An Investigation of the Design and Development of a Multi Frequency Inductive Power Transfer System

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It is good to have an end to journey toward; but it is the journey that matters, in the end.

- ERNEST HEMINGWAY
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Abstract

With the current surge in the development and use of Electric Vehicles (EVs) due to increased environmental consciousness, methods to increase the driving range and improve the battery charging conditions of these vehicles have become important research areas. Inductive Power Transfer (IPT) is one of the solutions to the challenges currently faced by EV manufacturers and customers. IPT is a method that is used to transfer power over large air gaps and has proven to be efficient, reliable and convenient in the charging of electric vehicles and also for factory automation systems and charging of consumer devices such as mobile phones.

In some cases, it necessary to have transmitter and receiver topologies having multiple coils especially in a three phase IPT highway system or a table top IPT system for charging many consumer devices or mobile phones at the same time. Multi coil topologies are also used in lumped charge pad topologies to enable better misalignment tolerances. In such cases it is necessary to mutually decouple these multiple coils to avoid interactions that could affect the efficiency of power transfer. The current state of the art systems offer solutions that lead to the decoupling of multiple coils by their relative spatial position, and are hence limited. Multi coil systems operating at the same frequency also generate magnetic fields that add or cancel out each other, thereby changing the air gap magnetic field orientation depending on which coil is powered.

A multi frequency IPT system developed in this thesis aims to provide a solution to the above challenges. A multi coil primary system with two coils carrying currents of different frequencies can be designed to transfer power independently without interactions between the two different frequency systems.

This thesis develops this concept by first investigating the need for decoupling multi coil transmitter and receiver designs. Multi frequency IPT is then introduced and the design considerations are developed in order to achieve a frequency decoupled IPT system. Experimental results of the proposed multi frequency method using two transmitter coils are also presented. Finally, a comparison is provided between this proposed multi frequency method using two transmitter coils and another recently published method that amplifies the third harmonic along with the fundamental from the input inverter, thereby achieving multi frequency IPT using one transmitter coil.
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CHAPTER 1

Introduction and Research Goals

An Inductive Power Transfer (IPT) system is one wherein wireless power transfer is achieved between a \textit{transmitter} and a \textit{receiver} over large air gaps. This concept of transferring power is an outcome of Maxwell’s electromagnetic equations and was first investigated by Nicola Tesla in the late 19th century but was limited in application due to limitations of the available technology at that time. IPT has now emerged as a practical method for recharging electric vehicles, providing solutions to existing challenges such as limited driving range and unreliable charging methods. This chapter provides an overview of the concept of IPT and the advantages that it brings and highlights the research goals and contributions of this MSc. thesis work.

1.1 A Technology for Today

![Figure 1.1: Inductive Power Transfer for Charging Electric Vehicles](image)

The development of high power (10 - 100 kW) IPT systems go hand in hand with advances in electric and hybrid electric vehicle (EV/HEV) technology. A current challenge faced by EV and HEV developers is the limited driving range of these vehicles which is currently directly related to the size and weight of its battery pack, requiring very large battery packs to reach the driving distance of traditional vehicles (at least 200 km) \cite{1}. Achieving high driving ranges in these vehicles thus leads to high initial costs, an increase in the weight of the vehicle and also an additional space requirement.
IPT systems provide many advantages over contact based power transfer systems. As there is no physical connection between the primary and secondary, electrical isolation is achieved, enabling operation in wet and dirty environments. These systems are also reliable, maintenance free and do not produce any environmental contaminants and are ideal for extending the driving range of EVs [2]. Figure 1.1 shows a schematic of an electric vehicle receiving power via a magnetic link from a transmitter in the road to a receiver placed under the car.

IPT systems are grouped as either distributed or lumped topologies. Distributed systems have applications in areas such as 'IPT Highways' where power is transferred continuously from the transmitter to the receiver and can potentially lead to electric vehicles with no battery pack or a very small battery pack. Lumped IPT systems, on the other hand, are better suited when power transfer is required only at certain fixed positions and can be used for electric vehicle charging at fixed locations such as packing lots or traffic signals. IPT as a technology is valid today more than ever and systems that are ten times more powerful, more tolerant of misalignment, safer, and more efficient may be achievable. [3].

IPT systems are not only promising for electric vehicle charging but are also applicable to charging of low power (5 W - 10 W) devices such as mobile phones and kitchen appliances and are witnessing widespread use due to relatively low prices and the increase in convenience that they provide.

1.2 Components of an IPT System

![Block diagram of a loosely coupled IPT system](image)

Figure 1.2: Block diagram of a loosely coupled IPT system

Figure 1.2 shows a block diagram schematic of a general IPT system. The transmitter is referred to as the primary winding and the receiver is referred to as the secondary winding. An IPT system transfers power via an alternating magnetic field which is produced by a high frequency current in the primary winding and are classified on the basis of the quality of coupling between the primary and secondary winding as closely coupled IPT systems and loosely coupled IPT systems. Transformers with a ferrite core, for example, are closely coupled IPT systems. In this work, loosely coupled IPT systems with an air gap of 100 mm to 400 mm are considered. Due to the large air gaps, IPT systems have coupling coefficients between 0.01 - 0.4 compared to transformers which have coupling coefficients over 0.95.

A general IPT system consists for the following components [4]:

1. A power supply that takes electric power from the grid and generates a high frequency current that flows through the primary inductor.
2. An elongated track or a charge pad that is driven by the power supply and produces a magnetic field.

3. A pickup that intercepts all or some of the magnetic field and converts it into electrical energy.

4. Electrical loads that are driven by this electrical energy.

An important component of IPT systems are the compensation capacitors which compensate for the high leakage inductances in order to increase power transfer and efficiency.

1.3 Research Goals

The main research goal of this thesis is to investigate the design and development of a multi frequency Inductive Power Transfer system with emphasis on the effect of multiple frequencies on decoupling multi coil primary and secondary systems.

In addition, the effect of multi frequency IPT on the following are also investigated: misalignment between the transmitter and receiver, inter-operability of a transmitter with different types of receivers, emitted field strength and losses in the inverter, transmitter and receiver coils.

In order to satisfy the above research goals, this work is structured as follows:

- A theoretical development of the fundamentals of IPT and a study of the evolution of magnetics design for IPT.

  Chapter 2 contains a theoretical development of the fundamentals of the design of IPT systems, highlighting important design equations, circuits and necessary conditions for the design of reliable and stable IPT systems.

  Chapter 3 contains a literature review of the development of IPT systems and a description of the current state of art and challenges in the development of IPT systems for stationary as well and distributed IPT systems.

- Description of the problem of Mutual Inductance in Multi Coil Primary and Secondary IPT systems.

  Chapter 4 introduces the problems and challenges faced in the design of IPT systems due to mutual inductance between coils in multi coil primary and secondary systems and introduces the concept of multi frequency IPT as a possible solution to these challenges.

- Study of multi frequency IPT and development of a novel multi frequency IPT system.

  Chapter 5 introduces the design and development of multi frequency IPT systems and highlights the design considerations and provides an analysis of the viability of multi frequency systems. As a first step, a dual frequency IPT system is considered. This Chapter also provides a comparative analysis between the method proposed in this work and another recently proposed method for multi frequency IPT.

  Chapter 6 contains a discussion of the conclusions of this thesis and recommendations for future work.
1.4 Contribution

The main contribution of this MSc. thesis is to provide an initial investigation into multi frequency IPT as a means to decouple multi coil topologies. The challenges faced in the design of multi coil IPT magnetic topologies are highlighted and a solution to solve these challenges is proposed. A design methodology is developed and preliminary laboratory results are presented. In addition, a comparative analysis between the method proposed in this thesis and another recently published multi-frequency IPT method is carried out.
CHAPTER 2

IPT Fundamentals

This chapter aims to introduce and highlight the fundamental concepts of Inductive Power Transfer. As IPT ‘transformers’ are very different from conventional transformers, primarily because of the large air gaps and the fact that the coupling between the primary and secondary need not be constant, it is important to study the design considerations of such systems.

2.1 Reactive Power Compensation

In its simplest form, an uncompensated IPT System can be visualised as a set of two coupled inductors, the primary inductor of inductance \( L_p \) and the secondary inductor of inductance \( L_s \) and as shown in Figure 2.1 [5]. The mutual inductance between the primary inductor and the secondary inductor is \( M \).

![Figure 2.1: Equivalent Circuit of two coupled inductors](image)

The voltage induced in the secondary coil due to primary current \( I_p \) with frequency \( \omega \) is given by

\[
V_s' = j\omega MI_p
\]  

(2.1)

and the voltage induced back to the primary coil due the secondary current \( I_s \) is

\[
V_p' = -j\omega MI_s
\]  

(2.2)

Due to the large primary to secondary air gaps of ICPT systems, a large part of the magnetic flux lines produced by the primary inductor do not couple with the secondary inductor giving
rise to large leakage inductances which drastically reduce the efficiency of such systems. It is hence necessary to compensate for these leakage inductances by adding capacitors to the primary and secondary circuits. This technique of reactive power compensation ensures that the system operates at resonance and only real power is transferred from the primary to the secondary as will be further elucidated in this section. This resonating system also has the advantage of acting as a tuned filter, only allowing currents at the resonance frequency to transfer power.

The compensation capacitors can be connected in series or parallel to the primary and secondary inductors, giving rise to four different compensation topologies: Series - Series (SS), Series - Parallel (SP), Parallel - Series (PS) and Parallel - Parallel (PP) as shown in Figure 2.2. The source voltage is denoted by $V_1$ and the load is denoted by $R_L$. The primary side inductances and capacitances are denoted by $L_p$ and $C_p$ respectively while the secondary side inductances and capacitances are denoted by $L_s$ and $C_s$ respectively. $R_p$ and $R_s$ are the series resistances of the primary and secondary side inductances respectively. $M$ is the mutual inductance between the primary and secondary inductors.

The values of the compensation capacitances are determined as follows. The secondary capacitance, $C_s$, is chosen so as to operate in resonance with the secondary inductance, $L_s$, thereby ensuring that maximum power transfer via the secondary is achieved, and the primary capacitance, $C_p$, is chosen so as to cancel the reactive part of the circuit as seen by the source. Reactive power compensation thus ideally achieves maximum power transfer at unity power factor.

$C_s$, for all the compensation topologies of Figure 2.2 is determined as follows:

$$C_s = \frac{1}{\omega_0^2 L_s}$$  \hspace{1cm} (2.3)

where $\omega_0$ is the frequency of operation (the resonance frequency) of the IPT system. In order to determine $C_p$, the total impedance, $Z_T$, as seen by the source, $V_1$, for all the compensation topologies of Figure 2.2 needs to be calculated. $C_p$ is then determined by equating the reactive (imaginary) part of $Z_T$ to zero. The source voltage, $V_1$, can therefore be represented by:

$$V_1 = Z_T I_1$$  \hspace{1cm} (2.4)
The equations for $Z_T$ for the different compensation topologies of Figure 2.2 will now be developed. The total impedance of the secondary is calculated as a lumped impedance, $Z_s$ and is given by

$$Z_s = \begin{cases} 
R_s + j \left( \omega L_s - \frac{1}{\omega C_s} \right) + R_L & \text{series compensated secondary} \\
R_s + j\omega L_s + \frac{R_L}{1 + j\omega C_s R_L} & \text{parallel compensated secondary}
\end{cases}$$  \hspace{1cm} (2.5)

**Series Compensated Primary**

The impedance of the primary circuits of a series compensated primary topology (SS or SP) is given by

$$Z_{p,s} = R_p + j \left( \omega L_p - \frac{1}{\omega C_p} \right)$$  \hspace{1cm} (2.6)

and the source voltage is given by

$$V_1 = I_p Z_{p,s} + V'_p$$  \hspace{1cm} (2.7)

From (2.1) and (2.2)

$$I_s = \frac{V'_s}{Z_s} = \frac{j\omega M I_p}{Z_s}$$  \hspace{1cm} (2.8)

$$\Rightarrow V'_p = \frac{\omega^2 M^2 I_p}{Z_s}$$  \hspace{1cm} (2.9)

From (2.9), the reflected impedance [5] of the secondary on the primary circuit is given by

$$Z_r = \frac{\omega^2 M^2}{Z_s}$$  \hspace{1cm} (2.10)

$Z_r$ is the loading effect of the secondary on the primary and is dependent on the operating frequency and the mutual coupling between the primary and the secondary.

Substituting (2.9) in (2.7)

$$V_1 = \left( Z_{p,s} + \frac{\omega^2 M^2}{Z_s} \right) I_p$$  \hspace{1cm} (2.11)

Hence, from (2.11), for series compensated primary ICPT systems, the total impedance as seen by the source is given by

$$Z_{T,s} = Z_{p,s} + \frac{\omega^2 M^2}{Z_s}$$  \hspace{1cm} (2.12)

where $Z_s$ is given by (2.5) depending on whether the secondary is series or parallel compensated.

**Parallel Compensated Primary**

In the case of parallel compensated primary topologies (PS and PP), the source voltage is given by

$$V_1 = Z_{L_p} I_p + V'_p$$  \hspace{1cm} (2.13)
where \( Z_{Lp} = R_p + j\omega L_p \) is the impedance of the primary coil.

Substituting (2.2) in (2.13)

\[
V_1 = \left( Z_{Lp} + \frac{\omega^2 M^2}{Z_s} \right) I_p
\]  
\tag{2.14}

The total impedance, \( Z_p \), of a parallel compensated primary system is given by

\[
Z_{T,p} = \frac{V_1}{I_1} = \frac{V_1}{I_{Cp} + I_p}
\]  
\tag{2.15}

Hence, from (2.14) and (2.15)

\[
Z_{T,p} = \frac{1}{j\omega C_p + \frac{1}{Z_Lp + \frac{\omega^2 M^2}{Z_s}}}
\]  
\tag{2.16}

where \( Z_s \) is given by (2.5) depending on whether the secondary is series or parallel compensated. The above equations are a summary of well known equations from literature [6], [7].

**The primary compensation capacitance (\( C_p \))**

As stated above, the primary compensation capacitance, \( C_p \), is determined by setting the reactive part of the expression for \( Z_T \) of the respective topology to zero [8]. This ensures that maximum power is transferred to the load at unity power factor. The expressions for \( C_p \) determined using (2.5), (2.12) and (2.16) are tabulated in Table 2.1. These expressions neglect the winding resistances of the primary and secondary coils, as they are assumed to be very small.

From the expressions of \( C_p \) tabulated in Table 2.1, it can be seen that \( C_p \) is independent of the load, \( R_L \), in the SS and SP topologies but is load dependant in the PS and PP topologies. This load dependant characteristic of \( C_p \) makes the PS and PP topologies unsuitable for IPT as the power transfer efficiency will drop as the load is changed. Also, the SS topology is the only one wherein \( C_p \) is independent of the mutual inductance, \( M \), between the primary and the secondary. The fact that \( C_p \) is load independent in the SS and SP topologies make them attractive options for IPT system design, especially in the case of distributed IPT systems where one primary can power multiple loads at the same time and these loads can connect and disconnect without a drop in efficiency.

Although, parallel compensated secondary designs have been proposed in literature and are used in some practical IPT systems, series compensated secondary designs will be considered in this thesis. This is due to the fact that in a series compensated secondary design, \( C_p \) is independent of \( M \) (coupling between the primary and the secondary) and hence, the primary will be correctly compensated even if the secondary is misaligned with it. Another reason for considering series compensated secondary designs is that, as will be described in Section 2.3, a series compensated secondary with a current controlled primary behaves like a voltage source and this is a desirable characteristic if the IPT secondary output is connected to the DC bus of the Electric Vehicle.

As will be highlighted in Section 2.4, parallel compensated secondary designs can have a \( M \)-independent \( C_p \) if an LCL type resonant topology is used in the receiver and these designs are used when a current source behaviour is required at the output of the secondary. Parallel compensated secondary designs are usually used if the receiver is directly connected to the battery of an electric vehicle which requires a constant current to charge, as will be explained in Section 2.3.


2.2 Power Transfer and Efficiency Equations

Of importance in IPT system design are the expressions for output power and efficiency and these will be derived in this section.

From (2.1), $V_s'$ can also be referred to as the open circuit voltage, $V_{oc}$, of the coupled system.

$$V_{oc} = j\omega MI_p$$ (2.17)

If the secondary is short circuited, the short circuit current, $I_{sc}$, is expressed as follows:

$$I_{sc} = \frac{V_{oc}}{j\omega L_s} = \frac{MI_p}{L_s}$$ (2.18)

The product of $V_{oc}$ and $I_{sc}$ is referred to as $P_{su}$ and is the maximum VA rating of the secondary.

It is also referred to as the uncompensated power transferred.

$$P_{su} = V_{oc}I_{sc} = \frac{\omega M^2 I_p^2}{L_s}$$ (2.19)

Due to reactive power compensation, practical IPT systems operate at resonance and in the secondary circuit, in the case of a series compensated secondary, the entire voltage $V_s'$ will
appear across the load $R_L$. The output power, $P_{out}$, can be calculated as follows:

$$P_{out} = \frac{V_s^2}{R_L} = \frac{(\omega M I_p)^2}{R_L} = \frac{\omega M^2 I_p^2}{L_s} \left( \frac{\omega L_s}{R_L} \right)$$

$$\Rightarrow P_{out} = \frac{\omega M^2 I_p^2 Q_L}{L_s} \quad (2.20)$$

From (2.19) and (2.21)

$$P_{out} = P_{su} Q_L \quad (2.22)$$

Here, $Q_L$ is the loaded quality factor of the tuned secondary system and $Q_L = \frac{\omega L_s}{R_s + R_L} \approx \frac{\omega L_s}{R_L}$ as $R_L \gg R_s$.

The output power equation (2.21) is derived for a series compensated secondary here but the same result is achieved for a parallel compensated secondary where $Q_L = \frac{R_s + R_L}{\omega L_s}$. In fact, (2.20) is more revealing and highlights the fact that in order for power to be transferred to the secondary all that is required is some coupling, $M$, between the primary and the secondary along with an alternating primary current. As will be highlighted in Section 2.4, the primary current, $I_p$, is assumed to be controlled and this highlights the fact that higher frequency IPT systems require a smaller primary current than lower frequency system to transfer a rated amount of power. This tends to reduce the conduction losses in the coils for higher frequency systems although the coil ac resistances could increase due to proximity and skin effects.

As only real power is transferred from the primary to the secondary in IPT systems, the only sources of power loss are in the series resistances of the inductor windings and in the equivalent series resistances (ESR) of the compensation capacitors. The capacitor ESR are very small when compared to the inductor series resistances and are neglected in the following derivations.

From (2.5) and (2.8), the efficiency, $\eta_{SS}$, of a SS IPT topology is given by

$$\eta_{SS} = \frac{I_s^2 R_L}{I_p^2 R_p + I_s^2 R_s + I_s^2 R_L} \Rightarrow \eta_{SS} = \frac{R_L}{(R_L + R_s) \left( 1 + R_p \frac{R_s + R_L}{\omega_0^2 M^2} \right)} \quad (2.23)$$

As can be seen in (2.23), $\eta_{SS}$ is a function of the the inductor series resistances as well as the product $(\omega_0 M)$. As will be explained in Section 2.4, the primary inductor excitation current, $I_p$, is maintained constant and therefore a system with a low frequency will have to be driven with a higher current in order to generate the same secondary excitation as compared to a system with a high frequency leading to lower efficiencies in low frequency systems. Similarly, an IPT system with a higher primary-secondary mutual inductance, $M$, will have a higher secondary induced voltage as compared to a system with a lower value of $M$ and will hence be able to generate the same secondary current with a lower primary current thereby increasing the system
efficiency. Hence, $\eta_{SS}$ increases with an increase in $(\omega_0M)$ and/or with a decrease in the series resistances of the inductors.

A Figure of merit (FOM) has been derived for IPT systems in [1], [9], [4] as

$$FOM = \kappa Q$$

(2.24)

where $\kappa$ is the coupling coefficient, $k = M/\sqrt{L_pL_s}$ and $Q = \sqrt{Q_pQ_s}$. $Q_p = (\omega L_p)/R_p$ and $Q_s = (\omega L_s)/R_s$ are the intrinsic quality factors of the primary and secondary coils. The above Figure of Merit is used for inductor selection and for optimizing of IPT magnetics and corresponds to the above statement that the efficiency of an IPT system can be increased with an increase in $(\omega_0M)$ and/or with a decrease in the series resistances of the inductors.

### 2.3 Current and Voltage Source behaviour of SS systems

The output of SS topology of Figure 2.2a has the property that it can operate as a current source as well as a voltage source. It is a current source if the input $V_1$ is a voltage source and it is a voltage source if the input current, $I_p$, is generated from a current source.

From (2.8) and (2.11), the output current, $I_{out} = I_s$, is given by

$$I_s = \left( \frac{j\omega M}{R_s + R_L} \right) \left( \frac{V_1}{R_p + \frac{\omega^2M^2}{R_s + R_L}} \right)$$

(2.25)

The series resistances, $R_p$ and $R_s$ can be neglected as they are usually very low. $I_s$ is therefore given by

$$I_s = j \left( \frac{V_1}{\omega M} \right)$$

(2.26)

It is therefore evident from (2.26) that if the input, $V_1$, is a voltage source, the output current, $I_s$, is load independent thereby behaving as a current source (assuming $R_p$ and $R_s$ are negligible in magnitude). Figure 2.3 shows the output characteristics of a SS system driven by a voltage source and where $R_p = R_s = 0.1\Omega$ and it can be seen that although $I_{out}$ is not really constant over a wide range of $R_L$.

![Graphs](a) Output Voltage vs Load Resistance (b) Output Current vs Load Resistance

Figure 2.3: Output characteristics of the SS topology with a voltage source input
From (2.1), which is repeated below, it is evident that if $I_p$ is generated from a current source, the output voltage, $V_s'$, is load independent thereby behaving as a voltage source as the voltage drop across the series resistances of the coil is negligible.

$$V_s' = j\omega MI_p$$

Hence, in the case of a SS system, if the primary system is powered by a voltage source, the secondary operates as a current source and if the primary coil is powered by a current source, the secondary operates as a voltage source.

As will be highlighted in the next section, it is more advantageous with respect to an optimal IPT system design to operate with a load independent primary current and these type of current source primary designs will be used in this thesis work.

### 2.4 Source and output behaviour

#### Importance of a Current Controlled Primary System

IPT systems are loosely coupled systems and unlike closely coupled systems, are normally designed to have a constant primary current \[10\]. Closely coupled systems, such as ferrite core transformers, are designed to fully utilise the core by having a flux density, $B(t)$, in the core that is not too high so as to saturate it but also not too low so as to under utilise it. $B(t)$ is determined by the voltage, $v(t)$, across the primary windings as shown in (2.27) and (2.28) where, $L_M$ is the magnetizing inductance, $n$ is the number of turns of the primary and $A_c$ is the cross sectional area of the core.

$$i_M(t) = \frac{1}{L_M} \int v(t) \, dt \quad (2.27)$$

$$B(t) = \frac{1}{nA_c} \int v(t) \, dt \quad (2.28)$$

IPT systems, on the other hand are air - cored transformers wherein the majority of the magnetomotive force is across the large air gap and its design is determined by the current in the primary winding. Therefore, in IPT systems, the primary can be fully utilised by operating at the rated current of the coil by using a power supply that delivers a constant primary current and acts as a current source. Ferrites are sometimes used to shape the flux lines in primary topologies and the primary coil current is chosen so as to not saturate the ferrites used.

Constant primary current IPT systems (current controlled primary systems) have many advantages as listed below:

- The advantages of a SS topology for IPT has been highlighted in Section 2.1 but a SS system with a primary supplied with a voltage source has an inherent disadvantage. It can be seen from (2.12), at the no-load situation ($Z_s \to \infty$) the primary voltage source will see the least possible resistance and will therefore supply the highest possible current. This is an undesirable situation which could lead to high losses and even worse a current run off at no load. An SS system with a current controlled primary can avoid this situation.

- The system design procedure is simpler and the primary can be optimally designed as the (copper) cost of the primary system directly depends on the primary current. The design of the secondary and primary systems can therefore, in principle, be treated independently.

- IPT systems with multiple secondary pickups above an elongate track benefit from having a primary with constant current as this implies that all the pickups would have the same induced emfs \[3\].
• It is important for the primary current to be known and controlled as this guarantees that the coupling to the secondary can be defined at each relative primary-secondary position position [11].

• Strict guidelines exist for the magnitude of magnetic field that humans beings can be exposed to determined by the International Commission on Non-Ionizing Radiation Protection (ICNRP) [12]. As the magnitude of the air gap magnetic field is directly related to the current flowing in the primary coil, it can be easily controlled by controlling the primary current. A brief description of the ICNRP guidelines on Magnetic Field Exposure is provided in Section 2.8.

A current controlled input power supply can be realised by two broad methods. The first possible method is to incorporate the SS topology discussed above with an additional current controller in the primary to control and limit the primary current. A second possible method is to implement the primary resonant tank with multiple reactive components, such as an LCL - T resonant circuit, which ensures a constant primary current without additional control circuitry. An introduction to the LCL topology is provided in Section 2.6. This work is concerned with the design procedure of multi-frequency IPT systems which, as will be explained in Chapter 5 is dependent on the resonant tanks used. Hence, an investigation of IPT using both of the above two methods is carried out in this work in order to determine which resonant topology could be better suited for multi-frequency IPT.

Assuming a constant primary current, the secondary of an IPT system acts as a voltage source when the secondary inductor is series compensated and as a current source when the secondary inductor is parallel compensated. These characteristics can be easily derived from the equations of Section 2.1. Figure 2.4 highlights the voltage source characteristic of a series compensated secondary and Figure 2.5 highlights the current source characteristic of a parallel compensated secondary.

![Output Voltage vs Load Resistance](image-a)

![Output Current vs Load Resistance](image-b)

Figure 2.4: Output Characteristics of a Series Compensated Secondary with a Current Controlled Primary
2.5 Requirements of an IPT Primary Power Supply

IPT systems need to be powered by a power supply that satisfy the following criteria:

1. The power supply should deliver rated power for the rated track loading at a pre-defined constant frequency.
2. The current in the primary inductor can be controlled control and maintained a constant irrespective of the load resistance.
3. The primary circuit should be tuned in order to eliminate the reactive component in the total impedance seen by the source, hence minimizing the VA rating of the power supply.

The first condition as stated above is fairly easy to satisfy and is usually done by setting an operating frequency and rated primary current for delivering the required rated power. Satisfying the second and the third condition requires an investigation of the resonant topology used in the primary and the secondary.

2.6 LCL - T Primary Resonant Topology

As the name suggests, a LCL - T based power supply has a LCL - T resonant tank in the primary circuit consisting of an inductor, $L_1$, the primary inductor of the IPT system (or track inductor), $L_p$, and the primary compensation capacitor, $C_p$ [13], [14], [15]. $V_{in}$ is the square wave voltage output from a full-bridge or half-bridge inverter. $R_{eq}$ is the equivalent resistance of the rest of the IPT system and includes the reflected resistance from the secondary. If a reactance is
reflected from the secondary (as in the case of a parallel compensated secondary), it is included in the reactance of $L_p$. The primary excitation current, $I_p$, is given by

$$I_p = \frac{V_{in}}{j\omega L_1 + (R_{eq} + j\omega L_p)(1 - \omega^2 L_1 C_p)}$$ (2.29)

If the converter is operated at the resonance frequency $\omega_0$ such that the reactance of $L_1$ is equal to the reactance of $C_1$ ($\omega_0 = 1/\sqrt{L_1 C_p}$), the primary current in (2.29) simplifies to

$$I_p = \frac{V_{in}}{j\omega_0 L_1}$$ (2.30)

Hence, from (2.30) a constant primary current, controlled by $V_{in}$ is maintained in the primary inductor and the sufficient condition to maintain this load independent current is to ensure that $L_1$ is resonating with $C_p$. This satisfies the second condition Section 2.5 for a good IPT primary power supply. In order to satisfy the third condition of eliminating the reactive component of the total impedance as seen by the source, the following analysis is carried out.

The total impedance as seen by the inverter output, $Z_{t,LCL}$ is given by [16]

$$Z_{t,LCL} = R_{t,LCL} + jX_{t,LCL}$$ (2.31)

where

$$R_{t,LCL} = \frac{R_{eq}}{(\omega C_p R_{eq})^2 + (1 - \omega^2 C_p L_p)^2}$$

$$X_{t,LCL} = \omega L_1 + \frac{\omega L_p - \omega C_p (\omega^2 L_p^2 + R_{eq}^2)}{(\omega C_p R_{eq})^2 + (1 - \omega^2 C_p L_p)^2}$$ (2.32)

At $L_1 = L_p = L$ and at $\omega_0 = 1/\sqrt{L_1 C_1}$

$$X_{t,LCL} = 0$$

$$R_{t,LCL} = \frac{(\omega L)^2}{R_{eq}}$$ (2.33)

Hence, (2.33) shows that the LCL - T primary, if properly tuned will only transfer real power to the load and hence also satisfies the third condition of Section 2.5 for a good IPT primary power supply.

### 2.6.1 Frequency Selective Properties of an LCT - T Resonant Tank

An LCL tank has two resonant frequencies as will be derived in this subsection. (2.32) can also be expressed as

$$Z_{t,LCL} = \frac{j\omega(L_1 + L_p - \omega^2 L_p C_p L_1) + R_{eq}(1 - \omega^2 L_1 L_p)}{1 - \omega^2 L_p C_p + j\omega C_p R_{eq}}$$ (2.34)

If the equivalent resistance, $R_{eq}$ is assumed to be zero (no load case), the total impedance, $Z_t$ and the total admittance, $Y_t$ seen by the source is given by

$$Z_t = \frac{j\omega(L_1 + L_p - \omega^2 L_p C_p L_1)}{1 - \omega^2 L_p C_p}$$

$$Y_t = \frac{1 - \omega^2 L_p C_p}{j\omega(L_1 + L_p - \omega^2 L_p C_p L_1)}$$ (2.35)
It can be seen that (2.35) indicates the absence of a real part and hence, two resonant frequencies of the LCL - T tank can be obtained by equating $Z_t$ and $Y_t$ to zero. These are represented as [15]

$$\omega_{resZ} = \sqrt{\frac{L_p + L_1}{L_p C_p L_1}} \tag{2.36}$$

$$\omega_{resY} = \frac{1}{\sqrt{L_p C_p}} \tag{2.37}$$

Figure 2.7 shows the typical impedance characteristics of an LCL resonant tank with a resonant frequency of 20 kHz. Here, $Z_{in}$, $R_{in}$ and $X_{in}$ are the input impedance, resistance and reactance of the LCL tank.

![Impedance Characteristics](image)

Figure 2.7: Typical Impedance Characteristics of an LCL Resonant Tank

Hence, the LCL tank, similarly as a series RLC circuit, has fixed resonance frequencies and its behaviour to induced currents will be further highlighted in Section 4.2.

### 2.7 Operating frequency of an IPT System

The selection of an operating frequency for power transfer in an IPT system is one of the most important design parameters. In single frequency IPT systems, switches of the input power supply switch at the operating frequency but as will be described in Chapter 5, a multi-frequency system could switch at a lower frequency than that at which power is being transferred. Hence, frequency selection is an important parameter in the design of the input power supply in order to minimise switching losses. As, the main focus of this work is not on the input power supply design, this aspect of the switching frequency will not be dealt with in detail. Operating frequencies in the range of 20 kHz to 150 kHz will be considered in this work and it is a fair assumption that within this frequency range, switching losses are sufficiently contained with the use of current state of the art power switches and resonant power supplies.

As is noted from (2.21), the power transferred is proportional to the operating frequency. An upper limit of 150 kHz is considered in this work as the input power supply and litz wire used have proven high efficiencies at this frequency and also because for frequencies above 150 kHz, a
broadcast license is required in most countries [17]. Early systems have been developed for sub 40 kHz frequencies due to cost and availability restrictions of power electronic components for higher frequencies but currently companies are designing and testing systems at 85 kHz [17].

From (2.21), it is clearly evident that as the operating frequency, $\omega$, of an IPT system increases, a larger amount of power is transferred with a given value of primary current and coupling. But, increasing the operating frequency in order to increase power transfer is considered to be a forced solution leading to adverse secondary losses (due to skin effect and proximity effect) in the copper coils as well as leading to an increase in switching losses in the power supply switches, if the frequency of operation is increased beyond 150 kHz.

In IPT systems, it is common to use Litz wire for the construction of the primary and secondary coils, thereby dramatically reducing losses due to skin effect when compared to a copper wire of the same diameter. In this study, for the laboratory work, readily available Litz wire of 2.2 mm diameter with 600 strands, each with a diameter of 0.071 mm (a $600 \times 0.071$ mm Litz wire) was used.

The expression for skin depth in a conductor is given by

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_r\mu_0}}$$  \hspace{1cm} (2.38)

where $\rho$ is the resistivity of the material, $\mu_r$ is the relative permeability of the material and $\omega$ is the angular frequency of the current flowing in the conductor. Figure 2.8 shows Skin Effect by a plot of the current density ($A/mm^2$) in a copper wire of 2.2 diameter and it can be clearly seen that as the frequency is increased from 20 kHz to 150 kHz, more of the current is concentrated towards the edges of the conductor, thereby increasing the AC conduction losses in the conductor.

In order to reduce the adverse effects of loss in efficiency due to Skin Effect, Litz wire of the correct diameter has to be chosen and as can be seen from Table 2.2, the chosen Litz wire is suitable within the frequency range of interest (20 kHz - 150 kHz). Although Skin Effect is reduced, it is interesting to note that the coil AC resistance may not remain constant as the frequency of the current flowing in the coils increases. This is due to the Proximity Effect and depends on how close the adjacent windings of the coils are wound [18]. The effect of an increase in the AC resistance of the coil built in the laboratory as the operating frequency is increased is shown in Figure 2.9. Although, it may appear that high frequency IPT
systems have very high coil losses, this is not really the case because as the operating frequency increases, the primary current magnitudes that are required to transfer rated power decrease. This is an inference of (2.21) and is further elaborated in Chapter 5.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>δ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.485</td>
</tr>
<tr>
<td>50</td>
<td>0.307</td>
</tr>
<tr>
<td>70</td>
<td>0.259</td>
</tr>
<tr>
<td>100</td>
<td>0.217</td>
</tr>
<tr>
<td>150</td>
<td>0.177</td>
</tr>
</tbody>
</table>

Table 2.2: Skin depth as a function of frequency for a Copper wire

2.8 ICNIRP Guidelines on Magnetic Field Exposure

As IPT systems can have a large amount of leakage magnetic fields, it is important to design such systems such that they meet the guidelines set by the International Commission on Non-Ionizing Radiation Protection (ICNRP) for exposure to time-varying magnetic fields [12].

Figure 2.10 shows the ICNRP reference magnetic field levels as a function of frequency and it can be seen that at frequencies above 3 kHz, which is where IPT operating frequencies lie, it is constant at 0.1 mT. This value is rather low but it only pertains to the leakage magnetic field as this is the field that humans could be exposed to. These strict guidelines for magnetic field exposure highlight the need for a current controlled primary system as the peak magnetic field in the air gap is directly proportional to the peak current in the primary coil.
2.9 Design of an IPT System

IPT systems are complex and have many design variables and in order to simplify the design process it is therefore necessary to have a certain number of input variables that have been pre-defined. In this work, the following are considered as input variables:

- The primary inductance, $L_p$
- The secondary inductance, $L_s$
- The Mutual inductance between the primary and the secondary, $M$
- The series resistances of the inductors, $R_p$ and $R_s$
- An initial frequency of operation, $\omega_i$. This initial frequency can be chosen, for example, in the initial design of the input power supply.
- The required output voltage, $V_{out}$, and the rated output power, $P_{out}$

As can be seen above, it is recommended to design the magnetic system separately and the reason for this is as follows. The amount of magnetic structures that are possible for IPT are virtually unlimited although a few important and accepted topologies are highlighted in Chapter 3. The designer therefore, has to first select a particular magnetic topology that suits the project application. Due to the complexity of some of the magnetic designs it could be difficult to derive analytical expressions for the respective primary and secondary inductance and the primary-secondary mutual inductance and Finite Element Analysis (FEM) is therefore used in order to get an approximate value for these inductances. Equation (2.20) highlights the fact the the primary-secondary mutual inductance, $M$, is proportional to the power delivered to a rated load and (2.23) highlights the fact that the efficiency is also proportional to $M$. Hence, the magnetics should be designed such that the rated power is delivered with a desired efficiency.

The design process of an IPT system with a current controlled primary and a secondary that is series compensated is shown in the flowchart of Figure 2.11.
2.10 Discussion

This chapter provides an overview of the fundamentals of IPT system design. Of importance is the fact that the series-series (SS) compensation is the preferred compensation topology for this thesis. This is due to the fact that the primary compensation capacitance is independent of the primary-secondary coupling as well as the output load. The LCL compensation topology is also studied but as will be explained in Chapter 4, will not be implemented in this thesis due to the fact that the input voltage is dependent on the primary coil inductance.

The importance of a current controlled primary is also highlighted and hence, a current controlled primary system will be assumed for the rest of this thesis, even in the case of SS compensation.
Evolution of IPT Charge Pads

IPT charge pads, also known as lumped charging systems, have seen considerable development in the last 20 years and have proved viable for high power transfer over air gaps of 50 mm to 500 mm. These charge pads began as plug-in inductive paddle systems and have now evolved to charge pads with complex magnetic designs with high coupling and operate at frequencies between 20 kHz and 150 kHz. This chapter provides a literature survey of IPT charge pad development and highlights the key parameters that were improved in each new charge pad design thus providing a clear picture of the requirements of a good IPT charge pad. A comparison of the various charge pad topologies is also provided.

3.1 Requirements of an IPT Charge Pad

IPT charge pads can have a virtually unlimited number of different magnetic designs. The main considerations while designing a Lumped Charge Pad are listed below.

- **The Coupling Factor, \( \kappa \), should be maximised.**

  The coupling factor \( \kappa = \frac{M}{\sqrt{L_p L_s}} \) gives a direct indication of the amount of flux generated from the primary that couples with the secondary. IPT systems are characterised by having leakage inductances that are much greater than the mutual inductance between the primary and the secondary and hence charge pad design is focused on improving the coupling factor between the primary and the secondary. The coupling factor provides a good measure in order to compare different charge pad designs and a value of \( \kappa > 0.2 \) is considered good.

- **The magnetic flux lines from the primary should be single-sided.**

  An IPT charge pad with a single-sided flux pattern has flux lines that are present only on the top side of the charge pad surface and negligible flux lines on the bottom side. This kind of flux distribution is desirable and increases the coupling between the primary and the secondary and avoids unnecessary leakage flux under the charge pad which doesn’t couple to the secondary. As will be highlighted in this chapter, consecutive improvements in IPT charge pads have resulted in primary designs that have a single-sided flux distribution.
• **The charge pad design should be scalable**

The relationship between the size of a charge pad and its ability to couple with a secondary pad placed above it is studied using the concept of fundamental flux path height [3], [19]. The relation between the fundamental flux path height and the pad diameter should be such that in order to increase the air gap the dimensions of the charge pad should not be increased by a *large* amount. This is the relationship between the size of the pad and its ability to throw flux to a receiver pad above it and is an important parameter as the space available below an electric vehicle is often limited. It is also an important parameter that determines copper saving in the windings of a charge pad.

• **The charge pad should have good tolerance to misalignment in order to allow easier parking.**

A common disadvantage that plagues IPT charge pads is that they can transfer maximum power when the secondary is perfectly aligned to the primary but the amount of power transferred decreases if the secondary is misaligned. IPT charge pads with *good* misalignment tolerances can work upto misalignments of about 200 mm albeit with much lower power transferred as compared to a perfectly aligned case.

• **A charge pad used in the secondary system should be able to interoperate with different primary charge pads**

As there are many different types of primary topologies used in IPT systems, it is important that secondary systems are able to interoperate with these different primary systems maintaining high efficiencies.

• **The charge pad should be as thin as possible to enable for ground clearance and fitting.**

As mentioned above, space under an electric vehicle (especially in the chassis) is limited and compact charge pad designs are therefore preferred.

• **The charge pad should be lightweight in order to minimise vehicle energy requirements.**

If the energy requirements of an electric vehicle are minimised, the driving range can be increased and hence the charge pad designs should be as lightweight as possible. This is one reason why charge pads without ferrites are often preferred [1].

### 3.2 Classification of IPT Charge Pads

IPT charge pads can be classified as non-polarised or polarised and single-sided or double-sided. These classifications are further explained below:

• **Non-polarised charge pads:** A primary that is non-polarised generates a symmetrical flux distribution and hence can couple optimally with a similar secondary which can approach it from any direction. Typical examples of non-polarised charge pads are the Circular charge pad, highlighted in Section 3.3, and the single-sided Circular charge pad, highlighted in Section 3.4.
• **Polarised charge pads:** A primary that is polarised generates a non-symmetrical flux distribution and hence can only couple optimally with a similar secondary if the secondary approaches it in a particular direction. Typical examples of polarised charge pads are the Flux pipe charge pad, highlighted Section 3.5, and the Double - D charge pad, highlighted in Section 3.6, which generate a parallel type flux along the length of the pad and can only optimally couple with a secondary that approaches it along the length of the pad.

• **Double - sided charge pads:** Double - sided charge pads generate flux lines on both sides of the charge pad surface. These charge pads, if used as a primary, are not ideal, as the flux lines under the charge pad do not couple with the secondary. Hence, double - sided charge pads are usually not preferred for primary designs. Typical examples of double - sided charge pads are the Circular charge pad, highlighted in Section 3.3, and the Flux pipe charge pad, highlighted Section 3.5.

• **Single - sided charge pads:** Single - sided charge pads, contrary to double - sided charge pads, generate a single - sided flux when used as a primary [20]. These charge pads therefore do not have any magnetic flux under the charge pad and are usually preferred over double - sided charge pad for primary charge pad designs. Typical examples of single - sided charge pads are the single - sided Circular charge pad, highlighted in Section 3.4 and the Double - D charge pad, highlighted in Section 3.6.

### 3.3 Circular Charge Pad

![Figure 3.1: Top View of a Circular Charge Pad](image)

The circular charge pad is one of the first charge pads to be developed for IPT applications and are one of the most common coupler topology used for electric vehicle charging [3]. In its simplest form it looks like an archimedian spiral from the top view as shown in Figure 3.1, and as a result generates a 'fountain' type flux distribution as shown by the FEM plots in Figure 3.2.

Figure 3.2 also clearly highlights the fact that this design of the Circular Charge Pad generates a double-sided flux distribution which is not ideal for IPT applications as the magnetic field under the charge pad does not couple with a secondary that is above the charge pad. This major disadvantage of this type of a Circular Charge Pad lead to the development of Single-sided Circular Charge Pads (Section 3.4) which use ferrites in order to shape the magnetic field.
3.4 Single-sided Circular Charge Pad

A Single-sided Circular Charge Pad topology, shown in Figure 3.3, has a ferrite base that shapes the magnetic flux and thereby reduces the unwanted magnetic field under the charge pad. These charge pads have been thoroughly investigated in literature and either have a ferrite disk or ferrite spokes [21] that shape the magnetic flux. The ferrite base also has another advantage of reducing the amount of leakage flux, thereby containing the majority of the flux within the area of the charge pad and leading to an increase in the primary-secondary mutual inductance.

The flux distribution is shown in Figure 3.4 and it can be clearly seen that this charge pad is single sided with negligible magnetic flux density under the charge pad. In order to understand the conditions for scalability of such a charge pad, the concept of fundamental flux path height [19] is used. In a Single-sided Circular Charge Pad, the fundamental flux path height is roughly proportional to one quarter of the pad diameter [3], [19]. Hence, if $P_z$ is the fundamental flux path height and $P_d$ is the pad diameter:

$$\Delta P_z \propto \frac{1}{4} \Delta P_d$$

This means that the pad diameter must increase by four times the increase in the air gap. This fact can result in large and heavy circular charge pads making them impractical for systems with large air gaps.
Figure 3.4: Magnetic Flux density plot of a Single-sided Circular Charge Pad (2 mm wire, 5 A current)

3.5 Flux Pipe Charge Pad

The Flux pipe (also called the Solenoidal Coil) charge pad was designed in order to overcome the undesirable fact that the fundamental flux path height of the magnetic field lines of a circular charge pad were proportional to a quarter of the pad diameter [19]. The Flux Pipe Charge Pad consists one or two coils would around a I core type (bar) ferrite and has also been widely studied and used in literature [19], [22]. The flux pipe is a polarised coupler and results in a fundamental flux path height that is proportional to half of the pad length. Hence, if $P_z$ is the fundamental flux path height and $P_l$ is the pad length:

$$\Delta P_z \propto \frac{1}{2} \Delta P_l$$

This means that the pad diameter must increase by two times the increase in the air gap and is a major improvement from the circular charge pad. This is the reason that charge pads with a flux pipe type flux distribution are being recently researched and developed for use in electric vehicles charging [3], [4].
The polarised nature of the flux pipe magnetic field distribution is not a hindrance to its use in charging electric vehicles as they can be positioned accordingly in parking lots and at traffic signals but a major disadvantage of this charge pad is that it generates a double-sided magnetic flux distribution.

### 3.6 Double-D charge pad

![Figure 3.6: Structure of a DD charge pad with flux distribution](image)

The Double - D (DD) charge pad, shown in Figure 3.6, is a polarised, single - sided charge pad that combines advantages of both the circular and flux pipe designs [23]. The two coils of the DD charge pad are magnetically in series by virtue of their winding arrangements and generate a parallel type flux. The two D coils can be wound electrically in series using the same Litz wire or in parallel. The DD structure a flux pipe in the centre which is made as long as possible and the remaining length of the coil is minimised in order to save copper and lower the AC resistive losses.

Figure 3.6 also shows the magnetic flux distribution of the DD charge pad which is similar to that of the Flux Pipe charge pad of Section 3.5 with the important exception that it is single-sided. Having a flux pipe type flux distribution implies that the fundamental flux path height, $P_z$, is approximately proportional to half of the pad length, $P_L/2$, which is indeed a desirable characteristic.

### 3.7 Double-D Quadrature charge pad

The Double-D Charge Quadrature (DDQ) Pad as the name suggests is a Double-D charge pad (Section 3.6) with an additional Quadrature coil that is spatially decoupled with the DD coil as shown in Figure 3.7. These pads were primarily designed for the receiver system in order to improve misalignment tolerances [23] but have only recently been studied for use in the primary coil system [24] in order to improve interoperability between different charge pads.
3.8 Bipolar Pad

The Bipolar Pad, introduced in [25] and [26], is a pad that was designed for use in the primary system [24]. It consists of two spatially decoupled rectangular coils (similar to the D coils in Section 3.6). Figure 3.8 shows a schematic of the Bipolar pad and it can be seen that it is a multi-coil topology with the two individual coils decoupled due to their spatial position. In [26] it was used to study interoperability between different primary and secondary designs. The Bipolar Pad was compared against the performance of a Double-D Quadrature Pad, when used with a DD or circular transmitter, and was found to have comparable performance while using less copper.

The Bipolar Pad was also proposed in [27] for use in the secondary system. Here, it was shown to be advantageous in order to increase misalignment tolerances in electric vehicles while ensuring that the leakage magnetic field that pedestrians are exposed to are well within the ICNRP guidelines.

3.9 Interoperability of Charge Pads

Interoperability of charge pads is an essential requirement for the commercial adoption of IPT for charging applications as this means that a given receiver topology can be used with different transmitter topologies with high efficiency. Interoperability of charge pads has been studied in [24] and [17] and is a challenge due to the fact that current state of the art charge pads are designed to transmit or receive either a perpendicular or parallel flux at the perfectly aligned position.
Figure 3.9 shows a simplified view of transmitters that transmit a perpendicular and parallel flux pattern. Perpendicular flux is transmitted by circular charge pads while the DD and Bipolar charge pads transmit a parallel flux pattern.

Figure 3.9: Simplified view of transmitters that transmit a perpendicular and parallel flux pattern

3.10 Discussion

A summary of the evolution of IPT Charge Pads is provided in this chapter as a literature survey. Of prime importance was the motivation behind the development of various charge pads, which highlight the challenges faced by IPT. In the early developmental stages, the goal was to improve the coupling between the primary and the secondary given a particular primary charge pad area.

Once this goal was sufficiently met, the focus gradually shifted toward designs that were tolerant to primary-secondary misalignment and then to secondary systems that could interoperate with different primary systems with high efficiencies.

In order to meet the above goals of misalignment tolerance and interoperability, multi-coil designs were introduced. These multi-coil charge pads were designed in a way that the multiple coils were spatially decoupled, highlighting the fact that mutual inductance between coils in a multi-coil primary and secondary is not a desirable feature.

Perhaps, with current research being conducted in the design of multi coil topologies and the introduction of multi-frequency IPT [28], [29], another asset of IPT systems would soon be their ability to also operate efficiently even though they are mutually coupled to adjacent transmitters and receivers.
CHAPTER 4

Mutual Inductance in Multi-coil Transmitters and Receivers

IPT systems often have more than one coil transmitting and/or receiving power. Such systems are called multi-coil topologies and have to be designed such that the mutual inductance between the coils is negligible. This chapter explains why this is required and highlights the adverse problems that are created due to intra-coil mutual inductance in multi-coil primary and secondary magnetic designs of IPT systems. This chapter begins with an introduction to multi-coil topologies, their advantages and applications and proceeds to develop a theoretical understanding of the interactions between the coils due to a finite mutual inductance. Finally, multi-frequency IPT is proposed as a solution to decouple power transfer in multi-coil topologies.

4.1 Multi-coil IPT Magnetic Topologies

Multi-coil magnetic topologies are topologies that have more than one independent coil and are used in transmitter as well as receiver designs. Chapter 3 highlights as examples the Double-D Quadrature Pad and the Bipolar Pad which have been proposed to improve misalignment tolerances [23] as well as interoperability of a receiver with different transmitter pads [24]. As explained in Section 3.7 and Section 3.8, the two coils in these topologies are spatially arranged such that the mutual inductance between them is negligible and currents in one coil therefore do not induce currents in the other coil. Although, there is virtually no mutual inductance between these coil, the magnetic fields produced by the two coils interact with each other and add or subtract each other depending on the relative phase of the currents in the coils. Hence, although electrically decoupled, the magnetic field distribution in the air gap changes depending on whether one or both of the coils are powered. This could be advantageous or disadvantageous depending on the specific application.

A multi-coil E core pickup has also been studied in [8] and [18] and is shown in Figure 4.1. Here, the quadrature coil is wound perpendicular to the main winding and hence the two coils are spatially decoupled. In this application, the main winding intercepts the primary flux in the perfectly aligned position while the quadrature winding intercepts the primary flux in a situation wherein the secondary is misaligned with the primary. Hence, in the above mentioned E-core topology as well as in the DDQ receiver topology, the main and the quadrature coils are not
fully utilised together implying an under utilisation of copper in the receiver.

Parallel and perpendicular transmitter flux patterns are highlighted in Section 3.9 and the reason for the under utilisation of copper is the fact the in the DDQ system the DD coils intercept a parallel flux and the Q coil intercepts a perpendicular flux at a misaligned position, and it is not always possible for single frequency systems to generate both type of flux patterns. In the E core pickup of Figure 4.1 too, the main winding is designed to intercept a perpendicular flux while the quadrature winding intercepts a parallel flux at a misaligned position. As will be highlighted in Section 5.9, multi-frequency systems have the potential to effectively solve this issue.

![Figure 4.1: E core pickup with decoupled main and quadrature windings](image)

Multi-coil primary topologies are also used in charging of consumer devices such as mobile phones and laptops [30] and kitchen appliances and Figure 4.2 [31] shows a photo of a charge surface that can be used to charge many mobile phones at the same time. These coils have a finite mutual inductance hence are designed such that many coils are powered by the same inverter and that charging points are sufficiently far apart.

![Figure 4.2: A multi-coil charge pad topology used for charging consumer devices](image)

Another example of multi-coil system are three phase system which are used for dynamic charging of electric vehicles on roadways [32]. These systems improve misalignment tolerances as there are three primary loops and provide a constant power profile for moving loads such as electric vehicles [16]. Three phase systems suffer from the effects of inter-phase mutual inductance and have to be designed to incorporate corrections that negate this inter-phase mutual inductance [18].

Of interest in this section are multi-coil systems that do have a finite inter-coil mutual inductance. Figure 4.3 shows an example of a multi-coil system which contains to rectangular coils and can
be used in the primary and/or secondary system.

![Figure 4.3: A multi coil topology with a finite mutual inductance between the two coils](image)

### 4.2 Mutual Inductance within multi-coil Primary and Secondary systems

IPT systems often require multiple coils in the primary and secondary systems. Multiple coils provide benefits such as the ability of a primary to transfer power via many phases [16] or to improve the lateral tolerance of secondary systems [27]. If the multiple coils in a primary and secondary system are not decoupled, adverse effects such as detuning of the primary and secondary system and power transfer within the primary and secondary circuits in addition to between a primary and a secondary system will occur. These concepts will be developed in this section with the aim of highlighting the effects of mutual inductance within multi-coil primary and secondary systems.

**Multi-coil Secondary Systems**

Multi-coil secondary systems provides the receiver with the ability to interoperate with different transmitters [26] and/or increase the lateral tolerance of the receiver to misalignment [27]. Such receivers consist of coils that are spatially decoupled due to their relative positioning and hence do not interact with each other as is the case with the Bipolar Receiver Pad and the Double-D Quadrature (DDQ) Receiver Pad. Multi-coil receivers with non decoupled coils interact and exhibit different resonant frequencies depending on the currents flowing in the secondary coils, thereby drastically reducing the efficiency in the IPT system. This can be explained as follows. Figure 4.4 shows the equivalent circuit of two mutually coupled receiver coils. Here, $V'_{s1}$ and $V'_{s2}$ are the induced voltages on the secondary due to the primary current. Assuming that the same primary current ($I_p$) induces both these voltages, from (2.1)

$$V'_{s1} = \omega M_1 I_p$$

$$V'_{s2} = \omega M_2 I_p$$

Where, $M_1$ and $M_2$ are the mutual inductances between the primary and $L_{s1}$ and $L_{p1}$ respectively, which depends on the relative alignment between the primary and the secondary systems. It can be assumed that, when the primary and secondary systems are perfectly aligned, $M_1 = M_2$, which implies that $V_{s1} = V_{s2}$.

$V'_1$ is the induced voltage in the circuit of $L_{s1}$ due to $I_{s2}$ and $V'_2$ is the induced voltage in the
Figure 4.4: Equivalent circuit of two mutually coupled (Series Compensated) Receiver Coils

The induced voltages in (4.3) and (4.4) have a positive or negative sign depending on the relative magnetic orientation of the two receiving coils. Again, it can be assumed that, when the primary and secondary systems are perfectly aligned, $I_{s1} = I_{s2}$ and hence, $V'_1 = V'_2$. The voltage equations of the equivalent circuit of Figure 4.4 are therefore

\[ V'_{s1} = \left[ R_{L1} + j \left( \omega L_{s1} - \frac{1}{\omega C_{s1}} \right) \right] I_{s1} \pm j\omega M'_s I_{s2} \] (4.5)

\[ V'_{s2} = \pm j\omega M'_s I_{s1} + \left[ R_{L2} + j \left( \omega L_{s2} - \frac{1}{\omega C_{s2}} \right) \right] I_{s2} \] (4.6)

The resonant frequency of the receiver can be obtained by setting the imaginary part of equation (4.5) and (4.6) to zero and these equations show that the receiver resonant frequency is dependant on the currents $I_{s1}$ and $I_{s2}$, which is an undesirable characteristic in the case of IPT systems as the secondary current magnitudes are directly dependant on the coupling between the primary and the secondary, which varies depending on the relative alignment between the primary and the secondary. If the two secondary coils are mutually decoupled due to their spatial positions ($M'_s = 0$), the resonant frequencies of the two receiver coils always remain the same and are independent of the currents $I_{s1}$ and $I_{s2}$. It is therefore important to design decoupled multi-coil receivers for IPT systems.

Two special cases arise: a case when the secondary system is perfectly aligned with the primary system and a case when the secondary system is misaligned to the extent that only one of the secondary coils (say $L_{s1}$) couples with the primary, implying that the induced voltage due to the primary in the other coil ($V'_{s2}$) is zero. From (4.5) and (4.6), it can be easily seen that for the perfectly aligned case and after applying all of the above assumptions, the resonant frequency of the receiver, $\omega_{r1}$, is

\[ \omega_{r1} = \frac{1}{\sqrt{(L_{s1} \pm M'_s)C_{s1}}} = \frac{1}{\sqrt{(L_{s2} \pm M'_s)C_{s2}}} \] (4.7)
$L_{s1}$ is usually equal to $L_{s2}$ but (4.7) is valid even if that is not the case.

In the misaligned special case described above ($V'_s = 0$), (4.5) and (4.6) reduce to

$$V'_{s1} = \left[ R_{L1} + j \left( \omega L_{s1} - \frac{1}{\omega C_{s1}} \right) \right] I_{s1} + j \omega M'_1 I_{s2}$$

$$0 = \pm j \omega M'_1 I_{s1} + \left[ R_{L2} + j \left( \omega L_{s2} - \frac{1}{\omega C_{s2}} \right) \right] I_{s2}$$

$$\Rightarrow V'_{s1} = \frac{\left( \omega M'_1 \right) I_{s1}}{R_{L1} + j \left( \omega L_{s1} - \frac{1}{\omega C_{s1}} \right)}$$

(4.8)

From (4.9) it can be clearly seen that the resonant frequency of the receiver for this misaligned special case is now

$$\omega_{r2} = \frac{1}{\sqrt{L_{s1}C_{s1}}} = \frac{1}{\sqrt{L_{s2}C_{s2}}}$$

(4.10)

(4.7) and (4.10) are indeed special cases but they highlight the fact that the resonant frequency of a multi-coil receiver with independent and non decoupled coils can vary greatly and is also directly dependent on the currents in the respective receiver coils. Currently, a receiver of this kind would have spatial decoupled coils as in the case of a Bipolar Receiver Pad or a DDQ Pad. As will be explained in this Chapter, multi-frequency IPT systems can provide the decoupled feature to multi-coil receivers with more freedom in the coil design, which can also possibly lead to a saving of copper.

Multi-coil Primary Systems

The primary coils in IPT systems often have multiple coils which transfer power to the secondary. If both these coils are independently excited, the mutual inductance between them will cause power to be transferred between them by inducing voltages, much like how the mutual inductance between a primary and secondary causes power transfer [16]. These induced voltages will cause induced currents to flow, which can be dangerous to the normal operation of an IPT primary system, as will be explained in this section. The effects the mutual inductance between two coils of a multi-coil primary will be examined for the case of a SS and a LCL - S system as follows.

The problem with mutual inductance within multi-coil primary IPT systems has been acknowledged and studied in literature [16]. With respect to lumped charge pads, multi-coil systems (such as the DDQ pad) have conventionally been developed to improve misalignment tolerances and hence are found in secondary system designs. It is only recently, that multi-coil systems have been studied for primary system design and have been found to be advantageous in ensuring interoperability of the primary with various different secondary designs [24]. But of course, these multi-coil systems have coils that are spatially decoupled and hence have a negligible inter-coil mutual inductance.

Unlike the case of multi-coil secondary systems, mutual inductance in multi-coil primary systems can adversely affect the input power supply. These effects will be analysed in this section and requirements for a decoupled multi-coil primary design will be developed.

Similar to the case of multi-coil secondary systems, Figure 4.5 shows the equivalent circuit of two mutually coupled Transmitter Coils. Here, $V_{in,1}$ and $V_{in,2}$ are the output voltages from the
power supply. These are square wave waveforms but Fundamental Mode Analysis [33] is used and these are replaced with a sinusoidal voltage of frequency and magnitude equal the fundamental component of the inverter output. $R_p$ and $R_{p2}$ are the the summation of the series resistances of the primary inductor, capacitor as well as and equivalent resistance of the internal losses in the inverter. $R'_L$ and $R'_{L2}$ are the reflected resistances from the secondary onto the primary. As this is a series compensated primary the input power supply is designed to provide rated primary currents.

Due to the the inter coil mutual inductance, $M'_p$, the voltage equations are

\[
V_{in,1} = \left[ R'_L + R_p + j \left( \omega L_p - \frac{1}{\omega C_p} \right) \right] I_{p1} \pm j \omega M'_p I_{p2}
\]

\[
V_{in,2} = \pm j \omega M'_p I_{p1} + \left[ R'_L + R_p + j \left( \omega L_p - \frac{1}{\omega C_p} \right) \right] I_{p2}
\]

(4.11)

Hence, similar to the multi-coil receiver case, (4.11) highlights the fact that the resonance frequency of mutually coupled multi-coil primaries depend on the currents flowing in the respective circuits. It can also be seen that $M'_p$ can cause a reverse current that flows into the inverter, being rectified by the anti - parallel diodes of the inverter switches. These induced currents can therefore cause the voltage across the input DC bus capacitors to rise and therefore cause problems in the normal control of the inverter.

Let $\kappa_p$ be the coupling factor that defines the coupling between the two primary coils and be defined as:

\[
\kappa_p = \frac{M'_p}{\sqrt{L_p L_{p2}}}
\]

(4.12)

Figure 4.6 shows a simulation plot of $V_{in,1}$ and $I_{p1}$ for the case when $\kappa_p = 0$ for a series compensated primary tuned at 100 kHz. It can be clearly seen that the input voltage and current waveforms are in phase and hence at resonance at 100 kHz. Figure 4.7 on the other hand shows a simulation plot of $V_{in,1}$ and $I_{p1}$ for the same system when $\kappa_p = 0.11$ and it can be
clearly seen that the two waveforms are out of phase. This implies that the resonance frequency has changed from 100 kHz thereby reducing the system efficiency.

Figure 4.6: Simulated $V_{in,1}$ (green) and $I_{p1}$ (red) in 100 kHz tuned series compensated Primary with $M'_p = 0$ ($\kappa_p = 0$)

Figure 4.7: Simulated $V_{in,1}$ (green) and $I_{p1}$ (red) in 100 kHz tuned series compensated Primary with $\kappa_p = 0.11$

**Multi-coil LCL Primary**

Mutual inductance in multi-coil primary LCL systems has been studied in [16] with respect to three phase primary systems where each phase was powered by an independent inverter leg. In a LCL system, the inter-coil mutual inductance does not cause a detuning of the transmitter but causes a reverse current into the power supply, which is rectified by the body diodes of the inverter switches and in turn causes a higher than nominal bus voltage across the input capacitor. This can cause problems with the inverter control.
The equivalent circuit of a LCL primary coil is shown in Figure 4.8. This coil is mutually coupled with another coil in the primary topology (not shown) with a mutual inductance, \( M'_p \). \( V' \) is the induced voltage due to currents in the other primary coil. In order to examine the effects of inter-coil mutual inductance in LCL primary systems, the following transfer functions must be calculated:

\[
Z' = \frac{V'}{I'_p} = \frac{j\omega(L_1 + L_p - \omega^2L_1L_pC_p)}{(1 - \omega^2L_1C_p)} \tag{4.13}
\]

\[
G'_t = \frac{I'_{\text{inv}}}{V'} = \frac{1}{j\omega(L_1 + L_p - \omega^2L_1C_pL_p)} \tag{4.14}
\]

\( Z' \) highlights the effect of inter-coil mutual inductance on the coil current and \( G'_t \) highlights its effect on the reverse inverter current. Figure 4.9 plots \( I'_{p1} \) and \( I'_{\text{inv},2} \) for 100 kHz induced currents for the IPT system of Appendix A. It can be therefore seen that if the frequency of the resonant tank is also 100 kHz, implying that both the primary coils are tuned to the same frequency, the induced coil current, \( I'_{p1} \), is completely attenuated but the reverse inverter current, \( I'_{\text{inv},2} \), is not. Also it can be seen that, as expected, the magnitude of the induced currents depend on the magnitude of the currents, \( I_{p1} \) in the other primary coil.

The expression for the primary current in a LCL tank, (2.30), is derived in Section 2.6 and is repeated below.

\[
I_p = \frac{V_{\text{in}}}{j\omega_0L_1}
\]

Therefore, in order to maintain a constant primary current, the input voltage is directly proportional to the primary coil inductance. As will be further elaborated in Section 5.4.1, the design of multi frequency systems with multi-coil primary topologies involves increasing the inductances in the primary and secondary circuits. This is not ideal in the case of LCL systems as this would imply an increase in the primary source voltage which might increase to large values for high power systems. Hence in this thesis, the study of multi frequency systems is limited to compensation topologies that result in a input voltage that is independent of the primary coil inductance, such as the SS compensation topology.

The adverse effects of mutual inductance within non decoupled and independent multi-coil primary and secondary systems have been highlighted in this section. It is important to note that although two coils have a specific mutual inductance due to their relative spatial positioning, it is the induced currents that lead to the above mention adverse effects due to inter coil mutual
inductance. Hence, if the magnitude of the induced currents are reduced, these effects can be contained irrespective of the relative spatial positioning of the same two coils. A dual frequency IPT system has the potential to achieve a general design of a decoupled multi-coil primary or secondary, as currents of a particular frequency are generally attenuated when induced in a circuit tuned to a different resonant frequency. The design considerations for the design of a multi frequency IPT system will be elaborated in Chapter 5.

### 4.3 Flux Addition and Subtraction in Multi-coil Topologies

Multi-coil topologies, such as the DD type coil shown in Figure 4.3, when used in the primary of IPT systems will generate a magnetic field in the air gap depending on the relative phase of the currents in the two coils [24]. If the two coil currents are anti-phase, flux addition occurs producing a parallel type flux distribution in the air gap and if the two coil currents are in phase flux cancellation occurs in the center of the primary as shown in Figure 4.10.
Hence, multi-coil primary topologies operating at the same frequency inevitably produce air gap magnetic fields that interact with each other and are therefore operated only in a particular way with respect to the phase of currents in the coils. The DD primary charge pad, for example, is operated with currents 180° out of phase in coil 1 and coil 2. The moment the phase relative difference of the currents is changed or one of the coils is switched off, the orientation of the air gap magnetic field changes.

4.4 An indication of inter-coil Mutual Inductance

Figure 4.11 shows a multi-coil primary that was used in the laboratory prototype. The distance between the centres of the two coils is $x$ and Table 4.1 highlights the variation of the inter-coil mutual inductance with $x$. Each coil has a diameter of 12 cm.

![Multi-coil primary system](image)

**Figure 4.11: Multi-coil primary system**

<table>
<thead>
<tr>
<th>$x$ (cm)</th>
<th>$M'(\mu H)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.8</td>
</tr>
<tr>
<td>15</td>
<td>0.4</td>
</tr>
<tr>
<td>18</td>
<td>0.2</td>
</tr>
<tr>
<td>9</td>
<td>1.8</td>
</tr>
<tr>
<td>6.5</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>10.6</td>
</tr>
<tr>
<td>0</td>
<td>17.6</td>
</tr>
</tbody>
</table>

Table 4.1: Intercoil Mutual Inductance as a function of distance between the centres of two coils

Table 4.1 highlights the fact that the mutual inductance is very high when the two coils are stacked one over the other and that spatial decoupling is achieved when $x = 6.5$ cm which is a partial overlapped position. It should be noted that although the two coils are spatially decoupled, it might not be possible to power them independently as the air gap magnetic field distribution would change depending on which coil is powered as is highlighted in Section 4.3.

The spatially decoupled position of the two coils could be beneficial in multi-frequency systems too and will be further explained in Chapter 5.
4.5 Multi-frequency IPT as a means to decouple Multi-coil Topologies

A multi-frequency IPT method is proposed in this thesis as a means to decouple multi-coil topologies. If the two coils of a multi-coil primary topology are powered by currents of different frequencies, there is a possibility of decoupling the two coils as:

- A current of one frequency is attenuated when induced in a circuit which is tuned to a different frequency. Hence, if the induced currents are suppressed, the adverse effects of inter-coil mutual inductance as highlighted in this chapter are controlled.

- The air gap magnetic field is directly proportional to the currents flowing in the primary coils. As shown in Figure 4.12, a magnetic field produced by currents of two different frequencies is a superposition of the two frequencies and as such cannot be completely cancelled out as in the case when the two frequencies are equal. Hence, if intercepted properly by a receiver that is tuned to the correct frequency, orthogonal power flow as described in [34] can be achieved.

![Figure 4.12: A primary current that is the superimposition of a 20 kHz and a 100 kHz current.](image)

Chapter 5 explains the concept of multi frequency IPT as is proposed in this thesis.

4.6 Discussion

This chapter introduces the problems and challenges faced in the design of IPT systems due to mutual inductance between coils in multi coil primary and secondary systems. The inter-coil mutual inductance in the secondary detunes the secondary system and the resonant frequency of the secondary is, in fact, a function of the secondary induced currents. This is extremely undesirable, as the secondary induced currents are a function of the primary-secondary mutual inductance which in turn is a function of the relative primary-secondary alignment.

In the primary system, the effect of mutual inductance also causes a detuning of the primary depending on the respective primary currents but sometimes of greater concern is the fact that
induced currents due to an adjacent primary coil cause reverse currents into the primary power supply, over charging the input capacitor, which can cause problems for the inverter control. These effects are highlighted for a series compensated as well as a LCL compensated primary system.

The fact that the magnetic field distribution in the air gap changes when the primary has multiple coils is also highlighted in this chapter. The magnetic field distribution is a function of the relative phases of the currents in the two coils.

In conclusion, the concept of multi frequency IPT as a possible solution to these challenges is proposed. Multi-frequency IPT has the potential to decouple two primary coils that otherwise have a finite mutual inductance. This is due to the fact that a system tuned to one frequency can attenuate induced currents of another frequency thereby considerably minimising the adverse effects caused by inter-coil mutual inductance that are highlighted in this section. The magnetic field in the air gap can also be thought of as a superimposition of the field caused by the two frequencies, thereby creating a system that can operate efficiently independent of whether one or both the coils are powered.
This chapter provides the basis for the design of a multi-frequency IPT system wherein power is transferred from the transmitter to the receiver via multiple frequencies. The chapter begins by describing the methodology used to investigate this concept of multi-frequency IPT followed by an extensive study of how multi frequency IPT can be used to decouple multi-coil topologies, which is a continuation of the introduction provided in Section 4.5. A comparative investigation into the two different possible approaches for realising multi frequency IPT is then provided. The respective advantages and disadvantages of the two methods are analysed and design consideration and limitations are highlighted. As a first step, a dual frequency IPT system, transferring power from a transmitter to a receiver via two different frequencies is analysed.

5.1 Multi-frequency IPT: Analysis Method

The current state of art wireless power transfer systems, although frequency dependent, are designed to only transfer power at a single frequency without a signal multiplex. Multiplexed signals are extremely common in wired and wireless communication (digital and analogue) and enables multiple independent information signals to be transferred and intercepted via the same communication link, but the concept has only just begun to be researched for inductive wireless power transfer systems. In order to investigate the viability of multi-frequency IPT, this study has been carried out with emphasis on its effect on the important parameters of IPT systems and on the current challenges faced by IPT system design as mentioned in Chapter 3.

This chapter describes the design and development of multi-frequency IPT systems with emphasis on the effect of multiple frequencies in IPT on:

- Decoupling multi-coil primary and secondary systems
- Misalignment between the transmitter and receiver
- Inter-operability of a transmitter with different types of receivers
- Emitted leakage field strength
- Conduction losses in the inverter, transmitter and receiver coils
- Copper saving in the transmitter and receiver coils
Multi-frequency IPT can be analysed by the following two approaches:

- **Multi-frequency IPT using one transmitter coil**: In this approach, a multi-frequency IPT system is realised by *amplifying* the fundamental and the third harmonic of the inverter output voltage, hence exciting the primary coil with a current that is a superposition of the two frequencies [28], [29]. Here, the inverter switches operate at the lower fundamental frequency but the power transfer is shared between the fundamental and the third harmonic frequencies.

- **Multi-frequency IPT using multiple transmitter coils**: This approach of multi-frequency IPT is the main focus of this work and consists of a primary system that consists of multiple transmitter coils. As a first step, a system with two transmitter coils is considered. Each transmitter coil carries an AC current of a frequency, that is different from that carried in the other coil. These two primary coils are powered independently by two separate inverter legs from the same DC link and in general have a finite mutual inductance between them. This method has the advantage of electrically decoupling two coils in a multi-coil topology which is the major focus of this work.

A multi-frequency IPT system can have a single receiver coil tuned to resonate at both the transmitting frequencies [29] or two or more independent receiving coils each tuned to resonate at one or both of the transmitting frequencies. As mentioned above the major focus of this work is to study multi-frequency IPT as a means to decouple multi-coil topologies and therefore the receiver topology that will be given prime importance is that which has two coils tuned to different frequencies, implying that the receiver coils are also electrically decoupled.

### 5.2 Dual frequency IPT using one Transmitter Coil

A dual frequency system was recently proposed by Z. Pantic *et al* in [28] and [29], wherein, both the fundamental and third harmonic of the inverter current are fed to the transmitter and the receiver intercepts a magnetic field produced by the superimposition of the currents of both the frequencies. The power transferred to the load is shared by the two frequencies and is determined by a power sharing ratio, \( m \). This section provides a concise description and analysis of this method.

![Figure 5.1: Schematic of a dual frequency IPT system with one transmitter - receiver pair](image-url)
Figure 5.1 shows the general schematic of a dual frequency IPT system with one transmitter - receiver pair. The primary inductor is excited by two equivalent power sources, \( V_{in,1} \) and \( V_{in,3} \), which are considered to be the fundamental and the third harmonic of the inverter. \( G_t \) and \( G_r \) are the transconductances of transmitter and receiver compensation blocks which are designed so that the first and third harmonics are resonant frequencies. Assuming that the inverter produces a square wave output voltage, it can be decomposed into a Fourier series of odd harmonics:

\[
v_{in}(t) = \frac{2V_{dc}}{\pi} \sum_{k=1,3,5,...}^{+\infty} \frac{\sin(k\omega t)}{k} \tag{5.1}
\]

Hence, from (5.1), the RMS values of the first and third harmonics of the input voltage are

\[
V_{in,1} = V_{in}(\omega = \omega_1) = \frac{2\sqrt{2}V_{dc}}{\pi} \\
V_{in,3} = V_{in}(\omega = \omega_3) = \frac{V_{in,1}}{3} \tag{5.2}
\]

As can be noted from (5.2), the third harmonic voltage is attenuated and is three times less than the magnitude of the fundamental. Hence, when the same power is transferred only by this frequency to a given load, it will draw a current from the inverter that is three times higher as compared to an IPT system with an input voltage \( V_{in} \) and operating at a frequency equal to this third harmonic. This concept of power sharing will be further developed in detail in this Chapter.

The subsequent odd harmonics after the fundamental are further attenuated by by the factor \( k \) as can be easily calculated from (5.1) and hence are not considered. Hence, this method fixes power transfer only to the fundamental and third harmonic. A major advantage of this method is the fact that some power can be transferred by the third harmonic while the inverter switches operate at the fundamental harmonic. As further explained in this Chapter, although it could be said that the switching losses are less compared to an inverter switching at the third harmonic; the conduction losses increase when more power is transferred at the higher frequency.

Also as will be highlighted in this Chapter, the third harmonic can provide benefits such as decreasing the field emissions around the transmitter, but this comes at a price of transmitting only 20 % of the power via the higher third harmonic frequency.

As shown in Figure 5.1, the compensation blocks of the transmitter and receiver are represented by their transconductances \( G_t \) and \( G_r \). \( I_p \) is the primary coil current and \( I_s \) is the current flowing in the secondary coil. \( V_{oc} \) is the open circuit secondary induced voltage and \( I_{out} \) is the current flowing into the load. The transconductances are defined as the ratio of the current variation at the output to the voltage variation at the input and are given by

\[
G_t(\omega) = \frac{I_p(\omega)}{V_{in}(\omega)} \tag{5.3} \\
G_r(\omega) = \frac{I_{out}(\omega)}{V_{oc}(\omega)} \tag{5.4}
\]

The power sharing ratio, \( m \), which defines the ratio with which power is shared between the two frequencies is defined as follows

\[
m = \frac{P_{out,1}}{P_{out}} \\
1 - m = \frac{P_{out,3}}{P_{out}} \tag{5.5}
\]
Here, $P_{\text{out},1}$ and $P_{\text{out},3}$ are the output powers delivered by the first and third harmonics respectively.

For the case of a dual frequency system, a non linear load can be modelled as a frequency and power dependent resistance as

$$R_{\text{ac},1} = R_{\text{ac}}(\omega = \omega_1, m) = R_{\text{ac},0}^* R_{\text{ac},1}(m)$$
$$R_{\text{ac},2} = R_{\text{ac}}(\omega = \omega_3, m) = R_{\text{ac},0}^* R_{\text{ac},3}(m)$$

(5.6)

where, $R_{\text{ac},1}(m)$ and $R_{\text{ac},3}(m)$ are the normalised ac load resistances, normalised to the single frequency resistance, $R_{\text{ac},0}$. The power transferred to the load in a single frequency system, $P_{\text{out},s}$, and via the first and third harmonic, $P_{\text{out},1}$ and $P_{\text{out},3}$ respectively, are given by

$$P_{\text{out},s} = I_{\text{out},s}^2 R_{\text{ac},0} = (V_{oc} G_r)^2 R_{\text{ac},0} = (I_p M \omega_1 G_r)^2 R_{\text{ac},0}$$
$$P_{\text{out},1} = (V_{in,1} M \omega_1 G_{t,s} G_{r,s})^2 R_{\text{ac},1}$$
$$P_{\text{out},3} = (V_{in,3} M \omega_3 G_{t,3} G_{r,3})^2 R_{\text{ac},3}$$

(5.7)

The power equations in (5.7) basically express the output power as a function of the input voltage. From (5.2), (5.5) and (5.7)

$$(G_{t,1} G_{r,1})^2 R_{\text{ac},1} = m (G_{t,s} G_{r,s})^2 R_{\text{ac},0}$$
$$(G_{t,3} G_{r,3})^2 R_{\text{ac},3} = (1 - m) (G_{t,s} G_{r,s})^2 R_{\text{ac},0}$$

(5.8)

The normalised forms of (5.8), where $G_{t,s}$, $G_{r,s}$ and $R_{\text{ac},0}$ are used as base values, are more indicative and are represented as

$$G_{t,1}^* G_{r,1}^* = \sqrt{\frac{m}{R_{\text{ac},1}^*}}$$
$$G_{t,3}^* G_{r,3}^* = \sqrt{\frac{1 - m}{R_{\text{ac},1}^*}}$$

(5.9)

The equations derived above are the groundwork for an extensive analysis of Multi-frequency IPT as stated in Section 5.1 and will be used throughout this chapter to develop the concept of Multi-frequency IPT and to highlight the major differences between the two methods of using one transmitter coil and multiple transmitter coils.

The above expression (5.9) highlights the fact that the power sharing ratio is determined by setting the frequency dependent transconductance values. The transconductance at a particular frequency depends on the value of the inductances and capacitances and hence, once set for a particular design cannot be changed. This implies that the power sharing ratio also remains fixed. This is in stark contrast to the proposed dual frequency IPT system with two transmitter coils where the power sharing ratio is dependent on the currents flowing in the coils and can be changed and controlled even when the IPT system is operating. This is the system proposed in this work and will be introduced in Section 5.3.

**Design of multi-resonant transmitter and receiver**

As this one transmitter system uses just one inverter and one transmitting and possibly one receiving coil, a multi-resonant transmitter and receiver is required. The design of such a
resonant tank is explained in [28] as a L-C-L-C ladder configuration. Such a system is a fourth order system and the values of the inductors and capacitor can be chosen such that it resonates at the first and third harmonic of the inverter frequency.

5.3 Dual frequency IPT with two Transmitter Coils

In this section, a dual frequency system using two transmitter coils will be introduced and relevant equations that will be required for the subsequent analysis and design will be derived. In general, a dual frequency system of this kind, can have one receiver coil that is tuned to resonate at both the frequencies of the transmitter or can have two independent and decoupled receiver coils that are each tuned to one of the transmitting frequencies. These two frequencies can be represented as $\omega_1$ and $\omega_n$ where

$$\omega_n = n\omega_1$$  \hspace{1cm} (5.10)

and $n$ is, in general, any real number. From (5.10), it may be perceived that the choice of frequencies could be virtually unlimited. This is not true, although the designer has more freedom in choosing the two frequencies as compared to the one transmitter dual frequency system of Section 5.2. These conditions for frequency selection will be highlighted in Section 5.4.1. This is in stark contrast to the dual frequency IPT system of Section 5.2, where the third harmonic of the fundamental is always the higher frequency and $\omega_n = \omega_3 = 3\omega_1$. Figure 5.2 shows a schematic block diagram of a two transmitter dual frequency system.

Given this nomenclature, the power sharing ratio, $m$, for a dual frequency IPT system with two transmitter coils is represented by

$$m = \frac{P_{out,1}}{P_{out}}$$

$$1 - m = \frac{P_{out,n}}{P_{out}}$$  \hspace{1cm} (5.11)

and (5.8) becomes
\[(G_t,1 G_r,1)^2 R_{ac,1} = m(G_t,s G_r,s)^2 R_{ac,0}\]
\[n^2 (G_t,n G_r,n)^2 R_{ac,n} = (1 - m)(G_t,s G_r,s)^2 R_{ac,0}\]

The normalised forms of (5.12), where \(G_t,s\), \(G_r,s\) and \(R_{ac,0}\) are used as base values are

\[G_{t,1}^* G_{r,1}^* = \sqrt{\frac{m}{R_{ac,1}^*}}\]
\[G_{t,n}^* G_{r,n}^* = \frac{1}{n} \sqrt{\frac{1 - m}{R_{ac,1}^*}}\]

As can be seen in (5.13), unlike in (5.9), the product of the normalised transconductances of the higher frequency system is now divided by the factor \(n\).

Figure 5.3: Schematic of a dual frequency IPT system with two transmitter coils and two receiver coils

As mentioned above, the power sharing ratio in a dual frequency system with two transmitter coils is dependent on the currents flowing in the two individual coils. Hence, instead of representing \(m\) as a function of the transconductances as in (5.13), it is more revealing to represent it as a function of the currents in the primary coil as in this system the two frequencies can be considered as being of independent systems.

As is seen in (2.21), the output power is directly proportional to the square of the primary current. From (5.11), \(P_{out}\) is the total real power that is delivered to the rated load and is the sum of the real powers delivered by both the transmitting frequencies. The value of \(P_{out}\) is hence independent of frequency and can be considered to be a reference. Let \(I_{eq}\) be the primary current of a conventional single frequency system delivering \(P_{out}\) to the rated load at a reference frequency equal to the lower frequency. Also, let \(I_1\) and \(I_n\) be the lower frequency and the higher frequency currents respectively. These assumptions are used in the analysis of transmitter and receiver conduction losses in Section 5.8.3. From (5.11) and (2.21)
\[ P_{out,1} = mP_{out} \]
\[ \Rightarrow I_{p,1} = \sqrt{m}I_{eq} \]  

(5.14)

and for the higher frequency

\[ P_{out,n} = (1 - m)P_{out} \]
\[ \Rightarrow I_{p,n} = \sqrt{1 - \frac{m}{n}}I_{eq} \]  

(5.15)

As can be seen from (5.14) and (5.15) the power sharing ratio, \( m \), can be controlled by controlling the primary currents. It can also be seen that when \( m = 1 \), \( I_{p,1} = I_{eq} \) and all the power is transferred via the lower frequency and when \( m = 0 \), \( I_{p,n} = I_{eq}/n \) which is expected when all the power is transferred via the higher frequency. This leads to the well known result that as the operating frequency increases, the primary current decreases by the factor of increase in the operating frequency.

Hence, unlike the single transmitter dual frequency system described in Section 5.2, a dual frequency IPT using multiple transmitter coils, will have to be adequately designed so as to decouple the two transmitter coils. The design procedures for this will be described in Section 5.4. For the analysis in Section 5.8, it is assumed that the two dual frequency coils in the primary are already decoupled.

### 5.4 A Dual frequency SS system

As mentioned before, this work is focussed on a dual frequency, two transmitter coil system and the design considerations of such a system will be developed in this Section. A SS system is considered because of the fact that \( C_p \) is only dependant on the primary inductance and operating frequency which are assumed to remain constant during the system operation and also because of the fact that in SS systems, the input voltage is independent of the primary inductance value.

In the analysis of a Dual Frequency IPT system with two transmitter coils a consistent symbol nomenclature is used which is described in Table 5.1.

In addition to the symbols defined in Table 5.1, the mutual inductances between the various coupled coils are defined as follows:

- \( L_{p1} - L_{p2} \) Mutual Inductance: \( M'_p \)
- \( L_{s1} - L_{s2} \) Mutual Inductance: \( M'_s \)
- \( L_{p1} - L_{s2} \) Mutual Inductance: \( M'_{p1,s2} \)
- \( L_{p2} - L_{s1} \) Mutual Inductance: \( M'_{p2,s1} \)

In the case of identical coils being used in the the primary and secondary systems of both the frequencies \( (L_{p1} = L_{p2} = L_{s1} = L_{s2}) \), as shown in Figure 5.3, the above mutual inductance symbols are defined as follows:

- \( L_{p1} - L_{p2} \) Mutual Inductance: \( M' \)
- \( L_{s1} - L_{s2} \) Mutual Inductance: \( M' \)
- \( L_{p1} - L_{s2} \) Mutual Inductance: \( M'' \)
- \( L_{p2} - L_{s1} \) Mutual Inductance: \( M'' \)
### Table 5.1: Symbol nomenclature in a Dual Frequency IPT System with two transmitter coils

<table>
<thead>
<tr>
<th></th>
<th>Higher Frequency System</th>
<th>Lower Frequency System</th>
</tr>
</thead>
<tbody>
<tr>
<td>System Name</td>
<td>System 1</td>
<td>System 2</td>
</tr>
<tr>
<td>Operating Frequency</td>
<td>$\omega_1$</td>
<td>$\omega_2$</td>
</tr>
<tr>
<td>Primary Inductance</td>
<td>$L_{p1}$</td>
<td>$L_{p2}$</td>
</tr>
<tr>
<td>Primary Compensation Capacitance</td>
<td>$C_{p1}$</td>
<td>$C_{p2}$</td>
</tr>
<tr>
<td>Secondary Inductance</td>
<td>$L_{s1}$</td>
<td>$L_{s2}$</td>
</tr>
<tr>
<td>Secondary Compensation Capacitance</td>
<td>$C_{s1}$</td>
<td>$C_{s2}$</td>
</tr>
<tr>
<td>Primary Coil Current</td>
<td>$I_{p1}$</td>
<td>$I_{p2}$</td>
</tr>
<tr>
<td>Secondary Coil Current</td>
<td>$I_{s1}$</td>
<td>$I_{s2}$</td>
</tr>
<tr>
<td>Primary-Secondary Mutual Inductance</td>
<td>$M_1$</td>
<td>$M_2$</td>
</tr>
<tr>
<td>Impedance of Primary equivalent circuit</td>
<td>$Z_{p1}$</td>
<td>$Z_{p2}$</td>
</tr>
<tr>
<td>Impedance of Secondary equivalent circuit</td>
<td>$Z_{s1}$</td>
<td>$Z_{s2}$</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
    V_{in,1} \\
    0 \\
    0 \\
    0
\end{bmatrix} =
\begin{bmatrix}
    Z_{p1} & j\omega_1 M_1 & j\omega_1 M' & j\omega_1 M'' \\
    j\omega_1 M_1 & Z_{s1} & j\omega_1 M'' & j\omega_1 M' \\
    j\omega_1 M' & j\omega_1 M'' & Z_{p2} & j\omega_1 M_2 \\
    j\omega_1 M'' & j\omega_1 M' & j\omega_1 M_2 & Z_{s2}
\end{bmatrix}
\begin{bmatrix}
    I_{p1} \\
    I_{s1} \\
    I_{p2} \\
    I_{s2}
\end{bmatrix}
\]

(5.16)

\[
\begin{bmatrix}
    V_{in,2} \\
    0 \\
    0 \\
    0
\end{bmatrix} =
\begin{bmatrix}
    Z_{p1} & j\omega_2 M_1 & j\omega_2 M' & j\omega_2 M'' \\
    j\omega_2 M_1 & Z_{s1} & j\omega_2 M'' & j\omega_2 M' \\
    j\omega_2 M' & j\omega_2 M'' & Z_{p2} & j\omega_2 M_2 \\
    j\omega_2 M'' & j\omega_2 M' & j\omega_2 M_2 & Z_{s2}
\end{bmatrix}
\begin{bmatrix}
    I_{p1} \\
    I_{s1} \\
    I_{p2} \\
    I_{s2}
\end{bmatrix}
\]

(5.17)

As can be seen from (5.17) and (5.17) the principle of superposition is used to analyse the induced currents by the two frequency systems.

#### 5.4.1 Frequency Selection

![Figure 5.4: Impedance vs. Frequency in a Series RLC circuit with $L = 20\mu H$](image_url)
Frequency selection plays a major role in the design process of a SS multi-frequency system. As the aim is to decouple two coils, it is important that the two circuits present a high impedance to currents of the other frequency while resonating at their respective resonance frequencies. The required magnitude of this impedance, depends on the the currents in the coils and the mutual inductance between the two coils as will be highlighted in this section. In a general RLC circuit, the impedance, $Z$, and the quality factor, $Q$, are given by

$$Z = R + j \left( \omega L - \frac{1}{\omega C} \right) \quad (5.18)$$
$$Q = \frac{\omega L}{R} \quad (5.19)$$

The Quality factor gives an indication of a resonators bandwidth relative to its centre frequency and is directly proportional to the resonant frequency $\omega$. Figure 5.4 shows the variation of $|Z|$ vs frequency for RLC circuits, with $L = 20 \ \mu H$, resonating at 20 kHz, 50 kHz, 100 kHz and 150 kHz and it is clearly evident that the higher frequency resonance circuits have a better quality factor as compared to the lower frequency resonance circuits. A fallout of this is that high frequency RLC circuits invariably present a much higher impedance to lower frequency currents than vice versa. Figure 5.4 shows that the 100 kHz circuit presents an impedance to 20 Khz currents that is close to 10 times that presented by the 20 kHz circuit to 100 kHz currents. As can be seen from (5.18), the impedance is directly proportional to the inductance. Figure 5.5 shows the impedance vs frequency plots when the inductance is increased to 70 $\mu H$ and it can be clearly seen that the impedances at off resonance points are higher than those of Figure 5.4.

![Figure 5.5: Impedance vs. Frequency in a Series RLC circuit with $L = 70\mu H$](image)

Hence, in multi-frequency SS IPT designs, the impedance is directly proportional to the inductances of the respective circuit and (5.18) can be written as

$$Z = R + jL \left( \omega - \frac{\omega_0^2}{\omega} \right) \quad (5.20)$$

where $\omega_0$ is the resonant frequency.

From the above discussion, the following two points are important in while considering frequency selection in a dual frequency SS IPT system:
1. Two frequencies that are far apart are considered more suitable than two that are close as the further the two frequency magnitudes are from each the higher the impedance presented by one frequency system to the other.

2. In order for both the systems to be electrically decoupled for rated power transfer, an increase in the inductance of one or both of the systems could be required. The amount of increase in inductance is dependent on the rated power as well as the the impedances presented by one frequency system to the other frequency system. This increase in inductance is carried out by adding an inductor in series with the coil of the primary or secondary system, resulting in a proportional decrease in the capacitance, in order to maintain the same resonant frequency as shown in Figure 5.7. This series inductor is independent and does not interact with the rest of the IPT system and could therefore be a ferrite wound inductor. Its sole function is to increase the inductance of the required primary or secondary circuit. Now, when inductances of the primary and secondary of IPT circuits are increased it is important to keep the following two points under consideration:

   - The quality factor of series resonant circuits is directly proportional to the inductance and as explained in Appendix B, the voltage across the resonating capacitor gets scaled up by the quality factor. Hence, it is important to adequately design the system so as to ensure not very low values of capacitance, resulting in high blocking voltages.

   - The choice of the resonant tank should ideally be such that the input voltage is independent of the primary coil inductance. This is true for the case of a series compensated primary. A primary with a LCL-T resonant tank, on the other hand, has an input voltage given by

     \[ V_{in} = j\omega L_p I_p \]

     The input voltage is therefore directly proportional to the primary inductance and hence can increase to large values as the primary inductance is increased. Another disadvantage of using an LCL-T resonant tank is that when the inductance is increased two separate series inductors have to be added, thereby increasing the losses in the resonant tank.

A dual frequency system can have efficiencies that are comparable to a single frequency system. This is due to the fact that the efficiency curve of an ideal IPT system 'saturates' as frequency is increased and in an ideal case, two frequencies can be chosen where the system has the same efficiency. Figure 5.6 shows the efficiency vs frequency plot for a SS system derived from (2.23). Here, the coupling coefficient, \( \kappa \), is increased by decreasing the primary-secondary air gap while maintaining the respective inductances (and number of turns) of the primary and secondary constant.

Figure 5.6 also highlights the fact that efficiencies at lower frequencies can be increased by decreasing the air gap, but this might not be the ideal thing to do in some cases when an IPT system has to be designed for a particular air gap. As the efficiency is directly proportional to the primary-secondary mutual inductance, \( M \), the efficiencies at lower frequencies can be increased by increasing the primary-secondary mutual inductance. This can be done by increasing the number of turns of the primary and secondary, thereby increasing the respective inductances of the primary and secondary coils. Alternatively, the magnetic design can be further improved in order to increase \( M \).

Due to the fact that the efficiency curves of Figure 5.6 'saturate' as frequency is increased [35], efficiency is an important factor in the design of multi frequency systems and is considered as an input parameter. Figure 5.6 is plotted by assuming a frequency independent resistance for
the litz wire equal to 0.6 Ω. This is not true as can be seen from Figure 2.9 but the assumption holds as the losses in the litz wire are dominated by the magnitude of current that flows due to its negligible resistance.

Figure 5.7: Schematic of a dual frequency SS IPT system with two transmitter coils and two receiver coils. $L_{series,1}$ and $L_{series,2}$ are additional series inductors that may be needed to be added in order to decouple the two frequency systems.

**5.4.2 Capacitor Blocking Voltage**

As explained above, the inductances of a multi-frequency IPT system can be higher than a single frequency system and as a result of this, the capacitor blocking voltages can also be considerably higher. This is further explained in Appendix B. Figure 5.7 shows the circuit diagram of the dual frequency SS system with the added series inductances, $L_{series,1}$ and $L_{series,2}$ which results in the increase on the inductance of the primary and secondary circuits. In a SS system at resonance, the capacitor blocking voltage is given by
5.4.3 Design Procedure

In this section a generalised design procedure for a Dual Frequency SS IPT System will be developed. A major advantage of this proposed design methodology is that it builds upon the design method of individual single frequency systems and therefore does not add much complexity to already existing design procedures. In Chapter 2, a design procedure to determine the rated primary current and load resistance is explained in Figure 2.11, assuming that the magnetics have already been chosen. In this section a method to determine the choice of magnetics is also described.

In a dual frequency system, it is important that the system has a high efficiency for power transfer at both the frequencies and hence a suitable magnetic design has to be chosen. This is done in the following way. In dual frequency SS systems, as explained in Section 5.4.1, it is beneficial to chose frequencies that are further apart and hence, it would be advantageous to have the lower frequency in the range of 20 kHz - 50 kHz so that a higher frequency can be chosen around 100 kHz. Hence, if it is ensured that the magnetics of the IPT system are very efficient at the lower frequencies, high efficiency is also ensured at the higher frequency as is highlighted in Figure 5.6. The efficiency of the IPT system at the lower frequency, therefore determines the selection of the magnetic topology.

![Graphs showing the variation of self-inductance and mutual inductance with number of turns](image)

Figure 5.8: Variation of inductances as number of turns are increased for a Circular coil of inner diameter = 10 cm at a fixed air gap of 10 cm calculated using COMSOL

As shown in Figure 5.6, the efficiency of an IPT system at lower frequencies (20 kHz - 50 kHz) is proportional to the primary secondary mutual inductance. The primary-secondary mutual inductance can be increased in two ways as was highlighted in Chapter 3. One is to improve the magnetic design of the existing charge pad so as to increase coupling between the primary and the secondary keeping the dimensions of the charge pad constant and the other way is to in fact increase the dimensions of the charge pad. Figure 5.8 shows the variation of the inductance (via FEM analysis) of a circular coil used in the design of Section 5.6 and highlights the fact the the primary-secondary mutual inductance can be increased by increasing the number of turns of the coils and hence the size of the coil. It is assumed that both the primary and the secondary
consist of identical coils.

A design procedure for the selection of the magnetics is shown in Figure 5.9 and can be implemented using a FEM software package. As outlined in Figure 5.9 an initial magnetics topology has to be chosen and also the required air gap has to be set. This could be any of the topologies highlighted in Chapter 3 or another topology as decided by the designer. The initial lower frequency is then set. This should be initially set as low as possible and will be increased if

Figure 5.9: Design procedure for choosing the magnetics for ensuring high efficiency for the lower frequency

has to be chosen and also the required air gap has to be set. This could be any of the topologies highlighted in Chapter 3 or another topology as decided by the designer. The initial lower frequency is then set. This should be initially set as low as possible and will be increased if
the diameter, \( d \), of the magnetic system is larger than maximum allowed diameter, \( d_{\text{max}} \). The system is then designed using the design procedure of Figure 2.11 for a rated \( P_{\text{out}} \) and \( V_{\text{out}} \) and hence the primary and secondary rated currents, \( I_p \) and \( I_s \), are determined along with the rated load. The efficiency can therefore be calculated using equation (2.23). If the efficiency is below the reference efficiency (in this case 90 %), the magnetics has to be changed in order to increase the primary-secondary mutual inductance. This is done by increasing the size of the coil as has been highlighted earlier in this sub section.

The reference efficiency is chosen to be 90 % in Figure 5.9, but can also be set by the designer to a higher value such as 95 % as IPT systems are designed to have these kind of efficiencies [4]. In this case the reference efficiency has been kept to an initial low value of 90 % so as to not have too high inductance of coils in the first design process. High coil inductances would imply higher mutual inductance values with the adjacent coils in the IPT system. If required, another run of this procedure can be done with a higher efficiency.

After selecting the magnetics for required efficiencies, the next steps in the design sequence are outlined in Figure 5.10. The next step is to select the higher operating frequency. IPT systems traditionally operate at 20 kHz but now there have been developments that are pushing the standard frequency to 85 kHz and above [4]. In this design procedure therefore, frequencies between 20 kHz and 100 kHz are considered as is also done in other studies such as in [1].

As has been explained above, the efficiency curve of an IPT system ‘saturates’ and it is therefore possible to have systems operating at the maximum efficiency over a wide range of frequencies. In the design of a dual frequency SS IPT system, therefore, it is possible to select two operating frequencies that enable the system to operate at high efficiencies. These two frequencies have to be selected as far apart as possible because as explained in Section 5.4.1, this would imply that the two RLC resonance circuits present a high impedance to the currents of the other frequency.

After the two frequencies are determined, each of the frequency systems is first separately designed using the design procedure of Figure 2.11 for the rated output power, \( P_{\text{out}} \), and the rated
output voltage, $V_{out}$. Hence, the rated primary currents for the higher and lower frequency system, $I_{p1}$ and $I_{p2}$ respectively, are determined. The rated load resistance is also determined.

The next step is to design the dual frequency system following the design procedure of Figure 5.11. This design procedure ensures that there is negligible interaction due to the mutual inductance between the systems by calculating the induced currents and determining the series inductances that need to be added if needed. The design procedure assumes the following initial inputs: Inductances of the primary and secondary coils, the mutual inductances between the coils and the two (initial) operating frequencies. It is also assumed that the IPT systems are initially designed according to the procedure of Section 2.9.

**Setting the value of $I_{thres}$**

The threshold value for the induced currents, $I_{thres}$, is also an input parameter to the design procedure of Figure 5.11 and can be set as follows. $I_{thres}$ has to be set such that the induced currents of one frequency system in the other frequency system are negligible. An initial solution can first be obtained by setting the induced current in the other frequency system, $I_{thres}$, to be 5% of the inducing current. The algorithm will then compute the required series inductances that should be added to achieve this. If the correct series inductances are added, the end result would be that the rated currents flow in the primary and secondary, even though all the four coils are coupled. If not, the value of $I_{thres}$ can be further decreased. $I_{thres}$ should not have a very low value as this would imply very high and possibly non-realistic values of the series inductances and resonating capacitances as after all, a small induced current would always be present. Although IPT systems are current controlled, (5.16) and (5.17) show a voltage source. This is to allow the currents to be drawn from the respective voltage sources according to the impedances seen by the source and when the correct value of $L_{series}$ is added, the drawn current converges to the rated magnitude.

Figure 5.11 shows the design procedure for an SS dual frequency System. The aim is for the two systems operating at different frequencies to be electrically decoupled implying that one frequency induces currents of a very small magnitude in the other system.

**Assumptions**

The design procedure mentioned in this section is not an optimised one and the following assumptions are made:

- All the transmitting and receiving coils are assumed to be identical. This is done so as to have lesser variables for inter-coil mutual inductance. IPT systems usually have identical receivers and transmitters but non-identical primary and secondary coils can also be used in this design.

- The added $L_{series}$ values, as shown in Figure 5.7 are assumed to be equal in the primary and the secondary of each system. This is not a required condition, but assuming them to be equal allows the algorithm to converge quickly to a first initial solution. If the induced currents are still high, the $L_{series}$ values can be further changed as needed.

- The incremental change in $L_{series}$ was initially set as 20 $\mu$H.

The design procedure described here is not an optimised one, but this could be the first step in creating an optimised design algorithm.
Start

Inputs: \( L, M, \omega_1, \omega_2, I_{p1} \& I_{s1}, I_{p2} \& I_{s2} \)

Set threshold value for induced current in System 1(2) due to System 2(1): \( I_{thres,1(2)} \)

Set input of System 1 to zero and determine induced currents in System 1 due to System 2 using (5.17)

Increase \( L_{series,1} \) No

\( I_{ind1} < I_{thres,1} \)?

Yes

Set input of System 2 to zero and determine induced currents in System 2 due to System 1 using (5.16)

Increase \( L_{series,2} \) No

\( I_{ind2} < I_{thres,2} \)?

Yes

Design Complete

Figure 5.11: Design procedure for a dual frequency SS system by increasing the series inductance
5.5 A 10 W SS Dual Frequency System

A 10 W IPT system can be considered to be a *low power* system which is used for charging consumer devices such as mobile phones. These devices can be placed on a table top and as such have air gaps that are less than 1 cm and have primary and secondary coils that have dimensions of the order of 3 cm to 5 cm [36]. A side view of the simulated system is shown in Figure 5.12.

![Figure 5.12: Side View of the SS Dual frequency system with circular coils (dimensions in mm)](image)

The system has been designed such that both frequency systems transfer the same amount of power, that is each system transfer 5 W. The power sharing ratio, \( m \), is therefore

\[
m = 0.5
\]

Such a system was simulated in using the FEM software package COMSOL Multiphysics and the system parameters are listed in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>20.7 ( \mu )H</td>
</tr>
<tr>
<td>( M_1 )</td>
<td>11.8 ( \mu )H</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>11.8 ( \mu )H</td>
</tr>
<tr>
<td>( M' )</td>
<td>2.2 ( \mu )H</td>
</tr>
<tr>
<td>( M'' )</td>
<td>1.9 ( \mu )H</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>110 kHz</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>50 kHz</td>
</tr>
<tr>
<td>( I_{p1} )</td>
<td>0.6 A</td>
</tr>
<tr>
<td>( I_{p2} )</td>
<td>1.4 A</td>
</tr>
<tr>
<td>( R_L )</td>
<td>4.9 ( \Omega )</td>
</tr>
</tbody>
</table>

Table 5.2: System parameters for the 10 W SS Dual Frequency System

The chosen frequencies of 50 kHz and 110 kHz were decided based up the calculation of the efficiencies of this magnetic system assuming only resistive losses in the windings. The variation of the efficiency of the system with frequency is plotted in figure 5.13 and it can be clearly seen that if the primary-secondary mutual inductance, \( M \), is increased the efficiencies at lower frequencies go up. At \( M = 11.2 \mu H \), the efficiency for \( f_1 = 110 \) kHz is 98.6 % and for \( f_2 = 50 \) kHz is 97 %.

![Figure 5.13: Efficiency variation with frequency](image)

Each of the frequency systems are individually designed and the rated primary currents are also tabulated in Table 5.2. The dual frequency system is then designed using the method of Figure 5.11 and Figure 5.14 shows the frequency spectrum of the output current of the two systems when no additional series inductances are added in the two circuits. The induced components of
Figure 5.13: Efficiency vs. Frequency Plot used to select the two operating frequencies

the other frequency are found to be less than 7 % and highlights the fact that IPT systems can be decoupled just by virtue of them operating at different frequencies and without the need of adding high series inductances depending on the amount of power being transferred and therefore the currents in the transmitter and receiver coils.

Figure 5.14: Frequency Spectrum of Output Current

It is of interest to study the system as power transferred to the rated load is increased as this implies increasing the power transferred while keeping the loaded (secondary) quality factor constant. In a dual frequency system, the secondary quality factor will not remain the same because as the power transferred is increased, the currents in the primary and secondary increase causing larger series inductances to be added. Hence, the primary quality factor will not remain constant too. As the quality factor increases, the capacitor blocking voltage increases as is described in Section 5.4.2 and Appendix B. It is therefore important to study the variation of the capacitor blocking voltages as power transfer is increased.

Figure 5.15 shows the increase of capacitor blocking voltages as the added series inductance
increases when power transferred is increased. The primary and secondary quality factors, from Appendix B are:

\[ Q_p = \frac{L_p R_L}{\omega M^2} \]

\[ Q_s = \frac{\omega L_s}{R_L} \]

For higher power transfer the primary-secondary mutual inductance, \( M \), as well as the rated load \( R_L \) are usually increased too, as is highlighted in Section 5.6, and this will tend to decrease the value of the primary and secondary quality factors which is beneficial to the design of dual frequency IPT systems.

In order to further reduce the effects of the inter-coil mutual inductances, a series inductance was added to the low frequency system 2. The value for \( I_{thres, 2} \) was set to be less than 5% of the currents of System 1 and a series inductance of 78 \( \mu \)H was therefore determined. Figure 5.16 shows the frequency spectrum of the Primary and Secondary currents of System 2.
currents from one frequency system induce negligible currents in the other frequency system.

(a) System 1 (110 kHz) and System 2 primary short circuited

(b) System 2 (50 kHz) and System 1 primary short circuited

Figure 5.17: FEM plots of the Magnetic Flux Density of System 1 and System 2 for a 10 W system

5.6 A 100 W SS Dual Frequency System

Similar to the design of a 10 W SS Dual Frequency System of Section 5.5, this section highlights the design of a 100 W system. The aim of this section is to highlight major changes that occur as the power rating of the system increases.

The power sharing ratio is again kept to $m = 0.5$ and therefore each frequency system has a $P_{out} = 50W$ and it is assumed that the output voltage, $V_{out} = 50V$. If the magnetic system is kept the same as in Section 5.5, the efficiency for lower frequencies is extremely low. In particular, the efficiency for an operating frequency of 30 kHz is only 64% at $\kappa = 0.56$ ($M_2 = 11.2 \mu H$) as is shown in Figure 5.18. The primary-secondary mutual inductance of the lower frequency System 2 has to therefore be increased for higher efficiencies.

![Efficiency vs. Frequency Plot](image)

Figure 5.18: Efficiency vs. Frequency Plot used to select the two operating frequencies

Figure 5.19 shows the variation of efficiency as the primary secondary mutual inductance of System 2, $M_2$, is increased and it can be seen that if $M_2$ is greater than 29$\mu H$, efficiencies of above 90% can be expected with an operating frequency of 30 kHz.
The diameter of the charge pad is increased and analysed in COMSOL and the system parameters are tabulated in Table 5.3. The increase of the coil inductance with the increase in the diameter (number of turns) is shown in Figure 5.8. Also, different from the analysis of Section 5.5, the air gap is now increased to 10 cm.

\[
\begin{align*}
L &= 144.4 \, \mu H \\
M_1 &= 33.1 \, \mu H \\
M_2 &= 33.1 \, \mu H \\
M' &= 23.9 \, \mu H \\
M'' &= 6.2 \, \mu H \\
f_1 &= 100 \, \text{kHz} \\
f_2 &= 30 \, \text{kHz} \\
I_{p1} &= 2.4 \, \text{A} \\
I_{p2} &= 8 \, \text{A} \\
R_L &= 49.9 \, \Omega
\end{align*}
\]

Table 5.3: System parameters for the 100 W SS Dual Frequency System

The results after running the algorithm of Figure 5.11, are that \( L_{\text{series},1} = 44 \, \mu H \) and \( L_{\text{series},2} = 498 \, \mu H \) and Figure 5.20 and Figure 5.21 show the frequency spectrum of the currents in System 1 and System 2 derived from a FEM analysis.

Figure 5.22 shows the FEM output during a frequency domain simulation, highlighting the fact that currents of one frequency system induce negligible currents in the other system.

Figure 5.23 shows the increase in the \( L_{\text{series}} \) required to keep the induced currents below 1 % as power transferred is increased to 4 kW at a power sharing ratio of \( m = 0.5 \) using the same magnetics of this section. Of course, as the power transfer increases, the efficiency of the current magnetics would reduce.

As mentioned in Section 5.4.2, the capacitor voltage ratings increase as power transfer and \( L_{\text{series}} \) increase. This is due to the fact that the secondary quality factor increases and Figure 5.24 highlights this fact. Hence, as power transfer is increased, adequate measures for the design of the resonance capacitors must be taken. As compared to the secondary system, the primary capacitor voltage rating does not increase by a large amount.
5.7 Experimental Results

This section provides a description and highlights the preliminary results obtained from a laboratory set-up built to demonstrate a 10 W two transmitter dual frequency IPT system. As part of this thesis work a half bridge inverter was built and used to power one of the primary coils. The other coil was powered by the full bridge inverter developed in the thesis work of D. Voglitsis [15]. Figure 5.25 shows a photo of the experimental setup of the four coupled coils.

The higher frequency system 1 has an operating frequency of 100 kHz and the lower frequency system 2 has an operating frequency of 20 kHz. Figure 5.26a shows the oscilloscope waveforms of the induced current of 29.1 mA in system 2, when a rated current of $I_{p1} \approx 712$ mA flows in
Figure 5.23: Increase in $L_{\text{series}}$ as the power transferred increases

Figure 5.24: Increase in the voltage rating of the secondary capacitors as power transfer and hence $L_{\text{series}}$ is increased

System 1. The induced current in System 2 is therefore about 4 % of the current in System 1. This induced current in System 2 can be further reduced by adding a series inductance and Figure 5.26b shows the oscilloscope waveforms of the induced current of 11.03 mA when a 40 $\mu$H series inductance is added to the lower frequency System 2. Hence, the induced current in System 2 has now dropped to about 1.5 % of the current in System 1.

Figure 5.25: Photo of the experimental setup of the four coupled coils

Figure 5.27 shows the oscilloscope waveforms when a rated current is flowing through both the coils of the primary. It can be seen that the two primary coils are sufficiently decoupled and
the induced currents of the other frequency can barely be noticed on the oscilloscope. It should also be noted that as System 1 is operating at a frequency that is five times that of System 2, the current $I_{p2}$ is about five times $I_{p1}$. Exact values of the current in System 2 could not be obtained here due to a current limit on the input power supply.

This laboratory system was not optimally designed as per the design procedures of this section but was constructed as a proof of concept of a dual frequency IPT system.

5.8 Power Sharing in Dual Frequency IPT Systems

In a dual frequency system, as power is transferred to the load via two frequencies, it could be possible to share the power effectively within the two frequencies in a power sharing ratio as defined in (5.5). In this section, power sharing is investigated for the two methods of Section 5.1 of designing a dual frequency systems with emphasis on:

- Conduction losses in the Inverter
- Emitted leakage field strength
- Conduction losses in the Transmitter and Receiver coils

5.8.1 Conduction losses in the Inverter

The conduction losses in the Inverter of an IPT system is proportional to the current, $I_{in}$, flowing through the inverter switches during normal operation. The following analysis describes the effect power sharing, between two frequencies in an IPT system, has on $I_{in}$.
Dual frequency IPT using one Transmitter Coil

Assuming an IPT system that transfers power from the transmitter to the receiver at unity power factor and ignoring all the other losses in the system, the output power delivered by the inverter is the power received by the load:

\[ P_{\text{inv}} = P_{\text{out}} \]  

(5.22)

where, \( P_{\text{inv}} = V_{\text{in}} I_{\text{in}} \) and \( P_{\text{out}} \) is the power delivered to the load. From (5.7) and (5.22), the inverter currents for the equivalent single frequency system and dual frequency system are

\[
I_{\text{in},s} = V_{\text{in},1}(M\omega_1 G_{t,s}G_{r,s})^2R_{ac,0}, \\
I_{\text{in},1} = V_{\text{in},1}(M\omega_1 G_{t,1}G_{r,1})^2R_{ac,1}, \\
I_{\text{in},3} = V_{\text{in},3}(M\omega_3 G_{t,3}G_{r,3})^2R_{ac,3} \\
\]  

(5.23)

From (5.8) and (5.23), the normalised value of the inverter current in this dual frequency system is [29]

\[
I_{\text{in},d} = \frac{I_{\text{in,d}}}{I_{\text{in,s}}} = \sqrt{\frac{I_{\text{in},1}^2 + I_{\text{in},3}^2}{I_{\text{in},s}}}
\]

\[
\Rightarrow I_{\text{in},d} = \sqrt{((G^*_{t,1}G^*_{r,1})^2R_{ac,1})^2 + 9((G^*_{t,3}G^*_{r,3})^2R_{ac,3})^2} \\
\Rightarrow I_{\text{in},d} = \sqrt{m^2 + 9(1 - m)^2}
\]  

(5.24)

Figure 5.28: Normalised Inverter Current vs Power Sharing Ratio, \( m \), in a one transmitter coil dual frequency system

Figure 5.28 shows the variation of the normalised Inverter current, \( I_{\text{in,d}} \), with the power sharing ratio. For \( m > 0.8 \), the dual frequency system has a smaller inverter current, with the minimum of \( I_{\text{in,d}} = 0.95 \) at \( m = 0.9 \). This implies that, if 90% of the power is transferred via the fundamental harmonic frequency (and 10% via the third harmonic), the inverter current is 5% lower than it is in an equivalent IPT system operating at the fundamental frequency only.

For \( m < 0.8 \) however, the normalised inverter current increases and is \( I_{\text{in,d}} = 3 \) at \( m = 0 \), implying that the inverter current in a dual frequency system, where all the power is transferred via the third harmonic, is three times larger that that in an equivalent IPT system operating...
at the lower frequency. The normalized inverter current increases for $m < 0.8$ as the magnitude of the third harmonic voltage is attenuated by a factor of three as given by (5.2) and hence, requires a high magnitude of input current in order to sustain the power flow at the third harmonic frequency. Hence, at $m = 0$, when all the power is transferred via the third harmonic, although the switching losses are less than a single frequency system switching at the third harmonic, the conduction losses are nine times more.

**Dual frequency IPT using two Transmitter Coils**

It should be noted here that, in this case, the dual frequency equivalent inverter current is the vector sum of the rms currents in the two inverter legs unlike the one transmitter case wherein the dual frequency equivalent inverter current was the equivalent current through the same inverter leg. One of the main differences between the two methods is that in the proposed two Transmitter Coil case, the two transmitter coils are driven independently and hence require two independent inverter inputs as compared to the one Transmitter coil case wherein only one inverter is required.

In the case of a two transmitter system, the input voltage for both the frequencies is the same ($V_{in}$) and let $I_{in}$ be the equivalent input current or the equivalent single frequency input current. From (5.22), the power sharing ratio can therefore be expressed as

$$m = \frac{I_{in,1}V_{in}}{I_{in}V_{in}} = \frac{I_{in,1}}{I_{in}}$$

$$1 - m = \frac{I_{in,n}V_{in}}{I_{in}V_{in}} = \frac{I_{in,n}}{I_{in}}$$

From (5.25), the normalised value of the inverter current in this dual frequency system is

$$I_{in,d}^* = \frac{I_{in,d}}{I_{in}} = \sqrt{\frac{I_{in,1}^2 + I_{in,n}^2}{I_{in}}}$$

$$\Rightarrow I_{in,d}^* = \sqrt{m^2 + (1 - m)^2}$$

Figure 5.29 shows the variation of the normalised Inverter current, $I_{in,d}^*$, with the power sharing ratio and it can be seen that when power is transferred via two frequencies, $I_{in,d}^* < 1$ always. This is a major advantage of such a system and it will lead to lower conduction losses in the inverter. It can be seen that for a power sharing ratio $m = 0.5$, where 50% of the power is transferred via each frequency, $I_{in,d}^*$ attains a minima and the inverter currents in this dual frequency system are 29.3% less than that in a single frequency system with one input inverter.

**5.8.2 Emitted Leakage Field Strength**

The emitted magnetic field in the air gap of an IPT system is directly proportional to the magnitude of the current in the transmitter. An important design consideration in IPT system design is the magnitude of the emitted magnetic field which has to be within the maximum levels as stated by the International Commission on Non-Ionizing Radiation Protection (ICNRP) in [12]. As highlighted in Section 2.8, in the operating frequency range under consideration (20 kHz - 150 kHz), the maximum allowable magnetic field exposure does not vary with frequency. As the magnetic field intensity is directly proportional to the current flowing in the coil (Ampere’s Law), hence, methods that reduce the peak current in the transmitter and receiver coils would also reduce the magnetic field in the air gap.
Dual frequency IPT using one Transmitter Coil

The instantaneous value of the transmitter current is given by

\[ I_{t,d}(t) = I_{t,1}(t) + I_{t,3}(t) = \sqrt{2}|G_{t,1}|V_{in,1}\sin(\omega_1 t) + \sqrt{2}|G_{t,3}|V_{in,3}\sin(3\omega_1 t + \phi_3) \]  \hspace{1cm} (5.26)

From (5.2) and by setting the phase angle \( \phi_3 \) to zero:

\[ I_{t,d}(t) = \sqrt{2}|G_{t,1}|V_{in,1}\sin(\omega_1 t) + \sqrt{2}|G_{t,3}|\frac{V_{in,3}}{3}\sin(3\omega_1 t) \]  \hspace{1cm} (5.27)

In order to obtain the maximum value of the transmitter current, the derivative of (5.27) with respect to \( \omega_1 t \) is calculated and set to be equal to zero. The maximum transmitter current is hence

\[ I_{t,d,max} = \begin{cases} \sqrt{2}V_{in,1}\left(|G_{t,1}|-\frac{|G_{t,3}|}{3}\right) & \text{if } |G_{t,3}| < \frac{|G_{t,1}|}{3} \\ \frac{\sqrt{3}}{3}V_{in,1}\sqrt{1+\frac{|G_{t,1}|}{|G_{t,3}|}(|G_{t,1}|+|G_{t,3}|)} & \text{if } |G_{t,3}| > \frac{|G_{t,1}|}{3} \end{cases} \]  \hspace{1cm} (5.28)

Now, taking the derivative of (5.28) with respect to \( G_{t,3} \) and setting it to zero, the condition resulting in the minimum peak value of the transmitter is

\[ \min(I_{t,d,max}) = \sqrt{2}\frac{\sqrt{3}}{2}V_{in,1}|G_{t,1}| \quad \text{when } |G_{t,3}|_{opt} = \frac{|G_{t,1}|}{2} \]  \hspace{1cm} (5.29)

Hence, from (5.29) and (5.27), in order for the condition of \( \min(I_{t,d,max}) \) to hold

\[ \frac{I_{peak,3}}{I_{peak,1}} = \frac{1}{6} \]  \hspace{1cm} (5.30)
where, \( I_{\text{peak,3}} \) and \( I_{\text{peak,1}} \) are the peak current magnitudes of the third and first harmonic respectively. Hence, peak current of the fundamental has to be six times higher than that of the third harmonic making the power sharing ratio, \( m = 0.8 \). Also,

\[
\min(I_{t,d,\text{max}}) = \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} (V_{\text{in,1}} |G_{t,s}|) |G_{t,1}^*| \\
= \frac{\sqrt{3}}{2} (I_{\text{peak,s}}) |G_{t,1}^*| \tag{5.31}
\]

where, \( I_{\text{peak,s}} \) is the peak current in an equivalent single frequency system operating at the fundamental frequency. Consider that \( G_{t,1} = 1 \), implying that the current magnitude of the fundamental in this dual frequency system is equal to that of the equivalent single frequency system operating at the fundamental frequency. Hence, from (5.31), the peak current in a dual frequency system is reduced to 86.6% of the original; when the ratio of the first to the third harmonic in the transmitter coil is 1:6. Figure 5.30 shows a plot of the normalised currents at this required condition of \( I_{\text{peak,1}} = 6 I_{\text{peak,3}} \).

![Figure 5.30: Normalised Current where \( I_{\text{peak,1}} = 6 I_{\text{peak,3}} \)](image)

**Dual frequency IPT using two Transmitter Coils**

The equation for power transferred to the load in an IPT system has been derived in (2.21) and is repeated below:

\[
P_{\text{out}} = \frac{\omega M^2 I_p^2}{L_s} Q_L \\
= \frac{(\omega M I_p)^2}{R_L}
\]

Hence, it can be clearly seen and also intuitively deduced that, a higher frequency current flowing in the primary coil transfers the same amount of power to a load, \( R_L \), at a lower primary peak current magnitude as compared to a lower frequency current. Therefore, if the aim is to reduce the emitted field strength in a dual frequency system using two transmitter coils, the power will have to be predominantly transferred via the higher frequency. This is possible as in this dual frequency IPT method, both the frequency systems can be operated independently.

5.8.3 Conduction losses in the Transmitter and Receiver coils

Conduction losses in the transmitter and receiver coils depend on the currents flowing through the coils as well as the resistance offered by the Litz wire to the particular current frequency
flowing through the coil. Litz wire minimises the increase in AC resistance of the cooper wire as frequency increases, but as highlighted in Section 2.7, it does increase by a small amount as the frequency of the current flowing through it increases owing to proximity effects. In this section, conditions for achieving minimum conduction losses in the transmitter and receiver coils will be developed and it will be examined whether power sharing between the two frequencies can improve conduction losses.

**Dual frequency IPT using one Transmitter Coil**

The transmitter currents can be represented by

\[
I_{t,s} = V_{in,1}G_{t,s} \\
I_{t,1} = V_{in,1}G_{t,1} \\
I_{t,3} = V_{in,3}G_{t,3}
\]  \(5.32\)

where \(I_{t,1}\) and \(I_{t,3}\) are the transmitter currents corresponding to the fundamental and the third harmonic respectively and \(I_{t,s}\) is the transmitter current of an equivalent single frequency system operating at the fundamental frequency.

Assuming negligible losses in the system and a unity power factor between the induced voltages and currents in the receiver

\[
P_{out,1} = V_{oc,1}I_{r,1} = M\omega_1G_{t,1}V_{in,1}I_{r,1} \\
P_{out,3} = V_{oc,3}I_{r,3} = M\omega_3G_{t,3}V_{in,3}I_{r,3}
\]  \(5.33\)

From (5.7) and (5.33), the expressions for the receiver currents are

\[
I_{r,s} = V_{in,1}M\omega_1G_{t,s}G^2_{r,s}R_{ac,0} \\
I_{r,1} = V_{in,1}M\omega_1G_{t,1}G^2_{r,1}R_{ac,1} \\
I_{r,3} = V_{in,3}M\omega_3G_{t,3}G^2_{r,3}R_{ac,3}
\]  \(5.34\)

From (5.32) and (5.34), the relation between the transmitter and receiver currents is

\[
I_{r,s} = M\omega_1G^2_{t,s}R_{ac,0}I_{t,s} = CI_{t,s} \\
I_{r,1} = CG^*_{r,1}R^*_{ac,1}I_{t,1} \\
I_{r,3} = CG^*_{r,3}R^*_{ac,3}I_{t,3}
\]  \(5.35\)

where the constant \(C\) is represented as

\[
C = M\omega_1G^2_{t,s}R_{ac,0}
\]

In this analysis of transmitter and receiver power loss, it is revealing to compute the ratio of the power losses in a dual frequency system to that of an equivalent single frequency system operating at the lower frequency (in this case, the fundamental frequency). This ratio is represented as \(P_{\gamma}^*\):

\[
P_{\gamma}^* = \frac{P_{\gamma,d}}{P_{\gamma,s}} = \frac{I_{t,1}^2R_{t,1} + I_{t,3}^2R_{t,3} + I_{r,1}^2R_{r,1} + I_{r,3}^2R_{r,3}}{I_{t,s}^2R_{t,s} + I_{r,s}^2R_{r,s}}
\]  \(5.36\)
Assuming frequency independent resistive loads \( R_{a_1} = R_{a_3} = R_{a_0} \) and from (5.9), (5.36) simplifies to

\[
P^*_{\gamma} = \frac{(G^*_{t,1})^2 \left( R_{t,1} + \frac{m^2 C^2}{(G^*_{t,1})^4} R_{r,1} \right) + \frac{(G^*_{t,3})^2}{9} \left( R_{t,3} + \frac{9(1-m)^2 C^2}{(G^*_{t,3})^4} R_{r,3} \right)}{R_{t,1} + C^2 R_{r,1}} \tag{5.37}
\]

Now, in order to minimise the losses in the two coils, the optimal transmitter gain can be found by setting the derivative of \( P^*_{\gamma} \) with respect to \( G^*_{t,1} \) and \( G^*_{t,3} \) to zero:

\[
\frac{\partial P^*_{\gamma}}{\partial G^*_{t,1}} = 0 \Rightarrow G^*_{t,1,\text{opt}} = (mC)^{0.5} \left( \frac{R_{r,1}}{R_{t,1}} \right)^{0.25}
\]

\[
\frac{\partial P^*_{\gamma}}{\partial G^*_{t,3}} = 0 \Rightarrow G^*_{t,3,\text{opt}} = (3(1-m)C)^{0.5} \left( \frac{R_{r,3}}{R_{t,3}} \right)^{0.25} \tag{5.38}
\]

Substituting (5.38) into (5.37), the minimum value of \( P^*_{\gamma} \) is obtained:

\[
P^*_{\gamma,\text{min}} = \frac{2C}{1 + C^2 (R_{r,1}/R_{t,1})} \left( m \sqrt{\frac{R_{r,1}}{R_{t,1}}} + \frac{R_{t,3}}{R_{t,1}} \frac{(1-m)}{3} \sqrt{\frac{R_{r,3}}{R_{t,3}}} \right) \tag{5.39}
\]

If it is assumed that the transmitter and receiver coils are identical and that the coil resistances are frequency independent, (5.39) can be simplified to

\[
P^*_{\gamma,\text{min}}(R_{t,1} = R_{r,1} = R_{t,3} = R_{r,3}) = \frac{2m + 1}{3} \frac{2C}{1 + C^2} \tag{5.40}
\]

As can be seen in Figure 5.31, the power losses in the two coils are minimum at \( m = 0 \), when all the power is transferred via the third harmonic. This is due to the fact that, as can be inferred from (2.21), at higher frequencies the same amount of power can be transferred to a load at lower current magnitudes. It can also be seen that the power loss decreases as \( C \) increases. This is due to the fact that an increase in \( C \) implies an increase in the mutual inductance, \( M \), between the transmitter and the receiver.

Transferring power at \( m = 0 \) is not a good option in a dual frequency system with one transmitter coil, as highlighted in Section 5.8.1. In that case, the inverter currents would be three times that of an equivalent single frequency system operating at the fundamental (Figure 5.28) but also as can be seen in Figure 5.31, as \( m \) is increased from 0 to 1, transmitter and receiver conduction losses increase. Hence, a trade off between coil losses and the inverter losses will have to be made.
Dual frequency IPT using two Transmitter Coils

The coil power losses in a dual frequency system using two transmitter coils can be analysed in a similar way as the one transmitter coil case by evaluating the expression for $P_\gamma^*$:

$$P_\gamma^* = \frac{P_{\gamma,d}}{P_{\gamma,s}}$$

Here $P_{\gamma,s}$ is the power loss when $I_{eq}$ as described in Section 5.3 flows in the primary coil.

$$P_\gamma^* = \frac{I_{t,1}^2 R_{t,1} + I_{t,3}^2 R_{t,3} + I_{r,1}^2 R_{r,1} + I_{r,3}^2 R_{r,3}}{I_{t,s}^2 R_{t,s} + I_{r,s}^2 R_{r,s}}$$  \hspace{1cm} (5.41)

In a series compensated secondary, the relation between receiver and transmitter currents for the lower frequency system are

$$I_{r,1} = \frac{\omega_1 M}{Z_s} I_{t,1}$$
$$= k I_{t,1}$$  \hspace{1cm} (5.42)

where $k = (\omega_1 M)/Z_s$. An increase in $k$ implies an increase $(\omega M)$ which as explained in Section 2.2 leads to an increase in efficiency. Hence, the relation between the receiver and transmitter currents for the higher frequency system are

$$I_{r,n} = n k I_{t,n}$$  \hspace{1cm} (5.43)

Hence, from (5.14), (5.15), (5.41), (5.42) and (5.43)

$$P_\gamma^* = \frac{m(R_{t,1} + k^2 R_{r,1}) + \frac{(1 - m)}{n^2}(R_{t,n} + n^2 k^2 R_{r,n})}{R_{t,1} + k^2 R_{r,1}}$$
\[
\begin{align*}
\frac{1}{1 + k^2 \frac{R_{r,1}}{R_{t,1}}} & + \frac{1 - m}{n^2} \left( \frac{R_{t,n}}{R_{t,1}} + n^2 k^2 \frac{R_{r,n}}{R_{t,1}} \right) \\
= & \frac{m \left( 1 + k^2 \frac{R_{r,1}}{R_{t,1}} \right) + \left( 1 - m \right) n^2 \left( 1 + n^2 k^2 \right)}{1 + k^2} 
\end{align*}
\] (5.44)

If \( R_{r,1} = R_{t,1} = R_{r,n} = R_{t,n} \)

\[
P_\gamma^* = \frac{m \left( 1 + k^2 \right) + \left( 1 - m \right) \left( 1 + n^2 k^2 \right)}{1 + k^2}
\] (5.45)

Figure 5.32 shows a plot of \( P_\gamma^* \) vs. \( m \) and highlights that, as expected, the transmitter and receiver conduction losses are minimum at \( m = 0 \) but unlike the one Transmitter Case, the inverter input currents do not increase above that of a single frequency system.

5.9 Charge Pad Interoperability and Misalignment Tolerance Improvement

This section provides a short note on how multi-frequency IPT could possibly improve charge pad interoperability and misalignment tolerances.

Section 3.9 highlights the fact that the current state of the art charge pads are design to transmit of receive either a perpendicular or a parallel flux at the perfectly aligned position. This means that a transmitter that is designed to transmit a perpendicular flux will not operate well with a receiver that is designed to receive a parallel flux. A multi-frequency IPT system has the potential to address this issue as a charge pad can potentially be designed that can transmit a perpendicular and a parallel flux at different frequencies. Therefore, parallel and perpendicular flux receivers also should have their respective operating frequencies pre defined.

Multi-frequency IPT could also potentially prove beneficial in improving misalignment tolerances with respect to electric vehicle charging. In fact, a self-decoupled multi-coil pick-up based on the Bipolar Pad of Section 3.8 is used to improve misalignment tolerances in [27] while ensuring low leakage magnetic fields (due to the fact that the two coils on the receiver are decoupled).
A multi-frequency system as proposed in this chapter could further improve misalignment tolerances by using the fact that the power sharing ratio can be changed while the system is operating. At the perfectly aligned case, equal amount of power can be transferred via both the frequencies and at a misaligned case, even all of the power can be transferred via one frequency. This concept is highlighted in Figure 5.33.

The design of the secondary system has to done in such a way that the secondary can always intercept both the transmitting frequencies. There therefore is a need to have multiple coils in the secondary. The coils could also be design to be overlapped so as to decrease the inter-coil mutual inductance, much like was seen in Section 4.4.

### 5.10 Comparison between the One and Two Transmitter Coil Dual Frequency Systems

Table 5.4 highlights the major comparisons between the One and Two Transmitter Coil Dual Frequency Systems with respect to the advantages that they offer.

<table>
<thead>
<tr>
<th>One Transmitter Coil</th>
<th>Two Transmitter Coils</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main Advantage:</strong></td>
<td><strong>Main Advantage:</strong></td>
</tr>
<tr>
<td>Some power transferred at a frequency that is three times the switching frequency.</td>
<td>Possibility of decoupling multi coil primary and multi coil secondary systems.</td>
</tr>
<tr>
<td><strong>Other Advantage:</strong></td>
<td><strong>Other Advantages:</strong></td>
</tr>
<tr>
<td>- Peak Current Reduced to 86.6 % of the original if $I_{peak,2}/I_{peak,1} = \frac{1}{3}$ ($m = 0.8$)</td>
<td>- Inverter conduction losses reduced by 29.3 % at $m = 0.5$</td>
</tr>
<tr>
<td>- Independent control of coil currents and $m$.</td>
<td>- Possibility to improve misalignment tolerances as well as to save Cu in the windings.</td>
</tr>
<tr>
<td><strong>Disadvantage:</strong></td>
<td></td>
</tr>
<tr>
<td>Complex compensation topologies for the transmitter and the receiver.</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: Comparison between the One and Two Transmitter Coil Dual Frequency System
An advantage of the two transmitter coil system, as mentioned in Table 5.4 is that it has the potential to save copper in the windings. This is due to the fact that two coils can be decoupled by virtue the frequency of the current that is flowing in them. Hence, complex structures to create a primary with spatially decoupled coils is not required which could lead to simpler designs with less use of copper.

5.11 Discussion

This chapter introduces a multi frequency IPT system and two methodologies of achieving multi-frequency IPT are highlighted. The first, a one transmitter coil method, uses only one transmitting coil and one input inverter and amplifies the first and third harmonic of the input current.

The second method, a two transmitter coil method, is proposed in this thesis and it is shown that this method is capable of decoupling multiple coils in primary and secondary circuits. A design methodology is shown in this chapter for the design of a multi frequency IPT system with two transmitter coils, which involves the selection of magnetics, frequency and determination of the value of a series inductance required for suitably decoupling two systems.

The main motivation for the one transmitter method is that it can transfer power at a frequency that is three times the switching frequency but in order for it to be more efficient that a single frequency system, less then 20 % of the power has to be transferred via the third harmonic frequency, which tends to negate the above mentioned advantage. Also the design of the compensation tank is more complex than the design for existing single frequency systems and leads to a condition where the power sharing ratio is fixed for a particular design.

The two transmitter dual frequency system on the other hand presents a possibility to decouple power transfer via IPT systems that are otherwise coupled. As the power sharing ratio is only dependent on the primary coil currents, this can be changed even during system operation for example at times when misalignment tolerances need to be increased or at times when the air gap magnetic field intensity needs to be decreased. The extent to which decoupling can be achieved depends on the inter-coil mutual inductance, the rated currents in the two coils and the value to the series inductance and it is shown, via simulations and experiments that the induced currents in a frequency decoupled system can be below 1% of the original, resulting in two systems with negligible interactions.
CHAPTER 6

Conclusion

6.1 Conclusions

The major focus of this MSc. thesis was the study of Inductive Power Transfer via multiple frequencies. The primary motivation behind conducting this investigation was to determine whether multiple frequency IPT could be beneficial in designing IPT systems which have reduced interactions with adjacent coils. The conclusions of this work are as follows.

An account of the development of IPT magnetic systems and the motivation behind each new development was given in Chapter 3 and it was noted that recent research is pointing towards IPT systems that are more tolerant to misalignment between the primary and the secondary and also towards systems that enable interoperation of a secondary with different primary designs. Multi-coil transmitters have also gained importance for the charging of consumer devices. In order to meet these goals, multi-coil primary and secondary systems are being researched, which require that the multiple coils in these topologies are decoupled.

The challenges in the design of multi-coil primary and secondary systems was introduced in Chapter 4. It was explained how induced currents due to inter-coil mutual inductance in these topologies cause design issues such as detuning of the respective primary and secondary circuits as well as causing reverse currents to flow back into the input power supply. Operating the two coils at different frequencies was proposed as a solution to this problem as induced currents are attenuated in a frequency system that is tuned to a different frequency. As the adverse effects of inter-coil mutual inductance are caused by the induced currents, the attenuation of these induced currents solves this problem. Multi-Frequency IPT also provides a solution to systems wherein interactions between the magnetic fields generated by these two coils is not desired.

The design considerations and equations of the proposed two transmitter dual frequency system were provided in Chapter 5. A method of effectively determining the two frequencies for high efficiency is described, wherein system that is highly efficient for the lower operating frequency implies that it is highly efficient for the higher operating frequency too. Decoupling of the two frequency systems often require the addition of a series inductance, and this procedure is also explained. The proposed method results in attenuating unwanted induced currents to about 2% of the rated current in the other coil. As a proof of concept, preliminary experimental results
are provided.

A major advantage of the two frequency system developed in this thesis is that the power sharing ratio can be changed with an effective control strategy while the system is operating. This enables control of the air gap magnetic field intensity as well gives the possibility of suitably powering a primary coil to enable better misalignment tolerance. In comparison to another recently proposed dual frequency system using one transmitter coil, the method developed in this thesis provides the possibility of reducing the inverter input current by 29.3% in comparison to the 5% reduction provided by the other method. The method of this thesis also as the advantage of a simple design procedure, building on existing IPT design methods in comparison to a complex design procedure that is need for the other method. The fact that this proposed multi-frequency IPT method can potentially prove beneficial in increasing misalignment tolerances as well as improving interoperability between charge pads is also highlighted.

6.2 Recommendations for Future Work

This thesis work provides a first look at a two transmitter multi frequency system that can decouple multi-coil IPT topologies. The design process is not optimised and therefore a more detailed optimisation analysis for the transmitter, receiver and the added series inductances should be carried out in order to obtain more efficient designs.

As the design of these multi frequency systems is directly dependent on the type of resonant topology used for primary and secondary compensation, another topic that could be looked into is the use of other resonant tanks with multiple passive components. The LCL resonant tank was studied in this thesis but was not recommended due to the fact that the input voltage is directly proportional to the primary inductance but a more optimised design for a LCL primary and/or secondary could also be developed.

The control of multi-frequency IPT systems with respect to power sharing between the two frequencies to improve, for example, misalignment tolerances can be studied. Suitable control strategies that take advantage of the fact that the power sharing ratio can be changed during system operation should also be investigated.


Appendices
A case study IPT system with the following specifications was used for plotting various graphs in this thesis, and also provided a starting point for the development of the laboratory prototype.

\[
\begin{align*}
L_p &= 20.68 \, \mu H \\
L_r &= 20.68 \, \mu H \\
M &= 7.2 \, \mu H \\
M' &= 2.17 \, \mu H \\
M'' &= 1.41 \, \mu H \\
\end{align*}
\]

The above symbols are as listed in the List of Symbols at the beginning of this thesis and in Table 5.1.
Capacitor Voltage Rating in Series RLC Circuits

A series RLC circuit is shown in Figure B.1 and the impedance, $Z$ is given by

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \quad (B.1)$$

and at resonance,

$$\omega = \frac{1}{\sqrt{LC}} \quad (B.2)$$

The quality factor of a series resonant circuit is given by

$$Q = \frac{\omega L}{R} = \frac{1}{\omega CR} \quad (B.3)$$

Hence, from (B.1) and (B.2), the impedance is minimum for a current of the resonant frequency and resonance, the current peaks to a value $i_{res}$ given by

$$i_{res} = \frac{V}{R} \quad (B.4)$$

It is of interest to examine the voltage across the capacitor, $V_c$, at resonance. Using (B.3), $V_c$ is
given by

\[ |V_c| = |X_c|i_{res} = \frac{1}{\omega C R} V \]

\[ \Rightarrow |V_c| = QV \] (B.5)

It can be seen from (B.5) that the voltage across the capacitor at resonance is directly proportional to the quality factor. As quality factor in turn is directly proportional to the inductance in the RLC circuit, it is crucial in the design of Multi-frequency IPT systems, where the series inductances can be higher than single frequency systems, to ensure that the compensation capacitors used have appropriate rated blocking voltages.

Of interest in this work are the primary and secondary quality factors, \( Q_p \) and \( Q_s \), of an SS compensated IPT system. The primary quality factor, \( Q_p \), is given by

\[ Q_p = \frac{\omega L_p}{R_r} \] (B.6)

where, \( R_r \) is the reflected impedance. As the secondary is also series compensated, from (2.10), \( R_r \) is purely resistive and is given by

\[ R_r = \frac{\omega^2 M^2}{R_L} \] (B.7)

From (B.6) and (B.7), \( Q_p \) is given by

\[ Q_p = \frac{L_p R_L}{\omega M^2} \] (B.8)

The secondary quality factor, \( Q_s \), is given by

\[ Q_s = \frac{\omega L_s}{R_L} \] (B.9)
IEEE Paper: Multi-Frequency Inductive Power Transfer as a means to decouple Multi-Coil Primary and Secondary Topologies

Authors: Jose Ralino Prazeres, Venugopal Prasanth, Pavol Bauer
Multi-Frequency Inductive Power Transfer as a means to decouple Multi-Coil Primary and Secondary Topologies

J.R.E.G. Prazeres, V. Prasanth, P. Bauer

Abstract—With the current surge in the development and use of Electric Vehicles (EVs) due to increased environmental consciousness, methods to increase the driving range and improve the battery charging conditions of these vehicles have become important research areas. Inductive Power Transfer (IPT) is one of the solutions to the challenges currently faced by EV manufacturers and customers.

In some cases, it is necessary to have transmitter and receiver topologies that have multiple coils, especially in a three phase IPT highway system or a table top IPT system for charging many consumer devices or mobile phones at the same time. Multi coil topologies are also used in lumped charge pad topologies to enable better misalignment tolerances. In such cases it is necessary to mutually decouple these multiple coils to avoid interactions that could affect the efficiency of power transfer. The current state of the art systems offer solutions that lead to the decoupling of multiple coils by their relative spatial position, and are hence limited. Multi coil systems operating at the same frequency also generate magnetic fields that add or cancel out each other, thereby changing the air gap magnetic field orientation depending on which coil is powered, which could be disadvantageous depending on the application.

This paper presents a multi frequency IPT system that aims to provide a solution to the above challenges. A multi coil primary system with two coils carrying currents of different frequencies can be designed to transfer power independently without interactions between the two different frequency systems. This paper presents this concept by first investigating the need for decoupling multiple coil transmitter and receiver designs. Multi frequency IPT is then introduced and the design considerations are developed in order to achieve a frequency decoupled IPT system.

Index Terms—Inductive Power Transfer (IPT), Multi-frequency systems.

I. INTRODUCTION

An Inductive Power Transfer (IPT) system is one wherein wireless power transfer is achieved between a transmitter and a receiver over large air gaps. This concept of transferring power is an outcome of Maxwell’s electromagnetic equations and was first investigated by Nicola Tesla in the late 19th century but was limited in application due to limitations of the available technology at that time. IPT has now emerged as a practical method for recharging electric vehicles, providing solutions to existing challenges such as limited driving range and unreliable charging methods.

The development of high power (10 - 100 kW) IPT systems go hand in hand with advances in electric and hybrid electric vehicle (EV/HEV) technology. A current challenge faced by EV and HEV developers is the limited driving range of these vehicles which is currently directly related to the size and weight of its battery pack, requiring very large battery packs to reach the driving distance of traditional vehicles (at least 200 km) [1]. Achieving high driving ranges in these vehicles thus leads to high initial costs, an increase in the weight of the vehicle and also an additional space requirement.

Figure 1 shows a block coupled IPT system

\[ \text{Fig. 1. Block diagram of a loosely coupled IPT system} \]

Current challenges in the development of IPT systems are sufficient misalignment tolerances between the receiver and the transmitter, the ability for a receiver to interoperate with various transmitter topologies [3] and low leakage electromagnetic field as determined by ICNRP guidelines [4]. Multi-coil primary and secondary topologies, with spatially decoupled coils, such as the Double-D Quadrature (DDQ) Charge Pad [5], [6] and the Bipolar Pad [7] have been proposed to solve these challenges. Multi-Frequency IPT has been dealt with in [8] and [9] but in this paper a novel method of multi-frequency IPT enabling decoupling of multiple coils in an IPT system will be discussed.

II. NEED FOR DECOUPLING MULTI-COIL PRIMARY AND SECONDARY TOPOLOGIES

IPT systems often require multiple coils in the primary and secondary systems. Multiple coils provide benefits such as the
ability of a primary to transfer power via many phases [10] or to improve the lateral misalignment tolerance of secondary systems [11]. If the multiple coils in a primary and secondary system are not decoupled, adverse effects such as detuning of the primary and secondary system and power transfer within the primary and secondary circuits in addition to between a primary and a secondary system will occur. These concepts will be developed in this section with the aim of highlighting the effects of mutual inductance within multi-coil primary and secondary systems by considering a multi-coil topology with a finite mutual inductance, $M'$ between the two coils.

\[ M' = \frac{1}{C_p} \]

Fig. 2. A multi-coil primary or secondary topology with mutual inductance $M'$ between the two coils

Figure 2 shows a schematic of an example of a multi-coil topology, the two coils being mutually coupled. In order to enable efficient power transfer, a series-series (SS) compensation topology is used in this paper due to the fact that primary compensation capacitor, $C_p$, is independent of the load and the primary-secondary mutual inductance. Also, the input voltage in SS systems is independent of the primary coil inductance.

\[ V_{v1} = V_{v2} \]

The induced voltages in (3) and (4) have a positive or negative sign depending on the relative magnetic orientation of the two receiving coils. Again, it can be assumed that, when the primary and secondary systems are perfectly aligned, $I_{s1} = I_{s2}$ and hence, $V_{v1} = V_{v2}$. The voltage equations of the equivalent circuit of Figure 3 are therefore

\[ V_{v1} = \left[ R_{L1} + j \left( \omega L_{s1} - \frac{1}{\omega C_{s1}} \right) \right] I_{s1} \]

\[ V_{v2} = \left[ R_{L2} + j \left( \omega L_{s2} - \frac{1}{\omega C_{s2}} \right) \right] I_{s2} \]

The resonant frequency of the receiver can be obtained by setting the imaginary part of equation (5) and (6) to zero and these equations show that the receiver resonant frequency is dependant on the currents $I_{s1}$ and $I_{s2}$, which is an undesirable characteristic in the case of IPT systems as the secondary current magnitudes are directly dependant on the coupling between the primary and the secondary, which varies depending on the relative alignment between the primary and the secondary. If the two secondary coils are mutually decoupled due to their spatial positions ($M' = 0$), the resonant frequencies of the two receiver coils always remain the same and are independent of the currents $I_{s1}$ and $I_{s2}$. It is therefore important to design decoupled multi-coil receivers for IPT systems.

Two special cases arise: a case when the secondary system is perfectly aligned with the primary system and a case when the secondary system is misaligned to the extent that only one of the secondary coils (say $L_{s1}$) couples with the primary, implying that the induced voltage due to the primary in the other coil ($V_{v2}$) is zero. From (5) and (6), it can be easily seen that for the perfectly aligned case and after applying all of the above assumptions, the resonant frequency of the receiver, $\omega_{r1}$, is

\[ \omega_{r1} = \frac{1}{\sqrt{(L_{s1} \pm M'_s)C_{s1}}} = \frac{1}{\sqrt{(L_{s2} \pm M'_s)C_{s2}}} \]
$L_{s1}$ is usually equal to $L_{s2}$ but (7) is valid even if that is not the case.

In the misaligned special case described above ($V_{s2} = 0$), (5) and (6) reduce to

$$V_{s1}' = \left[ R_{L1} + j \left( \omega L_{s1} - \frac{1}{\omega C_{s1}} \right) \right] I_{s1} \pm j \omega M' I_{s2}$$

$$0 = \pm j \omega M' I_{s1} + \left[ R_{T2} + j \left( \omega L_{s2} - \frac{1}{\omega C_{s2}} \right) \right] I_{s2}$$

$$\Rightarrow V_{s1}' = \left[ R_{L1} + j \left( \omega L_{s1} - \frac{1}{\omega C_{s1}} \right) \right] I_{s1}$$

$$\pm \frac{R_{T2} + j \left( \omega L_{s2} - \frac{1}{\omega C_{s2}} \right)}{\left[ R_{L1} + j \left( \omega L_{s1} - \frac{1}{\omega C_{s1}} \right) \right]}$$

From (9) it can be clearly seen that the resonant frequency of the receiver for this misaligned special case is now

$$\omega_{r2} = \frac{1}{\sqrt{L_{s1} C_{s1}}} = \frac{1}{\sqrt{L_{s2} C_{s2}}}$$

(7) and (10) are indeed special cases but they highlight the fact that the resonant frequency of a multi-coil receiver with independent and non decoupled coils can vary greatly and is also directly dependent on the currents in the respective receiver coils. Currently, a receiver of this kind would have spatial decoupled coils as in the case of a Bipolar Receiver Pad or a DDQ Pad. As will be explained in the next chapter, multi-frequency IPT systems can provide the decoupled feature to multi-coil receivers with more freedom in the coil design, which can also possibly lead to a saving of copper.

B. Multi-coil Primary Systems

The primary coils in IPT systems often have multiple coils which transfer power to the secondary. If both these coils are independently excited, the mutual inductance between them will cause power to be transferred between them by inducing voltages, much like how the mutual inductance between a primary and secondary causes power transfer [10]. These induced voltages will cause induced currents to flow, which can be dangerous to the normal operation of an IPT primary system, as will be explained in this section for a series compensated primary.

The problem with mutual inductance within multi-coil primary IPT systems has been acknowledged and studied in literature [10]. With respect to lumped charge pads, multi-coil systems (such as the DDQ pad) have conventionally been developed to improve misalignment tolerances and hence are found in secondary system designs. It is only recently, that multi-coil systems have been studied for primary system design and have been found to be advantageous in ensuring interoperability of the primary with various different secondary designs [3]. But of course, these multi-coil systems have coils that are spatially decoupled and hence have a negligible inter-coil mutual inductance.

Unlike the case of multi-coil secondary systems, mutual inductance in multi-coil primary systems can adversely affect the input power supply. Similar to the case of multi-coil secondary systems, Figure 4 shows the equivalent circuit

of two mutually coupled Transmitter Coils. Here, $V_{in,1}$ and $V_{in,2}$ are the output voltages from the power supply. These are square waveforms but Fundamental Mode Analysis is used and these are replaced with a sinusoidal voltage of frequency and magnitude equal the fundamental component of the inverter output. $R_{p1}$ and $R_{p2}$ are the summation of the series resistances of the primary inductor, capacitor as well as and equivalent resistance of the internal losses in the inverter. $R_{T1}'$ and $R_{T2}'$ are the reflected loads due to the output load on the primary. As this is a series compensated primary the input power supply is designed to provide rated primary currents.

Due to the inter coil mutual inductance, $M'_{p}$, the voltage equations are

$$V_{in,1} = \left[ R_{T1}' + R_{p1} + j \left( \omega L_{p1} - \frac{1}{\omega C_{p1}} \right) \right] I_{p1} \pm j \omega M'_{p} I_{p2}$$

$$V_{in,2} = \pm j \omega M'_{p} I_{p1} + \left[ R_{T2}' + R_{p2} + j \left( \omega L_{p2} - \frac{1}{\omega C_{p2}} \right) \right] I_{p2}$$

Hence, similar to the multi-coil receiver case, (11) highlights the fact that the resonance frequency of mutually coupled multi-coil primaries depend on the currents flowing in the respective circuits. It can also be seen that $M'_{p}$ can cause a reverse current that flows into the inverter, being rectified by the anti-parallel diodes of the inverter switches. These induced currents can therefore cause the voltage across the input DC bus capacitors to rise and therefore cause problems in the normal control of the inverter.

III. A DUAL FREQUENCY IPT SYSTEM

The adverse effects of inter-coil mutual inductance in multi-coil primary and secondary designs are highlighted in Section II. A multi-frequency IPT system that can decouple multi-coil systems that are otherwise mutually coupled, is introduced in this section. As a first case, a dual frequency system is considered.

A. Introduction

A dual frequency system using two transmitter coils each powered by a current of a different frequency is introduced in this section. The receiver is assumed to also have two
independent receiver coils that are each tuned to one of the transmitting frequencies. These two frequencies can be represented as \( \omega_1 \) and \( \omega_n \) where

\[
\omega_n = n\omega_1
\]  

and \( n \) is, in general, any real number. From (12), it may be perceived that the choice of frequencies could be virtually unlimited. This is not really true and the conditions for frequency selection will be highlighted in Section III-B. Figure 5 shows a schematic block diagram of a two transmitter dual frequency system.

Figure 5. Schematic of a two transmitter Dual Frequency System

In general, the power transferred from the transmitter to the receiver can be shared between the two transmitting frequencies and the power sharing ratio, \( m \), is given by

\[
m = \frac{P_{\text{out},1}}{P_{\text{out}}} \quad \text{and} \quad 1 - m = \frac{P_{\text{out},n}}{P_{\text{out}}}
\]  

the total output power. The power sharing ratio is dependent on the primary currents in the respective frequency systems and therefore can be changed while the system is operating. Figure 6 shows the circuit schematic of this dual frequency IPT system. As can be seen, it could be necessary to include additional series inductances \( L_{\text{series},1} \) and \( L_{\text{series},2} \) to the primary and secondary circuits in order to decouple the two frequency systems.

B. Frequency Selection

Frequency selection plays a major role in the design process of a SS multi-frequency system. As the aim is to decouple two coils, it is important that the two circuits present a high impedance to currents of the other frequency while resonating at their respective resonance frequencies. The required magnitude of this impedance, depends on the the currents in the coils and the mutual inductance between the two coils. In a general RLC circuit, the impedance, \( Z \), and the quality factor, \( Q \), are given by

\[
Z = R + j \left( \omega L - \frac{1}{\omega C} \right) \\
Q = \frac{\omega L}{R}
\]

The Quality factor gives an indication of a resonators bandwidth relative to its centre frequency and is directly proportional to the resonant frequency \( \omega \). Figure 7 shows the variation of \( |Z| \) vs frequency for RLC circuits, with \( L = 20 \mu \text{H} \), resonating at 20 kHz, 50 kHz, 100 kHz and 150 kHz and it is clearly evident that the higher frequency resonance circuits have a better quality factor as compared to the lower frequency resonance circuits. A fallout of this is that high frequency RLC circuits invariably present a much higher impedance to lower frequency currents than vice versa.

where, \( P_{\text{out},1} \) and \( P_{\text{out},n} \) are the powers transferred by the lower and higher frequencies respectively and \( P_{\text{out}} \) is
Figure 7 shows that the 100 kHz circuit presents an impedance to 20 Khz currents that is close to 10 times that presented by the 20 kHz circuit to 100 kHz currents. As can be seen from (14), the impedance is directly proportional to the inductance and hence in order to have frequency decoupled IPT systems, it is necessary to increase the inductance of a RLC circuit. This is shown in Figure 6 by the addition of a series inductance, $L_{\text{series}}$.

\[ Z_j = \begin{bmatrix} Z_{p1} & j\omega_1M_1 & j\omega_1M' & j\omega_1M'' \\ j\omega_1M_1 & Z_{s1} & j\omega_1M' & j\omega_1M'' \\ j\omega_1M' & j\omega_1M'' & Z_{p2} & j\omega_1M_2 \\ j\omega_1M'' & j\omega_1M' & j\omega_1M_2 & Z_{s2} \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{s1} \\ I_{p2} \\ I_{s2} \end{bmatrix} \]

(16)

\[ V_{in,1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

(17)

As can be seen from (16) and (17) the principle of superposition is used to analyse the induced currents by the two frequency systems. Here all the four coupled coils are considered to be identical and the symbols are as defined in Figure 6.

The design procedure for the dual frequency IPT system is highlighted in Figure 8 where it is initially assumed that the magnetics, the two operating frequencies as well as the rated primary and secondary currents of both the frequency systems are already determined.

**C. Design Procedure**

The governing equations of the four coupled circuits of Figure 6 are

\[ V_{in,1} = \begin{bmatrix} Z_{p1} & j\omega_1M_1 & j\omega_1M' & j\omega_1M'' \\ j\omega_1M_1 & Z_{s1} & j\omega_1M' & j\omega_1M'' \\ j\omega_1M' & j\omega_1M'' & Z_{p2} & j\omega_1M_2 \\ j\omega_1M'' & j\omega_1M' & j\omega_1M_2 & Z_{s2} \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{s1} \\ I_{p2} \\ I_{s2} \end{bmatrix} \]

(16)

\[ V_{in,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

(17)

As can be seen from (16) and (17) the principle of superposition is used to analyse the induced currents by the two frequency systems. Here all the four coupled coils are considered to be identical and the symbols are as defined in Figure 6.

The design procedure for the dual frequency IPT system is highlighted in Figure 8 where it is initially assumed that the magnetics, the two operating frequencies as well as the rated primary and secondary currents of both the frequency systems are already determined.

**IV. Simulation Results**

A multi-coil dual frequency system with a primary and secondary topology as in Figure 2 is simulated for a power transfer of 100 W. The primary-secondary air gap was set to 10 cm. The higher and lower frequencies were selected to be 100 kHz and 30 kHz and Figure 9 - 12 shows the frequency spectrum of the primary and secondary currents after the design procedure of Figure 8 was applied.

FEM model results are shown in Figure 13 - 14 which highlight the fact that one frequency system induces negligible currents in the other frequency system.
VI. CONCLUSION

The two transmitter dual frequency system described in this paper presents a possibility to decouple power transfer...
via IPT systems that are otherwise coupled. As the power sharing ratio is only dependent on the primary coil currents, this can be changed even during system operation for example at times when misalignment tolerances need to be increased or at times when the air gap magnetic field intensity needs to be decreased. The extent to which decoupling can be achieved depends on the inter-coil mutual inductance, the rated currents in the two coils and the value to the series inductance and it is shown, via simulations and experiments that the induced currents in a frequency decoupled system can be below 1% of the original, resulting in two IPT systems that operate with negligible interactions.

REFERENCES


