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Measurements of the flow field in a rotating annular flume

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FLOW FIELD IN A
ROTATING ANNULAR FLUME

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ABSTRACT

Annular flumes are used for studying erosion and/or deposition of cohesive sediments in the laboratory. These flumes have advantages over straight recirculating flumes in that effects of inflow and outflow conditions are avoided and that there are no pumps which break down the suspended flocculated sediment. A disadvantage of annular flumes is that, because of the curvature secondary flow, velocities are generated in the flowing water, yielding a complex 3-D flow field that is not well known up to now.

The present report describes experiments in a rotating annular flume, i.e. an annular flume in which not only the top lid rotates but the flume itself as well. The investigation is aimed at the accurate optimization of the ratio of rotational speeds of top lid and flume. Two different optimizing criteria were considered. The first criterion (criterion I) is the minimal intensity of secondary flow (especially in the lower part of the flume) and the second criterion (criterion II) a uniform distribution of tangential velocity and of near bottom shear stress over the flume width.

The measurements were done in a rotating annular flume with a mean diameter of 3.7 m. Flow velocity measurements were done with a laser-Doppler velocimeter. The water depth, the rotational speeds of top lid and flume and the top lid width were varied.

Minimal secondary flow circulations near the bottom of the flume are found to occur in conjunction with nearly uniform distributions, in the radial direction, of near bottom shear stresses. The investigation also shows that in situations where the near-bottom secondary flow circulations are minimal, the remaining secondary flow velocities are generally not small compared with cohesive-sediment fall velocities. This has implications for future studies of erosion, flocculation and deposition of cohesive sediments in rotating annular flumes.

Another topic discussed is the generalization of the results obtained from the experiments. Conditions for (strict or only geometrical) similarity of flows in carousels are investigated. In case of similar flows, relations between optimum conditions can be deduced. An experimental verification of these relations is presented.

A simplified analysis of the flow in carousels is given. This analysis provides a method to obtain several relations for carousel flow. With these relations it is possible to estimate the bottom shear stress and to compare non-similar carousel flows, for example. The relations obtained in this way are confronted with the experimental evidence.
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1. INTRODUCTION

1.1 ROTATING ANNULAR FLUMES OR CAROUSELS

Annular flumes are used for studying erosion and/or deposition of cohesive sediments in the laboratory. In these flumes the flow is driven by a rotating top lid. Annular flumes have advantages over straight recirculating flumes in that effects of inflow and outflow conditions are avoided (the entire flow is contained over the bed), and that there are no pumps which break down the suspended flocculated sediment. However, a disadvantage of annular flumes is that due to the curvature secondary flow velocities are generated in the flowing water, yielding a complex 3-D flow field instead of a 2-D one as assumed in many erosion/deposition conceptions. The secondary flow is caused by the differences of the tangential flow velocity over the cross-section. The fluid with higher tangential velocity near the lid is driven away from the axis of rotation and a secondary flow circulation in the cross-section is created. The secondary flow circulations can be counteracted by rotating the flume (i.e. bottom and side walls) and the top lid in opposite directions as shown by Partheniades et al. (1966) in a pioneering paper. In such a rotating annular flume (or carousel) an optimum value for the ratio \( \omega_t/\omega_b \) of the rotational speeds of top lid (\( \omega_t \)) and bottom (\( \omega_b \)) has to be determined before starting the cohesive sediment experiments.

The present investigation is aimed at the optimization of the ratio \( \omega_t/\omega_b \) in order to get minimal secondary circulations in a cross-section, in particular in the lower part of the cross-section, (criterion I, important for deposition studies, Mehta and Partheniades (1973a, 1973b)) or the optimization of the ratio \( \omega_t/\omega_b \) in order to get uniform distributions of tangential velocity and near bottom shear stress across the flume width (criterion II, important for erosion studies, Mehta and Partheniades (1979), Parchure and Mehta (1985)). Generally it is assumed that the secondary flow velocities in the optimum situation are small enough to use the 2-D erosion/deposition conceptions. As the fall velocities of silt and cohesive sediment ranges from 10 mm/s, say, to many orders of magnitude smaller, this assumption should be checked. This is made possible by the detailed measurements in this investigation.

The measurements were executed in the rotating annular flume of the Faculty of Civil Engineering of Delft University of Technology (see Fig. 1.1). The velocity field was measured with a backscatter laser-Doppler velocimeter system. Detailed measurements of tangential velocity, vertical velocity and near bottom shear stress were carried out for different values of water depth, rotational speeds of top lid and bottom and for two top lid widths.
An alternative for carrying out measurements would be the use of a computer model. To obtain the desired detailed knowledge of the flow field from flow computations requires an advanced turbulence model because of the curvature of the flow and its influence on the turbulent structure and turbulent transports. Work on this modelling is in progress and will be reported on later. The present experimental results will be used for validation of the model.

The rotating annular flumes in use vary with respect to their radii, \( R \), and flume widths, \( b \). To be able to predict the flow in other situations or carousels the possibility of generalization of the results obtained from the experiments described is investigated.

Fig. 1.1  Rotating annular flume of Delft University of Technology.
1.2 PREVIOUS INVESTIGATIONS

First, results obtained in previous investigations are summarized. In none of these investigations secondary flow circulations in a cross-section were measured in detail.

The first investigators who used a rotating annular flume for measuring the erosion and deposition of cohesive sediment were Partheniades et al. (1966). As described before, Partheniades et al. (1966) showed that the simultaneous rotation of top lid and bottom in opposite directions was necessary to minimize secondary circulations in their carousel (mean diameter 0.82 m, width 0.10 m, depth 0.30 m) and to cause sediment to deposit and erode uniformly across the flume width.

Mehta and Partheniades (1973a, 1973b) used a flume of somewhat larger dimensions (mean diameter 1.50 m, width 0.20 m, depth 0.46 m) and determined the optimal ratios $\omega_f/\omega_b$ for different values of water depth and $\omega_b$ by observing visually the displacement of plastic beads with a density slightly larger than the water density. Mehta and Partheniades (1973a, 1973b) argued that secondary circulations in a cross-section were minimal when these beads moved along the centre line of the carousel bottom. Their optimal values for $\omega_f/\omega_b$ are shown in

![Graph showing optimal ratios $\omega_f/\omega_b$ for rotating annular flume of University of Florida (Mehta and Partheniades, 1973a, 1973b).](image)

*Fig. 1.2 Optimum ratios $\omega_f/\omega_b$ for rotating annular flume of University of Florida (Mehta and Partheniades, 1973a, 1973b).*
Fig. 1.2. Some typical values for $\omega_b \geq 1.5 \text{ rpm}$ (rotations per minute) are: $\omega_l/\omega_b = -3.1$ for a height of the water column (or water depth) $h = 30 \text{ cm}$, $\omega_l/\omega_b = -2.5$ for $h = 20 \text{ cm}$ and $\omega_l/\omega_b = -2.3$ for $h = 15 \text{ cm}$.

![Diagram of flow components](image)

**Fig. 1.3** Tangential flow velocity component $u$, radial flow velocity component $v$ and vertical flow velocity component $w$ in a rotating annular flume.

Sheng (1989) calculated the secondary flow velocities $v$ and $w$ (see Fig. 1.3 for definitions) in an annular flume by using an integral boundary layer model and measured the tangential velocities $u$. By examining the cause of secondary flow, Sheng (1989) recognized the possibility of a two-cell secondary flow in a rotating annular flume, see Fig. 1.4.

![Diagram of secondary flows](image)

**Fig. 1.4** Secondary flows in a rotating annular flume according to Sheng (1989).
Karelse (1990) measured, in the rotating annular flume of Delft Hydraulics (mean diameter 2.10 m, width 0.20 m, depth 0.30 m), the tangential velocity $u$ and radial velocity $v$ just above the bottom with an electromagnetic current meter placed in a point at the centre line of the flume bottom. Varying $\omega_1/\omega_b$, Karelse (1990) found distinct zero-crossings of the time-averaged radial velocity $\bar{v}$ at $\omega_1/\omega_b = -2.7$ for $h = 30$ cm and at $\omega_1/\omega_b = -2.3$ for $h = 23$ cm. An example of Karelse’s (1990) data is shown in Fig. 1.5 (where the overbar in $\bar{u}$ and $\bar{v}$ denotes time-averaging, $u'$ and $v'$ are the turbulent velocity fluctuations and $U_1$ is the velocity difference between top lid and bottom in the centre of the flume. The optimal ratios $\omega_1/\omega_b$ found with the electromagnetic current meter agree well with the values following from the application of the visual method of Mehta and Partheniades (1973a, 1973b) in the carousel of Delft Hydraulics.

![Graph](image)

**Fig. 1.5** Variation of $\bar{u}/U_1$, $\sqrt{\langle u'^2 \rangle}/\bar{u}$, $\sqrt{\langle v'^2 \rangle}/\bar{u}$ and $\bar{v}/\bar{u}$ as functions of the ratio $\omega_1/\omega_b$ for $h = 0.30$ m and $\omega_1 = 10$ rpm measured by Karelse (1990) in the carousel of Delft Hydraulics.

The National Water Research Institute in Burlington, Ontario, Canada has recently constructed a very large rotating annular flume (mean diameter is 5.0 m, width is 0.30 m, depth is 0.30 m). Using laser-Doppler velocimetry Krishnappan (1991) measured flow velocity components in tangential (see Fig. 1.6) and vertical directions for two situations, one where $\omega_i = -2.0$ rpm and $\omega_b = 0$ and a second where $\omega_i = -1.0$ rpm and $\omega_b = 1.0$ rpm, both with a water depth $h = 15.5$ cm. With a Preston tube Krishnappan (1991) measured distributions of bed friction velocity for the above situations and for values of $\omega_1/\omega_b$ of -1.17
and $-1.25$ and water depths ranging between 0.10 m and 0.20 m. Unfortunately Krishnappan (1991) did not determine optimal values for $\omega_t/\omega_b$ explicitly and did not execute accurate measurements of the flow circulations in a cross-section.

![Tangential velocity $\bar{u}$ [cm/s]](image)

$\omega_t = 2.0$ rpm  
$\omega_b = 0$

Tangential velocity $\bar{u}$ [cm/s]

![Tangential velocity $\bar{u}$ [cm/s]](image)

$\omega_t = 1.0$ rpm  
$\omega_b = -1.0$ rpm

Tangential velocity $\bar{u}$ [cm/s]

Fig. 1.6 Krishnappan's (1991) semi-log plots of tangential velocity distributions $\bar{u}$ over a number of verticals (vertical 1 is near the inner wall, vertical 9 is near the outer wall).

Hydraulics Research Ltd in Wallingford, England has an annular flume, which is stationary, see Graham et al. (1992). The mean diameter of this flume measures 5.6 m, the width is 0.40 m and the depth is 0.35 m. Graham et al. (1992) measured the velocity field (with laser-Doppler velocimetry) and side-wall and bed shear stresses (with hot-film stress probes) in this flume and compared the data with numerical predictions (using the computational fluid
dynamics program HARWELL-FLOW3D). Very good agreement was found. Graham et al. (1992) also compared the bed shear stress predictions with experimentally obtained contours of eroded sediments. The non-uniform erosion across the width in their flume shown in Fig. 1.7 indicates that stationary annular flumes are not so appropriate for studies of erosion and deposition of fine sediment.

![Graph showing depth of eroded material vs. distance from outer wall for different RPMs.](image)

**Fig. 1.7** Experimental eroded bed profiles for Fawley mud measured by Graham et al. (1992).
2. GENERAL CONSIDERATIONS ON THE FLOW IN CAROUSELS

2.1 REYNOLDS EQUATIONS IN A CYLINDRICAL CO-ORDINATE SYSTEM

The rotating annular flumes in use vary with respect to their radii, \( R \), and flume widths, \( b \). To execute a set of measurements as reported here for different water depths, \( h \), and rotational speeds, \( \omega_t \) and \( \omega_b \), for a rotating annular flume is very demanding and often not possible. The generally desired detailed knowledge of the flow field could in principle also be obtained from flow computations. At first sight the geometrical configuration looks very well suited for flow computations. However, because of the curvature of the flow and its influence on the turbulent structure and transports, an advanced turbulence model which describes the behaviour of the Reynolds stresses in curved flow, for example a Reynolds stress model, is required (Leschziner, 1993). Ample validation of the used model for the concerned flow based on experimental results is necessary. Work in this direction is in progress and will be reported on later. Here we try to derive ways to generalize the results obtained from the experiments by inspection of the flow equations and use of general knowledge of the flow along moving walls.

![Co-ordinate system](image)

**Fig. 2.1** Co-ordinate system.

The appropriate co-ordinate system for the description of the flow in a rotating annular flume is the cylindrical (polar) co-ordinate system, see fig. 2.1. The continuity equation and the
momentum equations (Navier Stokes equations) in the cylindrical co-ordinate system read for steady axisymmetric flow (Aris, 1962, and Batchelor, 1967)

\[
\frac{\partial r v}{\partial r} + \frac{\partial w}{\partial z} = 0
\]  
(1)

\[
\frac{\partial r u}{\partial r} + \frac{\partial u w}{\partial z} + \frac{u v}{r} - v \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right) - f_\phi
\]  
(2)

\[
\frac{\partial r v^2}{\partial r} + \frac{\partial w}{\partial z} - \frac{u^2}{r} + \frac{\partial p/\rho}{\partial r} - v \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right) - f_r
\]  
(3)

\[
\frac{\partial r u w}{\partial r} + \frac{\partial w^2}{\partial z} + \frac{\partial p/\rho}{\partial z} - v \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) - f_z
\]  
(4)

where \( \phi, r \) and \( z \) are the non-rotating cylindrical co-ordinates in tangential, radial and vertical direction (see fig. 2.1); \( u, v \) and \( w \) are the corresponding velocity components; \( \rho \) the fluid density, which is assumed constant here, and \( \nu \) the kinematic viscosity; \( f_\phi, f_r \), and \( f_z \) are the components of external body forces per unit of mass in \( \phi, r \) and \( z \) direction. All three components \( f_\phi, f_r, \) and \( f_z \) can be taken zero for the carousel flow if the gravity is included in the pressure terms: \( p \) is then the deviation from the hydrostatic pressure \( p = P - \rho g z \), where \( P \) is the fluid pressure.

Decomposition of variables in time-averaged values, denoted by an overbar, and turbulent fluctuations, denoted by a prime, for example

\[
u = \overline{\nu} + \nu'
\]  
(5)

and averaging equations 1-4 over time yields the time-averaged continuity equation and the Reynolds equations (Hinze, 1975)

\[
\frac{\partial r \overline{v}}{\partial r} + \frac{\partial \overline{w}}{\partial z} = 0
\]  
(6)

\[
\frac{\partial r \overline{u} \overline{v}}{\partial r} + \frac{\partial \overline{u} \overline{w}}{\partial z} + \frac{\overline{u} \overline{v}}{r} - v \left( \frac{\partial^2 \overline{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \overline{u}}{\partial r} + \frac{\partial^2 \overline{u}}{\partial z^2} - \frac{\overline{u}}{r^2} \right) + 
\]

\[
+ \frac{\partial r q_{\phi r}}{\partial r} + \frac{\partial q_{\phi z}}{\partial z} + \frac{q_{\phi r}}{r} = 0
\]  
(7)
\[
\frac{\partial r v v}{\partial r} + \frac{\partial v w}{\partial z} - \frac{u u}{r} + \frac{\partial p / \rho}{\partial r} - \nu \left( \frac{\partial^2 v}{\partial r^2} + \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2} \right) + \\
+ \frac{\partial r q_{rr}}{\partial r} + \frac{\partial q_{rz}}{\partial z} - \frac{q_{\phi \phi}}{r} = 0
\] (8)

\[
\frac{\partial r v w}{\partial r} + \frac{\partial w w}{\partial z} + \frac{\partial p / \rho}{\partial z} - \nu \left( \frac{\partial^2 w}{\partial r^2} + \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) + \\
+ \frac{\partial r q_{rz}}{\partial r} + \frac{\partial q_{zz}}{\partial z} = 0
\] (9)

Here the turbulence stresses per unit mass, \(q_{\phi \phi}\) etc., are defined by

\[
q_{\phi \phi} = u' u' ; \quad q_{rr} = v' v' ; \quad q_{zz} = w' w' \\
q_{\phi r} = u' v' ; \quad q_{rz} = v' w' ; \quad q_{r \phi} = w' u'
\] (10)

Assumption of gradient type turbulent momentum transport and a turbulent eddy viscosity \(\mu_t\) that is independent of the transport and momentum direction is not correct in curved flow, in particular not in the strongly curved flow in carousels.

Appropriate boundary conditions for a carousel flow are:
- at the lid

\[
u = r \omega , \quad v = w = 0 \quad \text{and so} \quad \bar{u} = r \omega , \quad \bar{v} = \bar{w} = 0
\] (11)

- at the bottom and sidewalls

\[
u = r \omega , \quad v = w = 0 \quad \text{and so} \quad \bar{u} = r \omega , \quad \bar{v} = \bar{w} = 0
\] (12)
2.2 STRICT SIMILARITY

The flow in a carrousel is completely determined by the characteristic quantities \( \omega_b, \omega_f, b, h, R, \nu \), where \( R \) is the mean radius of the flume, and \( b \) and \( h \) are its width and the water depth, respectively. Dimensional analysis shows that similar flows in (different) carrousels require identical values of the following non-dimensional numbers:

- \( N_f = \frac{\omega_f}{\omega_b} \), the ratio of the rotational velocities;
- \( N_2 = \frac{h}{b} \), the aspect ratio of the flume;
- \( N_3 = \frac{b}{R} \), the relative curvature of the carrousel;
- \( N_4 = \frac{\omega_b R b}{\nu} \), a Reynolds number of the carrousel flow. Here \( \omega_b R \) is a characteristic velocity and \( b \) a characteristic length.

Identical values of \( N_2 \) and of \( N_3 \) imply that similar flows can only be found in geometrically similar carrousels. Identical values of \( N_f \) and of \( N_4 \) mean that identical ratios of rotational speeds are required together with equal Reynolds numbers. The pressure terms in the momentum equations do not lead to extra requirements as the pressures over the flume adapt automatically.

The similarity of the carrousel flows can be understood from non-dimensional equations. These can be constructed by writing all variables in equations (6-9) as a product of a scaling constant and a non-dimensional variable. The scaling constant used for all lengths is \( b \), the scaling constant for all velocities, included the turbulent velocities in the Reynolds stresses, is \( \omega_b R \), for example

\[
r = b r \text{ and } \vec{v} = \omega_b R \vec{v}
\]

Here non-dimensional variables are notated by bold symbols. Dividing the continuity equation (6) by \( \omega_b \), the Reynolds equations (7-9) by \( \omega_b^2 b \) and the boundary conditions (11-12) by \( \omega_b b \) yields a set of non-dimensional equations. The form of the non-dimensional continuity and Reynolds equations is exactly equal to equations (6-9) but in non-dimensional (bold) variables. Only the inverse of the Reynolds number \( N_4 \) appears instead of the viscosity \( \nu \), which shows that the non-dimensional equations are exactly the same for carrousel flows with identical Reynolds number. The other requirements come from the non-dimensional boundary conditions which read

\[
\text{lid: } \quad \left[ \vec{u} \right]_{z=(h+z_b)/b} = -\frac{\omega_f}{\omega_b} \frac{b}{R} \vec{r} \quad , \quad \left[ \vec{v} \right]_{z=(h+z_b)/b} = \left[ \vec{w} \right]_{z=(h+z_b)/b} = 0 \tag{14}
\]
bottom: \[ \begin{bmatrix} \bar{u} \end{bmatrix}_{z = z_b / b} = \frac{b}{R} r, \quad \begin{bmatrix} \bar{v} \end{bmatrix}_{z = z_b / b} = \begin{bmatrix} \bar{w} \end{bmatrix}_{z = z_b / b} = 0 \quad (15) \]

inside wall: \[ \begin{bmatrix} \bar{u} \end{bmatrix}_{r = r_i / b} = 1 - \frac{1}{2} \frac{b}{R}, \quad \begin{bmatrix} \bar{v} \end{bmatrix}_{r = r_i / b} = \begin{bmatrix} \bar{w} \end{bmatrix}_{r = r_i / b} = 0 \quad (16) \]

outside wall: \[ \begin{bmatrix} \bar{u} \end{bmatrix}_{r = (r_i + b) / b} = 1 + \frac{1}{2} \frac{b}{R}, \quad \begin{bmatrix} \bar{v} \end{bmatrix}_{r = (r_i + b) / b} = \begin{bmatrix} \bar{w} \end{bmatrix}_{r = (r_i + b) / b} = 0 \quad (17) \]

Here \( z_b \) and \( r_i \) are the co-ordinates of the bottom and the inside wall of the flume, respectively. The boundary conditions correspond exactly for carousel flows with geometrical similarity and equal ratio of rotational velocities, that is for identical \( N_i = \omega_i / \omega_b, N_2 = h / b \) and \( N_3 = b / R \).
2.3 NEGLECTING THE REYNOLDS-NUMBER DEPENDENCE DUE TO THE VISCOUS SHEAR STRESSES

The strict similarity discussed in the previous paragraph gives only very limited possibility to compare carousel flows. If the condition of equal Reynolds number $N_x$ connected with the molecular viscosity $\nu$ could be dropped, for example, this would lead to a decoupling of the rotational velocities and the length measures. The molecular viscosity $\nu$ plays a role in the viscous sublayer directly along the walls only. The thickness, $\delta_v$, of this viscous sublayer which is about $\delta_v = 5 \nu / u_*$, where $u_*$ is the friction velocity, is generally of the order of .01 mm to .05 mm for carousel flows. At distances larger then .06 mm to .30 mm from the wall the viscous shear stresses can be completely neglected yielding Reynolds equations of the form

$$\frac{\partial r\bar{u}v}{r\partial r} + \frac{\partial \bar{u}w}{\partial z} + \frac{\bar{u}v}{r} + \frac{\partial q_{\varphi r}}{r\partial r} + \frac{\partial q_{\varphi \varphi}}{\partial z} + \frac{q_{\varphi r}}{r} = 0$$

(18)

$$\frac{\partial r\bar{v}v}{r\partial r} + \frac{\partial \bar{v}w}{\partial z} - \frac{\bar{u}u}{r} + \frac{\partial \bar{p}/\rho}{\partial r} + \frac{\partial q_{rr}}{r\partial r} + \frac{\partial q_{rz}}{\partial z} - \frac{q_{\varphi \varphi}}{r} = 0$$

(19)

$$\frac{\partial r\bar{w}w}{r\partial r} + \frac{\partial \bar{w}w}{\partial z} + \frac{\bar{p}/\rho}{\partial r} + \frac{\partial q_{rz}}{r\partial r} + \frac{\partial q_{zz}}{\partial z} = 0$$

(20)

The region where the viscous stresses play a role can be left out of consideration if for the boundary conditions the law of the wall is used

$$\frac{\bar{v} - v_w}{u_*} = \frac{1}{\kappa} \ln \left( \frac{a}{a_0} \right)$$

(21)

Here $\kappa$ is the Von Karman constant, $a$ is the distance to the wall, $a_0$ the roughness length of the wall, $v$ is the velocity at the distance $a$ from the wall in the local flow direction and $v_w$ is the velocity of the wall in the same direction. For a hydraulically rough wall $a_0$ depends only on the wall structure. If the wall structure elements should scale with the other length measures of the carousel strict similarity would apply again.

Most carousels, however, have hydraulically smooth walls on all sides. For example, for the carousel used in the measurements described here, glass sidewalls had to be used to allow laser-Doppler measurements. For hydraulically smooth walls a wall Reynolds number $u_* a_0 / \nu = .11$ determines the roughness length $a_0$ and therewith viscosity still plays a role in
the flow problem, now via the roughness length in the boundary conditions. In practice the influence of the roughness length on the carousel flow is very small. The thickness of the turbulent boundary layer along the wall, \( \delta_w \), can be shown to be of the order of one tenth to one half times the width or the height of the flume. The thickness of the inner turbulent wall region in the boundary layer in which equation (21) applies is about \( 1/6 \delta_w \). The ratio between this thickness and the roughness length \( \delta_w/6a_0 \) is of the order of five thousand. The influence of variations of the friction velocity or of the flume measures on the logarithm of this value is small. Realistic variations in the values of those variables between carrousels are generally below a factor of 2 to 3, which lead to \( \ln(\delta_w/6a_0) \approx 8.5 \pm 1 \). Equation (21) shows that these small variations cause only small variations in the relation between flow velocity and friction velocity. Neglect of these small variations leads to boundary conditions of the following form. (The first boundary conditions given are those at the lid.)

\[
\begin{align*}
[\bar{u}]_{z=(h+z_{b,0})/b} &= \frac{\omega}{\omega_b} \frac{b}{R} r - cu_* \cos \psi, \\
[\bar{v}]_{z=(h+z_{b,0})/b} &= -cu_* \sin \psi, \\
[\bar{w}]_{z=(h+z_{b,0})/b} &= 0
\end{align*}
\]  

(22)

The (nearly) constant \( c = \kappa \ln(a/a_0) \). It's value depends on \( a \), the distance to the wall of the place where the boundary condition is applied, which should be rather small. The variable \( \psi \) is the angle between the local flow direction and the direction tangential to the flume. As \( \psi \) is small (of the order of \( 5^\circ \)), \( \cos \psi \approx 1 \) and \( \sin \psi \approx \bar{v}/\bar{u} = \bar{v}/\bar{u} \) can be substituted in equation (22), yielding

\[
\begin{align*}
[\bar{u}]_{z=(h+z_{b,0})/b} &= \frac{\omega}{\omega_b} \frac{b}{R} r - cu_* \cos \psi, \\
[\bar{v}]_{z=(h+z_{b,0})/b} &= -cu_* \frac{\bar{v}}{\bar{u}}, \\
[\bar{w}]_{z=(h+z_{b,0})/b} &= 0
\end{align*}
\]  

(23)

and at the bottom

\[
\begin{align*}
[\bar{u}]_{z=(z_{b,0})/b} &= \frac{b}{R} r + cu_* , \\
[\bar{v}]_{z=(z_{b,0})/b} &= cu_* \frac{\bar{v}}{\bar{u}} , \\
[\bar{w}]_{z=(z_{b,0})/b} &= 0
\end{align*}
\]  

(24)

The friction velocity \( u_* \) in equations (23) and (24) can be expressed in the shear stresses near the lid and the bottom in the following way

\[
\begin{align*}
\frac{u_*}{\bar{u}} | u_* | &= \sqrt{\left[ \frac{u'w'}{\bar{u}} \right]^2 + \left[ \frac{v'w'}{\bar{u}} \right]^2} \approx -\frac{u'w'}{\bar{u}} \cos \psi = -\frac{u'w'}{\bar{u}}
\end{align*}
\]  

(25)
For the sidewalls analogous expressions apply

inside wall: \[ \bar{u}_{r-(r+\delta r)/a}/b = 1 - \frac{1}{2} \frac{b}{R} + cu_*, \quad \bar{v}_{r-(r+\delta r)/a}/b = cu_* \frac{\bar{w}}{\bar{u}}, \]

\[ \left[ \bar{w} \right]_{r-(r+\delta r)/a}/b = 0 \] \hspace{1cm} (26)

outside wall: \[ \bar{u}_{r-(r+\delta r-b)/a}/b = 1 + \frac{1}{2} \frac{b}{R} - cu_*, \quad \bar{v}_{r-(r+\delta r-b)/a}/b = -cu_* \frac{\bar{w}}{\bar{u}}, \]

\[ \left[ \bar{w} \right]_{r-(r+\delta r-b)/a}/b = 0 \] \hspace{1cm} (27)

where the friction velocity \( u_* \) is now expressed in the shear stresses along the inside and outside wall \( u_* \mid u_* \mid = -u'v'/u \). Similarity of carousel flow based on equations (6) and (18-20) and boundary conditions (23-24) and (26-27) does not require equal Reynolds numbers \( N_d \) any more, but only geometrical similarity and equal ratio of rotational velocities.

Boundaries for the applicability of similarity for carousel flows in this case are not easy to give. For similarity of two flows equal values of \( c \) are required. Very close to the wall there will always be an appreciable influence of the dimension of the carousel on the value of \( c \) for equal relative distances from the wall (equal \( a/h \) or \( a/b \)). However, assume for example that similarity (to a sufficient approximation) of carousel flows is desired over the complete cross-section except for a small layer, 1/20 \( h \) thick, along the walls. A difference of, say, 10% in the values of \( c \) is accepted at \( a = h/20 \). The ratio \( a/a_0 \) in the expression for \( c = \kappa \ln(a/a_0) \), can be related to a Reynolds number of the carousel flow \( hU_1/\nu \), where \( U_1 \) is the velocity difference between (the centres of) the lid and the flume of the carousel.

Obtaining \( a_0 \) from \( u_* a_0/\nu = .11 \) for hydraulically smooth walls, in which expression \( u_* = 40 U_1 \) can be used as \( \delta_1/6a_0 \) is of the order of 5000 for most carousels, yields the relation \( a/a_0 = .0114 hU_1/\nu \) for \( a = h/20 \). For example, in the standard case in this investigation (see paragraph 3.2) the Reynolds number is \( hU_1/\nu = 5.4 \times 10^5 \), leading to \( a/a_0 = 6.1 \times 10^3 \) and \( c = 8.7 \). A difference of 10% in \( c \) then allows a difference in the Reynolds numbers of the carousel flows of a factor of about 2.2. A carousel flow with a Reynolds number between \( 2.4 \times 10^5 \) and \( 1.3 \times 10^6 \) is then similar (in the desired approximation) to the standard case of this investigation, when geometrical similarity and equal ratio of rotational velocities between the carousel flows apply. Even larger differences in Reynolds numbers will not lead to large differences in \( c \) and consequently not to large deviations from similarity.
2.4 DEVIATION FROM GEOMETRIC SIMILARITY

Carousels can also differ in geometrical form, because of a difference in relative curvature $b/R$ and/or a difference in aspect ratio $h/b$. In many carousels the aspect ratio can be adjusted, but geometrical similarity between different carousels is often prevented by the difference in relative curvature. The relation between flows in carousels can be discussed with a step-by-step procedure in which different aspects
- difference in aspect ratio,
- difference in relative curvature,
- difference in measure but geometrical similarity (discussed above),
are considered separately.

The concept of similarity of carousel flows is not very useful in case of deviation from geometric similarity, as is shown below. The scaling of $p$ and the scaling of the Reynolds stresses pose difficulties that are not easy to surmount, except in case of small differences in relative curvature and very small differences in aspect ratio.

**Difference in aspect ratio**

To obtain similarity of carousel flows in case of difference in aspect ratio a different scaling of vertical and horizontal length scales is required. An identical non-dimensional continuity equation requires then also different velocity scales. If the horizontal length measures are scaled with $b$, the vertical length measures with $h$, the horizontal velocity components $u$ and $v$ (including the turbulent velocity components in the Reynolds stresses $u'$ and $v'$) as before with $\omega_b R$, but the vertical velocity components $w$ (and $w'$) with $\omega_b Rh/b$, the non-dimensional equations (6) and (18-20) and the boundary conditions (23-24) and (26-27) become independent of the aspect ratio of the carousel after dividing by the appropriate constant. Only the pressure terms need a slightly different approach. The pressure in the radial momentum equation requires scaling with $\rho \omega_b^2 R^2$ and in the vertical momentum equation with $\rho \omega_b^2 R^2 (h/b)^2$. This can be a problem unless we can approximate $p$ by $p = P_0 + p_r + p_z$, where $P_0$ is a constant, $p_r$ is a part of $p$ which varies in horizontal direction, mainly connected with differences in horizontal velocity components and with the centripetal acceleration, and $p_z$ varies in vertical direction, which variation is connected with differences in vertical velocities. In that case similar flows between carousels with different aspect ratios would appear possible for equal ratios of rotational velocities $\omega_z/\omega_b$. Such an approximation in which the variation of $p$ in the different directions can be completely separated is however not justified generally.

The approach used in the scaling of the Reynolds stress terms was also not correct. Scaling the different turbulent velocity components with the corresponding time-averaged velocity components is incompatible with the knowledge of the 3-dimensional structure of turbulent
flow. All turbulent velocity components are of the same order of magnitude and are mainly determined by the velocity component that shows the most important velocity gradient in the flow. Hence all turbulent velocity components should scale with $\bar{u}$. The most important consequence of this scaling is the impossibility to bring $q_{z\phi}$ in equation (18) into the required non-dimensional form. The shear stress $q_{z\phi}$, which plays an important role in carousel flow, should scale to this end with $\omega_b^2 R^2 (h/b)$, instead of $\omega_b^2 R^2$, which means that only very small variations in aspect ratio $h/b$ are allowed for approximate similarity.

It can be argued that the scaling of the shear stresses can be chosen independently of the scaling of the turbulent velocity components, as it is essentially the covariance of the turbulent velocity components that is scaled. However, the correlation between the velocity components will not behave differently in similar flows, leaving no extra freedom in the scaling parameters.

**Difference in relative curvature**

For investigation of the possibility of similarity of carousel flows in case of difference in relative curvature we first restrict ourselves to flows of so low relative curvature $b/R$ that $\partial \bar{v} / \partial r \gg 1/r$, for example $\partial \bar{v} / \partial r \gg \bar{v}/r$. In that case we can use $r = R$ all over the flume. Equations (6) and (18-20) then simplify to

\[
\frac{\partial \bar{v}}{\partial r} + \frac{\partial \bar{w}}{\partial z} = 0
\]

\[
\frac{\partial \bar{u} \bar{v}}{\partial r} + \frac{\partial \bar{uw}}{\partial z} + \frac{\partial q_{\phi r}}{\partial r} + \frac{\partial q_{z\phi}}{\partial z} = 0
\]

\[
\frac{\partial \bar{v} \bar{v}}{\partial r} + \frac{\partial \bar{w} \bar{w}}{\partial z} - \frac{\bar{u} \bar{u}}{R} + \frac{\partial q_{rr}}{\partial r} + \frac{\partial q_{rz}}{\partial z} = 0
\]

\[
\frac{\partial \bar{v} \bar{w}}{\partial r} + \frac{\partial \bar{w} \bar{w}}{\partial z} + \frac{\partial q_{rz}}{\partial r} + \frac{\partial q_{zz}}{\partial z} = 0
\]

The only remainder of the cylindrical co-ordinate system is the centripetal acceleration term $\bar{u} \bar{u}/R$. In the boundary conditions (23-24) and (26-27) the expressions for the tangential velocity component simplify to

\[
\bar{u}_{z} = \frac{\omega_l}{\omega_b} - cu_\ast, \quad \bar{u}_{z} = 1 + cu_\ast,
\]

\[
\bar{u}_{r} = 1 + cu_\ast, \quad \bar{u}_{r} = 1 - cu_\ast.
\]
2.5 INFLUENCE OF THE WALL BOUNDARY LAYER

As the similarity concept is only of limited use between the flows in not geometrically similar carousels, other ways have to be found to develop relations between them. Information about the dependence of an optimum ratio of rotational velocities on the aspect ratio may be obtainable without knowledge of the exact flow field in the carousel, for example. Therefore we investigate here some physical mechanisms that determine the flow in the carousel and try to derive some estimates.

Turbulent carousel flows are characterized by limited regions along the wall boundaries with important gradients in the tangential velocity component and a large central region with only a slight variation in this component. The turbulent boundary layers along the carousel wall are relatively thick, because of the small angle between flow direction and tangential direction, allowing a long development stretch for the boundary layer. Their development is generally limited by spatial constraints, for instance connected with the presence of the other walls. This allows us to estimate the tangential velocity component in the central region, \( u_{av} \). Analogously to equation (21) we can put

\[
\frac{u_{av} - u_w}{u_*} = \frac{1}{\kappa} \ln \left( \frac{\delta_i}{a_0} \right) = c_t
\]

where \( \delta_i \) is the thickness of the turbulent wall boundary layer, and \( u_w \) is the velocity of the wall. As \( \delta_i \) is large compared to the roughness length \( a_0 \) and varies relatively little, \( c_t \) is nearly constant. As the flow is steady the net moment of the shear forces executed by the lid and by the flume (bottom and walls) of the carousel around the axis of rotation on the water in the carousel should cancel, which with equation (33) leads for small \( b/R \) to

\[
\frac{(u_{av} - \omega R)^2}{(u_{av} - \omega_p R)^2} = \frac{A_c}{A_l} = \beta
\]

in which \( A_c \) and \( A_l \) are the surface area of the flume and of the lid of the carousel respectively and \( \beta \) is their ratio, \( \beta = (b+2h)/b \). So \( u_{av} \) is

\[
u_{av} = \omega_p R \frac{1}{1 + \sqrt{\beta}} + \omega_p R \frac{\sqrt{\beta}}{1 + \sqrt{\beta}}
\]

For two secondary flow cells to be present the flow at the bottom and at the lid have both to be driven outward. Therefore both \( | \omega_p R | \) and \( | \omega_l R | \) have to be larger than \( | u_{av} | \), which leads to the condition for the presence of two secondary cells:

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We consider two carousels with equal \( b \) and \( h \), but different \( R \), which means equal aspect ratio \( b/h \), but different relative curvature \( b/R \). Scaling of the variables to obtain similar solutions would be easy if there was no centripetal acceleration term. The scaling of all lengths can be equal, \( b \), but the centripetal acceleration term necessitates again different velocity scales. Using \( \omega \mu R \) as a scale for the tangential velocity, the scale for the other velocity components has to become \( \omega \mu R(b/R)^{1/2} \). This scaling requirement can be understood if the larger centripetal acceleration in more strongly curved flow and its rôle in the generation of secondary flow in carousels is considered. This way of scaling the velocity components leads to the same kind of problems with the scaling of \( p \) and of the Reynolds stresses as in the case of difference in aspect ratio.

The approach in the scaling of the Reynolds stresses is again not correct because of the different velocity scales used, but in this case the consequences are less serious. All three turbulent velocity components should scale equally with \( \omega \mu R \). The Reynolds stresses \( q_{\theta r} \), \( q_{\theta \phi} \), \( q_{rr} \) and \( q_{\phi \phi} \) should scale with \( \omega \mu^2 R^2 \). The last two of these stresses play a minor rôle in carousel flow and can generally be neglected. Only for very low curvatures these two stresses are important again, as they can give rise to secondary flow like they do in straight flumes (Nikuradse, 1926). For the third Reynolds shear stress \( q_{rz} \) the scaling is more complicated. The secondary flow caused by the curvature of the flume rotates the flow direction and with it the more important shear stresses \( q_{\theta r} \) and \( q_{\theta \phi} \) over a small angle \( \psi \). This means that \( q_{rz} \) is proportional to a combination of \( q_{\theta r} \) and \( q_{\theta \phi} \), scaling with \( \omega \mu^2 R^2 \), and to \( \sin \psi \), with \( \psi \) the angle between local flow direction and the tangential direction. Here \( \sin \psi \) is a combination of \( \bar{v}/u \) and \( \bar{w}/u \), scaling with \( (b/R)^{1/2} \). This means that \( q_{rz} \) becomes larger with curvature not because of an increase in the turbulent velocity components \( v' \) and \( w' \), but by an increase of the correlation between them, whereas the correlation between the velocity components in the other Reynolds stresses remains more or less constant. So, \( q_{\theta r} \) and \( q_{\theta \phi} \) scale with \( \omega \mu^2 R^2 \), where similarity requires scaling with \( \omega \mu^2 R^2(b/R)^{1/2} \), and \( q_{rz} \) scales with \( \omega \mu^2 R^2(b/R)^{1/2} \), where similarity requires \( \omega \mu^2 Rb \). Again no strict similarity can be found, but the square root in the difference in the scaling values \( (b/R)^{1/2} \) means that not to large deviations of similarity are to be expected for small differences in relative curvature.
\[
- \left( \frac{1}{1 + 2/\sqrt{\beta}} \right) > (\omega/\omega_b) > - (1 + 2/\sqrt{\beta})
\]  

(36)

An attempt to derive the relative dimensions of both secondary flow cells can yield a rough estimate for the optimum ratio of the rotational velocities ($\omega/\omega_b$). As the flow in both cells is stationary, the forces driving the circulations have to be in equilibrium for both cells. Both cells are driven by the centripetal acceleration near the lid or the bottom and are counteracted by the cross flow component of the shear force along the walls. (Properly speaking it is the insufficiency of the pressure gradient near the lid and the bottom to bring about the centripetal acceleration which causes outward flow; or the centrifugal forces in a rotating coordinate system, which is less appropriate here because of the differences in rotation velocity). The nearly constant value of $c_t$ suggests that the thickness of the part of the boundary layers directly along the wall where the velocity deviates appreciably from $u_{av}$ is more or less equal everywhere. This makes the regions where this centripetal acceleration effect drives the secondary flow along the bottom and the lid of equal magnitude. The net effect of the centripetal acceleration, which is proportional to the velocity squared, is given by the difference to the centripetal acceleration in the central part of the flume. Therefore the ratio of the influences of the driving forces along the lid and the bottom on their respective secondary flow cells, $I$, can be estimated by

\[
I = \frac{(\omega_RB^2 - u_{av}^2)}{(\omega_BR^2 - u_{av}^2)} = \frac{(\omega/\omega_b)^2(1 + \sqrt{\beta})^2 - ((\omega/\omega_b) + \sqrt{\beta})^2}{(1 + \sqrt{\beta})^2 - ((\omega/\omega_b) + \sqrt{\beta})^2} = \left( \frac{\omega_t}{\omega_b} \right)^2
\]

(37)

where the last approximation applies if $(\omega/\omega_b + \sqrt{\beta})/(1 + \sqrt{\beta}) \ll 1$ which is anyhow the case for flows with rotation ratios not to far from optimum.

The counteracting shear force components on the two cells must then have the same ratio, $I$. The estimate of these shear force components is quite complicated (and not very accurate), in particular for the upper flow cell. The cross shear stress component, $\tau_{cr}$, for small cross velocities is proportional to the wall shear stress, $\tau = \rho u_s | u_s |$, itself and to $\tan \psi$, where $\psi$ is the angle between the local flow direction and the tangential direction (see chapter 2.3), or $\tan \psi = v_{cr}/(u_{av} - u_w)$ (Here $v_{cr}$ is the cross-flow velocity component, with $v_{cr} = v$ along the bottom or the lid and $v_{cr} = w$ along the vertical walls of the flume.):

\[
\tau_{cr} = \rho u_s | u_s | v_{cr}/(u_{av} - u_w)
\]

(38)

In the computation of the shear forces the cross-flow velocity component, $v_{cr}$, is assumed constant along the rigid boundaries of a cell. It can be estimated from the driving force along lid or bottom. The cross-flow velocity component of the upper cell, $v_{cr,t}$, can hereto be taken as the outward acceleration by the outward driving force times the time $t_p$ that is needed for a parcel of water to pass the lid ($t_p = b/v_{cr,t}$). Then $v_{cr,t}$ is proportional to $\sqrt{1 - (\omega_b R)^2 - u_{av}^2}$.
For the cross-flow velocity of the lower cell, $v_{cr,b}$, a similar argumentation leads to a proportionality with $\sqrt{\frac{(\omega_b R)^2 - u_{av}^2}{(\omega_b R)^2 - u_{av}^2}}$. The ratio between the cross-flow velocity components of the upper and the lower cell is then

$$\frac{v_{cr,l}}{v_{cr,b}} = \frac{(\omega_l R)^2 - u_{av}^2}{(\omega_b R)^2 - u_{av}^2} \approx \frac{\omega_l}{\omega_b}$$  \hspace{1cm} (39)

To obtain the net shear forces on both cells, the contributions (shear stress times boundary surface) of lid or bottom and appropriate part of the walls have to be added. In the wall contribution for the lower cell and the upper cell the wall surface in contact with these cells, $2\lambda h$ and $2(1-\lambda)h$ respectively, times the length of the flume, has to be considered. The shear stresses are obtained from equation (38), where along the lid $u_w=\omega R$ and $u_w$ is proportional to $(u_{av}-\omega R)$ and along the bottom and the walls $u_w=\omega R$ and $u_w$ is proportional to $(u_{av}-\omega R)$. The ratio of the shear forces on the upper and lower cell then becomes

$$\frac{(\omega_l R - u_{av}) b - (\omega_b R - u_{av}) 2(1-\lambda)h}{(\omega_b R - u_{av})(b+2\lambda h)} \frac{\omega_l}{\omega_b} = I = \left(\frac{\omega_l}{\omega_b}\right)^2$$ \hspace{1cm} (40)

Substitution of equation (34) and simplification yield

$$\frac{\omega_l}{\omega_b} = \frac{2(\lambda-1)}{2\lambda h + 1} \frac{\lambda h}{b} - \sqrt{\beta} \hspace{1cm} \text{with} \hspace{1cm} \beta = 1 + 2\frac{h}{b}$$ \hspace{1cm} (41)

With expression (41) the ratio of the rotational velocities for different relative magnitudes of the two cells can be estimated for different aspect ratios of the flume, $h/b$.

To obtain an estimate of the optimum ratio of the rotational velocities of the carousel, the relative magnitudes of the two cells for a certain aspect ratio, expected at this optimum ratio can be inserted. The relative magnitudes of the cells at the optimum ratio will, however, not be independent from the aspect ratio as carousel flows are not completely similar for different aspect ratios. For very small aspect ratios the height of the flume appears to be of minor influence and similarity may apply, leading to

$$\frac{\omega_l}{\omega_b} \approx -1 - 3\frac{h}{b} + 4\lambda \frac{h}{b} \quad \text{or} \quad \frac{\omega_l}{\omega_b} + 1 \approx -\left(3 - 4\lambda\right) \frac{h}{b}$$ \hspace{1cm} (42)

For very large aspect ratios of the flume the height of the flume can be assumed not to be very important for the magnitude of the bottom cell. In that case $2\lambda bh$ in the wall surface in contact with the lower cell can better be replaced by $2\lambda y b$ as the height of this part of the walls scales with $b$ instead of $h$. Here $\lambda b = \lambda h/b$ is a constant and consequently $\lambda$ is
proportional to \((h/b)^{-1}\) for large aspect ratios, leading to

\[
\frac{\omega_i}{\omega_b} = -\frac{2}{2\lambda_b + 1} \frac{h}{b} \tag{43}
\]

In general carousels are used with intermediate aspect ratios, for which the dependence of \(\lambda\) will be between these two extremes: \(\lambda\) proportional to \((h/b)^{-1}\) and constant, i.e. proportional to \((h/b)^0\). Assuming \(\lambda = c_\lambda (h/b)^{-1/2}\) for these intermediate aspect ratios yields

\[
\frac{\omega_i}{\omega_b} = \frac{2[c_\lambda - (h/b)^{1/2}](h/b)^{1/2} - (1 + 2(h/b)^{1/2}}{2c_\lambda(h/b)^{1/2} + 1} \tag{44}
\]

The many assumptions needed for these estimations make them less reliable than desired.
3. EXPERIMENTAL SETUP AND MEASURING PROGRAMME

3.1 ROTATING ANNULAR FLUME AND LASER-DOPPLER VELOCIMETER EQUIPMENT

The rotating annular flume used for the present experiments was constructed at the Laboratory of Fluid Mechanics of the Faculty of Civil Engineering of Delft University of Technology for cohesive sediment transport studies (see Figures 1.1 and 3.1). The carousel has a rectangular cross-section, 0.304 m wide and 0.47 m deep, and a mean diameter of 3.70 m. The flume has large glass windows for observation and laser-Doppler measurements. Top lid, bottom and side-walls are hydraulically smooth.

The play between the 0.30 meter wide top lid and both side-walls varies between 1 mm and 4 mm. The tangential velocity of the top lid can be accurately controlled over a range of 0.05 m/s - 2.0 m/s, that of the flume over a range of 0.05 m/s - 1.0 m/s. The direction of the rotation of both assemblies can be changed independently. The power of the top lid motor is 0.55 kW, that of the bottom motor 2.2 kW. The horizontal level of the flume bottom varies within ± 1.0 mm, that of the top lid within ± 1.5 mm. Electrical power is provided to the rotating flume through a set of slip rings to power instruments such as a laser-Doppler velocimeter system and a personal computer.

The laser-Doppler velocimeter (LDV) used in the present experiments consisted of an air-cooled 300 mW ILT argon-ion laser, a 2-dimensional (two colour) back scatter fibre optics DANTEC optical system with a DANTEC Flow Velocity Analyzer (FVA). The 2-dimensional probe was mounted on a home-made traversing system, which allowed radial and vertical transposition of the probe. All the instruments: laser, optics (beam separator, optical fibres, probe, photomultipliers, etc.), traversing system, FVA, a personal computer (PC) were placed around the driving shafts and rotated with the flume.

The large amount of measurements were executed automatically, so without the necessity to stop the rotation of top lid and flume between two observations. The traversing system was controlled by the PC, which also stored the large amount of data. The memory on the hard disc of the PC and the number of measuring files produced allowed automatic measuring sessions of about half a day. The determination of the flow in about 300 measuring points in a cross-section for each flow case did cost about two full days. The measuring time in each point of 6 minutes was required to obtain reliable results for the average velocity, the turbulence energy components and the turbulent shear stress.
Fig. 3.1  Schematic drawing of rotating annular flume of Delft University of Technology.
The focal distance of the front lens of the probe was 310 mm, creating a measuring volume of 1.8 mm length and .11 mm width. The refraction of the light by the very slightly curved glass window of the flume already interfered with the creation of one measuring volume, by displacing one of the two points of intersection of the beams up to 14 mm. This could however easily be corrected for by introducing the beams in the water through a flat glass plate normal to the optical axis of the probe. The so created space between glass plate and flume was filled with water. For the observations within 3 cm of the bottom, one of the beams would have been blocked. Therefore the probe was placed under a small angle of about 8 degrees. The much worse refraction problem in this case could be solved in the same way (see fig. 3.2).

Fig. 3.2  Drawing of the optical arrangement of the LDV.
3.2 PROCESSING PROCEDURE AND MEASURING PROGRAMME

The processing procedure.

The FVA is a processor that correlates a time-delayed signal and the original signal to give the Doppler frequency of the scattered laser-light and therewith the velocity of small particles that follow the water movement. The FVA distributes the observed velocity-signal over 256 (= $2^8$) velocity slots, which yields sufficient accuracy for the present application.

The LDV-system was used to measure the instantaneous values of the tangential velocity $u$ and the vertical velocity $w$ (see Fig. 1.3). These instantaneous velocities were analyzed by the FVA to give the time-averaged values $\bar{u}$ and $\bar{w}$, the turbulence energy components $(\overline{u'})^2$ and $(\overline{w'})^2$ and one of the Reynolds stresses $q_{z\phi} = -\overline{u'w'}$.

The time-averaged radial velocity component $\overline{v}$ was determined from the time-averaged vertical velocity component $\overline{w}$ by applying the continuity equation for axisymmetric flow (6), which can be written as

$$\frac{\partial \overline{v}}{\partial r} + \frac{\overline{v}}{r} + \frac{\partial \overline{w}}{\partial z} = 0$$

(45)

Differentiation of $\overline{w}$ in vertical direction and a subsequent integration in radial direction yields $\overline{v}$.

The procedure for determining $\overline{v}$ and $\overline{w}$ was somewhat more complicated for the near-bottom measurements with the tilted probe. In principle it can be done by an integration in the direction of the optical axis. This appeared to yield inaccurate results unless the traversing would also have been done along this axis, which was however not preferred because of the desired grid of measuring points. The used procedure for determining $\overline{v}$ and $\overline{w}$ was based on (45) and an iteration procedure was used to correct for the tilt angle. It is not possible to use a similar procedure to obtain $v'$ from $w'$, as the solution of $v'$ from an equation analogous to (45) for $v'$ and $w'$ requires the simultaneous measurement of the turbulent velocity components at several places. Therefore the shear stress was measured as:

$$q_{z\phi} = -\overline{u'(w'\cos \alpha + v'\sin \alpha)}$$

(46)

where $\alpha$ is the tilt angle of the probe (about 8°). The error in expression (46) because of the non-zero angle $\alpha$ is, however, negligible as $\cos \alpha \approx 0.99$ and $\overline{u'v'} \sin \alpha$ is very small compared to $\overline{u'w'}$.

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The differentiation of $\bar{w}$ in vertical direction and the subsequent integration in radial direction introduce errors in $\bar{v}$ related to small deviations in the exact placing of the probe, for example between the radial co-ordinate at two different horizontal traverses. The greatest part of these errors could be eliminated by a simple smoothing procedure over the first (vertically) neighbouring points. A typical result of the measured secondary flow velocities in a cross-section is shown in fig. 3.3 and it's smoothed version in fig. 3.4.

The measuring programme of this study was aimed at the determination of the value of the optimal ratio $\omega_p/\omega_b$ for different flow cases, for example different aspect ratios (i.e. different water depths as the flume width can not be varied). Two criteria can be formulated for the determination of the optimum value of the ratio $\omega_p/\omega_b$ for a rotating annular flume, depending on what has to be investigated in the flume.

I. If the topic is the distribution of suspended sediment in a free surface flow or the deposition of that sediment on the bottom, then the optimum situation would be achieved with the lowest secondary flow velocities, in particular near the bottom.

II. For an erosion study, however, the uniformity in radial direction of the turbulent shear stress near the bottom is the most important factor.

The measurements were executed for two different widths of the top lid. In cluster A the top lid covered as well as feasible the upper side of the flume, leaving only very small free surfaces of some millimetres at most in the play at both sides between lid and flume. In cluster B a smaller top lid was used. The top lid in this cluster covered the inner half (0.15 m) of the flume, leaving a free surface in the outer half. The advantage of a top lid which is (significantly) smaller than the flume width is that stationary instruments can be put into the flume. This smaller top lid can have two disadvantages: first, the driving of the secondary flow near the upper surface is less uniformly divided, which presumably causes a stronger secondary flow over the cross-section; secondly the larger free surface can give rise to flow disturbances. The measurements with the 0.15 m wide top lid were done to investigate whether such a smaller top lid can be used to drive the flow in a rotating annular flume without deteriorating the optimum secondary flow condition and the uniform distributions in radial direction of the turbulent shear stresses. These observations were restricted to the near-bottom area (the lowest 6 cm of the water column).

The measurements are divided in different groups according to the height of the water column $h$ and subdivided again in series with different rotational velocity of the flume $\omega_b$. In the series the ratio of the rotational velocities $\omega_p/\omega_b$ varies to determine the optimum ratio. The measurements are grouped around a standard case (A1.1, see below) with $h = 29.7$ cm and $\omega_b = -3.0$ rpm. The measured flow cases in each series included the optimum value of the ratio $\omega_p/\omega_b$ (criterion I). Several cases included measurements over the complete flow field with the horizontal probe as well as near bottom measurements with the inclined probe. To save time many measurements were restricted to one of the two optical arrangements.
Fig. 3.3  Originally determined secondary flow velocities in a cross-section.

Fig. 3.4  Smoothed secondary flow velocities in a cross-section.
The measuring programme.

The measuring programme of the present experimental study can be summarized as follows:

(A) Width top lid = 30 cm

- Height water column = 29.7 cm

A1.1 $\omega_b = -3.0$ rpm : $\omega/\omega_b = -1.5, -1.9, -2.0, -2.1, -2.2, -2.3, -2.7, -3.1$

A1.2 $\omega_b = -4.5$ rpm : $\omega/\omega_b = -2.1, -2.2, -2.3$

A1.3 $\omega_b = -1.5$ rpm : $\omega/\omega_b = -2.1, -2.2, -2.3$

- Height water column = 19.7 cm

A2.1 $\omega_b = -3.0$ rpm : $\omega/\omega_b = -1.6, -1.7, -1.8, -1.9$

A2.2 $\omega_b = -4.5$ rpm : $\omega/\omega_b = -1.7, -1.8, -1.9$

A2.3 $\omega_b = -1.5$ rpm : $\omega/\omega_b = -1.7, -1.8, -1.9$

- Height water column = 15.7 cm

A3.1 $\omega_b = -3.0$ rpm : $\omega/\omega_b = -1.4, -1.5, -1.6, -1.7$

(B) Width top lid = 15 cm

- Height water column = 29.7 cm

B1.1 $\omega_b = -3.0$ rpm : $\omega/\omega_b = -2.7, -2.8, -2.9, -3.0$

- Height water column = 19.7 cm

B2.1 $\omega_b = -3.0$ rpm : $\omega/\omega_b = -2.0, -2.1, -2.2, -2.3$

B2.2 $\omega_b = -4.5$ rpm : $\omega/\omega_b = -2.3$

- Height water column = 15.7 cm

B3.1 $\omega_b = -3.0$ rpm : $\omega/\omega_b = -1.8, -1.9, -2.0, -2.1$
4. MEASURING RESULTS

4.1 OPTIMUM RATIO OF ROTATIONAL VELOCITIES

Minimum intensity of secondary flow.

The first criterion (Criterion I) for the optimum value of the ratio $\omega_r/\omega_b$ of a rotating annular flume is the minimum intensity of secondary flow, in particular near the bottom. This criterion is recommended when the topic is the distribution of suspended sediment in a free surface flow or the deposition of that sediment on the bottom. For the determination of the optimum value of the ratio $\omega_r/\omega_b$, according to this first criterion the experimentally obtained secondary velocity data are collected in vectorplots. These vectorplots are reproduced in Appendix A. If for the same flow case measurements were executed with horizontal and inclined probe, the information is combined in one vectorplot.

In series A1.1 (fig. A1-A8) an extensive range of flows with increasing ratio $\omega_r/\omega_b$ was measured. Fig. A2 demonstrates that for certain values of $\omega_r/\omega_b$ two secondary flow circulations can be distinguished in a cross-section, one cell driven by the high velocities near the lid and one by the high (negative) velocities near the flume. This confirms Sheng’s (1989) suggestion of a possible two-cell secondary flow pattern occurring in a rotating annular flume. Fig. A7 shows a clockwise secondary flow circulation for relatively high lid velocity ($\omega_r/\omega_b = -2.7$) where the driving of the secondary flow by the high velocities near the lid is apparently so large that the upper secondary flow cell predominates. Fig. A1 shows a situation with relatively low lid velocity ($\omega_r/\omega_b = -1.5$) in which the anti-clockwise secondary flow circulation driven by the flow along the bottom and the sidewalls predominates.

The lowest secondary flow velocities near the bottom (criterion I) were measured in Experiment series A1.1 for $\omega_r/\omega_b = -2.1$ to -2.2, see Figures A4-A5. To find a more precise optimum condition obviously requires more measurements between those values and an exact value is even then not easy to define. An estimate of a more precise optimum condition is $\omega_r/\omega_b = -2.17$. Sometimes the plots from this series of measurements show a very complicated flow, for example in fig. A8 for $\omega_r/\omega_b = -3.1$. This flow pattern illustrates the reliability of the velocity measurements, as the velocities were measured in horizontal sweeps through the flume and the flow pattern looks very regular in vertical direction. For several ratios of $\omega_r/\omega_b$ more than two secondary flow circulations can be distinguished in a cross-section, sometimes because of a division of the lower secondary flow cell (see figures A5-A6) and in other cases, for example where a small secondary flow cell appears in an upper corner (see for instance figures A4-A8), probably connected with anisotropic normal
Reynolds stresses, corresponding to the secondary flow cells observed in straight flumes (Nikuradse, 1926).

Experiments with different rotational velocities of the flume, series A1.2 and A1.3, yield about equal optimum values $\omega_r/\omega_b = -2.1$ to -2.2 (see figures A9 through A14), indicating that this optimum value is not strongly dependent of the values of $\omega_r$ and $\omega_b$ themselves. Estimates for slightly more precise values can be -2.13 for series A1.2 and -2.19 for series A1.3.

The aspect ratio of the flume shows an influence on the optimum value for $\omega_r/\omega_b$. The experiments with different water depth $h$ show lower optimum values for smaller depths:
- $\omega_r/\omega_b = -1.8$ for A2.1, A2.2 and A2.3 with $h = 19.7$ cm see figures A15 through A24,
- $\omega_r/\omega_b = -1.6$ for A3.1 with $h = 15.7$ cm see figures A25 through A28.

In all figures for these series the upper cell can be seen to be displaced to the outer side of the flume and the lower cell to the inner side, giving a slightly different character to the optimum flow. For these slightly shallower flows it is very difficult to indicate an optimum condition. Around this optimum condition a small eddy appears to have separated itself from the top eddy and is found located between two bottom eddies (see for example fig. A20). This arrangement of several contra-rotating eddies resembles the secondary flow in wide open channels (Nezu and Nakagawa, 1993).

The experiments with smaller top lid show a hardly more irregular secondary flow pattern, compared to the experiments with complete lid. (Compare figure A35 with figure A17.) The pattern of optimum values of the ratio $\omega_r/\omega_b$ is comparable but at higher values, caused by the smaller driving region of secondary flow near the lid:
- $\omega_r/\omega_b = -3.0$ for B1.1 with $h = 29.7$ cm see figures A29 through A32,
- $\omega_r/\omega_b = -2.3$ for B2.1 and B2.2 with $h = 19.7$ cm see figures A33 through A37,
- $\omega_r/\omega_b = -2.1$ for B3.1 with $h = 15.7$ cm see figures A38 through A41.

**Uniform bottom shear stress**

The second criterion (Criterion II) for the optimum value of the ratio $\omega_r/\omega_b$ of a rotating annular flume, the uniformity in radial direction of the tangential velocity component near the bottom and in particular the bottom shear stress, which is virtually equal to the Reynolds shear stress $q_{2\phi}$ at the bottom, can be of importance for erosion studies. In Appendix B plots of horizontal profiles of the tangential velocity component are reproduced. Only the measurements with a horizontal probe are included. In Appendix C plots of horizontal profiles of the Reynolds shear stress $q_{2\phi}$ are reproduced. Here only measurements with an inclined probe are included.

The velocity plots of series A1.1 (see figures B1-B7) show an important influence of the
convection of the tangential velocity component by the secondary flow on the distribution of the tangential flow over the cross-section. A dominating top cell convects the tangential flow strongly clockwise over the cross-section, leading to very unequal velocities over the width at all depths, see figures B6-B7. At a lower ratio of $\omega_f/\omega_b$ the convection in the upper cell of the higher tangential velocities downwards along the outer wall and to the inner wall makes the horizontal profiles of this velocity component quite uniform in the lower part of the flume, see figures B2-B4. The optimum condition for uniform tangential flow in the lower half is at $\omega_f/\omega_b = -1.9$ (see fig. B2). At lower ratios the uniformity at about half-depth decreases but the more important uniformity near the bottom remains good (see fig. B1). The other measurement series confirm that the uniformity near the bottom is good when the ratio of the rotational velocities is 5% or more below the optimum value for the first criterion (see figures B8-B22). An optimum value for the uniformity of the tangential velocity in the lower part of the flume will be about 10% below the value according to the first criterion. However the optimum for the uniformity of the tangential velocity component is much less sharp, especially when the flow near the bottom is considered.

The uniformity of the bed shear stress is directly related to the uniformity of the tangential velocity near the bed. The horizontal profiles of the Reynolds shear stress $q_{xy}$ show that the optimum value of $\omega_f/\omega_b$ for the uniformity of this Reynolds shear stress (criterion II) is again slightly below (.2) the optimum value for the presence of secondary flow (criterion I), see figures C1-C25. However, the optimum, considering criterion II, appears to be quite broad again, so the set-up of the flow conditions for erosion studies is not very critical.

Usefulness of carousel flow for sedimentation and erosion studies.

In an rotating annular flume the velocity of the outer wall is larger than the velocity of the inner wall because of the difference in radius (for the present flume about 18%). This causes a larger tangential velocity near the outer wall than near the inner wall. Taking this effect into account, it can be concluded from the figures in appendices B and C that the uniformity in radial direction of the tangential velocity component and the bed shear stress is good and not very sensitive to the exact flow conditions. Therefore the carousel proves to be an appropriate apparatus for erosion studies.

However, for a velocity difference $U_f$ between lid and flume of the considered carousel of about 2 m/s the (vertical) secondary flow velocities do reach values of about 2 cm/s near the bottom and up to about 5 cm/s elsewhere, even in the optimal situation according to criterion I. A related problem appears in straight flumes. In vertexes of straight flumes anisotropy of the normal Reynolds stresses causes secondary flow that locally reaches velocities of the order of 3% of the velocity in the flume direction. For example a velocity in the flume direction of 0.6 m/s leads to maximum secondary flow velocities of the order of 2 cm/s. So secondary flow velocities near the bottom of the same order of magnitude as the secondary
flow velocities in carousels occur also in angular points of straight flumes. Hence it is hardly possible to reduce the secondary flow velocities further. The fall velocities of fine sediment can be of the order of magnitude of the secondary flow velocities mentioned above to several orders below these velocities. Therefore the use of carousels for studies of the distribution of suspended sediment in a free surface flow or the deposition of that sediment on the bottom appears to be limited.

The improvement concerning the optimum flow in a carousel when compared to the flow in a stationary annular flume with rotating lid is remarkable. No measurements with a stationary flume were executed in this investigation, but the plots of flows with large \( \omega_l/\omega_R \) ratios show already a very strong secondary flow (see series A1.1: \( \omega_l/\omega_R = -2.7 \) and -3.1). A stationary annular flume is therefore even less useful for studies of the distribution or the deposition of suspended sediment and of limited use for erosion studies because of the non-uniform bed shear stress.

Not only the secondary flow field in a carousel restricts the usefulness of the carousel flow as a model for the free surface flow in rivers or channels. Also the turbulence pattern and with it the transport coefficients, for example the turbulence diffusion coefficient, differ appreciably. The measured turbulence energy distribution for the optimum flow \( \omega_l/\omega_R = -2.2 \) of series A1.1, shown in fig. 4.1, differs in several respects from the turbulence energy distribution measured in free surface flow. (In figures 4.1 to 4.5 the zero levels of the plotted quantities for the various profiles are indicated at the left side and the scale of the plotted quantities at the lower right side. The left side (\( y = 0 \) mm) represents the inner side of the flume and the right side (\( y = 304 \) mm) the outer side. The width of the flume is not exactly 304 mm everywhere and the sidewalls are not exactly vertical, partly because of the influence of the water pressure on the glass walls.)

The measured distribution of turbulence energy in free surface flow can be described by (Nezu and Nakagawa, 1993)

\[
\frac{k}{u^*_*} = 4.78 \exp\left(-2 \frac{z}{h}\right) \quad (47)
\]

where \( z \) is the distance above the bottom and \( k \) is the (kinematic) turbulence energy, defined by \( k = \frac{1}{2} \left[ (u'^2) + (v'^2) + (w'^2) \right] \). Expression (47) describes a turbulence energy that decreases monotonously with the distance to the bottom, whereas the measurements in the carousel indicate a higher turbulence energy level near the lid and along the sidewalls of the carousel. Also a convection of the turbulence energy by the secondary flow is apparent. Because of the 2-dimensional measurements only \( u' \) and \( w' \) could be measured in the used optical configuration, so \( k \) in fig. 4.1 is defined as \( k = \frac{1}{2} (u'^2 + (w'^2) \right), in which case expression (47) remains valid when the constant is changed from 4.78 to 3.45 (Nezu and Nakagawa, 1993).
Series A1.1:
Width top lid: 30.0 cm
$h = 29.7$ cm
$\omega_b = -3.0$ rpm

Case:
$\omega_i = 6.6$ rpm
$\omega_i/\omega_b = -2.2$

inside wall at $y = 0$ mm,
outside wall at $y = 304$ mm,
zeros of the velocity profiles at different $z$ are indicated at left,
energy scale at lower right.

Fig. 4.1. Profiles of the measured turbulence energy, $k = \frac{1}{2}[(u')^2 + (w')^2]$. Series A1.1, case $\omega_i/\omega_b = -2.2$. 

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Series A1.1:
Width top lid: 30.0 cm
\( h = 29.7 \) cm
\( \omega_b = -3.0 \) rpm

Case:
\( \omega_t = 6.6 \) rpm
\( \omega_t/\omega_b = -2.2 \)

inside wall at \( y = 0 \) mm,
outside wall at \( y = 304 \) mm,
zeros of the velocity profiles at different \( z \) are indicated at left,
energy scale at lower right.

Fig. 4.2. Profiles of the measured turbulence energy component \( \langle u^2 \rangle \).
Series A1.1, case \( \omega_t/\omega_b = -2.2 \).
Series A1.1:
Width top lid: 30.0 cm
h = 29.7 cm
\omega_b = -3.0 \text{ rpm}

Case:
\omega_t = 6.6 \text{ rpm}
\omega_t/\omega_b = -2.2

inside wall at y = 0 mm,
outside wall at y = 304 mm,
zeros of the velocity profiles at different z are indicated at left,
energy scale at lower right.

Fig. 4.3. Profiles of the measured turbulence energy component \overline{(w')^2}.
Series A1.1, case \omega_t/\omega_b = -2.2.
Series A1.1:

Width top lid: 30.0 cm
h = 29.7 cm
\( \omega_b = -3.0 \) rpm

Case:
\( \omega_t = 6.6 \) rpm
\( \omega_t/\omega_b = -2.2 \)

inside wall at \( y = 0 \) mm,
outside wall at \( y = 304 \) mm,
zeros of the velocity profiles at different \( z \) are indicated at left,
intensity scale at lower right.

Fig. 4.4. Profiles of the measured turbulence intensity component \( \sqrt{\langle u^2 \rangle} \).
Series A1.1, case \( \omega_t/\omega_b = -2.2 \).
Series A1.1:

Width top lid: 30.0 cm
\( h = 29.7 \text{ cm} \)
\( \omega_b = -3.0 \text{ rpm} \)

Case:
\( \omega_t = 6.6 \text{ rpm} \)
\( \omega_t/\omega_b = -2.2 \)

inside wall at \( y = 0 \text{ mm} \),
outside wall at \( y = 304 \text{ mm} \),
zeros of the velocity profiles at different \( z \) are indicated at left,
intensity scale at lower right.

Fig. 4.5. Profiles of the measured turbulence intensity component \( \sqrt{(w')^2} \).
Series A1.1, case \( \omega_t/\omega_b = -2.2 \).
Remarkable is the maximum turbulence energy along the outside wall of the carousel. This maximum energy is caused by the flow curvature. Along a concave wall the flow can be shown to be more unstable leading to an enhanced turbulence production, whereas along the convex inner wall the flow is relatively stable, causing a relatively low turbulence level. From figures 4.2 and 4.3, where the components of the turbulence are given separately, it is obvious that in particular the tangential turbulence component is enhanced along a concave wall. This can be understood by considering the influence of the centrifugal force on water parcels with different turbulent tangential velocities. In figures 4.4 and 4.5 the turbulence intensities of both components are plotted.

Because of the differences between the flow fields and turbulence patterns in a carousel and in prototype free surface flows, in suspended sediment research results concerning the sediment distribution and sedimentation pattern, obtained in carousels, should be handled with care. In general sedimentation research in carousels will have to be limited to the investigation of the mechanisms of sediment transport and sedimentation. For the numerical computation of secondary flow patterns and sediment transport in carousels the special behaviour of the curved flow and in particular the turbulence characteristics along the concave wall have to be taken into account. To this end an advanced turbulence model (for example a Reynolds stress model) is required.
4.2 DISCUSSION OF THE RESULTS OF CAROUSEL FLOW MEASUREMENTS

Similarity

Strict similarity between carousel flows requires identical values of several non-dimensional numbers, see chapter 2.2. The most important requirement for the comparison of the flows in different carousels is equality of \( N_2 = b/R \), the relative curvature of the carousels. The other requirements, equality of \( N_1 = \omega_1/\omega_b \) (the ratio of the rotational velocities), \( N_2 = h/b \) (the aspect ratio of the flume) and \( N_3 = \omega_b R b/\nu \) (a Reynolds number of the carousel flow), are more easy to fulfil, as \( h \), \( \omega_b \) and \( \omega_1 \) are generally variable. The various carousels mentioned all show different relative curvatures:

- Mehta and Partheniades \( b/R \approx .27 \);
- Karelse \( b/R \approx .19 \);
- Krishnappan \( b/R \approx .12 \);
- This investigation \( b/R \approx .16 \).

Neglecting the viscous shear stresses removes the requirement of equal Reynolds numbers of the carousel flows, \( N_3 = \omega_b R b/\nu \). Equal relative curvature between carousels is still required, but now flows in the same carousel with different \( \omega_b \) can be considered similar as long as the requirements of equal ratio of rotational velocities, \( N_1 = \omega_1/\omega_b \), and of equal aspect ratio, \( N_2 = h/b \), are fulfilled. In this investigation this indeed leads to virtually similar flows and therewith to practically equal optimum conditions for \( \omega_1/\omega_b \) for different values of the rotational velocity of the flume of the carousel, \( \omega_b \):

- For series A1.1, A1.2, A1.3 (\( h = 297 \text{ mm} \)) the optimum is at \( \omega_1/\omega_b = -2.1 \) to -2.2;
- For series A2.1, A2.2, A2.3 (\( h = 197 \text{ mm} \)) the optimum is at \( \omega_1/\omega_b = -1.8 \).

A very small variation that can be observed in the optimum condition (see chapter 4.1) appears to be caused by a small influence of the viscous shear stresses remaining:

- In A1.2 for \( \omega_b = -4.5 \text{ rpm} \) the optimum can be estimated at \( \omega_1/\omega_b = -2.13 \);
- In A1.1 for \( \omega_b = -3.0 \text{ rpm} \) the optimum can be estimated at \( \omega_1/\omega_b = -2.17 \);
- In A1.3 for \( \omega_b = -1.5 \text{ rpm} \) the optimum can be estimated at \( \omega_1/\omega_b = -2.19 \).

This small influence can be understood considering equation (33) in chapter 2.5 and the discussion in chapter 2.3. Complete similarity requires equal \( c_f \), which means an equal relation between \( u_\ast \) and \( u_{qv} - u_w \). A smaller \( u_\ast \), due to smaller rotational velocities, leads to a larger \( a_0 \) (see chapter 2.3) and therewith to slightly smaller values for \( c_f \) and \( (u_{qv} - u_w)/u_\ast \). Because the values of \( u_{qv} - u_w \) and \( u_\ast \) along the flume are smaller than along the lid of the carousel, due to the larger surface area of the flume, this effect is stronger along the flume than along the lid. The larger decrease of \( c_f \) and the consequently relatively larger increase of the wall shear stress along the flume for smaller rotational velocities has to be
compensated for by a larger rotational velocity of the lid to yield the optimum flow situation. Consequently a slight increase of the $N_r = \omega_1/\omega_b$ for lower rotational velocities should indeed be observed.

The results of Mehta and Partheniades (1973a, 1973b), see fig. 1.2, show also nearly constant values of the optimum ratio of $\omega_1/\omega_b$ for different values of the rotational velocity of the flume of the carousel, $\omega_b$. Constant values would mean that for each water depth, $h$, the measuring points could be connected by a straight line through the origin of the plot. In fig. 4.6 this is checked for the results of Mehta and Partheniades. The assumption of straight lines is confirmed, but the lines do not go exactly through the origin. Although deviations of the linear relationship can be expected at low rotational velocities because of viscous effects, their influence at higher rotational speeds was shown in chapter 2.3 to be small, suggesting nearly similar flows and constant values for the optimum ratio of $\omega_1/\omega_b$. The deviation is probably due to the measuring method used by Mehta and Partheniades. The determination of the optimum ratio is based on the movement of beads. The beads have a diameter of about 1 cm, which is quite large compared to the boundary layer structure at the bottom, especially near optimum flow. The condition measured in this way is not completely representative for the flow at the bottom. Another effect can be the requirement of a certain velocity of the top lid $\omega_1$ to give a begin of movement for the beads. Mehta and Partheniades mention the measurement of the deposition of kaolinite. Equal deposition over the cross-section suggested a reduction with about 10% of the ratio for optimum flow. It is not clear

![Fig. 4.6 Linear relation between $\omega_1$ and $\omega_b$ at optimum ratio $\omega_1/\omega_b$ for the rotating annular flume of the University of Florida (Mehta and Partheniades, 1973a, 1973b).](image_url)
for how many and for which situations they executed these deposition experiments. If a constant value of about 1.5 rpm is subtracted from the top lid rotational velocity, which corresponds with a correction of about -10\% over an important part of the measured conditions, the measured optimum conditions lie almost on straight lines through the origin, except for those with a very small water depth.

**Deviation from geometric similarity**

Carousels differ generally in geometrical form. The carousels mentioned in this investigation have different relative curvatures $b/R$. Moreover the considered flows show a large variation in aspect ratio $h/b$. In chapter 2.4 it was shown that only very small variations in the aspect ratio are allowed for approximate similarity. This is confirmed by the result of this and other investigations (see fig. 4.7) were the optimum ratio of the rotational velocities is found to depend strongly on the flow depth. It was also shown in chapter 2.4 that differences in the relative curvature lead to less large deviations of similarity of the flows and therewith to less large differences in the optimum ratio of the rotational velocities.

Differences in relative curvature appear only between different carousels. The measuring methods and the criteria used in the determination of the optimum ratio of the rotational velocities differ however between the different investigators. A difference obtained between the optimum condition between flows in two carousels with equal aspect ratio can therefore not be attributed unambiguously to the difference in relative curvature.

![Fig. 4.7 Optimum ratio $\omega_i/\omega_b$ as a function of the aspect ratio $h/b$.](image_url)
The measured optimum flow conditions in fig. 4.7 show a similar, more or less linear, dependence on the aspect ratio for all investigations considered. The measurements of Karelse (1990) fit exactly to the measurements of this investigation. The difference of relative curvature between the carousels used in the two investigations is small (.19 and .16, see page 40). On the other side the optimum ratios found by Mehta and Partheniades (1973a and 1973b) (corrected for the bead size, see fig. 4.6) are slightly high compared to those of Karelse and this investigation. It is not clear whether this is an effect of the measuring method or of the stronger relative curvature. The carousel of Mehta and Partheniades had a much smaller radius and therewith a much stronger relative curvature \((b/R = .27)\). This stronger curvature may increase the influence of the flume, for example because of the higher turbulence level along the concave wall, which effect is not considered in the simplified analysis in chapter 2.5. For optimum flow this increased influence of the flume would then have to be compensated by a larger rotational velocity ratio.

Also given in fig. 4.7 is a curve based on the estimated values of the optimum ratio of rotational velocities from equation (44), with \(c_\lambda = .25\) (obtained by fitting of the optimum ratio in the standard case, series A1.1, case \(\omega_f/\omega_b = -2.2\)). This value for \(c_\lambda\) corresponds to the assumption of a lower cell that is in contact with about one quarter of the sidewalls in the standard case, which agrees well with the measurements in fig. A4. It is clear from figure 4.7 that the assumption of \(\lambda = c_\lambda (h/b)^{1/2}\) of chapter 2.5 leads to acceptable results for the aspect ratios used generally in carousel flow. For these aspect ratios the relation between the optimum ratio of rotational velocities and the aspect ratio can also be approximated (see fig. 4.7) by the linear relation

\[
\frac{\omega_f}{\omega_b} + 1 = -1.17 \frac{h}{b}
\]

(48)

For very small aspect ratios and for very large aspect ratios different relations were derived, equations (42) and (43), which correspond well to relation (48). Equation (42) is identical to equation (48) for \(\lambda = .46\), which is near to the .5 (equal influence) expected for very small aspect ratios.

However, the predictive possibilities of the equations (42-44) for the ratio of the rotational velocities \(\omega_f/\omega_b\) can be discussed. The measured flow fields at small aspect ratios, with two cells lying not above but slightly beside each other (see for example fig. A23) make it doubtful if the description used and the assumptions made in chapter 2.5 can be used there. Based on equation (44) equal magnitude of both cells \((\lambda = .5)\) would apply at about \(\omega_f/\omega_b = -1.4\) for series A1.1, whereas in reality, see the measured flow field of fig. A2, it is found to take place at about \(\omega_f/\omega_b = -1.9\).

**Central tangential velocity component and bottom shear stress**
<table>
<thead>
<tr>
<th>Case</th>
<th>ratio of rotational velocities</th>
<th>Estimated central velocity (m/s) eq.(35)</th>
<th>Measured velocity in flume centre (m/s)</th>
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<tr>
<td></td>
<td>$\omega/\omega_b$</td>
<td>$u_{av}\omega_b R$</td>
<td>$u_c\omega_b R$</td>
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<tr>
<td>A1.1</td>
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<td>.48</td>
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<td></td>
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<td>B3.1</td>
<td>-2.1</td>
<td>.60</td>
<td>.58</td>
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</tbody>
</table>

*Table 1: Estimated central velocity and measured velocity in the middle of the flume.*

44
Fig. 4.8  Vertical profiles of the tangential velocity component, $\bar{u}$, relative to the bottom. Series A1.1, case $\omega_1/\omega_b = -2.2$.

Fig. 4.9  Vertical profiles of the tangential velocity component, $\bar{u}$, relative to the bottom. Series A1.1, case $\omega_1/\omega_b = -3.1$. 

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The simplified analysis of the flow in carousels given in chapter 2.5 provides a method to obtain several relations in the flow when geometric similarity does not apply, for example the estimate of a tangential velocity component in the central region, $u_{av}$, by means of equation (35). In table I the velocity measured (directly or by interpolation) in exactly the centre of the flume, $u_c$, is compared to this estimated central velocity component, $u_{av}$. The flow cases given in the table are those in which the velocity in the centre of the flume is measured. For both velocities the difference with the bottom velocity (in the centre of the flume bottom), $u_c - \omega_b R$ and $u_{av} - \omega_b R$, is given as this last difference plays a part in the estimate

![Graph](image)

**Fig. 4.10** Semi logarithmic plot of vertical profiles of the tangential velocity component, $\bar{u}$, relative to the bottom, for the determination of the bottom shear stress.

Series A1.1, case $\omega/\omega_b = -2.2$.  

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<table>
<thead>
<tr>
<th>Case</th>
<th>$h, \omega_b$</th>
<th>ratio of rot. velocities $\omega/\omega_b$</th>
<th>Estimated shear stress $(10^{-3} \text{ m}^2/\text{s}^2)$</th>
<th>Measured shear stress $(10^{-3} \text{ m}^2/\text{s}^2)$</th>
<th>Shear stress from profile method $(10^{-3} \text{ m}^2/\text{s}^2)$</th>
</tr>
</thead>
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<tr>
<td>A1.1</td>
<td>$h=297\text{mm}$; $\omega_b=-3.0$</td>
<td>-1.9 \hspace{1em} -2.0 \hspace{1em} -2.1</td>
<td>.61 \hspace{1em} .65 \hspace{1em} .69</td>
<td>.48 \hspace{1em} .56 \hspace{1em} .61</td>
<td>.53 \hspace{1em} .54 \hspace{1em} .56</td>
</tr>
<tr>
<td>A2.1</td>
<td>$h=197\text{mm}$; $\omega_b=-3.0$</td>
<td>-1.6 \hspace{1em} -1.7 \hspace{1em} -1.8</td>
<td>.57 \hspace{1em} .62 \hspace{1em} .68</td>
<td>.50 \hspace{1em} .58 \hspace{1em} .67</td>
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</tr>
<tr>
<td>A2.2</td>
<td>$h=197\text{mm}$; $\omega_b=-4.5$</td>
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<td>A2.3</td>
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<td>$h=157\text{mm}$; $\omega_b=-3.0$</td>
<td>-1.4 \hspace{1em} -1.5 \hspace{1em} -1.6</td>
<td>.53 \hspace{1em} .58 \hspace{1em} .62</td>
<td>.60 \hspace{1em} .60 \hspace{1em} .62</td>
<td>.56 \hspace{1em} .54 \hspace{1em} .56</td>
</tr>
<tr>
<td>B1.1</td>
<td>$h=297\text{mm}$; $\omega_b=-3.0$; half lid</td>
<td>-2.7 \hspace{1em} -2.8 \hspace{1em} -2.9</td>
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</tr>
<tr>
<td>B2.1</td>
<td>$h=197\text{mm}$; $\omega_b=-3.0$; half lid</td>
<td>-2.0 \hspace{1em} -2.1 \hspace{1em} -2.2</td>
<td>.49 \hspace{1em} .52 \hspace{1em} .56</td>
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<tr>
<td>B3.1</td>
<td>$h=157\text{mm}$; $\omega_b=-3.0$; half lid</td>
<td>-1.8 \hspace{1em} -1.9 \hspace{1em} -2.0</td>
<td>.46 \hspace{1em} .50 \hspace{1em} .53</td>
<td>.59 \hspace{1em} .61 \hspace{1em} .65</td>
<td>.27 \hspace{1em} .45 \hspace{1em} .54</td>
</tr>
</tbody>
</table>

Table II  Comparison between the bottom shear stress estimated by means of equation (33), the measured bottom shear stress and the bottom shear stress obtained with the profile method. (The rotational velocities are in rpm.)
of the bottom shear stress. The estimated velocity appears to be more or less correct. A
difference of up to about 10% is found, where the estimated value is always higher than the
measured one. This difference can be understood in the following way. The estimation
method is based on an assumed constant tangential velocity in the central part of the flow.
However in reality the velocity increases towards the outer wall (see fig. 4.8) and this
increase is not linear over the field but is slightly faster towards the outer wall. This yields
a tangential velocity component in the middle of the flume that is below the average velocity
in the central part of the flume. This effect is often amplified by a convection of faster
flowing fluid from the lid around the central part where the tangential velocity component
is more influenced by the bottom and inside wall (see fig. 4.9). It can be concluded that the
estimated central velocity component, $u_{av}$, may yield a useful value for the average tangential

![Graph showing the relationship between bottom shear stress and velocity difference](image)

**Fig. 4.11** Relation between the bottom shear stress $q_{z0}$ and the velocity difference $U_1$
between the lid and the flume of the carousel. Measurements by Mehta and Partheniades (1973a and 1973b).

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velocity component in the central part of the flume, at least for a flow not to far from the optimum condition, and even better than is suggested by comparison with the velocity, $u_c$, measured in exactly the middle of the flume.

The bottom shear stress, which is important for erosion studies, per unit mass can be estimated with equation (33), where for $u_{ew}$ the estimated central velocity component can be used and for $u_w$ the bottom velocity (in the middle of the flume), $\omega_p R$. In equation (33) $c_s$ is nearly constant. When the bottom can be considered hydraulically smooth, its value for the used carousels and rotational velocities can be taken as about 25. In table II the shear stresses

![Graph](image)

**Fig. 4.12** Relation between the bottom shear stress $q_{es}$ and the velocity difference $U_j$ between the lid and the flume of the carousel. Series A2.1 to A2.3.
estimated in this way are compared with the directly measured shear stresses \( q_{ob} = \bar{w'}/u' \) and with the shear stresses which can be obtained from the tangential velocity profiles directly above the bottom, see fig. 4.10. The flow cases given in the table are those in which measurements near the bottom with the inclined probe were executed.

The measured shear stresses appear to be sufficiently reliable. The values in table II are the average values of the measured shear stress values over the middle half of the width and between 2 mm and 1 cm from the bottom. The variation of the measured values in this region is not very large (see appendix C). The agreement between the estimated shear stress values and the measured values is fair, considering the very rough estimation method used. The difference is generally of the order of 10% and is partly positive and partly negative. The only consistent larger differences found are obtained for the larger \( \omega_1/\omega_2 \) ratios of the deep flow case (series A1.1 with \( h = 29.7 \text{ cm} \)), where larger values of the shear stress are presumably present in reality along the outside wall. The estimated value, which gives an average value over the complete flume, will then be high compared to the measured bottom shear stress. Another case where large differences are found is the very shallow flow case with half lid (case B3.1 with \( h = 15.7 \text{ cm} \)), where the flow is very distorted and high velocity flow can reach the middle section of the bottom. The profile method gives less reliable values. Deviations can originate in several ways. In the first place there can be a difficulty to determine the exact distance to the bottom. A small correction in the exact place of the bottom can lead to an important change in the obtained bottom friction velocity. In the second place the thickness of that part of the bottom boundary layer in which a undisturbed logarithmic layer is formed is generally small because of the convection by the secondary flow and the influence of the corners. Often this thickness varies also over the width. This small logarithmic layer makes the determination of the bottom friction velocity problematical. Moreover the bottom shear stress is obtained from the square of the bottom friction velocity, leading to a relative error in the bottom shear stress that is twice as large as the error in the bottom friction velocity.

Measurements of the bottom shear stress by Mehta and Partheniades (1973a and 1973b) with a preston tube did yield relations between the bottom shear stress and the velocity difference between the lid and the flume, \( U_f \), of the form

\[
q_{ob} = c_r U_f^\gamma
\]  

(49)

where \( c_r \) is a constant and the exponent \( \gamma \) is for different depths of the order of 1.15 for \( h = 6 \text{ inch} \) to 1.37 for \( h = 12 \text{ inch} \) (see fig. 4.11). The estimation method used here leads to \( \gamma = 2.0 \), which value is confirmed in the only measurement series in which a sufficient variation of \( U_f \) is obtained (series A2.1 to A2.3), see fig. 4.12. The low value of \( \gamma \) found by Mehta and Partheniades is probably due to the measuring method. Measurement of the bed shear stress by means of a preston tube does presumably give too large errors because of the small thickness of the logarithmic bottom layer, in particular for flows far from the
optimum condition. This explains also the larger deviation from $\gamma = 2.0$ for shallower flumes. Mehta and Partheniades expressed also some doubt when discussing the measuring method. The value $\gamma = 2.0$ is also better understandable for dimensional reasons. Even if different values would be correct, then they would only apply for a narrow field of flow cases.

The estimation method for the bed shear stress appears to give sufficiently reliable estimates of the bed shear stress to be useful for erosion predictions, especially around the optimum condition for the secondary flow.
7 CONCLUSIONS

The following conclusions can be drawn from the present study:

1. The experimental results show that optimal values of $\omega_f/\omega_b$ for the carousel of Delft University of Technology are:
   - Based on lowest secondary flow velocities (in radial direction) near the bottom (criterion I):
     (A) width top lid is 0.30 m: $d = 29.7$ cm : $\omega_f/\omega_b = 2.1$ to $2.2$
     $d = 19.7$ cm : $\omega_f/\omega_b = 1.8$
     $d = 15.7$ cm : $\omega_f/\omega_b = 1.6$
     (B) width top lid is 0.15 m: $d = 29.7$ cm : $\omega_f/\omega_b = 3.0$
     $d = 19.7$ cm : $\omega_f/\omega_b = 2.3$
     $d = 15.7$ cm : $\omega_f/\omega_b = 2.1$
   - Above values become of the order of 10% lower if the criterion of uniformity of turbulent (near)bottom shear stresses (criterion II) is applied. The uniformity across the flume width of the tangential velocities $\bar{u}$ is also better for these 10% lower values of $\omega_f/\omega_b$.

2. Even in situations with optimal values for $\omega_f/\omega_b$, secondary flow velocities never become much smaller than the fall velocities of fine sediment, and in many cases are much larger. This has implications for future experimental work on the deposition of fine sediment in rotating annular flumes. The use of carousels for studies of the distribution of suspended sediment in a free surface flow or the deposition of that sediment on the bottom appears to be limited. It is not concluded here that these experiments are impossible in rotating annular flumes, but the complex 3-D velocity pattern should be taken into account when making such experiments.

3. The tangential velocities and near bottom shear stresses are almost uniformly distributed across the flume width in situations with optimal (criterion II) values for $\omega_f/\omega_b$.

4. The near-bottom shear stresses are not very sensitive for the value of $\omega_f/\omega_b$. This and conclusions 1 and 3 indicate that a rotating annular flume is well suited to experiments on erosion and entrainment of fine sediment.

5. The turbulence pattern and with it the transport coefficients, for example the turbulence diffusion coefficient, deviate appreciably from the pattern and the coefficients in free surface flow in rivers or channels, which restricts the usefulness of the carousel flow as a model for free surface flow.
6. In the optimum situations, the smaller top lid does not produce significantly larger radial flow velocities near the bottom than the wider top lid. Also the uniformity in radial direction of the turbulent shear stress and the tangential velocity, both in the near-bottom region, are comparable for both top lid widths.

7. Similarity considerations leading to the notion that the optimal $\omega_1/\omega_b$ is independent of $\omega_1$ and $\omega_b$ for the same water depth and only slightly dependent on the relative curvature $b/R$ are confirmed by the experiments.

8. A simple relation between the optimum ratio of the rotational velocities $\omega_1/\omega_b$ (based on criterion 1) and the aspect ratio is determined experimentally.

9. A simplified analysis of the flow in carousels provides useful estimates for the bottom shear stress, for the average velocity in the central part of the flume, and for the optimum ratio of $\omega_1/\omega_b$ (based on criterion 1). These estimates show a fair agreement with the experimental results.

10. The values of the power $\gamma$ in the relation between the bottom shear stress and the velocity difference between lid and flume reported by Mehta and Partheniades (eq. 49) deviate strongly from the single value obtained in this investigation, based both on experimental evidence and on theoretical considerations. The lower values of $\gamma$ found by Mehta and Partheniades is probably due to their measuring method, in which a Preston tube was used.

11. The smooth operation and reliability of the DANTEC optical system and processor (FVA) allowed automatic measuring procedures (Booij et al., 1993). Without these it would have been impossible to execute the large amount of measurements in a reasonable time.

12. The FVA did yield sufficiently accurate results to be able to extract the time-averaged radial velocity component from the measured time-averaged vertical velocity component, applying the continuity equation for axisymmetric flow.
ACKNOWLEDGEMENT

The author would like to thank L.M.M. Melis for the accurate execution of the many measurements and the data processing, P.J. Visser for the overview of the previous investigations of carousel flow and the staff of the Laboratory for Fluid Mechanics for the skilful construction of the carousel and preparation of the measuring set up and for the enthusiastic support and fruitful discussions during the measurements and the preparation of this manuscript.
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NOTATION

\( a \)  distance to the wall
\( a_0 \)  roughness length
\( A_c \)  surface area of the flume
\( A_l \)  surface area of the lid
\( b \)  width of the flume
\( c, c_i \)  (nearly) constants in velocity-friction velocity relations
\( c_\lambda \)  constant in contact surface between lower cell and walls
\( c_r \)  constant in the relation between bottom shear stress and the velocity
difference between lid and flume
\( f_{\phi}, f_r, f_z \)  components of external body forces per unit of mass
\( h \)  height of the water column (water depth)
\( I \)  ratio of influences of lid and flume
\( k \)  measured turbulence energy
\( N_J, N_2, N_3, N_4 \)  non-dimensional numbers
\( p \)  deviation of hydrostatic pressure
\( p_\phi, p_r, p_z \)  components of \( p \)
\( P \)  fluid pressure
\( q_{\phi\phi}, q_{rr}, q_{zz} \)  normal Reynolds or turbulence stresses
\( q_{\phi r}, q_{rz}, q_{z\phi} \)  Reynolds or turbulence shear stresses
\( r \)  radial co-ordinate
\( r_i \)  radial co-ordinate of the inside wall of the flume
\( R \)  mean radius of the flume
\( t \)  time
\( t_p \)  time for a parcel of water to pass the lid
\( u \)  velocity component in tangential direction
\( u_{av} \)  tangential velocity in central region of flume
\( u_c \)  measured velocity in middle of flume
\( u_w \)  velocity of the wall
\( u_* \)  friction velocity
\( U_J \)  velocity difference between bottom and lid in centre of flume
\( v \)  velocity component in radial direction
\( v_{cr} \)  cross-flow velocity component
\( v_{cr,t} \)  cross-flow velocity component of the upper cell
\( v_{cr,b} \)  cross-flow velocity component of the lower cell
\( w \)  velocity component in vertical direction
\( z \)  vertical co-ordinate
\( z_b \)  vertical co-ordinate of the bottom
\[ \alpha \] tilt angle of probe
\[ \beta \] ratio of surface areas of flume and lid
\[ \gamma \] exponent in the relation between bottom shear stress and the velocity difference between lid and flume
\[ \delta_t \] thickness of turbulent boundary layer
\[ \delta_v \] thickness of viscous sublayer
\[ \kappa \] Von Karman constant
\[ \lambda_1, \lambda_b \] constant in contact surface between lower cell and walls
\[ \mu_t \] turbulent eddy viscosity
\[ \nu \] kinematic viscosity
\[ \rho \] fluid density
\[ \tau \] wall shear stress
\[ \tau_{cr} \] cross component of wall shear stress
\[ v \] velocity in local flow direction at distance \( a \) from wall
\[ v_w \] velocity of wall in local flow direction at distance \( a \) from wall
\[ \phi \] tangential co-ordinate
\[ \psi \] angle between local flow direction and tangential direction
\[ \omega_b \] rotational velocity of flume
\[ \omega_l \] rotational velocity of lid

\[ \cdot' \] (prime) turbulent fluctuation
\[ \overline{\cdot} \] (overbar) time averaged value
bold symbol non-dimensional variable
APPENDIX A

VECTOR PLOTS OF SECONDARY FLOW IN A CROSS-SECTION

The measured secondary velocity data are collected in the vectorplots of this appendix. The data on which these vectorplots are based are reproduced in tabular form in Booij and Melis (1994). The information is also obtainable on floppy discs.

If for the same flow case measurements were executed with horizontal and inclined probe, the information is combined in one vectorplot. This is the case in
- series A1.1, cases $\omega_f/\omega_b = -1.9, -2.1, -2.2, -2.3$
- series A2.1, cases $\omega_f/\omega_b = -1.7$ and $-1.8$
- series A2.2 and series A2.3, case $\omega_f/\omega_b = -1.8$

Only measurements with a horizontal probe were executed in
- series A1.1, cases $\omega_f/\omega_b = -1.5, -2.3, -2.7, -3.1$
- series A1.2 and A1.3, all cases
- series A2.1, case $\omega_f/\omega_b = -1.9$
- series A2.2 and series A2.3, case $\omega_f/\omega_b = -1.7$ and $-1.9$

Only measurements with an inclined probe were executed in
- series A1.1, cases $\omega_f/\omega_b = -2.0$
- series A2.1, case $\omega_f/\omega_b = -1.6$
- series A3.1, all cases
- all measurements with smaller top lid (cluster B)

The plots have all the same outline: the walls, bottom, lid (and free surface in cluster B) are represented by lines, at the left is the inner side of the flume, at the right the outer side. The secondary velocity vectors are indicated by arrows, which point away from the measuring points. The measure of the secondary velocity arrows is given at right below the vectorplots.

The radial coordinate of the measuring points for the inclined probe and the horizontal probe do not correspond generally. The transition between the velocities measured with the horizontal probe and with the inclined probe is not always completely smooth. This indicates the sensitivity of the flow pattern for the exact flow situation, which can have been slightly different because the measurements with horizontal and with inclined probe were executed in different periods. A same flow case had to be set up separately for both arrangements.
Series A1.1: Width top lid: 30.0 cm
h = 29.7 cm
$\omega_b = -3.0$ rpm

$\rightarrow = 0.05 \text{ m/s}$

Case:
$\omega_t = 4.5$ rpm
$\omega_t / \omega_b = -1.5$

Fig. A1. Vectorplot of secondary flow in a cross-section. Series A1.1, case $\omega_t / \omega_b = -1.5$. 

A2
Series A1.1: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 5.7 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.9 \]

Fig. A2. Vectorplot of secondary flow in a cross-section. Series A1.1, case $\omega_t/\omega_b = -1.9$. 

A3
Series A1.1: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_i = 6.0 \text{ rpm} \]
\[ \omega_i/\omega_b = -2.0 \]

Fig. A3. Vectorplot of secondary flow in a cross-section. Series A1.1, case \( \omega_i/\omega_b = -2.0 \).
Series A1.1: Width top lid: 30.0 cm
    \( h = 29.7 \text{ cm} \)
    \( \omega_b = -3.0 \text{ rpm} \)

\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
    \( \omega_t = 6.3 \text{ rpm} \)
    \( \omega_t/\omega_b = -2.1 \)

Fig. A4. Vectorplot of secondary flow in a cross-section. Series A1.1, case \( \omega_t/\omega_b = -2.1 \).
Series A1.1: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_l = 6.6 \text{ rpm} \]
\[ \omega_l/\omega_b = -2.2 \]

Fig. A5. Vectorplot of secondary flow in a cross-section. Series A1.1, case \( \omega_l/\omega_b = -2.2 \).
Series A1.1: Width top lid: 30.0 cm
h = 29.7 cm
ω_b = -3.0 rpm

Case:
ω_t = 6.9 rpm
ω_t/ω_b = -2.3

→ = 0.05 m/s

Fig. A6. Vectorplot of secondary flow in a cross-section. Series A1.1, case ω_t/ω_b = -2.3.
Series A1.1: Width top lid: 30.0 cm
h = 29.7 cm
ω_b = -3.0 rpm

→ = 0.05 m/s

Case:
ω_l = 8.1 rpm
ω_l/ω_b = -2.7

Fig. A7. Vectorplot of secondary flow in a cross-section. Series A1.1, case ω_l/ω_b = -2.7.
Series A1.1: Width top lid: 30.0 cm
\begin{align*}
h &= 29.7 \text{ cm} \\
\omega_b &= -3.0 \text{ rpm} \\
\rightarrow &= 0.05 \text{ m/s}
\end{align*}

Case:
\begin{align*}
\omega_t &= 9.3 \text{ rpm} \\
\omega_t/\omega_b &= -3.1
\end{align*}

Fig. A8. Vectorplot of secondary flow in a cross-section. Series A1.1, case \( \omega_t/\omega_b = -3.1 \).
Series A1.2: Width top lid: 30.0 cm

\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]

Case:
\[ \omega_i = 9.0 \text{ rpm} \]
\[ \omega_i/\omega_b = -2.0 \]

Fig. A9. Vectorplot of secondary flow in a cross-section. Series A1.2, case \( \omega_i/\omega_b = -2.0 \).
Series A1.2: Width top lid: 30.0 cm
    \[ h = 29.7 \text{ cm} \]
    \[ \omega_b = -4.5 \text{ rpm} \]
    \[ \rightarrow = 0.05 \text{ m/s} \]

Case:
    \[ \omega_i = 9.45 \text{ rpm} \]
    \[ \omega_i/\omega_b = -2.1 \]

Fig. A10. Vectorplot of secondary flow in a cross-section. Series A1.2, case \( \omega_i/\omega_b = -2.1 \).
Series A1.2: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 9.9 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.2 \]

Fig. A11. Vectorplot of secondary flow in a cross-section. Series A1.2, case $\omega_t/\omega_b = -2.2$. 

A12
Series A1.3: Width top lid: 30.0 cm
\[
\begin{align*}
h &= 29.7 \text{ cm} \\
\omega_b &= -1.5 \text{ rpm} \\
\omega_t/\omega_b &= -2.1 \\
\rightarrow &= 0.05 \text{ m/s}
\end{align*}
\]

Case: \[\omega_t = 3.15 \text{ rpm}\]

Fig. A12. Vectorplot of secondary flow in a cross-section. Series A1.3, case \(\omega_t/\omega_b = -2.1\).
Series A1.3: Width top lid: 30.0 cm

\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -1.5 \text{ rpm} \]
\[ \longrightarrow = 0.05 \text{ m/s} \]

Case:

\[ \omega_l = 3.3 \text{ rpm} \]
\[ \omega_l/\omega_b = -2.2 \]

Fig. A13. Vectorplot of secondary flow in a cross-section. Series A1.3, case \( \omega_l/\omega_b = -2.2 \).
Series A1.3: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -1.5 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 3.45 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.3 \]

Fig. A14. Vectorplot of secondary flow in a cross-section. Series A1.3, case \( \omega_t/\omega_b = -2.3 \).
Series A2.1: Width top lid: 30.0 cm
\[ h = 19.7 \, \text{cm} \]
\[ \omega_b = -3.0 \, \text{rpm} \]
\[ \rightarrow = 0.05 \, \text{m/s} \]

Case:
\[ \omega_t = 4.8 \, \text{rpm} \]
\[ \omega_t/\omega_b = -1.6 \]

Fig. A15. Vectorplot of secondary flow in a cross-section. Series A2.1, case \( \omega_t/\omega_b = -1.6 \).
Series A2.1: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_i = 5.1 \text{ rpm} \]
\[ \omega_i/\omega_b = -1.7 \]

Fig. A16. Vectorplot of secondary flow in a cross-section. Series A2.1, case \( \omega_i/\omega_b = -1.7 \).
Series A2.1: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 5.4 \text{ rpm} \]
\[ \omega_t / \omega_b = -1.8 \]

Fig. A17. Vectorplot of secondary flow in a cross-section. Series A2.1, case \( \omega_t / \omega_b = -1.8 \).
Series A2.1: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \] \[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_l = 5.7 \text{ rpm} \]
\[ \omega_l/\omega_b = -1.9 \]

Fig. A18. Vectorplot of secondary flow in a cross-section. Series A2.1, case \( \omega_l/\omega_b = -1.9 \).
Series A2.2: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_i = 7.65 \text{ rpm} \]
\[ \omega_i/\omega_b = -1.7 \]

Fig. A19. Vectorplot of secondary flow in a cross-section. Series A2.2, case \( \omega_i/\omega_b = -1.7 \).
Series A2.2: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]
\[ \vec{\text{----}} = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 8.1 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.8 \]

Fig. A20. Vectorplot of secondary flow in a cross-section. Series A2.2, case \( \omega_t/\omega_b = -1.8 \).
Series A2.2: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 8.55 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.9 \]

Fig. A21. Vectorplot of secondary flow in a cross-section. Series A2.2, case \( \omega_t/\omega_b = -1.9 \).
Series A2.3: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -1.5 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 2.55 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.7 \]

Fig. A22. Vectorplot of secondary flow in a cross-section. Series A2.3, case \( \omega_t/\omega_b = -1.7 \).
Series A2.3: Width top lid: 30.0 cm
h = 19.7 cm
ω_b = -1.5 rpm

Case:
ω_t = 2.7 rpm
ω_t/ω_b = -1.8

Fig. A23. Vectorplot of secondary flow in a cross-section. Series A2.3, case ω_t/ω_b = -1.8.
Series A2.3: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -1.5 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_l = 2.85 \text{ rpm} \]
\[ \omega_l/\omega_b = -1.9 \]

Fig. A24. Vectorplot of secondary flow in a cross-section. Series A2.3, case \( \omega_l/\omega_b = -1.9 \).
Series A3.1: Width top lid: 30.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 4.2 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.4 \]

Fig. A25. Vectorplot of secondary flow in a cross-section. Series A3.1, case \( \omega_t/\omega_b = -1.4 \).
Series A3.1: Width top lid: 30.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow \quad = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 4.5 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.5 \]

Fig. A26. Vectorplot of secondary flow in a cross-section. Series A3.1, case \( \omega_t/\omega_b = -1.5 \).
Series A3.1: Width top lid: 30.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_i = 4.8 \text{ rpm} \]
\[ \omega_i/\omega_b = -1.6 \]

Fig. A27. Vectorplot of secondary flow in a cross-section. Series A3.1, case \( \omega_i/\omega_b = -1.6 \).
Series A3.1: Width top lid: 30.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 5.1 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.7 \]

Fig. A28. Vectorplot of secondary flow in a cross-section. Series A3.1, case $\omega_t/\omega_b = -1.7$. 
Series B1.1: Width top lid: 15.0 cm
$h = 29.7$ cm
$\omega_b = -3.0$ rpm

Case:
$\omega_i = 8.4$ rpm
$\omega_i/\omega_b = -2.8$

Fig. A29. Vectorplot of secondary flow in a cross-section. Series B1.1, case $\omega_i/\omega_b = -2.8$. 

A30
Series B1.1: Width top lid: 15.0 cm
   \( h = 29.7 \text{ cm} \)
   \( \omega_b = -3.0 \text{ rpm} \)

Case:
   \( \omega_i = 8.7 \text{ rpm} \)
   \( \omega_i/\omega_b = -2.9 \)

\[ \rightarrow = 0.05 \text{ m/s} \]

Fig. A30. Vectorplot of secondary flow in a cross-section. Series B1.1, case \( \omega_i/\omega_b = -2.9 \).
Series B1.1: Width top lid: 15.0 cm
    \( h = 29.7 \text{ cm} \)
    \( \omega_b = -3.0 \text{ rpm} \)
    \[ \rightarrow = 0.05 \text{ m/s} \]

Case:
    \( \omega_r = 9.0 \text{ rpm} \)
    \( \omega_r/\omega_b = -3.0 \)

Fig. A31. Vectorplot of secondary flow in a cross-section. Series B1.1, case \( \omega_r/\omega_b = -3.0 \).
Series B2.1: Width top lid: 15.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case: \[ \omega_t = 6.0 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.0 \]

Fig. A32. Vectorplot of secondary flow in a cross-section. Series B2.1, case \( \omega_t/\omega_b = -2.0 \).
Series B2.1: Width top lid: 15.0 cm
h = 19.7 cm
$\omega_b = -3.0$ rpm

$\rightarrow = 0.05$ m/s

Case:
$\omega_i = 6.3$ rpm
$\omega_i/\omega_b = -2.1$

Fig. A33. Vectorplot of secondary flow in a cross-section. Series B2.1, case $\omega_i/\omega_b = -2.1$. 
Series B2.1: Width top lid: 15.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_i = 6.6 \text{ rpm} \]
\[ \omega_i/\omega_b = -2.2 \]

Fig. A34. Vectorplot of secondary flow in a cross-section. Series B2.1, case \( \omega_i/\omega_b = -2.2 \).
Series B2.1: Width top lid: 15.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case: \[ \omega_t = 6.9 \text{ rpm} \]
\[ \omega_t / \omega_b = -2.3 \]

Fig. A35. Vectorplot of secondary flow in a cross-section. Series B2.1, case \( \omega_t / \omega_b = -2.3 \).
Series B3.1: Width top lid: 15.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 5.4 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.8 \]

Fig. A36. Vectorplot of secondary flow in a cross-section. Series B3.1, case \( \omega_t/\omega_b = -1.8 \).
Series B3.1: Width top lid: 15.0 cm  
\[ h = 15.7 \text{ cm} \]  
\[ \omega_b = -3.0 \text{ rpm} \]  
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:  
\[ \omega_t = 5.7 \text{ rpm} \]  
\[ \omega_t/\omega_b = -1.9 \]

Fig. A37. Vectorplot of secondary flow in a cross-section. Series B3.1, case \( \omega_t/\omega_b = -1.9 \).
Series B3.1: Width top lid: 15.0 cm
h = 15.7 cm
$\omega_b = -3.0 \text{ rpm}$

Case:
$\omega_i = 6.0 \text{ rpm}$
$\omega_i/\omega_b = -2.0$

Fig. A38. Vectorplot of secondary flow in a cross-section. Series B3.1, case $\omega_i/\omega_b = -2.0$. 
Series B3.1: Width top lid: 15.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]
\[ \rightarrow = 0.05 \text{ m/s} \]

Case:
\[ \omega_t = 6.3 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.1 \]

Fig. A39. Vectorplot of secondary flow in a cross-section. Series B3.1, case \( \omega_t/\omega_b = -2.1 \).
APPENDIX B

HORIZONTAL PROFILES OF TANGENTIAL VELOCITY COMPONENT

This appendix contains plots of horizontal profiles of the tangential velocity component. Only the measurements with a horizontal probe are included. The data on which these vectorplots are based are reproduced in tabular form in Booij and Melis (1994). The information is also obtainable on floppy discs.

The measured data points in the plots are connected by straight lines. The zero levels of the tangential velocity for the various profiles are indicated in the plots at the left side and the scale of the tangential velocity at the lower right side. The left side ($y = 0$ mm) represents the inner side of the flume and the right side ($y = 304$ mm) the outer side. The width of the flume is not exactly 304 mm and the sidewalls are not exactly vertical, partly because of the influence of the water pressure on the glass walls.

The horizontal plots of the tangential velocity component are mainly interesting in connection with the uniformity of the flow over the carousel width and the influence of the side wall shear stress and the convection by the secondary flow on this velocity component.

The second criterion for the optimum value of the ratio $\omega_t/\omega_h$ of a rotating annular flume, the uniformity in radial direction of the tangential velocity component near the bottom and in particular the bottom shear stress, which is virtually equal to the Reynolds shear stress $q_{z\phi}$ directly above the bottom, can be of importance for erosion studies.
Series A1.1: Width top lid: 30.0 cm
   \[ h = 29.7 \text{ cm} \]
   \[ \omega_b = -3.0 \text{ rpm} \]

Case:
   \[ \omega_t = 4.5 \text{ rpm} \]
   \[ \omega_t / \omega_b = -1.5 \]

Fig. B1. Profiles of the tangential velocity component. Series A1.1, case \( \omega_t / \omega_b = -1.5 \).
Series A1.1: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case:
\[ \omega_t = 5.7 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.9 \]

Fig. B2. Profiles of the tangential velocity component. Series A1.1, case \( \omega_t/\omega_b = -1.9 \).
Series A1.1: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case:
\[ \omega_t = 6.3 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.1 \]

Fig. B3. Profiles of the tangential velocity component. Series A1.1, case $\omega_t/\omega_b = -2.1$. 
Series A1.1: Width top lid: 30.0 cm
   \( h = 29.7 \text{ cm} \)
   \( \omega_b = -3.0 \text{ rpm} \)

Case:
   \( \omega_t = 6.6 \text{ rpm} \)
   \( \omega_t/\omega_b = -2.2 \)

Fig. B4. Profiles of the tangential velocity component. Series A1.1, case \( \omega_t/\omega_b = -2.2 \).
Series A1.1: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case:
\[ \omega_t = 6.9 \text{ rpm} \]
\[ \omega_t / \omega_b = -2.3 \]

Fig. B5. Profiles of the tangential velocity component. Series A1.1, case \( \omega_t / \omega_b = -2.3 \).
Series A1.1: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case: \[ \omega_t = 8.1 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.7 \]

Fig. B6. Profiles of the tangential velocity component. Series A1.1, case \( \omega_t/\omega_b = -2.7 \).
Series A1.1: Width top lid: 30.0 cm
    h = 29.7 cm
    $\omega_b = -3.0 \text{ rpm}$

Case:
    $\omega_t = 9.3 \text{ rpm}$
    $\omega_t/\omega_b = -3.1$

Fig. B7. Profiles of the tangential velocity component. Series A1.1, case $\omega_t/\omega_b = -3.1$. 
Series A1.2: Width top lid: 30.0 cm
h = 29.7 cm
ω_b = -4.5 rpm

Case: ω_t = 9.0 rpm
ω_t/ω_b = -2.0

Fig. B8. Profiles of the tangential velocity component. Series A1.2, case ω_t/ω_b = -2.0.
Series A1.2: Width top lid: 30.0 cm
  \( h = 29.7 \text{ cm} \)
  \( \omega_b = -4.5 \text{ rpm} \)

Case:
  \( \omega_t = 9.45 \text{ rpm} \)
  \( \omega_t/\omega_b = -2.1 \)

Fig. B9. Profiles of the tangential velocity component. Series A1.2, case \( \omega_t/\omega_b = -2.1 \).
Series A1.2: Width top lid: 30.0 cm
   \[ h = 29.7 \text{ cm} \]
   \[ \omega_b = -4.5 \text{ rpm} \]

Case:
   \[ \omega_i = 9.9 \text{ rpm} \]
   \[ \omega_i/\omega_b = -2.2 \]

Fig. B10. Profiles of the tangential velocity component. Series A1.2, case \( \omega_i/\omega_b = -2.2 \).
Series A1.3: Width top lid: 30.0 cm
  \( h = 29.7 \text{ cm} \)
  \( \omega_b = -1.5 \text{ rpm} \)

Case:
  \( \omega_t = 3.15 \text{ rpm} \)
  \( \omega_t/\omega_b = -2.1 \)

Fig. B11. Profiles of the tangential velocity component. Series A1.3, case \( \omega_t/\omega_b = -2.1 \).
Series A1.3: Width top lid: 30.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -1.5 \text{ rpm} \]

Case:
\[ \omega_t = 3.3 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.2 \]

Fig. B12. Profiles of the tangential velocity component. Series A1.3, case $\omega_t/\omega_b = -2.2$. 

B13
Series A1.3: Width top lid: 30.0 cm
  \[ h = 29.7 \text{ cm} \]
  \[ \omega_b = -1.5 \text{ rpm} \]

Case:
  \[ \omega_t = 3.45 \text{ rpm} \]
  \[ \omega_t / \omega_b = -2.3 \]

Fig. B13  Profiles of the tangential velocity component. Series A1.3, case \( \omega_t / \omega_b = -2.3 \).
Series A2.1: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case: \[ \omega_t = 5.1 \text{ rpm} \]
\[ \omega_t / \omega_b = -1.7 \]

Fig. B14. Profiles of the tangential velocity component. Series A2.1, case \( \omega_t / \omega_b = -1.7 \).
Series A2.1: Width top lid: 30.0 cm
h = 19.7 cm
ω_b = -3.0 rpm

Case:
ω_t = 5.4 rpm
ω_t/ω_b = -1.8

Fig. B15. Profiles of the tangential velocity component. Series A2.1, case ω_t/ω_b = -1.8.
Series A2.1: Width top lid: 30.0 cm
h = 19.7 cm
$\omega_b = -3.0$ rpm

Case:
$\omega_t = 5.7$ rpm
$\omega_t/\omega_b = -1.9$

Fig. B16. Profiles of the tangential velocity component. Series A2.1, case $\omega_t/\omega_b = -1.9$. 

B17
Series A2.2: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]

Case:
\[ \omega_t = 7.65 \text{ rpm} \]
\[ \omega_t / \omega_b = -1.7 \]

Fig. B17. Profiles of the tangential velocity component. Series A2.2, case $\omega_t / \omega_b = -1.7$. 

B18
Series A2.2: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]

Case:
\[ \omega_i = 8.1 \text{ rpm} \]
\[ \omega_i/\omega_b = -1.8 \]

Fig. B18. Profiles of the tangential velocity component. Series A2.2, case \( \omega_i/\omega_b = -1.8 \).
Series A2.2: Width top lid: 30.0 cm

\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]

Case:
\[ \omega_t = 8.55 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.9 \]

Fig. B19. Profiles of the tangential velocity component. Series A2.2, case \( \omega_t/\omega_b = -1.9 \).
Series A2.3: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -1.5 \text{ rpm} \]

Case: \[ \omega_t = 2.55 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.7 \]

Fig. B20. Profiles of the tangential velocity component. Series A2.3, case \( \omega_t/\omega_b = -1.7 \).
Series A2.3: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -1.5 \text{ rpm} \]

Case:
\[ \omega_i = 2.7 \text{ rpm} \]
\[ \omega_i/\omega_b = -1.8 \]

Fig. B21. Profiles of the tangential velocity component. Series A2.3, case \( \omega_i/\omega_b = -1.8 \).
Series A2.3: Width top lid: 30.0 cm
    \[ h = 19.7 \text{ cm} \]
    \[ \omega_b = -1.5 \text{ rpm} \]

Case:
    \[ \omega_t = 2.85 \text{ rpm} \]
    \[ \omega_t / \omega_b = -1.9 \]

Fig. B22. Profiles of the tangential velocity component. Series A2.3, case \( \omega_t / \omega_b = -1.9 \).
APPENDIX C

HORIZONTAL PROFILES OF REYNOLDS SHEAR STRESS

This appendix contains plots of horizontal profiles of the Reynolds shear stress $\tau_{xy}$. Here only measurements with an inclined probe are included. The data on which these vectorplots are based are reproduced in tabular form in Booij and Melis (1994). The information is also obtainable on floppy discs.

The measured data points in the plots are connected by straight lines. The zero levels of the Reynolds shear stress for the various profiles are indicated in the plots at the left side and the scale of the Reynolds shear stress at the lower right side. The left side ($y = 0$ mm) represents the inner side of the flume and the right side ($y = 304$ mm) the outer side. The width of the flume is not exactly 300 mm everywhere and the sidewalls are not exactly vertical, partly because of the influence of the water pressure on the glass walls.

The horizontal profiles of the Reynolds shear stress are mainly important in connection with the second criterion for the optimum value of the ratio $\omega_r/\omega_b$ of a rotating annular flume, uniformity of the bottom shear stress, which is of importance for erosion studies.
Series A1.1: Width top lid: 30.0 cm
  \[ h = 29.7 \text{ cm} \]
  \[ \omega_b = -3.0 \text{ rpm} \]

Case:
  \[ \omega_t = 5.7 \text{ rpm} \]
  \[ \omega_t/\omega_b = -1.9 \]

Fig. C1. Profiles of the Reynolds shear stress. Series A1.1, case \( \omega_t/\omega_b = -1.9 \).
Series A1.1: Width top lid: 30.0 cm
   \[ h = 29.7 \text{ cm} \]
   \[ \omega_b = -3.0 \text{ rpm} \]

Case:
   \[ \omega_i = 6.0 \text{ rpm} \]
   \[ \omega_i/\omega_b = -2.0 \]

Fig. C2. Profiles of the Reynolds shear stress. Series A1.1, case \( \omega_i/\omega_b = -2.0 \).
Series A1.1: Width top lid: 30.0 cm
h = 29.7 cm
\( \omega_b = -3.0 \text{ rpm} \)

Case: \( \omega_l = 6.3 \text{ rpm} \)
\( \omega_l/\omega_b = -2.1 \)

Fig. C3. Profiles of the Reynolds shear stress. Series A1.1, case \( \omega_l/\omega_b = -2.1 \).
Series A1.1: Width top lid: 30.0 cm
    \( h = 29.7 \text{ cm} \)
    \( \omega_b = -3.0 \text{ rpm} \)

Case:
    \( \omega_t = 6.6 \text{ rpm} \)
    \( \omega_t / \omega_b = -2.2 \)

Fig. C4. Profiles of the Reynolds shear stress. Series A1.1, case \( \omega_t / \omega_b = -2.2 \).
Series A2.1: Width top lid: 30.0 cm
  h = 19.7 cm
  $\omega_b = -3.0$ rpm

Case: $\omega_t = 4.8$ rpm
  $\omega_t/\omega_b = -1.6$

Fig. C5. Profiles of the Reynolds shear stress. Series A2.1, case $\omega_t/\omega_b = -1.6.$
Series A2.1: Width top lid: 30.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case: \[ \omega_i = 5.1 \text{ rpm} \]
\[ \omega_i/\omega_b = -1.7 \]

Fig. C6. Profiles of the Reynolds shear stress. Series A2.1, case $\omega_i/\omega_b = -1.7$. 

C7
Series A2.1: Width top lid: 30.0 cm
   h = 19.7 cm
   $\omega_b = -3.0$ rpm

Case: $\omega_t = 5.4$ rpm
   $\omega_t/\omega_b = -1.8$

Fig. C7. Profiles of the Reynolds shear stress. Series A2.1, case $\omega_t/\omega_b = -1.8$.  

C8
Series A2.2: Width top lid: 30.0 cm

\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -4.5 \text{ rpm} \]

Case:

\[ \omega_t = 8.1 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.8 \]

Fig. C8. Profiles of the Reynolds shear stress. Series A2.2, case \( \omega_t/\omega_b = -1.8 \).
Series A2.3: Width top lid: 30.0 cm  
\[ h = 19.7 \text{ cm} \]  
\[ \omega_b = -1.5 \text{ rpm} \]

Case:  
\[ \omega_t = 2.7 \text{ rpm} \]  
\[ \omega_t/\omega_b = -1.8 \]

Fig. C9. Profiles of the Reynolds shear stress. Series A2.3, case \( \omega_t/\omega_b = -1.8 \).
Series A3.1: Width top lid: 30.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case:
\[ \omega_t = 4.2 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.4 \]

Fig. C10. Profiles of the Reynolds shear stress. Series A3.1, case \( \omega_t/\omega_b = -1.4 \).
Series A3.1: Width top lid: 30.0 cm
   \[ h = 15.7 \text{ cm} \]
   \[ \omega_b = -3.0 \text{ rpm} \]

Case: \[ \omega_t = 4.5 \text{ rpm} \]
   \[ \omega_t/\omega_b = -1.5 \]

Fig. C11. Profiles of the Reynolds shear stress. Series A3.1, case \( \omega_t/\omega_b = -1.5 \).
Series A3.1: Width top lid: 30.0 cm
    h = 15.7 cm
    \omega_b = -3.0 \text{ rpm}

Case: \omega_t = 4.8 \text{ rpm}
    \omega/\omega_b = -1.6

Fig. C12. Profiles of the Reynolds shear stress. Series A3.1, case \omega/\omega_b = -1.6.
Series A3.1: Width top lid: 30.0 cm
    \( h = 15.7 \) cm
    \( \omega_b = -3.0 \) rpm

Case:
    \( \omega_t = 5.1 \) rpm
    \( \omega_t / \omega_b = -1.7 \)

Fig. C13. Profiles of the Reynolds shear stress. Series A3.1, case \( \omega_t / \omega_b = -1.7 \).
Series B1.1: Width top lid: 15.0 cm
   \( h = 29.7 \, \text{cm} \)
   \( \omega_b = -3.0 \, \text{rpm} \)

Case:
   \( \omega_l = 8.1 \, \text{rpm} \)
   \( \frac{\omega_l}{\omega_b} = -2.7 \)

Fig. C14. Profiles of the Reynolds shear stress. Series B1.1, case \( \frac{\omega_l}{\omega_b} = -2.7 \).
Series B1.1: Width top lid: 15.0 cm
\[ h = 29.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case:
\[ \omega_t = 8.4 \text{ rpm} \]
\[ \omega_t / \omega_b = -2.8 \]

Fig. C15. Profiles of the Reynolds shear stress. Series B1.1, case \( \omega_t / \omega_b = -2.8 \).
Series B1.1: Width top lid: 15.0 cm
h = 29.7 cm
\( \omega_b = -3.0 \text{ rpm} \)

Case: 
\( \omega_t = 8.7 \text{ rpm} \)
\( \omega_t/\omega_b = -2.9 \)

Fig. C16. Profiles of the Reynolds shear stress. Series B1.1, case \( \omega_t/\omega_b = -2.9 \).
Series B1.1: Width top lid: 15.0 cm
   h = 29.7 cm
   \( \omega_b = -3.0 \text{ rpm} \)

Case:
   \( \omega_l = 9.0 \text{ rpm} \)
   \( \omega_l/\omega_b = -3.0 \)

Fig. C17. Profiles of the Reynolds shear stress. Series B1.1, case \( \omega_l/\omega_b = -3.0 \).
Series B2.1: Width top lid: 15.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case: \[ \omega_t = 6.0 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.0 \]

Fig. C18. Profiles of the Reynolds shear stress. Series B2.1, case \( \omega_t/\omega_b = -2.0 \).
Series B2.1: Width top lid: 15.0 cm
h = 19.7 cm
\( \omega_b = -3.0 \text{ rpm} \)

Case: \( \omega_t = 6.3 \text{ rpm} \)
\( \omega_t / \omega_b = -2.1 \)

Fig. C19. Profiles of the Reynolds shear stress. Series B2.1, case \( \omega_t / \omega_b = -2.1 \).
Series B2.1: Width top lid: 15.0 cm
h = 19.7 cm
\( \omega_b = -3.0 \text{ rpm} \)

Case:
\( \omega_i = 6.6 \text{ rpm} \)
\( \omega_i/\omega_b = -2.2 \)

Fig. C20. Profiles of the Reynolds shear stress. Series B2.1, case \( \omega_i/\omega_b = -2.2 \).
Series B2.1: Width top lid: 15.0 cm
\[ h = 19.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case:
\[ \omega_t = 6.9 \text{ rpm} \]
\[ \omega_t/\omega_b = -2.3 \]

Fig. C21. Profiles of the Reynolds shear stress. Series B2.1, case \( \omega_t/\omega_b = -2.3 \).
Series B3.1: Width top lid: 15.0 cm
   \[ h = 15.7 \text{ cm} \]
   \[ \omega_b = -3.0 \text{ rpm} \]

Case:
   \[ \omega_i = 5.4 \text{ rpm} \]
   \[ \omega_i / \omega_b = -1.8 \]

Fig. C22. Profiles of the Reynolds shear stress. Series B3.1, case \( \omega_i / \omega_b = -1.8 \).
Series B3.1: Width top lid: 15.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case:
\[ \omega_t = 5.7 \text{ rpm} \]
\[ \omega_t/\omega_b = -1.9 \]

Fig. C23. Profiles of the Reynolds shear stress. Series B3.1, case \( \omega_t/\omega_b = -1.9 \).
Series B3.1: Width top lid: 15.0 cm
\[ h = 15.7 \text{ cm} \]
\[ \omega_b = -3.0 \text{ rpm} \]

Case: \[ \omega_i = 6.0 \text{ rpm} \]
\[ \omega_i/\omega_b = -2.0 \]

Fig. C24. Profiles of the Reynolds shear stress. Series B3.1, case \( \omega_i/\omega_b = -2.0 \).
Series B3.1: Width top lid: 15.0 cm
    h = 15.7 cm
    ω_b = -3.0 rpm

Case:
    ω_t = 6.3 rpm
    ω_t/ω_b = -2.1

Fig. C25. Profiles of the Reynolds shear stress. Series B3.1, case ω_t/ω_b = -2.1.