INTRODUCTION:
Removal of surface-related multiples and internal multiples can be considered as the first step in the inversion of seismic data. The removal of surface-related multiples can be formulated by means of Rayleigh's reciprocity theorem, which leads to an integral equation of the second kind for the reflected pressure wavefield (Fokkema and van den Berg, 1993). The method requires no a priori information about the subsurface geology, nor structural nor material. In this paper a method is proposed to remove internal multiples of the first layer. Application of Rayleigh's reciprocity theorem leads to two integral relations that must be satisfied simultaneously: the first requires consistency in the first layer, implemented as an integral equation, while the other requires consistency with the measured data in the form of an integral representation. The result of simultaneous manipulation of the two integral relations is that the homogeneous domain has been extended downwardly, removing the first layer. This process can be continued to deeper levels to remove internal multiples from deeper layers. The method presented requires a priori information on the material properties in the layer from which the internal multiples are to be removed.

THEORY:
Rayleigh's reciprocity theorem describes the interaction of two non-identical states in one domain. We consider the following configuration: Let $D_{g}$ be the inhomogeneous scattering domain. In $D_{w}$, the material is homogeneous with material constants $\rho$ and $\kappa$. It is assumed that the contrasting domain $D_{g}$ has no contrast in the volume density $\rho$ but only in the compressibility $\kappa$, i.e. $\kappa^{g} = \kappa^{h}(x)$. The domain to which the reciprocity theorem is to be applied is $D_{sc}^{(0)}$. One state, state "0", is identified as the state which is homogeneous up to depth level $x_{3}^{(0)}$ (see Figure 1), while in the other state, state "1", the homogeneous layer is extended to $x_{3}^{(1)} = x_{3}^{(0)} + \Delta x_{3}$ (see Figure 2). In this case, the homogeneous domain is $D_{sc}^{(0)} \cup D_{w}$. Source and receivers are both located on the plane $x_{3} = 0$. Application of the reciprocity relation to the domain $D_{sc}^{(0)}$ with the above mentioned states leads integral equation

$$\int_{x \in D_{sc}^{(0)}} \hat{p}^{(0)}(x|x') \chi(x') \hat{p}^{(0)}(x'|x) dV,$$

with $x \in D_{sc}^{(0)}$, (1)
where \( \hat{W}(\omega) \) is the source signature and \( \chi \) is the known contrast function given by

\[
\chi(x') = 1 - \frac{\kappa^g(x')}{\kappa} = 1 - \left( \frac{c}{c^g(x')} \right)^2, \quad c^g(x') = (\rho \kappa^g(x'))^{-1/2}.
\] (2)

Consistency with the measured data on plane \( M \{ x \in \mathbb{R}^3 | -\infty \prec x_1, x_2 \prec \infty, x_3^M \prec x_3^0 \} \) yields the integral representation

\[
\hat{\rho}^{(0)}(x'x^S) - \hat{\rho}^{(1)}(x'x^S) = \frac{-\omega^2}{\hat{W}(\omega)} \int_{x' \in D_{se}^{(0)}} \hat{\rho}^{(1)}(x'x^S) \chi(x') \hat{\rho}^{(0)}(x'x^S) dx' , \quad \text{with } x \in M.
\] (3)

After solving \( \hat{\rho}^{(1)} \) from Eq.(3), by using Eq.(1) as constraint, the homogeneous domain is extended to level \( x_3^{(1)} \). As a consequence the primary of the contrasting layer has been removed, plus all the internal multiples that are caused by this interface. In principle this process can be continued to deeper levels.

**CONCLUSIONS:**

A method to remove inhomogeneous internal multiples is derived. By extending the homogeneous layer downwards the primary and the internal multiples caused by this reflecting interface are removed. The method requires a priori information of the layer causing the internal multiples.

**REFERENCES:**

J.T. Fokkema, and P.M. van den Berg, Seismic Applications of acoustic Reciprocity, 1993, Elsevier

**Figure 1:** State "0"

**Figure 2:** State "1"