Survey of and suggestions for experiments of nonlinear vibration of thin-walled cylindrical shells

Third report on the nonlinear vibration of imperfect thin-walled cylindrical shells

February 1985

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INTRODUCTION

Since the first pioneering paper of Reissner (1955) considerable efforts have been devoted to the theoretical investigations of the non-linear vibration of thin elastic cylindrical shells. Despite all this work it still remains as one of the disputed problems in applied mechanics where no satisfactory agreement has yet been achieved between theoretical predictions and experimental results. Thus certain fundamental questions persist. For example, hardening nonlinear behaviours have been predicted for certain modes, but these predictions have never been confirmed by experiments. Virtually all experiments to date show nonlinear softening behaviour.

In comparison with the many theoretical studies only very few experimental investigations have been devoted to nonlinear shell vibration up to now.

In this paper we shall review the historical development of nonlinear vibration experiments beginning with Evensen's work (1964) and continuing to the present date. The paper is divided into three parts. The first part is the survey of the development of nonlinear shell vibration experiments, the second part describes the major theoretical results obtained to date and in the last part the author makes some suggestions for new experiments.

1. DEVELOPMENT OF EXPERIMENTS OF NONLINEAR SHELL VIBRATION

The first experiment was the pioneering work of Evensen's (1964) but it dealt with only nonlinear vibration of rings rather than shells. To the writer's knowledge only four papers about the experiment of the nonlinear shell vibration were published since then. The authors of these papers are Olson (1965), Yuji Matsuzaki and Shigeo Kobayashi (1968), J. Chen (1972) and N. Yamaki (1983), respectively.

A: EVENSEN'S EXPERIMENT

- Test Specimen

The specimen was a ring which was supported by four very thin
suspension threads equally spaced around the circumference. Its geometric properties were as follows:

\[
R = 10.16 \text{ cm} \\
L = 2.52 \text{ cm} \\
h = 0.013 \text{ cm}
\]

Experimental Set-up

The experimental set-up is shown in Fig. 1. The exciter was an electrodynamic shaker. It was connected to the ring by a fine tungsten drive wire (0.003 cm diameter) which served as a very soft coupling spring. The magnitude of the input force was held constant for the individual response curves by maintaining a fixed displacement of the shaker, during each run.

An inductance-type pickup was used to measure the amplitude of the response.

![Diagram]

Fig. 1 Scheme of ring vibration setup (Evensen, 1964).

Results

The experimental response curves for different amplitudes of force are shown in Fig. 2 and Fig. 3. One can find from these figures that:

1. The nonlinearity was softening;
2. The well-known jump phenomena were detected. They depend on the amplitudes of force;
3. The companion mode was detected. Its amplitude and region depend on the amplitude force.

Another important result was that "double-frequency contraction" associated with circumferential inextension was detected (see
also p. 49 of Ref. 1).

Fig. 2  Ring vibration in n = 4 mode (Evensen, 1964).
Fig. 3 Ring vibration in mode $n = 4$ (Evensen, 1964).
B: OLSON'S EXPERIMENT

. Test specimen

The test specimen was made of copper by an electroplating process. Its Radius/thickness was 1820. length was 15 3/8 in. and thickness was 0.0044 in. It was mounted on the flutter model.

. Experimental Set-up

The set-up is shown in Fig. 4.

![Diagram of shell vibration setup](image)

Fig. 4 Scheme of shell vibration setup.

Driver was an acoustic driver whose acoustic output was focused through a conical nozzle with a 0.25-in-diam exit hole. It was positioned midway between the ends of shell. The pressure output from the driver was calibrated with one of the pressure transducers. It was essentially sinusoidal for driver-to-transducer spacings δ up to 0.03 in.

Pickup was inductance type that could be traversed both axially and circumferentially along the shell without touching it.

. Results

The amplitude response of mode n = 10 and m = 1 are plotted in Fig. 5. The mode shapes at peak response are plotted in Fig. 6.

Its major results were:
(1) The well-known jump phenomena were detected;
(2) The nonlinearity was softening and quite small;
(3) The double-frequency nodal contraction, which was detected in nonlinear ring vibration by Evensen, was found also in nonlinear shell vibration.
(4) The vibration would become unstable at some critical amplitudes as any mode was driven to ever increasing amplitude by increasing the driver voltage;
(5) The ultraharmonics up to 1/8 were obtained for many modes.

Fig. 5 Response-frequency relationship
Fig. 6 Mode shape at peak response.

in mode \( n = 10 \).

C: MATSUZAKI AND KOBAYASHI'S EXPERIMENT

Test Specimen

The specimen was made by winding a sheet of super-invar around a pair of ring frames rigidly connected by four columns. The cylinder was bonded to the ring frames over 20 mm width, and had a lapped seam of 15 mm bonding width in the axial direction. "Super-cemedine" was used for bonding. This bonding provided practically the clamped conditions corresponding to

\[ w = \frac{dw}{dx} = u = v = 0 \]

The geometric and material properties of the specimen were as follows:

\[
\begin{align*}
R &= 55.0 \text{ mm} \\
L &= 110.0 \text{ mm} \\
E &= 1.28 \times 10^4 \text{ kg/mm}^2 \\
\rho &= 0.813 \times 10^{-9} \text{ kg mm}^{-4} \text{ sec}^2 \\
\nu &= 0.25 \\
h &= 0.052 \text{ mm for specimen I} \\
     &= 0.055 \text{ mm for specimen II}
\end{align*}
\]

One very important fact which should be noted is that the initial deviation from the reference circle was about five times the thickness of the shell in the vicinity of the seam.

Experimental Set-up
The set-up is shown in Fig. 7. The electro-magnetic shaker fixed to the lower ring was placed midway between the ends of the cylinder. The vibration was picked up by a capacitance type deflection meter and the frequency was counted by an electronic counter.

Fig. 7 Scheme of shell vibration set-up (Matsuzaki and Kobayashi, 1968).

Results

The amplitude response for modes \( n = 8, m = 1 \) and \( n = 11, m = 1 \) are shown in Fig. 8 and Fig. 9, respectively.

![Fig. 8](image1.png)  
**Fig. 8** Experimental data and theoretical resonance curves of the driven mode for the \( n=8 \) and \( m=1 \) vibration mode of test cylinder I.

![Fig. 9](image2.png)  
**Fig. 9** Experimental data and theoretical resonance curves of the driven mode for the \( n=11 \) and \( m=1 \) vibration mode of test cylinder II.
The experimental results were as follows:
(1) Response curves were of softening type;
(2) As the driving frequency was decreased the steady-state vibration of a small amplitude changed into vibration with beating near the natural frequency.

D: CHEN'S EXPERIMENT

Chen's experiment is the most careful and successful one done up to now.

. Test Specimen

The specimen was made of aluminium alloy 7075-T6. It was machined from a seamless tube to its desired dimensions. The end rings were an integral part of the shell and they were chosen so as to simulate the simply supported boundary conditions.

\[
w = w_{xx} = N_x = ν = 0
\]

The geometric and material properties of the shell were as follows:

- \( R = 4.004865 \) in
- \( L = 8.065 \) in
- \( E = 10.3 \times 10^6 \) PSI
- \( ρ = 0.101 \) lb./in\(^3\)
- \( ν = 0.31 \)
- \( h = 0.00973 \) in

. Experimental Set-up

The Fig. 10 describes the block diagram of the experimental set-up.

The exciter was an acoustic driver. The motion of the shell skin was measured with a non-contact reluctance-type pickup. The magnitude of the force output of the acoustic driver was kept constant by applying a constant voltage across the driver.

![Diagram](image-url)

Fig.10 Scheme of vibration set-up(Chen,1972).
Results

The major results of Chen's experiment can be seen in Figures 11-13.

Fig. 11 Response-frequency relationship of driven mode m=1, n=6.

Fig. 12a Response-frequency relationship of driven mode m=1, n=6.

Fig. 12b Response-frequency relationship of companion mode m=1, n=6.
Fig. 13 Response-frequency relationship of driven mode \( m=1, n=6 \).

1. For small external force the single mode and jump phenomenon associated with it were detected, as shown in Fig. 11. The nonlinearity was softening.

2. With the magnitude of the acoustic input increased further, a different response-frequency relationship was obtained as shown in Fig. 12. To the right of the single mode resonance the secondary peak occurred, that indicated the companion mode's participation, and to the left of the secondary peak the response became "nonstationary" in the sense that the response would not remain at one amplitude, rather it drifted slowly from one amplitude to another.

3. With the magnitude of the acoustic input further increased, the response-frequency relationship is shown in Fig. 13. At both sides of \( \Omega = 1 \) the response became "nonstationary".

4. Rotating vibration of the shell was detected at certain amplitudes and frequencies. It was repeatable.

**E: YAMAKI'S EXPERIMENT**

**Test Specimen**

The test specimen was made of a commercial polyester film of nominal thickness 0.25 mm by lap-jointing along the longitudinal seam, and attaching 11 mm thick duraluminium end plates along both edges to provide the clamped boundary conditions. The geometric properties were as follows:

\[
\begin{align*}
R &= 100.00 \text{ mm} \\
L &= 226.00 \text{ mm} \\
h &= 0.247 \text{ mm}
\end{align*}
\]
Experimental Set-up

The experimental set-up diagram can be seen in Fig. 14.

Fig. 14 Scheme of shell vibration set-up (Yamaki, 1983).

Results

The major results can be seen in Fig. 15 and Fig. 16. As shown in these figures, a weak nonlinear effect of softening type was detected. No other significant nonlinear phenomena were observed in the range of the experiment, where the vibration amplitudes were limited to those due to small capacity of the exciter B&K 4809 with a rated force of 44.5 N.

Fig. 15. Effect of the vibration amplitude on the natural frequencies: (a) vibrations with the modes (1, 8) and (1, 9); (b) vibration with the mode (1, 7); (c) vibration with the mode (1, 10).
E: CONCLUSIONS

The conclusions one could obtain from these experiments are as follows:

I: For single mode response the agreement of the experimental results to those predicted by theoretical analysis was quite good. The general behaviour of nonlinear shell vibration, such as jump phenomena have been observed clearly;

II: The softening behaviour of some response modes was detected by all experiments. According to the theoretical results the nonlinear vibration of these modes are softening; No hardening behaviour was detected by any experiment, which was predicted by theoretical analysis for those modes where $\xi > 2$.

III: For coupled-mode response, generally, the degree of the agreement of the experimental results to those of theoretical analysis was not satisfactory. Considerable disagreements still persist. Some of these disagreements can be attributed to the imperfect theory used; some of them were caused directly by crude specimen and experimental set-up. For example, in Matsuzaki and Yamaki's experiments the specimens with seam used could significantly alter the vibration behaviour. Even in Chen's experiment, whose specimen was machined carefully, up to 4% deviation of the shell wall thickness was found. As an example of a simple experimental set-up one should note the control of the input to the shell by using acoustic driver in Olson and Chen's experiments. It is very clear that the input force to the shell would depend on the distance $\delta$ between the shell and the acoustic driver, which should be adjusted continuously to the different $\delta$ in order to keep the input constant at each frequency. However, $\delta$ was kept constant for the whole period of experiments.

As mentioned before the most successful experiment is the one done by Chen. It is a pity that in his theoretical analysis he used a far bigger damping than the one his test specimen exhibited. This fact contributed to the considerable disagreement between his theoretical and experimental results. Chen did not investigate the influence of damping on his results.
2. DEVELOPMENT OF THE THEORY SINCE 1960'S

An important contribution to the theory of nonlinear shell vibration was made by Evensen in 1964, who introduced for the first time the companion mode in the vibration analysis of rings. Evensen's major theoretical results can be seen in Figures 17-19.

![Fig. 17 Single-mode response.](image1)

![Fig. 18 Coupled mode response (Evensen, 1964).](image2)

![Fig. 19 Influence of large amplitudes on vibration frequency for various aspect ratios. Free vibration, single mode case.](image3)
As shown in Fig. 17 and Fig. 18, Evensen's results indicated that there were two possible responses in the nonlinear ring vibration, one was a single mode response and another was the coupled-mode response, although only one mode was directly driven by the forcing function.

It follows from Fig. 19 that nonlinearity of shell vibration is either softening or hardening, depending upon the aspect ratio $\xi$, where

$$\xi = \frac{m\pi}{L} \frac{L}{R}$$

Small values of $\xi$ generally result in softening characteristics, and large values of $\xi$ give rise to hardening effects. Similar results have been obtained by Chen using perturbation procedure in 1972.

A more meticulous analysis was performed by Bleich and Ginsberg in 1970. Their major results can be seen in Fig. 20 and Fig. 21. Bleich and Ginsberg's results indicated that damping had a pronounced influence on the single mode response as well as the coupled-mode response. As one can see from Fig. 20 the addition of damping altered the solutions drastically indeed.

Fig. 20  Response-frequency relationship (Bleich and Ginsberg, 1970).
Fig. 21 Response-frequency relationship for small damping, \( \beta = 0.4 \times 10^3 \).

Fig. 22 Response-frequency relationship for large damping (Ginsberg).
One can clearly see from Figures 21 and 22 the considerable influence that the magnitude of damping has on the coupled mode response, called the "unusual response" by Ginsberg. Increasing damping will quickly round off the peak of the coupled mode response.

One important result which should be specially noted in Fig. 21 is the existence of a small region to the left of $\tilde{\Omega} = 1$, where neither the single mode response nor the double mode response are stable.

An alternative approach to the problem was taken by J. Chen in 1972, who applied a systematic perturbation to the governing partial differential equations. His theoretical and experimental results are shown in Fig. 23.

As mentioned before the damping used in Chen's theoretical analysis was too big, a fact which caused the disagreement between theoretical and experimental results.

In a recent theoretical analysis the author used Galerkin's method and the Method of Averaging to obtain solutions for cases where besides axisymmetric and asymmetric imperfections also the effect of axial loads is included. Both undamped and damped cases of single mode as well as coupled-mode responses were analysed. Some of the results are shown in Figures 24-28.

As can be seen from these figures the influence of damping on the shape of response curves computed is similar to the results obtained by Ginsberg.

The results shown in Figs. 26-28 indicate that both axisymmetric and asymmetric imperfections can have considerable influence on the coupled mode response. If certain conditions between the response
modes and the imperfection shapes are satisfied. Thus initial imperfections can alter the solutions drastically just the same as the presence of damping.

Fig. 24a Response-frequency relationship of driven mode for small damping ($\gamma = 5 \times 10^{-5}$).
Fig. 24b Response-frequency relationship of companion mode for small damping ($\gamma=5\times10^{-5}$).
Fig. 25a Response-frequency relationship of driven mode for large damping ($\gamma=1.35\times10^{-5}$).
Fig. 26a Response-frequency relationship of driven mode of perfect shell.
Fig. 26b Response-frequency relationship of companion mode of perfect shell.
Fig. 27a Response-frequency relationship of driven mode of shell with the axisymmetric imperfection ($\xi_1 = -0.02$).
Fig. 27b Response-frequency relationship of companion mode of shell with axisymmetric imperfection ($\bar{\xi}_1 = -0.02$).
Fig. 28a Response-frequency relationship of driven mode of shell with the asymmetric imperfection ($\bar{C}_g = 0.07$).
Fig. 28b. Response-frequency relationship of companion mode of shell with the asymmetric imperfection ($\bar{\zeta}_2 = 0.07$).
3. THE PROBLEMS AND SUGGESTIONS FOR NEW EXPERIMENT

A new experiment on the nonlinear shell vibration is necessary to prove the new theoretical results, and to suggest modifications to the theory.

In the new experimental investigation, it is expected that the following aspects can be achieved, namely:

1. To try to observe the hardening nonlinearity;
2. To determine the range of companion mode participation and the characteristics of the companion mode for different level of damping;
3. To determine the stable and unstable range of coupled-mode response and to study carefully the bifurcation and beating phenomena;
4. To observe the influence of initial geometric imperfections on the vibration behaviour;
5. To investigate the effect of boundary conditions;
6. To make a general comparison between the theoretical and experimental results and to observe the characteristics of nonlinear response which were not predicted by the theoretical analysis and to suggest modifications to the theory.

For aspect (1), the difficulty is that high order responses along the axial direction must be excited in order to get large values of $\xi$. These modes probably are not stable or cannot be isolated.

For aspect (2), one problem is how to produce different levels of damping in the same test-specimen artificially without altering its geometric and material properties. A possible answer is to change the damping at the edges of the specimen by using different fillers along the ends of the shell. Another problem associated with aspect (2) is how to measure the damping in the specimen. From the author's experience it appears better to measure the logarithmic decrement $\delta$.

The third and really difficult problem is how to measure the amplitudes of the companion mode accurately, which will shift with time.

As to aspect (4), it is obvious that the problem is how to measure the initial geometric imperfections in a specimen and how to make a given initial geometric imperfection on a specimen.

So far as the author was able to verify nobody has attempted up to now to investigate experimentally the effect of boundary conditions on the nonlinear vibration of shells. In the proposed new experiments special effort will be devoted to model those boundary conditions which frequently occur in praxis.
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