DIRECT NUMERICAL SIMULATION OF DEFORMABLE BUBBLES IN WALL-BOUNDED SHEAR FLOWS

F. S. Sousa∗†, L. M. Portela∗, R. F. Mudde∗ and N. Mangiavacchi††

∗Delft University of Technology, Faculty of Applied Sciences
Prins Benhardlaan 6, 2628 BW Delft, The Netherlands
e-mail: L.Portela@tudelft.nl, R.F.Mudde@tudelft.nl
†Universidade de São Paulo, Instituto de Ciências Matemáticas e de Computação
Av. Trabalhador São Carlese 400, 13560-970 São Carlos-SP, Brazil
e-mail: fsimeoni@icmc.usp.br
††Universidade do Estado do Rio de Janeiro, Faculdade de Engenharia
Rua São Francisco Xavier 524, 20550-900 Rio de Janeiro-RJ, Brazil
e-mail: norberto@uerj.br

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Abstract. We present a method for fully-resolved simulations of bubbly flows using a front-tracking/front-capturing technique. The method is a modification of a marker-and-cell method developed previously for free-surface flows. The basic approach is somehow similar to the front-tracking method of Tryggvason: the continuity and Navier-Stokes equations are solved in a single Eulerian grid, and the interface is represented by an unstructured Lagrangian grid moving through the Eulerian grid, with the velocities at the Lagrangian grid obtained by interpolation from the Eulerian grid. However, contrary to Tryggvason’s method, our method uses a sharp interface between the gas and the liquid, since it “captures” the interface within one cell of the Eulerian grid. The surface tension is expressed as a body-force in the Eulerian grid, and is computed using a least-squares fit, together with a mass-conserving filter to remove “sub-grid oscillations”. The Navier-Stokes equations are solved using a finite difference scheme on the Eulerian grid and the continuity is enforced using a standard projection technique, with the resulting Poisson equation being solved by a conjugate gradient method. We present some results of the simulations of single bubbles of different sizes in laminar wall-bounded uniform-shear flows at moderate Reynolds numbers (bubble-Reynolds-number in the range of 20-50). In wall-bounded shear flows, it has been observed experimentally that depending on the bubble size the lift force can push the bubble either towards or away from the wall. Our simulations show a good agreement with the experiments, both qualitatively and quantitatively, and an explanation for the lift inversion mechanism is provided based on the analysis of the forces acting on the bubble.
1 INTRODUCTION

In spite of the importance of bubbly flows, the fundamental understanding of the interactions between the bubbles and the flow is still rather poor. Many industrial situations involve complex flows with a large number of bubbles of different sizes and several phenomena occurring simultaneously. One of these situations is motivated by the oil industry, specifically in the transport of raw oil from the deeps in pipelines. In order to reduce the weight of the oil column, a technique called “gas-lift” is applied. In this technique, gas is injected into the oil pipe forming a bubble column, reducing considerably the amount of power needed to pump this oil up. However, it was observed that under certain circumstances bubbles tend to cluster in the middle of the pipe forming long slugs of gas, compromising the use of this technique. Under other circumstances they tend to cluster close to the pipe wall. This change of behavior is due to an inversion of the lift force acting on the bubbles.

In order to better understand the mechanisms behind lift force inversion, many authors have researched the lift force of bubbles in several situations. A number of experiments have been carried out in order to explain the interactions between bubbles and flow. Takemura & Magneaudet\(^1\) studied the mechanisms behind the lateral migration of bubbles rising in a quiescent viscous liquid near the wall. They have found that both clean and contaminated bubbles move away from the wall for low Reynolds numbers. For \(Re > 1\) contaminated bubbles are repelled from the wall, while the lift force of clean bubbles is directed away from the wall for \(Re < 35\) and toward it for higher \(Re\). They build empirical correlations to derive practical expressions for the lift force as function of \(Re\) and the separation between bubble and wall. Magneaudet et al.\(^2\) have investigated theoretically the rise of a single bubble in both quiescent and linear shear flow situations. They derived analytical expressions for the drag and lift forces acting on a boyant drop moving in a viscous fluid at rest or in a linear shear flow near a wall. In both works, the deformation of the bubble is not important for the direction of the lift force, but only the influence of the wall onto the bubble.

In a recent work, Sakata et al.\(^3\) have studied experimentally the dynamics of an air bubble rising in a linear shear flow using PIV (Particle Image Velocimetry). They measured the forces acting on a bubble and the lateral motion induced by vortex shedding from its wake. They have found correlations between the shape, horizontal forces, and the wake structures of the bubble for slightly high Reynolds numbers.

Tomiyama et al.\(^4\) have also concluded that shape determines the direction of lift force for rising bubbles in a linear shear flow. They carried out experiments with bubbles of different diameters rising in a glycerol-water solution. They have found that small bubbles have their lift force directed towards the stationary wall, resulting in a wall-peaking regime, while large bubbles have their lift force directed away from the stationary wall, resulting in a core-peaking regime. Other authors have also reported this behavior.\(^5, 6\) Tomiyama have also derived a correlation for the lift coefficient, with Reynolds number
for small bubbles (diameter $D \leq 4.4 \, \text{mm}$) and with Eötvös number for large bubbles ($D > 4.4 \, \text{mm}$).

Even though there is a large number of experimental works carried out for single bubbles rising in a shear flow, a lack of numerical simulations in this field is noticeable. Accurate numerical simulations can improve the understanding about the lift force inversion, due to the amount of data that can be produced and analyzed in details. However, accurate numerical simulations are not easy to achieve, due to the discontinuities arising from representation of the interface.

Among the few works available in the literature, the work of Ervin & Tryggvason\textsuperscript{7} is perhaps the first successful attempt to simulate the lift force inversion. The authors implemented the Tryggvason’s front-tracking method to simulate rising bubbles in shear flow induced by two opposite moving walls. This study was done in two steps: first a 2D simulation was performed in order to get a better understanding about the phenomenon, and finally a 3D simulation was carried out to validate the assumptions based on the 2D results. In order to perform longer simulations they have implemented a periodic boundary condition at the top and bottom of the domain. This could have some influence over the results if the wake of the bubble is greater than the domain. The results showed the same behavior reported by Tomiyama, where bubbles with higher surface tension migrate in the same direction as solid particles while bubbles with smaller surface tension have an opposite lateral migration. In their numerical results, they have found that asymmetric deformations are caused by buoyancy and shear, forcing a circulation around the bubble which opposes the mean shear of the flow. They have compared only one simulation against experimental lift and drag coefficient provided by Shridar & Katz\textsuperscript{8} with certain agreement.

Deen and co-workers\textsuperscript{9} have also used the front-tracking method from Tryggvason to simulate the rise of a single bubble in a vertical shear flow. In their work, they use the front-tracking methodology to obtain closure information (drag, lift and added mass forces) for the simulation of bubbly columns using Euler-Lagrange model. They obtained good results for drag and added mass forces, however, due to a mass conservation problem in their implementation they could not perform longer simulations, consequently compromising their lift force prediction.

Recently, a work by Sousa et al.\textsuperscript{10} describes a front-tracking/front-capturing technique for the simulations of multiphase flows. It uses the same marker particles as Tryggvason’s method does, but with additional information about the phases in the background Eulerian mesh. This “front-capturing” part of the code allows for the representation of sharp interfaces while keeping its improved mass conservation properties and avoiding some problems encountered in Tryggvason’s code. The method used for the solution of the Navier-Stokes equations is a projection scheme based on the GENSMAC method developed by Tomé & McKee,\textsuperscript{11} and extended in many other works.\textsuperscript{12, 13, 14, 15} It is a MAC-like method,\textsuperscript{16, 17} with finite differences discretization over a staggered grid. The method has already been validated for a number of simple test cases.
Motivated by the work done by Tomiyama et al.\textsuperscript{4}, we use the front-tracking/front-capturing technique by Sousa et al.\textsuperscript{10} to carry out simulations of single bubbles rising in a linear shear flow. The results are compared with Tomiyama’s work in order to validate the code for this type of flow both qualitatively and quantitatively. Also, using the numerical data provided by our simulations, we discuss the lift force inversion mechanisms and its relation to the shape of the deformable bubble.

2 NUMERICAL METHOD

The equations modelling this multiphase flow problem are the Navier-Stokes equations: momentum and mass conservation equations. Considering density and viscosity as variables over the whole domain, but as constants inside each specific fluid, these equations can be written as:

\[
\rho \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)) + \rho g + \mathbf{f}_\sigma
\]

\[
\nabla \cdot \mathbf{u} = 0
\]  

where \( \mathbf{u} = \mathbf{u}(x, t) \) denotes the velocity field, \( p = p(x, t) \) is the pressure field, \( \rho \) and \( \mu \) are the density and viscosity of the fluid. Here \( g \) denotes the gravitational field and \( \mathbf{f}_\sigma \) is the interface tension force given by

\[
\mathbf{f}_\sigma = \sigma \kappa \nabla H
\]

where \( \sigma \) is the surface tension coefficient, \( \kappa \) is the local curvature and \( H \) is a Heaviside function which is non-zero inside a specific phase.

To represent the interface, a front-tracking/front-capturing technique described in Sousa et al.\textsuperscript{10} is employed. This technique is somehow similar to the Tryggvason’s method,\textsuperscript{18} using an unstructured 2D mesh to link the marker particles, that are transported over a background Eulerian mesh. The interface is explicitly represented by the marker particles, and its curvature is geometrically computed from the position of these particles by a highly accurate fitting scheme.

The main difference between the two methods is the technique used to identify the fluid regions. While in the Tryggvason’s method, a Poisson equation must be solved every time-step to obtain the correct distribution of density and viscosity, in our method this is achieved by classifying the cells of the background Eulerian grid according to the position of the marker particles representing the interface. This classification is done only once, and then it is updated only close to the interface every time-step according to the new position of the marker particles. By using this procedure, a discrete Heaviside function is automatically provided by the cell classification, which is then used for the distribution of the forces on the interface. Additionally, no Poisson equation has to be solved to transfer information from the interface to the background Eulerian grid. Therefore, problems like inaccuracies in the far field and smearing of the properties at the interface are avoided, and the interface is kept as sharp as one grid cell resolution.
Figure 1: Position of the variables in a cell. The velocities are computed on the faces of the cell, and the pressure ($p$), density ($\rho$), viscosity ($\mu$) evaluated at the cell center.

The procedure for the solution of the equations (1)-(2) is based on a projection method described in details in Sousa et al.$^{10}$ Shortly, given a velocity field $u(x, t_0)$ at $t = t_0$, the updated velocity field $u(x, t)$ at $t = t_0 + \Delta t$ is computed by the following procedure:

1. Given an arbitrary pressure field $\tilde{p}(x, t)$, an intermediate velocity field $\tilde{u}(x, t)$ is computed by the explicitly discretized form of (1). It possesses the correct vorticity at time $t$, but, as $\tilde{p} \neq p$, it does not satisfy (2);

2. The correction function $\psi(x, t)$ is computed solving the elliptic equation

$$\nabla \cdot \frac{1}{\rho} \nabla \psi(x, t) = \nabla \cdot \tilde{u}(x, t)$$

3. Finally, the updated velocity is given by

$$u(x, t) = \tilde{u}(x, t) - \frac{1}{\rho} \nabla \psi(x, t)$$

4. The positions of the marker particles are updated using the new velocity field.

Similarly to the MAC method, all the equations are discretized by finite differences on a staggered grid. Figure 1 shows an example of a staggered grid cell and the position of the variables: the velocities are calculated on the faces, and the other variables (pressure, density and viscosity) are computed at the cell center.

The convective terms of (1) are discretized by the high order upwind scheme VONOS.$^{19}$ Details about the implementation of this scheme can be also found in Ferreira et al.$^{20}$ The other derivatives are approximated using standard central scheme. Additionally, for the computation of $\rho$ and $\mu$ at the interface, simple and harmonic averaging are employed.

The curvature is computed by a highly accurate fitting scheme, which locally approximates the surface of the bubble by quadrics. As this computation is done at a cell level, a mass-conserving filter called TSUR (Trapezoidal Sub-grid Undulation Removal) is employed to prevent unphysical sub-grid oscillations to appear.$^{10, 21, 22}$
3 VALIDATION

The employed method was already validated for a number of simple test cases (e.g. static, oscillating and rising bubbles) and for more complex simulations like coalescence of bubbles and splashing drop. In this work, we validate this method for some of the experimental data found in Tomiyama’s work.

The bubble is released initially in spherical shape inside a shear flow, which is obtained enforcing appropriate velocity boundary conditions at the walls. In order to perform longer simulations, we implemented a moving reference frame, moving with the rising velocity of the bubble. Thus, the bubble stays stationary in this reference frame, allowing it to rise for many diameters and using a short domain. To accomplish that, inflow and outflow boundary conditions are enforced at the top and bottom of the domain. The velocity of the bubble is subtracted from the whole domain in order to keep it stationary in the moving reference frame. A schematic drawing can be seen in fig. 2.

The simulations were performed for the following parameters, found in Tomiyama et al.: \( \rho_L = 1166 \text{ kg/m}^3 \), \( \mu_L = 0.022 \text{ kg/m} \cdot \text{s} \) and \( \sigma = 0.061 \text{ N/m} \) are respectively the density, viscosity and surface tension coefficient for the liquid phase. We consider two different bubble diameters, \( D = 3.52 \text{ mm} \) and \( D = 5.53 \text{ mm} \), hereinafter called respectively as small and large bubbles. The intensity of the shear flow used was \( \omega = 6.2 \text{ s}^{-1} \).

The Morton number, defined as

\[
M = \frac{g(\rho_L - \rho_G) \mu_L^4}{\mu_L^2 \sigma^3},
\]

is a property group of the two phases, and the Eötvös number, defined as

\[
Eo = \frac{g(\rho_L - \rho_G) D^2}{\sigma},
\]
is the ratio of buoyancy to surface tension forces. Here, $g$ is the gravitational constant and $\rho_G$ is the density of the gas phase. The resulting theoretical Morton number for both simulations is $M = 10^{-5}$, and resulting Eötvös numbers are $Eo = 2.3$ for the small bubble and $Eo = 5.7$ for the large one. Figure 3 displays the relative Reynolds number for both small and large bubbles, computed as

$$Re = \frac{\rho_L |u_b| D}{\mu_L}$$

where $u_b$ is the relative velocity of the center of mass of the bubble, obtained subtracting the shear field from the total velocity of the bubble.

Similarly to the approach used by Tryggvason,\textsuperscript{7,18} the density ratio between gas and liquid can be reduced in order to perform faster simulations. As we are dealing with an explicit code, reducing the density ratio decreases the computational cost by allowing
larger time-steps to be used. A comparison between trajectory and relative Reynolds number for the large bubble simulated with real density ratio and density ratio nearly 10 times smaller can be seen in figure 4. The results are quite similar, showing that there is no difference in simulating with a 10 times smaller density ratio. Despite the effect caused by the viscosity inside the bubbles, which is observed by small pressure gradients, we keep the viscosity ratio in all of our simulations equal to 1.

A qualitative comparison of the bubble shape obtained experimentally, and the respective shape obtained in the numerical simulations can be seen in figure 5. The photos from experiments are taken from Tomiyama et al.\textsuperscript{4} It can be seen in this figure that numerical simulations predicted well the shape of both small and large bubbles.

A comparison of the bubble trajectories can be seen in fig. 6. Here, the trajectories of the bubbles rising with and without shear flow are displayed and compared with experimental data from Tomiyama et al.\textsuperscript{4} While for the large bubbles the results are in good agreement with experiments, results for small bubbles have a discrepancy from the experimental data.

A resolution study (fig. 7) reveals that the path of the small bubble have converged, however to a different angle. On the other hand, for the large bubble a resolution of 11 cells in the diameter of the undeformed bubble ($D/h \approx 11$) is enough to produce reasonable results. For the sake of comparison, using a $1 \times 1 \times 1$ domain, this resolution would correspond to $60^3$ cells, while for the small bubble, the same domain corresponds to a resolution of $80^3$ for $D/h \approx 9$ and $100^3$ cells for $D/h \approx 12$, sharply increasing the amount of computational power needed to simulate this problem.

A quantitative comparison can be seen in fig. 8, where the evolution of transverse lift coefficient in time is displayed. The transverse lift force is given by

$$F_T = -C_T V \rho_L (u_G - u_L) \times (\nabla \times u_L)$$

(4)

where $C_T$ is the transverse lift force coefficient, $V$ is the volume of the bubble, $\rho_L$ is the density of the liquid, and $u_G$ and $u_L$ are respectively the velocities of gas and liquid phase.

As shown in fig. 8, the lift coefficient matches the experimental value for the large bubble. Considering the fully developed bubble in the shear flow, the average value for the lift force is computed as being $C_T = -0.292$ against $C_T = -0.28$ found in Tomiyama’s
Figure 6: Trajectories for both small and large bubbles, with and without shear flow: lines represent numerical solutions.

Figure 7: Resolution study for small and large bubbles, where $D$ is the diameter of the bubble and $h$ is the grid spacing. The ratio $D/h$ represents how many cells are present in the diameter of the bubble in one direction.
work. Similarly, the transverse lift force coefficient for small bubble was found to be $C_T = 0.176$ against $C_T = 0.29$ from Tomiyama’s work.

4 DISCUSSION

Both our simulations and the experiments of Tomiyama et al.\(^4\) show that the small and large bubbles have a lift force in opposite directions, associated with transverse migrations in opposite directions. This lift-force inversion is linked with the differences in the flow around the bubble; in particular, with the differences in the bubble wake. In figure 9 are shown the streamlines of the numerical simulations, in a frame of reference moving with the vertical velocity of the bubble. From the figure it is clear that for the small bubble the wake is attached to the surface of the bubble, without any recirculation pattern behind it, whereas for the large bubble the wake is not attached to the bubble, and there exists a recirculation pattern on the right-side. This asimmetry in the wake is responsible for the lift force inversion.

The asymmetry of the wake is due to both the ellipsoid bubble-shape and the differences in velocities in the frame of reference of the bubble on both sides. For the large bubble, a higher relative velocity on its right-side is enough to produce a recirculation pattern, whereas on the left-side, due to the lower relative velocity, the streamlines tend to get more attached to the surface of the bubble. The small bubble has a shape that is closer to a sphere and since the Reynolds number is lower the wake remains attached to the bubble on both sides.

In the past, the lift force inversion for the large bubble has been attributed to an
inversion in the circulation inside and around the bubble, associated with an asymmetry in its shape, and an analogy has been made with the lift in 2D airfoils in potential flow. Here, we do not observe any inversion in the circulation, showing that the lift inversion phenomenon cannot be understood simply by analogy with the lift in airfoils.

5 CONCLUSION

Direct numerical simulations of deformable bubbles rising in wall-bounded shear flow were performed. The front-tracking/front-capturing technique to represent sharp interfaces developed by Sousa et al. produced accurate results when compared to experimental data found in literature. The numerical simulations showed good agreement when compared to experimental data, both qualitatively and quantitatively. The results showed that finer grids are needed to capture the phenomenon for the small bubble, where interaction between the wake structures and shear flow are more involving.

Our results indicate that the inversion of the lift force for the large bubble can be explained by the asymmetry in the wake associated with both its ellipsoid shape and the difference in velocities in the two sides of the bubble. The inversion of the lift force was not associated with an inversion of the circulation inside and around the bubble, indicating that the phenomenon cannot be explained by a simple analogy with potential flow in 2D airfoils.
REFERENCES


