Stellingen

behorende bij het proefschrift

"Superconducting Transmission Lines in Microwave Filters"
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Sven Olav Wallage, Delft, 27 mei 1997

1. Passieve bandstopfilters kunnen in hoge temperatuur supergeleiders het meest compact worden gerealiseerd met behulp van coplanaire golfgeleiders.

2. De coplanaire golfgeleiderresonator is voor het bepalen van de London indringdiepte geschikter dan microstrip-resonatoren.

3. Nauwkeurige afstemming van supergeleidende filters is slechts experimenteel mogelijk.

4. 'Full wave' klinkt goed, maar 'quasi-TEM' ontwerpt beter.

5. De twee meest succesvolle roeiverenigingen op de Varsity moeten overwegen terug te gaan naar het Spaarne om daar te strijden voor de overwinning.

6. De groeiende populariteit bij jazzmuzikanten voor oude en nieuwe handgemaakte Selmer saxofoons, suggereert dat een perfect instrument niet tot de mooiste muziek leidt.

7. De kosten die gepaard gaan met de invoering van computersystemen zullen nog vele jaren de besparingen overtreffen.
8. Een mondiale 2-kinderen-per-ouderpaar-politiek is de enige effectieve milieumaatregel.


11. Eerlijkheid is allang het duurst.

12. Het streven naar louter winstmaximalisatie van bedrijven doet de immateriële welvaart dalen.

13. Bij het streven naar mondiale vrijhandel dient terdege rekening te worden gehouden met de daarmee gepaard gaande grotere politieke macht die multinationale ondernemingen kunnen uitoefenen.
Superconducting Transmission Lines in Microwave Filters

Sven Wallage
Superconducting Transmission Lines in Microwave Filters

PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR
AAN DE TECHNISCHE UNIVERSITEIT DELFT,
OP GEZAG VAN DE RECTOR MAGNIFICUS PROF. DR IR J. BLAAUWENDRAAD
IN HET OPENBAAR TE VERDEDIGEN TEN OVERSTAAN VAN EEN COMMISSIE,
DOOR HET COLLEGE VAN DEKANEN AANGEWIEZEN,
OP DINS Dag 27 MEI 1997 TE 13:30 UUR
DOOR

Sven Olav WALLAGE

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geboren te Winsum.
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Universiteit van Gent
Rijksuniversiteit Groningen

P. Hadley PhD. heeft als begeleider in belangrijke mate aan de totstandkoming van dit proefschrift bijgedragen

Published and distributed by:
Delft University Press
Mekelweg 4
2628 CD Delft, The Netherlands
Telephone +31 15 2783254
Fax +31 15 2781661

CIP-DATA KONINKLIJKE BIBLIOTHEEK, DEN HAAG
Wallage, S.O.
Superconducting Transmission Lines in Microwave Filters / S.O. Wallage. - Delft: Delft University Press - III.
Ph.D. Thesis Delft University of Technology. - With ref. - With summary in Dutch.
ISBN 90-407-1444-4
NUGI 831
Subject headings: Superconductivity, Microwave, Electromagnetism

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Cover: With permission of Bols Benelux B.V. and PMS.

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Voor mijn vader
## Contents

### Introduction

1.1 The usage of the electromagnetic spectrum ............................................. 1

1.2 At high frequencies ............................................................................. 2

1.3 Planar microwave transmission lines .................................................. 3

1.4 Microwave filters ............................................................................. 4

1.5 Inside the superconductor .................................................................. 6

1.5.1 The first London equation ............................................................... 6

1.5.2 The second London equation ......................................................... 8

1.6 Properties and models of YBa$_2$Cu$_3$O$_{7-\delta}$ ..................................... 8

1.6.1 The material properties of YBa$_2$Cu$_3$O$_{7-\delta}$ ................................ 8

1.6.2 Models for YBa$_2$Cu$_3$O$_{7-\delta}$ ...................................................... 9

1.6.3 High frequency behaviour of a d-wave superconductor .................. 10

1.6.4 Future prospects for high $T_c$ superconductors ............................ 12

1.7 Goals of the project ......................................................................... 13

### Cryogenic microwave measurements and characterisation techniques

2.1 Introduction ..................................................................................... 15

2.2 The cold wafer prober ..................................................................... 16

2.3 The material properties at microwave frequencies .......................... 17

2.3.1 Resonators: a scattering matrix approach .................................... 19

2.3.2 Dielectric properties .................................................................... 23

2.3.3 Superconducting properties: the parallel-plate waveguide .......... 24

2.3.4 Superconducting properties: the coplanar waveguide ............... 26

2.4 Coplanar waveguide measurements ................................................. 30

2.4.1 Interpretation ............................................................................... 30

2.4.2 Discussion ................................................................................... 34

### Transmission line models

3.1 Introduction ..................................................................................... 41

3.2 Formulation of the problem ............................................................. 41
3.2.1 Lossy dielectrics ................................................. 44
3.2.2 Metals ............................................................... 45
3.2.3 Superconductors .................................................. 46
3.3 The high $T_c$ superconductors ...................................... 46
3.4 Maxwell’s equations .................................................. 48
3.5 The transform-domain equations ..................................... 49
3.6 General solution of the system’s matrix ............................ 49
3.7 Two cases ............................................................... 50
  3.7.1 Uniaxial layer: extra-ordinary axis // x-axis ................. 50
  3.7.2 Isotropic layer .................................................. 51
3.8 The scattering matrix formalism ..................................... 52
3.9 The integral equation ............................................... 53
3.10 Numerical considerations .......................................... 54
  3.10.1 Inverse Fourier transformation ............................... 54
  3.10.2 The matrix elements .......................................... 55
  3.10.3 Guided wave search ......................................... 56
3.11 Numerical results .................................................. 57
  3.11.1 Numerical results ........................................... 57
4 Bandstop filter design ............................................... 71
  4.1 Introduction ...................................................... 71
  4.2 Lumped element design ......................................... 72
  4.3 Coplanar waveguide distributed line filters ...................... 78
    4.3.1 A prototype at 3 GHz ................................... 79
    4.3.2 Filter fabrication ......................................... 80
    4.3.3 Measurement results ...................................... 81
  4.4 Bandstop filter at 1.53 GHz .................................... 82
    4.4.1 Finetuning the response .................................. 83
    4.4.2 Measurements ............................................... 84
A Deposition and patterning of high $T_c$ thin films .................. 93
  A.1 Introduction ..................................................... 93
  A.2 Deposition ...................................................... 94
  A.3 Patterning ....................................................... 95
B The cryogenic microwave on-wafer measurement system .............. 97
  B.1 Introduction .................................................... 97
  B.2 The vacuum system ............................................. 98
  B.3 Inside the vacuum .............................................. 99
  B.4 Measurement ................................................... 100

Nomenclature
Summary

Summary

Inzetting

Dankwoord

Curriculum Vitae

Page ix

Over:
Modified advertisement of 'Jägermeister'.
Chapter 1

Introduction

1.1 The usage of the electromagnetic spectrum

The global need for information transmission is increasing at an unprecedented rate. Electromagnetic transmission systems employing for example twisted pair cable, optical fibers or free space are used. The choice may be based on the distance, the quantity to be transmitted and the fidelity of transmission required. For example, plain old telephone services utilize 3 kHz bandwidth and a two wire twisted pair cable, spanning many kilometers, may be used. In contrast, video-conferencing based on the MPEG-2 standard cannot be transmitted over long distances through twisted pair cables, because the dispersion is too high for bitrates of 6 Mbps. The perceived need for an ever increasing mondial telecommunication transport capacity has resulted in a series of paradigm shifts from the introduction of satellite communication systems in the sixties, the widespread installation of television cable systems in the seventies, the growth of optical networks, which gathered momentum in the eighties, to the unprecedented expansion in the use of personal wireless transmission systems in the nineties. It is apparent that the transport capability of the telecommunication infrastructure must increase. This can be achieved by shifting to higher carrier frequencies and/or different waveguides. Optical fibers offer the largest bandwidth and are being used for broadband signals. The optical frequencies in use are certainly high enough and transport capability is presently limited by the speed of the electronics. For greater flexibility, however, carrier frequencies in the microwave regime can be used for signal bandwidths up to ~200 MHz. For example, Personal Communication Systems (PCS) are now available with frequencies ranging from 400 MHz to 3 GHz. Efficient use of frequencies up to 10 GHz has become an important issue with many groups competing for frequency assignments of channels inside the 1-10 GHz band.

Radio astronomers have measured at microwave frequencies for many years, but measurements are currently threatened by technological developments, which have made frequencies channels inside the 1-10 GHz band commercially interesting. In the first place global positioning
systems are using channels in this band. At the moment there are two systems: GPS from the United States (1.2 GHz) and Glonass from Russia (1.6 GHz). The purpose of these systems is to have position detection all around the world. Secondly, the number of mobile communications systems is increasing rapidly. With prices of semiconductors going down, the consumer market for flexible voice and data services is opening. In addition, in developing countries the telecommunication infrastructure is expanding using wireless techniques. Examples are Motorola's Iridium project, that is aiming at world coverage of their mobile communication service using low orbiting satellites, and the European standard on digital cordless telephone (DECT-1900). Finally, fixed satellite up- and downlinks and fixed point-to-point terrestrial links are using frequencies in the microwave and millimeter wave regime. So separate channels must be assigned depending on the type of application. Depending on the channel spacing it is possible that one type of application interferes with another.

One specific case is radio astronomy low noise measurements. Weak signals from outer space are being measured using huge radio telescopes. With a large telescope the weak signals can be focussed onto a receiver. In this way, the signal to noise ratio can be improved. For the radio astronomers it might be desirable to suppress an adjacent channel to improve signal quality. Several developments are currently endangering the measurements of radio astronomers: Glonass (1560-1610 MHz) and Motorola's Iridium project (1616-1626.5 MHz). Both applications lie close to spectral line at 1612 MHz of the hydroxyl (OH) radical. The atomic hydrogen line (1420 MHz) is another useful source of microwave radiation for radio astronomers indicating that suppression is desirable in the band ranging from 1430-1600 MHz. Within the research described in this thesis a bandstop filter was developed for this range.

1.2 At high frequencies

For frequencies in the megahertz regime, the phase of a plane electromagnetic (EM) wave can start to influence the characteristics of an electrical circuit. Obviously with increasing frequency, the wavelength becomes commensurate with the dimension of the circuit and its components. Another important aspect of high frequencies are the losses along the line. Due to the skin effect of metallic transmission lines, the field is concentrated near the boundary of the transmission line. As a result, the resistive losses increase with the square root of the frequency. Intuitively this can be understood by considering a 1D situation: a plane EM-wave incident upon a non-perfect conductor [1, pp. 42-45]. The wave will decay exponentially inside the metal with a typical decay of $\delta_e$, the skin depth of the metal,

$$\delta_e = (\pi f \mu \sigma)^{-\frac{1}{2}}. \tag{1.1}$$

Apparently with increasing frequency the decay is getting shorter. By integrating over the thickness of the metallic plate the losses can be calculated. However, one arrives at the same loss assuming that the metallic plate is as thin as a skin depth and the currents inside the metallisation are uniformly distributed over a thickness of one skin depth. So from a loss point
of view, the strip is effectively one skin depth thick, increasing the losses at higher frequencies. Due to this effect the concept of surface resistance is very useful and is introduced as

$$R_s = \frac{1}{\sigma_0} \text{, } \Omega \text{ per square. (1.2)}$$

With $R_s$, the high frequency losses along the line can be calculated by multiplication of $R_s$ with the length of the line and division by the width of the line, i.e. equivalent to DC-resistance calculations using resistivity per square.

One way to overcome the losses at microwave frequencies is to use different types of transmission lines, like metallic or dielectric waveguides [2]. Another way is to apply different materials like superconductors. An important drawback of superconductors is the need for cooling. However, with the discovery of high temperature superconductors (HTS) the cooling requirement has been relaxed: at liquid nitrogen temperatures (77 Kelvin) one can apply the HTS compounds Tl-Ca-Ba-Cu-O having a transition temperature ($T_c$) close to 120 degrees Kelvin and YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) with a transition temperature of 92 degrees Kelvin [3]. For specific instances cryogenic cooling may already be used in, for example, extremely low-noise radio astronomical measurements. Complete systems designed for use at temperatures below approximately 100 degrees Kelvin can have dramatically enhanced overall performance, justifying the extra expenses of cooling and of increased power consumption [4][5][6]. The commercial potential of higher transition temperatures is evidenced by these companies who recently started to sell high $T_c$ superconductor based products: Conductus markets a YBa$_2$Cu$_3$O$_{7-\delta}$ based mobile telephone base station and Conductus, DuPont, and STI sell high frequency superconducting filters. The YBa$_2$Cu$_3$O$_{7-\delta}$ based mobile telephone base station is now in a beta-testing phase. If they prove to be reliable, superconductivity will gain a foothold in the mobile communications arena. DuPont has demonstrated superconducting dual-channel receivers, low-noise oscillators and other hybrid-circuits [7].

### 1.3 Planar microwave transmission lines

Different types of planar microwave transmission lines are depicted in Fig. 1.1. In this thesis we will concentrate on coplanar waveguides (CPW) and microstrip transmission lines. The coplanar waveguide transmission line is very well suited for measurements with microwave probes and high integration can be obtained by scaling down the dimensions of the coplanar waveguide lines without affecting the impedance (the ratio of the width of the signal strip versus the distance to the ground metallisation determines the impedance, when the substrate is thick compared to that distance). Another advantage is the fact that both signal and ground strips are positioned on the top of the substrate, so active components can easily be grounded. A major disadvantage is the multi-mode character of the coplanar waveguide: a coplanar waveguide line will guide both the symmetric and anti-symmetric mode of the waveguide. In the latter case the ground strips will have an opposite phase on either side of the signal line. When the transmission line makes a sharp turn, the anti-symmetric mode is especially likely to be excited. In
the majority of the cases, single mode propagation is desired and this is obtained through short bonding wires or metallic airbridges. In case of microstrip lines, the anti-symmetric mode is effectively suppressed by the large, continuous ground plane.

The range of impedance values in coplanar waveguide technology for commercial high $T_c$ substrates is limited to approximately $30 - 150 \Omega$. This is comparable to microstrip impedance values on identical substrates. The coplanar waveguide lines tend to be more sensitive to conductor losses and edge effects than microstrip lines, because in coplanar waveguide lines the currents are concentrated near the edges of the strips. For a microstrip line the ground currents are smeared over a large plane, in this way reducing the losses. Etching effects are expected to play a minor role in the propagation characteristics of superconducting coplanar waveguide and microstrip transmission lines: the thickness of a typical high temperature superconducting thin film is approximately 300 nm, while widths of transmission lines are larger than 10 $\mu$m. So trapezoidal shapes will not influence the propagation characteristics of coplanar waveguide and microstrip transmission lines too much. The losses inside the superconductor are insignificant for applications relevant to this thesis.

1.4 Microwave filters

At higher frequencies phase effects along a transmission line play a dominant role in the design process of the electrical circuit. The designer can either compensate or take advantage of these phase effects. However, devices based on these phase effects are in general narrowband, because they depend on a desired change of phase, which in turn linearly on frequency. But this is not true for microwave transmission line filters. The bandwidth can be close to 100%
of the center frequency (due to the repetitive nature of the phase along a transmission line). A transmission line of length \( l \) is depicted in Fig. 1.2. The impedance seen from the input depends on the output connection, either open or shorted. Richards developed a transformation that can be applied to TEM-transmission line in the following way [8]

\[
S = j\Omega = j\tan\left(\frac{\pi f}{2f_r}\right),
\]

(1.3)

where \( f_r \) represents the frequency where the transmission lines are one quarter of a wavelength (\( \beta l = \pi/2 \)). The new frequency variable \( \Omega \) ranges from zero to infinity over the interval zero to \( f_r \), so the entire frequency response is reproduced over a finite portion of the frequency band. So in the \( \Omega \)-domain an open circuit behaves like a capacitance and the shorted circuit like an inductor. So commensurate line elements (all elements having the same length \( \frac{1}{4}\lambda \)) can be used to obtain a response similar to a lumped element filter (e.g. Butterworth or Chebyshev) over a finite frequency range. In general, higher quality factors \( Q = \Delta f_{3dB}/f_r \) can be obtained using transmission lines rather than filters based on lumped elements. Unfortunately the losses on a stripline limit the application of planar transmission lines to filters with quality factors below approximately 500 and at low frequencies the transmission line filters are getting extremely large (if the filter is based upon \( \frac{1}{4}\lambda \) synthesis). Obviously microwave filter design can benefit from superconductors if the quality factors of the filter are limited by the losses of conventional normal conductors or if extremely low insertion loss is required. Extreme narrowband filters with \( Q \)'s exceeding 10,000 are reported for microstrip based circuits [9]. For even higher selectivity, cavity resonators are the right choice: \( Q \)'s of several millions have been obtained using a circular waveguide with a sapphire rod inside at both ends shorted by superconducting plates [10]. For high power applications the cavity resonators show a better performance than the microstrip circuits [11]. However, the cavity resonators are larger
and less flexible to apply than the planar circuits, so if extreme $Q$'s or high powers are not necessary, planar circuits are more popular.

1.5 Inside the superconductor

The best known property of superconducting materials is the absence of a voltage drop across a superconductor when subjected to a direct current (DC). This property was first observed by Kamerlingh Onnes in 1911. He measured the resistance of mercury and discovered that the resistance dropped to zero, when cooled below 4.2 Kelvin. Additional experiments ruled out experimental errors and he published his results. In the same year he also discovered that the superconducting state could be destroyed by subjecting the sample to high current densities (known as the critical current). Not surprisingly high magnetic fields are also capable of reverting the material to its normal (i.e. ohmic) state as well. One less known phenomenon is the Meissner-effect: a superconductor expels all magnetic flux, when subjected to a magnetic field below a critical value. This effect is in contrast with a perfect conductor, a material that tries to conserve the amount of flux. So superconductors are perfect diamagnets. This property is clearly demonstrated by levitating magnets above a superconductor.

In the search for a model for superconductors Heinz and Fritz London used the Drude model to formulate a model, which could explain the macroscopic phenomena connected with superconductors. Within the Drude model electron-electron interactions are neglected and the justification of this assumption are the remarkably good results with this (simple) model. The following two subsections are a kind of overview of the paragraphs from the book of Orlando and Delin [12, pp. 53-87]

1.5.1 The first London equation

From Newton's law the force on a particle changes its velocity. Two forces act on an electron with mass $m$ and velocity $v$: electromagnetic ($f_{em}$) and drag forces ($f_{dr}$). We obtain

$$m \frac{dv}{dt} = f_{em} + f_{dr}. \quad (1.4)$$

In not too high magnetic fields the electromagnetic force on charged particles is predominant over the electric field, so

$$f_{em} = qE. \quad (1.5)$$

Drude reasoned that the scatter processes inside a conductor result macroscopically in a viscous drag of the electron in a 'sea'. On average the electrons scatter after some time $\tau_{sc}$. This force is proportional to the velocity, i.e.

$$f_{dr} = -\frac{m}{\tau_{sc}}v. \quad (1.6)$$
but, apparent from the minus sign, directed in opposite direction. Combining equations (1.4), (1.5) and (1.6) we find

$$m \frac{dv}{dt} + \frac{m}{\tau_{sc}} v = qE.$$  \hspace{1cm} (1.7)

With a time-harmonic excitation we obtain

$$v = \left( \frac{q \tau_{sc}}{m} \right) \frac{1}{1 + j \omega \tau_{sc}} E.$$  \hspace{1cm} (1.8)

Because the total flow of charge per unit of time is determined by the amount of charge per volume \( n \), the amount of charge per particle \( q \) and the velocity \( v \), we find for the current

$$J = nqv.$$  \hspace{1cm} (1.9)

Evidently equations (1.8) and (1.9) lead us to a dispersive form of Ohm’s law,

$$J = \sigma_0 \frac{1}{1 + j \omega \tau_{sc}} E,$$  \hspace{1cm} (1.10)

where

$$\sigma_0 = \frac{nq^2 \tau_{sc}}{m}.$$  \hspace{1cm} (1.11)

Instead of stating that for superconductors the \( \sigma \) would go to infinity, the London brothers postulated that the scattering time \( \tau_{sc} \) goes to infinity. Furthermore they envisioned the superconductor as a conductor capable of carrying currents through two channels: a normal conducting channel and a superconducting channel; the normal channel with a finite scattering time and the superconducting channel with an infinite scattering time. The voltage across an inductor is linked to the current in a familiar way as the electric field to the supercurrent density (provided by the superconducting channel) through a non-dissipative, purely imaginary conductivity. That is why this term is often called the kinetic inductance. Kinetic, because energy is stored in the lossless motion of the super-electrons. A lumped element model of this idea is presented in Fig. 1.3. So when we apply a time-varying (AC) electromagnetic field to our superconductor we see that a voltage drop is present over the superconducting (inductive) channel, inducing losses in the normal channel. So, apparently losses are still present inside the superconductor. However, the larger part of the current is conducted through the inductive channel in this way providing a short circuit from input to output. Increasing the scattering time in equation (1.7) to infinity, i.e. scattering events take an infinite amount of time, we obtain the first London equation

$$E = \frac{\partial}{\partial t} (\Lambda J),$$  \hspace{1cm} (1.12)

where \( \Lambda \) is a material dependent parameter (see section 3.2.3). According to BCS theory (named after Bardeen, Cooper and Schrieffer) the superconducting channel is formed by paired electrons [13]. The pairing is a phonon-mediated process introducing a energy gap between paired and un-paired electrons. These so-called Cooper pairs are said to be frozen into a superconducting state. Only energies larger than twice the gap-energy can break the Cooper pairs into separate un-paired electrons, the quasi-particles.
1.5.2 The second London equation

If we combine Faraday’s law, i.e.

$$\nabla \times E = -\frac{\partial}{\partial t} B,$$

(1.13)

with the first London equation (1.12) we arrive at

$$\nabla \times \frac{\partial}{\partial t} \Lambda J = -\frac{\partial}{\partial t} B.$$  

(1.14)

This equation is important, because in this way we can incorporate the Meissner effect in our model. This can be understood by integrating the latter equation with respect to time,

$$\nabla \times (\Lambda J(t) + \Lambda J(0)) = -B(t) - B(0).$$

(1.15)

Suppose now that when the superconductor is cooled down below its critical temperature and before the superconducting state has been reached a static magnetic field is applied to the material. We know that, according to the Meissner effect, all magnetic flux must be expelled from the superconductor. At $t = 0$ the superconducting state is reached, so that we have only one way of assigning a value to the field components at $t = 0$, namely taking it equal zero. This results in London’s second equation,

$$\nabla \times \Lambda J(t) = -B(t).$$

(1.16)

1.6 Properties and models of YBa$_2$Cu$_3$O$_{7-\delta}$

1.6.1 The material properties of YBa$_2$Cu$_3$O$_{7-\delta}$

A large number of Perovskite type superconductors have been fabricated since the discovery of the high temperature superconductor La$_2$BaCuO$_4$ in 1986 [3]. All these superconductors sha
6 Properties and models of YBa$_2$Cu$_3$O$_{7-\delta}$

a layered CuO$_2$ structure and especially the number of layers for a certain system strongly
fluences the value of $T_c$. For example, in the Bi-systems the $T_c$ increases if the number
of CuO$_2$-layers is increased from 1 to 2, but then decreases again if the number of layers is
further increased from 2 to 3. Normally CuO$_2$ is an insulator, but the other elements in the Per-
ovskite unit cell dope the CuO$_2$-layers resulting in a finite conductivity above $T_c$. Due to the
layered nature of the high temperature superconductors, the material is strongly anisotropic:
anisotropic resistances, current densities and magnetic penetration depths.

The superconductor YBa$_2$Cu$_3$O$_7$ is very commonly used and also within this research we have
cussed on the properties of this superconductor. The family of Perovskite materials share the
chemical formula ABX$_3$ and in this respect YBa$_2$Cu$_3$O$_{7-\delta}$ is called oxygen-deficient, because
only 7 oxygen atoms are present in the unit cell. By changing the oxygen content from 6 up to
5 the material changes from an insulating anti-ferromagnet into metallic material. Increasing
the content up to 6.64 results in a phase that exhibits superconductivity. Depending on the
application, the oxygen content can be slightly modified to either increase $J_c$ or $T_c$. In the su-
perconducting compound the unit cell is orthorhombic, with an almost identical a- and b-axis
of 0.388 nm and 0.384 nm respectively. The c-axis is roughly 3 times as large as the other two
axes, 1.163 nm. The material is very sensitive to twinning: along a twinning plane the a- and
axis are interchanged on either side of this plane. Possibly this twinning can enhance the
surface resistance of the superconductor.

The high $T_c$ superconductors are extreme type-II superconductors, because the magnetic pen-
etration depth is much larger than the coherence length. Type-II superconductors can be used
high magnetic field applications [12, pp. 393-474].

6.2 Models for YBa$_2$Cu$_3$O$_{7-\delta}$

The pairing mechanism for YBa$_2$Cu$_3$O$_{7-\delta}$ is still not understood. For several years a dis-
sion has been going on between two groups: supporters of an anisotropic s-wave model
and those in favour of a d-wave model. Many experiments, both at high and low frequencies,
suggest that the s-wave model does not hold for high $T_c$ superconductors. Within the s-wave
model, the pairing occurs between electrons with opposite spins and momentum. The d-wave
model still has pairing with opposite spins, but not necessarily opposite momentum. The result
is that the gap between Fermi-surface and the quasi-particle energy level varies between zero
and $\Delta$ (the superconducting energy gap) depending on the direction of the momentum. An
isotropic s-wave superconductor has a finite energy gap between the Fermi-surface and the
quasi-particle energy state, leading to zero quasi-particle states at the Fermi-surface at $T = 0$
[14].

Several measurements have been performed to validate the d-wave pairing model. An elegant
and precise measurement was performed at the Watson Research Center of IBM. A tricrystal
substrate was used to grow grain boundary tunnel junctions. D-wave theory predicts that for
specific orientations of the junctions spontaneous fluxes will be generated in a loop within a su-
perconducting ring consisting of three junctions. This has indeed been observed using a scan-
ning SQUID-probe [15]. So the measurements are consistent with d-wave theory, but cannot reveal all details of the superconducting order parameter. Unfortunately, the reproducibility of high \( T_c \) tunnel junctions is not good enough, so probably many extrinsic effects hinder correct interpretation of the measurement data of tunnel junctions. Several attempts have been undertaken to use combinations of high and low \( T_c \) material. In one experiment a lead contact was connected to a bulk \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) crystal on different parts of the \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) crystal. D-wave superconductivity predicts different junction behaviour (as a function of magnetic field) if the lead is connected on different parts of the crystal (i.e. on different crystallographic orientations). Again the results were consistent with d-wave superconductivity. The linear behaviour of the London penetration depth at low temperatures (\( T<0.3T_c \)) is another issue and has been investigated by many groups [16][17]. The observation does support the d-wave theory, but an s-wave superconductor with sufficient magnetic impurities shows this type of behaviour as well. Also the large residual surface resistance reported by all groups favours a d-wave pairing model. The s-wave supporters attribute this surface resistance to defects in the thin film, but a striking result is that bulk \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) crystals, supposed to have a small amount of defects, exhibit an even higher surface resistance. It is believed that the scattering time of the quasi-particles increases as the temperature decreases, resulting in a higher surface resistance \( (R_s \propto \sigma_n, \text{if} \sigma_n \uparrow \text{then} R_s \uparrow) \). The enhancement of the scattering time is reduced in thin films due to the defects, in this way lowering the surface resistance. This result is consistent with the dirty d-wave picture for \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) thin films. Both clean and dirty d-wave superconductor regimes have been calculated by Hirschfeld [18][19]. The observations are not conclusive for the answer of d- versus s-wave superconductivity.

### 1.6.3 High frequency behaviour of a d-wave superconductor

Assuming the d-wave model is valid, what are then the consequences for high frequency applications? A microwave engineer is interested in loss and propagation characteristics of transmission line, so to what extent can d-wave superconductivity influence these numbers.

#### Surface resistance

The d-wave model could explain the low temperatures residual surface resistance at high frequency fields: a d-wave superconductor has many energy states for quasi-particles available at the Fermi-surface, allowing for scattering processes at frequencies below the gap frequency \( f_G = \Delta / h \) (\( \Delta \) is the superconducting gap energy and \( h \) is Planck's constant) [20]. Lee has estimated the \( \sigma_{min} \) resulting in a lower limit for the surface resistance \( ((\mu_0 \omega)^2 \lambda^3 \sigma_{min}/2) \sim 5 \mu \) for \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \) at 10 GHz [21]. The best reported values are an order of magnitude away from this value [20]. This implies that extremely high-Q resonators are limited by this value. Extrapolating to higher frequencies, assuming a \( f^2 \)-dependence for the superconductor and a
1.6 Properties and models of YBa$_2$Cu$_3$O$_{7-\delta}$

<table>
<thead>
<tr>
<th>Group</th>
<th>Year</th>
<th>Type &amp; Freq (Growth)</th>
<th>$R_s, 10$ GHz @ 77K $\mu \Omega$</th>
<th>$R_s, 10$ GHz @ 20K $\mu \Omega$</th>
<th>Average $\lambda_0$ $\text{nm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DuPont [9]</td>
<td>1991</td>
<td>$\frac{1}{2}\lambda @ 5$ GHz (M)</td>
<td>2000</td>
<td>440</td>
<td>680</td>
</tr>
<tr>
<td>B. Columbia [23]</td>
<td>1993</td>
<td>c @ 2 GHz (C)</td>
<td>250</td>
<td>250</td>
<td>n.a.</td>
</tr>
<tr>
<td>Stanford [17]</td>
<td>1993</td>
<td>p @ 10 GHz (S)</td>
<td>300</td>
<td>50</td>
<td>190</td>
</tr>
<tr>
<td>Siemens [24]</td>
<td>1993</td>
<td>$\frac{1}{2}\lambda @ 6.2$ GHz (S)</td>
<td>390</td>
<td>130</td>
<td>200</td>
</tr>
<tr>
<td>Birmingham [16]</td>
<td>1995</td>
<td>$\frac{1}{2}\lambda @ 8$ GHz (M)</td>
<td>140</td>
<td>47</td>
<td>220</td>
</tr>
<tr>
<td>Kyushu [25]</td>
<td>1995</td>
<td>$\frac{1}{2}\lambda @ 3.8$ GHz (M)</td>
<td>240</td>
<td>140</td>
<td>260</td>
</tr>
<tr>
<td>VS-SG film</td>
<td>1995</td>
<td>$\frac{1}{2}\lambda @ 5.5$ GHz (L)</td>
<td>520</td>
<td>260</td>
<td>230</td>
</tr>
<tr>
<td>Wuppertal film</td>
<td>1996</td>
<td>$\frac{1}{2}\lambda @ 5.5$ GHz (S)</td>
<td>460</td>
<td>80</td>
<td>215</td>
</tr>
<tr>
<td>Kinder film</td>
<td>1997</td>
<td>$\frac{1}{2}\lambda @ 5.5$ GHz (M)</td>
<td>240</td>
<td>160</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 1.1 Surface resistance value and London penetration depth for YBa$_2$Cu$_3$O$_{7-\delta}$ from various groups rescaled to 10 GHz assuming a $f^2$ dependence. The abbreviation 'M' stands for molecular beam epitaxy, 'S' for sputtering, 'L' for laser ablation, 'C' for single crystal, 'c' for cavity and 'p' for parallel plate.

The dependence for copper, the following values for the surface resistance $R_s$ are found

$$R_{s,Cu@4.2K} = 2 \sqrt{f} \, \text{m} \Omega, \ (f \text{ in GHz}), \quad (1.17)$$

$$R_{s,YBCO@4.2K} = 50 f^2 \, \text{n} \Omega, \ (f \text{ in GHz}). \quad (1.18)$$

The crossover frequency is observed at 1 THz. Clearly this frequency is too close to the gap frequency (so strictly speaking this reasoning is not valid), but the d-wave limit indicates that for frequencies up to 100 GHz the superconductor exhibits significant lower loss than conventional conductors. One should be careful however, because the density of states for quasi-particles increases rapidly with increasing frequency (energy) if the d-wave picture is valid [21]. Measurement in the 1 THz-region revealed surface resistances of 0.01 up to 0.1 $\Omega$ per square still in good agreement with equation (1.18) [22].

The extremely low losses will not influence the phase velocity for microwave signals in such a way that a microwave engineer needs to take them into account. If losses are an issue, the quasi-particle conductivity can be taken into account by using tabulated values and not the (standard) temperature dependence for high temperature superconductors (equation (3.20)).

The film to film parameters are still not reproducible enough, so the safest way is using the surface resistance measurement of the film and to use this number within simulations. Some results of various groups can be seen in table 1.1.

London penetration depth

For low phase-noise oscillators and long delay lines an accurate knowledge of the penetration of the electromagnetic field into the superconductor is important. The extra phase delay
due to the Cooper pairs should be taken into account. The superconductivity is directly related to the London penetration depth (equation (3.34)), so the discussion can focus on the temperature dependence of the London penetration depth. For these types of applications the operation temperature is well below $T_c$ (less than $\sim 0.3 \ T_c$) and in this temperature region the s- or d-wave discussion might be important. The first thin film measurements revealed a linear T-dependence of London penetration as a function of temperature at low temperatures [26]. However, later on other groups reported $T^2$-dependence, a result consistent with a dirty d-wave superconductor. A thin film of the thallium-compound $2212 \ \text{TlBaCaCuO}$ and $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ single crystals do show a linear T-dependence at low temperature, but this can be explained by assuming that these two materials are d-wave superconductors in the clean limit. Furthermore, it is expected that the CuO-chain and CuO-plane structure of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ plays some role in the differences between $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ and the other high $T_c$-compounds.

For high stability oscillators and accurate delay lines $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ should be a better candidate than $\text{TlBaCaCuO}$, because the London penetration depth of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ thin films is less sensitive to temperature fluctuations. From a strictly practical point of view, the deviations in London penetration depth due to small temperature changes ($\sim 5$ Kelvin) are close to 10 nm. Such values are impossible to take into account during simulations of the microwave circuits. Only experimental methods can guarantee proper (i.e. stable) operation of the device. The spread in London penetration depth values has improved over the last 5 years. According to table 1.1 the spread in $\lambda_L$ is about 100 nm. For high-Q microwave devices this spread can still result in deviations from a desired operation frequency. Proper operation can only be guaranteed by using half wavelength resonator measurements to retrieve the phase velocity and attenuation of the transmission line.

1.6.4 Future prospects for high $T_c$ superconductors

As stated before, the first commercial applications of the high $T_c$ superconductors are being marketed today: a front-end for a base-station with increased sensitivity. With the final sub-system a provider of telecommunication services can save money by reducing the number of cells in a certain area. It is difficult to compete with Si-industry, so the functionality must be dramatically increased (apart from offering low resistance). At the moment many digital circuits can be fabricated with superconducting technologies: analog to digital converters (A/D), high speed data switches, shift registers and interface circuits (to be published in the proceedings of the Applied Superconductivity Conference 1996). Some of the circuits operate up to 20 GHz at 65 Kelvin (extrapolated, not measured) and are based upon single flux quantum logic. At the moment six layers can be fabricated on top of each other, demonstrating that great complexity is already possible. For active microwave devices the only option is a hybrid technology, where the active elements are made of Gallium Arsenide (GaAs) field effect transistors. Complete systems are presented in a special issue of IEEE Transactions on Microwave Theory and Techniques [27]. Also the use of other members of the Perovskite family can increase the functionality: many other Perovskite materials have interesting physical properties.
1.7 Goals of the project

A number of these properties could be useful in electronic circuits. There is a renewed interest in Perovskite ceramic material initiated by the discovery of the high $T_c$ superconductors. The compatibility with the $\text{XTiO}_3$ and $\text{XNbO}_3$ families of materials (where X can be elements like Pb, Sr, Ba, Li and Bi) opens the possibility for a number of applications: (small) dielectric resonators, tunable phase shifters, compact overlap capacitors, optical switches, surface acoustic wave (SAW) devices and thin film circulators [28]. This compatibility could be the key to open the market for cool(ed) electronics.

1.7 Goals of the project

Within this project, which was sponsored by the university’s governing board, we wanted to know to what extend superconductivity could contribute to current microwave (sub-)systems. In some applications extremely low-loss or extreme narrowband systems are essential for proper operation. For these systems superconductivity is an interesting technology. One specific application was anticipated and has been tested: a bandstop filter for use in radio astronomical observatories. The increased usage of the electromagnetic spectrum had made it desirable to suppress signals as to avoid distortion, overload or damage of the receiving front-end. High level out of band signals must be significantly suppressed. For modern radio astronomical sites the use of superconductivity is a small step: cryocoolers are already present, since the radio telescope receivers are already cooled to 20 Kelvin to reduce the noise contribution of the semiconductor electronics. Initially our work concentrated on bandpass filters for use at such sites. Actual developments at the major synthetic aperture telescope at Westerbork in the Netherlands prompted us to shift to bandstop filters in response to the interference to astronomical observations caused by a cluster of Russian navigation satellites known as the Glonass system [29]. A bandstop filter was designed for and tested in one of the receiving telescopes.

Simultaneously a full-wave electromagnetic simulation package was developed to retrieve the propagation constant of the superconducting transmission line. Because the electromagnetic field still penetrates the superconductor (in contrast to a perfect conductor) the quasi-TEM mode will propagate at a slightly slower speed. In order to get the center frequency at the proper position and the right values for characteristic impedances, superconducting design information must be incorporated in the design process.

The constituent parameters of the superconductor can be obtained experimentally using transmission line resonators. Several types of the resonators have been tested: the simple half wavelength resonator proved to be easiest to measure and was an excellent tool for obtaining parameters of the superconductor.
Chapter 2

Cryogenic microwave measurements and characterisation techniques

2.1 Introduction

After the first publications of superior microwave devices made of HTS materials, it was evident that new models and accurate verifications tools were needed to take full advantage of the potential of the superconductor. Evidently accurate and calibrated microwave measurements at low temperatures must be performed for both material and device characterisation. A good overview of surface impedance measurements of high Tc superconductors has been carried out by Piel [30].

In this chapter we will elaborate on the cold wafer prober that has been built to measure passive superconducting devices at microwave frequencies. In principle this cryogenic setup can perform both material and device characterisation. We will mention briefly different ways of measuring the superconducting high frequency properties, but we will go into more detail on the half-wavelength resonator. The half-wavelength resonator proved to be an accurate tool to test the quality of the high Tc thin films. We will present several measurement results of superconducting films grown in different ways from different manufacturers. Films were both dry and wet etched. Finally we will try to resolve the relevant parameters of the superconductor: the London penetration depth and the normal conductivity.
2.2 The cold wafer prober

For accurate measurements flow cryostats are the right choice, because they avoid the vibrations associated with closed-cycle cryocoolers. Many research groups use this type of cryostat, but unfortunately He is consumed. Closed-cycle cryocoolers offer flexibility and can cool down to 10 Kelvin [31]. He-gas is compressed in a compressor and expanded in the cold finger, in this way supplying the cooling power. A vacuum system around the cooled object is needed to avoid heating of the object through convection of the surrounding air.

With respect to accurate, calibrated on-chip measurements, small coplanar waveguide probes offer an easy and fast way of measuring superconducting, passive circuits. Because there is no need for packaging, immediately after patterning the chip a measurement can be performed. A standard calibration substrate accounts for the influence of the coaxial lines connecting the network analyzer (NWA) with the device under test (DUT). In this way the transition from coaxial mode into a planar transmission line mode is taken into account. This reduces the problem to on-chip de-embedding which is much easier than de-embedding with launchers, spring-loaded contacts or packaged devices. For accurate measurements up to 50 GHz, coplanar probes and suitable high frequency connectors must be used. For example Pico-probes are outfitted with "K"-connectors and those of Cascade with "K"-connectors to 40 GHz and APC-2.4 to 50 GHz. Initially the Pico-probes were selected, because the flexible probe tips are more robust than the original CPW probes from Cascade. More recently Cascade has been marketing a new series of probes with mechanical properties similar to those of Pico. Most recently Cascade provided our group with some experimental stainless steel cryogenic probes. Coaxial cables with high thermal isolation (but increased electrical attenuation) facilitate a small heat-load on the cold-head and can be used up to 50 GHz. Hermetic glass beads were used in vacuum tight feedthroughs for connection of the coaxial cables from inside the vacuum chamber to atmospheric pressure. A schematic layout can be seen at Fig. 2.1.

Variable temperature measurements are more difficult than fixed temperature measurements, but the use of probes minimizes the temperature change in the coaxial cables, because of the high thermal resistance of the probes [32]. Still the large temperature gradient will affect the electrical properties of the probes, so if a room-temperature calibration is performed the absolute accuracy is questionable. However, on-chip through lines can be fabricated, enabling low temperature 'thru-line' measurements. In this way insertion loss at extremely low temperatures can be determined accurately. Determination of the phase is much more difficult and the use of a standard matched load is different at low temperatures [33]. Inside the vacuum system an option exists to connect the probes to the 80 degrees Kelvin heat shield. However, this will result in a dramatic change of the properties of the microwave probes, in contrast to connecting the probes to a 20 degrees Kelvin substrate holder with only an extremely small contact area. We have performed measurements on the probes and simultaneously have connected the probes to the substrate holder. When we connected the probes we could see the substrate holder temperature going slightly up (≈ 0.1°) and only a small (≈ 0.1°) temperature decrease of the probe. We believe that leaving the probes at room temperature will result in more accurate measure
ments. After patterning the superconductor, the substrate is mounted on the cold finger of
the cryogenic cooler. Before pumping and cooling down, a 'Line-Reflect-Match'-calibration
(LRM) is performed at room temperature with a standard calibration substrate from Cascade.
The vacuum chamber is closed (by placing the window holder on the top cover of the vacuum
chamber) and both turbo and prevacuum pump are started. The vacuum chamber is heated
up to ~ 40°C and the cold finger up to ~ 30°C. After one night of pumping and heating
(12 hours) the cooler is started and after 90 minutes the minimum temperature of 20 Kelvin is
reached. The heating procedure is performed to minimize the amount of condensation of water
onto the substrate.

2.3 The material properties at microwave frequencies

A designer of passive microwave structures is interested in the following data:

1. The relative dielectric constant and loss tangent of the dielectric layers involved.

2. The electrical behaviour of the metallisations, i.e. the losses and inductances.

3. The characteristic impedances of the transmission lines.

The concept of characteristic impedances is useful if one assumes that a quasi-TEM calculation
of the transmission line is justified. If so, then the capacitance $C$, inductance $L$, resistance $R$
and conductance $G$ per unit length can be calculated and used for the characteristic impedance

$$Z_0 = \sqrt{\frac{j\omega L + R}{j\omega C + G}}.$$  \hspace{1cm} (2.1)

In case of superconductors on microwave substrates, the losses in both conducting and insulating material can be neglected with respect to $\omega L$ and $\omega C$. The dielectric and metallic behaviour of the structure must be incorporated in the calculation of the propagation constant. This propagation constant in turn can be used in the calculation of the characteristic impedance, because for a low-loss transmission line in the quasi-TEM regime the propagation constant depends on the capacitance and inductance per unit length as well, i.e.

$$\nu_{ph} = \frac{1}{\sqrt{LC}}.$$  \hspace{1cm} (2.2)

The latter equation can be used to eliminate the inductance, while the capacitance can be calculated using standard (e.g. conformal mapping) techniques [34].

Two techniques can be used to determine the electrical properties of materials at microwave frequencies: accurate phase-shift measurements or determination of the absolute value of the transfer function of the device under test. For the phase-shift measurements accurate and reproducible calibration steps are required. However, the phase resolution of a network analyzer limits this measurement technique considerably. In general this technique is based upon assigning an extra phase-shift after calibration to the material properties of the device under test. An example is the pin-probe measurement technique of Hewlett Packard, where a coaxial probe is positioned on the dielectric material. For variable temperature measurement this method is less suitable, because the change in temperature requires a different calibration at different temperatures.

The most popular and most extensively used technique is the determination of the absolute value of the transfer function of the device under test. This can either be a reflection or a transmission type of measurement. For a high measurement resolution the resonator technique is the most suitable one. Unfortunately the resonator technique is a narrowband method, so the relevant data can only be extracted at discrete frequencies of the microwave spectrum. For the reflection type of measurement only one port measurements are performed, while for the transmission type of measurements two ports are measured. However, the one port measurements are much more sensitive to non-ideal feedlines, so the transmission configuration is a better choice for variable temperature measurements.

A large amount of publications can be found on resonator measurements with high temperature superconductors. The first resonators were based upon a cavity replacement method: a cylindrical cavity operating in $TE_{011}$-mode with superconductors as one or both of the endplate [35][36]. The sensitivity of this method is limited by the losses in the cylindrical wall and the normal conducting endplate of the cavity with respect to the losses in the superconducting endplate. To increase sensitivity a niobium cavity at 4.2 degrees Kelvin can be used with the
2.3 The material properties at microwave frequencies

![Diagram showing resonances at $n\lambda_T$ and length $l$](image)

**Figure 2.2** Schematic representation of a resonator.

High temperature superconductor mounted on a carrier which can be heated up to 100 degrees Kelvin. In 1990, two other methods with increased sensitivity were proposed, both using two superconducting thin films: a parallel-plate resonator and a sapphire-loaded microwave resonator. The parallel-plate resonator uses two superconducting thin films separated by a low loss dielectric sheet [37]. The sapphire-loaded microwave resonator consists of a sapphire rod at both ends shorted by a superconducting thin film [38].

A conical cavity was proposed by Mayer and co-workers. This cavity avoids mode-degeneration [39]. Because the superconductors are 300 nm thick, part of the microwave power will leak through the superconductor. A correction for this effect was presented by Klein *et al.* [40]. Recently, thick film YBa$_2$Cu$_3$O$_{7-\delta}$ cavities have been measured by Button *et al.* [41]. Many people report measurements on half wavelength transmission lines, which are more accurate with respect to measurement of the London penetration depth than the cavity measurements (the parallel plate resonator is more accurate). The measurements were performed on microstrip, coplanar waveguide and stripline transmission lines.

### 2.3.1 Resonators: a scattering matrix approach

The dielectric behaviour of the insulating material and the inductive behaviour of the metallisation present in a configuration can be extracted from transmission lines at resonance. If a plane electromagnetic wave travels with propagation constant $\beta$ along a transmission line of finite length $l$, as depicted in Fig. 2.2, resonances will occur if

$$\beta l = n\pi, n \in \{1, 2, \ldots\}. \tag{2.3}$$

In the equation (2.3), the boundary conditions at both ends of the lines are neglected, but in some way the electromagnetic signal must be excited at one end and measured at the other end, schematically depicted in Fig. 2.2. There are two possible ways to describe the problem: a lumped element model and a scattering matrix approach. An excellent description of the scattering matrix concept can be found in the thesis of Klopman [42, pp. 96–102] from which we will summarize the results useful for our measurements. The general concept of scattering matrices is depicted in Fig. 2.3 [1, pp. 541–548]. For simplicity we will assume that a finite length
2. Cryogenic microwave measurements and characterisation techniques

![Diagram of a two port network using the scattering matrix concept.](https://example.com/diagram)

**Figure 2.3** A two port network using the scattering matrix concept.

of transmission line is symmetrically coupled to its surrounding, so the scattering matrices describing the coupling are identical at both sides of the resonator. The latter discontinuities are assumed to be lossless and reciprocal and can be represented as

\[
\bar{S} = \begin{pmatrix}
Re^{j\phi_R} & Te^{j\phi_T} \\
Te^{j\phi_T} & Re^{j(2\phi_T - \phi_R \pm \pi)}
\end{pmatrix}.
\]

The 22-element of the $S$-matrix reflects the lossless and reciprocal nature of the discontinuity. Apparently the discontinuity is reflecting an incident wave with a fraction $R$, while a fraction $T$ is transmitted. The phase of a wave is shifted $\phi_R$, if it reflects at one of the discontinuities, while a phase shift $\phi_T$ is associated with a transmission of the wave through one of the discontinuities. The transmission line can be described mathematically as

\[
\bar{S} = \begin{pmatrix}
0 & e^{-j\beta l} e^{-\alpha l} \\
e^{-j\beta l} e^{-\alpha l} & 0
\end{pmatrix}.
\]

A resonance can be observed if a wave crossing the discontinuity interferes constructively with a wave that has already entered and traveled up and down the resonator, i.e.

\[
-2\beta l + 2\phi_T \pm \pi = 2k\pi, \ k \in \{0, 1, 2, \ldots\}.
\]

For those frequencies where $\beta l - \phi_T = (k + \frac{1}{2})\pi$, the transmission line starts to resonate. If the resonator was coupled weakly ($\lim R \to 1$), a clear peak in the transmission can be observed. In many cases, like small capacitive coupling, the $\phi_T$ approaches $-\pi/4$ resulting in the well known resonance condition $\beta l = k\pi$, i.e. resonances will occur at multiples of half the wavelength of the transmission line. So by measuring a weakly coupled resonator an accurate estimation of the $\beta$ of a transmission line can be made. If the propagation constant is calculated, the effective wavelength $\lambda_{eff}$, phase velocity $v_{ph}$ and effective refraction index $N_{eff}$ are found using

\[
\lambda_{eff} = \frac{2\pi}{\beta},
\]

\[
v_{ph} = \frac{2\pi f_c}{\beta},
\]

\[
N_{eff} = \frac{v_{ph}}{c_0}.
\]
2.3 The material properties at microwave frequencies

![Figure 2.4 S21 of capacitor-resonator-capacitor circuit.](image)

Extending the discussion to measurements, the $|S_{11}|^2$ and $|S_{21}|^2$ elements of the scattering matrix of the resonator and the coupling sections equal [42, p. 98]

\[
|S_{11}|^2 = 1 - T^2 \frac{e^{2\alpha t} - R^2 e^{-2\alpha t}}{2R^2(\cos\phi + 1) + (e^{\alpha t} - R^2 e^{-\alpha t})^2}, \tag{2.10}
\]

\[
|S_{21}|^2 = \frac{T^4}{2R^2(\cos\phi + 1) + (e^{\alpha t} - R^2 e^{-\alpha t})^2}, \tag{2.11}
\]

where $\phi = 2\beta l - 2\phi t$. A maximum in $|S_{21}|^2$ occurs if $\cos\phi = -1$, in agreement with the resonance condition. In the lossless case $\alpha$ equals zero and at resonance all the power is transmitted to the load. We will consider the case where we can represent the coupling to the resonator by a capacitor. The scattering matrix for a series element can be substituted in equation (2.10) and (2.11) [1, p. 547]. For different coupling capacitors both $S_{11}$ and $S_{21}$ are plotted in Fig. 2.4. In this figure the dispersion of the resonator around the resonant frequency is neglected and the losses on the line depend linearly on the frequency. We have considered a realistic case with $\gamma_{th} = 0.2809 c_0$, $l = 7.55$ mm, $\alpha = \frac{\beta}{2Q_0}$ and $Q_0 = 9200$ in a 50 $\Omega$ environment. Clearly visible is the effect of stronger coupling, i.e. larger capacitance values, to the resonator. This plot also demonstrates that, for very low loss and weakly coupled resonators, the reflection type of measurements is much more difficult than the transmission measurement. An important parameter that can be deduced from measurements is the quality factor $Q_0$. The quality factor itself is defined as

\[
Q_0 = \frac{2\pi}{\frac{\text{energy stored per cycle}}{\text{energy dissipated per cycle}}}. \tag{2.12}
\]

However, this is the intrinsic quality factor without coupling effects. The losses of the resonator are attributed to conductor losses, dielectric losses and radiation losses, i.e. $\alpha = \alpha_c + \alpha_d + \alpha_r$. 
2. Cryogenic microwave measurements and characterisation techniques

By coupling electromagnetic energy into the resonator and transferring part of this energy to a load, the width of the resonant peak broadens, as depicted in Fig. 2.4. In this respect the load quality factor \( Q_L \) can be introduced as

\[
Q_L = \frac{f_r}{\text{3 dB Bandwidth}}.
\]  
(2.12)

In Fig. 2.4 the intrinsic quality factor was taken to be 9200, but the responses show loaded quality factors of 2042, 4865 and 7450, for the capacitance values of 10 fF, 5 fF and 2.5 fF, respectively. So even very weak coupling can have a dramatic influence on the measured quality factor. The time constant associated with a loaded parallel RLC-circuit is \( \tau_1 = 2Q_{L,lumped}/\omega_r \). The time constant associated with a traveling wave along a resonant transmission line can be expressed as

\[
\tau_2 = \frac{l}{\nu_{ph}} + 2 \left( \sum_{n=1}^{\infty} \frac{R^2 e^{-2a l}}{1 - R^2 e^{-2a l}} \right) = \frac{l}{\nu_{ph}} \frac{1 + R^2 e^{-2a l}}{1 - R^2 e^{-2a l}}.
\]  
(2.14)

With each cycle the amplitude of the wave is reduced by \( R^2 e^{-2a l} \), because of transmission line and coupling losses. The two time constants \( \tau_1 \) and \( \tau_2 \) can be compared resulting in the quality factor

\[
Q_L = \frac{1}{2} \beta l \frac{1 + R^2 e^{-2a l}}{1 - R^2 e^{-2a l}}.
\]  
(2.15)

So for low loss, weakly coupled resonators, the \( Q_L \) approximates \( Q_0 \) and then the measured \( Q \) can be used to estimate the losses on the line through

\[
\alpha = \frac{\beta}{2Q_0}.
\]  
(2.16)

If a proper through-line calibration is performed, then the unloaded \( Q \) can be determined using the handbook of Ginzton,

\[
Q_0 = \frac{Q_L}{1 - \alpha}, \quad \alpha = 10^{-IL[\text{dB}]/20},
\]  
(2.17)

where \( IL \) stands for insertion loss in dB at resonance [43, pp. 403-405]. Turning back to the scattering matrix around resonance, close to resonance the cosine in the denominator of \( |S_{21}|^2 \) becomes approximately \( -1 + 2(\beta l \delta \omega/\delta \omega_r)^2 \), where the cosine is expanded around the resonant frequency \( \omega_r \). On a log-scale the \( |S_{21}|^2 \) equals the power transmitted to a load with a parallel RLC-circuit (apart from an offset), \( \left[ 1 + (2Q_{L,lumped} \delta \omega/\delta \omega_r)^2 \right]^{-1} \), where \( Q_{L,lumped} = \omega_r L/(R + 2Z_0) \) [1, p. 415]. Both \( Q \)’s can be compared using equation (2.15)

\[
Q_{L,lumped} = \sqrt{Q_L^2 - \left( \frac{\beta l}{2} \right)^2}.
\]  
(2.18)

Clearly a lumped element analysis is allowed if \( \beta l \) is small. So measurements of a resonator for \( Q_L \) determination can be performed in a similar way as in the lumped case. A standard procedure is the determination of the half power points of the parallel RLC-circuit as in equation
2.3 The material properties at microwave frequencies

By measuring at different frequencies the dispersion of the line can be found, which can be attributed to the dielectric material(s), the metallisation and modal dispersion. For conventional conductors the frequency dependence of the skin effect leads to dispersion of the line, because the waves see, with changing frequencies, a change in the geometry. For superconductors, operated at frequencies not too close to the gap-energy and in moderate electromagnetic fields, the London penetration depth is independent of the frequency, in this way reducing part of the dispersion along a transmission line (for microstrip lines the contribution to the dispersion due to the frequency dependence of the skin depth is very small).

2.3.2 Dielectric properties

To determine the properties of the dielectric material in the structure, in most cases a dielectric resonator is constructed of the material. From the resonance the dielectric constant and the losses can be extracted [44][45]. Unfortunately, the standard type of substrates for high Tc thin film do not have the right geometry for obtaining high quality resonators, so it is not possible or very difficult to measure the dielectric constant of the substrate that is going to be used. One attempt was made to measure dielectric properties of a substrate of $10 \times 10 \times 1 \text{ mm}^3$. It was expected that TE$_{m0n}$ resonances could be excited using a tapered coplanar waveguide structure on top of a aluminium oxide substrate. Simulations were performed using the finite element package HFSS from Hewlett Packard. With the simulation we could set up a model to mimic the behaviour of the coplanar to rectangular waveguide transition. Within HFSS the simulation data was deembedded in such a way that within the microwave design system of Hewlett Packard, MDS, standard coplanar waveguide and rectangular waveguide models can be used. The simulation indicated that the coupling was strong enough for microwave measurements. From literature it was found that LaAlO$_3$ has a relative dielectric constant of approximately 23.7 [44] [45]. For the given geometry and dielectric behaviour resonances were expected at [1, p. 439]

$$f_r = 3.079 \sqrt{m^2 + n^2} \text{ (GHz)}.$$  \hspace{1cm} (2.19)

Assuming that the electric losses of the copper metallic enclosure are dominant over dielectric losses, the quality factor of the resonance can be found as [1, p. 439]

$$Q_0 = \frac{\pi}{24} \frac{\eta}{R_s} \sqrt{1 + n^2},$$ \hspace{1cm} (2.20)

where $\eta = \sqrt{\mu/\epsilon}$. The surface resistance of metallic material equals ($\mu_r = 1$)

$$R_s = \sqrt{\frac{\pi f_0 \mu_0}{\sigma}}.$$ \hspace{1cm} (2.21)

Using the values of copper at room temperature, the estimated quality factor was 770 at approximately 4.35 GHz. A measurement showed that this geometry of the substrate cannot be
measured in a flexible way: no clear resonance was observed from 4 up to 20 GHz. Even cooling down (increasing the conductivity by a factor of 10) did not yield the desired result. The values for the substrate were to be taken from literature [44] [45].

2.3.3 Superconducting properties: the parallel-plate waveguide

The resonator technique is an excellent vehicle for determination of the superconducting properties. Both the inductive and resistive parts of the superconductivity can be determined at different temperatures. In principle, characterisation is possible with and without patterning the superconducting film. But if one can extract the superconducting parameters non-destructively a designer can use this information immediately to incorporate the superconducting effects within the design.

The most sensitive type of measurement is the parallel plate waveguide as depicted in Fig. 2. The structure consists of five layers: a substrate, a lower superconductor, a dielectric spacer, an upper superconductor and a superstrate. A two dimensional model can be used if \( w \gg t \)

and is thus valid for thin dielectric spacer material. In this regime, a large part of the electromagnetic power propagates through the superconductors and the fields penetrate into the sub- and superstrate. One consequence of this is that the waves propagate much slower than they would in a structure consisting of thick superconducting and thick dielectric layers. The electromagnetic field is strongly affected by the superconductor's properties and in this way enabling accurate determination these properties. Using a program that calculates the propagation of electromagnetic waves in layered structures, we have investigated the parallel plate transmission line composed of uniaxial superconductors. Several other articles report results for similar approaches, but none of them addresses the issue of the validity of the quasi-TEM approximation or the uniaxiality of the YBa$_2$Cu$_3$O$_{7-\delta}$-superconductor [46][47] [48][49]. To check the program, we started with the isotropic case and we compared the results with the quasi-TEM formula of Swithart [50],

\[
N_{\text{eff}} = \left\{ \varepsilon_r [1 + (\lambda_1/t_d) \coth(t_1/\lambda_1) + (\lambda_2/t_d) \coth(t_2/\lambda_2)] \right\}^{-\frac{1}{2}},
\]

where \( t_d \) and \( \varepsilon_r \) are the thickness and dielectric constant of the dielectric layer and \( t_n \) and \( \lambda_n \) are the thickness and London penetration depth of superconducting layer \( n \). This formula assumes a slowly varying E-field and does not take loss or the refractive index of the sub-superstrate into account. Nevertheless, the quasi-TEM formula gives the same results as the five layer full wave analysis to within 0.5% for superconducting layers as thin as 5 nm. Only if considerable losses are present in either the dielectric or the superconducting layers, the quasi-TEM formula fails. To demonstrate the lossy case one structure has been tested numerically. We focused on the parameters applicable to devices made from YBa$_2$Cu$_3$O$_{7-\delta}$-material. The right hand side of Fig. 2.6 shows the effective refraction index and the attenuation as a function of the penetration depth for a five layer configuration consisting of two 300 nm YBa$_2$Cu$_3$O$_{7-\delta}$ c-axis films separated by 200 nm of MgO (loss tangent tan\( \delta = 10^{-3} \)) on a LaAlO$_3$ substrate (tan\( \delta = 10^{-6} \)). The slow wave characteristics are apparent from the large value of the effective
2.3 The material properties at microwave frequencies

![Diagram of a parallel plate waveguide with labels for Gold, Substrate, Dielectric, and Superconductor.]

**Figure 2.5** The parallel plate waveguide if \( w \gg t_d \). The reference frame coincides with crystallographic axes: the \( x \)-axis with the \( c \)-axis and the \( y, z \)-plane with the \( a, b \)-plane of the high temperature superconductor.

![Graph showing the effective refraction index as a function of the London penetration depth.]

**Figure 2.6** The effective refraction index as a function of the London penetration depth.

Experimentally one varies the penetration depth by adjusting the temperature. This has the undesirable side effect that the temperature dependence of the dielectric constant and the thermal expansion of the structure are included in the measurement. These side effects can be minimized by using thin superconducting layers. The propagation of waves in a parallel plate configuration of thin superconducting layers is more sensitive to variations in the penetration depth than the same configuration using thick layers making the thin layer configuration a more sensitive probe of the penetration depth. Unfortunately the parallel plate waveguide exhibits a very low impedance (similar to a capacitor), so a lot of the microwave power will be reflected. Also the fabrication of the parallel plate waveguide requires many fabrication steps. To avoid the complex fabrication steps an alternative approach could be the fabrication of a thick gold
layer on top of one superconducting thin film. The gold layer can act as a separator and a resonator structure with capacitive coupling will be patterned in the superconducting thin film. On top of the gold layer another superconducting film will be placed upside down acting as ground plane. If a gold layer of $\sim 5\mu m$ can be fabricated than an excellent dielectric spacer material is present, vacuum. However, the gold layer should be as thin as possible to force the electromagnetic field into the superconductor, otherwise the sensitivity will decrease.

A full wave analysis is necessary to take the uniaxiality of the superconductors into account. The films can either be oriented with the $c$-axis normal to the substrate and the direction of poor conductivity perpendicular to the propagation direction, or a $a$-axis orientation with the direction of poor conductivity parallel to the propagation direction. In either case, Maxwell's equations decouple into TE- and TM-waves. For thin layers and low frequencies, $t\omega \sqrt{\varepsilon_r} / c_0 \ll 2\pi$, only the TM$_0$-mode will be guided by the structure and single-mode solutions can be calculated. The scattering matrix method is used to calculate the source-free solution of the structure (chapter 3.8). With a complex zero-searching routine (Muller's method), source-free solutions can be found in the complex plane for some value of the wavenumber $k = \beta - j\alpha$.

A thin five layer configuration was studied for both $a$-axis and $c$-axis oriented films. In the quasi-TEM approximation, the currents flow along the direction of propagation. Thus the currents flow along the $c$-axis for an $a$-axis oriented film and in the $a$-$b$-plane for a $c$-axis oriented film. The full wave analysis shows that the currents do flow primarily in the $a$-$b$-plane for $c$-axis oriented films, but that current has components both along the planes and perpendicular to the planes for $a$-axis films. This is due to much higher (super)conductivity in the $a$-$b$-plane. Consequently the quasi-TEM approximation works very well for $c$-axis oriented films. The deviations in propagation characteristics of a uniaxial superconductor compared to an isotropic superconductor and the quasi-TEM formula are illustrated in Fig. 2.7. Similar results were obtained using three uniaxial layer configurations. Especially with thin layers one should be able to measure the anisotropy of the superconductor.

### 2.3.4 Superconducting properties: the coplanar waveguide

The cold wafer prober can be used to measure coplanar waveguide resonators. In many cases when coplanar probes are used, the coplanar waveguide structure is changed into a microstrip transmission line using via-holes to a conducting (ground-)plane. It is difficult to make via-holes in the substrates used for high $T_c$ superconductors, so we have used coplanar waveguide transmission lines. In the subsequent subsections we will elaborate on the consequences of variable temperature measurements of superconducting coplanar waveguide transmission lines.
2.3 The material properties at microwave frequencies

![Figure 2.7](image)

Uniaxial versus isotropic (1 GHz and $\tan \delta_{MGO} = 10^{-3}$).

**Temperature dependence of the losses**

The overall quality factor, as stated before, can be measured by measuring the 3 dB points of $S_{21}$. This loaded $Q$ contains several loss contributions

$$\frac{1}{Q_L} = \frac{1}{Q_{sc}} + \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r},$$

(2.23)

where $sc$ denotes conductor losses, $c$ the coupling losses, $d$ the dielectric losses and $r$ the radiation losses. We will start with an estimations of the various $Q$ values, in this way validating the measurement's interpretation and finding the accuracy for the parameter extraction. We start with the temperature dependence of the coupling, which can be estimated if an estimation of the expansion coefficient can be found. This can be done by assuming a linear dependence on the resonant frequency due to a linear thermal expansion term [51]. If this is a valid approach, then values in the range of 25-65 parts per million per degree are found. Assuming the worst case of 65 parts per million for the thermal expansion coefficient, the gap distance will change by 325 parts per million for a temperature change of 50 degrees. A rough estimate for the resulting change in capacitance value shows, using a static finite difference technique and linearizing around the gap value, a change of 0.03 fF [52]. Now two remarks must be made. In the first place is the estimation of thermal expansion questionable, because d-wave superconductivity predicts a linear London penetration depth, which in its turn changes the resonant frequency. In the second place, if the temperatures is increasing, the couple gap is increasing and hence the frequency will increase. However, in the experiment we find a decreasing frequency, so at least this effect is not dominant. It is not expected that the temperature dependence of the coupling will change the measured quality factor significantly. Using the fitted
value for the capacitance change and calculating the 3 dB points of $S_{21}$ from equation (2.11) we find for a coupling capacitance of $8 \, \text{fF}$ a loaded $Q$ of 2848 and for a coupling capacitance of $8.03 \, \text{fF}$ a loaded $Q$ of 2834. It is emphasized, that the measured quality factors were high as 9200, so the coupling capacitance must have been lower. This value was obtained by positioning the probes above the resonator and not on the film, so the thermal effect on the capacitance for this type of measurement can be neglected (simulated quality factors for 1 and 1.03 $\text{fF}$ are 8840 and 8700).

Radiation losses are hard to estimate. The approximate solutions are spectral domain solutions and result in dependencies on relative dielectric constant, thickness of the substrate and propagation constant [53, p. 41]. No sensible conclusion can be drawn from this approximation with respect to temperature changes. With respect to radiation losses in general, the couple gap is expected to be the dominant effect for radiation. At these positions all electromagnetic field components are present and can give rise to radiation in all directions. Only approximate formulas are found, but no definite upper limit can be found in literature (e.g. [54] [55]). It can only be concluded that by keeping the distance of the two ground planes small compared to the substrate thickness, the radiation losses can be minimised. Apart from radiation at the edges, there is also radiation along the line. However, with finite metalisation and a non-grounded conducting plane below the substrate the $TE_0$-mode will be guided by the structure if $h > 0.1\lambda_d$, where $h$ equals the height and $\lambda_d$ the wavelength of the substrate [56]. At 5 GHz, relative dielectric constant $\varepsilon_r = 23.7$ and substrate thickness of 0.5 mm, the $TE_0$-mode is still below cut-off, so radiation into guided modes is effectively suppressed. For this substrate, frequencies above 12.3 GHz are expected to radiate part of the guided wave power into the $TE_0$-mode, so then increased losses can be expected. So radiation losses can put an upper limit on the quality factors, but these losses are assumed to be independent of temperature.

For the change in dielectric losses we can estimate worst case scenarios using values from several articles. Konopka has measured loss tangent values for LaAlO$_3$ of $2 \times 10^{-4}$ at 4 degrees Kelvin up to $7 \times 10^{-4}$ at 100 degrees Kelvin [45]. Krupka and co-workers report values of $1 \times 10^{-5}$ at 22.5 degrees Kelvin going up to $1 \times 10^{-4}$ at 80 degrees Kelvin. They also report different values for different growth methods resulting in a measured loss tangent as low as $7 \times 10^{-7}$ [44]. Both articles claim strong temperature dependencies covering almost one order of magnitude in the temperature range from 20 up to 80 degrees Kelvin. The measurements of Krupka et al. are more careful than Konopka, so we will rely on the loss tangent values Krupka. The quality factor due to dielectric losses can be estimated as [53, p. 41]

$$Q_d = \frac{1}{\tan \delta}. \quad (2.2)$$

A loss tangent of $10^{-4}$ is reached at 60 degrees Kelvin. At this value still high $Q$ values are measured reducing the overall quality factor below $10^4$. Because no clear picture of temperature dependent losses are available, the upper limit of the loss tangent of $10^{-4}$ could be used within the interpretation of the measurements. This is one of the problems with the interpretation of the quasi-particle (normal) conduction of the superconductor. Suppose that the substrate
substrate losses decrease one order of magnitude with the temperature dropping from 60 degrees Kelvin down to 20 degrees Kelvin. The measured overall quality factor for one sample increased from 6500 up to 9300. If there are no other loss mechanisms (coupling and radiation) than the conductor quality factor equals 10,250 at 20 degrees Kelvin and 17,000 at 60 degrees Kelvin. Since this result is extremely unlikely (see also the next section with the high impedance line measurement), it is concluded that the loss tangent must already be lower than 10^{-4} at 60 degrees Kelvin. A proper approach, that has been tried, would be the determination of the losses in the substrate before deposition of the superconductor. Subsequently growing the high Tc-film and pattern it for R_s determination. If the high temperature deposition of the superconductor has not affected the substrate, then both loss contribution can be distinguished. It is evident that at high Q-values the dielectric losses can play a role, but unfortunately there is no simple way to take this into account.

Temperature dependence of the resonant frequency

Apart from the losses on the line, we are also interested in the phase velocity of the superconducting transmission lines. Referring to the resonance condition $\beta l - \phi_T = (k + \frac{1}{2}) \pi$, the various temperature effects can be estimated by looking at the separate components of the latter resonance condition: phase coefficient $\beta$, the length $l$ and the phase change of the coupling $\phi_T$. The phase coefficient will change due to changes in superconductivity and dielectric behaviour, while both length and coupling will change with temperature. To start with the changing coupling, a similar reasoning holds for the determination of the losses. Over a temperature change of 50 degrees we estimated that changes in the order of several percent can result. This in its turn is a negligible value with respect to changes in other parameters involved in the resonance condition. A similar reasoning holds for a change in dielectric constant. The two articles that tried to resolve the temperature dependence of the dielectric constant are quite consistent for values and temperature stability: $\varepsilon_r = 23.7 \pm 0.05$ [44] [45]. Suppose the worst case scenario holds, i.e. a change of 0.05 in the dielectric constant after a temperature change of 60 degrees. The approximate formula for coplanar waveguide lines will be employed: $N_{eff} = \sqrt{(1 + \varepsilon_r)/2}$. This change will result in a relative change of the phase coefficient of 0.1%. By looking at measurements this value can make a difference. This change reduces the phase coefficient, i.e. lowers the resonant frequency. It might be used as a correction to the temperature dependence of the London penetration depth. A smaller contribution is expected to result from changes in physical length of the resonator. By taking the same value for the thermal expansion of the LaAlO_3-substrate as in the situation where we estimated the losses, we end up with 325 parts per million over a temperature range of 50 degrees. So, a change in line length results a change in $\beta$ of less than 0.03 % and will be neglected during interpretation of the measurements. In conclusion, the change in superconductivity dominates the shift of the resonant frequency as a function of temperature. The only significant correction could be the change in dielectric constant.
2.4 Coplanar waveguide measurements

Within a quasi-TEM approximation, the superconducting parameters can be found assuming that the kinetic inductance is the dominant temperature dependent effect. Several films have been measured. For the laser ablation system in Delft, the film-to-film quality is still not good enough; the behaviour was probably dominated by extrinsic effects, not by intrinsic superconductivity. The DuPont film showed disappointing results as well, but the films made in Wuppertal, Germany, exhibited excellent superconducting properties. From the Wuppertal films the results did correspond well with results from several other articles.

2.4.1 Interpretation

In order to retrieve the superconducting parameters the following reasoning has been used. The dimensions of the coplanar waveguides are much smaller than the wavelength at 5.5 GHz, so the structure can be approximated with a quasi-TEM analysis. Because the couple $Q$ is large, the physical length is directly related to the resonant frequency (equation (2.8)). Also, in the quasi-TEM regime the resonant frequency is related to the capacitance and inductance per unit length

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$  \hspace{1cm} (2.29)

The dominant factor for the change in resonant frequency is the superconducting penetration depth $\lambda_L$. So to find the temperature dependence of this penetration depth, a relative change in frequency can be determined using

$$\frac{f_r(T)}{f_r(T_0)} = \sqrt{\frac{L_{geo} + L_{kin}(\lambda_L(T_0))}{L_{geo} + L_{kin}(\lambda_L(T))}},$$  \hspace{1cm} (2.26)

where $L_{geo}$ is the geometrical inductance and $L_{kin}$ the kinetic inductance due to field penetration into the superconductor. For fitting purposes it is more convenient to rewrite equation (2.26) into

$$\bar{L}_{kin}(\lambda_L(T)) = \frac{1 + \overline{L}_{kin}(\lambda_L(T_0))}{\bar{f}_r(T)} - 1,$$  \hspace{1cm} (2.27)

where $\overline{L}_{kin}(\lambda_L(T)) = L_{kin}(\lambda_L(T))/L_{geo}$ and $\overline{f}_r(T) = f_r(T)/f_r(T_0)$. A partial wave analysis reveals the different contributions of both inductances [53, pp. 42-56]. The geometrical and kinetic inductance can then be calculated as

$$L_{geo} = \frac{\mu_0 K(k')}{4K(k)},$$  \hspace{1cm} (2.28)

$$L_{kin} = \frac{\mu_0}{4ADK(k)} \left( \frac{1.7}{\sinh(t/2\lambda_L)} + \frac{0.4}{\sqrt{[(B/A)^2 - 1][1 - (B/D)^2]}} \right),$$  \hspace{1cm} (2.29)
2.4 Coplanar waveguide measurements

where

$$k = \frac{w}{d},$$  \hspace{1cm} \text{(2.30)}

$$k' = \sqrt{1 - k^2},$$  \hspace{1cm} \text{(2.31)}

$$A = \frac{t}{\pi} + \frac{1}{2} \sqrt{(2t/\pi)^2 + w^2},$$  \hspace{1cm} \text{(2.32)}

$$B = \frac{w^2}{4A},$$  \hspace{1cm} \text{(2.33)}

$$C = \frac{B - \frac{t}{\pi} + \frac{1}{2} \sqrt{(2t/\pi)^2 + (d - w)^2/4},}{\pi + C},$$  \hspace{1cm} \text{(2.34)}

$$D = \frac{2t}{\pi} + C,$$  \hspace{1cm} \text{(2.35)}

where $t$ is the thickness, $w$ the width of the signal line and $d$ the ground-ground line distance of the coplanar waveguide line. The elliptic functions $K(k)$ and $K(k')$ are tabulated in the Handbook of mathematical functions [57]. For the geometry that we have used, a geometrical inductance of 408 nH/m was obtained. By using different models for the temperature dependence of the London penetration depth in equation (2.29), a best fit with the measurements using equation (2.27) can be made. In this way the London penetration depth at $T = 0$ Kelvin can be determined as well. The well-known Gorter-Casimir model for the London penetration depth can be written as

$$\lambda_L = \frac{\lambda_0}{\sqrt{1 - \Upsilon^2}},$$  \hspace{1cm} \text{(2.36)}

where $\Upsilon = T/T_c$. Several experiments on thin films indicate that the temperature dependence of the London penetration depth differs from the Gorter-Casimir model, i.e.

$$\lambda_L = \frac{\lambda_0}{\sqrt{1 - \Upsilon^2}}.$$  \hspace{1cm} \text{(2.37)}

It is emphasised that this model, which we will call the HTS thin film model, is still not understood from a theoretical point of view (see also section 1.5). Valenzuela suggest the addition of a linear term of $\Upsilon$ to improve the fit [53, p. 92],

$$\lambda_L = \frac{\lambda_0}{\sqrt{1 - 0.1\Upsilon - 0.9\Upsilon^2}}.$$  \hspace{1cm} \text{(2.38)}

The different models can be compared to measurements by using the different temperature dependences of $\lambda_L$ in equation (2.29), which in it's turn is used in equation (2.26). In Fig. 2.8 the kinetic inductance is plotted as a function of the London penetration depth from equation (2.29), where $w = 40 \mu m$ and $d = 170 \mu m$. The surface resistance is also different from normal conductors. As can be shown in the one dimensional case, the electromagnetic field decays exponentially inside a conductor. The fields will stay near the edges of the conductors on a
2. Cryogenic microwave measurements and characterisation techniques

![Graph showing kinetic inductance vs London penetration depth for various film thicknesses.]

**Figure 2.8** The kinetic inductance versus London penetration depth for a coplanar waveguide transmission line of various thicknesses from equation (2.29) \((w = 40 \, \mu m, \, d = 170 \, \mu m)\).

Typical scale \(\delta_s\), the skin depth \([1, \, p. \, 45]\)

\[
\delta_s = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma}}.
\]  \hspace{1cm} (2.39)

The surface resistance \(R_s\) of a metal at high frequencies is given by

\[
R_s = \frac{1}{\sigma \delta_s}.
\]  \hspace{1cm} (2.40)

Because field penetration inside a superconductor is determined by the London penetration depth, also its surface resistance differs from a normal conductor. It can be shown that now the surface resistance is given by \([12, \, pp. \, 111-113]\)

\[
R_s = \frac{2}{\delta_s \sigma_n} \left(\frac{\lambda_L}{\delta_s}\right)^3.
\]  \hspace{1cm} (2.41)

Clearly if the field penetration is small inside the superconductor, the losses are reduced considerably. Substitution of equation (2.39) in (2.41) shows that the surface resistance depends quadratically on the frequency

\[
R_s = \frac{1}{2} \mu_0^2 \omega^2 \lambda_L^3 \sigma_n.
\]  \hspace{1cm} (2.42)

The losses inside the superconductor increase faster than inside a normal conductor, so at a certain frequency below the gap frequency it might be unfavourable to use superconductors. The surface resistance increases linearly with increasing normal conductivity. This remarkable result is easily understood by realizing that the normal and superconductor channel are in
24 Coplanar waveguide measurements

parallel (Fig. 1.3). So if the resistance of the normal channel decreases, more power will be
tied by the lossy channel, in this way increasing the losses.

In fact, to calculate the surface resistance, one has to determine the normal conductivity. There
is a way to circumvent this by using the circuit-equivalent of the half wavelength resonator,
which is a valid equivalent picture near resonance (section 2.3.1). The unloaded $Q_0$ of a series
resonating circuit equals

$$Q_0 = \omega \frac{L_{geo} + L_{kin}}{R}. \quad (2.43)$$

For the two-fluid model all conductive losses are inside the normal channel parallel to the
superconducting channel, so

$$R = \left( \frac{\sigma_n}{\sigma_s} \right) \omega L_{kin}. \quad (2.44)$$

By using equation (2.44) in (2.43), we obtain an expression for the $\sigma_n$ to be used in equation
(2.42). Subsequently we can eliminate $\sigma_n$ using equation (3.24) resulting in the following
expression for the surface resistance [25] [24]

$$R_s = \frac{\mu_0 \lambda L \omega (L_{geo} + L_{kin})}{2Q_0 L_{kin}}. \quad (2.45)$$

In this way, after determining the London penetration depth, we can find $R_s$ by using the mea-
sured (and corrected) quality factor in (2.45). Also the $\sigma_n$ can be determined if the $R_s$ has
been calculated. However, instead of directly using equation (2.42), the surface resistance can
be calculated by measuring a thin film at different harmonics of the resonator. Because the
dependence on the frequency is known through equation (2.42), the $\sigma_n$ can be determined by
fitting the conductivity for the different frequencies. The results proved to be quite consistent,
but variable frequency measurements are much more time consuming.

From Fig. 2.13 our surprising finding is that the conductivity goes up with decreasing tempera-
ture. This is in sharp contrast with the two-fluid model, where we expect the quasi-particle
density to go down with decreasing temperature, in this way reducing the normal conductivity
(and enhancing superconductivity). A possible explanation, proposed by Bonn et al., is an
increasing scattering time [23]. Within the Drude model (see section 1.5) the scattering time $\tau$
can be determined using

$$\sigma_n(T, \omega) = \frac{1}{\mu_0 \lambda^2} \frac{\tau(T)}{1 + \omega^2 \tau^2(T)}. \quad (2.46)$$

For practical values of $\tau$, the quadratic term in the nominator can be neglected. The depen-
dence on temperature of the quasi-particle density is described using $\tau$ assuming a quadratic
dependence. Implicitly the equivalence between the electron density at $T_c$ and the London
penetration depth at zero Kelvin has been used. The quasi-particle density decreases, while
the scattering time increases, so the normal conductivity is expected to have a peak at some
temperature. Due to the amount of defects in thin films, the scattering time is expected to
increase less than in case of single crystals. Several measurements demonstrate that this peak
2. Cryogenic microwave measurements and characterisation techniques

![Graph showing resonant frequency versus temperature for different films.](image)

Figure 2.9 The resonant frequency versus temperature for different films.

can indeed be observed. For single crystals, articles report values around 40 Kelvin [17]. The peak is not always present and definitely depends on film quality. Due to its simplicity, the Drude model has been used to model the quasi-particle conductivity, but is strictly speaking not correct for materials below the superconducting transition.

2.4.2 Discussion

All transmission measurements were performed using a HP-8510B network analyzer from Hewlett Packard with source levels of 0 dBm. At 5 GHz, the cables, feedthroughs and K to APC-2.4 transitions add another 2.5 dB attenuation. It is not expected that such power levels drive the superconductor into a non-linear regime [58]. A standard resonator length has been selected based on the size of a superconducting substrate (10×10 mm) and has been set at 7.15 mm length. An approximate 50 Ω transmission line has been selected, because this line geometry fits best to the coplanar probes. Simulations indicated that, if the unloaded quality factor does not change with respect to the impedance of the line, the measured $Q$ will not change significantly (see equation (2.11)). However, this is a naive picture of the resonant structure.

By looking at the coplanar waveguide line in terms of capacitance and inductance per unit length, it is evident that by increasing the impedance (i.e. increasing inductance and reducing capacitance) the resonator will store more of its energy in magnetic fields increasing current densities. So one would expect, if the conductor losses are dominant, that by increasing the impedance the resonator $Q$ will go down. This has indeed been observed, supporting the idea that conductor losses are dominant.
2.4 Coplanar waveguide measurements

The measurements were performed on YBa$_2$Cu$_3$O$_{7-δ}$ samples from Delft, DuPont (U.S.A) and Cryoelectra (Germany). Film growth details of the DuPont films are not known, because this is confidential information. The Wuppertal films are grown using a high pressure on-axis DC sputtering process. In Delft, the superconductor thin films were grown using a Lambda Physik Excimer laser. Different layer thicknesses were grown on LaAlO$_3$ at 765° C and 750 mTorr. The laser power was 90 mJ at a repetition rate of 9 Hz. The sample was annealed one hour at 500° C in atmospheric pressure O$_2$. For the Delft films, a 300 nm gold layer was deposited in-situ at room temperature, while for the DuPont film the gold was deposited ex-situ. Subsequently standard photoresist (S1813) was used to define the pattern for both gold and YBa$_2$Cu$_3$O$_{7-δ}$ layers. The gold was wet-etched, while dry-etching (Ar-ion milling) was used for the superconductor. Several remarks must be made with respect to the gold layer deposition and the Ar-ion etching. First of all an ex-situ gold layer is worse with respect to an in-situ process: higher contact resistances and weaker adhesion of the gold to superconductor. In the second place the Ar-ion etcher is not really stable or homogeneous, resulting in poor reproducibility. However, with the measured sample we are confident that both contacts and etching were successful.

Let’s turn to the measurements performed by positioning the probes on the sample. Clearly this way of measuring loads the resonator much stronger than measuring with the probes just above the sample. The results are shown in Fig. 2.9. Film thicknesses that were grown are indicated in the figure, 150, 250 and 300 nm. The relative difference between the highest and lowest resonant frequency at 25 Kelvin is 3% and must be explained by the differences in phase velocity. It was expected that thinner films would reduce the propagation constant of the coplanar waveguide line. However, the 150 nm film has a higher resonant frequency than the 250 nm film.
2. Cryogenic microwave measurements and characterisation techniques

![Graph showing London penetration depth as a function of temperature.]

**Figure 2.11** London penetration depth as a function of temperature.

nm film. This indicates that the film-to-film reproducibility is (still) not good enough. To get a better understanding of the differences, the unloaded $Q$ as a function is plotted in Fig. 2.10. At the lowest temperature the DuPont film has the highest loaded $Q$. It is very difficult to translate into the unloaded $Q$, because then an accurate through-calibration is needed. Fortunately we have remeasured the DuPont film with the probes above the sample resulting in a $Q$ at the lowest temperature of 2200. If we neglect the couple $Q$ in that case, then we can estimate the couple $Q$ in case of probes on the sample. For the couple $Q$ we obtain 7500 and we will keep this value as a correction to the loaded $Q$ values. Clearly the $Q$-values around 2000 are not to high, so we conclude that the VS-SG films were not state-of-the-art films available. This did not surprise us, because the Delft films were always optimized for $T_c$ and not for microwave applications. The DuPont film did disappoint us however, because they reported much better results for their films [9]. We think that we either had a bad film or our etching damages the film. To compare the quality factor with the resonant frequencies we observe that the 250 nm VS-SG film has the lowest phase velocity combined with rapidly increasing quality factor suggesting that the film has a large penetration depth and many defects reducing the enhancement in normal conductivity.

By using the frequency shift information, the London penetration depth can be calculated with the following steps:

1. Calculate the relative change in kinetic inductance using equation (2.27).
2. Estimate the relative kinetic inductance at $T_0$ using Fig. 2.8.
3. Calculate all relative kinetic inductance values with the latter value and the measured (normalized) frequency values.

4. Using a polynomial fit of the plot from Fig. 2.8, the London penetration depth can be calculated.

5. Fix the value of $\lambda_0$ and $T_c$ and compare this result with the result of the thin film model in equation (2.37).

The results coincide with the London penetration depth at zero Kelvin and the temperature dependence are found. Subsequently, assuming the couple $Q$ equals 7500, the unloaded $Q$ can be calculated followed by a calculation of $R_s$. The results are presented in Fig. 2.11 and Fig. 2.12. The surface resistance of $1 \times 10^{-4}\Omega$ is about 20 times as low as cooled copper ($\approx 10^{-3}\Omega$).

The $\sigma_n$ can be calculated using (2.42) and the result is shown in Fig. 2.13. We do observe that the 250 nm thick VS-SG film has a large penetration depth and low normal conductivity. For high-$Q$ resonators in coplanar waveguide technology both London penetration depth and normal conductivity should be minimised. A striking observation is how close the normal conductivity gets close to $T_c$ for the different samples, so minimising the normal conductivity above $T_c$ may not yield the desired result. Because the surface resistance increases with the third power of $\lambda_L$ and scales only linearly with the normal conductivity, the best approach is reducing $\lambda_L$. In none of the plots is there a peak in the normal conductivity, which has been by others [17] [22][23]. It is emphasized, that we only measured as low as 20 Kelvin, so
which has been seen by others [17][22][23]. It is emphasized, that we only measured as low as 20 Kelvin, so it is still possible that the normal conductance drops below this value (indeed, is in the articles of Bonn and Ma a significant drop in normal conductance below this value) [23][17].

We will treat the Wuppertal film in a different way for two reasons: 1) we have measured with the probes above the resonator, 2) the $Q$-values are much higher. Without the electrical and mechanical contact of the probe and the superconductor, the measurement are less affected by vibrations and heating effects, so it is expected that the Wuppertal measurement show more intrinsic details of the superconductor, than the VS-SG and DuPont film measurement. Unfortunately there was no way for us to measure or estimate the unloaded $Q$, so for the Wuppertal films we only have loaded $Q$ values. The resonant frequency, loaded $Q$ values and an inset of the normalized frequency versus temperature are plotted in Fig. 2.14. Because the relative frequency change is a measure for the London penetration depth, it would appear that both resonators, which are made on one substrate, exhibit different magnetic penetration depths. A normalized kinetic inductance versus London penetration depth has been calculated for the high impedance line as well, but this can not explain the normalized frequency plot. The quasi-TEM analysis predicts a higher (relative) frequency shift for the high impedance line. Such different phase velocities could prove to be a difficult issue for complex designs, where many different impedances are necessary. All design tools should incorporate the impedance dependent phase velocity for proper operation, but this can only work if an accurate knowledge of the London penetration depth is available.
The different $Q$-values differ in a way that was already anticipated: for higher impedances the losses will increase. This difference indicates that for these $Q$-values the superconductor losses are dominant. In the article of Rauch, the difference in surface resistance of various resonators with equal impedances is attributed to inhomogeneous distributed weak links [24]. For perfect conductors the impedances are equal, but in the case of the superconductors the kinetic inductance changes the impedance even if the $w/d$-ratio is kept constant (equation (2.29)). In addition, if the strip is cut in half, the conductor losses increase faster than the inductance. In this way more power is lost in the resistor (conductive losses) than stored in the inductor (magnetic flux and kinetic inductance).

We will look into detail at the 50 $\Omega$-line. A similar calculation procedure as the VS-SG and DuPont films has been applied to the Wuppertal film. The London penetration depth shows a reasonable fit with a 215 nm penetration at zero Kelvin. Closer to $T_c$, the thin film model deviates from the measurements, but this is a trend observed by many others. From a physics point of view, this is an interesting range, but the microwave engineer will probably operate the device at temperatures below half $T_c$. If one compares the value with the DuPont and VS-SG films, the London penetration depth is in the majority of the cases in the range of 150–250 nm. This value is also in good agreement with other measurements, for example from
2. Cryogenic microwave measurements and characterisation techniques

![Graph](image)

**Figure 2.15** The London penetration depth as a function of temperature.

![Graph](image)

**Figure 2.16** Surface resistance and normal conductance as a function of temperature.

Valenzuela, Bonn, Ma and Yoshida [53][23][17][25]. If we look at the surface resistance and $\sigma_n$-results, we do see that minimizing the normal conductivity and not the London penetration depth, resulted in an increased quality factor of about 5. Clearly the reduction in normal conductivity can be obtained by optimizing the film quality, while the London penetration depth appears to be more intrinsic or much more sensitive to (small) defects. The surface resistance results correspond reasonably well with results of the university of Wuppertal [59].
Chapter 3

Transmission line models

1 Introduction

In this chapter we will give a short overview of different techniques to model the superconducting waveguide structure that forms the transmission lines. We will describe the configuration and discuss the basic assumptions concerning the electromagnetic properties of the layered structure and transmission line(s). We will introduce the equations governing the superconducting behaviour of the transmission lines. We can rewrite Maxwell’s equations as four first-order differential equations by employing a spatial Fourier transform. The resulting system can be written in matrix form. The basic properties of this matrix depend on the configuration, other isotropic or uniaxial layers. With the aid of the scattering matrix formalism we can calculate the Green’s tensor. Once the Green’s tensor is known, we can formulate the transmission line model in terms of a domain integral equation, when considering the transmission line(s) contrasting domain(s) in the layered configuration.

3.2 Formulation of the problem

To take full advantage of the superconductors, the design criteria for these devices become tighter. Accurate design tools, in addition to standard CAD-software, are needed and should take the superconducting properties into account [12][60][61]. Ghazaly et al. used a finite difference approach [62]. They simplified the calculations assuming currents are only flowing in the direction of propagation, corresponding to the quasi-TEM mode (actually the TM₀-mode) of the microstrip. A true quasi-TEM approximation was employed by people from MIT [63]. They formulated the problem by dividing the strips and groundplane in small, non-uniform sections. They concentrated on calculation of the inductance and resistance per unit length. The current distribution found with the latter approach, however, did not show current crowding at the lower side of the strip. Ghazaly addresses this as a possible current limiting problem.
of superconducting strips on high dielectric substrates. The coplanar waveguide was studied by Kessler and co-workers [64]. They choose a partial wave analysis and restricted the discussion to isotropic media. A method based on surface currents (thin superconductors) was proposed by Nghiem and co-workers [65]. Inherently they neglected the anisotropy of the superconducting strips. Their results are applicable to parallel plate like structures. Full wave analysis and anisotropy were studied by Lee et al. [66][67]. In the first paper a domain integral equation has been employed, where the anisotropy of the superconductor is taken into account. It was concluded that the difference in superconductivity along $ab$-plane and in $c$-direction does not affect the propagation of the electromagnetic waves along the transmission line. The latter crystallographic axes coincide with the Cartesian reference frame, as depicted in Fig. 3.1: the $c$-direction aligns with the $x$-axis and the $ab$-plane runs parallel to the $yz$-plane. In the second paper, a surface current was assumed and the influence of an anisotropic substrate is investigated. The results are useful if superconducting films on, for example, sapphire are used. An overview is given for different propagation modes on the anisotropic substrate. In the thesis of Klopfman, a spectral domain approach was used, where the configuration was assumed to be enclosed by a non-perfect conducting box [42, pp. 27-56]. He assumed that the strip can be represented by surface currents. The superconducting and dielectric losses were evaluated using a perturbation technique. In this way complex calculations are circumvented. Very recently an article by Mao et al. was published [68]. They used Legendre polynomials to represent the currents inside the superconducting strips. They claim to have obtained accurate results with respect to both propagation and attenuation constants.

In the present thesis we have used a domain integral equation method. This technique was successfully used for integrated optical configurations and Lee reported good results with a similar approach, so we decided to use the full wave integral equation method.

The transmission line $D_t$ is embedded in a layered medium. This embedding consist of the subdomains $D_2$, $D_3$, ..., $D_{N-1}$ which have finite thicknesses $t^{(2)}$, $t^{(3)}$, ..., $t^{(N-1)}$ and are sandwiched between the semi-infinite metallic substrate $D_1$ and the semi-infinite superstrate $D_N$ (see Fig. 3.1). We adopt the optics convention and assume that the planes are parallel to the $(y,z)$-axis of a right-handed Cartesian coordinate system $(x,y,z)$. The layers are homogeneous, instantaneously reacting, uniaxial dielectric media and the transmission line is assumed to be cylindrically parallel to the $z$-axis and made of superconducting material. The permeability of the entire configuration is equal to $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.

Time-harmonic solutions of the source-free Maxwell equations are sought that represent guided mode solutions propagating in the positive $z$-direction. So the electromagnetic field constituents of angular frequency $\omega$ and axial wavenumber $k_z$ have the form

$$\{E,D,H,B\} = \{\hat{E},\hat{D},\hat{H},\hat{B}\} (x,y;k_z) \exp(-jk_zz).$$  \hspace{1cm} (3.1)

The complex time factor $\exp(j\omega t)$ is omitted throughout this thesis. If we regard the $D_t$ as perturbation of the layered configuration, we need to solve

$$-\nabla^2 \hat{H} + j\omega \hat{D} + \hat{J} = 0,$$  \hspace{1cm} (3.2)
3.2 Formulation of the problem

![Diagram of transmission line configuration]

**Figure 3.1** The transmission line configuration.

\[ \nabla^T \times \hat{\mathbf{E}} + j\omega \hat{\mathbf{B}} = 0, \quad (3.3) \]

where

\[ \nabla^T = (\partial_x, \partial_y, -j k_z). \quad (3.4) \]

The electromagnetic fields satisfy the following boundary conditions:

\[ n \times \hat{\mathbf{E}} \text{ and } n \times \hat{\mathbf{H}} \text{ are continuous across an interface,} \quad (3.5) \]

and

\[ \lim_{\sqrt{x^2+y^2} \to \infty} \{ \hat{\mathbf{E}}(x,y;k_z), \hat{\mathbf{H}}(x,y;k_z) \} = 0, \quad (3.6) \]

where \( n \) denotes the vector normal to an interface.

We have to supplement Maxwell’s equations with constitutive relations. The electromagnetic constitutive relations constitute the relationship between \( \{\hat{\mathbf{D}}, \hat{\mathbf{B}}, \mathcal{J}\} \) and \( \{\hat{\mathbf{E}}, \hat{\mathbf{H}}\} \). These relations are representative for the local, macroscopic electromagnetic properties of the materials present in the configuration. We are interested in three types of *electric* behaviour:

- lossy dielectric behaviour
- metallic behaviour
- superconducting behaviour

Details of the constitutive relations of these materials will be considered in the next subsections.
3.2.1 Lossy dielectrics

In case of uniaxial anisotropic dielectric media we have

\[
\begin{align*}
\hat{D}(x,y;k_z) &= \varepsilon(x,y;k_z), \\
\hat{J}(x,y;k_z) &= \sigma(x,y;k_z), \\
\hat{B}(x,y;k_z) &= \mu_0 \hat{H}(x,y;k_z).
\end{align*}
\] (3.7) (3.8) (3.9)

In our configuration, the dielectric materials are slightly lossy, so in the microwave regime we have for all components of the \(\varepsilon\) and \(\sigma\) tensors that \(\omega \varepsilon_{k,l} \gg \sigma_{m,n}\), for all \(\{k,l,m,n\} \in \{x,y,z\}\). Losses in dielectrics are commonly expressed in terms of the loss-tangent for isotropic dielectric media, which is given by

\[
\tan\delta = \frac{\sigma}{\omega \varepsilon}.
\] (3.10)

It is clear that this loss-tangent represents a small number, resulting in small attenuation of electromagnetic waves through these types of media. The loss-tangent for low-loss dielectrics are less than \(10^{-3}\) at 3 GHz and 293 Kelvin.

3.2.2 Metals

For metals we normally take the permittivity equal to the permittivity of vacuum, so then we obtain for isotropic metals

\[
\begin{align*}
\hat{D}(x,y;k_z) &= \varepsilon_0 \hat{E}(x,y;k_z), \\
\hat{J}(x,y;k_z) &= \sigma \hat{E}(x,y;k_z), \\
\hat{B}(x,y;k_z) &= \mu_0 \hat{H}(x,y;k_z).
\end{align*}
\] (3.11) (3.12) (3.13)

In the microwave regime we find for good conductors \(\sigma \gg \omega \varepsilon_0\).

3.2.3 Superconductors

In the next section will deal with the high \(T_c\) superconductors, but now we will start with the two-fluid model for the classical superconductors. Within this model one assumes that the total current consist of a normal conducting and a superconducting part, i.e.

\[
\hat{j} = \hat{j}_n + \hat{j}_s,
\] (3.14)

where the subscripts \(n\) and \(s\) denote normal and supercurrent, respectively. The total amount of charge carriers is assumed to be independent of temperature and equals the sum of normal current and supercurrent charge carriers, i.e.

\[
n = n_n + n_s.
\] (3.15)
3.2 Formulation of the problem

In the latter equations $n_c$ equals the normal current charge carrier density and $n_s$ the supercurrent charge carrier density. The temperature dependence of these two "fluids" is given by

$$\frac{n_s}{n} = 1 - \gamma^4, \quad (3.16)$$

$$\frac{n_n}{n} = \gamma^4, \quad (3.17)$$

where the relative temperature $\gamma$ is introduced as

$$\gamma = \frac{T}{T_c}, \text{if } T \leq T_c. \quad (3.18)$$

In the latter equations $T$ equals the absolute temperature and $T_c$ the critical temperature at which the superconducting transition occurs. With Ohm's law one can write

$$\hat{j}_n = \sigma_n \hat{E}. \quad (3.19)$$

Within the Drude model, the normal conductivity is related to the charge carrier density in the following way (see section 1.5)

$$\sigma_n = \frac{n_n q^2 \tau_{sc}}{m}, \quad (3.20)$$

where $q$ equals the elementary charge, $\tau_{sc}$ the scattering or transport time and $m$ the mass of the charge carrier. This relation only holds for pure superconductors, where the conductivity is dominated by intrinsic properties and not by pair-breaking impurities. Evidently we assume a frequency independent conductivity and this approximation holds for the frequencies of interest.

In order to obtain an expression similar to Ohm's law for the supercurrent the London brothers postulated in the steady-state limit [69][14],

$$\hat{E} = j\omega \Lambda \hat{j}_s, \quad (3.21)$$

where the supercurrent is locally related to the electric field. In the latter equation we introduced the material dependent parameter $\Lambda$, which can be expressed as

$$\Lambda = \mu_0 \lambda_L^2. \quad (3.22)$$

The London penetration depth $\lambda_L$ is an important parameter and denotes the typical length scale over which a magnetic field penetrates the superconducting material. In a similar way as we expressed the normal conductivity in terms of more fundamental quantities, we can relate the London penetration depth to the supercurrent charge carrier density in the following way

$$\lambda_L = \sqrt{\frac{m}{n_s 4e^2 \mu_0}}, \quad (3.23)$$
where the factor 4 in the denominator accounts for paired electrons, the Cooper pairs. In the steady-state analysis we can introduce a superconductivity in order to obtain \([12],[60]\),
\[
\dot{J}_s = (j\omega)^{-1}\sigma_s \dot{E}.
\]
(3.24)

It is clear that the London penetration depth is linked to the superconductivity in the following way,
\[
\lambda_L = \frac{1}{\sqrt{\mu_0 \sigma_s}}.
\]
(3.25)

Similar to the normal, i.e. not superconducting, metals we assume that the permittivity of the material is equal to the permittivity of vacuum, so for the constitutive relations we obtain
\[
\dot{D} = \varepsilon_0 \dot{E},
\]
(3.26)
\[
\dot{J} = \sigma_{sc}\dot{E},
\]
(3.27)
\[
\dot{B} = \mu_0 \dot{H},
\]
(3.28)

where we defined
\[
\sigma_{sc} \overset{\text{def}}{=} \sigma_n + (j\omega)^{-1}\sigma_s.
\]
(3.29)

Now we have the constitutive relations for dielectric, metallic and superconducting media, the materials that are present in the configuration.

### 3.3 The high \(T_c\) superconductors

For the high \(T_c\) superconductor some modifications on the classical superconductivity model must be made in order to get an improved fit with experimental data. In the first place the temperature dependence of the two fluids is more like a square behaviour, so we obtain
\[
\frac{n_s}{n} = 1 - \gamma^2,
\]
(3.30)
\[
\frac{n_n}{n} = \gamma^2.
\]
(3.31)

Secondly, the material exhibits strong anisotropic behaviour, resulting in a matrix for the London penetration depth, i.e.
\[
\overline{\lambda}_L = \sqrt{\frac{\overline{m}^*}{n_n 4q^2 \mu_0}},
\]
(3.32)

where we introduced an effective, anisotropic mass \(\overline{m}^*\). Fortunately, for the high \(T_c\) superconductors of practical interest the anisotropic behaviour is restricted to uniaxial anisotropy with the "optic" axis parallel to the \(x\)-axis. We obtain for the normal conductivity and superconductivity
\[
\overline{\sigma}_n = n_n q^2 \tau_r \overline{m}^{-1},
\]
(3.33)
\[
\overline{\sigma}_s = \frac{1}{\mu_0 \overline{\lambda}_L}.
\]
(3.34)
3.4 Maxwell’s equations

In order to write Maxwell’s equations in a convenient form we introduce the transverse admittance per length $\vec{n}$.

$$\vec{n} = j\omega \vec{e} + \vec{\sigma},$$  \hspace{1cm} (3.35)

where $\vec{\sigma}$ can either be the normal conductivity or the superconductivity. So we rewrite Maxwell’s equations as

$$-\nabla^T \times \vec{H} + \vec{n} \vec{E} = 0,$$  \hspace{1cm} (3.36)

$$\nabla^T \vec{E} + j\omega \mu_0 \vec{H} = 0.$$  \hspace{1cm} (3.37)

In the next chapter we continue with a representation of the fields in a stratified configuration.

3.5 The transform-domain equations

We take advantage of the shift-invariance in the horizontal direction by employing a Fourier transformation in the $y$ directions, i.e.

$$\{ \vec{E}, \vec{H} \} (x; k_y, k_z) = \int_{-\infty}^{\infty} \{ \vec{E}, \vec{H} \} (x,y; k_z) \exp(jk_y(y-y'))dy,$$  \hspace{1cm} (3.38)

Now Maxwell’s equations transformed into

$$j\partial_y \vec{H}_z = k_z \vec{H}_x + \eta_y \vec{E}_y,$$  \hspace{1cm} (3.39)

$$-j\partial_y \vec{H}_y = k_y \vec{H}_z + \eta_z \vec{E}_z,$$  \hspace{1cm} (3.40)

$$j\partial_x \vec{E}_z = k_z \vec{E}_x + j\omega \mu_0 \vec{H}_y$$  \hspace{1cm} (3.41)

$$-j\partial_x \vec{E}_y = -k_z \vec{E}_x + j\omega \mu_0 \vec{H}_z.$$  \hspace{1cm} (3.42)

Through elimination of $\{ \vec{E}_x, \vec{H}_x \}$ using the following algebraic equations

$$k_y j \vec{H}_z - k_z j \vec{H}_y + \eta_y \vec{E}_x = 0,$$  \hspace{1cm} (3.43)

$$k_y \vec{E}_z - k_z \vec{E}_y + j\omega \mu_0 \vec{H}_x = 0,$$  \hspace{1cm} (3.44)

we can rewrite the Maxwell’s equations for each layer as four first-order ordinary differential equations, i.e.

$$\partial_y F^{(n)} = \vec{A}^{(n)} F^{(n)},$$  \hspace{1cm} (3.45)

we assume that all layers are source-free. In (3.45) we have introduced the field vector

$$F^{(n)} = \left( \vec{E}_y^{(n)}, \vec{E}_z^{(n)}, j\vec{H}_x^{(n)}, -j\vec{H}_y^{(n)} \right)^T,$$  \hspace{1cm} (3.46)
and the system matrix

$$\mathbb{A}^{(n)} = \begin{pmatrix} \mathbb{A}^{(EE)} & \mathbb{A}^{(EH)} \\ \mathbb{A}^{(HE)} & \mathbb{A}^{(HH)} \end{pmatrix}, \quad (3.47)$$

where \(\mathbb{A}^{(EE)}, \mathbb{A}^{(EH)}, \mathbb{A}^{(HE)}, \mathbb{A}^{(HH)}\) are 2-by-2 submatrices and the superscript \((n)\) refers to the \(n\)th layer. One can find general expressions for the system matrix \(\mathbb{A}^{(n)}\), where in our case a simplification occurs due to the dielectric uniaxial behaviour with extraordinary axis parallel to the \(x\) axis [70, pp. 198–200]. We find

$$\mathbb{A}^{(EE)} = \mathbb{A}^{(HH)} = 0, \quad (3.48)$$

and

$$A_{1,1}^{(EH)} = j \frac{k_x^2}{\eta_x} - \omega \mu_0, \quad (3.49)$$

$$A_{1,2}^{(EH)} = j \frac{k_y k_z}{\eta_x}, \quad (3.50)$$

$$A_{2,1}^{(EH)} = j \frac{k_y k_z}{\eta_x}, \quad (3.51)$$

$$A_{2,2}^{(EH)} = j \frac{k_x^2}{\eta_x} - \omega \mu_0, \quad (3.52)$$

and

$$A_{1,1}^{(HE)} = - (\omega \mu_0)^{-1} k_z^2 - j \eta_y, \quad (3.53)$$

$$A_{1,2}^{(HE)} = (\omega \mu_0)^{-1} k_y k_z, \quad (3.54)$$

$$A_{2,1}^{(HE)} = (\omega \mu_0)^{-1} k_y k_z, \quad (3.55)$$

$$A_{2,2}^{(HE)} = - (\omega \mu_0)^{-1} k_y^2 - j \eta_z. \quad (3.56)$$

### 3.6 General solution of the system’s matrix

For each layer \((n)\) we have to solve the homogeneous system

$$\partial_x \mathbf{F} = \mathbb{A} \mathbf{F}, \quad (3.57)$$

This is normally carried out in the following way (see [71, pp. 29-40]). Let \(W_N\) be the wave vector related to \(F_j\) through the transformation

$$\mathbf{F} = \mathbb{C} \mathbf{W}. \quad (3.58)$$
3.6 General solution of the system's matrix

$\bar{C}$ is commonly known as the composition matrix. If $\bar{C}$ is non-singular, the decomposition matrix $\bar{D}$ exists, so then we can write

$$ W = \bar{D}F = \bar{C}^{-1}F. \quad (3.59) $$

Using (3.58) in (3.57) and premultiplication by $D$ yields

$$ \partial_x W = \bar{\Lambda}W, \quad (3.60) $$

where $\bar{\Lambda}$ is given by the matrix product

$$ \bar{\Lambda} = \bar{D} \bar{A} \bar{C}. \quad (3.61) $$

If $\bar{\Lambda}$ is a diagonal matrix, the differential equation (3.60) can be solved. Once we have $W$, we can use (3.58) to obtain $F$. Furthermore, $\bar{C}$ is the eigencolumn matrix of $\bar{\Lambda}$ and $\bar{D}$ the eigenrow matrix of $\bar{A}$. If we denote the eigenvalues by $\lambda^{(\pm r)}$ and the eigenvectors by $v^{(\pm r)}$ ($r$ does not denote the layer!) we can write for $\bar{C}$

$$ \bar{C} = (v^{(1)}, v^{(2)}, v^{(-1)}, v^{(-2)}). \quad (3.62) $$

In order to calculate $\bar{D}$ we define a matrix $\bar{H}$ as

$$ \bar{H} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \quad (3.63) $$

The latter matrix satisfies the following property

$$ \bar{H} \bar{H}^T = \bar{I}. \quad (3.64) $$

For distinct eigenvalues or independent eigenvectors of $\bar{\Lambda}$ the associated eigenvectors obey the following relations

$$ v^{(\pm r)} \bar{H} v^{(\pm r)} = z^{(r)}, \quad (3.65) $$

$$ v^{(-\pm r)} \bar{H} v^{(-\pm r)} = -z^{(r)}, z^{(r)} \in C. \quad (3.66) $$

We can write down the following orthogonality relation,

$$ \bar{C}^T \bar{H}^{\dagger} \bar{C} = \bar{L}, \quad (3.67) $$

in which

$$ \bar{L} = \begin{pmatrix} z^{(1)} & 0 & 0 & 0 \\ 0 & z^{(2)} & 0 & 0 \\ 0 & 0 & -z^{(1)} & 0 \\ 0 & 0 & 0 & -z^{(2)} \end{pmatrix}. \quad (3.68) $$
If we normalize the eigenvectors \( \psi^{(\pm r)} \) on \( \sqrt{\zeta^{(r)}} \) then matrix \( \overline{L} \) contains plus or minus one on the diagonal. Postmultiplication of equation (3.67) by \( \overline{D} \) and premultiplication by \( \overline{L}^{-1} \) leads to

\[
\overline{L}^{-1} \overline{C}^T \overline{H} = \overline{D}.
\] (3.69)

In this way a direct computation of \( \overline{D} \) is possible. Premultiply equation (3.59) with \( \overline{C} \) result for field vector \( \vec{F} \) in

\[
\vec{F} = \sum_{i=1}^{2} r^{(+i)} \psi^{(+i)} \exp \left[ \lambda^{(+i)} (x - x^{(ref)}) \right] + \sum_{i=1}^{2} r^{(-i)} \psi^{(-i)} \exp \left[ \lambda^{(-i)} (x - x^{(ref)}) \right],
\] (3.70)

if

\[
\vec{r}_M = \left( r^{(+1)}, r^{(+2)}, r^{(-1)}, r^{(-2)} \right)^T = \overline{D} \vec{F}(x^{(ref)}),
\] (3.71)

where \( \vec{F}(x^{(ref)}) \) denotes the value of \( \vec{F} \) at some reference position \( x^{(ref)} \). Once we solve \( \vec{F} \) for each layer we can use the formalism of section 3.8 in order to calculate the fields in the entire configuration. Using this formalism we will see that the plus sign for the eigenvalue and eigenvectors correspond to upward propagating waves and the minus sign to downward propagating waves.

### 3.7 Two cases

We are interested in two types of layers:

1. isotropic layer
2. uniaxial layer with the extra-ordinary axis parallel to x-axis

We will start with the uniaxial layer with the extra-ordinary axis parallel to the x-axis.

### 3.7.1 Uniaxial layer: extra-ordinary axis // x-axis

All off-diagonal elements are equal to zero and \( \eta_{xx} = \eta_x, \eta_{xy} = \eta_{yz} = \eta_y \). In this case there is decoupling into \( TE \) and \( TM \) waves. For the eigenvalues we find [70, p. 52]. We find

\[
\lambda^{(-1)} = \left( \eta_y/\eta_x \right)^{1/2} \left[ j \omega \mu_0 \eta_x + k_y^2 + k_z^2 \right]^{1/2},
\] (3.72)

\[
\lambda^{(-2)} = \left[ j \omega \mu_0 \eta_y + k_y^2 + k_z^2 \right]^{1/2},
\] (3.73)

\[
\lambda^{(+1)} = -\lambda^{(-1)}
\] (3.74)

\[
\lambda^{(+2)} = -\lambda^{(-2)}.
\] (3.75)
3.7 Two cases

and the corresponding eigenvectors

\[
\begin{align*}
\psi^{(-1)} & \sim \begin{pmatrix}
-k_y \left[ j \omega_0 \eta_x + k_y^2 + k_z^2 \right]^{\frac{1}{2}} \\
-k_z \left[ j \omega_0 \eta_x + k_y^2 + k_z^2 \right]^{\frac{1}{2}} \\
j k_y \left( \eta_x \eta_y \right)^{\frac{1}{2}} \\
j k_z \left( \eta_x \eta_y \right)^{\frac{1}{2}} \\
\omega_0 k_z \\
-\omega_0 k_y
\end{pmatrix}, \quad (3.76) \\
\psi^{(-2)} & \sim \begin{pmatrix}
-k_z \left[ j \omega_0 \eta_y + k_y^2 + k_z^2 \right]^{\frac{1}{2}} \\
k_y \left[ j \omega_0 \eta_y + k_y^2 + k_z^2 \right]^{\frac{1}{2}}
\end{pmatrix}, \quad (3.77) \\
\psi^{(+1)} & \sim \begin{pmatrix}
k_y \left[ j \omega_0 \eta_x + k_y^2 + k_z^2 \right]^{\frac{1}{2}} \\
k_z \left[ j \omega_0 \eta_x + k_y^2 + k_z^2 \right]^{\frac{1}{2}} \\
j k_y \left( \eta_x \eta_y \right)^{\frac{1}{2}} \\
j k_z \left( \eta_x \eta_y \right)^{\frac{1}{2}} \\
-\omega_0 k_z \\
\omega_0 k_y
\end{pmatrix}, \quad (3.78) \\
\psi^{(+2)} & \sim \begin{pmatrix}
-k_z \left[ j \omega_0 \eta_y + k_y^2 + k_z^2 \right]^{\frac{1}{2}} \\
k_y \left[ j \omega_0 \eta_y + k_y^2 + k_z^2 \right]^{\frac{1}{2}}
\end{pmatrix}, \quad (3.79) \\
& \quad (3.80)
\end{align*}
\]

One can easily infer from equations (3.72)-(3.75) that for anisotropic materials there are either four distinct eigenvalues or two eigenvalues equal to zero. We are looking for a propagation constant \( k = \beta - j \alpha \), with \( \alpha \geq 0 \) and \( \beta > 0 \). For real \( k \), and lossless or lossy dielectric materials it can be verified that two eigenvalues can be equal to zero. However, for practical values of \( k \), \( \alpha \) and \( \beta \) the diagonal decomposition can be applied.

3.7.2 Isotropic layer

In this case

\[
\eta_m = \eta. \quad (3.81)
\]

Again there is neither electric-electric nor magnetic-magnetic coupling in these types of layers resulting in \( TE \) and \( TM \) waves. Using the expressions for the uniaxial case (see (3.72)-(3.75)) and employing \( \eta_m = \eta \), the eigenvalues can be found as

\[
\lambda^{(\pm \ell)} = \mp \rho = \mp \left( j \omega_0 \eta + k_y^2 + k_z^2 \right)^{\frac{1}{2}}. \quad (3.82)
\]

There exist two different eigenvalues each with algebraic multiplicity two. The eigenvectors are obtained using the expressions for the eigenvector in the uniaxial case and employing
\[ \eta_{n,m} = \eta \delta_{n,m}. \] This results in

\[ \begin{align*}
\nu^{(-1)} & \sim (-\rho k_y, -\rho k_z, j\eta k_y, j\eta k_z)^T, \\
\nu^{(-2)} & \sim (\omega \mu_0 k_z, -\omega \mu_0 k_y, -\rho k_z, \rho k_y)^T, \\
\nu^{(+1)} & \sim (\rho k_y, \rho k_z, j\eta k_y, j\eta k_z)^T, \\
\nu^{(+2)} & \sim (-\omega \mu_0 k_z, \omega \mu_0 k_y, -\rho k_z, \rho k_y)^T.
\end{align*} \] (3.8)

### 3.8 The scattering matrix formalism

In order to formulate an integral representation for the field, we have to solve the following system

\[ \partial_x F = \overline{A} F + S \exp(\text{j}k_y y^{(s)}) \delta(x - x^{(s)}), \] (3.87)

the Green's function in the background configuration due to a point source at source level \( x_i = x_i^{(s)} \). The source vector \( S_I \) is defined as

\[ S = j\eta_x^{-1} \begin{pmatrix} k_x f_x^{(con)} \\ k_y f_y^{(con)} \\ -\eta_x f_y^{(con)} \\ -\eta_y f_x^{(con)} \end{pmatrix}, \] (3.88)

where we have introduced a contrast source, which will be defined in chapter 3.9. We take advantage of the continuity of the field across a source-free interface and of the jump of the field across a source at level \( x^{(s)} \). We start with the downward recursive procedure. We first define a propagator matrix \( \overline{P} \) of layer \( (n) \) as

\[ \overline{P}^{(n)} = \overline{C}^{(n)} \exp \left[ -\Lambda^{(n)} t^{(n)} \right] \overline{D}^{(n)}, \] (3.89)

where \( t^{(n)} \) denotes the thickness of layer \( (n) \). Implicitly we have employed that the inverse of the propagator matrix can be obtained by changing the sign of the argument in the exponent. The propagator matrix enables us to propagate the field from one interface to the other due to the continuity of the field, i.e.

\[ F(x^{(N-1)}) = \overline{P}^{(N-1)} F(x^{(N)}). \] (3.90)

We can rewrite the latter equation using equation (3.71) as

\[ t^{(N-1)} = \overline{P}^{(N-1)} \overline{Q}^{(N-1)} t^{(N)}, \] (3.91)

if

\[ \overline{Q}^{(N-1)} = \overline{Q}^{(N-1)} \overline{P}^{(N-1)} \overline{C}^{(N)} = \exp \left[ + \text{mat} \Lambda^{(N-1)} t^{(N-1)} \right] \overline{D}^{(N-1)} \overline{C}^{(N)}. \] (3.92)
3.8 The scattering matrix formalism

From equation (3.70) it is evident that the vector \( r \) denotes the amplitudes of the waves of our system matrix \( \overline{A} \). We can repeat this process down to just above the source level \( s^+ \), resulting in

\[
\overline{r}(s^+) = \overline{Q}^{(s^+,N)} r^{(N)},
\]

(3.93)

where

\[
\overline{Q}^{(s^+,N)} = \overline{Q}^{(s^+)} \overline{Q}^{(s^+ + 1)} \cdots \overline{Q}^{(N-1)}.
\]

(3.94)

If the top layer we only have up-going waves resulting in \( r_3^{(N)} \) and \( r_4^{(N)} \) equal to zero (see (3.71)), if we define for the eigenvalues

\[
\text{Re}\{\lambda^{(-j)}\} \geq 0,
\]

(3.95)

\[
\text{Re}\{\lambda^{(+j)}\} \leq 0.
\]

(3.96)

We can rewrite equation (3.93) as

\[
\overline{r}^{(N;+)} = \overline{T}^{(s^+,N)} r^{(s^+;+)} ,
\]

(3.97)

\[
\overline{r}^{(s^+; -)} = \overline{R}^{(s^+,N)} r^{(s^+;+)} ,
\]

(3.98)

If we define

\[
\overline{r}^{(n)} = \begin{bmatrix} r_1^{(n;+)} , r_2^{(n;+)} , r_1^{(n;-)} , r_2^{(n;-)} \end{bmatrix}^T ,
\]

(3.99)

\[
\overline{T}^{(s^+,N)} = \begin{bmatrix} \overline{Q}^{(s^+,N)} \end{bmatrix}^{-1} ,
\]

(3.100)

\[
\overline{R}^{(s^+,N)} = \begin{bmatrix} \overline{Q}^{(s^+,N)} \end{bmatrix}^{-1} ,
\]

(3.101)

where \( \overline{Q}_{ij} \) denotes the 2-by-2 submatrix of \( \overline{Q}^{(s^+,N)} \).

Instead of direct multiplication of the \( Q \) matrices one can improve the stability of the calculation by employing the scattering matrix and immediately arrive at the \( R \) and \( T \) matrices of the stack of layers. This can be done by recursion ((70, pp. 60-66)), i.e.

\[
\overline{T}^{(n,N)} = \overline{T}^{(n+1,N)} \left[ \overline{Q}^{(n)}_{11} + \overline{Q}^{(n)}_{12} \overline{R}^{(n+1,N)} \right]^{-1} ,
\]

(3.102)

\[
\overline{R}^{(n,N)} = \left[ \overline{Q}^{(n)}_{21} + \overline{Q}^{(n)}_{22} \overline{R}^{(n+1,N)} \right] \times \left[ \overline{Q}^{(n)}_{11} + \overline{Q}^{(n)}_{12} \overline{R}^{(n+1,N)} \right]^{-1} ,
\]

(3.103)

The recursive relations are completed by the initializations

\[
\overline{T}^{(N,N)} = \overline{I}, \overline{R}^{(N,N)} = 0.
\]

(3.104)
In a similar fashion we can perform the upward recursive procedure. We have
\[ r^{(s^-)} = \overline{Q}^{(s^-,1)} r^{(1)}, \]  \hspace{1cm} (3.10)\]
where again we have defined
\[ \overline{Q}^{(s^-,1)} = \overline{Q}^{(s^-)} \overline{Q}^{(s^- - 2)} \ldots \overline{Q}^{(1)}, \]  \hspace{1cm} (3.10a)\]
and
\[ \overline{Q}^{(n)} = \overline{D}^{(n)} \left[ \overline{P}^{(n)} \right]^{-1} \overline{C}^{(n-1)} = \exp \left[ +\Lambda^{(n)} t^{(n)} \right] \overline{D}^{(n)} \overline{C}^{(n-1)} \]  \hspace{1cm} (3.10b)\]
It is clear that we only have down-going waves into the bottom layer, so \( r_1^{(1)} \) and \( r_2^{(1)} \) are equal to zero. Again we can formulate a reflection and transmission matrix, i.e.
\[ r^{(s^-;+)} = \frac{\overline{Q}^{(s^-,1)}}{\overline{R}} r^{(s^-;-)}, \]  \hspace{1cm} (3.10c)\]
\[ r^{(1;+)} = \frac{1}{\overline{T}^{(s^-,1)}} r^{(1;-)}, \]  \hspace{1cm} (3.10d)\]
if we define
\[ r^{(n)} = \begin{bmatrix} r_1^{(n;+)} & r_2^{(n;+)} & r_1^{(n;-)} & r_2^{(n;-)} \end{bmatrix}^T, \]  \hspace{1cm} (3.10e)\]
\[ \overline{T}^{(s^-,1)} = \begin{bmatrix} \overline{Q}_{22} \end{bmatrix}^{-1}, \]  \hspace{1cm} (3.10f)\]
\[ \overline{R}^{(s^-,1)} = \begin{bmatrix} \overline{Q}_{12} \end{bmatrix} \begin{bmatrix} \overline{Q}_{22} \end{bmatrix}^{-1}. \]  \hspace{1cm} (3.10g)\]
Again we can calculate \( R \) and \( T \) directly by recursion, i.e.
\[ \overline{R}^{(n,1)} = \left[ \overline{Q}_{11} \overline{R}^{(n-1,1)} + \overline{Q}_{12} \right] \times \]  \hspace{1cm} (3.10h)\]
\[ \left[ \overline{Q}_{21} \overline{R}^{(n-1,1)} + \overline{Q}_{22} \right]^{-1}, \]  \hspace{1cm} (3.10i)\]
\[ \overline{T}^{(n,1)} = \overline{T}^{(n-1,1)} \left[ \overline{Q}_{21} \overline{R}^{(n-1,1)} + \overline{Q}_{22} \right]^{-1}. \]  \hspace{1cm} (3.10j)\]
The recursive relations are completed by the initializations
\[ \overline{R}^{(1,1)} = 0, \overline{T}^{(1,1)} = \overline{I}. \]  \hspace{1cm} (3.10k)\]
We can couple the solution for \( r^{(s^+)} \) and \( r^{(s^-)} \) by using equation (3.87), i.e.
\[ r^{(s^+)} - r^{(s^-)} = \overline{S}, \]  \hspace{1cm} (3.10l)\]
where
\[ \overline{S} = \overline{D}^{(s)} S \exp(ik_y x_2^{(s)}) \delta(x - x^s). \] (3.117)

Equation (3.116) can be rewritten if we distinguish three different possibilities for the location of the point source:

1. In the intermediate layer
2. In the top layer
3. In the bottom layer

In the first case we can calculate the four components from
\[ r^{(s:-)} = \left[ \overline{R}^{(s:+,N)} \overline{R}^{(s:-,1)} - \overline{S} \right]^{-1} \left[ \overline{S}^{(-)} - \overline{R}^{(s:+,N)} \overline{S}^{(+)}, \right], \] (3.118)
\[ r^{(s:+)} = \overline{R}^{(s:-,1)} r^{(s:-)}, \] (3.119)
\[ r^{(s:+)} = r^{(s:+)} + \overline{S}^{(+)}; \] (3.120)
\[ r^{(s:-)} = r^{(s:-)} + \overline{S}^{(-)}. \] (3.121)

where we used a short notation for \( \overline{S} \)
\[ \overline{S} = \left[ \overline{S}^{(+)}, \overline{S}^{(-)} \right]^T. \] (3.122)

If the point source is positioned in the top layer (case 2) we obtain
\[ r^{(s:+)} = 0, \] (3.123)
\[ r^{(s:-)} = \overline{S}^{(-)}, \] (3.124)
\[ r^{(s:+)} = -\overline{R}^{(s:-,1)} \overline{S}^{(-)}, \] (3.125)
\[ r^{(s:+)} = r^{(s:+)} + \overline{S}^{(+)}. \] (3.126)

For case 3 the result is
\[ r^{(s:+)} = 0, \] (3.127)
\[ r^{(s:+)} = \overline{S}^{(+)}; \] (3.128)
\[ r^{(s:+)} = \overline{R}^{(s:+,N)} \overline{S}^{(+)}, \] (3.129)
\[ r^{(s:-)} = r^{(s:+)} - \overline{S}^{(-)}. \] (3.130)

In our configuration the transmission line is positioned in one layer, so the four amplitude factors in the source layer are sufficient for implementation of an integral equation. A check on the program can be made by solving the homogeneous version of the equations, i.e. by setting \( S = 0 \). For some values of \( k_y \) and \( k_z \) guided waves of the layered configuration can be found.
3.9 The integral equation

For practical reasons the discussion will be restricted to uniaxial materials with the optic axis normal to the layer interfaces. The transverse admittance tensor can then be written as a diagonal matrix

$$\bar{\eta} = \delta_{\alpha,\beta} \eta_{\beta}, \{\alpha,\beta\} \in \{x,y,z\}. \quad (3.13)$$

In layer $D_N$ (see Fig.(3.1)) the contrast with respect to a background medium is introduced as

$$\Delta\bar{\eta}(x,y) = (\bar{\eta}_{D_N}^{D_t}(x,y) - \bar{\eta}_{B}), \quad (3.132)$$

in which $\bar{\eta}_{D_t}$ denotes the transverse admittance of the transmission lines, while $\bar{\eta}_{B}$ denotes the transverse admittance of the background, being layer $D_N$ without the strips. The transmission line is assumed to be homogeneous. Therefore a contrast source $j^{\text{con}}$ can be introduced as

$$j^{\text{con}}(x,y) = \begin{cases} \Delta\bar{\eta}\bar{E}(x,y) & \text{if } (x,y) \in D_t, \\ 0 & \text{otherwise}. \end{cases} \quad (3.133)$$

The electric field can be expressed in terms of a domain-integral representation as

$$\bar{E}(x,y;k_z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk_y \exp(-jk_y(y-y')) \int_{D_t} \bar{G}(x,y;x',y';k_y;k_z)j^{\text{con}}(x',y';k_z)dx'dy', \quad (3.134)$$

in which

$$\bar{G}(x,y;x',y';k_y) = G_{\alpha,\beta}(x,y;x',y';k_y) + \delta_x \delta_y \delta(x-x'), \quad (3.135)$$

where $\bar{G}$ is the Green’s function [70, pp. 206-220]. The other part of $\bar{G}$, $G_{\alpha,\beta}$, is calculated using the scattering matrix method for the layered background. When $(x,y) \in D_t$, equation (3.134) constitutes a homogeneous Fredholm integral equation of the second kind. Non-trivial solutions exists for discrete values of $k_z$. The corresponding fields $\bar{E}(x,y)$ constitute the guide modes of the transmission line configuration.

In order to get a finite set of algebraic equations the domain integral of equation (3.134) is discretised. A computational domain $D_c$, consisting of (non-uniform) grid elements, will enclose the transmission line area $D_t$ (see Fig.(3.2)). To reduce computation time the elements will be chosen smaller near the skin than at the interior of the strips: due to the skin effect in metals and the flux expulsion in superconductors, the electric field will change more rapidly near the boundary of the strips than at their centre. Furthermore the configuration is assumed to be symmetrical, so the $x$-axis of the Cartesian reference frame coincides with the symmetry plane of the strips.

The computational domain $D_c$ is imposed on the superconducting strips $D_t$ and consists (non-uniform) rectangular grid elements, which are indexed by a number pair

$$(n_x', n_y') \in N_c, \quad (3.13)$$
where \( N_c \) is a two dimensional subspace containing all grid element index pairs. For the rectangular grid that is applied in this study

\[
N_c = N_x \times N_y, \quad \text{with } N_x, N_y \subset \mathbb{Z}. \tag{3.137}
\]

Furthermore a barycentre is defined in each grid element as

\[
(b'_{x}, b'_{y}) \in B_c, \tag{3.138}
\]

where

\[
B_c = \{(b_{x}(n'_{x}), b_{y}(n'_{y})) | (n'_{x}, n'_{y}) \in N_c \}. \tag{3.139}
\]

In order to obtain a matrix equation from the integral equation, expansion and weighting functions will be introduced. The numerical solution of the integral equation (3.134) poses some problems due to the existence of a singularity in the Green's function and the high contrast between the superconductor and the layered background. For a realistic configuration with a high \( T_c \) superconductor (see chapter 2), we have a London penetration depth of 200 nm. The resulting contrast function becomes \( \Delta \eta = 1/(\omega \mu_0 \lambda \gamma^2) = 5.76 \cdot 10^8 \). One approach to tackle problems connected with this singularity is to introduce a weak form of the domain-integral equation [72, pp. 36–38]. In this research we have followed the approach of Lee, Ali and Lyons [66], in which the integral equation was solved using the method of moments (Galerkin technique). In the Galerkin method we have chosen both pulse and rooftop functions (depending on the component of the electric field strength) for the expansion and weighting procedure. In addition we have applied the collocation technique, where delta-functions were selected as weighting functions in order to simplify the implementation. We have found that no improvements could be achieved for calculation of the attenuation coefficient by using a Galerkin technique.

The expansion function used in this research are rooftop and pules functions. Rooftop functions are introduced as

\[
\Phi(x_m, n_m) = \begin{cases} 
1 + \Delta_m/l_m(n_m), & -l_m(n_m) \leq \Delta_m \leq 0, \\
1 - \Delta_m/r_m(n_m), & 0 \leq \Delta_m \leq r_m(n_m), \\
0, & \text{otherwise},
\end{cases} \tag{3.140}
\]
while the pulse functions are defined as

$$
\Pi(x_m, n_m) = \begin{cases} 
1, & -d_m(n_m) \leq \Delta_m \leq d_m(n_m), \\
0, & \text{otherwise},
\end{cases}
$$

(3.141)

where

$$
\Delta_m = [|x_m| - b_m(n_m)].
$$

(3.142)

It is noted that the rooftop functions are not symmetrical around their barycentres \(b_m(n_m)\) and extend to the nearest-neighbour barycentre, i.e. \(b_m(n_m) - l_m(n_m)\) and \(b_m(n_m) + r_m(n_m)\). The pulse functions are symmetrical around their barycentres \(b_m(n_m)\) and the support is determined by \(d_m(n_m)\). The components of the electric field can therefore be expanded as

$$
\hat{E}_\alpha(x, y) \simeq \sum_{(n_x, n_y) \in \mathcal{N}_c} e_\alpha(n_x, n_y) X_\alpha(x, n_x) Y_\alpha(y, n_y),
$$

(3.143)

where \(e_\alpha(n_x, n_y)\) denotes the value of the \(x, y, z\)-component of the electric field at the barycentre of the grid element indexed by \((n_x, n_y)\). The functions \(X_\alpha\) and \(Y_\alpha\) are introduced as

$$
X_\alpha(x, n_x) = \begin{cases} 
\Phi(x, n_x), & \text{if } \alpha \in \{x\}, \\
\Pi(x, n_x), & \text{if } \alpha \in \{y, z\},
\end{cases}
$$

(3.144)

$$
Y_\alpha(y, n_y) = \begin{cases} 
\Phi(y, n_y), & \text{if } \alpha \in \{y\}, \\
\Pi(y, n_y), & \text{if } \alpha \in \{x, z\}.
\end{cases}
$$

(3.145)

Together with the contrast function \(\Delta \eta_\alpha\) we can use these expressions in the integral representation (3.134). The domain integral equation relates an electric field component in one specific barycentre to the three electric field components in all the barycentres. In order to obtain the set of equations, expressions have to be derived which relate an electric field component in one barycentre to an electric field component in a second barycentre. For the collocation technique the spatial integration can be performed in the \(k_y\)-domain. These expressions can be obtained by substitution of equations (3.144) and (3.145) in equation (3.134), which yields

$$
\hat{E}_m(n_1, k_2) = \sum_{(n'_x, n'_y) \in \mathcal{N}_c^k} \tilde{M}_{m,n}(n_x, n'_x, k_y, n'_y) \hat{E}_n(n'_x, n'_y),
$$

(3.146)

in which

$$
\tilde{M}_{1,1}(n_x, n'_x, k_y, n'_y) = -\Delta \eta_1 \eta_{11}^{-1} \sum_{i=1}^4 (k_2 v_3^{(i)} + k_3 v_4^{(i)}) \times
\int_{1,1}^{\text{ext}} (n_x, n'_x) I_2^{\text{puls}}(n'_y) +
-\delta_{n_x, n'_x} \Delta \eta_2 \eta_{12}^{\text{puls}}(n'_y)
$$

(3.147)

$$
\tilde{M}_{1,2}(n_x, n'_x, k_y, n'_y) = -\Delta \eta_2 \eta_{11}^{-1} \sum_{i=1}^4 (k_2 v_3^{(i)} + k_3 v_4^{(i)}) \times
$$

(3.148)
3.9 The integral equation

\[ \tilde{M}_{1,3}(n_x, n'_x, k_y, n'_y) = -\Delta \eta_3 \eta_1 \sum_{i=1}^{4} (k_2 v_{3}^{(i)} + k_3 v_{4}^{(i)}) \times I_{1,2}^{\text{puls},(i)}(n_x, n'_x) I_{2}^{\text{ent},(i)}(n'_y) \]  

(3.148)

\[ \tilde{M}_{2,1}(n_x, n'_x, k_y, n'_y) = \Delta \eta_1 \sum_{i=1}^{4} v_{1}^{(i)} I_{1,1}^{\text{ent},(i)}(n_x, n'_x) I_{2}^{\text{puls},(i)}(n'_y) \]  

(3.149)

\[ \tilde{M}_{2,2}(n_x, n'_x, k_y, n'_y) = \Delta \eta_2 \sum_{i=1}^{4} v_{1}^{(i)} I_{1,2}^{\text{ent},(i)}(n_x, n'_x) I_{2}^{\text{puls},(i)}(n'_y) \]  

(3.150)

\[ \tilde{M}_{2,3}(n_x, n'_x, k_y, n'_y) = \Delta \eta_3 \sum_{i=1}^{4} v_{1}^{(i)} I_{1,3}^{\text{ent},(i)}(n_x, n'_x) I_{2}^{\text{puls},(i)}(n'_y) \]  

(3.151)

\[ \tilde{M}_{3,1}(n_x, n'_x, k_y, n'_y) = \Delta \eta_1 \sum_{i=1}^{4} v_{2}^{(i)} I_{1,1}^{\text{ent},(i)}(n_x, n'_x) I_{2}^{\text{puls},(i)}(n'_y) \]  

(3.152)

\[ \tilde{M}_{3,2}(n_x, n'_x, k_y, n'_y) = \Delta \eta_2 \sum_{i=1}^{4} v_{2}^{(i)} I_{1,2}^{\text{ent},(i)}(n_x, n'_x) I_{2}^{\text{puls},(i)}(n'_y) \]  

(3.153)

\[ \tilde{M}_{3,3}(n_x, n'_x, k_y, n'_y) = \Delta \eta_3 \sum_{i=1}^{4} v_{2}^{(i)} I_{1,3}^{\text{ent},(i)}(n_x, n'_x) I_{2}^{\text{puls},(i)}(n'_y) \]  

(3.154)

\[ \tilde{M}_{3,3}(n_x, n'_x, k_y, n'_y) = \Delta \eta_3 \sum_{i=1}^{4} v_{2}^{(i)} I_{1,3}^{\text{ent},(i)}(n_x, n'_x) I_{2}^{\text{puls},(i)}(n'_y) \]  

(3.155)

In these expressions, the functions \( I^{\text{ent}} \) and \( I^{\text{puls}} \) reflect the 'smearing' of the point source over its grid element due to the discretisation with rooftop and pulse functions respectively. Their mathematical expressions can be found in the report of van Eekelen [73]. In case of the spline technique (using a Galerkin technique), the spatial integration is performed numerically and can be implemented in a straightforward way (see also the next section). It is noted that the above expressions for \( \tilde{M}_{m,n} \) are still in the \( k_y \)-domain and have to be transformed back to the \( y \)-domain. This transformation together with the application of point matching will yield \( \tilde{M}_{m,n} \) in which \( b_y(n_y) \), the \( y \)-coordinate of the point of observation, is introduced. The final expression for the matrix equation is

\[ \tilde{E}_n(n_x, n_y) = \sum_{(n_x', n_y') \in \Omega} \tilde{M}_{m,n}(n_x, n'_x, n_y, n'_y) \tilde{E}_m(n'_x, n'_y). \]  

(3.156)

In this way the problem has been reduced to an homogeneous linear system of algebraic equations with system matrix \( \tilde{B}(\omega, k_y) \). Non-trivial solutions of that system are found for discrete values \( k_z = k_{z,m}(\omega) \) for which

\[ \det[\tilde{B}(\omega, k_z)] = 0. \]  

(3.157)

The corresponding electromagnetic field \( \{ \tilde{E}, \tilde{H} \}_m \) constitute a guided mode of the transmission line structure, where

\[ k_{z,m} = \beta_m(\omega) - j\alpha_m(\omega), \]  

(3.158)
where $\beta_m > 0$ is the phase coefficient of the $m$-th guided mode, while $\alpha_m \geq 0$ denotes its attenuation coefficient. Often the effective refraction index (of the mode) is introduced through

\[ N_{\text{eff},m} = \frac{\beta_{\epsilon,m}}{k_0}. \]  

(3.159)

### 3.10 Numerical considerations

Several steps are needed before the determinant can be calculated. In the first place, all Green tensor components are in the $k_y$-domain and thus inverse-Fourier transformation is needed. In the second place, the elements of the matrix are small with a large dynamic range. To avoid underflow in the calculation of the determinant, scaling must be performed. Finally, once the matrix has been filled, the zero's of (3.157) must be found in the complex plane. These steps are elaborated in this subsection.

#### 3.10.1 Inverse Fourier transformation

If the inverse Fourier transformation along the real $k_y$-axis is performed, poles and branch points are encountered. In the transmission line under consideration (see Fig. 3.1) two sets of branch points are found; one related to the top layer $D_N$, 

\[ k_y = \pm j\sqrt{k_2^2 - k_0^2}, \]  

and one related to the substrate $D_1$, 

\[ k_y = \pm j\sqrt{k_2^2 - k_0^2 \epsilon_r}. \]  

(3.16)

The lines for which the real part of $\sqrt{k_y^2 - (k_0^2 - k_2^2)}$ and $\sqrt{k_y^2 - (k_0^2 \epsilon_r - k_2^2)}$ equal zero are chosen to be the branch cuts of the configuration. For a lossless configuration the complex $k_y$-plane is shown in Fig. 3.3. The propagation constants $k_{z,m}$ satisfy $k_z > k_0 \sqrt{\epsilon_r} > k_0$ (for details see [74]). In the lossless case, poles may exist on the real $k_y$-axis. However, when losses are present, as is the case in our configuration, there will be no poles on the latter axis. Thus the inverse Fourier transformation can be performed along the real $k_y$-axis.

The analysis of the $k_y$-spectrum shows different behaviour for the $G_{z,z}$ component from the others. This can easily be seen by comparing a dipole solution to a line-source solution: the range of a dipole is much smaller than that of a line-source. So the spectrum of a dipole is much wider than the spectrum of a line-source. The tensor element $G_{z,z}$ behaves like a line source element, while the other elements exhibit dipole-like behaviour. Many checks have been performed on the calculation of the Green's function:

1. The scattering matrix implementation was checked by setting $k_z = 0$ and performing a root search in the complex $k_y$-plane. In this way poles of the layered background can be
10 Numerical considerations

\[ \text{Figure 3.3 Branch points in the complex } k_y\text{-plane for a lossless configuration.} \]

...found. These values were compared with results of an optical mode solvers for layered structures.

2. The tensor element \( G_{z,z} \) was compared with a Hankel function, both in the spatial and spectral domain.

3. The elements \( G_{x,x} \) and \( G_{y,y} \) were compared with dipole solutions. Only the shape of the functions could be verified, not the absolute values.

In this way the proper calculation of the tensor elements has been verified.

Fast Fourier transform

A first approach was the use of a Fast Fourier Transform (FFT). However, taking a smaller spectrum for \( G_{z,z} \), results in large steps in the spatial \( y \)-domain if an inverse fast Fourier transform (FFT) is used. This immediately leads to a problem: a typical coplanar waveguide is 500 \( \mu \)m wide, while the smallest grid element equals a fraction of \( \lambda_L \), i.e., 50 \( \text{nm} \). This leads to a number of FFT-points of 10,000. To avoid aliasing, this number should be doubled, resulting in an enormous demand on computer memory in order to fill the matrix with all tensor components. The transformation was performed along a deformed contour to avoid any singularities, but no improvements were found. With the implementation of the FFT, point matching was used as weighting functions. The use of an FFT proved to be feasible and some results will be presented in the next section. Afterwards it has been concluded that point matching did not lead to accurate results concerning the loss calculations of the transmission line.
Cosine and sine transformation

To avoid the large arrays needed for an FFT, a direct cosine and sine transformation was considered. Together with this implementation a Galerkin technique was employed. With the Galerkin technique, the resulting functions showed strong oscillatory behaviour. Functions of the following form occurred as matrix elements: \( \hat{f} = g(k_y) \cos(k_y \Delta_1) \cos(k_y \Delta_2) \), where \( g(k_y) \) is a smooth function of \( k_y \). Due to the non-uniform grid, the elements \( \Delta_1 \) and \( \Delta_2 \) can differ by a factor of 10,000 (i.e. \( \Delta_1 \) is the smallest grid element, while \( \Delta_2 \) equals the largest grid element). A cosine and sine integration of such functions can be extremely time-consuming and can lead to inaccurate results. Only narrow microstrip lines can be calculated with this inverse Fourier transformation. Another major drawback of a non-uniform grid is that all elements are different in the matrix, i.e. all points need to be calculated separately.

This inverse transformation did not result in satisfactory solutions, was slow and the determinant calculations did not show convergence. No results will be presented with respect to the cosine and sine transformation. From the article of Lee and co-workers, it is not clear what type of integration they used [75]. The widest strip they calculated is relatively narrow when compared to measurements. This could indicate similar problems associated with the fast oscillations.

Spline interpolation

A final attempt was made to obtain accurate results for the complex propagation constant. To avoid fast oscillations associated with spatial integration in the \( k_x \)-domain, the integration is performed numerically in the \( x, y \)-plane. The spectrum of all Green's tensor components behave reasonably smooth on the real \( k_y \)-axis, so they can in principle be approximated by splines. The inverse transformation is then a matter of integration of the splines times a cosine or sine function and can be done analytically. The spectrum is divided into three parts: a part around the branch point of the dielectric layer, a part for high spectral values and an intermediate part. In this way the functions can be recovered on a very small scale and a very wide scale.

When source (integration patch) and receiver (observation patch) height coincide, a singularity is encountered which is not smeared over a finite area. This problem was circumvented by extrapolation: if source and receiver height coincide, an extra value of half a grid element in height is calculated. Subsequently an extrapolation is performed using the function value at one grid element difference and the value at half a grid element difference. Calculations showed that this a stable method.

For small distances in \( y \)-direction even the spline calculations suffered from numerical errors. For low spectral \( k_y \)-values a Taylor expansion must be used to circumvent rounding errors in the analytical expressions for the inverse transformation.
3.10 Numerical considerations

Figure 3.4 Sketches of the Green's tensor components and different regions for spline interpolation. Note the different spectral behaviour of the \( G_{zz} \) tensor element. Below is the extrapolation if heights of source and receiver overlap. The values at \( R^1 \) and \( R^2 \) are calculated and extrapolated to a value at \( R \).

3.10.2 The matrix elements

Small distances from source to observation that can occur in a grid are a problem for accurate calculation of the Green's tensor elements. To accurately approximate the behaviour of the electrical field inside the strip, distances as small as one fifth of the London penetration depth are needed. In contrast, points at the other side of the strip can be separated one million times the London penetration depth. Evidently, the values of the Green's function change very fast. This results in large elements near or on the diagonal of the matrix and, by moving away from the diagonal, the values decrease rapidly.

Two approaches have been employed to scale the matrix: 1) calculate the determinant once and use the absolute value for scaling subsequent matrices, 2) scale all rows or column separately and calculate the determinant. If the Crout factorisation is used the matrix \( \overline{B} \) is rewritten as \( \overline{P} = \overline{LU} \). The product of the diagonal elements of matrix \( \overline{L} \) equals the determinant of matrix \( \overline{B} \), where the sign depends on the permutation matrix \( \overline{P} \). Subsequent matrices can be scaled using the factor \( \kappa = \exp \left( \sum_{i=1}^{N} \ln |l_{ii}| \right) / N \), where \( l_{ii} \) are the latter diagonal elements and \( N \) is the order of the matrix. The other method scales the rows of the \( G_{xx} \) and \( G_{yy} \) elements on
\[ \sqrt{G_{x,x} G_{x,x+2}}, \text{ while the row of the } G_{z,z} \text{ elements are scaled on } \sqrt{G_{z,z} G_{z,z-2}}. \] The notation \( x + 2 \) denotes two position to the right of the diagonal, while \( z - 2 \) denotes two position to the left of the diagonal. Because the first method scales once, the root search can always find a solution, if the first scaling is a very large number. However, if the starting point is very close to the solution, a relatively small scaling is used, resulting in no root at all. A major drawback of the second method is that, by increasing the number of discretisation steps, the absolute value of the determinant can get very large. However, with every next step in the iteration process, the determinant is calculated in a 'fair' way: the determinant approaches zero if a linear combination of the rows or columns equals zero. The first way was used with the inverse FFT approach, while the second method has been applied with the spline interpolation method.

### 3.10.3 Guided wave search

Basically two strategies can be used to find the guided waves of the transmission line configuration. Firstly, a certain area in the complex \( k_3 \)-plane can be defined in which all guided wave solutions are located. A routine can then be used to search this area and find the root(s) of the equation. This method has the advantage that, in principle, all guided wave solutions can be found independently, without presupposed knowledge of their approximate location. In general, however, a lot of function evaluations are required to search the area. A second approach uses one or more initial values for \( k_3 \), and the solution is searched starting from these values. If the initial values are well chosen, this method requires only a few function evaluations, thus using less computation time than the first method. The fact that another simulation tool is needed to supply the initial values is a disadvantage of this method.

The programs require considerable computation time to compute the determinant of the matrix for a certain \( k_3 \)-value. Therefore the latter of the two methods is used: the mathematical routine "Root". This routine requires two initial values, which can be taken from other simulation tools, that give a good approximation to the solution. "Root" assumes the function of which a zero is to be found, to be reasonably smooth in the area between the two initial values. If this is not the case, convergence problems might occur.

### 3.11 Numerical results

In this section numerical results will be presented. With the FFT approach two types of superconducting transmission lines will be presented: the microstrip and the coplanar waveguide. For the spline interpolation method more results will be presented.

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3.11.1 Numerical results

To reduce computation time, we have always started with microstrip transmission lines. The program was checked using various values for the London penetration depth and dielectric constants of the intermediate layer. However, two values for the dielectric constants correspond to the high $T_c$ substrates, i.e. $\varepsilon_r = 10$ represents an MgO substrate, while $\varepsilon_r = 23.7$ corresponds to a LaAlO$_3$ substrate.

Results

An exponential grid for the FFT approach was used. This reduces the number of points to be calculated. The grid lines can be seen in Fig. 3.5 for the discretisation in y-direction. Each line center is positioned in the middle of two grid lines. The number of points for the subsequent examples was selected to be 4 divisions in x-direction and 10 in y-direction per strip. The exponential decay was set to three times the London penetration depth. The results of a superconducting microstrip transmission line on top of an MgO substrate can be viewed in Fig. 3.6. Remarkable is the result of the full-wave package compared to the program 'LinceCalc' (a package within MDS); the wave along a superconducting strip is faster than along a perfect conductor and exhibits less dispersion. Normally, when there is some field penetration in a conductor, the phase velocity will go down. The program predicts that anisotropy does affect the propagation, in contrast with the result of Lee and co-workers [66]. More results will be presented with the spline interpolation approach.

The results of a superconducting coplanar waveguide transmission line on top of a LaAlO$_3$ substrate is shown in Fig. 3.6. For small values of the London penetration depth, the values are very close to the results of 'LineCalc'. However, if $\lambda_0$ increases, the wave propagation becomes much slower. The program predicts that for the coplanar waveguide transmission line, the influence of the anisotropy should be measurable. The strange behaviour at higher frequencies (i.e. phase velocity going up) contradicts the measurements: the phase velocity is approximately constant up to 20 GHz. More results will be presented with the spline interpo-
3. Transmission line model

![Graphs showing effective refraction index as a function of frequency for a microstrip and coplanar waveguide transmission line. The superconductor and dielectric material are assumed to be lossless. London penetration depths are indicated in the plots. For anisotropic superconductors, the London penetration depth is assumed to be five times as large in x than in y, z-direction. Geometry values are in micrometer.]

**Figure 3.6** Effective refraction index as a function of frequency for a microstrip and coplanar waveguide transmission line. The superconductor and dielectric material are assumed to be lossless. London penetration depths are indicated in the plots. For anisotropic superconductors, the London penetration depth is assumed to be five times as large in x than in y, z-direction. Geometry values are in micrometer.

In conclusion, for this approach, the results are not convincing enough and do not correspond to measurements performed on the coplanar waveguide transmission lines. No checks on the accuracy of the inverse Fourier transformation can be performed, which is a major drawback of the FFT method. For the $G_{zz}$ component of the Green's tensor, a Hankel-type function, the decay is not fast enough near the boundaries of the FFT-window. The result is significantly aliased, increasing the error in this component. The inverse Fast Fourier Transform is not suitable for metallic structures solved using a domain integral equation method.

**Spline results**

To reduce the complexity of the grid, a uniform grid in x-direction was selected and two grid distances in y-direction. A uniform grid in x-direction is possible, because for typical high-temperature superconductor thin films the thicknesses are in the same range as the London penetration depth (i.e. $\approx 250$ nm). In y-direction the small grid elements at the edge of the strip(s) are typically a fraction of the London penetration depth, while the rest of the strips is subdivided into large elements. This grid is shown in Fig. 3.7.

The spatial integration for the small elements is multiplication by the area, while the larger elements are subdivided into a number of sub-elements that must be passed to the program by the user. It was found that subdivision into at least 25 elements was needed to get convergence. All subsequent examples were calculated using 25 subdivisions. In x-direction the strips were subdivided into two cells.
Despite several approaches to solve the domain integral equation, the attenuation proved to be difficult to calculate. For typical losses along superconducting lines, the imaginary part of the propagation constant is $10^5$ smaller than the real part (at 5 GHz for lossless dielectric materials). This poses strong demands on the calculation of the determinant of the matrix. Because large matrices must be filled and calculated, unacceptable large computation times are needed; quadrupole precision is used.

The configuration presented in left hand side of Fig. 3.8 has been calculated by Olyslager [76]. The results coincide with results from the program 'LineCalc'. The size of the small elements near the boundaries was set to the value of the London penetration depth. Convergence was checked by increasing the number of small elements at both boundaries of the superconducting strip as indicated by the labels in the figure. By increasing the number of discretisations, the value for the real part of the propagation constant hardly changes. It was found that only one subdivision at the edge was sufficient. It is noted that 4 discretisations at both edges of the strip did not converge anymore. The difference between the 'LineCalc' answer is due to the kinetic inductance of the superconducting strip ($\lambda_0 = 200$ nm).

The configuration presented in Fig. 3.8 was calculated using the FFT approach. It was calculated with the spline method as well. First of all, the full-wave package predicts the same dispersion as a quasi-static approximation using 'LineCalc'. And in the second place, the offset with respect to the 'LineCalc' answer indicates a measurable kinetic inductance in case of London penetration depths of 150 nm. Convergence was checked by increasing the number of small elements at both boundaries of the superconducting strip as indicated by the labels in the figure. Again no significant change was found by increasing the number of small elements at the edge of the microstrip.

The anisotropy was also checked for the microstrip transmission lines. For an anisotropic, axis film, where the superconductivity is 25 times smaller in $x$ than in $y,z$-direction, the propagation constants were almost identical to the numbers calculated if the film was assumed isotropic. This results corresponds with the results of the MIT group [66]. Together with the other results of the FFT calculations, it is believed that the FFT approach did not result in
Figure 3.8 The effective refraction index of a lossless superconducting strip on top of a lossless dielectric as a function of frequency. In the left plot was the London penetration depth set to 200 nm. The number of discretisations is indicated at the labels of plot. In the right plot was the London penetration depth set to 150 nm.

accurate numbers.

We will now turn to the coplanar waveguide. The same coplanar waveguide as in Fig. 3.6 has been simulated. This transmission line has been measured with various film thicknesses, so the thickness was varied using increasing values for the London penetration depth. Concerning computation time, the coplanar waveguide takes a long time to calculate. Time required for one iteration is approximately 6 hours on a HP 9000/C100 workstation. The numerical integration over one cell subdivided into 25 subcells requires many calculation steps. The coplanar waveguide could only be simulated with one element at the edge of the strips to avoid excessive computation time. All simulations were performed with a frequency of 5.5 GHz. This frequency corresponds with the measurement frequency. The relative kinetic inductance $\bar{L}_{kin}$ was deduced from the simulation using

$$\bar{L}_{kin} = \frac{N^2_{eff,\, sid}}{12.35} - 1,$$  \hfill (3.162)

where 12.35 is the average dielectric constant of the substrate LaAlO$_3$ (23.7) and vacuum (1.0). The difference between a perfect conductor and the calculation is attributed to the kinetic inductance.

The partial wave results were obtained by solving equations (2.28) and (2.29). Within the partial wave analysis the ground lines extend to infinity. The full wave method has been employed with a finite width of 190 $\mu$m. The full wave solutions were only obtained at three London penetration depth values per film thickness. Unfortunately no convergence check could be performed due to the long computation times. The behaviour for the 150 nm coplanar waveguide line follows a similar trend as the partial wave analysis, but the values are slightly different. The difference could be a result of finite ground lines. The results of a 300 nm
3.11 Numerical results

Figure 3.9 The relative kinetic inductance as a function of the London penetration depth for a lossless, superconducting coplanar waveguide of 150 nm and 300 nm thickness on top of a lossless dielectric substrate. All geometric values are micrometers.

Thick superconductor reveal a larger difference between partial and full wave solutions. The full wave simulation predicts a stronger dependence on the London penetration depth than the partial wave analysis.

The results of the domain integral equation indicate that the large contrasts and the singularity of the Green’s function should be solved with a different approach. Especially the losses of superconducting transmission lines could not be simulated. The weak form of the domain-integral equation could be a method to improve the results [72].
3. Transmission line models
Chapter 4

Bandstop filter design

1.1 Introduction

In this chapter we consider the design of bandstop filters for the low GHz-range for the radioastronomic site in Westerbork, the Netherlands. The Dutch radio astronomical research unit ASTRON in Dwingeloo provided the specifications for a bandstop filter to suppress signals coming from the Russian navigation satellites supporting the Glonass frequencies [29]. The filter template is shown in Fig. 4.1. Due to the extreme weak signal levels, the insertion loss must be kept at an absolute minimum. The Glonass signal (1560-1610 MHz) is very close to the hydroxyl radical line (1612 MHz), so that steep skirts are imperative. The steps taken in fulfilling these requirements using the phenomenon of high temperature superconductivity are reviewed in the following section.

![Figure 4.1 Specifications of the bandstop filter.](image-url)
Several articles illustrating lumped element design applied to the realisation of superconducting bandpass filters, for example from STI [77] and ComDev [78]. Multiplexers have also been demonstrated in superconducting technology [79]. Zhang et al. have a clever approach for narrowband lumped element microstrip filters: frequency dependent inductors using a high inductance line in parallel with an interdigital capacitor [80]. A combination of superconducting lumped and distributed line elements was presented by Shipton [81]. Hybrid technology, combining GaAs-transistors with high T_c passive components, has been demonstrated by DuPont, Hittite Microwave Corporation and Bhasin et al. [7][82][83]. Complete systems are presented in a special issue of IEEE Transactions on Microwave Theory and Techniques [27].

In literature not much could be found on coplanar waveguide bandstop filters or bandstop filters in general. Worthwhile reference dealing with superconducting bandstop filters is the design by STI [5]. In this reference Fenzi and co-workers report a bandstop filter using microstrip rings capacitively coupled to a transmission line and intersected by a light sensitive GaAs switch. Light coupled to the switch can be used to select the frequency of oscillation of one of the rings. Based on the amount of coupling a frequency band can be rejected, when the switch is in the 'on'-state. Using multiple rings several bands can be blocked or left unaffected. Lancaster et al. [84] used a lumped element approach for a bandrejector: with an interdigital capacitor one of the digits is shorted from input to output, realising in effect a parallel LC-resonator. A model is proposed for this type of circuit, but the desired accuracy is questionable and the resonant frequency too high (∼5 GHz).

We will first demonstrate that a lumped element design is not suitable for the Westerbork filter, even when using superconducting material and active components. We will then consider commensurate transmission line filter. Reflecting the heavy requirements imposed on the filter, many other aspects, apart from filter theory, have dealt with as well. In conclusion we will present the results of a bandstop prototype at 3 GHz and the final design at 1.53 GHz.

4.2 Lumped element design

The combination of high fall-off rates in the stopband and a Butterworth or Chebychev response would result in an extremely high order filter. However, by introducing transmission zeros at either side of the resonant frequency f_r very sharp skirt performance is feasible. Filters having ripples in both stop and passband are called elliptic function filters. In contrast to Butterworth and Chebychev filters no closed-form expressions are available for elliptic function filters. Fortunately, extensive catalogues do exist, for example [85, pp. 169-189]. We arrive at an elliptic filter with ρ = 15% [85, p. 143] and θ = 46° as can be seen in Fig. 4.2. The variable Θ is used to tabulate the small changes in the bandstop parameter Ω_s = \frac{f_s}{f_r} = \frac{\theta}{\sin\theta} in a gradual way. According to the tables a five pole design almost matches the specifications. One can transform the normalized low-pass values into it's bandstop equivalence using the transformation of Fig. 4.3 [85], where parameter a equals the bandstop parameter of the filter (i.e. \frac{f_r}{f_1-f_2}). From the specifications in Fig. 4.1 the bandpass parameter equals 9.55, resulting...
extremely different inductance values (actually the extreme values are 58 nH and 0.4 nH!). Another major problem is that the components are not broadband. Interdigital capacitances and high impedance lines are used to mimic the impedance behaviour of the capacitances and inductances. However, only 0.2 dB mismatch loss is allowed in the passband, imposing strong demands on the separate components in the filter. Simulations indicated that broadband fits of planar capacitances and inductances are very difficult, if not impossible. Especially the large values cannot be fabricated in standard planar technology. Values go up to 2 pF for capacitances and 6 nH for inductances. One way to work around this problem is by using impedance transformers and using overlap capacitances. By decreasing the internal impedance, the inductance values will decrease as well, simultaneously increasing the capacitances. If overlap capacitances can be fabricated the high capacitance values can be dealt with. It is difficult to use impedance transformers if a bandstop filter is required, because parts of the filter will be used in the impedance transformer. In the case of a bandstop filter, the circuit normally starts with either a series resonant circuit to ground or a parallel resonant circuit in series. These subcircuits are not suitable for inclusion in impedance transformers. In general a bandstop filter gives much less design freedom than bandpass filters. If we use the lowpass filter as presented in Fig. 4.2 and transform it using the design rules of Fig. 4.3, then we observe that we can use the capacitance at the input and output terminals in a Norton transformation. However, for realisable values of inductances we need at least a factor 10 for the impedance transformation. A transformer of 1:4 is acceptable, reducing the impedance by a factor of 16. Unfortunately this transformation leads to a negative capacitance at the input and output terminal. This latter capacitance cannot be compensated. Another possibility is transforming both branches to ground. If large capacitances can be fabricated then the series resonant circuit in the branch to ground, containing the largest inductive value, must be transformed into a large capacitance and small inductance. The large capacitance can be fabricated using a high dielectric material compatible with the superconductor. Compatible means that the lattice of the superconductor \( \text{La}_{2}\text{Cu}_{3}\text{O}_{7-\delta} \) matches with the high dielectric material, for example \( \text{SrTiO}_3 \). In general, the layers grown on top of the superconductor are of lesser quality (surface impedance), so we have selected the type of capacitor as presented in Fig. 4.4. This type of capacitor avoids lines running on top of the \( \text{SrTiO}_3 \), so the highest quality superconductor can be used for the induc-
4. Bandstop filter design

\[
\begin{align*}
L' &= \frac{LR}{\omega r} \\
C' &= \frac{a}{LR \omega r} \\
C' &= \frac{a}{a\omega r} \\
L' &= \frac{aR}{C\omega r}
\end{align*}
\]

**Figure 4.3** Transforming the low-pass into a bandstop design.

**Figure 4.4** Overlap capacitors using YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7-5} electrodes and SrTiO\textsubscript{3} as dielectric.

tances and interconnects. Obviously this way of fabricating capacitors increases the area of capacitance by a factor of 2, but the dielectric constant of SrTiO\textsubscript{3} is high enough, so that this increase is acceptable (if we assume that the dielectric constant of SrTiO\textsubscript{3} equals 1000 having a thickness of 100 nm, we end up with a value of 88.5 fF/\mu m\textsuperscript{2}) [86][87][88]. Also with the configuration for the capacitance only one side of the bottom electrode needs to be dry etched under a small angle. In this way the next dielectric layer can grow smoothly from the bottom electrode to the substrate enabling the top electrode to contact left and right overlapping regions. In this way two capacitances in series can be fabricated.

One drawback of this lumped element approach is that the response of the circuit is extremely sensitive to small deviations of the separate element values. This results in a strong demand for reproducibility of the dielectric layers grown on top of the superconductor. The laser ablation setup has been extensively used for growth of films with a strong emphasis on physics, not technology. From the start we knew that a bandstop filter matching the specifications would be extremely difficult to fabricate, but still it was interesting to check the performance of SrTiO\textsubscript{3} dielectric layer, both its relative dielectric constant and leakage. Especially with the cold wafer prober we can perform calibrated measurements of many capacitances in one cool down. Measurement of several overlap capacitors can be seen in Fig. 4.5.
For an ideal capacitor the plot should start at 1, corresponding to an ideal open at the lowest frequency. By increasing the frequency the trace should follow the lowest circle approaching the −1 corresponding to an ideal short. From the measurement we do observe a capacitive behaviour, but at low frequencies the capacitor is not a true capacitance. Apparently the top and bottom electrode are not separated by an ideal dielectric: at low frequencies a large portion of the signal is carried to the output (if all power was transferred to the output the trace would start in the center of the circle). From an apparatus available at our fabrication facility, the x-stepper [89], we could measure the smoothness of a 300 nm YBa$_2$Cu$_3$O$_{7-δ}$ film. We found that the film thickness ranged from 250 up to 450 nm (we can better speak of roughness!). This type of film can easily have shorts from top to bottom electrode and we believe that this is indeed the case with the films we measured. We can draw one important conclusion from the measured results: the overlap capacitors, as we fabricate them, are not suitable for low loss, high pole and accurate bandstop filters.

For the lumped element approach two options remained: 1) a low-pass and high pass filter in parallel, 2) a hybrid filter combining a low-noise field effect transistor (FET) with a bandstop
filter as feedback. With the first option we started with a low-pass and high-pass prototype and transformed the values to the desired frequency. The next step was connecting both filter and optimising the transfer of the filter by adapting the coupling and the component values of the filters. This resulted in acceptable component values and frequency response of the filter (however, at the high pass side of the filter the insertion loss was close to $-1$ dB). Still the most extreme values of the inductances are $8.1$ nH and $3.35$ pF for the capacitances. To check the performance of such devices in planar technology we have fabricated interdigital capacitances and long, high inductive lines in superconducting thin films. The performance of a $300$ nm thick, $40$ digit interdigital capacitor on top of a LaAlO$_3$ ($\varepsilon_r = 23.7$) substrate is demonstrated in Fig. 4.6. Clearly visible are the sharp resonances in the structure. We did not expect such a behaviour, especially not at low frequencies. The model is available within Hewlett Packard Microwave Design System (MDS) and predicts the trend of the trace reasonably well. The sharp resonances can be simulated using a quasi-static calculation of $40$ parallel coupled lines. With this model all but the lowest resonance can be predicted. Similar behaviour of coupled conductors can be found in ceramic capacitors: by increasing the number of plates, the resonance frequency decreases [90]. This $40$ digit capacitor was necessary in the design, but to employ this type of capacitor new models should be developed and tested to exploit this type of device. Also the long high inductive lines did not perform the way they should and especially the models for the high inductive lines could not predict the inductance accurate enough. Broadband inductors are not available in case of the extreme low insertion-loss, high pole bandstop filter.
4.2 Lumped element design

The final option was a combined low-noise FET (Field Effect Transistor) bandstop filter. The idea is presented in Fig. 4.7 and is recently demonstrated in GaAs-technology [91]. With this type of feedback we can design a bandpass filter combined with an active device. Because no connections need to be made between the filter and the low-noise amplifier, the demand on the insertion loss can be relaxed (in a coax to coax connection 0.1 dB is easily lost). A strong out of band signal can still overload the amplifier stage, but measurement should clarify if this option is workable. Previous research on low-noise amplifiers resulted in instable devices [92] [93], but this time the technology could be controlled in a much better way (bond-wires and stripline width). Using the cold wafer probe we can measure the $S$-parameters at low temperatures so that a stable design may more easily be realised. Unfortunately the feedback complicates the stability issue. We first started with the calibrated measurement of the cold FET. Low and high temperature results are presented in Fig. 4.8. A standard model could fit the measurement only if we incorporated the bonding wires attached to the die. A quasi-static model is available for bonding wires, but the same results could be obtained if a series-inductance of 1.4 nH was connected from gate to die, drain to die and a series-inductance of 0.5 nH from source to die. The test carrier exhibited considerable conductors losses and these losses were incorporated with 2.1 mm lossy transmission lines of $50 \Omega$, an effective relative dielectric constant of 9.8 and a series resistance of 3 $\Omega$/mm. The latter transmission lines were connected in series with both gate and drain. Only the $S_{21}$ has been plotted, because the other parameters differed only slightly over temperature ($< 0.1$ dB). However for the $S_{21}$ the gain increased by 1 dB at 20 Kelvin.

Subsequently a superconducting hybrid circuit was fabricated in 300 nm thick YBa$_2$Cu$_3$O$_{7-\delta}$ film on top of a LaAlO$_3$ substrate. Feedback as depicted in Fig. 4.7 was employed by means of a high inductive line and interdigital capacitors (now used as a narrowband device). Measurements showed that this device started to oscillate at 6.54 GHz and it's harmonics. Because we performed this measurement simultaneously with the measurements of the inductive lines and interdigital capacitances, we did not know that the capacitance would start to oscillate at frequencies as low as 1.5 GHz. Simulations with the dataset from the interdigital capacitance
measurement and the dataset of the cold FET demonstrated that the amplifier is not unconditionally stable. In this hybrid circuit using a 60 digit capacitance is likely to give oscillation at even lower frequencies. Having tried this final option, we concluded that with the given specifications, we could not build a lumped element bandstop filter. The only possibility that remained, were distributed line filters.

4.3 Coplanar waveguide distributed line filters

Commensurate line filters consist of transmission line sections having a length of $\frac{1}{4}\lambda$ at the design frequency. Several types of transmission lines can be used, but in the majority of the cases microstrip lines are employed. With our cold wafer probe setup one automatically ends up with coplanar waveguide design if vias cannot be fabricated. Coplanar waveguide structures allow for increased packing density without the need for thin substrates as in the case of microstrip transmission lines. In addition, high performance microstrip transmission lines require a superconducting groundplane and integration with active elements should be possible without the need for vias. Unfortunately the multi-mode character of coplanar waveguide is a major drawback and can only be solved using airbridges or grounded coplanar waveguide. Furthermore, the microwave simulation programs have extensive libraries for microstrip type of transmission lines and only limited support for coplanar waveguide structures.
4.3 Coplanar waveguide distributed line filters

<table>
<thead>
<tr>
<th>Filter element</th>
<th>Schematic</th>
<th>$T$ (transfer) matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit element</td>
<td>$\frac{\lambda}{4}$</td>
<td>$(Z \overline{A} + Z^{-1} \overline{A} S + I) \sqrt{1 - S^2}$</td>
</tr>
<tr>
<td>$C$</td>
<td></td>
<td>$CS \left[ I + \frac{1}{\overline{A} S} + \frac{\overline{A}}{CS} \right]$</td>
</tr>
<tr>
<td>$L$</td>
<td></td>
<td>$LS \left[ I + \frac{1}{\overline{A} S} + \frac{\overline{A}}{LS} \right]$</td>
</tr>
</tbody>
</table>

Figure 4.9 Transfer matrix corresponding with low pass $LC$ and unit elements. The $\overline{A}$-matrix is defined in equation 4.3 and $I$ is the identity matrix.

### 4.3.1 A prototype at 3 GHz

Initially we designed a prototype bandreject filter at 3 GHz, since this would fit on a $10 \times 10$ mm$^2$ substrate of LaAlO$_3$. A hairpin-like design was selected, because of its compact size. Coplanar waveguide design contributes to decreased coupling between the different $\lambda/4$-sections, in this way making the structure even more compact than microstrip design.

The starting point was a low-pass Chebychev filter with a 94.75% bandwidth and 0.1 dB ripple using an expert system implemented within the Microwave Design System of Hewlett Packard [94]. The synthesis algorithm used is based upon the theory of Horton and Wenzel [95]. First the complex frequency variable $S$ is introduced as [8]

$$S = j\Omega = j \left( \frac{\pi f}{2 f_r} \right), \quad (4.1)$$

where $f_r$ represents the frequency where the transmission lines are one quarter of a wavelength long ($\beta l = \pi/2$). The new frequency variable ranges from zero to infinity over the interval zero to $f_r$, so the entire frequency response is mapped to a finite portion of the frequency band.

In the $\Omega$-domain an open circuited line element behaves like a capacitance and a shorted line element as an inductor. Standard elements for low pass element are presented in Fig. 4.9, together with the transfer ($T$) matrix form. The transfer matrix relates waves at the input ($a_1$ and $b_1$) to waves at the output ($a_2$ and $b_2$) of a two port and can be introduced as

$$\begin{pmatrix} b_1 \\ a_1 \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}. \quad (4.2)$$

The matrix $\overline{A}$ in Fig. 4.9 is defined by

$$\overline{A} = \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}. \quad (4.3)$$
4. Bandstop filter design

Clearly, by having a mixed cascade of \( m \) low pass elements and \( n \) unit elements, the overall \( T \)-matrix has the form

\[
\begin{pmatrix} \bar{T}_{m+n} \end{pmatrix} = \left( \frac{1}{S} \right)^m \left( \frac{1}{\sqrt{1-S^2}} \right)^n \begin{pmatrix} t_{m+n;11} & t_{m+n;12} \\ t_{m+n;21} & t_{m+n;22} \end{pmatrix},
\]

where the elements of \( \bar{T}_{m+n} \) are polynomials of \((m+n)\)th degree in \( S \). Matrix element \( t_{m+n;12} \) represents the ratio of input reflected wave (\( \rho \)) to the transmitted wave into the load (\( \tau \)). For a lossless filter the total power is conserved

\[
|\rho|^2 + |\tau|^2 = 1
\]

which can be rewritten as

\[
|\tau|^2 = \frac{1}{1 + |t_{m+n;12}|^2}.
\]

By introducing the \( S \)-plane cut-off frequency \( S_c^2 = (j \tan \theta_c)^2, \theta_c = \pi \omega_c/2 \omega_r \), the prototype response ratio of reflected to transmitted power is given by

\[
\frac{|\rho|^2}{|\tau|^2} = \frac{-S_c^2}{-S_c^2} \frac{-S_c^2(1-S_c^2)}{-S_c^2(1-S_c^2)} Q_{m+n} \left( \frac{-S_c^2}{-S_c^2} \right),
\]

where \( Q_{m+n} \) is an \((m+n)\)th degree polynomial in \((S_c^2)/(-S_c^2)\). Horton and Wenzel demonstrate that this form can be realized with Butterworth or Chebychev polynomials. They show that Richard’s theorem and pole removing techniques can be used to determine the unit element and \( LC \) values.

Within the filter design system, the user has to input the filter response (Butterworth or Chebychev; low pass or high pass), topology (stubs or coupled lines), ripple, frequency, bandwidth, impedance and the combination of elements (a valid sequence of unit elements and \( S \)-plane capacitors and inductors). The system returns values for the unit elements and \( S \)-plane capacitors and inductors. If the element values are out of range, the user can add unit elements and rerun the application. With 0.1 dB ripple in the passband and skirt selectivity 2 (=BW_{0.1 dB}/BW_{-35dB}) a five poles design exhibits at least 35 dB attenuation in the stopband [1, pp. 480–482]. This skirt selectivity is a compromise between the ripple, the stopband attenuation and the order of the filter.

By using the non-redundant algorithm we obtain a transmission line filter with five unit elements and five series (transmission line) inductances. This type of filter exhibits many transmission zeros in the stopband, but unfortunately does not improve the skirt selectivity. Due to the asymmetry of the filter the input and output impedance are unequal. Simulations demonstrated that this increases the insertion loss by 0.05 dB and shifts the center frequency by 1.6%. Since we wanted to have space on the sample for the measurement of separate line sections, we kept the number of section at five and accepted the deteriorated filter response. Applying the \( S \)-plane equivalence of the cascade of a unit element and a series \( S \)-plane inductance, as depicted
With coplanar waveguide transmissions lines the shorted elements are very simple to realise and this transformation leads to realistic impedance values. The step from impedance values to geometries can be found, for example, in [98] and [34]. In our design we restricted ourselves to a resolution of 2.5 μm, so the values in the table 4.1 are not the optimal values, but very close to the values obtained from Eq. (4.8) and (4.9). This again results in a slightly deteriorated bandstop behavior: 0.05 dB extra ripple in the pass band and 0.65% increase in the bandwidth. By including the end effects of the open coupled section another 0.05 dB ripple was added and the bandwidth increased by 0.5%. By incorporating the coupling sections in the simulations an increase of 0.05 dB insertion loss added. We noted that especially near the edges of the copband the filter characteristic is very sensitive for changes in line impedance, end effects, coupling sections and impedance mismatches.

To check the minimum width of the ground plane between the two coupled 1/4 λ sections a 1-pole Chebychev filter has been simulated. Using a quasi-static model with seven coupled lines. By reducing the width of the ground strip a degradation of the transfer was observed if the width was reduced below 350 μm. This was taken as the minimum distance between two sections for this type of substrate.

Within the design the modelling was performed assuming ideal coupled lines, so that neither coupling or dispersion was included. To validate this approximation an electromagnetic simulation must be performed. Unfortunately, in a coplanar waveguide structure the metallisation...
Table 4.1 Impedance values and coplanar waveguide geometry

<table>
<thead>
<tr>
<th>UE nr.</th>
<th>Impedance (Ω)</th>
<th>Geometry (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Z_{OE}$</td>
<td>$Z_{OO}$</td>
</tr>
<tr>
<td>1</td>
<td>58.20</td>
<td>32.98</td>
</tr>
<tr>
<td>2</td>
<td>62.90</td>
<td>30.02</td>
</tr>
<tr>
<td>3</td>
<td>62.35</td>
<td>29.59</td>
</tr>
<tr>
<td>4</td>
<td>61.44</td>
<td>29.79</td>
</tr>
<tr>
<td>5</td>
<td>56.54</td>
<td>31.50</td>
</tr>
</tbody>
</table>

Figure 4.12 Ideal coupled lines versus Momentum.

almost completely covers the substrate, so a standard electromagnetic simulator will run out of computer power. HP’s planar simulator Momentum can model a slotline instead of the metallisation, where the results of the simulation can be seen in Fig. 4.12. Within the simulation tool we modelled the metallisation as a perfect conductor and at 3 GHz the grid was 30 copper wavelength on a LaAlO$_3$ substrate ($\varepsilon_r = 23.7$). Apparently the ideal coupled lines are a reasonably good approximation of the coplanar waveguide bandreject structure, although Momentum predicts a slightly larger bandwidth. We tried to simulate a structure without bonding wires, but this resulted in a worthless response. The coplanar waveguide design trades off the advantage of increased packing density and the technological complications with placement of short distance bonding wires.
4.3 Coplanar waveguide distributed line filters

Figure 4.13 Layout of the filter with bonding wires.

4.3.2 Filter fabrication

The superconductor YBa$_2$Cu$_3$O$_{7-\delta}$ was grown using a Lambda Physik Excimer laser. A 300 nm layer was grown on LaAlO$_3$ at 765°C and 750 mTorr. The laser power was 90 mJ at a repetition rate of 9 Hz. The sample was annealed one hour at 500°C in atmospheric pressure O$_2$. In situ a gold layer of 300 nm gold layer was deposited at room temperature. Standard photoresist (Shipley S1813) was used to define the pattern for both gold and LaAlO$_3$ layers. The gold was wet-etched, while dry-etching (Ar-ion milling) was used for the superconductor. Finally the sample was bonded using a Kajjo FB118CH hybrid goldwire bonder. The filter and an magnification of a small part of the filter with bonding wires can be viewed in Fig. 4.13. Not only the corners were bonded, but along the coupled lines two equally spaced extra bonding pads were defined to assure equal phase for both ground planes.

4.3.3 Measurement results

A calibration substrate was mounted on the cold finger and at room temperature a standard Line-Reflect-Match calibration was performed. Although at low temperatures the probes contact an extremely cold object the thermal resistance of the probe is high enough to avoid cooling of the probe or heating of the sample [32]. This allows room temperature calibration data to be used during the low temperature measurements.

The sample was mounted on the cold finger of the cryostat using silver paste. One of our standard procedures before cooling down is to pump over night, while simultaneously heating both cold finger and vacuum chamber. As soon as we start cooling we maintain our cold finger at 310 degrees Kelvin, until the temperature starts to drop due to the limited power of the heater. In this way we prevent condensation of water on our sample.
The cold wafer prober was cooled down to 35 Kelvin. We kept the sample at this temperature for 15 minutes to avoid temperature gradients. The filter measurement result is shown in Fig. 4.14. The shape of both traces is similar, demonstrating that Momentum can accurately simulate the behaviour of the slotlines. The influence of kinetic inductance was demonstrated by measuring at 77 Kelvin. The center frequency decreased to 2.74 GHz accompanied by an increase in insertion loss of 0.6 dB.

Separate measurements were carried out on a coupled line pair as depicted in Fig. 4.15. Because of the limited area of the sample, the measurement data was collected from 4 to 8 GHz. The model incorporates compensation for the difference in phase velocity. It would appear that the behaviour of a coupled line section can be accurately predicted. Small deviations are attributed to the room temperature calibration and the difference in dielectric constant of the superconductor substrate ($\varepsilon_r = 23.7$) and the calibration substrate ($\varepsilon_r = 9.8$). End-effects and the 90 degree angles at the input and output of the coupled line section were also neglected during the simulation.
4.4 Bandstop filter at 1.53 GHz

The success of the 3 GHz prototype augured well for the realisation of more complex filters. Increasing the number of coupled lines to 7, resulted in a skirt selectivity of 1.53, i.e. −40 dB down at 1.477 GHz. Furthermore, a symmetric circuit was selected to obtain equal input and output impedances. An extra coupled line is needed, so the inductance in the middle of the filter is distributed over the unit elements left and right of this inductance. In this way the 7 pole design requires 8 coupled lines. With a ripple of 0.1 dB, center frequency at 1.53 GHz, cut-off frequency at 1.45 GHz and 7 inductances and 8 unit elements the values depicted in Fig. 4.16 were obtained. Transformation of the unit element in series with an inductance result in odd and even impedances for the parallel coupled line sections. The values are presented in Table 4.2 (see also Fig. 4.11).
Table 4.2 Impedance values and coplanar waveguide geometry

<table>
<thead>
<tr>
<th>UE nr.</th>
<th>Impedance (Ω)</th>
<th>Geometry (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Z_{0e}$</td>
<td>$Z_{0o}$</td>
</tr>
<tr>
<td>1 = 8</td>
<td>58.865</td>
<td>32.695</td>
</tr>
<tr>
<td>2 = 7</td>
<td>62.81</td>
<td>29.61</td>
</tr>
<tr>
<td>3 = 6</td>
<td>63.59</td>
<td>29.07</td>
</tr>
<tr>
<td>4 = 5</td>
<td>52.92</td>
<td>31.22</td>
</tr>
</tbody>
</table>

4.4.1 Finetuning the response

Finite lithographic resolution

The masks were fabricated with a resolution of 1 µm, resulting in impedance values close to, but not exactly, the values derived from the prototype. Unfortunately there is no general rule for predicting the influence of limited resolution. But in the 8 coupled line filter the average deviation was 0.086 Ω for the even impedance and 0.038 Ω for the odd impedances. Simulations indicated that this increases the insertion loss in the passband at some ripple by less than 0.008 dB, a negligible amount.

Coupling lines and end effects

The coupling section of 350 µm 50Ω coplanar waveguide line contributed an additional 0.008 dB. In order to optimize the design, we can compensate for their connection by adding a highly inductive line between the coupled line sections and including this line in the transformation from a unit element in series with an inductance to a parallel coupled line section. In practice, the phase-shift due to this coupling section is so small (at 1.5 GHz the wavelength on LaAlO$_3$ is 55.8 mm, so that the line is $\sim 1/160$ of a wavelength), that the coupling has almost no influence on the transfer of the bandstop filter. Similarly to the lumped element design, the inductive connection is not broadband, so the result with this compensation was worse than the 'normal' coupling 50Ω coplanar waveguide line.

The next step is the reduction and inclusion of the end-effects of the coupled lines and coupling lines. Simulations were performed using the planar electromagnetic simulator Momentum, available via MDS. This simulator has successfully been used during the design of the 3 GHz bandstop filter. Several boundary conditions must be mentioned:

- The airbridge model within MDS is based on planar metalisation, not circular wires.
- We must resolve micrometer dependencies at frequencies with wavelengths on the order of 55.8 mm to optimize the signal transport at the 0.01 dB level, so that it remains questionable how accurate and how applicable the results are going to be. In addition, the exact geometry of the bonding wires is not known in advance.
4.4 Bandstop filter at 1.53 GHz

Figure 4.17 The package for the coplanar waveguide bandstop filter.

It was concluded that simulation of such effects would not be productive.

4.4.2 Measurements

Three filters have been fabricated and measured. The first two 1.5 GHz filters were fabricated from one 2-inch LaAlO$_3$ wafer coated with 300 nm of YBa$_2$Cu$_3$O$_{7-\delta}$, which was purchased from Cryoelectra, Wuppertal, Germany. The wafer was fabricated using a high pressure on-axis DC sputtering process. The wafer was cut into 4 pieces from which the first filter was fabricated using wet etching. Goldwire bonding was performed to ensure equal phasing of the ground planes using a Kajio FB118CH hybrid goldwire bonder. The filter was packaged in a copper box. Connection to ground was established by connecting the top cover to large ground strips on the substrate. A tapered structure facilitates connection of microstrip launchers to the coplanar waveguide signal lines. A conceptual drawing of the package can be seen in Fig. 4.17. After cooling down to 20 Kelvin the packaged filter was measured. The measurement result is shown in Fig. 4.18. Clearly visible is the reduced band rejection and the extra ripple near the stopband. Measured rejection of -28 dB is less than the simulated -40 dB and is believed due to direct and parallel plate coupling from input to output. The extra ripple may be due to underetching (estimated $\approx 1.5 \mu$m) of the YBa$_2$Cu$_3$O$_{7-\delta}$ layer. A very weak etch solution of Cl (0.04 % Vol.) was used. YBa$_2$Cu$_3$O$_{7-\delta}$ etches faster in the ab-plane than along the c-axis resulting in an underetch. [99] This underetch increases impedance values excessively, deteriorating the bandstop response and increasing ripple in the passband. The $S_{11}$ slightly exceeds 0.1 in the stopband, probably due to the use of a room temperature calibration. An accurate value for the phase velocity of the coplanar waveguide transmission line was not known in advance resulting in a frequency shift from 1.53 up to 1.56 GHz. To eliminate the influence of connectors and box the filter must be measured with the coplanar probes. Unfortunately the filter designed for probing was damaged during fabrication. In conclusion, to better approximate the design dimensions, dry etching should be carried out of the YBa$_2$Cu$_3$O$_{7-\delta}$ layer. The packaged filter should be redesigned, so that the in and output are placed diagonally on
the substrate and parallel plate modes are suppressed, see Fig. 4.19. The alternating distance between coplanar waveguide ground plane and the copper box results in a series of impedance steps reducing the contribution of the parallel plate mode.

The next filter was fabricated from the same wafer as the wet-etched one. This time dry etching was employed. Unfortunately, technology changes often lead to new and unexpected results. This time only the half wavelength resonator was successfully etched. Although optical inspection indicated that the whole wafer was etched successfully, subsequent room temperature
4.4 Bandstop filter at 1.53 GHz

DC-measurements demonstrated that a thin layer was still present, shorting the lines to ground. The substrate is continually rotated during dry-etching leaving the homogeneity of the film thickness as the most likely problem. The filters were positioned near the center of the wafer, so that it appears that the deposition of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ near the center is higher than near the edge with the sputtering process. Since dry etching is essential to get the right impedance values for the coupled lines, the next quarter wafer was also dry etched. Unfortunately the filter broke during packaging, leaving only the wafer-probe filter for measurement. The probed result is shown in Fig. 4.20. The measurement supports the thesis that the reduced bandwidth resulted from packaging and not from the filter itself. Two observations complicate the measurement's interpretation: the small peak in the middle of the stopband and the range behaviour of the $S_{22}$. The behaviour of the $S_{22}$ must be due to problems with probing. This could clarify the strong ripple in the transmission as well. Despite the fact that the phase velocity of the previous bandstop filter was used, the center frequency remained 30 MHz too high. From the same fabrication run a half wavelength resonator was measured as well. The phase velocity calculated from the measurement corresponded very well with the phase velocity of the filter, i.e. $N_{\text{eff}} = 3.56$. The film homogeneity must be adequate, because the half wavelength resonators (fabricated in both dry etch runs) showed almost the same phase velocity ($N_{\text{eff}}$ of 3.56 and 3.559). Apparently the wet-etching changes the phase velocity. Part of the difference can be explained due to the different geometry, because this changes the kinetic inductance slightly (see equation (2.29)). The 2% difference, however, is much larger than the change in kinetic inductance.

Figure 4.20 Wafer probe measurement of the 1.53 GHz bandstop filter.
The next filter run was made on a new wafer from Cryoelectra, Wuppertal, Germany. The quality of the gold adhesion to the superconductor was to be improved, since it appeared to be weak for the first Cryoelectra-wafer. After processing, it appeared that the gold adhesion had not improved at all. It was concluded, that the Wuppertal films were not suitable for bonding. The superconducting properties were excellent: at 5.5 GHz and 35 Kelvin, the measured quality factor was as high as 8800.

A last wafer was purchased from the group of professor Kinder, University of München, Germany. They use a molecular beam epitaxy technique and are able to grow superconducting thin films on 9" wafers of LaAlO$_3$, 30×30 mm$^2$ MgO substrates and 3" and 4" wafers of sapphire and GaAs [100]. All films have at least a transition temperature of 85 Kelvin and exhibit low surface resistance at microwave frequencies. They have the possibility of in-situ gold deposition, so the adhesion of the gold is expected to be much better than the Wuppertal wafers. Simulations with Momentum indicated that the characteristics changed, if bonding wires at the corners were not connected.

We have fabricated from the wafer five bandstop filters suitable for packaging, one bandstop filter suitable for wafer probe measurements and one half wavelength resonator. A photo of the wafer can be seen in Fig. 4.21.

The half wavelength resonator Kelvin revealed a surface resistance of 160 µΩ (scaled to 0 GHz) at 25 degrees Kelvin, London penetration $\lambda_0$ of 150 nm and $N_{eff}$ of 3.54213.

The measurements on the packaged filter and wafer probe filter can be seen in Fig. 4.22. For the packaged filter a line length of 13.908 mm resulted in a center frequency of 1.52 GHz. Using $N_{eff}$ from the half wavelength resonator measurement, a length of 13.920 mm corresponds to a quarter of a wavelength at 1.52 GHz. Apparently the half wavelength resonator is an excellent way to determine the propagation characteristics of a coplanar waveguide transmission line. A remarkable phenomena is the peak in the stopband in both packaged and wafer probe
Figure 4.22 Wafer probe and packaged filter measurement of the Kinder-wafer. The packaged filter had coupled line length of 13.908 mm, while the lines of the wafer probe filter were 400 μm shorter.

A similar peak is visible in the measurement in Fig. 4.20. A quasi-static analysis has been performed on the geometry of 27 coupled lines. The result did demonstrate a bandstop response, but not the peak in the stopband. The measurement on the five pole design (Fig. 4.14) showed a broader peak in the stopband. Possibly this peak becomes sharper if more coupled lines (from 5 to 8) are used. The distance between the coupled line sections can be increased to reduce the coupling. The extra dip in the passband for the wafer probe measurement is not understood.

Finally the packaged filter was tested one of the Westerbork antennas. Noise measurements were performed on two polarisations available in the front-end. One of the polarisations was equipped with the bandstop filter, while the other was directly connected to the antenna without filtering. Firstly, the telescope was aimed at a 'cold' part of the sky and the noise temperatures of both polarisations were measured. Subsequently the telescope was aimed at a source in the universe with a known spectral noise density. With this latter value the absolute values of the measurements were determined and are plotted in Fig. 4.23. This result merely confirms the measured bandstop characteristic of Fig. 4.22, but now on a noise temperature scale. The extra dip around 1350 MHz is attributed to the front-end and not to the filter. Remarkable is the reduced noise temperature above 1590 MHz measured on the channel with the filter. From the
Figure 4.23 Measured noise temperature as a function of the frequency. Clearly visible is the high noise temperature associated with bandstop frequencies. The gain stability of the measurement is given together with a schematic drawing of the measurement.

From the plot we infer that the filter does not deteriorate the performance of the telescope at frequencies of interest.

To check if the radio-astronomy site can take advantage of high $T_c$ filters, an interference source was positioned near the telescope aimed at the front end. At a frequency of 1575 MHz the source was transmitting 200 mW power and simultaneously the noise temperature was measured. The channel without the filtering exhibited a gain stability of approximately 15%, while the channel with the high $T_c$ filter maintained the normal gain stability of about 1.5%. It is noted that the reduction in gain stability occurred at the back-end of the measurement system and not in the front-end. Clearly the telescopes can benefit from the use of high temperature superconducting filters, which reduce the influence of interference sources near the radio astronomy bands without seriously influencing the sensitivity.
Appendix A

Deposition and patterning of high $T_c$ thin films

A.1 Introduction

In this appendix, we will describe the basic methods of growth and patterning of high $T_c$ thin films. The deposition techniques available at applied physics are pulsed laser deposition (PLD) and molecular beam epitaxy (MBE). Both systems have been used to study the physical properties and to fabricate devices in superconducting films [101][102][103][104]. Two other techniques are sputtering and metal organic chemical vapour deposition (MOCVD). Their operation principles will be presented in this chapter. Subsequently we will describe how high $T_c$ thin films are patterned.

A.2 Deposition

For both PLD and sputtering a sintered pellet of the material of the right composition is made. With PLD, a pulsed laser is used to shoot material from the pellet on a substrate. A sputtering system uses Ar-ions, which are accelerated towards the target. In this way material will come off and will be deposited on a substrate. Clearly, the stochiometry can only be varied by changing the target or changing the background oxygen pressure. Both MBE and MOCVD are more flexible systems than PLD and sputtering, because they have separate sources for and accurate tuning of the different elements to obtain the desired composition of the superconductor. However, to find the right parameters (like oxygen pressure, temperature, separate element tuning etc.) is a much more difficult task than PLD or sputtering systems. If the right parameters are found, the MBE and MOCVD are very well suited for increased growth capacity. With the MBE technique different elements are heated in crucibles with either e-guns or Knudsen
cells. Beams of molecules start to evaporate towards the substrate. With metal organic chemical vapour deposition a precursor can be spinned on the substrate. Subsequently, the substrate is brought into the vacuum chamber and heated up to the desired temperature. The metals (Y, Ba, Cu) are incorporated in an organic molecule that carries the metal through a gas flow to the substrate. On the substrate the metals become detached from the organic carrier through a chemical reaction. The organic material is pumped away, while the superconductor is formed on the substrate. The different systems are drawn in Fig. A.1.

Due to the simplicity of the PLD setup, this system became a popular way of growing high T<sub>c</sub> thin films. The diagram in Fig. A.1 shows schematically the way the laser removes material from the sintered pellet. In Delft a XeCl-eximer laser was set up in 1990 and modified in 1992. It contains a rotating multi-target holder (e.g. YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>−δ, SrTiO<sub>3</sub> and gold), a heated substrate holder and a shutter. The diameter of the targets is about 20 mm. Substrates are glued to the front side of the stainless steel heater block. During deposition, the substrate temperature is kept at 700-800 °C and can be controlled manually to within one degree. Typical parameters used for deposition are: 45° incident angle between laser beam and target surface, pulse repetition of 9 Hz, energy 2.5 J/cm<sup>2</sup>, substrate temperature of 760 °C and an oxygen pressure of 750 mTorr in the chamber. Cooling down takes places in oxygen at 1 bar for about 30 m-
utes. The deposition rate can only be determined after film growth. With the laser system, we have grown multilayer structures for quasi parallel plate waveguiding structures and overlap capacitors. With the parallel plate waveguide, we used MgO as an insulator, while SrTiO₃ was used for the overlap capacitor. For both structures the film roughness was a major problem. To optimize for film roughness takes a lot of time. At applied physics it was decided to stop high Tc in Delft, so we concluded that film growth should not be optimized within this research. The second system available in Delft, is molecular beam epitaxy. The system is equipped with ozone inlets for low pressure oxidation. Four ozone inlet tubes are pointing towards the substrate. Before deposition, the evaporations are independently tuned using either mass-spectrometres or a quartz crystal monitor. This latter quartz crystal is also used to calibrate the deposition rate of the superconducting compound. With a 1250 Watt quartz lamp, the temperature can be controlled within 5 °C up to 1000 °C. The elements Ba, Ca, Sm and Sr were in the Knolls and Au, Ce, Cu, Ti and Y were in the e-gun crucibles. After deposition, an ozone flux is maintained and the films are slowly cooled down to room temperature in about 30 minutes. Several type of high Tc thin films and multilayers have been grown with the MBE system. For microwave devices, it has been tried to grow YBa₂Cu₃O₇−δ on top of a sapphire substrate. Sapphire is one of the best microwave substrate available, but the material exhibits anisotropy, which makes it more difficult to apply. A BaTiO₃ template layer of about 25 nm was grown for an improved lattice match and to avoid interdiffusion of Al (sapphire is Al₂O₃). Some of the films showed transition temperatures of 83 Kelvin. However, to optimize film growth many films must be grown. Due to lack of time and other priorities, the growth on sapphire was stopped. The group of professor Kinder has demonstrated YBa₂Cu₃O₇−δ growth on large substrates made of LaAlO₃, MgO, silicon and GaAs [100]. They use CeO₂ and MgO buffer layers. With their set up, they can grow YBa₂Cu₃O₇−δ on 9” LaAlO₃ wafers or 12 2” wafers.

A.3 Patterning

Before any superconducting material is removed, photoresist is deposited (spinning) on the superconducting layer. With a Cr-mask a pattern is defined in the photoresist layer. After illumination with UV-light, the layer is developed. In this way certain parts of the superconducting layer are covered with resist. For patterning of high Tc thin films, essentially two methods can be used: 1) dry etching and 2) wet etching. The wet etching can be done with a weak acid, for example diluted HCl (0.1-0.2%) or a saturated water solution of ethylene diamine tetraacetic (EDTA). Unfortunately YBa₂Cu₃O₇−δ is extremely sensitive to acids, so many times a severe underetch was observed. It is also known, that YBa₂Cu₃O₇−δ etches faster along the ab-plane (parallel to the substrate), than in c-direction [99]. Almost all etchants are water diluted which will degrade the YBa₂Cu₃O₇−δ. So if line geometries must be defined carefully dry, anisotropic etching must be used. Argon ion beam milling is such a process. The principle is the same as the sputtering deposition technique, but now the target is replaced by the thin film. The Ar-ions are accelerated towards the film and superconducting material.
is removed at places where the film is not covered by photoresist. If the photoresist is thick enough, the film can be etched down to the substrate. The process is drawn schematically in Fig. A.2. The Ar-ion millin system at applied physics could etch YBa$_2$Cu$_3$O$_{7-\delta}$ films at a rate of $\sim$360 nm per hour. This system was not capable of etching substrates significantly larger than 10$\times$10 mm$^2$. A larger system was available at the Delft Institute of Micron and Submicrontechnologies (DIMES). However, this system etches much slower: a 300 nm film should be etched for 5 hours. In most cases, a gold layer was deposited on top of the superconductor. The gold was etched using a mixture of potassium iodide and iodine in water (200 gr. KI + 100 gr. I$_2$ in 400 ml H$_2$O). A more diluted solution must be used to reduce the etch rate.
Appendix B

The cryogenic microwave on-wafer measurement system

1. Introduction

At the start of the project, no possibility existed for high frequency measurement at low temperatures. A flexible measurement setup is built to perform measurements down to 10 Kelvin. Coplanar probes were selected to perform calibrated on wafer measurements. New high frequency coaxial vacuum feedthroughs were designed for operation up to 50 GHz. Precautions were taken to suppress the influence of vibrations of the closed cycle cryocooler. The specifications for the setup can be summarized as:

- Variable temperature measurements down to 10 Kelvin.
- Multiple measurements at one cool down.
- Microwave measurements up to 50 GHz.
- Cable and connector insertion loss: -1.5 dB 1 GHz and -14 dB at 40 GHz.
- Substrate size up to 40 mm.
- Range of micropositioners: ±5 mm in x,y,z-direction.

B. The vacuum system

To avoid heating due to convection, the cold finger must be placed inside a vacuum system. Apart from measurements on superconductors, noise measurements on semiconductors will be performed in the future. The electronics for noise measurement must be as close as possible
B. The cryogenic microwave on-wafer measurement system

Figure B.1 The hermetic coaxial feedthrough for operation up to 50 GHz. On the left is a schematic drawing of the feedthrough. On the right a photo of both feedthroughs mounted on the vacuum system. Also visible are the electrical feedthroughs for the heater and the temperature sensors.

to the device under test. This can be achieved by placing the micro-positioners inside the vacuum, so that the noise electronics can be mounted on the micro-positioner’s arm. In the base plate of the vacuum system, two holes were made: one for the cryocooler, the other for the turbo pump. Four standard flanges at 90° angle of each other were welded to the cylindrical vacuum chamber. The flanges facilitate a flexible way of mounting rotation and electrical feedthroughs. The top plate has an opening to enable a microscope’s view on the sample. A special feedthrough was fabricated using a hermetic glass bead from Omnispectra (M/A-com, U.S.A.). In Fig. B.1 is demonstrated how the glass bead is soldered in a copper holder. The holder can be clamped on a standard ISO-KF-16 flange.

B.3 Inside the vacuum

Coplanar probes were mounted on arms connected to Karl Süss micropositioners. The positioners can be controlled from outside the vacuum using rotation feedthroughs. The coplanar probes were thermally isolated from the arms using nylon and delrin spacers. Precision Tapes
B.3 Inside the vacuum

Figure B.2 Close up of the coplanar probes and the substrate holder. Clearly visible are the nylon pins, the leads for the heater and the planarity adjustment.

(C.S.A.) sells coaxial cables with high thermal isolation (but increased electrical attenuation). These cables facilitate a small heat-load on the cold-head and can be used up to 50 GHz.

The substrate holder has been connected to the cryocooler's cold head with braided copper wire and is held by nylon pins (see Fig. B.2). In this way a flexible but low-thermal resistance connection to the cold head was established. Unfortunately the cryocooler itself is rigidly connected to the vacuum chamber, so the measurements still suffer from vibrations. A better solution is to build a smaller vacuum system with the positioners outside the vacuum and use a rigid arm, connected with a bellows into the vacuum chamber, to mount the probes. Bellows should also be used for the cryocooler to isolate more effectively it's vibrations.

Recently pulse-tube system have become available producing much less vibrations than the present cryocooler. Depending on the base-temperature a pulse tube system might be a much better choice.

The temperature is measured using a standard PT-100 platinum resistor. A Lakeshore temperature controller can measure the temperature at two points, with various types of sensors. The output of the controller is connected to a 25 Watt heater. The heater consists of supply and return wires that are running in parallel, in this way minimizing the magnetic fields associated with high currents. The temperature can be controlled up to 100 Kelvin (not higher due to the limited power of the heater). An 80 Kelvin heat shield surrounds the cold finger. This reduces the heat load on the second stage of the cold finger.
B.4 Measurement

A sample is glued with silver paste on a special holder enabling rotation to guarantee proper alignment with the probes. The use of silver paste can not guarantee the planarity of the sample with respect to the probes. This planarity can be adjusted by tilting part of the arms on which the probes are mounted. Before pumping and cooling down, a 'Line-Reflect-Match' calibration (LRM) is performed at room temperature with a standard calibration substrate from Cascade. The vacuum chamber is closed (by placing the window holder on the top cover of the vacuum chamber) and both turbo and prevacuum pump are started. The vacuum chamber is heated up to ~ 40° C and the cold finger up to ~ 30° C. After one night of pumping and heating (12 hours) the cooler is started and after 90 minutes the minimum temperature of 20 Kelvin is reached. The heating procedure is performed to minimize the amount of condensation of water onto the substrate.

At the base temperature, a measurement is performed by positioning the coplanar probes on the substrate's contact pads. The network analyzer is set to step mode, single sweep. For short amount of time, the cryocooler is stopped, a measurement taken and subsequently, the cooler is started. A sweep takes roughly 3 seconds. By stopping the cryocooler, the vibrations are eliminated and in this way accurate measurements can be made. This procedure is repeated at the desired temperatures. One serious drawback of the setup, is damage of the contact pads caused by 'skating' probes: when the cryocooler is running, the probes move with respect to the substrate. If the measurement takes too long, the probes must be lifted and, as soon the temperature is reached, be lowered. In this way one can circumvent the destruction of the gold pads.
References


### Nomenclature

#### List of symbols

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
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<td>$t_0$</td>
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109
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<td>superconductivity</td>
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Summary

With the discovery of superconductivity at liquid nitrogen temperature, the cost of exploiting superconducting components is significantly lower than with low $T_c$ superconductors, where the material must be cooled below 10 Kelvin. One of the more prominent applications is the realisation of superconducting transmission lines in microwave subsystems. Due to the low loss and low dispersive nature at GHz-frequencies, the superconductor can be used in complex filter structures, delay lines and as high-speed interconnects.

In this thesis, the feasibility of superconducting microwave filters has been examined. Microwave filters can be made in different ways: very small (lumped) components or quarter wavelength transmission line segments. With superconductivity, lumped elements can be employed in high-order filters, without seriously affecting the insertion loss in the passband. Within this project several lumped components have been fabricated in high $T_c$ material, but they proved not to be suitable for wideband operation. Transmission line filters are less compact than the lumped element approach, but they can be used in wideband filters.

A bandstop filter has been designed to suppress signals close to radio astronomy bands. With the increasing demand by commercial users for more channels in the GHz-regime, the sensitive measurements of radio astronomers in Westerbork, the Netherlands, are currently endangered. In the cooled receiving front ends of radio telescopes, high $T_c$ superconductors can suppress interference from non-galactic sources. A highly complex filter can be used, while the insertion loss is reduced to a minimum. A parallel coupled line approach in coplanar waveguide technology was used to obtain the desired bandstop response. With a cryogenic wafer prober, build during the course of the project, an unpackaged filter was measured. A packaged filter was successfully tested at the radio astronomy site in Westerbork, the Netherlands.

With superconducting microwave transmission lines, the electromagnetic field penetrates into the superconductor over a typical length scale $\lambda_L$, the London penetration depth. Consequently, the propagation properties depend on this material dependent parameter. The penetration depth is frequency independent, in contrast to normal conductors, where the field penetrates over a frequency dependent length $\delta$, the skin depth. This results in lower dispersion in the case of superconducting transmission lines. In order to use superconducting transmission lines, the modelling should be tailored to facilitate accurate design of passive microwave circuits. At higher frequencies losses in the superconductor scale with the square of the frequency, while...
for normal conductors the losses scale with the square-root of the frequency. The highest frequency, for which superconductors offer significant improvements over normal conductors, is an important value. Both aspects, propagation characteristics and losses, are dealt with in this thesis. These parameters have been determined using half-wavelength resonators.
Samenvatting

Het ontdekking van supergeleiding bij de temperatuur van vloeibaar stikstof, zijn de kosten gekoppeld aan het gebruik van supergeleidende componenten aanzienlijk afgenomen. Vroeger was voor klassieke supergeleiders dure koeling van beneden 10 graden Kelvin noodzakelijk. Eén van de eerste toepassingen zijn supergeleidende componenten in microgolfsubsystemen. Door lage verliezen en weinig dispersie kan de supergeleider gebruikt worden in complexe filterstructuren, tijdvertragingslijnen en elektrische aansluitelementen geschikt voor hoge schakelsnelheden.

Dit proefschrift is gekeken naar de haalbaarheid van supergeleidende microgolffilters. Supergeleidende filters kunnen op verschillende manieren gemaakt worden: zeer kleine, discrete componenten of transmissielijnen met een lengte van een kwart golflengte. Complex filterstructuren zijn mogelijk, terwijl toch de verliezen acceptabel blijven. In het kader van het onderzoek zijn verschillende discrete componenten vervaardigd in hoge $T_c$ supergeleidend materiaal. De componenten bleken echter niet geschikt te zijn voor breedbandige ontwerpen. Transmissielijnfilters zijn minder compact dan discrete componentenfilters, maar kunnen gemakkelijker in breedbandige filterontwerpen worden toegepast.

Een bandstopfilter is ontworpen om storende signalen in de buurt van radio-astronomische banen te onderdrukken. Momenteel lopen radio-astronomische metingen in Westerbork gevaar door het toenemende gebruik van het elektromagnetische spectrum in het GHz-gebied. De gekoelde ontvangers in de radio-telescopen kunnen worden voorzien van hoge $T_c$ supergeleiders om storende bronnen, zoals mobiele telefoons en satellieten, te onderdrukken. Een complex filter met zeer weinig overdrachtsverliezen is mogelijk met supergeleiders. De zeer lage verliezen zijn noodzakelijk om de gevoelige metingen van de radio astronomen voort te kunnen zetten. Met behulp van parallel-gekoppelde lijnen is het bandstop-filter gerealiseerd. Met een cryogene wafer-prober, gebouwd tijdens het onderzoek, is het filter getest. Vervolgens is een ander filter, voorzien van een metalen behuizing voor montage in een radio-telescoop, in Westerbork gemeten.

Transmissielijnfilters dringen de elektromagnetische velden de supergeleider binnen over een materiaal-afhankelijke lengte $\lambda_L$, de London-indringdiepte. Dit resulteert in een propagatiesnelheid die afhankelijk is van deze indringing. De indringing is onafhankelijk van de frequentie in tegenstelling tot normale metalen. Dus om de supergeleider te kunnen gebruiken
in microgolfschakelingen, moeten de modellen aangevuld worden met de supergeleidende eigenschappen. Voorts zijn zelfs supergeleiders verliezend bij GHz-frequenties. Die verlieze schalen met het kwadraat van de frequentie, terwijl de verliezen in normale metalen met de wortel toenemen. Het is daarom van belang om te weten in welk frequentiebereik de lagere verliezen in de supergeleiders een voordeel bieden boven normale geleiders. Beide aspecten, propagatie en verliezen, zijn geanalyseerd in de context van dit onderzoek. Beide parameters zijn verkregen met behulp van resonatoren met een lengte van een halve golflengte.
Dankwoord


Ik wil Hans Mooij danken voor de mogelijkheid binnen VS-SG onderzoek te verrichten. Hoewel het woord 'toepassing' niet echt een dominante rol speelt, waardeer ik zeer sterk de manier waarop hij de groep leidt. Ook Gie Han Tan, van de radiosterrenwacht in Dwingeloo, dank ik voor de prettige samenwerking, de hulp vanuit Dwingeloo en de metingen in Westerbork.

Speciaal wil ik Bram Huis noemen, *de-man-met-de-gouden-handen*. Zonder hem had ik me nemen zaken niet uit kunnen voeren, waren zeer veel zaken te laat voor elkaar gekomen en waren bepaalde ideeën nooit praktisch uitvoerbaar geweest. De andere handige handen waren eveneens zeer belangrijk: Leo Lander voor de laser ablatie en hulp bij de cryopomp en vacuum- systemen (en daarbij vaak geholpen door Gerard van der Gaag) en Bram van der Enden voor het bonden, meedenken, zagen etc. etc. Gerrit van Dijk ben ik erkentelijk voor het lijmen van de connectoren op de supergeleider.


Voor het bonden ben ik Wim van der Vlist en David van der Weg zeer erkentelijk. De minimale afhaken die ze de bonddraden kunnen laten overbruggen waren van essentieel belang voor de filters. Speciaal de service van David, die de filters twee keer thuis kwam brengen, heb ik
zeer op prijs gesteld. Ook wil ik Marc O'Shea bedanken voor de snelle fabricage van maskers, zodat ik snel door kon gaan met lithografie.

Ik heb het erg interessant gevonden om de manier van werken bij VS-SG te kunnen vergelijken met de manier van werken bij MCG. Zelf geloof ik dat bij natuurkunde meer kwalitatief, dan kwantitatief wordt gewerkt. Die houding levert regelmatig zeer goede resultaten op, maar geen bandstop-filter met inzet verliezen kleiner dan 0.2 dB. De groep was bijzonder leuk en ik zag het waarderen dat er iets vaker achter het beeldscherm vandaan wordt gekomen dan bij MCG. Bij SG wil ik de samenwerking met Zi-Wen Dong en Vladimir Matijasevic noemen. Beide hebben mij veel geleerd over supergeleiders, zowel de groei als ook de eigenschappen. Van de niet-hoge-\text{T}_c'ers wil ik de samenwerking met Marco Matters op computergebied noemen. Hoeveel ellende wij niet 'op' het netwerk vonden...

De prettige werksfeer in en om MCG en de uitstekende meetapparatuur waren belangrijke factoren in het slagen van dit onderzoek. Speciaal in MCG-context wil ik Koen Mouthaan, Frank van Vliet en Wouter Pieper bedanken; Koen voor zijn ondersteuning op filter en modelleringsgebied en Frank en Wouter voor het meedenken op zowel microgolftechnisch, als ook op 'maatschappelijk' gebied. Voor Unix en \texttt{X} men ik Xavee Leijtens zeer dankbaar. Hoewel computer-vragen hij op een dag ook krijgt, hij blijft even geduldig zonder te vaak 'RTFM' te roepen. Vooral zijn hulp bij Linux was perfect.

Ten slotte wil ik Mike Helderman en Tim van Eekelen bedanken voor hun bijdrage aan het onderzoek. Behalve dat zij een deel van het onderzoek hebben verricht, was het ook erg leuk en leerzaam om twee studenten te kunnen begeleiden tijdens hun afstuderen.