Robust design in structural engineering
MSc Thesis

Robust design in structural engineering

by

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Preface
This thesis marks the end of my master’s study Hydraulic Engineering at the Delft University of Technology. The thesis discusses the applicability of the concept of ‘robust design’ on design procedures in civil engineering. ‘Robust design’ or ‘robustness’ has recently become a popular term in structural and hydraulic engineering, especially when considering the effects of climate change. The term appears more and more in design requirements, but much ambiguity exists as to what it actually means. In this thesis, I have tried to capture the meaning of ‘robust design’ and to show its applicability in structural engineering. I hope this thesis will help to make the concept more comprehensible for all that are interested.

During my 6 years as a student, at two different universities, I have studied many challenging subjects. However, I can say rightfully that none of those subjects were as challenging as writing this thesis. Therefore, I would like to thank all members of my graduation committee for their help in finding the definition of robustness. Especially Arno Willems and Wouter van der Wiel have my gratitude for their enthusiasm for the subject and for all ideas they have supplied me with during our brainstorm sessions. Moreover, I would like to thank my parents for supporting me in my studies over the past 24 years. Also, I would like to thank Martijn, for keeping out of the house at the times that I was busy writing this thesis.

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Abstract

Robustness has recently become a popular term in structural and hydraulic engineering. However, the concept of ‘robust design’ or ‘robustness’ has no clear definition. The aim of this thesis was to find a definition for a robust system and to use this definition to search for an added value of robust design in structural engineering. The literature on robust design has been extensively studied to establish a definition for a robust system. It was concluded that a robust system can be found by assuming that the additional risk that arises due to discrepancies between the reality and the model, is smaller for a robust system than for a less robust system. When designing a system, it is however always the priority to find a costs-optimal design alternative. The choice for a robust design alternative will therefore only be justified if the expected lifecycle costs, as calculated from the model, are less than or equal to the expected lifecycle costs of any other design alternative. In other words, economic optimization has priority over robustness. The reason for this it that robustness focusses on hazards for which no value of the risk can be defined, so that it cannot be quantified what may be spent on robustness.

This definitions of a robust system was tested in several case studies. From these case studies it seemed that robustness of a system increases with a decreasing expected value of the risk. This means that when two design alternatives have equal expected lifecycle costs, the choice should be for the alternative that has the lowest expected risk. It is recommended that this conclusion is further investigated theoretically.
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Summary

Introduction

In this summary, first the literature on standard design procedures and on the concept of robustness will be discussed. The theory as proposed by Taguchi is an important aspect in this discussion. Following, several ‘robust’ optimization tasks as introduced in literature, will be reflected upon. At last, some practical measure for designing a robust structural system will be discussed.

The literature study has been used to find a definition of a robust system and to find a method for selecting a robust design during the design phase. Both aspects are enclosed in this summary. Also a recommendations for implementing robustness in the system will be proposed. These recommendations have been tested in the case studies. The results of these case studies will be discussed in this summary. At last a conclusion is given as to what the expected added value of robust design is in structural engineering.

Standard procedure: Limit state losses

The standard procedure for design in civil engineering is based on limit states. When such a limit state is exceeded, a loss will occur. A distinction is made between serviceability limit states (SLS) and ultimate limit state (ULS). An example of an SLS is the amplitude of the oscillation of a bridge deck. An increase in amplitude results in a continuously increasing loss since the main function of the bridge is at stake, namely to act as an overpass for cars. This will lead to an economic loss due to transport hindrance, which is a functional loss. An example of an ultimate limit state of a bridge is a maximum stress constraint in the beams. If this constraint is exceeded, it is expected that a loss will occur as a result of structural failure (collapse). The probability that a limit state is exceeded can be calculated from the probability density functions of the load and the strength parameters. The risk over the system can then be determined by multiplying this probability with the corresponding loss, and summing over all limit states. This risk can be added to the investment costs to find the lifecycle costs of a system. These lifecycle costs can be minimized in an economic optimization task.

Standard procedure: Optimization tasks

In structural engineering, the values of the design variables are often determined from an optimization task. Such an optimization task is performed if the preliminary design of the system is finished, but when the dimensions and the material strengths (the design variables) have not yet been quantified. In the standard procedure, the objective of the optimization is generally to minimize the lifecycle costs. Such a minimization is shown in Figure 1 and is called an economic optimization. Another important objective may be to minimize the loss of life during the lifetime of the system. In this study, it is assumed that a minimum in the lifecycle costs is the prior decision factor for choosing a design alternative.
Robust design: Taguchi’s Approach

Taguchi proposed an approach to robust design that was aimed at designing high-quality products. This means that this approach is not readily applicable in structural engineering. In this study, this approach is investigated to find the links with structural engineering.

“a product whose performance is minimally sensitive to factors causing variability (at the lowest possible cost)”.

Taguchi’s definition of a robust product is:
Taguchi claims that a robust product is beneficial since, the losses of a product are quadratically proportional to the deviation of the performance. Taguchi therefore introduced a general quadratic loss function, that he claimed to be true for every product and for every definition of the performance. Such a loss function is shown in Figure 2.
When using a Taguchi loss function, the risk of the system can be reduced by minimizing the standard deviation of the performance function. This is illustrated in an example on caramel candy.

**Robust design: Example Caramel candy**

In 1948, Taguchi developed a recipe for caramel candy for the company Morinaga Seika Co. The company gave him the assignment to develop a recipe for caramel candy so that its chewability, or plasticity, was less sensitive to temperature changes. Namely, this would result in a better quality of the candy and therefore in better sales. The design problem is illustrated in Figure 3 and Figure 4. In Figure 3 it is shown how the ‘robust’ recipe results in a smaller standard deviation in the plasticity, given the probability density function of the temperature. Figure 4 shows the loss function that is assumed by Taguchi, from which it can be concluded that a decrease in the standard deviation of the plasticity is beneficial for reducing the risk. If a design with such minimized standard deviation is found, Taguchi’s claims that a successful robust design with minimized losses is found.
**Figure 3** PDF of the plasticity as a result of the temperature PDF, for two different recipes.

**Figure 4** Taguchi loss function and the PDF's of the plasticity, for two different recipes.
Robust design: Drawbacks Taguchi’s approach

There are two major drawbacks of Taguchi’s approach when comparing it to the standard design procedures. The first drawback is the quadratic quality loss function, since this loss function is simply not realistic for most design problems. Opposed to this, the limit state approach considers all structural and functional failure modes separately for every design problem. This approach is therefore much more sophisticated. The second drawback is that Taguchi only minimizes the risk when searching for a robust design. This means that investment costs are not considered, which may lead to an uneconomical design.

For example, consider the loss function that is based on limit states as given in Figure 5. For this loss function, the risk can be minimized by either shifting the mean of the plasticity distribution, by minimizing the standard deviation or by a combination of both. Which option is chosen depends on the investment costs associated with the option. The optimum design is then the design that has minimal lifecycle costs. Whether this is achieved by minimizing the standard deviation of the performance function or by shifting the mean, is not important for this economic optimization.

**Figure 5** Limit state loss function and pdf’s of the plasticity, for two different recipes.
Robust design: ‘Robust’ optimization tasks

In literature, several ‘robust’ optimization tasks have been introduced, that are claimed to result in robust designs. In this study, it has been investigated if these tasks have an added value compared to the standard procedure, in which an economic optimization is done based on a probabilistic analysis. The optimization tasks that are investigated in this study, are based on limit state loss functions. In this summary, they are illustrated with the example problem of a column under axial loading, see Figure 6.

A column that is subject to an external axial loading, will have as its ultimate limit state a maximum stress constraint. If this constraint is exceeded, the column will collapse. The distribution of the external axial loading cannot be influenced, this load is therefore the uncertain parameter. The distribution of the stress can be influenced by two design variables, namely the cross-sectional area of the column and the amount of prestressing.
Loss functions
The loss function in this problem is a limit state loss function and is shown in Figure 7.

![Limit state loss function for beam under axial loading](image)

**Figure 7 Limit state loss function for a beam under axial loading**

Optimization task: Non-probabilistic uncertainty
In many articles, the ‘robust’ optimization task is based on ‘non-probabilistic uncertainty’. This means that the uncertainty of a parameter is described by interval bounds, between which the parameter can take any value. An example of an approach that uses ‘non-probabilistic uncertainty’, is the robust counterpart approach. In this approach, a design is searched for, that performs best under the worst combination of the uncertain parameters. The optimization task is then to find the values of the design variables for which the limit states are never exceeded, under any possible combination of the uncertain parameters. For a column under axial loading, the approach is shown in Figure 8. The design that will be selected, is the one with a cross-sectional area of 200 mm$^2$. The reason for this is that the stress in a column of this design, will not exceed the maximum stress constraint, under any value of the uncertain external loading.
A different approach to robustness, that is based on ‘non-probabilistic uncertainty’, is proposed by Yakov Ben-Haim. Ben-Haim has written an enormous amount of articles in which he introduces non-probabilistic design approaches to achieve a robust system. He defines robustness as the maximum amount of parameter uncertainty for which the limit states are not exceeded. This uncertainty is described by interval bounds, so this approach is in essence not different from the robust counterpart approach. Robustness as described by Ben-Haim is in structural engineering equal to reliability. Ben-Haim also claims that there is a trade-off between robustness and the performance. He argues that an optimal design is never robust since it is only based on the information that is known, and that it does not account for unexpected variations. In civil engineering, the optimal design is the design with the least lifecycle costs. When applying Ben-Haim’s strategy in civil engineering, the tradeoff between optimality and robustness translates into a tradeoff between reliability and investment cost. This is agrees with the standard procedure for the design of structural systems.

**Optimization task: Multiobjective optimization**

In literature that is based on a probabilistic description of uncertainty, multiobjective optimization is often used to define a ‘robust’ optimization tasks. A multiobjective optimization task has more than one objective, and will therefore always have more than one solution. Multiobjective optimization is thought to be helpful in robust design, since it allows the designer to optimize for an additional objective, additional to minimizing the lifecycle costs. An example is a task in which the investment costs and the deviation in the performance are simultaneously minimized. Now consider applying this task to the beam under axial loading, as shown in Figure 9. It can be concluded that minimizing the standard deviation of the stress, means maximizing the cross-sectional area $A$ of the beam.
Maximizing $A$ does however result in a maximization of the investment costs, since a greater cross-sectional area costs more. Another measure that is shown in Figure 9 is to use prestressing. This measure will shift the stress distribution away from the limit states, which results in a lower risk. However, the standard deviation of the stress stays unchanged under this measure. If prestressing is less expensive than increasing the cross-sectional area of the column, the prestressing option will be favourable in view of the first objective, since the investment costs are less. However, the standard deviation is not reduced with this option, which means that it is unfavourable in view of the second objective. For a design in which no prestressing is used, the opposite is true.

**Figure 9 Stress distribution for a beam under random axial loading, for different design alternatives. The load is normally distributed with a mean value of 50 kN and a standard deviation of 5 kN**

The main difficulty in this multiobjective optimization task, is to define the mutual weights of both objectives. A solution to this problem is to express both objectives in monetary units so that they can easily be added. The result is then a single objective optimization of the lifecycle costs. It can be concluded that this optimization approach is not of any added value compared to a minimization of the lifecycle costs.

**Discrepancy between model and reality**

In the above paragraphs, it was concluded that the alternative optimization tasks as proposed in literature, are not of any added value compared to the standard procedures. This means that an added value of ‘robust design’ has not been found from the literature on ‘robust’ optimization.

Definitions of a robust system that have been found in literature, do however often relate to loads and events that have not been accounted for in the design phase. The expected lifecycle costs are based on the expected behaviour of the loads and the material strength. But what happens if the
future does not keep to what is expected during the design phase? An example is the drastically increased traffic intensity over the Dutch steel bridge decks, which resulted in a much higher risk over the lifetime of the bridge decks than expected. Another example is the sea level rise, that is predicted by models in which great uncertainty exists. A robust system should be a system that is less sensitive to such errors in the models that are used for designing the system.

One of the major difficulties in this definition of a robust system, is that money is always spend according to the risk that is expected. It is not cost-efficient to spent money to prevent an event from happening, if you have no clue whether it actually has a probability of happening. A robust system will therefore only be chosen over a less robust system, if the latter has equal or higher expected lifecycle costs. Another difficulty is that any uncertainty in the model that is thought of during the design phase, can be taken into account in a full probabilistic economic optimization. For example, if we know that the predictions of the sea level rise are prone to large uncertainty, this should be included in the analysis. In this case, there is no need to separately do an analysis to check the robustness of the system to a higher sea level rise. Consequently, in robust design the focus is on the discrepancy between the reality and the model that is used in the design phase, including the model uncertainties. A robust system should be insensitive to any increase in the considered hazards, even if during the design phase there is no clue as to why such an increased hazard will occur.

In Figure 10, the column design problem is shown, for a load distribution that has a higher mean and a large standard deviation than in Figure 9. The load distribution in Figure 10 represents the actual load distribution, while that in Figure 9 represents the distribution as assumed in the design phase. From this figure it seems that the design with the smallest standard deviation in case of the assumed distribution, will have the least increase in the risk when considering the actual load distribution. This hypothesis is checked in the case studies. The design alternative with the least increase in the risk is the most robust alternative.
Robust design: Preliminary design stage

In the thesis, it was concluded that robustness is preferably implemented in the preliminary design, when the shape of the system is still under consideration. This can be done as proposed in the Eurocode, by using measures that avoid progressive collapse. In this phase, it is important to consider all hazards and to design a system that is least sensitive to these hazards. It should also be noted that these hazards may be larger than expected from the statistics. A system that is influenced least by such changes in the hazards, is a robust system. For example, a robust flood protection system is designed such that an unexpected extreme climate scenario will have a minimized influence on the lifecycle costs. All measures that are used to make the system more robust should be judged in an economic optimization. A design alternative will only be selected if:

1) The design has a minimum in the lifecycle costs
2) The design is more robust to discrepancies between the model and the reality, than a different design with the same lifecycle costs.
Definition of a robust system

A robust system is defined as follows:

\[ \text{The additional risk that arises due to discrepancies between the reality and the model, is smaller for a robust system than for a less robust system.} \]

In order to choose the robust system during the design phase, the expected lifecycle costs may not be larger than the costs of any other alternative system.

Consider that the designer has to choose between two alternatives that have exactly the same expected lifecycle costs, according to a full probabilistic analysis. This is shown in the upper two graphs of Figure 11. The most robust alternative is the one that has an expected lifecycle cost that is least sensitive to changes in the distribution function of the performance parameter. In Figure 11, this is true for the blue graphs. Another possibility is that there are two alternatives, of which one has higher expected lifecycle costs, but is also more robust, as shown in the lower two graphs of Figure 11. Now, the decision maker will have to make a decision between robustness and expected lifecycle costs. If the higher expected lifecycle costs of the robust system are the result of a higher expected risk, the choice will not be for this system. If these higher lifecycle costs are the result of higher investment costs, there is a trade-off between robustness and investment costs.

\[ \text{Figure 11 Cost as a function of the performance parameter for a robust and a less robust system. The axes are the same for each graph.} \]
Selecting a robust design alternative

In this paragraph, it will be described how a robust design alternative is chosen from many different alternatives. The following definitions is for a robust system is the basis of this paragraph:

- The additional risk that arises due to discrepancies between the reality and the model, is smaller for a robust system than for a less robust system.

Different design alternatives

Consider a system that has to be designed, by determining the optimal values of the design variables $x_1$ and $x_2$. The system is subject to the uncertain parameter $\theta$, that has a distribution function as shown on the $x$-axis of the graph in Figure 12. The different designs result in different distributions of the performance parameter $p(x,\theta)$, as shown on the $y$-axis of the graph in Figure 12.

**Figure 12 Design alternatives**

Loss function

For certain values of the performance parameter $p(x,\theta)$, losses will occur. Taguchi’s describes these losses by a quadratic function and in the standard approach these losses are often described by limit states. The most accurate solution is to define the loss function as precise as possible, for every separate case. Examples of fictive options for the loss function are shown in Figure 13.

**Figure 13 Different loss functions**
Risk

Now consider that the limit state loss function is chosen. The risk of a certain design alternative can then be determined by integrating the probability density function of the performance parameter over the loss function. The expression for the risk is: 

\[ R = E[\text{Loss}] = \int_{-\infty}^{\infty} pdf(p(x, \theta)) \cdot f_{loss} dp \]

**Figure 14** Loss function and probability density function of \( p \) for different alternatives

Economic optimization

In an economic optimization the lifecycle cost is minimized, which is the sum of the risk and the investment cost. This will result in a cost minimum combination of the design variables, which represents the cost optimum design. Consider that there are two design variables \( x_1 \) and \( x_2 \) and that two different combinations of \( x_1 \) and \( x_2 \) exist that have equal minimal costs. These two combinations represent two cost optimum design alternatives. The resulting probability density functions of the performance parameter \( p(x, \theta) \) are shown in Figure 15. At this point, the designer is indifferent with regard to choosing one of the alternatives, since both have the same lifecycle costs. However, one of the two alternatives may be more robust. This alternative will have a smaller increase in lifecycle costs in case the distribution of \( \theta \) turns out to be different.

**Figure 15** The probability density functions of \( p(x, \theta) \), for two different designs with equal and minimum lifecycle costs.
Discrepancy in the distribution of $\theta$

In Figure 16, two distributions functions of $\theta$ are shown. The grey line represents the distribution that is taken into account to choose an alternative with the economic optimization. The red line represents the actual distribution function to which the system is subjected. This actual distribution function will result in actual distribution function of the performance parameter, than used in the economic optimization. These distribution functions are shown on the y-axis of Figure 16 and in Figure 17. From Figure 17, it can be concluded that the risk corresponding to these distribution functions differ from the risk as used in the economic optimization. Clearly, alternative 2 will have a higher additional risk due to the errors in the distributions of $\theta$. The reason for this may be sought in the difference in standard deviation or in the difference in the initial expected risk. For this loss function and the assumed change in the distribution of $\theta$, alternative 1 is the most robust design. It should however be noted that this result is strongly dependent on the loss function, on the way in which the distribution of $\theta$ changes and on the initial expected risk of both alternatives. A cost-analysis should always be done to select the robust alternative.

**Figure 16** Change in distribution of the performance parameter due to a change in the distribution of $\theta$, for two design alternatives.

**Figure 17** Loss function and the assumed and actual distribution functions
Discrepancy in parameter distributions
In the definition of a robust design as given in this summary, it is assumed that the distribution functions of the parameters as used in the design phase, may be erroneous. This may be the case if not enough statistical data is present or if the past is not a good predictor for the future. However, if such thing is suspected, it can always be included in the economic optimization by increasing the uncertainty in the parameters. Still it is possible that two alternative system have the same expected lifecycle costs under all uncertainty that can be thought of, but that one of these alternatives is more robust. The reason for this is that some discrepancies in the load distributions can simply not be quantified by an uncertainty. Examples are changing discharges due to climate change and increasing traffic intensity due to increasing welfare. These trends can be thought of during the design phase, but their magnitude cannot be predicted since they simply

Case studies
The conclusion and recommendations as drawn in the study, have been checked in three case studies. The results of these case studies will hereby be discussed.

Dike height problem
In this problem the only hazard is the water level, and the only design variable is the dike height. In this case, a robust dike is always a high dike. However, it is also possible to adopt a robust heightening strategy. This strategy entails inspection during the lifetime of the system, so that the dike can be improved if the sea level rise is higher than expected in the design phase. If the averted risk is larger than the improvement costs, the lifecycle costs will be reduced.

Sea dike
In this case study it was investigated whether a revetment dike or a grass-covered dike is the most robust design alternative for a sea dike at the Frisian coast. Both alternatives have the same expected lifecycle costs. In the case study, it was concluded that the revetment dike is the most robust alternative, since its lifecycle costs are least sensitive to a discrepancy between the assumed parameters and the actual parameters of the water level distribution. This is shown in Figure 18. It should be noted that the revetment dike initially has an expected risk that is twice as small as that of the grass-covered dike. Since the risk increases exponentially, the increase from a small risk will be less than the increase from a larger risk. The robust alternative is therefore also the alternative with the smallest risk, under the expected water level distribution.
In this case study it was investigated whether a prestressed column or a non-prestressed column is the most robust design alternative. It was concluded that both alternatives are equally robust, since the additional lifecycle costs under changing parameters are equal. It is expected that the reason for this result is that both alternatives had the same risk to start off with. Also both alternative have normal stress distributions. Any change in the load distributions function will therefore result in an equal increase in risk for both alternatives. The symmetry of the loss function may also play a role in this result. This conclusion agrees with the conclusion of the case study in which a robust sea dike was designed. Namely, in this case study it was concluded that the design alternative with the lowest expected risk, is the most robust alternative.
Conclusion

It can be concluded that the following definition of robustness may result in an added value to the standard design procedures:

\[ \text{The additional risk that arises due to discrepancies between the reality and the model, is smaller for a robust system than for a less robust system.} \]

Robustness can be implemented in the preliminary design phase by making the system less sensitive to its hazards. Robustness can also be implemented in the parameter design phase, in which the dimensioning of the system is done. The objective for robustness is to minimize the increase in lifecycle costs in case of discrepancies between the parameter distributions as used for the economic optimization, and the actual parameter distributions. Ideally, when designing a system, the robust system has the same expected lifecycle costs as the less robust alternative system. When the prior design objective is to minimize the lifecycle costs, this means that the designer is indifferent in which alternative to choose. In this case, choosing the robust system will always be beneficial. Namely, the additional lifecycle costs due to a range of unexpected loads, will be less for the robust system.

In the case studies, it was concluded that the robust system is the system with the lowest risk in the economic optimization. If both system have the same expected risk, they are equally robust. It is recommended to investigate this conclusion theoretically.
1. Introduction

1.1. Robustness

Robustness is a term that intuitively refers to something that is strong, solid or reliable. When we are offered a ‘robust’ coffee, we expect a strong flavour. When we travel with a ‘robust’ railway system, we expect that the trains will be on time even if something goes wrong. A ‘robust house’ is expected to be strong and safe during for example hurricanes. Undoubtedly, robustness is a very attractive term. Namely, nothing can go wrong if everything is robust!

Hence, robustness has become a popular concept in civil engineering. Many different definitions and methods to implement robustness have been proposed. However, when trying to apply robustness to civil engineering problems, many difficulties arise. For example, consider that you are planning to build a house in an area that is prone to flooding. A house that is ‘robust’, at least against flooding, will be built on a high terp so that the water can never reach the house. However, the investment costs for the construction of such a terp may be much higher than the losses associated with flooding. So robustness may not always be cost-efficient. Also, the probability that a flood will actually occur in the years that the house is in use may be very small. This raises questions on what may be the added value of robustness, especially when considering the cost-efficiency. Investigating this, brings us back to the 1950’s, when Genichi Taguchi introduced his approach to ‘robust design’.

Taguchi introduced robust design, as a method for designing products with a high quality. He proposed that the quality loss for the user of the product, is proportional to the square of the deviation from the target performance. Therefore, a product should always be designed so that it performs on the target. However, a product is always subject to ‘noise factors’ that cannot be controlled by the designer. An example of a noise factor in civil engineering is wave height or wind speed. These noise factors can cause the product performance to deviate from its target. Taguchi proposed a method to minimize this deviation. The method is based on experimental design, which means that experiments are done to find a product design that has the smallest deviation under the noise factors. This is done by testing the performance of several designs, imposing different values of the noise factors. The results can be used to find an optimal design.

An example of Taguchi’s method for robustness is the manufacturing of microwave popcorn. The manufacturer can control the volume inside the bag, the amount of corn kernels in the bag and the material of the bag. These are the design variables. The manufacturer cannot control the cooking time and the temperature that the customer will actually use and the positioning of the bag inside the microwave, so these are noise factors. The manufacturer wishes to design the bag such that the amount of edible popcorn in the bag is least influenced by the cooking time and the temperature that the customer uses. Popcorn is edible if the kernel has popped and if it is not burned. Following Taguchi’s method this can be done by testing alternative bags of different volumes, different materials and with different amounts of corn kernels. For each alternative, tests are run in which the temperature, the cooking time and the position are varied. The resulting amounts of edible are used to find the alternative for which the amount of edible popcorn is least influenced by the noise factors. This alternative is the most robust alternative.

From the example, it follows that Taguchi’s approach for finding robust designs, is an experimental design method that searches for the design that is least influenced by the noise factors. The added value of such a design is that the quality of the product will have relative small deviations under
uncertain factors, so that customers dissatisfaction is reduced. This will reduce the loss of the manufacturer due the reparation, replacement and loss of customers. Taguchi’s method will therefore reduce indirect costs. This should be weighed against the initial cost of the design alternative to determine whether it is a cost efficient alternative.

Also, Taguchi’s method for experimental design can be used to design more efficient processes. Using the example of the popcorn, Taguchi’s method can be used to design a bag that produces the largest amount of popcorn from the smallest amount of kernels. When assuming that the bag does not pose extra investment costs, this will result in a direct profit, since the manufacturer will have to use less corn kernels.

Since using Taguchi’s method for robust design was presented as a method that leads to more cost efficient design, robustness has become a very popular concept in the years after his first publication. The concept has been applied in many disciplines, ranging from chemical engineering, to electrical engineering, automotive engineering and structural engineering. However, Taguchi’s method cannot readily applied to every engineering problem. Consequently, in literature, there is a large scatter in the definition of robustness and in the methods for assessing robustness. In civil engineering, a key reason that Taguchi’s method cannot be applied is that it is not feasible to do experimental design on buildings. For example, a building that has to be able to withstand seismic loads cannot just be built to test of it works. The tests will cost more than the project itself. Therefore, theoretical and empirical formulas are done to investigate the behaviour of the structure. A certain design can be ‘tested’ by doing a model run, instead of by doing a physical experiment. The noise factors can be implemented in the model run just by changing the values of the parameters. However, in civil engineering, often a distribution function of such a noise factor is known. Implementing the distribution functions in the model run will result in a distribution function of the behaviour that is investigated. The robust design is than the design for which this distribution function has the smallest standard deviation. However, doing a model run and implementing all uncertain parameters is computationally very time consuming. Therefore, many simpler methods are proposed to implement robustness when designing a civil structure.

When following Taguchi’s method for robust design, robustness seems to be a pretty straightforward concept. In reality, the opposite is true. Robustness has become a fashionable term, for example, imagine yourself in a furniture shop looking for a living room table. One table has a label that says ‘robust table’. Does this have anything to do with Taguchi? Does the robust table have the best performance, even if you use it as a podium? Maybe. Was the robust table designed by the manufacturer such that it is cost-efficient? Probably not.

The same problem arises in civil engineering, where a robust system is often interpreted as a system that is reliable under uncertainty. This does however not in any way distinguish a robust system from a system that is designed according to the standard design procedures for civil engineering. Namely, these standard procedures are already focussed on designing reliable structures and taking into account all uncertainties. So what is the added value of robustness to the standard design procedures? Moreover, the standard design procedures search for a minimum in the costs, a different ‘robust’ design will always be more costly. How can this be justified?

The aim of this thesis is to investigate how the concept of robustness can be described, and what can be the added value in civil engineering.
1.2. Objectives and Research Questions

The objective of this thesis is to investigate the added value of the robustness concept in the field of civil engineering. In literature, there is a large scatter in definitions of robustness and methods to assess robustness. The majority of them will be discussed, investigated and criticised in the view of their applicability to engineering problems. This will be done by performing case studies on several civil engineering problems. An attempt will be made to define a framework for implementing robustness in design procedures. This framework will be illustrated by a more extensive case study on a realistic design problem. The following research question will be answered:

1) How can robustness be defined?
2) What are the applications of robust design?
3) What methods are available for assessing robustness?
4) How can these methods be applied to structural engineering problems?
5) How do these methods compare to the standard design procedures?
6) What can be the added value of these methods in structural engineering?
7) What can be the added value of robustness in structural engineering?
8) How can robustness be implemented in the design procedures for structural engineering?

1.3. Outline

Part 1 of this thesis is an introduction on the standard design procedures as used in civil engineering problems. This standard design procedure is an important starting point for searching for the added value of robustness. Part 2 of this thesis is a literature review on the concept of ‘robust design’ as introduced by Genichi Taguchi. Part 3 discusses the theoretical optimization methods for robust design that have been proposed in articles on structural design. These approaches are investigated with three case studies. Part 4 investigates more practical approaches for robust design that have been proposed in literature on structural and hydraulic engineering. These approaches are investigated with the same case studies as in Part 3. The information that is gained in Parts 1 to 4 is used to discuss the concept of robustness in Part 5. In this part, recommendations are done for implementing robustness in a system during the design phase. Also a final definition of a robust system is given. This definition is investigated in the case studies in Part 6. Finally, Part 7 gives the conclusions of this study and the recommendations for further study.
Part 1. Standard design procedures in civil engineering

Literature Study

Conclusion

“The optimal design is the design that has a minimum in the lifecycle costs.”
2. Design stages

2.1. Process stages
A design project is always initiated by a problem, a vision or a dream. For example, ports need to be extended as shipping companies strive after larger container ships. Climate change causes changes in sea level and river discharge, increasing the need for flood protection systems. Growing cities expand towards the sea which triggers the wish for land reclamation.

In the initiative stage, the problem, vision or dream is captured in a design problem. The design problem is described by a definition of the goals and the scope of the project. Following on the initiative stage is the feasibility stage, in which the functional requirements and the wishes and conditions of the stakeholders are defined. In this stage, also the feasibility of the project is estimated, based on studies on reference projects. In the feasibility stage, also the functional requirements are defined, after which the project is handed to the engineer. The engineer manages the technical design phase and will make a technical design, based on the functional requirements, the budget and the boundary conditions. This technical design is used for construction. After the construction has finished, the use and maintenance phase starts. In this phase the structure fulfils its functions and is inspected to check its conditions. If a structure is rejected after inspection, maintenance will be done to increase the reliability of the structure. The use phase ends when the structure is upgraded, reused or demolished. When maintenance and possibility for upgrading or reuse have been accounted for in the design, the design approach is a lifecycle approach.

2.2. Technical design stage
In this study, the focus will mainly be on the technical design of the structure. The technical design can be divided in three stages; the preliminary design, the final design stage and the detailed design stage. In the preliminary design stage, several variants are investigated. For example, when designing storm surge barrier, the variants under consideration may be a mitre gate, sliding doors and a bellow barrier. When a variant is chosen, a specific location can be defined, with the corresponding specific boundary conditions. Once the variant and the location are definite, the design for the variant can be further investigated. For example, the choice for the number of doors is made based on the functional requirements. When the main properties of the structure have been defined, the dimensions can be determined in the final design stage. In the final design stage, all loads and failure mechanisms are taken into account for calculating the dimensions. The design is further elaborated in the detailed design stage. In this design stage the technical details are designed, such as the connections, the rails and the joints. It should be noted, that in practice, three design steps will overlap.

2.3. Lifecycle approach
A lifecycle approach essentially means that the use, the maintenance and options for reuse and upgrading are taken into account in the technical design stage. For example, following a lifecycle approach, a dike will built at a location where horizontal space is abundant, so that upgrading of the dike is possible. Another example is the choice for vertical lift gates instead of sinking gates, since the maintenance of the first is easier. In modern design procedures, aspects of the lifecycle approach are used when they are specifically added to the requirements. Developments towards a full lifecycle approach are based on considering lifecycle costs.
3. Limit state design

In the final design stage of the technical design, the required dimensions of the structure have to be determined. To ensure the safety, this is done based on a reliability analysis. In this analysis, reliability of the structure to perform its functions is investigated. The first step is to define the functions of the structures. From this, the ways of malfunctioning can be described, also called the failure modes. These failure modes can be described mathematically by a limit state equation. The general form of a limit state equation is:

$$ Z = g(X) = R(X) - S(X) $$

In which:

- $X$ is the set of random variables
- $R$ = strength, resistance to failure
- $S$ = load, solicitation to failure

Failure will occur when the load exceeds the strength, so that $Z < 0$. Failure due to different failure modes will lead to different consequences. For example, exceedence of the wind load limit state of a house may have as a consequences that some roof tiles have to be replaced, while collapse due to an earthquake will have as a consequence that the whole house has to be replaced. Three types of limit states have been defined in literature; the Serviceability Limit State (SLS), the Ultimate Limit State (ULS) and the Accidental Limit State (ALS). The SLS corresponds to a state in which the functions can barely be fulfilled, an example of exceeding of the SLS is overtopping of a dike due to high water levels. In this case the function of the dike, namely retaining the water, is temporarily not fulfilled. The ULS corresponds to a state in which the structure has collapsed and lost its function permanently, for example breaching of the dike and the structural loss of its water retaining function. The ALS describes failure under accidental loads, such as collision or explosion.

The input of the limit state equation is a set of random variables, with corresponding probability density functions. Consider a limit state equation with random variables $R$ and $S$ that are both described by a probability distribution, giving the probability that a certain value will occur. The probability that failure will occur can be calculated from the limit state equation. For example, consider the general limit state function as in equation (1).

With:

- $R: \mu_R = 8, \ \sigma_R = 0.5$ Normally distributed
- $S: \mu_S = 5, \ \sigma_S = 1.0$ Normally distributed

The probability distributions of $R$ and $S$ are shown in Figure 19. The overlap between them means that there is a probability that the load $S$ is larger than the strength $R$, which corresponds to failure. The probability of failure is equal to the joint probability that $S$ is greater than $R$. This probability is illustrated in Figure 20 by the area of the density function that is inside the failure space.
**Figure 19** Probability distributions of $R$ and $S$

**Figure 20** Probability of failure for $Z = R - S$
Obviously, a larger $R$ would result in a smaller probability of failure. It can be stated that the reliabilities of the elements are dependent on the margin between the loads and the resistance to failure. This margin can be achieved using different calculation approaches. The Joint Committee on Structural Safety proposed the following classification of the calculation approaches [1]:

- **Level III:** The reliability of an element is expressed by its probability of failure. To calculate the probability of failure, the probability density function of all uncertain strength and load variables are considered. This requires simulation calculations.

- **Level II:** This level includes methods for determining an approximated probability of failure, based on the assumption that all uncertain parameters are normally distributed. The failure probability is found by linearizing the limit state function in the point where failure will most likely occur.

- **Level I:** No failure probabilities are calculated in this method. The margin between the loads and the resistance to failure is created by using partial safety factors. This method is used in the standards.

The Level III and Level II methods are complex and cost-intensive and are not generally used by designers. The Level I method is the standard that is prescribed in design codes, but gives no unambiguous quantification of the reliability of the structure. In the next chapter, the above mentioned calculation approaches will be reviewed thoroughly. Also, they will be mathematically defined such that they are directly applicable to the case studies.
4. Reliability analysis

In the field of probabilistic design, the reliability of a system is measured by the probability of no failure. The uncertain parameters are described by a probability density function. Evaluating the limit state functions for these probability density functions will result in a probability of failure for the corresponding failure mechanism.

4.1. Level III method

The level III failure probability calculation is a full probabilistic method that takes into account the probability densities of all load and resistance variables. The failure probability can be defined as the fraction of the probability space, for which the limit state is exceeded, which means that \( Z \leq 0 \). For example, if the joint probability of strength \( R \) and load \( S \) is known, the probability of failure can be calculated by integration over the space in which failure occurs. This is illustrated in the formula below and in Figure 21.

\[
P_f = \int \int_{Z<0} f_{R,S}(R,S) dRdS = \int_{-\infty}^{\infty} \int_{-\infty}^{S} f_{R,S}(R,S) dRdS
\]  

(2)

For a limit state function with more random variables, a general formulation of the probability of failure can be expressed.

\[
Z = g(X_1, X_2, ..., X_n)
\]  

(3)

\[
P_f = \int \int \ldots \int f_{X_1, X_2, ..., X_N}(X_1, X_2, ..., X_N) dX_1 dX_2 \ldots dX_N
\]  

(4)

Since this integral can only seldom be determined analytically, the solution is calculated with numerical methods or simulation method. Since numerical methods are computationally intensive and not commonly used, in this study only a simulation method called Monte Carlo analysis will be discussed.

![Figure 21 Joint probability density function and failure space](image)

The Monte Carlo method is a simulation method, that evaluates the limit state function for possible values of the uncertain parameters. In the Monte Carlo method, every uncertain parameter can be...
assigned any distribution function. From these distribution functions a realisation is taken, so that for every uncertain parameter a value is defined. A set of realisations of all uncertain parameters is called a simulation. Combining the realisations in the limit state function results in either failure or no failure. The probability of failure is the fraction of the total number of simulations that resulted in failure. The required number of calculations depends on the error that is tolerated in the calculation of the failure probability. The relative error of the Monte Carlo simulation is proportional to the difference between the calculated probability of failure and the actual probability of failure:

$$\varepsilon = \frac{n_f - P_f}{P_f}$$

(5)

The error has an expected value of zero and a standard deviation of:

$$\sigma_{\varepsilon} = \sqrt{\frac{1 - P_f}{n \cdot P_f}}$$

(6)

If n is sufficiently large, the error is normally distributed, based on the central limit theorem. The probability that the relative error is smaller than a given value E is:

$$P(\varepsilon < E) = \Phi\left(\frac{E}{\sigma_{\varepsilon}}\right)$$

(7)

This indicates that for a reliability of $\Phi(k)$, the relative error should be smaller than $E = k\sigma_{\varepsilon}$. Consequently:

$$n > \frac{k^2}{E^2} \left(\frac{1}{P_f} - 1\right)$$

(8)

Now the number of required simulations can be calculated from an acceptable reliability $\Phi(k)$ that the maximum relative error is equal to E. The required number of simulations is also dependent on the probability of failure. For a small failure probability, the required number of simulations is large.

Methods are available to reduce the number of simulations. The methods are based on defining the probability distribution of the limit state function, during the simulations. The method of Importance Sampling, for example, searches for simulations that have results close to the failure space. This way, the failure space can be defined accurately.

In practice, the Monte Carlo method has to be performed using a programming language. MATLAB has a standard procedure for drawing random numbers from a specified distribution function.

### 4.2. Level II method

If the limit state function is normally distributed, the failure probability can be calculated as follows:

$$P(Z < 0) = \Phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right) = \Phi\left(-\frac{\mu_Z}{\sigma_Z}\right) = \Phi(-\beta)$$

(9)

In which $\beta$ is the reliability index. The aim of the level II method is to find the value of the reliability index so that the probability of failure can be calculated. For a linear limit state function with normally distributed random variables, finding $\beta$ is straightforward. First consider the standard limit
state function, $Z = R - S$. If $R$ and $S$ are normally distributed, they can be transformed to standard normally distributed variables by:

$$U_R = \frac{R - \mu_R}{\sigma_R}, \quad U_S = \frac{S - \mu_S}{\sigma_S}$$

The limit state function with the standard normally distributed variables is:

$$Z = (\mu_R + \sigma_R U_R) - (\mu_S + \sigma_S U_S) = \sigma_R U_R - \sigma_S U_S + \mu_R - \mu_S$$

Now the failure space is defined by:

$$\sigma_R U_R - \sigma_S U_S + \mu_R - \mu_S \leq 0$$

The minimum distance from the origin to the edge of the failure space is:

$$d_{\text{min}} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\mu_Z}{\sigma_Z} = \beta$$

From this, it can be concluded that the reliability index is the distance from the origin to failure space, for a linear limit state function and normally distributed random variables. This is the definition of the reliability index according to Hasofer and Lind [3].

![Figure 22 Definition of reliability index [2]](image)

The above introduced reliability index can easily be calculated for a normally distributed limit state function. However, the distribution of a for example a non-linear limit state function of normally distributed random variables, is not normal. Therefore, a linearization of the limit state function is necessary:

$$Z = g(\bar{X}) \approx g(\bar{X}_0) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(\bar{X}_0)(X_i - X_0)$$
From this linearized limit state function, $\mu_Z$ and $\sigma_Z$ can be calculated so that $\beta$ can be determined:

$$
\mu_Z \approx g(X_0) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(X_0)(\mu_{X_i} - X_0) 
$$

(15)

$$
\sigma_Z \approx \sqrt{\sum_{i=1}^{n} \left( \frac{\partial g}{\partial X_i}(X_0)\sigma_{X_i} \right)^2}
$$

(16)

$$
\beta = \frac{\mu_Z}{\sigma_Z} \approx \frac{g(X_0) + \sum_{i=1}^{n} \frac{\partial g}{\partial X_i}(X_0)(\mu_{X_i} - X_0)}{\sqrt{\sum_{i=1}^{n} \left( \frac{\partial g}{\partial X_i}(X_0)\sigma_{X_i} \right)^2}}
$$

(17)

An important feature in this method, is that the value for $\beta$ is dependent on the linearization point, as illustrated in Figure 23. For an accurate value of $\beta$, the linearization should be performed in the design point. The design point is the point on the edge of the failure area, that has the smallest distance to the origin. This is illustrated in Figure 24. The design point can be defined by iteration between the design point and the reliability index. If the deviation of the design points between subsequent iteration becomes small, an approximation of the design point has been found.
The design point is given by:

\[ X_i^* = \mu_{x_i} + U_i^* \sigma_{x_i}, \quad (U_1^*, U_2^*, \ldots, U_N^*) = (\alpha_1 \beta, \alpha_2 \beta, \ldots, \alpha_n \beta), \quad (18) \]

In which \( \alpha \) is the influence coefficient that represents the influence of a certain uncertain parameter on the failure probability. The \( \alpha \) values are calculated with:

\[ \alpha_i = -\frac{\partial}{\partial X_i} \sum_{j=1}^{n} (\bar{X}^* \sigma_{x_i}) \quad (19) \]

The new design point can be calculated with:

\[ X_i^* = \mu_i - \alpha_i \beta \sigma_{x_i} \quad (20) \]

The above equations have to be iteratively solved until the design point values are converged. The corresponding reliability index is an approximation of the actual reliability index.

The level II procedure is also applicable to limit state functions with non-normally distributed random variables or dependent random variables. This is done through transformations and linearization. However, since various approximations are necessary, the accuracy will decrease.

4.3. Calibration of partial safety factors from level II method

In appendix A, an extensive summary is given on the design standard as defined in the Eurocode [4]. The reliability assessment in this standard is employed by using partial safety factors. Design values for resistances are defined as a characteristic value of the resistance divided by a partial safety factor (larger than 1) and design values for load effects are introduced as characteristic values multiplied by a partial safety factor (larger than 1). Also load combination factors are set on one or more variable loads, to account for the effect of simultaneously occurring variable load effects. The reliability is ensured since partial safety factors are based on calibration from probabilistic calculations. This calibration procedure is based on probabilistic design and will be described in this paragraph.
4.3.1. Eurocode
The values for the partial factors in the Eurocode, have been obtained by calibration from historical and empirical methods and are further developed using calibration from a level II probabilistic method. The calibration from the level II method is based on a target reliability that should be reached. A design is sufficient if limit states are not reached, so that the constraint is:

\[ S_d < R_d \] (21)

Now the design values should be determined for which the design reaches the target reliability. An approximation is done for the probability of exceedence:

\[ P(S > S_d) = \Phi \left( + \frac{S_d - \mu_S}{\sigma_S} \right) \approx \Phi(+\alpha_s \beta) \] (22)

\[ P(R < R_d) = \Phi \left( - \frac{R_d - \mu_R}{\sigma_R} \right) \approx \Phi(-\alpha_R \beta) \] (23)

Where \( \alpha_s \) and \( \alpha_R \) are the values of the FORM sensitivity factors. The factors have been calibrated using a First Order Reliability Method. According to NEN 1990-2000 the values of these factors may be taken as -0.7 and 0.8 respectively, provided that \( 0.16 < \frac{\sigma_R}{\sigma_S} < 7.6 \).

Now the design values \( R_d \) and \( S_d \), that correspond to the target reliability, can be calculated from the probability distribution function belonging to the parameters. From this design value and the characteristic value, the required partial factor can be calculated.

4.4. Discussion
In this discussion, the ability of the different approaches to assess the reliability of a structure will be discussed. First the level II and level III approaches will be discussed and they will be compared. Following, the calibration of partial safety factors from the level II method will be discussed.

As stated in 4.2, the level II method only results in an exact value of the reliability index if the uncertain parameters are normally distributed and if the limit state function is linear. If this is not the case, an approximation of the limit state function can be done by using linearization techniques. The level II method is also not applicable when the failure space is discontinuous. The level III method can evaluate the limit state function regardless of the form of the function or the character of the probability distributions. Therefore, the level III method is a more accurate method. The accuracy does however strongly depend on the number of simulations that is done. For complex problems with a low acceptable failure probability, this can often result in a very expensive and time-consuming calculation process. A greater accuracy for a smaller number of simulations can be obtained by applying Importance Sampling or Adaptive Sampling. Both methods are based on using information of preceding simulations to choose samples that will have a result near the failure space.

Another advantage of Monte Carlo simulation is that it can easily applied to systems with multiple limit state functions. For every sample of parameters, all limit state functions can be calculated to check if failure occurs. In this way, the statistical dependence of the functions is implicitly taken into account. This means that the total failure space is considered. For a level II calculation, the limit state functions can only be evaluated separately. The total failure space must be considered a serial system, since the statistical dependence of the limit state functions is unknown. The boundaries for
the failure probability of the system can then be defined from the failure probabilities of the separate elements.

In the level I method, as prescribed by the Eurocode, partial safety factors and characteristic values are used to ensure the reliability of the structure. The partial safety factor that has to be imposed on a parameter depends on the classification of the load, strength or material parameter. For every situation within such a classification, the partial safety factor will have to result in a sufficiently reliable design. Therefore, the partial safety factor is often a conservative value which will result in a design that has a reliability that is higher than the target reliability, which means that the investment costs are not optimal. In chapter 6, alternative calibration methods for partial safety factors are discussed, of which one is based on a cost minimization.
5. System Analysis

5.1. Fault tree
In the theory as described in chapter 3 and 4, the focus has been on the reliability of a single failure mode with a single limit state equation. In reality, even for a simple structure, multiple failure modes are relevant. These failure modes can be combined in a fault tree, that gives a logical succession of events that lead to failure. This logical succession comes to expression by combining the failure modes in a series system, a parallel system or a combination. In Figure 25 [2] a fault tree is given in which the ‘OR’ port represents a series system and the ‘AND’ port represents a parallel system. The failure event is defined as failure of the structure and is called the ‘top event’.

![Fault tree diagram]

**Figure 25 Fault tree [5]**

Since every failure mode corresponds to a limit state equation, for every failure mode a probability of failure can be calculated, given the values for the random variables. The probability of failure of the whole system, which equals the probability of the top event, can be calculated by combining the separate probabilities of failure. This can be done analytically when the failure modes have either zero correlation or full correlation. If this is not the case, it is necessary to do an approximation of the system failure probability. Calculation procedures for determining the system probability of failure are explained in the next paragraph.

5.2. Probability of failure of a system
The system probability of failure due to multiple failure modes, depends on two aspects; the correlation between the failure modes and the connection of the failure modes in either a serial system or a parallel system.
5.2.1. Series system
In a series system, the whole system fails if one element fails. The system failure probability can therefore be defined as the union of the failure probabilities of all elements as shown in Figure 26, for two elements are shown.

**Figure 26** EVENTS $E_1$ AND $E_2$ AND THEIR INTERSECTION $\cap$ AND UNION $\cup$

The probability of failure of a serial system with two elements can now be defined by:

$$P_{f_{\text{system}}} = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1) \cdot P(E_2|E_1)$$

If the failure modes of the two elements are independent:

$$P(E_1) \cdot P(E_2|E_1) = P(E_1) \cdot P(E_2)$$

$$P_{f_{\text{system}}} = P(E_1) + P(E_2) - P(E_1) \cdot P(E_2)$$

If the failure modes are not independent, the failure of one elements also implies the failure of the other element. The probability of failure of an element, given failure of the other element, is then equal to the smallest probability of failure. This results in:

$$P(E_1) \cdot P(E_2|E_1) = P(E_2) \cdot P(E_1|E_2) = \min(P(E_1), P(E_2))$$

$$P_{f_{\text{system}}} = P(E_1) + P(E_2) - \min(P(E_1), P(E_2)) = \max(P(E_1), P(E_2))$$

For a serial system with unknown dependencies between the failure modes it can be concluded that:

$$\max(P(E_i)) \leq P_{f_{\text{system}}} \leq \sum_{i=1}^{N} P(E_i) \quad i = 1, 2, ..., N \quad (24)$$

5.2.2. Parallel systems
A parallel systems only fails if all elements have failed. The failure space is defined by the intersection of the failure spaces of all elements. For a system with two elements:

$$P_{f_{\text{system}}} = P(E_1 \cap E_2) = P(E_1) \cdot P(E_2|E_1)$$

If the failure modes for element 1 and 2 are statistically independent:

$$P_{f_{\text{system}}} = P(E_1) \cdot P(E_2)$$
If they are statistically dependent:

$$P_{f, system} = \min(P(E_1), P(E_2))$$

For a parallel system with unknown dependencies between the failure modes it can be concluded that:

$$\prod_{i}^{N} P(E_i) \leq P_{f, system} \leq \min(P(E_1), P(E_2)) \quad i = 1, 2, \ldots, N$$ (25)

5.2.3. Boundaries

The probability of failure of the system can now be defined by combining the separate failure probabilities according to the fault tree. The system failure probability can now be calculated using the upper and lower boundaries in equation (29) and (30). For a conservative calculation, it is favourable to use upper bounds. This will however lead to a less economical design. A more optimized design can be found when boundaries can be defined that approximate the ‘true’ failure probability to a better extent. Ditlevsen proposed a method to obtain more accurate boundaries [6]. The Ditlevsen boundaries are based on an approximation method and they require input on the correlation between the failure modes.
6. Optimization

In chapter 4 and 5, it is explained how the failure probability of a system can be defined. These failure probabilities are a function of the values of the design variables. Design variables are for example the dimensions of the elements or the strength of the material. Higher values of these design variables correspond to higher investment costs for constructing the system. Since a designer always aims at reducing the investment costs, the values should therefore be kept as small as possible. This will however generally lead to a higher failure probability, which is unfavourable. The optimal values of the design variables, for which the investment costs as well as the probability of failure are sufficiently low, depend on the design objective. This design objective depends on the procedure that is chosen with regard to socio-economic aspects. Obvious socio-economic aspects are loss-of-life, construction costs and lifecycle costs of the structure. Another important aspect is the assessment on environmental effects (in Dutch: ‘Milieu effect rapportage’), that is often a compulsory task. Also aspects such as multi-functionality, landscape friendliness and sustainability may play a role. It is common practice to consider these aspects in a Multi Criteria Analysis, in which alternatives are compared according to their scores on the different aspects. If the design objective can be expressed in a function, the objective function, an optimization task can be performed to find the optimal values of the design variables. The general expression for an optimization task is:

\[
\begin{align*}
\text{Find} & \quad x \\
\text{Optimize} & \quad f(x) \\
\text{Such that} & \quad g_i(x, \theta) \leq 0, \quad i = 1, \ldots, l
\end{align*}
\]  

(26)

In which:
- \( x \) is the vector of design variables.
- \( \theta \) is the vector of random variables.
- \( f(x, \theta) \) is the objective function.
- \( g_i(x, \theta) \) is the set of constraints.

An extensive theoretic description on optimization tasks is given in chapter 10. In this chapter, the principle of economic optimization as well as optimization procedures addressing loss of life will be discussed. Also, two optimization tasks will be described that have been proposed for the calibration of partial safety factors.

6.1. Economic optimization

When loss of life is not relevant or only of small importance, the acceptable failure probability can be determined from a cost optimization. The costs come into the design problem twofold; through the investment costs and through the risk. The sum of the investment costs and the risk is the lifecycle cost of a system. The values of the design variables should be chosen such that these lifecycle costs are minimized. The risk of failure of a system is determined by:

\[
Risk = \text{Probability of failure} \times \text{Damage} = P_f \cdot D
\]

(27)

In structural engineering, \( P_f \) often denotes the annual probability of failure. The risk that is calculated using \( P_f \cdot D \) is therefore annual risk.
The net present value of the risk over the entire lifetime is calculated by:

$$
\sum_{t=1}^{T} \frac{P_f \cdot D}{(1 + r' - g)^t}
$$

(28)

The net present value of the total costs can then be expressed by:

$$
C = I_0 + \sum_{i=1}^{N} I_i \cdot x_i + \sum_{t=1}^{T} \frac{P_f \cdot D}{(1 + r' - g)^t}
$$

(29)

In which:

- $C$ is the net present value of the total cost
- $I_0$ is the initial investment cost
- $I_i$ is the cost of constructing $x$ units of parameter $i$
- $P_f$ is the annual probability of failure of the system
- $D$ is the damage that occurs due to failure
- $r'$ is the annual discount
- $g$ is the annual growth
- $t$ is the time in years
- $T$ is the considered lifetime of the system in years

The optimization task is then:

Find $x_i$, $i = 1, 2, ..., N$

Minimizing $C = I_0 + \sum_{i=1}^{N} I_i \cdot x_i + \sum_{t=1}^{T} \frac{P_f \cdot D}{(1 + r' - g)^t}$

(30)

The damage includes economic damage but may also include loss of life. Loss of life is in this case expressed by a monetary value. The graph that corresponds to equation (30) is given in Figure 27.

The total cost is plotted against the probability of failure to find an optimum in costs. The corresponding failure probability is the acceptable failure probability. In the 1960’s, the required flood protection level of the Netherlands was defined in this manner. A cost optimization for determining the accepted probability of failure, is based on the assumption that it makes no sense to spend money on a safety measure if the risk reduction associated with this measure is smaller than the investment costs.
6.2. Maintenance costs

As an addition to the cost function that is given in 6.1, maintenance costs during the lifetime can also be considered during the design phase. By taking into account the maintenance costs, a more complete figure of the lifecycle costs is used for decision making. The maintenance costs are the sum of the inspection and the reparation costs. These costs depend on the maintenance strategy that is adopted. This maintenance strategy can be load-based and condition-based. When a load-based strategy is adopted, maintenance is done after the system has experienced a certain amount of loading. In a condition-based strategy, inspections are done to determine the condition of a system. If the condition does not fulfil the demands, maintenance is done. In the design phase, the probability that maintenance is necessary during the design phase can be determined, and the corresponding risk can be added to the lifecycle cost function in the optimization task. In this way, all expected costs due to maintenance, repair and inspection can be described as a risk and implemented in the lifecycle costs function. The optimization task then becomes more accurate, since the risk is described more accurately.

6.3. Loss of life

The risk of loss of life can be assessed by taking the individual accepted risk to define the target reliability. The individual accepted risk is the probability of loss of life in case of system failure that is accepted by an individual person. This target probability is set at $10^{-4}$ per year, or smaller, in Western countries. This target poses and upper boundary on the probability of failure of the system.

Another way to assess the risk of loss of life is by considering the societal accepted risk. This risk is dependent on the relative importance of the economic damage and the total number of lives lost in case of failure. If the total number of lost lives is large, reducing the risk of loss of life is the prior design objective. The design variables will be determined according to an acceptable risk of loss of life. On the other hand, if the economic damage is dominant, an economic optimization can be used to find the optimal values of the design variables.
An option that was already proposed in paragraph 61, is to express the loss of a life in a monetary value. The monetary risk of loss of life can then be added to the lifecycle costs function, to find an ‘economic’ optimum set of design variables.

### 6.4. Calibration of partial safety factors

The partial safety factors as given in the Eurocode, are based on the calibration procedure as given in paragraph 4.3. In literature, alternative methods for calibration of safety factors have been introduced. These methods are based on optimization tasks. The Joint Committee on Structural Safety discussed several optimization methods for calibration of partial safety factors [7]. In this paragraph a method based on maximizing the total expected cost-benefits and a method based on the minimization of the error in reliability index are explained.

#### 6.4.1. Maximize cost-benefit

Sørensen [8] introduced a method to maximize the total expected cost-benefits for a given class of structures. The design variables in the method are the partial safety factors. The formula is expressed by:

Find $\gamma$

Maximizing $W(\gamma) = \sum_{j=1}^{l} w_j [B_j - C_{ij}(\gamma) - C_{Rj}(\gamma) - C_{Fj}P_{Fj}(\gamma)]$

Such that $\gamma_i^l \leq \gamma_i \leq \gamma_i^u$, $i = 1, ..., m$

Where:

$\gamma = (\gamma_1, ..., \gamma_m)$ is the set of partial safety factors to be calibrated

$w_j$ is a factor indicating the relative frequency of failure mode $j$

$B_j$ is the expected benefits

$C_{ij}$ is the investment cost

$C_{Rj}$ is the repair and maintenance cost

$C_{Fj}$ is the damage due to failure

$P_{Fj}$ is the probability of failure of failure mode $j$ for given partial safety factors

This optimization is aimed at finding the set values for the partial safety factors that maximizes the expected value of the benefits minus the costs.
6.4.2. Closeness measure

In this method, the partial safety factors are found by minimizing the deviation between the target reliability index and the reliability index of the structure under a certain load combination. This approach is called the closeness measure and is expressed by:

\[
\min_{\gamma} W(\gamma) = \sum_{j=1}^{L} w_j (\beta_j(\gamma) - \beta_t)^2 \quad (32)
\]

Or:

\[
\min_{\gamma} W'(\gamma) = \sum_{j=1}^{L} w_j (P_{Fj}(\gamma) - P_{Ft})^2 \quad (33)
\]

Where:

- \( \beta_j \) is the reliability index for the \( j^{th} \) load combination.
- \( \beta_t \) is the target reliability index.
- \( P_{Fj} \) is the probability of failure due to the \( j^{th} \) load combination.
- \( P_{Ft} \) is the target probability of failure.
- \( w_j \) is the weight factor that depends on the importance or frequency of a load combination.

The closeness measures can be solved for the partial safety factors, if the target reliability index is determined and if the reliability indexes \( \beta_j \) are calculated. The same holds for the closeness measure based on the failure probability. The reliability indexes \( \beta_j \), as a function of the values of the partial safety factors, can be calculated as in chapter 4. This is only possible if the structural model and the values of the design variables are known. The partial safety factors must however work for a range of design variable values. Therefore, it is best to make sure that the partial safety factors work for the worst set of design variable values. Once this worst set is determined, the reliability indexes can be estimated by a probabilistic method as given in chapter 4. These reliability indexes can then be inserted in the closeness measure to determine the partial safety factors. These values for the partial safety factors function as a tool for determining the final partial safety factors. This is done taking into account engineering expert judgement and tradition.
7. Conclusion Part 1

The process for designing a civil system exists of an initiative stage, a feasibility stage and a technical design stage. After the technical design stage has finished, the system, can be constructed after which the use and maintenance stage starts. At the end of the lifetime, the system will be upgraded, reused or demolished.

In the technical design stage, the shape and topology are defined and the optimal values of the design variables are determined. This is done by finding an optimum combination for the investment costs and the reliability. This reliability can be determined using a level III or level II calculation method. In a level I calculation, the values of the design variables are determined without explicitly determining the reliability. The reliability enters the level I approach through the partial safety factors, of which the values are based on experience and calibration.

Using the level II or level III method, the reliability of the separate elements can be determined. In order to determine the reliability of the system as a whole, system analysis should be applied. In system analysis, the probability of failure of a system is determined using a fault tree, in which the connections between the structural elements are given. For example, the calculation for the probability of failure of a serial system differs from that for the probability of failure of a parallel system. Since the connections between the elements are generally not known exactly, lower and upper bounds for the system failure probability are given. For a conservative design, the upper bound should be used.

The acceptable reliability of a system depends on the optimization strategy used. In this thesis, a strategy based on minimizing loss of life and on minimizing lifecycle costs are described. Minimizing either the lifecycle costs or the loss of life will result in optimal values for the design variables, with a corresponding acceptable failure probability. For such an optimization strategy, a level III or level II probabilistic approach is necessary to define the failure probability for a certain set of design variables. The partial safety factors can however also be based on calibration from an economic optimization task. It can be concluded that an optimization task is always the basis for finding the values of the design variables.

In the proceedings of this study, the focus will be on the economic optimization for finding the values of the design variables. The choice will always be for the design that has a minimum in the lifecycle costs.
Part 2. Taguchi

Literature Study

Conclusion

“A robust product is a product whose performance is minimally sensitive to factors causing variability”
8. Taguchi’s approach to robust design

8.1. Taguchi

Genichi Taguchi was the pioneer of the concept of robust design in the 1980’s. Taguchi’s definition of a robust product is:

“a product whose performance is minimally sensitive to factors causing variability (at the lowest possible cost)”.

He introduced robust design as a method for developing a high-quality product. According to Taguchi, a higher quality results in lower costs, as opposed to the general assumption that improvement of quality responds to higher costs. This difference lies in the alternative definition that Taguchi uses for quality. Namely, Taguchi defines quality as the loss a product causes to society after being shipped, which means that the performance of the product over its lifetime is considered. A smaller loss corresponds to a higher quality. Opposing, the traditional definition of quality refers to a judgement of the ability of a product to perform its functions, before it is actually used.

In the definition by Taguchi, “society” means that both the losses to the manufacturer and to the customer are considered. The “loss” can be divided in loss caused by variability of the functions and loss caused by harmful side effects. Loss caused by variability of the functions includes energy, time and money spend by the customer as a result of problem fixing or failure of the product. Losses due to harmful side effects refer for example to the decrease of market shares of the manufacturer as a result of customer dissatisfaction. Losses due to harmful side effects may also refer to physical harm imposed on the customer by the products, for example due to a car crash because the breaks are of poor quality.

In the approach of Taguchi, a product always has a target performance for which the loss is zero. Performance functions refer to the functions of the system. For example the target performance of a thermostat may be that the inside air temperature is kept exactly at 20°C. According to Taguchi, any deviation in this target will lead to a quadratic increase in the loss to society. To quantify this, Taguchi developed a quality loss function which gives the quality loss as a quadratic function of the deviation from the target performance of the product, see Figure 28. This quality loss function is purely empirical and in reality it is very unlikely that a loss function has such a symmetric and continuous shape . However, the loss function as defined by Taguchi was a step forward from the traditional consideration of quality, in which it is assumed that a loss will only occur if the performance violates certain tolerances. This is shown in Figure 29.
Figure 28 Taguchi's Quality Loss Function

Figure 29 Comparison of Old and New Measure of Loss Function [9]
Following the consideration as described above, Taguchi’s philosophy was summarized in three concepts by Roy [10]:

- **Quality should be designed into the product, not inspected into it.** This means that a product should be designed such that its performance is as close as possible to its target performance. If the quality is assessed through inspection, products are inspected after they have been produced, and thrown away when they do not fulfil the demands.
- **Quality is best achieved by minimizing the deviation from the target.** This means that the product should be designed such that it is immune to uncontrollable environmental factors.
- **The cost of quality should be measured as a function of the deviation from the standard.** This is illustrated in Figure 28.

Based in these concepts, Taguchi introduced an approach to implement quality into a design. This approach consists of three steps; system design, parameter design and tolerance design. System design refers to the development of a preliminary design that fulfils the functional requirements under the nominal values of the uncertain parameters. Parameter design is aimed at determining values of the design variables for which the system is functional and robust against noise factors that cause variability in the performance. The last step that can be used to implement quality into a design is tolerance design. Tolerance design is necessary when parameter design is not sufficient for reducing the variability of the performance under noise parameters. The factors that have a negative influence on the variability are then assigned smaller tolerances. This means that better and more expensive components and processes are needed, which increases the investment costs. Therefore, parameter design should be employed preceding on tolerance design and is the most important step for increasing the quality.

Taguchi’s approach to parameter design can be referred to as ‘robust design’. According to Taguchi, the aim of robust design is to minimize the standard deviation of the performance, to reduce the quality loss. This can be achieved by using the method for robust design as proposed by Taguchi. This method is based on experimental design, which means that experiments are done to investigate the response of a system. In Taguchi’s approach, design variables are called control factors, while noise factors correspond to the uncertain parameters. For robust design, values of the control factors should be found for which the performance of the system is least sensitive to variations in the noise factors. This will reduce the standard deviation of the performance. It is only possible to find such values when there is an interaction between the control factors and the noise factors. In other words, a robust system can only be found if the variance of the performance can be influenced by the designer.

Taguchi proposed an experimental design approach to study this interaction. This approach is referred to as ‘cross array’ and consists of an inner orthogonal array and an outer orthogonal array. The design variables are varied according to the inner array. For each design variable setting, the noise variables are varied according to an outer array. The result of all experiments is a set of response data for every set of design variables. These data are used to estimate the mean and the variance of the response for a certain set of design variables. From this a performance measure can be defined for every set of design variables, namely the **signal-to-noise-ratio (SNR)**. Optimizing the SNR will result in the set of design variables that yields the least noise and is therefore the most robust design. Three expressions for the SNR have been found in literature:
\[ SNR = -10 \cdot \log \left( \frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \right) \text{[dB]} \quad \text{the-bigger-the-better} \quad (34) \]

\[ SNR = -10 \cdot \log \left( \frac{1}{n} \sum_{i=1}^{n} y_i^2 \right) \text{[dB]} \quad \text{the-smaller-the-better} \quad (35) \]

\[ SNR = 10 \cdot \log \left( \frac{\mu_y^2}{\sigma_y^2} \right) \text{[dB]} \quad \text{nominal-the-best} \quad (36) \]

Only the nominal-the-best expression gives a measure of the deviation around the target performance. Therefore, minimizing this measure will result in a minimized loss when using Taguchi’s quality loss function. The two other expressions relate to different loss functions that are shown in Figure 30 and Figure 31. Only the nominal-the-best SNR is fits Taguchi’s concept of robust design. This SNR has to be maximized in order to minimize the standard deviation of \( y \), so that the loss is minimized.

**Figure 30**: Loss function for which a bigger value of the performance function will result in a smaller loss.
The experimental method as developed by Taguchi can be explained by the example on microwave popcorn. This problem has three control variables, namely the volume of the bag, the material of the bag and the amount of corn kernels in the bag. The noise variables are the cooking temperature, the cooking time and the positioning of the bag in the microwave. The objective function is the amount of popcorn, that should be least sensitive to changes in the noise variables. Now assume that all variables can have two different values, that are called levels. In order to determine the amount of popped corn kernels in every combination of the variables, $2^6 = 64$ experiments have to be performed. In order to reduce the amount of experiments, Taguchi proposed to use orthogonal arrays. In an orthogonal array all combinations of the levels of any two design variables appear exactly once. An orthogonal array is not possible for every amount of variables and levels. For example, if the variables have two levels, an orthogonal array is possible for a set of three or seven variables but not for a set of four variables. The most simple orthogonal array is the $L_4$ orthogonal array for three variables that can take two levels. This orthogonal array is shown in Table 1.

<table>
<thead>
<tr>
<th>Experimental set-up 1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental set-up 2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Experimental set-up 3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Experimental set-up 4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 $L_4$ orthogonal array for 3 variables that can take 2 levels
The $L_4$ orthogonal array can be used to define the experimental set-ups that should be used to investigate the robust design of a microwave popcorn bag. The corresponding orthogonal array is shown in Table 2.

<table>
<thead>
<tr>
<th>Experimental set-up</th>
<th>Volume [dm$^3$]</th>
<th>Material</th>
<th>Amount of kernels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>Material 1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>Material 2</td>
<td>130</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>Material 1</td>
<td>130</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>Material 2</td>
<td>100</td>
</tr>
</tbody>
</table>

**Table 2 Orthogonal Array for Design Variables of Microwave Popcorn Bag**

According to Taguchi, this orthogonal array of design variables is the ‘inner orthogonal array’. Each of the experiments in this array has to be done under different values of the noise factors, to investigate the performance of the design under noise factors. The sets of noise factors that is imposed on the experimental set-ups is also determined by an orthogonal array. In the case of the microwave popcorn bag, this array is as shown in Table 3. This array is by Taguchi defined as the ‘outer orthogonal array’.

<table>
<thead>
<tr>
<th>Experimental set-up</th>
<th>Temperature [Watt]</th>
<th>Cooking time [s]</th>
<th>Positioning [m from centre]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>700</td>
<td>210</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>700</td>
<td>150</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>900</td>
<td>210</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>900</td>
<td>150</td>
<td>0.01</td>
</tr>
</tbody>
</table>

**Table 3 Orthogonal Array for Noise Factors of Microwave Popcorn Bag**

The set of experiments that should be performed can be determined by imposing the outer array on the inner array as shown in Table 4. Every experiments results in a value $y_{ij}$ that represents the amount of corn kernels that are popped. From these values, the signal to noise ratio can be calculated.
For example, consider that the results of the experiments are given in Table 5.

\[
\begin{array}{cccccc}
\text{Temp} & 700 & 700 & 900 & 900 \\
\text{Time} & 210 & 150 & 210 & 150 \\
\text{Position} & 0,01 & 0,05 & 0,05 & 0,01 \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{Temp} & 700 & 700 & 900 & 900 \\
\text{Time} & 210 & 150 & 210 & 150 \\
\text{Position} & 0,01 & 0,05 & 0,05 & 0,01 \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{Volume} & \text{Material} & \text{Amount} & Y_{1,1} & Y_{1,2} & Y_{1,3} & Y_{1,4} \\
4 & \text{Material 1} & 100 & \text{Y} & \text{Y} & \text{Y} & \text{Y} \\
4 & \text{Material 2} & 130 & \text{Y} & \text{Y} & \text{Y} & \text{Y} \\
5 & \text{Material 1} & 130 & \text{Y} & \text{Y} & \text{Y} & \text{Y} \\
5 & \text{Material 2} & 100 & \text{Y} & \text{Y} & \text{Y} & \text{Y} \\
\end{array}
\]

TABLE 4 SET OF EXPERIMENTS THAT SHOULD BE DONE WHEN USING THE Taguchi APPROACH.

From these results, the signal to noise ratios can be calculated from equation 41:

\[
\text{SNR} = 10 \cdot \log \left( \frac{\mu^2}{\sigma^2} \right)
\]

The results are given in Table 6. From these results, the mean signal-to-noise ratio for each level of the design variables can be determined. These mean signal-to-noise ratios are given in Table 6 and in Figure 32, Figure 33 and Figure 34. From these figures it can be concluded that the first level is the best for all design variables, in terms of the signal-to-noise ratio. Using the first level for all design variables will however resulted in the smallest mean amount of popped corn kernels, which is 83,75. This means that this method may result in a design that does not have the best performance, but that does have the smallest deviation in the performance. From the figures, it can also be concluded that the signal-to-noise ratio is more sensitive to a 1 dm$^3$ change in the volume than to a change of material or to a change of 30 in the amount of corn kernels.
### Table 6: Signal to Noise Ratios for Each Experimental Set-Up

<table>
<thead>
<tr>
<th>Volume</th>
<th>Material</th>
<th>Amount</th>
<th>$\mu_y$</th>
<th>$\sigma_y$</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Material 1</td>
<td>100</td>
<td>83,75</td>
<td>4,79</td>
<td>24,85</td>
</tr>
<tr>
<td>4</td>
<td>Material 2</td>
<td>130</td>
<td>96,00</td>
<td>7,53</td>
<td>22,11</td>
</tr>
<tr>
<td>5</td>
<td>Material 1</td>
<td>130</td>
<td>93,75</td>
<td>10,87</td>
<td>18,71</td>
</tr>
<tr>
<td>5</td>
<td>Material 2</td>
<td>100</td>
<td>85,50</td>
<td>9,33</td>
<td>19,24</td>
</tr>
</tbody>
</table>

### Table 7: Mean Signal-to-Noise-Ratios Corresponding to the Level of the Design Variable

<table>
<thead>
<tr>
<th>Level</th>
<th>Volume</th>
<th>Material</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23,48</td>
<td>21,78</td>
<td>22,05</td>
</tr>
<tr>
<td>[4, material 1, 100]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18,98</td>
<td>20,68</td>
<td>20,41</td>
</tr>
<tr>
<td>[5, material 2, 130]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 32: Mean SNR as a Function of the Volume of the Bag

**Figure 32: Mean SNR for both values of the volume of the bag**
Figure 33: Mean SNR for both materials of the bag

Figure 34: Mean SNR for both values of the amount of corn kernels in the bag
Although Taguchi’s approach is introduced for experimental design, it is can also be applied on simulation based design. Simulation based design is useful in structural design, where it is often not feasible to study the interaction between noise and control factors experimentally. In structural design, empirical and theoretical formulas can be used to define the relations between the control factors, the noise factors and the response. Taguchi’s approach using orthogonal arrays is much faster than a Monte Carlo analysis, in which all options are evaluated. However, much information is eliminated by the orthogonal array approach since all statistical information on the noise parameters is neglected. Taguchi’s approach can therefore not be used to define a failure probability, as discussed in chapter 4. In paragraph 8.4 a further discussion is given on the applicability of Taguchi’s approach on structural design problems.

8.2. Robust design in chemical engineering

In chemical engineering, experimental design is an important tool. Therefore, Taguchi’s approach for robust design has found many applications in this field of study. An example of robust design in chemical engineering has been given as a case study [11] in the book by Taguchi [12]. The case study was aimed at finding a formula for the chemicals in body warmers, such that the temperature of the body warmers was least sensitive to environmental temperature changes. Also, there was a need for more cost-efficient designs of body warmers as a result of the severe competition in the market. These goals can be combined in one goal, namely to reduce the heat-generation of the body warmer if the environmental temperature is sufficiently warm. The temperature inside the body warmer should be kept at its target under temperature changes, which means the system should be robust. Also, the heat-generation of the body warmer can be less, which corresponds to a more cost-efficient system.

The chemicals used for heat-generating in body warmers are iron powder, active carbon, water, salt and a water-retaining agent. An optimum combination of the concentration and amount of these chemicals in the body warmers had to be found. Therefore, seven control factors were identified, representing these properties. The uncontrollable noise factor was the change in environmental temperature. This was simulated by covering the body warmer with flannel clothes, in absence of tools to influence the air temperature. An additional noise factor was the deterioration of the substance that was simulated by exposure to air. The performance of the system was defined by the amount of heat-generation of the system.

The experiments were done for 18 different combinations of the control factors. In each of the 18 experimental runs, the signal factor was measured for two different combinations of the noise factors. The results were the amount of heat-generation in every experiment, considering the noise factors. From these results, for each combination the signal-to-noise ratio and hence the expected value and the standard deviation of the heat generation was calculated. From this, the optimum combination of the control factors was determined, corresponding to the optimum concentration and amount of the chemicals in a body warmer.

It was calculated that for this optimum design “a gain of 3.91 dB in the signal to noise ratio and a 34% increase in heat-generation efficiency were achieved without cost increase”[11]. This was beyond expectation and it was now possible to reduce the size and the weight of the body warmer while maintaining the performance.
It can be concluded that Taguchi’s approach the robust engineering was very useful in this case. The body warmers could now be produced at equal cost but with the better performance. The amount of experiments that was done was relatively small, which makes it a feasible method for experimental design. In many design problems, it is however too cost-intensive to use an experimental design problem. For example, when designing a bridge, it is not feasible to test a number of bridges for finding the most optimum design. In such cases, the tests should be done using models. An example of such a design problems is given in the next paragraph, that deals with robust design in automotive engineering.

8.3. Robust design in automotive engineering

Robust design methods have been applied in automotive engineering ever since this industry exists. When designing a car, the objectives are often conflicting. For example, the car must have a high resistance but the weight should be minimized. The engine should be efficient but investment costs should be minimized. A robustness study of a bumper system has been done at Volvo Cars [13]. With a ‘robustness study’ it is meant that the response of the bumper is investigated under different sets of input parameters. The bumper system has to protect the car against accidental loads due to crashes. The initial problem in this study was that, although 70% of the crashes are low speed crashes, damage due to a low speed crash is equal to damage due to a high speed crash. This can be assigned to the fact that the rear part of the cars is highly integrated. To minimize the risk, the bumper beam must be designed to absorb the energy of a low speed crash, while the costs and safety at high speed crashes is unchanged. Therefore, the bumper needs to be optimized with respect to safety and costs at low speed crashes. The damage of the system will then be less sensitive to low speed crashes, that have the highest probability of occurring.

The uncertain parameters in this problems are the material properties, the geometry, the friction, the tilt angle of the bumper, the impact angle of the action and the speed. These parameters are the input of the analysis. The output consists of values on the strain, the deformation and the internal energy in the bumper. This optimization study was called a robustness study and result was a list of ranked effects of the input parameters on the output parameters. In this way, the dominant parameters could be selected. For example the internal energy in the bumper is mostly influenced by the tilt angle. This means that most efficient way to achieve a reduction in the internal energy is by optimizing the tilt angle. The robustness study also revealed a set of designs giving extreme results. An optimization on several objectives was done to find the optimal bumper properties. The result was a bumper with a both a decreased mean value and a decreased standard deviation of the maximum strain under low speed crash loading. The drawback of the result was that the bumper turned out to be not necessarily reliable for high speed load cases, which undermines the usefulness of the study, and the robustness of the solution.

The robustness study that was done for the design of a rear bumper was very expensive and time-consuming. The result was however promising since a bumper was designed that has a lower possible maximum strain than the original bumper. Also the uncertainty in the maximum strain was reduced. In the study, only low speed load cases were taken into account. For high speed load cases, the bumper may be less applicable. This means that the resulting design was not robust, since a variance in the load (noise factor) could cause large variability in the performance of the bumper.

This example illustrates the methodology of robust design in automotive engineering as well as the advantages and disadvantages. The advantage is that the worst load cases are identified and that the
uncertain parameters can be ranked according to their influence on the performance. These advantages are however not exclusively properties of robust design. The resulting design has a smaller variance in the consequences under low speed crashes than the original design. This is exactly the aim of robust design. The drawbacks are the large calculation time and the fact that the resulting design may be incompetent for load cases that have not been considered in the design.

8.4. Taguchi versus limit state design

8.4.1. Case study: caramel candy

The comparison between robust design as proposed by Taguchi and limit state design, can be explained by an example concerning caramel candy. In 1948, Taguchi developed a recipe for caramel candy for the company Morinaga Seika Co. The company gave him the assignment to develop a recipe for caramel candy so that its chewability, or plasticity, was less sensitive to temperature changes. Taguchi’s method for robust design is well applicable to this problem. Namely, the problem has a target performance that is defined as a target plasticity for which the customers satisfaction is the highest. Also, both a larger and a smaller plasticity will lead to customer dissatisfaction. The plasticity is influenced by an uncontrollable factor, namely the temperature. The customer dissatisfaction can therefore be reduced by reducing the sensitivity of the plasticity to temperature changes. In Figure 35, the distribution of the plasticity is determined as a function of the distribution of the temperature and the relation between plasticity and temperature. This figure is based on fictive equations and distribution functions. The robust recipe results in a plasticity that is less sensitive to the temperature and has therefore a plasticity distribution with a smaller standard deviation.

![Figure 35 Distribution of plasticity of caramel candy as a result of the temperature distribution, for the original recipe and the robust recipe.](image)
The plasticity distribution can be translated in an expected loss by imposing it on the loss function. The expression for the expected loss is:

\[
E[\text{Loss}] = \int_{-\infty}^{\infty} p \cdot \text{pdf}(p) \cdot f_{\text{loss}} \, dp
\]  

(37)

In which:

- \(p\) is the plasticity
- \(\text{pdf}(p)\) is the probability density function of the plasticity

The Taguchi loss function and the probability density functions of the plasticity are shown in Figure 36. In Figure 37 the resulting risk is visualized. The area under the graph represents the total expected of the loss. This expected value is equal to 0.56 for the original recipe and equal to 0.06 for the robust recipe. This means that in this example, a 10 times reduction in the loss is achieved by developing a robust recipe. It should however be noted that all variables used in this example are fictive. Also, no unity for the loss is given so that no validation of the assumed loss function is possible.

![Figure 36 Taguchi loss function and distribution of plasticity](image-url)
The risk is calculated by multiplying the probability of occurring of a certain plasticity with the corresponding loss.

From this example on Taguchi approach to the robust design of caramel candy, the link can be made to limit state design. According to limit state design, a loss will occur if the limit state function is smaller than zero. When considering the caramel candy, different limit state functions can be defined that all correspond to a different type of loss. Consider the following fictive limit states:

\[
Z_9 = p - 9 > 0 \quad \text{if} \quad p > 9 \quad \text{opening the candy will result in smudging (SLS), resulting in loss L9}
\]

\[
Z_{12} = p - 12 > 0 \quad \text{if} \quad p > 12 \quad \text{the candy is not edible anymore and is thrown away (ULS), resulting in loss L12}
\]

\[
Z_5 = 5 - p > 0 \quad \text{if} \quad p > 5 \quad \text{candy is not edible temporarily (SLS), resulting in loss L5}
\]

These limit states can be represented in the loss function as given in Figure 38. The total expected value of the loss, also called the risk, can again be calculated by integrating the loss over the plasticity distribution. The result for the original recipe is given in Figure 39. From this figure it can be concluded that the serviceability limit state function \(Z_9\) has the largest influence on the risk. The risk can be significantly reduced by reducing the probability that the plasticity will be larger than 9. This can be done by minimizing the standard deviation of the plasticity distribution but also by shifting the mean of the plasticity distribution towards a lower value. For example, shifting the mean plasticity to a value of 7 without changing the standard deviation, results in a significant smaller total risk. This is illustrated in Figure 39. The general conclusion that can be drawn from this example is that the best approach to minimize the risk, strongly depends on the shape of the loss function.

**Figure 37** Risk of caramel candy. The risk is calculated by multiplying the probability of occurring of a certain plasticity with the corresponding loss.
**Figure 38** Limit state loss function for the plasticity of caramel candy, with probability density functions of the plasticity for two different recipes.

**Figure 39** Risk of caramel candy as a function of the plasticity, using the limit state loss function and two different recipes.
In this caramel candy example, the risk has been defined using Taguchi’s loss function and using a limit state loss function. Taguchi claims the optimal design is found by minimizing this risk. In the standard approach however, the risk forms only part of the cost function to be optimized. The cost function is defined as the sum of the investment costs and the net present value of the risk. Taguchi’s approach to robust design is based on the costs that occur after shipping of the product. The investment costs are neglected, which means that Taguchi’s robust optimum is not necessarily an economic optimum. Both Taguchi’s approach and the standard approach can be captured in a formula, from which their difference and similarities are clearly visible.

**Taguchi**

Find $x$

Minimizing $\int_{-\infty}^{\infty} pdf(x) \cdot f_{loss,tag} \, dx = \int_{-\infty}^{\infty} pdf(x) \cdot k \cdot (x - m)^2 \, dx$  \hspace{1cm} (38)

**Standard**

Find $x$

Minimizing $I(x) + \int_{-\infty}^{\infty} pdf(x) \cdot f_{loss,ls} \, dx = I(x) + \sum_{i=1}^{T} \frac{P_{f,i} D_i}{(1+r)^t}$  \hspace{1cm} (39)

In which:

$n$ is the number of limit states considered

Clearly, the standard approach is much more attractive since it results in an economic optimum. Also, the standard approach is more sophisticated since the losses are better described by limit states than by the empirical quadratic cost function.

### 8.4.2. Differences and similarities

The main difference between limit state design and Taguchi’s robust design, is the definition of the loss function. Taguchi proposed a loss function that increases quadratically with increasing deviation from a certain performance target. This performance target is often described by a function of the system, for example the chewability of the caramel or the water level in a navigation channel. Any deviation of the target chewability or the target water level will lead to functional losses. The total of this functional loss has to be reduced which means that the deviation from the target should be reduced. The quadratic loss function is not applicable to structural failure, since this will lead to a jump in the loss function. The losses are much better described by the limit states.

In limit state design, the loss function is a step function that changes value at the limit states. The limit state may be a serviceability or an ultimate limit state, that respectively describes functional losses and loss due to structural failure. Both the Taguchi loss function and the limit states are a model of the losses that will occur in reality. The limit states are more realistic than the Taguchi loss function but the limit states cannot be used to describe gradual losses. A more realistic loss function can be constructed by extensively investigating the behaviour of the structure to be designed. This loss function is different for every structure and should therefore be investigated separately for every design problem. Other than the definition of the loss function, the Taguchi approach also differs from the standard approach in the definition of the optimization task. Taguchi’s approach is aimed at minimizing the risk while the standard procedure is aimed at minimizing the sum of the
investment costs and the net present value of the risk. The latter results in a design alternative that is economically most attractive. Opposed, Taguchi’s approach may result in infeasible designs. The conclusion as done in this paragraph are summarized in Table 8.

<table>
<thead>
<tr>
<th></th>
<th>Taguchi robust design</th>
<th>Standard design approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loss function</strong></td>
<td>Loss increases quadratically with the deviation from a performance target.</td>
<td>Loss occurs if a limit state is exceeded.</td>
</tr>
<tr>
<td><strong>Loss</strong></td>
<td>Loss is user-defined but should fit the loss function. This function is continuous and is therefore best applicable to functional losses.</td>
<td>Exceedance of an ultimate limit state results in losses due to failure. Exceedance of a serviceability limit state results in loss due to loss of functionality.</td>
</tr>
<tr>
<td><strong>Optimization task</strong></td>
<td>Minimization of overall risk.</td>
<td>Minimization of the sum of the investment costs and the overall risk.</td>
</tr>
</tbody>
</table>

**Table 8 Comparison of approaches**

8.5. Robust against model errors

In the preceding paragraphs, it has been concluded that Taguchi’s loss function is not general applicable in civil engineering. Still, there are some aspects in Taguchi’s approach that may be of use to enhance the standard design approach. Taguchi for instance states that the performance function should be relative insensitive to noise factors. In the standard approach, these noise factors, or uncertain parameters, are described statistically. From this statistical distribution an overall risk can be calculated that is used for the economic optimization. This approach is watertight if we assume that these statistical distributions are determined accurately. However, the past has shown that during the lifetime, trends and events can occur that have not been accounted for in the design phase. An example are the steel bridges in the Netherlands, that are typically designed for a lifetime of 100 to 120 years. In reality, many bridge decks already showed cracks after 30 years, partly as a result of the fast increasing traffic intensity over the bridges. This resulted in unexpected costs due to inspection, repair and maintenance.

Unexpected costs occur if the performance probability density function as assumed in the design phase is different from the actual probability density function (pdf). This may be the result of statistical errors or of a trend that has occurred during the lifetime. When using the example of the caramel candy, an error in the assumed temperature distribution, will lead to an error in the plasticity distribution. For example, consider that the actual mean temperature and the standard deviation of the temperature have a higher value than assumed in the design phase. The resulting probability density function of the plasticity is shown in Figure 40. In Figure 41, the resulting risk is shown. The actual risk is higher than expected in the design phase. This risk increase can be reduced if the shape of the plasticity distribution is less sensitive to the shape of the temperature distribution. The performance of the system, or in this case the chewability of the caramel candy, will then be robust against future changes.
FIGURE 40 LIMIT STATE LOSS FUNCTION WITH DIFFERENT PROBABILITY DENSITY FUNCTIONS OF THE PLASTICITY.

FIGURE 41 RISK AS A FUNCTION OF PLASTICITY FOR DIFFERENT PROBABILITY DENSITY FUNCTIONS OF THE PLASTICITY. THE TOTAL RISK IS RESPECTIVELY 0.18, 3.08 AND 1.43.
9. Conclusion part 2

Taguchi developed a method to design products that have a performance that is minimally sensitive to factors causing variability. When translating this to civil engineering problems, the performance is associated to the functional requirements of the system. For example, the performance of a dike is related to its capability to retain the water. The performance of a quay is related to its capability to moor ships. The factors that are causing variability in this performance are the uncertain conditions, such as wave height and water level, but also construction errors and the operational use of the structure. According to Taguchi the design variables should be chosen such that the performance is least influenced by these uncertain conditions. He states that any deviation in the performance will lead to quadratic proportional loss. Under this assumption, the lifetime losses can be reduced by reducing the standard deviation of the performance, that arises from the uncertain conditions.

Taguchi proposed an experimental method to find the set of design variables for which the standard deviation of the performance is minimal; the orthogonal array method. When using the orthogonal array method, only a limited number of experiments have to be done to find the optimal combination of design variables. In structural design, it is often not feasible to do experiments to predict the behaviour of the structure. Therefore, formulas are used to run simulations from which the value of the performance can be determined as a function of the design variables. The orthogonal array approach allows the designer to find an optimal design by only doing a small number of simulations. The drawback is that the probability of failure cannot be calculated using the orthogonal array approach. Also, the exact optimum cannot be defined, since only certain levels of the design variables are analysed. The orthogonal array approach is therefore especially useful for getting a first idea of the problem, but not for finding the exact values of the optimum design variables. Also, it is much more sophisticated to use a Monte Carlo analysis than to just do a small amount of simulations. Namely, when using a Monte Carlo analysis, the probability density function of the performance can be accurately defined. This makes it easier to find the design variables that correspond to a minimum in the standard deviation of the performance.

The reason that Taguchi proposed to minimize the standard deviation of the performance, is that he believed that the loss of a product increases quadratic with the deviation from the target performance. Minimizing the standard deviation of the performance then results in reduced lifetime losses. The quadratic loss function is however not realistic, since a loss function is case-specific and often not symmetric. Still, if the loss function is two-sided, minimizing the standard deviation may be useful to minimize the lifetime losses, or the risk.

An example of Taguchi’s approach on caramel candy is given in paragraph 8.4.1., in which it is shown that minimizing the standard deviation of the chewability is beneficial when using the quadratic loss function as well as when using the limit state loss function. However, in this example, no notion is given on the investment costs associated with the ‘robust’ recipe of the candy. In chapter 6, it was explained that, in the standard procedures, a system is generally designed according to an economic optimum. This economic optimum represents a minimum in the sum of the investment costs and the net present value of the risk. The loss function as proposed by Taguchi is said to represent the loss after shipping, which means that investment costs are not considered. It can be concluded that Taguchi’s approach to robust design may lead to uneconomic designs.
An economic optimization is based on finding the values of the design variables, for which the sum of the net present value of the investment and the risk is minimum. When using a full probabilistic approach this is done by implementing all uncertain parameters into the simulation, varying the design variables and selecting the solution that corresponds to the cost minimum. In this process, the standard deviation in the performance is not separately considered. The standard deviation may have any value, as long as the total costs are minimum. This approach makes sense if all uncertainty can be accounted for in the design phase. However, the past has shown that not everything can be accounted for in the design phase. An example is the increasing traffic intensity over bridges. Therefore, it may be beneficial if the performance of the system does not change tremendously if unpredicted events and trends occur. A system has to be robust against discrepancies between the model and the reality. In 8.5, it has been shown that it is possible to find a design for which the additional lifecycle costs are least, in case of a discrepancy between the assumed load distribution and the actual distribution. It is recommended that this benefit is weighed against the increase in costs.

The conclusions in this paragraph are based on a study on Taguchi’s approach, and a comparison with the standard procedures. However, many more different definitions of robustness and methods to assess robustness have been described in literature. These will be discussed in chapter 9 and 10. It will be investigated whether the systems designed using these methods, are indeed robust systems. The definition of a robust system that will be used is the one that is given by Taguchi:

\[
\{ "a product whose performance is minimally sensitive to factors causing variability (at the lowest possible cost)". \}
\]

In this expression, the product is the system that is to be designed. The performance is the ability of the system to maintain its functions. The factors causing variability are all uncertain parameters that describe the environment, the operational use and the construction errors. In this study, the focus is on uncertain environmental loads.
Conclusion

"An added value of robust design may be found in minimizing the additional lifecycle costs that arise due to a discrepancy between reality and the model."
10. Optimization theory

In paragraph 6.2, the economic optimization is introduced to find values of the design variables that correspond to a minimum in the lifecycle costs. Such an optimization is done if the shape and the topology of a system have been defined, but the dimensions still have to be found. In literature on robust design, much attention is given to optimization methods. Several optimization tasks are proposed that are said to result in a robust system design. Since optimization tasks play such an important role in the literature on robust design, in this chapter the theoretical background of optimization tasks will be described. A selection of the proposed optimization tasks for robust design is given in chapter 13, after the philosophy behind these tasks has been described in chapter 11.

10.1. General optimization task

In this paragraph, the properties of a general optimization task will be described. Following to this paragraph, the theory will be extended for use in problems with probabilistic and convex descriptions of uncertainty.

A general optimization task is described by the following mathematical model:

\[
\begin{align*}
\text{Find} & \quad x \\
\text{Optimize} & \quad f(x) \\
\text{Such that} & \quad h_l(x) \leq 0, \quad l = 1, ..., L
\end{align*}
\]

In which:

- \( x \) is the vector of design variables.
- \( f(x) \) is the objective function.
- \( h_l \) is the set of deterministic constraints.

This optimization task searches for the set of design variables for which the objective function is minimum, maximum or attains a target value. The resulting set of design variables should fulfil the constraints.

The objective function, or the performance function, is the function for which the design should be optimized. An example of an objective function is the mass of a structural element. The optimization step is often to minimize the mass. Other examples are minimization of the costs or maximization of strength.

10.2. Uncertain parameters

In structural engineering, the variable parameters in the design problem are not only the design variables. Namely, many uncertain parameters influence the problem, that cannot be described by a single deterministic value. These uncertain parameters have to be considered separately in the optimization task. The optimization task is then defined by:
Find $\mathbf{x}$

Optimize $f(\mathbf{x})$

Such that $g_i(\mathbf{x}, \mathbf{\theta}) \leq 0, \quad i = 1, \ldots, I$

$s_j(\mathbf{x}, \mathbf{\theta}) = 0, \quad j = 1, \ldots, J$

$h_l(\mathbf{x}) \leq 0, \quad l = 1, \ldots, L$

In which $\mathbf{\theta}$ is the vector of uncertain parameters. In this vector, all uncertain parameters are included. In general, uncertainties in a design process can be classified in objective uncertainties or subjective uncertainties, also called epistemic uncertainties. Objective uncertainties are of irreducible stochastic nature and are for example present in wind loading, wave loading, seismic loading and production tolerances. This uncertainty is often described by a probability distribution. Subjective uncertainties are related to a lack of information, this kind of uncertainty can in principle be reduced by obtaining extra information. Examples of subjective uncertainties are model errors and errors due to numerical approaches for finding a solution.

When a probabilistic approach is used to find a solution to a design problem, the values of all uncertain parameters are described by a probability distribution. The overall assumption in this method is that it is accurate to describe every uncertain parameter by a probability distribution, if the uncertainty in this distribution is taken into account. In paragraph 3.2, it is shown that the uncertainty of a probability distribution resulting from a Monte Carlo analysis, can be taken into account by setting an acceptable probability that the error has a certain value.

A probability distribution for the loading is always based on statistical data. When no statistical data are available, bounded uncertainty may be used. The approach in which upper and lower bounds for the value of the uncertain parameters is defined, is called interval analysis. The advantage of this approach is that the designer only has to define lower an upper bounds for the values of the parameter, instead of having to define the distribution functions. Ben-Haim and Elishakoff [14] suggested such a bounded-uncertainty approach for structural optimization. They have also shown that the domain of uncertainty is convex, for most practical problems. This simplifies the mathematical analysis to find the critical load case. The theory of convex sets for uncertainty, will be explained in the paragraph on convex optimization.

Another way to handle design problems in which no statistical data are available, is Bayesian updating. Bayesian updating is based on the fact that the uncertainty in the reliability of a system can be reduced by analysing observations. For example, consider a dike that is designed for a probability of overtopping of once in 10 year. If observations in the first year show 5 events of overtopping, the probability distribution that describes overtopping should be altered.
10.3. Optimization under a reliability constraint

In literature, it is often proposed to design a system under a reliability constraint. This means that one of the constraints in the optimization problem is a maximum failure probability. The optimization task is then defined by [15]:

\[
\begin{align*}
\text{Find} & \quad x \\
\text{Minimizing} & \quad f(x) \\
\text{Such that} & \quad P[g_i(x, \theta) \leq 0] \leq P_i & i = 1, 2, \ldots, p
\end{align*}
\]

In which:

- \( x \) is the vector of design variables.
- \( \theta \) is the vector of uncertain parameters.
- \( f(x) \) is the objective function.
- \( g_i \) is the set of inequality constraints.
- \( P_i \) is the allowed probability of failure of constraint \( g_i \).

In this optimization task, the constraint can also be expressed by a target reliability \( \beta_{\text{target}} \) so that the constraint becomes:

\[
\beta_i(x) \geq \beta_{i,\text{target}} & \quad i = 1, 2, \ldots, p
\]

In which the reliability index can be determined as described in paragraph 3.3. for every limit state function \( g_i(x, \theta) \).

In the economic optimization task as described in paragraph 6.2, the probability of exceedence of the limit states \( g_i \) is present in the objective function in the form of a risk. In the economic optimization, the failure probability is part of the optimization task and does not pose a constraint on the task. The optimization task as proposed in this paragraph is applicable if a failure probability has been prescribed. It should however be noted that such a prescribed failure probability is often determined from an economic optimization.

10.4. Convex optimization

Doing an optimization task may lead to complex mathematical problems, especially if the objective function or the uncertain parameters are not continuous. In the last two decades, tremendous progress is made in the field of optimization problems. Thanks to the developments in the area of convex programming, it is now possible to theoretically optimize complex real-world problems. Namely, the definition of a convex set is that it does not contain discontinuities. Another advantage of a convex optimization problem is that it is computationally tractable. This means that “the computational effort required to solve the problem with given accuracy ‘grows moderately’ with the dimensions of the problem and the required number of accuracy digits.” [16]
Most of the mathematical methods for robust design, that will be discussed in chapter 12, are based on convex optimization. Therefore, the basics of the concept of convex optimization will be explained in this paragraph.

10.4.1. Convex optimization task
A convex optimization task is no different from the general optimization task:

\[
\begin{align*}
\text{Find} \quad & x \\
\text{Optimize} \quad & f(x) \\
\text{Such that} \quad & g_i(x, \theta) \leq 0, \quad i = 1, \ldots, I \\
& s_j(x, \theta) = 0, \quad j = 1, \ldots, J \\
& h_l(x) \leq 0, \quad l = 1, \ldots, L
\end{align*}
\]

This problem is considered to be a convex optimization problem if:

- The domain of \( x \) is a closed convex subset of \( \mathbb{R}^n \)
- The domain of \( \theta \) is a closed convex subset of \( \mathbb{R}^m \)
- The objective function \( f(x) \) and the constraints are convex functions of \( x \) and \( \theta \)

In which \( \mathbb{R}^n \) is the finite-dimensional Euclidean space. The conditions for a convex optimization problem can only be understood with some knowledge on the mathematics of convex models. Therefore the essentials of the theory of convexity will be explained in the next paragraph.

10.4.2. Convexity
A set \( B \) is called convex, if for any two points \( x \) and \( y \) in \( B \), the straight line segment joining \( x \) and \( y \) lies completely inside \( B \). Consequently circles and triangles are convex, but stars are not. Intuitively, convex sets do not have holes or dips. In Figure 42, the definition of a convex set is visualized in a two-dimensional drawing.

![Figure 42 Convex set and non-convex set](image)

Convexity is a powerful concept in applied mathematics. The advantage of convex optimization can be explained by the fact that if a local minimum is found, it is always a global minimum. This can be proven mathematically but also follows from the intuitive assumption that no dips and holes are present in a convex set. Since this study will focus on the engineering applications of convex
optimization, the mathematics will not be discussed thoroughly. However, to be able to define a convex optimization problem for engineering purposes, it is important to have some knowledge on the mathematical definition of convex set and functions.

The mathematical definition of a convex set is that a set $C$ in $B$ is said to be convex if, for all $x$ and $y$ in $C$ and for all $t$ in $[0,1]$, the point $(1 - t) \cdot x + t \cdot y$ is within $C$. This is illustrated in Figure 42.

The mathematical definitions of a convex function is similar, and reads: A real valued function, defined on a convex set $X$ is called convex if, for any two points $x_1$ and $x_2$ and any $t$ in $[0,1]$: 
\[
 f(t \cdot x_1 + (1 - t) \cdot x_2) \leq t \cdot f(x_1) + (1 - t) \cdot f(x_2)
\]

The function is called strictly convex if:
\[
 f(t \cdot x_1 + (1 - t) \cdot x_2) < t \cdot f(x_1) + (1 - t) \cdot f(x_2)
\]

The definitions for convex functions are illustrated in Figure 43 and Figure 44, using examples of convex and strictly convex functions. The convex function is defined by $f(x) = 4 \cdot x^2 + 3 \cdot x + 5$ and the strictly convex function read $f(x) = x$. Both figures correspond to a value of $t$ of 0.5.

**Figure 43 Convex function, $t=0.5$**
It can be concluded that the uncertain parameters, the objective function and the constraints have to be defined according to these rules, so that the optimization problem is a convex optimization problem. This make it easier to solve the problem numerically using computer programming.

The convex set of a single parameter is given by:

\[ X = \{ x : x_{\text{min}} \leq x \leq x_{\text{max}} \} \]  \hspace{1cm} (45)

This expression represents an interval in one-dimensional space, bounded by \( x_{\text{min}} \) and \( x_{\text{max}} \).

In most design problems, more than one uncertain parameter needs to be accounted for. Consider for example two uncertain parameters, \( x \) and \( y \), in \( \mathbb{R}^2 \) space. The convex set can be expressed by:

\[ X = \{ (x, y) : x_{\text{min}} \leq x \leq x_{\text{max}}, \ y_{\text{min}} \leq y \leq y_{\text{max}} \} \]  \hspace{1cm} (46)

The objective functions and constraints have to be a convex function of the convex uncertain parameters. Generally, a function is convex if it is at least to the first power with respect to the uncertain parameters. For example, \( f(x, y) = x^2 + y^3 \) is convex but \( f(x, y) = x^{-2} + y^3 \) is not. Also, there exist a number of operations that preserve convexity. When such an operation is detected in a function, and the operation is performed on convex functions, the function is also convex.
10.4.3. Programming for convex optimization

In optimization problems for engineering purposes, the demand is often to minimize the objective function. Examples are minimization of the total mass, minimization of the maximum stress and minimization of the deflection. Therefore, convex optimization problems are often convex minimization problems. The advantages of a convex minimization problems are:

- If a local minimum exists, it is always a global minimum
- The minimum of a strictly convex function is unique.

These properties of a convex minimization problem highly reduce the computational effort required to solve the problem, since the computing algorithms do not have to account for the possibility that a minimum is a local minimum. The field of programming of convex minimization problems is called Convex Programming. Many algorithms are available for finding the minimum in a convex optimization task. The programming algorithm that is chosen, depends on the characteristics of the optimization problem.

10.5. Conclusion

An optimization task is aimed at finding the set of design variables that optimizes a certain objective function. The set of optimal design variables is only a solution if it satisfies the constraints in the optimization task. Optimization tasks can be performed to find the optimum set of design variables, under uncertain parameters. These uncertain parameters can be described by a probability distribution or by a convex set. If the uncertain parameters are described by a convex set and the objective function is also a convex set, the optimization task is computationally tractable. In chapter 12, convex optimization tasks for robust design as well as optimization tasks based on probabilistic uncertainty will be introduced. These optimization tasks are based on the philosophies for robust design as discussed in chapter 11.
11. Optimization methods for robust design

In this chapter, several definitions of robustness will be discussed that are found in literature. Most definitions are based on the approach as proposed by Taguchi and therefore they are focussed on robust parameter design.

11.1. Definitions

Eric Sandgren [17] used Taguchi’s definition of robustness as a basis and illustrated the definition of a robust system with Figure 45. In this figure, the right vertical bound represents the performance constraint. Each of the three designs satisfies the requirement that the performance value remains below the limit with a certain probability. The first design has a wide distribution so that a low mean is necessary for the distribution to remain below the limit. The second design has the same mean, but a much tighter distribution so that the probability that the constraint value comes above the limit is much smaller. For a tight distribution, the constraint will also be fulfilled if the value of the mean is increased, as shown by design 3. Sandgren states that the aim of a calculation method for robust design would be to locate design 3, since it allows for wider variation in actual conditions. Also, design 3 is less sensitive to errors in the distributions functions that are used to determine the failure probability. In short, according to Sandgren, design 3 will have a smaller increase in the failure probability when the predictions turn out to be erroneous. However, it is much more likely that design 2 has a smaller increase in failure probability. Moreover, such a statement should be investigated in an analysis of the distribution function. Also, Sandgren did not take into account the investment costs of the designs, which will in practice highly influence the choice for a design. Moreover, the cost of a design will influence the acceptable failure probability, when following the economic optimization.

Figure 45 Robust design according to Sandgren [17]
In 2008, G.I. Schuëller [18] published an overview of computational methods for optimization under uncertainties. In the article he classifies three specific research areas; reliability based optimization, robust design optimization and model updating. He defines a robust design as a design that is:

"relatively invariant with respect to parameter changes, production tolerance, model sensitivities and other uncertain conditions".

Robust design optimization is favourable over deterministic design optimization, since the latter may result in optimal deterministic solutions that become unfeasible under large variability of the parameters of the system. In the article, Schuëller discusses several approaches for achieving robust designs, all based on optimization tasks. Examples are the robust counterpart approach and the multi-objective optimization approach. A selection of the approaches as described by Schuëller will be discussed in paragraph 9.2.

Doltsinis [19] states that:

"robust structural design aims rather to reduce the variability of structural performance caused by regular fluctuation than to avoid occurrence of catastrophe in extreme events".

This statement has been illustrated by Figure 46. In this figure it is shown that reliability is associated with extreme events and that robustness is associated with regular fluctuations around the mean performance. When applying this definition to a sea-dike, a regular fluctuation is the water level variation due to tides and swell and an extreme event is storm surge. The overall assumption in this article is that the structural performance of a system may be subject to large scatter during the service life-cycle. Such scatter may worsen the structural quality and cause deviations from the desired performance. This may result in additional lifecycle costs for maintenance, inspection and repair. A robust system will have less scatter which results in reduced lifecycle costs. From this definition of robust design it can be concluded that there is a potential for a robust design to be more cost-effective design. It should however be noted that the construction costs are likely to increase with robustness. Also the cost-effectiveness can only be quantified if the distribution of the scatter is known. In his article, Doltsinis gives several numerical examples. However, in none of these examples it is investigated whether the robust design is actually more cost efficient. Moreover, in none of the examples the graph in Figure 46 is used to explain how the multiobjective optimization changes the distribution function of the performance.
Fragiadakis [20] proposed robust design optimization as a method for performance-based earthquake engineering. He states that robust design optimization in structural engineering entails minimizing both the initial costs as well as the coefficient of variation of the response. This minimization needs to be done under the limit state constraints, without taking into account failure probabilities. The difference with a deterministic design optimization, where only the initial costs are minimized under the limit state constraints, is given in Figure 20. In Figure 20, represents the vector of random variables. The robust optimum will have a smaller deviation in the response function under random variation of the variables in . However, this comes at the tradeoff that the robust system does not have a minimum in the objective function.

An alternative definition of robustness, specifically for structural engineering, has been introduced by Yakov Ben-Haim [21]. He defines robust reliability as:

\[ \text{"the amount of parameter uncertainty consistent with no failure".} \]
This ‘amount of uncertainty’ relates to the boundaries of uncertainty around the nominal value of the uncertain parameter. These boundaries are defined by convex models for uncertainty. Ben-Haim proposed a calculation approach in which the amount of uncertainty that is consistent with no failure is described by a robustness function. The robustness function is to be maximized. This approach will be discussed in paragraph 9.2.

11.2. Optimization approaches

Different approaches for robust parameter design have been introduced in literature. In this chapter, two approaches for structural robust parameter design will be introduced, that are based on convex models for uncertainty. The reason for this is that robust design optimization is often linked to optimization under non-probabilistic uncertainty. This means that the uncertainty of the parameters is not described by a probability distribution. The use of a non-probabilistic approach has been defended by Ben-Haim [22]. He argued that in some cases, the inability to verify the details of the probability distribution of the uncertain parameters, will lead to significant inaccuracy in the reliability analysis. Examples of such cases are presented in the article by Ben-Haim[22]. In the state-of-the-art approaches, such uncertainty in the distribution functions is dealt with using Bayesian statistics. Ben-Haim however pleads to describe uncertainty without using distributions functions. He proposes to use convex models, in which the uncertainty in the parameters is described by an interval of values that the parameter can take. Convex models are known for their computational simplicity, since discontinuities are not allowed in these models. Ben-Haim therefore introduced convex modelling as a tool to deal with uncertainties in case there is a lack of information. However, convex modelling may also be useful as an easy alternative to deal with uncertainty, for example if the designer lacks knowledge to work with probability distribution functions. Since only information on the boundaries of the uncertain parameters is needed, the method has potential to become a quick and comprehensible tool for obtaining more optimized designs. The drawback of the use of non-probabilistic uncertainty is that valuable information on the uncertain parameters is neglected. Therefore, in this chapter, also a third optimization approach is discussed, that is based on a probabilistic description of uncertainty, namely ‘multiobjective optimization’.

11.2.1. Robust counterpart approach

11.2.1.1. Description

When designing a structure, the design variables are found by optimizing an objective function. Examples of an objective function are the weight, the displacement or the cost. When the input parameters are uncertain, an extra step is required to account for these uncertainties. In the robust counterpart approach, this step includes deriving a robust counterpart of the original objective function. The problem is then reduced to optimizing the robust counterpart function, in which all uncertainties are implemented. The advantage of this method is that uncertainties are incorporated in the design problem in the very beginning.

The way in which the uncertainty is taken into account is called the robustness measure. This robustness measure is determined by the definition of the robust counterpart. In this paragraph, a robustness measure is introduced that states that the design should be feasible in all situations. For a design problem with bounded uncertainty, given an objective function $f(x)$ to be minimized, the robust counterpart is defined as [15]:

$$f(x) = \min \{ f(x) : x \in [a,b] \}$$
\[ F_B(x; \epsilon) = \sup_{\xi \in X(x, \epsilon)} [f(\xi)] \]  

(47)

In which \( \sup \) denotes the supremum of the set of functions \( f(\xi) \). The supremum is defined as the smallest real number that is greater than or equal to every number in the set \( f(\xi) \). In this expression, \( \xi \) is a set of design variables that is present in the neighbourhood of nominal design variables \( x \). The neighbourhood is defined by the uncertain bounds, denoted by \( \epsilon \). The robust counterpart for bounded uncertainty can now also be denoted as:

\[ F_B(x; \epsilon) = \max_{\xi} \{ f(\xi) | x - \epsilon \leq \xi \leq x + \epsilon \} \]  

(48)

The optimization task is defined as:

\[
\begin{align*}
\text{Minimize} & \quad F_B(x; \epsilon) \\
\text{Such that} & \quad g_i(x; \epsilon) \geq 0, \ i = 1, ..., n_g
\end{align*}
\]

(49)

In which \( g_i \) are the constraint functions. This expression is true for an objective function that has to be minimized. In short, the robust counterpart is the function that gives the maximum possible value of the objective function \( f \) for a given set of design variables \( x \). The maximum possible value is the worst value, determined by the worst combination of the uncertain parameters. The optimum design under uncertainty can be determined by minimizing the robust counterpart, given the constraints. This design will be feasible, even if the uncertain parameters are at their most extreme unfavourable values. In the example, this approach is used to design a column under axial loading.

The robust counterpart approach can be illustrated by considering a column with axial loading. The axial loading poses a compressive stress on the column. This stress can be reduced by increasing the cross-sectional area of the column. The goal of this example is to define the amount of axial loading under which the maximum stress is not exceeded. The uncertainty in the amount of axial loading that is applied is 10%. This problem can be defined mathematically by:

\[
\begin{align*}
f(\xi) &= \xi / A & \text{axial stress} \\
P - \epsilon &\leq \xi \leq P + \epsilon & \text{neighbourhood of } P \\
\frac{P}{A} &< s_{\text{max}} & \text{constraint}
\end{align*}
\]

Following the definition by Beyer [15], the robust counterpart is:

Concerning the maximum stress constraint, the maximum allowable value for \( P \) is \( A \cdot s_{\text{max}} - \epsilon \).

Result: \( P \leq A \cdot s_{\text{max}} - \epsilon \)

In the example, the robust counterpart function is linear, so that no minimum can be found and a constraint has to be used to find a value for the optimal value of the load. In design problems where the robust counterpart function is not linear, a minimization can be employed that is similar to minimizing the original objective function. In general, the optimum set of design variables will then depend on the uncertain bounds as denoted by \( \epsilon \). It should be noted that taking \( \epsilon \) too large may result in robust solutions with poor performance, because the optimum design is based on situations
that have only a very small chance of occurring. In such cases, it is better to consider robustness measures based on probability.

A more general expression for the optimization task using the robust counterpart is (rewritten from [16]):

Find \( x \)

Minimizing \( f(x, \theta) \)

Such that \( g_i(x, \theta) \geq 0, \; i = 1, \ldots, n_g \), \( \forall \theta \in C_\theta \)

In which \( \forall \theta \in C_\theta \) means that the constraints have to be fulfilled for all sets of uncertain parameters in the bounded region \( C_\theta \). In other words, a set of design variables has to be found that minimize the objective function and for which the constraints are always satisfied, given a set of uncertain parameters. This definition will be used in the case studies.

A more generally used term that can be used for this approach is the ‘minmax’ approach. A minmax optimization, maximizes the objective function for one set of variables, and minimizes it for a different set of variables. This is exactly the robust counterpart approach as discussed in this section. Moreover, the robust counterpart approach is also often called the optimization-with-anti-optimization approach. Herein, the anti-optimization step searches for values of the uncertain parameters for which the objective function is maximized. The optimization step entails finding design variables for which the objective function is minimized, given the uncertain parameters. The minmax optimization approach and optimization with anti-optimization will not be separately discussed in this study.

11.2.1.2. Discussion

The robust counterpart approach is based on finding the design that has feasible performance under all values of the uncertain parameters. This approach is considered to result in robust systems since the system’s performance is insensitive to all uncertainty. The strategy is shown in Figure 48.
In Figure 48, the performance function is the limit state function, and the performance is considered to be feasible if no failure occurs. This approach is comparable to the standard approach, in the sense that the limit state forms the design constraint. The difference is that the standard approach uses probabilistic uncertainty, which makes it possible to demand a probability that failure occurs instead of demanding that failure will never occur. Since in the standard approach more information is used and because a cost optimization is used, it is a more sophisticated approach. The robust counterpart approach does not use statistical data, so that the designed system may be adapted to a set of uncertain parameters that has a very small probability of occurring. Since costs are not taken into account, this may lead to a design that is highly cost-inefficient.

The link between the robust counterpart approach and the methodology as proposed by Taguchi is not obvious. Taguchi’s approach focusses on minimizing the losses by making sure that a product will have target performance under all uncertain parameters. The robust counterpart is aimed at making a design that will violate the constraints under all uncertain parameters, however cost-efficiency is not taken into account in this approach. Moreover, the robust counterpart approach is likely to result in very expensive designs, that can resist even the most extreme and improbable loads.

However, we have stated earlier, that other than stress or displacement, cost can also be used as an objective function. The robust counterpart approach can then be used to find the design that is the least expensive, under all values of the uncertain parameters. However such an approach poses many problems on the designer. At first, the cost function as defined in paragraph 6.2 is dependent on the probability of failure. However, the robust counterpart approach is a non-probabilistic method which means that the probability of failure cannot be determined. An option is to eliminate the probability of failure from the costs function, which leaves only the investment costs. These will
increase with increasing design variables, which means that a minimum is not possible. Moreover, any uncertainty in the load parameters will not have any influence on the investment costs. The uncertainty in the load parameters has to come into the calculation by a constraint function. This constraint function will pose a minimum on the values of the design variables. This set of design variables represents the minimum in the costs, under the constraints. Consequently, the values of the design variables are entirely determined by the constraints or the limit state functions, as already shown in Figure 48.

11.2.2. Robustness function

11.2.2.1. Description

The robustness function is introduced by Ben-Haim [21] and is substantially different from the robust counterpart approach. Ben-Haim describes the robustness function as “the greatest horizon of info-gap uncertainty within which the performance is guaranteed to meet the aspirations”. In which info-gap is defined by: “Info-gap uncertainty is ignorance or incomplete understanding of the systems and processes being optimized.” Summarized, robustness is defined as the largest uncertainty for which a design is still feasible.

According to Ben-Haim, the info-gap model for the designer’s uncertainty about uncertain parameter \( \theta \) is described by:

\[
\mathcal{U}(\alpha, \theta) = \{ \theta: |\theta - \tilde{\theta}| \leq \alpha, \quad \alpha \geq 0 \}
\]  

(51)

In which \( \mathcal{U}(\alpha, \theta) \) is the set that contains all values that \( \theta \) can obtain. Parameter \( \tilde{\theta} \) is the nominal estimate of \( \theta \) and \( \alpha \) is the horizon of uncertainty in \( \theta \). In the info-gap model, the horizon of uncertainty \( \alpha \) is the unknown parameter. This means that \( \mathcal{U}(\alpha, \theta) \) is not a bounded set, so no worst case can be identified as in the robust counterpart approach. This feature distinguishes the robustness function theory from the robust counterpart approach. A general expression for the robustness function is:

\[
\hat{\alpha}(x) = \max\{\alpha: \min_{\phi \in \mathcal{U}(\alpha, \tilde{\theta})} g(x, \theta) \geq 0\}
\]

(52)

In which \( g(x, \theta) \) is the design requirement, generally expressed by a limit state function. From expression (36) it can be concluded that \( \hat{\alpha}(x) \) is the maximum value of the uncertainty in the load, for which the limit state is just satisfied. Maximizing \( \alpha \) results in a system that fulfils the limit state, even under large variability in uncertain parameters. However, this high reliability comes at the cost that the overall performance is not optimal. Consequently, there is a trade-off between the performance and the robustness.

An example was given by Ben-Haim [21]. Consider a cantilever beam under uniform load density \( \phi \), as illustrated in Figure 49. The cross-section of the beam is rectangular and the width is uniform along the length. The design variable is the thickness profile \( T(x) \) of the beam in the load plane. The aim of the designer is to choose the thickness profile that minimizes the mass of the beam and simultaneously minimizes the maximum stress in the beam. The performance is good if the beam has a minimum mass and a minimum stress, since this will minimize the cost and maximize the safety. In
this example it will be illustrated that maximizing the performance corresponds to minimizing the robustness. The first step in the analysis is to do a performance optimization.

![Figure 49 Cantilever beam under uniform load](image)

**Figure 49 Cantilever beam under uniform load**

The performance can be maximized by fulfilling two design criteria. The first design criterion is to minimize the mass $\theta$ of the beam, that is dependent on $T(x)$. The second design criterion is to minimize the maximum value of the absolute bending stress in the beam. These design criteria are in conflict, since decreasing the mass for equal uniform load requires decreasing $T(x)$. Decreasing $T(x)$ will cause an increase in the maximum absolute stress, which conflicts the requirement of stress minimization.

In Figure 50, the trade-off between the minimum stress maximum and minimum mass is given. Each point on the curve represents an equally optimal thickness profile $T_m$ that has an optimal combination of minimal stress maximum ($T_m$) and minimum mass $m$. The expression of the curve is found by an analysis of the thickness profiles that have a minimum stress maximum at a minimal mass. The resulting expression is:

$$R(T_\theta) = \frac{3 \cdot \phi \cdot L^4}{4 \cdot w \cdot m^2}$$

In Figure 50, point P represents an optimal design. Since a decrease in the maximal stress corresponds to an increase in the mass, point P is Pareto optimal and the curve is called a Pareto-optimal curve. Point R and point Q represents sub-optimal designs for the thickness profile. Sub-optimal designs can be explained as being mass-or stress-excessive. Mass-excessive means that for the maximum stress level, a thickness profile is possible that corresponds to a lower mass. Stress-excessive means that for the mass of the beam, a thickness profile is possible that corresponds to a lower maximum stress.

![Figure 50 Pareto optimal curve: $\theta$ is mass parameter, $R(T_\theta)$ is maximal stress](image)
Now the performance optimization step has been explained, the same problem is considered under an uncertain load profile. The info-gap model for the designer’s uncertainty about uncertain parameter $\phi$ is described by:

$$\mathcal{U}(\alpha, \phi) = \{ \phi : |\phi - \bar{\phi}| \leq \alpha \}, \quad \alpha \geq 0$$  \hfill (53)

Using this expression, the stress corresponding to thickness profile $T_\theta$ can be expressed as follows:

$$R(T_\theta) = \frac{3 \cdot (\bar{\phi} + \alpha) \cdot L^4}{4 \cdot w \cdot m^2} \leq s_c$$

Now the robustness function can be defined as:

$$\alpha(T_\theta, s_c) = \frac{4 \cdot w \cdot \theta^2 \cdot \sigma_c}{3 \cdot L^4} - \bar{\phi}$$

If a Pareto optimal design is chosen, the following expression is valid:

$$R(T_\theta) = s_c = \frac{3 \cdot \bar{\phi} \cdot L^4}{4 \cdot w \cdot \theta^2} \rightarrow \alpha = 0$$

Thus, for a Pareto optimal design, the robustness function is equal to zero. This means that performance-optimization results in zero robustness. If the maximum stress-requirement $s_c$ is increased for constant mass $m$, $\alpha$ will be larger than zero. In other words, choosing a suboptimal design will increase the robustness. In Figure 51, it is illustrated that robustness decreases as the performance-requirement becomes more stringent.

![Robustness function versus maximum stress requirement](image)

**Figure 51 Robustness function versus maximum stress requirement [21]**

It can be concluded that optimizing a design always leads to the least robust solution. Therefore, Ben-Haim [21] strongly recommended the designer to “satisfice the performance and maximize the robustness”.

A more comprehensible example of the robustness function, applied to the design of a two-bar truss, was given by Kanno and Takewaki [23].
Consider the two-degree of freedom truss shown in Figure 52. The uncertain parameters are forces $f_1$ and $f_2$, the design variables are the cross-sectional areas of bar one and bar 2, denoted by $a_1$ and $a_2$.

![Figure 52 Truss with two degrees of freedom](image)

The stresses in the bars can be expressed by:

$$s_1 = \frac{f_1 + f_2}{A_1}$$

$$s_2 = \frac{f_2}{A_2}$$

The stress constraint is given by:

$$|s_i| \leq s^c \quad (i = 1,2)$$

The info-gap model for $f_1$ and $f_2$ are given by:

$$f_1 = f_2 = \bar{f} + \alpha, \quad \alpha \geq |\zeta|, \quad \alpha \geq 0$$

The robustness function can now be expressed by:

$$\hat{a}(A_i) = \max\{\alpha: |s_i| \leq s^c\}$$

Which can be transcribed to:

$$\hat{a}(A_1, s^c) = \{\alpha: \frac{2(\bar{f} + \alpha)}{A_1} = s^c\}$$

$$\hat{a}(A_2, s^c) = \{\alpha: \frac{\bar{f} + \alpha}{A_2} = s^c\}$$

From this, it can be obtained that:

$$\hat{a}(A_1, s^c) = \frac{s^c \cdot A_1}{2} - \bar{f}$$

$$\hat{a}(A_2, s^c) = s^c \cdot A_2 - \bar{f}$$

When taking the following values:

$$s^c = 300 \frac{N}{mm^2}$$

$$A_1 = A_2 = 125 \ mm^2$$

$$\bar{f} = 10 \cdot 10^3 \ N$$
The robustness is:
\[ \hat{a}(A_1, s^c) = 8750 \text{ N} \]
\[ \hat{a}(A_2, s^c) = 27500 \text{ N} \]

When assuming that a structure is as robust as its least robust member, the robustness is 8750 N. This means that the worst case corresponds to \( f_1 = f_2 = 18,75 \cdot 10^3 \text{ N} \).

This example does not consider a trade-off, it can however easily be concluded that there is a trade-off between robustness and the maximum weight of the structure. Reducing the weight corresponds to reducing the cross-sectional areas \( A_1 \) and \( A_2 \), which will lead to a smaller value of the robustness. It can be concluded that a true example of the robustness function requires a second performance requirement, namely minimizing the weight. For the numerical example, minimizing the weight can already be achieved by reducing the robustness of bar 2 so that it is equal to the robustness of bar 1. This results in the following value for \( A_2 \):
\[ \hat{a}(A_2, s^c) = s^c \cdot A_2 - \hat{f} = 8750 \text{ N} \rightarrow A_2 = 62.5 \text{ mm}^2 \]

This corresponds to an minimum in the weight under a robustness of 8750 N and is therefore a Pareto optimal solution. The set of Pareto optimal solutions is given in Figure 53.

![Pareto optimal curve for the cross-sectional areas of a 2-dof truss](image)
under any increase in the stress, this beam is not robust. When the designer decides that the beam should be more robust, this is traded off against a minimum in the mass. The beam is made heavier in order to be more robust.

The same trade-off can be found when performing an economic optimization, where reliability is traded against investment cost. By representing reliability as a monetary value, this trade-off can easily be described by a minimization problem. The best combination of investment cost and reliability corresponds to a minimum in the net present value.

It can be concluded that there is no actual difference in the method as proposed by Ben-Haim and the standard approach. What is presented as ‘robustness’ by Ben-Haim, is actually just the ability to resist increasing loads. In the standard approach this ability is defined as ‘reliability’. Ben-Haim also does not use probability distributions to describe uncertainty, which means that a whole lot of knowledge on the behaviour of a certain system is ignored. He state that robustness equals the amount of uncertainty that a system can resist without failure, however he ignores the fact that much is already known about the amount of uncertainty that the system will actually have to resist.

The conclusion that Ben-Haim’s approach to robustness closely resembles the standard approach, leaves the question if his approach has anything to do with Taguchi’s methodology. The answer to this question is that there is no link whatsoever between the two. The reason for this is that the loss function and the wish for target performance do not apply to the problems as investigated by Ben-Haim. Also, the literature by Ben-Haim does certainly not focus on cost-efficiency but rather on spending money to make systems more reliable under unexpected events.

11.2.3. Multiobjective optimization

11.2.3.1. Description

As stated in 8.5, the definition of a robust design is that the performance is relatively invariant with respect to parameter changes, production tolerance, model sensitivities and other uncertain conditions. This means that the design variables should be chosen such that the variability of the performance is minimized. However, the initial objective of optimizing the expected value of the performance should not be neglected. This results in an optimization problem with two objectives, namely optimization of the expected value and minimization of the variance of the performance. Such a problem can be solved by choosing one objective to optimize and incorporating the other objective as constraints. This approach does however limit the choices of the design, making it a difficult optimization task. The optimization task can also be solved by combining all objectives in one objective function, for example by adding them. The drawback of this approach is that the relative importance of the objectives is not taking into account, so that a further sensitivity analysis of the solutions is necessary. In an article by Coello [24], a number of methods are described that deal with multiobjective optimization problems. Coello describes the multiobjective optimization problem as follows (rewritten from [24]):
Find \( x \)

Optimizing \( \vec{f}(x) = [f_1(x), f_2(x), \ldots, f_k(x)] \)

Such that \( g_i(x) \leq 0, \quad i = 1, \ldots, I \)
\( s_j(x) = 0, \quad j = 1, \ldots, J \)

The greatest challenge in multiobjective optimization is that the meaning of the ‘optimum solution’ is not well defined. The reason for this is that there rarely exists a situation in which all objectives have their minimum at the same vector of design variables. Consequently, the problem is a Pareto optimal problem, which means that making one objective more stringent, releases the other objective. Pareto optimality can be illustrated by the short blanket problem; pulling up the blanket to cover your chest, will leave your feet uncovered. A solution to this problem is to search for a vector of design variables that is Pareto optimum. This is the case if there is no other feasible vector of design variables, which would decrease an objective without simultaneously increasing another objective. For most problems, a set of equally Pareto optimal solutions is possible.

An alternative to the Pareto optimum is the min-max optimum. This optimum is found by comparing relative deviations of the objective functions from their individual minima. In other words, first every objective is analysed separately to find the minimum. Then the optimal vector of design variables is searched for, that minimizes the sum of the deviations of the objective functions from their individual minima. Other alternatives to find an optimum are for example to use a weighted sum of the objectives or to rank the objectives according to their importance.

In many studies, the genetic algorithm has been introduced to perform the multiobjective optimization task [24]. Different forms of the genetic algorithm have been described that all define the optimum solution differently. The aim of these studies was to find an algorithm that results in suitable solutions.

The multiobjective optimization approach is a useful tool when searching for robust designs, since the expected value as well as the variance of the performance can be optimized at once. An appealing definition of general multiobjective optimization for robust design is introduced by Doltsinis et al. [19].

Find \( x \)

Minimizing \( \{E(f(x, \theta), \sigma(f(x, \theta))\} \)

Such that \( E(g_i(x, \theta)) + \beta_i \cdot \sigma(g_i(x, \theta)) \leq 0 \quad (i = 1, 2, \ldots, k) \)
\( \sigma(h_j(x, \theta)) \leq \sigma_j^+ \quad (j = 1, 2, \ldots, l) \)
\( x_a^- \leq x_a \leq x_a^+ \quad (a = 1, 2, \ldots, n) \)

In which:
\( x \) is the set of design variables \( x_a \) with \( a = 1, 2, \ldots, n \)
\( \theta \) is the set of uncertain parameters
\[ f(x, \theta) \] is the objective function

\[ g_i(x, \theta) \] is the set of reliability constraints

\[ \beta_i \] is a prescribed reliability index for the \( i^{th} \) constraint

\[ h_j(x, \theta) \] represents the structural performances with constraints on the standard deviations

The probability that constraint \( g_i \) is fulfilled can be prescribed by choosing a value for the reliability index \( \beta_i \).

As explained above, a multiobjective optimization problem cannot be solved straightforwardly. Doltsinis [19], used a weighted sum approach by introducing a weighting factor \( \omega \) for the tradeoff between minimizing the mean performance and its standard deviation.

Find \( x \)

Minimizing \( (1 - \omega) \cdot \frac{E(f(x, \theta)) - \mu^*}{\mu^*} + \omega \cdot \frac{\sigma(f(x, \theta))}{\sigma^*} \)

Such that \( E(g_i(x, \theta)) + \beta_i \cdot \sigma(g_i(x, \theta)) \leq 0 \) \( (i = 1,2, \ldots, k) \)

\[ \sigma(h_j(x, \theta)) \leq \sigma_j^+ \] \( (j = 1,2, \ldots, l) \)

\[ x_a^- \leq x_a \leq x_a^+ \] \( (a = 1,2, \ldots, n) \)

\[ 0 \leq \alpha \leq 1 \]

In this problem, all values of \( \alpha \) correspond to a Pareto optimal solution. The normalisation factors \( \mu^* \) and \( \sigma^* \) are introduced to take into account the difference in the absolute value of the mean and the standard deviation of \( f \). This weighted sum optimization method is more straightforward than the multiobjective optimization and will therefore be used in the case studies in chapter 12 to 14.

However, it is computationally intensive to find the mean value and the standard deviation of the objective function. The Monte Carlo method may be used if this is not computationally too expensive, otherwise an approximation is needed. Approximating the mean and standard deviation of \( f \) linearly by the first-order Taylor series is done as follows:

\[ E(f(x)) = f(\mu_x) \]

\[ \sigma^2(f(x)) = \sum_{a=1}^{n} \left( \frac{\partial f}{\partial x_a} \right)^2 |\mu_x| \cdot \sigma_a^2 \] \( \quad (57) \)

Another way of estimating the probability distribution of the objective function is by using a meta model, that creates an approximation of the deterministic response. The idea of a meta model is to generate a simpler model that describes the response sufficiently well. For simple case studies, the weighted sum optimization in combination with Monte Carlo Analysis or Taylor approximation will be used. For more complex case studies, it may be necessary to use a meta model.
The example of the column under axial loading can again be used to illustrate the concept of the weighted sum optimization in combination with a Taylor approximation. The Taylor approximation of the expected value and the standard deviation of the objective function are defined by:

\[ E(f(x)) = f(\mu_P) = \frac{\mu_P}{A} \]

The standard deviation is the square root of the variance and can be defined by:

\[ \sigma(f(x)) = \sqrt{\left( \frac{\partial f}{\partial P} \right)^2 \mid_{\mu_P} \cdot \sigma_P^2} = \sqrt{\frac{1}{A^2} \cdot \sigma_P^2} = \frac{\sigma_P}{A} \]

Now the optimization problem is as follows:

Find \[ P \]

Minimizing \[ (1 - \alpha) \cdot \frac{\mu_P}{A} + \alpha \cdot \frac{\sigma_P}{A} \]

Such that \[ E\left(\frac{P}{A} - s_{\text{max}}\right) + \beta_i \cdot \sigma \left(\frac{P}{A} - s_{\text{max}}\right) \leq 0 \quad (i = 1, 2, \ldots, k) \]

and \[ 0 \leq \alpha \leq 1 \]

The solution of this optimization problem can be found using for example a gradient based algorithm in Matlab. The reliability index \( \beta_i \) can be chosen according to a prescribed probability of failure, assuming that the constraint function is normally distributed.

11.2.3.2. Discussion

The multiobjective optimization can be performed on any two or more objectives. Doltsinis et al. [19] assumed that a robust design can be obtained by minimizing the expected value as well as the standard deviation of a certain objective function. This corresponds closely to Taguchi’s approach, in which it is stated that the performance should be at the target and with a minimized standard deviation. However, the applicability of this concept clearly depends on the definition of the performance and the losses. In Doltsinis et al. [19] it is stated that less scatter around the mean performance will reduce the lifecycle cost. However, no explanation is given as to how a smaller standard deviation of the performance will reduce lifecycle costs. This means that there is a difficulty quantifying the that is averted by robustness, so that it is impossible to determine what robustness may cost. In Taguchi’s approach, an empirical quality loss function is used to determine the losses as a function of the standard deviation in the performance. Such a loss function would make the multiobjective optimization approach better applicable to practical problems.

It can be concluded that the multiobjective optimization approach has high potential for designing robust systems. For a better practical applicability of the approach, it is important that a quality loss function is defined that describes the lifecycle cost as a function of the standard deviation of the performance. The investment costs of a robust system can then be justified by the averted lifecycle costs.
11.3. Discussion robust parameter design

11.3.1. Definitions

In this chapter, several definitions of robust design or robust systems have been found. These definitions will be compared with the definition of a robust product as given by Taguchi:

“A robust product is a product whose performance is minimally sensitive to factors causing variability (at the lowest possible cost).”

Sandgren [17] states that the aim of a calculation method for robust design would be to locate a design with a minimized standard deviation of the performance distribution function. Such a design would allow for wider variation in actual conditions and is less sensitive to errors in the distribution functions that are used to determine the failure probability.

This definition can be interpreted by Figure 35 and can therefore be considered to fit Taguchi’s definition. Figure 45 is confusing since it does not show that the distribution of the performance is influenced by the uncertain parameters, which is an essential aspect of this approach.

Schüeller [18] defines a robust design as a design that is:

“relatively invariant with respect to parameter changes, production tolerance, model sensitivities and other uncertain conditions”.

This definition agrees with Taguchi’s definition of robustness. In his article, Schüeller [18] summarizes the optimization approaches that can be used for designing a robust system. Among them are the robust counterpart approach and the multiobjective optimization approach. These approaches do however not necessarily fit this definition, as will be shown in the case studies.

Doltsinis [19] states that:

“robust structural design aims rather to reduce the variability of structural performance caused by regular fluctuation than to avoid occurrence of catastrophe in extreme events”.

This means that Doltsinis aims at reducing functional losses and not losses due to structural failure. With ‘factors causing variability’, Taguchi did not make any distinction between regular fluctuations and extreme events. Also, in the quadratic loss function, losses due to rework, delay, replacement, reparation inspection and so on are taken into account. This means that functional losses as well as losses due to structural failure are considered. Concluding, the distinction as made by Doltsinis has no background in Taguchi’s theory. Moreover, such a distinction is not necessary, since in the loss function, functional losses as well as losses due to structural failure can be included.
Yakov Ben-Haim [21] defines robust reliability as:

\[ \text{“the amount of parameter uncertainty consistent with no failure”} \]

This definition does not have any link with Taguchi’s theory and is for structural design problems equal to the reliability. Ben-Haim describes a tradeoff between robust reliability and performance, which is the same as the trade-off between reliability and investment costs in structural design problems.

With exception of the theory by Ben-Haim, these definitions all follow the idea of Taguchi that the variability of a certain ‘performance function’ due to the uncertain parameters, should be minimized. In chapter 8, it has already been concluded that the performance is related to the functions of the system. Variability in the performance will lead to losses. The function that describes this loss has to be defined separately for every design problem. Losses may occur due to exceedence of ultimate limit states (structural failure) but also due to exceedence of serviceability limit states (functional losses). Doltsinis [19] clearly aims at reducing functional losses, while the other definitions may also apply to structural failure. All sorts of loss should be considered to do an accurate optimization.

11.3.2. Optimization approaches

In this paragraph, three optimization approaches to robust parameter design have been given; the robust counterpart approach, the robustness function approach and the multiobjective optimization approach.

The robust counterpart approach is aimed at minimizing a certain objective function, under the worst situation of the uncertain parameters. In paragraph 9.2.1. it has been concluded that in structural engineering, the investment cost should be chosen as the objective function. The limit states enter the optimization problem through the constraints. The robust counterpart approach is then aimed at minimizing the investment costs, under the constraint that no limit state is exceeded under any possible value of the uncertain parameters. These possible values are described by interval bounds for uncertainty. The resulting system will not exceed the limit states under any variation of the uncertain parameters and can therefore be considered as a robust approach. The case studies will be used to find out if the approach does have any added value compared to a full probabilistic economic optimization.

The robustness function was proposed by Ben-Haim. It has however already been concluded that Ben-Haim’s interpretation of robustness has no added value for structural design purposes. The case studies will be used to proof this statement.

Multiobjective optimization approaches can be used to simultaneously minimize two objectives. Several suggestions have been done as to which objectives to use for robust design. Doltsinis proposed minimizing the investment costs and the variance of the performance function. Fragiadakis proposed to minimize the expected value and the variance of the performance function. Both alternatives will be tested in the case studies. It is expected that a multiobjective optimization will always result in a single objective optimization, namely minimizing the lifecycle costs.

In the following chapters, the optimization approaches as discussed in this chapter, will be applied in case studies. The results of these case studies will be used to check if the conclusion as drawn in this
paragraph are true. The case studies will first be solved according to the limit state design approach and an economic optimization, as described in chapters 1 to 7. Following, the optimization approaches as discussed in this chapter will be applied. The results will be compared. It should be noted that the case studies all have one limit state function, which means that the loss function exists of a single step at the limit state.
12. Case Study Column under axial loading
12.1. Introduction
In this case study, the applicability of robust design to a column under axial loading will be investigated. The design variable is the cross-sectional area of the column, which is a random variable. The column is subject to an axial loading which is also a random variable.

First the optimal cross-sectional area of the column will be designed using the standard level III and level I approaches. Following, the robust counterpart approach and the robustness function will be applied to the design problem, to investigate if they are indeed comparable to the level I approach. The second part of the case study will be based on the robust design of the column.

12.2. Design problem
Consider a simple limit state function of a column with cross-sectional area $A$ [$mm^2$] under axial loading $S$ [$N$] and with a compressive strength $s_0$ [$\frac{N}{mm^2}$].

$$Z = s_0 - \frac{S}{A} \quad s_0 = 300 \frac{N}{mm^2}$$

Failure will occur if the loading exceeds the strength, so failure occurs if $Z < 0$.

Now assume the parameter distributions as given in Table 9.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Normal</td>
<td>$\mu_A$ [$mm^2$]</td>
<td>$0.1 \cdot \mu_A$ [$mm^2$]</td>
</tr>
<tr>
<td>$S$</td>
<td>Normal</td>
<td>150 [$kN$]</td>
<td>$0.1 \cdot 150$ [$kN$]</td>
</tr>
</tbody>
</table>

**Table 9** Distribution functions of uncertain parameters

The cost function is defined as:

$$C = I_0 + I_A \cdot A + \frac{p_f \cdot D}{r' - g}$$

In which:

$I_0 = 5$ [$€$]

$I_A = 0.05$ [$€$]

$D = 30$ [$€$]

$r' - g = 0.015$ [-]

The required nominal value of the strength of the element can be determined using the calculation approaches as introduced in the literature study.
12.3. Standard approach: Level III
The optimization task is:

Find \( A \)

Minimizing \( C = l_0 + I_A \cdot A + \frac{p_{TD}}{f - g} \)

A Monte Carlo simulation is performed to find the failure probability \( P_f \) as a function of \( A \). The minimum in the cost is €18,0 and corresponds to a target cross-sectional area of 800 mm\(^2\).

Result:

\[ \mu_A = 800 \quad [\text{mm}^2] \]
\[ P_f = 5,6 \cdot 10^{-4} \quad [-] \]
\[ C = 46,5 \quad [\text{€}] \]

12.4. Standard approach: Level I
In a level I calculation, the required characteristic value of the strength can be calculated using the following formulas:

\[ s_o > \frac{s_d}{A_d} \quad \left[ \frac{N}{\text{mm}^2} \right] \]
\[ A_d = \frac{A_k}{\gamma_M} \quad [\text{mm}^2] \]
\[ S_d = \gamma_S \cdot S_k \quad [N] \]

The partial safety factors \( \gamma_M \) and \( \gamma_S \) are unknown since they are respectively dependent on the manufacturing process and on the characteristics of the load. Assume an ultimate limit state and an unfavourable and permanent load \( S \):

\[ \gamma_S = \gamma_{G,j} = 1,10 \]

For the material factor, a value of 1,10 is taken.

\[ \gamma_M = 1,10 \]

The characteristic value of the load is an upper bound that has a probability of non-exceedence of 95%:

\[ S_k = S_{95\%} \rightarrow \Phi(U) = 0,95 \rightarrow U = 1,65 = \frac{S_k - \mu_S}{s_S} \rightarrow S_k = 1,65 \cdot 0,1 \cdot 150 + 150 = 174,75 \text{ kN} \]

Now the required characteristic value of the cross-sectional area \( A \) can be calculated:

\[ A_k = \gamma_M \cdot A_d = \gamma_M \cdot \frac{\gamma_S S_k}{s_o} = 1,10 \cdot \frac{1,10 \cdot 174,75 \cdot 10^3}{300} = 704,83 \text{ mm}^2 \]

And the characteristic value of the strength is a lower bound that is considered to have a probability of non-exceedence of 5%:

\[ A_k = A_{5\%} \rightarrow \Phi(U) = 0,05 \rightarrow U = -1,65 = \frac{A_k - \mu_A}{s_A} \rightarrow \mu_A = A_k + 1,65 \cdot s_A = 844 \text{ mm}^2 \]
Result: \[ \mu_A = 844 \text{ mm}^2 \]

This result corresponds to a probability of failure of \( 0.9 \cdot 10^{-4} \) and a cost of 18,4, when evaluating using a Monte Carlo analysis.

Result: \[ \mu_A = 844 \quad [\text{mm}^2] \]
\[ P_f = 2.2 \cdot 10^{-4} \quad [-] \]
\[ C = 47.6 \quad [\text{e}] \]

12.5. Robust counterpart approach

The robust counterpart approach optimization task is expressed as follows:

Find \[ x \] (59)

Minimizing \[ f(x, \theta) \]

Such that \[ g_i(x, \theta) \geq 0, \quad i = 1, \ldots, n_g \quad \forall \theta \in \mathcal{C}_\theta \]

For the column under axial loading, the objective function \( f(x, \theta) \) can be seen as the cost function. This function exists merely of the investment costs, since the probability of failure is not determined in this approach.

The cost function can be defined as:

Minimizing \[ f(x, \theta) = f(A) = C = I_0 + I_A \cdot A \]

There is one limit state function, namely the maximum stress constraint. This means that \( i = 1 \) and:

\[ g(x, \theta) = g(S, A) = \sigma_0 - \frac{S}{A} \]

The convex bounds of \( S \) and \( A \) are considered to be equal to three standard deviations.

\[ \mathcal{C}_S = \{ S: S_{\text{nom}} - 3 \cdot s_S \leq S \leq S_{\text{nom}} + 3 \cdot s_S \} \]
\[ \mathcal{C}_A = \{ A: A_{\text{nom}} - 3 \cdot s_A \leq A \leq A_{\text{nom}} + 3 \cdot s_A \} \]

The optimization task is now:

Find \[ A \] (60)

Minimizing \[ I_0 + I_A \cdot A \]

Such that \[ s_0 - \frac{S}{A} \geq 0, \quad \forall S \in \mathcal{C}_S, \quad \forall A \in \mathcal{C}_A \]

The values of \( S \) and \( A \) that corresponds to the worst case for the stress constraint, are:

\[ S = S_{\text{nom}} + 3 \cdot s_S \]
\[ A = A_{\text{nom}} - 3 \cdot s_A \]

The resulting optimization task is:
Find \( A \) \\
Minimizing \( I_0 + I_A \cdot A \) \\
Such that \( s_0 - \frac{s_{nom} + 3 \cdot s_S}{A_{nom} - 3 \cdot s_A} \geq 0 \)

The result is the smallest value of \( A \) for which the constraint is fulfilled. This value can be easily calculated.

\[
H = 0 \Rightarrow \frac{s_{nom} + 3 \cdot s_S}{A_{nom} - 3 \cdot s_A} \leq s_0 \Rightarrow A_{nom} \geq \frac{s_{nom} + 3 \cdot s_S}{s_0} + 3 \cdot s_A \Rightarrow A_{nom} \geq 929 \text{ mm}^2
\]

This result corresponds to a probability of failure of \([-\] and a cost of \([\text{€}]\), when evaluating using a Monte Carlo analysis.

Result: 
\[
\mu_A = 929 \quad [\text{mm}^2] \\
P_f = 0.2 \cdot 10^{-4} \quad [-] \\
C = 51.5 \quad [\text{€}]
\]

12.6. Robustness function

The expression of the robustness function is:

\[
\hat{a}(A, S) = \max\{ \alpha : \min_{\theta \in \mathcal{U}(A, S)} g(x, \theta) \geq 0 \}
\]

When applied to the limit state function, this robustness function can be denoted as:

\[
\hat{a}(A, S) = \max\{ \alpha : \min_{(A, S) \in \mathcal{U}(A, S)} (s_0 - \frac{S}{A}) \geq 0 \}
\]

In which:

\[
\mathcal{U}(A, S) = \{ A : |A - A_{nom}| \leq \alpha_A, S : |S - S_{nom}| \leq \alpha_S \}, \quad \alpha_A, \alpha_S \geq 0
\]

Then:

\[
\hat{a}(A, S) = \max\{ \alpha : \left( s_0 - \frac{s_{nom} + \alpha_S}{A_{nom} - \alpha_A} \right) \geq 0 \}
\]

Now there are three unknowns, namely the value for \( A_{nom} \) and the values for \( \alpha \). The amount of uncertainty that \( S \) and \( A \) can have at the level III design can be calculated by taking \( A_{nom} = 800 \text{ mm}^2 \). The values \( \alpha_S \) of \( \alpha_A \) depend on each other. This dependency is shown in Figure 54.
From this figure it can be concluded that for example for an uncertain interval of $0,20 \cdot 150 = 30 \text{kN}$ around the nominal load, the uncertain interval around the cross-sectional area may be $0,25 \cdot 800 = 200 \text{mm}^2$.

### 12.7. Multiobjective optimization

#### 12.7.1. Investment costs and variance of response

Following Fragiadakis [20], robust design entails simultaneously minimizing the investment costs and the coefficient of variation of the response. The latter is expected to result in a minimization of the lifecycle costs. The response function can in this case be defined as the stress in the element. The optimization task for the column under axial loading, is as follows:

Find $\mathbf{A}$

Minimizing $[I_0 + I_A \cdot A, \ \sigma(S/A)]$

Such that $E \left( \frac{s}{A} - s_0 \right) + \beta \cdot \sigma \left( \frac{s}{A} - s_0 \right) \leq 0$

The multiobjective optimization task will be solved using the weighted sum approach. The objectives in this task can however not be directly added, since the first term represents cost while the second term represents the deviation in the stress. Ideally, the stress deviation should be translated into a monetary value. It is however not known how exactly the deviation in the stress will cause lifecycle costs. Therefore, in this case study, the objectives are weighted without considering the unities.
Find $A$

Minimizing $(1 - \alpha) \cdot (I_o + I_A \cdot A) + \alpha \cdot (\alpha(S/A))$

Such that $E \left( \frac{S}{A} - s_0 \right) + \beta \cdot \sigma \left( \frac{S}{A} - s_0 \right) \leq 0$

The result of this minimization task is the optimum cross-sectional area as a function of the weight factor $\alpha$, as shown in Figure 55. The reliability $\beta$ is chosen equal to 3.

**Figure 55** Optimum cross-sectional area for multiobjective optimization I

From Figure 55, it can be concluded that the optimum cross-sectional area is determined by the constraint for an $\alpha$ smaller than 0.5. For values of $\alpha$ that are larger than 0.5, the optimum cross-sectional area corresponds to a minimum in the weighted sum. The optimum increases exponentially with growing weight of the standard deviation. No statement can be made as to which value of $\alpha$ should be used. For this, the monetary value corresponding to the stress deviation should be known. An acceptable result is found for an $\alpha$ of 0.6, since this corresponds to an optimum cross-sectional area of 810 mm$^2$.

Result: $
\mu_A = 810 \quad [mm^2]$

$P_f = 5.9 \cdot 10^{-4} \quad [-]$  

$C = 46.7 \quad [\text{€}]$

12.7.2. Expected value and variance of response

The multiobjective optimization approach was proposed by Doltsinis [19] to simultaneously minimizes the expected value and the standard deviation of a performance function. Doltsinis
expected that minimizing the standard deviation of an arbitrary performance function would reduce the lifecycle cost. In the multiobjective optimization approach as proposed by Doltsinis, the reliability of the system is ensured by demanding a target reliability. The optimization task for the dike height problem is as follows:

Find \( x \)

Minimizing \( (1 - \alpha) \cdot \frac{E(f(x,y))}{\mu^2} + \alpha \cdot \frac{\sigma(f(x,y))}{\sigma^2} \)

Such that \( E(g) + \beta \cdot \sigma(g) \leq 0 \)

In which \( f(x, y) \) is the performance function that needs a minimized standard deviation, for reducing the lifecycle costs. For the column under axial loading, the stress in the column can be regarded as the performance function. The normalization factors \( \mu^* \) and \( \sigma^* \) are considered to be equal to 1. The optimization task is as follows:

Find \( A \)

Minimizing \( (1 - \alpha) \cdot E(S/A) + \alpha \cdot \sigma(S/A) \)

Such that \( E \left( s_0 - \frac{S}{A} \right) + \beta \cdot \sigma \left( s_0 - \frac{S}{A} \right) \leq 0 \)

Both norms in the function that has to be minimized, decrease with increasing \( A \). The constraint poses a minimum on the value of \( A \), namely 730 mm\(^2\). The result is an infinitely large value of \( A \). It can be concluded that this optimization task is of no use for finding an optimum value of the cross-sectional area. The reason for this is that \( E(S/A) \) as well as \( \sigma(S/A) \) decrease with increasing \( A \). A minimum can only be found if they show opposite trends under an increase of \( A \).

12.8. Discussion

This case study was aimed at finding the optimal cross-sectional area \( A \) of a column under axial loading. The result for the different approaches is shown in Table 19.

<table>
<thead>
<tr>
<th>Approach</th>
<th>( A ) [mm(^2)]</th>
<th>( P_1 ) [yr(^{-1})]</th>
<th>Cost [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level III economic optimization</td>
<td>800</td>
<td>5,6 ( \times 10^{-4} )</td>
<td>46,5</td>
</tr>
<tr>
<td>Level I</td>
<td>844</td>
<td>2,2 ( \times 10^{-4} )</td>
<td>47,6</td>
</tr>
<tr>
<td>Robust counterpart</td>
<td>929</td>
<td>0,2 ( \times 10^{-4} )</td>
<td>51,5</td>
</tr>
<tr>
<td>Multiobjective optimization I</td>
<td>810</td>
<td>5,9 ( \times 10^{-4} )</td>
<td>46,7</td>
</tr>
</tbody>
</table>

**Table 10** Results of designing column under axial loading with different approaches

**Standard approaches**

First \( A \) was determined using the standard approach, which is a full probabilistic cost minimization. The result was a cross-sectional area of 800 mm\(^2\). The level I approach resulted in a somewhat
greater value of $A$ of 844 $mm^2$, corresponding with lower failure probability but higher costs. This result agrees with the expectation that the level I approach gives a more conservative solution than a full probabilistic approach.

**Robust counterpart approach**

The first ‘robust’ approach that was applied, was the robust counterpart approach. In this approach, the uncertain parameters are described by an interval. The approach is aimed at minimizing the robust counterpart of a certain objective function, under the constraint that the limit states are not exceeded. The objective function was given by the investment costs. The single limit state function was the stress constraint. Following the robust counterpart approach, this constraint may not be exceeded, even under the worst values of the uncertain parameters. The resulting optimal cross-sectional area, is the smallest value of $A$ that satisfies this constraint. This means that this value is entirely determined by the limit state and by the interval bounds. The value of $A$ is found in the exact same way in the level I approach, where safety factors and critical values are used instead of interval bounds. Since the values of the safety factors are based on experience and on calibration, the level I approach is more sophisticated and therefore preferable over the robust counterpart approach.

The ‘robust’ counterpart approach does not minimize the losses under all deviations in the expected loading. It is possible that failure will occur immediately if the loading exceeds the boundaries that have been defined. Therefore, this approach does not lead to a ‘robust’ system as defined by Taguchi.

**Robustness function**

The robustness function approach is aimed at finding the convex bounds for which the limit state is exceeded. The difference with the level I approach and the robust counterpart approach, is that the input is the cross-sectional area and the output is the set of convex bounds. Ben-Haim [22] states that the value of these convex bounds represents robustness. This ‘robustness’ has been calculated for a cross-sectional area of 800 $mm^2$. The result was a graph that gives the acceptable convex bound of $S$ as a function of the convex bound of $A$. Relaxing the uncertainty in the loading, will make the uncertainty in the cross-sectional area more stringent. The convex bounds ($\alpha_S$ and $\alpha_{S'}$) have a value that is larger than zero, since uncertainty was taken into account to determine the $A$ of 800 $mm^2$. It can be concluded that in this case study, the robustness function just represents the acceptable uncertainty in the parameters. This acceptable uncertainty corresponds to a certain reliability. Except for the reversal of input and output, this approach is no different from the level I or the robust counterpart approach. Therefore, also this approach is not fit for finding a ‘robust’ system.

**Multiobjective optimization I**

A multiobjective optimization task exists of two or more objective functions that have to be minimized. The choice of these objective functions determines whether the optimization task will result in robust solutions. In this case study, two different multiobjective optimization tasks have been defined. One of them minimizes the investment costs, as well as the standard deviation of a certain performance function, in this case the stress. This minimization is done under a reliability constraint. A weighted sum approach is chosen to minimize both objective functions simultaneously. The main difficulty is to find the normalization factors and the weight factor $\alpha$ that should be used. This difficulty arises because the investment costs and the deviation of the stress have a different
unity, namely \( [\varepsilon] \) and \( \left[ \frac{N}{\text{mm}^2} \right] \). When assuming a normalization factor of 1, \( A \) is entirely determined by the reliability constraint if \( \alpha \) is smaller than 0.5. For values of \( \alpha \) that are larger than 0.5 and smaller than 1, the optimum cross-sectional area corresponds to a minimum in the weighted sum. In this range, the optimum of \( A \) is strongly dependent on the value of \( \alpha \). As long as there is no clue on which value to use for \( \alpha \) and the normalization factors, no justified value of \( A \) can be found using this approach. The approach would be useful if the deviation in the stress can be monetized, so that it is directly comparable to the investment costs. The monetization of this deviation can be done by considering maintenance costs and increase in risk due to for example fatigue. The multiobjective optimization task is then aimed at minimizing the investment cost as well as lifecycle costs due to deviations in the performance. This corresponds to robustness as proposed by Taguchi.

**Multiobjective optimization II**

The second multiobjective optimization task that has been evaluated, aims at minimizing the expected value of the stress simultaneously with the standard deviation of the stress. This is again done under a reliability constraint. The result is a value of \( A \) that is infinitely large, since the expected value as well as the standard deviation of the stress increases with increasing \( A \). It can be concluded that this approach is not applicable to find an optimal solution. It is recommended to always include the investment costs, since minimizing the investment costs often conflict with minimizing the risk.

12.9. **Conclusion**

It can be concluded that for this case study, the robust counterpart approach and the robustness function approach have no added value compared to the level I approach. The multiobjective approach may be of added value if both objectives can be expressed in a monetary value. When using the weighted sum approach, this will then result in a single-objective lifecycle cost optimization. Such an optimization is no different from the optimization approach in the standard design procedures. Concluding, from this case study, no added value of robust optimization approaches compared to the standard approaches was found.
13. Case Study three-bar truss
13.1. Introduction
In this case study, the applicability of robust design to a three-bar truss will be investigated. The set of design variables is the set of cross-sectional area of the column. The truss is subject to two external loads, that are random variables.

First the optimal cross-sectional area of the column will be designed using the standard level III and level I approaches. Following, the robust counterpart approach will be applied to the design problem, to investigate if the approach is indeed comparable to the level I approach. If this is the case, it is considered that this is also valid for the robustness function. Namely, in the case study of the column it has been concluded that these approaches are the same. The second part of the case study will be based on the robust design of the column. A multiobjective optimization will be done. Also the vulnerability and the robustness index will be evaluated.

13.2. Design Problem
In this case study, a three-bar truss is considered that is statically indeterminate to the first degree. Two external loads $P_1$ and $P_2$ are imposed on the truss. The problem is illustrated in Figure 67. The goal of the designer is to find the cross-sectional areas of the bars, which are the design variables.

\[ \text{FIGURE 56 THREE-BAR TRUSS} \]

The loads are the uncertain parameters, with nominal values of:

$P_1 = 100 \text{ kN}$  
$P_2 = 200 \text{ kN}$

The maximum allowable stress and compression is set to:

$s_0 = 300 \text{ N/mm}^2$

The limit state function can be formulated by:

$Z_i = s_0 \frac{N_i}{A_i} \geq 0$

The design variables are the cross-sectional areas of the bars. This means that there are 3 unknowns. For every bar, a stress constraint is available, so 3 limit state equations are present. Using the expressions for the axial forces, it is possible to calculate the required cross-sectional area of each bar. For the probabilistic calculations, distribution functions for the loads should defined. A yearly normal distribution is assumed, with a standard deviation of 10 kN.
The cost function is defined as:

\[ C = I_0 + \sum_{i=1}^{3} I_d \cdot A_i + \frac{P_f \cdot D}{r' - g} \]

In which:

\[ I_0 = 10000 \text{ [€]} \]
\[ I_d = 40 \text{ [€]} \]
\[ D = 60000 \text{ [€]} \]
\[ r' - g = 0.015 [-] \]

The probability of failure of the system can be calculated using the system analysis as introduced in chapter 5. In this example, the correlation coefficients are assumed unknown, so that the elementary boundaries have to be used. For a conservative calculation, the upper boundary is used.

\[ P_f \leq P(E_1) + P(E_2) + P(E_3) \]

The required nominal value of the cross-sectional areas of the element can be determined using the calculation methods as introduced in the literature study.

13.3. Standard approach: Level III

The optimization task is:

Find \[ A_i, \quad i = 1, 2, 3 \]

Minimizing \[ C = I_0 + \sum_{i=1}^{3} I_d \cdot A_i + \frac{P_f \cdot D}{r' - g} \]

The input of the cost function is the set of cross-sectional areas of the beams and the corresponding failure probabilities. Since the stress in a single bar depends on the cross-sectional areas of all bars, every combination of cross-sectional areas should be evaluated separately. A minimization algorithm in MATLAB is used to determine the combination of cross-sectional areas that correspond to a minimum in the cost. The optimal combination is given in Table 11, and corresponds to a total cost of €57400 and a failure probability of \(2.9 \cdot 10^{-4} \text{ yr}^{-1}\).

<table>
<thead>
<tr>
<th>Bar</th>
<th>Cross-sectional area [mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>211</td>
</tr>
<tr>
<td>2</td>
<td>640</td>
</tr>
<tr>
<td>3</td>
<td>306</td>
</tr>
<tr>
<td>Total</td>
<td>1157</td>
</tr>
</tbody>
</table>

**Table 11 Cost optimal truss design**
13.4. Standard approach: Level I

In a level I calculation, a partial safety factor has to be assigned to the external loads. The value of this safety factors depends, among others, on the load being favourable or unfavourable. At first sight, all loads on the truss system seem unfavourable. The partial safety factor for unfavourable, permanent loads in a transient load combination is:

\[ \gamma_{G,j} = 1.10 \]

This result in the following design values for the external loads:

\[ P_{1,D} = \gamma_{G,j} \cdot P_{1,k} \rightarrow P_{1,k} = P_{1,95\%} = 1.65 \cdot 120 \text{ kN} + 116.5 \text{ kN} \rightarrow P_{1,D} = 128 \text{ kN} \]

\[ P_{2,D} = \gamma_{G,j} \cdot P_{2,k} \rightarrow P_{2,k} = P_{2,95\%} = 1.65 \cdot 120 \text{ kN} + 216.5 \text{ kN} \rightarrow P_{2,D} = 238 \text{ kN} \]

The required cross-sectional areas of the bars can be calculated using the constraint that the axial stress or compression is not larger than 300 N/mm². This is done by searching for converged values of the cross-sectional areas. The result is given in Table 12. A Monte Carlo analysis on the truss as designed in this paragraph, results in a failure probability of 0.026 yr⁻¹ and a cost of €157220.

<table>
<thead>
<tr>
<th>Bar</th>
<th>Cross-sectional area [mm²]</th>
<th>Axial force [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>170</td>
<td>51</td>
</tr>
<tr>
<td>2</td>
<td>622</td>
<td>187</td>
</tr>
<tr>
<td>3</td>
<td>309</td>
<td>93</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1101</td>
</tr>
</tbody>
</table>

Table 12 Level I approach

13.5. Robust counterpart approach

For the robust counterpart, convex models for uncertainty are used. As convex bounds, three times the standard deviation is used.

\[ C_{P_1} = \{P_1: 70 \leq P_1 \leq 130\} \text{ kN} \]

\[ C_{P_2} = \{P_2: 170 \leq P_2 \leq 230\} \text{ kN} \]

The robust counterpart optimization task for the three-bar truss can be defined by:

Find \( A_i \)

Minimizing \( I_0 + \sum_{i=1}^{3} l_d \cdot A_i \)

Such that \( s_0 - \frac{N_i(P/A_i)}{A_i} \geq 0, \forall P_1 \in C_{P_1}, \forall P_2 \in C_{P_2}, j = 1,2, \ i = 1,2,3 \)

The values of \( P_1 \) and \( P_2 \) that corresponds to the worst case for the stress constraint, are:

\[ P_1 = 130 \text{ kN} \]

\[ P_2 = 230 \text{ kN} \]
The resulting optimization task is:

\[
\begin{align*}
\text{Find} & \quad A_i \\
\text{Minimizing} & \quad I_0 + \sum_{i=1}^{3} J_d \cdot A_i \\
\text{Such that} & \quad S_0 - \frac{N_i(P_{1}=130, P_{2}=230, A_i)}{A_i} \geq 0, \quad i = 1,2,3
\end{align*}
\]

(63)

The resulting set of cross-sectional areas is the set with the smallest \(\sum_i A_i\) for which the constraint is just fulfilled. The resulting set of cross-sectional areas is given in Table 13. The result corresponds to a cost of €150420 and a failure probability of 0.024.

<table>
<thead>
<tr>
<th>Bar</th>
<th>Cross-sectional area ([\text{mm}^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>178</td>
</tr>
<tr>
<td>2</td>
<td>596</td>
</tr>
<tr>
<td>3</td>
<td>307</td>
</tr>
<tr>
<td>Total</td>
<td>1081</td>
</tr>
</tbody>
</table>

**Table 13 Design with reliability under both governing load combinations**

### 13.6. Multiobjective optimization

**13.6.1. Investment costs and variance of response**

Following Fragiadakis [20], robust design entails simultaneously minimizing the investment costs and the coefficient of variation of the response. The latter is expected to result in a minimization of the lifecycle costs. The response function can in this case be defined as the stress in the element. The optimization task for the column under axial loading is as follows:

\[
\begin{align*}
\text{Find} & \quad A_i \\
\text{Minimizing} & \quad I_0 + \sum_{i=1}^{3} J_d \cdot A_i, \quad \sum_{i=1}^{3} \sigma(N_i/A_i) \\
\text{Such that} & \quad E \left( S_0 - \frac{N_i}{A_i} \right) + \beta \cdot \sigma \left( S_0 - \frac{N_i}{A_i} \right) \leq 0, \quad i = 1,2,3
\end{align*}
\]

When using a weighted sum to solve the multiobjective optimization, the task is as follows:

\[
\begin{align*}
\text{Find} & \quad A_i \\
\text{Minimizing} & \quad (1 - \alpha) \cdot (I_0 + \sum_{i=1}^{3} J_d \cdot A_i) + \alpha \cdot \left( \sum_{i=1}^{3} \sigma(N_i/A_i) \right) \\
\text{Such that} & \quad E \left( S_0 - \frac{N_i}{A_i} \right) + \beta \cdot \sigma \left( S_0 - \frac{N_i}{A_i} \right) \leq 0, \quad i = 1,2,3
\end{align*}
\]

The multiobjective optimization task is solved using the weighted sum approach. The objectives in this task can however not be directly added, since the first term represents cost while the second term represents the deviation in the stress. Ideally, the stress deviation should be translated into a monetary value. It is however not known how exactly the deviation in the stress will cause lifecycle costs. For this case study, the deviation in the stress is in the order of tens and the investment costs are in the order of tens of thousands. For good comparison, a normalization factor should be used.
However, since the solution highly depends on the normalization factors and the weights, the solution only makes sense if these are well defined. This problem can be solved if an expression for the maintenance and inspection costs as a function of the deviation in the stress can be found.

### 13.6.2. Expected value and variance of response

The multiobjective optimization approach was proposed by Doltsinis [19] to simultaneously minimize the expected value and the standard deviation of a performance function. Doltsinis expected that minimizing the standard deviation of an arbitrary performance function would reduce the lifecycle cost. In the multiobjective optimization approach as proposed by Doltsinis, the reliability of the system is ensured by demanding a target reliability. The optimization task for the dike height problem is as follows:

Find \( x \)

Minimizing \( (1 - \alpha) \cdot \frac{E(f(x,y))}{\mu^*} + \alpha \cdot \frac{\sigma(f(x,y))}{\sigma^*} \)

Such that \( E(g) + \beta \cdot \sigma(g) \leq 0 \)

In which \( f(x,y) \) is the performance function that needs a minimized standard deviation, for reducing the lifecycle costs. For the column under axial loading, the stress in the column can be regarded as the performance function. The normalization factors \( \mu^* \) and \( \sigma^* \) are considered to be equal to 1. The optimization task is as follows:

Find \( A \)

Minimizing \( \sum_{i=1}^{3} (1 - \alpha) \cdot E(N_i/A_i) + \alpha \cdot \sigma(N_i/A_i) \)

Such that \( E(s_0 - \frac{N_i}{A_i}) + \beta \cdot \sigma(s_0 - \frac{N_i}{A_i}) \leq 0 \quad i = 1,2,3 \)

The function that has to be minimized, decreases with increasing \( A \). The result of the minimization task is therefore an infinitely large value of every \( A \). This optimization task is of no use for finding an optimum design. The reason for this is that \( E(N_i/A_i) \) as well as \( \sigma(N_i/A_i) \) decrease with increasing \( A \). A minimum can only be found if these norms show opposite trends under an increase of \( A \).

### 13.7. Discussion

This case study was aimed at finding the optimal cross-sectional areas \( A_i \) of the bars of a three-bar truss. The result for the different approaches is shown in Table 22.

<table>
<thead>
<tr>
<th>Approach</th>
<th>( A_1 ) [mm(^2)]</th>
<th>( A_2 ) [mm(^2)]</th>
<th>( A_3 ) [mm(^2)]</th>
<th>( P_f \cdot 10^{-4} ) [yr(^{-1})]</th>
<th>Cost [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level III</td>
<td>211</td>
<td>640</td>
<td>306</td>
<td>( 2.9 \cdot 10^{-4} )</td>
<td>57400</td>
</tr>
<tr>
<td>Level I</td>
<td>170</td>
<td>622</td>
<td>309</td>
<td>( 260 \cdot 10^{-4} )</td>
<td>157220</td>
</tr>
<tr>
<td>Robust counterpart</td>
<td>178</td>
<td>596</td>
<td>307</td>
<td>( 240 \cdot 10^{-4} )</td>
<td>150420</td>
</tr>
</tbody>
</table>

Table 14 Results of designing column under axial loading with different approaches
Standard approaches

First, the cross-sectional areas were determined using the standard approach, which is a full probabilistic cost minimization. The result was a set of cross-sectional areas that corresponds to a cost of €57400 and a failure probability of 2.9∙10^{-4} yr^{-1}. The level I approach resulted in a somewhat greater values of $A$, that correspond to a failure probability of 0.026 yr^{-1} and a cost of €157220. This result agrees with the expectation that the level I approach gives a more conservative solution than a full probabilistic approach.

Robust counterpart approach

The robust counterpart approach was applied with convex bounds of three times the standard deviation around the mean value of the external load. The result was a set of cross-sectional areas that is very close to the result of the level I approach. The result corresponds to a cost of €150420 and a failure probability of 0.024 yr^{-1}. For this case study, the calculation process of the cross-sectional areas using the robust counterpart approach was no different than the process of the level I approach. It can be concluded that for this case study, the robust counterpart approach is of no additional value. This was also concluded for the case study of the column.

Multiobjective optimization I

The multiobjective optimization approach was evaluated for two sets of objective functions. The task in which the investment costs and the standard deviation of the stress are simultaneously minimized, poses difficulties in determining the respective weight of the two objectives. It would be useful to monetize the standard deviation of the stress, so that it can be easily compared to the investment costs. The multiobjective optimization will then become a single-objective optimization.

Multiobjective optimization II

The second multiobjective optimization task was based at simultaneously minimizing the expected value and the standard deviation of the stress. Since increasing the cross-sectional areas reduces the expected value as well as the standard deviation of the stress, the optimum set of cross-sectional areas would exist of infinitely large values. This optimization task is not useful for finding an optimized design.

13.8. Conclusion

It can be concluded that for this case study, the robust counterpart approach has no added value compared to the level I approach. The multiobjective approach may be of added value if the deviation of the performance function can be monetized. The multiobjective optimization will then become a single-objective optimization. It is advised to search for an accurate expression of the lifecycle costs.
14. Case study Dike height
14.1. Introduction
In this case study, the possibility to design a dike with a ‘robust’ dike height will be investigated. For reference, the dike height will be determined using the standard approach. This approach is based on a cost optimization and a level III Monte Carlo analysis for determining failure probabilities. From paragraph 14.3 and on, the ‘robust’ approaches as described in the literature study will be used to design a robust dike. In the conclusion the results will be discussed extensively. The focus in this conclusion is on finding the added value of the ‘robust’ approaches compared to the standard approaches.

14.2. Design problem
In this case study, the required height of a dike has to be calculated. In the Netherlands, the required dike heights are defined according to a minimum in the total cost. The total cost is a function of the investment costs and the risk and can be expressed as follows[2]:

\[ C = I_0 + I'(h - h_0) + \frac{D \cdot e^{\frac{h - \nu}{\kappa}}}{r' - g} \]

\[ I = I_0 + I'(h - h_0) \]

\[ P_f = P(z > h) = e^{\frac{h - \nu}{\kappa}} \]

In which:
- \( I_0 \) is the fixed cost
- \( I' \) is the cost per meter heightening
- \( h \) is the dike height after improvement
- \( h_0 \) is the initial dike height
- \( D \) is the damage given failure
- \( r' \) is the annual discount
- \( g \) is the annual growth
- \( I \) is the investment cost
- \( R_{PV} \) is the present value of the risk
- \( z \) is the storm surge level
- \( P_f \) is the annual probability of failure

This expression for \( C \) can be used to find an optimum in the cost, as illustrated in Figure 68.
For the Dutch Delta Plan, the following values of the parameters were used (originally in guilders):

\[ I_0 = 110 \times 10^6 \quad [\text{€}] \]
\[ I' = 40,1 \times 10^6 \quad [\text{€}/\text{m}] \]
\[ h_0 = 3,25 \quad [\text{m}] \]
\[ D = 24,2 \times 10^9 \quad [\text{€}] \]
\[ r' - g = 0,015 \quad [-]\]
\[ \nu = 1,96 \quad [\text{m}] \]
\[ \kappa = 0,33 \quad [\text{m}] \]

The dike height corresponding to a minimum in the total cost is the design dike height. The corresponding failure probability is the acceptable probability of overtopping. It should be noted that overtopping is not the only failure mechanism. However, in this case study it is assumed to be the most important threat to dikes. In the following paragraphs, the standard approach of this cost optimization problem will be compared to several robust approaches.
14.3. Standard approach: Level III
The standard approach is aimed at finding a dike height that corresponds to a minimum in the cost function. The optimization task is:

Find \[ h \]

Minimizing \[ C = I_0 + I'(h - h_0) + \frac{D e^{-\frac{h - \nu}{\xi}}}{r^{1-g}} \]

The graph of the total cost as a function of the accepted failure probability is given Figure 58. In Figure 59 the total cost is given as a function of the dike height. The cost optimum is at a dike height of 5.83 m. This corresponds with a failure probability of \( 8 \cdot 10^{-6} \) yr\(^{-1} \) and a total cost of € \( 2.27 \cdot 10^8 \).

![Figure 58 Cost as a function of P, for Dutch Delta Plan](image-url)
**Figure 59 Cost as a function of the dike height for the Dutch Delta Plan**

### 14.4. Standard approach: Level I

In the level I approach it is demanded that the design value of the dike height is larger than the design value of the water level. The design value of a random parameter can be determined from the characteristic value and the safety factor. It is assumed that the dike height is deterministic.

\[ h > z_d \]

\[ z_d = \gamma_z \cdot z_k \ [m] \]

The value of the partial safety factors \( \gamma_z \) is in this case study assumed to be equal to 1.1.

\( \gamma_z = 1.10 \)

For the characteristic value of the water level, the water level corresponding to a probability of exceedence of \( 10^{-4} \) is used. This is the probability as prescribed by the Delta Committee for the Dutch coastal zones.

\[ z_k = z_{99.99\%} = 5.00 \ m \]

Now the required dike height \( h \) can be calculated:

\[ h > z_d = \gamma_z \cdot z_k = 1.10 \cdot 5.00 = 5.50 \ m \]
14.5. Robust counterpart approach

14.5.1. Calculation

When following the robust counterpart approach, the robust counterpart of the objective function has to be defined. The robust counterpart of a function, is the same function but then with the uncertain bounds implemented in it. The objective function in this case study, is the cost function. In this cost function, uncertainty is already included, in the form a probability of failure. For the robust counterpart, the function to be minimized should be a deterministic function. Therefore, only the investment costs can be minimized. The reliability is ensured by setting a limit state constraint. The optimization task is:

Find \( h \)

Minimizing \( I_0 + I'(h - h_0) \)

Such that \( h - z \geq 0, \quad \forall z \in C_z \)

Damage occurs if the height of the dike \( h \) is smaller than or equal to the expected value of the water level. Defining the robust counterpart of this function would entail taking the worst case values for the random variables. When considering only \( z \) as a random variable, and taking its worst case value as the expected value plus three times the standard deviation, the bounded region \( C_z \) can be expressed by:

\[
C_z = \{ z : E(z) - 3 \cdot \sigma(z) \leq z \leq E(z) + 3 \cdot \sigma(z) \}
\]

From \( P_f = P(z > h) = e^{-\frac{h - \nu}{\kappa}} \), it follows that the expected value of the water level \( z \) is equal to:

\[
E(z) = \kappa + \nu = 2,29 \, m
\]

The standard deviation of the water level \( z \) is equal to:

\[
\sigma(z) = \beta = 0,33 \, m
\]

The optimization task can now be expressed by:

Minimize \( I_0 + I'(h - h_0) \)

Such that \( h - (E(z) + 3 \cdot \sigma(z)) \geq 0 \)

The dike height \( h \) that results from the optimization task is the minimum dike height for which the constraint is fulfilled. The resulting dike height is 3,28 m.

14.5.2. Conclusion

In the robust counterpart approach, the worst case for the water level is assumed to be at the expected value plus three times the standard deviation. This results in a required dike height of 3,28 m. This dike height corresponds to a probability of failure of 0,02 and a cost of €297 \cdot 10^8. This is not a reliable, neither a cost-efficient solution. It can be concluded that the robust counterpart approach is not applicable to the dike height problem, if it is assumed that the worst case is described by the means plus three times the standard deviation. This result is the same as the result of the level I approach. It can be concluded that more investigation is needed on how to define the safety factors or the convex bounds for the dike height problem. It should be noted that in practice, for every dike...
ring a prescribed failure probability per year is defined. This prescribed probability is the result of an economic optimization.

14.6. Robustness function
The robustness function in this case study can be defined as the amount of uncertainty in the water level for which the dike is still safe. This amount of uncertainty increases with increasing distance between the dike height and the expected value of the water level. Since the latter is a fixed value, the robustness function can only be increased by increasing the dike height, which also means an increase in investment costs. This means that there is a tradeoff between the robustness and the investment costs.

14.7. Multiobjective optimization approach
Multiobjective optimization is a general term for the simultaneous optimization of several objective functions. Both Fragiadakis [20] and Doltsinis [19] proposed a different multiobjective optimization approach for designing robust systems. Both approaches will be discussed in this section.

14.7.1. Investment costs and variance of response
Following Fragiadakis [20], robust design entails simultaneously minimizing the investment costs and the coefficient of variation of the response. The latter is expected to result in a minimization of the lifecycle costs. The optimization task is as follows:

Find $x$

Minimizing $[C_{in}v \cdot \frac{\sigma f(x)}{E(f(x))}]$

Such that $E(z - h) + \beta \cdot \sigma(z - h) \leq 0$

In which $f(x)$ is the performance function under consideration. The performance of a dike corresponds to its ability to retain the water. This function can therefore be defined as the dike height minus the water level, which is exactly the limit state function $Z = h - z [m]$. The standard deviation of the limit state function cannot be minimized, so that minimizing $\frac{\sigma f(x)}{E(f(x))}$ means that $E(f(x)) = E(Z)$ needs to be maximized. This has to be done simultaneously with minimizing the investment costs. This essentially comes down to simultaneously minimizing the risk and the investment costs, which is exactly the economic optimization as used in the standard approach.

14.7.2. Expected value and variance of response
The multiobjective optimization approach was proposed by Doltsinis [19] to simultaneously minimize the expected value and the standard deviation of a performance function. Doltsinis expected that minimizing the standard deviation of an arbitrary performance function would reduce the lifecycle cost. In the multiobjective optimization approach as proposed by Doltsinis, the reliability of the system is ensured by demanding a target reliability. The optimization task for the dike height problem is as follows:

Find $x$

Minimizing $(1 - \alpha) \cdot \frac{E(f(x,y))}{\mu^*} + \alpha \cdot \frac{\sigma f(x,y)}{\sigma^*}$

Such that $E(z - h) + \beta \cdot \sigma(z - h) \leq 0$
In which \( f(x, y) \) is the performance function that needs a minimized standard deviation, so that the lifecycle costs are reduced. As stated before, the performance of the dike can be expressed by the limit state function. The standard deviation of this limit state function is a constant. The optimization task results in minimization of the expected value of the limit state function, which will lead to an infinite value of the dike height. It can be concluded that this multiobjective optimization task is not applicable to the dike height problem.

14.1. Discussion

This case study was aimed at finding the optimal height of a dike. The result for the different approaches is shown in Table 22.

<table>
<thead>
<tr>
<th>Approach</th>
<th>h [m]</th>
<th>( P_l ) [yr(^{-1})]</th>
<th>Cost [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level III economic optimization</strong></td>
<td>5.83</td>
<td>8.0 ( \times 10^{-6} )</td>
<td>2.3 ( \times 10^8 )</td>
</tr>
<tr>
<td><strong>Level I</strong></td>
<td>5.50</td>
<td>100 ( \times 10^{-6} )</td>
<td>3.6 ( \times 10^8 )</td>
</tr>
<tr>
<td><strong>Robust counterpart</strong></td>
<td>3.28</td>
<td>0.02</td>
<td>297 ( \times 10^8 )</td>
</tr>
</tbody>
</table>

**Table 15 Results of optimizing a dike height**

*Standard approaches*

First, the cross-sectional areas were determined using the standard approach, which is a full probabilistic lifecycle cost minimization. The result was a dike height of 5.83 m that corresponds to a cost of €2.3 \( \times 10^8 \) and a failure probability of 8.0 \( \times 10^{-6} \) yr\(^{-1}\). For the level I approach the prescribed probability of exceedence of 10\(^{-4}\) yr\(^{-1}\) was used. The result does not agree with the expectation that the level I approach gives a more conservative solution than a full probabilistic approach. This may be the result of the choice of the safety factor.

*Robust counterpart approach*

The robust counterpart approach was applied with convex bounds of three times the standard deviation around the mean value of the external load. The result was a dike height that is much smaller than the dike height calculated using an economic optimization. The result corresponds to a cost of €297 \( \times 10^8 \) and a failure probability of 0.02 yr\(^{-1}\). Clearly, in this case, using three times the standard deviation does not give a sufficient result.

*Multiobjective optimization I*

The multiobjective optimization approach was evaluated for two sets of objective functions. The first task was to simultaneously minimize investment costs and the standard deviation of the performance function. The performance function is in this case the limit state function for the water level. Analysis of this optimization task resulted in the conclusion that it comes down to an economic optimization, as in the standard approach.
**Multiobjective optimization II**

The second multiobjective optimization task was based at simultaneously minimizing the expected value and the standard deviation of the limit state function. Since the standard deviation is constant for all values of the dike height, this comes down to minimizing the expected value of the limit state function. The result is an infinite dike height. It can be concluded that this optimization task is not applicable to the dike height problem.

### 14.2. Conclusion

In the standard approach, the required dike height is determined according to a minimization of the sum of the investment costs and the risk. The result was a dike height of 5.83 m. This corresponds with a failure probability of $8 \cdot 10^{-6}$ yr$^{-1}$ and a total cost of € $2.27 \cdot 10^8$. The level I as well as the robust counterpart approach, resulted in lower dike heights of 5.5 and 3.28 m. The latter corresponds to a probability of failure of 0.02, which means that the risk is very high. It was concluded the robust counterpart approach is not applicable to the dike height problem. Finding an accurate expression for the convex bounds is necessary to make this approach better applicable.

The multiobjective optimization approach is investigated for two different optimization tasks. The main difficulty was that the performance function of the dike has a constant standard deviation. Minimizing the standard deviation of the performance function, is only possible if this deviation is dependent on the design variables. For the dike height problem, the risk can only be minimized by minimizing the *expected value* of the performance function. This corresponds to an infinite dike height. When simultaneously minimizing the investment costs, an optimum dike height can be found. This approach is exactly the same as the economic optimization approach.
15. Conclusion part 3

15.1. Robust counterpart approach and robustness function
The robust counterpart approach and robustness function approach are in essence the same as the level I approach, since they are all aimed at finding a design that fulfils the constraints under extreme values of the parameters. The advantage of the level I approach is that the safety factors are based on years of experience and on calibration from probabilistic analyses. For the ‘robust’ approaches, such a standard for determining the interval bounds is not available.

15.2. Multiobjective optimization
Multiobjective optimization can be used to optimize different objectives. In literature it was proposed that a ‘robust’ system could be designed by:

- By minimizing the initial cost and the deviation of the response and using the target reliability as a constraint.
- By minimizing the expected value and the deviation of the response and using the target reliability as a constraint.

The main conclusion arising from the case studies is that the first multiobjective optimization task can be useful if the standard deviation in the response is representative for the maintenance and inspection costs. The difficulty in the optimization task is that using a different weighting factor will result in different solutions. A solution to this difficulty is to describe the standard deviation in the response by the corresponding costs. The function that has to be minimized is then the weighted sum of the investment costs and the risk due of failure, functional losses and inspection and maintenance costs. The multiobjective optimization task does then however become a single objective optimization task, with as an objective the lifecycle costs. Such an optimization task does not have an added value compared to the standard optimization approach, as described in 6.2.

This standard optimization task is however not useful for minimizing extra costs due to unexpected changes in the uncertain parameters, during the lifetime. A system that has minimized expected lifecycle costs according to the economic optimization, but does also have minimized additional costs due to unexpected events, is preferable. Such a system may be called a ‘robust’ system. This definition of robustness will be applied to the dike height problem in chapter 18.

15.3. Conclusion
The robust optimization approaches as discussed in this chapter do not show any added value to the design procedures as described in chapter 1 to 6. Eventually, the most important design criterion is to minimize the overall lifecycle cost. An added value of robust design may however be found in minimizing the additional lifecycle costs due to a discrepancy between the assumed distribution parameters and the actual distribution parameters. This may not increase the expected lifecycle costs. Such a robust design can therefore not be found in an optimization task. It is recommended to already search for robust systems during the preliminary design stage, in which the shape of the system is still under consideration. In the next part, a literature review is given on such measures. Also, many recently developed definitions for robust design in structural engineering will be discussed.
Part 4. Practical measures for robustness
Literature study and case studies

Conclusion

“A system that is robust against unexpected future changes will only be chosen if the expected lifecycle costs are minimal.”
16. Robust design in structural engineering

In the definitions as given in chapter 9, the design of a robust system is achieved through optimization the parameters, this is called parameter design. In robust parameter design, the robustness assessment is implemented in the detailed technical design stage, when the preliminary design has already been decided upon. The design variables, such as dimensions and material properties, are then optimized. In part 3 it was concluded, that the optimization task should always be aimed at minimizing the expected lifecycle costs. An extra, ‘robust’ objective will always lead to an increase in these lifecycle costs.

An approach that does not focus solely on robust parameter design, but tries to implement robustness during the system design phase, has been introduced by Bergman [25] and is named ‘Robust Design Methodology’. Moreover, in the Eurocode, it is advised that robustness is achieved through the structural form. This means that robustness should be taken into account already in the feasibility stage and the preliminary technical design phase and is not limited to parameter design. In the next paragraph, first the definition of Robust Design Methodology will be dealt with. Thereafter, the robustness assessment as described in the Eurocode will be summarized and some recent advances in the quantification of structural robustness will be described. This chapter ends with two paragraphs on robust structural systems and robust flood defence systems.

16.1. Robust Design Methodology

In the book by Bergman [25], it is stated that robustness can be implemented in the design in all stages. It is stated that:

“Robust Design Methodology means systematic efforts to achieve sensitivity in noise factors. These efforts are founded on an awareness of variation and can be applied in all stages of product design.” [25]

Applying this definition to the design process of hydraulic structures, means that robust design can also be applied in the initiative stage, the feasibility stage, and the preliminary technical design stage. In this paragraph, the concepts described in the book will be introduced.

Bergman states that:

“The objective of Robust Design Methodology is to create insensitivity to existing sources of variation, without elimination of these sources.” [25]

This corresponds to robustness as defined by Taguchi, who states that a robust product is “a product whose performance is minimally sensitive to factors causing variability”. However when the system is very sensitive to a specific noise factor, it may be more sensible to redesign the system to be independent of that noise factor. This cannot be done through parameter design but entails finding a creative solution.

In the preliminary technical design stage, a creative solution can be searched for that makes the important noise factors irrelevant. This means that the concerned failure mode is made irrelevant or at least less probable. For example, consider a marble rolling from one end of a slat to the other. The failure mode is that the marble falls down the side of the slat before reaching the other end. Robust
parameter design would include increasing the width of the slat. A creative solution would include adding sides to the slat so that it forms a gutter. Now the failure mode of falling down the side is made irrelevant. Another example is a building under an uncertain earthquake load. Robust parameter design would include making the structure very flexible so that it does not lose stability under very large horizontal accelerations. A creative solution that leads to a building that is robust against earthquakes, would be to develop a foundation that does not transfer horizontal accelerations, so that the building is insensitive to earthquake loads.

Creativeness is expected to be a helpful aspect when designing flood protection systems. While robust parameter design would come down to heightening the dikes, a creative robust solution would include adding for example flood plains or mounds. The concept of multi-layered safety [26], which was introduced in 2009 in the Netherlands, is based on quantifying the benefits of such creative design elements. The general procedure for assessment of robustness in the Eurocode is also focussed on creative solution, instead of parameter design. This will be discussed in the next paragraph on definitions of robustness.

16.2. Robustness in the Eurocode

In the Eurocode, some measures of robustness are given that will help to prevent progressive collapse from happening. A well-known example of a progressive collapse is the accident in the Ronan Point apartment tower that happened in 1968. At the 18th floor of the 22 story building, a gas explosion happened in a kitchen in a corner apartment. The woman standing in the kitchen survived the explosion, but some of the outer walls blew out. This resulted in progressive collapse of the corner bays in the entire height of the building which caused the death of four persons. In this example, a small accident with low probability of occurrence caused disproportionally large damage. With ‘disproportionally’ it is meant that the damage is not expected to occur due to the original event, but is caused by the result of the original event. In building codes, a reliability-based design procedure is prescribed, so that events with a small probability of occurring but with large consequences are not considered. A risk-based analysis would be able to prevent such accidents, since it takes into account the risk that is defined as the probability multiplied by the damage. Also, in building codes, the safety is examined on element level instead of on system level, so that the overall safety of the structure is not investigated. It would be best to perform a full risk analysis to assess the safety. This is however very time consuming task with many parameters, so it has no potential to become a standard procedure. Therefore, in modern codes, robustness requirements are defined that have to be fulfilled as an addition to the design on element level.

In the Eurocode it is required that:

“A structure shall be designed and executed in such a way that it will not be damaged by events such as:

- explosion,
- impact, and
- consequences of human errors,

to an extent disproportional to the original cause.” [4]
This statement continues as follows:

“Potential damage shall be avoided or limited by appropriate choice of one or more of the following:

- avoiding, eliminating or reducing the hazards to which the structure can be subjected;
- selecting a structural form which has low sensitivity to the hazards considered;
- selecting a structural form and design that can survive adequately the accidental removal of an individual member or a limited part of the structure, or the occurrence of acceptable localised damage;
- avoiding as far as possible structural systems that can collapse without warning;
- tying the structural members together.” [4]

The Eurocode does not specify the above requirements as being requirements for robustness, but the requirements do lead to a design that is insensitive to explosion and impact events and consequences of human error. Following the definition of robustness in the Eurocode and the example of the Ronan Point apartment tower, structural robustness can be defined as:

“The structural insensitivity to local failure” [41].

This is only a qualitative definition of robustness. Recently, several attempts have been done to find a quantitative description of robustness. A selection of these will be described in the next paragraph.

16.3. Vulnerability and Robustness Index

The definition of structural robustness in the Eurocode that focuses on redundancy and ductility has recently been accompanied by focus on system performance under the removal of members. Research on this topic has been done by the Joint Committee of Structural Safety [27]. The strategy is that a system should have acceptable performance, even under a small level of damage, that is expressed by the removal of members. The removal of members due to an exposure is described by the vulnerability of the system. The robustness is therefore partly dependent on the vulnerability. This is expressed in Figure 60; the local damage due to exposure is described by the vulnerability and the system damage due to the local damage is described by the robustness. In robustness assessment approaches, the local damage is often defined as the direct consequence and the subsequent system damage is defined as the indirect consequence. In the example of the Ronan Point apartment tower, the direct consequence was the blow out of the outer walls of one apartment and the indirect consequence was the progressive collapse of the corner bays.
Recent advances in the field of robustness assessment have been described by the COST Action TU0601. For this Action, Baker [28] proposed some robustness criteria to be implemented in design methods. He claims that optimizing the structure for maximum reliability and minimum cost would be sufficient, if the structural system and its environment was known perfectly, in a stochastic sense. However, the experience is that factors such as human error, unusual loads and progressive failure, are not adequately captured by the calculation models. Therefore, it may be valuable to define additional criteria related to robustness. These criteria are aimed at designing the structure such that it has an acceptable performance under unanticipated loading. Two examples of robustness criteria that can be used in a probability-based robustness assessment are summarized by Baker. The first criterion is introduced by Lind [29], who defined the vulnerability of a system as:

$$ V = \frac{P(r_d, S)}{P(r_0, S)} $$  \hspace{1cm} (65) 

In which:

- $r_d$ is the resistance of the damaged system
- $r_0$ is the resistance of the integer (non-damaged) system
- $S$ is the expected loading on the system
- $P$ is the probability of failure of the system, as a function of the resistance and the loading
A small vulnerability relates to a small difference in the probability of failure of the integer system and of the damaged system. This means that the system has acceptable performance under an unanticipated situation, which means that the system has sufficient robustness. In other words, following this definition, a system is robust if a small level of damage does not significantly increase the probability of failure. This assessment of robustness can be regarded as being reliability-based.

Baker [28] introduced a different criterion for the robustness of a system, that takes into account the consequences of failure and is therefore risk-based. The approach distinguishes between direct consequences and indirect consequences. For the robustness assessment an index of robustness was defined:

\[ I_{rob} = \frac{R_{dir}}{R_{dir} + R_{ind}} \]  

In which:

- \( R_{dir} \) is the direct risk
- \( R_{ind} \) is the indirect risk

These risks are defined using an event tree as shown in Figure 61.

Figure 61: An event tree for robustness quantification [28]

In which:

- \( EX_{BD} \) is the exposure
- \( D \) is a damage state
- \( \bar{D} \) means that no damage occurs under the exposure
- \( F \) is a system failure state
- \( \bar{F} \) means that no system failure occurs

Each branch of the event tree relates to a certain probability of occurrence, the result of the event tree analysis will therefore be a measure of the direct and indirect consequences and their probability of occurrence. In the approach as introduced by Baker, the consequences include repair costs, reduced functionality or temporary loss of functionality, injuries, fatalities, socio-economic losses and damage to the environment. This means that a lifecycle approach is required.
Multiplying the consequences with the corresponding probability of occurring, will result in a measure of the direct risk and the indirect risk. If the indirect risk is much higher than the direct risk, the robustness index will approach zero, which corresponds to low robustness. A value of the robustness index that approaches one, refers to high robustness.

The robustness index approach as described above requires a risk-based lifecycle approach, which means that the performance of the system as a whole, over the whole lifetime, is taken into account. This is an aspect that often lacks in modern reliability-based approaches, that take into account only the reliability of the separate elements and the construction costs. It can be concluded, that for a robustness assessment, a risk-based lifecycle approach is promising. It should be noted that such an approach does not fulfil all requirements for robustness, additional robustness measures are necessary.

16.4. Robustness of coastal structures
Although much has been written on the concept of robustness, the practical application in structural engineering has not yet been investigated thoroughly. In this study, the focus will be on practical applicability in hydraulic engineering, which has been even less addressed in literature. In the field of hydraulic engineering, the focus is on reliability-based design and on cost optimization, as described in chapter 2. Research concerning the applicability of the concept of robustness in this field has only recently started. Few reports have been written on robustness of coastal structures. For example, in the ‘Leidraad Rivieren’ robust designing has been interpreted as follows:

“Accurate (robust) designing means: taking into account future developments and uncertainties, such that the system will maintain its function during its lifetime without drastic and valuable adaptions being necessary and such that the system can be extended if that is economically justified.” (Translated from ‘Leidraad Rivieren’ [33])

In a Rijkswaterstaat report on robust design for coastal structures[30] this definition of robustness was explained as consisting of two requirements:

- Taking into account future developments and uncertainties.
- Ensuring adaptability of the system.

The first requirement is expected to lead to a robust design if unfavourable uncertainties in the predictions of the future developments are taken into account. The uncertainties correspond to errors in the statistical description of the boundary conditions, errors in the calculation models and consequences of adaptions of the norms. Fulfilling the requirement will correspond to making the system more conservative, but will also result in higher construction costs. It is noted in the report that the robustness requirements will only be applied for designing but not for inspection. This means that the bandwidth between design performance and acceptable performance will increase.

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1 Rijkswaterstaat is the executive arm of the Dutch ministry of Infrastructure and the Environment
The second requirement can be implemented in the design stage and will for example entail giving a dike a wider foundation so that it is easier to heighten the dike. Adaptability makes the system less sensitive against variability in uncertain parameters and therefore increases robustness. It should be noted that adaptability is only useful when considering slowly developing uncertain parameters, so that there is enough time to adapt the system.

Recently, Vrijling et al. [31] have written an article on how to implement robustness in design approaches for coastal structures. In this article, the concept of robust design was split in three requirements:

1) “All uncertainties should be accounted for in the design in a probabilistic manner.”
2) “A structure should only be improved in case the cost of heightening is lower than the risk reduction.”
3) “It should be studied if structures can be made adaptable.”

The first requirement means that additional to inherent uncertainty on the main hydraulic variables, also model uncertainties and statistical uncertainties should be accounted for. According to the first requirement, all uncertainties have to be taken into account. This is only possible if the uncertainties are known. ‘Unknown uncertainties’ or ‘unanticipated situations’ are not taken into account. In this point, the approach differs from the approach as described by [30]. Taking into account all known uncertainties does not make this approach significantly different from the state-of-the-art approach as introduced in chapter 2 to 6. The difference will be that the design is more conservative.

The second requirement means that structures that do not fulfil the safety standards are only improved if the cost of improvement is equal to or less than the benefits of risk reduction. This requirement is illustrated in Figure 62, that shows the cost as a function of the height of a dike structure. Heightening is profitable if the investment combined with the residual risk is lower than the actual risk, which is if.

![Figure 62: Heightening is profitable](image)

In the situation where, as in the left graph in Figure 62, the investment should be postponed until the risk corresponding to dike height has increased, or until has decreased.
The result of using this approach is a bandwidth between the design crest level and the rejection level. The heightening scheme of the dike is stepwise in time, as shown in Figure 63.

![Figure 63 Stepwise heightening scheme in time](image)

The third requirement on adaptability was also given by the ‘Leidraad Rivieren’ and will not be discussed any further.

In a report by Deltares [32] the information from the ‘Leidraad Rivieren’[33] and the article by Vrijling et al. [31] has been used to find an approach to robust design for water retaining structures. Two basic premises have been identified in this report:

- The basis for robust design is to build a water retaining structure that does not need to be adapted during the lifetime. This will reduce cost and inconvenience for the environment and the people living near the structure.
- A standard for robust design does not have to give a decisive answer about the length of the lifetime. This length follows from an optimization task.

It is assumed that all known random variables are already included in the state-of-the-art design process. The added value of robustness is translated in a robustness surcharge on the design dike height. This robustness surcharge is a function of the uncertainty in the predicted load variables. Everything that is not known or unsure is not taken into account in the design. It can be concluded, that in the report by Deltares [32] it is proposed that unknown unknowns are not taken into account and robustness solely means that uncertainty in the predicted load variables is taken into account.

It can be concluded, that attempts to implement robustness in design of coastal structures, resulted in the following robustness requirements:

- Ensure the adaptability of the system.
- Take into account all uncertainties in predicted load variables.
- Only improve the structure if this is economically justifiable.

In other words, a robust structure is reliable under all known uncertainty, under all known future developments and the uncertainty in the prediction of these developments. Also, a robust structure should be easily adaptable so that under unanticipated changes in the uncertain parameters, it is relatively cheap to improve the structure. Moreover, the structure is only improved if this is economically justifiable. This means that the total cost will be minimized.
It is doubtable if these requirements of robustness are in accordance with the other definitions of robust design that have been given in this chapter. Clearly, the definition of robust design is very ambiguous and differs between different research areas. An attempt to find a general definition that is applicable in the field of hydraulic structures will be done in chapter 11. First, in the next paragraph the use of robustness in flood defence systems will be discussed. This is a topic that is very important when considering robustness in the field of hydraulic structures.

## 16.5. Robustness of flood defence systems

In this paragraph, recent attempts to designing robust flood defence systems will be described.

### 16.5.1. Super levees

An important uncertainty when designing flood defence system is the variability of natural parameters and the future developments herein. In other words, flood defence systems should be designed to be resistant against climate changes such as sea level rise. This can be done by building higher and stronger flood barriers, but recent publications on flood defence systems show an interest in ‘robust’ flood defences. In a publication by several Dutch ministries and institutions, robust flood defences are defined as:

> "dikes and barriers that do not fail suddenly when the normative water level is exceeded, but still provide a certain amount of protection" [42].

A good example of this definition are the so-called ‘super levees’ in Japan. These can have a base width of up to 300 meter and therefore have a reduced back slope. The width of the dike prevents failure mechanisms such as piping and sliding to happen. The reduced back slope drastically decreases the erosion due to overtopping. The result is that the dike is virtually unbreakable, even when overtopping occurs. The drawback is the extra construction cost that increases with the soil volume used and the additional use of space in the horizontal direction. A solution to the latter is to make the dike multifunctional, by using the extra space for building, as shown in Figure 64.

A study on the technical feasibility of climate dikes has been done by Larissa Smolders [34]. She concluded that it is technically possible to design an unbreachable dike. However, constructing climate dikes is not the most efficient measure to increase the safety level of a dike section.
16.5.2. Response curve for system robustness

As a part of the programme ‘Knowledge for Climate’, M.J.P Mens [36] published an article in which robustness is considered to be related to the system response to flooding. An important feature of the article is that it distinguishes between system robustness and decision robustness. System robustness refers to “the ability of systems to maintain desired system characteristics when subjected to disturbance” [36]. Decision robustness is a criterion for making decisions under uncertainty. The decision robustness requirement is that the decision, or the solution, is still desirable under future changes. The measures as described in 10.1 to 10.4, such as adaptability and redundancy, refer to decision robustness because they give a measure on how the system will respond in the future. Also, in decision robustness, lifecycle aspects such as reparation and maintenance can be taken into account, as well as the risks over the lifetime. Since decision robustness refers to the behaviour if the structure in the lifetime, this most closely relates to the state-of-the-art approach. Consequently, the focus in the study is on decision robustness. However, this paragraph on the response curve refers to system robustness.

In the article by M.J.P, Mens, a response curve for flood risk systems is introduced, that gives the system response as a functions of the disturbance magnitude. In figure Figure 65 a theoretical curve is given and in Figure 66 response curves for the Westerschelde estuary are given.
The response curve is characterized by a resistance threshold, the severity of the response, the proportionality of the response and the point of no recovery. It is assumed that a robust system has a response curve that “stays far from the recovery threshold for a large range of disturbance magnitudes”. For flood risk systems, the disturbance magnitude is the water level and the system response can be either the economic damage, the number of casualties or the number of displaced people. A robust system has a high resistance threshold, a low severity of the response, a good proportionality of the response and a high recovery threshold. A high resistance corresponds to a low risk, since damage will only start to occur at high water levels. The proportionality refers to the change in the response relative to the change in the disturbance. A high proportionality means that the response to the disturbances in the water level is gradual and not sudden. This corresponds to a high robustness. When using the definition that a robust system should be insensitive to disturbances, proportionality is an important measure for robustness.

This approach to robustness of a flood risk system is inspired by the assumption that a system is robust if it is relatively insensitive to changes in the uncertain parameters. However, in the view of protection against flooding, it makes no sense to worry about the point of no recovery. The most important criterion for a flood protection system is to keep the expected value of the economic damage and the number of fatalities as low as possible, at the lowest cost. When considering Figure 65 this corresponds to having a resistance threshold that is as high as possible. Since a value of the water level near the resistance threshold is much more probable than a value at the no recovery threshold, it is always more feasible to spend money on heightening the resistance threshold. More realistic response curves are given in Figure 66. Figure A shows that the resistance threshold is much larger when building a storm surge barrier, which makes it the best option from the view of safety. However, the cost of such a storm surge barrier may not weigh up to the reduced risk. The other options have the same resistance threshold, but result in different values of the damage when flooding occurs. An economic optimization taking into account the probability of occurrence of a water level and the cost of the option, will have to point out which option is most feasible.

Concluding, the response curve approach is not fit as an alternative for decision making, which has also been pointed out by M.J.P. Mens. The response curve is actually just a loss function, that may be more accurate than the limit states as used in the standard approach. In the standard approach. The expected value of the damage can then be obtained by integration of the curve over the water level distribution.

**Figure 65 Theoretic response curve for flood risk system [36]**

![Response Curve Diagram](image)
16.5.3. Conclusion flood defence systems

In this paragraph, the available literature on robust design of flood defence systems has been introduced. First, the use of super levees was discussed. It can be concluded that super levees may indeed be more resistance against piping and back slope erosion. However, building super levees is not a very cost-efficient solution. Furthermore, the building that are on the dike will still be prone to flooding at high water levels.

A total different approach to robustness of flood defence systems is the use of the response curve, that describes the damage due to flooding of the system as a function of the water level. The response curves for different flood protection alternatives can be compared to find the alternative that will result in the most robust system. The drawback of this approach is that the costs of the alternatives and the probability distribution of the water level are not taken into account. Also, the article does not give a clue on how to quantify the robustness from the response curve. Concluding, the response curve may only be useful as an alternative to the limit state functions when quantifying damage and risk.
16.6. Discussion robustness in structural engineering

From the literature as discussed in this chapter, the following recommendations can be made for the design of a robust system:

1) Robustness can already be implemented in the preliminary design phase by searching for a system that eliminates or reduces the hazards.
2) Robustness can already be implemented in the preliminary design phase by searching for a system that is adaptable.
3) Robustness can also be implemented in the preliminary design by reducing the consequences of local damage of the system. The Eurocode gives measures to avoid such ‘progressive collapse’. The vulnerability factor and the robustness index can be used to quantify the response of the system to local damage.
4) Robustness means that the consequences of model errors are reduced. In other words, a system should be robust against unexpected values of the loading or the strength. An example of such unexpected loading is the increase in traffic intensity on steel bridges in the Netherlands, in the past decennia.

In the case studies, it will be investigated if it is possible to design a system according to these recommendations. The case studies will be a continuation of the case studies in part 3. For every case study, the following steps will be taken.

- Search for a system that is more robust, according to recommendation 1.
- Search for a system that is adaptable.
- Calculate the robustness indexes of the solutions in part 3.
- Calculate the vulnerability factors of the solutions in part 3.
- Investigate how the design can be made robust against the discrepancies between the model and the reality.
17. Case Study Column under axial loading
17.1. Design problem
In this case study, the optimal value of the cross-sectional area of a column under axial loading is investigated. Since this case study only discusses a single element, the preliminary system design cannot be considered. For this reason, only the vulnerability, the robustness index and the robustness against the unexpected will be discussed.

Consider a simple limit state function of a column with cross-sectional area $A \ [mm^2]$ under axial loading $S \ [N]$ and with a compressive strength $s_0 \ \left[\frac{N}{mm^2}\right]$.

$$Z = s_0 - \frac{S}{A} \quad s_0 = 300 \ \frac{N}{mm^2}$$

Failure will occur if the loading exceeds the strength, so failure occurs if $Z < 0$. The parameter distributions are given in Table 16.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Normal</td>
<td>$\mu_A \ [mm^2]$</td>
<td>$0.1 \cdot \mu_A \ [mm^2]$</td>
</tr>
<tr>
<td>$S$</td>
<td>Normal</td>
<td>150 $[kN]$</td>
<td>0.1 $\cdot$ 150 $[kN]$</td>
</tr>
</tbody>
</table>

**Table 16 Distribution functions of uncertain parameters**

The cost function is defined as:

$$C = I_0 + I_A \cdot A + \frac{P_f \cdot D}{r' - g}$$

In which:

- $I_0 = 5 \ [€]$  
- $I_A = 0.05 \ [€]$  
- $D = 30 \ [€]$  
- $r' - g = 0.015 \ [-]$  

The required nominal value of the strength of the element can be determined using the calculation approaches as introduced in the literature study.

17.2. Vulnerability
The vulnerability factor is a measure of the behaviour of the column in case of damage. Assume that damage can be described as a reduction of the cross-sectional area of the column, for example due to corrosion or impact. A reduction in the cross-sectional area will linearly increase the stress due to axial loading $S$. Assume a reduction of the cross-sectional area of 10%. The vulnerability factor is:

$$V = \frac{P(0.9 \cdot \mu_A, S)}{P(\mu_A, S)} = \frac{P(A_{90\%}, S)}{P(A, S)}$$

And the Monte Carlo approach for defining the failure probabilities, the vulnerability of the different designs can be compared. The results are shown in Table 17.
From the table it can be concluded that the vulnerability increases with increasing cross-sectional area of the column. This conflicts with our intuition that a larger column is less vulnerable. The reason for this unexpected result is that the 10% reduction of the cross-sectional area will be larger in absolute value, for a larger initial cross-sectional area. The absolute decrease of the cross-sectional area is therefore larger for larger cross-sectional areas. This leads to a larger increase in the failure probability. Consequently, the vulnerability gives a distorted view of the behaviour of the column.

In order to make the results more applicable, the consequences of the damage should be considered. Therefore, the increase in risk due to the damage has been added in Table 17. It can be seen that this increase is largest for smaller initial cross-sectional areas. This means that from a cost perspective, a column with a large cross-sectional area will always lead to less cost increase under unexpected damage.

It can be concluded that the robustness cannot be assessed by the vulnerability, since the vulnerability does not account for the consequences. When assessing the robustness, the consequences have to be taken into account. This is done in the calculation of the robustness index.
17.3. Robustness index
The expression for the robustness index is:

\[ I_{rob} = \frac{R_{dir}}{R_{dir} + R_{ind}} \]

In this expression:

- \( R_{dir} = \) is the cost of reparation multiplied by the probability that the damage occurs.
- \( R_{ind} = \) is the damage due to failure of the system multiplied by the probability of failure.

Consider:

\[ R_{dir} = 0,1 \cdot I_A \cdot A \]
\[ R_{ind} = D \cdot \frac{P(0,9 \cdot A_S)}{r_i^g} \]

The results are given in Table 17.

<table>
<thead>
<tr>
<th>Method</th>
<th>( A ) [mm²]</th>
<th>( P(A,S) ) [-]</th>
<th>( P(A_{90%},S) ) [-]</th>
<th>( R_{dir} )</th>
<th>( R_{ind} )</th>
<th>RI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard: Level III</td>
<td>800</td>
<td>7,6 \cdot 10^{-4}</td>
<td>60 \cdot 10^{-4}</td>
<td>4,0</td>
<td>12,0</td>
<td>0,25</td>
</tr>
<tr>
<td>Standard: Level I</td>
<td>844</td>
<td>2,2 \cdot 10^{-4}</td>
<td>22 \cdot 10^{-4}</td>
<td>4,2</td>
<td>4,4</td>
<td>0,49</td>
</tr>
<tr>
<td>Robust counterpart</td>
<td>929</td>
<td>0,2 \cdot 10^{-4}</td>
<td>2,9 \cdot 10^{-4}</td>
<td>4,6</td>
<td>0,6</td>
<td>0,88</td>
</tr>
<tr>
<td>Multiobjective</td>
<td>810</td>
<td>5,9 \cdot 10^{-4}</td>
<td>50 \cdot 10^{-4}</td>
<td>4,1</td>
<td>10,0</td>
<td>0,29</td>
</tr>
<tr>
<td>optimization I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 18 Robustness Index of Column under Axial Loading**

It can be concluded that the robustness index increases with increasing cross-sectional area. This is mainly caused by the decrease of the indirect risk, with increasing cross-sectional area. This decrease is caused by the fact that probability of failure increases exponentially with decreasing cross-sectional area. A 10% reduction of a large cross-sectional area will therefore lead to a relatively small increase in the probability of failure, which corresponds to a relatively small increase in the risk.

In this case study, the robustness index can be described as the ratio between the reparation costs and the failure probability after damage. The reparation costs are a measure for the severity of the damage, so that the robustness index can be seen as a measure describing whether the risk after damage is proportional to the damage. The closer the robustness index is to 1, the better the proportionality. It can be concluded that this proportionality increases with increasing cross-section.

Robustness as introduced by Taguchi aims at reducing the lifecycle costs by reducing the deviation of the performance, under uncertainty. The robustness index aims at reducing the increase in risk in case damage occurs. The robustness index does not refer to reducing cost due to maintenance and reparation but it will reduce the risk increase in case of unexpected damage. Therefore, the robustness index can be used as a measure for robustness.
17.4. Robust against model errors
The column is robust if the additional lifecycle costs due to unexpected changes in the random variables, are minimized. The lifecycle cost function is defined as:

\[ C = I_0 + I_A \cdot A + \frac{P \left( \frac{S}{A} > s_0 \right) \cdot D}{r - g} \]

The random variables in this expression are the cross-sectional area \( A \) and the load variable \( S \). Changes in the expected value of \( A \) may occur due to degradation of the material. Changes in the distribution of \( S \) may occur due to changing environmental conditions or changing use. Now assume that the distribution of \( S \) changes during the lifetime. The lifecycle costs can be reduced by reducing the increase in the failure probability \( P \left( \frac{S}{A} > s_0 \right) \). The only way to do this is by increasing the cross-sectional area \( A \). Increasing the cross-sectional area will increase the investment costs. This means that there is a tradeoff between the investment costs and the robustness against the unexpected, for this particular case study.

17.5. Discussion
This case study was aimed at finding the optimal cross-sectional area \( A \) of a column under axial loading. The result for the different approaches is shown in Table 19.

<table>
<thead>
<tr>
<th>Approach</th>
<th>( A ) [mm(^2)]</th>
<th>( P_f ) [#/yr]</th>
<th>Cost [€]</th>
<th>( V ) [-]</th>
<th>( RI ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level III economic</td>
<td>800</td>
<td>7,6( \times 10^{-4} )</td>
<td>46,5</td>
<td>8,2</td>
<td>0,25</td>
</tr>
<tr>
<td>optimization</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level I</td>
<td>844</td>
<td>2,2( \times 10^{-4} )</td>
<td>47,6</td>
<td>10,0</td>
<td>0,49</td>
</tr>
<tr>
<td>Robust counterpart</td>
<td>929</td>
<td>0,2( \times 10^{-4} )</td>
<td>51,5</td>
<td>14,5</td>
<td>0,88</td>
</tr>
<tr>
<td>Multiobjective optimization</td>
<td>810</td>
<td>5,9( \times 10^{-4} )</td>
<td>46,7</td>
<td>8,5</td>
<td>0,29</td>
</tr>
</tbody>
</table>

**Table 19 Results of designing column under axial loading with different approaches**

**Vulnerability**

Other than parameter design tasks, also system design tasks have been considered. The vulnerability of the column was defined as the ratio of the probability of failure of the damaged and the undamaged column. It was assumed that a damaged column has a cross-sectional area that is 10% smaller. The results are misleading, since it appears that columns with a larger cross-sectional area are more vulnerable. The increase in risk due to the damage is however significantly larger for columns with smaller cross-sectional areas. From this, it was concluded that an assessment of the robustness should be risk-based.

**Robustness index**

The robustness index was defined as the ratio of the reparation costs and the risk, of a damaged column. A column with a larger robustness index, will have a smaller risk after damage and is therefore more robust. The robustness index also assesses proportionality of the risk to the damage,
disproportionality may correspond to progressive collapse and will also result in a small robustness index. The index is especially useful for comparing different alternatives, since no general rules are available as to what value is acceptable. The robustness index matches with robustness as defined by Taguchi, since the index will increase if the risk due to unexpected damage is reduced.

**Robust against the unexpected**

In this case study, the robustness against the unexpected can only be increased by increasing the reliability. This will result in extra investment costs which means that there is a tradeoff between the investment costs and the robustness against the unexpected.

17.6. Conclusion

When considering system design, the calculation of the vulnerability may lead to misleading results. It is advised to consider the consequences when assessing the robustness. The robustness index has shown to be useful when comparing different alternatives on their robustness.

For this case study there is a tradeoff between the investment costs and the robustness against the unexpected.
18. Case Study three-bar truss
18.1. Design Problem
In this case study, a three-bar truss is considered that is statically indeterminate to the first degree. Two external loads $P_1$ and $P_2$ are imposed on the truss. The problem is illustrated in Figure 67. Clearly, in this case study, the preliminary design is already given. The underlying functional and technical requirements are not given, which means that no alternative preliminary design can be defined. For this reason, only the vulnerability, the robustness index and the robustness against the unexpected will be discussed.

**Figure 67 Three-bar truss**

The loads are the uncertain parameters, with nominal values of:

$P_1 = 100 \text{ kN}$

$P_2 = 200 \text{ kN}$

The maximum allowable stress and compression is set to:

$s_0 = 300 \text{ N/mm}^2$

The limit state function can be formulated by:

$$Z_i = \left| s_0 - \frac{N_i}{A_i} \right| \geq 0$$

The design variables are the cross-sectional areas of the bars. This means that there are 3 unknowns. For every bar, a stress constraint is available, so 3 limit state equations are present. Using the expressions for the axial forces, it is possible to calculate the required cross-sectional area of each bar. For the probabilistic calculations, distribution functions for the loads should be defined. A normal distribution is assumed, with a standard deviation of 10 kN.

The cost function is defined as:

$$C = I_0 + \sum_{i=1}^{3} I_d \cdot A_i + \frac{P_f \cdot D}{v - g}$$

In which:

$I_0 = 10000 \text{ [€]}$

$I_d = 40 \text{ [€]}$
The probability of failure of the system can be calculated using the system analysis as introduced in chapter 5. In this example, the correlation coefficients are assumed unknown, so that the elementary boundaries have to be used. For a conservative calculation, the upper boundary is used.

\[ P_f \leq P(E_1) + P(E_2) + P(E_3) \]

The required nominal value of the cross-sectional areas of the element can be determined using the calculation methods as introduced in the literature study.

### 18.2. Vulnerability

The vulnerability of a truss system can be determined by assuming that the damaged system is the truss with one bar removed. In this paragraph, the vulnerability will be compared of the trusses designed using the economic optimization approach and the robust counterpart approach. The failure probability of these trusses is respectively $2,9 \times 10^{-4}$ and $260 \times 10^{-4}$. In this paragraph, the failure probabilities will be calculated, for all situations in which the cross-sectional area of one bar is set to zero. This is done using a Monte Carlo analysis. Now the vulnerability can be calculated for all situations in which one bar is removed, using the expression:

\[ V = \frac{P(r_d,S)}{P(r_0,S)} \]

<table>
<thead>
<tr>
<th>Level III</th>
<th>Level I</th>
<th>Robust counterpart</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All bars</td>
<td>$2,9 \times 10^{-4}$</td>
<td>$260 \times 10^{-4}$</td>
</tr>
<tr>
<td>Bar 1 removed</td>
<td>1</td>
<td>3448</td>
</tr>
<tr>
<td>Bar 2 removed</td>
<td>1</td>
<td>3448</td>
</tr>
<tr>
<td>Bar 3 removed</td>
<td>1</td>
<td>3448</td>
</tr>
</tbody>
</table>

**Table 20 Vulnerability of three bar truss**

From Table 20 it can be concluded that the three bar truss is extremely vulnerable to the removal of one member. Namely, removal of a member will always lead to failure.
18.3. Robustness Index

For calculating the robustness index, the direct risk and the indirect risk due to a certain event should be known. When considering that this event is the removal of one bar, the direct risk refers to this bar and the indirect risk refers to system failure. The expressions for the direct and indirect risk are:

\[ R_{dir} = \sum_{i=1}^{3} P_{f,\text{bar} i} \cdot C_{\text{bar} i} \]

\[ R_{indir} = \sum_{i=1}^{3} P_{f,\text{bar} i} \cdot P_{f,\text{sys}|P_{f,\text{bar} i}} \cdot C_{\text{sys}} = \sum_{i=1}^{3} P_{f,\text{bar} i} \cdot P_{f,\text{sys}|\text{bar i removed}} \cdot C_{\text{sys}} \]

Assume:

\[ C_{\text{bar} i} = 40 \cdot A_i \ [\epsilon] \]

\[ C_{\text{sys}} = D = 60000 \ [\epsilon] \]

The robustness index can be calculated using:

\[ I_{rob} = \frac{R_{dir}}{R_{dir} + R_{indir}} \]

<table>
<thead>
<tr>
<th>Approach</th>
<th>( R_{dir} )</th>
<th>( R_{indir} )</th>
<th>( RI [-] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level III</td>
<td>5,3</td>
<td>17,1</td>
<td>0,24</td>
</tr>
<tr>
<td>Level I</td>
<td>207</td>
<td>1560</td>
<td>0,12</td>
</tr>
<tr>
<td>Robust counterpart</td>
<td>422</td>
<td>1440</td>
<td>0,23</td>
</tr>
</tbody>
</table>

**Table 21 Robustness Index Three-Bar Truss**

The robustness index has the same order of magnitude for every design which means that there is no design that is evidently more or less robust. The robustness index is dependent on the probabilities of failure of the separate bars and on the cost of reparation of one bar, which depends on the cross-section of the bar. The trusses that are designed according to a level I approach and an RC approach have almost the same probability of failure. However, the latter has a robustness index that is almost twice as great. The reason for this is that the direct risk of the RC truss is almost twice as high, while the indirect risk is almost equal for both designs. Since the probability of failure after removal of one bar is equal to one for all designs, the value of the robustness is in this case dominated by the indirect risk. This does not agree with the definition of robustness, in which it is stated that the structure should not collapse under the removal of one member. It can be concluded that the values of the robustness index as calculated in this paragraph cannot be used to search for a design that is more robust, since the difference are only caused by indirect risk. A more robust design is only possible if the probabilities of failure of the damaged systems, as in Table 20, are smaller than 1.
18.4. Robust against model errors

The column is robust if the additional lifecycle costs due to unexpected changes in the random variables, are minimized. The lifecycle cost function is defined as:

\[ C = I_0 + \sum_{i=1}^{3} I_d \cdot A_i + P_f \cdot \frac{D}{r'} - g \]

\[ P_f = \sum_{i=1}^{3} P \left( \frac{N_i (P_1, P_2)}{A_1} > s_0 \right) \]

The random variables in this expression are the cross-sectional areas \( A_i \) and the external load variables \( P_1, P_2 \). Changes in the expected value of \( A_i \) may occur due to degradation of the material. Changes in the distribution of the external loads may occur due to changing environmental conditions or changing use. Now assume that the distribution of \( P_1 \) changes during the lifetime. The lifecycle costs can be reduced by reducing the increase in the failure probability. The only way to do this is by increasing the cross-sectional areas. Increasing the cross-sectional areas will increase the investment costs. This means that there is a tradeoff between the investment costs and the robustness against the unexpected, for this particular case study.

18.5. Discussion

This case study was aimed at finding the optimal cross-sectional areas \( A_i \) of the bars of a three-bar truss. The result for the different approaches is shown in Table 22.

<table>
<thead>
<tr>
<th>Approach</th>
<th>( A_1 ) [mm(^2)]</th>
<th>( A_2 ) [mm(^2)]</th>
<th>( A_3 ) [mm(^2)]</th>
<th>( P_f \cdot 10^{-4} ) [yr(^{-1})]</th>
<th>Cost [€]</th>
<th>V [-]</th>
<th>RI [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level III economic optimization</td>
<td>211</td>
<td>640</td>
<td>306</td>
<td>2.9 ( \cdot 10^{-4} )</td>
<td>57400</td>
<td>3448</td>
<td>0.24</td>
</tr>
<tr>
<td>Level I</td>
<td>170</td>
<td>622</td>
<td>309</td>
<td>260 ( \cdot 10^{-4} )</td>
<td>157220</td>
<td>38</td>
<td>0.12</td>
</tr>
<tr>
<td>Robust counterpart</td>
<td>178</td>
<td>596</td>
<td>307</td>
<td>240 ( \cdot 10^{-4} )</td>
<td>150420</td>
<td>42</td>
<td>0.23</td>
</tr>
</tbody>
</table>

**TABLE 22 RESULTS OF DESIGNING COLUMN UNDER AXIAL LOADING WITH DIFFERENT APPROACHES**

Vulnerability

Robustness can also be implemented in a design, in the system design phase. In this case study, it was investigated if this can be done by reducing the vulnerable of the structure. The vulnerability of the structures, designed with the different parameter design approaches, was determined. The result is that these structures are very vulnerable. Namely, removal of one bar will always lead to failure. The three-bar truss is very vulnerable and should not be chosen in the system design phase.

Robustness index

The robustness index was calculated as the ratio between risk that a bar has to be replaced and the sum of this risk and the risk of failure due to removal of this bar. Because the probability of failure...
after removal of one bar is always equal to one, the robustness index takes values that are entirely dependent on the replacement costs. These value are the smallest values possible, which means that the robustness is very small. Concluding, the vulnerability of the three-bar truss is high and the robustness is low, due to the design of the system. For better results, the vulnerability and the robustness index have to be considered during the system design phase.

*Robust against the unexpected*

In this case study, the robustness against the unexpected can only be increased by increasing the reliability. This will result in extra investment costs which means that there is a tradeoff between the investment costs and the robustness against the unexpected.

### 18.6. Conclusion

When considering system design, the calculation of the vulnerability and the robustness index may be useful for finding a robust topology. It is advised that during the system design phase, the robustness of the system is taken into account. This can be done by investigating the response after removal of an element. For this case study there is a tradeoff between the investment costs and the robustness against the unexpected.
19. Case study Dike height
19.1. Design problem

In this chapter, the ‘robust’ approaches as described in the literature study will be used to design a robust dike. In the conclusion the results will be discussed extensively. The focus in this conclusion is on finding the added value of the ‘robust’ approaches compared to the standard approaches.

In the case study, the required height of a dike has to be calculated. In the Netherlands, the required dike heights are defined according to a minimum in the total cost. The total cost is a function of the investments costs and the risk and can be expressed as follows[2]:

\[
C = I_0 + I'(h - h_0) + \frac{D \cdot e^{-\frac{h - \nu}{\kappa}}}{r' - g}
\]

\[
I = I_0 + I'(h - h_0)
\]

\[
P_r = P(z > h) = e^{-\frac{h - \nu}{\kappa}}
\]

Minimizing this expression for \( C \) will result in a cost minimum solution, as illustrated in Figure 68.

For the Dutch Delta Plan, the following values of the parameters were used (originally in guilders):

\[
l_0 = 110 \cdot 10^6 \quad [\text{€}]
\]

\[
l' = 40,1 \cdot 10^6 \quad [\text{€/m}]
\]

\[
h_0 = 3,25 \quad [\text{m}]
\]

\[
D = 24,2 \cdot 10^9 \quad [\text{€}]
\]

\[
r' - g = 0,015 \quad [-]
\]
\[ \nu = 1.96 \quad [m] \]

\[ \kappa = 0.33 \quad [m] \]

The dike height corresponding to a minimum in the total cost is the design dike height. The corresponding failure probability is the acceptable probability of overtopping. It should be noted that overtopping is not the only failure mechanism. However, in this case study it is assumed to be the most important threat to dikes. In the following paragraphs, the standard approach of this cost optimization problem will be compared to several robust approaches.

### 19.2. System design: Eliminate hazards

Robustness can be implemented in the preliminary system design by reducing or eliminating the hazards. In this case study, the only hazard that is considered is flooding. This hazard cannot be eliminated by the design of the dike since the only function of the dike is to prevent flooding. An example of a hazard that can be reduced is piping. Piping can, for example, be reduced by increasing the base width of the dike or by keeping the ditch far away from the dike.

It can be concluded that robust system design can only be applied when several hazards are considered. Moreover, it is best to consider the full design problem so that making the system robust to a single hazard will not result in complications for other hazards.

### 19.3. Robust to model errors

In part 3 it is concluded that the multiobjective optimization tasks as proposed by Fragiadakis and Doltsinis is not of added value. In this paragraph it is investigated whether a multiobjective optimization on the expected value and the standard deviation of the cost function, is of added value. The expression of the robust optimization task is then as follows:

Find \( h \)

Minimizing \[ (1 - \omega) \cdot \frac{E(C(h))}{\mu^*} + \omega \cdot \frac{\sigma(C(h))}{\sigma^*} \]  \hspace{1cm} (68)

Such that \[ E(z - h) + \beta \cdot \sigma(z - h) \leq 0 \]

In which the cost function is defined as:

\[ C(h) = I_0 + I'(h - h_0) + \frac{D \cdot e^{\frac{h-v}{\kappa}}}{r' - g} \]

This cost function does not contain any random variables, which means that the cost is not a random variable. Consequently, the cost function does not have an expected value and a standard deviation.

The optimization task in equation 47 can however be useful for minimizing the deviation of the costs under future changes of the parameters. In the future, costs may be influenced by a change in distribution function of the water level, by a change in the damage cost and by changes in the annual discount and the annual growth. The cost function can now be expressed as:

\[ C_r = C(h, D, \alpha, \beta, r' - g) = I_0 + I'(h - h_0) + \frac{D \cdot e^{\frac{h-v}{\kappa}}}{r' - g} \]  \hspace{1cm} (69)
In this function, $h$ is deterministic and the other variables are random. The aim of the multiobjective optimization is to minimize the costs and to simultaneously minimize the deviation in the costs due to a deviation in the random variables. The solution of the multiobjective optimization will not be at a cost minimum, but at a Pareto optimum in which both the cost and the standard deviation are minimized as described in chapter 9. Many Pareto optima are possible, depending on the relative importance of minimizing either the cost or the standard deviation of the cost. The result of the multiobjective optimization will therefore not be at the cost optimum as found in 12.1, but at a less cost optimum solution with a smaller standard deviation.

19.3.1. Calculation example
Consider the following optimization task:

Find $h$

Minimizing $(1 - \omega) \cdot \frac{E(C_r)}{\mu} + \omega \cdot \frac{\sigma(C_r)}{\sigma^*}$

Subject to $E(z - h) + \beta \cdot \sigma(z - h) \leq 0$

In this minimization task, $E(C_r)$ is the expected value of the total costs and $\sigma(C_r)$ represents the standard deviation of the total costs due to the randomness of the input variables. The normalization factors $\mu^*$ and $\sigma^*$ are necessary so that both objectives are of the same order. In this case study the normalization parameters are chosen as respectively the expected value and the standard deviation at $h_0 = 3,25 \text{ m}$. This is the initial height of the dike. The minimization can then be explained in words by:

$$(1 - \omega) \cdot \left[ \frac{\text{Lifecycle costs after improving to dikeheight } h}{\text{Initial risk for } h = 3,25 \text{ m}} \right]$$

$$+ \omega \cdot \left[ \frac{\text{Standard deviation of lifecycle costs after improving to dikeheight } h}{\text{Initial standard deviation for } h = 3,25 \text{ m}} \right]$$

The random variables are assumed to be normally distributed and to have a coefficient of variation of 10%, which means that they have a standard deviation of 0,1 times the nominal value. The optimization task is evaluated for every random variable, while keeping the other variables deterministic. The minimization of the expected value and the standard deviation are given equal weight, which means that an $\omega$ of 0,5 is chosen. The results are given in Table 23.

<table>
<thead>
<tr>
<th>Random variable $h$ [m]</th>
<th>$D$</th>
<th>$v$</th>
<th>$\kappa$</th>
<th>$r^* - g$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5,95</td>
<td>5,99</td>
<td>6,23</td>
<td>5,86</td>
</tr>
</tbody>
</table>

**Table 23 Optimal dike height**

From Table 23 it can be concluded that the multiobjective optimization task always results in a dike height that is higher than when using the costs minimization. This extra dike height causes higher investment costs. These extra costs are justified by reduced lifecycle costs in case changes occur in the variables that are otherwise considered deterministic. The averted lifecycle costs cannot be determined using this method. This is done in the next paragraph for a future change in $\kappa$. 
19.4. Improvement costs due to future changes
The lifecycle costs of a dike consist of improvement costs and risk. As proposed in [31] a dike should only be improved if the improvement costs are less than the averted risk.

Consider that the dike that is constructed with a height of 5.83 m. There is a possibility that in the lifetime of the dike, the probability distribution function of the water level will change. This could for example be caused by rising sea water levels due to climate change. The frequency of exceedence of a certain water level will then increase. Consequently, the values of ν and κ in the probability distribution function of the water level will change. It should be noted that altering the probability distribution function is mainly a political question, but this will not be considered in this study.

Consider that twenty years after the dike was constructed, the mean sea water level has increased with 0.5 m and that there is an uncertain expected rise in the future. In this case, ν has to be increased with 0.5 m and κ is for example raised to 0.45. The probability of failure of a dike with a height of 5.83 m is then increased from $8.00 \times 10^{-6}$ yr$^{-1}$ to $5.6 \times 10^{-4}$ yr$^{-1}$. Heightening may be necessary to reduce the risk. In Figure 69 the reduction in cost as a function of the improved dike height is shown. For the damage and the construction costs, the same values have been used as in paragraph 18.1.

\[ \text{Figure 69 Cost reduction for dike heightening} \]

It can be concluded that for the assumed change in the probability distribution function of the water level, improvement is always beneficial. The cost reduction is maximal for an improved dike height of 7.6 m, which corresponds to a total cost reduction of € 7.04 \times 10^{8}. The probability of failure for this dike height is $1.0 \times 10^{-5}$ yr$^{-1}$ and the improvement costs are € 1.65 \times 10^{7}.

If the dike would have been designed at 7.6 m in the initial design, in this case only the extra initial costs of € 110 \times 10^{6} would have been averted. This means that a cost of € 55 \times 10^{6} would have been
made to avert the € \(110 \times 10^6\) in case improvement is necessary. Spending € \(55 \times 10^6\) in the beginning, would avert having higher costs at a later stage. Also making the dike easily adaptable will reduce the costs at a later stage. However, it is unlikely that money will be spent to avoid costs of which the probability of occurring is not known. It is expected that only a measure that reduces the uncertainty drastically at low costs, has potential. Another option is a measure that not necessarily has low costs but that does reduce the probability of failure drastically. An example is the ‘superlevee’ or a sheet pile wall in the dike.

19.5. Adaptability
As concluded above, making the dike more adaptable would be a step towards robustness. It should however be kept in mind that this adaptability should be at low cost, since the averted future costs cannot be quantified. Also, adaptability is only useful when considering slow future developments, such as climate change. Extreme events and accidental loads generally occur without warning so there is not time to adapt the dike to be resistant against such events.

Making a dike adaptable for climate change means that it should be relatively simple to adapt the dike so that it retains increased water levels. The most straightforward measure is to heighten the crown of the dike. Heightening the dike by a meter does however correspond to increasing the base width with at least 10 m. This has recently caused problems in for example ‘Krimpen aan de Lek’ in the Netherlands, where the dike was inside a built-up area so that heightening of the dike would require moving houses [37]. An adaptable dike is therefore a dike with an increased base width, so that heightening the dike would not require widening the dike. This measure could in the future avoid more costs than only the construction costs, since buying out the owners of the houses or divert a dike is much more expensive. An adaptable dike would avoid such problems, which is a great advantage.

When the dikes where build in Krimpen, some decennia ago, it was not predicted that the area would become so densely populated. This population growth is an example of an unknown uncertainty, that could not be taken into account in the design stage since it was simply not known. If at that stage the dikes would have been made robust, by making them higher or better adaptable, the costs of improving them would have been averted.

19.6. Vulnerability
The robustness of a dike or a dike ring system can also be assessed using quantified measures for vulnerability and robustness. In this paragraph these vulnerability measure and the robustness index will be investigated for dikes. In Figure 71 vulnerability is classified as being a component of robustness. Decreasing the vulnerability will help to increase the robustness. A quantified measure of the vulnerability was introduced by Lind [29] and reads:

\[
V = \frac{P(r_d, S)}{P(r_0, S)}
\]

According to this expression, the vulnerability of a dike is equal to the ratio between the failure probability of the damaged dike and the failure probability of the undamaged dike. The vulnerability depends on the definition of damage and the failure mode that is considered. For this case study, overtopping is the considered failure mode. The damage is unpredicted settling of a part of the dike ring. The retaining height at this location is lower than the design value of the retaining height. When considering the dike ring as being a series system, the dike is as strong as its weakest part. The
damaged dike will thus have a failure probability corresponding to the lowest retaining height. Consider a design dike height of 6.0 m, a settled dike height of 5.5 m and the following probability distribution function:

\[ P_f = P(z > h) = e^{-\frac{h-1.96}{0.33}} \]

Now:

\[ P(r_d, S) = e^{-\frac{5.5-1.96}{0.33}} = 2.19 \cdot 10^{-5} \]

\[ P(r_0, S) = e^{-\frac{6.0-1.96}{0.33}} = 0.48 \cdot 10^{-5} \]

The vulnerability is given by:

\[ V = \frac{P(r_d, S)}{P(r_0, S)} = \frac{2.19 \cdot 10^{-5}}{0.48 \cdot 10^{-5}} = 4.5 \]

A dike ring system has a vulnerability of 4.5 when settling of 0.5 m occurs. It is unclear what this means for the system, since there is no reference value and only one damage mode has been taken into account and the magnitude of this damage is purely fictive. The vulnerability factor can however be useful when comparing different systems on future hazards.

**19.7. Robustness index**

The expression of the robustness index is:

\[ I_{rob} = \frac{R_{dir}}{R_{dir} + R_{ind}} \]

In a dike ring system, the direct consequence of a dike breach is the cost of reparation of the dike. The indirect consequence is the flooding of the hinterland. The probability of these events is equal, the robustness index is therefore equal to ratio of the total consequences and the consequence of dike reparation. Assuming the consequence of flooding to be $S = 24.2 \cdot 10^9$ [€] and the consequence of dike reparation to be $I_0 + 5.83 \cdot 10^6$ [€], the robustness index is:

\[ I_{rob} = \frac{344 \cdot 10^6}{344 \cdot 10^6 + 24.2 \cdot 10^9} = 0.014 \]

This is a small value, which means that the robustness of a dike ring system, solely existing of a ring of the design dike, is small. This does not necessarily mean that the design should be altered, since it is still the cost optimal solution. In the state-of-the-art approach, the lack of robustness will be accepted since it is still the most cost-efficient solution. Clearly there is a tradeoff between robustness and cost.
19.8. Conclusion

In chapter 14, it was concluded that the added value of a robust system is that it has a minimal increase in lifecycle costs under unexpected values of the loading. These unexpected values arise during the lifetime and can be expressed by changing the distribution function for the loading. Changing this distribution function may result in the system not being a cost-optimal system anymore. In paragraph 18.4, it is assumed that a change in the parameter $\kappa$ occurs during the lifetime of the dike. The dike will have to be improved, if the risk reduction is larger than the improvement costs. The improved dike height is determined based on a maximum in the total cost reduction (risk reduction – investment costs). Following, it is investigated what the costs would have been if this improvement was done during the initial construction. It is concluded that at least the initial investment costs ($I_0$) would have been averted. However, since no probability can be given that parameter $\kappa$ will change during the lifetime, the additional initial building costs cannot be justified. It is recommend to evaluate past projects to investigate whether these unexpected changes will indeed occur. Also, for an accurate analysis, the net present value of the improvement should be used.

A special case of making a dike more robust is by making it adaptable. The obvious way to make a dike adaptable is by increasing its base width. This will in the future avoid cost as a consequence of a lack of space for extension of the dike. Increasing the base width will however increase the initial construction costs.

It can be concluded that it is not possible to design dikes robustly, when following the definition as adopted in this study. The reason for this is that robustness is always traded against extra investment costs. In chapter 11, alternative measures have been introduced that may make the structure more cost-effective and better resistant against uncertainty. These measures can be summarized as:

- Take into account all uncertainties, including uncertainties in the loading and the strength but also uncertainties in the structural and the numerical model and manufacturing tolerances. Also take into account future developments, such as sea level rise.
- Decide for improvements based on a cost-benefit analysis.
- Make structures adaptable, so the structure can be adapted if future changes occur.

The above measures do not represent a concept that is very different from the state-of-the-art approach. In fact, they are measures to making this approach more accurate and more focussed on economic optimization in the lifecycle. Only the measure that states that a structure should be adaptable, can be thought of as being a robust measure. After all, it will reduce the cost and risk under changing parameters. It should be noted that this is only valid for slow changes in the parameters, so that anticipation is possible. Risk due to accidental loads or extreme events cannot be avoided by making a structure adaptable, this can only be done by making the structure more resistant and thus choosing robustness over economic optimization.
20. Conclusion part 4

20.1. System design
Robustness can be implemented in the design of the system, that is done during the preliminary design stage. This can be done by reducing or eliminating the hazards to the system, by reducing the consequences of local damage and by making the structure adaptable. For these measures, it is important that the system is considered as a whole. This means that all hazards and failure modes should be considered, to make sure that a robustness measure is beneficial for the whole system. Another important aspect of system design is that the investment costs should be considered. Optimization of the lifecycle costs is always an important aspect when choosing a design. Another important aspect is loss of life, this aspect will however not be considered in this study.

20.2. Vulnerability and robustness index
The vulnerability and the robustness index have shown to be useful when designing a system in the system design phase. To avoid misleading results, it is advisable to keep a good eye on how the vulnerability and the robustness index are exactly defined. Both should give a measure on the probability of failure or the risk of the system, after local damage has occurred. A robust system will not be subject to a large increase in risk, due to damage of only one element. Namely, this would mean a large deviation in the response after a small deviation in the design variables or the topology. It can be concluded that robustness should already be implemented in the system design phase.

20.3. Robust against the unexpected
A system that is robust against the unexpected, has minimum additional lifecycle costs in case of unexpected changes in the distribution functions of the random variables. This resembles the definition of robustness as defined by Taguchi.

In standard design procedures, the priority is however to reduce the overall expected lifecycle costs of a system. Therefore, a system that is robust against unexpected future changes will only be chosen if the expected lifecycle costs are minimal. This is only possible if two different alternatives can be thought of, that both have the same minimum lifecycle costs in the optimization, but of which one is more robust. This is illustrated in Figure 70.

\[ \text{Figure 70: Cost as a function of the performance parameter for a robust and a less robust system. Both have the same lifecycle costs.} \]
Part 5. Robustness in practice
Review and recommendations

Conclusion

“It is recommended to make the system robust already in the preliminary design stage.”
21. Introduction
Many different definitions of robustness are available, corresponding to different measures of robustness. For a better understanding of the meaning of robustness, in this part a comparison will be made with comparable concepts, such as vulnerability, redundancy and resilience. Following on this attempt to define robustness, the measures of robustness will be discussed according to their applicability in the field of civil engineering.

22. Summary of robustness definitions

22.1. Taguchi
In chapter 8, it was concluded that Taguchi’s approach to robust design is based on the assumption that the loss of a product will increase with increasing deviation from a target performance. The robust design method is introduced to minimize these losses. Taguchi introduced a three step method to achieve a robust design, existing of system design, parameter design and tolerance design. He claims that the parameter design step is the most important step for implementing robustness. This step exists of an experimental method in which for every design alternative the expected value and the standard deviation of the performance are determined. The alternative with the smallest standard deviation in the performance is the most robust alternative. In short, Taguchi proposed a minimization of the standard deviation of the performance to reduce the losses.

There are three major drawbacks of Taguchi’s approach when comparing it to the standard procedures, in which limit state design is combined with an economic optimization. The first drawback is the quadratic quality loss function, since this loss function is simply not realistic for most design problems. Opposed to this, the limit state approach considers all structural and functional failure modes separately for ever design problem. This approach is therefore much more sophisticated. The second drawback is that all statistical information is neglected in the orthogonal array approach, which makes that it is not an accurate method to defined standard deviations of the performance. This orthogonal array approach can however simply be replaced by a Monte Carlo analysis without affecting Taguchi’s approach. The third drawback is that Taguchi only minimizes the risk when searching for a robust design. This means that investment costs are not considered, which may lead to an uneconomical design.

It was concluded that more investigation is needed to find out if Taguchi’s approach may be of added value in civil engineering design. It is expected that this may be the case in problems where a target performance is demanded. Also the approach to robust parameter design may be of added value to design for unexpected changes in the uncertain parameters that have not been considered in the design phase.

22.2. ‘Robust’ optimization tasks
In chapter 9, alternative approaches to robust parameter design in structural design are discussed. These approaches are different from the Taguchi approach in the sense that they are not based on experimental design. Also, the quality loss function is not discussed in these approaches, but losses are avoided by setting constraints (limit states). The first approach that was discussed is the robust counterpart approach. It was concluded that this approach resembles the standard approach with bounded uncertainty instead of probabilistic uncertainty. This bounded uncertainty can also be
described by safety factors, which means that the robust counterpart approach is in essence no different than the level I approach.

The second approach that was discussed in chapter 9 was the robustness function. This approach is based on the assumption that there is always a trade-off between optimal performance and robustness. When applying the approach to civil engineering, this trade-off is translated to a trade-off between cost and reliability, just as in the standard approach. This approach has therefore no significant different philosophy than the standard approach.

In chapter 9, it was found that the approach to robustness as proposed by Doltsinis [19] closely resembles the Taguchi approach. Doltsinis assumed that a smaller standard deviation in performance functions as stress or displacement, will lead to less losses over the lifetime. This standard deviation can be reduced by doing a multiobjective optimization on the expected value and the standard deviation of the performance function. Doltsinis does not give a formula for the losses as a function of the deviations. It is recommended to find such a formula. However, if such a formula is known, the concept of multiobjective optimization is not useful anymore, since there is just a single objective left, namely minimizing the lifecycle costs.

22.3. Robust design in structural engineering
Practical approaches to robust design that have been proposed in structural engineering have been discussed in chapter 10. The first approach that was discussed is creative design for robustness. In the Taguchi approach, this refers to the system design step. In this step, the preliminary design is made robust by making sure that the influence of the uncertain parameters on the lifecycle costs is reduced. This involves changing the topology and the shape of the design, instead of optimizing the values of the design variables as in parameter design. The measures as given in the Eurocode are also fit for implementation in the preliminary design stage. In the Eurocode, a structure is robust if it will experience reduced damage due to explosion, impact and the consequences of human errors. Other measures that can be used to assess robustness of structures in the preliminary stage are the vulnerability factor and the robustness index. All the measures that are given in this paragraph refer to the preliminary design or the system design. For this reason they are not comparable to Taguchi’s approach for robust parameter design. The measures can however be used to make the design more robust in the system design stage, by making sure that the performance of the structure is least sensitive to uncertain parameters or unexpected changes in the parameters.

Other alternative approaches to robust design refer to flood defence structures. In this field of study, robust design is seen as an addition to the standard economic optimization approach. This addition entails that all uncertainties are taken into account, including model uncertainties and uncertainties in the predicted variables. An example is the uncertainty in the predicted climate change and the result this has on the hydraulic boundary conditions. If the structure is designed taking into account all sorts of uncertainty, this will reduce the probability that the structure will exceed the limit states during its lifetime. However, this approach is not essentially different from the standard design procedures. Another aspect of a robust design is assumed to be adaptability. If a structure is made adaptable, this may reduce the costs in case future adaption is necessary. Apparently, in the field of flood defences, robust design means that all known uncertainty should be taken into account to make an economic optimization. A design that is not economically optimal will not be chosen, even if it has high reliability. For example, a super levee is unable to breach and therefore robust against all uncertainties. However, the super levee is has great dimensions and is therefore not an economic
optimum design. Adaptability is not a separate measure but can implicitly be included in the economic optimization task.

In the above paragraphs, many definitions of robust design as given in literature are discussed. The overall conclusion is that robust design involves reducing the losses over the lifetime of a system by reducing the deviation in the performance distributions. The performance parameters are case-dependent and correspond to the functions and the limit states of the system. For example, for a cantilever, the performance parameters are the internal stress and the displacement. For a caramel candy, the performance parameter is the chewability. The deviation in the performance parameters is caused by the uncertain parameters, that are assumed to be unknown by Taguchi but that are described probabilistically in the standard approach. Since the standard approach makes it possible to find the most cost-efficient solution under all known uncertainty, robust design does not seem to be of any added value. However, robust design may be of added value when assuming that a model always has errors that cannot be described mathematically. The errors can then not be included in the calculation of the expected lifecycle costs. An example of such an error is a higher than expected increase in traffic loading, or a higher than expected sea level rise.

**SUMMARY OF METHODS TO ASSESS ROBUSTNESS**

In the methods to assess robustness as discussed in chapter 9 and chapter 10, have been summarized. For extra clarification, a more extensive discussion on every method is given below.

**22.4. Robust counterpart approach**

The key difference between the robust counterpart approach and the standard approach, is that interval uncertainty is used instead of probabilistic uncertainty. This has as a consequence that in the robust counterpart approach, it is impossible to determine failure probabilities. The reliability has to be ensured by the demand that the limit state may not be exceeded in any case. Under this demand, the design with the lowest investment costs has to be found. It is concluded that this approach has no added value compared to the level I approach. Moreover, the approach does not result in ‘robust’ design, since a small deviation in the expected values of the uncertain parameters may immediately lead to exceedence of the limit states. Consequently, high losses may occur due to a small change in the expected value of the parameters.

**22.5. Robustness function**

The robustness function has been applied in many different fields and represents the amount of uncertainty for which the system still fulfils the constraints. The higher the possible uncertainty, the higher the robust reliability. Ben-Haim claims that robust reliability is always traded off against the performance. When applying the approach on structural design problems, this tradeoff is nothing more than a tradeoff between investment costs and reliability. The reliability is again described by non-probabilistic interval bounds, as in the robust counterpart approach. It was concluded that the robustness function has no added value for structural design problems.

**22.6. Multiobjective optimization**

In literature, the multiobjective optimization is used for designing more robust truss structures by minimizing the expected value as well as the variance of a certain performance function. This performance function can be weight, stress, structural compliance or manufacturing costs. The reliability is ensured by demanding that the expected value of the constraint function added to a
number of times the standard deviation of the constraint function, is smaller than zero. In the
multiobjective optimization, a probability description of uncertainty can be used.

It was concluded that multiobjective optimization in structural engineering, is only applicable if both
objectives can be expressed in monetary values. This will however immediately result in a single
objective optimization, that aims at minimizing the lifecycle costs.

22.7. Eurocode
In the Eurocode, measures are given that prevent a structure from being damaged disproportionally
by events such as explosion, impact and consequences of human errors. These measures are useful
for designing a robust system, already in the preliminary design phase.

22.8. Vulnerability
The vulnerability factor gives a measure of the decrease in reliability of a system after damage has
taken place. The factor can be calculated using a probabilistic approach and is in literature only
applied to buildings. This approach cannot be used to design a system, but if handled well, is a good
tool to judge the robustness of the system to local damage.

22.9. Robustness index
The robustness index gives a measure of the amount of indirect damage compared to the direct
damage of an event. A low robustness index means that a structure is prone to progressive collapse.
The factor can be calculated using a probabilistic approach and is in literature only applied to
buildings. This approach cannot be used to design a system, but if handled well, is a good tool to
judge the robustness of the system to local damage.

22.10. Adaptability
Making a structure adaptable will reduce the costs of adaptation during the lifetime. Adaptation is a
measure that can be used under slowly changing load conditions so that there is enough time to
adapt the structure to these changes. Adaptability can be implemented in the preliminary design
stage, but will only be decided upon if it reduces the expected lifecycle costs.

22.11. Response curve
The response curve shows the consequences as a function of changing load parameters. The
probabilistic character of the consequences is not taken into account. This response function can
therefore be seen as a loss function, from which the risk can be determined by integrating over the
probability density function of the performance. According to the article on the response curve, a
system is robust if large changes in the parameters can occur before the system reaches a ‘point of
no recovery’. When using the definition that a robust system should have a minimum in the
additional lifecycle costs, it makes more sense to use the response curve as a tool to define the risk.
The response curve is a loss function that is more accurate than the limit state loss function. The loss
function is however case-dependent and should be determined separately for every design problem.
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**Table 2.4: Summary of Methods to Assess Robustness**
23. Robustness and comparable concepts

The vague definition of robustness raises the question if it can be seen as a new concept. Concepts such as resistance, vulnerability, resilience and redundancy have definitions that closely resemble that of robust design. In this paragraph, these concepts will be discussed and compared to the robustness concept.

**Resistance** can be defined as the ability of a system to withstand events without reduction of the stability of the system. The resistance of the system is measured by the exceedence probability of the limit states. For example, a dike that is designed on a failure probability of $10^{-4}$ yr$^{-1}$ will resist all water levels that have an exceedence frequency of less than $10^{-4}$ year.

**Vulnerability** has already been described in 10.3, based on the definitions as given by the JCSS [27]. It is assumed that vulnerability is a measure of the direct damage that occurs due to an event. For a vulnerable system, the direct damage due to an event will be disproportionately large compared to the size of the event. According to the JCSS [27] robustness relates to damage that occurs as a result of the direct damage. If a system is robust, there will be few indirect damage.

Lind [29], gives a definition of **vulnerability** that is based on the probability instead of on the consequence: “vulnerability is the ratio of the failure probability of the damaged system to the failure probability of the undamaged system”. Following this definition, a system is vulnerable if the failure probability increases drastically after direct damage of the system. This conflicts with the definition from the JCSS [27], that relates vulnerability to the direct damage.

**Redundancy** refers to the ability of the structure to create alternate load paths when one member is removed. A redundant system will therefore increase the robustness of a system, since small damage (variation) will not result in large consequences. Redundancy is therefore a tool for achieving robustness, but a redundant system is not necessarily robust and vice versa.

**Resilience** is a term that is used mainly in the US and refers to the ability of a system to quickly restore to full functionality after an extreme event has occurred. The resiliency of a structure will benefit from a robust structural system, since the damage due to an event will be relatively small.

When considering **robustness**, there is always a performance function that has to be made invariant against changes. This will result in a reduced additional system risk under variation of the uncontrollable conditions. System risk can easily be related to the above described concepts. In Figure 71 the classification of the different concept with regard to system risk is given schematically.
Figure 71 Classification of structural concepts

System robustness cannot be implemented in this scheme since it is a measure of how the system risk will develop under variation in the uncontrollable conditions. Since resistance, redundancy, vulnerability and resilience influence the system risk, they will also influence the system robustness. So robustness can be determined by imposing variation on the system and then calculate the variation in the system risk. This is made visual in Figure 72.

Figure 72 System robustness connected to comparable concepts
24. Recommendations

A robust system will have reduced lifecycle costs under unexpected changes in the uncertain parameters. The lifecycle cost is the sum of the investment costs and the net present value of the risk. The risk is determined by the loss function and by the probability density of the uncertain parameters. In Taguchi’s approach, investment costs are not explicitly taken into account. Investment costs play however an important role in civil engineering, since the investor of a project always aims at an economic optimum design. This economic optimum lies at the minimum of the investment costs and the risk. The risk is the integral of the loss function multiplied with the probability density function of the performance parameters. In this loss function, costs due to failure, functional losses and inspection and maintenance should be included. According to Doltsinis [19], the total risk is proportional to the expected fluctuations in performance functions such as the displacement and the stress. It is recommended that this proportionality is investigated. This can be done by defining the shape of the loss function. If this is a two-sided function, the total risk can be reduced by reducing the standard deviation, as proposed by Taguchi. If the loss function is one sided, the risk can also be reduced by keeping the probability density function of the performance far away from the area in which losses occur. In a full probabilistic economic optimization, the lifecycle costs are reduced without explicitly assigning the standard deviation or the mean value of the performance parameter. There is no need for separately reducing standard deviations.

Investigating the response of a system if it is subjected to unexpected variations of the loads, is essentially different from the standard design procedures. In the standard approach, the lifecycle cost function is based on the statistical data that are present. The design that is chosen, has a minimum in this cost function. However, it is not investigated how this system will perform if the statistical data are bad predictors for the future situations. Since a probability of occurring of these unexpected future situations is unknown, they cannot be considered in a full probabilistic approach.

It may be possible to design a system of which the lifecycle costs are less sensitive to discrepancies in the assumed load distributions than an alternative system. It is not expected that this is possible in the parameter design phase, in which the values of the design variables are optimized. Namely, such an optimization will generally result in only one cost minimum, if no discontinuities are present. It is therefore recommended to make the system robust to unpredicted future changes in the preliminary, or system design stage. The measures that can be used to do have been discussed in this part 4.
25. Recommendations for ‘robust’ optimization

It is recommended to define an overall lifecycle cost function that is as realistic as possible and that includes all uncertainties. Minimizing this lifecycle costs function will result in a system that is economic optimal, given the knowledge that is available in the design phase. However, not everything that happens in the lifetime of a system, can be accounted for in the design phase. For example, traffic intensity may increase tremendously and sea level rise may be higher than expected. Also, the expected discount and growth rates may change significantly due to the economic crisis. These cannot be taken into account when doing a full probabilistic approach. Therefore, it may be preferable to design a system that is minimally sensitive to such changes.

‘Robust’ optimization can be of added value if it is used to reduce the additional risk of the system in case of discrepancies in the assumed distribution functions. In chapter 18, it was concluded that this corresponds to a higher reliability for design problems with only one limit state, such as the dike height problem. For design problems with a nominal-is-best loss function, the additional risk may be reduced by reducing the standard deviation of the performance function. This is however only possible if the performance function is dependent on the uncertain parameters and if this dependency can be influenced. Also, since unpredicted parameter changes cannot be assigned a probability of occurring, higher investment costs cannot be justified. If the robust system has higher expected lifecycle costs than an alternative, less robust system, the choice will always be for the less robust system. To truly embrace robustness, a change in philosophy may be necessary. The new philosophy is to accept extra costs if this increases the robustness to discrepancies between the model and the reality. This philosophy can be justified by looking at past projects. In this study, it will be assumed that a robust system will only be chosen if it does not have higher expected lifecycle costs than any alternative less robust system.
26. Recommendations for robustness in the preliminary design

Robustness of the system can already be considered in the preliminary design of the system. This can be done by reducing or avoiding the hazards of the system, which may require out-of-the-box thinking. Also, when a system is designed, the event tree should investigated to prevent that local damage will result to large losses. This fits the definition of robustness since local damage is often unexpected. Keeping the losses of such an unexpected event low is equal to making the system robust. It can thus be said that a system is robust if the removal or damage of one member will not disproportionately increase the risk of the system. This is especially useful when designing truss systems, as is concluded in the case study on the three-bar truss. Another example of making the system robust in the preliminary design stage is the construction of a house on piles. Using multiple piles to carry the load is always more robust than only using one pile. Namely, in the latter case, the removal of one pile will lead to immediate failure.

27. Implementation of robustness

27.1. System design

In chapter 27, it was concluded that robustness can best be implemented in a system in the preliminary design stage. The available measures to implement robustness in the preliminary design stage, have been well summarized in the Eurocode:

“Potential damage shall be avoided or limited by appropriate choice of one or more of the following:

- avoiding, eliminating or reducing the hazards to which the structure can be subjected;
- selecting a structural form which has low sensitivity to the hazards considered;
- selecting a structural form and design that can survive adequately the accidental removal of an individual member or a limited part of the structure, or the occurrence of acceptable localised damage…” [4]

The first measure will reduce the risk due to changes in the avoided hazards and is therefore certainly a measure that will increase the robustness. Avoiding these hazards requires creativity in the preliminary design stage. The second measure will reduce the response of the system if changes in the parameters occurs. It is however unclear how this ‘insensitivity’ can be obtained. The measure has to be considered from case to case. The third measure has been investigated in the case studies, trough the robustness index and the vulnerability. It was concluded that the event tree should be closely investigated to understand the behaviour of the system under local damage, and to reduce the increase in risk in case local damage occurs.

In short, the following measures should be considered when designing a robust system:

- The risk may not increase disproportionally due to local damage or due to removal of one member. The event tree is a good tool to assess this measure.
- Hazards that greatly influence the probability of failure of the system should be avoided, eliminated or reduced. This requires a creative solution.
27.2. Optimization of design variables
Robustness can be implemented in a design by minimizing the risk increase due to discrepancies between the reality and the model as used in the design phase. However, the set of design variables corresponding to such a robust system, should also have a minimum in the expected lifecycle costs. It is therefore recommended to only optimize the expected lifecycle costs, which is the standard procedure. If several design alternatives have equal minimum lifecycle costs, the most robust design can be chosen from an analysis of the lifecycle costs when discrepancies are present. This does not require a ‘robust’ optimization task. Moreover, robust optimization task do not actually exists. All tasks that have been proposed in literature can be analysed back to an economic optimization.

27.3. Conclusion
From this chapter, it can be concluded that a robust system is characterised by:

- Minimal lifecycle costs.
- Minimal increase in lifecycle costs due discrepancies between the model and the reality.

In short, it can be concluded that:

Robust design is the process of designing a system that has a minimum increase in expected lifecycle costs if the reality is different from the model that was used in the design phase. The system also has minimum expected lifecycle costs, when evaluated in the design phase.

In the case studies, it will be investigated if it is possible to design a robust system, when using this definition. First the robust dike height problem is further investigated, to find what robustness means for dike heightening strategies. In the case study, sea level rise is used as a hazard that may have a different trend than assumed during the design phase. The hazard of sea level rise has recently become a popular topic and is seen as the most important future hazards for the Dutch dikes.

The second case is aimed at designing a sea dike. In this case study, the design variables are the dike height and the revetment thickness. The hazards are overtopping and outer slope erosion. Also, there is an option to either use a block revetment or a grass cover for the outer slope of the dike. It is investigated which of the two is the most robust option. This case study illustrates that it is possible to find a robust design alternative, as long as all options are considered. When designing a real dike, many more failure modes and design variables have to be considered. A system approach is necessary to find the robust combination of these design variables.

The third case study is aimed at designing a column under axial loading. This column has two design variables, namely the cross-sectional area and the prestressing. In the case study it is shown that two different design alternatives, with equal expected lifecycle costs, can be equally robust.
Part 6. Case studies

Conclusion

“The design alternative with the lowest expected risk in the design phase, is the most robust alternative.”
28. Robust dike
28.1. Introduction
In this case study, it is investigated how robustness can be implemented in the dike height problem. From the case studies in part 3, it has been concluded that a robust system has reduced overall lifecycle costs under unexpected variation of the loading. In this case study, the uncertain loading is the sea level rise. A robust dike has reduced expected lifecycle costs under this uncertain sea level rise. The case study is a continuation of the dike height case study in chapter 14. In this chapter, it was shown that the cost optimum dike height is 5.8 m, when using the standard approach. In this chapter, it will be investigated whether it is possible to design a dike that is robust to sea level rise.

When a dike is prone to an uncertain sea level rise, it is advisable to take into account improvement costs that may arise during the lifetime. These improvement costs arise due to degradation of the relative strength of the dike, due to an increase in the hydraulic loading. In this case study it is assumed that the storm surge level increases due to sea level rise. This increase may have as a result that improvement of the dike height is necessary. In the following paragraphs, first the loss function of the dike height problem is discussed. Following to this, the expected sea level rise will be discussed. This expected sea level rise will be used to define a lifecycle cost function that includes improvement costs. This lifecycle cost function will be used to find the optimum dike height. In the further study, it will be checked if this optimum is robust against unexpected changes. Also, the influence of the fixed investment costs $I_0$ will be investigated.

28.2. Loss function
In this design problem, the performance function is the difference between the dike height and the water level. It is assumed that structural failure due to breaching will occur immediately if the water level exceeds the dike height. In reality, other factors, such as the duration of the water level, determine the degree of failure. The loss function as used in this case study is shown in Figure 73.
The performance function is a function of the dike height, which is the design variable, and the water level, which is a random variable. The probability density function of the annual maximum water level is expressed by:

\[ p(z = z') = \frac{1}{\kappa} e^{-\frac{xz - \nu}{\kappa}} \]

The probability density function of the performance function \( h - z \) can then be expressed by:

\[ p(z = z') = \frac{1}{\kappa} e^{-\frac{xz - \nu}{\kappa}} \rightarrow p(h - z = z') = \frac{1}{\kappa} e^{-\left(\frac{z' + h - \nu}{\kappa}\right)} = pdf(h - z) \]

The probability density function of the performance, as a result of the water level distribution, is shown in Figure 74. From this figure, it can be concluded that the dike height influences the mean of the performance distribution, but not the standard deviation. In Figure 75, the probability density function of the performance and the loss function are shown in one graph.
**Figure 74** Probability density function of the performance parameter $h-z$, for different values of the dike height.

**Figure 75** Loss function of the dike height problem.
The total annual risk of exceedence of the water level limit state is then defined by:

\[ R = \int_{-\infty}^{\infty} p d f(h - z) \cdot f_{\text{loss}}(h - z) \, dh \]

In which:

\( p d f(h - z) \) is the probability density function of the value of the performance function

\( f_{\text{loss}}(h - z) \) is the loss as a function of the value of the performance function

For the dike height problem, the annual risk is then defined by:

\[ R(h) = \frac{D}{\kappa} \cdot \int_{-\infty}^{0} e^{-\frac{(z'+h)-\nu}{\kappa}} \, dz' = D \cdot e^{-\frac{h-\nu}{\kappa}} \]

This risk decreases with the dike height. The annual risk as a function of the dike height is shown in Figure 76. When following Taguchi, the aim would be to reduce the risk over the lifetime, which equals reducing the annual risk. From Figure 76 it can be concluded that the risk is minimum for an infinite dike height. This means that the robustness increase with the dike height.

\[ \times 10^{12} \]

**Figure 76 Annual risk as a function of the dike height**

In this study, it has already been recommended to minimize the overall lifecycle costs instead of only the risk. This is also the general approach in civil engineering. The overall lifecycle costs are the sum of the investment costs, the risk and in this case the improvement costs. In chapter 12 it was already shown that the sum of the investment costs and the risk is minimum for a dike height of 5.8 m, when using a full probabilistic approach. There is no added value for robustness in this case, since a more robust dike will always be less economical.
The definition of a robust system is that it does not only have minimum lifecycle costs, but also a minimum deviation in the lifecycle costs under a discrepancy between the reality and the model that was used to design the system. For the dike height problem, uncertain sea level rise is a good example of loading which may show a different trend during the lifetime of the dike, than expected in the design. The dike can be made robust to uncertain sea level rise by increasing the height of the dike. It is however also possible to adopt an improvement strategy that will make the lifecycle costs less sensitive to changes in the sea level rise. Both aspects will be investigated in the rest of this case study.

28.3. Sea level rise
Melting of the ice caps and expansion of the seawater causes a rise of the sea level. This rise has shown an acceleration in the last decennia, which is presumably due to the greenhouse effect. The future sea level rise can be predicted by using climate models that are based on data from the past. Because of the lack of knowledge on the mechanisms that cause climate change, the predictions are highly uncertain. For the sea level rise, this results in a bandwidth of the expected sea level rise in the future.

The KNMI used different scenarios to predicted the sea level rise at the Dutch coast. The result is given in Figure 77. This result is based on the scenarios as defined by the IPCC, the Intergovernmental Panel on Climate Change.

![Figure 77 KNMI climate scenario, with reference to 1990 [38]](image)

In September 2008, the Delta Committee published an advice [38] in which the predicted sea level rise was based on an additional research by the KNMI. This research was aimed at finding ‘plausible upper limits’ for the future sea level rise. The result is shown by the red area in Figure 78.
The prediction as shown in Figure 78 will be used in this study to investigate the robustness of the dike against sea level rise. The uncertainty in the predicted values is merely described by a bandwidth. In this study it is assumed that the bandwidths represent the 95% confidence interval and that the distribution of the sea level rise is Gaussian. The result is as follows:

- In the period 2000-2050, the rise is normally distributed with a mean value of 4.2 mm and a standard deviation of 1.0 mm.
- In the period 2050-2100, the rise is normally distributed with a mean value of 10.4 mm and a standard deviation of 2.3 mm.

These values for the sea level rise will be implemented in the maintenance cost function to find an optimum initial dike height.

### 28.4. Lifecycle cost function under sea level rise

When assuming that the water level distribution is prone to sea level rise, the annual risk in year $t$ can be expressed by:

$$R(h, t) = D \cdot P(h - z < 0, t) = D \cdot e^{-\frac{h-(v+\nu t)}{\kappa}}$$

In which $\nu$ represent the sea level rise in m/yr and $t$ denotes the year in which the water level is determined.

The probability density functions of $h - z$ in respectively year 1, 20 and 50 are shown in Figure 79. The probability density function shifts towards the failure area which means that the annual risk will grow with time. The sea level rise does not influence the shape of this density function, in this case study. In reality, the shape will change due to the correlation between water depth and for example wave set-up or storm surge.

Since the loss function is one-sided, reduction of the losses over the lifetime can be done by keeping the probability density function away from the failure space. It is not strictly necessary to reduce the standard deviation of the density function. The density function can be kept from the failure space, by increasing the dike height.
Now when considering the definition of robustness, a robust dike will have a smaller increase in lifecycle costs under changing conditions, than a less robust dike. When considering Figure 79, a dike can only be robust against sea level rise by shifting the probability density function to the right. This can be done by increasing dike height \( h \). In other words, a robust dike is a high dike. In this case, robustness is explicitly connected to higher investment costs.

![Loss function of the dike height problem](image)

**Figure 79** PDF of the performance function. The pdf shifts towards the failure area due to sea level rise but the shape remains unchanged.

In this case study, the additional costs under sea level rise will be further investigated. This can be done by investigating the net present value of the lifecycle costs for different construction heights.

The net present value of the risk is expressed by:

\[
NPV[R(h, t)] = \sum_{t=1}^{T} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (v + d_v t)}{\kappa}}
\]

In which \( T \) is the lifetime that is considered and \( d_v \) is the amount of sea level rise in meters in time \( t \) (years). Assume that \( d_v \) is equal to \( 2.0 \cdot 10^{-3} \) m.

The lifecycle cost function is the sum of the investment costs and the risk. The net present value of these costs is expressed by:

\[
C = I_0 + I_h \cdot (h - h_0) + \sum_{t=1}^{T} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (v + d_v t)}{\kappa}}
\]
28.5. Improvement

A robust system will have the least increase in lifecycle costs under conditions that are different from the expected conditions. For the dike height problem under sea level rise, this means that the robustness of a dike increases with its height. This is explained in paragraph 26.5. An increasing dike height corresponds to increasing investment costs, which means that there is a tradeoff between the robustness of the dike and the investment costs. In this case, the choice will always be for the economic optimum. Another strategy to make the dike robust against future changes in the sea level rise, is to do inspection during the lifetime and to improve only if necessary. This may save investment costs since improvement is not always necessary. However, in case improvement is indeed necessary, the fixed investment costs \( I_0 \) will have to be paid twice. Namely, for the initial construction and for the improvement.

In this case study, it is investigated whether an inspection strategy is more robust under uncertain sea level rise than a strategy in which the dike height that is constructed at the beginning in the lifetime is not improved during the lifetime. This is done by choosing an inspection moment, for example year 20 of the lifetime, at which it is decided whether the dike is heightened and to which level it is heightened. The criteria that should be fulfilled to make heightening necessary are:

- The actual dike height at the decision moment is smaller than the optimum dike height at that moment. This optimum is based on a minimum in the lifecycle cost, calculated from that moment on with the information that is present at that moment.
- The costs of improving the dike to its optimum, are less than the net present value of the risk that is averted by heightening the dike.

The corresponding lifecycle costs are analysed for a decision moment in year 20 and a decision moment in year 50. A total lifetime of 100 years is considered.

28.5.1. Inspection in year 20

Assume that inspection is done in year twenty. From this inspection, it is decided whether improvement is needed. The probability that improvement is needed is:

\[
P_{\text{imp}}(t) = P \left( \Delta R(t = 20) > I_0 + I_h \cdot (h_{\text{opt},20} - h) \right)
\]

In which:

\[
\Delta R(t = 20) = \sum_{t=1}^{80} \frac{D}{(1+r)^t} \left( e^{-\frac{h-(v+dv,20)}{k}} - e^{-\frac{h_{\text{opt},20}-(v+dv,20)}{k}} \right)
\]

\[
h_{\text{opt},20} = h_{\text{opt}} \min \left[ I_0 + I_h \cdot (h_{\text{opt}} - h) + \sum_{t=1}^{80} \frac{D}{(1+r)^t} \left( e^{-\frac{h_{\text{opt},20}-(v+dv,20)}{k}} \right) \right]
\]

In these expressions, \( dv \) is a random value that is normally distributed with the parameters as given in paragraph 18.2. and \( h \) is the dike height as initially constructed. The optimum dike height in year 20 depends on the total sea level rise that has occurred until year 20. Using a Monte Carlo analysis, the expected value of the optimum dike height in year 20 can be determined, as a function of the initial constructed dike height.
The cost function can now be expressed by:

\[ C = I_0 + I_h \cdot (h - h_0) + \sum_{t=1}^{20} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (\nu + d \cdot v \cdot t)}{\kappa}} + \]

\[ p_{\text{imp}} \left[ \frac{I_0 + I_h \cdot (h_{\text{opt},20}(h) - h)}{(1 + r)^{20}} + \sum_{t=21}^{100} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h_{\text{opt},20}(h) - (\nu + d \cdot v \cdot t)}{\kappa}} \right] + \]

\[ (1 - p_{\text{imp}}) \cdot \sum_{t=21}^{100} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (\nu + d \cdot v \cdot t)}{\kappa}} \]

The probability that improvement is needed in year 20, as a function of the initial constructed dike height is shown in Figure 80.

The probability that improvement is needed in year 20, as a function of the initial constructed dike height is shown in Figure 80.

\[ F_{IGURE \ 80 \ PROBABILITY \ THAT \ IMPROVEMENT \ IS \ NECESSARY \ IN \ YEAR \ 20} \]

This means that the probability that improvement is necessary in year 20, is non-zero if the initial dike height is smaller than 5,9 m. In Figure 81, the lifecycle cost function is shown as a function of the initially constructed dike height. The separate parts of this function are shown in Figure 82. A dike height of 6,1 m is the optimal initial dike height. For this initial dike height, no improvement is necessary in year 20. The optimum dike height is therefore entirely dependent on the initial construction costs and the risk, but not on the improvement costs. It can be concluded that both strategies are equally cost optimal, when assuming that inspection does not cost anything. This is shown in Figure 83. The total cost corresponding to a dike height of 6,1 m is €2,38\cdot10^8.
Figure 81 Expected value of the lifecycle costs of the dike under sea level rise. The lifetime of the dike is 100 years and after 20 years it is decided whether the dike height should be improved.
**Figure 82** Expected value of all lifecycle costs of the dike under sea level rise. The lifetime of the dike is 100 years and after 20 years it is decided whether the dike height should be improved.

**Figure 83** Expected value of the lifecycle costs depending on the improvement strategy that is chosen.
28.5.2. Inspection in year 50

Now consider that inspection is not done after 20 years but after 50 years. When assuming that the lifetime starts in 2015, the dike has been subject to 35 years of 4,2 mm/yr sea level rise and 15 years of 10,4 mm/yr sea level rise. It is expected that this will result in higher probabilities that improvement is needed for all dike heights, than when inspection is done in year 20. The lifecycle cost function is as follows:

\[ C = I_0 + I_h \cdot (h - h_0) + \sum_{t=1}^{50} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (\nu + d \cdot \nu \cdot t)}{k}} + \]

\[ P_{imp,50} \cdot \left[ \frac{I_0 + I_h \cdot (h_{opt,50} - h)}{(1 + r)^{50}} + \sum_{t=51}^{100} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h_{opt,50} - (\nu + d \cdot \nu \cdot t)}{k}} \right] + \]

\[ (1 - P_{imp,50}) \cdot \sum_{t=51}^{100} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (\nu + d \cdot \nu \cdot t)}{k}} \]

The result is shown in Figure 85, from which it can be concluded that shifting the decision moment to year 50, will not influence the optimum dike height. Improvement will be necessary after fifty years if the initial constructed dike height is 5,7 m or smaller, as shown in Figure 84. In The cost optimum is again entirely dominated by the initial investment costs and the risk over the lifetime, but not by the improvement costs. This shown in Figure 85, Figure 86 and Figure 87. The resulting optimum dike height for construction is: \( h = 6,1 \) m and the total cost corresponding to this dike height is \( €2,38 \cdot 10^8 \).

![Figure 84 Probability that improvement is necessary in year 50](image)
**Figure 85** Expected value of the lifecycle costs of the dike under sea level rise, if inspection is done after 50 years.

**Figure 86** Expected value of all lifecycle costs of the dike under sea level rise, if inspection is done after 50 years.
28.5.3. Conclusion
Using an inspection strategy will not lead to improvement if an economic optimum dike height was already constructed in the design phase. This means that both strategies result in the same lifecycle costs, and that the inspection strategy may actually be more expensive in case inspection costs money. An inspection strategy will however reduce the additional lifecycle costs if the initially constructed dike height is not at the economic optimum. This may be due to errors in the construction phase or errors in the predictions. In the next paragraph, it will be investigated what the influence is of a sea level rise that is higher than predicted. It is expected that an inspection and improvement strategy will reduce the lifecycle costs in such a situation.
28.6. Unexpected sea level rise

In paragraph 26.6, it was concluded that the most optimal strategy for constructing the dike, is making it 6.1 m high from the beginning. In this case, inspection is not necessary. This conclusion is based on the prediction of the mean sea level rise and the standard deviation in this rise. However, if the actual sea level rise turn out higher than expected, this strategy may not be optimal.

In this paragraph it is assumed that the actual sea level rise is three times higher than expected. This will be found out during the inspection. The initially constructed dike height is 6.1 m, based on the cost optimum under the expected sea level rise. If inspection is done, the dike height can be adjusted to the increase sea level rise. If no inspection is done, the dike will have a height of 6.1 throughout its lifetime and will be subject to a higher risk than expected. The robust strategy is the strategy that results the smallest value overall ‘expected’ lifecycle costs over the lifetime.

28.6.1.1. Inspection in year 20

The resulting cost function for inspection in year 20, is:

\[
C = I_0 + I_h \cdot (h - h_0) + \sum_{t=1}^{20} \frac{D}{(1 + r)^t} \cdot e^{\frac{h-(v+d \cdot v \cdot t)}{\kappa}} + \\

P_{imp,20} \cdot \left[ \frac{I_0 + I_h \cdot (h_{opt,20} - h)}{(1 + r)^{20}} + \sum_{t=21}^{100} \frac{D}{(1 + r)^t} \cdot e^{\frac{h_{opt,20}-(v_{20}+d \cdot v \cdot t)}{\kappa}} \right] + \\

(1 - P_{imp,20}) \cdot \sum_{t=21}^{100} \frac{D}{(1 + r)^t} \cdot e^{\frac{h-(v_{20}+d \cdot v \cdot t)}{\kappa}}
\]

In which \( A_{20} \) represents the sea level in year 20 and \( d v_3 \) represent the 3 times higher sea level rise in the future.

In Figure 88, the probability that improvement is necessary in year 20 is shown. When comparing this figure to Figure 80, it becomes clear the probability that improvement is necessary in year 20 has greatly increased due to the additional sea level rise. For a dike height of 6.1 m, the probability that improvement is necessary in year 20 is 0.999. In Figure 89, it can be seen that an initial constructed dike height of 6.1 is no longer the optimal solution, if no inspection is done during the lifetime. This is the result of the increased risk due to the additional sea level rise. In Figure 89, the expected lifecycle costs are shown for both strategies under the expected climate scenario and the three times higher sea level rise. The additional expected costs due to the three times higher sea level rise are €1.2 \( \cdot 10^8 \) if an inspection is performed. The additional expected costs are €4.4 \( \cdot 10^8 \) if no inspection is performed. Clearly, the inspection strategy is the most robust option, since it is subject to the smallest increase in lifecycle costs, in case of a discrepancy in the sea level rise prediction.
**Figure 88** Probability that improvement is necessary in year 20, for a sea level rise that is three times higher than expected.

**Figure 89** Lifecycle cost functions in case of expected sea level rise and 3 times higher sea level rise, for a strategy with and without inspection in year 20.
28.6.2. Inspection in year 50
The resulting cost function for inspection in year 50, is:

\[ C = I_0 + I_h \cdot (h - h_0) + \sum_{t=1}^{50} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (v + dV_3 t)}{K}} + \]

\[ P_{imp,50} \cdot \left[ I_0 + I_h \cdot (h_{opt,50} - h) + \sum_{t=51}^{100} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h_{opt,50} - (v_{50} + dV_3 t)}{K}} \right] + \]

\[ (1 - P_{imp,50}) \cdot \sum_{t=51}^{100} \frac{D}{(1 + r)^t} \cdot e^{-\frac{h - (v_{50} + dV_3 t)}{K}} \]

In which \( A_{50} \) represents the sea level in year 50 and \( dV_3 \) represent the 3 times higher sea level rise in the future.

In Figure 91, the probability that improvement is necessary in year 50 is shown. When comparing this figure to Figure 84, it becomes clear the probability that improvement is necessary in year 50 has greatly increased due to the additional sea level rise. For a dike height of 6,1 m, the probability that improvement is necessary in year 20 is equal to 1. In Figure 92, it can be seen that an initial constructed dike height of 6,1 is no longer the optimal solution, if no inspection is done during the lifetime. This is the result of the increased risk due to the additional sea level rise. In Figure 93, the expected lifecycle costs are shown for both strategies under the expected climate scenario and the three times higher sea level rise. When the dike is initially constructed at 6,1 m, the expected...
The lifecycle cost under the predicted sea level rise is equal to €2.36 \times 10^8. The additional expected costs due to a three times higher sea level rise are €0.8 \times 10^8 if an inspection is performed. The additional expected costs are €4.3 \times 10^8 if no inspection is performed. Clearly, the inspection strategy is the most robust option, since it is subject to the smallest increase in lifecycle costs, in case of a discrepancy in the sea level rise prediction.

**Figure 91** Probability that improvement is necessary in year 50, for 3xSLS.
Figure 92 Lifecycle cost functions in case of expected sea level rise and 3 times higher sea level rise, for a strategy with and without inspection in year 50.

Figure 93 Lifecycle cost functions in case of expected sea level rise and 3 times higher sea level rise, for a strategy with and without inspection in year 50. The black line is at $h=6.1$ m.
28.6.3. Conclusion
In the above paragraph it is shown that using an inspection strategy will decrease the additional lifecycle costs of an unpredicted high sea level rise occurs during the lifetime of the dike. It can be concluded that an inspection and improvement strategy is a robust strategy if the following criteria are fulfilled:

- The actual dike height at the decision moment is smaller than the optimum dike height at that moment.
- The costs of improving the dike to its optimum, are less than the net present value of the risk that is averted by heightening the dike.

28.7. Influence of $I_0$
When assuming that the sea level rise is accurately predicted, it is most cost-efficient to initially construct a high dike so that improvement is not necessary during the lifetime. This means that the extra construction costs at the beginning of the lifetime, are less than the risk of having to improve. It is expected that this result will change if the improvement costs are reduced. In this paragraph, it is investigated whether this hypothesis is true. The choice for waiting with the improvement may reduce lifecycle costs since improvement may not be necessary at the inspection moment, if sea level rise is smaller than expected.

28.7.1. Inspection in year 20
In this paragraph, the lifecycle costs for the inspection strategy are again analysed, for a fixed investment cost of €1·10^6 instead of €40·10^6. The results of this analysis are shown in Figure 94, Figure 95 and Figure 96.

![Figure 94: Probability that improvement is necessary in year 20, for $I_0=1·10^6$.](image-url)
Figure 95 Expected lifecycle costs for a strategy with inspection in year 20, for \( I_o = 1 \cdot 10^6 \).

Figure 96 Expected value of the lifecycle costs for a strategy with and without inspection in year 20.
In Figure 95, it is shown that the expected lifecycle cost has minimum of \(1,27 \times 10^8\) at a dike height of 5.8 m, for the inspection and improvement strategy. The probability that improvement is necessary in year 20, for an initial construction height of 5.8 m, is 0.996. This is shown in Figure 94. The height to which the dike has to be improved in this year is 6.2 m. When using a strategy without inspection, the minimum lifecycle costs are \(1,27 \times 10^8\) and correspond to an initially constructed dike height of 6.15 m. This means that there is no difference in the lifecycle costs between both strategies.

28.7.2. Inspection in year 50
In this paragraph, the lifecycle costs for the inspection strategy are again analysed, for a fixed investment cost of \(1 \times 10^6\) instead of \(40 \times 10^6\). The results of this analysis are shown in Figure 97, Figure 98 and Figure 99.

![Figure 97: Probability that improvement is necessary in year 50, for \(I_0 = 1 \times 10^6\).](image-url)

**Figure 97** Probability that improvement is necessary in year 50, for \(I_0 = 1 \times 10^6\).
FIGURE 98  EXPECTED LIFECYCLE COSTS FOR A STRATEGY WITH INSPECTION IN YEAR 50, FOR $I_0 = \text{€}1 \cdot 10^6$.

FIGURE 99  EXPECTED VALUE OF THE LIFECYCLE COSTS FOR A STRATEGY WITH AND WITHOUT INSPECTION IN YEAR 50, FOR $I_0 = \text{€}1 \cdot 10^6$. 

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In Figure 98, it is shown that expected lifecycle cost has minimum of €1,27\times 10^8 at a dike height of 5,9 m, for the inspection and improvement strategy. The probability that improvement is necessary in year 50, for an initial construction height of 5,9 m, is 0,932 (Figure 97). The height to which the dike has to be improved in year 50 is 6,2 m. When using a strategy without inspection, the minimum lifecycle costs correspond to an initially constructed dike height of 6,15 m . The corresponding lifecycle costs are €1,27\times 10^8. This means that there is no difference in the lifecycle costs between both strategies.

### 28.7.3. Conclusion

It can be concluded that for an $I_0$ of €1\times 10^6, both strategies result in equal costs. In 26.6 it was however concluded that an inspection and improvement is more beneficial under large unexpected increase of the sea level rise. If the value of $I_0$ is negligible compared to other lifecycle costs, there is no difference in the overall expected lifecycle between the two strategies. In this case, a robust solution is to choose the inspection and improvement strategy, since this strategy will result in reduced additional lifecycle costs under unexpected changes in the sea level rise.

### 28.8. Conclusion: Risk as a function of higher sea level rise

In this study, two dike heightening strategies have been investigated. In one strategy, the dike is constructed at once and no inspection and improvement take place during the lifetime. In the other strategy, the dike is inspected after 20 or 50 years and improvement is done at that moment if necessary. It was concluded that the latter strategy will reduce the additional lifecycle costs due to unexpected high sea level rise. In this paragraph, the exact influence of the strategy on the additional lifecycle costs is investigated.

In Figure 100, the risk over the lifetime is shown as a function of the multiplication factor of the sea level rise. This multiplication factor denotes how much higher the sea level rise is than expected during the design phase. It can be concluded that the inspection strategy results in lower additional lifecycle costs. Moreover, this trend increases with increasing sea level rise multiplication factor.

In Figure 101 the overall lifecycle costs are shown for both strategies with an initial dike height of 6,1 m. The inspection strategy becomes beneficial at if improvement is necessary in year 20. This is the case if the actual sea level rise is equal to 2,36 times the expected sea level rise. If the actual sea level rise is less than 2,36 times the expected sea level rise, improvement will not be necessary in year 20, which means that the total lifecycle costs equal the costs in case no inspection is done.

In Figure 102, the same graph is shown as in Figure 101, but then for a fixed investment costs of €1\times 10^6. From this graph it becomes clear, that for a low $I_0$, improvement will already be performed in year 20, for a multiplication factor of 1,2. It can be concluded, that with the decrease of $I_0$, it becomes more beneficial to use an inspection strategy. The strategy will reduce the lifecycle costs already for a small discrepancy between the expected and the actual sea level rise, which means that it is a robust strategy.
**Figure 100** Risk as a function of the multiplication factor for the sea level rise.

**Figure 101** Lifecycle costs as a function of the factor of unexpected sea level rise.
From the above figures it can be concluded that an inspection and improvement strategy is preferable since it will result in lower lifecycle costs under changing conditions. The strategy becomes more beneficial with decreasing $I_0$. 

**Figure 102 Lifecycle costs as a function of the factor of unexpected sea level rise, $I_0 = €1 \cdot 10^6$.**
28.9. Conclusion robust dike

In paragraph 26.2 it was concluded that the loss of the dike can be reduced by making the dike higher. For this design problem, a robust dike is a high dike. Also, if the water levels are higher than expected during the design phase, the additional lifecycle costs will be reduced by increasing the initial construction height. It can be concluded that in the dike heightening problem, a robust dike is always a high dike. However, it is also possible to adopt a maintenance strategy that will reduce the expected lifecycle costs of the dike. In this case study, a strategy is chosen in which the dike is inspected at one moment during its lifetime. The dike will be heightened after this inspection if:

- The actual dike height at the decision moment is smaller than the optimum dike height at that moment.
- The costs of improving the dike to its optimum, are less than the net present value of the risk that is averted by heightening the dike.

In 26.5 it was concluded that such a strategy is not beneficial if the mean sea level rise during the lifetime develops as predicted during the design phase. Namely, in this case, the sea level rise is already taken into account in the design phase and no extra inspection is needed. Only if the dike is constructed at a height that is much lower than the optimum, inspection and improvement is needed. This is however certainly not the most cost-efficient solution.

In 26.6 it was concluded that an inspection and improvement strategy is beneficial if the sea level during the lifetime turns out to be higher than expected in the design phase. In this case, the risk will be reduced because the dike height can be adjusted to the actual occurring sea level rise.

In paragraph 26.7, the influence of the fixed investment cost $I_0$ was investigated. If these costs are high, it is expensive to improve the dike during its lifetime, since that entails making the cost $I_0$ twice. However, an inspection and improvement strategy may save costs since improvement is not always necessary. In 26.7 it was concluded that for a small $I_0$, the minimum expected lifecycle costs are equal for the inspect and construct strategy and for the initial construction strategy. The robust choice would be to choose the inspect and construct strategy since the expected lifecycle costs are the same but the additional lifecycle costs due to extra sea level rise are reduced.

The inspection and improvement strategy can be called a robust strategy, since it will result in reduced lifecycle costs in case of unexpected variations in the sea level rise. The strategy becomes more beneficial if $I_0$ is smaller.

The overall conclusion from this case study, is that a dike can only be made robust in the design stage, by making the dike higher. In order to increase the robustness over the lifetime, it is also an option to use an inspection and improvement strategy. Such a strategy will result in reduced additional lifecycle costs if the sea level rise is higher than expected in the design phase.

It should be noted that in a realistic design problem, there are more design variables than only the dike height. Also, there are many more failure modes than only overtopping. This means that many more loss functions and performance parameters have to be taken into account, and that it may be possible to find a combination of design variables for which the expected lifecycle costs are reduced. In other words, to truly find a robust design during the design stage, it is important to do an analysis of the full system and all its failure modes.
29. The robust design of a sea dike along the Frisian coast
29.1. Introduction
This case study is inspired by an article by M. Hussaarts [39], in which the optimal thickness of a dike revetment is determined using a standard economic optimization. In this case study, it is investigated whether a dike design with a revetment or with a grass cover is more robust. The design variables are the dike height and the thickness of the cover. These variables are optimized for both design alternatives, so that the resulting two design have the same expected lifecycle costs. Following to this economic optimization, the additional costs in case of changes in the water level distribution and the wave height distribution occur. The design with the least additional lifecycle costs is the most robust design.

The case study will start with a definition of the scope. Following, the design problem will be concretized by defining the design variables and the failure modes. Then, the necessary hydraulic boundary conditions will be defined. First the standard economic optimization approach will be used to find an optimum dike design for a dike with a grass cover and with a revetment. At last, it is investigated which design is the most robust design.

29.2. Scope
A sea dike has to be built to protect the Frisian coast from flooding. In this study, it is assumed that the sea dike fails if flooding of the hinterland occurs. This flooding can be caused by overtopping of the dike or by breaching of the dike. It is assumed that the governing mechanism for breaching, is erosion of the outer slope. Breaching due to erosion occurs if the revetment layer as well as the underlying clay layer have eroded.

The design variables in this case study are the dike height and the revetment thickness. The uncertain parameters are the hydraulic boundary conditions. All other parameters are assumed to be deterministic.

29.3. Failure Modes
29.3.1. Overtopping
The limit state function for overtopping is:

\[ Z = h - z \]

In which:

\[ h \] = dike height [m]

\[ z \] = storm surge level [m]

29.3.2. Erosion of grass cover
The limit state equation for the erosion of a grass cover is:

\[ z = t_{RG} - t_s = \frac{d_G}{\gamma_G \cdot c_E \cdot H_s} - t_s \]

In which:

\[ t_{RG} \] = duration of grass erosion [h]

\[ t_s \] = storm duration [h]
29.3.3. Erosion of revetment

The limit state equation for erosion of a revetment by waves is:

\[ z = D_{n50,\text{actual}} - D_{n50} \]

In which:

- \( D_{n50,\text{actual}} \) = actual stone diameter of revetment elements
- \( D_{n50} \) = required stone diameter of revetment elements

In this study, Hudson’s formula for the required stone diameter is used:

\[ D_{n50} = \frac{1,27 \cdot H_s}{\Delta(K_D \cdot \cot(\alpha))^{\frac{1}{3}}} \]

In which:

- \( H_s \) = significant wave height [m]
- \( \Delta \) = relative density of revetment = \( \frac{\rho_{\text{rev}} - \rho_w}{\rho_w} \) [\( \cdot \)] = 1,65
- \( \alpha \) = outer slope angle [°]
- \( K_D \) = 4,0 for non-breaking waves

The limit state equation for erosion of a clay cover is:

\[ z = t_{RK} - t_s = 0,4 \cdot \frac{L_K \cdot c_{RK}}{r^2 \cdot H_s^2} - t_s \]

In which:

- \( d_K \) = thickness of clay cover
- \( L_K = \frac{d_K}{\sin(\alpha)} \)
- \( r = 1 \), \( \beta \leq \beta_r \)
- \( r = \frac{(110^\circ - \beta)}{(110^\circ - \beta_r)} \), \( \beta > \beta_r \)
- \( c_{RK} = 54 \cdot 10^3 \) good clay
29.4. Hydraulic boundary conditions

The hydraulic boundary conditions that have to be inserted in the limit states are the storm surge level and the significant wave height. According to [39], the storm surge level is Gumbel distributed with parameter values $v = 2.91$ m and $\kappa = 0.36$ m. This distribution is expressed by:

$$P(z > h) = 1 - \exp\left(-e^{-\frac{h-v}{\kappa}}\right) [\text{yr}^{-1}]$$

It is assumed that the following is true for the relation between the wave height and the water level:

$$H_s = 0.6 \cdot z [\text{m}]$$

This results in the following distribution for the probability of exceedence of the wave height:

$$P(H_s > H) = 1 - \exp\left(-e^{-\frac{H_s}{\frac{v}{\kappa}}}\right) [\text{yr}^{-1}]$$

29.5. Standard approach

The system as illustrated in the fault tree is a series system. The probability of failure of a series system with two statistically independent elements has been given in paragraph 5.2.1.

$$P_{f,\text{system}} = P(E_1) + P(E_2) - P(E_1) \cdot P(E_2)$$

The failure probability of the sea dike is then:

$$P_f = P_{ov} + P_{er} - P_{ov} \cdot P_{er}$$

In which:

- $P_f$ = the annual probability of failure of the sea dike [-]
- $P_{ov}$ = the annual probability of overtopping of the sea dike [-]
- $P_{er}$ = the annual probability of breaching due to erosion of the sea dike [-]
29.5.1. System design
In this paragraph, the shape of the dike will be determined. A sea dike generally has the smallest slope at the sea side to reduce the wave impact. In Figure 104 the typical shape of a sea dike is given. This shape will be used in the parameter design stage.

![Figure 103 Fault tree for dike design](image)

**Figure 103 Fault tree for dike design**

29.5.2. Parameter design
In this stage, the values of the following design variables will have to be determined:

- Height of dike $h$ [m]
- Thickness of revetment blocks $d_r$[m] or thickness of clay layer $d_K$[m]

This will be done using a cost optimization. The cost function is expressed by:

$$C = I_0 + I_{body} \cdot f(h) + I_{cover} \cdot f(h, d_r, d_K) + \frac{D \cdot P_f}{r' - g}$$

In which:

$$P_f = P_{ov} + P_{er} - P_{ov} \cdot P_{er}$$

And:

$$P_{ov} = P(z > h) = 1 - \exp\left(-e^{\frac{h - \nu}{\kappa}}\right) \, [yr^{-1}], \quad \nu = 2,91 \quad \kappa = 0,36$$
\[ P_{er} = P_{er,D} \cdot P_{breaching|er,D} = P_{er,D} \cdot P_{er,d_K} \]

\[ P_{er,D} = P(D_{n50} > d_r) \text{ revetment} \]

\[ P_{er,d_K} = P(t_s > t_{RK}) \text{ clay cover} \]

Evaluation of the limit state equation and the hydraulic boundary conditions for the clay and revetment dike results in:

\[ P_{er,str} = 1 - \exp\left(-e^{-\frac{d_r/(0.105-0.6) \cdot \nu}{\kappa}}\right) \text{ [yr}^{-1}] \]

\[ P_{er,d_K} = 1 - \exp\left(-e^{-\frac{0.4 \cdot L_K \cdot c_{RK}}{t_s \cdot 0.6^2 \cdot \nu} \cdot \kappa}\right) \text{ [yr}^{-1}] \]

So that the annual probability of erosion of the revetment is defined by:

\[ P_{er,rev} = \left(1 - \exp\left(-e^{-\frac{d_r/0.105-A}{B}}\right)\right) \cdot \left(1 - \exp\left(-e^{-\frac{0.4 \cdot L_K \cdot c_{RK}}{t_s \cdot 0.6^2 \cdot \nu} \cdot \kappa}\right)\right) \text{ [yr}^{-1}] \]

Evaluation of the limit state equation and the hydraulic boundary conditions for the clay and grass dike results in:

\[ P_{er,grass} = 1 - \exp\left(-e^{-\frac{C_1/(t_s \cdot 0.6) - \nu}{\kappa}}\right) \text{ [yr}^{-1}] \]

In which:

\[ C_1 = \frac{d_G}{y_G \cdot c_e} + \frac{0.4 \cdot L_K \cdot c_{RK}}{r^2} \]

**Block revetment and clay layer**

For a dike with a block revetment on a clay layer, the design variables are:

- Height of the dike \( h \) [NAP + m]
- Thickness of revetment \( d_r \) [m]

The uncertain parameters are:

- Water level \( z \) [NAP + m]
- Significant wave height \( H_s \) [m]

The deterministic parameters are:

- velocity coefficient \( y_G = 1,0 \)
- duration of storm \( t_s = 4,6 \) h
- coefficient for erosion resistance \( c_{RK} = 54 \cdot 10^3 \) for good clay
- reduction factor for oblique wave attack \( r = 1 \) for normal wave attack
The cost function has to be minimized. This can be done if a complete expression of the cost function is defined. For this, the shape and the material of the soil body and the revetment has to be considered. Also the unity prices of the material should be known. In this case study, it is assumed that the soil body exists entirely of sand. The cost of the dike can therefore be determined from the volume price of the sand. When assuming that the sand is obtained from dredging, the price of applying the sand can be set at €9/m$^3$. The inner slope is assumed to be 1:3 and the crest is assumed to have a width of 3 m.

The volume under the inner slope and the crest per meter length of the dike is:

$$V_{in} + V_c = \frac{1}{2} \cdot h^2 \cdot 3 + 3 \cdot h \left[ \frac{m^3}{m} \right]$$

The volume under the outer slope of the dike per meter length of the dike, is:

$$V_{out} = \frac{1}{2} \cdot 4 \cdot h^2 \left[ \frac{m^3}{m} \right]$$

The cost for the revetment is €100/m$^3$. Under a revetment a clay layer is always applied, it is assumed that this clay layer has a thickness of 0,05 m. The cost of applying the clay is assumed to be €50/m$^3$. The area per meter length of the dike that has to be covered with a revetment and clay cover is:

$$A_{gr} = \frac{h}{\sin(\alpha)} \approx 4 \cdot h \left[ \frac{m^2}{m} \right]$$

From the above equations, the following cost function can be defined.

$$C = I_0 + I_{body} \cdot \left( \frac{1}{2} \cdot h^2 \cdot 3 + 3 \cdot h + 2 \cdot h^2 \right) + (I_{clay} + I_r) \cdot 4 \cdot h + \frac{D}{\rho - g} \cdot (P_{ov} + P_{er} - P_{ov} \cdot P_{er})$$

In which:

- $I_{body} = 9$ [€/m$^3$]
- $I_{clay} = 50$ [€/m$^3$]
- $I_r = 100$ [€/m$^3$]

Now assume:

- $I_0 = 200$ €/m
- $D = 2500$ €/m

In MATLAB, a combination of the dike height $h$ and the revetment thickness was found that corresponds to a minimum in the costs. This minimum is at a dike height of 5,3 m and a revetment thickness of 0,38 m. The corresponding cost per meter dike is 2331 €/m$^3$ and the failure probability is 0,0013. This result is shown in Figure 105.
Figure 105 Optimal combination of revetment thickness and dike height, the red square is located at the optimum.
Grass and clay cover

For a dike with a grass cover on a clay layer, the design variables are:

- Height of the dike \( h \) [NAP + m]
- Thickness of clay cover \( d_K \) [m]

The uncertain parameters are:

- Water level \( z \) [NAP + m]
- Significant wave height \( H_s \) [m]

The deterministic parameters are:

- Coefficient for erosion resistance \( c_e = 1.5 \cdot 10^{-6} \) (m s)\(^{-1}\) for good to average quality
- Velocity coefficient \( \gamma_G = 1.0 \)
- Duration of storm \( t_s = 4.6 \cdot h \)
- Thickness of grass cover \( d_G = 0.2 \) m
- Slope of the seaward face of the dike \( \alpha = 14^\circ \)
- Obliqueness factor \( r = 1.0 \)

The cost for a grass cover with a thickness of 0,2 m is €0,6/m\(^2\). Under a grass cover, a clay layer is always applied. The thickness of this clay layer is the thickness under consideration in the erosion limit state. The cost of applying the clay is assumed to be €50/m\(^3\). The area per meter length of the dike that has to be covered with a clay-grass cover is:

\[
A_{gr} = \frac{h}{\sin(\alpha)} \text{[m}^2/\text{m]}\]

From the above equations, the following cost function can be defined.

\[
C = I_0 + I_{body} \cdot \left( \frac{1}{2} \cdot h^2 \cdot 3 + 3 \cdot h + \frac{1}{2} \cdot h^2 \cdot \frac{1}{\tan(\alpha)} \right) + I_{cover} \cdot \frac{h}{\sin(\alpha)} + \frac{D \cdot P_f}{r' - g}
\]

\( I_{body} = 4 \text{ [€/m}^3\] \)

\( I_{cover} = (0.6 + 50 \cdot d_K) \text{ [€/m}^3\] \)

Now assume:

\( I_0 = 200 \text{ €/m} \)

\( D = 2500 \text{ €/m} \)

In MATLAB, a combination of the dike height \( h \) and the clay layer thickness was found that corresponds to a minimum in the costs. This minimum is at a dike height of 5,3 m and a clay layer thickness of 0,58 m. The corresponding cost per meter dike is 2331 €/m\(^1\) and the failure probability is 0,0027. This result is shown in Figure 106.
Figure 106: Optimal combination of clay layer thickness and dike height, for a grass covered dike. The red square is located at the optimum.
29.6. Selection of robust alternative

In the preceding paragraphs, two dike design alternatives have been proposed that both have the same lifecycle costs. Both alternatives perform therefore equally well from a lifecycle cost point of view. Now a decision between the two can be made, based on robustness. The alternative that has the least additional lifecycle costs due to changing load distributions, is the most robust alternative.

The uncertain loads in this design problem are the water level and the wave. Since the wave height is a function of the water level distribution. Changes in the load distributions are a result of changes in the parameters $\nu$ and $\kappa$ of the water level distribution. Therefore, it is investigated what is the result of a 10% increase and decrease of the parameters. The result is given in Figure 107, in which the green graph represent the dike with a grass cover and the blue graph represents the dike with the revetment. Clearly, a discrepancy between the assumed and the actual distribution parameters has the largest influence on the lifecycle costs of the grass-covered dike. This large influence works in both the positive (cost-reducing) and negative (cost-increasing) direction, but is larger in the negative direction. The dike with the revetment is for that reason the most robust design alternative.

The reason that the revetment design alternative, is the most robust alternative, can be found in the influence of the parameter distributions on the total risk. The revetment dike is higher than the grass dike, which means that the risk of overtopping will have a faster increase for the revetment dike. Since the risk of a revetment dike shows a smaller overall influence of the parameter changes, this must be the result of the difference between the revetment cover and the grass cover. It can be concluded that the increase in risk due to the discrepancy in the assumed and the actual water level distribution, is in this case dominated by the erosion risk.

![Figure 107 Lifecycle costs, in case actual distribution parameter values are different than the values assumed in the design phase. The green graph denotes the grass covered dike and the blue graph denotes the revetment dike.](image-url)
29.7. Conclusion
In this case study it was investigated whether a revetment dike or a grass-covered dike is the most robust design alternative for a sea dike at the Frisian coast. In 29.5 it was concluded that the revetment dike is the most robust alternative, since its lifecycle costs are least sensitive to a discrepancy between the assumed distribution parameters and the actual distribution parameters. It should be noted that the revetment dike initially has a failure probability that is twice as small as that of the grass-covered dike. Since the failure probability increases exponentially, the increase from a small failure probability will be less than the increase from a larger failure probability. The robust alternative is therefore also the alternative with the smallest failure probability.
30. Case study column under axial loading

**Figure 108** Column under axial loading, the small arrows denote the prestressing.
30.1. Introduction
Consider a column under axial loading as shown in Figure 108. The column has to resist a variable axial tensile loading $Q$. The yearly mean of the loading is normally distributed with the following parameters:

$$\mu_Q = 50 \text{ kN}$$
$$\sigma_Q = 5 \text{ kN}$$

The loss function of this problem, is two-sided and determined by a upper axial stress constraint of 400 N/mm$^2$ and a lower axial stress constraint of 50 N/mm$^2$. The upper bound refers to breaking of the column. The lower bound may for example be related to fitting problems that arise due to the length of the column, that is influenced by the tensile stress. The designer has two options to reduce the axial stress in the column, namely by increasing its cross-sectional area and by applying prestressing. The investment costs associated with these options are:

$$I_A = 5,0 \left( \frac{\text{€}}{\text{mm}^2} \right)$$
$$I_{Pr} = 12,5 \left( \frac{\text{€}}{\text{kN}} \right)$$

The lifecycle cost function is defined as:

$$C = I_0 + I_A \cdot A + I_{Pr} \cdot P_f + \frac{D \cdot P_f}{r' - g}$$

In which:

$$I_0 = 100 \text{ €}$$
$$D = 2000 \text{ €}$$
$$r' - g = 0,015 [-]$$

30.2. Economic optimum design
In this paragraph, two design alternatives are checked. The first alternative is a column without prestressing. The second alternative is a column with a fixed cross-sectional area of 125 mm$^2$ and additional prestressing.

**Alternative 1**

If no prestressing is used, the lifecycle cost function simplifies to:

$$C = I_0 + I_A \cdot A + \frac{D \cdot P_f}{r' - g}$$

This function can be evaluated by using a Monte Carlo analysis to determine the probability of failure of the column, for a certain cross-sectional area. A minimum in the cost function was found for a cross-sectional area $A$ of 171 mm$^2$. The corresponding lifecycle costs is €970. The risk of this system is 15,7 €/yr.
Alternative 2

For a column with a cross-sectional area of 125 mm$^2$ and prestressing, the lifecycle cost function simplifies to:

$$C = l_0 + I_A \cdot 125 + I_{Pr} \cdot P_r + \frac{D \cdot P_f}{y'} - g$$

This function is evaluated using a Monte Carlo analysis. The lifecycle cost minimum corresponds to a prestressing of 147 N/mm$^2$, and has value of €970. The risk of this system is €16,0 €/yr, this is approximately the same as the risk of the column without prestressing. This means that the investment costs are almost equal for both design alternatives.

30.3. Influence of discrepancy in load distribution

In the preceding paragraph, two design alternatives have been proposed that have equal expected lifecycle cost, given that the load is normally distributed with a mean of 50 kN and a standard deviation of 5 kN. In this paragraph, it will be determined which of the alternatives is the most robust alternative. This is done by investigating the influence of changes in the distribution parameters, on the lifecycle costs of the system. In Figure 109, the stress distributions are shown that correspond to the two design alternatives. The distribution of alternative 1 has the smallest standard deviation. When following Taguchi’s approach, it is expected that the alternative with the smallest standard deviation in the stress distribution, is the most robust alternative. This hypothesis is tested by investigating the increase in the lifecycle costs as a result for changes in the load distribution, for
both alternatives.

![Diagram of stress distributions for two design alternatives](image)

**Figure 109 Stress distributions for the two design alternatives, as a result of the expected load distribution.**

*Increase in mean loading*

If the actual mean value of the load distribution is equal to 50 kN instead of 55 kN as assumed in the design phase, the actual stress distributions for both alternatives are as shown in Figure 110. The lifecycle costs that corresponds to both design, under this load distribution:

\[ C_1 = €1444 \]
\[ C_2 = €1450 \]

From this result it can be concluded that alternative 1 is most robust to a discrepancy between the assumed mean value of the load distribution and the actual mean value.
Increase in the standard deviation of the loading

If the actual standard deviation of the load distribution is equal to 5,5 kN instead of 5,0 kN as assumed in the design phase, the actual stress distributions for both alternatives are as shown in Figure 111. The lifecycle costs that corresponds to both design, under this load distribution:

\[ C_1 = €1009 \]

\[ C_2 = €1009 \]

From this result it can be concluded that alternative 1 is most robust to a discrepancy between the assumed standard deviation of the load distribution and the actual standard deviation.
30.4. Conclusion

In this case study, it is shown that it is possible that during the design phase different design alternatives are possible, that have equal expected lifecycle costs. In such case, it is an option to decide between the options based on robustness. Namely, robust system will have the least additional costs in case the assumed distribution parameters are different from the actual distribution parameters to which the system is subject during the lifetime. There is however not a tradeoff between robustness and expected lifecycle costs, since the robust system also has minimum lifecycle costs.

It should be noted that in this case study, both alternative are equally robust, since the additional lifecycle costs under changing parameters are equal. It is expected that the reason for this result is that both alternatives had the same risk to start off with. Also the stress distributions of both alternatives are normal. Any change in the load distributions function will therefore result in an equal increase in risk for both alternatives. The symmetry of the loss function may also play a role in this result. This conclusion agrees with the conclusion of the case study in which a robust sea dike was designed. Namely, in this case study it was concluded that the design alternative with the lowest expected risk, is the most robust alternative.
“The additional risk that arises due to discrepancies between the reality and the model, is smaller for a robust system than for a less robust system.”
31. Conclusions

31.1. Final definition
The final definition for a robust system that follows from this study is:

The additional risk that arises due to discrepancies between the reality and the model, is smaller for a robust system than for a less robust system.

31.2. Taguchi
The concept of robustness or robust design was first introduced by Taguchi. The method that Taguchi developed to design a robust product was based on the assumption that the loss of a product increases quadratically with the deviation from the target performance. The performance function of a robust product has a relatively small standard deviation and will therefore have lower losses during the lifetime. The main conclusions on Taguchi’s approach that have been drawn in this thesis are:

- The quadratic loss function is not a realistic function. The loss function, or functions, should be defined separately for every design problem.
- Taguchi’s approach is aimed at minimizing the losses, while in the standard approach the loss as well as the investment costs are minimized. The latter will result in an economic optimal solution, which is always the preferred solution in civil engineering.

31.3. Robust parameter design
The robust counterpart approach and the robustness function approach are optimization methods that are aimed at finding a system that will not exceed the constraints under any possible value of the uncertain parameters. This uncertainty is described by interval bounds of the uncertain parameters. By designing this way, it is considered that losses will not occur during the lifetime. In reality, there is however always a probability that a loss will occur. For an economic optimization, this risk has to be weighed against the investment costs. Such a consideration is not made in the robust counterpart approach and the robustness function approach. It can be concluded that the approaches are less sophisticated than a probabilistic approach. The approaches are claimed to be useful if no statistical data are available. However, some knowledge should be available as to which interval bounds to choose, since unrealistic large bounds may results in very high investment costs. In civil engineering, a lack of statistical data is solved by assuming a distribution function, based on expert’s opinions. This distribution is then adjusted when statistical data become available. This way, all information and knowledge is used to make a well-founded decision, which makes it a more sophisticated approach.

The multiobjective optimization approach simultaneously optimizes two objectives. The choice of these objectives depends on the aims of the designer. In literature, it was proposed to minimize the investment cost and the standard deviation of the performance function or to minimized the expected value and the standard deviation of the performance function. Optimizing two objective function simultaneously will lead to a range of Pareto-optimum solutions, of which none is the best. This problem can be solved by using a weighted-sum approach, which means that both objectives are assigned a relative weight and that the sum of both weighted objectives is minimized. This results in nothing more than a single objective optimization. In the thesis is has shown that the two objectives
can best be added by expressing them both in monetary values, which means that the single-objective is the lifecycle cost. The conclusion is that multiobjective optimizations are not of added value to the standard procedures, which means that they are not fit for designing ‘robust’ systems.

31.4. Robustness in structural engineering
In the design process of a structure, robustness can already be applied in the preliminary or system design stage. This can be done by making the behaviour of the structure insensitive to hazards or to local damage. This requires creativity, but in the Eurocode a list of measures are given to design such an insensitive structure. Moreover, recently the JCSS introduced the robustness index and the vulnerability factor, that can be used to investigate the response of a structure to local damage. The resulting values should however be treated carefully. A more accurate view on the response of the structure can be found by separately investigating the event tree, that has to be constructed anyway to find the robustness index.

For robustness in coastal structures, it has been proposed in literature to take into account all uncertainties in predicted load variables in the economic optimization. By doing this, all hazards that can be thought of are accounted for in the design phase. Following this recommendation will result in a design that performs better under all considered uncertainty, but is not specifically aimed at increasing the robustness. Another recommendation that is done in literature is to make a system adaptable, in order to reduce improvement costs in case this turns out to be necessary in the future. Making the system adaptable may indeed reduce the lifecycle costs in case actual load and parameter distributions are different than assumed. The benefits of making a system adaptable should be investigated in a cost benefit analysis.

31.5. Expected added value of robust design
A robust system can be defined as a system that will have the least additional expected lifecycle costs if the actual uncertain parameter distribution differ from the distributions as assumed during the design phase. A robust system will only be chosen during the design phase, if the expected lifecycle costs are equal to or less than the lifecycle costs of any other design less robust alternative. The difference between the expected lifecycle costs and the additional expected lifecycle costs is that the latter are due to the error in the assumed distribution functions. It must be noted that with this it is meant that the error is also not described by the assumed errors in the distribution function. Robustness focusses on the difference between everything that is taken into account in the design phase, and the things that simply cannot be predicted or captured in an uncertainty. Robustness does not focus on black swans, but on the difference between the actual distribution functions and the distribution functions of the parameters that have been assumed in the design phase. Moreover, robustness only focusses on the difference that has not been captured in an uncertainty during the design phase. An example is the tremendous increase in traffic intensity over the Dutch steel bridges. Namely, an increasing traffic intensity was assumed, with a given uncertainty, but the actual increase could not be grasped during the design phase. In the future, the same may be true for the effect of climate change on the sea level and on river discharges. Since robustness is aimed at reducing lifecycle costs for cases for which no probability of occurring is available, making a system robust may not result in extra investment costs.

In the case study on the dike height problem, it was shown that an inspection and improvement strategy for the dike height problem, is more robust than a strategy in which the dike is brought to its optimal height already at the beginning of its lifetime. Both strategies, result in the same expected
lifecycle costs, but the robust strategy will have smaller lifecycle cost in case that what is expected, is not what actually happens in the future. However, this result does not correspond to a robust system, but to a robust strategy. In the dike height problem, the only way to make the dike more robust, is by increasing the height. The reason for this is that there is only one design variable.

In the case study on the design of a sea dike, two design variables were considered. Namely, the dike height and the revetment thickness. In the case study, a grass-covered as well as a revetment were designed, and both had the same expected lifecycle costs. It was concluded that the revetment dike is the most robust design alternative. It is therefore expected that the difference in robustness between the two alternatives, lies in the difference in the risk as expected in the design phase. The alternative with the smallest expected risk will also have a small increase in the risk in case of discrepancies in the uncertain parameters. This result was underpinned in the case study on the column under axial loading. In this case study, it was shown that two design alternatives that have equal expected risk, also have equal robustness. The performance (stress) distributions of the both alternatives in this case study, had different standard deviations. This fact did not result in a different in the robustness of both alternatives.

32. Recommendations

A robust system is less sensitive to the discrepancy between the assumed distribution and the actual distributions of the uncertain parameters. An example of such a discrepancy is the increase in traffic intensity that was assumed, during the design of the steel bridges, and the actual increase in traffic intensity, that was much higher. This discrepancy was not captured in the uncertainties in the model that were taken into account in the design phase. It is recommended to find more of such examples and to analyse those to find out whether the additional lifecycle costs due to these discrepancies would have been less if robustness was implemented in the design phase. From this, it would be possible to draw a conclusion on the practical applicability of robustness.

From the case studies, it was concluded that the robustness of a system is proportional to the expected risk of the system in this design phase. This conclusion is however only based on two case studies. It is recommended to investigate if this conclusion can be underpinned theoretically. Part of this investigation should entail checking if the conclusion holds for all distribution functions that are used in civil engineering.
References


[16] A. Ben-Tal and A. Nemirovsk, Convex Optimization in Engineering, Modeling, Analysis,


i. List of symbols

$X$ set of random design variables

$x$ set of random design variables

$\theta$ set of random uncertain variables

$R$ strength, resistance to failure

$S$ load, solicitation to failure

$Z$ limit state function

$g(X, \theta)$ limit state function

$\mu_{X_1}$ mean value of random variable $X_1$

$E(X_1)$ expected value of random variable $X_1$

$\sigma_{X_1}$ standard deviation of random variable $X_1$

$P_f$ failure probability

$f_{X_1, X_2}$ joint probability density function of $X_1$ and $X_2$

$n_f$ number of simulations in which failure occurs

$n$ total number of simulations

$\varepsilon$ value of the error

$\Phi$ operator for normal distribution

$\beta$ reliability

$U_{X_1}$ standard normally distributed variable of random variable $X_1$

$\alpha_1$ influence coefficient, represents the influence of $X_1$ on the failure probability

$E_i$ event representing failure of element $i$

$C$ the net present value of the total cost

$I_0$ the initial investment cost

$I_i$ the cost of constructing $x$ unities of parameter $i$

$P_f$ the annual probability of failure of the system

$D$ the damage that occurs due to failure
\( r' \) the annual discount
\( g \) the annual growth
\( t \) the time in years
\( T \) the considered lifetime of the system in years
\( SNR \) signal-to-noise ratio
\( pdf(X_1) \) probability density function of random variable \( X_1 \)
\( f_{loss} \) loss function
\( F_B \) robust counter part of objective function \( f \)
\( \xi \) set of design variables in the neighbourhood of set \( x \)
\( \epsilon \) uncertain bound around variable \( x \)
\( A \) cross-sectional area
\( P \) external loading
\( s \) stress
\( s_{max} \) maximum stress
\( C_\theta \) bounded region for values of uncertain variable \( \theta \)
\( U(\alpha, \theta) \) set of all values that \( \theta \) can obtain if the \( \alpha \) is the horizon of uncertainty
\( \alpha \) horizon of uncertainty in the info-gap model
\( \hat{a}(x) \) robustness function in the info-gap model
\( \phi \) external uniform load density
\( T(x) \) thickness as a function of the position on the x-axis
\( m \) mass
\( w \) width
\( L \) length
\( f(x) \) objective function
\( s_j(x) \) equality constraint
\( \omega \) weighting factor for objective functions
\( \gamma_M \) material partial safety factor
\( \gamma_S \) load partial safety factor
\( S_k \) characteristic value of the load
\( S_{95\%} \) 95\% non-exceedence value of the load
\( S_{nom} \) nominal value of the load
\( N \) normal force
\( h \) dike height constructed at beginning of lifetime
\( h_0 \) is the initial dike height, before construction
\( R_{PV} \) present value of the risk
\( z \) storm surge level
\( \nu \) factor for exponential distribution
\( \kappa \) factor for exponential distribution
\( C_{inv} \) investment costs
\( V \) vulnerability factor
\( r_d \) resistance of the damaged system
\( r_0 \) resistance of the integer (non-damaged) system
\( I_{rob} \) robustness index
\( R \) annual risk
\( R_{dir} \) direct risk
\( R_{ind} \) indirect risk
\( C_{opt} \) optimum sum of investment cost and residual risk
\( C_0 \) actual risk
\( P_{imp} \) probability that improvement is necessary
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Appendix A

Summary of Eurocode
A.1. Background
The Structural Eurocode is an European Standard for the design of construction works. The initiative for the code was taken in 1975, by the Commission of the European Community. The aim was to eliminate technical obstacles to trade and the harmonisation of technical specifications. The Structural Eurocode was given the status of national standard in 2002 and by March 2010, all conflicting national standards have been withdrawn. The Eurocode consists of ten parts, starting with a part called ‘Basis of Structural Design’. The nine remaining parts treat specified areas of structural design such as ‘Design of Concrete Structures’, ‘Design of Steel Structures’ or ‘Actions on Structures’. This chapter will be mainly based on the NEN 1990-2002: ‘Basis of Structural Design’.

A.2. Design philosophy

A.2.1 Reliability
The Eurocode defines basic requirements for structures. The basic requirements relate to the structural reliability of the structure. This reliability is to be reached by choice of suitable material, appropriate design and detailing and by specifying control procedures for design, production, execution and use. The level of reliability that a certain structure should have should be chosen according to the possible causes of failure, the possible consequences of failure, the public aversion to failure and the expense of reducing the risk of failure. The level of reliability can be achieved by preventive and protective measures, measures relating to quality management and applying partial factors. The level of reliability should be present throughout the lifetime of the structure.

A.2.2 Limit state design
The Eurocode is based on limit state design, as introduced in chapter 2. Two limit states are defined; the Serviceability Limit State and the Ultimate Limit State. These limit states are related to design situations, that can be classified for being persistent, transient, accidental or seismic. For example, it is possible that for a seismic design situation the Serviceability Limit State does not have to be fulfilled, but the Ultimate Limit State should be fulfilled. That is, the structure may lose its functions and appearance during a seismic event, but not its structural safety. The Eurocode demands that “Design for limit states shall be based on the use of structural models and load models for relevant limit states”. These models are defined by the partial factor method, or alternatively, the probabilistic design method may be used.

A.2.3 Actions
According to the Eurocode, actions should be classified by their variation in time in this way:

- Permanent actions (G)
- Prestressing action (P)
- Variable actions (Q)
- Accidental actions (A)

The actions should also be classified by their:

- Origin
- Spatial variation
- Nature and the structural response
The characteristic value of an action is its main representative value and is denoted by $F_k$. The value should be determined by the designer and refers to a mean value, a lower or upper value or a nominal value. These values can be extracted from the probability distributions of the value of the actions. These distributions are based on expert judgment and statistical data. The lower and upper value are defined as respectively the 5% and the 95% fractile value.

### A.2.4 Material and product properties
Properties of materials or products should be represented by their characteristic values. Depending on whether a high or a low value is favourable, the characteristic value is determined by the 5% and the 95% fractile value. The statistical data on the properties should be obtained from standardised tests. When no statistical data are available, nominal values are used. In the Eurocode, also information is included referring to specific cases.

### A.2.5 Geometrical data
Geometrical data have to be represented by their characteristic value or directly by their design values. Geometrical data are often design variables, so that their characteristic values are the dimensions as specified in the design. If a statistical distribution around the nominal value is known, the fractiles may be used for lower and upper characteristic values.

### A.2.6 Structural analysis
The structural analysis is based on structural models, that have to be appropriate for solving the design problem. The modelling for static actions is based on force-deformation relationships applied to the structure. Dynamic actions are evaluated by dynamic analysis where necessary. If possible, quasi-static theory may be used.

### A.2. Partial factor method
Partial factors are used to determine design values from characteristic values. Characteristic values for loads are generally multiplied by a partial factor large than 1, while characteristic values for the strength are divide by a partial factor large then 1. These partial factors represent the margin between the load and the strength. Increasing the partial factors equal increasing this margin, which results in a greater reliability. Using the resulting design values, it should be verified that no relevant limit state is exceeded for all relevant design situations. This ensures that failure will not occur as long as the uncertain parameters stay within the margin. The Eurocode states that design values calculated from a full probabilistic method, should at least have the same level of reliability as the design values based on partial factors given in the standard.

### A.3.1 Design values of actions
The design value of an action $F$ is defined by:

$$F_d = \gamma_f \cdot F_{rep} \tag{71}$$

Where:

$$F_{rep} = \psi \cdot F_k \tag{72}$$
And:

\[ F_k \] is the characteristic value of the action.

\[ F_{rep} \] is the relevant representative value of the action.

\[ \gamma_f \] is a partial factor for the action.

\[ \psi \] is either 1,00, \( \psi_0, \psi_1 \) or \( \psi_2 \) and depends on the limit state to be verified.

### A.3.2 Design values of effects of actions

The design value of the effects of actions for a specific load case can be expressed as:

\[
E_d = \gamma_{sd} \cdot E\{\gamma_f, i F_{rep,i}; a_d\} \quad i \geq 1
\]  

(73)

Where:

\[ a_d \] is the set of design values of the geometrical data.

\[ \gamma_{sd} \] is a partial safety factor taking into account uncertainties in modelling of effects and actions.

The formula can be simplified by:

\[
E_d = E\{\gamma_f, i F_{rep,i}; a_d\} \quad i \geq 1
\]  

(74)

Where:

\[ \gamma_{f,i} = \gamma_{sd} \cdot \gamma_{f,i} \]  

(75)

### A.3.3 Design values of material or product properties

The design value of a material or product property can be expressed as:

\[
X_d = \eta \cdot \frac{X_k}{\gamma_m}
\]  

(76)

Where:

\[ X_k \] is the characteristic value of the material or product property.

\[ \eta \] is the mean value of the conversion factor for volume, moisture and temperature effects.

\[ \gamma_m \] is the partial factor of the material or product property.

### A.3.4 Design values of geometrical data

The design values of geometrical data may be represented by their nominal values:

\[
a_d = a_{nom}
\]  

(77)
When effects of deviation of the geometrical data are significant for the reliability of the structure, the design values should be defined by:

\[ a_d = a_{\text{nom}} \pm \Delta a \]  \hspace{1cm} (78)

Where \( \Delta a \) takes account of the possibility of unfavourable deviations and the cumulative effect of simultaneous occurring geometrical deviations.

**A.3.5 Design resistance**

The design resistance is expressed by:

\[ R_d = \frac{1}{\gamma_{R_d}} R \left( X_{d,i}; a_d \right) = \frac{1}{\gamma_{R_d}} R \left( \eta_i \frac{X_{k,i}}{\gamma_{m,i}}; a_d \right) \hspace{1cm} i \geq 1 \]  \hspace{1cm} (79)

Where:

\[ \gamma_{R_d} \] is a partial safety factor for uncertainty in the resistance model.

The formula can be simplified by:

\[ R_d = R \left( \eta_i \frac{X_{k,i}}{\gamma_{m,i}}; a_d \right) \]  \hspace{1cm} (80)

Where:

\[ \gamma_{M,i} = \gamma_{R_d} \cdot \gamma_{m,i} \]  \hspace{1cm} (81)

Alternatively, the design value of the strength may be determined directly from the characteristic value of material or product resistance:

\[ R_d = \frac{R_k}{\gamma_{M}} \]  \hspace{1cm} (82)

### A.3.6 Combinations of actions for Ultimate Limit States

The design values of the action effects should be determined for every critical load case. This is done by combining the values of actions that are considered to occur simultaneously. Each combination should include a leading variable action or an accidental action.

The expression for the effects of load combinations with leading variable actions is:

\[ E_d = \gamma_{Sd} \cdot E \left\{ \gamma_{g,j} G_{k,j}; \gamma_{p}; \gamma_{q,1} Q_{k,1}; \gamma_{q,1} Q_{k,1} \psi_{0,1} Q_{k,1} \right\} \hspace{1cm} j \geq 1; i \geq 1 \]  \hspace{1cm} (83)

The expression for the effects of load combinations with accidental actions is:

\[ E_d = E \left\{ G_{k,j}; P; A_d; (\psi_{1,1} \text{or } \psi_{2,1}) Q_{k,1}; \psi_{2,1} Q_{k,1} \right\} \hspace{1cm} j \geq 1; i \geq 1 \]  \hspace{1cm} (84)
The expression for the effects of load combinations with seismic actions is:

\[ E_d = E\{G_{k,j}; P; A_{Ed}; \psi_i Q_{k,i}\} \quad j \geq 1; i \geq 1 \] (85)

In which:

- \(G_{k,j}\) is the characteristic value of a permanent load
- \(P\) is the characteristic value of a prestressing load
- \(Q_{k,1}\) is the characteristic value of the leading variable action
- \(Q_{k,i}\) is the characteristic value of an accompanying variable action
- \(A_d\) is the design value of an accidental action
- \(A_{Ed}\) is the design value of a seismic action
- \(\psi_0\) is a factor for the combination value of a variable action
- \(\psi_1\) is a factor for the frequent value of a variable action
- \(\psi_2\) is a factor for the quasi-permanent value of a variable action

### A.3.7 Combination of actions for Serviceability Limit States

The expression for the effects of the characteristic load combination is:

\[ E_d = E\{G_{k,j}; P; Q_{k,1}; \psi_0 Q_{k,i}\} \quad j \geq 1; i \geq 1 \] (86)

The expression for the effects of the frequent load combination is:

\[ E_d = E\{G_{k,j}; P; \psi_1 Q_{k,1}; \psi_1 Q_{k,i}\} \quad j \geq 1; i \geq 1 \] (87)

The expression for the effects of the quasi-permanent load combination is:

\[ E_d = E\{G_{k,j}; P; \psi_2 Q_{k,i}\} \quad j \geq 1; i \geq 1 \] (88)

The partial factors \(\gamma_M\) for the properties of materials are always taken to be 1.0 for serviceability limit state.

### A.3.8 Partial factors

The partial factors are dependent on the type of structure and the type of loads considered. In the Eurocode, tables with the values for all categories are given. In the National annexes, different values may be given, these are always more conservative than the values given in the Eurocode. When designing a structure for a particular country, the values in the National annex of that country should be used.