A.F. Pruijssers

AGGREGATE INTERLOCK AND DOWEL ACTION UNDER MONOTONIC AND CYCLIC LOADING
AGGREGATE INTERLOCK AND DOWEL ACTION UNDER MONOTONIC AND CYCLIC LOADING
AGGREGATE INTERLOCK AND DOWEL ACTION
UNDER MONOTONIC AND CYCLIC LOADING

PROEFSCHRIFT
ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus Prof.dr. J.M. Dirken,
in het openbaar te verdedigen ten overstaan van
een commissie aangewezen door
het College van Dekanen
op dinsdag 14 juni 1988
te 16.00 uur door

ADRIANUS FRANS PRUIJSSERS,
geboren te Rotterdam,
civiel ingenieur

Delft University Press / 1988
ACKNOWLEDGEMENT

The experimental part of this research was performed in the Stevin Laboratory of the Delft University of Technology with financial support and under the supervision of the CUR (Netherlands Centre for Civil Engineering, Research, Recommendations and Codes), which is greatly appreciated.

The author wishes to record his thanks to all members of the "Concrete Structures Group", who have contributed to this research project.

I would like to express my gratitude to Dirk Verstoep b.v. for giving the opportunity to complete this thesis.

The financial support received from the "Stichting Professor Bakkerfonds" for this publication is gratefully acknowledged.

CIP GEGEVENS KONINKLIJKE BIBLIOTHEEK, DEN HAAG

A.F. Pruijssers

ISBN 90-6275-451-1

Copyright © 1988 by A.F. Pruijssers.

All rights reserved. Published 1988.

No part of the material protected by this copyright notice may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording or by any Information storage and retrieval system, without written permission from the publisher: Delft University Press.

Printed in The Netherlands
CONTENTS

1. INTRODUCTION
1.1 Scope of research. 1
1.2 Aim of the research program. 3

2. SURVEY OF THE LITERATURE
2.1 Introduction. 5
2.2 Aggregate interlock; monotonic loading. 6
2.3 Aggregate interlock; cyclic loading. 17
2.4 Dowel action; monotonic loading. 22
2.5 Dowel action; cyclic loading. 28
2.6 Contribution of axial steel stress. 29
2.7 Shear strength of cracked reinforced concrete; monotonic loading. 30
2.8 Shear strength of cracked reinforced concrete; cyclic loading. 35
2.9 Conclusions. 37

3. EXPERIMENTAL STUDY
3.1 Introduction. 39
3.2 Reinforced specimens; repeated loading. 39
3.2.1 Test arrangement. 39
3.2.2 Test variables. 41
3.2.3 Experimental results. 43
3.3 Externally reinforced specimens; repeated loading. 51
3.3.1 Test arrangement. 51
3.3.2 Test variables. 52
3.3.3 Experimental results. 52
### 4. THEORETICAL MODELLING OF THE RESPONSE OF CONCRETE TO MONOTONIC SHEAR LOADING

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>59</td>
</tr>
<tr>
<td>4.2</td>
<td>The mechanism of aggregate interlock</td>
<td>59</td>
</tr>
<tr>
<td>4.3</td>
<td>The mechanism of dowel action</td>
<td>64</td>
</tr>
<tr>
<td>4.4</td>
<td>The combined mechanism of aggregate interlock and dowel action</td>
<td>81</td>
</tr>
<tr>
<td>4.5</td>
<td>Influence of the normal restraint stiffness upon the shear stiffness</td>
<td>89</td>
</tr>
<tr>
<td>4.6</td>
<td>Additional detailed tests</td>
<td>93</td>
</tr>
<tr>
<td>4.7</td>
<td>Concluding remarks</td>
<td>95</td>
</tr>
</tbody>
</table>

### 5. THEORETICAL MODELLING OF THE RESPONSE OF CRACKED CONCRETE TO REPEATED AND REVERSED SHEAR LOADING

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>97</td>
</tr>
<tr>
<td>5.2</td>
<td>The mechanism of aggregate interlock</td>
<td>97</td>
</tr>
<tr>
<td>5.3</td>
<td>The mechanism of dowel action</td>
<td>111</td>
</tr>
<tr>
<td>5.4</td>
<td>The combined mechanism of aggregate interlock and dowel action</td>
<td>120</td>
</tr>
<tr>
<td>5.5</td>
<td>Influence of the normal restraint stiffness upon the shear stiffness for the case of repeated loading</td>
<td>129</td>
</tr>
<tr>
<td>5.6</td>
<td>Concluding remarks</td>
<td>131</td>
</tr>
</tbody>
</table>

### 6. IMPLEMENTATION OF THE CYCLIC AGGREGATE INTERLOCK MODEL INTO NUMERICAL PROGRAMS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>134</td>
</tr>
<tr>
<td>6.2</td>
<td>Simplified expressions for the static two-phase model</td>
<td>136</td>
</tr>
<tr>
<td>6.3</td>
<td>Rheological model for an element with the smeared out crack concept</td>
<td>138</td>
</tr>
<tr>
<td>6.4</td>
<td>The stress-strain relation for the case of cyclic loading</td>
<td>148</td>
</tr>
<tr>
<td>6.5</td>
<td>Implementation of the dowel action mechanism</td>
<td>151</td>
</tr>
<tr>
<td>6.6</td>
<td>Concluding remarks</td>
<td>153</td>
</tr>
</tbody>
</table>
Stellingen behorende bij het proefschrift:
"Aggregate interlock and dowel action under monotonic and cyclic loading"
van A.F. Pruijssers

1. De openingsrichting van een scheur in gewapend beton belast door een
monotoon toenemende schuifkracht wordt aanvankelijk bepaald door de
vervorming van de wapeningsstaven. Na het volledig ontwikkelen van
plastische scharnieren in deze staven, wordt het scheureningspad
opgelegd door de haakweerstand van de toeslagkorrels.

2. Het gedrag van een scheur in gewapend beton onderworpen aan een zeer
groot aantal lastwisselingen met een kleine amplitude van de schuif-
spanning, kan quasi-statisch worden beschreven.

3. Het twee-fasen model van Walraven voor de beschrijving van de haak-
weerstand van de toeslagkorrels onder monotoon toenemende belasting
kan op eenvoudige wijze worden aangepast voor het geval van herhaal-
de- en wisselende schuifbelastingen.

4. De samenwerking van staal en beton leidt tot een verhoging van de
deuvelsterkte van op afschuiving belaste wapeningsstaven.

5. Ten aanzien van de haakweerstand van de korrels onder wisselende
schuifbelasting met constante amplitude kan worden gesteld dat de
belastingsgeschiedenis volledig ligt besloten in de eindscheurver-
plaatsingen van de laatste wisseling.

6. Er treedt geen herverdeling van de belasting op tussen de mechanis-
men van de haakweerstand van de korrels en van de deuvelwerking ten
gevolge van het wisselen van belasting.

7. Het gebruik van een schuif-reductiefactor met een constante waarde
gaat voorbij aan het fysische gedrag van een scheur, maar leidt on-
der monotoon toenemende belastingen niet tot onrealistische schuif-
spanningen in een gescheurd element. Indien de richting van de
hoofdspanningen zich sterk wijzigt gedurende het belasten, bijvoor-
beeld door het wisselen van de belasting, wordt met een constante
reductiefactor een fysisch on juist scheurgedrag verkregen.
8. Een goede toets voor de toepasbaarheid van een numeriek elementenprogramma is wellicht het simuleren van een proef met een vooraf bekende, doch gefingeerde en fysisch onmogelijke uitkomst.

9. Het afschuifdraagvermogen van niet op afschuiving gewapende liggers berust nagenoeg geheel op de som van de schuifspanningen in de drukzone, de ongescheurde trekzone en de tension-softening zone.


11. De te verwachten zeespiegelrijzing noopt het laaggelegen en dichtbevolkte Nederland tot een zeer actief beleid ten aanzien van de Europese eenwording.

12. De juiste oplossing voor een probleem is vaak zo eenvoudig dat het niet meevalt om te verklaren waarom deze niet eerder gevonden is.

13. Het is niet dom om iets slims niet zelf te bedenken, wel om het om die reden niet te gebruiken.
1. INTRODUCTION

1.1. Scope of the research.

Today's offshore industries demand offshore platforms enabling the exploitation of large oil and gas reservoirs in the Arctic and the deep sea up to about 300 m. Heavily reinforced high-strength concrete structures are very effective in withstanding the severe loading conditions in the Arctic environment, dominated by icefields and icebergs, and by the deep sea, characterized by extreme wave and wind attacks. The safety against failure of such complex structures is analysed by idealizing the structure as an assemblage of basic elements. The interactions of these elements and their redistribution of the applied loads and deformations can be simulated in advanced finite element programs. As a consequence, the problem of designing a large-scale structure with sufficient safety against failure is shifted towards a thorough understanding of the material behaviour of the basic elements and towards efficient numerical solution techniques.

It was for this reason that the Netherlands Centre for Civil Engineering Research, Recommendations and Codes (CUR) started the project 'Concrete Mechanics'. This project comprises experimental research and material modelling on the one hand and implementation of these models in numerical programs on the other hand. The Concrete Mechanics project is a cooperation of a division of the Netherlands Ministry of Transport and Public Works (Rijkswaterstaat), the Institute for Applied Scientific Research on Building Materials and Building Structures (IBBC-TNO) and the Universities of Technology of Delft and Eindhoven.

Due to the applied loads and deformations, structural elements are subjected to tensile stresses causing cracking of the concrete. Although offshore structures are generally designed to remain uncracked under service conditions, colliding ships or icebergs might cause cracks. In 1980 five severe ship collisions were reported in the English part of the North Sea [59], resulting in damage of the structure.

Apart from the 'special circumstances' such as collisions, a structure can possibly be designed in a more economic way when the stiffness of cracked reinforced concrete, which is still considerable, is utilized in withstanding the applied loads and deformations. In bridge design, it
appeared that such an approach is especially favourable for the case of large settlements. A partially prestressed concrete structure can easily follow such settlements, whereas a fully prestressed structure cannot. As a consequence of the development of cracks, the response of the elements to severe loading conditions becomes highly non-linear with large irreversible deformations. This non-linear material behaviour must be thoroughly understood and modelled. Therefore, the first phase of the 'Concrete Mechanics' project focussed upon the experimental and theoretical investigation of the static shear strength and stiffness of cracked concrete, the bond behaviour of the reinforcing bars and the fundamental material behaviour, such as multiaxially loaded concrete.

As numerical tools, two basically different non-linear finite element programs were developed. In the first program (MICRO), the development of cracks is taken into account by defining additional crack displacements within an element, the so-called 'discrete crack' concept. This program is particularly suitable for analyzing structural details. The second program (DIANA) is based upon the concept of 'smeared-out' cracks, in which the effect of cracking is accounted for by reducing the stiffness of the 'cracked' elements.

![Typical offshore structure and loads acting upon the structure.](image)

Fig. 1.1. Typical offshore structure and loads acting upon the structure.

Offshore structures are designed in such a way as to transfer the cyclic loads due to wave and wind attacks to the subsoil by means of in-plane
stresses [25,26]. The walls of the base of such a structure will be subjected to in-plane shear, see Fig. 1.1. Thermal deformations due to the storage of hot oil and unequal settlements might cause additional cracking of the walls of the base. For this reason, the current study, which forms part of the second phase of the Concrete Mechanics project, focuses upon the response of cracked reinforced concrete to cyclic in-plane shear loads. Experiments with cyclic in-plane shear loading provide vital information on the response degradation of the cracked elements due to cycling.

A large number of tests [33,37,43,78] has been conducted with a rather large initial crack width and a relatively high shear load, the so-called 'high-intensity low-cycle' experiments. These tests especially reflected the case of a nuclear containment vessel, which is cracked due to an internal explosion and subsequently subjected to earthquake motions. For the case of offshore structures, those tests provide information on the response of the structure to severe loading conditions. However, offshore structures are generally subjected to millions of load cycles due to wind, wave and ice attacks. These load cycles have a relatively low amplitude with respect to the static strength, but might cause gradually increasing irreversible deformations, thus influencing the strength and stiffness of the structure in the case of subsequent higher loads. Therefore, apart from the 'high-intensity low-cycle' tests, experiments of the 'low-intensity high-cycle' type are of special interest for offshore structures.

1.2. Aim of the research program.

The aim of the research program is the determination of the relationship between the stresses and displacements occurring in the crack plane. The results of previous experimental investigations [45,76,81] showed that the transfer of stresses across a crack in concrete depends upon the mechanisms of the axial and lateral stiffness of the bars crossing the crack and upon the roughness of the crack faces. With regard to the roughness of the crack faces, the first phase of the Concrete Mechanics project yielded a physical model describing the response of cracked plain concrete to monotonic shear loading [81]. According to this model, this roughness is caused by the contact between the matrix material and
the aggregate particles protruding from the crack faces. Because of the nature of this mechanism, the particle distribution, the maximum particle size, the strength of the matrix material and the coefficient of friction between the particles and the matrix affect the shear stiffness of the crack.

The contribution of the bars to the transfer of shear stress across the crack is characterized by a strong interaction of the axial steel force and the lateral force (dowel force). Therefore, a physically sound description of the shear stiffness must incorporate the interaction with the normal restraint stiffness of the crack. Hence, the relationship between the stresses and displacements in a crack has to be expressed by:

\[
\begin{pmatrix}
\Delta \sigma \\
\Delta \tau
\end{pmatrix} =
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
\Delta \delta_n \\
\Delta \delta_t
\end{pmatrix}
\]

1.1

with \( \Delta \delta_t, \Delta \delta_n \) = increments of crack displacements. (see Fig. 1.2.)

![Stresses and displacements in the crack plane.](image)

However, in numerical programs generally only \( S_{11} \) is taken into account. Therefore, the existing static model [81] will first be simplified for a proper implementation of eq. (1.1) in numerical programs. Second, the effect of cyclic loading on the crack response will be experimentally investigated and incorporated in the existing model.

The response of cracked concrete to shear loading has been subject of numerous experimental studies. Therefore, the information obtained in these surveys concerning shear transferring mechanisms will be briefly reviewed in the following Chapter. Furthermore, additional tests have been carried out since it appeared that there was a lack of experimental information.
2. SURVEY OF THE LITERATURE

2.1. Introduction.

The major mechanisms affecting the transfer of stresses across cracks in reinforced concrete are:

a. Aggregate interlock of the crack faces; Due to the roughness of the crack faces, stresses can be transferred from concrete to concrete. This mechanism, denoted aggregate interlock by Fenwick [18], is based upon the fact that in low to medium strength gravel concrete the particles have a much higher strength than the matrix material. Therefore, a crack runs through the matrix and along the interface between particles and matrix, see Fig. 2.1a. As a consequence, the stiff particles are protruding from the crack plane, thus providing a contact with the matrix material of the opposing crack face when shear sliding occurs.

b. Dowel action of the reinforcing bars; Dowel action is based upon the response of the concrete supporting a steel bar, which is forced to a lateral displacement, see Fig. 2.1b.

c. Axial steel stress in the reinforcing bars; The reinforcing bars generally cross the crack plane at different angles. The component of the steel stress parallel to the crack plane contributes to the shear stress transfer across the crack, see Fig. 2.1c.

These mechanisms will be discussed separately. Finally the interaction of the mechanisms is reviewed. In this Chapter most attention is paid to the experimental results reported in the literature. Information on the available empirical and physical models will be reviewed in Chapter 4.
2.2 Aggregate interlock; monotonic loading.

Taylor (in 1959, [67]) and Moe (in 1962, [48]) paid some attention to the role of aggregate interlock in the load transfer in cracked concrete. It was, however, Fenwick [18], who first carried out a detailed experimental study into the aggregate interlock mechanism. The scope of this investigation was to determine the relationship between the shear resistance of cracked plain concrete and the crack displacements. The variables were the crack width, ranging from 0.06 mm to 0.38 mm, and the concrete strength varying from 19 MPa to 56 MPa. Fig. 2.2. presents the test specimen and testing rig used by Fenwick. The specimens were pre-cracked providing a relatively small shearing area of 7900 mm$^2$. The influence of the crack width was investigated in the first test series,
in which the crack width was kept constant during the stepwise shear load application. Unfortunately, the normal force, which was used to adjust the crack width, was not measured during the tests. For this test series with a concrete cylinder strength of 33 MPa, the mean test results are shown in Fig. 2.3. All the specimens failed due to secondary cracking. Each test was repeated five to six times to reduce the scatter of the readings.

A second test series with a constant crack width of 0.19 mm and varying concrete grade was used to determine the influence of the concrete strength upon the shear resistance. The average experimentally obtained curves are shown in Fig. 2.4. The following empirical expression was derived from the experimental results:

\[
\tau_a = \left( \frac{0.928}{\delta_n} - 0.658 \right) \left( \sqrt{f_{ccyl}} - 1.447 \right) (\delta_t - 0.0446) \text{ [MPa]} \quad (2.1)
\]

with \( \tau_a \), \( f_{ccyl} \) in [MPa], \( \delta_n \), \( \delta_t \) in [mm]

In addition to this experimental study, Houde and Mirza [31] performed 32 experiments in a testing rig, which was quite similar to the equipment used by Fenwick. Apart from the crack width and the concrete strength, the maximum particle size was a variable, ranging from 9.5 mm to 19 mm. The extremities of the concrete specimens were reinforced preventing any secondary cracks. Fig. 2.5 shows some average test results, which are comparable with Fenwick's results. It was found that the maximum particle size hardly influenced the shear resistance. The shear strength was found to be proportional to \( \sqrt{f_{ccyl}} \) for concrete strengths ranging from 16.5 MPa to 51 MPa. The experimental results of Fenwick and Paulay [18] and Houde and Mirza [31] might be influenced by the test set-up allowing flexural cracking and the relatively small shearing area, giving rise to a considerable scatter.

Therefore, Paulay and Loeber [50] performed tests with an improved type of specimen, see Fig. 2.6. Now, the shear plane of the pre-cracked push-off specimen was 21660 mm². The upper part of the specimen could slide along the shear plane of the lower part, which was fixed. The authors performed 44 tests exploring the nature of shear transfer. A part of this test series was carried out with a cyclic shear load and will be
Fig. 2.5. Test results of Houde and Mirza [31].
Fig. 2.6. Test specimen used by Paulay and Loeber [50].

discussed in Section 2.3. The variables were the type of aggregate (9.5 mm and 19 mm round maximum size and 19 mm crushed maximum size), the crack width (0.13 mm, 0.25 mm and 0.51 mm) and the way of load application. A concrete cylinder strength of 37 MPa was used. The experimentally obtained relationship between the monotonically increased shear stress and the shear displacements is shown in Fig. 2.7 for constant crack widths. It appeared that neither the aggregate size nor its shape strongly influenced the shear resistance.

Because of the improved type of specimen, the shear strength exceeded the maximum values obtained by Fenwick [18] and Houde [31]. During the constant crack width tests the magnitude of the normal restraining force was measured, see Fig. 2.8. The test results yielded an average coefficient of friction equal to 1.7. An important observation was the insensitivity of this value to the crack width and the aggregate type.

A second test series focussed upon the influence of an increasing crack width upon the shear transfer in cracked plain concrete. During these tests, the ratio of the shear load to the crack width was kept constant at a value of 1.38 MPa to 0.1 mm. The experimental results of these tests are compared with the results of the constant crack width tests, see Fig. 2.9. The dotted line in Fig. 2.9 represents the theoretical results according to the tests with constant crack width. This curve has the same shape as the mean experimental curve for the variable crack.
Fig. 2.7. Relation between shear stress and shear displacement [50].

Fig. 2.8. Shear stress as function of normal stress [50].

Fig. 2.9. Comparison of tests with constant and with variable crack width [50].

width tests. This indicated that the load history or crack opening path hardly influenced the crack response to shear loads.

Taylor [66] pointed out that a crack actually opens simultaneously with the shear sliding. Therefore, he performed tests with a constant ratio of the crack width to the shear displacement. A schematic presentation of the test equipment is shown in Fig. 2.10a.

The specimens were pre-cracked with a shearing area of 17780 mm².
A total of 32 tests was carried out, exploring the influence of type of aggregate, the aggregate size, the concrete strength and the ratio of crack width to shear displacement. The influence of the crack opening direction is shown in Fig. 2.10b for crack width to shear displacement ratios ranging from 0.27 to 2.15. The influence of the concrete strength is presented in Fig. 2.10c showing a nearly linear relation between the shear strength and the concrete strength, although a large scatter is observed. From a test series with weak aggregate particles, it appeared that the particle strength with respect to the matrix strength strongly influenced the shear strength. A relatively weak aggregate particle compared with the matrix material will allow the crack to run through the particles, thus yielding a smooth crack plane. During these tests, the crack opened simultaneously with the shear sliding. It must, however, be doubted whether the constant ratio of crack width to shear slip provided a suitable description of the actual crack behaviour. From [80] it is known, that for beams the ratio of crack width to shear slip increased with increasing crack width. Therefore, Walraven [81] performed tests on precracked push-off specimens with external restraint bars, see Fig. 2.11.

For these specimens the crack opening was restrained passively, so that the crack opened according to the internal equilibrium in the crack plane. The crack displacements were measured by means of plate spring gauges. The displacements were recorded at three locations on both sides of the specimen. The external normal force was measured using strain gauges stuck to the external bars. The tests were performed in a dis-

Fig. 2.10. Test arrangement and experimental results of Taylor [66].
Taylor [66] pointed out that a crack actually opens simultaneously with the shear sliding. Therefore, he performed tests with a constant ratio of the crack width to the shear displacement. A schematic presentation of the test equipment is shown in Fig. 2.10a.

The specimens were pre-cracked with a shearing area of 17780 mm².
placement-controlled manner, so that the post-peak behaviour could be recorded. The variables were the concrete strength and composition, the external restraint stiffness and the initial crack width.

The mix composition was varied using gap-graded mixes and mixes according to Fuller's ideal curve, normal and lightweight concrete and varying the maximum particle size (16 mm and 32 mm). The cube crushing strength was ranging from 13.4 MPa to 59 MPa. For the tests the initial crack width varied between 0.01 mm and 0.40 mm. The Figs. 2.12a-c present some typical test results for a 150 mm cube strength equal to 37.6 MPa and a maximum particle diameter of 16 mm.

Fig. 2.11. Test specimen as used by Walraven [81].

Fig. 2.12. Typical test results of Walraven for the plain concrete specimens [81].
The identifying number of the individual specimens consists of the mix number, the initial crack width and the restraint stress at a crack width equal to 0.6 mm respectively. Due to the increasing normal force, there still was a slight increase in shear strength for very large shear displacements. Although the crack was allowed to open simultaneously with the shear sliding, the curves for constant crack width were derived in the same manner as was done in Fig. 2.9. These curves are shown in Fig. 2.13. It appeared that the maximum particle size had only a slight influence upon the shear strength in the range tested. Therefore, simple bilinear expressions were derived empirically from test results, ignoring the influence of the maximum particle size:

\[
\tau_a = -\frac{f_{\text{ccm}}}{30} + \left(\frac{1.8}{\delta_n^{0.80}} + \left(-0.20\right)f_{\text{ccm}}\right)\delta_t \quad (\tau > 0)
\]

\[
a = -\frac{f_{\text{ccm}}}{20} + \left(\frac{1.35}{\delta_n^{0.63}} + \left(-0.15\right)f_{\text{ccm}}\right)\delta_t \quad (a > 0)
\]

with \(\tau_a, a, f_{\text{ccm}}\) in [MPa], \(\delta_n, \delta_t\) in [mm].

Fig. 2.13. Comparison of the experimental results and eqs. (2.2)-(2.3) [81].

These bilinear relations are compared with the experimental results in Fig. 2.13.

Daschner and Kupfer [12] performed 52 tests on normal- and lightweight concrete specimens varying the concrete cube strength \(f_{\text{ccm}} = 25\) and \(55\) MPa and the maximum particle diameter \(8\) mm and \(16\) mm. The test equipment was an improved version of the test arrangement used by Fenwick et al. [18] and Houde et al. [31]. In a first test series, the crack width
was kept constant on a preset value ranging from 0.05 mm to 0.40 mm. A second test series focussed on tests with a constant normal restraint stress during shear sliding. It emerged from the test results that for very high initial normal stresses the crack width changed sign, which indicated that the readings of the displacement transducers were influenced by the deformation adjacent to the crack. Indeed, Daschner and Nissen [13] suggested that the high normal restraint force had caused elastic and plastic deformations of the test specimens thus influencing the deformation between the measuring points. Because of the questions left open, these tests will not be discussed here. In addition to these tests, Nissen [49] improved the test arrangement, which was used by Daschner and performed 42 push-off tests, see Fig. 2.14a. He investigated the influence of the crack opening path upon the shear stiffness of the crack. Tests with constant crack width and tests with constant normal stress were performed. Some typical results are shown in Fig. 2.14b.

![Diagram](image_url)

**Fig. 2.14.** Test arrangement and test results of Nissen [49].

The cube concrete strength was varied between 27-31 MPa and between 54-57 MPa (cube 200x200x200 mm³). The water cement ratio was rather high (w/c 0.51-0.80). The maximum diameter of the gravel particles was varied between 8, 16 and 32 mm. Nissen found that the ratio \( \tau_a / f_{ccm} \) was hardly affected by the concrete strength and maximum particle diameter. It appeared that the stresses transferred across a crack for any given combination of the crack displacements are strongly influenced by the
crack opening path followed during the tests.

Millard and Johnson [45] carried out tests on pre-cracked specimens of the push-off type. The equipment used was very much alike the testing rig used by Walraven [81]. However, now the normal restraint bars could be tensioned before application of the shear load, see Fig. 2.15. The test variables were the initial crack width ranging from 0.063 mm to 0.75 mm, the cube crushing strength varying between 29 MPa and 52 MPa and normal restraint stiffness. The Figs. 2.16a-c present some typical test results for a cube crushing strength of 36 MPa and a normal restraint stiffness of 6.2 MPa.

The experimentally obtained results are in agreement with the test results found by Paulay and Loeber [50] and Walraven [81].

![Test arrangement used by Millard and Johnson [45].](image)

Vintzeleou [76] carried out push-off experiments exploring the influence of the surface roughness (smooth, sand blasted, rough) upon the shear strength. Furtheron, the concrete cylinder strength was varied between 16 and 40 MPa and the normal restraint stress was kept constant at values of 0.5, 1.0 and 2.0 MPa. Fig. 2.17 shows the test arrangement used by Vintzeleou. For a cylinder strength equal to 25 MPa and maximum particle diameter of 30 mm, some typical test results are presented in Fig. 2.18a-b. Note that the crack opened faster the higher the normal restraint stress. Vintzeleou stated that this was due to the large scatter. Apart from the crack displacements, the roughness of the shear plane was measured before and after the actual shear test. For the rough
interface, the roughness, defined as half the height of the protruding aspertities, was 1.75 mm before and 1.45 mm after testing due to the deterioration of the crack faces.

Divakar and Shah [14] also performed push-off tests with constant normal stress. Using a dead-weight, see Fig. 2.19a, a constant normal stress was applied to the crack plane. For displacement-controlled tests, it was found that for increasing normal stress, the increment of the shear displacement becomes larger relative to the crack width increment, see Fig. 2.19b. The concrete strength was 35 MPa. Note that the shear area
Fig. 2.18. Some typical test results of Vintzeleou [76].

was very small with respect to the maximum particle diameter of 12.7 mm (crushed angular aggregate). In fact, the results showed a remarkable consistency related to those small specimen dimensions.

Fig. 2.19. Test specimen and results of Divakar [14].
2.3. Aggregate interlock; cyclic loading.

Colley and Humphrey [9] performed cyclic loading experiments on joints in plain concrete in pavements. The specimen consisted of two slabs based upon a subsoil, see Fig. 2.20. The test variables were the joint width, the load level, the aggregate type and the quality of the subsoil. The applied load simulated the approach and departure of a wheel by subsequently unloading the approach slab and loading the departure slab. Fig. 2.20b presents the loading rate and joint response in time. The joint resistance to shear load was expressed by the effectiveness as defined by Teller and Sutherland [68].:

\[
\text{Effectiveness} = \frac{2\delta_{\text{departure slab}}}{\delta_{\text{approach}} + \delta_{\text{departure}}} \times 100 \, \% \quad (2.4)
\]

An effectiveness less than 100 percent indicated that shear slip occurred in the joint. Some test results are shown in Fig. 2.21a-c. It appeared that the aggregate type influenced the joint effectiveness. The tests were of the 'high-cycle low-intensity' type with a low shear stress (0.1–0.2 MPa) and a high number of cycles (up to one million cycles).

Other experimental work focussed on the 'low-cycle high-intensity' behaviour, exploring the response of cracked nuclear containment vessels subjected to shear. Such tests with a relatively high stress intensity were conducted by White and Holley [88]. A total of sixteen precracked specimens was loaded as to transmit shear by the crack roughness. The
shearing area was 180645 mm². The parameters investigated were the size and gradation of the aggregate, the normal restraint stiffness provided by external bars, the shear stress level, the number of cycles and the initial crack width. The tests were used to try out the test equipment and to make a first assessment of the crack response to cyclic shear loading.

On the basis of these results, further tests were performed by Laible, White and Gergely [37]. The type of specimen used was similar to the specimen as used by White et al. [88], see Fig. 2.22. Now, the shearing area was 194000 mm². The concrete cylinder strength for the major series was 20.7 MPa, the maximum particle size was equal to 38 mm. Apart from the variables in White’s test series [88], the specimen geometry and the strength and the age of the concrete were varied in the tests. The specimen was precracked by applying line loads halfway the specimen. Next, the crack width was set to the desired value of 0.25, 0.51 or 0.76 mm by adjustment of nuts on the restraint bars. The applied shear stress of 1.24 MPa was fully reversed. Fig. 2.23a–c presents a test result, which is representative of the generally observed behaviour. The number of cycles was 25. For the cycles No. 1 and No. 15 the load was applied stepwise, during the other cycles the full load was applied in one step.

The first loading cycle showed a nearly linear relationship between the crack displacements and the shear stress, whereas this relation became
highly non-linear for the later cycles. During unloading the recovery of the shear displacement was about 20 percent of the maximum slip, which was probably due to local irreversible deformation of the contact areas. Fig. 2.23a shows that the stiffness increases with increasing shear displacement, which supports the assumption of deformed contact areas. Due to the crushing of the matrix material in the previous cycles, the initial stiffness is very low, because a 'contactless' free slip can occur before any contact between the opposing crack faces is possible.

Paulay and Loeber [50] carried out both static (see Section 2.2) and repeated shear loading tests. Fig. 2.24a-c shows the experimental results for a maximum shear load of 6 MPa. The crack width was kept constant during the tests. A surprising result was the low stiffness during unloading compared with the stiffness during loading. This result deviated from the low recovery in shear displacement during unloading found in the tests of Laible [37]. The major difference between both test series was the constant crack width in Paulay's tests, where the crack width
increased with increasing shear sliding in Laible's tests. The high normal stress required to maintain the constant crack width, probably influenced the unloading of the specimen in Paulay's test series.

In addition to the static test series, Vintzeleou [75] performed cyclic tests with a fully reversed shear displacement. Due to the large applied displacements only a few cycles were used. For various normal stresses, the test results are presented in Fig. 2.25. It was found that for a high normal stress no degradation of the response occurred. The following empirical expression was derived describing the decrease in shear strength:

\[
\frac{\tau_n}{\tau_{tu}} = 1 - 0.12 \left(1 - \frac{f_{cy1}}{f_{tu}}\right)^{\frac{3}{2}} \left(\delta - \delta_{tu}\right)
\]  

(2.5)

with \(n = \) number of cycles and \(\delta_{tu} = 2\) mm.

Chung [8] carried out impact tests on push-off specimens with a shearing plane of 18750 mm\(^2\), which consisted of a joint between precast and cast in situ concrete. Apart from a test series with a single impact load, a test series was performed, in which the specimens were preloaded with a low intensity shear load during two million cycles. For a load intensity of 55 percent of the static strength no degradation of the response was recorded. For an intensity of 66 percent a decrease of 14-20 percent was observed. It was found that the impact shear strength for a loading rate of 12000 MPa/s was 80 percent higher than the static a shear strength.
Fig. 2.25. Test results of Vintzeleou [75].
2.4. Dowel action; monotonic loading.

Reinforcing bars crossing a crack will counteract the crack displacements. For bars perpendicularly crossing the crack plane, this response can be subdivided in an axial and a lateral stiffness of the bar. The axial stiffness is provided by the bond between steel and concrete. The lateral stiffness is due to the reaction stresses of the surrounding concrete and is called dowel stiffness.

![Diagram of dowel action](image)

**Fig. 2.26. Failure modes for dowels.**

Several failure modes can occur in dowel action, including splitting failure of the concrete cover, see Fig. 2.26a. This type of failure generally occurs in the case of bottom bars in a beam, when the concrete cover is too small to make equilibrium with the dowel force. This failure mode will not be discussed here. If adequate confinement is provided to prevent splitting of the concrete, concrete crushing around the bar may occur, see Fig. 2.26b. Now, the concrete reaction force is relatively high with respect to the concrete strength due to the multi-axial stress condition in the surrounding concrete. Fig. 2.26c presents the difference in response for both failure modes.

As for the aggregate interlock mechanism, the first dowel tests focused on joints in concrete pavements based upon a silt loam subgrade [65,68]. Teller and Sutherland [68] showed that the effectiveness in load transfer of a dowel depends on the slab thickness, the joint width, the dowel spacing and the load application with respect to the location of the dowels. From [65] it was found that the slab deflection was directly proportional to the magnitude of the load on the slab, see Figs. 2.27a-b.

Paulay, Park and Philips [51] performed dowel action tests on a fixed corbel, which was connected to a concrete block by means of reinforcing
bars perpendicularly crossing the smooth contact area, see Fig. 2.28a. Now, there was no subsoil influencing the response of the dowels to the shear load. The test results are presented in Fig. 2.28b. Rasmussen [57] performed tests on dowels perpendicularly protruding from a large concrete block, see Fig. 2.29a. He found that plastic hinges developed in the bar accompanied by a considerable crushing of the concrete under the bar. The experimentally obtained ultimate dowel force is presented in Fig. 2.29b and can be expressed by the following relation:

\[ F_{du} = C \phi^2 \sqrt{f_{ccyl} f_{sy}} \text{ [N]} \]  

(2.6)

with \( C = 1.3 \) for the case of no load eccentricity.

\[ \phi \text{ in [mm], } f_{ccyl}, f_{sy} \text{ in [MPa]} \]
Fig. 2.29. Test arrangement and results of Rasmussen [57].

Rasmussen's test results were in agreement with experimental results obtained by Bennett and Banerjee [4]. The specimen used is shown in Fig. 2.30a. Tests were performed with bottom bars or top bars only and with the combination of top and bottom bars. For the tests with bottom bars, the results showed that the dowel strength is directly proportional to the cross-sectional area of the bar, see Fig. 2.30b.

In practice, bars cross the crack plane at various angles. Dulacska [15] explored the influence of the angle of inclination upon the dowel strength. The specimen used was of the push-off type, in which the aggregate interlock mechanism was prevented by means of two 0.2 mm thick brass plates, see Fig. 2.31a. The experimental results are shown in Fig.
2.31b. The ultimate dowel force can be expressed by the following empirical relation:

\[
F_{du} = 0.2 f_{sy} \sin(\theta) \left( \sqrt{1 + \frac{f_{ccm} \sin(\theta)}{0.03 f_{sy}} - 1} \right) \quad \text{[N]} \quad (2.7)
\]

with \( \theta = \) angle of inclination (normal to the crack plane \( \theta = 0^\circ \)).

\[
f_{ccm}, f_{sy} \text{ in [MPa]}
\]

![dowel force](image)

\[\text{ultimate dowel force } F_{du} \text{[kN]}\]

\[\text{concrete strength } f_{ccm} \text{[MPa]}\]

\(\theta\) \& \(f_{ccm}\) \text{[MPa]}

- 10° 10 295
- 10° 14 257
- 30° 10 295
- 30° 10 295
- 30° 10 295

**Fig. 2.31. Testing rig and experimental results of Dulacska [15].**

For \( \theta = 0^\circ \), Rasmussen's formula is obtained with \( C = 1.25 \). The experimentally found shear displacement as a function of the applied dowel load, can be expressed by:

\[
\delta = \frac{11.35 \times 10^{-4} F_d}{\phi} \sqrt{\frac{F_{du}}{F_{du}^2} \tan(\frac{\pi}{2})} \quad \text{[mm]} \quad (2.8)
\]

with \( \phi, \delta \text{ in [mm]}, F_d, F_{du} \text{ in [N]} \)

Mills [47] performed three dowel tests with an angle of inclination of 45°. For a bar with a diameter of 38 mm and yield strength of 210 MPa and a concrete cylinder strength of 36 MPa, an average dowel strength of 76 kN was obtained.

Utescher and Herrmann [73] performed a large number of dowel tests, exploring the influence of the bar diameter and load eccentricity upon the dowel strength. Fig. 2.32a-b presents the experimental results. The load
eccentricity was varied by applying the load at distances of 5, 10, 20 and 50 mm from the concrete surface. It was found that the load eccentricity strongly influenced the ultimate dowel force. The testing rig used was very similar to Rasmussen's test arrangement. Therefore, it must be doubted whether Rasmussen's tests were carried out with zero-eccentricity, as was reported in [57]. Fig. 2.32b shows that the dowel strength was proportional to the steel area. Utescher et al. observed a considerable crushing of the concrete close to the crack plane, see Fig. 2.32c.

![Graph showing ultimate dowel force versus eccentricity](image)

![Graph showing ultimate dowel force versus steel area](image)

![Graph showing spalling-off of the concrete](image)

Fig. 2.32. Experimental results of Utescher and Herrmann [73].

In practice, dowels cross cracks. Therefore, the load eccentricity is caused by the crack width. Due to the bond between the steel bar and the concrete, the load eccentricity is accompanied by an axial steel stress. Eleiott [16] performed dowel tests with pretensioned bars. A cyclic dowel force was applied. It appeared, that already in the first static-cycle the dowel stiffness was strongly decreased by the axial steel force, see Fig. 2.33. Unfortunately, no detailed information on the crack width was reported in [16].

![Diagram showing Dowel action specimen](image)

![Graph showing dowel load versus dowel displacement](image)

Fig. 2.33. Influence of the axial stress upon the dowel force [16].
Vintzeleou [75] carried out dowel action tests with a reinforced version of the specimen shown in Fig. 2.17. The bars perpendicularly crossed a joint of 4 mm, thus preventing the aggregate interlock mechanism. Fig. 2.34 presents the experimental results for a steel yield strength of 420 MPa, showing that the dowel strength is approximately proportional to the square root of the concrete strength.

![Fig. 2.34. Experimental results of Vintzeleou [75].](image)

Millard [45] performed dowel action tests, exploring the influence of bar diameter, concrete strength and axial steel stress upon the dowel strength.

The testing rig shown in Fig. 2.15 was used. The experimental results as shown in Fig. 2.35 were in agreement with those of Rasmussen [57], Bennett [4], Utescher [73] and Vintzeleou [75].

![Fig. 2.35. Test results of Millard et al. [45].](image)
2.5. Dowel action; cyclic loading.

Numerous cyclic dowel action tests are performed at Cornell University. In [33] experimental results of Eleiott, Stanton and Jimenez [32] are briefly reviewed. Eleiott [16] carried out tests exploring the influence of axial steel stresses upon the dowel stiffness. Fig. 2.36a presents a test result for a bar diameter of 12.6 mm and a concrete strength of 21 MPa. As a result of the steel stress of 175 MPa, the crack width increased, thus reducing the dowel stiffness by up to fifty percent with respect to a test with an unstressed bar (see Fig. 2.33). In cycle No.16, the steel stress was increased to 350 MPa, which again strongly increased the crack width and reduced the dowel stiffness. Stanton [64] and Jimenez [32] performed tests on large concrete blocks interconnected by several bars perpendicularly crossing the crack plane. Fig. 2.36b presents the experimental result for a specimen with four 29 mm diameter bars. It was found that the energy absorption capacity decreased with cycling (up to 50 cycles). During these tests, the load was fully reversed, showing a similar response in both loading directions.

![Fig. 2.36. Experimental results of cyclic dowel action tests [33,78].](image)

Vintzeleou and Tassios [77,78] performed tests focusing on structures subjected to earthquakes. As earthquakes cause cyclically imposed displacements, the tests were performed in a displacement-controlled manner. The test arrangement was similar to the one described for the static tests, see Section 2.2. Fig. 2.36c presents a test result for a bar with a cover of 260 mm in the positive direction and a cover of 40 mm in
the negative direction. Obviously, the response of the bar to lateral displacements is asymmetrical due to the splitting failure in the negative direction. The decrease in dowel force at maximum shear displacement can be expressed by the following expression:

$$P_d, \frac{n=n}{P_d, n=1} = 1 - \frac{1}{a} \sqrt{n-1}$$  \hspace{1cm} (2.9)

with \( n = \) number of cycles (\( n < 7 \) cycles)

\( a = \)

- 7 for fully reversed loads
- 14 for repeated loads


Reinforcing bars generally cross a crack at different angles. Shear stress is transferred across the crack by means of the component of the steel stress parallel to the crack plane. This contribution to the shear stress transfer can easily be determined when the axial steel stress and the angle of inclination are known. The magnitude of the axial steel stress depends upon the bond characteristics.

For bars perpendicularly crossing a crack, the relationship between the magnitude of the axial steel stress and the crack width is known from pull-out experiments. However, the bond characteristics obtained in these tests cannot be applied to the case of bars at different angles to the crack plane or to bars subjected to both axial and lateral displacements. Due to the lateral displacement, the bond between the steel bar and the concrete is broken. Therefore, it is expected that for these cases the bond capacity will decrease with decreasing angle of inclination. This was experimentally confirmed by Klein et al. [35], who performed displacement-controlled tests with bars at various angles to the crack plane, see Fig. 2.37a. Each bar was prepared with strain gauges stuck to the bar over a length of 360 mm, thus recording the variation of steel strains over the bond length. Test variables were the bar diameter (10-16 mm) and the angle of inclination (45°, 60° and 90°). Some typical results are presented in Fig. 2.37b, showing that no systematic variation in bond behaviour for several angles of inclination was obtained in these tests. Due to the lack of proper bond characteristics, the magnitude of the axial steel stress must be derived from the equi-
2.7. Shear strength of cracked reinforced concrete; monotonic loading.

For design purposes, a simple shear-friction model was introduced by Mast [39] and Birkeland [5]. According to this model the shear strength of cracked reinforced concrete was provided by the friction in the shear plane. The shear strength can then be calculated by multiplying the normal compressive stress due to the reinforcement by the tangent of the angle of friction $\psi$. The ultimate shear stress is reached at the onset of yielding of the reinforcing bars, thus:

$$\tau_u = \rho f_{sy} \tan(\psi) \quad \text{[MPa]}$$

(2.10)

The angle of friction was empirically derived from tests yielding $\psi$ equal to 55° ($\tan(\psi) = 1.4$) for bars normal to the shear plane. From tests on corbels [39], it was found that an applied normal tensile stress could be subtracted from the contribution of the steel $\rho f_{sy}$, yielding: (tensile stress has a negative sign)
with $\tau_u$, $\sigma_n$, $f_{sy}$ in [MPa]

$$\tau_u = (\rho f_{sy} + \sigma_n) \tan(\psi) \quad [\text{MPa}] \quad (2.10a)$$

This was confirmed by tests performed by Hofbeck, Ibrahim and Mattock [29], who performed an experimental study, exploring the applicability of the shear-friction analogy. They carried out tests on uncracked and precracked reinforced push-off elements. Fig. 2.38a shows some test results indicating that the failure mechanism for the uncracked specimens was basically different from the failure mode of the cracked specimens. Fig. 2.38b shows that the uncracked specimens failed forming short diagonal cracks across the shear plane. For heavily reinforced precracked specimens, the shear plane locked up and a similar type of failure was found. For moderately reinforced precracked specimens, the shear strength was determined by the response of the crack plane and could be expressed by the shear-friction analogy. However, it was found that the angle of friction was $39^\circ$ ($\tan(\psi) = 0.8$). Also a cohesion was added, representing the dowel action. Therefore, the average ultimate shear stress can be expressed by:

$$\tau_u = 2.8 + 0.8(\rho f_{sy} + \sigma_n) \quad [\text{MPa}] \quad (2.11)$$
with \( \tau_u, \sigma_n, f_{sy} \) in [MPa]

![Graphs showing experimental results of tests with inclined bars.](image)

**Fig. 2.39.** Experimental results of tests with inclined bars [40].

In further tests, Mattock [40] investigated the shear capacity of cracks crossed by parallel and orthogonal reinforcement with an angle of inclination to the shear plane. Fig. 2.39a-b presents the experimental results, showing that there was little influence of the bar inclination for the specimens with orthogonal bars. However, a strong influence of the bar inclination on the shear capacity was obtained for the parallel bars. A maximum was found for \( \theta \) equal to approximately 120°. Apart from tests exploring the restrictions of the shear-friction analogy, Mattock [41] performed tests to obtain information on the fundamental crack response to shear loads. The aggregate type was varied thus yielding a sand-gravel concrete, a sanded lightweight and an all-lightweight concrete. The number of bars perpendicularly crossing the crack plane was varied yielding reinforcement ratios of 0.4% to 2.3%. For a concrete cylinder strength of 28 MPa and an average initial crack width of 0.25 mm, test results are presented in Fig. 2.40a-c. Mattock found that the sand-gravel and the sanded lightweight concrete exhibited the same crack opening path, whereas the all-lightweight concrete exhibited a steeper crack opening path. Therefore, the small particles must have a large influence upon the crack opening direction.

In another test series, Mattock [42] performed experiments with a ten-
sile force perpendicular to the crack plane. No systematic differences in crack opening paths were found for the tensile forces investigated, see Fig. 2.41.

Fig. 2.40. Experimental results of tests Fig. 2.41. Results of tests with a normal force [42].

Walraven [81,85] conducted tests on push-off elements similar to the specimens used by Mattock. In addition to the tests on plain concrete (see Section 2.2), Walraven carried out displacement-controlled tests on reinforced specimens. Fig. 2.42a presents the specimen used. The test variables were the bar diameter ranging from 6 to 16 mm, the reinforcement ratio varying between 0.56% and 3.36% and the concrete cube strength ranging from 19.9 MPa to 56.1 MPa. The maximum particle diameter was 16 mm, except for mix 5, in which a maximum particle of 32 mm was used. The initial crack width remained small (< 0.1 mm). The steel yield strength was 460 MPa. The measured crack opening paths appeared to be insensitive for variations of the bar diameter and reinforcement ratio, see Fig. 2.42b. However, the number of bars strongly influenced the ultimate shear strength, see Fig. 2.42c.

In addition to the tests on plain concrete and on dowels, Millard [46] performed push-off tests on cracked reinforced concrete. The test variables were the initial crack width and the bar diameter ranging from 8 to 16 mm. The steel yield strength was 485 MPa, the concrete cube strength was in the range of 25.5 MPa to 45.4 MPa for a maximum particle diameter of 10 mm. The variation of the initial crack width yielded a
variation in the initial axial steel stress. The test results are shown in Fig. 2.43a-b. The experimentally obtained crack opening paths showed a constant angle to the crack plane. However, this crack opening direction deviated from the average crack opening path found by Walraven, see dotted line in Fig. 2.43b.

![Fig. 2.42. Experimental results of Walraven [85].](image1)

![Fig. 2.43. Experimental results of Millard [46].](image2)
2.8. Shear strength of cracked reinforced concrete; cyclic loading.

Apart from tests on plain concrete and on dowel action, Eleiott [16] and Jimenez [33] performed cyclic push-off tests on cracked reinforced concrete. Fig. 2.44a-c presents some test results of Jimenez, showing that the crack response to cyclic loading depends on the initial crack width and the applied shear stress level. It must be noted that an increase in the initial crack width is accompanied by an increasing axial steel stress. Fig. 2.44d presents an experimental result, for which the shear stress was increased in the 15th cycle. It can be seen that the response in the 15th cycle tended to the static envelope, which would have been obtained if the shear load reached this level in the first cycle.

Mattock [43] carried out cyclic tests on the specimen shown in Fig. 2.45a with a reinforcement ratio ranging from 0.60% to 1.32%. The concrete strength was approximately 41.6 MPa for the normal weight concrete and 28.3 MPa for the lightweight concrete with a maximum particle diameter of 16 mm and 13 mm respectively. Some schematic presentations of the relations between the shear stress and the shear displacement are shown in Fig. 2.45b. During unloading the response was characterized by a retention of almost all the shear displacement under maximum shear stress until the shear stress was reduced to approximately 50 percent of its maximum value. In all precracked specimens a decrease in crack width at zero shear stress was observed. This decrease accounted 0.08-0.13 mm with respect to an initial crack width of 0.25 mm. This crack width
remained constant until shortly before failure. Fig. 2.46a-b shows some experimental results for both normal weight and lightweight concrete.

Fig. 2.46. Shear stress versus shear displacement for tests of Mattock [43].

The result for a cyclic loading test is compared with the static test performed with a similar specimen. Fig. 2.46a shows that for the case of normal gravel concrete, the maximum slip during cycling with a low shear stress level was approximately equal to the shear displacement occurring in the monotonic test at the same stress. However, at a shear stress level of 80 percent of the static shear strength, the slip rapidly increased. The same held true for the crack width.

For the cyclic test on lightweight concrete, the shear slip was larger and the crack width was smaller than those occurring in the monotonic test at the same shear stress.
2.9. Conclusions.

With regard to the mechanism of aggregate interlock, it was found that a parallel displacement of the crack faces does not only generate shear stresses $\sigma_{nt}(\tau)$ but also normal stresses $\sigma_{nn}$. The dependency between stresses and displacements can be expressed by the following relation:

$$
\begin{pmatrix}
\sigma_{nn} \\
\sigma_{nt}
\end{pmatrix} =
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
\delta_n \\
\delta_t
\end{pmatrix}
$$

(2.12)

The static shear stress of plain concrete was found to be proportional to the square root of the concrete compressive strength. Furthermore, the type of aggregate influences both the shear strength and the crack opening path (for a constant normal restraint stiffness). The maximum particle diameter, ranging from 8 to 32 mm hardly influences the static shear strength, but affects the crack opening direction.

For cracks in plain concrete subjected to cyclic loading, it was found that there is a basic difference in the behaviour in the first and in the subsequent cycles. The response in the subsequent cycles becomes highly non-linear with an initially low stiffness, which is abruptly changed in a high stiffness approaching the maximum shear slip obtained in the previous cycle. The shear stress level was found to be an important parameter.

Most of the experimental work focussed upon the response to a high intensity loading. No tests with a very large number of cycles ($10^3$-$10^6$) were carried out yet.

The static shear stress-shear displacement relationship for dowel action is characterized by a linear line up to approximately 40 percent of the dowel strength. Next, the relation between dowel displacement and dowel force becomes nonlinear until the maximum dowel capacity is reached. Now, the concrete strength largely influences the ultimate dowel strength. The type of failure was found to be characterized by the formation of plastic hinges in the bar and local crushing of the concrete under the bar.

The dowel strength is strongly influenced by the initial crack width and the initial axial steel stress, although these parameters might interfere.
As for the aggregate interlock mechanism, the dowel response to cyclic loading becomes highly non-linear for subsequent cycles with respect to the response in the first cycle, which is approximately linear. All dowel action tests found in literature were of the 'high-intensity low-cycle' type.

For the contribution of the axial steel stress to the static shear stress transfer for inclined bars, it was found that the ordinary bond characteristics obtained in pull-out experiments can not be applied to the case of inclined bars or bars subjected to the combined action of axial and lateral forces. For bars subjected to a lateral displacement, the bond between the bar and the concrete is broken at one side of the bar, thus decreasing the over-all bond capacity.

The angle of bar inclination and the number of bars were found to be important parameters for the static shear strength of cracks in reinforced concrete.

The type and size of the aggregate have some influence upon the crack opening direction, whereas no influence of the number of bars was found. The response of cracked reinforced concrete to cyclic shear stresses is influenced by the initial crack width and the shear stress level. As for the tests on plain concrete and on dowel action, the available experimental knowledge is restricted to the range of high-intensity loads.

Therefore, it can be concluded that there is still a lack of experimental information on the response of cracked concrete subjected to a very large number of load cycles with a relatively low shear stress with respect to the static shear strength.
3. EXPERIMENTAL STUDY

3.1. Introduction.

The survey of the literature yielded the conclusion that there still is a lack of experimental information with respect to the response of cracks in concrete to 'low-intensity high-cycle' loading. Therefore, an experimental study focussing on 'low-intensity high-cycle' fatigue of offshore structures was started in the Stevin laboratory. For offshore structures, the Arctic and deep sea environments provide intense dynamic forces with a loading frequency of approximately 1 cycle per second [25]. In a substructure placed on the seabed, there generally is a static load apparent, to which the cyclic load is superimposed. In consequence, the shear walls in the base are predominately subjected to repeated loads. Therefore, the first test series was performed with a repeated shear load on precracked push-off specimens. In the second test series, the dowel action was eliminated by using external restraint bars, enabling a quantification of the contribution of the aggregate interlock mechanism to the shear transfer. The magnitude of the contribution of the dowel action is then known. These experiments will be briefly described in this Chapter. A full survey of all the experimental results has been given in [56].

3.2. Reinforced specimens; repeated loading.

3.2.1. Test arrangement

The geometry of the test specimen in this experimental program was similar to the push-off specimen used by Walraven in his static tests [81]. The shear area was 36000 (120 x 300 mm²), see Fig. 3.1. Previously, shear tests were performed on specimens with a similar shear area, so the test results can be compared without scale-factors. The specimens were cast in a steel mould placed horizontally, so that at the time of casting the shear plane was in a vertical position. The cantilevers of the specimen were prestressed preventing preliminary failure of these cantilevers due to secondary cracking. Prior to the actual shear test,
the crack was made in a three-point bending test by pushing a steel
knife-edge into a V-shaped groove along the shear plane. Both sides of
the specimen were subsequently cracked in this manner, resulting in an
initial crack width ranging from 0.01 mm to 0.08 mm. Next, the specimen
was centred in the test frame, see Fig. 3.3. The specimen was supported
by a roller bearing thus preventing restraining forces being transmitted
during crack opening. The shear load was provided by means of a 1000 kN
hydraulic jack placed on the foot of the frame. A knife hinge induced
the load at the top of the specimen.

Fig. 3.1. Test specimen.

Fig. 3.2. Measuring system.

Fig. 3.3. Testing rig.

The crack displacements were recorded by means of linear voltage dis­placement transducers, attached to steel footings glued on the concrete
on both sides of the specimen, see Fig. 3.2. The transducers, Hewlett-
Packard type 7-DCDT-100, had a 0.01 mm measuring accuracy at 5 mm range.
The shear load was measured by means of a load cell with a measuring ac­
Accuracy of 0.25 kN. All the signals were led to a micro-computer for storage and monitor display. In order to reproduce the sinusoidal signals each measured cycle was scanned nine times. A trigger level was adjusted to the maximum load by means of a special circuit. By sampling this trigger level it was possible to start the first scan after each call on the peak value of the applied load.

During the actual test, the specimen was subjected to a shear load, which alternated between a minimum shear stress level \( \tau_0 \) and a maximum shear stress level \( \tau_m \). The crack displacements were not recorded for the first few cycles due to the adjustment of the shear stress levels, the load frequency and the trigger level.

### 3.2.2. Test variables.

The test variables were the reinforcement ratio and steel yield strength, the concrete compressive strength, the initial crack width, the number of cycles and the applied shear stress level.

The normal restraint stiffness depends on the reinforcement ratio \( \rho \). For the tests, four and six 8 mm diameter stirrups were used, yielding a reinforcement ratio of 1.12% and 1.68% respectively. The steel yield strength of the ribbed bars \( f_{sy} \) was 460 MPa (denoted low-strength) and 550 MPa (denoted high-strength) with a rib coefficient \( f_R \) (approximately the rib height/rib distance) equal to 0.050 and 0.059 respectively. The use of two steel grades provided an opportunity to investigate whether the reinforcement yielded or not.

The concrete grade \( f_{ccm} \) had an average 28-day cube crushing strength of 50 MPa (Mix A) and 70 MPa (Mix B) reflecting the high-strength concrete used in offshore structures. It can be expected that with increasing concrete strength an increasing number of particles is fractured during cracking of the concrete, thus reducing the aggregate interlock mechanism. Both mixes had a maximum particle diameter of 16 mm and almost complied with the Fuller grading curve. Detailed information is given in Appendix I.

The initial crack width \( \delta_{no} \) varied from 0.01 mm to 0.08 mm to ensure a small crack width (< 0.25 mm) in order to simulate offshore service conditions. For the tests, the initial crack width was not an adjusted, but a measured parameter.
The number of cycles ranged from 118 to 931731 cycles for the test series. The large number of cycles interfered with a good planning of the tests, in general the experiments did not start at an age of 28 days. Therefore, the concrete strength at the start of the test was obtained from Fig. 3.4.

![Graph](image)

Fig. 3.4. The concrete strength versus the concrete age.

The applied shear stress level is referred to the static shear strength obtained in the static tests of Walraven [81]. According to the shear friction analogy, the shear strength can be expressed as a function of the yield strength and the reinforcement ratio [84]:

\[ \tau = a pf_{sy}^b \]  

with \( a = 0.822 f_{ccm}^{0.406} \) and \( b = 0.159 f_{ccm}^{0.303} \) in [MPa]

\( \tau_u, f_{ccm}, f_{sy} \) in [MPa]

During the tests, the shear stress level \( \tau_m \) was in range of 46 to 90 percent of the static strength and a minimum shear stress \( \tau_0 \) equal to 0.3 MPa. Despite of the shear stress of up to 90 percent of the static strength, the tests were still of the 'low-intensity' type because of the very small crack width.
3.2.3. Experimental results.

A total of 42 repeated push-off tests was carried out. The specimens have been assigned an identifying code consisting of 5 characteristics. The first character denotes the concrete grade (Mix A or Mix B), the second the number of 8 mm diameter stirrups and the steel yield strength (Low or High). The next character represents the shear stress level $\tau_m/\tau_u$, followed by the actually applied shear stress. Finally, the initial crack width forms part of the identifying code.

The Tables 3.1 and 3.2 list a review of the experimental results.

Table 3.1. Test results of Mix A.

<table>
<thead>
<tr>
<th>Code</th>
<th>$f_{ccm}$ [N/mm$^2$]</th>
<th>$\tau_u$ [N/mm$^2$]</th>
<th>$\tau_m$ [N/mm$^2$]</th>
<th>$\tau_m/\tau_u$</th>
<th>No. of cycles</th>
<th>failure during cycling</th>
</tr>
</thead>
<tbody>
<tr>
<td>A/4L/.61/6.1/03</td>
<td>54.47</td>
<td>10.00</td>
<td>6.1</td>
<td>.610</td>
<td>455000</td>
<td>no</td>
</tr>
<tr>
<td>A/4L/.63/6.0/06</td>
<td>50.20</td>
<td>9.47</td>
<td>6.0</td>
<td>.634</td>
<td>263337</td>
<td>no</td>
</tr>
<tr>
<td>A/4H/.64/7.0/06</td>
<td>54.30</td>
<td>10.97</td>
<td>7.0</td>
<td>.638</td>
<td>368000</td>
<td>no</td>
</tr>
<tr>
<td>A/4L/.65/6.0/01</td>
<td>46.83</td>
<td>9.29</td>
<td>6.0</td>
<td>.646</td>
<td>34000</td>
<td>no</td>
</tr>
<tr>
<td>A/4H/.66/6.9/02</td>
<td>50.99</td>
<td>10.50</td>
<td>6.9</td>
<td>.657</td>
<td>550000</td>
<td>no</td>
</tr>
<tr>
<td>A/4L/.70/7.0/01</td>
<td>54.49</td>
<td>10.00</td>
<td>7.0</td>
<td>.700</td>
<td>2410</td>
<td>no</td>
</tr>
<tr>
<td>A/4L/.73/7.2/05</td>
<td>53.70</td>
<td>9.90</td>
<td>7.2</td>
<td>.727</td>
<td>14000</td>
<td>yes</td>
</tr>
<tr>
<td>A/4L/.74/7.0/01</td>
<td>49.85</td>
<td>9.42</td>
<td>7.0</td>
<td>.743</td>
<td>435</td>
<td>no</td>
</tr>
<tr>
<td>A/4L/.76/7.0/02</td>
<td>48.20</td>
<td>9.20</td>
<td>7.0</td>
<td>.761</td>
<td>5925</td>
<td>yes</td>
</tr>
<tr>
<td>A/4H/.76/7.7/03</td>
<td>48.50</td>
<td>10.15</td>
<td>7.7</td>
<td>.759</td>
<td>4762</td>
<td>yes</td>
</tr>
<tr>
<td>A/4L/.77/7.2/04</td>
<td>49.48</td>
<td>9.38</td>
<td>7.2</td>
<td>.768</td>
<td>1785</td>
<td>yes</td>
</tr>
<tr>
<td>A/4H/.78/8.0/04</td>
<td>49.30</td>
<td>10.26</td>
<td>8.0</td>
<td>.780</td>
<td>5198</td>
<td>yes</td>
</tr>
<tr>
<td>A/4L/.79/8.6/02</td>
<td>53.47</td>
<td>10.86</td>
<td>8.6</td>
<td>.792</td>
<td>895</td>
<td>yes</td>
</tr>
<tr>
<td>A/4L/.80/7.3/03</td>
<td>47.20</td>
<td>9.09</td>
<td>7.3</td>
<td>.803</td>
<td>1192</td>
<td>yes</td>
</tr>
<tr>
<td>A/4L/.80/7.5/05</td>
<td>49.60</td>
<td>9.39</td>
<td>7.5</td>
<td>.799</td>
<td>299450</td>
<td>yes</td>
</tr>
<tr>
<td>A/4L/.82/7.4/05</td>
<td>46.43</td>
<td>8.99</td>
<td>7.4</td>
<td>.823</td>
<td>996</td>
<td>yes</td>
</tr>
<tr>
<td>A/4L/.90/9.0/05</td>
<td>54.50</td>
<td>10.00</td>
<td>9.0</td>
<td>.900</td>
<td>118</td>
<td>yes</td>
</tr>
<tr>
<td>A/6L/.51/6.0/04</td>
<td>51.37</td>
<td>11.89</td>
<td>6.0</td>
<td>.505</td>
<td>508000</td>
<td>no</td>
</tr>
<tr>
<td>A/6L/.56/6.7/05</td>
<td>51.41</td>
<td>11.90</td>
<td>6.7</td>
<td>.563</td>
<td>386000</td>
<td>yes</td>
</tr>
<tr>
<td>A/6L/.58/6.8/01</td>
<td>50.40</td>
<td>11.73</td>
<td>6.8</td>
<td>.578</td>
<td>194000</td>
<td>no</td>
</tr>
<tr>
<td>A/6L/.61/7.2/04</td>
<td>51.30</td>
<td>11.88</td>
<td>7.2</td>
<td>.606</td>
<td>160300</td>
<td>no</td>
</tr>
<tr>
<td>A/6L/.62/8.0/04</td>
<td>57.30</td>
<td>12.89</td>
<td>8.0</td>
<td>.621</td>
<td>410781</td>
<td>no</td>
</tr>
<tr>
<td>A/6H/.66/7.9/03</td>
<td>45.10</td>
<td>11.84</td>
<td>7.9</td>
<td>.663</td>
<td>1750</td>
<td>yes</td>
</tr>
<tr>
<td>A/6L/.66/8.6/02</td>
<td>58.60</td>
<td>13.10</td>
<td>8.6</td>
<td>.656</td>
<td>40722</td>
<td>yes</td>
</tr>
<tr>
<td>A/6L/.67/8.2/08</td>
<td>53.20</td>
<td>12.20</td>
<td>8.2</td>
<td>.672</td>
<td>21000</td>
<td>yes</td>
</tr>
<tr>
<td>A/6L/.68/8.0/03</td>
<td>51.04</td>
<td>11.83</td>
<td>8.0</td>
<td>.676</td>
<td>1000</td>
<td>yes</td>
</tr>
</tbody>
</table>
Table 3.2. Test results of Mix B.

<table>
<thead>
<tr>
<th>Code</th>
<th>( f_{ccm} ) ([N/mm^2])</th>
<th>( \tau_u ) ([N/mm^2])</th>
<th>( \tau_m ) ([N/mm^2])</th>
<th>( \tau_m/\tau_u )</th>
<th>No. of failure cycles during cycling</th>
</tr>
</thead>
<tbody>
<tr>
<td>B/4L/.57/7.0/.03</td>
<td>73.54</td>
<td>12.27</td>
<td>7.0</td>
<td>0.570</td>
<td>665000  no</td>
</tr>
<tr>
<td>B/4L/.59/7.0/.20</td>
<td>69.90</td>
<td>11.85</td>
<td>7.0</td>
<td>0.591</td>
<td>1331    yes</td>
</tr>
<tr>
<td>B/4L/.60/7.0/.06</td>
<td>69.46</td>
<td>11.74</td>
<td>7.0</td>
<td>0.596</td>
<td>512660  no</td>
</tr>
<tr>
<td>B/4L/.60/7.4/.08</td>
<td>73.54</td>
<td>12.27</td>
<td>7.4</td>
<td>0.603</td>
<td>82500   yes</td>
</tr>
<tr>
<td>B/4L/.61/7.3/.04</td>
<td>70.80</td>
<td>11.95</td>
<td>7.3</td>
<td>0.611</td>
<td>23500   yes</td>
</tr>
<tr>
<td>B/4H/.61/8.5/.04</td>
<td>75.34</td>
<td>13.89</td>
<td>8.5</td>
<td>0.612</td>
<td>355716  no</td>
</tr>
<tr>
<td>B/4L/.63/7.3/.04</td>
<td>67.89</td>
<td>11.61</td>
<td>7.3</td>
<td>0.629</td>
<td>931731  yes</td>
</tr>
<tr>
<td>B/4L/.65/8.0/.07</td>
<td>73.47</td>
<td>12.26</td>
<td>8.0</td>
<td>0.653</td>
<td>62000   yes</td>
</tr>
<tr>
<td>B/4H/.66/9.0/.04</td>
<td>74.00</td>
<td>13.69</td>
<td>9.0</td>
<td>0.657</td>
<td>2224    yes</td>
</tr>
<tr>
<td>B/4L/.75/8.4/.05</td>
<td>65.10</td>
<td>11.28</td>
<td>8.4</td>
<td>0.745</td>
<td>219029  yes</td>
</tr>
<tr>
<td>B/4L/.79/8.8/.08</td>
<td>63.88</td>
<td>11.11</td>
<td>8.8</td>
<td>0.792</td>
<td>1150    yes</td>
</tr>
<tr>
<td>B/4L/.81/9.1/.04</td>
<td>64.70</td>
<td>11.23</td>
<td>9.1</td>
<td>0.810</td>
<td>1441    yes</td>
</tr>
<tr>
<td>B/6L/.46/6.9/.04</td>
<td>70.70</td>
<td>15.11</td>
<td>6.9</td>
<td>0.457</td>
<td>550000  no</td>
</tr>
<tr>
<td>B/6L/.52/7.9/.02</td>
<td>71.70</td>
<td>15.26</td>
<td>7.9</td>
<td>0.518</td>
<td>290000  no</td>
</tr>
<tr>
<td>B/6L/.53/8.0/.06</td>
<td>71.40</td>
<td>15.21</td>
<td>8.0</td>
<td>0.526</td>
<td>250000  no</td>
</tr>
<tr>
<td>B/6L/.56/8.9/.02</td>
<td>75.10</td>
<td>15.81</td>
<td>8.9</td>
<td>0.563</td>
<td>325900  no</td>
</tr>
</tbody>
</table>

Because of the large number of load cycles, it is hardly possible to show the crack response for each measured cycle. Fig. 3.5 presents a typical relation between the crack displacements in a specific cycle as a function of the shear stress related to the maximum applied shear stress. For this test, a maximum of 1785 cycles till failure was obtained. Fig. 3.5 shows that even for this relatively small number of cycles the increase in crack displacements between two subsequent cycles remained smaller than the 0.01 mm accuracy of the displacement transducer.

![Fig. 3.5. Typical crack response during cycling.](image-url)
Furthermore, it was observed that the shear displacement increase was initially smaller than the crack width increase, but exceeded the crack width increment with increasing crack displacements.

Fig. 3.6. Influence of the steel yield strength.

The influence of the steel yield strength upon the crack response during cycling is shown in Fig. 3.6a-b. The specimens Nos. A/4L/.77/7.2/.04 and A/4H/.78/8.0/.04 were subjected to a nearly equal percentage of the static shear strength (77% and 78% respectively). Both specimens exhibited a similar behaviour with respect to the crack displacement increase during cycling and followed an identical crack opening path. This indicates that the influence of the steel yield strength can be properly taken into account by eq. (3.1) for the static strength. The actual maximum shear stress was 7.2 MPa for specimen No. A/4L/.77/7.2/.04 and 8.0 MPa for specimen No. A/4H/.78/8.0/.04 with a calculated static shear strength of 9.38 MPa and 10.26 MPa respectively.

Fig. 3.7a-b presents the effect of the variation of the reinforcement ratio. It was found for this parameter, that specimens with a different number of bars exhibited a nearly similar response to cycling. The specimens shown in Fig. 3.7, Nos. A/4L/.61/6.1/.03 and A/6L/.61/7.2/.04, had four and six 8 mm stirrups respectively and were both loaded at 61 percent of their static strength. Therefore, it can be concluded that the influence of reinforcement ratio is accounted for by the eq. (3.1).

It is obvious that an increasing concrete strength yields a stiffer
crack response to cyclic loading. In Fig. 3.8a-b it is shown that also the effect of the concrete strength upon this response is satisfactorily accounted for by the influence of the concrete strength upon the static shear strength. The specimens Nos. A/4L/.65/6.0/.05 and B/4L/.65/8.0/.07 exhibited a similar relation between the crack width and the number of cycles and a similar crack opening path. It must be noted that the relation between the shear displacement and the number of cycles had the same shape as the relation shown in Fig. 3.8a.
Contrary to the previously mentioned parameters, no influence of the magnitude of the initial crack width was found within the range investigated. The specimens Nos. A/6L/.66/8.6/.02 and A/6L/.67/8.2/.08 had a difference in the initial crack width of 0.06 mm. Both specimens followed nearly the same crack opening path except for the small crack displacements.

For mix A, the Fig. 3.10a shows the relationship between the crack width and the number of cycles for various shear stress levels. Similar relations were obtained for the shear displacement as a function of the number of cycles, see Fig. 3.10b. The shear stress level is referred to the static shear strength according to eq. (3.1). Fig. 3.11 presents similar results for mix B.

As expected, the increments of the crack displacements increased with increasing shear stress level. The influence of the parameters investigated in this test series are satisfactorily taken into account by means of the magnitude of the static shear strength. Therefore, the relations presented in the Figs. 3.10-3.11 can be approximated by the following empirical expressions, see Fig. 3.12:

$$\delta_n = 0.10\left(\frac{t_m}{t_u}\right)^2 + [0.15\left(\frac{t_m}{t_u}\right)^3 + 0.40\left(\frac{t_m}{t_u}\right)^9]\log(n) + 0.20\left(\frac{t_m}{t_u}\right)^2[\log(n)]^{10} \quad (3.2a)$$
\[ \delta_t = 0.10 \left( \frac{\tau_m}{\tau_u} \right)^2 + 0.07 \left( \frac{\tau_m}{\tau_u} \right)^3 + 0.70 \left( \frac{\tau_m}{\tau_u} \right)^9 \log(n) + 1.17 \left( \frac{\tau_m}{\tau_u} \right)^{26} \log(n)^5 \]  
(3.2b)

with \( \tau_u \) according to eq. (3.1) and \( \delta_t, \delta_n \) in [mm].

**Fig. 3.10.** The influence of the applied shear stress level for mix A.

**Fig. 3.11.** The influence of the applied shear stress level for mix B.

**Fig. 3.12.** The crack response acc. to eq. (3.2).

In the expressions (3.2) for the crack displacements, the minimum shear stress \( \tau_0 \) is not apparent. One should assume that the ratio of the ap-
plied maximum shear stress $\tau_m$ to the applied minimum shear stress $\tau_o$ will strongly influence the relations expressed by the eqs. (3.2a-b). However, no information on this variable was obtained in this test series.

![Graph showing shear stress level versus number of cycles till failure](image)

Fig. 3.13. The shear stress level versus the number of cycles till failure.

Fig. 3.13 presents the relationship between the applied shear stress level and the number of cycles till failure. Despite of the large scatter, the following mean relation was derived by means of a regression analysis:

$$\frac{\tau_m}{\tau_u} = 1.00 - 0.0736 \log(n_f)$$  \hspace{1cm} (3.3)

The specimens, which did not fail during cycling, were subsequently sheared-off in a static test. Table 3.3 lists the ratio of the shear strength according to eq. (3.1) to the experimentally obtained strength. An average ratio of 1.01 with a coefficient of variation of 0.08 was found. Therefore, it can be concluded that pre-loading with a low shear stress has no measurable influence upon the crack response to higher static shear loads. A similar conclusion can be drawn with respect to higher repeated shear loads. The specimens Nos. B/4L/.57/7.0/.03 and B/6L/.53/8.0/.06 were firstly loaded to 57 and 53 percent of their static strength respectively. Both specimens endured more than 250000 cycles. Subsequently, they were subjected to a repeated load of 65 per-
cent of their static strength. Shear failure occurred after 88500 and 20631 cycles respectively. These numbers of cycles were in reasonable agreement with the 57500 cycles according to eq. (3.3).

Table 3.3. Comparison of the theoretical and the experimental static shear strength.

<table>
<thead>
<tr>
<th>Code</th>
<th>$\tau_u$ [N/mm²]</th>
<th>$\tau_{u,th}$ [N/mm²]</th>
<th>$\tau_u/\tau_{u,th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A/4L/.61/6.1/.03</td>
<td>10.00</td>
<td>10.17</td>
<td>0.98</td>
</tr>
<tr>
<td>A/4L/.63/6.0/.06</td>
<td>9.47</td>
<td>10.10</td>
<td>0.94</td>
</tr>
<tr>
<td>A/4H/.64/7.0/.06</td>
<td>10.97</td>
<td>11.77</td>
<td>0.93</td>
</tr>
<tr>
<td>A/4H/.66/6.9/.02</td>
<td>10.50</td>
<td>12.44</td>
<td>0.84</td>
</tr>
<tr>
<td>A/6L/.51/6.0/.04</td>
<td>11.89</td>
<td>12.10</td>
<td>0.98</td>
</tr>
<tr>
<td>A/6L/.58/6.8/.01</td>
<td>11.73</td>
<td>11.21</td>
<td>1.05</td>
</tr>
<tr>
<td>A/6L/.61/7.2/.04</td>
<td>11.88</td>
<td>12.30</td>
<td>0.97</td>
</tr>
<tr>
<td>A/6L/.62/8.0/.04</td>
<td>12.89</td>
<td>12.48</td>
<td>1.03</td>
</tr>
<tr>
<td>B/4L/.60/7.0/.06</td>
<td>11.74</td>
<td>10.60</td>
<td>1.11</td>
</tr>
<tr>
<td>B/4H/.61/8.5/.04</td>
<td>13.89</td>
<td>12.66</td>
<td>1.10</td>
</tr>
<tr>
<td>B/6L/.46/6.9/.04</td>
<td>15.11</td>
<td>13.50</td>
<td>1.12</td>
</tr>
<tr>
<td>B/6L/.52/7.9/.02</td>
<td>15.26</td>
<td>14.27</td>
<td>1.07</td>
</tr>
<tr>
<td>B/6L/.56/8.9/.02</td>
<td>15.81</td>
<td>15.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

average ratio = 1.01, $s = 0.08$
3.3. Externally reinforced specimens; repeated loading.

3.3.1. Test arrangement

Contrary to the previous test series, the normal restraint stiffness was not applied by means of embedded reinforcement, but by means of four external restraint bars. This test series comprised 14 repeated loading tests focusing upon the aggregate interlock mechanism. The specimen is shown in Fig. 3.14.

![Fig. 3.14. Test specimen with external restraint bars.](image)

The dimensions of the specimen were the same as in the previous series. Now, no bars crossed the crack plane. The auxiliary reinforcement was still apparent preventing preliminary failure of the specimen. At the small sides of the specimen steel plates were placed interconnected by four 20 mm diameter bars. A thin layer of rapidly hardening sand-cement paste placed between the steel plates and the concrete surface of the specimen ensured an almost linear interaction between crack-opening and restraint force. However, the restraint stiffness remained low compared with the specimens having embedded reinforcement. To ensure a small crack width during the first cycles all the specimens were prestressed with an initial normal stress on the crack plane ranging from 0.8-3.6 MPa.

The instrumentation used in this part of the experimental program was nearly the same as used for the reinforced specimens, see Section 3.2.1. The addition made for this test series was the recording of the steel strains of the external restraint bars by means of strain gauges.
3.3.2. Test variables

The concrete strength, the initial crack width, the number of cycles and the applied maximum shear stress level were variables as already described in Section 3.2.2. Now, the initial normal stress was an additional variable instead of the reinforcement ratio for the specimens with embedded reinforcement. The initial normal stress was relatively high (0.8-3.6 MPa) to ensure very small crack widths. Unfortunately, Walraven [81] performed static tests on similar specimens without such a high initial normal stress. Therefore, the static shear strength obtained in his tests provided only global information for the static shear strength in the present test series. Furthermore, for plain concrete no actual shear failure occurred due to the increasing normal stress. In consequence, no shear stress level could be determined without performing static shear tests with a high normal stress. Because of the better fundamental insight in the mechanism of aggregate interlock as a result of Walraven's theoretical work (see Chapter 4 and [81]), it was decided to perform repeated loading tests with various maximum shear loads in spite of a lack of knowledge about the actual static shear strength. As for the previous test series, the applied minimum shear stress was 0.3 MPa.

3.3.3. Test results

A total of 14 tests was performed. The identifying code, which was assigned to the specimens consisted of the concrete grade (mix A or mix B), the initial normal stress, the applied maximum shear stress and the initial crack width. Contrary to the experiments on specimens with embedded reinforcement, the applied maximum shear stress \(T_m\) was not the initially applied shear stress. Due to the fact that for low shear stress levels no significant crack displacements were recorded, the applied shear stress was raised to such a level that significant crack displacements occurred. This stress level was then subsequently maintained and is adopted in the code of the specimen. Table 3.4 lists a review of the experimental results.
Table 3.4. Experimental results of plain concrete specimens.

<table>
<thead>
<tr>
<th>Code</th>
<th>$f_{ccm}$ [N/mm²]</th>
<th>$\sigma_0$ [N/mm²]</th>
<th>$\tau_m$ [N/mm²]</th>
<th>No. of cycles</th>
<th>failure during cycling</th>
</tr>
</thead>
<tbody>
<tr>
<td>A/1.3/4.0/.12</td>
<td>51.53</td>
<td>1.30</td>
<td>4.0</td>
<td>30800</td>
<td>yes</td>
</tr>
<tr>
<td>A/1.2/5.0/.01</td>
<td>52.06</td>
<td>1.22</td>
<td>5.0</td>
<td>20326</td>
<td>no</td>
</tr>
<tr>
<td>A/1.3/5.0/.01</td>
<td>54.60</td>
<td>1.27</td>
<td>5.0</td>
<td>3080</td>
<td>yes</td>
</tr>
<tr>
<td>A/1.9/5.0/.19</td>
<td>48.40</td>
<td>1.90</td>
<td>5.0</td>
<td>9756</td>
<td>yes</td>
</tr>
<tr>
<td>A/0.8/5.5/.01</td>
<td>49.56</td>
<td>0.80</td>
<td>5.5</td>
<td>1520</td>
<td>yes</td>
</tr>
<tr>
<td>A/2.1/6.1/.01</td>
<td>54.53</td>
<td>2.14</td>
<td>6.1</td>
<td>8640</td>
<td>yes</td>
</tr>
<tr>
<td>A/1.3/6.2/.01</td>
<td>48.09</td>
<td>1.34</td>
<td>6.2</td>
<td>283549</td>
<td>yes</td>
</tr>
<tr>
<td>A/1.3/6.2/.02</td>
<td>52.57</td>
<td>1.26</td>
<td>6.2</td>
<td>3120</td>
<td>yes</td>
</tr>
<tr>
<td>B/1.1/5.0/.04</td>
<td>71.64</td>
<td>1.07</td>
<td>5.0</td>
<td>254369</td>
<td>no</td>
</tr>
<tr>
<td>B/1.2/5.6/.01</td>
<td>68.36</td>
<td>1.15</td>
<td>5.6</td>
<td>89500</td>
<td>no</td>
</tr>
<tr>
<td>B/1.0/6.2/.04</td>
<td>72.25</td>
<td>0.99</td>
<td>6.0</td>
<td>2736</td>
<td>yes</td>
</tr>
<tr>
<td>B/2.0/6.5/.01</td>
<td>69.14</td>
<td>2.02</td>
<td>6.5</td>
<td>343</td>
<td>yes</td>
</tr>
<tr>
<td>B/3.6/6.9/.19</td>
<td>70.20</td>
<td>3.58</td>
<td>6.9</td>
<td>82378</td>
<td>no</td>
</tr>
<tr>
<td>B/1.5/7.7/.01</td>
<td>67.54</td>
<td>1.99</td>
<td>7.7</td>
<td>3970</td>
<td>yes</td>
</tr>
</tbody>
</table>

Fig. 3.15 presents some typical relations of the crack displacements versus the shear stress related to the applied maximum shear stress, showing the crack response during a cycle. As for the reinforced specimens, the hysteretic loop indicated that dissipation of energy occurred in the shear plane. Again it was observed that the shear displacement increment was initially smaller than the crack width increment. For increasing crack displacements, the shear slip increment exceeded the crack width increment. In fact, no shear failure occurred during the repeated tests, because even for large crack displacements ($\delta > 2$mm) the shear plane was still capable of transferring the applied shear load. This was probably due to the increase in normal force with crack-opening. The specimens were unloaded when the shear displacement exceeded 2 mm.

The influence of the concrete strength upon the shear stress was proportional to $f_{ccm}^{0.56}$ for the static tests of Walraven [81]. If this proportionality could be applied to the case of a repeated shear loading, this must be reflected in the range of the applied maximum shear stress $\tau_m$. The range for which significant crack displacements were recorded, was 5.0-6.2 MPa for mix A and 5.6-7.7 MPa for mix B. The average ratio of the range for mix A to the range for mix B was equal to 1.18, which was in good agreement with $(70/51)^{0.56} = 1.19$. 
The relationship between the normal stress and the crack width was in the same range for both mixes, see Fig. 3.16.

The influence of the magnitude of the initial crack width upon the crack response to repeated shear loading is shown in Fig. 3.17a. The specimens Nos. A/1.3/5.0/.01 and A/1.9/5.0/.19 had an initial crack width equal to 0.01 mm and 0.19 mm respectively.

The last mentioned specimen had a very large crack width during pre-cracking due to a bad fitting of the steel restraint plates to the concrete surface. Although a high normal stress was applied to the crack plane, it was not possible to completely re-close the crack. Therefore, this specimen had an initial crack width, which was not in the range re-
fleeting offshore conditions. However, the result is of interest for the influence of the initial crack width. The normal stress as a function of the crack width was similar for both specimens, see Fig. 3.17b. Fig. 3.17a shows that the initial crack width hardly influenced the crack response for the range investigated. The influence of the normal stress is shown in Fig. 3.18. The specimens Nos. A/0.8/5.5/01, A/1.3/5.0/01 and A/2.1/6.1/01 had initial normal stresses of 0.8 MPa, 1.3 MPa and 2.1 MPa respectively. There was a slight difference in the applied shear stress. Fig. 3.18a shows that the increase in crack width with cycling was similar for both the specimens with the high initial normal stresses. The specimen with the relatively low initial normal stress exhibited initially smaller crack widths, which increased much more rapidly than for the other specimens. Obviously, an increase of the initial normal stress up to about 1.3 MPa influenced the crack response. A further increase of the normal stress however, had little effect on the increase in crack width. The effect of the normal stress upon the crack opening path was only slight. Fig. 3.18b shows that there was no systematic variation of the crack opening path for the initial normal stress investigated.

![Graphs](image)

Fig. 3.18. The influence of the initial normal stress.

The influence of the magnitude of the applied shear stress \( \tau_m \) is shown in Fig. 3.19a-b, representing the crack width versus the number of cycles for both mixes and various shear stresses. Contrary to the specimens with embedded reinforcing bars, the increase in crack displace-
ments appeared to be hardly affected by the applied shear stress. An exception was obtained for the specimen with the relatively low initial normal stress of 0.8 MPa, as is already shown in Fig. 3.18.

![Fig. 3.19. The influence of the applied maximum shear stress.](image1)

Shear failure occurred when the crack faces became unable to transfer the applied shear stress and was characterized by an abrupt increase in the crack displacements, instead of the gradual increase observed for the reinforced specimens. The crack opening paths for mix A varied only within a small range, see Fig. 3.20a. For mix B, a somewhat wider range was found, with a crack opening path slightly deviating from the mean.

![Fig. 3.20. Crack opening paths for both mixes.](image2)
crack opening path of mix A, see Fig. 3.20b. The number of cycles till failure was not only influenced by the applied shear stress, but also by the normal stress. Although no ultimate shear stress is clearly defined for experiments with an increasing normal stress, a rough approximation of the static shear strength can be made using the experimental results of Walraven [81] and Daschner [12]. Daschner's results were questionable [13] with respect to the measured crack displacements, but the relation between the (constant) normal stress and the static shear strength was properly recorded, see Fig. 3.21. The following expression for the static shear strength was derived by means of a regression analysis [82]:

\[
\tau = 1.647 \cdot 0.321 f_{ccm}^{0.427} \cdot n^{0.56}
\]

(3.4)

Fig. 3.21. Shear strength versus normal stress [82].

Fig. 3.22. Shear stress level versus number of cycles till failure

For the present test series, the normal stress at the onset of shear failure was presented in [56]. Inserting this value of the normal stress into eq. (3.4), the number of cycles till failure is related to the (approximated) shear stress level, see Fig. 3.22. Despite of the scatter of the results, it appeared that most of the plain concrete specimens endured less cycles till failure than the reinforced specimens at the same shear stress level. In Fig. 3.22 this is shown by means of the
dashed line according to eq. (3.3).

As discussed in Section 3.2.2., the aggregate interlock mechanism is strongly influenced by the amount of particles with interfacial bond fracture. For four plain concrete specimens, the area of fractured particles after pre-cracking and load-cycling was determined. Typical results are shown in the Figs. 3.23a-b. The mean ratio of the fractured area of the particles to the total cross-sectional area of the particles in the crack plane was 20.4 percent for mix A and 25.1 percent for mix B. This was in reasonable agreement with the expectation, although the amount of particles fractured in mix A was larger than expected. The difference between the areas of fractured particles of the two mixed was not significant.

![Fractured area of particles](image)

**Fig. 3.23.** Fractured area of particles.
4. THEORETICAL MODELLING OF THE RESPONSE OF CRACKED CONCRETE TO MONOTONIC SHEAR LOADING

4.1. Introduction.

In the Chapters 4 and 5, all the attention will be devoted to the physical understanding of the shear transfer mechanisms in cracks in plain and reinforced concrete. Therefore, models referred to in the literature, which are mainly based upon empirically derived relations between the stresses and displacements in the crack plane, such as the shear friction-analogy, are not discussed here.

Physical models properly describing the response of the mechanisms of aggregate interlock and dowel action are basically derived for the case of increasing static shear loads [22, 57, 81]. These models will be extended to the case of a repeated or reversed shear load with a constant amplitude. Therefore, the existing models will be briefly discussed and their presentation will be adapted to the case of constant shear loads. Furthermore, newly observed material behaviour is incorporated into these models. Next, the response of cracks in reinforced concrete to static shear loads is described on the basis of the crack opening path. It is shown how the transfer mechanisms affect the crack opening direction.

Finally, in Chapter 5 the existing models will be adapted to the case of repeated and reversed shear loads. A distinction is made between the description of 'high-intensity low-cycle' fatigue on the one hand and 'low-intensity high-cycle' fatigue on the other hand.

4.2. The mechanism of aggregate interlock.

Walraven [81] developed a physical model, based upon the assumption that concrete can be conceived as a composition of two basically different materials; the strong and stiff glacial river aggregate particles and the matrix material consisting of hardened cement paste with a much lower strength and stiffness. If a crack is formed in the concrete, it will run through the matrix material and along the interface of the matrix and the particles. Therefore, the crack plane exhibits a global undulation caused by the irregular shape of the crack faces and a local
roughness due to the particles protruding from the crack plane. The roughness due to the protruding particles dominates the roughness caused by the global undulation. The crack plane can therefore be approximated by a flat plane intersected by stiff particles. Next, the irregularly shaped particles are randomly oriented. The most accurate simplification of the particles is to consider them as spheres. The crack plane according to Walraven's assumptions is shown in Fig. 4.1. His schematic two-phase presentation of the actual crack plane provides a physically close approximation of the experimentally observed crack response to static shear loads. The particles are regarded as rigid spheres embedded in the matrix material, which is considered as a rigid material with crushing strength $\sigma_{pu}$. In consequence, the particles are undeformably crushing the matrix during shear sliding.

Fig. 4.1. Crack plane according to Walraven's two-phase model [81].

Whether a particle makes contact with the opposing crack face depends upon the particle size, its embedment depth, the crack width and the shear displacement. An interesting aspect of Walraven's schematic presentation of the crack plane is that the total contact area of all the particles in a unit area of the crack plane can be determined analytically. Considering a gradation according to Fuller's ideal curve, Walraven quantified the projected contact areas $a_x$ and $a_y$ for any given particle during shear sliding, see Fig. 4.2a. For a thin slice of the crack plane the particles reduce to circles, which appearance in the crack plane is described by a probability density function. The projected contact areas in this thin slice can be determined analytically.
being the distance between the intersection point of two circles and the intersection point of a straight line and a circle. All the projected contact areas $a_x$ and $a_y$ are summed up numerically yielding the total projected contact areas $A_x$ and $A_y$ for a unit area of the crack plane. Now, the equilibrium condition for this unit area can be described by, see Fig. 4.2b:

$$
\tau_a = \sigma_a (A + \mu A_x) \\
\sigma_a = \sigma (A - \mu A_y)
$$

(4.1a)

(4.1b)

in which

- $\tau_a =$ shear stress
- $\sigma_a =$ normal stress
- $\sigma_{pu} =$ strength of matrix material
- $A_x =$ total projected contact area per unit area of the crack plane parallel to the crack plane
- $A_y =$ total projected contact area per unit area of the crack plane normal to the crack plane

The strength of the matrix material $\sigma_{pu}$ and the coefficient of friction were derived from the experimental results of Walraven's static tests [81]:

$$
\sigma_{pu} = 6.39 f_{ccm}^{0.56} \text{ [MPa]}
$$

(4.1c)
Fig. 4.3 presents a comparison of the model with some typical test results [81]. Apart from his own experimental results, Walraven's model provides good predictions for tests of Paulay et al. [50] and Millard et al. [45].

The two-phase model as presented in Fig. 4.3 describes a unique relationship between the stresses and the displacements in the crack plane. For the use in this document, it was preferred to present this relationship as crack opening paths for constant shear or constant normal stresses. These stresses are related to the matrix strength according to eq. (4.1c), see Figs. 4.4a-b. According to the model the maximum particle size has a slight influence upon the crack response. This is shown in the Fig. 4.4 for particle diameters of 10 mm, 16 mm and 32 mm. The lower the size of the maximum particle the steeper the crack opening path ($\Delta_1$-increment $> \Delta_n$-increment).

It should be noted that the two-phase model is based upon the assumption that the contact areas between the particles and the matrix material for a given combination of the crack displacements can be calculated neglecting the previously followed crack opening path, see Fig. 4.5a. Generally, the actually followed crack opening path causes a gradual increase of the contact areas, thus incorporating the previously formed contact areas in the newly formed contact areas, see Fig. 4.5b.

**Fig. 4.3.** Comparison of the model with experimental results [81].

**Fig. 4.4.** Crack opening path acc. to the model.
However, the condition of the gradually increasing contact areas must be fulfilled for the applicability of the two-phase model. Fig. 4.6a shows that for a given particle diameter this condition is just fulfilled when the maximum value of the slope of the crack opening path in each point is equal to the tangent of the upper intersection point of the contact area. Now, for a given combination of the crack width and the shear displacement, an average crack opening direction can be determined taking
into account all the particles intersecting the crack plane. In fact,
this calculation is performed similarly to the numerical solution of the
total contact areas of the two-phase model of Walraven using the proba-
bility density function. For a maximum particle diameter of 16 mm, the
additional condition to Walraven's model is presented in Fig. 4.6b.
Other maximum particle diameters yield only slightly different results.
The model is valid when the crack opening direction is according to Fig.
4.6b or steeper. The experimentally observed crack opening paths in Wal-
raven's static test series generally fulfilled this condition. It must
be realized that the matrix strength and the coefficient of friction
were determined from these results, thus accounting for some influence
of the previously followed crack opening path.
For the case of a less steep crack opening direction the transferred
stresses will be less than according to the two-phase model.

4.3. The mechanism of dowel action.

The mechanism of dowel action is based upon the response of a bar and
the surrounding concrete to a lateral bar displacement. As described in
Section 2.4, failure of a dowel occurs due to crushing of the concrete
and yielding of the bar when the concrete cover of the bar is suffi-
ciently large to prevent splitting failure. For the case of crushing
failure three mechanisms can be distinguished, according to Paulay [51]:
a. bending; the dowel force is transmitted due to bending of the bar.
   For this mechanism the ultimate load is reached due to yielding of
   the bar.

b. pure shear; it is expected that the transfer of dowel force by means
of pure shear is unlikely because of the deterioration of the con-
crete at the vicinity of the bar. Therefore, the resulting dowel
forces at both sides of the crack plane have a relatively large ec-
centricity resulting in yielding of the bar due to bending.

c. kinking; for a considerable lateral bar displacement the axial bar
force in the crack plane has a component parallel to the crack direc-
tion, see Fig. 4.7. For the case of cracked concrete the crack width
remains small relatively to the bar diameter. Hence, the effect of
kinking of the bar will be small (except for the case of large crack
widths in combination with small bar diameters).
Fig. 4.7. Kinking of a bar according to Paulay [51].

The ultimate dowel force is reached when plastic hinges develop in the bar. Therefore, this mechanism is affected by both the properties of concrete and steel. The most important parameters are the bar diameter, the concrete strength, the axial steel stress and the steel yield strength.

Fig. 4.8. Bar considered as a beam on elastic foundation.

The dowel load-lateral displacement relation can be described using the theory of beams on elastic foundation, as published by Timoshenko and Lessels [70]. Now the bar is considered as a flexible beam of infinite length supported on an elastic foundation, see Fig. 4.8. This mechanism is first used by Friberg [22] and accounts for the bar diameter and the concrete strength.

According to this mechanism, the following relation can be derived:

$$\delta = \frac{F \cdot d}{283 \cdot EI} \quad [\text{mm}]$$  \hfill (4.2)
with \( g = \sqrt{\frac{K_f \phi}{4EI}} \)

\( K_f \) = foundation modulus of concrete [MPa/mm]

\( \phi \) in [mm], \( E \) in [MPa], \( I \) in [mm^4]

According to Finney [19] the value of the foundation modulus of concrete is in the range of 200 to 2400 MPa/mm with an average value of approximately 700 MPa/mm, thus showing a large scatter. This scatter is probably due to some deterioration of the concrete at the vicinity of the bar, although it is assumed that for the overall behaviour the concrete is still uncracked. In consequence, the way of load application and the eccentricity of the dowel force (distance of dowel load to the concrete surface) may influence the magnitude of the foundation modulus obtained in an experimental program.

Finney described the bar displacement accounting for the lateral displacement caused by the deformation of the 'free' length of the bar. This free length comprises the crack width for bars perpendicularly crossing the crack plane. However, for bars with an inclination to the crack plane, Schäfer [63] suggested to enlarge the free length to take account for the inclined cracking of the concrete at the vicinity of the crack plane.

In [81, page 42] Walraven combined the analytical solutions of Friberg [22], Finney [19] and Schäfer [63], thus yielding:

\[
F_d = 3.56 \times \phi^{1.75} \times K_f^{0.75} \times \delta_t [N]
\]

(4.3)

with \( K_f \) = foundation modulus of concrete [MPa/mm]

\( \phi, \delta_t \) in [mm]

So the dowel force is proportional to \( \phi^{1.75} \). Jimenez [34] based a similar relation also on a beam on elastic foundation:

\[
F_d = 190 \phi^{1.75} \delta_t [N]
\]

(4.4)

with \( \phi, \delta_t \) in [mm]

Vintzeleou [77] derived an expression for the foundation modulus assum-
ing that the concrete supporting the bar is deformed by the dowel force up to a distance of twice the bar diameter. Assuming linear elastic material behaviour it was found that:

\[
K_f = \frac{E}{2\phi} \quad \text{[MPa/mm]} \quad (4.5)
\]

![Diagram showing foundation modulus as function of lateral bar displacement.](image)

**Fig. 4.9.** Foundation modulus as function of the lateral bar displacement. [51]

Equation (4.5) appeared to be valid for dowel loads less than 50 percent of the ultimate dowel force. In fact, for very low dowel forces, the concrete stressed by the dowel load is situated close to the bar. For that case the distance will be smaller than twice the bar diameter. In consequence, the foundation modulus will increase. Indeed, according to experimental observations reported by Paulay [51] the foundation modulus must decrease with increasing lateral bar displacements, see Fig. 4.9.

Millard [45] found experimentally that the initial foundation modulus of concrete was equal to 750 MPa/mm for moderate strength concrete. For high-strength concrete the value of the foundation modulus was found to be proportional to the square root of the concrete strength.

It must be noted that the stress distribution according to Timoshenko's theory does not agree with the real distribution of the reaction stresses in the concrete, see Fig. 4.10.

With increasing dowel force the concrete stresses at the vicinity of the bar exceed the uniaxial compressive strength. However, the surrounding concrete provides a considerable confining pressure, thus yielding a triaxial compressive zone under the bar. Therefore, the concrete strength can be several times as high as the uniaxial strength.
Now, the bar itself becomes the weakest link and the ultimate dowel force is reached when the bar yields.

Rasmussen [57] performed tests on dowels protruding from a large concrete block, see Section 2.4. Apart from his experimental study, Rasmussen modelled the dowel action according to the behaviour of the steel dowels in timber structures. Fig. 4.11 presents the failure mechanism in which a plastic hinge is situated at some distance to the 'crack plane'. In the plastic hinge the plastic moment of the bar is reached, which is equal to $0.167 f_{\text{ccyl}} \phi^3$. Now, the equilibrium condition yields:

$$F_d = B \phi^2 \sqrt{\frac{f_{\text{ccyl}}}{f_{\text{sy}}}} [N]$$  \hspace{1cm} (4.6a)

with

$$B = C(\sqrt{1+\epsilon C}^2 - \epsilon C)$$  \hspace{1cm} (4.6b)

$$\epsilon = 3 \frac{e}{\phi} \sqrt{\frac{f_{\text{ccyl}}}{f_{\text{sy}}}}$$  \hspace{1cm} (4.6c)

$e$ = eccentricity of the dowel load [mm]

$C$ = empirical constant

$f_{\text{ccyl}}$, $f_{\text{sy}}$ in [MPa], $\phi$ in [mm]

It was found experimentally that $C$ was equal to 1.3, assuming a zero-eccentricity of the dowel load.

Dulacska [15] also derived an expression for the ultimate dowel force, see eq. (2.7).

Vintzeleou [77] derived an expression for the ultimate dowel force, which is in fact similar to Rasmussen's formula. However, the derivation
is based upon a failure criterion, which was used by Broms [7] to describe the ultimate lateral force for a pile in a cohesive soil. In Vintzeleou's approach, no empirical constant is used for the calculation of the ultimate dowel force. Fig. 4.12 presents Broms' failure mechanism. The soil at the vicinity of the surface reacts less stiff than the soil situated at some distance from the surface, where the compressive strength of approximately five times the uniaxial strength is obtained. According to Vintzeleou, there is no decrease in stiffness at the vicinity of the crack plane for a bar embedded in concrete.

For the failure mechanism presented in Fig. 4.13, Vintzeleou derived the following expression for the ultimate dowel force:

\[
P_{du} + (10 \frac{f_{ccyl}}{e \phi}) F_{du} - 1.7 \frac{f_{ccyl}}{f_{sy}} = 0
\]

For zero-eccentricity eq. (4.7) becomes equal to Rasmussen's formula. Despite the close fit to experimental results it must be doubted whether the failure mechanism shown in Fig. 4.13 is valid. Experimental observations of Dulacska [15] and Utescher [73] showed a considerable spalling-off of the concrete close to the crack plane, see Section 2.4. Due to this spalling-off of the concrete, Rasmussen's and Broms' descriptions seemed to be in closer agreement with the actual stress dis-
reaction stresses

approximated reaction stresses.

c = cohesion = 0.5 f_{c_{cyl}}

Fig. 4.12. Failure mechanism for a pile in a cohesive soil according to Broms [7].

Fig. 4.13. Failure mechanism for a dowel in concrete according to Vintzeleou [77].

distribution than Vintzeleou's approach. Therefore, Rasmussen's stress distribution will be used for a further analysis of the dowel action mechanism.

Fig. 4.14 presents a bar protruding perpendicularly from a concrete block. At a distance X from the 'crack plane' a plastic hinge had been developed. The magnitude and the distribution of the reaction stresses in the concrete under the bar are not known beforehand. According to the theory of plasticity the plastic moment of the bar is equal to 0.166 $\phi^3 f_{sy}$. However, this plastic moment can only be applied to a bare bar. For the case of a dowel embedded in concrete, the concrete supporting the bar in the cross-section situated at the plastic hinge of the bar is deformed due to the lateral bar displacement. In consequence, the bond between the bar and the concrete above the bar must be broken. Contrary to this, the bond between the bar and the concrete supporting the bar will be extremely good due to the high reaction stresses. For the cross-section, in which the plastic hinge is situated, the equilibrium condition is shown in Fig. 4.15.

The resulting force due to the bond stresses between the bar and the concrete supporting the bar is situated at a distance $z_n$ from the neutral axis of the bar. Due to the bond force the neutral axis of the
Fig. 4.14. Failure mechanism of bar due to plastification.

Fig. 4.15. The equilibrium condition for the plastic hinge.

Bar is shifted over a distance $z$, see Fig. 4.16. To obtain equilibrium the bond force $N$ must be equal to $2A_a f_{sy}$. Now, according to the equilibrium condition it follows:

$$\sum N = 0: (A + A_c) f_{sy} = (A + 2A_a) f_{sy} \tag{4.8a}$$

$$\sum M = 0: (A + A_c) f_{sy} z = A_b f_{sy} z + 2A_a f_{sy} (z - z) \tag{4.8b}$$

With $A_a = \frac{\pi}{2} r^2 - A_b \tag{4.8c}$

$$A_b = r^2 [\theta - 0.5 \sin(2\theta)] \tag{4.8d}$$

Fig. 4.16. Shift of the neutral axis.
The plastic moment can now be calculated as a function of $z_n$ according to eqs. (4.8a-g). This is shown in Fig 4.17, in which the distance $z_n$ is related to the radius of the bar $r$. The shift of the neutral axis of the bar can be determined if the eccentricity of the bond force is known. The eccentricity of the bond force is related to the distribution of the bond stresses. This distribution is not exactly known, but can be determined with reasonable accuracy.

![Diagram](image)

**Fig.4.17.** The plastic moment as function of the eccentricity of the bond force $z_n$.

First, the bond stress is related to the steel strain. Therefore, the bond stress is proportional to $r \cos(\alpha)$, see Fig. 4.18. Second, the bond stress is influenced by the normal pressure on the bar. Untrauer [72] found that the bond stress is proportional to $\sqrt{\sigma_n}$. According to a linear elastic response, the normal stress can be approximated by:

$$\sigma_n = \frac{\Delta D}{\pi r} \cos(\alpha)$$  \hspace{1cm} (4.9)
Thus yielding for the bond stress:

$$\tau_{\text{bond}} = f(\cos^{1.5}(a))$$  \hspace{1cm} (4.10)

The magnitude of the eccentricity of the bond force can now be calculated according to, see Fig. 4.18:

$$z_n = r \frac{\int_{0}^{\pi/2-\beta} \tau_{\text{bond}} \cos(\theta) d\theta}{\int_{0}^{\pi/2-\beta} \tau_{\text{bond}} d\theta}$$

$$= r \frac{\int_{0}^{\pi/2-\beta} \cos^{1.5}(a) \cos(\theta) d\theta}{\int_{0}^{\pi/2-\beta} \cos^{1.5}(a) d\theta}$$  \hspace{1cm} (4.11)

with \( \alpha = \frac{\pi}{\pi - 2\beta} \theta \)

Fig. 4.18. The distribution of the bond stresses.

Fig. 4.19. The eccentricity of the bond as function of \( \gamma \).

The steel close to the shifted neutral axis of the bar will slip relatively to the concrete due to the reduced bond as a result of the low normal stress. Therefore, the concrete at the vicinity of the shifted neutral axis will provide only a minor contribution to the bond force. This is shown by the dashed line in Fig. 4.18, assuming that for an angle \( \gamma \) there is little bond between steel and concrete. Now, eq. (4.11) becomes:

$$\frac{z_n}{r} = r \frac{\int_{0}^{\pi/2-\beta-\gamma} \cos^{1.5}(a) \cos(\theta) d\theta}{\int_{0}^{\pi/2-\beta-\gamma} \cos^{1.5}(a) d\theta}$$  \hspace{1cm} (4.12)
Eq. (4.12) is shown in Fig. 4.19 for $\theta$ equal to 10 degrees. With the eccentricity of the bond force taken equal to the average value of 0.93$r$, it is found in Fig. 4.17, that the plastic moment is equal to:

$$M = 1.78 \, r^3 \, f = 0.223 \, \phi^3 \, f_{sy} \tag{4.13}$$

Note that the plastic moment is 34 percent higher than the plastic moment of the bare bar according to the theory of plasticity. For this case the neutral axis is shifted over a distance equal to 0.14$r$. In consequence $\theta$ is equal to 8 degrees, which is in good agreement with the assumption of 10 degrees.

Now, the equilibrium according to Fig. 4.14 becomes:

$$F_{du} (e + aX) = 0.223 \, \phi^3 \, f_{sy} \tag{4.14}$$

and

$$F_{du} = X f_{ccyl}^* \phi \tag{4.15}$$

with $f_{ccyl}^*$ = mean compressive strength of the multi-axially loaded concrete

Combining eqs. (4.14)-(4.15) yields:

$$F_{du} = 0.5 \, C \left[ \sqrt{4 \times 0.223 + (Cc)^2} - Cc \right] \phi^2 \sqrt{f_{sy} f_{ccyl}} \tag{4.16} \text{[N]}$$

with $C = \sqrt{n/a}$ = empirical constant

$$n = f_{ccyl}^*/f_{ccyl}$$

$$e = \frac{\phi}{f_{ccyl}^*/f_{sy}}$$

$\phi$, $e$ in [mm], $f_{ccyl}$, $f_{sy}$ in [MPa]

The constant $C$ can be solved empirically by means of the experimental results of Rasmussen. Rasmussen found from his test results, assuming zero-eccentricity:
Thus \( C \) is equal to 2.75. However, as stated in Chapter 2, some eccentricity was inevitable during these tests. Therefore:

\[
0.5 \, C \sqrt{4x0.223} = 1.3
\]  

(4.17)

According to Rasmussen's formula the ultimate dowel force is equal to:

\[
F_{du} = 1.3 \phi^2 \frac{f_{ccyl}}{f_{sy}} \quad [N] 
\]  

(4.18)

Assuming that the contact zone between the bar and the loading frame is loaded beyond its yielding strain up to the ultimate strain, the length of the contact zone can be determined. For the steel used in Rasmussen's tests the ratio between the yield strength and ultimate strength was approximately 0.6. Now, the eccentricity becomes, see Fig. 4.20:

\[
e = 0.5 \, L = \frac{0.5 \, F_{du}}{\phi \, f_{sy} / 0.6} \quad [mm] 
\]  

(4.19)

thus

\[
e = 0.39 \phi \frac{f_{ccyl}}{f_{sy}} \quad [mm]
\]

The average value of the ratio between steel strength and concrete strength was equal to 10.77 for the tests of Rasmussen. Now, combination of the eqs. (4.17a) and (4.19) yields:

\[
C = 3.1
\]

Thus:

\[
F_{du} = 1.55 \left[ \sqrt{0.892 + (3.1\epsilon)^2} - 3.1\epsilon \right] \phi^2 \frac{f_{ccyl}}{f_{sy}} \quad [N] 
\]  

(4.20a)

with

\[
e = \frac{1}{\phi} \frac{f_{ccyl}}{f_{sy}}
\]
F_{du} = 1.35 \left[ \frac{1 + 9e^2 - 3e}{\phi^2} \right] \sqrt{f_{sy}} f_{ccm} \ [N] \quad (4.20b)

with

\varepsilon = \frac{e}{\phi} \sqrt{\frac{f_{ccm}}{f_{sy}}}

Fig. 4.20. Load eccentricity in Rasmussen's tests.

In Fig. 4.21 eq. (4.20b) is compared with available test results of Bennett [4], Paulay [51], Rasmussen [57], Vintzeleou [75] and Utescher [73]. It was found that for 76 experimental results the average ratio between the theoretical and the experimental dowel force was equal to 0.998 with a coefficient of variation of 17.2 percent. Appendix II presents detailed information on these tests.

Now, the model will be extended to the case of a combined axial and lateral load. In practice, axial steel stresses develop because embedded reinforcing bars crossing the crack plane are strained due to the crack opening during shear sliding. In consequence, this increasing axial steel tensile force will influence the equilibrium of forces and bending moments in the bar. The axial steel force will make equilibrium with axial stresses at the vicinity of the (shifted) neutral axis of the bar, see Fig. 4.22.

Fig. 4.22 is similar to Fig. 4.16 except for the influence of the axial force. The contribution of the axial force to the equilibrium can be easily taken into account in the eqs. (4.8a-g). Fig. 4.23 presents the influence of the axial force upon the magnitude of the ultimate dowel force. For small values of the axial force the shift of the neutral axis is hardly influenced. For increasing axial force the shift of the neutral axis decreases, to become zero for an axial force equal to the yield force, see Fig. 4.24.
The interaction between the axial steel force and the ultimate dowel force can be satisfactorily predicted by the eq. (4.21) proposed by Vintzeleou [75] with \( n \) and \( m \) equal to 2. See Fig. 4.23:

\[
\frac{F_d}{F_{du}}^n + \left(\frac{F_{sy}}{F_{sy}}\right)^m = 1 \quad \text{or} \quad \gamma_d = \frac{n}{1 - \left(\frac{F_{sy}}{F_{sy}}\right)^m} \tag{4.21}
\]

with \( F_d \) = dowel force
\( F_{du} \) = ultimate dowel force according to eq. (4.20b)
\[ F_s = \text{axial steel force} \]
\[ F_{sy} = \text{steel yield force} \]
\[ \gamma_d = \frac{F_d}{F_{du}} \]

Eqs. (4.20b) and (4.21) can be combined for the ultimate dowel force, thus yielding an expression for bars with an axial steel force. This expression is compared with experimental results of Millard [45], see Fig. 4.25. The experimental result of test 25L is obviously disturbed. This was probably due to the fact that for the small 8 mm diameter bar used in this test, the strain gauges were stuck to the surface of the bar thus influencing the bond of the bar to the concrete. Neglecting this result, an average ratio of the predicted to the experimentally obtained dowel strength equal to 1.02 with a coefficient of variation of 8.1% is found. Detailed information is presented in Appendix II.

Bars generally cross a crack plane at different angles. For inclined bars, the angle of inclination influences the magnitude of the ultimate dowel force. Two cases of inclined bars can be distinguished. First, for small angles of inclination, the concrete supporting the bar will respond less stiff to lateral bar displacements than is the case for bars perpendicular to the crack plane. This is caused by the less favourable shape of the concrete, which might cause inclined cracking. Second, for large angles of inclination, the concrete will provide a stiff response.
Fig. 4.25. The experimental and theoretical results for Millard's tests [45].

to lateral bar displacements. However, the axial steel force, which is inevitable for this case, will influence the magnitude of the dowel force according to eq. (4.21).

Next, both types of bar inclinations will be treated separately.

a. Angle of inclination in the range 0° to 90°.

Fig. 4.26 presents a bar with an angle of inclination 0 to the crack plane. The bar is subjected to a load $F_{\text{ext}}$ parallel to the crack plane. According to Vintzeleou [75], the concrete reaction force is provided by a layer of concrete with a depth equal to twice the bar diameter. For small bar inclinations, the concrete supporting the bar at the vicinity of the crack plane has a depth, which is far less than twice the bar diameter. Apart from that, inclined cracking can occur in the concrete, see Fig. 4.26.

In consequence, the ultimate dowel force will be less than the dowel resistance in the case of a bar perpendicular to the crack plane. The actual stress distribution in the concrete is not accurately known, so that the influence of the angle of inclination upon the concrete response to a lateral bar displacement must be roughly estimated. To account for this influence, the dowel resistance is related to the inclination according to:

$$F_{\text{du}} = F_{\text{du},0}\sin(\theta)$$  \hspace{1cm} (4.22)
In consequence the externally measured force is equal to $F_{du,90}$. Furthermore, the dowel force in the crack has an eccentricity $e$ to the concrete. This eccentricity is equal to:

$$e = 0.5 \phi \cotan(\theta) + 0.5 \delta_n / \sin(\theta) = \text{approx.} 0.5 \phi \cotan(\theta) \quad (4.23)$$

Now, this eccentricity can be inserted in eq. $(4.20b)$. The result is shown in Fig. 4.27 for the ratio $f_{sy/ccm}$ equal to 10. The dowel force is taken proportional to the dowel force for a bar perpendicular to the crack plane with zero-eccentricity. It is shown, that for the range of $\theta$ investigated by Dulacska [15], eq. $(4.23)$ provides a reasonable prediction of the experimental results. Eq. $(4.23)$ is in close agreement with the reduction according to $\sin^2(\theta)$ as proposed by Mattock [40]. It can be concluded that the influence of small inclinations can be taken into account in eq. $(4.20b)$ with an additional eccentricity and multiplying the resulting dowel force with $\sin(\theta)$.

b. Angle of inclination in the range of $90^\circ$ to $180^\circ$.

Fig. 4.28 presents a bar with an angle of inclination $\theta$ in the range of $90^\circ$ to $180^\circ$. Now, the additional eccentricity has a negative sign. It can be expected, that the response of the concrete to lateral bar displacements is stiffer than for bars perpendicularly crossing the crack plane. However, because of the fact that this increase is limited to the concrete close to the crack plane, the increase in stiffness is less pronounced than the decrease in stiffness for small angles of inclination. The minor increase of concrete stiffness will be neglected here.
The eccentricity of the dowel load can be expressed by:

\[ e = -0.5 \phi \tan(\theta - 0.5 \pi) \]  \hspace{1cm} (4.24)

Simultaneously with the dowel force, an axial force develops in the bar, influencing the magnitude of the dowel force according to eq. (4.14). The axial force is equal to \( F_d \tan(\theta - 0.5 \pi) \). Thus an implicit expression is obtained for the dowel force by inserting this axial force into eq. (4.21) and combining it with eq. (4.20b). Therefore, this expression is solved numerically. Fig. 4.29 presents the dowel force as a function of the angle of inclination. The ratio \( f_{sy}/f_{ccm} \) is taken equal to 10. The dowel force is related to the dowel resistance of a bar perpendicularly crossing the crack plane with zero-eccentricity. The externally measured force consists of contribution of the dowel force and axial steel force. It can be concluded, that the dowel force of a bar with a large angle of inclination can be calculated according to eq. (4.20b) with an additional eccentricity.

4.4. The combined mechanism of aggregate interlock and dowel action.

Vintzeleou [75] and Millard [46] have already demonstrated that the combined mechanism of aggregate interlock and dowel action is suitable for predicting the shear resistance of cracked concrete according to the equilibrium condition shown in Fig. 4.30. However, for push-off experiments with a very small initial crack width as performed by Walraven...
[85], their models underestimate the measured shear strength of the crack.

Fig. 4.30. The equilibrium condition for cracked reinforced concrete.

Furthermore, for practical use the equilibrium presented in Fig. 4.30 can only be determined if the relation between the axial bar force and the crack width is known. The magnitude of the axial bar force can be calculated by means of the equilibrium with the normal force due to aggregate interlock. The bond characteristics for a bar subjected to the combined action of axial and dowel forces are not yet determined experimentally. Due to the lack of knowledge about the actual bond behaviour, empirical relations are still used describing the crack opening path.

In this Section, the previously described mechanisms will be combined taking into account the magnitude of the initial crack width. In order to accurately predict the crack response, the effect of these mechanisms and of their interactions upon the crack opening direction must be known.

The crack opening path obtained in push-off tests on pre-cracked specimens showed a large scatter, see Section 2.7. However, it was found by Walraven [85], that the crack opening path is hardly influenced by the reinforcement ratio, nor by the bar diameter. There was some influence of the concrete strength on the direction of the crack opening (see Fig. 2.41b; mixes 1, 3 and 4).

Furthermore, some influence of the maximum particle diameter was observed (mixes 1 and 5). This indicates that the aggregate interlock mechanism determines (partially) the crack opening path, because the dowel action is not related to the maximum particle size. Therefore, it is important to know how the aggregate interlock mechanism influences
the crack opening direction in a plain concrete push-off specimen. For the experimental test series on cracked plain concrete performed by Walraven [85], some typical results are shown in Fig. 4.31. In these tests a small initial crack width (< 0.1 mm) was used. It appeared that the crack opening path followed during the tests resulted in a more or less constant shear stress after some shear sliding. The tests were carried out load-controlled. Apparently, the increase of the crack displacements is accompanied by a decreasing increase of the shear stress. A slight decrease of the shear stress is observed only in a few tests.

Assuming that this holds true for reinforced specimens, the crack opening path followed during the tests will provide a constant contribution of the aggregate interlock to the transferred shear stress. According to this, the crack opening path for different shear stress levels (transferred by aggregate interlock) can be drawn, see Fig. 4.4 in Section 4.2.

Indeed, the crack opening paths obtained in Walraven's tests fit reasonably with the calculated crack opening paths, see Fig. 4.32. Furthermore, the calculated crack opening direction explains the difference in crack opening paths obtained in the experiments of Walraven [85] and Millard [46]. The maximum particle diameter used by Walraven and by
Millard was 16 mm and 10 mm respectively. Accounting for the particle diameter, the crack opening paths can be calculated, see Fig. 4.33, which is in good agreement with Fig. 2.42.

However, it appeared from the test results that the aggregate interlock mechanism provides a prediction for the crack opening direction for constant shear stress contribution only. For the phase with an increasing contribution of the aggregate interlock to the shear transfer, the plastic hinge in the reinforcing bars is still developing. Therefore, the bars will also influence the crack opening direction. To account for this influence, the deformation of the bar due to the dowel force is roughly approximated as presented in Fig. 4.34, in which the lateral bar displacement (shear sliding) is assumed to be caused by a rotation of the bar around the plastic hinge. Now, the crack opening direction can be expressed by:

\[ \frac{\Delta \delta}{\Delta \delta_n} = \frac{X + 0.5\phi}{0.5 \phi} \approx \frac{2X}{\phi} \]  

(4.25)

The magnitude of X is expressed empirically on basis of Rasmussen's experiments.
Thus,  

$$X = 0.318 \phi \sqrt{f_{sy}/f_{ccm}}$$ \hfill (4.26)  

$$\frac{\Delta \delta}{\Delta \delta_n} = 0.736 \sqrt{f_{sy}/f_{ccm}}$$ \hfill (4.25a)  

Eq. (4.25a) is a rough approximation of the actually occurring crack opening direction. It is obvious, that for instance the initial crack width or the initial steel stress will influence the crack opening path. Therefore, an empirical relation is derived describing the crack opening path for increasing shear stress. Indeed, it was found that the crack opening path is determined by the ratio $f_{sy}/f_{ccm}$ and by the initial crack width:  

$$\delta_t = \frac{\delta_{no}}{2 f_{ccm}} (\delta_n - \delta_{no})^{0.667} \ [\text{mm}]$$ \hfill (4.27)  

with $\delta_{no}$ = initial crack width  

$\delta_t, \delta_{no}, \delta_n$ in [mm], $f_{ccm}, f_{sy}$ in [MPa]  

Fig. 4.35. Theoretical and experimental crack opening paths.
ing direction is fully determined by the aggregate interlock mechanism, as is shown in Fig. 4.35a. During this phase, the equilibrium condition in the bar is determined by the axial steel force. The bar itself does not influence the crack opening path, due to the plastic hinge, which is now fully developed. In the plastic hinge each combination of the axial bar force and the dowel force according to eq. (4.21) is possible. Fig. 4.35c presents a comparison of the calculated crack opening paths with experimentally obtained opening paths of Millard [46], Mattock [41] and Walraven [85]. There is a reasonable agreement between the calculated and experimental crack opening paths. During the phase of increasing shear stress, the contribution of the dowel action mechanism to the shear also increases. An empirical relation is given by:

\[
\frac{F_d}{F_u} = 3 \sqrt{\frac{\delta_t}{\delta_{t,e}}} < 1 \quad (4.28)
\]

with \( \delta_{t,e} \) = maximum slip occurring during the elastic deformation.

For very small values of the initial crack width a minimum value of 0.1 mm is used, accounting for the larger crack width, which occurs during pre-cracking the specimen. From the experimental results of Vintzeleou [75], it was found that \( \delta_{t,e} \) was obtained for a dowel force equal to 41 percent of the ultimate dowel force. Inserting this value into eq. (4.2) yields:

\[
\delta_{t,e} = \frac{2 F_d}{\beta^3} \left[ \frac{\delta}{\sqrt{K_f \phi}} \right] \quad [\text{mm}] \quad (4.29)
\]

with \( \beta = \frac{\sqrt{K_f \phi}}{4E_I} \)

According to Fig. 4.9 the foundation modulus of the concrete can be approximated by: (Note that \( \delta_{t,e} = 2\delta_t \) in Fig. 4.9)

\[
K_f = 390 \delta_{t,e}^{-0.78} \quad [\text{MPa/mm}] \quad (4.30)
\]

Now, eq. (4.29) becomes:

\[
\delta_{t,e} = 1.31 \times 10^{-7} \delta_{t,e}^{0.60} \left( E_{ccm} f_{sy} \right)^{1.2} \quad [\text{mm}] \quad (4.31)
\]
For a large initial crack width, eq. (4.31) becomes of minor importance proportional to the crack width.

Now, the crack opening path can be described according to the transfer mechanisms. However, the bars crossing the crack plane restrain the crack opening due to the normal stress caused by the mechanism of aggregate interlock. For increasing crack displacements this normal stress can become so high, that the restraining force in the bars is equal to the yield force. A further increase in crack displacements should fulfill the equilibrium condition according to Fig. 4.29. Therefore, it was expected that the crack opening direction was now determined by a constant contribution of the aggregate interlock mechanism to the normal stress. However, it was found experimentally [85], that the crack opening path was hardly affected by the yielding of the bars. This can be easily explained by the additional condition to the two-phase model as presented in Fig. 4.6. In the case of yielding of the bars, the increase in crack width exceeds the increase in shear slip, thus causing a dramatic decrease in the magnitude of the contact areas. In consequence, the normal stress decreases and makes equilibrium with the yield strength in the bars. Due to the decrease in contact areas the shear stress decreases also. According to the two-phase model this decrease is less pronounced than the decrease of the normal stress. However, for an easy calculation method, the reduction in shear stress is related to the decrease in normal stress. This yields:

\[
\gamma_a = \frac{f_{sy}}{\sigma_a} = \frac{\rho f_{sy}}{\sigma_a}
\]

Now, the combined mechanism will be shown by means of an example of Walraven's test specimen No. 110208t. For this test, the cube compressive strength was 35.9 N/mm². The maximum particle diameter was 16 mm. The steel yield strength was equal to 460 N/mm². Four 8 mm diameter bars were used, corresponding to a reinforcement ratio of 0.0056. The externally measured shear stress consists of the contributions of dowel action and aggregate interlock according to:

\[
\tau_{cal} = \gamma_a \tau_a + \gamma_d \tau_d
\]

\[
\sigma_{cal} = \gamma_a \sigma_a
\]
with $\tau_a$ according to eq. (4.1a), $\sigma_a$ according to eq. (4.1b), $\gamma_a$ according to eq. (4.32), $\tau_d$ according to eq. (4.20b) ($F_d$/shear area), $\gamma_d$ according to eq. (4.21)

Table 4.1 lists the results of the calculation for test No. 110208t. The theoretical crack opening path is calculated according to eq. (4.27) and according to Fig. 4.4 for $\tau_a$ equal to 3.8 MPa. The theoretical results are in reasonable agreement with the experimental results. This holds true for both the crack opening path and the shear stress - crack width relation. Both relations are shown in Fig. 4.36a-b.

Table 4.1. The calculated results of test No. 110208t.

<table>
<thead>
<tr>
<th>$\delta_n$</th>
<th>$\delta_t$</th>
<th>$\tau_a$</th>
<th>$\sigma_a$</th>
<th>$\tau_d$</th>
<th>$\gamma_d$</th>
<th>$\gamma_a$</th>
<th>$\tau_{cal}$</th>
<th>$\tau_{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>mm</td>
<td>N/mm$^2$</td>
<td>N/mm$^2$</td>
<td>N/mm$^2$</td>
<td>N/mm$^2$</td>
<td>N/mm$^2$</td>
<td>N/mm$^2$</td>
<td>N/mm$^2$</td>
</tr>
<tr>
<td>.02 .00</td>
<td>.00 .00</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>.05 .03</td>
<td>.03 .09</td>
<td>0.9</td>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>1.4</td>
<td>2.5</td>
</tr>
<tr>
<td>.10 .07</td>
<td>.07 .33</td>
<td>3.3</td>
<td>0.2</td>
<td>0.7</td>
<td>.99</td>
<td>1.0</td>
<td>4.0</td>
<td>3.8</td>
</tr>
<tr>
<td>.20 .17</td>
<td>.17 .38</td>
<td>3.8</td>
<td>0.7</td>
<td>1.1</td>
<td>.96</td>
<td>1.0</td>
<td>4.9</td>
<td>4.5</td>
</tr>
<tr>
<td>.30 .28</td>
<td>.28 .38</td>
<td>3.8</td>
<td>1.0</td>
<td>1.2</td>
<td>.92</td>
<td>1.0</td>
<td>4.9</td>
<td>5.0</td>
</tr>
<tr>
<td>.40 .40</td>
<td>.40 .38</td>
<td>3.8</td>
<td>1.1</td>
<td>1.2</td>
<td>.90</td>
<td>1.0</td>
<td>4.9</td>
<td>5.1</td>
</tr>
<tr>
<td>.50 .57</td>
<td>.57 .38</td>
<td>3.8</td>
<td>1.3</td>
<td>1.2</td>
<td>.86</td>
<td>1.0</td>
<td>4.8</td>
<td>5.0</td>
</tr>
<tr>
<td>.60 .71</td>
<td>.71 .38</td>
<td>3.8</td>
<td>1.5</td>
<td>1.2</td>
<td>.81</td>
<td>1.0</td>
<td>4.8</td>
<td>4.9</td>
</tr>
<tr>
<td>.70 .88</td>
<td>.84 .38</td>
<td>3.8</td>
<td>1.7</td>
<td>1.2</td>
<td>.75</td>
<td>1.0</td>
<td>4.7</td>
<td>4.7</td>
</tr>
<tr>
<td>.80 1.10</td>
<td>1.00 .38</td>
<td>3.8</td>
<td>1.8</td>
<td>1.2</td>
<td>.72</td>
<td>1.0</td>
<td>4.7</td>
<td>4.4</td>
</tr>
<tr>
<td>.90 1.30</td>
<td>1.20 .38</td>
<td>3.8</td>
<td>2.1</td>
<td>1.2</td>
<td>.58</td>
<td>1.0</td>
<td>4.5</td>
<td>3.9</td>
</tr>
<tr>
<td>1.00 1.50</td>
<td>1.40 .38</td>
<td>3.8</td>
<td>2.2</td>
<td>1.2</td>
<td>.52</td>
<td>1.0</td>
<td>4.4</td>
<td>---</td>
</tr>
</tbody>
</table>

Fig. 4.36c presents the comparison between a few experimental and calculated results. The calculations for these tests are carried out in the same way as has been done for specimen No. 110208t. The agreement between the calculated and experimental results is quite satisfactory.
4.5. Influence of the normal restraint stiffness upon the shear stiffness.

In the previous Section, it is shown that for reinforced push-off specimens the crack opening path and the shear stresses transferred across the crack can be calculated according to the model of the combined action of aggregate interlock and dowel action. However, the equilibrium condition in such a specimen generally deviates from the equilibrium condition in practical cases, see Fig. 4.37a–c.
Case a in Fig. 4.37a represents the restraint condition of a push-off specimen as used by Walraven [85] and Millard [46]. Case b represents a shell element, which is cast between two large elements. Due to shrinkage and thermal deformation, cracks will arise in the element. When a shear load is subsequently applied to the element, shear displacements will occur in the cracks. Due to the action of the bars, the cracks will simultaneously open. Due to this crack opening, the concrete between two cracks is unloaded, reducing the steel stress in the bars. As a consequence, the crack opening has a crack closing effect.

Case c represents a large constant normal force on a crack. In fact this is a special situation of case b.

Because of the fact, that case b represents most of the practical cases, it will be shown in what way the model can be applied to this case.

Fig. 4.38. Wall cast between two storage tanks.

Fig. 4.38 presents a wall, which is cast between two storage tanks. Due to shrinkage, the strain in the element is $300 \times 10^{-6}$. A subsequent drop in temperature of 20 degrees causes an additional strain of $240 \times 10^{-6}$. This thermal deformation is not followed by the tanks, due to the temperature of the fluid in the tanks.

Due to this shortening, the element will have an almost fully developed crack pattern, see Fig. 4.39. The model, which is used to describe the bond behaviour, is not discussed here, because of its minor importance. The calculated crack width is 0.106 mm due to an axial steel stress of 152 MPa. The mean crack distance is equal to 190 mm.
After cracking, the wall is subjected to a top load causing shear stresses in the cracks. As a consequence, the crack faces slide with respect to each other. From the model presented in the previous sections, it is found that shear sliding in the crack is accompanied by a crack opening according to eq. (4.27). According to the mechanism (eqs. (4.28) and (4.31)), the plastic hinges in the bars are fully developed after a shear displacement equal to $0.106 + 0.088 = 0.194$ mm. Such a crack response is obtained for case a. However, for case b any increase in the crack width causes a decrease in the tensile stress in the concrete between the cracks.

To determine the crack response for case b, first the initial crack response for case a is determined, see Table 4.2. In Table 4.2 the decrease in steel stress $\Delta \sigma_{s,c}$ due to the decrease in concrete stress between the cracks is presented. This decrease is calculated according to Fig. 4.39a. Furthermore, the increase in steel stress $\Delta \sigma_{s,cr}$ due to $\sigma_a$ is presented.

For any crack width and constant bond characteristics of the reinforcing bars, the normal stress in the crack must be in equilibrium with $\Delta \sigma_{s,c} + \Delta \sigma_{s,cr}$. The equilibrium with $\Delta \sigma_{s,c}$ is necessary because otherwise the element is unloaded and the crack closes. The equilibrium with $\Delta \sigma_{s,cr}$ is necessary according to the model of dowel action. It must be noted that the bond behaviour of the bars changes during shear sliding, as is already shown in Section 2.6. Due to this, the axial steel stress in the cross-section of the bar situated in the crack decreases. A gen-
erally applicable description of the bond behaviour of a bar subjected to both axial and dowel forces is not yet derived. In the example presented here, the change in bond behaviour is not taken into account. For this particular case, only slight differences with the actual response will occur, due to the very high stiffness of the springs representing the concrete between the cracks. Because of this relationship between the crack width and the total change in steel stress as listed in Table 4.2, the crack opening path for case b can be calculated, see Table 4.3.

Table 4.2. Crack response for case a.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>0.02</td>
<td>0.32</td>
<td>-0.04</td>
<td>1.55</td>
<td>1.00</td>
<td>0.89</td>
<td>45</td>
<td>-2</td>
</tr>
<tr>
<td>0.12</td>
<td>0.05</td>
<td>1.42</td>
<td>-0.12</td>
<td>2.10</td>
<td>1.00</td>
<td>0.89</td>
<td>157</td>
<td>-8</td>
</tr>
<tr>
<td>0.13</td>
<td>0.07</td>
<td>2.24</td>
<td>-0.08</td>
<td>2.36</td>
<td>1.00</td>
<td>0.88</td>
<td>269</td>
<td>-6</td>
</tr>
<tr>
<td>0.14</td>
<td>0.08</td>
<td>2.82</td>
<td>0.06</td>
<td>2.47</td>
<td>1.00</td>
<td>0.86</td>
<td>382</td>
<td>4</td>
</tr>
<tr>
<td>0.15</td>
<td>0.10</td>
<td>3.28</td>
<td>0.17</td>
<td>2.65</td>
<td>1.00</td>
<td>0.85</td>
<td>494</td>
<td>11</td>
</tr>
<tr>
<td>0.16</td>
<td>0.11</td>
<td>3.59</td>
<td>0.27</td>
<td>2.74</td>
<td>1.00</td>
<td>0.85</td>
<td>606</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 4.3. Crack response for case b.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>0.10</td>
<td>45</td>
<td>-2</td>
<td>0.67</td>
<td>5.0</td>
<td>0.03</td>
<td>3.28</td>
</tr>
<tr>
<td>0.12</td>
<td>0.16</td>
<td>157</td>
<td>-8</td>
<td>2.24</td>
<td>8.0</td>
<td>0.04</td>
<td>3.30</td>
</tr>
<tr>
<td>0.13</td>
<td>0.24</td>
<td>269</td>
<td>-6</td>
<td>3.95</td>
<td>10.4</td>
<td>0.04</td>
<td>3.30</td>
</tr>
</tbody>
</table>

The decrease in external normal stress on the crack plane can be taken into account by reducing the initial crack width with Δδt,e. This change in initial crack width can be calculated because the crack displacement in Table 4.3 must be compatible with the crack displacements according to eq. (4.27). As a consequence, the ultimate dowel force is reached at a shear displacement equal to δt,e + δt,e - Δδt,e. According to the model, any further crack opening will be determined by the crack opening direction for a constant contribution of the aggregate interlock mechanism to the transfer of shear stress. However, in this case the influence of the
external normal stress is larger than the normal stress according to the dowel action mechanism ($\Delta \sigma_{s,cr}$). As a consequence, the crack opening path is determined according to the equilibrium between the internal normal stress due to aggregate interlock $\sigma_a$ and the external normal stress according to line A-B in Fig. 4.39. The crack response for the cases a and b is shown in Fig. 4.40.

![Fig. 4.40. Calculated crack response.](image)

According to the calculation method presented in this Section, the effect of the external springs in case b can be accounted for by means of a decrease or increase (for an increase in tensile stress) of the initial crack width. For most of the practical cases the stiffness of the external springs is so large that the crack opening path is fully determined by the equilibrium of the normal stresses in the crack.

It must be born in mind that the proper bond characteristics must be known.

### 4.6. Additional detailed tests.

In addition to the main test program described in Chapter 3, some push-off tests were performed to verify experimentally the dowel action mechanism, which is developed in Section 4.3. These tests are fully described in [21]. The specimen geometry and the test procedure were as described in Chapter 3.

However, now the steel strain of the reinforcing bars was recorded by means of a bolt gauge, type BTM-8 of Tokyo Sokki Kenkyojo. Half of all the bars was prepared with a gauge of this type, which was cemented at
the neutral axis of the bar near the crack plane, see Fig. 4.41. The overall dimensions of the gauge were 20 mm x 2 mm with a working length of 8 mm x 1 mm [71].

A total of six push-off tests was performed. Three series of two specimens were subjected to static, sustained and repeated shear loading. The results of the static tests will be briefly discussed here.

In theory, the bolt gauges, which were in the neutral axis of the bare bar, reflected the strain due to the equilibrium with the normal stress caused by aggregate interlock. However, according to the model of dowel action the neutral axis was shifted to the supported side of the bar. In consequence, the ends of the strain gauges were situated in regions of the bar, which were highly strained. According to the model, the plastic hinges are situated rather close to the crack plane, so that the ends of the gauge nearly reached the yield strain. Thus, the average recorded strain according to the model will be considerably larger than is necessary to make equilibrium with the normal force due to aggregate interlock. Indeed, this phenomena was observed in the experiments. Taking into account the yield strength at the ends of the gauge, the average normal stress according to the model agrees reasonably with the experimental results, see Fig. 4.42.

Due to the plastic hinges, which are apparent in the bar according to the dowel action model, a marked localization of the elongation of the steel occurred during shearing-off. Therefore, a microscopic examination of the steel crystals at the vicinity of the crack plane was performed. The theoretical shift of the neutral axis could be roughly determined by means of this localized orientation. The bars, which were not prepared with a bolt gauge, were carefully removed after testing. A total of 22 specimens was obtained from the six push-off specimens. Ignoring the
results indicating yield of the whole cross-sectional area of the bars, the majority of the observations (67 percent) supported the shift of the neutral axis and thereby the model developed. There is, however, more detailed information necessary for a solid physical proof of this mechanism.

4.7. Concluding remarks.

The transfer of static shear stress in cracked reinforced concrete depends upon the aggregate interlock mechanism and dowel action. The interaction between both mechanisms is determined by the equilibrium of forces normal to the crack plane. The dowel action is depending upon the bond between the bar and the concrete, thus causing a shift of the neutral axis of the bare bar. This results in a strong increase of the plastic moment relative to the plastic moment of the bare bar. For an increasing axial steel force, this shift decreases, thus reducing the dowel strength. The crack opening path is determined by the deformation of the bar until the plastic hinges in the bar have fully been developed. Subsequently, the crack opening direction is related to a constant contribution of the aggregate interlock mechanism to the shear transfer. The combined model presented in this Chapter can be applied to cracked reinforced concrete subjected to static shear loads irrespective of the magnitude of the initial crack width. The effect of external springs
normal to the crack plane can be accounted for by means of an increase or decrease of the initial crack width.
5. THEORETICAL MODELLING OF THE RESPONSE OF CRACKED CONCRETE TO
REPEATED AND REVERSED SHEAR LOADING

5.1. Introduction.

The static models of aggregate interlock and dowel action, which are de­
scribed in Chapter 4, will be adapted to the case of a repeated or a re­
versed shear load. Although the crack response remains essentially the
same, a distinction is made between 'high-intensity low-cycle' fatigue
and 'low-intensity high-cycle' fatigue. This distinction is made for
practical reasons. For 'low-intensity' tests as described in Chapter 3,
the increments of the crack displacements per cycle can be far less than
the numerical accuracy of any numerical program. Therefore, this type of
test cannot be analysed by calculating all subsequent load cycles. In
consequence, the physical models must describe the over-all response de­
gradation and irreversible deformations of the concrete due to cycling.
The same holds true for analyzing the response of large-scale structures
to millions of load cycles with a low amplitude. Over-all characteris­
tics, such as reduced shear stiffness and increased crack displacements,
will then be used in numerical programs for analyzing the response of
the structure to subsequent load cycles with a very high amplitude. For
these 'high-intensity' cycles, the displacement increments can be deter­
mined accurately in a numerical program. Therefore, in this Chapter
'low-intensity' fatigue will be treated by means of the description of
the over-all behaviour. On the other hand, 'high-intensity' fatigue will
be analysed in detail by means of the physical transfer mechanisms.
As in Chapter 4, first the transfer mechanisms will be dealt with sepa­
rately. Finally, the response of cracked reinforced concrete to repeated
and reversed shear loading is analysed.

5.2. The mechanism of aggregate interlock.

The analyses of the crack response to 'high-intensity' fatigue provides
a deep insight in the physical behaviour of cracked concrete, because of
the relatively large crack displacements in a load cycle. Therefore,
this type of fatigue will be discussed firstly. In 1980 [81], Walraven
already described qualitatively the response of a crack in plain con-
crete to cyclic shear loading. According to his two-phase model, the crack response can be roughly described monitoring the displacements of a single particle during sliding, see Fig. 5.1.

Fig. 5.1a shows that the ascending branch of the first cycle can be described with the static two-phase model. After reaching the maximum applied shear stress, the shear load is decreased. Consequently, the normal restraint force will re-close the crack. However, the friction in the contact area will counteract this displacement to a certain extent. Walraven derived the following expression for the reduction in shear stress before any displacement backwards can occur, see Fig. 5.1b:

\[
\frac{\tau}{\tau_m} = \frac{A_x - \mu A_y}{A_x + \mu A_y} \frac{A_x - \mu A_y}{A_x + \mu A_y}
\]

(5.1)

A further decrease in shear load forces the crack to close until the initial crack width is obtained (then the normal force becomes zero). Simultaneously, the shear displacement decreases, but reaches not its original value due to the deformed matrix material, see Fig. 5.1c. This shear displacement can be determined by means of the static two-phase model, inserting the initial crack width and zero-shear stress. Reducing the shear displacement to its original value only some friction due to
rubble in the crack plane causes a low shear stiffness. The crack response during sliding in the opposite direction is similar to the previously described, see Fig. 5.1d. During re-loading in the following cycle, the shear displacement increases without (hardly) any shear load, see Fig. 5.2a. This free slip is caused by the already deformed matrix material. A further increase in shear load brings the particle in firm contact with the opposing crack face, see Fig. 5.2b. The contact area can now, however, not be calculated according to the analytical two-phase model.

The numerical contact model.
Walraven modified the calculation of the contact areas of the particles (1986, [83]) by replacing the analytical solution by a numerical solution. Now, the shape of the contact area of each particle in the crack plane is monitored by means of several points situated on the surface of the deformed matrix material, see Fig. 5.3.
For a specific combination of the crack displacements, the highest and the lowest point, which are situated in the contact area, can be determined. Now, the projected contact areas $a_x$ and $a_y$ can be determined as the distance between those points in parallel and normal direction respectively. Taking into account several particle diameters and embedment depths, the total contact areas per unit area of the crack plane $A_x$ and $A_y$ can be calculated as in the original analytical two-phase model. Again, the presence of a specific particle is accounted for by a probability density function [81]:
Fig. 5.3. Representation of the contact area by means of points situated in the contact area [83].

\[ p(D_o) = \frac{p_k}{D_o} (0.532x^{0.5} - 0.212x^4 - 0.075x^6 - 0.036x^8 - 0.025x^{10}) \]  

(5.2)

with \( p_k = \text{volume of particles/total volume} \)

\[ x = \frac{D_o}{D_{\text{max}}} \]

This modification of the two-phase model will be denoted here as the numerical contact model. A listing of the program is presented in Appendix III.

With this model, Walraven simulated the cyclic push-off tests of Laible [36,37]. The particles used in this test series had a moderate strength. Consequently, the number of particles fractured during cracking of the concrete was large with respect to the number of fractured glacial river particles used by Walraven. Hence, a reduction of the total contact areas must be taken into account. Furthermore, the coefficient of friction must be adjusted to the proper value for this type of aggregate. From the experimental results of the first cycle of the test, Walraven derived a reduction factor of 0.75 and a coefficient of friction equal to 0.20. Inserting these values into the model, the subsequent cycles are simulated, see Fig. 5.4. For reasons of symmetry, only the results in the positive direction are shown. A good agreement between the exper-
imental and the calculated results is found. It appeared from the calculation that no reduction of the matrix strength due to cycling has to be taken into account. This is probably due to the fact that the high contact stresses are only apparent in the contact areas. In Laible’s tests, the displacement increments in each cycle are large enough to crush the previously loaded matrix material and to subsequently load new material.

Fig. 5.4. Comparison of the experimental result of test No. Al of Laible with the numerical contact model [83].

In the calculation process, ten different particle diameters are used, reflecting the gradation curve. Each particle diameter was embedded at ten different depths, thus yielding one hundred particles to be taken into account. The contact area of each particle was monitored by means of ten points. Hence a total of two thousand coordinates determines the total contact areas. The experimentally obtained normal restraint stiffness was input data for the program.

Walraven demonstrated that the numerical contact model is suitable for describing the crack response to cyclic and repeated shear loads. This model can be used to perform sensitivity studies, but is too complex for implementation in advanced finite element programs. Because of the fact that Walraven already solved the problem of simulating the crack response to cyclic 'high-intensity' shear loading, most effort will be devoted here to a sound simplification of the model in order to speed up the calculation process.
The analytical contact model.

As already stated in Section 5.1, the physical model must describe the over-all response degradation and the irreversible deformations due to 'low-intensity' load cycles. In consequence, only a part of the load history will be accounted for in subsequent 'high-intensity' load cycles. Therefore, it is assumed that the load history is fully incorporated in the contact areas, which are formed in the last load step of the previous cycle. This assumption is shown in Fig. 5.5.

Fig. 5.5. The load history incorporated in the end-deformation of the crack plane.

An important consequence of this assumption is that it holds true for all the load steps in a cycle, see Fig. 5.6.

When this assumption is valid, the magnitude of the contact areas can be determined analytically by means of the intersection points of three circles, see Fig. 5.6. Now, only three pairs of coordinates of the origin of the circles determine the contact area. Again ten particle diameters and ten embedment depths are used. This version of the model is denoted as the analytical contact model, which is listed in Appendix III. For test No. Al of Laible [36], the model is compared with the experimental results in Fig. 5.7.

Fig. 5.6. The assumption applied to a given load step.
Input in the program was the end displacement of the first load cycle. Therefore, the first cycle is not simulated. It appeared that the crack response is satisfactorily described even if the three pairs of coordinates of the three circles remain the same for all particle diameters. Therefore, six coordinates determine the total contact areas in normal and parallel direction. The calculation process is about twice as fast as for the numerical contact model. However, the model is still too complex for implementation in finite element programs.

The reduced contact model.
A further simplification can be obtained by reducing the number of particle diameters and embedment depths. There is, however, a more simple method. During cycling, the stresses in the crack plane for a given combination of the crack displacements are as large as or less than in the case of a static test. This is due to the reduced size of the contact areas, thus:

\[
\tau = \sigma \left( \lambda A + \mu \lambda A \right)_{x \times y} \quad (5.3a)
\]

\[
\sigma = \sigma \left( \lambda A - \mu \lambda A \right)_{y \times y} \quad (5.3b)
\]
with $A_x, A_y = $ total contact areas for the static case.

$\lambda_x, \lambda_y = $ reduction factors.

\[ \lambda_x = 0.8 \left( \frac{\delta_t - \delta_0}{\delta_{tm} - \delta_0} \right)^2 \]  
\[ \lambda_y = 0.7 \left( \frac{\delta_t - \delta_0}{\delta_{tm} - \delta_0} \right)^3 \]  
\[ \delta_0 = \delta_{nm} - \sqrt{\delta_{nm}^2 - \delta_{no}^2} < 0.67 \delta_{tm} \]  
\[ \delta_{nm}, \delta_{tm}, \delta_{no} = $ end displacements of previous cycle. 

With the eqs. (5.4a-c), the calculation process becomes quasi-statically. Now, the problem is shifted towards an easy calculation method of the contact areas for the static case. Because the derivation of these expressions forms part of the numerical implementation, these expressions are presented in Chapter 6 (eqs. (6.1a-i)).

Substitution of these equations into the model yields a simple version of the model, which is denoted the reduced contact model. Again test No. Al is simulated, showing good agreement with the experimental results, see Fig. 5.9.
The calculation process is now approximately one hundred times as fast as for the numerical contact model. The Figs. 5.10-5.11 show that the reduced contact model also provided good predictions of the tests with smaller crack widths and higher normal restraint stiffnesses.
In Fig. 5.12, the results of test No. El of Laible is shown. In this test, twenty load cycles with a maximum shear stress up to 0.69 MPa were applied to the specimen, followed by five cycles with a maximum shear stress of 1.24 MPa. Because of the assumption, that the end deformation of the contact areas incorporates the load history, only the cycles Nos. 21 to 25 are simulated. Although the calculated crack response is somewhat too stiff, the over-all behaviour is satisfactorily predicted.
tact model are only valid for load cycles, in which the maximum shear stress is at least equal to the maximum shear stress in any previous cycle. However, cyclic deformation-controlled shear tests, such as performed by Vintzeleou [75], can be easily simulated with these models. Now, the calculation is performed similarly to the case of a stress-controlled test. Afterwards, the last part of each cycle is neglected, see Fig. 5.13a. There is of course a difference with the actually obtained deformation of the contact areas. This difference has, however, only a very small effect upon the observed crack response. The case of repeated loading can also be treated as a fully-reversed shear loading. Now, the first part of the cycle is neglected, see Fig. 5.13b. Again, the actual response is slightly different. For the case of repeated loading, the crack width will not reduce to its initial value, because of the fact that some normal stress can be transferred due to the friction in the remaining contact areas. In consequence, re-loading will cause an initially stiff response of the crack, which is satisfactorily simulated by the method presented in Fig. 5.13b.

![Shear Stress vs Shear Displacement](image)

**Fig. 5.13.** The crack response for the case of deformation-controlled tests and of repeated shear loading tests.

Simulating the above mentioned shear tests, it emerged from the calculation that there was no decrease in matrix strength due to cycling. This is probably due to the fact that the crack displacement increments are relatively large for the case of 'high intensity' fatigue. The high stresses causing fatigue of the matrix material are restricted to the volume close to the contact areas. Due to the increasing displacements,
the previously loaded matrix material is crushed and the matrix material lying behind is then subjected to high contact stresses. This matrix material was subjected to low stresses in the previous cycles, so no fatigue of the material has occurred. For the case of 'low-intensity high-cycle' fatigue, the crack response can theoretically be simulated with the proposed models. Decreasing the initial crack width to 0.15 mm and using a normal restraint stiffness of 7.5 MPa/mm, a shear stress of 3 MPa can be transferred by the crack plane. The crack displacement increase in each cycle rapidly diminishes to become smaller than the numerical accuracy, indicating that the actual displacement increase is even smaller. For this case, the crack displacement increments are very small, so now a reduction of the matrix strength occurs due to cycling. On the other hand, 'high-cycle' tests are generally performed with a rather high loading frequency, which might cause rate-effects and thus increases the matrix strength.

Because of the small displacement increment, the fatigue of the suggested matrix material and the rate effects, empirical expressions must be used to describe the increase in crack displacements due to 'low-intensity' cycles.

For the test series on cracked plain concrete described in Section 3.3, no straightforward relationship between the stress level, the number of cycles and the crack displacements was obtained. Generally, the crack response was initially stiffer than could be expected on basis of the two-phase model, even if the repeated shear load was treated quasi-statically, see Fig. 5.14. In this figure shaded area corresponds to the $\tau_a / \sigma_{pu}$ ratios used in the experiments.

This phenomenon is probably due to the relatively high initial normal stress, which was used in this test series. During cracking of the concrete prior to the actual push-off test, some matrix material and small particles will be completely torn out of the crack faces, as is shown theoretically by Termonia and Meakin [69]. Due to this material, the crack cannot be re-closed to its original value. This rubble transfers the initial normal stress and is thereby pushed into the crack faces. A subsequent shear sliding will force this rubble to act like stiff struts transferring both normal and shear stresses. Five different stages can be distinguished. Initially, the struts only transfer normal stress, see Fig. 5.15a. A shear displacement will cause a rotation of these struts.
In consequence, the struts provide a positive contribution to the transfer of shear stresses and determine the crack opening direction, see Fig. 5.15b. Simultaneously, the transfer of stresses due to aggregate interlock increases. For a given shear displacement, the average orientation of the struts is perpendicular to the crack plane, see Fig. 5.15c. Now, only normal stress is transferred by the struts. A further increase in the shear displacement will cause a rapidly decreasing negative contribution of the struts to the shear transfer. A steep crack opening path is then obtained, see Fig. 5.15d. Finally, the normal stress due to aggregate interlock is in equilibrium with the externally applied normal stress. Now, the struts become inactive, similarly to the case of a very low initial normal stress.

Fig. 5.15. The strut mechanism due to the initial normal stress.
However, the crack opening direction is already determined by the struts. According to the additional condition to the two-phase model, see Section 4.2, a less steep crack opening direction will cause a sharp decrease of the shear stress transferred by the crack plane. Therefore, the steep crack opening path is followed despite the inactivity of the struts.

Some experimental proof of this mechanism can be obtained from the tests on cracked plain concrete. According to the strut mechanism, the ratio of the shear stress to the normal stress transferred by the struts as a function of the shear displacement must be similar for all tests. Indeed, such a constant relationship was found, see Fig. 5.16.

The calculated average length of the struts was approximately 0.7 mm, which is a realistic value for the small particles and matrix material in the crack plane.

The higher the initial normal stress, the deeper the particles are pushed into the crack faces. In consequence, the initial crack opening direction will become less steep for higher normal stresses. For static tests, this phenomenon was observed by Vintzeleou [76], see Fig. 2.17.

For cyclic and repeated shear loads, the struts will cause a nearly constant crack opening path with small crack displacement increments in each cycle. However, when the struts become inactive, the aggregate interlock has to transfer the full shear load, thus causing a rather abrupt and strong increase in crack displacements, as was indeed observed experimentally, see Fig. 3.19.

It can be concluded that for the case of cracked plain concrete with a relatively high initial normal stress, stiff struts contribute to the

![Fig. 5.16. The ratio of the shear stress to the normal stress as function of the shear slip for the plain concrete specimens.](image-url)
stress transfer and initially determine the crack opening direction. Therefore, this type of test cannot be simulated with the proposed cyclic aggregate interlock models alone.

For cracked plain concrete with a low initial normal stress, empirical expressions for the relationship between the shear stress, the number of cycles and the crack displacements can be derived on basis of the tests on cracked reinforced concrete specimens.

5.3. The mechanism of dowel action.

Although numerous cyclic dowel action tests are performed by among others, Eleiott [16], Jimenez [32] and Vintzeleou [75], the large number of observations did not yield a fundamental insight in the physical behaviour of dowels embedded in concrete and subjected to cyclic loads. Therefore, empirical expressions for the dowel stiffness are derived for practical use. Vintzeleou derived an expression for the maximum dowel force for the case of imposed shear displacements, see eq. (2.9). On basis of this relation, she proposed a formalistic model for fully reversed shear displacements as presented in Fig. 5.17. [77].

![Fig. 5.17. Formalistic model for fully reversed shear displacements as proposed by Vintzeleou [77].](image)

Vintzeleou's formalistic model can be easily implemented into numerical programs. There still is, however, a need for a more theoretical approach of the behaviour of a dowel subjected to cyclic loads. Therefore, an attempt is made to derive a model, which is to a large extent based
upon material behaviour. Because of the fact, that previously mentioned dowel action tests did not provide information of the local response degradation of the concrete underneath the bars, this model will also be to some extent based upon empirically derived expressions.

For the case of a monotonic loading, the dowel response can initially be predicted by regarding the bar as a beam on elastic foundation, see Section 4.3. With increasing dowel force a plastic hinge develops. This plastic hinge is situated at a distance to the crack plane, which is approximately equal to the bar diameter. Now, the bar displacement is largely determined by the response of this part of the bar. Underneath the bar, high triaxial stresses support the bar. After reaching the maximum dowel force, the force decreases. Now, the triaxial stress state first rapidly loses its confinement. In consequence, the dowel force can decrease without (hardly) any restitution of the concrete deformation, see Fig. 5.18.

The drop in concrete stress is strongly related to the loading path of the concrete. For triaxially loaded concrete, Van Mier [44] found that at the moment of unloading a sudden stress drop occurs. This will cause a drop in dowel force, which appeared to be approximately 25 percent of the maximum dowel force. After that, the shear displacement decreases with decreasing dowel force. Some non-recoverable shear displacement remains for zero-dowel force, which is about 25 percent of the maximum shear displacement. For a fully reversed dowel force, the deformed matrix material after unloading is shown in Fig. 5.19.

Upon re-loading in the positive direction, the bar initially responds as a beam partially fixed at one side, see Fig. 5.20. For this loading
case, the dowel stiffness is determined by linear elastic material behaviour and can be expressed by:

\[
K_i = \frac{9 \phi^4 E_g}{120 \phi L^2 + 40 L^3} \quad \text{[N/mm]} \quad (5.5)
\]

Unfortunately, the magnitude of \( L \) cannot be derived theoretically. Therefore, Vintzeleou [75], performed dowel action tests on specimens having a cylinder strength of 30 MPa, a steel yield strength of 420 MPa and maximum dowel force equal to 80 percent of the dowel strength. For imposed displacements, the response degradation of the concrete causes an increase of the length \( L \) with cycling. Eq. (2.9) accounts for this decrease. The following empirical relation was derived for \( L \):

\[
L = 3.5 \phi \sqrt{\delta_{t,\text{max}} (n-1)^{0.1}} \quad \text{[mm]} \quad (5.6)
\]

with \( \delta_{t,\text{max}} \) = maximum shear displacement in previous cycle.
\( n \) = number of cycle; Note that the first load cycle in the negative direction is cycle 2. In consequence, the second
cycle in the positive direction is cycle 3. For the case, that the imposed displacement in the negative direction is for instance 50 percent of the displacement in the positive direction, the cycle in the negative direction is cycle 1.5. The second cycle in the positive direction is then cycle 2.5 and so on.

The term \((n-1)^{0.1}\) in eq. (5.6.) accounts for the fatigue of the concrete underneath the bar. Because of the fact that for increasing dowel displacements, previously strained matrix material is crushed and unaffected material is loaded, the number of cycles can be reset to 1 for the case of load-controlled tests.

The stiffness \(K_1\) determines the dowel response until the shear displacement \(\delta_{t,o}\) is reached, see Fig. 5.20. For this shear displacement, the bar is supported by the concrete over the length \(L-\delta\), see Fig. 5.21.

![Fig. 5.21. The dowel stiffness \(K_2\).](image)

Due to the support of the concrete, the dowel stiffness increases to become equal to stiffness \(K_2\). The response of the concrete is approximated regarding the bar as a beam on an elastic foundation. According to eq. (4.30), the coefficient of the subgrade reaction is:

\[
K_f = 390 \ (\delta^*_t)^{-0.78} \quad \text{[MPa/mm]}
\]

However, in this expression the shear displacement does not include the slip related to \(K_1\). Substitution of \(\delta^*_t\) by an average value of 0.3 \(\delta_{t,\text{max}}\) yields:

\[
K_f = 998 \ (\delta_{t,\text{max}})^{-0.78} \quad \text{[MPa/mm]} \quad (5.7a)
\]
Because the subgrade reaction is proportional to the modulus of elasticity, which is approximately proportional to the square root of the concrete grade, the following expression is obtained:

\[
K_f = 168 \sqrt{f_{ccm}} (\delta_{t, \text{max}})^{-0.78} \quad [\text{MPa/mm}] \quad (5.7b)
\]

Combination of eq. (5.7b) and eq. (4.3) yields:

\[
F_d = 166 \phi f_{ccm}^{0.75} \delta_{t, \text{max}}^{0.375} \Delta \delta_t \quad [\text{N}] \quad (5.8)
\]

However, according to Fig. 5.21, there is an eccentricity \( e \) equal to the bar diameter. For this case, the shear displacement is expressed by:

\[
\delta_t = \frac{F_d}{3 B^2 E I} (3 + 68e + 68^2 e^2 + 28^3 e^3) \quad [\text{mm}] \quad (5.9)
\]

For an average value of \( K_f = 300 \text{ MPa/mm} \), it was found that \( 8e \) is approximately equal to 0.6. Now, the total shear displacement is about three times the slip according to eq. (5.8). Accounting for this, the dowel stiffness \( K_2 \) becomes:

\[
K_2 = 55 \phi f_{ccm}^{0.75} \delta_{t, \text{max}}^{0.375} \Delta \delta_t \quad [\text{N/mm}] \quad (5.10)
\]

This stiffness is valid until the plastic hinge develops. Because of the fact, that in this stage there is not yet a contribution of the concrete to the section modulus of the bare bar, the dowel force at the onset of yielding is:

\[
F_{d, \text{sy}} = M / e = 0.1 \phi^2 f_{sy} \quad [\text{N}] \quad (5.11)
\]

Due to the development of the plastic hinge, the bar makes contact with the concrete close the crack plane, see Fig. 5.22. For the coefficient of subgrade for this part of the concrete again eq. (5.7b) can be used. This part of the concrete is now determining the dowel stiffness. According to the observation, that about 25 percent of the maximum shear displacement is non-recoverable, it is assumed that the deformation of the concrete over the length \( L - \phi \) contributes 25 percent of the total shear displacement. Thus, the dowel stiffness \( K_3 \) is expressed by:
Fig. 5.22. The dowel stiffness $K_3$.

An additional condition is that the total dowel force is always less than 85 percent of the dowel force for the monotonic case at the same shear displacement. The total loading path according to the proposed model is shown in Fig. 5.23. Unloading causes a drop in dowel force of 25 percent of the maximum value. Next, the unloading stiffness is equal to $K_3$ until $F_d,s_y$ is reached. Finally, the stiffness $K_2$ is used to obtain zero-dowel force. For the case of a repeated dowel force, the response is calculated according to the model for reversed dowel loads. However, the stiffness $K_3$ is used to connect the zero-stress state to the re-loading branch, see Fig. 5.23a. Apart from that, another restriction should be made. The dowel force is not allowed to exceed the magnitude of 85 percent of the dowel force for the monotonic case, see Fig. 5.23b. The stiffness is then equal to the stiffness $K_u$, which was found for the monotonic case for the given shear displacement.

Fig. 5.23. Loading and unloading according to the model.
Fig. 5.24 presents a comparison of this model with experimental results of Vintzeleou [75] and Jimenez [32]. There is a reasonable agreement with the experimental results. Jimenez's test result shows that the model can also be applied to the case of imposed dowel loads.

From the monotonic dowel tests of Eleiott [16], it is known that the dowel stiffness is strongly influenced by the magnitude of the axial steel stress, see Fig. 2.32. For the case of an axial steel force, the following expression is proposed to reduce the dowel stiffness:

\[ \gamma_k = (1 - \frac{F}{F_{sy}})^{0.5} \]  

(5.13)

![Diagram](image)

**Fig. 5.24.** Comparison of experimental results with the proposed model.

For the case of cyclic dowel action tests with an initial axial steel force, the dowel stiffnesses \( K_1, K_2 \) and \( K_3 \) are simply multiplied by the dowel stiffness reduction factor \( \gamma_k \). In Fig. 5.25 a cyclic dowel test of Eleiott (see Fig. 2.35) is simulated with the proposed model. Cycle No. 16 showed a underestimation of the energy-dissipation. The over-all response is, however, satisfactorily simulated.

It must be born in mind that this model is partially based upon empirical expressions, thus limiting its application.

With respect to the proposed model, a special situation arises when the...
dowel strength $F_{du}$ is obtained in a cycle. Now, the whole cross-sectional area of the bar will yield. For the case of a reversed dowel force, the only restriction is that the force cannot exceed the dowel strength. However, for the case of a repeated dowel force, the yield of the bar strongly influences the bar stiffness. For this case, the bar remains in close contact with the supporting concrete, thus approximating the situation in the first load cycle. Because of the already deformed bar, this can be regarded as a shift of the initial stiffness of the bar, see Fig. 5.26. Now, the force-displacement relation for the static case must be applied.

Fig. 5.26. The model in case of reaching the dowel strength.
For very large fully reversed shear displacements, the bond between the bar and the concrete will diminish. The magnitude of the ultimate dowel force is, however, strongly related to the cooperation of the bar and the supporting concrete. Generally, the high radial contact stresses will prevent slip of the bar relative to the concrete. For the case of large reversed shear displacements, the residual elongation due to the decrease in bond increases. In consequence, the bar shape and the shape of the supporting concrete will be different, thus reducing the cooperation of the bar and the concrete. Although there is no experimental proof found yet, the magnitude of the ultimate dowel force can decrease due to this 'lack of fit', see Fig. 5.27. According to the proposed mechanism presented in Chapter 4, a strength reduction up to approximately 15 percent is possible. For the case of repeated shear loads, the bar remains more or less in close contact with the concrete, thus preventing this reduction.

![Inclined cracks and 'lack of fit'](image)

**Fig. 5.27.** Strength reduction due to 'lack of fit'.

Till now, no cyclic dowel tests have been performed with a 'low-intensity' dowel force. Because of the fact, that for 'high-intensity' dowel tests the response degradation during cycling is very similar to the behaviour for the aggregate interlock mechanism, it is expected that this holds true for 'low-intensity high-cycle' fatigue. For practical use, however, the response of both interacting mechanisms in cracked reinforced concrete is of much more interest. The combination of both the transfer mechanisms will be discussed in Section 5.4.
5.4. The combined mechanism of aggregate interlock and dowel action.

The mechanism of aggregate interlock for the case of cyclic shear loads, as presented in Section 5.2 and the mechanism of dowel action for the cyclic loads as presented in Section 5.3, will be combined to describe the case of cyclic shear loads applied to cracked reinforced concrete. In Chapter 4, it was shown how these mechanisms influence the crack opening direction and the relation between the stresses and displacements in the crack. It appeared that the bond characteristics obtained in an ordinary pull-out experiment, cannot be applied to this case. Fortunately, the static crack opening path can be determined without exactly knowing the bond characteristics of the reinforcing bars.

In Section 5.2, Laible's cyclic aggregate interlock tests have been satisfactorily described by the proposed aggregate interlock model. However, the relationship between the normal stress and the crack width was input in the calculations. Therefore, this relationship must also be known for the case of cyclically loaded cracked reinforced concrete. The normal stress was not recorded during the tests reported in the literature.

In order to obtain information on this normal restraint stiffness for reinforced concrete, some additional tests were performed with bars prepared with a bolt gauge situated in the crack plane. The testing procedure is roughly discussed in Section 4.5 and in detail in [21]. Apart from the static tests, two specimens were subjected to a repeated shear

\[
\begin{align*}
\sigma_n &= 50 \text{ MPa} \\
\delta_n &= \text{crack width}
\end{align*}
\]

Fig. 5.28. The relation between normal stress and crack width during cycling [21].
load. From these tests, it appeared that the relation between the normal stress and the crack width is almost linear, see Fig. 5.28. However, as stated before in Section 4.5, the magnitude of the measured normal stress is largely influenced by the yield strength in the plastic hinges. Therefore, a linear relationship between the normal stress and the crack width is assumed. The magnitude of the normal restraint stiffness is, however, derived from Fig. 4.38b. It was found, that the normal stress can be approximately related to the crack width according to:

\[ \sigma_n = a f_{sy} (\delta_n - \delta_{n,0}) \quad \text{[MPa]} \]  

(5.14)

with \( a = 0.25 \) \text{[mm}^{-1}] \) for 'low-intensity high-cycle' fatigue.

For the case of 'high-intensity' fatigue, the normal stiffness will be initially higher due to the large displacements, but will decrease in the subsequent cycles to the value of 0.25 \( f_{sy} \). Cyclic pull-out tests on similar bars [21], show a decrease of the bond behaviour during cycling, see Fig. 5.29. Assuming a similar response degradation of the bond for the bars used in the push-off elements, \( a \) is expressed by:

\[ a = \frac{3}{(n+2)} > 0.25 \]  

(5.15)

with \( n = \) number of cycles (fully reversed)
The normal stiffness according to the eqs. (5.14)-(5.15) was applied to the tests of Jimenez et al. [37]. The cylinder compressive strength of the concrete used was approximately 23 MPa, the steel yield strength was 455 MPa. In test No. 5, the initial axial steel stress was 331 MPa. The specimen was reinforced by means of four 22 mm diameter bars (ρ = 1.08%).

The experimentally obtained and the calculated response for cycle number 15 is shown in Fig. 5.30. The calculated response is in reasonable agreement with the experimental result, although the calculated response is a little bit too soft. The end-displacements in each cycle are satisfactorily predicted, see Fig. 5.30. The load was fully reversed, but only the response in the positive direction is shown. The crack response in the opposite direction might be slightly different due to the position of the bars with respect to the casting direction. Because of this, the bars can have a different support of the concrete in both directions.

![Graphs showing experimental and calculated responses](image)

**Fig. 5.30.** Test No. 5 of Jimenez [37] compared with the proposed model.

The calculation yielded information about the contributions of both transfer mechanisms to the externally applied load. It appeared that the contributions of both mechanisms remained nearly constant during cycling. For this experiment, the aggregate interlock mechanism transfers approximately two-third of the total shear load.

An important criterion for a model is its sensitivity to the magnitude
of the load or displacement increments used in the calculation process. For the calculation shown in Fig. 5.30, the displacement increment was 0.01 mm. The sensitivity of the model was investigated by performing the same calculation with two different displacement increments, \( \Delta \delta = 0.02 \text{ mm} \) and \( \Delta \delta = 0.002 \text{ mm} \) respectively. The results are shown in Fig. 5.31. The differences between both calculations are in fact negligible.

**Fig. 5.31. Test No. 5 with different displacement increments.**

Although the model is apparently rather insensitive for the magnitude of the displacement increment \( \Delta \delta \), the increment must be small with respect to the actual maximum displacements. Fig. 5.32 shows the comparison of the model with the experimental result of test No. 6 of Jimenez. Now, the initial crack width was 0.51 mm. The bar diameter was equal to 29 mm (\( \rho = 1.82\% \)), the initial steel stress was 227 MPa. Because of the expected small increase in displacement in each cycle, the displacement increment \( \Delta \delta \) was chosen equal to 0.005 mm. Again, the end-displacements are satisfactorily predicted by the model. However, the crack response in the 15th cycle was initially too stiff. For this test, the contribution of the dowel action mechanism to the transfer of the externally applied shear stress was approximately 60 percent and remained nearly constant during cycling. Due to the small contribution of the aggregate
interlock mechanism, the calculation process became even more insensitive for changes in the displacement increment than was the case for test No. 5.

Fig. 5.32. Test No. 6 of Jimenez [37] compared with the model.

A further decrease of the initial steel stress will provide a stiffer response of the crack to cyclic shear loads. Fig. 5.33 presents the comparison of the experimental and calculated result of test No. 7 of Jimenez. The initial steel stress in the 29 mm diameter bars was 151 MPa. The calculated crack response is in good agreement with the experimentally obtained response. The contribution of the dowel action mechanism is 50 percent of the total shear stress. Again, this contribution remained approximately constant during cycling. Because of the very small displacements, a factor \((n-1)^{0.1}\) was applied to the dowel stiffness to account for the fatigue of the supporting concrete. Each fully reversed cycle was counted as one cycle.

The combined model proposed in this Section can of course be applied to the case of 'low-intensity high-cycle' fatigue. This type of experiments is described in Chapter 3. The experimentally obtained crack displacements must however be larger than the smallest displacements, which can accurately be predicted by the model. Therefore, some cycles approaching shear failure are simulated to ensure a sufficient increase in the crack displacements in each cycle. Again, the normal restraint stiffness according to eq. (5.14) was used in the calculation.
Fig. 5.33. Test No. 7 of Jimenez [37] compared with the model.

First, the cycles 640 and 1620 of test No. A/6H/66/7.9/.03 are considered (page 57, [56]), see Fig. 5.34. The maximum applied repeated shear stress during cycling was 7.9 MPa. The cube compressive strength was 48.0 MPa. The crack plane was reinforced by means of twelve 8 mm diameter bars with a yield strength of 550 MPa. For the cycles considered, the shear displacement largely exceeds the shear displacement, for which the ultimate dowel strength is obtained for a monotonie increasing shear load. Therefore, it is expected that a large plastic deformation has occurred in the plastic hinges in the reinforcing bars. For the calculation process, this plastic deformation is accounted for by applying the static dowel action model to the measured shear displacement at zero stress according to Fig. 5.26. Furthermore, the measured crack width at zero stress was input in the calculation. The theoretical results are also presented in Fig. 5.34, showing a reasonable agreement with the experimentally obtained results. Because of the large contribution of the dowels to the transfer of shear stress, restitution of the shear slip during unloading starts at 75 percent of the shear load, as is predicted by the dowel action model.
shear stress $\tau$ (MPa)

$$\text{Fig. 5.34. Test No. A/6H/.66/7.9/.03 compared with the proposed model [56].}$$

$\text{Fig. 5.35 presents the comparison between the model and the experimental result for test No. B/4L/.81/9.1/.04 (page 72, [56]). Now, the cube compressive strength was 68.0 MPa. Eight 8 mm diameter bars with a yield strength of 460 MPa were used. The maximum applied repeated shear load was 9.1 MPa. Again, it was found that the dowel stress reaches its ultimate value. As for the computations on plain concrete test results, it appeared that there is no decrease in matrix strength. For this specimen, the restitution of the shear slip during unloading started at 55 percent of the shear load being the average of 75 percent according to the dowel action mechanism and 40 percent of the aggregate interlock mechanism.}$

For this type of tests, in which the contribution of the dowel mechanism to the shear transfer is equal to its ultimate value, an interesting scenario for the tests can be found. During the first few cycles, the mechanism of aggregate interlock transfers the difference between the applied shear stress and the ultimate dowel stress. The combination of the end-displacements in these cycles will be determined by the matrix strength and the maximum particle size according to Fig. 4.4. Because of the fact, that the contribution of aggregate interlock to the shear stress transfer remains constant during cycling, it can be expected that the crack opening path is determined by a constant ratio of $\tau_a$ to $\sigma_{pu}$. 
With this assumption, it can be easily checked whether the tests described in Chapter 3 are in agreement with the proposed model. For all the tests, the crack opening path should follow the theoretical opening path for a constant ratio \((\tau - \tau_{du})/\sigma_{pu}\). In Fig. 5.36 a few typical test results are compared with this assumption. It was found that for a total of 42 repeated loading tests, 16 experimental crack opening paths are in close agreement with the theoretical opening path. There is a reasonable agreement for three tests, while the difference between the result of six tests and the model can easily be explained. The measured displacements of 12 experiments were too small to draw any conclusions. In consequence, only five measured crack opening paths showed large deviations from the theoretical crack opening paths. From these results, it can be concluded, that also in the case of 'high-cycle' repeated shear loading the crack opening direction is determined by a constant ratio between \(\tau_a\) and the matrix strength \(\sigma_{pu}\).

According to the model, it can be concluded that the crack displacement increments are in fact related to \(\tau_a/\sigma_{pu}\) rather than to \(\tau_m/\sigma_{pu}\), which was used in the eqs. (3.3a-b). Therefore, empirical results similar to the eqs. (3.3a-b) are derived relating to the ratio of \(\tau_a\) to the matrix strength:
Fig. 5.36. The experimental crack opening path compared with the opening path according to the model.

\[
\delta_n = 3.34 \left( \frac{\tau}{\sigma_{pu}} \right)^{1.6} + (130 \left( \frac{\tau}{\sigma_{pu}} \right)^{2.9} + 2.6 \times 10^{6} \left( \frac{\tau}{\sigma_{pu}} \right)^{6.9}) \log(n) \\
+ 2.1 \times 10^{-33} \left( \frac{\tau}{\sigma_{pu}} \right)^{34} (\log(n))^{10} \ [\text{mm}] 
\]  
\[ (5.16a) \]

\[
\delta_t = 3.34 \left( \frac{\tau}{\sigma_{pu}} \right)^{1.6} + (61 \left( \frac{\tau}{\sigma_{pu}} \right)^{2.9} + 1.2 \times 10^{6} \left( \frac{\tau}{\sigma_{pu}} \right)^{6.9}) \log(n) \\
+ 2.2 \times 10^{-19} \left( \frac{\tau}{\sigma_{pu}} \right)^{19.4} (\log(n))^{5} \ [\text{mm}] 
\]  
\[ (5.16b) \]

Fig. 5.37. The relationship between the number of cycles and the crack displacements as a function of the stress level.
with \( \sigma_{pu} = 4.5 f_{ccm}^{0.67} \) [MPa] \( (5.17) \)

Eq. (5.17) is a slight modification of eq. (4.1c), to ensure that the matrix strength is equal or higher than the compressive strength of the concrete for very high concrete strengths. Fig. 5.37 presents the relation between the stress level and the crack displacements during cycling according to the eqs. (5.16a-b).

5.5. Influence of the normal restraint stiffness upon the shear stiffness for the case of repeated loading.

In this Section, a description will be given of the influence of the normal restraint stiffness upon the shear stiffness of cracks in reinforced concrete subjected to a repeated shear loading.

In addition to the example presented in Section 4.5, the same case will be used here. Because of the small increase of the crack width during the first load cycle (\( \delta_n : 0.106 \text{ mm to } 0.122 \text{ mm} \)), the development of the inclined cracks is ignored. It is assumed that unloading starts at a crack width of 0.13 mm. Now, it is possible to investigate the effect of a crack width increment during re-loading.

The top load placed upon the wall between the two storage tanks is removed. Due to the elastic deformation of the bars and the high normal stress upon the cracks, the crack faces will partially slide back. For the static case (cycle 1), it was shown that the plastic hinges in the reinforcing bars crossing the cracks have fully been developed. Because of the large shear displacement, which occurred after these hinges have been developed, the residual shear displacement according to eq. (5.4c) cannot be applied to this case. The shear displacement at zero shear stress is therefore estimated to be equal to \( 0.16 \text{ mm} \) (approximately equal to \( \delta_{t,\text{max}} - \delta_{t,e} \)). Because of the plastic hinges, the static dowel action model is applied with a reduced value of the shear displacement \( (\delta_t - 0.16 \text{ mm}) \) according to Fig. 5.26. The crack width at zero shear stress is taken equal to the initial crack width. Furthermore, it is assumed that the normal restraint stiffness is not influenced by the unloading and re-loading of the wall. Therefore, the normal stress - crack width relation is the same as for the static case.
Table 5.1 presents the calculated results for the crack response during re-loading the wall. The reduced contact model is applied for the aggregate interlock mechanism. The shear stress - crack width relationship and the crack opening path are shown in the Figs. 5.38a-b.

Table 5.1. Crack response according to the reduced contact model.

<table>
<thead>
<tr>
<th>( \delta_n ) [mm]</th>
<th>( \delta_t ) [mm]</th>
<th>( \tau_a ) [MPa]</th>
<th>( \sigma_a ) [MPa]</th>
<th>( \tau_d ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>0.19</td>
<td>0.57</td>
<td>0.64</td>
<td>2.03</td>
</tr>
<tr>
<td>0.12</td>
<td>0.22</td>
<td>4.55</td>
<td>2.24</td>
<td>2.49</td>
</tr>
<tr>
<td>0.13</td>
<td>0.25</td>
<td>10.35</td>
<td>3.95</td>
<td>2.86</td>
</tr>
</tbody>
</table>

![Shear stress-crack width relation.](image1)

**a.** Shear stress-crack width relation.  
**b.** Crack opening path.

Fig. 5.38. Calculated response of the cracks in the wall during re-loading.

Table 5.2. Crack response according to the analytical contact model.

<table>
<thead>
<tr>
<th>( \delta_n ) [mm]</th>
<th>( \delta_t ) [mm]</th>
<th>( \tau_a ) [MPa]</th>
<th>( \sigma_a ) [MPa]</th>
<th>( \tau_d ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>0.20</td>
<td>0.58</td>
<td>0.69</td>
<td>2.03</td>
</tr>
<tr>
<td>0.12</td>
<td>0.23</td>
<td>2.37</td>
<td>2.44</td>
<td>2.49</td>
</tr>
<tr>
<td>0.13</td>
<td>0.24</td>
<td>8.91</td>
<td>3.95</td>
<td>2.86</td>
</tr>
</tbody>
</table>

The increments of the crack displacements during re-loading deviated from the results, on which the empirical relations for the retention
factors $\lambda_x$ and $\lambda_y$ are based. In order to investigate the sensitivity of these retention factors for different crack opening paths, the crack response is also calculated according to the analytical contact model, see Table 5.2. The agreement between the results of both methods is satisfactory.

5.6. Concluding remarks.

In this Chapter, the static aggregate interlock and dowel action models are adapted to the case of repeated and reversed shear loads. Walraven already showed that the two-phase aggregate interlock model can be applied to the case of cyclic shear tests on cracked plain concrete. Therefore, all effort is paid to a sound simplification of the two-phase model in order to speed up the calculation process. For increasing crack displacements during cycling, the proposed reduced contact model provides good predictions of the crack response during cycling. However, a simplified model cannot be used being as general as the original two-phase model.

For practical use, the response of cracked reinforced concrete is of much more interest than the response of cracked plain concrete. Now, the contribution of the dowels to the shear stress transfer must be taken into account. However, even the simple reduced contact model is rather sophisticated with respect to the available models describing the cyclic dowel action behaviour. Therefore, the use of the reduced contact model to describe reinforced concrete tests will introduce smaller deviations from the actual response than the deviations caused by the dowel action model.

The existing cyclic dowel action models are formalistic. In Section 5.3, a dowel action model is proposed, which is to some extent based upon the physical dowel behaviour. Due to a severe lack of detailed information on the actual bar behaviour, this model is still very simple and therefore limited in its applications.

The reduced contact model and the proposed dowel action model are combined to describe the response of cracked reinforced concrete. The combined model satisfactorily predicts the experimentally obtained crack response for the case of 'high-intensity low-cycle' fatigue and for the case of 'low-intensity high-cycle' fatigue.
All the simulated tests were subjected to shear loads with a stress ratio $R (\tau_{\min} / \tau_{\max})$ being equal to 0 (repeated load) and -1 (fully reversed load) respectively. It was found from the calculations, that there was no strength degradation due to fatigue of the matrix material. This can be explained by the large increase in crack displacements. The previously loaded matrix material is then crushed. For stress ratios $R$ in the range of -1 to 1, fatigue of the matrix material will affect the crack response. A sustained shear load ($R = 1$) in the range of 60 to 90 percent of the static shear strength causes a gradual increase of the crack displacements in time, Frenay [20]. This increasing displacements must be caused by strength reduction and material flow in the contact areas between the particles and the matrix material.

A stress ratio of 0.4–0.5 will not cause a restitution of the displacements during unloading, see Fig. 5.1. Any increase in the displacements due to re-loading must then be caused by strength reduction due to fatigue.

![Graphs showing crack resistance vs. number of cycles](image)

Fig. 5.39. The fatigue strength of concrete [38].

The phenomenon of concrete fatigue is investigated by among others Van Leeuwen and Siemes [38]. For various types of concrete grade, curing condition and age, Wöhler curves were determined for several stress levels and stress ratios $R$, see Fig. 5.39a–c. It was found that the fatigue strength increases with increasing concrete strength, but to a lesser extent than the concrete grade. The frequency of the stress
cycles was found to affect the fatigue strength, see Fig. 5.38a and c. The lower the frequency, the lower the fatigue strength. For very high stress levels (> 75% of the static strength) the strength reduction increases progressively. This phenomenon is probably caused by creep effects. These findings are in agreement with the experimental results of Holmen [30] and Graf et al. [27]. It must however be noted that the experimentally obtained material characteristics of concrete cannot be applied directly to the matrix material. Fatigue of concrete is partially due to the interfacial bond between the stiff particles and the matrix material. The matrix material has a relatively large amount of air voids and water inclusions, which affects fatigue. However, the overall characteristics of concrete and matrix material will be similar.

Even if a proper description of the fatigue of the matrix material due to the high cyclically applied compressive stresses is implemented in the model for the crack response to cyclic shear loads, it is doubted whether such a model can be used to describe the crack response degradation for small stress ratios R. The increase in crack displacements during cycling will decrease with decreasing shear stress level and with decreasing stress ratio R. These small crack displacements cannot be simulated with the proposed model. Therefore, further experimental investigations into this field are necessary.
6. IMPLEMENTATION OF THE CYCLIC AGGREGATE INTERLOCK MODEL INTO NUMERICAL PROGRAMS

6.1. Introduction.

In the previous chapters, the mechanisms of aggregate interlock and dowel action are described for both cases of monotonic and cyclic shear loads on cracked reinforced concrete. Now, an attempt is made to implement the models developed into advanced finite element programs. It must be noted that the implementation of the mechanism of dowel action is strongly related to the way reinforcing bars are treated in numerical programs. In these programs, the bars are tied to plain concrete elements. This can be done directly or by using a boundary layer between the bars and the ordinary concrete elements [61,79]. The numerical description of the static bond stresses between the steel bar and the concrete is rapidly improving [61], but a very fine mesh is needed for such a detailed description. For practical use in elements intersected by reinforcing bars, the element strain is also applied to the bars. In such programs as proposed by among others, Bazant and Gambarova [1] and Vecchio and Collins [74], the combined stiffness of steel and concrete is accounted for by a tension-stiffening parameter. The contribution of dowel action to shear stiffness of the cracked element is completely neglected. Models developed by Fardis and Buyukozturk [17] and by Perdikaris et al. [54,74] account for dowel action by means of a dowel stiffness based upon the concept of a beam on an elastic foundation. The interaction between an axial steel force and a dowel force is, however, not yet considered. Apart from that, the applied bond characteristics are derived for bars subjected to an axial force only. The bond capacity is, however, reduced by a shear displacement of the crack faces, the same holds true for bars inclined to the crack direction, if the crack only widens. The static dowel action model described in Chapter 4 can easily be implemented in any numerical program, when an appropriate bond model is used. The cyclic dowel action model developed in Chapter 5 has been already expressed by means of various dowel stiffnesses, which can directly be implemented. Obviously, the implementation of the dowel action model must be accompanied by the implementation of a proper bond model for bars subjected to both axial and lateral forces. Such a bond
model is, however, still lacking. Furthermore, for the case of monotonic loading, the crack opening path is no longer related to the bond stresses after the plastic hinges in the bars have fully been developed, see Chapter 4.

Finally, the plain concrete on the one hand and the reinforcing bars on the other hand represent two different mechanisms, which must be described by means of two different stiffness relations. Therefore, the aggregate interlock mechanism and the dowel action mechanism will be treated separately. For the mechanism of dowel action a rough description of the stiffness relation will be given because of its dependency on the (unknown) bond characteristics.

Although the static two-phase aggregate interlock model satisfactorily describes the physical behaviour, it is still too complex for implementation into advanced finite element programs. Therefore, first empirical relations will be derived based on the two-phase model. These expressions will be used to describe the stiffness relation between the stresses and the displacements in a concrete element. Numerical programs can be subdivided into two types. The first type, denoted as discrete crack program, accounts for the development of cracks by defining new element boundaries along the crack or by adding the crack displacements to the displacements of the whole element. The second type of program uses the smeared crack approach. Now, the entire cracked element is still considered as a continuum. With this concept crack displacements are converted into strains of the entire element or Gauss point. Therefore, the expressions, which are based upon crack displacements, should also be converted to strain based formulas.

In a finite element program, the calculation starts linear elastically. With increasing external loads, the tensile strength is reached in a given Gauss-point. Then the uncracked element becomes partially cracked due to the development of micro-cracks. On increasing tensile strain the damage will affect the whole area; the element is then fully cracked. In this chapter, an attempt will be made to describe all these three stages with only one stiffness relation. It will be shown how the interaction between tension-softening and shear-softening affects the element stiffness for the case of monotonic loads. Finally, the stress-strain relations are adapted to the case of cyclic loads.
6.2. Simplified expressions for the static two-phase model.

In order to keep close to the physical model of Walraven [81] and to keep the expressions as simple as possible, expressions are derived for the contact areas instead of for the stresses. With the magnitudes of the contact areas and eqs. (4.1a-d), the stresses in the crack can be determined. The curves, which fitted closely to the theoretical results according to the two-phase model are described by the following expressions:

\[
P_k \left( \frac{1+\exp(-K)}{\exp(-K)/K+1} + b \cdot p \right) \quad \text{[mm}^2/\text{mm}^2] \quad (6.1a)
\]

with \( K = \frac{b}{a} \cdot \delta_t \) \( (6.1b) \)

for \( A_x : a = 4 \)

\[
b = 7.00 \quad D_{\max}^{0.056} \quad 1.07 \quad (6.1c)
\]

\[
p = 0 \quad (6.1d)
\]

for \( A_y : a = 2 \)

\[
b = 3.00 \quad D_{\max}^{0.280} \quad (6.1f)
\]

\[
m = -1.47 \quad D_{\max}^{-0.063} \quad (6.1g)
\]

\[
p = 0.5 \quad (\delta_t - \delta_n - \delta_t) \quad \text{abs} [\delta_t - \delta_n] \quad \exp(-1-D_{\max}/32-0.5 \cdot \delta_n^2) \quad (6.1i)
\]

The limitations of the equations are:

\(- \delta_t < 1.2 \quad \delta_n\)

- Particle distribution according to Fuller.

Because of the fact, that in finite element calculations, the crack displacements are very small, the eqs. (6.1a-i) are adapted to provide a closer fit at small shear displacements. The eqs. (6.1d) and (6.1g) are altered:

\[
A_x : b = 7.74 \quad D_{\max}^{0.06} \quad (6.1j)
\]
Now, the limitations are:

- \(\delta_t < 0.2\) mm
- \(\delta_t < \delta_n\)

In the case of settlements large shear displacements can occur. Then, the original formules according to the two-phase model must be used.

Next, the eqs. (6.1a-m) will be converted into strains in order to use these expressions in numerical programs based upon the concept of smeared out cracks.

The strains due to cracking can be expressed as:

\[
\varepsilon_{nn,cr} = \delta / h \quad \text{(6.2a)}
\]

\[
\gamma_{cr} = \delta_t / h \quad \text{(6.2b)}
\]

with \(h = \) size of the element normal to the crack.

The smeared out deformation can represent one large crack, but also two or more smaller cracks, see Figs 6.1a-c. Both systems transfer the same stresses. So, the concept of smeared cracks implies constant stresses \(\tau_a\) and \(\sigma_a\) for a given ratio of \(\varepsilon_{nn,cr}\) to \(\gamma_{cr}\). This can be expressed by:

\[
\frac{\delta_n}{\delta_t} = \frac{\varepsilon_{nn,cr} \cdot h}{\gamma_{cr} \cdot h} = \frac{\varepsilon_{nn,cr}}{\gamma_{cr}} = \text{constant} \quad \text{(6.3)}
\]

then \(\tau_a = \) constant and \(\sigma_a = \) constant
Therefore, it is quite essential that the relations for $\tau_a$ and $\sigma_a$ are dependent upon the ratio between the normal strain $\varepsilon_{nn,cr}$ and the shear sliding $\gamma_{cr}$, so that they will be independent of the element size. With this restriction, it was found empirically on basis of the two-phase model, that:

\[
\text{For } A_x: \ K = 2.17 \frac{D_x^{0.06}}{\varepsilon_{\max,n,c}} \frac{\gamma_{cr}}{\varepsilon_{nn,cr}} \quad (6.4a)
\]

\[
\text{For } A_y: \ K = 3.74 \frac{D_y^{0.13}}{\varepsilon_{\max,n,c}} \frac{\gamma_{cr}}{\varepsilon_{nn,cr}} \quad (6.4b)
\]

The eqs. (6.4a-k) can directly be used in programs of the discrete crack concept. For elements of the smeared-out crack type, the eqs. (6.4a-b) have to be implemented into the stiffness of the whole element. Therefore, a rheological model will be presented in the next Section, in which the crack strain can be related to the element strain.

6.3. Rheological model for an element with the smeared out crack concept.

In an element a crack zone is formed, when the strain in this element exceeds the tensile fracture strain of concrete. The behaviour of this crack zone is determined by the development of small micro-cracks and can be described by a tension-softening model. Due to this crack zone, the stiffness of the whole element is strongly reduced. In order to adapt the stiffness of the element to this reduction a normal retention factor $\nu$ is applied to the modulus of elasticity of concrete $E_c$, see
Fig. 6.2. Actually, the softening of the element is localized at the crack zone, see Fig. 6.3. The shape of the descending branch depends upon the element size \( L \). Therefore, the stiffness of the descending branch \( E_c \) and the normal retention factor \( \mu \) are no real material parameters. In fact, the observed softening probably is a structural effect due to non-homogeneous deformations during cracking [6]. The stiffness \( E_c \) is used to describe the incremental stress-stain relation of a partially cracked element, the normal retention factor is used to account for the reduced stiffness of the element. \( E_c \) can be expressed as a function of \( \mu \).

![Fig. 6.2. Tension-softening behaviour](image)

![Fig. 6.3. The rheological model.](image)

The reduction factor for the crack zone can be obtained from the rheological model shown in Fig. 6.3b:

\[
L \Delta \epsilon_{nn} = L_{cr} \Delta \epsilon_{nn, cr} + L \Delta \epsilon_{nn, cr}
\]  

(6.5)

with \( L = L_{co} + L_{cr} \), see Fig. 6.3.

Now, the normal retention factor for the crack zone \( \eta \) can be expressed by:

\[
\eta = \frac{L_{cr}}{L_{co}} \left( \frac{L_{co} + L_{cr}}{L_{co} + L_{cr}} \right) \mu
\]  

(6.6)

However, the length of the crack zone \( L_{cr} \) is not accurately known. A length of approximately 3 \( D_{max} \) is suggested by Bazant et al. [3]. This length will be used here.
It must be born in mind that eq. (6.6) is no longer valid when the length of the crack zone is nearly equal to the length \( L_{co} \) (this can be the case when very small elements are used).

Using the factor \( \eta \), the weakening of the element is numerically localized in the crack zone. Now, the rheological model will be extended using the philosophy of smeared out cracks. According to this philosophy, the crack strain caused by the integrated action of the micro cracks can be considered as the strain caused by a single large crack, so that the crack zone can be regarded to be divided in a fully cracked part and an uncracked linear elastic part with a reduced cross-sectional area, see Fig. 6.4.

![Fig. 6.4. Extended rheological model.](image)

For this extended version of the rheological model, the incremental stress-strain relation for each part of the element will be described; finally the relation for the entire element will be derived.

**Stiffness matrix for the uncracked section of the element.**

To the uncracked part of the element, the linear elastic theory can be applied, yielding:

\[
\begin{pmatrix}
\Delta \sigma_{nn}
\end{pmatrix} =
\begin{pmatrix}
E & \frac{ue}{1-u^2} & 0 \\
\frac{ue}{1-u^2} & \frac{E}{1-u^2} & 0 \\
0 & 0 & 2(1+u^2)
\end{pmatrix}
\begin{pmatrix}
\Delta \epsilon_{nn,co}
\end{pmatrix}
\]

(6.7)

**Stiffness matrix for the uncracked part of the crack zone.**

The stiffness matrix for the uncracked part of the crack zone corresponds to the matrix of the uncracked section adapted to the softening
by means of the parameter $n$:

\[
\begin{pmatrix}
\Delta \sigma_{nn,cr,co} & \Delta \sigma_{tt,cr,co} & \Delta \sigma_{nt,cr,co}
\end{pmatrix} =
\begin{pmatrix}
\frac{\nu \eta E}{1-\eta^2} & \frac{\nu \eta E}{1-\eta^2} & 0 \\
\frac{\nu \eta E}{1-\eta^2} & \frac{\nu \eta E}{1-\eta^2} & 0 \\
0 & 0 & \frac{\eta E}{2(1+\nu^2)}
\end{pmatrix}
\begin{pmatrix}
\Delta \varepsilon_{nn,cr} \\
\Delta \varepsilon_{tt,cr} \\
\Delta \varepsilon_{nt,cr}
\end{pmatrix}
\]  

(6.8)

The normal retention factor $n$ is related to the normal retention factor $\mu$ of the whole element. This relation is expressed by eq. (6.6).

### Stiffness matrix for the cracked part of the crack zone.

As shown in Fig. 6.1, the external shear sliding and normal strain can be caused by a single large crack as well as a large number of very small (micro-)cracks. According to this philosophy, the micro-cracks are replaced by a single crack. Now, the two-phase model can be applied to this crack. Because of the rather complex relations describing this behaviour, the actual matrix will be represented by the following matrix for use here:

\[
\begin{pmatrix}
\Delta \sigma_{nn,cr,cr} & \Delta \sigma_{tt,cr,cr} & \Delta \sigma_{nt,cr,cr}
\end{pmatrix} =
\begin{pmatrix}
S_{11} & 0 & S_{13} \\
0 & 0 & 0 \\
S_{31} & 0 & S_{33}
\end{pmatrix}
\begin{pmatrix}
\Delta \varepsilon_{nn,cr} \\
\Delta \varepsilon_{tt,cr} \\
\Delta \varepsilon_{nt,cr}
\end{pmatrix}
\]  

(6.9)

in which $S_{11}$, $S_{13}$, $S_{31}$ and $S_{33}$ represent the stiffness relations according to the two-phase model.

### The stiffness relation for the entire cracked element.

Both springs in the rheological model representing the crack zone are subjected to the same state of deformation. Hence, the eqs. (6.8) and (6.9) can be directly added, thus yielding:

\[
\begin{pmatrix}
\Delta \sigma_{nn,cr} & \Delta \sigma_{tt,cr} & \Delta \sigma_{nt,cr}
\end{pmatrix} =
\begin{pmatrix}
\psi S_{11} & \frac{\eta E}{1-\eta^2} & \psi S_{13} \\
\psi S_{13} & \frac{\psi \eta E}{1-\eta^2} & \psi S_{31} \\
\psi S_{31} & 0 & \psi S_{33} + \frac{\eta E}{2(1+\nu^2)}
\end{pmatrix}
\begin{pmatrix}
\Delta \varepsilon_{nn,cr} \\
\Delta \varepsilon_{tt,cr} \\
\Delta \varepsilon_{nt,cr}
\end{pmatrix}
\]  

(6.10)

in which $\psi$ is a reduction factor for the cracked area, which is expressed by:
\[
\psi = \frac{(f_{\text{ctm}} - \sigma_{\text{nn}})}{f_{\text{ctm}}} < 1 \quad \text{for partially cracked element} \quad (6.11)
\]
\[
\psi = 0 \quad \text{for uncracked element}
\]

In [55], a detailed description of the derivation of the stiffness matrix of the whole element is given. This stiffness matrix, which includes the stiffness relation of the uncracked section and of the crack zone is presented by eq. (6.12). This matrix is derived by directly adding the strain increment of the uncracked zone to the strain increment of the crack zone. This yields the strain increment of the entire cracked element \( \Delta \varepsilon_{\text{nn}} \):

\[
\begin{align*}
\Delta \sigma_{\text{nn}} &= \begin{bmatrix}
\alpha_1 E & \nu \alpha_1 E & \alpha_2 E \\
\frac{1 - \alpha_1 v^2}{1 - \alpha_1 v^2} & \frac{1 - \alpha_1 v^2}{1 - \alpha_1 v^2} & \alpha_2 E \\
\beta_1 G & \beta_1 G & \beta_2 G
\end{bmatrix} \Delta \varepsilon_{\text{nn}} \\
\Delta \sigma_{\text{tt}} &= \begin{bmatrix}
\alpha_1 E & \nu \alpha_1 E & \alpha_2 E \\
\frac{1 - \alpha_1 v^2}{1 - \alpha_1 v^2} & \frac{1 - \alpha_1 v^2}{1 - \alpha_1 v^2} & \alpha_2 E
\end{bmatrix} \Delta \varepsilon_{\text{tt}} \\
\Delta \sigma_{\text{nt}} &= \begin{bmatrix}
\beta_1 G & \beta_1 G & \beta_2 G
\end{bmatrix} \Delta \gamma
\end{align*}
\]

The factors \( \alpha_1, \alpha_2, \beta_1 \) and \( \beta_2 \) are expressed by the following (approximated) relations:

\[
\begin{align*}
\alpha_1 &= \frac{nE + \psi S_{\text{11}}}{(1 + n)E + \psi S_{\text{11}}} \\
\beta_2 &= \frac{nG + \psi S_{\text{33}}}{(1 + n)G + \psi S_{\text{33}}} \\
\alpha_2 &= \frac{1 - \beta_2}{1 - \beta_1} \frac{\epsilon_{\text{nn}}}{y (1 + n)E + \psi S_{\text{11}}} \\
\beta_1 &= \frac{1 - \alpha_2}{1 - \alpha_1} \frac{\epsilon_{\text{nn}}}{y (1 + n)G + \psi S_{\text{33}}}
\end{align*}
\]

Now, the stiffness relation for the entire plain concrete element is described, except for the exact description of the terms \( S_{\text{11}}, S_{\text{13}}, S_{\text{31}}, \) and \( S_{\text{33}} \). In [55], the derivation of proper expressions for these terms is given according to the two-phase model. The following expressions are obtained:

\[
\begin{align*}
S_{\text{11}} &= \frac{d \sigma_{\text{nn}}}{d \epsilon_{\text{nn}}} \\
S_{\text{13}} &= \frac{d \sigma_{\text{nn}}}{d \gamma}
\end{align*}
\]
This yields:

\[ S_{33} = \sigma_{nt} / d\gamma \]  

(6.14d)

This yields:

\[ S_{33} = \sigma_{nt} / d\gamma - \mu \frac{d\Delta}{d\gamma} \]  

(6.15d)

\[ S_{33} = \sigma_{pu} (d\Delta /d\gamma + \mu d\Delta /d\gamma) \]  

(6.15d)

in which: \( a, K \) according to the eqs. (6.19 and (6.4)).

\[ \mu = \text{coefficient of friction} = 0.4 \]

\[ \frac{d\Delta}{d\gamma} = -a \frac{\exp(-K)[K-1+\exp(-K)]+K^2 \gamma}{[(\exp(-K)+K)^2]} \frac{C}{\epsilon_{nn}} \]  

(6.15e)

\[ \frac{d\Delta}{d\gamma} = a \frac{\exp(-K)[K-1+\exp(-K)]+K^2 \gamma}{[(\exp(-K)+K)^2]} \frac{C}{\epsilon_{nn}} \]  

(6.15f)

For \( A_x : C = 2.17 \) \( D^{0.06}_{max} \)  

(6.15g)

For \( A_y : C = 3.74 \) \( D^{0.13}_{max} \)  

(6.15h)

Now, an accurate description of the real crack behaviour can be made. In the following example, the interaction between tension-softening and shear-softening is shown. An element is subjected to a given normal tensile strain. Reaching a specific post-peak normal stress \( \sigma_{nn} \) (3.0, 2.0, 1.0 MPa), the normal strain is kept constant at increasing shear sliding. The simple normal tensile stress strain relation used is presented in Fig. 6.5a. The calculated results for the shear stress-relative strain relation are shown in the Figs. 6.5b-d. In the figures, two different curves are drawn. The first is based upon the calculation according to the expressions (6.15a-h). Second, the response is calculated with the advanced models reported in the literature. In fact, till now the interaction between tension-softening and shear-softening is ne-
glected in the numerical programs. The shear softening is described by a constant shear retention factor $\beta_2$, with a value between 0.05 and 0.20. More sophisticated expressions for $\beta_2$ are derived by Rots [60], see the eqs. (6.16a-b). The first expression is derived empirically from the experimental results of Paulay and Loeber [50]. The second is derived on basis of the theoretical model of Bazant and Gambarova[3]:

$$\beta(2) = \frac{1}{1 + 4447 \varepsilon_{nn}}$$

(6.16a)

$$\beta(2) = \frac{1}{4762 \varepsilon_{nn}} - \frac{1}{1346 \sqrt{\varepsilon_{nn}}}$$

(6.16b)

Fig. 6.5. The calculated response of a partially cracked element to shear sliding.

Fig. 6.6. The shear retention factor $\beta$ [60].
Both expressions are shown in Fig. 6.6. The calculated response of the cracked element according to these formulas is also shown in the Figs. 6.5b–d. The shear stress is still reasonably well predicted. However, as is shown in Fig. 6.5d, the description according to Rots still allows the transfer of a constant tensile stress across the element. The model presented in this chapter indicates that the stress normal to the crack plane decreases for increasing shear sliding.

Walraven [80] performed shear tests on reinforced beams without stirrups. During the tests, cracks developed in the beam. The displacements in these cracks were recorded and a typical result is shown in Fig. 6.7a. The calculated response of one crack is presented in Fig. 6.7b. The recorded crack displacements are converted into strains using a measuring length of 100 mm. First, the shear stress–shear sliding relation is determined according to Walraven's two-phase model, thus according to the matrix presented in eq. (6.12). Second, the crack response according to the eqs. (6.16a–b) is shown. It is found that the response according to Rots overestimated the shear stiffness of the element. Finally, the response according to the strain based formulas (eqs. (6.4a–b)) is shown, which only slightly deviated from the result of Walraven's model. The overestimation of the result according to the formulas of Rots is mainly caused by the fact that the expressions neglect the effect of the off-diagonal terms in the stress–strain relation upon the transferred shear stress. Therefore, contrary to the statement of De Borst [17, page 115], the off-diagonal terms have to be taken into account even if small crack strains are considered.

The eqs. (6.16a–b) cannot provide a proper description of the real crack behaviour because they are not related to a possible increase in crack width. For a strong increase in crack width without shear sliding, the physical two-phase model predicts a sharp drop in the transferred shear stress, whereas the eqs. (6.16a–b) only predict a less strong increase of the transferred shear stress. These formulas fit, however, very well with the solution methods, which are commonly used in numerical programs. Therefore, an attempt is made to derive an expression for a shear retention factor based upon the physical two-phase model instead of upon experimental results.
Fig. 6.7. The crack response for a measured crack opening path [3].

Re-arranging eq. (6.13b), it was found that the following expression can be used for the case that the ratio of normal strain to shear strain decreases with increasing deformations:

$$
\beta^{(2)} = \frac{\eta P \epsilon_{nn} + \psi}{(1 + \eta) P \epsilon_{nn} + \psi}
$$

(6.17)

with

$$
P = \frac{2500}{D_{\max} \left[ 0.76 - 0.16 \frac{\epsilon_{nn}}{\gamma} (1 - \exp[-6\gamma/\epsilon_{nn}]) \right]}
$$

Fig. 6.8. The shear retention factor $\beta$ according to eq. (6.17).
An important conclusion is that $\beta$ is independent of the concrete quality, so $\beta$ is a real damage parameter. Eq. (6.17) is shown in Fig. 6.8 for a fully cracked element ($\eta = 0; \psi = 1$). For $D_{\text{max}}$ equal to 19 mm and the ratio of the normal strain to the shear sliding equal to 3, eq. (6.17) fits with the expressions derived by Rots.

The physical crack behaviour indicates that there is hardly any increase in the magnitude of the transferred stresses for constant ratios of the normal strain to the shear strain. If the increase in normal strain is larger than the increase in shear sliding, the transferred normal and shear stress must become smaller. To account for this, eq. (6.17) is extended to the following expression:

$$\beta(z) = \frac{\left[\eta P \epsilon_{nn} + \psi\right]}{(1+\eta) P \epsilon_{nn} + \psi} \frac{\text{abs}(X_{n-2} - X_{n-1})}{(X_{n-2} - X_{n-1})}$$

(6.18)

with $X_{n-2} = \epsilon_{nn} / \gamma$ for step $n-2$ in the calculation.

$$X_{n-1} = \epsilon_{nn} / \gamma$$ for step $n-1$ in the calculation.

Fig. 6.9. The shear retention factor for mixed mode fracture problems.

For the crack opening path shown in Fig. 6.7, the shear stress - shear sliding relationship according to the eqs. (6.17-6.18) is shown in Fig. 6.9b. Now, there is a reasonable agreement between the calculated result and the result obtained with the two-phase model of Walraven. Variation of the measuring length (element length) yielded only very small differences. In Fig. 6.9c, the response of the element is calculated with a
constant $\beta$-value of 0.05 for the cracked element. Although there is a rather strong dependency upon the measuring length, the calculated result agrees reasonably well with the result according to the physical model. The very low constant value of the shear retention factor averages the slightly increasing shear stress according to the physical model. Because of the fact, that the crack opening path shown in Fig. 6.7a is rather typical for shear tests on beams, it is obvious that for this case a constant value of $\beta$ yields good results. A constant value of $\beta$ has, however, no general applicability.

This was also recognized by Rots and De Borst [62] for mixed-mode fracture problems. They found that a constant positive value for $\beta$ yields an overestimation of the shear strength of such an element. Therefore, they proposed a shear softening behaviour as shown in Fig. 6.10. Although the shear stiffnesses $D_1$ and $D_2$ are largely based upon trial-and-error methods, the over-all shear stiffness is to some extent similar to the relation shown in Fig. 6.9b. However, they also proposed a linear relation between shear stress and shear strain for unloading and re-loading of the element. According to the cyclic model presented in Chapter 5, this relation cannot be true either for unloading or for re-loading the element. Therefore, an improved expression for the shear stiffness for the case of cyclic shear loads will be derived in the next Section.

![Fig. 6.10. Shear softening relation according to Rots and De Borst [62].](image)

6.4. The stress-strain relation for the case of cyclic loading.

Unloading can be described by means of a linear relation between the shear stress and the shear strain. However, the shear strain must be reduced by the residual shear sliding $Y_0$, which can be calculated
according to eq. (5.4c). In this expression the shear displacement can be replaced by the shear sliding.

Re-loading the element, the crack response is determined by the cyclic aggregate interlock model, as is developed in Chapter 5. In this Section the simplest model, the reduced contact model, will be implemented. In this model, the projected contact areas, which would develop in case of a monotonic loading, are reduced by means of the reduction factors $\lambda_x$ and $\lambda_y$, see eqs. (5.4a-b). These reduction factors are determined empirically from the calculated results based upon the analytical contact model and are therefore limited in their applications. With regard to the average increments in the crack displacements, it is found that the reduction factors can be applied when the crack width increment is approximately equal to the increment of the shear displacements. Conversion to strains yields the assumption:

$$d\varepsilon_{nn} = d\gamma$$  \hspace{1cm} (6.19)

For the case of cyclic loading, the eqs. (6.15a-d) become:

$$S_{11} = \sigma_{pu} \left( \frac{d(\lambda_x A_x)}{d\varepsilon_{nn}} - \mu \frac{d(\lambda_y A_y)}{d\varepsilon_{nn}} \right)$$  \hspace{1cm} (6.20a)

$$S_{13} = \frac{\varepsilon_{nn}}{\gamma} S_{11}$$  \hspace{1cm} (6.20b)

$$S_{31} = -\frac{\gamma}{\varepsilon_{nn}} S_{33}$$  \hspace{1cm} (6.20c)

$$S_{33} = \sigma_{pu} \left( \frac{d(\lambda_x A_x)}{d\gamma} + \mu \frac{d(\lambda_y A_y)}{d\gamma} \right)$$  \hspace{1cm} (6.20d)

in which:

$$\frac{d\lambda A}{d\varepsilon_{nn}} = \frac{d\lambda}{d\varepsilon_{nn}} A + \lambda \frac{dA}{d\varepsilon_{nn}}$$  \hspace{1cm} (6.20e)

$$\frac{d\lambda A}{d\gamma} = \frac{d\lambda}{d\gamma} A + \lambda \frac{dA}{d\gamma}$$  \hspace{1cm} (6.20f)

However, $\lambda_x$ and $\lambda_y$ are not related to the normal strain. Because of eq. (6.19), it can be stated that: $d\lambda/d\varepsilon_{nn} = d\lambda/d\gamma$

Now, the eqs. (6.20e-f) become:
\[ \frac{d\lambda A}{d\epsilon_{nn}} = \frac{d\lambda}{d\gamma} A + \lambda \frac{dA}{d\epsilon_{nn}} \]  
\[ \frac{d\lambda A}{d\gamma} = \frac{d\lambda}{d\gamma} A + \lambda \frac{dA}{d\gamma} \]  
(6.20e)  
(6.20f)

with \( A \) expressed in the eqs. (6.1) and (6.4).
\( \frac{dA}{d\gamma} \) and \( \frac{dA}{d\epsilon_{nn}} \) expressed in the eqs. (6.15e-f).
\( \lambda \) expressed in the eqs. (5.4a-b).

Finally, the expressions for the reduction factors must be converted to strains and differentiated with respect to \( \gamma \). Using the eqs. (5.4a-b), the following expressions are obtained:
\[ \frac{d\lambda x}{d\gamma} = \frac{(\gamma - \gamma_0)}{(\gamma_m - \gamma_0)^2} \]  
\[ \frac{d\lambda y}{d\gamma} = 2.1 \frac{(\gamma - \gamma_0)^2}{(\gamma_m - \gamma_0)^2} \]  
(6.20g)  
(6.20h)

With these expressions, the incremental stress - strain relation for a cracked element can be described. It must be noted, that for the case of unloading and subsequent re-loading of an element, the off-diagonal terms of the stiffness matrix are as important as the diagonal terms. This is due to the fact that upon re-loading the direction of the principal tensile stress will generally strongly deviate from the direction normal to the crack plane. To reduce the calculation process, the effect of the off-diagonal terms can be partially accounted for when \( \psi \) in eq. (6.17) is reduced. Whereas the shear stress largely depends upon the projected contact area in the \( y \)-direction, \( \psi \) is reduced according to eq. (6.20h). This is, however, a very rough approximation. For the previously given example, the calculated response of the crack in the second cycle is shown in Fig. 6.11 (in the experiment no cyclic loading was applied). It is obvious, that this cyclic behaviour strongly deviates from the response suggested by Rots and De Borst [62], see Fig. 6.10.
6.5. Implementation of the dowel action mechanism.

The implementation of the mechanism of dowel action is strongly related to the description of the bar-elements in a numerical program. A physically sound description is provided by using three independent elements, see Fig. 6.12:
- a steel bar element,
- the interface element or slip layer element,
- and the plain concrete element.

Such an approach allows for slip between the steel bar and the concrete. The major disadvantage of this method is the fine mesh, which is required to account for the effect of the splitting cracks close to the bar.

In this section, the slip layer element and the bar element will be con-
sidered as being one element. In the Chapters 4 and 5, it is shown that the dowel action mechanism depends upon the cooperation of the steel bar and the concrete under the bar. The empirical and theoretical expressions derived for this mechanism are based upon this cooperation of the bar element and the slip layer element. This element will be denoted here as the dowel element.

The dowel element will provide the normal restraint force for the (cracked) plain concrete elements.

The stiffness matrix for the dowel element is presented schematically by eq. (6.21):

\[
\begin{bmatrix}
\Delta \sigma_{nn} \\
\Delta \sigma_{tt} \\
\Delta \sigma_{nt}
\end{bmatrix} =
\begin{bmatrix}
D_{11} & D_{13} & \Delta \varepsilon_{nn} \\
0 & 0 & \Delta \varepsilon_{tt} \\
D_{31} & D_{33} & \Delta \gamma
\end{bmatrix}
\]  

(6.21)

In this matrix \( D_{33} \) represents the dowel stiffness. For the case of a monotonic dowel load, the dowel stiffness \( K_0 \) can be used, which can be derived from eq. (4.28):

\[
K_0 = 0.33 \frac{F_{du}}{\sqrt[3]{\delta^2 \left( \delta_{no} + \delta_{te} \right)}}
\]  

(6.22)

For the case of cyclic loading, the dowel stiffnesses \( K_1 - K_u \) as derived in Chapter 5, can be used for \( D_{33} \).

The term \( D_{11} \) represents the axial bond characteristics of the bar. Unfortunately, this stiffness is related to both the normal strain and the shear sliding. Therefore, the bond characteristics obtained in a pull-out experiment cannot be used here. Detailed tests in this field are necessary. The same holds true for the terms \( D_{13} \) and \( D_{31} \). The first represents the decrease in bond capacity with increasing shear sliding. The second represents the decrease in dowel capacity with increasing normal strain. Because of the fact, that the stiffness relation presented in eq. (6.21) is related to both the dowel mechanism and the (unknown) bond mechanism, no further description of the stiffness relation is possible here.
6.6. Concluding remarks.

The implementation of the reduced contact model, which describes the cyclic aggregate interlock mechanism, yielded complex expressions for the stiffness relations of a partially cracked element. It appeared that both the diagonal and off-diagonal terms of the matrix are non-zero. For a monotonically increasing load on an initially uncracked element, it is expected that the assumption of zero off-diagonal terms is rather close to reality. The direction of the principal stresses for this type of loading is only slightly deviating from the stress direction during cracking.

However, for closing and re-opening cracks (cyclic loading), the effect of the off-diagonal terms must be accounted for directly or indirectly by means of the eqs. (6.17) and (6.20h).

The dowel action model is not valid for implementation in numerical programs. This mechanism is strongly related to the bond characteristics of the bar. Unfortunately, a generally applicable bond mechanism is not yet described in the literature.
7. RETROSPECTIVE VIEW AND CONCLUSIONS

The safety of large-scale structures, such as offshore platforms, might depend upon the stiffness of cracked reinforced elements subjected to in-plane loads. Previous studies [34,40,43,76,81] showed that the transfer of in-plane static and cyclic shear stresses across cracks in reinforced concrete largely depends upon the combined mechanism of aggregate interlock, dowel action and components of the axial steel stress in the reinforcing bars. In fact, the experimental studies concerning cyclic shear loads were restricted to a relatively small number of cycles with a very large load amplitude.

![Fig. 7.1. Scheme of the load cycles for an offshore structure.](image)

An offshore structure endures millions of load cycles during its economical life-time. Generally, the amplitude of these cycles is far less than the magnitude of the static load, see Fig. 7.1. The subsequent few cycles with a very large amplitude, representing a super-storm, are of special interest for the designer. The (partially) cracked structure has to withstand this severe loading conditions, even if there is a stiffness degradation due to the millions of 'low-intensity'-cycles. There are no numerical tools available to simulate all these cycles in order to determine the response degradation. Furtheron, there was no experimental knowledge with respect to 'low-intensity high-cycle' fatigue.

Therefore, first an experimental program was devoted to this type of test, yielding empirical expressions for the increase in crack displacements due to cycling. It is shown that 'high-cycle' fatigue can be treated quasi-statically in order to obtain the crack displacements at the onset of the design load ('high-intensity low-cycle' fatigue).
Second, the static models for aggregate interlock and dowel action are adapted to the case of 'high-intensity' cyclic loading. Both mechanisms are described as to fit to the crack displacements obtained with the empirical relations for 'low-intensity' load cycles. An important conclusion is that for the aggregate interlock mechanism, the load history is fully incorporated in the end crack displacements of the previous cycle. Furthermore, it is shown, that the contributions to the shear transfer of both dowel action and aggregate interlock remain nearly constant during cycling. This observation was used to prove that the cyclic models are also valid for the case of 'high-cycle' fatigue.

Although valuable experimental and theoretical information is obtained with respect to 'low-intensity high-cycle' fatigue, further study is necessary in this field. First, only one stress ratio (R = 0) was investigated in combination with a constant loading frequency. Both parameters might largely influence the stiffness degradation of the crack due to cycling.

Second, only reinforcing bars perpendicularly crossing the crack plane were used. In practice, orthogonal reinforcing webs cross cracks at various angles to the crack plane. For this case, the contribution of the axial steel stresses and the bond strength degradation also influence the crack response. Furthermore, the cyclic aggregate interlock model is strongly simplified with respect to the physical reality. This model is, however, still applicable to a wide range of tests. Contrary to this, the cyclic dowel action model is to a large extent based upon empirical relations, thus limiting its application. Therefore, further theoretical work into this field is necessary.

Finally, according to the static dowel action mechanism there is no longer a relationship between the axial steel stress and the crack width when the plastic hinges in the bar fully have been developed. Because of this lack of a relation between the crack width or normal strain and the normal restraint stiffness, the implementation of the dowel action mechanism into numerical programs is not yet fully described in this report. With respect to the mechanism of aggregate interlock, further experimental study is necessary in the following fields:

a. 'low-intensity high-cycle' experiments with a reversed shear load (R < 0).

b. cyclic push-off tests on pre-cracked specimens with water or oil in
the crack. Due to the opening and re-closing of the crack water (oil) is pumped in and out the crack, thus transporting crushed matrix material. Furthermore, the pressure of the fluid transfers stresses normal to the crack plane.

The mechanism of dowel action can provide an important contribution to the transfer of shear stress across cracks in reinforced concrete. In most cases, however, the direction of the principal tensile stress after cracking only slightly deviates from its direction during cracking. For these cases, the dowel action mechanism is of minor importance due to the relatively high axial steel stress.

With respect to the mechanism of dowel action, some questions remained unanswered:

a. The dowel capacity decreases with increasing initial crack width. Yet, it is not known to what extent this is caused by the crack width itself or by the initial steel stress necessary to obtain this crack width.

b. The dowel mechanism is strongly related to the bond mechanism. This interaction is, however, poorly understood.

An experimental study on push-off elements in which the aggregate interlock mechanism is prevented by means of smooth crack faces, can provide the detailed information needed in this field. In such a test series, the dowel capacity must be measured for several combinations of the initial crack width and axial steel stress. Furthermore, several load paths must be investigated, such as:

- first a dowel force is applied, subsequently an axial steel force is increased monotonically until failure occurs.
- for an initial axial steel stress, the dowel force is increased. Then the axial steel stress is removed. What will be the crack displacements due to this unloading.

It must be noted that especially any improvement in modelling the bond between the steel bar and the concrete will also improve the physical understanding of the dowel action mechanism. It is the author's opinion that bond tests are necessary to make the dowel model generally applicable.
8. SUMMARY

Offshore platforms, used for the exploitation of the oil and gas reservoirs in the Arctic and the deep sea, are designed to withstand severe loading conditions, characterized by wave and wind attacks. Such structures are so configured as to transfer the applied cyclic loads to the subsoil by means of in-plane shear and normal stresses. The walls of the base of such a structure might be cracked due to unequal settlements and thermal deformations. As a consequence, these cracked reinforced panels will respond highly nonlinear to the applied stresses.

The transfer of in-plane stresses across cracks in reinforced concrete is based upon the interaction of several mechanisms:

a. the axial stiffness of the reinforcing bars crossing the crack,
b. the lateral stiffness of the bars, called dowel action, and
c. the interlocking of the aggregate particles protruding from the crack faces, denoted as aggregate interlock.

For the case of cyclic loads, usually a distinction is made between on the one hand 'low-intensity high-cycle' loading, reflecting the load history of millions of small wave attacks. On the other hand, high-intensity low-cycle' loading is considered, which forms another severe loading condition a structure has to withstand. From literature research it has become clear that there was a lack of experimental knowledge on the response of cracked concrete subjected to shear loading, especially for the case of a large number of cycles with a low shear stress relative to the static shear strength.

Therefore, first an experimental study was carried out on push-off specimens. For reinforced concrete specimens, a repeated shear load was applied ranging from 46 to 90 percent of the static shear strength. The number of cycles ranged from 118 to 931731 cycles. The increase in crack displacements due to cycling was recorded and expressed in empirical relations. Apart from this test series, similar tests were performed on plain concrete specimens in order to determine the contribution of the aggregate interlock mechanism alone.

Second, the crack response under monotonic loading was discussed. Whereas the aggregate interlock mechanism was satisfactorily described by means of the two-phase model of Walraven, the mechanism of dowel action
was not yet fully understood. Therefore, a physical description of the static dowel action mechanism based upon cooperation of the steel bar and the supporting concrete was given. It was found that the combined mechanism of aggregate interlock and dowel action could be used to simulate static shear tests on reinforced concrete push-off specimens. Initially, the deformation of the bars determines the crack opening direction. After plastic hinges in the bar have fully been developed, the crack opening path is determined by the aggregate interlock mechanism.

Next, it was shown that Walraven's extended version of his two-phase model could be applied to the case of cracked plain concrete subjected to cyclic shear loading. This numerical model was then simplified yielding an analytical solution method, the analytical contact model. The results of this model were used to derive empirical expressions for retention factors, which could be applied to the contact areas according to the static two-phase model. Several 'high-intensity low-cycle' experiments were simulated with this reduced contact model.

As for the static dowel action mechanism, the response of a bar to cyclic dowel forces was not yet described by a physical model. Therefore, a rather simple model is proposed, which is based upon physical material behaviour. Because of a lack of detailed experimental information, this model is still to a large extent based upon empirical expressions. It enables the determination of the effects of bar diameter, steel and concrete strength and initial crack width. The results of test specimens subjected to a small number of cycles with a large amplitude were satisfactorily simulated with the dowel action model.

For practical use, the combined model of aggregate interlock and dowel action under cyclic shear loading is much interest. Again, it was found that the calculated crack response according to the combined model agrees very well with the results of 'high-intensity low-cycle' experiments. Furtheron, a few cycles of 'high-cycle' tests were satisfactorily predicted. An important conclusion was that the contributions of both transfer mechanisms remained almost constant during cycling. With this observation, it was possible to give a reasonable prediction of the crack opening path of 'low-intensity high-cycle' experiments.

Finally, the aggregate interlock model for monotonic and cyclic shear loading was made valid for implementation in numerical programs. It was shown that the commonly used shear retention factor, which has a con-
stant value, has no general applicability and neglects the physical behaviour of the crack. However, for the case of a monotonic loading, a constant retention factor gives reasonable predictions of the shear stresses. Contrary to this, for the case of mixed-mode fracture problems or cyclic loading, a constant shear retention factor will largely overestimate the shear loading capacity of the crack for a given combination of the crack displacements. Furthermore, for this case the interaction between the tension-softening and the shear-softening behaviour must be accounted for.
SAMENVATTING

De booreilanden, welke worden gebruikt voor de exploitatie van de olie- en gas reserves in de Poolzee en de diepzee, zijn ontworpen om de extreme belastingen te weerstaan, welke worden gekenmerkt door golf en wind belastingen. Dergelijke constructies zijn zo samengesteld dat de opgelegde (wisselende) belastingen naar de ondergrond worden overgedragen door middel van spanningen in het vlak. De wanden van de funderingsplaat van een booreiland zijn mogelijk gescheurd ten gevolge van ongelijkmatige zettingen en temperatuurspanningen. Als een gevolg hiervan reageren deze gescheurde schijven sterk niet-lineair op de opgelegde spanningen.

De overdracht van de spanningen in het vlak over de scheuren in gewapende betonnen schijven berust op de interactie van verschillende mechanismen, te weten:

a. de axiale stijfheid van de staven, welke het scheurvlak doorsnijden,

b. de deuvelweerstand van de wapening en
c. de haakweerstand van de toeslagkorrels, die uit het scheurvlak steken.

Voor het geval van wisselbelastingen wordt er meestal onderscheid gemaakt tussen aan de ene kant belasting met een lage intensiteit en een groot aantal wisselingen, welke staan voor de belastinggeschiedenis van miljoenen golfaanvallen. Aan de andere kant worden belastingen beschouwd met een grote amplitude gedurende een beperkt aantal wisselingen. Deze wisselingen vormen feitelijk de ontwerpbelasting. Uit een literatuuronderzoek is gebleken dat experimentele gegevens ten aanzien van de reactie van gescheurd gewapend beton op wisselende schuifkrachten ontbreken, met name ten aanzien van een groot aantal wisselingen met een lage schuifspanning in verhouding tot de statische schuifsterkte.

Teneinde deze informatie te verkrijgen is een experimenteel programma uitgevoerd op afschuif-elementen. Op gewapende proefstukken is een herhaalde belasting varieerd van 46 tot 90 procent van de statische schuifsterkte aangebracht. Het aantal wisselingen varieerde daarbij van 118 tot 931731 wisselingen. De toename van de scheurverplaatsingen ten gevolge van de lastwisselingen is gemeten en uitgedrukt in empirische relaties. Daarnaast zijn soortelijke proeven uitgevoerd op proefstukken
van ongewapend beton teneinde de bijdrage van het mechanisme van de haakweerstand van de korrels te bepalen.

Vervolgens is de reactie van de scheur op monotoon stijgende belasting bestudeerd. Daar waar het mechanisme van de haakweerstand van de korrels goed werd beschreven door het twee-fasen model van Walraven, was het mechanisme van de deuvelwerking nog niet volledig verklaard. Daarom is een beschrijving van het fysische gedrag van de staaf onder een deuvelkracht gegeven, waarbij is uitgegaan van de samenwerking van de stalen staaf en de beton direct onder de staaf. Het bleek dat het gecombineerde mechanisme van deuvelwerking en haakweerstand kon worden toegepast op experimenten met gewapend betonnen proefstukken. Aanvankelijk wordt het scheuropeningspad bepaald door de vervorming van de staven. Echter na het ontwikkelen van plastische scharnieren in deze staven, bepaalt de haakweerstand de scheuropeningsrichting.

Daarna is aangetoond dat een uitgebreid twee-fasen model zoals dat is voorgesteld door Walraven, kan worden toegepast voor het geval van wisselende schuifspanningen op ongewapend beton. Dit model is vervolgens vereenvoudigd tot een model met een analytische oplossing voor de grootte van het kontaktvlak tussen de korrels en de matrix. De resultaten van de berekeningen met dit model zijn op hun beurt weer gebruikt om uitdrukkingen af te leiden voor reductie-factoren. De reductie-factoren kunnen worden toegepast op de kontaktvlakken volgens het statistische twee-fasen model. Dit gereduceerde kontakt-model is gebruikt om verschillende proeven met een grote amplitude en een beperkt aantal wisselingen door te rekenen.

Net als voor het statische deuvelmodel was de reactie van een staaf op wisselende deuvelkrachten nog niet volledig fysisch verklaard. Daarom is een relatief eenvoudig model voorgesteld, welke is gebaseerd op fysisch materiaal gedrag. Een ontbreken van gedetailleerde experimentele informatie veroorzaakte echter dat ook dit model tot op zekere hoogte is gebaseerd op empirische uitdrukkingen. Het model maakt het mogelijk om de effecten van staafdiameter, staal- en betonkwaliteit en initiële scheurwijdte in rekening te brengen. De resultaten van meerdere deuvelproeven met een grote amplitude gedurende een gering aantal wisselingen zijn redelijk gesimuleerd met dit model.

Voor toepassing in de praktijk is met name het gekombineerde model van de haakweerstand en deuvelwerking van belang. Ook nu werd gevonden dat
de berekende reactie volgens het gekombineerde model een goede voorspelling geeft voor het experimenteel verkregen scheurgedrag in proeven met een gering aantal wisselingen. Verder zijn enkele wisselingen van proeven met een zeer groot aantal wisselingen nagerekend met bevredigend resultaat. Een belangrijke conclusie was dat de bijdragen van de afzonderlijke mechanismen nagenoeg constante blijven gedurende de lastwisselingen. Op grond van deze waarneming was het mogelijk een betrouwbare voorspelling te doen van het te volgen scheuropeningspad voor proeven met een kleine amplitude en een groot aantal wisselingen.

Tenslotte is het model van de haakweerstand voor monotone en wisselende schuifbelasting geschikt gemaakt voor implementatie in numerieke programma's. Aangetoond is dat de vaak gebruikte reductiefactor voor de schuifweerstand, welke een constante waarde heeft, niet algemeen toepasbaar is en geen relatie heeft met het werkelijke scheurgedrag. Toch zal een constante waarde voor de reductiefactor in de regel geen overschatting van de schuifspanningen geven voor het geval van een monotoon stijgende belasting. Daarentegen wordt de schuifsterkte wel sterk overschat indien een constante waarde voor de reductiefactor wordt gehanteerd in het geval van mixed-mode scheurproblemen en in het geval van wisselbelastingen voor gegeven combinaties van de scheurverplaatsingen. Tevens moet voor dit geval worden meegenomen de interactie tussen de afnemende stijfheiden onder enerzijds de normaalspanning (tension-softening) en anderzijds de schuifspanning (shear-softening).
9. NOTATION

a, b  numerical constants

a_x  projected contact area in x-direction [mm^2]

a_y  projected contact area in y-direction [mm^2]

ej  eccentricity [mm]

f_{ccyl}  cylindrical concrete crushing strength [MPa]

f_{ccm}  cube concrete crushing strength [MPa]

f_{sy}  steel yield strength [MPa]

h  element size [mm]

m  numerical constant

n  number of cycles

n_f  number of cycles till failure

p_k  volume of the particles/total volume

r  radius [mm]

x  length [mm]

x  direction in the two-dimensional space

y  direction in the two-dimensional space

z  distance to neutral axis [mm]

A  cross-sectional area [mm^2]

A_x  total projected contact area in x-direction [mm^2]

A_y  total projected contact area in y-direction [mm^2]

C  numerical constant

D_{max}  maximum particle diameter [mm]

D  specific particle diameter [mm]

E_c  modulus of elasticity of concrete [MPa]

E_s  modulus of elasticity of steel [MPa]

F_d  dowel force [kN]

F_{du}  ultimate dowel force [kN]

G  shear modulus [MPa]

I  moment of inertia [mm^4]

K  numerical constant

K_{f_i,...,h}  dowel stiffness [N/mm]

K_f  foundation modulus of concrete [MPa/mm]

L  length [mm]

M_u  ultimate bending moment [Nm]

P  numerical constant
\( R \) cyclic stress ratio = \( \tau_{\text{min}} / \tau_{\text{max}} \)

\( S_{11,..,33} \) terms of the stiffness matrix

\( a \) numerical constant

\( \alpha \) angle of inclination

\( \alpha_{1,..,2} \) normal retention factors

\( \beta \) angle

\( \beta_{1,..,2} \) shear retention factors

\( \gamma \) shear sliding

\( \gamma_m \) maximum shear sliding

\( \gamma_0 \) residual shear sliding

\( \gamma_{cr} \) shear sliding of the crack zone

\( \gamma_{co} \) shear sliding of the uncracked zone

\( \gamma_a \) retention factor of the stresses due to aggregate interlock

\( \gamma_d \) retention factor of the stresses due to dowel action

\( \gamma_k \) retention factor of the dowel stiffness

\( \delta_t \) shear displacement [\( \text{mm} \)]

\( \delta_0 \) residual shear displacement [\( \text{mm} \)]

\( \delta_m \) maximum shear displacement [\( \text{mm} \)]

\( \delta_{t,e} \) shear displacement due to elastic deformation [\( \text{mm} \)]

\( \delta_{n,o} \) initial crack width [\( \text{mm} \)]

\( c \) numerical constant

\( \varepsilon_{nn} \) normal strain

\( \varepsilon_{nn,cr} \) normal strain of the crack zone

\( \varepsilon_{nn,co} \) normal strain of the uncracked zone

\( \eta \) normal retention factor

\( \lambda_{x,..,y} \) retention factors of the contact areas

\( \mu \) coefficient of friction

\( \mu \) normal retention factor

\( u \) Poisson's ratio

\( \rho \) reinforcement ratio

\( \sigma \) normal stress [\( \text{MPa} \)]

\( \sigma_a \) normal stress due to aggregate interlock [\( \text{MPa} \)]

\( \sigma_d \) normal stress due to dowel action [\( \text{MPa} \)]

\( \sigma_s \) steel stress [\( \text{MPa} \)]

\( \sigma_{pu} \) strength of material [\( \text{MPa} \)]

\( \sigma_n \) normal stress acting upon the crack plane [\( \text{MPa} \)]
\( \tau_a \) shear stress due to aggregate interlock [MPa]
\( \tau_b \) bond stress [MPa]
\( \tau_d \) shear stress due to dowel action [MPa]
\( \tau_o \) minimum applied shear stress [MPa]
\( \tau_m \) maximum applied shear stress [MPa]
\( \tau_u \) shear strength [MPa]
\( \psi \) angle of inclination
\( \phi \) retention factor of the cracked area
\( \Theta \) angle of inclination
\( \phi \) bar diameter [mm]
\( \psi \) angle of friction
10. LITERATURE

[1] Bazant, Z.P., Cambarova, P.G.,
Rough cracks in reinforced concrete,
ASCE, Journal of Structural Division, Vol. 106, No. 4,
April 1980, pp. 819-842.

Microplane model for concrete subject to tension and shear,
Int. Conference on Concrete under multiaxial conditions, Toulouse,

Crack shear in concrete, crack band microplane model,
ASCE, Journal of Structural Engineering, Vol. 110, No.9,

[4] Bennett, E.W., Banerjee, S.,
Strength of beam-column connections with dowel reinforcement,
The Structural Engineer, Vol. 51, No. 4, April 1976, pp. 133-139.

Connections in precast concrete constructions,

[6] De Borst, R.,
Non-linear analysis of frictional materials,

[7] Broms, B.B.,
Lateral resistance of piles in cohesive soils,
ASCE, Journal of soil mechanics, Vol. 90, No.2,
March 1964, pp. 27-59.

[8] Chung, H.W.,
Shear strength of concrete joints under dynamic loads,

[9] Colley, B.E., Humphrey, H.A.,
Aggregate interlock at joints in concrete pavements,

[10] Collins, M.P.,
Memorandum to the participants in the University of Toronto's International prediction competition,
October 1984.

Shear design of complex high strength concrete structures,
Proceeding of the conference on high strength concrete,

[12] Daschner, F., Kupfer, H.,
Versuche zur Schubkraftübertragung in Rissen von Normal- und Leichtbeton,
Bauingenieur 57, (1982), pp. 57-60.

[13] Daschner, F., Wissen, I.,
Schubkraftübertragung in Rissen von Normal- und Leichtbeton,

A constitutive model for shear transfer in cracked concrete,
submitted for publication, ASCE, Structural division 1987.

[15] Dulacska, H.,
Dowel action of reinforcement crossing cracks in concrete,
[16] Eleiott, A.F.,
   An experimental investigation of shear transfer across cracks in
   reinforced concrete,

[17] Fardis, M.N., Buyukozturk, O.,
   Shear stiffness of concrete by finite elements,
   ASCE, Journal of the Structural Division, Vol. 106, No. 6,

[18] Fenwick, R.C., Paulay, T.,
   Mechanisms of shear resistance of concrete beams,
   ASCE, Structural Division, Vol. 94, No. 10, Oct. 1968,
   pp. 2325-2350.

[19] Finney, E.A.,
   Structural design considerations for pavement joints,
   Subcommittee III, ACI-committee 325, ACI-journal,

[20] Frenay, J.W.,
   Shear transfer across a single crack in reinforced concrete
   under sustained loading, Part I. Experiments
   Report 5-85-5, Stevin Laboratory, Delft University of

[21] Frenay, J.W., Liqui Lung, C., Pruijsers, A.F.,
   Shear transfer across a single crack in reinforced concrete
   Additional detailed tests,
   Report 5-86-5, Stevin Laboratory, Delft University of

[22] Friberg, B.F.,
   Design of dowels in transverse joints of concrete pavements,

[23] Gambarova, P.G.,
   On aggregate interlock mechanism in reinforced concrete plates
   with extensive cracking, Transactions of IABSE colloquium Delft
   1981 on Advanced Mechanics of Concrete Delft, June 1981, pp. 99-
   120.

[24] Gambarova, P.G.,
   Crack shear in concrete: Rough crack model and micro-plane model,
   Presented at the 1983 meeting of the Italian Society for normal

[25] Gerwick, B.C.,
   High strength concrete, key to the Arctic and deep sea,
   Proceeding of the conference on high strength concrete,

[26] Gerwick, B.C.,
   High-Amplitude Low-cycle fatigue in concrete sea structures,

[27] Graf, O., Brenner, E.,
   Versuch zur Ermittlung der Wiederstandfähigkeit
   von Beton gegen oftmals wiederholte Belastung,
   Deutscher Ausschuss für Stahlbeton, Heft 76 und 83,
   Berlin, 1934 and 1936.

[28] Hansen, R.J., Nawy, E.G., Shah, J.M.,
   Response of concrete shear keys to dynamic loading,
   ASCE-journal of Structural Div., Vol. 32, No. 11, May 1961,
   pp. 1475-1490.
[29] Hofbeck, J.A., Ibrahim, I.O., Mattock, A.H.,
Shear transfer in reinforced concrete,

[30] Holmen, J.O.,
Fatigue of concrete by constant and variable amplitude
loading,
Bulletin No. 79-1, Division of Concrete Structures,
NTH-Trontheim, 1979, pp. 218.

[31] Houde, J., Mirza, M.S.,
A finite element analysis of shear strength of reinforced
concrete beams,
ACI-Special Publication 42, 'Shear in reinforced concrete',
pp. 103-128.

Shear transfer across cracks in reinforced concrete,
Report No. 78-4, dept. of Structural Eng., Cornell University,

[33] Jimenez, R., Perdikaris, P., Gergely, P.,
Interface shear transfer and dowel action in cracked reinforced
concrete subject to cyclic shear,

[34] Jimenez, R., White, R.N., Gergely, P.,
Cyclic shear and dowel action models in reinforced concrete,
ASCE, Journal of Structural Engineering, Vol. 108,

[35] Klein, D., Kristjansson, R., Link, J., Mehlhorn, G., Schaeffer, H.,
Zur Berechnung von dünnen Stahlbetonplatten bei Berücksichtigung
eines wirklichkeitsnahen Werkstoffverhaltens,
Forschungbericht No. 25, Inst. für Massivbau,

[36] Laible, J.P.,
An experimental investigation of interface shear transfer and
applications in the dynamic analysis of nuclear containment
vessels,

[37] Laible, J.P., White, R.N., Gergely, P.,
Experimental investigation of seismic shear transfer across
cracks in concrete nuclear containment vessels,
ACI-Special Publication 53, Reinforced concrete structures

[38] Van Leeuwen, J., Siemes, A.J.M.,
Miner's rule with respect to plain concrete,

[39] Mast, R.F.,
Auxiliary reinforcement in concrete connections,
ASCE, Journal of Structural Division, Vol. 94,
No. 6, June 1968, pp. 1485-1499.

[40] Mattock, A.H.,
Shear transfer in concrete having reinforcement at an angle
to the shear plane,
ACI-Special Publication 42, Shear in reinforced concrete,
[41] Mattock, A.H.,
Effect of aggregate type on single direction shear transfer strength in monolithic concrete,

[42] Mattock, A.H.,
Effect of moment torsion across the shear plane on single direction shear transfer strength in monolithic concrete,

[43] Mattock, A.H.,
Cyclic shear transfer and type of interface,

[44] Van Mier, J.G.M.,
Strain-softening of concrete under multi-axial loading conditions,

[45] Millard, S.C., Johnson, R.P.,
Shear transfer across cracks in reinforced concrete due to aggregate interlock and dowel action,

[46] Millard, S.C., Johnson, R.P.,
Shear transfer in cracked reinforced concrete,
Mag. of Concrete Research, Vol. 37, No. 130, March 1985, pp. 3-15.

[47] Mills, G.M.,
A partial kinking yield criterion for reinforced concrete slabs,

[48] Moe, J.,

[49] Nissen, I.,
Rissverzahnung des Betons, gegenseitige Rissuferverschiebungen und übertragene Kräfte,

[50] Paulay, T., Loebel, P.J.,
Shear transfer by aggregate interlock,

[51] Paulay, T., Park, R., Philips, M.H.,
Horizontal construction joints in cast in place reinforced concrete,

[52] Perdikaris, P.C., Hilmy, S., White, R.N.,
Extensional stiffness of precracked reinforced concrete panels,

[53] Perdikaris, P.C., White, R.N., Gergely, P.,
Strength and stiffness of biaxially tensioned reinforced concrete subjected to reversed shear loads,
[54] Perdikaris, P.C., White, R.W.,
Shear modulus of precracked reinforced concrete panels,
ASCE, Journal of Structural Engineering, Vol. 111,

[55] Pruijssers, A.P.,
Description of the stiffness relation for mixed-mode fracture
problems using the rough-crack model of Walraven,
Stevin Report 5-85-2, Delft University of Technology, 1985, pp.36.

[56] Pruijssers, A.P., Liqui Lung, G.,
Shear transfer across a crack in concrete subjected to repeated
loading, Experimental results, Part I,
Report 5-85-12, Stevin Laboratory, Delft University of Technology,
1985, pp. 178.

[57] Rasmussen, B.H.,
Strength of transversely loaded bolts and dowels cast into
concrete, Laboratoriet for Bugningastatik, Denmark Technical
University, Meddelelse, Vol. 34, No. 2, 1962, (in Danish).

[58] Reinhardt, H.W., Walraven, J.C.,
Cracks in concrete subject to shear,
ASCE, Journal of Structural Engineering, Vol. 108,

[59] Roland, B., Skare, E., Olsen, T.O.,
Ship impact on concrete shafts,

[60] Rots, J.C., Kuipers, G.M.A., Nauta, P.,
Variabele reductiefactor voor de schuifweerstand van gescheurd
beton,
TNO-IBBC report Bl-84-3, 1984, pp. 44.

[61] Rots, J.C.,
Bond-slip simulations using smeared cracks and/or
interface elements,
Report Delft University of Technology, 1985, pp. 56.

[62] Rots, J.C., De Borst, R.,
Analysis of mixed-mode fracture in concrete,

[63] Schaefer, H.,
Zur Berechnung von Stahlbetonplatten,
Dissertation, University of Technology Darmstadt, 1976.

[64] Stanton, J.F.,
An investigation of dowel action of the reinforcement of nuclear
containment vessels and their non-linear dynamic response to
earthquake loads,

[65] The structural design of concrete pavements,
Pt. 4, Public Roads, Sept. 1936.

[66] Taylor, H.P.J.,
Fundamental behaviour in bending and shear of reinforced concrete,

[67] Taylor, R.,
A note on the mechanism of diagonal cracking in reinforced
concrete beams without web reinforcement,
[68] Teller, L.W., Sutherland, E.J.,
A study of structural action of several types of transverse and longitudinal joint design,

[69] Termonia, Y., Meakin, P.,
Formation of fractal cracks in a kinetic fracture model,

[70] Timoshenko, S., Lessels, J.M.,
Applied elasticity,

[71] TML technical bulletin No. 1007,

[72] Untrauer, R.E., Henry, R.L.,
Influence of normal pressure on bond strength,

[73] Utescher, C., Herrmann, M.,
Versuche zur Ermittlung der Tragfähigkeit in Beton eingespannter Rundstahldollen aus nichtrostendem austenitischem Stahl,

[74] Vecchio, F.J., Collins, M.P.,
The modified compression-field theory for reinforced concrete elements subjected to shear,

[75] Vintzeleou, E.N.,
Mechanisms of load transfer along reinforced concrete interfaces under monotonic and cyclic actions,
Ph.D. Thesis (in Greek), Department of Civil Eng., National Technical University of Athens, December 1984, pp. 549.

[76] Vintzeleou, E., Tassios, T.P.,
Mechanisms of load transfer along interfaces in reinforced concrete, prediction of shear force versus shear displacement curves,
Studi e ricerche, Vol. 7, 1985, pp. 121-159.

[77] Vintzeleou, E., Tassios, T.P.,
Mathematical models for dowel action under monotonic and cyclic conditions,

[78] Vintzeleou, E., Tassios, T.P.,
Behaviour of dowels under cyclic deformations,
ACI, Structural journal, Vol. 84, Jan-Febr. 1987, pp. 18-30.

[79] Vos, E.,
Influence of loading rate and radial pressure on bond in reinforced concrete. A numerical and experimental approach,

[80] Walraven, J.C.,
Influence of depth on the shear strength of lightweight concrete beams without shear reinforcement,

[81] Walraven, J.C.,
[82] Walraven, J.C.,
The behaviour of cracks in plain and reinforced concrete subjected to shear,

[83] Walraven, J.C.,
Kornverzahnung bei zyklischer Belastung,
Mitteilungen aus dem Inst. für Massivbau der Techn.

[84] Walraven, J.C., Frenay, J., Pruijssers, A.F.,
Influence of concrete strength and load history on the shear friction capacity of concrete members,

[85] Walraven, J.C., Vos, E., Reinhardt, H.W.,
Experiments on shear transfer in cracks in concrete. Part I,
Description of results,
Report No. 5-79-3, Jan. 1979, Stevin Laboratory, Delft University of Technology.

[86] White, R.N.,
Interface shear transfer and dowel action in cracked reinforced concrete,

[87] White, R.N., Gergely, P.,
Design considerations for seismic tangential shear in reinforced concrete containment structures,

[88] White, R.N., Holley, M.J.,
Experimental study of membrane shear transfer,
APPENDIX I. Mix proportions

Mix code B1632550 strength $f'_c = 51 \text{ N/mm}^2$

(mix A)

<table>
<thead>
<tr>
<th>Components</th>
<th>Sieve analysis of aggregate</th>
<th>Sieve opening</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[kg/m$^3$]</td>
<td>[mm]</td>
</tr>
<tr>
<td>sand</td>
<td>877.2</td>
<td>8 – 16</td>
</tr>
<tr>
<td>gravel</td>
<td>1065.0</td>
<td>4 – 8</td>
</tr>
<tr>
<td>cement-B</td>
<td>325.0</td>
<td>2 – 4</td>
</tr>
<tr>
<td>water</td>
<td>162.5</td>
<td>0.5 – 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.25 – 0.5</td>
</tr>
<tr>
<td></td>
<td>2429.7</td>
<td>0.10 – 0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mix code B1642037.5 strength $f'_c = 70 \text{ N/mm}^2$

(mix B)

| sand        | 857.3      | 8 – 16 | 596.5 |
| gravel      | 1018.5     | 4 – 8  | 421.9 |
| cement-B    | 420.0      | 2 – 4  | 298.3 |
| water       | 147.0      | 1 – 2  | 212.0 |
| superpl.2%  | 10.5       | 0.5 – 1.0 | 148.6 |
|             |            | 0.25 – 0.50 | 105.0 |
|             | 2453.3     | 0.10 – 0.25| 93.5  |
|             |            |         | 1875.8|

Sieve analysis of aggregate

<table>
<thead>
<tr>
<th>[cum.%]</th>
<th>mix A</th>
<th>mix B</th>
<th>Fuller</th>
<th>sieve opening [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>8 – 16</td>
</tr>
<tr>
<td>67.9</td>
<td>68.2</td>
<td>70.7</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>45.2</td>
<td>45.7</td>
<td>50.0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>29.1</td>
<td>29.3</td>
<td>35.4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>17.7</td>
<td>18.5</td>
<td>25.0</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>9.7</td>
<td>10.6</td>
<td>17.3</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>4.0</td>
<td>5.0</td>
<td>12.5</td>
<td>0.1</td>
<td>0.25</td>
</tr>
</tbody>
</table>
APPENDIX II. Results of dowel action tests compared with the theoretical results.

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>bar diameter</th>
<th>eccentricity</th>
<th>$f_{ccm}$</th>
<th>$f_{sy}$</th>
<th>$F_{d,exp}$</th>
<th>$F_{d,cal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[mm]</td>
<td>[mm]</td>
<td>[MPa]</td>
<td>[MPa]</td>
<td>[kN]</td>
<td>[kN]</td>
</tr>
<tr>
<td>Experiments of Bennett and Banerjee [4]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>0</td>
<td>47</td>
<td>410</td>
<td>11.0</td>
<td>6.7</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td>47</td>
<td>410</td>
<td>11.0</td>
<td>6.7</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>0</td>
<td>47</td>
<td>410</td>
<td>11.0</td>
<td>6.7</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>0</td>
<td>47</td>
<td>410</td>
<td>11.0</td>
<td>6.7</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>0</td>
<td>36</td>
<td>410</td>
<td>23.3</td>
<td>27.7</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>0</td>
<td>36</td>
<td>410</td>
<td>24.0</td>
<td>27.7</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>0</td>
<td>36</td>
<td>410</td>
<td>27.6</td>
<td>27.7</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>0</td>
<td>39</td>
<td>410</td>
<td>39.5</td>
<td>43.7</td>
</tr>
<tr>
<td>9</td>
<td>19</td>
<td>0</td>
<td>36</td>
<td>410</td>
<td>43.7</td>
<td>59.2</td>
</tr>
<tr>
<td>Experiments of Paulay, Park and Philips [51]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.3</td>
<td>0</td>
<td>33</td>
<td>317</td>
<td>6.0</td>
<td>5.4</td>
</tr>
<tr>
<td>2</td>
<td>9.6</td>
<td>0</td>
<td>33</td>
<td>317</td>
<td>11.7</td>
<td>12.6</td>
</tr>
<tr>
<td>3</td>
<td>12.7</td>
<td>0</td>
<td>33</td>
<td>317</td>
<td>19.2</td>
<td>22.1</td>
</tr>
<tr>
<td>Experiments of Rasmussen [57] * See eq. (4.19)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>15.8</td>
<td>1.4*)</td>
<td>13</td>
<td>247</td>
<td>16.8</td>
<td>18.1</td>
</tr>
<tr>
<td>D2</td>
<td>25.1</td>
<td>2.2</td>
<td>12</td>
<td>225</td>
<td>38.5</td>
<td>41.4</td>
</tr>
<tr>
<td>D3</td>
<td>15.8</td>
<td>1.9</td>
<td>24</td>
<td>247</td>
<td>24.0</td>
<td>23.2</td>
</tr>
<tr>
<td>D4</td>
<td>25.1</td>
<td>3.7</td>
<td>32</td>
<td>225</td>
<td>62.5</td>
<td>61.0</td>
</tr>
<tr>
<td>D5</td>
<td>16.0</td>
<td>1.3</td>
<td>20</td>
<td>439</td>
<td>35.5</td>
<td>30.9</td>
</tr>
<tr>
<td>D6</td>
<td>25.9</td>
<td>2.0</td>
<td>16</td>
<td>408</td>
<td>70.5</td>
<td>69.9</td>
</tr>
<tr>
<td>D7</td>
<td>15.8</td>
<td>2.4</td>
<td>37</td>
<td>247</td>
<td>29.5</td>
<td>26.9</td>
</tr>
<tr>
<td>D8</td>
<td>25.1</td>
<td>3.8</td>
<td>35</td>
<td>225</td>
<td>69.5</td>
<td>62.8</td>
</tr>
<tr>
<td>D9</td>
<td>15.8</td>
<td>2.8</td>
<td>52</td>
<td>247</td>
<td>31.8</td>
<td>30.0</td>
</tr>
<tr>
<td>D10</td>
<td>25.1</td>
<td>4.7</td>
<td>52</td>
<td>225</td>
<td>79.2</td>
<td>70.2</td>
</tr>
<tr>
<td>Experiments of Vintzeleou [75]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DB-40,8/M</td>
<td>8</td>
<td>2</td>
<td>51</td>
<td>420</td>
<td>16.8</td>
<td>9.7</td>
</tr>
<tr>
<td>DB-40,14/M</td>
<td>14</td>
<td>2</td>
<td>28</td>
<td>420</td>
<td>21.0</td>
<td>25.5</td>
</tr>
<tr>
<td>DB-40,14/M</td>
<td>14</td>
<td>2</td>
<td>38</td>
<td>420</td>
<td>31.5</td>
<td>29.3</td>
</tr>
<tr>
<td>DB-40,14/M</td>
<td>14</td>
<td>2</td>
<td>55</td>
<td>420</td>
<td>32.5</td>
<td>34.5</td>
</tr>
<tr>
<td>DB-40,14/M</td>
<td>14</td>
<td>2</td>
<td>55</td>
<td>420</td>
<td>35.0</td>
<td>34.5</td>
</tr>
<tr>
<td>DB-40,14/M</td>
<td>14</td>
<td>2</td>
<td>55</td>
<td>420</td>
<td>37.5</td>
<td>34.5</td>
</tr>
<tr>
<td>DB-150,14/M</td>
<td>14</td>
<td>2</td>
<td>52</td>
<td>420</td>
<td>35.0</td>
<td>33.6</td>
</tr>
<tr>
<td>DB-40,18/M</td>
<td>18</td>
<td>2</td>
<td>49</td>
<td>420</td>
<td>42.8</td>
<td>55.9</td>
</tr>
</tbody>
</table>
Specimen No. | bar diameter | eccentricity | $f_{ccm}$ | $f_{sy}$ | $F_{d,exp}$ | $F_{d,cal}$
--- | --- | --- | --- | --- | --- | ---
109 | 25 | 5 | 26 | 270* | 55.4 | 58.7
110 | 25 | 5 | 24 | 270 | 51.7 | 56.8
111 | 25 | 5 | 26 | 270 | 48.8 | 58.7
112 | 25 | 5 | 24 | 270 | 48.8 | 56.8
141 | 14 | 5 | 33 | 293 | 15.0 | 18.3
142 | 14 | 5 | 33 | 293 | 15.6 | 18.3
143 | 14 | 10 | 33 | 293 | 11.9 | 13.3
145 | 14 | 10 | 33 | 293 | 12.2 | 13.3
147 | 14 | 20 | 33 | 293 | 8.4 | 8.2
201 | 20 | 5 | 33 | 280 | 36.5 | 40.2
202 | 20 | 5 | 33 | 280 | 38.8 | 40.2
203 | 20 | 10 | 33 | 280 | 31.8 | 31.7
204 | 20 | 10 | 33 | 280 | 29.0 | 31.7
205 | 20 | 20 | 33 | 280 | 24.9 | 21.1
206 | 20 | 20 | 33 | 280 | 23.8 | 21.1
251 | 25 | 5 | 33 | 270 | 57.6 | 64.7
253 | 25 | 5 | 33 | 270 | 60.1 | 64.7
254 | 25 | 10 | 33 | 270 | 53.6 | 53.0
257 | 25 | 10 | 33 | 270 | 51.6 | 53.0
258 | 25 | 20 | 33 | 270 | 37.7 | 37.1
259 | 25 | 20 | 33 | 270 | 37.7 | 37.1
3143A | 14 | 50 | 31 | 245 | 3.0 | 3.0
2141A | 14 | 20 | 31 | 245 | 8.2 | 6.9
2141B | 14 | 20 | 31 | 245 | 8.3 | 6.9
2143A | 14 | 50 | 31 | 245 | 3.2 | 3.0
2144A | 14 | 20 | 31 | 245 | 8.3 | 6.9
3141A | 14 | 5 | 32 | 245 | 14.6 | 16.1
3141B | 14 | 5 | 32 | 245 | 13.2 | 16.1
3143A | 14 | 5 | 32 | 245 | 15.9 | 16.1
3143B | 14 | 5 | 32 | 245 | 17.2 | 16.1
3144B | 14 | 5 | 32 | 245 | 13.3 | 16.1
3202A | 20 | 50 | 32 | 275 | 11.3 | 9.5
3203A | 20 | 20 | 32 | 275 | 24.1 | 20.6
3204A | 20 | 20 | 32 | 275 | 24.1 | 20.6
3253A | 25 | 50 | 32 | 280 | 21.5 | 18.6
4201A | 20 | 20 | 32 | 275 | 23.8 | 20.6
4202A | 20 | 20 | 32 | 275 | 23.8 | 20.6
4203B | 20 | 20 | 32 | 275 | 22.9 | 20.6
4251A | 25 | 20 | 32 | 280 | 40.0 | 38.0
4251B | 25 | 20 | 32 | 280 | 36.2 | 38.0
4252A | 25 | 50 | 32 | 280 | 21.2 | 18.6
4253A | 25 | 20 | 33 | 280 | 39.4 | 38.3
4253B | 25 | 20 | 33 | 280 | 36.9 | 38.3
4255A | 25 | 20 | 33 | 280 | 41.9 | 38.3
4255B | 25 | 20 | 33 | 280 | 39.2 | 38.3

Experiments of Utscher and Herrmann [73]
*) According to Utscher the steel yield strength was approximately 84% of the reported yield strength [73, page 60].
<table>
<thead>
<tr>
<th>No.</th>
<th>diam. [mm]</th>
<th>eccent. [mm]</th>
<th>stress [MPa]</th>
<th>f&lt;sub&gt;ccm&lt;/sub&gt; [MPa]</th>
<th>f&lt;sub&gt;sy&lt;/sub&gt; [MPa]</th>
<th>F&lt;sub&gt;d,exp&lt;/sub&gt; [kN]</th>
<th>F&lt;sub&gt;d,cal&lt;/sub&gt; [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>21L</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>38</td>
<td>485</td>
<td>28.5</td>
<td>26.3</td>
</tr>
<tr>
<td>22L</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>39</td>
<td>485</td>
<td>25.6</td>
<td>26.6</td>
</tr>
<tr>
<td>23L</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>54</td>
<td>485</td>
<td>30.5</td>
<td>31.5</td>
</tr>
<tr>
<td>24L</td>
<td>16</td>
<td>0</td>
<td>0</td>
<td>28</td>
<td>485</td>
<td>40.8</td>
<td>40.0</td>
</tr>
<tr>
<td>25L</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>32</td>
<td>435</td>
<td>5.5</td>
<td>10.2</td>
</tr>
<tr>
<td>26L</td>
<td>12</td>
<td>0.10</td>
<td>175</td>
<td>37</td>
<td>485</td>
<td>20.8</td>
<td>24.2</td>
</tr>
<tr>
<td>27L</td>
<td>12</td>
<td>0.31</td>
<td>344</td>
<td>40</td>
<td>485</td>
<td>19.1</td>
<td>18.6</td>
</tr>
</tbody>
</table>

Experiments of Millard and Johnson [46]
APPENDIX III. Contact models.

Numerical contact model.

'BEGIN'
'REAL' FC,R,RMAX, DMAX, Y,WO,W,DEL,SIGI,TAUI,SIGE,C,MU,SPU,PK
XCT,XSNO,XSNB,YSNO, YSNB,G,V,DO,PCDO,AAX,AAK,
N,POX,ROX,DOX,DDEL,TAUMAX,TOX, WROX,QROX,PUNK,
DELE,WMAX,WE,SOX,SICEX,TAUS,TAUO,KSI,KCQ,HAX,HAY,NN,Z,BD;
'INTEGER'A,B,T,RU,P,Q,U,CYC,NCYC,NMAX;
'REAL' ARRAY'XH(/1:10,1:20,0:20/),AX(/1:20/),AY(/1:20/),GAX(/1:10/),
GAY(/1:10/),PC(/1:10/);
PK:=0.75;
FC:=20.7;TAUMAX:=1.24;WO:=0.76;NMAX:=5;DW:=.03;DDEL:=0.03;c:=3000;
WMAX:=WO;DELE:=0;
MU:=0.23;
SPU:=0.85*0.77*6.39*FC**0.56;
AX:=38.01;
OUTSTRING(1,'('PARAMETERS')');LINE(1,1);
OUTSTRING(1,'('SPU=')');FIX(l,2,l,SPU);LINE(l,1);
OUTSTRING(1,'('DMAX=')');FIX(l,2,2,DMAX);LINE(l,1);
OUTSTRING(1,'('MU=')');FIX(1,1,2,MU);LINE(1,1);
OUTSTRING(1,*('PK=')');FIX(1,1,2,PK);LINE(1,1);
OUTSTRING(1,'(INMAX=')');FIX(1,3,0,NMAX);LINE(1,3);
OUTSTRING(1,'('')') ;LINE( 1,1);
OUTSTRING(1,'('W DELTA SIGMA TAU')');LINE(1,1);
RMAX:=0.5*DMAX;
'COMMENT' PROBABILITY DENSITY FUNCTION
'FOR'A:=1'STEP'1'UNTIL'10'DO " BEGIN'
DO:=(A/10-0.05)*DMAX;BD:=DO/DMAX;
PC(/A/):=0.532*BD**.5-0.212*BD**4-0.075*BD**6-0.036*BD**8;
PC(/A/):=(PC(/A/)-0.025*BD**10)*PK/DO;'END';
TOX:=0;WROX:=0;SOX:=0;QROX:=0;A:=0;
'COMMENT' UNDEFORMED CONTACT AREAS
HED:A:=A+1;R:=(A/10-0.05)*RMAX;B:=0;
OL:B:=B+l;T:=-1;
LIL:T:=T+1;RU:=T-B;'IF'RU<0.1'THEN'
'BEGIN' Y:=0.1*R*T;XH(/A,B,T/):=SQRT(2*Y*R-Y**2+.0000000001);
'END'
'ELSE'
'BEGIN'XH(/A,B,T/):=10000;XH(/A,B,T-1/):=XH(/A,B,T-1/)+R/20;
'END';
'IF'T<10'THEN' 'GOTO'LIL;
'IF'B<10'THEN' 'GOTO'OL;
'IF'A<10'THEN' 'GOTO'HED;
A:=0;B:=0;T:=0;NCYC:=0;
'COMMENT' LOB= START OF CYCLE NCYC
LOB:NCYC:=NCYC+1;LINE(1,2);W:=WO;POX:=0;TOX:=0;ROX:=0;
WROX:=0;DOX:=0;SOX:=0;DEL:=DELE;SIGI:=0;
OUTSTRING(1,'('CYCLE = ')');FIX(1,2,0,NCYC);
TAUI:=0;W:=WO-DW;
'COMMENT' CRACK WIDTH INCREMENT AND EXTERNAL NORMAL STRESS

\[ DEB: W = W + DW; \]

\[ IF 'NCYC = 1 THEN' SIGEX = 2.2 \times (W - W_0); \]

\[ IF 'NCYC > 1 THEN' SIGEX = 0.6 - 0.02 \times NCYC \times W^{3.4}; \]

'COMMENT' SHEAR DISPLACEMENT INCREMENT

\[ WIL: \text{DEL} = \text{DEL} + D\text{DEL}; \]

'COMMENT' DETERMINATION OF TOTAL CONTACT AREAS

\[ ANK: A = A + 1; R = 0.1 \times A - 0.5 \times R_{\text{MAX}}; \]

\[ FLOR: B = B + 1; XSNO = 0; YSNO = 0; P = 0; T = 0; \]

\[ ILSE: T = T + 1; Y = 0.1 \times R^T; IF Y < W THEN' XCT = 0 ELSE' \]

\[ XCT = \text{DEL} + \sqrt{R^2 - (Y - R - W)^2}; \]

\[ G = \text{XH}(A, B, T) - XCT; U = T - B; \]

'IF' G > 0 THEN'

'BEGIN' 'IF' P = 0 THEN'

'BEGIN'

\[ \text{IF' U < 0 THEN' GOTO' ILSE; END' \]

'END';

'IF' G < 0 THEN'

'BEGIN'

\[ \text{IF' P = 0 THEN' BEGIN' \]

\[ Q = T - 1; M = 0.1 \times R / (\text{XH}(A, B, T) - \text{XH}(A, B, Q)); \]

\[ NN = 0.1 \times R \times (T - 1) - M \times \text{XH}(A, B, Q); \]

\[ Z = NN - R - W; \]

\[ XSNB = -(M \times Z - \text{DEL}) \times \sqrt{(M \times Z - \text{DEL})^2 - (M \times M + 1) \times (\text{DEL}^2 + Z^2 - R^2)}; \]

\[ YSNB = XSNB / (M \times M + 1); \]

'END';

'END';

'IF' T < B THEN' 'GOTO' ILSE;

'END';

'Beginning of the logic for contact area calculations

'a = 0; aax = 0; aay = 0;

'calculation of internal stresses

\[ \tau_{11} = \text{SPU} \times (aay - mu \times aax); \]

\[ \sigma_{11} = \text{SPU} \times (aax - mu \times aay); \]

'END';
'COMMENT' FOR THE CASE THAT THE INTERNAL NORMAL STRESS 'COMMENT' EXCEEDS THE EXTERNAL STRESS, INTERPOLATION OF 'COMMENT' THE CRACK DISPLACEMENTS

'IF' SIGI>SIGEX'THEN' 'BEGIN'
DELS:=DEL-(SIGI-SIGEX)/(SIGI-POX+.00001)*DDEL;
TAUS:=TAU-(DEL-DELS)*(TAU-ROX+.000001)/DDEL;' END'

'ELSE' 'BEGIN'
TAUS:=TAUMAX+0.000001;DELS:=DEL-(TAU-TAUMAX)/(TAU-ROX+.00001)*DDEL;
'END';

'IF' TAUS<TAUMAX'THEN' 'BEGIN'

'DETERMINATION OF TOTAL CONTACT AREAS FOR THE 'COMMENT' INTERPOLATED CRACK DISPLACEMENTS

TOX:=TAUS; WROX:=W; SOX:=SIGEX; QROX:=DELS;
HEL:A:=A+1;R:=(A/10-0.05)*RMAX;B:=0;
AS:B:=B+1;T:=0;
NEL:T:=T+1;Y:=0.05*T*R;'IF' Y<W'THEN' XCT:=0'ELSE'
XCT:=DELS+SQRT(R*R-(Y-R-W)**2);
'IF' XCT>YH(A,B,T)'THEN' XCT:=YH(A,B,T);
'IF' T<B'THEN' 'GOTO' NEL;
'IF' B<20'THEN' 'GOTO' AS;
'IF' A<10'THEN' 'GOTO' HEL;
BLANK(1,3);FIX(1,1,2,W);BLANK(1,3);FIX(1,2,2,DELS);
BLANK(1,3);FIX(1,2,2,SIGEX);BLANK(1,3);FIX(1,2,2,TAUS);
BLANK(1,3);FIX(1,2,2,SIGI);LINE(1,1);
'GOTO' DEB;

'END'

'DETERMINATION OF END - DISPLACEMENTS

'ELSE' 'BEGIN' 'IF' SIGI>SIGEX'THEN' 'BEGIN'
DELE:=(TAUMAX-TOX)/(TAUS-TOX+.00001)*(DELS-QROX)+QROX;
WE:=(TAUMAX-TOX)/(TAUS-TOX+.00001)*(W-WROX)+WROX;
SIGE:=(TAU-TAUMAX)/(TAUS-TOX+.00001)*(SIGEX-SOX)+SOX;
'END' 'ELSE' 'BEGIN'

'END';

'DETERMINATION OF END - DEFORMATION OF CONTACT AREAS

HELEN:A:=A+1;R:=(A/10-0.05)*RMAX;B:=0;
AST:B:=B+1;T:=0;
NEL:T:=T+1;Y:=0.05*T*R;'IF' Y<WE'THEN' XCT:=0'ELSE'
XCT:=DELE+SQRT(R*R-(Y-R-WE)**2);
'IF' XCT>YH(A,B,T)'THEN' XCT:=YH(A,B,T);
'IF' T<B'THEN' 'GOTO' NEL;
'IF' B<20'THEN' 'GOTO' AS;
'IF' A<10'THEN' 'GOTO' HELEN;

'COMMENT' CALCULATION OF TAU AT THE MOMENT OF 'COMMENT' RESTITUTION OF THE SHEAR SLIP DURING UNLOADING

KSI:=(AAX-HU*AAY)/(AAX+HU*AAY+.00001)*(AAY-HU*AAX)/(AAY+HU*AAX+.00001);
TAUO:=KSI*TAUMAX;LINE(1,1);FIX(1,2,2,TAUO);LINE(1,2);
WMAX:=W;DELE:=0.2*DELE;
'IF' NCYC<NMAX'THEN' 'GOTO' LOB;
'END';

'END'
ANALYTICAL CONTACT MODEL

0 rem analytical contact model
1 n=12: qm=.005
3 dim x(15), w(15)
5 open 4, 4
10 read p$, dm, f, mu, w, wo
15 for u=1 to 0: read p(u): w(u)=w: x(u)=x: next u
20 su=6.39*f^0.5
100 w=wo: dx=.5*xm+.005: wa=wo+.02: da=0: rem start of cycle
102 print#4, " wisseling = n"
103 print#4, " scheurwijdte slip Labda-x Labda-y tau sigma-in sigma-
ex"
104 print#4, " -----------------------------------------------------------"
105 dx=.35
109 rem crack width increment and external normal stress
110 gosub 9500: w=w+.02: sb=0: qa=.02: se=(0.6-.02*n)*w(8-.33*n): al=1: kk=0
111 if n=1 then se=.98*w^0.48
114 if qa<qmand si>sethen 0.58
115 rem shear displacement increment
116 dx=dx+qa: lx=1: ly=1
117 for u=1 to 0: if u=1 then 0: x(u)=0: next u
118 if w(aa)=0 then gosub 9000
120 if da=0 and qa>0 then l057
121 if si<se and kk=1 then qa=qa/2: gosub 1015
122 if si<se then l015
123 if si/se<1.2 thank k=1: dx=dx-qa: qa=qa/2: gosub 1015
124 gosub 9900: k1=0: l1=0: next u:
125 w=w-xm: gosub 9000
126 w=w-wo: xx=dx-da: gosub 9000: ca=cc
127 l1=(ca*xw-5*xw-5*(ww+u)): tt=(c*dx-.5*w-uu): l1=-(ca*xw-5*(xx+xd)): l1=(.5*dx-c*u+.5*(dd-2-4*(uu+w)^2)^0.5): l1=0: Oortt=Othenny=0: gosub 9002
128 if ly<0 or lx<0: then 0: next u:
129 if dx<xm then 1050
130 if dx<xm then 9000
131 gosub 9000: w=w-xm: gosub 9000: ca=cc
132 l1=(ca*xw-5*xw-5*(ww+u)): tt=(c*dx-.5*w-uu): l1=-(ca*xw-5*(xx+xd)): l1=(.5*dx-c*u+.5*(dd-2-4*(uu+w)^2)^0.5): l1=0: Oortt=Othenny=0: gosub 9002
133 gosub 9900: k1=0: l1=0: next u:
134 if ly<0 or lx<0: then 0: next u:
135 if dx<xm then 1050
136 gosub 9000: w=w-xm: gosub 9000: ca=cc
137 l1=(ca*xw-5*xw-5*(ww+u)): tt=(c*dx-.5*w-uu): l1=-(ca*xw-5*(xx+xd)): l1=(.5*dx-c*u+.5*(dd-2-4*(uu+w)^2)^0.5): l1=0: Oortt=Othenny=0: gosub 9002
138 gosub 9900: k1=0: l1=0: next u:
139 gosub 9000: w=w-xm: gosub 9000: ca=cc
140 gosub 9000: w=w-xm: gosub 9000: ca=cc
141 gosub 9000: w=w-xm: gosub 9000: ca=cc
142 gosub 9000: w=w-xm: gosub 9000: ca=cc
143 gosub 9000: w=w-xm: gosub 9000: ca=cc
144 gosub 9000: w=w-xm: gosub 9000: ca=cc
145 gosub 9000: w=w-xm: gosub 9000: ca=cc
146 gosub 9000: w=w-xm: gosub 9000: ca=cc
147 gosub 9000: w=w-xm: gosub 9000: ca=cc
148 gosub 9000: w=w-xm: gosub 9000: ca=cc
149 gosub 9000: w=w-xm: gosub 9000: ca=cc
150 gosub 9000: w=w-xm: gosub 9000: ca=cc
151 gosub 9000: w=w-xm: gosub 9000: ca=cc
152 gosub 9000: w=w-xm: gosub 9000: ca=cc
153 gosub 9000: w=w-xm: gosub 9000: ca=cc
154 gosub 9000: w=w-xm: gosub 9000: ca=cc
155 gosub 9000: w=w-xm: gosub 9000: ca=cc
156 gosub 9000: w=w-xm: gosub 9000: ca=cc
157 gosub 9000: w=w-xm: gosub 9000: ca=cc
158 gosub 9000: w=w-xm: gosub 9000: ca=cc
159 x$=left$(str$(lx)^"000000"), 6)
160 w$=left$(str$(w)^"000000"), 6)
161 d$=left$(str$(dx)^"000000"), 6)
162 t$=left$(str$(ta)^"000000"), 6)
163 s$=left$(str$(si)^"000000"), 6)
170 print#6, w$, " d$" " y$" " t$" " s$, se
171 si=si/al
1074 rem taumax = 1.24 MPa.
1075 if ta > 1.24 and si > 1.20 then dx = dx - q: q = q / 2: goto 1015
1080 if ta > 1.24 then n = n + 1: goto 9600: x(n-1) = dx: w(n-1) = wx: mx = dx: w = w: goto 1000
1090 if si > se then wa = w: da = dx: goto 1010
5000 cc = 5 * (dd^2 / (ww^2 + xx^2) - 1) ^ .5: return
9000 lx = 0: ly = 0: for u = l to l0: ly = ly + ly(u) * p(u) / 10: lx = lx + lx(u) * p(u) / 10: next u: return
9500 for u = l to l0:
9510 if lx(u) > 0 then wa(u) = wa: da(u) = da:
9600 for u = l to l0:
9610 if lx(u) > 0 then w(u), x, y = w(u), x, y: goto 9620
9615 w(u) = wa(u): x(u) = da(u)
9620 print # 4, u, w(u), x(u): next u: return
9900 if w = wo + 0.02 then return
9980 rem determination of static contact areas and
9990 rem of retention factors
10000 for u = 0.05 to 0.96 step 0.1: p = (0.532 * u^5 - 0.212 * u^4 - 0.075 * u^3 - 0.036 * u^2 - 0.025 * u - 10)^0.5:
p = p * 0.75 / 38 / u^2: s = p: print u, p, s: print: stop
10330 if w < 0.2 then
10331 bx = 7 * dm^0.56: by = 3 * dm^0.28: bb = -1.47 * dm^0.63:
10332 kx = bx * w^(-1.07) * dx: goto 10340
10333 bx = 7.74 * dm^(-1.07 * d - 0.1): by = 4.5 * dm^0.21: bb = -1.21 * dm^0.03:
10334 kx = bx / 4 * dx
10340 ax = 0.01 * (4 * (kx - 1 + exp(-kx)) / (exp(-kx) / kx + 1))
10350 ky = by * w^bb / 2 * dx
10360 p = 0.5 * (w - dx) * abs(w - dx) * exp(-1 - dm / 32 - 0.5 * w^2)
10370 ay = 0.01 * (2 * (ky - 1 + exp(-ky)) / (exp(-ky) / ky + 1) + p * ky^2 / dx)
10375 ly = ly * 0.9: lx = lx * 1.25
10376 rem calculation of internal stresses
10380 ta = su * (ly + mu * ly)
10390 si = su * (lx - mu * ly): lx = lx / ax: ly = ly / ay
11000 return
50000 data "al", 38, 15.8, 22, 1.10, 899, 740
50010 data .120, .023, .0107, .0064, .0043, .0031, .0023, .0017, .0012, .0007
59999 end
REDUCED CONTACT MODEL

0 rem reduced contact model
1 n=1; gm=.001; du=.25; tm=0.69
5 open 4,4
10 read p$, dm, f, mu, wm, xm, wo
20 su=6.39*f**.56
1000 w=wo; wa=wo+.02; da=0
1002 print 4, " wisseling = " n
1003 print 4, " scheurwijdte slip Labda-x Labda-y tau sigma-in sigma-ex"
1004 print 4, "---------------------------------------"
1005 dx=du; vx=xm-du; if n=l then dx=0
1010 w=w+.02; qq=.02; se=(3.2-.01*n)*(w-.759); al=l: rem crack width increment
1011 if q<q mandated n then se=3.8*(w-.759)
1015 if q<q mandated n then
1016 dx=dx+qq; lx=l; ly=l; if n=l then l053: rem shear displacement increment
1017 vx=dx-du: lx=.8*(vx/vx): ly=.7*(vx/vx): 3: rem retention factors
1018 if ly>l or lx>l then ly=l: lx=l
1019 gosub 10330: print "w= " w: dx= "dx= print "ta= " ta: print "si= " si: print "se"
1020 if da=0 and ta>0 then l057
1025 if si<se and si>sethen 58
1026 if si<sethen 58 then
1027 if si/se>1.2 then dx=dx-qq: q=q/2: goto 1016
1030 if w<.2 then l0333: rem determination of static contact areas
1031 bx=7*dm*.056: by=3*dm*.28: bb=-1.47*dm-.063
1032 kx=bx*w-1.07/4: dx=goto 10340
1033 bx=7.74*dm*.06*w-1.07*dm-.01: by=4.5*dm-.21: bb=-1.21*dm-.03
1034 kx=bx/4*dx
1035 ax=0.01*(kx-1+exp(-kx))/kx/kx+1)
1036 ky=by*bb/2*dx
1037 p=5*((w-dx)*abs(w-dx))*exp(-1-dm/32-.5*w*.2)
1038 ay=0.01*(2*(ky-1+exp(-ky))/ky/ky+1)+p*ky/2/dx
1039 si=su*(1+ay*mu*lx*ax): rem calculation of internal stresses
1040 return
1045 if w=wo+.02 then return
1049 if a=al then return
1050 al=se=si=ta=kx=lx=1x=al: return
1055 if a=al then return
1058 if w<.2 then l0333: rem determination of static contact areas
1059 bx=7*dm*.056: by=3*dm*.28: bb=-1.47*dm-.063
1060 kx=bx*w-1.07/4: dx=goto 1030
1061 bx=7.74*dm*.06*w-1.07*dm-.01: by=4.5*dm-.21: bb=-1.21*dm-.03
1062 kx=bx/4*dx
1063 ax=0.01*(kx-1+exp(-kx))/kx/kx+1)
1064 ky=by*bb/2*dx
1065 p=5*((w-dx)*abs(w-dx))*exp(-1-dm/32-.5*w*.2)
1066 ay=0.01*(2*(ky-1+exp(-ky))/ky/ky+1)+p*ky/2/dx
1067 si=su*(1+ay*mu*lx*ax): rem calculation of internal stresses
1068 return
1070 if a=al then return
1075 if a=al then return
1080 if a=al then return
1090 if a=al then return
2000 goto 1015
9900 if w=wo+.02 then return
9910 al=se=si=ta=kx=lx=1x=al: return
10300 if a=al then return
10310 bx=7*dm*.056: by=3*dm*.28: bb=-1.47*dm-.063
10320 kx=bx*w-1.07/4: dx=goto 1030
10330 bx=7.74*dm*.06*w-1.07*dm-.01: by=4.5*dm-.21: bb=-1.21*dm-.03
10340 kx=bx/4*dx
10350 ax=0.01*(kx-1+exp(-kx))/kx/kx+1)
10360 ky=by*bb/2*dx
10370 p=5*((w-dx)*abs(w-dx))*exp(-1-dm/32-.5*w*.2)
10380 ay=0.01*(2*(ky-1+exp(-ky))/ky/ky+1)+p*ky/2/dx
10390 si=su*(1+ay*mu*lx*ax): rem calculation of internal stresses
10400 return
11000 return
50000 data "el",38,12.8,20.0,76,.000,740
59999 end
60000 save "01abda-korrelel", 8
Curriculum vitae

Adrianus Frans Pruijssers

10 december 1958
Geboren te Rotterdam

1971 - 1977
Van Oldebarneveldt
Scholengemeenschap te
Rotterdam

mei 1977
Diploma Atheneum - B

1977 - 1982
Technische Universiteit
Delft
Faculteit: Civiele Techniek
Afstudeerrichting:
Constructieve waterbouwkunde
Afstudeerproject:
Sluisverlenging te Maasbracht

november 1982
Diploma civiel ingenieur

15 augustus 1982 -
31 december 1987
Werkzaam als wetenschappelijk
ambtenaar bij de sectie
betonconstructies van de
Faculteit der Civiele
Techniek, Technische Univer-
siteit Delft.
Projekt: Aggregate interlock

1 januari 1988
Werkzaam bij ontwerpbureau
van Dirk Verstoep b.v.

25 augustus 1982
Gehuwd met Augustina
Josephina Maria van Wezel

25 augustus 1984
Geboren Francisca

23 juni 1986
Geboren Marianne