AN INTRODUCTION TO AQUEOUS HYDRAULIC CONVEYANCE OF SOLIDS IN PIPE LINES

by: DR. ARTHUR BREFNER

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AN INTRODUCTION TO AQUEOUS HYDRAULIC CONVEYANCE OF
SOLIDS IN PIPE-LINES

by

ARTHUR BREBNER

SYNOPSIS

The report deals with the basic parameters and relationships involved in the aqueous transport of solids in pipe-lines and is essentially a review of work done by others up to the present time.
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FOREWORD

The author has embarked on a programme of research into the aqueous transport of materials by pipe-line. The equipment consists essentially of a 400 ft. loop of 4 inch aluminum pipe with pumping, mixing and measuring facilities.

Two major areas are to be investigated, the conveyance of wood-chips whose specific gravity is approximately unity and the conveyance of mine products of varying specific gravities up to about 5.

This report is essentially one giving background to the problem and is intended to acquaint the reader with the parameters in use and the basic work which has already been most ably done by other workers in this field. Subsequent reports will contain the specific results from tests on the materials mentioned above.

The problem of the aqueous hydraulic transport of solids in pipe-lines has two distinct facets. The first consists of the mathematical relationship involved in the head losses due to friction in transporting such solids. The second consists in the technology of introducing such solids into the system and extracting them at the delivery end. This report deals essentially with the first problem which may be stated in broad terms as follows:

The water part of the mixture is the conveying medium, in much the same way as an empty vehicle or ship is the mode of transport, and the solids the "pay-load". All things being equal the greater the "pay-load" the more attractive is the proposition from an economic viewpoint. However, the type of "pay-load" or solid conveyed has a very marked effect on the economics of the situation. By type of "pay-load" is meant the size of the solid particle and its specific gravity, the shape (and associated with this its free fall or terminal velocity) and finally the ratio of amount of solid to amount of conveying water, that is the concentration. Moreover, the conveying vehicle or pipe-line has also an effect, its diameter and to a smaller extent its roughness being factors. Very small particles travel fully suspended in the water and the head losses associated with this type of homogeneous flow are fairly easily arrived at. Larger particles tend to settle out on the bottom of a horizontal pipe and the mechanism of conveyance and the associated head losses are much more complex. Associated with such solids is a minimum critical velocity below which the flowing water is incapable of transporting the solids in much the same way as a river in flood will sweep away large boulders it is incapable of moving under normal flow conditions.

The various facets affecting the head losses are dealt with separately in what follows and their overall effect brought together in a generalized relationship.

It must be emphasized that much of the basic empiricism of aqueous conveyance of solids is known and the majority of problems in this field of engineering hydraulics are now in areas of technique and practical engineering at the operating level.

ACKNOWLEDGEMENTS

The author wishes to acknowledge gratefully the financial assistance of the National Research Council of Canada and the Ontario Mining Association and the help given in equipment by Aluminum Laboratories, Canadian Bechtel and Allis-Chalmers. The author also wishes to thank his colleagues Professor R.J. Kennedy and R.R. Faddick for their help and criticism in writing this report.
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>$V$</td>
<td>Volume of solid particle.</td>
<td>$ft^3$</td>
</tr>
<tr>
<td>$w_s$</td>
<td>Specific weight of solid particle.</td>
<td>$lb/ft^3$</td>
</tr>
<tr>
<td>$w_w$</td>
<td>Specific weight of water.</td>
<td>$lb/ft^3$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Dimensionless Coefficient of Drag.</td>
<td>--</td>
</tr>
<tr>
<td>$A$</td>
<td>Projected area of the solid particle in a direction normal to $v_s$.</td>
<td>$ft^2$</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Free settling velocity related to still water of a single particle.</td>
<td>$ft/sec$</td>
</tr>
<tr>
<td>$N_R$</td>
<td>Dimensionless Reynolds' Number</td>
<td>--</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Kinematic viscosity.</td>
<td>$ft^2/sec$</td>
</tr>
<tr>
<td>$d$</td>
<td>Particle diameter.</td>
<td>$ft$ (or mm, micron, inch, etc.)</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Dimensionless concentration by volume.</td>
<td>--</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Dimensionless function.</td>
<td>--</td>
</tr>
<tr>
<td>$C_{d'}$</td>
<td>Dimensionless &quot;fictitious&quot; Coefficient of Drag.</td>
<td>--</td>
</tr>
<tr>
<td>$s_m$</td>
<td>Dimensionless specific gravity of mixture.</td>
<td>--</td>
</tr>
<tr>
<td>$s$</td>
<td>Dimensionless specific gravity of solid.</td>
<td>--</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress.</td>
<td>$lb/ft^2$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity.</td>
<td>$lb. sec/ft^2$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Yield value.</td>
<td>$lb/ft^2$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Coefficient of rigidity.</td>
<td>$lb/sec/ft^2$</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Limiting critical velocity for deposition.</td>
<td>$ft/sec$</td>
</tr>
<tr>
<td>$K$</td>
<td>A dimensionless function or constant.</td>
<td>--</td>
</tr>
<tr>
<td>$H_R$</td>
<td>Dimensionless hold-up ratio.</td>
<td>--</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure.</td>
<td>$lb/ft^2$</td>
</tr>
<tr>
<td>$f$</td>
<td>Dimensionless friction factor or function.</td>
<td>--</td>
</tr>
<tr>
<td>$D$</td>
<td>Internal Pipe diameter.</td>
<td>$ft$</td>
</tr>
<tr>
<td>$v$</td>
<td>Mean mixture or water velocity in pipe.</td>
<td>$ft/sec$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of pipe over which head-loss occurs.</td>
<td>$ft$</td>
</tr>
<tr>
<td>$i$</td>
<td>Dimensionless hydraulic gradient.</td>
<td>--</td>
</tr>
<tr>
<td>$\phi$</td>
<td>A function.</td>
<td>--</td>
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1. INTRODUCTION

The problem of hydraulic conveying is essentially one of estimating the minimum energy required to transport a maximum amount of material at a specified rate between two points.

Transporting fluids by pipe-line, e.g. oil, natural gas etc., is incontrovertibly the cheapest method of overland transport and there is good evidence to show that the conveyance of solids in a fluid medium through a pipe-line is also an attractive economic proposition.

The commonest fluids available are air and water, the main hydraulic difference between them being the lower dynamic viscosity and density of air. From a practical viewpoint separation of solids and fluids at delivery is usually easier with an air medium.

Since the fluid is acting as a transport medium only — unless one considers, for example, a coal-oil solid-fluid system where both can be processed at delivery — it is obvious from an economic viewpoint that there will be an optimum amount of fluid flowing through the system. Essentially the operating cost of a hydraulic conveying system depends on the relationship between the rate of solid transport versus the power required to achieve such a rate.

Moreland (1960) gives the following advantages made possible by conveying solids in pipe-lines:

a) No return of empty vehicles or cartons.
b) Less labour required.
c) Less dust and losses in transport.
d) Less degradation.
e) Less equipment maintenance.
f) Less product contamination.
g) Less space required.

On the other hand the many disadvantages are:

a) Pipe-lines which cannot be buried for maintenance purposes may be susceptible to freezing temperatures which greatly affect the fluid phase.
b) Profile of unburied pipe-lines is influenced by topography.
c) Power failures may produce line blockages, thus requiring diesel or gasoline standby pumping units or pipe-line drainage facilities.
d) High initial cost.
e) Erosion of pipe by materials such as sand and coal. (Rotation of the pipe-line may be required to partially offset this.)
f) Attrition of the transported material.
g) Long pipe-lines are not flexible and thus require an assured long-term source of supply and market.
h) Pipe-lines are only suitable for high capacity use. There is a minimum velocity and an optimum economic solids concentration which can be used.
i) Centrifugal pumps, which at present are the only pumps suitable for larger particles, are relatively inefficient. When handling slurries, efficiencies of greater than 70% cannot normally be expected. For long pipe-lines requiring high pressures the reciprocating positive displacement pump is the only practical solution at present and the wear on valves and cylinders is great.
j) Large volumes of water are usually required at the dispatching end and in some instances this may involve the necessity of recirculating water.
k) Product must not be spoiled by contact with water; (this may be overcome by "packaging" the solids).
l) A suitable slurry must be available for introduction into the pipe-line - the preparation of this slurry sometimes involves considerable cost.
m) In some cases (as for fine coal), the cost of separation of the product from the water at the receiving end is high.
2. TERMINOLOGY AND PARAMETERS AFFECTING THE PROBLEM OF HYDRAULIC CONVEYING

In this report only water transport systems will be referred to. The variables which are an integral part of the aqueous hydraulic transport problem are as follows:

a) Solid Particle Size, Shape and Specific Gravity

The specific gravity may be greater than unity (sand, gilsonite, mine tailings etc.) or less than unity (unsaturated wood-chips etc.). One single parameter which represents size, shape and specific gravity of the conveyed material and also represents the effect of the specific weight and viscosity of the conveying water is the settling or terminal velocity of the particle in water. Since this is a very important parameter in hydraulic conveying it will be examined more closely.

Settling or Terminal Velocity

As a particle falls (or rises if the specific gravity is less than the specific gravity of the surrounding water medium), it is subjected to a viscous drag by the water which opposes the gravity (or buoyant) force. Also, the particle continuously displaces water as it falls (or rises). The drag increases as the particle accelerates until an equilibrium state is reached when the gravity (or buoyant) force is balanced by the drag force. The velocity at which this equilibrium state is attained is called the free fall, settling or terminal velocity and is represented by the equation

\[ V(W_s - W_w) = C_d \frac{A \cdot w_s v_s^2}{2g} \]  
(1)

where 
- \( V \) = Volume of the solid particle
- \( W_s \) = Specific weight of the solid particle
- \( W_w \) = Specific weight of the fluid medium, in this case water
- \( C_d \) = Coefficient of Drag
- \( A \) = Projected area of the solid particle in a direction normal to \( V_s \)
- \( V_s \) = Free settling velocity relative to still water of a single particle.

For a single spherical particle of diameter \( d \) this may be re-written.

\[ \frac{\pi d^3}{6} \left[ \frac{W_s}{W_w} - 1 \right] = C_d \frac{\pi d^2}{4} \cdot \frac{v_s^2}{2g} \]

that is

\[ v_s^2 = \frac{4}{3} \frac{1}{C_d} \cdot g d (s - 1) \]  
(2a)

for particles heavier than water

or

\[ v_s^2 = \frac{4}{3} \frac{1}{C_d} \cdot g d (1 - s) \]  
(2b)

for buoyant particles.

where \( s \) is the specific gravity of the solid particle with respect to water.

The Coefficient of Drag, \( C_d \), is a function of \( N_R \), the Reynolds' Number, \( \frac{v d}{\nu} \); \( \nu \) is the kinematic viscosity of the water, the value of \( \nu \) at 60°F being about \( 1.2 \times 10^{-5} \) ft²/sec. With fully turbulent motion \( C_d \) is sensibly constant.

In general

\[ N_R < 0.2 \quad \quad C_d = \frac{24}{N_R} \quad \quad \text{Stokes Law} \]

\[ 0.2 < N_R < 500 \quad \quad C_d = \frac{24}{N_R} \left[ 1 + 0.15 N_R^{0.69} \right] \quad \quad \text{Schiller and Naumann} \]

\[ 500 < N_R < 2 \times 10^5 \quad \quad C_d = 0.44 \]
With these values it may be shown (Fig. 1), that particles greater in diameter than the order of \( d = 5 \text{ mm} \) (about 1/4 inch) have terminal velocities which are directly proportional to \( \sqrt{d} \)

\[ \text{i.e. } \frac{v_s}{\sqrt{gd}} = \text{constant} \]

whereas particles smaller in diameter than the order of \( d = 50 \text{ microns} \) (1 micron = \( 10^{-6} \) metres \( \approx 0.00004 \) inches) have terminal velocities, in the laminar range (\( C_d = 24/N_R \)), proportional to \( d^2 \), i.e. \( \frac{v_s}{\sqrt{gd}} \propto d^{3/2} \).

Examination of equations 2a and 2b shows that it is possible for a relatively light particle of say specific gravity \( s = 1.5 \) to fall at a rate equal to the rate of rise of a buoyant particle, having the same size, shape and surface texture, of \( s = 0.5 \). However, it is impossible to find a buoyant particle of sufficiently small \( s \) value to rise at the same rate of fall of a particle having an \( s \) value of 2.0 or greater.

For non-spherical particles the value of \( C_d \) can incorporate the shape effect assuming the particle to be a sphere of diameter \( d \) having the same volume as the non-spherical particle. Because of the difficulty in assigning mathematical values to shape for markedly non-spherical particles, e.g. wood-chips or metal turnings, laboratory tests must be performed to determine the value of \( v_s \) or \( C_d \); the non-spherical particle may then be considered to have a mean diameter corresponding to a spherical particle having the same value of drag coefficient.

Figure 1 shows the settling velocities of spherical particles of various sizes in water at normal temperature deduced from equations 2 with appropriate values of \( C_d \) as shown.

These values will be reduced by wall effects if the particle rises or falls in a non-infinite water medium and even more markedly reduced by concentration effects.

For non-spherical particles, the ratio

\[ \frac{\text{settling velocity of non-spherical particle}}{\text{settling velocity of spherical particle of same volume}} \approx \frac{d}{l} \]

where \( d \) is the diameter of a sphere of equal volume and \( l \) the greatest length of the non-spherical particle, is sensibly true if the greatest projected area of the particle is proportional to \( l^2 \).

As the number of particles falling or rising increases, that is the concentration of particles increases, the particles themselves restrict the area through which the displaced water flows, thus increasing this velocity and decreasing the settling velocity. The settling velocity of a concentration of particles is called the hindered settling velocity or the sedimentation velocity.

The relationship between the hindered settling velocity affected by concentration of particles, and the settling velocity of a single particle, has been shown by Maude and Whitmore (1958) to take the form

\[ \text{hindered settling velocity} = v_s (1 - C_T)^\beta \]  \hspace{1cm} (3)

where \( C_T \) is the volume concentration, i.e. the ratio volume of solid particles in a given volume of water-solid mixture and \( \beta \) is a function of particle shape, size distribution and Reynolds' Number given by Figure 2.

Equation 3 indicates that a 25% by volume concentration would have a hindered settling velocity of between 50% and 25% of the free settling velocity, \( v_s \), depending on the particle size etc.

Since particle size is an important parameter in settling velocity it is felt judicious to introduce at this time a comparison between Tyler Mesh Sizes, U.S. Standard Mesh Sizes and the usual engineering length units. These relationships are given in Table 1.
FIG. 1. SETTLING OR RISE VELOCITY OF SPHERES IN WATER AT NORMAL TEMPERATURES
FIG. 2. HINDERED SETTLING VELOCITY FUNCTION VERSUS REYNOLDS NUMBER

\[
N_R = \frac{v_s d}{\nu} \quad \text{(log SCALE)}
\]

<table>
<thead>
<tr>
<th>Tyler Mesh No.</th>
<th>Opening in Inches</th>
<th>Opening in Microns</th>
<th>U.S. Standard Mesh</th>
<th>Opening in Inches</th>
<th>Opening in Microns</th>
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<tr>
<td>400</td>
<td>0.0015</td>
<td>38</td>
<td>400</td>
<td>0.0015</td>
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<tr>
<td>200</td>
<td>0.0029</td>
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<td>100</td>
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<td>100</td>
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<td>65</td>
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<tr>
<td>3</td>
<td>0.263</td>
<td>6680</td>
<td>3</td>
<td>0.250</td>
<td>6350</td>
</tr>
</tbody>
</table>

1 metre x 10\(^{-6}\) = 1 micron:

1 mm = 1000 microns

TABLE I. STANDARD MESH SIZES
Equations 2a and 2b may be re-written in the form

\[ \frac{v_s}{\sqrt{gd}} = \left[ \frac{4}{3 C_d} (s-1) \right]^{1/2} \quad \text{or} \quad \left[ \frac{4}{3 C_d} (1-s) \right]^{1/2} \]

where \( \sqrt{gd} \) is a form of Froude Number. A plot of \( \frac{v_s}{\sqrt{gd}} \) against \( d \) using the appropriate values of \( C_d \) is given in Figure 3 and a plot of \( \sqrt{C_d} \) against \( d \) in Figure 3A.

For non-spherical particles \( d \) is the nominal diameter, measured as the mean of a large number of particles.

Equation 2a can be re-written as

\[ \frac{v_s}{\sqrt{gd}} = \frac{1}{\sqrt{C_d}} \left[ \frac{4}{3} (s-1) \right]^{1/2} = \frac{1}{\sqrt{C_d}} \]

where \( C_d \) is a so-called fictitious Drag Coefficient for a particular particle of given shape, roughness and specific gravity. The value of \( \sqrt{C_d} \) is readily obtained by carrying out tests to find the value of \( d \) and \( v_s \) and so finding the value of \( \sqrt{C_d} \) from the relationship 2c.

For pure spheres the value of \( C_d \) for various values of \( N_R \) may be used to determine \( C_d' \).

For example, for a sand, specific gravity 2.65, \( \frac{1}{\sqrt{C_d}} = 1.48 \frac{1}{\sqrt{C_d'}} \)

In a later part of this report the dimensionless fictitious Drag Coefficient \( C_d' \) and the Froude Number of settling \( \frac{v_s}{\sqrt{gd}} \) will be used extensively.

b) Definition of Concentration

The concentration of solids transported at the delivery end, \( C_T \), of the system may be defined as

\[ C_T \text{ by weight} = \frac{\text{solid weight (dry) delivered in unit time}}{\text{total weight of mixture delivered in unit time}} \]

OR

\[ C_T \text{ by volume} = \frac{\text{solid volume (dry) delivered in unit time}}{\text{total volume of mixture delivered in unit time}} \]

The latter relationship, that is, volume concentration is in more common usage and in ensuing references to concentrations volume concentration shall be implied. The solid volume delivered multiplied by the specific weight of the solid gives the weight of solid delivered in unit time.

The specific gravity of the mixture, \( s_m \), is given by

\[ s_m = \frac{\text{weight of mixture}}{\text{weight of equal volume of water}} \quad \text{and since the specific weight of water is 62.4 lb/ft}^3 \]

\[ s_m = \frac{(\text{volume of solid } \times s \times 62.4) + (\text{volume of water } \times 62.4)}{(\text{volume of mixture } \times 62.4)} \]

\[ = \frac{(\text{volume of solid } \times s) + (\text{volume of mixture } - \text{volume of solid})}{(\text{volume of mixture})} \]

that is,

\[ s_m = 1 + C_T (s-1) \]

and thus \( C_T \) (by volume) = \( \frac{s_m - 1}{s - 1} \)
FIG. 3. $\frac{v_s}{\sqrt{gd}}$ vs. $d$

Based on:

$$v_s^2 = \frac{4}{3} \frac{1}{C_d} \frac{gd}{s} (s - 1)$$

Where $C_d = \begin{cases} 
24/N_R & N_R < 0.2 \\
(24/N_R)(1 + 0.15N_R^{0.69}) & 0.2 < N_R < 500 \\
0.44 & N_R > 500 
\end{cases}$

$$N_R = \frac{v_s d}{\gamma} : \gamma \approx 10^{-5} \text{ ft}^{-2}/\text{sec}$$
ROUGH PLOT OF $\sqrt{C_d}$ vs $d$

FROM $v_s^2 = \frac{4}{3} \cdot \frac{1}{C_d} gd(s-1)$

WHERE $C_d = \begin{cases} 
\frac{24}{N_R} & \text{FOR } N_R < 0.2 \\
\frac{24}{N_R} \cdot (1 + 0.15N_R^{0.69}) & \text{FOR } 0.2 < N_R < 500 \\
0.44 & 500 < N_R < 2 \times 10^5 
\end{cases}$

FIG. 3a. DRAG COEFFICIENT VERSUS PARTICLE SIZE
Figure 4 shows the variation in specific gravity of mixture for various concentrations and various solid specific gravities. For example, a pipe-line system pumping 10% by volume of sand, $s = 2.65$, has an average mixture specific gravity, $s_m$ of 1.165.
c) Mode of Motion of Particles in Hydraulic Conveying

It is reasonable to assume that the mode of transport of the particles is directly related to the orientation in a vertical plane of the pipe-line. In a horizontal line heavier than water particles will tend, by gravity, to settle out and travel near the pipe invert whereas in a vertical line the particles should be symmetrically distributed across the pipe diameter. For an inclined line a combination of horizontal and vertical modes might be reasonably expected. In general the two commonest orientations will either be substantially horizontal or vertical as in a mine-shaft; a good example of the latter is the 1363 ft. vertical lift of minus 1/4" copper ore at a Northern Rhodesian mine and of the former the 108 mile line carrying finely crushed coal in Ohio.

Since horizontal conveying exhibits many more facets than vertical this orientation of line will now be dealt with fully before discussing vertical lines.

Consider a horizontal pipe-line conveying material particles of diameter \( d \) and specific gravity \( s \) at a concentration \( C_T \). In a glass length of such a pipe-line several different modes of conveyance are clearly visible with decreasing mixture velocities as follows (Newitt et al. (1955)).

1) Suspended Flow: at high velocities the particles move in a fully suspended state. Depending on the size and specific gravity the suspension can move either like a homogeneous suspension, eg. a soup or as a heterogeneous suspension having a concentration in a vertical plane normal to the flow direction which is non-uniform. (With increasing velocity a heterogeneous suspension tends to a homogeneous suspension.) At lower velocities in a heterogeneous suspension the vertical concentration increases near the bottom (for \( s > 1 \)) and the material there tends to move as a bed.

2) Suspended Flow with a Moving Bed: in the upper part of the pipe (for \( s > 1 \)) the particles move in heterogeneous suspension whilst on the bottom of the pipe a layer of particles slides or rolls along at a uniform rate.

3) Suspended Flow with a Stationary Bed: as in (2) heterogeneous suspensions occurs in the upper part of the pipe whilst on the bottom a layer of particles slides over a stationary deposit by saltation.

3a) Saltation with a Stationary Bed: ripples of material move slowly forward in the direction of flow:

4) Blockage of Pipe - Isolated or Complete

(For dilute suspensions of bleached sulphite pulp numerous experimenters have found further modes of travel, namely "plug flow" in various forms (Symposium on Flow of Suspensions, National Research Council of Canada, 1956.)

As has been pointed out previously the settling velocity is a most important criterion in hydraulic conveying and for all practical purposes mixtures can be classified into two distinct types, "non-settling" and "settling" which are functions of this settling velocity. Mixtures with low settling velocities of the order of roughly 0.005 ft. per second behave as "non-settling" pseudo-homogeneous fluids at sensibly all velocities whereas mixtures with settling velocities greater than 0.005 ft. per second behave as "settling" heterogeneous mixtures since even in very turbulent flow settling tends to take place in a horizontal pipe.

Non-settling mixtures behave as pseudo-homogeneous fluids, the specific gravity and the so-called viscosity or consistency of the mixture being the important parameters. Normally, non-settling mixtures do not behave as Newtonian fluids but as plastic or pseudo-plastic fluid. (In a Newtonian fluid, eg. water, \( \tau = \mu \frac{dv}{dy} \) where \( \tau \) is the unit shear stress, \( \mu \) the dynamic viscosity and \( \frac{dv}{dy} \) the velocity gradient. In a non-Newtonian fluid, eg. Bingham plastic, \( \tau - \tau_y = \eta \frac{dv}{dy} \) where \( \tau_y \) is the yield value and \( \eta \) the coefficient of rigidity.)

Settling mixtures exhibit the modes of transport itemized from 1 to 4 above and this report is concerned with such mixtures.
The transition from one mode to another takes place at mixture velocities which may be
defined by:

Transition from homogeneous to heterogeneous \( v_H \)
Transition from heterogeneous suspension to suspended flow with a moving bed, between cases (1) and (2) \( v_B \)
Transition from heterogeneous suspended flow with a moving bed to stationary bed and saltation, between cases (2) and (3) \( v_C \)
Transition from a stationary bed and saltation to partial or complete blockage, between cases (3a) and (4).

Knowledge of the limiting velocity characterizing the change from the regime of no-deposition to the regime of deposition, namely \( v_C \), is very important since it corresponds to the economic operating point and permits a correct choice of pipe diameter. (Condolios et al, 1961).

The various modes of flow are shown diagrammatically in Figures 5 and 6 and typical values of the limiting velocity in Table II.

![Figure 5](image1)

![Figure 6](image2)
### TABLE II. TYPICAL VALUES OF LIMITING VELOCITY

For values of $d$ greater than 1 mm. (i.e. about 1/25 of an inch or 20 mesh) the limiting velocity is apparently independent of concentration and size, and depends largely on $D$ and $s$. In view of the difficulty in assessing the value of $v_c$ by eye it is not surprising that there is some discrepancy amongst various sources for the value of $v_c$ for particles greater than 1 mm. However, it would appear reasonable to say that, for such particles,

$$v_c = K \sqrt{2gD(s-\ell)}$$

where $K$ is constant at about 1.5.

that is, $v_c \approx 1.5 \sqrt{2gD(s-\ell)}$ (in consistent units for $d > 1$ mm)

Since the necessary minimum velocity to maintain flow without stationary deposit appears to vary directly as the square root of the pipe size it follows that if, for example, the pipe size is increased from 3" to 12", the necessary limiting velocity is doubled.

For particles less than 1 mm. in size the relationship between $v_c$ and the other parameters is more complex. It has been suggested by Hughmark (1961) that the limiting velocity for particles below about 0.5 mm. in size decreases in a linear fashion with the particle diameter, that is $K$ decreases in a linear fashion with decreasing particle size. Durand and Condolios (1952) show a similar decrease in the value of $K$ (in $K = v_c/\sqrt{2gD(s-\ell)}$) but show that below around 1 mm. particle size the concentration also has an effect. Gibert (1960) for a 0.2 mm. sand gives $v_c \approx 1.0 \sqrt{2gD(s-\ell)}$ for $C_f > 7.5\%$.

#### d) Hold-up and Slip

In a homogeneous mixture the velocity of the mixture is the same as the velocity of the fluid and solid phases. However, with decreasing mixture velocities, a "settling" mixture flows as a heterogeneous suspension and there is a tendency for the fluid phase to slip past the solids in suspension or, in other words, there tends to be a "hold-up" of solids.

The ratio

$$\frac{\text{liquid/solid ratio (by volume) delivered}}{\text{liquid/solid ratio (by volume) at a given section}}$$

is known as the hold-up ratio $H_R$. 

---

<table>
<thead>
<tr>
<th>Material &amp; S.G.</th>
<th>Pipe D (inches)</th>
<th>Approx. Particle d (mm)</th>
<th>Max. $C_f$</th>
<th>Limiting Velocity $v_c$ (ft/sec)</th>
<th>$K = \frac{v_c}{\sqrt{2gD(s-\ell)}}$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sand (2.6)</strong></td>
<td>6&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>9.7</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>10&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>12.3</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>18&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>16.5</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>36&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>23.5</td>
<td>1.35</td>
</tr>
<tr>
<td><strong>Sand (2.6)</strong></td>
<td>1&quot;</td>
<td>1/25</td>
<td>0.5</td>
<td>25</td>
<td>5</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>1&quot;</td>
<td>1/8</td>
<td>0.5</td>
<td>25</td>
<td>5</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>1&quot;</td>
<td>3/16</td>
<td>4.8</td>
<td>15</td>
<td>4</td>
<td>1.38</td>
</tr>
<tr>
<td><strong>Sand (2.6)</strong></td>
<td>3&quot;</td>
<td>1/25</td>
<td>0.24</td>
<td>-</td>
<td>4.5</td>
<td>0.9</td>
</tr>
<tr>
<td><strong>Coal (1.4)</strong></td>
<td>1&quot;</td>
<td>1/16</td>
<td>1.6</td>
<td>35</td>
<td>3.5</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>1&quot;</td>
<td>1/8</td>
<td>3.2</td>
<td>20</td>
<td>3</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>1&quot;</td>
<td>1/6</td>
<td>4.8</td>
<td>20</td>
<td>3</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>Coal (1.5)</strong></td>
<td>6&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>5.4</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>10&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>7.1</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>18&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>9.3</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>36&quot;</td>
<td>1/25</td>
<td>1 mm</td>
<td>15%</td>
<td>13.1</td>
<td>1.35</td>
</tr>
<tr>
<td><strong>Perspex (1.185)</strong></td>
<td>1&quot;</td>
<td>1/50</td>
<td>0.5</td>
<td>40</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>1&quot;</td>
<td>1/16</td>
<td>1.6</td>
<td>30</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>1&quot;</td>
<td>1/8</td>
<td>3.2</td>
<td>30</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>Mang. Oxide (4.1)</strong></td>
<td>1&quot;</td>
<td>1/8</td>
<td>3.2</td>
<td>6</td>
<td>6</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Now $C_T$ is the delivered concentration, i.e. \( \frac{\text{solid volume delivered}}{\text{mixture volume delivered}} \) and if $C_T$ is defined as \( \frac{\text{solid volume at a given section}}{\text{mixture volume at that given section}} \) then

\[
H_R = \left[ \frac{1 - C_T}{C_T} \right] \left[ \frac{C_T}{1 - C_T} \right]
\]  

(6)

If the flow is a homogeneous one $C_T = C_T'$, that is, there is neither slip nor hold-up and the hold-up ratio is unity. As the mixture flow becomes heterogeneous $C_T$ becomes greater than $C_T'$ and the hold-up ratio, $H_R$, tends to increase above unity, its final value tending to infinity when all the solids remain stationary in the pipe and only liquid is delivered.

A plot of mean liquid velocity versus mean mixture velocity would appear as shown in Figure 7, after Govier and Charles (1961), illustrating the differing important velocities and the changing nature of $H_R$.

![Diagram](attachment:image.png)

**FIG. 7**

The hold-up ratio, $H_R$, may also be plotted against the mean mixture velocity and would reveal a relationship as shown in Figure 8.
3. HEAD LOSSES IN HYDRAULIC CONVEYING

a) Horizontal Pipe

Let $i_w$ be the head loss in feet of water per foot length of pipe, that is the hydraulic gradient, for water only flowing in the system.

Then, by the Darcy-Weisbach relationship,

$$\text{head loss in feet of flowing fluid} = f_w \frac{L}{D} \frac{v^2}{2g}$$

and since

$$i_w = \frac{h_l}{L}$$

$$i_w = f_w \frac{1}{D} \frac{v^2}{2g}$$

(7)

where $f_w$ is a function of pipe roughness and Reynolds' Number, $\frac{vD}{\nu}$, and is dimensionless. At high values of $N_R$, the flow is usually fully developed turbulent flow so that $f_w$ is sensibly independent of $N_R$.

The value of $f_w$ for clean smooth pipe is approximately as follows, e.g. aluminum

<table>
<thead>
<tr>
<th>$N_R$</th>
<th>$10^4$</th>
<th>$2 \times 10^4$</th>
<th>$4 \times 10^4$</th>
<th>$6 \times 10^4$</th>
<th>$8 \times 10^4$</th>
<th>$10^5$</th>
<th>$2 \times 10^5$</th>
<th>$4 \times 10^5$</th>
<th>$6 \times 10^5$</th>
<th>$8 \times 10^5$</th>
<th>$10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_w$</td>
<td>0.031</td>
<td>0.026</td>
<td>0.022</td>
<td>0.020</td>
<td>0.019</td>
<td>0.018</td>
<td>0.016</td>
<td>0.014</td>
<td>0.013</td>
<td>0.013</td>
<td>0.012</td>
</tr>
</tbody>
</table>
Let \( j \) be the head loss in feet of flowing mixture per ft., specific gravity \( s_m \), (see Figure 4 for relationship between \( s \), \( C_T \) and \( s_m \)), that is, the hydraulic gradient for a mixture flowing under any mode, saltation, heterogeneous or homogeneous, in the system.

\( j \) for homogeneous and heterogeneous mixtures will now be discussed separately.

**Head Loss for Homogeneous Suspensions**

Suspensions of fine particles may have a behaviour akin to a Newtonian fluid of specific gravity \( s_m \) and specific weight \( w_m \).

Then

\[
j = f_m \frac{1}{D} \frac{v^2}{2g}
\]  

(8)

where \( f_m \) is the dimensionless friction factor for the mixture, this factor being a function of pipe roughness and Reynolds' number. The latter is a function, amongst other things, of viscosity. If the viscosity, \( \nu \), of the mixture does not differ too greatly from that of water - and this will often be the case for low concentrations - \( f_m \) will be sensibly the same as \( f_w \) at the same velocity in the same pipe, internal diameter \( D \) and relative roughness \( k/D \).

Thus \( j = i_w \).

In words, for a homogeneous suspension, in theory the pressure drop expressed in feet of homogeneous suspension of specific weight \( w_m \), is the same as the pressure drop for clear water expressed in feet of water.

Let \( i_m \) be the pressure gradient of the homogeneous mixture expressed as feet of water per foot length of pipe-line.

Then

\[
i_m = j \frac{w_m}{w_w}
\]

But \( \frac{w_m}{w_w} = \frac{s_m}{s_w} = 1 + C_T (s - 1) \) from equation 4

Thus

\[
i_m = \left( (s - 1) i_w C_T \right) + i_w
\]  

(9)

That is,

\[
\frac{i_m - i_w}{i_w C_T} = (s - 1)
\]  

for homogeneous Newtonian Flow  

(10)

In words, the dimensionless parameter \( \frac{i_m - i_w}{i_w C_T} \) is, in theory for homogeneous flow, a constant whose value is solely a simple function of the specific gravity of the solid in suspension.

Further, it implies that, for a given concentration of a given homogeneous mixture,

\[
i_m = \text{(constant)}(i_w)
\]

where the constant is in excess of unity. (For a 10% by volume concentration of sand, Specific Gravity \( S = 2.6 \), the constant is 1.16.) Since \( i_w \) is sensibly proportional to \( \nu^2 \) in turbulent flow,

\[
i_m = \text{(constant)}(\nu^2)
\]

A plot of \( i_m \) versus \( \nu \) to log-log scales would indicate a theoretical behaviour as shown in Figure 9.

Experimental results of Newitt et al (1955) confirm the relationship of equation 10 for fine particles of sand, mean diameter about 0.004 inches (between 100 and 200 mesh) settling velocity 0.032 ft/sec.
Finer particles than those of Figure 10 did not confirm equation 10, indicating, in this instance, a non-Newtonian behaviour of the suspension akin to a plastic behaviour.

Larger particles tested by the same authors - Figure 11 illustrates test results for a sand having a mean diameter of 0.030 inches (20 mesh) and a settling velocity, \( v_s \), of 0.37 ft/sec - give differing results. Figure 10 indicates that \( i_m = \text{(constant)}(i) \) whereas Figure 11 illustrates a convergence of \( i_m \) and \( i_w \) with increasing velocity.

A similar relationship is obtained with even larger particles of coal, shown from tests by
Worster and Denny (1955) in Figures 12a and 12b.

Again, from Figure 12, a convergence of $i_m$ and $i_w$ is revealed, such a convergence being apparently a property of heterogeneous flow mixtures; thus equation 10 would appear to be only applicable to homogeneous mixtures of small particles. However, at high velocities the hydraulic gradient for heterogeneous flow becomes sensibly parallel with the clear water line thus indicating that when heterogeneous flow becomes homogeneous flow, at very high velocities for large particles of
SAND $s = 2.64$

$V_s = 0.37$ ft/sec.

d = 0.30 in.

(AFTER NEWITT ET AL.)

MEAN VELOCITY $v$ ft/sec

CONCENTRATION 25%

20%

15%

10%

5%

MEAN VELOCITY $v$ ft/sec

FIG. 11
FIG. 12

(a) 1/2" COAL - 3" PIPE
(AFTER WORSTER & DENNY)

(b) 1/2" COAL - 6" PIPE
(AFTER WORSTER & DENNY)
appreciable specific gravity, equation 10 may well hold true. (From Figure 12, at high velocities, taking \( s = 1.4 \) for 1/2" coal, at \( C_T = 10\% \) and \( v = 8 \text{ ft/sec.} \), \( i_m = 1.14 i_w \); equation 10 gives \( i_m = 1.04 i_w \)). Such a high velocity is uneconomic in practice and the relationship for \( \frac{i_m - i_w}{i_w C_T} \) is not as simple as indicated by equation 10 since the flow is heterogeneous and not homogeneous.

The velocity at which the flow is a homogeneous one, that is when the mixture gradient line is almost parallel to the clear water gradient line in a plot of \( i_m \) or \( i_w \) against mean velocity \( v \), is called the "standard velocity" (Spells, 1955).

A simple relationship for \( V_H \) suggested by Newitt et al (1955) is

\[
V_H = 38.6 \frac{3}{D} V_D (11)
\]

This equation was derived from tests on particles of various specific gravities and of size up to 3/16". Differing authors give similar types of equation for the value of \( V_H \). For any particular case the numerical value for \( V_H \) may vary quite an amount from the value predicted by equation 11.

As a rough rule, mixtures which behave as homogeneous ones in even weak turbulent conditions (as opposed to heterogeneous mixtures which tend to settle with a non-constant concentration across a vertical diameter in a horizontal pipe) are limited to mixtures of particles of less than 50 microns. That is, mixtures of particles greater than about 50 microns will exhibit heterogeneous before showing homogeneous behaviour with increasing velocity and do not follow the relationship of equation 10 until the velocity is probably much in excess of the economic velocity \( V_c \). The specific gravity of the particles will obviously be a factor too since low specific gravity particles of a given size can be kept in homogeneous suspension more readily than particles of high specific gravity. Thus fall velocity \( V_s \) is a factor in heterogeneous flow.

**Heterogeneous Suspension**

As seen from Figures 10 and 12 at velocities below the standard velocity \( V_H \) for homogeneous flow the relationship of equation 10, namely

\[
\frac{i_m - i_w}{C_T i_w} = (s - 1) \quad (10)
\]

is no longer valid except for very small particles.

For velocities in excess of the critical velocity where the flow is a heterogeneous one, with or without a moving bed, it would seem reasonable to amend equation 10 by introducing suitable additional parameters to the right-hand side of the equation whose values tend to unity at high velocities so that equation 10 again becomes applicable when the flow changes from heterogeneous to homogeneous.

Equation 10 may be re-written as

\[
i_m = (s - 1) C_T i_w + i_w
\]= \[
\left[(s - 1) C_T + 1\right] i_w
\]= \[
\phi_i i_w
\]

where \( \phi_i \) is a simple function of \( s \) and \( C_T \) for homogeneous flow. It is agreed that \( \phi_i \) should be modified for heterogeneous flow by additional parameters. The question arises as to what these parameters might be. For obvious reasons these parameters ought to be dimensionless so that, using consistent units, any relationship should have fairly universal application in any system of units.
In turbulent flow a parameter of great usefulness in model work where one extrapolates results to corresponding analogous situations is the Froude Number $N_F = \frac{v}{\sqrt{gD}}$. It will be recalled that this dimensionless Number appears in the pipe loss equation, namely

$$\frac{h_n}{L} = i = \frac{f}{2} \left(\frac{v}{\sqrt{gD}}\right)^2 = \frac{f}{D} \cdot \frac{v^2}{2g}$$

A further measure which will also be of importance is the size and shape of particle being conveyed. (Homogeneous flow has nothing to say about size, only specific gravity.) This can be taken care of by introducing either $\frac{v_s}{\sqrt{g d}}$, another form of dimensionless Froude Number (See Figure 3) or by introducing $\sqrt{C_d}$, which is also dimensionless and is inextricably linked with both $v_s$ and $d$. (See Figure 3A), or by introducing $\sqrt{C_d}$ from equation 2c.

It might reasonably be expected that an equation of the form

$$\frac{i_m - i_w}{i_w} = f_1(C_T), \ f_2(s-1), \ f_3\left(\frac{v}{\sqrt{gD}}\right), \ f_4\left(\frac{v_s}{\sqrt{g d}}\right)$$

would represent the typical result of Figure 11.

Experimental results of numerous authors would lead one to believe that the concentration term $f_1(C_T)$ is a linear one, at least within the range of tests reported. Hence equation 12 may be re-written in the form

$$\frac{i_m - i_w}{C_T i_w} = f_2(s-1), \ f_3\left(\frac{v}{\sqrt{gD}}\right), \ f_4\left(\frac{v_s}{\sqrt{g d}}\right)$$

(13)

From Figure 3 it is seen that $\frac{v_s}{\sqrt{g d}}\left(=\frac{1}{\sqrt{C_d}}\right)$ tends to a constant value at values of $d$ greater than about 2 mm. Thus it would appear that an increase in particle size over 2 mm. has no effect on the friction loss ratio $\frac{i_m - i_w}{C_T i_w}$. Such a conclusion is borne out very well by Durand's experimental work on sand and gravel.

The question now arises as to the values to be assigned to the functions $f_2$, $f_3$ and $f_4$ in equation 13 and as to whether they can be combined in any fashion since they are all dimensionless; here one must rely solely on experimental evidence.

It would appear that the commonest method of expressing these functions, and one well-known to hydraulic engineers, is in the form, for instance

$$f_3\left(\frac{v}{\sqrt{gD}}\right) = \text{Constant} \left(\frac{v}{\sqrt{gD}}\right)^a$$

where 'a' is a power determined experimentally as is the constant.

A relationship suggested by Durand and Condolios (1952) has the form

$$\frac{i_m - i_w}{C_T i_w} = \text{Constant} \left(\frac{\sqrt{gD}}{v}\right)^3 \left(\frac{1}{\sqrt{C_d}}\right)^{3/2}$$

for sand and gravel $s = 2.65$, that is $s$ constant.

The Constant in the above equation, from tests carried out at the Neyrpic Grenoble Laboratories and the Loire Maritime Department is found to be 180 according to Gibert (1960) and 176 according to Worster (1952).

That is, for the no-deposit regime of heterogeneous flow

$$\frac{i_m - i_w}{C_T i_w} = 180 \left[\frac{\sqrt{gD}}{v}\right]^3 \left[\frac{1}{\sqrt{C_d}}\right]^{3/2} = 180 \left[\frac{gD}{v^2} \cdot \frac{1}{\sqrt{v_d}}\right]^{3/2}$$

(14)

*Strictly speaking, since pipe-flow is a pressure flow, this is a pseudo Froude Number.*
\[
\frac{m - i_w}{C_T i_w} = 180 \left[ \frac{\sqrt{gD}}{v} \right]^3 \left[ \frac{v_s}{\sqrt{gd}} \right]^{3/2} = 180 \cdot \left[ \frac{gD}{v^2} \cdot \frac{v_s}{\sqrt{gd}} \right]^{3/2}
\]  

(15)

Both equations 14 and 15 are for sands and gravel of S.G. 2.65. However, Durand and Condolios (1952) agreed that the effect of specific gravity could be incorporated in the above relationship in the form

\[
\frac{m - i_w}{C_T i_w} = \text{Const} \left[ \frac{gD(s-1)}{v^2} \cdot \frac{1}{\sqrt{C_d'(s-1)}} \right]^{3/2}
\]

\[
\frac{m - i_w}{C_T i_w} = \text{Const} \left[ \frac{gD(s-1)}{v^2} \cdot \frac{v_s}{\sqrt{gd(s-1)}} \right]^{3/2}
\]

(In effect \( gD \) is replaced by \( gD(s-1) \) and \( \frac{1}{\sqrt{C_d'}} \) by \( \frac{1}{\sqrt{C_d'(s-1)}} \).)

Since \((s - 1)\) is 1.65 for sand the resulting equation for non-deposit heterogeneous flow becomes

\[
\frac{m - i_w}{C_T i_w} = 123 \left[ \frac{gD(s-1)}{v^2} \cdot \frac{v_s}{\sqrt{gd(s-1)}} \right]^{3/2}
\]  

(16)

\[
\frac{m - i_w}{C_T i_w} = 123 \left[ \frac{gD(s-1)}{v^2} \cdot \frac{1}{\sqrt{C_d'(s-1)}} \right]^{3/2}
\]  

(17)

Since \((s - 1)\) is 1.65 for sand the resulting equation for non-deposit heterogeneous flow becomes

\[
\frac{m - i_w}{C_T i_w} = 123 \left[ \frac{gD(s-1)}{v^2} \cdot \frac{v_s}{\sqrt{gd(s-1)}} \right]^{3/2}
\]  

(16)

\[
\frac{m - i_w}{C_T i_w} = 123 \left[ \frac{gD(s-1)}{v^2} \cdot \frac{1}{\sqrt{C_d'(s-1)}} \right]^{3/2}
\]  

(17)

Results for materials of \( s \) value ranging from 1.60 to 3.95 are in good agreement with the above formulae (Gibert 1960). Equations 16 and 17 are shown on Figure 13.

Another relationship for heterogeneous flow proposed by Newitt et al (1955) takes the form

\[
\frac{m - i_w}{C_T i_w} = \text{Const.} (s - 1) \frac{v_s}{v} \frac{gD}{v^2}
\]

which when plotted to experimental results reduces to

\[
\frac{m - i_w}{C_T i_w} = 1100 (s - 1) \frac{v_s}{v} \left( \frac{gD}{v^2} \right)
\]  

(18)

Comparing this latter equation with 16 (or 17) it is seen that

(1) \( \frac{m - i_w}{C_T i_w} \) is inversely proportional to the mean velocity of flow cubed in both equations

(2) in equation 16, \( \frac{m - i_w}{C_T i_w} \) is proportional to \( D^{3/2} \) whereas in 18 it is proportional to \( D \)

(3) in equation 16, \( \frac{m - i_w}{C_T i_w} \) is proportional to \((s - 1)^{3/4}\) whereas in 18 it is proportional to \((s - 1)\). (This difference is a minor one.)

(4) in equation 16, \( \frac{m - i_w}{C_T i_w} \) is proportional to \( \left[ \frac{v_s}{\sqrt{g}} \right]^{3/2} \) whereas in 18 it is proportional to \( v_s \).

Since Newitt et al were testing in a 1" diameter pipe only it would appear to be reasonable to give credence to Durand's relationship for \( D \) insofar as the latter tested many differing diameters. The discrepancies between the two expressions is not great except at low values of \( \frac{m - i_w}{C_T i_w} \) where experimental accuracy is not too good.
\[
\frac{i_m - i_w}{C_T i_w} = 123 \left[ \frac{gD(s-1)}{v^2} \sqrt{C_d(s-1)} \right]^3 = 123 \left[ \frac{gD(s-1)}{v^2} \frac{v_s}{\sqrt{g}(s-1)} \right]^3
\]

[After GIBERT, PONTS 8]
[CHAUSSEES 1960 p. 340]

\[
\frac{v^2}{gD(s-1)} \frac{\sqrt{C_d(s-1)}}{1} \quad \text{or} \quad \frac{v^2}{gD(s-1)} \frac{\sqrt{g}(s-1)}{v_s}
\]

FIG. 13
b) Vertical Pipe

Homogeneous mixtures can be treated as for horizontal pipes, the head losses being a simple function of the specific gravity of the mixture.

For heterogeneous mixtures at values of $v$ greater than $v_c$, the critical mean velocity for no deposition, Durand and Condolios (1952) and Gibert (1960) claim that the actual head losses are the same as with clear water.

Refer to Figure 14, which shows (a) a rising flow and (b) a descending flow using a water differential manometer.

\[ \frac{p_A - p_B}{w_w} \text{ (ft. of water)} = L s_m + L i_w \text{ for upward flow} \]
\[ = -L s_m + L i_w \text{ for downward flow} \]

where $s_m = \frac{w_m}{w_w}$ and $i_w = f \frac{L}{D} \frac{v^2}{2g}$

It may also be readily shown that, from the manometer, considering upward flow,

\[ p_z = p_z' \quad \text{(ignore $\Delta h$ of air)} \]
that is \[ p_0 = p_a - L w_w - \Delta h w_w \]

\[ \frac{p_a - p_0}{w_w} = L + \Delta h \]

But \[ \frac{p_a - p_0}{w_w} = L s_m + L i_w \]

\[ \Delta h = L s_m + L i_w - L \]

i.e. \[ \Delta h = L i_w + L (s_m - 1) \] for upward flow

and \[ \Delta h = L i_w + L (1 - s_m) \] for downward flow

These latter relationships make possible an easy method of measuring concentration since a knowledge of \( \Delta h \), \( i_w \) and \( L \) gives a value of \( s_m \). Knowing \( s_m \) and \( s \), \( C_T \) may be obtained from equation 4, namely \( s_m = 1 + C_T (s - 1) \).

The foregoing relationship for head loss in vertical conveying, namely that it is the same as for clear water, is somewhat surprising. The practical difficulties are fairly great, however, in measuring over any appreciable lengths in a vertical laboratory set-up and any effect of S.G. may be masked at small concentrations. For example, the \( s_m \) value of a 10% \( C_T \) of sand, \( s = 2.65 \) is 1.165 whereas \( i_w \) (in a 6 inch line), at say 8 ft/sec., is about 0.03, a small percentage of the \( s_m \) value.

When a concentration \( C_T \) is pumped vertically up a pipe, as in a mining operation, a distance \( L \), the following relationship may be obtained.

In an upwards vertical flow system the heavier than water particles will move downwards relative to the upwards moving water with a velocity equal to the hindered settling velocity. As a consequence the concentration in the pipe in vertical upwards flow will tend to be higher than the delivered concentration \( C_T \). However, this effect of additional concentration will be small if \( v \) is a good deal greater than the hindered settling velocity. Ignoring this effect the following efficiency of the vertical operation may be deduced.

Pressure at base of \( L \) feet of vertical pipe = \( w_m L + w_w h_L \) where \( w_m \) and \( w_w \) are the specific weights of the mixture and the water respectively and \( h_L \) is the water friction loss \( ( = \frac{f L}{D} \frac{v^2}{2 g}) \). The volume of solids raised per second is \( C_T \frac{n_0^2}{4} v \) and the useful work done/sec. in raising the solids - the object of the operation - is \( w_s C_T (\frac{n_0^2}{4}) v L \).

Thus the work done/sec. in circulating the water is \( w_w \frac{n_0^2}{4} v \cdot 2 h_L \).

The hydraulic efficiency, \( \varepsilon_H \), of the raising operation is given by,

\[ \varepsilon_H = \frac{w_s C_T \frac{n_0^2}{4} v \cdot L}{w_s C_T \frac{n_0^2}{4} v L + w_w \frac{n_0^2}{4} v 2 h_L} \]

\[ = \frac{s \cdot C_T}{s \cdot C_T + 2 \frac{h_L}{L}} \]

hence \( \varepsilon_H = \frac{s \cdot C_T}{s \cdot C_T + 2 i_w} \).
If the mechanical efficiency of the pump-motor set is $\xi_p$, then the overall efficiency of this mode of conveying is given by

$$\text{overall efficiency} = \xi_p \left[ \frac{s \cdot C_T}{s \cdot C_T + \frac{2i_w}{v}} \right]$$

For example, if an ore of $s = 4$ is delivered at 20% concentration by volume with an $i_w$ of 0.02 (6 ft/sec in a 6 inch pipe) with a pump efficiency of 50% the overall efficiency is 47%.

However, this is an idealized situation, since in practice additional work would have to be done in introducing the solids against pressure at the bottom of the line. It would seem that in any charging system an additional loss would be sustained - so adding a further term to the denominator and thereby reducing the overall efficiency. It may be that 30% is a more realistic overall efficiency: even at this figure hydraulic lifting may be an economic and attractive alternative to conventional hoisting methods.

c) Inclined Pipe

It was pointed out in the previous section that in a vertical situation that $i_m \approx i_w$. However, in an inclined situation, the friction loss is greater than in a horizontal line according to summarized replies on this question reported in a Hydraulics Institute questionnaire of April 1962 whereas Gibert (1960) gives the expression based (on horizontal pipes) of

$$\frac{i_m - i_w}{C_T i_w} = \text{Function of} \frac{v^2}{gD} \frac{\sqrt{C_d}}{\cos \alpha}$$

where $\alpha$ is the inclination of the line ( $\alpha = 0$ is horizontal). Such an expression reveals a decreasing value of $i_m$ from $\alpha = 0$ to $\alpha = 90^\circ$ such that at $\alpha = 90^\circ$ $i_m \approx i_w$.

In effect the horizontal pipe-line relationship, equation 16, is amended to

$$\frac{i_m - i_w}{C_T i_w} = 123 \left[ \frac{gD(s-1) \cdot \cos \alpha}{v^2 \sqrt{C_d(s-1)^3}} \right]$$

It may be that the upwards sloping pipe does require a greater critical velocity $v_c$, since it is reasonable to assume that the gravity force also must be overcome with sliding bed material. Fraser (1960) reporting on the test work preceding the installation of the International Nickel Co. of Canada tailings line found an increased friction head loss in uphill flow and a decreased friction loss in downhill flow. However, in the view of Costantini (1961) "there should be no significant increase in either minimum (i.e. critical) velocity or friction head loss up a sloping line."

In view of the paucity of experimental evidence no general conclusions can be drawn regarding this issue.
4. PUMP CHARACTERISTIC REQUIREMENTS

As has been seen from section 3 on friction or energy losses, the energy input at the pumping end of a horizontal system per unit weight of material delivered,

a) decreases with the concentration,

b) depends on the size of material being transported below a size of roughly 2 mm whereas at 2 mm and above it is at its maximum. The dependence on size of particle is such that it decreases with size,

c) increases with increasing density of material being pumped.

The following figures of Condolios et al (1961) give some idea of the powers involved in delivery at a rate of 100 Tons per hour of material of $s = 2.65$.

"to transport gravel larger than 2 mm diameter, provision must be made for power of about 6 H.P. per ton mile."

"for sand with a mean diameter of 0.4 mm, and included between 0.15 mm and 0.7 mm, the power required would be about 2.2 H.P. per ton mile."

"the power required would only be 0.12 H.P. per ton mile for raw cement paste with a mean diameter of the order of 40 to 50 microns."

Based on these figures, if power is bought at $50 per H.P. per year, the cost per ton-mile for overcoming the energy loss will vary from 3.5 cents per ton mile for large material to about 0.1 cents for small material on the basis of a continuous pumping operation.

(For vertical conveyance it is sufficient to adopt a transport velocity $\nu$ of about 2 to 3 times the fall velocity $v_f$. The power consumption has already been indicated on page 28.)

For safety of operation against jamming it is suggested that the pipe diameter be at least 2 to 3 times the size of the greatest particle being transported. (This will obviously not be applicable to "packaged" or "sausage" type of plug flow of materials.)

From a power economy viewpoint it will be advantageous to work near a velocity $v_c$.

However, with a constant-speed centrifugal pump with a flattish $H \nu Q$ curve a slight drop in pump speed or an increase in $C_T$ at the intake end will make it impossible for the two characteristics to meet and there will be a tendency for the flow rate to diminish and deposition and jamming to occur.

This is illustrated in Figure 15 where concentration $C_3$ is incapable of being transported and $C_2$ is also in the unstable region since a fall in $H$ due to a momentary speed decrease could cause a similar effect, or an increase in concentration start deposition and thus increase head-losses.

From a practical viewpoint with centrifugal pumps it is therefore preferable to control the feed rate of solids into the pump. The measurement of pressure drop on the initial length of pipe on the delivery side of the pump is a good indication of the variation in concentration. An increase in pressure drop can be used as a signal to decrease the feed rate.

On long lines the use of centrifugal pumps is not too practical at present since a single-stage pump usually delivers a maximum of 150 ft. of head and the number of pumps in series becomes great.

For long lines involving high pressures the reciprocating positive displacement pump is the only practical answer at present. Since such a pump has a vertical $H \nu Q$ characteristic this gives a most stable operation. (With a centrifugal pump it is preferably to work on the steeper falling part of the $H \nu Q$ curve for stability of operation.)

Starting and stopping of lines has been found to be best carried out - and the writer would fully concur with this from his experience in pumping wood-chips - by using water only. Further, the pump speed should be increased or decreased gradually if possible to obviate water hammer, a difficult problem with one fluid alone but much more complex with mixtures. If necessary pressurised surge chambers should be used to decrease water hammer effects.

For more detail regarding actual pumping installations the reader is referred to the Bibliography. Of particular interest to vertical lifting in mining operations is the paper by Condolios et al (1961).
FIG. 15. TYPICAL HvQ CURVE FOR CENTRIFUGAL PUMPING OPERATION
The following bibliography is far from complete and only contains references which were readily available to the writer in Queen's University Library. However, it is felt that they do give an adequate coverage of the problem of hydraulic conveying both from a theoretical and a practical viewpoint.

1. ANONYMOUS, "Advent of Big Coal Pipe-line Nearer", The Oil and Gas Journal, Nov. 13, 1961, pp 122-123.


33. MORELAND, C., "Pipeline Transportation of Solids", Research Council of Alberta (1960


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