tr>
<td>3.14</td>
<td>2D panel convergence study</td>
<td>37</td>
</tr>
<tr>
<td>3.15</td>
<td>Panel method comparison</td>
<td>39</td>
</tr>
<tr>
<td>3.16</td>
<td>RAE Wing planform</td>
<td>40</td>
</tr>
<tr>
<td>3.17</td>
<td>RAE airfoil design</td>
<td>40</td>
</tr>
<tr>
<td>3.18</td>
<td>Validation of $C_l_{\text{max}}$ computation</td>
<td>40</td>
</tr>
<tr>
<td>3.19</td>
<td>Comparison of aerodynamic properties the modified Q3D solver and experiment</td>
<td>41</td>
</tr>
<tr>
<td>4.1</td>
<td>Fokker 100 wing planform</td>
<td>43</td>
</tr>
<tr>
<td>4.2</td>
<td>Fokker 100 Lift over Drag ratio</td>
<td>44</td>
</tr>
<tr>
<td>4.3</td>
<td>Planform design variables</td>
<td>46</td>
</tr>
<tr>
<td>4.4</td>
<td>2D Airfoil shape design space</td>
<td>46</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.5</td>
<td>Extended Design Structure Matrix</td>
<td>48</td>
</tr>
<tr>
<td>4.6</td>
<td>Initial and optimized wing planforms</td>
<td>49</td>
</tr>
<tr>
<td>4.7</td>
<td>Combined optimization history</td>
<td>49</td>
</tr>
<tr>
<td>4.8</td>
<td>Initial and optimized wing deformed shapes under 2.5g pull up</td>
<td>50</td>
</tr>
<tr>
<td>4.9</td>
<td>Lift distribution on the initial wing and optimized wings in cruise condition</td>
<td>50</td>
</tr>
<tr>
<td>4.10</td>
<td>Lift distribution on the initial wing and optimized wings with flaps deployed in landing configuration</td>
<td>51</td>
</tr>
<tr>
<td>4.11</td>
<td>Initial and optimized flap configuration of the multi-element sections perpendicular to the sweep line in landing configuration</td>
<td>52</td>
</tr>
<tr>
<td>4.12</td>
<td>Initial and optimized airfoil shape on sections perpendicular to the sweep line</td>
<td>52</td>
</tr>
<tr>
<td>4.13</td>
<td>Initial and optimized pressure distribution on sections perpendicular to the sweep line</td>
<td>53</td>
</tr>
<tr>
<td>A.1</td>
<td>Pressure Difference Rule</td>
<td>64</td>
</tr>
</tbody>
</table>
## NOMENCLATURE

### LATIN SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>Speed of sound at standard sea-level conditions</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( b )</td>
<td>Wing span</td>
<td>[m]</td>
</tr>
<tr>
<td>( c )</td>
<td>Chord length</td>
<td>[m]</td>
</tr>
<tr>
<td>( c_f )</td>
<td>Flap chord length</td>
<td>[m]</td>
</tr>
<tr>
<td>( c_s )</td>
<td>Slat chord length</td>
<td>[m]</td>
</tr>
<tr>
<td>( c_r )</td>
<td>Root chord length</td>
<td>[m]</td>
</tr>
<tr>
<td>( c_t )</td>
<td>Tip chord length</td>
<td>[m]</td>
</tr>
<tr>
<td>( C_D )</td>
<td>Wing drag coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_{d_f} )</td>
<td>Section friction drag coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_{D_i} )</td>
<td>Induced wing drag coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_{dp} )</td>
<td>Section pressure drag coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_l )</td>
<td>Section lift coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_{ls} )</td>
<td>Section lift perpendicular to sweep line</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_{lsd} )</td>
<td>2D section lift coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_{I_a} )</td>
<td>Section lift curve slope</td>
<td>[1/rad]</td>
</tr>
<tr>
<td>( C_{I_{max}} )</td>
<td>Section maximum lift coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_L )</td>
<td>Wing lift coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_{L_{u}} )</td>
<td>Wing lift curve slope</td>
<td>[1/rad]</td>
</tr>
<tr>
<td>( C_{L_{max}} )</td>
<td>Wing maximum lift coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Pressure coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( \Delta C_{P_{crit}} )</td>
<td>Critical pressure difference (PDR)</td>
<td>[-]</td>
</tr>
<tr>
<td>( D )</td>
<td>Drag</td>
<td>[N]</td>
</tr>
<tr>
<td>( F )</td>
<td>Load</td>
<td>[N]</td>
</tr>
<tr>
<td>( F_{yield} )</td>
<td>Yield failure criteria</td>
<td>[-]</td>
</tr>
<tr>
<td>( F_{Euler} )</td>
<td>Euler buckling failure criteria</td>
<td>[-]</td>
</tr>
<tr>
<td>( F_{shear} )</td>
<td>Shear failure criteria</td>
<td>[-]</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravitational acceleration</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>( g_f )</td>
<td>Flap gap</td>
<td>[%]</td>
</tr>
<tr>
<td>( g_j )</td>
<td>Chebyshev polynomial</td>
<td>[-]</td>
</tr>
<tr>
<td>( G_j )</td>
<td>Chebyshev amplitude coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( h )</td>
<td>Height</td>
<td>[m]</td>
</tr>
<tr>
<td>( h_{cruise} )</td>
<td>Cruise altitude</td>
<td>[m]</td>
</tr>
<tr>
<td>( h_f )</td>
<td>Flap overlap</td>
<td>[%]</td>
</tr>
<tr>
<td>( h_s )</td>
<td>Screen height</td>
<td>[m]</td>
</tr>
<tr>
<td>( h_{t} )</td>
<td>Transition height</td>
<td>[m]</td>
</tr>
<tr>
<td>( K )</td>
<td>Stiffness</td>
<td>[N/m]</td>
</tr>
<tr>
<td>( L )</td>
<td>Lift</td>
<td>[N]</td>
</tr>
<tr>
<td>( M_{\infty} )</td>
<td>Free-stream Mach number</td>
<td>[-]</td>
</tr>
<tr>
<td>( M_{ff} )</td>
<td>Fuel fraction</td>
<td>[-]</td>
</tr>
<tr>
<td>( N )</td>
<td>Load factor</td>
<td>[-]</td>
</tr>
<tr>
<td>( R )</td>
<td>Range</td>
<td>[m]</td>
</tr>
<tr>
<td>( Re )</td>
<td>Reynolds number</td>
<td>[-]</td>
</tr>
<tr>
<td>( s )</td>
<td>Distance</td>
<td>[m]</td>
</tr>
<tr>
<td>( S )</td>
<td>Surface Area</td>
<td>[m²]</td>
</tr>
<tr>
<td>( S_a )</td>
<td>Aileron surface area</td>
<td>[m²]</td>
</tr>
<tr>
<td>( s_C )</td>
<td>Climb-out distance</td>
<td>[m]</td>
</tr>
</tbody>
</table>
\[s_{GR}\] Ground run distance [m]
\[s_{R}\] Rotation distance [m]
\[S_{ref}\] Reference surface area [m\(^2\)]
\[S_{s}\] Slat surface area [m\(^2\)]
\[s_{TR}\] Transition distance [m]
\[S_{sw}\] Wing surface area [m\(^2\)]
\[S_{swf}\] Flap surface area [m\(^2\)]
\[t_{u}\] Upper skin thickness [m]
\[t_{l}\] Lower skin thickness [m]
\[t_{fs}\] Front spar thickness [m]
\[t_{rs}\] Rear spar thickness [m]
\[T\] Thrust [N]
\[T_{TO}\] Takeoff thrust of critical engine [N]
\[U\] Displacement [m]
\[V_{\infty}\] Free stream velocity [m/s]
\[V_{\perp}\] Velocity perpendicular to sweep line [m/s]
\[V_{1}\] Takeoff decision speed [m/s]
\[V_{2}\] Takeoff climb speed [m/s]
\[V_{A}\] Landing approach speed [m/s]
\[V_{LOF}\] Liftoff speed [m/s]
\[V_{MC}\] Minimum control speed [m/s]
\[V_{MU}\] Minimum speed for safe flight with OEI [m/s]
\[V_{R}\] Rotation speed [m/s]
\[V_{1g}\] Stall speed at 1g [m/s]
\[W\] Weight [N]
\[W_{i}\] Initial fuel weight [N]
\[W_{f}\] Final fuel weight [N]
\[W_{fuel}\] Fuel weight [N]
\[W_{sec}\] Secondary wing weight [N]
\[W_{wing}\] Wing weight [N]
\[W_{wingbox}\] Primary structural wing weight [N]

**GREEK SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>Angle of attack</td>
<td>['']</td>
</tr>
<tr>
<td>(\alpha_0)</td>
<td>Zero lift angle of attack</td>
<td>['']</td>
</tr>
<tr>
<td>(\alpha_i)</td>
<td>Downwash angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\alpha_p)</td>
<td>Panel angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\alpha_{corr})</td>
<td>Viscous correction angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Flight path angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\Gamma)</td>
<td>Bound vortex strength</td>
<td>[m(^2)/s]</td>
</tr>
<tr>
<td>(\delta_f)</td>
<td>Flap deflection angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\delta_s)</td>
<td>Slat deflection angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\Delta n)</td>
<td>Shape perturbation</td>
<td>[m]</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>Wing twist angle, Error term</td>
<td>[''], [-]</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Taper ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>(\Lambda)</td>
<td>Wing sweep angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Normalized spanwise position</td>
<td>[-]</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Deflection angle</td>
<td>['']</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Dynamic viscosity</td>
<td>[N s /m(^2)]</td>
</tr>
<tr>
<td>(\mu_r)</td>
<td>Rolling friction Coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Air density</td>
<td>[kg/m(^3)]</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Normal stress</td>
<td>[N /m(^2)]</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Shear stress</td>
<td>[N /m(^2)]</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Adjoint vector</td>
<td>[-]</td>
</tr>
<tr>
<td>Acronyms</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>2D</td>
<td>Two-Dimensional</td>
<td></td>
</tr>
<tr>
<td>3D</td>
<td>Three-Dimensional</td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>Automatic Differentiation</td>
<td></td>
</tr>
<tr>
<td>AEO</td>
<td>All Engines Operative</td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>Aerodynamic Influence Coefficient</td>
<td></td>
</tr>
<tr>
<td>AR</td>
<td>Aspect Ratio</td>
<td></td>
</tr>
<tr>
<td>EASA</td>
<td>European Aviation Safety Agency</td>
<td></td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
<td></td>
</tr>
<tr>
<td>CR</td>
<td>Climb Rate</td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>Certification Specifications</td>
<td></td>
</tr>
<tr>
<td>CSRT</td>
<td>Class shape refinement Transformation</td>
<td></td>
</tr>
<tr>
<td>EASA</td>
<td>European Aviation Safety Agency</td>
<td></td>
</tr>
<tr>
<td>EOM</td>
<td>Equations Of Motion</td>
<td></td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aviation Administration</td>
<td></td>
</tr>
<tr>
<td>FAR</td>
<td>FAA Regulations</td>
<td></td>
</tr>
<tr>
<td>FD</td>
<td>Finite Difference</td>
<td></td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
<td></td>
</tr>
<tr>
<td>FM</td>
<td>Fowler Motion</td>
<td></td>
</tr>
<tr>
<td>FT</td>
<td>Full Throttle</td>
<td></td>
</tr>
<tr>
<td>GD</td>
<td>Gear Down</td>
<td></td>
</tr>
<tr>
<td>GU</td>
<td>Gear Up</td>
<td></td>
</tr>
<tr>
<td>KKT</td>
<td>Karush–Kuhn–Tucker</td>
<td></td>
</tr>
<tr>
<td>KS</td>
<td>Kreisselmeier-Steinhauser</td>
<td></td>
</tr>
<tr>
<td>LE</td>
<td>Leading Edge</td>
<td></td>
</tr>
<tr>
<td>LNG</td>
<td>Landing</td>
<td></td>
</tr>
<tr>
<td>LOF</td>
<td>Lift-Off</td>
<td></td>
</tr>
<tr>
<td>MDO</td>
<td>Multi-Disciplinary Design Optimization</td>
<td></td>
</tr>
<tr>
<td>MLW</td>
<td>Maximum Landing Weight</td>
<td></td>
</tr>
<tr>
<td>MTOW</td>
<td>Maximum Takeoff Weight</td>
<td></td>
</tr>
<tr>
<td>OEI</td>
<td>One Engine Inoperative</td>
<td></td>
</tr>
<tr>
<td>OL</td>
<td>Overlap</td>
<td></td>
</tr>
<tr>
<td>PDR</td>
<td>Pressure Difference Rule</td>
<td></td>
</tr>
<tr>
<td>Q3D</td>
<td>Quasi-Three-Dimensional</td>
<td></td>
</tr>
<tr>
<td>RAE</td>
<td>Royal Aircraft Establishment</td>
<td></td>
</tr>
<tr>
<td>RAM</td>
<td>Random Access Memory</td>
<td></td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds Averaged Navier Stokes</td>
<td></td>
</tr>
<tr>
<td>RHS</td>
<td>Right-Hand Side</td>
<td></td>
</tr>
<tr>
<td>SF</td>
<td>Safety Factor</td>
<td></td>
</tr>
<tr>
<td>SFC</td>
<td>Specific fuel consumption</td>
<td></td>
</tr>
<tr>
<td>SNOPT</td>
<td>Sparse Nonlinear Optimizer</td>
<td></td>
</tr>
<tr>
<td>SQP</td>
<td>Sequential Quadratic Programming</td>
<td></td>
</tr>
<tr>
<td>TE</td>
<td>Trailing Edge</td>
<td></td>
</tr>
<tr>
<td>TO</td>
<td>Take-off</td>
<td></td>
</tr>
<tr>
<td>VLM</td>
<td>Vortex Lattice Method</td>
<td></td>
</tr>
</tbody>
</table>
"One of the basic rules of the universe is that nothing is perfect. Perfection simply doesn't exist.....Without imperfection, neither you nor I would exist."

– Stephen Hawking
1 INTRODUCTION

1.1 BACKGROUND
Although knowledge of the physics of high-lift devices has come a long way since the fundamental paper of A.M.O Smith on high-lift aerodynamics in 1975 [1], analysis and optimization of high-lift devices still proves to be a difficult subject. Through the use of Computational Fluid Dynamics (CFD) and increased computing capabilities, extensive research on the subject has become possible. In early days, this research mainly focused on achieving high-lift requirements to satisfy take-off and landing performance requirements. However, over the past years the focus has switched to reducing weight and complexity [2] as aircraft manufacturers tend to use less complex high-lift devices [3]. The importance of weight and aerodynamic performance of high-lift devices in aircraft design is illustrated by Meredith [4]. According to Meredith, an increase of 0.1 in lift coefficient at constant angle of attack results in a reduction of approach attitude by about one degree, reducing landing gear length and thereby saving up to 1400 lb. Moreover, an increase of 1.5% in maximum lift coefficient ($C_{L_{max}}$) may result in an extra 6600 lb payload at fixed approach speed while an 1% increase in take-off lift over drag ratio (L/D) is equal to a 2800 lb increase in payload or a 150 nm range increase.

Even though numerous semi-empirical methods exist to predict the wing weight, drag and lift of multi-element wings [5–9], the accuracy of these methods does not yield the level of accuracy required by the industry, requiring e.g. a drag prediction accuracy of one drag count [10]. To achieve the required accuracy, more physics based methods are required such as CFD and Finite Element Methods (FEM) tools. The downside of these tools is that they require the use of high performance computational resources, making optimization problems too costly to solve. An alternative to the high-fidelity 3D aerodynamic solvers is the quasi-three-dimensional (Q3D) methodology, which combines two-dimensional (2D) viscous airfoil data with inviscid three-dimensional (3D) wing aerodynamic data. This methodology requires only a portion of the computational power required for high-fidelity tools while generating sufficiently accurate results and has been employed by Van Dam [2] and Mariens et al. [11]. Elham and Van Tooren developed a coupled-adjoint aerostructural analysis and optimization tool by coupling a Q3D method to a FEM [12]. This tool has been validated for drag prediction and twist deformation and using coupled-adjoint method, the tool is able to compute the derivatives of the outputs with respect to the inputs enabling gradient based optimization.

Current wing design processes are strongly governed by cruise design requirements. This approach to wing design leaves little room for high-lift device design optimization where trade-offs may be required between cruise wing and high-lift wing design [2, 3]. Moreover, the optimization of high-lift devices typically focuses on optimizing flap/slat setting, gap and overlap [13–16], without taking into account aeroelastic effects on high-lift performance and the effect of flap chord and span on wing weight. Ideally, the aerostructural optimization of the high-lift system and cruise wing should therefore be combined throughout the complete optimization process.

1.2 RESEARCH GOAL
In this research the previously mentioned aerostructural tool developed by Elham and Van Tooren [12] is extended to perform analysis and optimization of multi-element wings in high-lift conditions, as well as clean wings in cruise conditions. The extended tool is able to compute the derivatives of the additional outputs with
respect to the inputs using analytical methods, enabling gradient based optimization. The research objective of this thesis is formulated as follows:

- Develop a framework for the combined aerostructural optimization of cruise wing design and high-lift devices.

In order to achieve this research objective, the following three sub-objectives are identified:

1. Incorporate high-lift devices in the present aerostructural tool and enable high-lift aerodynamic analysis.
2. Validate the extended aerostructural tool.
3. Perform a full aerostructural test case optimization.

1.3 Thesis Outline
The structure of this thesis is as follows: First, in Chapter 2 the fundamentals of high-lift devices and their design methodology are described, followed by a description of the existing aerostructural analysis and optimization tool developed by Elham and Van Tooren [12]. Hereafter, a number of methods that can be used to perform high-lift aerodynamic analysis are described. In Chapter 3 the methodology of the current research is described, starting with the implementation of the high-lift aerodynamic analysis. Then the coupling of the high-lift aerostructural model is described followed by the method of computing the wing weight and airfield performance. Chapter 3 finishes with validation of the extended tool. Finally, in Chapter 4 a test case optimization is presented and in Chapter 5 conclusions and recommendations are provided.
In this chapter the theoretical background on which the research presented in this report is based is discussed. As mentioned in the introduction of this report, the research focuses on the combined optimization of cruise wing design and high-lift wing design. In order to assess the scope of this research, Section 2.1 starts off with a description of a typical mission profile, along with airfield regulations. In Section 2.2 the fundamentals of high-lift device aerodynamics and their design methodology are described, followed by an extensive review of the aerostructural analysis and optimization framework on which this research builds in Section 2.3. Finally, in Section 2.4 different methods of performing high-lift aerodynamic analysis are described.

2.1 MISSION REQUIREMENTS

Aircraft design typically starts with defining top-level design requirements such as range, cruise speed and payload. The range is defined as the distance traveled between take-off and landing and aircraft typically follow a specific mission profile within this range (see Figure 2.1). Current aircraft design processes mainly focus on cruise performance which determines fuel usage at large. The present research aims to extend this focus to take-off and landing performance, which in fact have a large impact on wing design.

Airfield performance is governed by take-off and landing performance, which are typically defined as take-off and landing distance. Take-off and landing distance are a function of wing loading, thrust loading and the aerodynamic efficiency of the wing. Take-off starts from standstill from which the aircraft accelerates to rotation speed during the pre-rotation phase at which point the nose wheel leaves the ground. At lift-off speed the aircraft lifts off and accelerates to the take-off safety speed at climb rate \( \gamma \). The total take-off distance is comprised of the ground run distance and the airborne distance to fly over an obstacle height of 35ft. A graphical representation of the take-off procedure is presented in Figure 2.2:

![Figure 2.1: Typical mission profile](image)
Landing starts with the approach at an altitude of 50ft. During the approach run, the aircraft flies at an approach speed $V_A$ at flight path angle $\gamma_A$. This approach is continued up to a point where the aircraft starts flaring and the nose is pulled up. The aircraft keeps flaring the aircraft up to touchdown while continuously reducing the climb rate. Upon touchdown, the aircraft breaks until standstill. Landing performance is typically measured by the approach speed required to achieve the required landing distance. Another aspect of importance is the pilot’s ground visibility, which is governed by the pitch angle of the fuselage and consequently the angle of attack. This puts limits on the angle of attack at which high-lift requirements are reached. In Figure 2.3 the take-off procedure is shown graphically.

Take-off and landing performance, also known as airfield performance, is regulated by the Federal Aviation Authority (FAA) and European Aviation Safety Agency (EASA). The regulations for transport category aircraft are specified in respectively FAA Regulations 25 (FAR-25) [18] and Certification Specifications 25 (CS-25) [19]. While different in name, the regulations as specified in these documents are the same and are listed in Table 2.1.
2.2 HIGH-LIFT DEVICES

In the previous section, the mission profile and regulations for airfield performance were described. In this section, it will be discussed why high-lift devices are required to both optimize cruise performance and airfield performance, and how the design of high-lift devices impacts wing design. First the relevant factors that play a role in cruise and airfield performance are described after which current wing design methodologies are assessed. This is followed by a description of current trends in high-lift device design. Finally, in order to obtain an understanding of the design and analysis of high-lift devices, the relevant flow physics are reviewed, followed by conventional design topology.

### 2.2.1 THE NEED FOR HIGH-LIFT DEVICES

In order to appreciate the need for high-lift devices, the historical development of aircraft cruise speed needs to be considered. As aircraft tended to achieve higher cruise speeds, swept wing designs with larger wing loadings and thus smaller wing areas were required to reduce wing drag and fuel usage. As a consequence, larger lift coefficients are required to meet take-off and landing performance requirements which becomes evident when considering Equation 2.1.

\[
C_L = \frac{1}{2} \rho V_\infty^2 \left( \frac{W}{S_w} \right) \quad (2.1)
\]

It becomes clear that aircraft optimized for cruise flight, may not meet take-off and landing performance requirements. To illustrate this fact Van Dam [2] used Equation 2.2, the Bréguet range equation, to compare the aerodynamic efficiency of two aircraft: one optimized for cruise, and one for take-off and landing. The results for the two aircraft are listed in Table 2.2.

\[
R = \frac{V_\infty L}{C_T D} \ln \left( \frac{W_i}{W_e} \right) \quad (2.2)
\]

<table>
<thead>
<tr>
<th>Table 2.2: Performance comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>( M_\infty )</td>
</tr>
<tr>
<td>( C_L )</td>
</tr>
<tr>
<td>( M_\infty \frac{L}{D} )</td>
</tr>
<tr>
<td>( \left( \frac{W}{S_w} \right)_{TO} )</td>
</tr>
<tr>
<td>( V_A )</td>
</tr>
<tr>
<td>( \left( \frac{W}{S_w} \right)_{Land} )</td>
</tr>
<tr>
<td>( C_{L_{max}} )</td>
</tr>
<tr>
<td>( C_{L_{max}} )</td>
</tr>
</tbody>
</table>

Aircraft B, being optimized for take-off and landing performance, has a much lower wing loading than Aircraft A but shows a huge drag penalty in cruise operation resulting in a reduction of range. A compromise is thus needed to achieve both efficient cruise performance and required airfield performance, for which
high-lift devices pose a solution.

High-lift devices are devices which are deployed during take-off and landing from the wing and are used to increase airfield performance. This allows a wing design to both have a large wing loading during cruise when high-lift devices are retracted and a small wing loading when they are deployed. High-lift devices can be categorized in leading edge devices and trailing edge devices. A detailed description of different types of high-lift devices is provided by Rudolph [20]. In general, trailing edge devices cause an upward shift of the lift curve. In other words, at the same angle of attack, a higher lift coefficient may be achieved when trailing edge devices are deployed. Leading edge devices on the other hand, do not generate more lift at the same angle of attack but allow for a postponement of flow separation. This allows for a higher usable angle of attack and also a higher maximum lift coefficient. In Figure 2.4 a general overview of these effects is shown. It can also be observed that with increased complexity of trailing edge devices, the maximum attainable lift coefficient increases. However, increasingly complex high-lift systems have a large weight penalty which needs to be taken into account during wing design.

![Figure 2.4: High lift device effects](image)

![Figure 2.5: Various types of high-lift devices](image)

### 2.2.2 HIGH-LIFT DEVICE DESIGN PROCESS

According to Flaig and Hilbig, the high-lift device design process can be categorized into three stages: pre-development, development and pre-flight [21]. A graphical representation of this design process is shown in Figure 2.6. In the pre-development phase typically analysis is performed on a range of configurations from which the best suitable configuration is chosen. This process has become increasingly fast and effective through the use of CFD. While during the analysis and optimization of high-lift systems, a lot of attention is put on achieving required wing maximum lift ($C_{l_{\text{max}}}$), from Figure 2.7 it becomes evident that the high-lift system design also needs to meet a large number of other requirements such as weight, cost and maintenance. Van Dam pointed out that the wing design process as depicted in Figures 2.6 and 2.7 is strongly governed by cruise design requirements [2], leaving little room for the optimization of the high-lift system design. Although the design process in Figure 2.6 dates back to 1993, not much has changed when considering the high-lift design process of the Airbus A380 wing, shown in Figure 2.8. In the presented design process, the high-lift system is very much dependent on the cruise wing design. Furthermore, present research on high-lift device optimization tends to focus on 2D design optimization for optimized cruise wing designs [13–16].
2.2. **High-Lift Devices**

- **Figure 2.6**: Airbus high lift design process (1993) [21]
- **Figure 2.7**: Boeing 777 high lift design considerations (1995) [22]
- **Figure 2.8**: Airbus A380 high-lift device design process (2002)[3]
When considering the different types of high-lift systems that have been employed by Boeing, Douglas and Airbus over the past years (see Figure 2.9), it becomes clear that there exists a tendency to achieve aerodynamic requirements with minimum system complexity. This is due to the fact that from a weight and cost point of view, the single-slotted flap is the best solution as a high-lift system [20]. The single-slotted flaps however pose two major obstacles. The first is that single-slotted flaps may produce maximum lift coefficients too low to perform landing and the second is that they would require a too large angle of attack to provide the required airfield performance. Rudolph proposes a number of solutions to these challenges which include both 2D high-lift design adjustments such as increasing flap deflection angle and 3D adjustments such as increasing flap span and wing area.

In order to achieve wing designs with minimal complexity and optimal performance, it is therefore crucial to implement a combined multidisciplinary optimization of the cruise and high-lift wing design, from the start of any design project.

![Figure 2.9: Trend in trailing edge high-lift device complexity][3, 23]

### 2.2.3 Flow Physics of High-Lift Configurations

The aerodynamic analysis of high-lift configurations has always been a challenge due to its complex nature and the dependency on viscous effects, as well as 3D effects. According to Reckzeh, the maximum lift of a well-designed high-lift system should be determined by flow separation on the main wing or leading edge device [3]. In reality local disturbances and viscous effects however have a large effect on the performance of high-lift devices which may cause flow separation before the maximum lift capability of the high-lift system is reached. According to Obert [24] there are three effects that determine the performance of high-lift devices:

1. **An increase in camber**  
   An increase in camber results in a higher maximum lift coefficient. This increase in camber may be achieved by deflecting the trailing-edge of an airfoil, resulting in more circulation and thus more lift at the same angle of attack. Leading edge deflection generally results in a decrease in lift due to leading-edge droop at low angles of attack. In the design of high-lift devices care must be taken to prevent excessive camber which produces excessive drag. This can be resolved by applying variable camber to high-lift devices.

2. **An increase in effective chord**  
   When a Fowler flap moves aft, it increases the effective chord. The same holds for a slat which moves down and forward. As the wing reference area usually remains unaltered due to this chord extension, the lift curve slope increases as a result of this chord extension.

3. **The mutual interaction effect**  
   An extensive analysis of the mutual interaction effects of multi-element wing configurations is done by A.M.O. Smith [1]. He described five main effects that occur due to the interaction of the various elements in high-lift configurations:
   - **Slat effect**: the slat, in essence a point vortex which incurs velocities that counter the velocities on the main element, reducing the pressure peak on the main element and with it the chance of flow separation (see Figure 2.10a).
   - **Circulation effect**: contrary to the slat effect, the circulation from a downstream element causes the adjacent upstream element to be in a high velocity region inclined to the mean line at the rear...
of the upstream element. In order to satisfy the Kutta condition, which states that flow should leave the airfoil at the trailing edge tangential to the trailing edge, an increased circulation is therefore induced on the upstream element which effectively increases the lift generated by that element (see Figure 2.10b).

- **Dumping effect**: as the trailing edge of an upstream element is in a region of velocity higher than freestream velocity, its boundary layer is “dumped” at a high velocity. As a result, the pressure recovery on the boundary layer is relieved and an increased lift is permitted (see Figure 2.10c).
- **Off-the-surface pressure recovery effect**: the boundary layer from forward elements is dumped at velocities higher than freestream velocity, and becomes a wake. The recovery to freestream velocity is thus done away from the element surface which is a far more effective as wakes can withstand larger adverse pressure gradients than boundary layers (see Figure 2.10d).
- **Fresh boundary layer**: each new element starts out with a fresh thin boundary layer at its leading edge, reducing the chance of flow separation as multiple thin boundary layers can withstand stronger adverse gradients than a single thick boundary layer.

![Figure 2.10: Mutual interaction effect [1, 25](a) Slat effect](image1)

![Figure 2.10: Mutual interaction effect [1, 25](b) Circulation effect](image2)

![Figure 2.10: Mutual interaction effect [1, 25](c) Dumping effect](image3)

![Figure 2.10: Mutual interaction effect [1, 25](d) Off-the-surface pressure recovery effect](image4)

Although understanding 2D effects is useful for optimization of the 2D multi-element airfoil design, as mentioned before, 3D effects also have a large effect on the performance of high-lift devices. Local separation regions such as behind the engine nacelle, fuselage juncture and wing tip need to be taken into account when designing the high-lift wing. In Figure 2.11 an overview of both 2D and 3D flow physics of a high-lift wing is presented.
2.2.4 DESIGN TOPOLOGY OF HIGH-LIFT-DEVICES

Typically, three parameters are used to define trailing edge and leading edge high-lift device settings, the Fowler motion (FM), gap (g) and overlap (h). For respectively leading edge and trailing edge devices, the Fowler motion is defined as:

\[
FM_{LE} = \frac{x_s}{c} \tag{2.3}
\]

\[
FM_{TE} = \sum_{n=1}^{N} x_{fn} \tag{2.4}
\]

The Fowler motion is thus a measure for the chord extension due to deployment of the high-lift devices. Large Fowler motions result in larger lift over drag (L/D) ratios as they come at low drag cost compared to increasing camber due to flap rotation. Two definitions of the gap can be used, the orthogonal and vertical definition, of which the orthogonal definition is most common. Both definitions are shown below in Figure 2.12.

To illustrate the importance of the gap and overlap settings on the performance of the high-lift device system, Figures 2.13a and 2.13b show the effect of the gap and overlap on local \( C_{l_{max}} \) for a flap deflection of respectively 35 and 40 degrees. From the figure it can be determined that for a flap deflection of 35 degrees, the optimum flap setting has an overlap of 1.0 % and a gap of 3.9 %. When the flap is deflected 40 degrees, these values change to -.7 and 3.8 % respectively. Considering the fact that in the design of a high-lift system, also aeroelastic effects and 3D effects need to be taken into account, proper design of the high-lift system becomes a complex task.
2.3 Aerostructural Analysis and Optimization Framework

The present research expands on the coupled-adjoint aerostructural analysis and optimization tool developed by Elham and van Tooren [12]. This section describes the fundamentals of this tool. First, the Q3D methodology is described followed by a description of the structural solver FEMWET. The method of coupling the aerodynamic and structural solvers is then presented along with a description of the method that is used to solve the coupled system. At the end of this section, a brief explanation on the coupled-adjoint method is given as it plays an essential part in determining the gradients required for gradient based optimization.

2.3.1 Q3D Aerodynamic Analysis

Aerodynamic analysis of a wing can be performed through numerous methods. These methods range from relatively simple empirical methods to complex 3D CFD solvers. While the use of (semi-)empirical methods [5–9] requires minimal amount of computational effort, the accuracy of these methods does not yield the level of accuracy required by the industry, which for instance requires a drag prediction accuracy of one drag count [10]. On the other hand, the computational effort required by 3D CFD solvers makes optimization too costly to solve.

In their research, Elham and Van Tooren used a methodology which combines 3D inviscid aerodynamic analysis with 2D viscous analysis following the Q3D methodology described by Mariens et al. [11]. This method has been proven to produce accurate results compared to CFD solvers, while requiring only a small portion of the computational effort. In the Q3D analysis, 3D inviscid analysis is performed using a Vortex Lattice Method (VLM). Then using strip theory [29], 2D viscous analysis is performed at a number of spanwise sections using the effective velocity and angle of attack. In Figure 2.15 the forces and angles of the 2D wing sections are shown.

A final aspect to take into account when determining optimum flap settings is the flap cove design. During analysis of the GA (W)-2 airfoil, it was found that for similar flap settings, a rounded entry lip as shown in Figure 2.14 resulted in a loss of $C_{\text{max}}$ of 0.21 compared to a sharp entry lip [28]. Care should therefore be taken during optimization of high-lift systems when flap sealing is considered to optimize cruise performance, for which sharp entry lips are required.

![Figure 2.13: Slot geometry effect on $C_{\text{max}}$ [27]](image)

![Figure 2.14: Flap cove design [28]](image)
From this figure, it can be seen that the total drag of a wing section depends on effective lift, drag and the induced angle of attack ($\alpha_i$). The effective sectional drag can be decomposed into three components: form drag, friction drag and induced drag. The total sectional drag may therefore be written as:

$$d_{\infty} = d_{p,\text{eff}} \cos \alpha_i + d_{f,\text{eff}} \cos \alpha_i + l_{\text{eff}} \sin \alpha_i$$

(2.5)

In this equation, the first term is called the pressure drag, also known as form drag. This drag component originates from the airfoil shape and the resulting flow over the airfoil [30]. The second term is the friction drag, which originates from the interaction between the airfoil surface and air that flows over it. The sum of these two terms is called the parasite drag. The final component in Equation 2.5 is called the induced drag, which is caused by the effective tilting of the lift vector due to the downwash angle ($\alpha_i$). Downwash results from the wing tip vortex and therefore becomes significant near the tip of the wing.

When computing wing drag in a 3D analysis, the induced drag is included in the pressure drag, resulting in just two drag components that need to be determined. This type of analysis is called a near-field analysis [31], in which the total wing drag is determined by integrating the pressure and friction drag around the wing body. In Q3D analysis, this method of computing the wing drag is not possible as the induced drag is not computed in strip theory. For this reason, the induced drag is determined through Trefftz Plane analysis using the VLM, which is a form of far-field analysis [32, 33].

The Q3D method is initialized by computing the lift distribution at a given angle of attack using a ring VLM, based on the method of Katz and Plotkin [34]. In this method, a ring vortex is placed around each collocation point, which is placed at the center of the panel three-quarter chord lines. The leading segment of the ring vortices are then placed at the quarter chord lines of the panels. The wing is followed by free wake vortices starting from the trailing edge, continuing to infinity. To take into account airfoil camber, the boundary conditions are applied on the wing camber line. A graphical representation of the ring VLM is shown in Figure 2.16.

Using wing geometry and angle of attack, the Aerodynamic Influence Coefficients (AIC) matrix and the Right Hand Side (RHS) vector are then computed. While the AIC depends purely on wing geometry, the RHS
2.3. Aerostructural Analysis and Optimization Framework

is computed through:

\[ \text{RHS} = -V_\infty (\cos(\alpha)\mathbf{n}_i + \sin(\alpha)\mathbf{n}_k) \]  

(2.6)

Where \( \mathbf{n}_i \) and \( \mathbf{n}_k \) are the normal vectors in respectively x and z direction (see Figure 2.16). For a complete derivation of the AIC and RHS vector the reader is referred to [34]. The strengths of the vortex rings (\( \Gamma \)) are then computed by Equation 2.7:

\[ \text{AIC} \Gamma = \text{RHS} \]  

(2.7)

The wing lift distribution is then calculated using the Kutta Joukowski theorem based on \( \Gamma \). This theorem states that the unit lift per span (\( L' \)) is equal to:

\[ L' = \rho V_\infty \Gamma \]  

(2.8)

The results are corrected for compressibility effects at high Mach numbers using the Prandtl Glauert compressibility correction. Applying this correction to e.g. the vortex strengths yields:

\[ \Gamma = \frac{\Gamma}{\sqrt{1 - M_\infty^2}} \]  

(2.9)

The next step in the Q3D analysis is to divide the wing into several sections for the 2D analysis. The flow properties at each section are determined from the 3D flow properties using two steps of transformation. First, sweep theory is applied to find the airfoil geometry and flow characteristics perpendicular to the sweep line:

\[ y_\perp = y_i \cos \Lambda \]  

(2.10)

\[ M_\perp = M_\infty \cos \Lambda \]  

(2.11)

\[ V_\perp = V_\infty \cos \Lambda \]  

(2.12)

\[ \alpha_\perp = (\alpha + \epsilon) / \cos \Lambda \]  

(2.13)

\[ C_{l_\perp} = C_l / \cos^2 \Lambda \]  

(2.14)

Here \( \epsilon \) is the local wing geometrical angle and the value of \( C_l \) is determined by interpolating the spanwise lift distribution computed by the VLM analysis. It should be noted that in the subsonic regime, the wing quarter-chord sweep angle should be used [24] and at transonic velocities the half-chord chord sweep [35].

The second step of transformation is performed to determine the airfoil effective angle of attack, Mach number and Reynolds number at each wing section in order to apply strip theory. This is done using the following transformations (Also see Figure 2.15):

\[ M_{\text{eff}} = M_\perp / \cos \alpha_i \]  

(2.15)

\[ \text{Re}_{\text{eff}} = \rho V_\perp c_\perp \cos \alpha_i / \mu \]  

(2.16)

\[ \alpha_{\text{eff}} = \alpha_\perp - \alpha_i \]  

(2.17)

In order to take into account downwash and wing elasticity, one last transformation is required to compute the 2D lift coefficient. This transformation is shown in Figure 2.17 and is performed as follows:

\[ C_{l_{2d}} = C_{l_{\text{eff}}} \cos \alpha_i - (C_{d_{p\text{eff}}} + C_{d_{\text{eff}}}) \sin \alpha_i \cos \theta \]  

(2.18)
The total sectional drag parallel to the free stream is computed by transforming Equation 2.5 into coefficients and applying sweep theory:

\[ C_d = C_{d_{\text{par}}} \frac{1}{\cos \alpha_i} + C_{d_{\text{f}}} \frac{\cos^3 \Lambda}{\cos \alpha_i} + C_{l_{\text{f}}} \frac{\cos^3 \Lambda \sin \alpha_i}{\cos^2 \alpha_i} \]  

(2.19)

As mentioned before, the induced drag is computed in the Trefftz plane analysis which is a more accurate method compared to the near field analysis proposed in equation 2.19. The total wing drag is therefore computed by:

\[ C_D = C_{D_{\text{par}}} + C_{D_i} \]  

(2.20)

2.3.2 FEMWET

The core of the structural analysis is performed by the 'Finite element based Elham Modified Weight Estimation Technique' (FEMWET) tool [36]. This tool is used for sizing of the wing structure and is capable of calculating the aeroelastic deformation and the resulting stresses, while providing an accurate wing weight estimation. The advantages of using FEMWET over a high-fidelity structural solver is that it is computationally efficient, and does not require detailed knowledge of the wing structure as it models the wing box as four equivalent panels, used as design variables. Furthermore, the tool is able to compute the sensitivities of the outputs with respect to the inputs, enabling efficient gradient based optimization.

Within FEMWET, the wingbox structure is modeled as a collection of beam elements along the wing span. In order to prevent unnecessary interpolation, the spanwise position of the nodes of the beam elements are at the same position as the airfoil sections analyzed in the Q3D analysis. These nodes are then positioned at the wing box shear center determined using the method from Megson [37]. A graphical representation of the node positioning is presented in Figure 2.18.
2.3. AEROSTRUCTURAL ANALYSIS AND OPTIMIZATION FRAMEWORK

After determining the node positions, the element stiffness, mass and force matrices are constructed using the shape functions for a 3D 2-node Timoshenko beam element [38]. For a complete explanation of the method, the reader is referred to [36]. With the stiffness matrix \( K \) and force matrix \( F \), the displacement vector \( U \) can be computed by solving:

\[
KU - F = 0 \tag{2.21}
\]

The displacement vector is then used to compute the stress distribution in the wingbox structure. In order to compute this stress distribution, the wing box four equivalent panels are divided into small elements of which each position is measured from the shear center (see Figure 2.19).

Using the computed stress distribution, failure criteria are then set up based on material yield stress, Euler buckling and shear buckling using Equations 2.22 to 2.24:

\[
F_{\text{yield}} = \frac{\sigma \times SF}{\sigma_{\text{yield}}} - 1 \tag{2.22}
\]

\[
F_{\text{Euler buckling}} = \frac{\sigma \times SF}{\sigma_{\text{buckling}}} - 1 \tag{2.23}
\]

\[
F_{\text{shear buckling}} = \frac{\tau \times SF}{\tau_{\text{buckling}}} - 1 \tag{2.24}
\]
Here $SF$ is the safety factor which is set to 1.5 for all analysis as defined by the FAA [18].

As mentioned before, the wing box structure is modeled using four equivalent panels of which the thicknesses are defined as design variables. Using this method, the total wing box weight is computed from the primary structural weight and the secondary weight. The primary weight includes all structural elements which transfer the loads acting on the wing. These include the equivalent panels, the ribs and the non-optimum structural elements, which include joints, connections and attachments. The upper and lower equivalent panels include skin, stiffeners and spar caps. The front and rear panels represent the spar webs of the wing box. The weight of the primary structure is then computed by equating the volume of the material with the material density. In Figure 2.20, a representation of the wing box internal structure modeling is presented.

In the research of Elham and Van Tooren [12], the weights not included in the FEM analysis are accounted for by multiplying the wing box primary structural weight with 1.5, based on the work of Kennedy and Martins [39]. The secondary weight, including high-lift devices, leading edge and trailing edge structure and control surfaces, are estimated to be equal to 15 times the wing surface area ($S_{ref}$), based on the work of Torenbeek [40]. The total wing weight is then computed by Equation 3.36.

$$W_{\text{wing}} = 1.5W_{\text{FEM Wingbox}} + 15S_{\text{ref}}$$ (2.25)

### 2.3.3 Aerostructural Coupling

In order to solve the aerostructural problem, four governing equations are defined. The first two governing equations relate to respectively the VLM analysis and FEM analysis and are given by Equations 2.7 and 2.21. The third equation states that the calculated lift should be equal to the aircraft weight and the fourth governing equation is used to equate the computed lift distribution from the VLM to the lift distribution from the strip theory. This system is solved for the four state variables: Vortex strengths ($\Gamma$), node deflections ($U$), angle of attack ($\alpha$) and section downwash angles ($\alpha_i$). The coupled aerostructural system can then be formulated as follows:

$$\begin{bmatrix} R1(\Gamma, U, \alpha, \alpha_i) \\ R2(\Gamma, U, \alpha, \alpha_i) \\ R3(\Gamma, U, \alpha, \alpha_i) \\ R4(\Gamma, U, \alpha, \alpha_i) \end{bmatrix} = \begin{bmatrix} AIC \Gamma - RHS \\ KU - F \\ L - nW_{des} \\ C_{1ld} - C_{1d} \end{bmatrix} = \delta$$ (2.26)

In order to solve this system efficiently, the Newton method for iteration is used where the updates on the state variables are determined using Equation 2.27:
2.4. **Maximum lift prediction methods**

While the method of Elham and Van Tooren described in Section 2.3 is suitable for computing the aerodynamic loads at cruise conditions, it is as of yet unable to compute the maximum wing lift coefficient ($C_{L \text{max}}$). One method of taking into account nonlinear effects of wing lift at high angles of attack is nonlinear lifting-line theory. Two implementations for this theory exist: the Gamma ($\Gamma$) methods which apply corrections on...
the circulation over the wing and alpha ($\alpha$) correction methods which apply the viscous correction through changes in angle of attack. An alternative to nonlinear lifting-line theory is using semi-empirical methods, such as the Pressure Difference Rule (PDR) which combines physical data with empirical data.

2.4.1 $\Gamma$ METHODS

Circulation based correction methods apply corrections over the wing lift distribution to take into account nonlinearity. A typical $\Gamma$ method starts with an initial assumed wing lift distribution after which the following iterative procedure is started:

1. The induced angle of attack of each section $y_n$ is computed using:

$$\alpha_i = \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{(d\Gamma/dy)dy}{y_n - y}$$

(2.34)

2. The viscous lift data is determined at the effective angle of attack using a 2D viscous analysis tool:

$$\alpha_{eff} = \alpha - \alpha_i$$

(2.35)

$$C_{l_{visc}} = 2D \text{ airfoil analysis tool}(\alpha_{eff}, M_{eff}, Re_{eff})$$

(2.36)

3. A new circulation distribution is computed using the Kutta-Joukowski theorem based on the freestream velocity $V_\infty$, local chord $c$ and the local viscous sectional lift $C_{l_{visc}}$:

$$\Gamma_{new} = \frac{1}{2} V_\infty c C_{l_{visc}}$$

(2.37)

4. The lift distribution is updated using a relaxation factor $\beta$:

$$\Gamma_{input} = \Gamma_{old} + \beta(\Gamma_{new} - \Gamma_{old})$$

(2.38)

5. This procedure is repeated until $|\Gamma_{new} - \Gamma_{old}| \leq \epsilon$, where $\epsilon$ is a user-selected convergence criteria.

A downside of this method is that it requires the under-relaxation factor $\beta$ to converge, and artificial viscosity needs to be implemented in order to converge in post-stall conditions.

2.4.2 $\alpha$ METHODS

Angle of attack based correction methods ($\alpha$ methods) apply corrections to the angle of attack of the inviscid analysis instead of the lift distribution. One of these methods has been developed by Van Dam in order to compute $C_{l_{max}}$ [43]. In his method 2D viscous flow data is obtained from either experimental data or CFD tools along a number of spanwise sections. From this analysis, the zero lift angle of attack $\alpha_0$ and lift-curve slope $C_{l_a}$ at each section is computed after which the following iterative procedure is started:

1. The effective local angle of attack for each spanwise section is computed using:

$$\alpha_{eff} = \frac{C_{l_{inv}}}{C_{l_a}} + \alpha_0 - \Delta_{corr}$$

(2.39)

2. At this effective local angle of attack, the viscous lift coefficient is computed through interpolation of the viscous data or from a viscous analysis tool:

$$C_{l_{visc}} = \text{airfoil analysis tool}(\alpha_{eff}, M_{eff}, Re_{eff})$$

(2.40)

3. Then if $|C_{l_{visc}} - C_{l_{inv}}| > \epsilon$, where $\epsilon$ is again a user-selected convergence criteria, the appropriate viscous correction angle is determined by:

$$\alpha_{corr} = \frac{C_{l_{visc}} - C_{l_{inv}}}{C_{l_a}}$$

(2.41)

4. Using $\alpha_{corr}$, the RHS of the 3D inviscid analysis is adjusted and a new lift distribution is computed.
2.4. **MAXIMUM LIFT PREDICTION METHODS**

This procedure is repeated until \( |C_{\text{disc}} - C_{\text{inv}}| \leq \epsilon \)

A graphical representation of the method is shown in Figure 2.21. This procedure needs to be performed for a complete range of angles of attack in order to find the stall angle and \( C_{L_{\text{max}}} \) from the resulting lift curve.

![Figure 2.21: a method routine](image)

In their research, Gallay and Laurendeau [44] suggest to replace \( C_{l_{\alpha}} \) in Equation 2.41 by \( 2\pi \), removing the dependency on the actual profile being used. This prevents numerical difficulties at maximum lift, where \( C_{l_{\alpha}} \) becomes zero and thus the equation does not hold. They concluded that this modification improves convergence in post-stall conditions. It should be noted however that in reality, the shape of the profile cannot be neglected and the actual and the actual sectional \( C_{l_{\alpha}} \) must be used.

A method similar to that of Van Dam has been developed by Mukherjee and Gopalarathnam. [45]. In their method besides a correction to angle of attack, also a correction is applied to the pitching moment \( C_{m_{\alpha}} \). By doing so, they effectively apply decambering to account for for boundary layer separation, which is assumed to be responsible for the change in viscous and potential theory results. This method was shown to produce accurate results for unswept wings, but lacked accuracy in predicting aerodynamic loads for swept wings.

### 2.4.3 **SEMI-EMPirical METHODS**

As mentioned before, semi-empirical methods combine physical data with empirical data to compute aerodynamic loads. While semi-empirical methods partially remove the restriction of being able to analyze only conventional designs, care should be taken in using these methods on novel design concepts.

**Pressure Difference Rule**

A promising semi-empirical method of computing the wing \( C_{L_{\text{max}}} \) has been developed by Valarezo and Chin [46] and is called the Pressure Difference Rule (PDR). They observed that for a given chord Reynolds number and free stream Mach number, there exists a relation between the pressure of the suction peak and trailing edge pressure, and wing stall. While in their research use was made of a higher order surface panel method, any other reliable method may be used such as the Q3D analysis mentioned in this report. The PDR is implemented as follows:

1. The effective Reynolds number is computed at several sections along the wing span, based on the local clean chord length:
   \[
   Re_{\text{eff}} = \frac{\rho V_{\text{eff}} c_{\text{local}}}{\mu} \quad (2.42)
   \]

2. Using the free stream Mach number the critical pressure difference at each spanwise section is determined from Figure 2.22.
3. With the critical pressure differences determined, an “allowable” pressure difference distribution is constructed.

4. Using an inviscid panel method, the pressure difference ($\Delta C_p$) between the leading edge (LE) suction peak and trailing edge (TE) is computed at each section.

$$\Delta C_{p_{\text{eff}}} = \text{panel method}(\alpha_{\text{eff}}, M_{\text{eff}})$$  \hfill (2.43)

5. Repeat this procedure for a range of angles of attack until at one of the sections: $|\Delta C_{p_{\text{eff}}} - \Delta C_{p_{\text{crit}}}| \leq \epsilon$, where $\epsilon$ is a user-selected convergence criteria, to find $C_{L_{\text{max}}}$.

Below in Figures 2.23 and 2.24, the implementation of the PDR is shown graphically and results are shown for a wing with combined leading edge and trailing edge devices. It can be seen that the computations agree remarkably well with experiment.
2.5 Optimization Algorithm

Considering that the number of design variables and design constraints may go up to the hundreds, the choice of an efficient optimization algorithm is critical in order to achieve an efficient design optimization method. The optimization algorithm is the core tool which ensures a local optimum is found, taking into account constraints and using sensitivities of the objective function and constraints with respect to the design variables. In [12], Elham and van Tooren used the Sparse Nonlinear Optimizer (SNOPT) [47]. This algorithm uses a Sequential Quadratic Programming (SQP) method which has proven to efficiently handle nonlinear programs with large numbers of constraints and variables of which the nonlinear functions are smooth and first derivatives are available. While an extensive description of the algorithm is presented in [47], a short summary will be presented in this section.

2.5.1 SNOPT

SNOPT incorporates an SQP algorithm through major and minor iterations. The major iterations converge to a point which satisfies the linear and nonlinear constraints and first-order optimality conditions. At each iteration, a Quadratic Programming (QP) subproblem is used to generate an appropriate search direction. The subproblems are solved using a reduced-gradient method and are defined as:

$$\min_x f_k + g_k^T(x - x_k) + \frac{1}{2}(x - x_k)^T H_k(x - x_k)$$

s.t. $l \leq \left( c_k + J_k \frac{x}{A_x} - x_k \right) \leq u$

Here $g_k$ is the gradient of the objective function $f$ and $H_k$ is the approximation of the Hessian, obtained through the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton update.

After solving the QP problem, a line search is performed towards the QP solution. This line search must achieve a sufficient decrease in the augmented Lagrangian Merit function $M$:

$$M_p(x, \pi, s) = f(x) - \pi^T(c(x) - s) + \frac{1}{2} \sum_{i=1}^{m} \rho_i (c_i(x) - s_i)^2$$

(2.44)

Here $\pi$ is the Lagrange multiplier, $s$ the slack variables and $\rho$ the penalty parameters, see [47] for further details.

Each minor iteration should result in a search direction which satisfies the aforementioned first-order optimality conditions. These optimality conditions, also referred to as the Karush-Kuhn-Tucker (KKT) optimality conditions, can be used to check if a design point represents an actual local minimum. For details on the determination of the KKT optimality conditions within SNOPT, the reader is referred to [47].
In this chapter the methodology applied to perform combined aerostructural analysis and optimization of cruise and high-lift wing design is described. In Section 3.1, the implementation of the physics-based $\alpha$ correction method and semi-empirical PDR for computing wing maximum lift are described in an effort to explore the possibilities of both methods. Then in Section 3.2, the required modifications to the existing aerostructural analysis and optimization tool developed by Elham and Van Tooren [12] as well as the development of additional tools are described which enable high-lift aerodynamic analysis and optimization. Hereafter, the developed aerodynamic tools are coupled to FEMWET in Section 3.3 to enable aerostructural analysis and optimization, followed by a description of the airfoil shape parameterization technique to be used during optimization in Section 3.4. In respectively Sections 3.5 and 3.6 the wing weight estimation model and aircraft performance model are described. Lastly, in Section 3.7, the developed analysis and optimization tool is validated against experimental data.

3.1 Two methods for the prediction of $C_{L_{\text{max}}}$

In Section 2.4, two suitable methods were described for the prediction of $C_{L_{\text{max}}}$: the physics based $\alpha$ correction method and the semi-empirical Pressure Difference Rule (PDR). In this section, the implementation of both methods are described in an effort to explore both possibilities in high-lift aerostructural analysis and optimization.

3.1.1 $\alpha$ method

While the general procedure of the $\alpha$ Method has been described in Section 2.4, the implementation into the Q3D analysis of Elham and Van Tooren [12] requires some attention. It includes applying a viscous correction angle to the effective angle of attack at which the 2D analysis is performed as shown below in Equation 3.1:

$$a_{\text{eff}} = \frac{C_{l_{\text{max}}}}{C_{l_{\alpha}}} + a_0 - a_{\text{corr}}$$

(3.1)

Using this effective angle of attack, 2D viscous analysis is then performed using MSES, a viscous analysis tool which will be described in Section 3.2.2. Where in the Q3D method employed by Elham and Van Tooren the induced angle of attack ($\alpha_i$) was used as state variable in the coupled aerostructural system, in the $\alpha$ correction method it is computed by Equation 3.2:

$$\alpha_i = \alpha_{\perp} - a_{\text{eff}}$$

(3.2)

The viscous correction angle should also be applied to the RHS of the 3D inviscid analysis, being the VLM analysis. The RHS then becomes:

$$\text{RHS} = -V_{\infty} (\cos(\alpha + a_{\text{corr}})\hat{n}_l - \sin(\alpha + a_{\text{corr}})\hat{n}_k)$$

(3.3)

3.1.2 Pressure Difference Rule

The implementation of the PDR in the existing Q3D analysis requires the development of a 2D inviscid analysis tool. This tool is referred to as 2DPANEL from now on and will be described in Section 3.2.3. Besides the development of 2DPANEL, also the implementation of the PDR in the Q3D method needs be considered
as sweep effects, downwash and wing elasticity are to be taken into account. The effective pressure differ-
ence and effective critical pressure difference are determined from respectively the analysis tool and the PDR
relations obtained from Figure 2.22:

\[
\Delta C_{p_{\text{eff}}} = 2\text{DPANEL}(\alpha_{\text{eff}}, M_{\text{eff}}) \quad (3.4)
\]

\[
\Delta C_{p_{\text{crit,eff}}} = \text{PDR}(M_{\infty}, Re_{\text{eff}}) \quad (3.5)
\]

Here the effective Mach number and Reynolds number can be computed from Equations 2.15 and 2.16. In order to account for downwash and wing elasticity as shown in Figure 2.17, the computed 2D pressure difference and critical pressure difference are then corrected by:

\[
\Delta C_{p_{\text{2d}}} = \Delta C_{p_{\text{eff}}} \cos \alpha_i \cos \theta \quad (3.6)
\]

\[
\Delta C_{p_{\text{crit,2d}}} = \Delta C_{p_{\text{eff}}} \cos \alpha_i \cos \theta \quad (3.7)
\]

Note that drag terms are omitted in this transformation as the PDR is a purely inviscid method. The complete procedure of solving the PDR is shown in Figure 3.2.
3.1. Two methods for the prediction of $C_{L_{\text{max}}}$

---

**Apply sweep theory to find $\alpha_{\perp}, M_{\perp}, V_{\perp}$:**

\[
\begin{align*}
\alpha_{\perp} &= \frac{\alpha + \epsilon}{\cos \Lambda} \\
M_{\perp} &= M_\infty \cos \Lambda \\
V_{\perp} &= V_\infty \cos \Lambda
\end{align*}
\]

**Compute $C_{L_{\perp}}$ from VLM after applying $\alpha_{\text{corr}}$ to RHS:**

\[
\text{RHS} = -V_\infty \left( \cos(\alpha_{\perp} + \epsilon) \right) \left( \mathbf{n}_i - \sin(\alpha_{\perp} + \epsilon) \mathbf{n}_k \right)
\]

**Use $\alpha_{\text{corr}}$ to compute effective 2D values and $\alpha_i$:**

\[
\begin{align*}
\alpha_{\text{eff}} &= \frac{C_{l_{\perp}}}{C_{l_{\infty}}} + \alpha_0 - \alpha_{\text{corr}} \\
\alpha_i &= \alpha_{\perp} - \alpha_{\text{eff}} \\
M_{\text{eff}} &= M_{\perp} \frac{\cos \alpha_{\text{eff}}}{\cos \alpha_i} \\
Re_{\text{eff}} &= \frac{\rho V_{\perp} c_{\perp}}{\mu}
\end{align*}
\]

**Compute sectional viscous correction angle through:**

\[
\alpha_{\text{corr}} = \frac{C_{l_{\infty}} - C_{l_{\perp}}}{C_{l_{\infty}}}
\]

and iterate until:

\[
|C_{l_{\infty}} - C_{l_{\perp}}| \leq \epsilon
\]

**Perform procedure for a sweep of $\alpha$ and determine $\alpha_{\text{stall}}$ and $C_{L_{\text{max}}}$ from resulting lift curve**

$\alpha_{\text{stall}}, C_{L_{\text{max}}}$

---

**Apply sweep theory to find $\alpha_{\perp}, M_{\perp}, V_{\perp}$:**

\[
\begin{align*}
\alpha_{\perp} &= \frac{\alpha + \epsilon}{\cos \Lambda} \\
M_{\perp} &= M_\infty \cos \Lambda \\
V_{\perp} &= V_\infty \cos \Lambda
\end{align*}
\]

**Use $\alpha_i$ to compute the effective 2D values:**

\[
\begin{align*}
\alpha_{\text{eff}} &= \alpha_{\perp} - \alpha_i \\
M_{\text{eff}} &= M_{\perp} \frac{\cos \alpha_{\text{eff}}}{\cos \alpha_i} \\
Re_{\text{eff}} &= \frac{\rho V_{\perp} c_{\perp}}{\mu}
\end{align*}
\]

**Compute effective pressure difference and critical pressure difference:**

\[
\begin{align*}
\left[ C_{l_{\text{eff}}}, \Delta C_{p_{\text{eff}}} \right] &= \text{MSES}(\alpha_{\text{eff}}, M_{\text{eff}}, Re_{\text{eff}}) \\
\Delta C_{p_{\text{crit}}} &= \text{PDR}(Re_{\text{eff}}, M_\infty)
\end{align*}
\]

**Correct for $\alpha_i$ and wing deflection $\theta$ through:**

\[
\Delta C_{p_{\text{2d}}} = (\Delta C_{p_{\text{eff}}}) \cos \alpha_i \cos \theta
\]

**At the determined $\alpha$, find $C_{L_{\text{max}}}$ using VLM analysis**

$\alpha_{\text{stall}}, C_{L_{\text{max}}}$

---

**Figure 3.1: Q3D $\alpha$ method procedure**

**Figure 3.2: Q3D PDR procedure**
3.2 **HIGH-LIFT AERODYNAMIC ANALYSIS**

While the aerodynamic analysis tool developed by Elham and Van Tooren [12] has proven to be effective at predicting aerodynamic loads in cruise conditions, modifications are required in order to enable high-lift analysis including high-lift devices. This section describes the necessary modifications to the VLM analysis and 2D viscous analysis, as well as the development of a 2D panel code required to solve the PDR method.

### 3.2.1 **HIGH-LIFT VORTEX LATTICE METHOD**

In the VLM analysis in [12], aileron deflection was simulated by rotating the vortex points on the flap in order to determine aileron effectiveness. Although rotating the vortex points is sufficient in the case of plain flaps, chord extension needs to be taken into account in the case of slotted and Fowler flaps. In this research, chord extension is simulated by shifting the vortex points. Furthermore, while a constant grid spacing is suitable in the case of small deflections, higher resolution near control surface edges is required at larger deflections such as in the case of trailing edge flaps. A cosine spacing is therefore implemented in both chordwise and spanwise direction. An identical cosine spacing is used for the nodes in the FEM analysis (displayed in red in Figure 3.13) described in Section 2.3.2 to prevent unnecessary interpolation. A comparison between the aerostructural mesh used in [12] and in this research is shown in Figure 3.13.

![Figure 3.3: Comparison of aerostructural High-lift aerostructural mesh](image)

#### 3.2.2 **MSES - MULTI-ELEMENT AIRFOIL DESIGN AND ANALYSIS SOFTWARE**

MSES, an interactive viscous/inviscid Euler solver, is used as the 2D viscous solver in the Q3D aerodynamic analysis. This tool is able to automatically generate a mesh by intersecting the inviscid flow streamlines and curves normal from the airfoil points (see Figure 3.4). The size of the grid, number of airfoil points and streamlines are provided by the user, as well as the boundary conditions and solver type. By coupling the inviscid and viscous equations using the Newton method for iteration, MSES is able to converge to a solution much faster than similar inviscid/viscous solvers.

![Figure 3.4: MSES automatic mesh generation](image)
3.2. High-Lift Aerodynamic Analysis

**Boundary layer calculation**

While MSES is able to perform 2D viscous aerodynamic analysis at an extremely fast rate, it encounters convergence problems when analyzing airfoils at high-angles of attack at low-speed conditions, especially in the case of multi-element airfoils where flow separation can occur quickly. In order to improve convergence behaviour of MSES near stall, the boundary calculation method has been modified according to the proposed method developed by Van Rooij [48]. In his research, he concluded that a variation in the shear lag coefficient $K_c$ is required in order to accurately predict near-stall lift properties. The shear lag coefficient controls how much the actual shear stress $C_\tau$ lags behind the equilibrium shear stress $C_{\tau, eq}$. A proper value for the lag coefficient is especially important for separated flows where the shape parameter of the boundary layer changes rapidly. In his research, Van Rooij proposes the following relation for the lag constant:

$$K_c = 4.65 - 0.95 \cdot \tanh(0.275 \cdot H_k - 3.5)$$  \hspace{1cm} (3.8)

Here $H_k$ is the kinematic shape parameter. This equation ensures a smooth transition of the lag constant when moving from the airfoil surface to the wake. Below in Figure 3.5, the improvement convergence behaviour of MSES is shown for the GA (W)-2 airfoil with 25% single slotted flap, deflected at 20° [28]. Tests were conducted at a Reynolds number of 2.2e6 and Mach number of 0.13 with free transition. It should be noted that discrepancies in the figure between computed and experimental values are mainly due to inaccuracies between the modeled and actual flap design.

![Figure 3.5: GA (W)-2 airfoil lift curve](image1)

![Figure 3.6: MSES modification results for GA (W)-2 airfoil [28]](image2)
FLOW TRANSITION

A second modification applied considers the prediction of MSES regarding transition from laminar to turbulent flow. In order to predict flow transition, MSES uses the “$e^n$ method” where $n$ is referred to as the n-factor [49]. This factor is the log of the amplification factor of the most-amplified frequency which triggers transition. Using this method, transition is assumed to take place as soon as this amplification factor reaches the value of $n$. Typical values for $n$ are shown below in Table 3.2:

Table 3.1: Typical n-factor values [50]

<table>
<thead>
<tr>
<th>Situation</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sailplane</td>
<td>12-14</td>
</tr>
<tr>
<td>Motorglider</td>
<td>11-13</td>
</tr>
<tr>
<td>Clean wind tunnel</td>
<td>10-12</td>
</tr>
<tr>
<td>Average wind tunnel</td>
<td>9</td>
</tr>
<tr>
<td>Dirty wind tunnel</td>
<td>4-8</td>
</tr>
</tbody>
</table>

Considering the fact that flow separation is the main mechanism which causes convergence problems within MSES, a solution was sought which prevents flow separation to occur in order to achieve robust high-lift analysis. One way of delaying flow separation is by inducing a turbulent boundary layer by means of artificial transition. Turbulent boundary layers are more capable of dealing with adverse pressure gradients and therefore are less likely to separate. While MSES allows for artificial transition locations to be specified, extensive analysis revealed that premature flow separation still occurred in low-speed multi-element airfoil analysis. An alternative to induce turbulence is through varying the n-factor, which has been employed by Benini et al. [13]. By default, the n-factor is set to 9 which is a good value if streamwise instability is assumed to arise from the viscous boundary layer. By lowering the n-factor, it is assumed that turbulence occurs in free stream which causes turbulence on the airfoil to occur right after linear instability begins, effectively reducing the transition region (see Figure 3.7). This in turns prevents flow separation and greatly enhances convergence behavior of MSES. Furthermore, Drela describes that in order to take 3D effects into account such as surface irregularities, the n-factor may be lowered to values lower than 1 [50]. In the present research the n-factor is set to 1 mainly in order to ensure convergence. While this results in an underestimated L/D ratio due to premature transition, this modification ensures a significantly more robust optimization scheme.

Figure 3.7: Turbulence development [51]

In order to assess the effect that the n-factor has on the aerodynamic analysis, an assessment has been performed on several values of $n$ for a Fokker 100 mid-airfoil at an angle of attack of $2.344^\circ$, a free stream Mach number of 0.1893 and a Reynolds number of 24 million with a single slotted flap deflected at $20^\circ$. The results are shown below in Table 3.2 and Figure 3.8:

Table 3.2: Effect of n-factor

<table>
<thead>
<tr>
<th>$n$</th>
<th>$C_l$</th>
<th>$C_d_p$</th>
<th>$C_d_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n=1$</td>
<td>1.7625</td>
<td>0.0067</td>
<td>0.0065</td>
</tr>
<tr>
<td>$n=4$</td>
<td>1.7921</td>
<td>0.0066</td>
<td>0.0058</td>
</tr>
<tr>
<td>$n=9$</td>
<td>1.7970</td>
<td>0.0066</td>
<td>0.0054</td>
</tr>
<tr>
<td>$n=12$</td>
<td>1.8025</td>
<td>0.0066</td>
<td>0.0053</td>
</tr>
</tbody>
</table>
3.2. HIGH-LIFT AERODYNAMIC ANALYSIS

It can be seen that indeed for lower values of $n$, friction drag increases and wing lift decreases, reducing L/D. From Figure 3.8a it can also be observed that when $n$ is set to 1, transition on the flap occurs at the flap leading edge which can be seen by the jump in friction drag coefficient. This means that when $n$ is set to 1, the complete flow over the flap is turbulent which will delay flow separation.

![Friction drag coefficient and Pressure coefficient](image)

Figure 3.8: Effect of n-factor on 2D friction drag and pressure coefficient distribution

While the combination of these adjustments improved convergence behavior of MSES, it proved to be very useful to introduce several backup runs in case MSES failed to converge. This was mainly due to the convergence sensitivity of MSES with respect to grid settings. For details on the grid settings the reader is referred to Appendix B.
3.2.3 **MULTI-ELEMENT LINEAR STRENGTH VORTEX PANEL METHOD**

In order to compute the pressure distribution used in the PDR, a 2D linear strength vortex panel method has been developed based on the method of Katz & Plotkin [34]. Being a higher-order paneling method, the linear strength vortex panel provides more accurate results compared to constant strength vortex panel methods and differs in the fact that the vortex strength ($\Gamma$) is different on either side of the panels (see Figure 3.9a).

![Diagram](image_url)

(3.9)

Using the nomenclature as depicted in Figure 3.9a, the perturbation velocity at any collocation point is defined at the center of the panels are given by Equations 3.9 and 3.10:

\[
\begin{align*}
 u_p &= \frac{z}{2\pi} \left( \frac{\Gamma_2 - \Gamma_1}{x_2 - x_1} \right) \ln \left( \frac{r_2}{r_1} \right) + \frac{\Gamma_1(x_2 - x_1) + (\Gamma_2 - \Gamma_1)(x - x_1)}{2\pi(x_2 - x_1)}(\theta_2 - \theta_1) \\
 w_p &= -\frac{\Gamma_1(x_2 - x_1) + (\Gamma_2 - \Gamma_1)(x - x_1)}{2\pi(x_2 - x_1)} \ln \left( \frac{r_1}{r_2} \right) + \frac{z}{2\pi} \left( \frac{\Gamma_2 - \Gamma_1}{x_2 - x_1} \right) \left( \frac{x_2 - x_1}{z} + (\theta_2 - \theta_1) \right)
\end{align*}
\]  

(3.10)

After transforming the perturbation velocities from panel to the global coordinate system, the coefficients in the Aerodynamic Influence Coefficients (AIC) matrix and the Right Hand Side (RHS) vector are computed through:

\[
a = (u, w) \cdot \mathbf{n}
\]

(3.11)

\[
\text{RHS} = -(U_{\infty}, W_{\infty}) \cdot \mathbf{t}
\]

(3.12)

Where $\mathbf{n}$ and $\mathbf{t}$ are the panel normal and tangential vectors (see Figure 3.9b). As in the VLM analysis, the strengths of the vortices can then be determined using Equation 3.13:

\[
\text{AIC } \Gamma = \text{RHS}
\]

(3.13)

Using the computed vortex strengths, the perturbation velocity at each collocation point is calculated with:

\[
V_t = \Gamma \cdot (u, w) \cdot \mathbf{t}
\]

(3.14)

The pressure coefficient at each collocation point is then determined through:

\[
C'_p = 1 - \left( \frac{V_t}{V_{\infty}} \right)^2
\]

(3.15)

And the lift coefficient is determined by summing the contributions to the lift coefficient as shown below:

\[
C'_l = -C_p \Delta l \cos \alpha
\]

(3.16)

\[
C_l = \sum_{j=1}^{N} C'_l
\]

(3.17)
3.3 AEROSTRUCTURAL COUPLING

In order to couple the modified high-lift aerodynamic analysis methods to the structural solver FEMWET, two systems of equations are defined: the $\alpha$ system which includes the $\alpha$ correction method and the PDR system, which solves the PDR in order to predict the wing $C_{l_{\max}}$. Through coupling of these systems, for the first time aerostructural effects are taken into account in the determination of high-lift aerodynamic loads and $C_{l_{\max}}$ prediction.

3.3.1 SYSTEMS OF EQUATIONS

Coupling of the $\alpha$ correction method to FEMWET is performed analogously to the coupling structure as described in Section 2.3.3, with the difference being that the viscous correction angle ($\alpha_{corr}$) replaces the induced angle of attack ($\alpha$) as state variable, and that the viscous correction angle is applied to the RHS of the VLM. In order to apply this kind of strong coupling, Equation 3.18 is defined as governing equation:

$$\alpha_{corr} = \frac{C_{l_{\text{visc}}} - C_{l_{\text{max}}}}{C_{l_a}} = 0 \quad (3.18)$$

While Gallay [44] employed a similar method of strong coupling of the $\alpha$ correction method, it should be noted that $C_{l_a}$ in this equation may be omitted, simplifying the formulation to: $|C_{l_{\text{visc}}} - C_{l_{\text{max}}}| = 0$. After applying the necessary transformations described in Section 2.3.1, the resulting system of equations becomes:

$$\text{Sys}_{\alpha} = \begin{bmatrix} R1(\Gamma, U, a, \alpha_{corr}) \\ R2(\Gamma, U, a, \alpha_{corr}) \\ R3(\Gamma, U, a, \alpha_{corr}) \\ R4(\Gamma, U, a, \alpha_{corr}) \end{bmatrix} = \begin{bmatrix} AIC \Gamma - \text{RHS} \\ KU - F \\ L - nW_{des} \\ C_{l_a} - C_{l_{\text{inv}}} \end{bmatrix} = \bar{0} \quad (3.19)$$

In order to efficiently compute the wing $C_{l_{\max}}$ using the implemented a method, it is proposed to replace the third governing equation in the viscous aerostructural system with: $C_{l_a} = 0$, where $C_{l_a}$ is determined through:

$$\frac{dC_l}{da}_{l_a} = \frac{1}{\Delta a} (C_l(\alpha + \Delta \alpha) - C_l(\alpha_0)) \quad (3.20)$$

The system of equations is then expanded to 7 governing equations where $R_5$, $R_6$ and $R_7$ are duplicates of respectively $R_1$, $R_2$ and $R_4$ which are solved for $a + \Delta a$. Although this adds additional analyses to be performed, this procedure ensures a more computationally efficient determination of $C_{l_{\max}}$ than solving the system for a sweep of angles of attack as has been proposed by Van Dam [43].

Coupling the PDR with FEMWET requires two modifications to the coupling structure in Section 2.3.3. The first modification is the replacement of the third governing equation by the Kreisselmeier-Steinhauser (KS) aggregation function [52]. This function is used in order to reduce the PDR convergence criteria: $\Delta C_{P_{adj}} = \Delta C_{P_{\text{visc}}}$, where $k$ denotes the spanwise section, to one governing equation. This method of aggregating the PDR convergence criteria has been employed in another research by Kennedy and Martins [53] where it was used as constraint in optimization. The KS aggregation function is defined as:

$$\text{KS} = f_{\text{max}} + \frac{1}{\rho} \log_e \sum_{k=1}^{K} e^{\rho(f_k(X) - f_{\text{max}})} = 0 \quad (3.21)$$

Here:

$$f_k = \Delta C_{P_{adj}} - \Delta C_{P_{\text{visc}}} \quad (3.22)$$

Where $k$ denotes the spanwise section, $f_{\text{max}}$ is the maximum value of the set of functions $f_k$ evaluated at $X$ and for $\rho$ a value of 80 is used [52].

The second modification is that the fourth governing equation states that the inviscid lift distribution computed by 2DPANEL (linear strength vortex panel method) is equal to the lift distribution from the VLM analysis. The system of equations becomes:
\[ \text{Sys}_{\text{PDR}} = \begin{bmatrix} R_1(\Gamma, U, \alpha, \alpha_i) \\ R_2(\Gamma, U, \alpha, \alpha_i) \\ R_3(\Gamma, U, \alpha, \alpha_i) \\ R_4(\Gamma, U, \alpha, \alpha_i) \end{bmatrix} = \begin{bmatrix} AIC \Gamma - \text{RHS} \\ KU - F \\ KS \\ C_{l,\text{adv}} - C_{l,1} \end{bmatrix} = \tilde{0} \] (3.23)

Both systems are again solved using the Newton method for iteration, of which the details are described in Section 2.3.3.

As mentioned before, coupling high-lift aerodynamic analysis to the structural solver FEMWET for the first time allows for the determination of high-lift aerodynamic loads and wing maximum lift while taking into account structural deflection as shown in Figure 3.10.

3.3.2 SENSITIVITY ANALYSIS

In order to solve the systems of equations described in the previous section, the partial derivatives of the governing equations with respect to the state variables are required. While a detailed derivation of these derivatives is presented in Appendix A, a summary of the derivatives is shown in Tables 3.3 and 3.4:

Table 3.3: Partial derivatives of \( \alpha \) system

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( U )</th>
<th>( \alpha )</th>
<th>( \alpha_{\text{corr}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>( AIC )</td>
<td>( \frac{\partial AIC}{\partial U} ) ( \Gamma - \frac{\partial \text{RHS}}{\partial U} )</td>
<td>( -\frac{\partial \text{RHS}}{\partial \alpha} )</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>( \frac{\partial F}{\partial \Gamma} )</td>
<td>( K )</td>
<td>0</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>( \frac{\partial C_{l,1}}{\partial U} )</td>
<td>( \tilde{0} )</td>
<td>0</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>( \frac{\partial C_{l,1}}{\partial \Gamma} )</td>
<td>( \frac{\partial C_{l,1}}{\partial U} )</td>
<td>( \frac{\partial C_{l,1}}{\partial \alpha} )</td>
</tr>
</tbody>
</table>

Table 3.4: Partial derivatives of PDR system

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( U )</th>
<th>( \alpha )</th>
<th>( \alpha_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>( AIC )</td>
<td>( \frac{\partial AIC}{\partial U} ) ( \Gamma - \frac{\partial \text{RHS}}{\partial U} )</td>
<td>( -\frac{\partial \text{RHS}}{\partial \alpha} )</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>( \frac{\partial F}{\partial \Gamma} )</td>
<td>( K )</td>
<td>0</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>( \frac{\partial C_{l,1}}{\partial U} )</td>
<td>( \tilde{0} )</td>
<td>0</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>( \frac{\partial C_{l,1}}{\partial \Gamma} )</td>
<td>( \frac{\partial C_{l,1}}{\partial U} )</td>
<td>( \frac{\partial C_{l,1}}{\partial \alpha} )</td>
</tr>
</tbody>
</table>

Again, as soon as the Newton method is converged, the partial derivatives matrix may be used in the coupled-adjoint method to compute the total derivatives of the functions of interest with respect to design variable \( X \). When computing the aerodynamic derivatives such as \( C_D \) and \( C_L \) with respect to \( X \) in take-off and landing conditions however, the nonlinear behavior of the stall speed \( V_{S_{1g}} \) should be considered. As described in Section 2.1, the approach speed and take-off speed need to be at least respectively 1.13 \( V_{S_{1g}} \) and 1.23 \( V_{S_{1g}} \), where \( V_{S_{1g}} \) is defined as:

\[ V_{S_{1g}} = \sqrt{\frac{W_{\text{des}}}{\frac{1}{2} \rho C_{l,\text{max}} S}} \] (3.24)

Here at take-off, \( W_{\text{des}} \) is the Maximum Take-Off Weight (MTOW) and at landing it is the Maximum Landing Weight (MLW). The derivative of \( V_{S_{1g}} \) with respect to design variable \( X \) is:
3.4 Airfoil Shape Parameterization

Airfoil parameterization plays an important role in efficiently changing the airfoil shape during optimization. While most parameterization techniques focus on specifying the airfoil shape using direct parameterization methods such as the PARSEC method [54] or CSRT method [55], MSES parameterizes the airfoil shape through perturbations over the reference airfoil shape using Chebyshev polynomials with basis functions $g_j$, and mode amplitudes $G_j$. Here the mode amplitudes may be defined as design variables. The Chebyshev polynomials perturb the airfoil surface by a distance $\Delta n$ normal to airfoil surface where $\Delta n$ is determined through Equation 3.27.

$$\Delta n(s) = \sum_{j=1}^{J} G_j g_j(s)$$  \hspace{1cm} (3.27)

Here $s$ is the fractional arc length of the segment that the perturbation is applied to and $J$ the total number of Chebyshev polynomials applied to each airfoil side. An exaggerated example of a perturbed airfoil using two Chebyshev modes is shown in Figure 3.11. When using a large number of Chebyshev polynomials (for example 10 per side), nearly arbitrary airfoil shapes can be obtained through this parameterization.

![Figure 3.11: Airfoil shape perturbation using Chebyshev modes](image)

3.5 Wing Weight Estimation

In Section 2.3.2, it was described that in the research of Elham and Van Tooren [12], the wing weight was computed following the weight prediction used by Kennedy and Martins [39]:

$$W_{\text{wing}} = 1.5 W_{\text{FEM}} + 15S_{\text{wing}}$$  \hspace{1cm} (3.28)

While this equation ensures an easy weight prediction for which no detailed knowledge is required of the wing design, it is not sufficient for the present research. As described in Section 2.2.2, the weight of the high-lift devices plays a large role in wing design, and should be therefore be included in any wing design optimization. The more detailed method of Torenbeek [40] enables this as it defines the wing secondary weight through the following elements:
### Table 3.5: Secondary weight components

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_{\text{fix}}\text{LE})</td>
<td>Fixed leading edge structure</td>
</tr>
<tr>
<td>(W_{\text{fix}}\text{TE})</td>
<td>Fixed trailing edge structure</td>
</tr>
<tr>
<td>(W_{\text{slat}})</td>
<td>Leading edge high-lift devices</td>
</tr>
<tr>
<td>(W_{\text{flap}})</td>
<td>Trailing edge high-lift devices</td>
</tr>
<tr>
<td>(W_{s})</td>
<td>Spoilers</td>
</tr>
<tr>
<td>(W_{a})</td>
<td>Ailerons</td>
</tr>
</tbody>
</table>

A simplified representation of the secondary weight components is shown in Figure 3.12:

![Figure 3.12: Secondary wing weights](image)

In the next sections, the details on the computation of the separate weight components are described.

#### Fixed leading and trailing edge structure

The fixed leading and trailing edge weights are computed using Equations 3.29 and 3.30.

\[
W_{\text{fix}}\text{LE} = 75k_{\text{LE}}S_{\text{LE}} \left(1 + \sqrt{\frac{\text{MTOW}}{10^6}}\right) \quad (3.29)
\]

\[
W_{\text{fix}}\text{TE} = 60S_{\text{TE}} k_{\text{TE}} \left(1 + 1.6\sqrt{\frac{\text{MTOW}}{10^6}}\right) + \Delta \quad (3.30)
\]

Here \(S_{\text{LE}}\) and \(S_{\text{TE}}\) are the areas of the fixed leading edge and trailing edge respectively (the area in front and aft of respectively the front and rear spar that remains after high-lift devices and controls surfaces are removed, including spoilers and lift dumpers that are not attached to the rear spar, see Figure 3.12). The factor \(k_{\text{LE}}\) accounts for leading edge device support structures. This value is equal to 1 when no leading edge devices are present and 1.4 when they are present. The value of \(\Delta\) is 0 for single slotted flaps, 45 for double slotted flaps and 105 for triple slotted flaps.

#### High lift devices

The leading and trailing edge high-lift device weights are computed using:

\[
W_{\text{slat}} = 160S_{\text{LE}} \left(1 + 0.7\sqrt{\frac{\text{MTOW}}{10^6}}\right) \quad (3.31)
\]

\[
W_{\text{flap}} = 100S_{\text{TE}} k_{\text{TE}} \left(1 + \sqrt{\frac{\text{MTOW}}{10^6}}\right) \quad (3.32)
\]

Here \(k_{\text{TE}}\) is a factor accounting for the flap complexity and can be obtained from Table 3.6.
### Ailerons and Spoilers

The aileron and spoiler weights are determined with the following equations:

\[
W_a = 125S_a \left[ 1 + 0.5 \left( \frac{MTOW}{10^6} \right)^{1/4} \right]
\]  
\[
W_s = 110S_s
\]

The spoiler surface may amount up to 6% of the total wing area. To account for structural weight of the support structure of the movables, the flap weight should be increased by 5% and aileron weight by 20%.

### Modified Wing Weight Estimation

The total secondary weight is then computed through:

\[
W_{sec} = W_{fix_{LE}} + W_{fix_{TE}} + W_{slat} + W_{flap} + W_a + W_s
\]

Using the wing secondary weight computed by the method of Torenbeek, the wing weight can then be computed using:

\[
W_{wing} = 1.5W^{FEM}_{wingbox} + W_{sec}
\]

### 3.6 Aircraft Performance Model

In the research of Elham and Van Tooren [12], aircraft performance was measured by fuel use during cruise. In the present research, both cruise and airfield performance need to be taken into account in order to optimize the wing design including high-lift devices. This section describes the methods used for the determination of cruise and airfield performance.

#### 3.6.1 Cruise

The mission fuel weight \((W_{fuel})\) is computed based on the method of Roskam [56]. The required fuel use for cruise is computed using the Bréguet range equation (see Equation 2.2), while statistical factors are used to determine the fuel weight of the remaining segments of the mission. In Table 3.7 the fuel fractions of a transport aircraft is listed for each segment of the flight mission.

<table>
<thead>
<tr>
<th>Mission segment</th>
<th>Fuel fraction ((M_{ff}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start &amp; warm-up</td>
<td>0.990</td>
</tr>
<tr>
<td>Taxi</td>
<td>0.990</td>
</tr>
<tr>
<td>Take-off</td>
<td>0.995</td>
</tr>
<tr>
<td>Climb</td>
<td>0.980</td>
</tr>
<tr>
<td>Cruise</td>
<td>(e^{-\left(\frac{\mu c L}{\tau_{\infty}}\right)})</td>
</tr>
<tr>
<td>Descent</td>
<td>0.990</td>
</tr>
<tr>
<td>Landing, taxi &amp; shutdown</td>
<td>0.992</td>
</tr>
</tbody>
</table>

The total mission fuel fraction can now be determined by the following equation:
The mission fuel weight is then computed using Equation 3.38, taking into account an additional 5% fuel weight as reserve fuel:

\[ W_F = 1.05(1 - M_R) \cdot MTOW \]  

### 3.6.2 Take-off

As described in Section 2.1, airfield performance is governed by take-off and landing distance. Regulations for take-off and landing performance are listed in FAR 25[18], which defines the take-off distance as the ground covered from standstill to the point where the aircraft is 50 ft above the ground. This distance is computed as the sum of ground roll, rotation, transition and climbing distance.

\[ s_{TO} = s_{GR} + s_R + s_{TR} + s_C \]  

These distances can be computed following the method described by Raymer [5], adjusted for current regulations.

\[ s_{GR} = \frac{1}{2gK_A} \ln \left( \frac{K_T + K_AV_f^2}{K_T + K_AV_i^2} \right) \]  

Here \( V_i \) is the initial velocity taken as 0 for the ground run, and \( V_f \) is the velocity at the start of rotation which may be no less than 1.1 \( V_{s-1g} \) according to Raymer. Coefficients \( K_A \) and \( K_T \) are defined as:

\[ K_T = \frac{T}{W} - \mu_r \]  
\[ K_A = \frac{\rho}{2(W/S)} (\mu_r C_L - D_{aero}) \]  

Here \( \dot{T} \) is the average thrust taken to be equal to the thrust at \( V = 1/\sqrt{\rho V_f} \). The rolling friction coefficient \( \mu_r \) is taken to be 0.03. Weight is equal to MTOW and the density is taken to be 1.225 kg/m³.

For large aircraft, the rotation time to lift-off attitude may be assumed to be three seconds. \( s_R \) can therefore be approximated to be 3.3\( V_{s-1g} \).

During transition, the aircraft accelerates from 1.1 \( V_{s-1g} \) to 1.13 \( V_{s-1g} \). The average speed during transition is therefore 1.12 \( V_{s-1g} \). The average lift coefficient during transition may be assumed to be 90% of \( C_{L_{max}} \)\( |_{TO} \). The average load factor during transition is equal to 1.2. This gives a rotation arc radius of 0.205 \( V_{s-1g} \). The climb angle at the end of transition is determined from:

\[ \gamma = \frac{T}{W} - \frac{D}{L} \bigg|_{1.2V_{s-1g}} \]  

The distance traveled and altitude gained during transition is computed by:

\[ s_{TR} = R \sin \gamma \]  
\[ h_{TR} = R(1 - \cos \gamma) \]  

The value of \( h_{TR} \) needs to be checked against the 50 ft obstacle height. If the obstacle height is cleared before the end of the transition segment, the following equation is to be used to compute the transition distance:

\[ s_{TR} = \sqrt{R^2 - (R - h_{TR})^2} \]  

Finally, the horizontal distance traveled during climb to clear the 50 ft obstacle height is found from:

\[ s_C = \frac{50\text{ft} - h_{TR}}{\tan \gamma} \]  

It should be noted that when the obstacle height is already cleared during transition, \( S_C \) becomes 0.
3.6.3 **LANDING**

Landing distance is computed the same way as take-off distance, taking into account that approach speed \( V_A \) must be at least 1.23 times higher than \( V_{S-1g} \), the approach angle should not be steeper than 3 deg and the touch down velocity \( V_{TD} \) is assumed to be 1.15 times \( V_{S-1g} \) according to Raymer. This results in an average flaring velocity \( V\text{FL} \) of 1.19 times \( V_{S-0} \). The load factor during landing can be taken as 1.2 and the rolling friction coefficient due to deployed brakes during the ground run can be taken to be 10 times higher than during take-off. It should be noted that typically, the aircraft rolls free for 1 to 3 seconds before the pilot applies the brakes.

3.7 **VERIFICATION AND VALIDATION**

While the aerostructural tool developed by Elham and Van Tooren has been validated for wing drag and wing deformation [12], the enhanced method needs to be validated in two additional areas: the accuracy of computing the maximum wing lift coefficient and the accuracy of computing wing lift over drag ratios in high-lift conditions. Also, the modified wing weight estimation method needs to be validated.

3.7.1 **VL M GRID CONVERGENCE STUDY**

An important aspect to be taken into account using a VLM is the panel density. While in [11] it was found that 312 panels are required for an accurate computation of the induced drag, computing \( C_{L\text{max}} \) using the Pressure Difference Rule requires more computational power and thus a convergence study has to be performed in order to check for an appropriate grid size. Five grid types have been evaluated and the results are shown in Table 3.8.

<table>
<thead>
<tr>
<th>Grid type</th>
<th>Direction</th>
<th># Panels</th>
<th>Total</th>
<th>Condition</th>
<th>( C_{L\text{max}} )</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>Spanwise</td>
<td>5</td>
<td>25</td>
<td>Clean</td>
<td>1.7193</td>
<td>4.22</td>
</tr>
<tr>
<td></td>
<td>Chordwise</td>
<td>5</td>
<td></td>
<td>Flaps Extended</td>
<td>1.5921</td>
<td>5.60</td>
</tr>
<tr>
<td>Medium Coarse</td>
<td>Spanwise</td>
<td>10</td>
<td>70</td>
<td>Clean</td>
<td>1.7184</td>
<td>11.07</td>
</tr>
<tr>
<td></td>
<td>Chordwise</td>
<td>7</td>
<td></td>
<td>Flaps Extended</td>
<td>1.6579</td>
<td>12.04</td>
</tr>
<tr>
<td>Medium</td>
<td>Spanwise</td>
<td>13</td>
<td>104</td>
<td>Clean</td>
<td>1.7107</td>
<td>23.94</td>
</tr>
<tr>
<td></td>
<td>Chordwise</td>
<td>8</td>
<td></td>
<td>Flaps Extended</td>
<td>1.9071</td>
<td>34.79</td>
</tr>
<tr>
<td>Medium Fine</td>
<td>Spanwise</td>
<td>20</td>
<td>200</td>
<td>Clean</td>
<td>1.7027</td>
<td>99.76</td>
</tr>
<tr>
<td></td>
<td>Chordwise</td>
<td>10</td>
<td></td>
<td>Flaps Extended</td>
<td>1.8983</td>
<td>102.44</td>
</tr>
<tr>
<td>Fine</td>
<td>Spanwise</td>
<td>24</td>
<td>312</td>
<td>Clean</td>
<td>1.7023</td>
<td>274.83</td>
</tr>
<tr>
<td></td>
<td>Chordwise</td>
<td>13</td>
<td></td>
<td>Flaps Extended</td>
<td>1.9170</td>
<td>285.45</td>
</tr>
</tbody>
</table>

From Table 3.8 it can be seen that the coarse grid results in a deviation in \( C_{L\text{max}} \) of 0.12% and 0.33% compared to a fine grid for respectively the clean and multi-element wing. When considering the computation time, it can be seen that compared to a fine grid, the coarse grid saves over 90% computation time, making it the preferred grid density for the present research.

![Figure 3.13: VLM grid convergence study](image-url)
Table 3.9: VLM grid convergence study

<table>
<thead>
<tr>
<th>Grid Type</th>
<th>Clean</th>
<th>Flaps Extended</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>Medium Coarse</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Medium</td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>Medium Fine</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>Fine</td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
</tbody>
</table>
3.7.2 2D PANEL METHOD VERIFICATION

The number of points used in any panel method has a large impact on the produced results. While larger amounts of points result in more reliable results, too many points also create unnecessarily long computation time. In order to determine the optimal amount of side points which both results in reliable results as acceptable computation time, a convergence study has been performed based on $C_l$ of which the results are shown below in Figure 3.14:

![Figure 3.14: 2D panel convergence study](image)

It can be seen that for 50 side points and up, no significant change in $C_l$ can be observed, which indicates that the least amount of points to be used is 50. Having determined the required number of points on each side of the airfoil, the results of the developed panel code are verified with the built-in panel method in MSES. Below, in Figure 3.15, the results for a multi-element airfoil at 2.344° angle of attack are compared and it can be seen that the developed panel code agrees with the results obtained from MSES, except for the pressure distribution over the flap cove, which is given special treatment in MSES.

![Figure 3.15: Panel method comparison](image)

When observing the results in Table 3.10, it can furthermore be concluded that the developed panel method is able to accurately predict the inviscid lift coefficient and pressure difference required for the PDR compared to MSES.

<table>
<thead>
<tr>
<th></th>
<th>MSES</th>
<th>2D panel</th>
<th>ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{p_{\text{min}}}$</td>
<td>-4.686</td>
<td>-4.441</td>
<td>5.23%</td>
</tr>
<tr>
<td>$</td>
<td>\Delta C_p</td>
<td>$</td>
<td>4.122</td>
</tr>
<tr>
<td>$C_l$</td>
<td>1.876</td>
<td>1.904</td>
<td>1.49%</td>
</tr>
</tbody>
</table>
3.7.3 **Pressure Difference Rule**

For validating the accuracy of the $C_{L_{\text{max}}}$ computation using the PDR, the 3D Royal Aircraft Establishment (RAE) experimental database [57] is used. For the current research, the basic body-off model was used without wing extension and with trailing edge Fowler flaps extending from 0.142 of the half span to the wing tip. Below in Table 3.11 the geometry data of the wing is shown and the wing planform and airfoil design are shown in Figures 3.16 and 3.17:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface area</td>
<td>$S_w$</td>
<td>0.5523 m$^2$</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>AR</td>
<td>8.351</td>
</tr>
<tr>
<td>Taper ratio</td>
<td>$\lambda$</td>
<td>0.35</td>
</tr>
<tr>
<td>Leading edge sweep</td>
<td>$\Delta_{\text{LE}}$</td>
<td>30$^\circ$</td>
</tr>
<tr>
<td>Flap chord</td>
<td>$C_f$</td>
<td>34%</td>
</tr>
<tr>
<td>Flap deflection angles</td>
<td>$\delta_f$</td>
<td>10$^\circ$, 25$^\circ$, 40$^\circ$</td>
</tr>
</tbody>
</table>

The analysis was conducted in the RAE 11.5- by 8.5-ft low-speed wind tunnel at a nominal Mach number of .223, corresponding to a Reynolds number of 1.35e6 based on the mean wing chord. Furthermore, in the PDR analysis it was assumed that the wing was rigid and thus no wing deflection occurs. Below, in Figure 3.18a the computed results are shown together with experimental values for $C_{L_{\text{max}}}$ from [57].

![Figure 3.16: RAE Wing planform](image)

![Figure 3.17: RAE airfoil design](image)

![Figure 3.18: Validation of $C_{L_{\text{max}}}$ computation](image)
The maximum error between the computed and experimental results is 4.38% at a flap deflection of 10°. A second test case has been performed to validate $C_{L_{\text{max}}}$ of a Fokker 100 class aircraft, of which the design parameters are described in Section 4.1.1. Using the PDR, the clean $C_{L_{\text{max}}}$ was predicted to be 1.71 (See Figure 3.18b). Compared to the actual value of 1.72 [24], this is an error of 0.58%. The largest error is 9.17% at a flap deflection of 24°, which can be contributed to inaccurate flap design as exact design data was unavailable for the present research.

### 3.7.4 $\alpha$ Method

In order to validate the computation of $C_{L_{\text{max}}}$ using the $\alpha$ correction method, again the RAE wing from section 3.7.3 is used. From Figure 3.19a and 3.19b, it can be seen that the results of the solver agree well to experiment for flap angles up to 10 degrees. As the flap angle increases, a decrease in accuracy results in an average error in computed lift over drag ratio of 11.2% at a flap angle of 25°. This can be attributed to the inability of the present VLM analysis to accurately predict the flow at large flap deflections. It can furthermore be seen that it was not possible to obtain results up to maximum lift values. This was due to the fact that despite the improvements made to MSES, severe convergence issues were experienced at larger angles of attack. It is for this reason that in the remainder of the research, the PDR is used to compute $C_{L_{\text{max}}}$.

![Figure 3.19: Comparison of aerodynamic properties the modified Q3D solver and experiment][57]

### 3.7.5 Wing Weight

The wing weight prediction method used by Elham and van Tooren in [12] (and described in Section 3.5) on an A320 class wing produced remarkably accurate results. Initial investigation in the wing weight prediction of a Fokker 100 class wing of which details are presented in Section 4.1.1 showed that this method under predicts the actual wing weight by 12%, which is likely due to the fact that the Fokker 100 design is a relatively old design and therefore far from optimal. When changing the factor which accounts for non-optimal weights of 1.5 to 2 in Equation 3.28, results agreed much better with actual wing weight data of the Fokker 100 wing [58]. The same factor is therefore applied to the modified wing weight prediction method, which includes specific secondary weights. Results of both the original and modified weight prediction methods are shown below in Table 3.12, together with actual wing weight data of the Fokker 100.

<table>
<thead>
<tr>
<th></th>
<th>Eq. (3.28)</th>
<th>Eq. (3.36)</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{\text{sec}}$</td>
<td>1431 kg</td>
<td>1543 kg</td>
<td>-</td>
</tr>
<tr>
<td>$W_{\text{wing}}$</td>
<td>4257 kg</td>
<td>4369 kg</td>
<td>4343 kg</td>
</tr>
</tbody>
</table>

The error of the weight estimation from respectively the original wing weight equation and modified equation compared to the actual wing weight is 1.89% and 0.59%. It can therefore be concluded that inclusion of the secondary wing weights according to the method of Torenbeek is suitable for optimization purposes.
3.7.6 Airfield Performance

As mentioned in Section 3.6, airfield performance is governed by take-off and landing distance. Using the method of Raymer [5], the total take-off distance is determined from the sum of ground roll, rotation, transition and climbing distance:

\[ s_{TO} = s_{GR} + s_R + s_{TR} + s_C \]  \hspace{1cm} (3.48)

Landing distance is determined from the sum of approach, flare, touchdown and breaking distance:

\[ s_{LNG} = s_A + s_F + s_{TD} + s_B \]  \hspace{1cm} (3.49)

Validation of the airfield performance method of Raymer is performed on a Fokker 100 class aircraft of which details are described in Section 4.1.1. The computed take-off and landing distance are compared to Fokker 100 performance data obtained from [59].

<table>
<thead>
<tr>
<th>Table 3.13: Fokker 100 Airfield performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>s_{TO} [m]</td>
</tr>
<tr>
<td>s_{LNG} [m]</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

3.7.7 Sensitivity Verification

Finally, the sensitivities computed by the developed aerostructural analysis methods are verified through Finite Differencing (FD). While FD is a relatively simple method for the verification of sensitivities, an appropriate FD step size should be defined in order to eliminate linearization or truncation errors. This step size is determined by analyzing the derivatives computed through FD for a range of step sizes. From the resulting analysis, it can be seen for which order of step sizes the derivatives remain constant and are thus unaffected by truncations or linearization errors. Then in order to take into account all aspects of the aerostructural analysis tool developed in this research, the derivatives of take-off distance with respect to a number of key design variables are verified for a Fokker 100 class wing (described in Section 4.1.1), of which the results are shown in Table 3.14:

<table>
<thead>
<tr>
<th>Table 3.14: Sensitivity Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>s_{LNG} [m]</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
4

TEST CASE OPTIMIZATION

In this chapter the developed tool is tested in an aerostructural optimization, focussing on combining the high-lift and cruise wing design by taking into account both cruise and airfield performance. The optimization setup, which includes the test case model, design variables and constraints and optimization architecture, is discussed in Section 4.1. The optimization results are then described in Section 4.2.

4.1 OPTIMIZATION SETUP

As a test case, the aerostructural optimization of a Fokker 100 class aircraft wing is considered [60]. In this section, the test model will be described first, followed by the optimization formulation, design variables and constraints. Finally the optimization architecture is described.

4.1.1 TEST CASE MODEL

Although in reality, the Fokker 100 wing consists of two double slotted flaps, in the present research it is assumed that the high-lift system consists of a single single slotted flap, spanning a distance equal to that of the original total flap span. In Table 4.2 the wing parameters of the Fokker 100 wing are listed and the planform is shown in Figure 4.1.

![Figure 4.1: Fokker 100 wing planform][61]

The aircraft performance characteristics such as range, cruise Mach number, altitude and specific fuel consumption (SFC) of the Fokker 100 are listed in Table 4.1.
Table 4.1: Flight characteristics of the Fokker 100 aircraft [61, 62]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Take-Off Weight</td>
<td>MTOW</td>
<td>43090 kg</td>
</tr>
<tr>
<td>Cruise altitude</td>
<td>( h_{\text{cruise}} )</td>
<td>10670 m</td>
</tr>
<tr>
<td>Cruise Mach number</td>
<td>( M_{\text{cruise}} )</td>
<td>0.77</td>
</tr>
<tr>
<td>Design Range</td>
<td>R</td>
<td>2389 km</td>
</tr>
<tr>
<td>Specific Fuel Consumption</td>
<td>SFC</td>
<td>17 g/(kN s)</td>
</tr>
</tbody>
</table>

Here the aircraft MTOW is assumed to be equal to the aircraft fuel weight, aircraft wing weight and the weight of the rest of the aircraft. The weight of the rest of the aircraft is computed from aircraft weight data [24] and is kept constant during optimization.

Table 4.2: Fokker 100 wing parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing span</td>
<td>( b )</td>
<td>14.04 m</td>
</tr>
<tr>
<td>Wing area</td>
<td>( S )</td>
<td>95.43 m²</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>( A )</td>
<td>8.26</td>
</tr>
<tr>
<td>Root chord</td>
<td>( C_r )</td>
<td>5.97 m</td>
</tr>
<tr>
<td>Tip chord</td>
<td>( C_t )</td>
<td>1.09 m</td>
</tr>
<tr>
<td>Taper ratio</td>
<td>( \lambda )</td>
<td>0.18</td>
</tr>
<tr>
<td>Dihedral</td>
<td>( \theta )</td>
<td>2.5°</td>
</tr>
<tr>
<td>Sweep angle</td>
<td>( \Lambda_{1/4c} )</td>
<td>18.52°</td>
</tr>
<tr>
<td>Root twist angle</td>
<td>( \epsilon_{\text{root}} )</td>
<td>0°</td>
</tr>
<tr>
<td>Kink twist angle</td>
<td>( \epsilon_{\text{kink}} )</td>
<td>-0.65°</td>
</tr>
<tr>
<td>Tip twist angle</td>
<td>( \epsilon_{\text{tip}} )</td>
<td>-5.4°</td>
</tr>
<tr>
<td>Flap span</td>
<td>( b_f )</td>
<td>6.5 m</td>
</tr>
<tr>
<td>Flap chord</td>
<td>( C_f )</td>
<td>0.32 c</td>
</tr>
<tr>
<td>Flap deflection angles</td>
<td>( \delta_f )</td>
<td>0°, 20°</td>
</tr>
<tr>
<td>Flap overlap (at ( \delta_f = 20° ))</td>
<td>( h_f )</td>
<td>0.05 c</td>
</tr>
<tr>
<td>Flap gap (at ( \delta_f = 20° ))</td>
<td>( g_f )</td>
<td>0.024 c</td>
</tr>
</tbody>
</table>

The total drag of the aircraft is based on Fokker 100 aircraft data [24] and is determined using Figure 4.2. It follows that the total drag coefficient of the aircraft is equal to 0.0291 at a L/D ratio of 16 and cruise lift coefficient of 0.47. Using this drag coefficient, the drag coefficient of the aircraft minus wings is computed based on an initial wing drag analysis. The drag coefficient of the aircraft minus wings is determined to be equal to 0.0146 and is kept constant during the optimization.

Figure 4.2: Fokker 100 Lift over Drag ratio [24]
4.1.2 Optimization formulation

In order to initiate the wing box structural design, an aeroelastic optimization is performed aiming to minimize structural weight while satisfying failure constraints such as material failure and fatigue under the load cases listed in Table 4.3 and a constraint on aileron effectiveness. Here fatigue is simulated by limiting the stress in the wing box lower panel to 42% of the maximum allowable stress of the material in a \(1.3g\) gust load case [63]. The aileron effectiveness in the critical roll case must be higher or equal to 0.52, based on data published by Boeing [64]. After this initial optimization a full aerostructural optimization is performed aiming to minimize the fuel weight while satisfying constraints on structural failure, aileron effectiveness and airfield performance. This final optimization is formulated as follows:

\[
\begin{align*}
\text{min} & \quad W_{\text{fuel}}^*(X) \\
\text{s.t.} & \quad \text{Failure}_k \leq 0 \\
               & \quad \frac{L_{\delta_0}}{L_\delta} - 1 \leq 0 \\
               & \quad \frac{s_{\text{TO}}}{s_{\text{TO}_0}} - 1 \leq 0 \\
               & \quad \frac{s_{\text{LNG}}}{s_{\text{LNG}_0}} - 1 \leq 0 \\
               & \quad W_{\text{fuel}} - 1 = 0 \\
               & \quad \frac{W_{\text{fuel}}}{W_{\text{fuel}}^*} - 1 = 0 \\
               & \quad MTOW - 1 = 0 \\
               & \quad X_{\text{lower}} \leq X \leq X_{\text{upper}}
\end{align*}
\]

Table 4.3: Load cases for wing aeroelastic optimization [61, 65].

<table>
<thead>
<tr>
<th>Load case</th>
<th>type</th>
<th>H [m]</th>
<th>M</th>
<th>n [g]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>pull up, (M_D)</td>
<td>7500</td>
<td>0.84</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>pull up, (V_D)</td>
<td>0</td>
<td>0.57</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>push down, (M_D)</td>
<td>7500</td>
<td>0.84</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>gust, (M_D)</td>
<td>7500</td>
<td>0.84</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>roll, (1.15V_D)</td>
<td>4000</td>
<td>0.81</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>cruise, (M_{\text{cruise}})</td>
<td>10670</td>
<td>0.77</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>takeoff, (V_2)</td>
<td>0</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>landing, (V_A)</td>
<td>0</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>
4.1.3 Design Variables and Constraints

In the full aerostructural optimization, design space $X$ is defined as: $X = [T, P, P_f, G_{1}, G_{2}, \xi, W_{fuel}^*, M_TOW^*]$. Here variables $T$, $P$ and $P_f$ represent respectively the equivalent panel thickness design vector, planform design vector and the high-lift device planform design vector. Below, a breakdown of these design vectors is presented.

- $T = \left[ t_{u_1}, t_{l_1}, t_{f_{si}}, t_{f_{ti}} \right]$
- $P = \left[ C_r, b_1, b_2, \lambda, A_1, A_2, \epsilon_{kink}, \epsilon_{tip} \right]$
- $P_f = \left[ b_f \right]$

The thicknesses $T$ are defined at 10 spanwise positions from root to tip. The wing planform geometry is parameterized using 8 design variables: root chord $C_r$, inboard span $b_1$, outboard span $b_2$, taper ratio $\lambda$, LE sweep angles $A_1$ and $A_2$, twist angle at kink $\epsilon_{kink}$ and twist angle at tip $\epsilon_{tip}$. As far as flap planform design variables go only the flap span is defined as design variable. Note that the inboard flap position is fixed to account for the landing gear. In Figure 4.3, a graphical representation of the planform variables is presented.

![Figure 4.3: Planform design variables.](image)

The fourth group of design variables is used to define the wing airfoils’ shapes as described in Section 3.4 and the fifth group of design variables is used to perturb the flap’s position using the mode amplitudes $G_{tk}$ (see Figure 4.4). The mode amplitudes consist of two translational modes. $G_{t1}$ controls the horizontal translation of the flap and $G_{t2}$ the vertical translation. The third mode amplitude $G_{t3}$ controls the flap deflection.

![Figure 4.4: 2D Airfoil shape design space.](image)

The author is aware that the proposed shape parameterization introduces a limitation in the design space of the airfoil shape due to the nature of the Chebyshev modes, which only perturb the surfaces in between the defined end points B, C, E and F. In order to include perturbations of these points in the design space, two
parameters ($\zeta_1$ and $\zeta_2$) may be added to design space $X$ which control the vertical position of respectively points B, E and C, F perpendicular to their adjacent surface (see Figure 4.4). As the gradient of the desired functions of interest with respect to these two parameters can only be determined through Finite Differencing and do not add to fulfilling the research objective, these parameters are not included in the present research. The final group of variables are used to avoid unnecessary iterations for aeroelastic analysis.

The final optimization problem is subject to a number of constraints. As for the aeroelastic optimization, the same load cases are used to compute the failure criteria and a constraint is put on the aileron effectiveness, as described in [12]. Two additional constraints are applied to the required take-off and landing distance in order to ensure airfield performance. A list of the applied constraints is presented in Table 4.4.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Equation</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression upper panel</td>
<td>$F_{\text{compression}} \leq 0$</td>
<td>128</td>
</tr>
<tr>
<td>Compression lower panel</td>
<td>$F_{\text{compression}} \leq 0$</td>
<td>64</td>
</tr>
<tr>
<td>Tension upper panel</td>
<td>$F_{\text{tension}} \leq 0$</td>
<td>64</td>
</tr>
<tr>
<td>Tension lower panel</td>
<td>$F_{\text{tension}} \leq 0$</td>
<td>128</td>
</tr>
<tr>
<td>Buckling upper panel</td>
<td>$F_{\text{buckling}} \leq 0$</td>
<td>192</td>
</tr>
<tr>
<td>Shear front spar</td>
<td>$F_{\text{shear}} \leq 0$</td>
<td>96</td>
</tr>
<tr>
<td>Buckling front spar</td>
<td>$F_{\text{buckling}} \leq 0$</td>
<td>96</td>
</tr>
<tr>
<td>Shear rear spar</td>
<td>$F_{\text{shear}} \leq 0$</td>
<td>96</td>
</tr>
<tr>
<td>Buckling rear spar</td>
<td>$F_{\text{buckling}} \leq 0$</td>
<td>96</td>
</tr>
<tr>
<td>Fatigue</td>
<td>$F_{\text{fatigue}} \leq 0$</td>
<td>64</td>
</tr>
<tr>
<td>Aileron Effectiveness</td>
<td>$1 - \frac{M_a}{M_{a_{\text{min}}}} \leq 0$</td>
<td>1</td>
</tr>
<tr>
<td>Takeoff distance</td>
<td>$\frac{\text{STO}}{S_{\text{TO0}}} - 1 \leq 0$</td>
<td>1</td>
</tr>
<tr>
<td>Landing distance</td>
<td>$\frac{\text{SNG}}{S_{\text{NG0}}} - 1 \leq 0$</td>
<td>1</td>
</tr>
<tr>
<td>Fuel weight</td>
<td>$W_{\text{fuel}} - W_{\text{fuel0}} = 0$</td>
<td>1</td>
</tr>
<tr>
<td>Maximum Take Off Weight</td>
<td>$\frac{M_{\text{TOW}}}{M_{\text{TOW0}}} - 1 = 0$</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1029</td>
</tr>
</tbody>
</table>
4. TEST CASE OPTIMIZATION

4.1.4 DESIGN STRUCTURE MATRIX

In an effort to develop a standard representation of multidisciplinary design optimization architectures, Lambe and Martins developed an Extended Design Structure Matrix (XDSM) which proves to be very effective in capturing complex architectures [66]. The aerostructural analysis and optimization tool developed by Elham and Van Tooren, on which the present research is based on, has been captured in a XDSM in [12]. In order to capture the optimization scheme proposed in this research, the XDSM presented in [12] is extended with the tools discussed in this report.

In order to capture the $C_{L\text{max}}$ prediction using the PDR in the XDSM, a loop was added to the MDA as can be seen in Figure 4.5. When computing takeoff and landing performance, the MDA now loops processes 2 to 4, being respectively the VLM analysis, the structural analysis and the PDR analysis, before proceeding to the second loop being the viscous aerostructural analysis. A description of the notation in this XDSM is presented in Table 4.5.

![Figure 4.5: Extended Design Structure Matrix](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>Vector of design variables</td>
</tr>
<tr>
<td>$x_s$</td>
<td>Vector of surrogate variables</td>
</tr>
<tr>
<td>$y^f$</td>
<td>Vector of coupling variable targets (inputs to a discipline analysis)</td>
</tr>
<tr>
<td>$y$</td>
<td>Vector of coupling variable responses (outputs from a discipline analysis)</td>
</tr>
<tr>
<td>$f$</td>
<td>Objective function</td>
</tr>
<tr>
<td>$c$</td>
<td>Vector of equality and inequality constraints</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of disciplines</td>
</tr>
<tr>
<td>()0</td>
<td>Functions or variables that are shared by more than one discipline</td>
</tr>
<tr>
<td>()i</td>
<td>Functions or variables that apply only to discipline $i$</td>
</tr>
<tr>
<td>()*</td>
<td>Functions or variables at their optimal value</td>
</tr>
</tbody>
</table>

Table 4.5: Mathematical notation for MDO problem data [66]
4.2 Optimization Results

In order to compare the proposed optimization procedure to conventional optimization schemes, two optimizations are performed for the Fokker 100 wing:

- Wing A (Combined optimization): Wing planform and high-lift devices are optimized for fuel weight from the start of the optimization, taking into account airfield performance constraints.

- Wing B (Sequential optimization): Wing planform is initially optimized for cruise performance without airfield performance constraints. Then high-lift devices are sized while fixing the optimized cruise wing planform to further minimize fuel weight taking into account airfield performance constraints.

Both optimizations were performed using 63GB of RAM and eight 3.50 GHz processors. The combined optimization was solved after 19 iterations and 103 function evaluations, each taking 7 minutes to complete, resulting in a total optimization duration of 12 hours.

Table 4.6: Initial and optimized wing geometry variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial</th>
<th>Wing A</th>
<th>Wing B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_r$ [m]</td>
<td>5.97</td>
<td>5.52</td>
<td>5.49</td>
</tr>
<tr>
<td>$\lambda$ [-]</td>
<td>0.18</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>$b_1$ [m]</td>
<td>4.70</td>
<td>3.97</td>
<td>4.39</td>
</tr>
<tr>
<td>$b_2$ [m]</td>
<td>9.34</td>
<td>11.73</td>
<td>10.42</td>
</tr>
<tr>
<td>$\Lambda_1$ [°]</td>
<td>25.5</td>
<td>21.26</td>
<td>21.46</td>
</tr>
<tr>
<td>$\Lambda_2$ [°]</td>
<td>21.5</td>
<td>18.23</td>
<td>20.91</td>
</tr>
<tr>
<td>$\epsilon_1$ [°]</td>
<td>-0.65</td>
<td>-0.80</td>
<td>-0.76</td>
</tr>
<tr>
<td>$\epsilon_2$ [°]</td>
<td>-5.40</td>
<td>-4.32</td>
<td>-4.34</td>
</tr>
<tr>
<td>$b_f$ [m]</td>
<td>6.50</td>
<td>5.06</td>
<td>5.50</td>
</tr>
</tbody>
</table>

In Figure 4.6 the planforms of both optimization cases A and B are shown. It can be seen that both optimized wings have a higher aspect ratio of respectively 10.65 and 9.74 compared to 8.26 of the initial wing. Quarter chord sweep angles are reduced to respectively $14.5^\circ$ and $16.9^\circ$ compared to the initial sweep angle of $18.5^\circ$. The difference in the planforms originate from the different optimization schemes as a pure high-speed wing design tends to have an increased sweep angle and larger wing loading to reduce pressure drag than a wing design which takes into account airfield performance. The fuel weight reduction of wing A is 9.65% and 9.20% for wing B while the wing weight is only marginally reduced by respectively 3.6% and 3.3% respectively. The optimization history of wing A is shown below in Figure 4.7.
From Table 4.9 it can be seen that the reduced fuel weight is a result of a reduced total drag of respectively 28.7% and 25.0% for wings A and B. While the induced drag is reduced by respectively 24.4% and 14.1% due to the increased aspect ratio which in turns ensures a more elliptical lift distribution (see Figure 4.9), the largest drag reduction is seen in pressure drag which have been reduced by respectively 57.1% and 60.3%. This reduction in pressure drag is a result of the optimized airfoil shapes which can be seen in Figure 4.12. Despite the increased normal Mach number due to the reduced sweep angle, the optimizer was able to weaken and at several spanwise positions completely remove the shock waves on the airfoils at cruise speed, which can be seen in Figure 4.13. The reduced shock wave strength over the wing results in reduced wave drag and thus pressure drag.

Table 4.7: Characteristics of the initial and the optimized aircraft.

<table>
<thead>
<tr>
<th></th>
<th>MTOW [kg]</th>
<th>$W_{\text{fuel}}$ [kg]</th>
<th>$W_{\text{wing}}$ [kg]</th>
<th>$S_{\text{wing}}$ [m$^2$]</th>
<th>$C_{L_{\text{cruise}}}$</th>
<th>$C_{D_{\text{cruise}}}$</th>
<th>$C_{D_{\text{i}}}$</th>
<th>$C_{D_{\text{p}}}$</th>
<th>$C_{D_{\text{f}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>43090</td>
<td>7260</td>
<td>4369</td>
<td>95.4</td>
<td>0.41</td>
<td>0.0188</td>
<td>0.0078</td>
<td>0.0063</td>
<td>0.0047</td>
</tr>
<tr>
<td>Combined</td>
<td>42487</td>
<td>6559</td>
<td>4468</td>
<td>92.6</td>
<td>0.42</td>
<td>0.0132</td>
<td>0.0059</td>
<td>0.0027</td>
<td>0.0046</td>
</tr>
<tr>
<td>Sequential</td>
<td>42492</td>
<td>6592</td>
<td>4446</td>
<td>90.1</td>
<td>0.43</td>
<td>0.0141</td>
<td>0.0067</td>
<td>0.0025</td>
<td>0.0049</td>
</tr>
</tbody>
</table>

As mentioned before, the wing weight of both wings A and B have been reduced. While a higher aspect ratio typically results in a larger structural weight due to increased bending moment, this weight penalty has been countered by reducing the wing sweep angle and increasing the wing flexibility. The initial vertical and twist deformation under 1g load of the initial wing tip are respectively 0.39m and -0.84°. For a 2.5g pull up these values are 1.12m and -2.24°. The vertical and twist deformation of wing A under 1g load are 0.94m and -1.29° and for wing B 0.75m and -1.25°. For a 2.5g pull up these deformations are 2.37m and -3.02° respectively for wing A and 1.93m and -2.99° for wing B. The increased flexibility of the wing can be clearly seen in Figure 4.8.

Figure 4.8: Initial and optimized wing deformed shapes under 2.5g pull up

(a) $C_{L_{\text{i}}}$ distribution
(b) $C_{L_{\text{i}}}$ distribution

Figure 4.9: Lift distribution on the initial wing and optimized wings in cruise condition
In Table 4.8 the flap designs for the initial and optimized designs are listed. Because wing B has a smaller wing span and wing area that wing A, it is expected to require a larger flap span to maintain airfield performance. Indeed, from Table 4.8 it follows that the flap span for wing A is reduced by 11.8% and for wing B only by 7.23%. The flap weight of wing A and B is also reduced by respectively 22.9% and 17.9%. Flap overlap ($h_f$), gap ($g_f$) and deflection ($\delta_f$) have all increased in landing configuration in order to increase flap efficiency to counter the reduced flap span and increased wing loading as can be seen in Figure 4.11. It is also notable that both optimization schemes result in similar flap settings, despite changes to wing geometry. While this fact suggests that a global optimum exists for flap settings, further research is necessary to confirm this.

<table>
<thead>
<tr>
<th></th>
<th>$W_{flap}$ [kg]</th>
<th>$S_{flap}$ [m$^2$]</th>
<th>$b_f$ [m]</th>
<th>$h_f$ [%]</th>
<th>$g_f$ [%]</th>
<th>$\delta_f$ [°]</th>
<th>$h_f$ [%]</th>
<th>$g_f$ [%]</th>
<th>$\delta_f$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>576</td>
<td>17.1</td>
<td>6.50</td>
<td>5.00</td>
<td>2.40</td>
<td>20.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td>443</td>
<td>13.0</td>
<td>5.06</td>
<td>5.69</td>
<td>2.75</td>
<td>28.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sequential</td>
<td>473</td>
<td>14.0</td>
<td>5.50</td>
<td>5.78</td>
<td>2.42</td>
<td>27.76</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The high-lift performance characteristics of the optimized wings are listed in Table 4.9. It can be seen that both wings A and B have increased $C_{L_{max}}$ in take-off and landing configuration as a result of both a reduced sweep angle and increased flap deflection. While both wings have a reduced wing area, the stall speed $V_{s_{1-g}}$ at MTOW and MLW remain similar to the initial design as a result of the increased $C_{L_{max}}$ values. Furthermore, when considering the lift distribution shown in Figure 4.10, it can be seen that due to the reduced weight of the aircraft, the total required lift for landing is reduced.

While wing A meets both airfield performance requirements, wing B is not able to achieve take-off performance with flaps retracted. Although this seems to be an infeasible solution, it is a result of the optimization formulation, in which take-off is always performed with flaps retracted. As a result, flap optimization was not performed for take-off conditions. When considering the design of wing B, it can be seen that its wing loading has increased due to the small wing area. While this is beneficial for cruise flight, it is undesirable for maintaining airfield performance.

<table>
<thead>
<tr>
<th></th>
<th>$C_{L_{max}}$</th>
<th>$C_{L_{max}}$</th>
<th>$V_{s_{1-g}}$</th>
<th>$V_{s_{1-g}}$</th>
<th>$\frac{L}{D}$</th>
<th>$\frac{L}{D}$</th>
<th>$s_{TO}$ [m]</th>
<th>$s_{LNG}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>1.71</td>
<td>1.91</td>
<td>65.03</td>
<td>51.91</td>
<td>19.99</td>
<td>12.84</td>
<td>1827</td>
<td>1436</td>
</tr>
<tr>
<td>Combined</td>
<td>1.75</td>
<td>2.12</td>
<td>64.69</td>
<td>50.81</td>
<td>23.39</td>
<td>11.15</td>
<td>1767</td>
<td>1432</td>
</tr>
<tr>
<td>Sequential</td>
<td>1.76</td>
<td>2.03</td>
<td>65.68</td>
<td>52.36</td>
<td>22.00</td>
<td>11.24</td>
<td>1835</td>
<td>1431</td>
</tr>
</tbody>
</table>

Figure 4.10: Lift distribution on the initial wing and optimized wings with flaps deployed in landing configuration
4. Test Case Optimization

Figure 4.11: Initial and optimized flap configuration of the multi-element sections perpendicular to the sweep line in landing configuration

Figure 4.12: Initial and optimized airfoil shape on sections perpendicular to the sweep line
4.2 Optimization Results

Figure 4.13: Initial and optimized pressure distribution on sections perpendicular to the sweep line

(a) $2y/b=0$
(b) $2y/b=0.14$
(c) $2y/b=0.29$
(d) $2y/b=0.43$
(e) $2y/b=0.57$
(f) $2y/b=0.71$
(g) $2y/b=0.86$
(h) $2y/b=1$
5

Conclusions and Recommendations

5.1 Conclusions

The present study was performed in an effort to develop a framework for the combined aerostructural optimization of the cruise and high-lift wing design. Since both designs require different requirements which are sometimes contradicting, but at the same time influence each other to a large extent, combining the designs in a single optimization offers great potential to wing design optimization. Three main objectives were set to achieve the research goal. The first objective was to incorporate the high-lift devices in an existing aerostructural analysis and optimization tool, focusing on keeping the overall structure of the tool intact. The second objective was to validate the enhanced model before performing a full aerostructural test case optimization to prove the potential of the proposed design methodology.

In order to compute the wing $C_{L_{\text{max}}}$, two methods have been researched and developed. The first method is an $\alpha$ correction method and is based on research performed by Van Dam. The second method is a semi-empirical method called the Pressure Difference Rule and is able to compute the wing maximum lift with minimal computational effort as it requires the use of only inviscid analyses. Both methods were implemented in a Q3D aerodynamic analysis and have been coupled to the structural solver FEMWET in order to include aeroelastic effects for the first time in high-lift aerodynamic analysis and maximum lift coefficient prediction. Both systems are solved using the Newton method for iteration and using the partial derivatives from the converged Newton method, the coupled-adjoint method was used to compute the total derivative of any function of interest with respect to the design variables.

Validation of the modified analysis tools showed that the implemented $\alpha$ method is very able to compute the aerodynamic loads of swept wings up to large angles of attack. However, due to the convergence issues of MSES, despite modifications applied to boundary layer computation and transition prediction, validation of $C_{L_{\text{max}}}$ prediction was not possible. Using the Pressure Difference Rule, a maximum lift prediction accuracy of 5% for a swept wing with clean flaps for flap deflections up to $40^\circ$ was obtained. For the remainder of the study the Pressure Difference Rule was therefore used as prediction method.

As test case, the aerostructural optimization of a Fokker 100 wing was performed using two optimization scheme, one being the combined optimization proposed in this thesis and the second being the classical scheme of optimizing a wing first for cruise conditions followed by an optimization to satisfy airfield performance, computed using the method of Raymer. The combined optimization resulted in a fuel weight reduction of 9.65% and the sequential optimization managed to reduce fuel weight by 9.20%. The reduced fuel weight was attributed to a reduction in pressure drag resulting from modified airfoil shapes, reducing shock waves over the wing. The optimized wings both had an increased aspect ratio and reduced sweep which resulted in a reduction of induced drag. To counter the weight penalty due to these modifications, the structural stiffness was reduced. The optimizer reduced flap weight of both optimized wings by respectively 22.9% and 17.9%, contributing to the difference in fuel weight reduction between the two wings.

Combining the optimization of the cruise and high-lift wing design from the start of the optimization process proves to be a promising methodology as it enables trade offs to be made between the various design aspects which influence both cruise and airfield performance. Although a semi-empirical method has been employed to determine the wing $C_{L_{\text{max}}}$, the present research has included a more physics based method.
which can be implemented as soon as a more robust 2D viscous solver becomes available. Once fully developed, the presented analysis and optimization tool will be able to perform combined aerostructural wing and high-lift system optimization of both conventional and novel wing design concepts.

5.2 LIMITATIONS AND RECOMMENDATIONS

As described in the previous section, the tool developed during this research poses a number of challenges which promote future research. In this section, a number of recommendations will be provided in order to further improve the analysis and optimization capabilities of the developed aerostructural analysis and optimization tool.

• The aerostructural analysis and optimization tool presented in this report uses a semi-empirical method to compute $C_{L_{\text{max}}}$, while yielding accurate results, it is recommended that MSES is replaced by a more robust 2D viscous solver in order to enable the use of the physics-based $\alpha$ method for $C_{L_{\text{max}}}$ prediction. This will enable the analysis and optimization of novel wing configurations, besides conventional wing designs.

• Flap variables in the presented research include flap span, gap, overlap and deflection as high-lift design variables. It is recommended that flap chord is included in the design space, enabling the optimization of the rear spar location, which in turn requires adding a constraint on fuel volume. It should be noted that in the methodology presented in this research, including flap chord as a design variable implies adding a Finite Difference step in order to acquire the required sensitivities, increasing computational cost.

• At present, the analysis and optimization tool is only capable of analyzing single-slotted flaps. It is recommended to include more complex trailing edge devices, as well as leading edge devices. While this can be easily achieved in the VLM analysis, extensive research should be performed on the capability of MSES to solve more complex high-lift system combinations.

• Besides different high-lift devices, also high-lift device support structure should be considered in future research in order to investigate the effect of different flap settings on structural weight. In order to do this, the flap support structure should be modeled in the FEM analysis as beams connecting the wing primary structure and high-lift devices.

• While the current take-off and landing performance model is suitable for investigation of possibilities of combined aerostructural wing and high-lift system optimization, a more extensive airfield simulation should be implemented to accurately predict airfield performance.

• In order to validate the effectiveness of the presented optimization tool, different wing designs should be considered in future research.

• As gradient based optimization results in local optima, it is recommended to perform a global optimization in order to check whether the produced results in this research are in fact global optima instead of local optima.


SENSITIVITY ANALYSIS

As described in Section 3.2.1, two systems of equations are defined in order to solve respectively the implementation of the $\alpha$ correction method and Pressure Difference Rule (PDR). In order to solve these systems, the Newton method for iteration is used for which the derivatives of the governing equations with respect to the input variables are required. In this appendix, a detailed derivation of the derivatives presented in Section 3.3.2 is presented.

A.1 $\alpha$ CORRECTION METHOD

The $\alpha$ correction method is integrated in the aerostructural analysis tool through the following set of equations:

\[
\begin{bmatrix}
R_1(\Gamma, U, \alpha, \alpha_{corr}) \\
R_2(\Gamma, U, \alpha, \alpha_{corr}) \\
R_3(\Gamma, U, \alpha, \alpha_{corr}) \\
R_4(\Gamma, U, \alpha, \alpha_{corr})
\end{bmatrix} =
\begin{bmatrix}
\text{AIC} \Gamma - \text{RHS} \\
K U - F \\
L - n W_{des} \\
C_{l_d} - C_{l_i}
\end{bmatrix}
= \vec{0} \tag{A.1}
\]

With partial derivative matrix:

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma$</th>
<th>$U$</th>
<th>$\alpha$</th>
<th>$\alpha_{corr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>AIC</td>
<td>$\partial \text{AIC} / \partial \Gamma$</td>
<td>$\partial \text{RHS} / \partial U$</td>
<td>$\partial \text{RHS} / \partial \alpha$</td>
</tr>
<tr>
<td>$R_2$</td>
<td>-$\partial \text{RHS} / \partial \alpha$</td>
<td>$0$</td>
<td>$0$</td>
<td></td>
</tr>
<tr>
<td>$R_3$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td></td>
</tr>
</tbody>
</table>

$R_1$: AIC $\Gamma$ - RHS $= 0$

Starting with the first row in Table A.1, the partial derivative of $R_1$ with respect to $\Gamma$ is the Aerodynamic Influence Coefficient (AIC) matrix. The partial derivatives of this matrix as well as the RHS vector with respect to the deflection $U$ are computes using Automatic Differentiation (AD). The derivatives of $R_1$ with respect to the angle of attack $\alpha$ and the viscous correction angle can be computed analytically through Equations A.2 and A.3:

\[
\frac{\partial R_1}{\partial \alpha} = \frac{\partial \text{AIC}}{\partial \alpha} \cdot \Gamma + \text{AIC} \cdot \frac{\partial \Gamma}{\partial \alpha} - \frac{\partial \text{RHS}}{\partial \alpha} = - \frac{\partial \text{RHS}}{\partial \alpha} \tag{A.2}
\]

\[
\frac{\partial R_1}{\partial \alpha_{corr}} = \frac{\partial \text{AIC}}{\partial \alpha_{corr}} \cdot \Gamma + \text{AIC} \cdot \frac{\partial \Gamma}{\partial \alpha_{corr}} - \frac{\partial \text{RHS}}{\partial \alpha_{corr}} = - \frac{\partial \text{RHS}}{\partial \alpha_{corr}} \tag{A.3}
\]

The derivative of the RHS vector with respect to $\alpha$ and $\alpha_{corr}$ can be computed analytically from Equation A.4.

\[
\text{RHS} = V_{\infty} \left( \cos(\alpha + \alpha_{corr}) \hat{i} - \sin(\alpha + \alpha_{corr}) \hat{k} \right) \tag{A.4}
\]

The derivative of the RHS vector with respect to $\alpha$ and $\alpha_{corr}$ become:
A. Sensitivity Analysis

\[
\frac{\partial \text{RHS}}{\partial \alpha} = \frac{\partial \text{RHS}}{\partial \alpha_{\text{corr}}} = -V_\infty \left( \sin(\alpha + \alpha_{\text{corr}}) \hat{I} + \cos(\alpha + \alpha_{\text{corr}}) \hat{k} \right)
\]  
(A.5)

**R2:** \( KU - F = 0 \)

Moving on to the second row in Table A.1, the derivative of \( R_2 \) with respect to \( \Gamma \) is equal to \( \frac{\partial F}{\partial \Gamma} \) which can be computed analytically when considering Equation A.6:

\[
F = \frac{\rho V_\infty \Gamma}{\sqrt{1 - M_\infty^2}}
\]

(A.6)

The derivative of \( F \) with respect to \( \Gamma \) becomes:

\[
\frac{\partial F}{\partial \Gamma} = \frac{\rho V_\infty}{\sqrt{1 - M_\infty^2}}
\]

(A.7)

The derivative of \( R_2 \) with respect to the deflection \( U \) is equal to the stiffness matrix \( K \) and from Equation A.6 it can be determined that the partial derivatives of \( R_2 \) with respect to \( \alpha \) and \( \alpha_{\text{corr}} \) are equal to \( \hat{0} \).

**R3:** \( L - nW = \hat{0} \)

The third governing equation relates to the lift computed from the VLM analysis. Its derivative with respect to \( \Gamma \) is computed analytically by deriving Equation A.8:

\[
L' = \rho V_\infty \Gamma dy
\]

(A.8)

Its derivative with respect to \( \Gamma \) becomes:

\[
\frac{\partial L'}{\partial \Gamma} = \rho V_\infty dy
\]

(A.9)

The derivatives of \( R_2 \) with respect to \( U \), \( \alpha \) and \( \alpha_{\text{corr}} \) are equal to \( \hat{0} \).

**R4:** \( C_{l_{2d}} - C_{l_{\perp}} = \hat{0} \)

The derivatives of \( R_4 \) require a bit more attention due to addition of viscous aerodynamic loads. Considering the fact that due to the implementation of the \( \alpha \) method both the 2D viscous lift distribution as the lift distribution from the VLM analysis are a function of \( \Gamma \) due to the formulation of the effective angle of attack used for the 2D analysis:

\[
a_{\text{eff}} = \frac{C_{l_{\perp}}}{C_{l_{2d}}} + \alpha_0 - \alpha_{\text{corr}}
\]

(A.10)

The derivative of \( R_4 \) with respect to \( \Gamma \) becomes:

\[
\frac{\partial R_4}{\partial \Gamma} = \frac{\partial C_{l_{2d}}}{\partial \Gamma} - \frac{\partial C_{l_{\perp}}}{\partial \Gamma}
\]

(A.11)

The derivatives of \( R_4 \) with respect to \( \alpha \) and \( \alpha_{\text{corr}} \) are defined as:

\[
\begin{align*}
\frac{\partial R_4}{\partial \alpha} &= \frac{\partial C_{l_{2d}}}{\partial \alpha} - \frac{\partial C_{l_{\perp}}}{\partial \alpha} = \frac{\partial C_{l_{2d}}}{\partial \alpha} \\
\frac{\partial R_4}{\partial \alpha_{\text{corr}}} &= \frac{\partial C_{l_{2d}}}{\partial \alpha_{\text{corr}}} - \frac{\partial C_{l_{\perp}}}{\partial \alpha_{\text{corr}}} = \frac{\partial C_{l_{2d}}}{\partial \alpha_{\text{corr}}}
\end{align*}
\]

(A.12)

(A.13)

In order to solve these derivatives, first Equation A.10 is considered. Below, the derivatives of \( a_{\text{eff}} \) with respect to \( \Gamma \), \( \alpha \) and \( \alpha_{\text{corr}} \) are defined:

\[
\begin{align*}
\frac{\partial a_{\text{eff}}}{\partial \Gamma} &= \frac{\partial C_{l_{\perp}}}{\partial \Gamma} \frac{1}{C_{l_{2d}}} \\
\frac{\partial a_{\text{eff}}}{\partial \alpha_{\text{corr}}} &= -\hat{1}
\end{align*}
\]

(A.14)

(A.15)
\[ \frac{\partial \alpha_{\text{eff}}}{\partial \alpha} = 0 \]  

(A.16)

Contrary to the research of Elham and van Tooren [12], the induced angle of attack in this research is computed analytically rather than being defined as state variable. The induced angle of attack is computed through:

\[ \alpha_i = \frac{\alpha + \epsilon}{\cos \Lambda} - \alpha_{\text{eff}} \]  

(A.17)

This yields the following derivative of \( \alpha_i \) with respect to \( \alpha_{\text{eff}} \):

\[ \frac{\partial \alpha_i}{\partial \alpha_{\text{eff}}} = -I \]  

(A.18)

Combining the derivatives defined above with the fact that the lift computed by MSES is a function of airfoil geometry, effective angle of attack, Mach number and Reynolds number, Equations A.11, A.13 and A.12 become:

\[
\frac{\partial R_2}{\partial \alpha} = \frac{\partial C_{\text{ld}}}{\partial \alpha} + \frac{\partial C_{\text{ld}}}{\partial C_{\text{eff}}} \left( \frac{dC_{\text{ld}}}{d\alpha_{\text{eff}}} + \frac{dC_{\text{ld}}}{dM_{\text{eff}}} \frac{d\alpha_{\text{eff}}}{d\alpha} + \frac{dC_{\text{ld}}}{dR_{\text{eff}}} \frac{d\alpha_{\text{eff}}}{d\alpha} \right) + \frac{\partial C_{\text{ld}}}{\partial M_{\text{eff}}} \frac{d\alpha_{\text{eff}}}{d\alpha} + \frac{\partial C_{\text{ld}}}{\partial R_{\text{eff}}} \frac{d\alpha_{\text{eff}}}{d\alpha} \]

(A.19)

The derivatives in the above equations are computed using a combined use of the coupled-adjoint method within MSES and Automatic Differentiation.
A.2 Pressure Difference Rule

The prediction of the wing $C_{l_{\text{max}}}$ using the implemented PDR is performed through the following system of governing equations:

$$\text{Sys}_{PDR} = \begin{bmatrix} R_1(\Gamma, U, \alpha, \alpha_i) \\ R_2(\Gamma, U, \alpha, \alpha_i) \\ R_3(\Gamma, U, \alpha, \alpha_i) \\ R_4(\Gamma, U, \alpha, \alpha_i) \end{bmatrix} = \begin{bmatrix} AIC \Gamma - \text{RHS} \\ KU - F \\ KS \\ C_{l_{\text{lim}}} - C_{l_{\perp}} \end{bmatrix} = \vec{0} \quad (A.22)$$

With partial derivative matrix:

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$U$</th>
<th>$\alpha$</th>
<th>$\alpha_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>$\frac{\partial AIC}{\partial \Gamma}$</td>
<td>$\frac{\partial \text{RHS}}{\partial U}$</td>
<td>$\frac{\partial \text{RHS}}{\partial \alpha}$</td>
</tr>
<tr>
<td>$R_2$</td>
<td>$\frac{\partial F}{\partial \Gamma}$</td>
<td>$K$</td>
<td>$0$</td>
</tr>
<tr>
<td>$R_3$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{\partial KS}{\partial \alpha}$</td>
</tr>
<tr>
<td>$R_4$</td>
<td>$\frac{\partial C_{l_{\perp}}}{\partial \alpha}$</td>
<td>$\frac{\partial C_{l_{\text{lim}}}}{\partial U}$</td>
<td>$\frac{\partial C_{l_{\text{lim}}}}{\partial \alpha}$</td>
</tr>
</tbody>
</table>

**$R_1$: $AIC \Gamma - \text{RHS} = 0$**

Starting again with the first row of Table A.2, the difference with Table A.1 is that no viscous correction angle is applied to the RHS of the VLM governing equation and that the derivative of $R_1$ with respect to $\alpha_i$ is equal to $0$.

**$R_2$: $KU - F = 0$**

The second row of Table A.2 is computed analogously to the second row of Table A.1.

**$R_3$: $KS = 0$**

Moving on to the third row, it can be observed that instead of equating wing lift to the design weight, the KS function is used as governing equation. The KS function is defined as:

$$KS = f_{\text{max}} + \frac{1}{\rho} \log e \sum_{k=1}^{K} e^{\rho(f_k(X)-f_{\text{max}})} = 0 \quad (A.23)$$

Here:

$$f_k = \Delta C_{p_{\text{crit}}k} - \Delta C_{p_{\text{crit}}} \quad (A.24)$$

Where $k$ denotes the spanwise section, $f_{\text{max}}$ is the maximum value of the set of functions $f_k$ evaluated at $X$ and for $\rho$ a value of 80 is used. The critical pressure difference at which maximum lift is occurs, $\Delta C_{p_{\text{crit}}}$, is computed from Figure A.1:

**Figure A.1: Pressure Difference Rule [46]**
The derivative of the KS function with respect to any variable \( x \) is defined as:

\[
\frac{\partial KS}{\partial x} = \sum_{k=1}^{K} e^{\phi(k)} \frac{\partial f_k}{\partial x} \frac{\partial f_k}{\partial \alpha} \frac{\partial \alpha}{\partial x}
\]  

(A.25)

Here, the derivatives of \( f_k \) with respect to \( \alpha \) and \( \alpha_i \) are computed analytically through:

\[
\frac{\partial f_k}{\partial \alpha} = \frac{\partial \Delta C_{pdx}}{\partial \alpha} \left[ \frac{d \Delta C_{pdx}}{d \alpha_{eff}} \frac{d \alpha_{eff}}{d \alpha} \right]
\]

(A.26)

\[
\frac{\partial f_k}{\partial \alpha_i} = \frac{\partial \Delta C_{pdx}}{\partial \alpha_{eff}} \frac{\partial \alpha_{eff}}{\partial \alpha_i} - \frac{\partial \Delta C_{pdx}}{\partial \alpha_i} - \frac{\partial \Delta C_{pdx}}{\partial \alpha_i} \left[ \frac{d \Delta C_{pdx}}{d \alpha_{eff}} \frac{d \alpha_{eff}}{d \alpha_i} + \frac{d \Delta C_{pdx}}{d M_{eff}} \frac{d M_{eff}}{d \alpha_i} \right]
\]

(A.27)

In order to compute the partial derivative of the critical pressure difference \( \Delta C_{pdx} \) with respect to the effective Reynolds number, a polynomial of order 9 has been used to fit the curves in Figure A.1. The polynomials take the form of:

\[ y(x) = p_1 x^9 + p_2 x^8 + p_3 x^7 + p_4 x^6 + p_5 x^5 + p_6 x^4 + p_7 x^3 + p_8 x^2 + p_9 x + p_{10} \]  

(A.28)

With derivative:

\[ y'(x) = 9p_1 x^8 + 8p_2 x^7 + 7p_3 x^6 + 6p_4 x^5 + 5p_5 x^4 + 4p_6 x^3 + 3p_7 x^2 + 2p_8 x + p_9 \]  

(A.29)

The coefficients \( p_1 \) to \( p_{10} \) for the curves of respectively Mach .15, .20 and .25 are shown below:

<table>
<thead>
<tr>
<th>( M_{inf} )</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
<td>9.363e-09</td>
<td>-3.23e-08</td>
<td>8.937e-09</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>-9.483e-07</td>
<td>3.152e-06</td>
<td>-8.061e-07</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>4.17e-05</td>
<td>-0.0001322</td>
<td>3.13e-05</td>
</tr>
<tr>
<td>( p_4 )</td>
<td>-0.001045</td>
<td>0.003107</td>
<td>-0.0006819</td>
</tr>
<tr>
<td>( p_5 )</td>
<td>0.01647</td>
<td>-0.04461</td>
<td>0.009096</td>
</tr>
<tr>
<td>( p_6 )</td>
<td>-0.1701</td>
<td>0.399</td>
<td>-0.0759</td>
</tr>
<tr>
<td>( p_7 )</td>
<td>1.161</td>
<td>-2.15</td>
<td>0.3924</td>
</tr>
<tr>
<td>( p_8 )</td>
<td>-5.12</td>
<td>6.188</td>
<td>-1.291</td>
</tr>
<tr>
<td>( p_9 )</td>
<td>13.47</td>
<td>-6.04</td>
<td>3.528</td>
</tr>
<tr>
<td>( p_{10} )</td>
<td>-2.355</td>
<td>8.644</td>
<td>4.436</td>
</tr>
</tbody>
</table>

The derivatives of the remaining terms in Equations A.26 and A.27 are computed using a combined use of Automatic Differentiation and the chain rule of differentiation within the in-house 2D panel code.

**R4: \( C_{ld} - C_{L1} = 0 \)**

The derivative of \( C_{L1} \) with respect to \( \Gamma \) can be computed through automatic differentiation (AD). Using the chain rule of differentiation, the derivatives of \( C_{ld} \) with respect to \( \alpha \) and \( \alpha_i \) are computed as follows:

\[
\frac{\partial C_{ld}}{\partial \alpha} = \frac{\partial C_{ld}}{\partial \alpha_{eff}} \left[ \frac{d C_{ld}}{d \alpha_{eff}} \frac{d \alpha_{eff}}{d \alpha} \right]
\]

(A.30)

\[
\frac{\partial C_{ld}}{\partial \alpha_i} = \frac{\partial C_{ld}}{\partial \alpha_i} + \frac{\partial C_{ld}}{\partial \alpha_{eff}} \left[ \frac{d C_{ld}}{d \alpha_{eff}} \frac{d \alpha_{eff}}{d \alpha_i} + \frac{d C_{ld}}{d M_{eff}} \frac{d M_{eff}}{d \alpha_i} \right]
\]

(A.31)
In this appendix examples of the relevant input files used to run MSES are presented as described in Section 3.2.2. Five input files are described for respectively the cruise analysis and high-lift analysis, of which the latter is being performed by MSIS, an alternative to the MSES routine specially developed for low-speed \( (M_\infty \leq 0.4) \) analysis. [67]:

- **blade.xxx**: contains information on the grid width and height and includes shape data points.
- **mses.xxx - clean airfoil**: contains solver type and flow information used for clean airfoil analysis
- **mses.xxx - multi-element airfoil**: contains solver type and flow information used for multi-element airfoil analysis
- **gridpar.xxx - clean airfoil**: contains information on grid settings for clean airfoil analysis
- **gridpar.xxx - multi-element airfoil**: contains information on grid settings for multi-element airfoil analysis

It should be noted that the “VAR” entries in the input files described in this appendix represent variable inputs which are used throughout the analysis and optimization. For a detailed description of the files’ inputs, the reader is referred to [67].

### B.1 Cruise analysis input files

This sections contains relevant input files used during cruise analysis which is ran by the MSES routine.

**BLADE.XXX**

```
-2 3 -3 3.5 | XINL  XOUT  YBOT  YTOP
X(1,1)  Y(1,1)
X(2,1)  Y(2,1)
X(3,1)  Y(3,1)
...    ...
...    ...
X(I,1)  Y(I,1)
```
**MSES.XXX**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>15</td>
<td>17</td>
<td>20</td>
</tr>
</tbody>
</table>

VAR 0 VAR | MACHIN CLIFIN ALFAIN
4 2 | ISMOM IFFBC
VAR 9 | REYNIN ACRIT
.01 .01 | XTRSI XTRP1
0.97 1.2 | MCRIT MUCON
0 0 | ISMOVE ISPRES
20 0 | NMODN NPOSN

**GRIDPAR.XXX**

141 | Airfoil side points
0.40 | Exponent for airfoil side points: \( n = N \cdot \text{chord}^e \)
37 | Inlet points on leftmost airfoil streamline
37 | Outlet points on rightmost airfoil streamline
21 | Number of streamlines in top of domain
15 | Number of streamlines in bottom of domain
7 | Maximum streamlines between two elements
1.3 | Smoothing parameter
2.5 | Aspect ratio of each cell at stagnation point
0.85 | X-spacing parameter
VAR | Angle of attack
0.4 0.8 1.0 | Spacing parameters element 1
0.03 0.07 1.00 1.00 1.00 0.00 | Spacing refinement element 1

**B.2 High-lift analysis input files**

This sections contains relevant input files used during high-lift analysis which is ran by the MSIS routine.

**BLADE.XXX**

-3.5 4.5 -4.5 4.5 | XINL XOUT YBOT YTOP
X(1,1) Y(1,1)
X(2,1) Y(2,1)
X(3,1) Y(3,1)
... ... 
... ...
X(I,1) Y(I,1)
999. 999.
X(1,2) Y(1,2)
X(2,2) Y(2,2)
X(3,2) Y(3,2)
... ... 
... ...
X(I,2) Y(I,2)
**MSES.XXX - CLEAN AIRFOIL**

3  4  5  7  10  15  20  
3  4  5  7  15  17  20  
VAR 0  VAR  |  MACHIN  CLIFIN  ALFAIN  
4  2  |  ISMOM  IFFBC  
VAR 1  |  REYNIN  ACRIT  
1  1  |  XTRS1  XTRP1  
0.99 1  |  MCRT  MUCON  
0  0  |  ISMOVE  ISPRES  
20  0  |  NMODN  NPOSN  

**MSES.XXX - MULTI-ELEMENT AIRFOIL**

3  4  5  7  10  15  20  30  
3  4  5  7  15  17  20  30  
VAR 0  VAR  |  MACHIN  CLIFIN  ALFAIN  
4  2  |  ISMOM  IFFBC  
VAR 1  |  REYNIN  ACRIT  
1  1  1  1  |  XTRS1  XTRP1  XTRS2  XTRP2  
0.99 1  |  MCRT  MUCON  
0  0  |  ISMOVE  ISPRES  
20  3  |  NMODN  NPOSN  

**GRIDPAR.XXX - CLEAN AIRFOIL**

231  |  Airfoil side points  
1.00  |  Exponent for airfoil side points: n = N · chord^E  
23  |  Inlet points on leftmost airfoil streamline  
23  |  Outlet points on rightmost airfoil streamline  
15  |  Number of streamlines in top of domain  
11  |  Number of streamlines in bottom of domain  
6  |  Maximum streamlines between two elements  
1.3  |  Smoothing parameter  
2.5  |  Aspect ratio of each cell at stagnation point  
0.85  |  X-spacing parameter  
VAR  |  Angle of attack  
0.8  0.8  0.8  |  Spacing parameters element 1  
1.00  1.00  1.00  1.00  0.00  |  Spacing refinement element 1  

**GRIDPAR.XXX - MULTI-ELEMENT AIRFOIL**

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>109</td>
<td>Airfoil side points</td>
</tr>
<tr>
<td>0.79</td>
<td>Exponent for airfoil side points: ( n = N \cdot \text{chord}^E )</td>
</tr>
<tr>
<td>23</td>
<td>Inlet points on leftmost airfoil streamline</td>
</tr>
<tr>
<td>23</td>
<td>Outlet points on rightmost airfoil streamline</td>
</tr>
<tr>
<td>15</td>
<td>Number of streamlines in top of domain</td>
</tr>
<tr>
<td>11</td>
<td>Number of streamlines in bottom of domain</td>
</tr>
<tr>
<td>6</td>
<td>Maximum streamlines between two elements</td>
</tr>
<tr>
<td>1.3</td>
<td>Smoothing parameter</td>
</tr>
<tr>
<td>2.5</td>
<td>Aspect ratio of each cell at stagnation point</td>
</tr>
<tr>
<td>0.85</td>
<td>X-spacing parameter</td>
</tr>
<tr>
<td>VAR</td>
<td>Angle of attack</td>
</tr>
<tr>
<td>0.4 0.8 0.05</td>
<td>Spacing parameters element 1</td>
</tr>
<tr>
<td>0.4 0.8 0.4</td>
<td>Spacing parameters element 2</td>
</tr>
<tr>
<td>1.00 1.00 1.00 1.00 0.00</td>
<td>Spacing refinement element 1</td>
</tr>
<tr>
<td>1.00 1.00 1.00 1.00 0.00</td>
<td>Spacing refinement element 2</td>
</tr>
</tbody>
</table>