Controller design for a magnetically levitated spindle system

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Abstract

In this thesis, a complete design strategy is presented for the control of a magnetically levitated spindle. The spindle is levitated by electromagnets located at the spindle ends. Two approaches are used in obtaining a stabilizing control system for this naturally unstable plant. The first approach is to model the spindle as two independently levitated masses, which results in two one-dimensional problems. For the control this means that two SISO (Single Input Single Output) controllers are used. The second is to model both degrees of freedom of the spindle, thus taking into account the mechanical coupling between the spindle ends, resulting in a MIMO (Multiple Input Multiple Output) controller. Two different controller types are used, viz. a lead compensator and a state feedback controller. The designed controllers are then implemented on a real-time system to be tested in an experimental setup.
Preface

This thesis is written as part of the bachelor graduation project of Electrical Engineering at the Technical University of Delft. A group of six persons has focussed on designing a setup to levitate a spindle. Three groups of two persons will focus on separate components of this system. In this thesis, the design of the controller, needed to stabilize the system, is described. The remaining two groups have put their efforts in the design of sensors and actuators.

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Summary

In this thesis, a complete design strategy is presented for the control of a magnetically levitated spindle. The spindle is levitated by electromagnets located at the spindle ends. Two approaches are used in obtaining a stabilizing control system for this naturally unstable plant. The first approach is to model the spindle as two independently levitated masses, which results in two one-dimensional problems. For the control this means that two SISO (Single Input Single Output) controllers are used. The second is to model both degrees of freedom of the spindle, thus taking into account the mechanical coupling between the spindle ends, resulting in a MIMO (Multiple Input Multiple Output) controller. Two different controller types are used, viz. a PID and a state feedback controller. The designed controllers are then implemented on a real-time system to be tested in an experimental setup.

The models were derived, by setting up the equations of motion for both the 1-DOF plant and the 2-DOF plant. Since only linear controllers were used, these nonlinear models were linearized first. The operating point was set taking into account the limitations of current through the coil. For each plant, the case of voltage control and current control were modeled. Since the setup uses a voltage to voltage amplifier to drive the coils, current control cannot be done without an extra controller. Lead compensation was used to obtain a satisfying current controller. This could be applied, because the system itself was already stable.

The controller design was based on assumption of the presence of an ideal voltage controlled current source used for driving the coils. The design of the voltage controlled current source was done separately. The choice to use current control was made to allow for better reference tracking and simpler modeling. In the design of the state feedback controllers, the advantage of a simpler model was apparent when using a second order reference system to determine the closed-loop dynamics. The poles of the reference system with specified settling time and overshoot were taken as closed-loop poles for the levitated spindle system. A PID controller was used only for 1-DOF control. This controller was designed using a pole placement method. The PID is only designed for stabilizing the system.

A PID controller is good for stabilizing the unstable plant, but for reference tracking the state feedback controller gave more satisfying results.
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Part I

System description & modelling

1 Introduction

This thesis is written to give insight in designing applications like active magnetic bearings. A setup was available to levitate a spindle with two electromagnets. This means that the spindle is to be suspended in air without mechanical support. The setup was equipped with optical sensors measuring displacement of the spindle. The goal is to find a stabilizing controller for this system. Various designs of control systems are presented to stabilize a magnetically levitated spindle. First, a more detailed description of the setup is given in chapter 2. In order to design a good controller, knowledge of the dynamics of the system to be controlled (further referred to as plant) is essential. Chapter 3 presents two approaches to modeling the plant: the spindle seen as two 1-degree of freedom (1-DOF) system (chapter 3.1) and the spindle modeled as a 2-degrees of freedom system (2-DOF) (chapter 3.2). Linearization of these models is done in chapter 5. Linearization is necessary since only linear controllers are considered in this thesis. Different means of actuation of the electromagnets and a choice of method is described in chapter 4. Voltage and current control are evaluated here. The design of the controllers is presented in chapter 7. Here, a PID controller and a state feedback controller are used to stabilize the plant modeled as two 1-DOF systems. For the 2-DOF model, a state feedback controller is designed. Simulations of the closed-loop systems are covered in chapter 8. Finally, the design procedure is evaluated and recommendations are given in chapter 9.
2 System description of a levitated spindle

In this chapter a walk through all the components used in the levitated spindle system will be taken. This way a good insight in what the system does and what all different parts of the system have to contribute to the final goal is given. In figure 1 the plant, which has to be controlled, is shown. The complete system is schematically shown in figure 2. All the parts are labelled and are shown in figure 2. Each of the parts will be highlighted in the next paragraphs. The individual components of the system are: spindle, sensors, actuators, real-time interface, amplification and control.

![Image 1: The plant under control](image1.png)

![Figure 2: Schematic view of the system](image2.png)
2.1 Spindle

A picture of the spindle to be used in the levitated spindle system is shown in figure 3. The specifications of the spindle are shown in table 2.1:

![Spindle Image]

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>115</td>
<td>gram</td>
</tr>
<tr>
<td>Length</td>
<td>130</td>
<td>mm</td>
</tr>
<tr>
<td>Diameter</td>
<td>12</td>
<td>mm</td>
</tr>
</tbody>
</table>

The properties mass, length and diameter are essential in deriving the spindle dynamics, which in turn are essential when designing the controller. The material used for the spindle is silversteel, which is a ferromagnetic material. This is important, since the spindle is to be levitated by a magnetic field.

2.2 Sensors

Two kinds of sensors are available to be used in the levitated spindle system. First, the optical sensors will be described. The second type of sensors, capacitive sensors, will be described after that.

2.2.1 Optical sensors

The available optical sensors are shown in figure 4. These sensors are of the shelf, industrial type sensors. Distance measurement can be done in the range of 60 to 250 mm, with a resolution of 10 micron. The measurements of the optical sensors are fed back to the control system (figure 2) to be processed further. For more knowledge about the specifications of the optical sensors, the reader is referred to the thesis [SB08].
2.2.2 Capacitive sensors

The second type of available sensors is a capacitive sensor. Four sensors are used to measure displacement of the spindle. The capacitive sensors will measure any deviation from the spindle with respect to its equilibrium point by measuring the current through each of the four plates. For more in-depth technical information, the reader is referred to [SB08].

2.3 Actuators

As described above, the actuators for the levitated spindle system are two electromagnets. These magnets will generate a magnetic field, which on his turn makes the spindle levitate. In figure 5 the actuators used in the levitated spindle system are shown. The specifications of the electromagnets are given in table 2.3 shown below.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point of acting force</td>
<td>2</td>
<td>mm</td>
</tr>
<tr>
<td>Max. continuous current</td>
<td>5</td>
<td>A</td>
</tr>
<tr>
<td>Peak current</td>
<td>6</td>
<td>A</td>
</tr>
<tr>
<td>Average inductance(^1)</td>
<td>5</td>
<td>mH</td>
</tr>
</tbody>
</table>

For more in-depth derivation of the specifications used for the electromagnets, the reader is referred to the following thesis [DHSJ08].

2.4 Real-time interface

For the communication between the sensors and the controller and for the communication between the controller and the actuators, a real-time dSPACE system is used. This system is shown in figure 6. The dSPACE system can communicate with a plant on a real time basis. The communication works
The six steps numbered in figure 7 are explained below.

1. The user creates a Matlab/Simulink model on the PC.
2. Matlab builds (Conversion to C and compilation).
3. Matlab loads the compiled model to dSPACE hardware and runs it.
4. The compiled model on the dSPACE hardware interacts with the setup.
5. Control-Desk reads/writes variables and parameters in compiled, running model.
6. The user reads data, values, etc. and/or changes parameters in the model.

The dSPACE system comes with a 32 input A/D-converter and a 6 output D/A-converter. With our system needing only 2 inputs and 2 outputs, there is redundancy in availability of I/O ports. The A/D-converter reads out the input ports by multiplexing all 32 inputs in pairs. In ControlDesk, the graphical user interface for the dSPACE system, the user can select which inputs are to be used. The A/D-converter will only multiplex between these selected inputs. The sample rate after the analog to digital conversion is user definable in ControlDesk. The bandwidth of the sensors is 500 Hz, so with nyquist in mind, the sample rate of the dSPACE system should at least 1000 Hz.
2.5 Amplification

An amplifier is used between the each control output and the input of the actuator. The amplifier is needed here because the output of the dSPACE D/A-converter can not deliver enough power to generate the magnetic field needed to levitate the spindle. The amplifiers used are voltage to voltage amplifiers, hence the control input to the actuators is a voltage.

2.6 Control

The control of the system can be compared with the brain of the human body. Here all the decisions are made. These decisions are based on the information obtained from outside the human body. Just like the control in this case getting his information from the sensors. With this information a decision is made whether the output voltage should increase or decrease.

For designing the control for the levitated spindle a Matlab/Simulink environment is used. This environment allows us to use the dSPACE system as described above. Also being familiar with this environment made us decide to use the Matlab/Simulink environment instead of Labview. Because the thesis is about the control and is already described several times, here it will be kept short. If an interest goes out to the control of the levitated spindle.
spindle system, the following chapters about the modeling linearizing and controller design should be read.
3 Modelling the physics of the plant

In this chapter a model will be derived for the levitated spindle system. For obtaining a good model for the levitated spindle a simplified model is derived first. This simplified model contains only one actuator. This means that only one magnetic field is applied on the spindle. The spindle will only be controlled in the translational direction. In other words the spindle will only be controlled in 1 degree of freedom. When this approach is done and the dynamics considering one electromagnet are clear, an extension is made with a second electromagnet. This means that the spindle will be actively controlled in two places. Now the translational degree of freedom and a rotational degree of freedom will be controlled. The system is evolved into the 2 degree of freedom system, which will have to be controlled for the setup as shown in figure 1. The models for a 1 degree of freedom and a 2 degrees of freedom plant will be derived in the following section.

3.1 1 degree of freedom

In this paragraph a non-linear model for a 1 degree of freedom system is created. It will be compared to a levitated ball system as shown in figure 8. For realizing the model, two forces have to be taken into account. The first force is the gravitational force, which works on the ball. To have a system in equilibrium the sum of the forces has to be zero. There must be another force opposing the gravitational force to obtain this requirement. This force is created by the magnetic field, which on his turn is created by the electromagnet. The electromagnet is placed directly above the levitated ball, because electromagnets can only attract a ferromagnetic metal, which is the material used for the ball. These two forces cancel each other out and should make it possible to levitate the ball. So, the total force given in equation (1) should equal zero.

![Electro magnent](image)

Figure 8: Mechanical implementation
The gravitational force is a constant force acting on the ball. The actual force is calculated by multiplying the gravitational constant times the mass of the ball. The magnetic force is a nonlinear force and depends on multiple factors. The distance between magnet and ball and the current through the coil of the electromagnet will have to be taken into account. How this will be done will be explained in the next paragraphs.

Distance  

The first non-linear factor on which the magnetic force is based is the distance ($x_s$). A predefined distance between the ball and the electromagnet defines the operating point. With this point the exact value for the current can be calculated for a system in equilibrium. The distance $x_s$ can easily be disrupted due to influences, like a small touch of a human hand, a blow of wind or even vibrations will be enough to disturb the equilibrium. When this event occurs and there is no accurate feedback the system will be unstable and the ball will drop.

The magnetic force decreases on a quadratic rate with an increasing distance. Because of the opposing directions of increase, the force is inverse quadratic with the distance. In figure 9 the dependence is shown.

Figure 9: Distance dependence

Figure 9 shows the operating point. Separate parts of the magnetic dependence are taken into account. The operating point works in the area, where a quadratic dependence gives a good approximation of the real plant. In the part, where $x_s$ approaches zero, the quadratic approximation is not valid anymore. The operating point will be in the second part and therefore will be approximated with the quadratic dependence.

Current  

The second non-linearity in the force of the magnets is the current. The current is used for active control. By increasing the current the magnetic force will increase as well. By adapting the current in the electromagnet at the moment a deviation in the distance is measured by the
sensors, a stable system can be created. Just like the distance, the current also has a quadratic dependence on the magnetic force. Here force and current will change in the same direction as shown in figure 10.

![Figure 10: Current dependence](image)

As shown in figure 10 the point of operation is in the area where a quadratic approximation is valid. What is not directly shown in the figure is that the magnetic force will saturate with an increasing current. For accurate control, it is clear that saturation of the magnet may never be reached.

The next three equations will show the forces of interest in our system for the levitated ball, the forces are the magnetic force, the gravitational force and the total force in equations (2), (3) and (4) respectively. In the first equation \( C \) defines a constant value, which will depend on the dynamics used for the electromagnet.

\[
F_m = C \left( \frac{i}{x} \right)^2 \tag{2}
\]
\[
F_x = mg \tag{3}
\]
\[
F_{tot} = m \ddot{x} \tag{4}
\]

By inserting equations (2), (3) and (4) into equation (1) the equation for the model of the levitated ball is created. The result is shown in equation (5).

\[
m \ddot{x} = mg - C \left( \frac{i}{x} \right)^2 \tag{5}
\]

Notice the minus sign in the equation (5). The minus sign imposes an opposite direction of gravitational and magnetic forces. In this model the gravitational force is positive, because the distance \( x_s \) increases when the ball drops. The equation can be used for creating a model of the plant in simulink.
Simulink model  The Simulink model showing the nonlinear relation between the force and both the distance and the current is shown in figure 11.

Figure 11: Simulink model of a nonlinear levitated ball

3.2 2 degrees of freedom

In this paragraph the model for a spindle, suspended in a magnetic field, is constructed. The magnetic field is generated by two electromagnets, exerting force on the spindle at its ends. Two electromagnetic forces $F_1$ and $F_2$ act on the spindle at distances $l_{f1}$ and $l_{f2}$ resp., from the centre of mass. Two sensors $s_1$ and $s_2$ measuring the distance to the spindle, are located at distance $l_{s1}$ and $l_{s2}$ resp. from the spindle’s centre of mass. The downward direction is chosen to be positive. See figure 12.

Figure 12: Levitated spindle

Equations of motion  The two forces acting on the spindle are cause of both a translational and a rotational displacement ($x$ and $\theta$, see figure 12).
The equation of translational motion is given by

\[ m \ddot{x} = mg - (F_1 + F_2) \]  \hspace{1cm} (6)

which means that the net force in the \( x \)-direction is equal to the difference between the gravitational force and the sum of the electromagnetic forces.

Rotational motion is described by

\[ I_z \ddot{\theta} = F_2 l_f 2 - F_1 l_f 1 \]  \hspace{1cm} (7)

\[ I_z = \frac{m}{12} (3r^2 + l^2) \]  \hspace{1cm} (8)

which means that the net torque on the centre of mass is equal to the difference between the two magnetic forces working from a distance on the centre of mass. The moment of inertia \( I_z \) about the \( z \)-axis is given by (8).

In matrix form, (6) and (7) are written as:

\[
\begin{bmatrix}
  m & 0 \\
  0 & I_z 
\end{bmatrix}
\begin{bmatrix}
  \ddot{x} \\
  \ddot{\theta}
\end{bmatrix} =
\begin{bmatrix}
  mg \\
  0
\end{bmatrix} +
\begin{bmatrix}
  -1 & -1 \\
  -l_f 1 & l_f 2
\end{bmatrix}
\begin{bmatrix}
  F_1 \\
  F_2
\end{bmatrix}
\]  \hspace{1cm} (9)

Sensor and bearing coordinates

Since the electromagnets and sensors are not necessarily at the same location along the \( z \)-axis, the sensors may measure a displacement different from the actual displacement of the spindle w.r.t. the electromagnets. Four new spindle-displacements are now introduced: \( x_{f1} \) for the displacement w.r.t. electromagnet 1, \( x_{f2} \) for the displacement w.r.t. electromagnet 2, \( x_{s1} \) for the displacement w.r.t. sensor 1 and \( x_{s2} \) for the displacement w.r.t. sensor 2. Since the sensors also may not be at the same location along the \( x \)-axis, a new variable denoting the distance from sensors to the spindle at initial position is introduced: \( x_{s0} \).

The displacements \( x_{f1,2} \) and \( x_{s1,2} \) follow from (10), (11), (12) and (13).

\[
x_{f1} = x_0 + x + l_f 1 \tan \theta \hspace{1cm} (10)
\]
\[
x_{f2} = x_0 + x - l_f 2 \tan \theta \hspace{1cm} (11)
\]
\[
x_{s1} = x_{s0} - x - l_{s1} \tan \theta \hspace{1cm} (12)
\]
\[
x_{s2} = x_{s0} - x + l_{s2} \tan \theta \hspace{1cm} (13)
\]

Since only small deviations from the equilibrium position \( x_0 \) and angle \( \theta_0 \) will be assumed, equations (10) to (13) can be simplified. Since for small angles \( \theta \), \( \tan \theta \approx \theta \). This will simplify equations (10) to (13) to:

\[
x_{f1} = x_{f0} + x + l_f 1 \theta \hspace{1cm} (14)
\]
\[
x_{f2} = x_{f0} + x - l_f 2 \theta \hspace{1cm} (15)
\]
\[
x_{s1} = x_{s0} - x - l_{s1} \theta \hspace{1cm} (16)
\]
\[
x_{s2} = x_{s0} - x + l_{s2} \theta \hspace{1cm} (17)
\]
Equations (14) to (17) are then put in matrix form as follows:

\[
\begin{bmatrix}
  x_{f1} - x_{f0} \\
  x_{f2} - x_{f0}
\end{bmatrix} = Z_f =
\begin{bmatrix}
  1 & l_{f1} \\
  1 & -l_{f2}
\end{bmatrix}
\begin{bmatrix}
  x \\
  \theta
\end{bmatrix} = T_f
\begin{bmatrix}
  x \\
  \theta
\end{bmatrix}
\] (18)

\[
\begin{bmatrix}
  x_{s1} - x_{s0} \\
  x_{s2} - x_{s0}
\end{bmatrix} = Z_s =
\begin{bmatrix}
  -1 & -l_{s1} \\
  -1 & l_{s2}
\end{bmatrix}
\begin{bmatrix}
  x \\
  \theta
\end{bmatrix} = T_s
\begin{bmatrix}
  x \\
  \theta
\end{bmatrix}
\] (19)

Since \( x_{s1} \) and \( x_{s2} \) are measured, but knowledge about \( x_{f1} \) and \( x_{f2} \) is desired, (18) and (19) are rewritten as

\[
\begin{bmatrix}
  x \\
  \theta
\end{bmatrix} = T_f^{-1} Z_f
\] (20)

\[
\begin{bmatrix}
  x \\
  \theta
\end{bmatrix} = T_s^{-1} Z_s
\] (21)

in order to obtain

\[
T_f^{-1} Z_f = T_s^{-1} Z_s
\]

\[
Z_f = T_f T_s^{-1} Z_s
\] (22)

Now, (22) is inserted in (9) to obtain the complete model in matrix form:

\[
M \begin{bmatrix}
  \ddot{x}_{f1} \\
  \ddot{x}_{f2}
\end{bmatrix} = MT_f T_s^{-1} \begin{bmatrix}
  \ddot{x}_{s1} \\
  \ddot{x}_{s2}
\end{bmatrix} = T_f \left\{ mg \begin{bmatrix} 0 \\
  0 \end{bmatrix} + \begin{bmatrix} -1 & -l_{f1} \\
  -l_{f2} & l_{f2}
\end{bmatrix} \begin{bmatrix} F_1 \\
  F_2 \end{bmatrix} \right\}
\] (23)

**Simulink model** The Simulink model derived out of the equations above is shown in figure 13.
4 Actuation of the electromagnets

This section will cover the control of the input quantity to the actuators (the electromagnets), viz. the current flowing through the coils. As could be seen in section 3, the electromagnetic force is proportional with $i^2$. Ideally, the electromagnetic force would be controlled by a current source connected to the electromagnets, providing the amount of current as determined by the controller of the levitated spindle system. Since a voltage controlled voltage amplifier is used in the levitated spindle system, there is no direct control over the current flowing through the coil. The first paragraph of this section will cover the case where the voltage over the coil is controlled. The second paragraph will describe the case where the current through the coil is controlled, by regulating the input voltage to the amplifier.

4.1 Voltage control

In this paragraph, a model will be derived for the case of a voltage controlled plant. The model will apply for both the 1-DOF and 2-DOF plant. The approach is to write the voltage over the coil in state-space representation, which is then to be combined with the state-space of the plant, to finally obtain a new state-space description of the plant under voltage control. The equivalent electrical circuit of a coil is shown in figure 14. It consists of an ohmage $R$ and inductance $L_c$. The relation between the voltage applied over the coil and the current flowing through the coil is

$$u(t) = L_c \frac{di(t)}{dt} + i(t)R$$

This equation is rewritten to

$$\frac{di(t)}{dt} = -i(t) \frac{R}{L_c} + \frac{1}{L_c} u(t)$$

Figure 14: Equivalent circuit of a coil
The latter can now be put into state-space form with the current \( i \) as state variable and the voltage \( u \) as input.

\[
\dot{x}_v = A_v x_v + B_v u_v
\]  

(26)

with \( A_v = -\frac{R}{L_c} \), \( B_v = \frac{1}{L_c} \) for the 1-DOF plant. For the 2-DOF plant \( A_v = \begin{bmatrix} -\frac{R}{L_c} & 0 \\ 0 & -\frac{R}{L_c} \end{bmatrix} \), \( B_v = \begin{bmatrix} \frac{1}{L_c} & 0 \\ 0 & \frac{1}{L_c} \end{bmatrix} \).

The input to the plant with state-space matrices \( A, B, C, & D \) is the state of (26). Now the model of the plant and coil voltage (24) are combined to obtain the following description of the plant under voltage control.

\[
\begin{bmatrix} \dot{x} \\ \dot{x}_v \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & A_v \end{bmatrix} \begin{bmatrix} x \\ x_v \end{bmatrix} + \begin{bmatrix} 0 \\ B_v \end{bmatrix} u_v
\]  

(27)

This state-space represents the 'new' plant with voltage control. Next, a simulink model of the voltage controlled plant is constructed. The voltage law (24) is transformed to the Laplace domain to obtain the transfer function \( G(s) \) (29)

\[
U(s) = LsI(s) + I(s)R
\]  

(28)

\[
G(s) = \frac{1}{Ls + R}
\]  

(29)

This transfer function is implemented in Simulink as shown in figure 15.

**Why voltage control?** The advantages of voltage control are listed here [SBT94]:

- More accurate modelling of the plant, which implies higher robustness;
- Weaker open-loop instability;
- Easy to implement very low stiffness;

![Simulink model](image-url)

Figure 15: Simulink model for eq. (29)
• A voltage amplifier is easier to realize than a current amplifier;
• The two-way property of electro-mechanical transducers can be used to make sensorless levitation devices.

A disadvantage to voltage control is explained next. Suppose a steady-state current of 1A is desired. For a steady-state it holds that $s = 0$. The voltage over the coil then follows from $IR$. A step input of $IR$ volts is applied to the system (29) and the response is plotted in figure 16. The desired current is reached after some time delay $\tau$. This delay is due to the phase lag introduced by the inductance $L$. The magnitude and phase response are plotted in figure ???. When controlling only the voltage over the coil, phase lag may deteriorate performance. Some disadvantages are listed below:

• Reference tracking will get less accurate with rising frequencies.;
• Higher order systems;

In the next paragraph, a controller is designed to compensate for this phase lag.

4.2 Voltage controlled current source

Now the amplifier and coil are placed in a feedback loop with a controller to compensate for the phase lag due to the inductance $L_c$ of the coil. The
Figure 17: Bode plot of plant $G(s)$

Phase compensation is done by means of a lead compensator. The transfer function of such a compensator is

$$K_l = K_c \frac{s/z + 1}{s/p + 1}$$  

where $z$ is the zero, $p$ the pole and $K_c$ the gain of the compensator. For the compensator to be of a leading type, it must hold that $z < p$, or $\frac{p}{z} = \alpha$ [FDPEN02]. The factor alpha can be determined using the following formulae:

$$\alpha = \frac{1 - \sin(\omega)}{1 + \sin(\omega)}$$  

The design of the compensator is aimed on obtaining a higher bandwidth than the open loop system (see figure 17). That is, the cross-over frequency of the plant $G(s)$ is to be raised. The following design procedure is used to find a suitable lead compensator:

- Determination of the gain $K_c$ to obtain the wanted cross-over frequency.
- Evaluation of the phase margin of the plant using the value $K_c$ obtained in step 1.
- Determination of the needed phase lead. Allow for 10° extra margin.
- Determination of $\alpha$ from (31).
• Calculation of the zero and pole of the compensator (30).
• Analysis of the closed-loop bode plot.
• If necessary, adjustment of the parameters.

The lead compensator for the current control loop is designed to give maximum phase at $\omega_c = 50 \times 2\pi \text{ rad/s}$. The value for $K_c$ was found as follows:

$$K_c = \left| \frac{1}{L_c j\omega + R} \right|^{-1} \quad (32)$$

A bode plot of the open loop system with added gain $K_c$ is given in the first plot in figure 18. The goal of the controller is to eliminate phase lag, so the desired phase margin is $-180^\circ$. The phase that the compensator must add is thus 180-PM, where PM is the phase margin of the uncompensated system with added gain $K_c$. Now the value for $\alpha$ is to be determined from (61c) by filling in the needed phase lead. From $\alpha$, the values for $p_c$ and $z_c$ can be calculated as follows:

$$p = \omega \sqrt{\alpha} \quad (33)$$

$$z = \frac{\omega}{\sqrt{\alpha}} \quad (34)$$

Filling in the values for $K_c$, $p$ and $z$ gives the following compensator $K_l$, with bode plot as shown in the third plot in figure 18. Now all that’s left
is to check for zero steady-state error in the closed-loop transfer function. First, the closed-loop transfer function is constructed:

\[ H_{cl} = \frac{K_i G}{1 + K_i G} \]  

(35)

The steady-state error is calculated by setting \( s \) to zero and the needed gain \( K_{dc} \) to compensate for the error, is given by

\[ K_{dc} = \frac{1}{H_{cl}(0)} \]  

(36)

Finally, a bode plot of the closed loop system is shown in the fourth plot in figure 18.

**Choice of actuation**  The choice is made to design the controllers based on voltage control. Reason for this is the easy practical implementation and the redundancy in off-the-shelf solutions.
5 Linear models of the plant

Before a solution is obtained for the non-linear model, a solution for the linear model, which is obtained by the nonlinear model in an operation point. The operation point is the point where the nonlinear plant has the same behaviour as the linear plant. For small deviations in the operation point the approximation of the linear model remains valid. First the operation point distance and bias current will be derived. These values will be taken into account for the 1 degree of freedom and the 2 degree of freedom linear models, which will also be derived in this chapter.

Operation point The distance between the magnet and the spindle in the operation point equals the position in which equilibrium is established. So the distance is chosen to be near the magnets, but far enough to see that levitation of the spindle is established. \( x_0 \) and \( i_0 \) are the values for the operation point to be determined. For determination of the \( i_0 \) equation (5) can be used. The forces will be in equilibrium, which means a resulting force of 0. Resulting in equation (37), which determines the current in the electromagnet during equilibrium. If a value for \( x_0 \) is chosen the value for \( i_0 \) can be calculated.

\[
i_0 = x_0 \sqrt{\frac{mg}{c}} \quad (37)
\]

5.1 1 Degree of freedom

In the 1 degree of freedom configuration only 1 magnetic force has to be taken into account. As mentioned before, the system will be linearized in the equilibrium point as calculated in the previous paragraph. Or in other words, the point indicated with \( x_0 \) and \( i_0 \) will be the working point, which indicates the centre of the operational field for the levitated ball. The linear model will be used to obtain the final control for the non-linear model. In the problem two non-linear factors will have to be linearized. The form of preference of the non-linear plant is given by equation (38) the general equation for the problem.

\[
f(x, i) = k_x x + k_i i \quad (38)
\]

Here the value of \( k_x \) shows the gradient in the point \( x_0 \) as given in figure 9. The gradient in point \( i_0 \) gives the value for \( k_i \) as shown in figure 10. To get the non-linear model in this form, a derivation is done in the operating point to both the distance and the current. The values for \( k_x \) and \( k_i \) are determined by the equations (39) and (40) respectively[SBT94].

\[
k_x = \frac{d}{dx} C \left( \frac{i_0}{x} \right)^2 = -2C \frac{i_0^2}{x_0^3} \quad (39)
\]
\[ k_i = \frac{d}{di} C\left( \frac{i}{x_0} \right)^2 = 2C \frac{i_0}{x_0^2} \]  
(40)

The total equation for the linear model of the levitated ball is given in equation (41).

\[ f(x, i) = 2C \frac{i_0^2}{x_0^3} x - 2C \frac{i_0}{x_0} i \]  
(41)

**Simulink model**  The simulink model derived by equation (41) is given in figure 19.

Figure 19: Simulink model for linear ball system

**State space representation**  The state space representation of the model is given in equations (42a) and (42b). For further information about obtaining a state space representation for a given model, the reader is referred to [FDPEN02].

\[
\begin{bmatrix}
\dot{x} \\
\ddot{x}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
\frac{k_x}{m} & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{k_i}{m}
\end{bmatrix} i
\]  
(42a)

\[ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \]  
(42b)

With the values for \( k_x \) and \( k_i \) calculated in equations 39 and 40 respectively.

**5.2 2 Degrees of freedom**

In this paragraph a linear state-space model of the levitated spindle will be derived. First, the nonlinear model as given in paragraph 23 will be linearized around a desired operating point. Then, the linear model will be put in state-space form, so it is ready to be put into the controller.
5.2.1 Linearization

The only nonlinearities are in the forces exerted by the electromagnets on the spindle\(^2\). The equation for the force is repeated here:

\[
F_m = c \left( \frac{i}{x} \right)^2
\]  

(43)

The linearization of (43) is exactly similar to the procedure as described in ?? and yields the same result.

\[
\Delta F_{m1}(i_1, x_1) = 2c_1 \frac{i_1}{x_{10}} i_1 - 2c_1 \frac{i_1^2}{x_{10}^2} x_1 + F_{m1}(i_{10}, x_{10})
\]  

(44)

\[
\Delta F_{m2}(i_1, x_1) = 2c_2 \frac{i_2}{x_{20}} i_2 - 2c_2 \frac{i_2^2}{x_{20}^2} x_2 + F_{m2}(i_{20}, x_{20})
\]  

(45)

In the operating points \(x_{10}\) and \(x_{20}\) it holds that there is zero net force, i.e.

\[
mg = F_{m1}(i_{10}, x_{10}) + F_{m2}(i_{20}, x_{20})
\]  

(46)

\[
l_{f1} F_{m1} = l_{f2} F_{m2}
\]  

(47)

The electromagnets are identical (i.e. \(c_1 = c_2\)) and \(x_{10} = x_{20} = x_0\), hence the bias currents are also equal, i.e. \(i_{10} = i_{20} = i_0\), and therefore

\[
F_{m1}(i_{10}, x_{10}) = F_{m2}(i_{20}, x_{20}) = F_m(i_0, x_0)
\]  

(48)

\[
mg = 2F_m(i_0, x_0)
\]  

(49)

\[
i_0 = x_0 \sqrt{\frac{mg}{2c}}
\]  

(50)

\[
2c_1 \frac{i_{10}^2}{x_{10}^2} = 2c_2 \frac{i_{20}^2}{x_{20}^2} = k_i
\]  

(51)

\[
-2c_1 \frac{i_{10}^3}{x_{10}^3} = -2c_2 \frac{i_{20}^3}{x_{20}^3} = k_x
\]  

(52)

Now, the linear equations for the electromagnetic forces are inserted into (23) to obtain

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 
\end{bmatrix} = T_{fM}^{-1} \begin{bmatrix}
mg \\
0 
\end{bmatrix} + S \left[ \frac{\Delta F_{m1}(i_1, x_1)}{\Delta F_{m2}(i_1, x_1)} \right]
\]  

(53)

which simplifies to (54) when filling in (46), (47), (51) and (52).

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 
\end{bmatrix} = T_{fM}^{-1} \left[ S \begin{bmatrix}
k_i i_1 + k_x x_1 \\
k_i i_2 + k_x x_2 
\end{bmatrix} \right]
\]  

(54)

\(^2\)In paragraph ??, the angle \(\theta\) was assumed to be small, so the nonlinear relation in equations (10)-(13) disappears, leaving \(F_{m1}\) and \(F_{m2}\) the only nonlinear equations in the system.
5.2.2 State-space representation

The next step is to put the linear model of the levitated spindle (54) into state-space form. This will give a model that can be used directly in the design of the controller, which is done in state-space. To make the transfer to state-space form somewhat easier, eq. (54) is rewritten.

\[
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\end{bmatrix} = T_f M^{-1} S \left\{ \begin{bmatrix}
k_i & 0 \\
0 & k_i \\
\end{bmatrix} \begin{bmatrix}i_1 \\
i_2 \end{bmatrix} + \begin{bmatrix}k_x & 0 \\
0 & k_x \end{bmatrix} \begin{bmatrix}x_1 \\
x_2 \end{bmatrix} \right\}
\]

(55)

The inputs to the system are the currents \(i_1\) and \(i_2\) and the outputs are the positions \(x_1\) and \(x_2\). The state-space with state vector \([x_1 \ x_2 \ \dot{x}_1 \ \dot{x}_2]^T\) is thus given as

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
T_f M^{-1} S K_x & 0 & 0 \\
0 & 0 \end{bmatrix} \begin{bmatrix}x_1 \\
x_2 \\
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} + \begin{bmatrix}0 & 0 \\
0 & 0 \\
T_f M^{-1} S K_i & 0 \\
0 & T_f M^{-1} S K_i \end{bmatrix} \begin{bmatrix}i_1 \\
i_2 \end{bmatrix}
\]

(56a)

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}x_1 \\
x_2 \\
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix}
\]

(56b)

where \(K_x = \begin{bmatrix}k_x & 0 \\
0 & k_x \end{bmatrix}\) and \(K_i = \begin{bmatrix}k_i & 0 \\
0 & k_i \end{bmatrix}\).
Part II
Controller design for the levitated spindle
6 List of requirements

Costs
- The maximum costs allowed for the system are 2000,-.

Physical requirements
- The parameters of the spindle to control are:
  1. Weight: 115 gram
  2. Length: 130 mm
  3. Diameter: 12 mm
  4. Material: Silver steel
- The parameters of the actuators to control are:
  1. Material core: High carbon steel
  2. Induction coefficient: 6 mH
  3. Force constant (C): 7.59e-6

Functional requirements
- The distance between the electromagnet and the levitated object in equilibrium will be 2 mm.
- The maximum in deviation with respect to the equilibrium point is 5 %.
- The closed loop levitated spindle system must have a specified step response with:
  1. Overshoot: 1%
  2. Settling time: 0.01 sec
- The system has to be adaptable for other sensors.

Production requirements
- Two persons will work on the concept.
- Seven weeks are available for completing the system.
- Documentation of operation is required.(English)
7 Controllers for the plant

In this chapter various controllers are designed. Both for 1-DOF and for 2-DOF. For 1-DOF a PID controller and a state feedback controller are designed. For 2-DOF only a state feedback controller is made. This is done because the 1-DOF state feedback obtained better test results then the 1-DOF PID controller.

7.1 1 Degree of freedom

Here the two controllers for the 1-DOF plant are designed.

7.1.1 PID controller

7.2 Proportional Integral and Derivative controller

In this section a Proportional, Integral and Derivative (PID) controller for the 1-DOF voltage controlled plant is created. The plant has a pole in the right half of the complex plane, which by definition is an unstable pole. By using a PID controller the poles should all be placed in the left half plane. The places for the poles are predefined and are chosen in such a way that bandwidth requirements are met. Equation (57) is used for the PID controller. This is a general model used for a PID controller.

\[ K_{pid} = K_p(1 + \alpha s + \frac{\beta}{\tau s + 1}) \]  

The values for \( K_p, \alpha, \beta \) and \( \tau \) will have to be derived. Recall that the 1-DOF voltage model is given as:

\[ G(s) = \frac{1}{Ls + R} \ast \frac{K_i}{ms^2 + K_x} \]  

All the parameters are determined in the previous sections. Equation (59) shows the normalized characteristic polynomial, which will have to be equal to the poles selected for a stable system. Also the bandwidth has to be taken into account, when selecting the poles. Since the characteristic polynomial has four poles, four poles will be in the reference system as well. These poles will all be in the left half plane. This is necessary for a stable system. The characteristic equation for the poles is shown in equation (60).

\[ CP_{K_{pid}G(s)} = s^4 + \left(\frac{Lm + mR\tau}{\tau Lm}\right)s^3 + \left(\frac{mR + \tau K_xL - K_iK_p\alpha\tau}{\tau Lm}\right)s^2 + \left(\frac{\tau RK_x + K_xL - K_iK_p(\alpha + \tau)}{\tau Lm}\right)s + \left(\frac{RK_x - K_iK_p(\beta + 1)}{\tau Lm}\right) \]  

26
\[ CP_{\text{poles}} = s^4 + (p_1 + p_2 + p_3 + p_4)s^3 + (p_1p_2 + p_1p_3 + p_1p_4 + p_2p_3 + p_2p_4 + p_3p_4)s^2 \\
+ (p_1p_2p_3 + p_1p_2p_4 + p_1p_3p_4 + p_2p_3p_4)s + p_1p_2p_3p_4 \\
= s^4 + Zs^3 + Ys^2 + Xs + W \quad (60) \]

CP stands for the characteristic polynomial. In the PID controller there are four unknown parameters. The parameters unknown are \( K_p, \alpha, \beta \) and \( \tau \). With equations (59) and (60) set equal to each other four new equations are obtained. These equations can be rewritten such that the unknown parameters of the controller can be derived. These equations are shown in equation (61)

\[
\tau = \frac{Lm}{ZLm - mR} \quad (61a)
\]

\[
K_p = \frac{YLm - mR - K_xL - X\tau Lm + \tau RK_x + K_xL}{K_i\tau} \quad (61b)
\]

\[
\alpha = \frac{Y\tau Lm - mR - \tau K_xL}{-K_iK_p} \quad (61c)
\]

\[
\beta = \frac{W\tau Lm - RK_x + K_iK_p}{-K_iK_p} \quad (61d)
\]

Now that all the parameters of the PID controller are known the poles of the reference system can be chosen. The values of the poles are taken so that the bandwidth requirements are met. Values for the poles are given to be

- \( p_1 = 400 \)
- \( p_2 = 500 \)
- \( p_3 = 600 \)
- \( p_4 = 800 \)

With these left half plane poles it should be possible to stabilize the 1-DOF voltage controlled system. As illustrated in figure 20 the minus one point is encircled in the counterclockwise direction. This means that the plant is stabilized. The bodeplot, figure 21 of the openloop system shows that with these values enough bandwidth is obtained. The cross-over frequency equals 742 rad/sec. And for bandwidth requirement only 628 rad/sec is needed.

**Simulink model** The simulink model for the PID controller is given in figure 22
7.2.1 State feedback controller

In this paragraph, control of the levitated ball problem using the principle of state feedback will be described. Then the state feedback and observer gains are to be determined. With determination of the observer and state feedback gains, the loop is closed and, finally, an analysis of the closed-loop is performed. The plant 42 combined with voltage control 27 is modelled as follows:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\dot{x}} \\
i
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
K & 0 & K_i \\
0 & 0 & -R/L
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x} \\
i
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1/L
\end{bmatrix} u 
\]

(62a)

\[
y = [1 \ 1 \ 0]
\begin{bmatrix}
x \\
\dot{x} \\
i
\end{bmatrix}
\]

(62b)

Closed-loop poles The closed-loop poles are selected by taking a standard reference model. This model represents a damped mass-spring system and is described by the following transfer function:

\[
H(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

(63)

with natural frequency \(\omega_n\) and damping ratio \(\zeta\). The natural frequency and damping ratio can be specified. The poles of eq. (63) are then the desired
Figure 21: Bode plot with bandwidth requirement

poles of the closed-loop system for the 1-DOF problem. Intuitively, it makes more sense to specify a desired settling time $T_{set}$ and overshoot $M_p$. The natural frequency $\omega_n$ and damping ratio $\zeta$ then follow from [FDPEN02]:

$$\omega_n = \frac{4.6}{T_{set}\zeta}$$  \hspace{1cm} (64)

$$\zeta = \sqrt{1 - \frac{1}{(\frac{1}{\pi} \ln M_p)^2 + 1}}$$  \hspace{1cm} (65)

The following values for $T_{set}$ and $M_p$ are chosen:

$T_{set} = 0.01$ sec

$M_p = 0.01 = 1\%$

From these values, $\omega_n$ and $\zeta$ are calculated (see eq. (64) and (65)). The poles of the reference system (63) can now be determined. The Matlab command `pole(sys)` is used and yields the following result:

$$p_1 = -460 + 313.81i$$
$$p_2 = -460 - 313.81i$$

Since the plant (62) is a type 3 system, three poles are to be specified. The third pole is chosen near $p_1$ and $p_2$. This results in the following poles to be placed:
Although this reference system is of lower order than the closed-loop system considered, it will give sufficient knowledge of the behaviour of the obtained closed-loop system dynamics. Some final tweaking of the parameter values may be done after analysis of the system with these initial parameter values. The poles $p_1, p_2$ and $p_3$ are put in a vector \( P \):

\[
P = [p_1 \ p_2 \ p_3]
\]  

(66)

**State feedback gains**

Now that the desired poles are selected, the state feedback and observer gains can be determined. For determination of the state feedback gains, full state information will be assumed. In this case, full state information means direct availability of the states as defined in eq. ???. The state feedback gains can be calculated with Ackermann’s formula (eq. 84). However, since numerical methods (Matlab) are used in designing the controller, Ackermann’s formula becomes unreliable. Therefore, a native function of Matlab, called place.m, is used. The state feedback gains are obtained with

\[
>> L=\text{place}(A,B,P),
\]

where \( A \) and \( B \) are the state-space matrices of the plant (62) under control and \( P \) is a vector with the desired closed loop poles. The result is a
1-by-3 matrix of gains, each belonging to one of the fed back states. A simple check is performed in Matlab to see if the eigenvalues of $A - BL$ are indeed as specified in $P$:

$$
\begin{align*}
\text{eig}(A-BL) \\
\text{ans} = \\
1.0e+002 \times \\
4.6000 + 3.1381i \\
4.6000 - 3.1381i \\
-5.00
\end{align*}
$$

These are indeed the eigenvalues as specified in $P$. The next step is to determine the observer gains.

**Observer gains** Full state information was assumed in determining the state feedback gains, but since only the position is measured, there will be need for an estimator, also called observer, to estimate the remaining two states. However, it is desirable that the observer is faster than the system $A - BL$, since the error will then converge quick enough to give reliable state information to the state feedback controller. The gains $K$ are determined with the place function in Matlab:

$$
\text{place}(A',B',5*P)'
$$

where the prime stands for the matrix transpose. With the observer gains known, the state-space matrices of the observer can be constructed. The result is a 3-by-1 matrix of gains, each representing the portion of $y - C\hat{x}$ ‘needed’ to give good state estimations. Again, a simple check is performed in Matlab to see if the eigenvalues of $A - KC$ are indeed as specified in $5*P$:

$$
\begin{align*}
\text{eig}(A-KC) \\
\text{ans} = \\
1.0e+003 \times \\
2.2999 + 1.56903i \\
2.2999 - 1.56903i \\
-2.50
\end{align*}
$$

These are indeed the eigenvalues as specified in $5*P$. 

31
Closing the loop  The state-space for the closed-loop system with controller and plant is now constructed. The state-space for the controller follows from (??).

\[ A_o = A - KC - BL \]  
\[ B_o = [B \ K] \]  
\[ C_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]  
\[ D_o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \]

(refer to state feedback control chapter). With determination of the controller state-space, the loop can be closed. The closed-loop state-space is given by

\[ A_{cl} = \begin{bmatrix} A - BL & BL \\ 0 & A - KC \end{bmatrix} \]  
\[ B_{cl} = \begin{bmatrix} 0 \\ 0 \\ -K \end{bmatrix} \]  
\[ C_{cl} = \begin{bmatrix} C & 0 & 0 \end{bmatrix} \]  
\[ D_{cl} = 0 \]

In the state-space systems 67 and 68 the matrices \(A, B\) and \(C\) are those as specified in ???. The closed-loop system, as built in simulink, is shown in figure 23.

Figure 23: closed-loop system in simulink
7.3 2 Degrees of freedom

Here the 2-DOF state feedback controller is designed. The state feedback controller is used here because of better performance in 1-DOF.

7.3.1 State feedback controller

This section will cover the design of a controller based on state feedback to stabilize the magnetically suspended spindle. For this, the model derived in paragraph 3.2 is used. The notation used for the voltage controlled plant is:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]  

(69)

In designing the controller, full state information is assumed. The state feedback gains are then determined to give closed-loop poles at a desired location. Then, since full state information is not available, the observer gains are determined. The observer poles are placed further in the left-half of the s-plane, to make the estimation error converge to zero quick enough to give reliable state estimations. For background information on the theory of state feedback control the reader is referred to appendix A.1.

Closed-loop poles  First, the desired closed-loop poles are determined. Again a reference system is used, for which a settling time \(T_{\text{set}}\) and an overshoot \(M_p\) can be specified. The poles of this reference system (63) are then used to design the controller. A second-order system is again used. The settling time and overshoot are the same as in section 7.2.1:

\[
\begin{align*}
T_{\text{set}} &= 0.01 \text{ sec} \\
M_p &= 0.01 = 1%
\end{align*}
\]

From these values, \(\omega_n\) and \(\zeta\) are calculated (see eq. (64) and (65)). The poles of the reference system (63) can now be determined. The Matlab command pole(sys) is used and yields the following result:

\[
\begin{align*}
p_1 &= -460 + 313.81i \\
p_2 &= -460 - 313.81i
\end{align*}
\]

These poles are used to determine the controller state-space matrices. However, since the plant (??\(\frac{}{??}\)) is a sixth order system, four extra poles are needed. For now, two poles will be copied and two others poles are chosen near these

\(^3\text{Note that the controller 'assumes' that the sensor outputs have been transformed to give displacements at the point of force acting on the spindle}\)
poles. This results in the following vector $P$:

$$P = \begin{bmatrix} p_1 & p_2 & p_1 & p_2 & -450 & -500 \end{bmatrix}$$  \hspace{1cm} (70)

Although the reference system is of lower order than the closed-loop system considered, it will give sufficient knowledge of the behaviour of the obtained closed-loop system dynamics. Some final tweaking of the parameter values may be done after analysis of the system with these initial parameter values.

**State feedback gains**  The state feedback gains $L$ are determined to give the system $A - BL$ the eigenvalues as specified in $P$. The following Matlab command is used to determine $L$:

\[
>> L = \text{place}(A, B, P)
\]

The result is a 2-by-6 matrix of gains, each belonging to one of the feedback states. A simple check is performed in Matlab to see if the eigenvalues of $A - BL$ are indeed as specified in $P$:

\[
>> \text{eig}(A - BL)\\
>> \text{ans} =
\]

\[
>> 1.0e+002 *
\]

\[
>> -4.6000 + 3.1381i\\
>> -4.6000 - 3.1381i\\
>> -4.6000 + 3.1381i\\
>> -4.6000 - 3.1381i\\
>> -4.50\\
>> -5.00
\]

These are indeed the eigenvalues as specified in $P$. The next step is to determine the observer gains.

**Observer gains**  The observer gains $K$ are determined to give the system $A - KC$ the eigenvalues as specified in $P$. However, it is desirable that the observer is faster than the system $A - BL$, since the error will then converge quick enough to give reliable state information to the state feedback controller. The gains $K$ are determined with the place function in Matlab:

\[
>> K = \text{place}(A', C', 5*P')
\]

The result is a 6-by-2 matrix of gains, each representing the portion of $y - C\hat{x}$ ‘needed’ to give good state estimations. Again, a simple check is performed in Matlab to see if the eigenvalues of $A - KC$ are indeed as specified in $5*P$:  

34
These are indeed the eigenvalues as specified in $5^*P$.

**Closing the loop**  The state-space for the closed-loop system with controller and plant is now constructed. The state-space for the controller follows from (71).

$$A_{\text{con}} = A - KC - BL \quad (71)$$

$$B_{\text{con}} = [B \ K] \quad (72)$$

$$C_{\text{con}} = [I] \quad (73)$$

$$D_{\text{con}} = [0] \quad (74)$$

where $C$ is an 6-by-6 identity matrix and $D$ is a 6-by-6 matrix with zeroes. The closed-loop state-space is then (see appendix A):

$$A_{\text{cl}} = \begin{bmatrix} A - BL & BL \\ 0 & A - KC \end{bmatrix} \quad (75)$$

$$B_{\text{cl}} = \begin{bmatrix} 0 \\ -K \end{bmatrix} \quad (76)$$

$$C_{\text{cl}} = \begin{bmatrix} C & 0 \end{bmatrix} \quad (77)$$

$$D_{\text{cl}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (78)$$

where the 0 in $A_{\text{cl}}$ is a 6-by-6 matrix with zeroes, the 0 in $B_{\text{cl}}$ is a 6-by-2 matrix with zeroes and the 0 in $C_{\text{cl}}$ is a 2-by-6 matrix with zeroes.

As with the design of the 1-DOF controller in section 7.2.1, a gain is calculated to ensure zero steady-state error. The calculation of this gain is the same as in the case of the 1-DOF controller, but is repeated nevertheless:

$$v = -C(A - BL)^{-1}B * L(A - KC)^{-1}B^{-1} \quad (79)$$

Input to the controller are the output of the plant $y$ and the reference input $r$. The plant and the controller are connected in simulink. The simulink model
is shown in figure 24. Both the linearized plant and nonlinear plant are included in the simulink model. Performance can then be easily compared. Simulation results can be found in section ??.

Figure 24: Simulink model of the MIMO closed-loop
Part III
Results
8 Closed-loop simulations of the levitated spindle

Here the simulations of the designed controllers are added. Just like the previous chapters distinction is made between 1 and 2 degrees of freedom.

8.1 1 Degree of freedom

In this paragraph, the simulation results of the state feedback controller with the linear and nonlinear plant are presented. The input to the controller is exited with a step function. The results obtained from this test are compared with design specifications, viz. desired settling time and overshoot.

**Linear simulation results**  To simulate the closed-loop system with the linear plant, the controller was connected as shown in figure 25. Figure 26 shows the step response when exiting the system with a unit step function. The control voltage needed to follow the step input is shown in figure 27. From figure 26 it can be seen that the design specification for the settling time was met. This parameter was set to 0.01 seconds. No overshoot occurs, while a maximum overshoot of 1% was specified. It can be seen that using a second order reference system, gives satisfying results.

**Nonlinear simulation results**  To simulate the closed-loop system with the nonlinear plant, the controller was connected as shown in figure 28. The nonlinear simulation was also carried out by exiting the controller input with a step function. The result is shown in figure 29. It’s clear that the nonlinear plant has an offset in its response. The cause of this may lie in a modelling error in Simulink or Matlab. Apart from this offset, the nonlinear simulation gives satisfying results. The settling time is as specified, and the
overshoot is less than 1%, viz. $\frac{1.045 - 1.038}{1.038} = 0.0067 = 0.67\%$, which can be seen in figure 30.

### 8.2 2 Degrees of freedom

In this paragraph, the simulation results of the MIMO state feedback controller with the linear and nonlinear plant are presented. Also, test results of the 2-DOF plant controlled by two SISO controllers are given. The inputs to the controller are exited with a step function. The results obtained from this test are compared with design specifications, viz. desired settling time and overshoot.

**Linear simulation results** To simulate the closed-loop system with the linear plant, the controller was connected as shown in figure 31. Figure 32 shows the step responses when exiting the system with a unit step function at each of the two inputs. On step is applied at $t = 0.02s$ and the other at $t = 0.05$. It clear that the system’s reference tracking is poor. The reason for this might be that the 2nd reference system was just too simple to determine performance for this 6th order system.

**Nonlinear simulation results** No nonlinear simulation results with the MIMO-controller are available, as it was not possible to eliminate recurring errors in simulink when trying to simulate. The errors probably have to do with numerical issues, as the ODE-solver in simulink crashed repeatedly. However, good results were obtained using two state feedback SISO-controllers to control the 2-DOF plant (see figure ??). The step responses are shown in figure 33. The settling time and overshoot are as specified in the controller design. Note how the mechanical coupling between the spindle ends is apparent as disturbances in the two positions.
9 Conclusions & Recommendations

9.1 Conclusion

From the simulations it could be seen that the use of two SISO state feedback controllers to stabilize the magnetically levitated spindle, based on voltage control, yielded the most satisfying results. Even with a simple 2\textsuperscript{nd} order reference system, the behaviour of the higher order closed-loop system could be accurately described in terms of settling time and overshoot. However, the MIMO-controller is nevertheless an interesting option, since it more accurately models the plant under control. Numerical issues of Matlab were most likely the cause of unreliable and in this case unpresentable simulations.

9.2 Recommendations

The SISO state feedback controllers used to stabilize the levitated spindle system, were based on full order observers, i.e. all states of the plant were reconstructed by the observer. Since sensor measurements are directly available, there is no need for the observer to estimate this state. Therefore, a reduced order observer based design is recommended [HB].

In determining the closed-loop poles, a second order reference system was used, of which settling time and overshoot could be specified. However, optimal control methods like LQ-control should also be looked upon, since
they allow the designer to put constraints on control effort, which might be interesting in for example low-power applications [Zhu00]. Also, robust control strategies like $H_{\infty}$-control [GL] might be of interest to the reader, since a robust control framework [Ast] allows for deviations in model parameters and disturbance rejection.
References


Figure 30: Overshoot in the step response of the nonlinear plant

Figure 31: Configuration for linear simulation of the 1-DOF with state feedback in simulink


A State-space design

The state-space method organizes the differential equations, which describe a dynamic system, as a set of first-order differential equations, in the form

\[ \dot{x} = Ax + Bu \]  
\[ y = Cx + Du \]  

where \( x \) is the state vector, \( y \) the output vector and \( u \) the input vector.

A.1 State feedback

State feedback controllers make use of the state-space approach to describe and regulate a plant. When designing a state feedback controller, one assumes availability of full state information (i.e. all states are directly available from the plant). This leads to the following control law [AW97]:

\[ u = -Lx \]  

In practice, full state information is rarely available, but for the design of the controller, this assumption is convenient to start with. In the next section, an estimator will be introduced to estimate the states from only the measured output. The matrix \( L \) contains \( n \) feedback gains, where \( n \) is the order of the system. Substituting eq. 81 in eq. 80, yields

\[ \dot{x} = Ax - BLx \]  

In order to obtain proper feedback gains, first the closed loop poles must be chosen.

The characteristic equation of the closed loop system becomes

\[ \det[sI - (A - BL)] = 0 \]  

From this equality the proper values for \( L \) can be determined, so that the desired closed loop poles are obtained. Another way to calculate the values in \( L \) is through the use Ackermann’s formula,

\[ L = (0 \ldots 0 1) W_c^{-1} P(A) \]  

Here, \( W_c \) is the reachability matrix of the plant to be controlled and \( P(A) \) a polynomial in \( A \). Note that \( W_c \) should have non-singular entries (i.e. the plant is reachable).
A.2 Observers

As mentioned in the previous section, in practice we have to estimate the states from a measured output. The state will have to be reconstructed from past input and output values. The estimation of the states is done through an observer. We write the estimated model as [AW97]:

\[
\dot{\hat{x}} = A\hat{x} + Bu, \tag{85}
\]

where \( \hat{x} \) is the estimate of the actual state \( x \). This estimate can be improved by also using the measured outputs. The difference between measured and estimates outputs, \( y - C\hat{x} \), is then fed back to the observer. Hence

\[
\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}), \tag{86}
\]

where \( K \) is a gain matrix. Now we need to determine the gains in matrix \( K \). In order to be able to do this, the reconstruction error

\[
e = x - \hat{x} \tag{87}
\]

is introduced. By subtracting eq. 86 from eq. 80a, the error dynamics now become

\[
\dot{e} = (A - KC)e \tag{88}
\]

The gains in \( K \) need be to chosen so that the system (88) is asymptotically stable. This means the error will always converge to zero. The determination of the matrix \( K \) is dual to determination of matrix \( L \), as described in the previous section, and can be solved using Ackermann’s formula, where

\[
L \rightarrow K^T \tag{89}
\]

or

\[
K = P(A)W_o^{-1} \begin{pmatrix} 0 & \ldots & 0 & 1 \end{pmatrix}^T \tag{90}
\]

where \( W_o \) is the observability matrix for the plant to be controlled. Note that \( W_o \) should have non-singular entries (i.e. the plant is controllable).

A.3 Closed-loop system with plant and state feedback controller

In this paragraph, the closed-loop state space model is derived for a plant under control of state feedback (see figure 35). This is done in terms of state dynamics and error dynamics. The error dynamics are given by eq. (88). Now, the dynamics of the state vector are expressed in terms of the states
and the error.

\[ \dot{x} = Ax + Bu \]

\[ = Ax - BL\dot{x} \]

\[ = Ax - BL\dot{x} + BLx - BLx \]

\[ = (A - BL)x - BL\dot{x} + BLx \]

\[ = (A - BL)x + BL(x - \dot{x}) \]

\[ = (A - BL)x + BL(e) \]

Eq. (88) and (91) are now used to construct the closed-loop state-space:

\[
\begin{bmatrix}
\dot{x} \\
\dot{e}
\end{bmatrix} = \begin{bmatrix}
A - BL & BL \\
0 & A - KC
\end{bmatrix} \begin{bmatrix}
x \\
e
\end{bmatrix} + \begin{bmatrix}
0 \\
-B
\end{bmatrix} r
\]  

(92a)

\[ y = \begin{bmatrix}
C & 0
\end{bmatrix} \begin{bmatrix}
x \\
e
\end{bmatrix} \]  

(92b)

### A.4 Zero steady-state error

In this paragraph, the gain needed at the input to obtain zero steady-state error is derived. The closed-loop state-space derived in paragraph ?? is used to work towards the solution.

In steady-state, the following equality holds:

\[
\dot{x} = (A - BL)x + BLe = 0 \]  

(93a)

\[
\dot{e} = (A - KC)e - Br = 0 \]  

(93b)

Eq. (93) is rewritten to obtain the error in steady-state:

\[ e = (A - KC)^{-1}Br \]  

(94)

Filling in eq. (94) into (??) yields

\[ x = -(A - KC)^{-1}BL(A - KC)^{-1}Br \]  

(95)

In order to have zero steady-state error, it must hold that

\[ y = Cx = r \]  

(96)

Eq. (95) is now inserted into (96) in order to obtain

\[ y = -C(A - KC)^{-1}BL(A - KC)^{-1}Br \]  

(97)

Finally, the gain \( v \) needed at the input to satisfy eq. (96), is given as

\[ v = [-C(A - KC)^{-1}BL(A - KC)^{-1}B]^{-1} \]  

(98)

### B M-files
Figure 32: Step response of linear SISO system

Figure 33: Step responses of 2-DOF plant controlled by two SISO state feedback controllers
Figure 34: 2-DOF plant controlled by two SISO state feedback controllers

Figure 35: Closed-loop configuration