Deterministic Price Setting Rules to Guarantee Profitability of Unbundling in the Airline Industry

G. van Diepen¹ & R. Curran²
Air Transport & Operations, Faulty of Aerospace Engineering, Delft University of Technology

Unbundling the traditional airfare is one of the airline industry’s practices to generate ancillary revenue in its struggle for profitability. However, unbundling might just as well negatively affect profit. In this paper deterministic price setting rules are established to guarantee profitability of unbundling irrespective of what product is unbundled. By comparing unbundling to a conventional price change, the least profitable scenarios of unbundling can be solved using the economic principle of price elasticity. The solution can be optimized dichotomously for profitability — profit guaranteed versus not guaranteed — by setting the unbundled prices such that the least profitable scenarios would still be profitable; profitability would then be guaranteed for any other scenario. The price setting rules this method renders are incorporated in a decision tree model and corresponding procedure for airline managers. As opposed to what customer research can produce, the model and corresponding procedure are generally applicable.

Nomenclature

- \( aob \) = add-on bundling
- \( C_A \) = cost of A
- \( C_b \) = cost of add-on product b
- \( D_{aob} \) = demand vector for add-on bundling
- \( D_{p} \) = demand vector for pure bundling
- \( D_{p1}\) = demand vector for pure bundling before a price change
- \( D_{p2}\) = demand vector for pure bundling after a price change
- \( D_{aob} \) = number of non-customers under add-on bundling
- \( D_{p} \) = number of non-customers under pure bundling
- \( D_{p1}\) = number of non-customers under pure bundling before a price change
- \( D_{p2}\) = number of non-customers under pure bundling after a price change
- \( D_{A} \) = demand for — or number of customers buying — only A under add-on bundling
- \( D_{A+b} \) = demand for — or number of customers buying — the bundle A+b under add-on bundling
- \( D_{A+b} \) = demand for — or number of customers buying — the bundle A+b under pure bundling
- \( D_{A+b} \) = demand for — or number of customers buying — the bundle A+b under pure bundling before a price change
- \( D_{A+b} \) = demand for — or number of customers buying — the bundle A+b under pure bundling after a price change
- \( G_{aob} \) = total gain from unbundling

¹ MSc. Student in Air Transport & Operations, Dept. of Aerospace Engineering, Delft University of Technology, email: G.vanDiepen@student.TUDelft.nl or gvd100@hotmail.com
² Head of Section/Chair of Air Transport & Operations, AIAA Senior Member

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\( G_{\text{unb}a} = \) minimal total gain from unbundling
\( k = \) positive constant
\( I_{\text{unb}} = \) total loss from unbundling
\( I_{\text{max}a} = \) maximal total loss from unbundling

\( P_{\text{A}} = \) market price of A (airline ticket: the bundle of products that remains after unbundling product b, at least comprising basic air transport) under add-on bundling
\( P_{\text{A}+b} = \) market price of the bundle A+b (airline ticket under pure bundling, airline ticket plus add-on product under add-on bundling)

\( P_{\text{A}+b}^{\text{pb}} = \) market price of the bundle A+b (airline ticket plus add-on product b) under add-on bundling
\( P_{\text{A}+b}^{\text{p}} = \) market price of the bundle A+b (airline ticket) under pure bundling
\( P_{\text{A}+b}^{\text{pb1}} = \) market price of the airline ticket (product bundle A+b) under pure bundling before a price change
\( P_{\text{A}+b}^{\text{pb2}} = \) market price of the airline ticket (product bundle A+b) under pure bundling after a price change

\( P_{\text{b}} = \) market price of add-on product b under add-on bundling
\( p_{\text{b}} = \) pure bundling
\( p_{\text{b1}} = \) pure bundling before a price change
\( p_{\text{b2}} = \) pure bundling after a price change

\( R_{\text{PA}} = \) reservation price for A
\( R_{\text{PA}+b} = \) conditional reservation price for A enhanced by b or the reservation price for A+b

\( \Delta = \) customer flow matrix for a price increase

\( q_{1i} = \) share of bundle buyers A+b under pure bundling (\( D_{\text{A}+b}^{\text{pb1}} \)) who remain bundle buyers A+b after a price change (\( D_{\text{A}+b}^{\text{pb2}} \))

\( q_{12} = \) share of non-buyers under pure bundling (\( D_{\text{b}}^{\text{pb1}} \)) who bundle buyers A+b after a price change (\( D_{\text{b}}^{\text{pb2}} \))

\( q_{21} = \) share of bundle buyers A+b under pure bundling (\( D_{\text{A}+b}^{\text{pb1}} \)) who become non-buyers after a price change (\( D_{\text{b}}^{\text{pb2}} \))

\( q_{22} = \) share of non-buyers under pure bundling (\( D_{\text{b}}^{\text{pb1}} \)) who remain non-buyers after a price change (\( D_{\text{b}}^{\text{pb2}} \))

\( \varepsilon_{\text{A}+b} = \) price elasticity of demand for the bundle A+b (negative constant)

\( i = \) market share

\( \tilde{\delta} = \) customer flow matrix for unbundling

\( \phi_{11} = \) share of bundle buyers A+b under pure bundling (\( D_{\text{A}+b}^{\text{pb}} \)) who remain bundle buyers A+b under add-on bundling (\( D_{\text{A}+b}^{\text{pb}a} \))

\( \phi_{21} = \) share of bundle buyers A+b under pure bundling (\( D_{\text{A}+b}^{\text{pb}} \)) who become buyers of only A under add-on bundling (\( D_{\text{A}}^{\text{pb}} \))

\( \phi_{22} = \) share of non-buyers under pure bundling (\( D_{\text{b}}^{\text{pb}} \)) who become buyers of only A under add-on bundling (\( D_{\text{b}}^{\text{pb}a} \))

\( \phi_{31} = \) share of bundle buyers A+b under pure bundling (\( D_{\text{A}+b}^{\text{pb}} \)) who become non-buyers under add-on bundling (\( D_{\text{b}}^{\text{pb}a} \))

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\[ \omega_b = \text{share of non-buyers under pure bundling} \left( D_{pb} \right) \text{ who remain non-buyers under add-on bundling} \left( D_{apb} \right) \]

I. Introduction

BUSINESS in the airline industry is a struggle for profitability. Due to erratically high fuel prices, fierce pricing competition and a pressing economic situation, exploiting the price inelasticity of business travelers (and the willingness of others to accept restrictions in return for discounts) or utilizing cost cutting capabilities do not seem to do the trick anymore. Ancillary revenue is considered a solution. Generally in the airline industry, four practices can be distinguished to generate ancillary revenue: dynamic packaging, product development, customer relationship management through the frequent flyer program, and unbundling the airfare.

A. Statement of problem

Dynamic packaging, product development and customer relationship management are all generally accepted ancillary revenue practices. They only complement airlines’ traditional base product and are unlikely to have a direct effect on demand, at least not a negative effect that is. Unbundling is less generally accepted however. It does not complement the base product but basically takes it apart. Therefore unbundling does have a direct effect on demand. And although it undoubtedly increases ancillary revenue and might even cut cost, unbundling having a direct effect on demand can also decrease profitability. In a deregulated airline industry the performance measure airlines are ultimately judged upon is profit-related, not ancillary revenue.

B. Literature

Bundling literature, for example Ref. 1-3, generally distinguishes between:

1) pure bundling, where only a bundle of products is offered, the products cannot be purchased individually;
2) mixed bundling, both the bundle and the individual products comprising the bundle are offered, i.e. a mixture of pure bundling and component pricing;
3) component pricing, where products are only offered individually (also referred to as separate pricing, unbundled sales, unbundling or no bundling).

About the latter strategy Ref. 1 notes: “typically, because this strategy is a base strategy for most firms, the strategy is called unbundling only when contrasting it with a bundling strategy.” Airlines’ traditional base fare is purely bundled however. It buys passengers a pure bundle of products at least comprising basic air transport.

Captivated by the success of ancillary revenue in the airline industry’s struggle for profitability, some airlines resort to unbundling previously included products from their airfare and offering them as optional add-on amenities. For example, a passenger of an airline that unbundled checked baggage b from its base fare A can choose to pay only the base fare, or the base fare plus an additional fee to check in baggage. Before, he only had to pay one airfare which allowed him (not obliged him) to check in baggage as well. Ref. 2 conveniently describes “a special case of [...] mixed bundling [...] simply take[s] the form of add-on services. The customer may purchase a single core service [...] or may select additional amenities [...] that are sold only with the core service at a single “bundled” price”. It refers to this form of mixed bundling as add-on bundling which is exactly the outcome of unbundling witnessed in the airline industry. Therefore unbundling (in the airline industry) is properly defined as going from pure bundling, where only the bundle A+b is offered, to add-on bundling, where a passenger can choose between A and A+b. Product b is offered and priced separately, not individually; it can only be bought in combination with base fare A. So before a product bundle is unbundled (that is under pure bundling), customers, or potential customers, can choose to buy (1) an airline ticket A+b or (2) not to buy a ticket. After unbundling (under add-on bundling), customers, or potential customers, can choose to buy (1) an airline ticket A and pay an additional fee for add-on product b (A+b), (2) to pay only the base fare A and not an additional fee for product b (A only), or (3) not to buy anything at all from the airline concerned.

What choice a (potential) customer makes is economically explained by the concept of reservation prices or willingness to pay. If the market price for a particular product (or bundle of products) is
higher than a potential customer’s reservation price for that product (or bundle of products), the potential customer will choose not to buy the product (or bundle of products). If the market price for a particular product (or bundle of products) is lower than a potential customer’s reservation price for that product (or bundle of products), the potential customer will choose to buy the product (or bundle of products); unless a competitor is offering a better deal. What a potential customer experiences as a better deal is explained by the concept of consumer surplus: the difference between what a consumer is willing to pay for a product and the market price actually paid. If buying from a competitor leaves a greater consumer surplus, buying from that competitor is experienced as a better deal. Customers always try to maximize their consumer surplus.

Ref. 2 also applies these two economic principles to explain customer behavior. However, it complements them with a third: complementarity. “[…] complementarity implies that the reservation price for one product […] is increased if another is purchased. That is, the conditional reservation price of A|b (A given b) may exceed the reservation price of A”\(^2\). Now we have:

\[
\text{reservation price } R_{PA} (\text{or willingness to pay}),
\]
\[
\text{the maximum price a consumer is willing to pay for product } A;
\]

\[
\text{consumer surplus } (R_{PA} - P_A),
\]
\[
\text{the difference between what a consumer is willing to pay for product } A \text{ and the market price } P_A \text{ actually paid for } A;
\]

\[
\text{and complementarity } (R_{PAb} > R_{PA}),
\]
\[
\text{when buying product } b \text{ enhances the utility, in this case represented by consumer surplus, of buying product } A.
\]

To comprehend the three economic principles, consider the following example: a gentleman in a cafeteria is thirsty for a cup of coffee. He reckons it is worth €2.50 and since the price is only €2 he buys it. His reservation price or willingness to pay for the cup of coffee is €2.50 and his consumer surplus from buying it is €2.50 minus €2 or €0.50. However, the gentleman is a real sweet tooth. He is only willing to pay €2.50 because he knows he will get all the sugar he wants. Had the cafeteria been out of sugar, he would have only given €1 for the cup of coffee. His conditional reservation price for a cup of coffee complemented with enough sugar \(R_{PAb}\) equals €2.50 (where A stands for coffee and b stands for sugar); €1.50 higher than his reservation price for a cup of coffee without sugar (\(R_{PA} = €1\)).

Ref. 2 subdivides mixed bundling (the bundling standard of which add-on bundling is a special form) into mixed-leader bundling, where “the price of one of two products is discounted when the other product is purchased at the regular price”, and mixed-joint bundling, where “a single price is set when two products are purchased jointly”. According to Ref. 2 “add-on bundling has the following key features.

1) Because the customer must choose between A and A + b (i.e., buying b only is not an option), \(P_b = 0\) and the mixed-joint form is the only feasible bundling approach.
2) Because b has no value independent of A, \(R_{Pb} = 0\). Moreover, \(R_{PBA} = R_{Pb}\) because complementarity must be unidirectional in the add-on case.
3) Most add-on products will be so intertwined with the lead product that split relationships among providers are not feasible. Thus […] , [competitor product] f can be purchased only with [competitor product] E, and b only with A.”

Normally, a customer’s reservation price for the bundle A+b equals the sum of his reservation prices for the individual products A and b (\(R_{PA} + R_{Pb} = R_{PAb}\)). However, when taking complementarity into account a customer’s reservation price for the bundle A+b equals the sum of his conditional reservation prices for product A enhanced by b and product b enhanced by A (\(R_{PAb} + R_{PBA} = R_{PAb}\)). According to the second feature of add-on bundling above, complementarity is unidirectional and \(R_{PBA} = R_{Pb} = 0\). Furthermore there are two types of customers: customers who are sensitive to the complementarity relationship, represented by \(R_{PAb} > R_{PA}\), and customers who are not, represented by \(R_{PAb} = R_{PA}\). If, for example, b stands for checked baggage, \(R_{PBA} = R_{Pb} = 0\) because checked baggage in itself is worthless; one needs to book a flight to appreciate it. That is, a customer going on a two-week holiday needs to check in a suitcase and is sensitive to the complementarity relationship, he might for example value the flight at €200 but complemented with checked baggage at €225.
For a customer going on an overnight business trip on the other hand, carry-on baggage suffices and his reservation price of €400 for the airline ticket is not enhanced by the option to check in baggage. To clarify this numerical example (the upper row represents the two-week holiday passenger and the lower row the overnight business trip passenger):

| RP_A | RP_b | RP_{A,b} | RP_{A|b} | RP_{b|A} | RP_{A+b} |
|------|------|-----------|----------|----------|----------|
| 200  | 0    | 225       | 0        | 400      | 225      |
| 400  | 0    | 400       | 0        | 400      | 400      |

C. Research question

Through this paper the reader is provided with an answer to the following research question: how should airlines set their prices to guarantee profitability of unbundling irrespective of what product is unbundled? The research question asks for the derivation of formulas for unbundled prices, such that they will not negatively affect profit, as generally applicable as possible, i.e. to any product once part of the traditional airline ticket – from checked to carry-on baggage and from peanuts to toilet use – and throughout the whole deregulated airline industry. The corresponding objective of the research is to develop a generally applicable model with which commercial passenger airlines of any kind can guarantee profitability of unbundling.

II. Methodology

To eventually establish price setting rules for which profitability of unbundling is guaranteed, irrespective of what product is unbundled, first the practice of unbundling and its effects on demand and thus on profit are to be modeled mathematically. For modeling, the definition of unbundling derived from bundling literature combined with industry practice and the economic principles of reservation price, consumer surplus and complementarity are used.

The shift in demand that results from the additional choice of only A and corresponding new price settings when A+b is unbundled can be solved stochastically. For data collection airlines are dependent on customer research since very few data on unbundled products in the airline industry are publically available. Customer research can be conducted to obtain (conditional) reservation price distributions from which customer choice at different unbundled prices (either keep buying A+b, start buying only A, or stop buying altogether; although accounting for competition poses some problems) can be modeled. Knowing how customers would respond to different unbundled prices, these prices can be continuously optimized for profit rendering price setting rules.

Although this method would be applicable irrespective of what product, the resulting price setting rules would not be. The optimized prices for unbundling checked baggage, for example, would be different from those for seat selection; the reservation price distributions are different. Besides, customer research is restricted to the specific group of customers researched, or, extrapolating, to the customer population, not necessarily meeting any carrier’s passenger profile. To conduct customer research here, for example by surveying passengers presenting hypothetical unbundled prices for (a) specific product(s) on a specific route flown by a specific airline, would render the ability to optimize those unbundled prices for profit. Maybe the airline could extrapolate the results to other routes. But every time it would consider unbundling a different product than the one(s) researched, the airline would have to conduct new customer research; a costly and time consuming business. And for other airlines the method would only serve as an example. The stochastic model would not be generally applicable.

In search of a method to establish a model that would be generally applicable, the similarity between unbundling and a conventional price change

<table>
<thead>
<tr>
<th>Table 1. Trade-off between research methods</th>
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<tr>
<td>Customer research</td>
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<tr>
<td>Costly and time consuming</td>
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<tr>
<td>Continuously optimizable for profit</td>
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<tr>
<td>Less reliable stochastic</td>
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<td>Not generally applicable model</td>
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((potential) customers are just not offered the additional choice of buying only A after a price change) offers a solution. The shift in demand arising from a conventional price change is entirely determined by the economic principle of price elasticity of demand. Price elasticities are often known in the airline industry. No customer research is required. The least profitable scenarios of unbundling can be solved deterministically by comparison to a conventional price change set equal to the prospective price increase of the bundle A+b when product b is unbundled. The solution can be optimized dichotomously for profitability – profit guaranteed versus not guaranteed – by setting the unbundled prices such that the least profitable scenarios would still be profitable; profitability would then be guaranteed for any other scenario. The price setting rules this self-developed method renders are generally applicable.

Choosing a method yields the following trade-off. The solution of the mathematical model of unbundling can be continuously optimized for profit – maximum profit – through customer research. It can also be dichotomously optimized for profitability – profit guaranteed versus not guaranteed – by comparison to a conventional price change and ensuring profitability of the least profitable scenarios. The latter method renders a less exact set of price setting rules, but on the other hand, they would be generally applicable, irrespective of what product is unbundled. And the procedural model (decision tree) in which they can be incorporated would be largely deterministic, requiring limited data validation. There public data on unbundled products in the airline industry is scarce, this is a welcome characteristic. The model would be cheap and fast to use, as opposed to conducting customer research every time one considers to unbundle a product from the base fare. (See Table 1). It being a requirement of the research objective, general applicability of the model was decisive for what method to use: comparison to a price change.

Therefore, not only the practice of unbundling and its effects on demand and thus on profit, but also the practice of price change and its effects on demand are modeled mathematically (in a similar fashion). From their comparison the least profitable scenarios are solved and dichotomously optimized for profitability exploiting price elasticity of demand. As already suggested, the price setting rules this yields are incorporated in a decision tree model with corresponding procedure.

III. Modeling: Unbundling versus Price Change

To start dichotomously optimizing unbundling for profitability, a model is to be developed first to graphically and mathematically depict unbundling in the airline industry. Because of the prospective method of comparing unbundling to a conventional price change, in a similar fashion a mathematical model is erected for such a price change.

A. Modeling unbundling

Industry practice shows that those airlines that unbundle a product b from their ticket fare and start charging for it separately, charge a fee for the unbundled add-on product b that, combined with their new ticket fare, is higher than their previous ticket fare. Their new ticket fare is either lower than the old one (a style initiated by European low cost carriers and copied by North American LCCs), or equally high (a style initiated by North American incumbents). But under add-on bundling (after unbundling) the product bundle A+b is always more expensive for the customer than the same product bundle under pure bundling (the airline ticket). Equivalently, the market price of the airline ticket after unbundling is either lower than or equally high as the market price of the airline ticket before unbundling; the market price for the airline ticket plus the add-on product after unbundling is always higher than the market price of the airline ticket before unbundling (representing the same product bundle). This can be mathematically depicted by:

\[ P_{A+b}^{unb} \leq P_{A+b}^{b} < P_{A+b}^{A+b} \]

where

- \( P_{A+b}^{unb} \) market price of the bundle A+b (airline ticket) under pure bundling
- \( P_{A+b}^{b} \) market price of A (airline ticket: now the bundle of products that remains after unbundling product b, at least comprising basic air transport) under add-on bundling

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Superscripts pb and aob stand for pure bundling and add-on bundling respectively. It was chosen not to describe the price a passenger buying both the base fare and the unbundled add-on is charged with under add-on bundling by \( p^{A+ob}_{aob} \), but by \( p^{A+ob}_{A+oB} \). Add-on product b is only sold separately, not individually; avoiding the \( p^{A+ob}_{A+oB} \) notation will prevent confusion. Besides, the number one key feature of add-on bundling\(^2\) sets \( p^{A+ob}_{A+oB} = 0 \) (see above). \( p^{A+ob}_{A+oB} < p^{A+ob}_{A+oB} \) says that the market price for the airline ticket plus the add-on product after unbundling is always higher than the market price of the airline ticket before unbundling (representing the same product bundle). This is consistent with bundling literature which says that when two products are bundled (basically the opposite of unbundling), the bundle is discounted (e.g. Ref. 1-3).

Now unbundling can be modeled both graphically and mathematically. Offering a certain airline ticket under pure bundling (product bundle A+b) brings about a certain market potential. The customer choice process will result in a number of buyers of (or demand for) the airline ticket – those potential customers whose reservation price \( R_{P_{A+b}} \) exceeds the ticket’s market price \( p_{A+b} \) – and in a number of non-buyers – those potential customers whose reservation price is less than the ticket’s market price or who find competition to offer a better deal. In accordance with generally accepted economic theory, (potential) customers are modeled as Homo Economici and thus are rational. Modeling the process of unbundling – going from pure bundling to add-on bundling – as an instant in time, i.e. the full effects kick in at that instant, will fix market potential (no additional routes are started flying and no potential customers can spontaneously emerge (Eqs. (3a-b)) and customer’s reservation prices (change in income effects are ruled out and no additional wealth can spontaneously emerge). After unbundling product bundle A+b the customer choice process will result in a number of customers paying both the airfare and an additional fee for the add-on product b (A+b), in a number of customers paying only the ticket price (A only), and in a number of non-buyers. That is, by unbundling your airline ticket A+b, part of your customers now become buyers of the bundled airline ticket A plus the add-on amenity b, part become buyers of only the unbundled ticket A, and part stops buying your product altogether. Part of your potential customers who did not buy your product before might now start buying only your unbundled airline ticket A since the unbundled ticket price might be less than the bundled ticket price \( (p^{A+ob}_{A+b} \leq p^{A+ob}_{A+b}) \). Non-buyers before unbundling becoming buyers of both the unbundled airline ticket A and the add-on amenity b is economically ruled out: non-buyers are those potential customers whose reservation price is less than the ticket’s market price or who find competition to offer a better deal; a price increase \( (p^{A+ob}_{A+b} < p^{A+ob}_{A+b}) \) will not change that. A graphical summary can be found in Fig. 1.

![Graphical Summary of Unbundling Process](image)
Figure 1. Unbundling graphically summarized

Mathematically translating Fig. 1 will result in a system of linear equations (Eqs. (2a-c)):

\[
\begin{align*}
\mathbf{D}^{ab}_b &= \mathbf{p} \mathbf{D}^b_b \\
\text{or} \quad
\begin{bmatrix}
\mathbf{D}^{ab}_A \\
\mathbf{D}^b_A \\
\mathbf{D}^{ab}_0 \\
\end{bmatrix}
&= 
\begin{bmatrix}
\varphi_{21} & 0 & 0 \\
\varphi_{21} & \varphi_{22} & 0 \\
\varphi_{21} & 0 & \varphi_{22} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{D}^{ab}_A \\
\mathbf{D}^b_A \\
\mathbf{D}^{ab}_0 \\
\end{bmatrix}
\end{align*}
\]

\[
\text{or}
\begin{align*}
\mathbf{D}^{ab}_A &= \varphi_{21} \mathbf{D}^b_A \\
\mathbf{D}^b_A &= \varphi_{21} \mathbf{D}^{ab}_A + \varphi_{22} \mathbf{D}^b_0 \\
\mathbf{D}^{ab}_0 &= \varphi_{21} \mathbf{D}^{ab}_A + \varphi_{22} \mathbf{D}^b_0 \\
\end{align*}
\]

where

- \( \mathbf{D}^b_b \) demand vector for pure bundling
- \( \mathbf{D}^{ab}_b \) demand for – or number of customers buying – the bundle \( A+b \) under pure bundling
- \( \mathbf{D}^b_A \) number of non-customers under pure bundling
- \( \mathbf{D}^{ab}_A \) demand vector for add-on bundling
- \( \mathbf{D}^{ab}_0 \) demand for – or number of customers buying – the bundle \( A+b \) under add-on bundling
- \( \mathbf{D}^b_0 \) number of non-customers under add-on bundling
- \( \mathcal{O} \) customer flow matrix for unbundling
- \( \varphi_{11} \) share of bundle buyers \( A+b \) under pure bundling (\( \mathbf{D}^{ab}_A \)) who remain bundle buyers \( A+b \) under add-on bundling (\( \mathbf{D}^{ab}_A \))
- \( \varphi_{21} \) share of bundle buyers \( A+b \) under pure bundling (\( \mathbf{D}^{ab}_A \)) who become buyers of only \( A \) under add-on bundling (\( \mathbf{D}^{ab}_A \))
- \( \varphi_{21} \) share of bundle buyers \( A+b \) under pure bundling (\( \mathbf{D}^{ab}_A \)) who become non-buyers under add-on bundling (\( \mathbf{D}^{ab}_A \))
- \( \varphi_{21} \) share of non-buyers under pure bundling (\( \mathbf{D}^b_0 \)) who become buyers of only \( A \) under add-on bundling (\( \mathbf{D}^{ab}_A \))
- \( \varphi_{21} \) share of non-buyers under pure bundling (\( \mathbf{D}^b_0 \)) who remain non-buyers under add-on bundling (\( \mathbf{D}^b_0 \))

The boundaries of the system are set by (Eq. (3a)) or equivalently by (Eq. (3b)):

\[
\begin{align*}
\mathbf{D}^{ab}_A + \mathbf{D}^b_0 &= \mathbf{D}^{ab}_A + \mathbf{D}^{ab}_A + \mathbf{D}^b_A + \mathbf{D}^{ab}_0 \\
\varphi_{11} + \varphi_{21} + \varphi_{21} &= \varphi_{22} + \varphi_{21} = 1 \\
\end{align*}
\]
These boundaries are the logical consequence of modeling the process of unbundling as an instant in time: market potential remains unchanged. $D_{A+b}^{bp}$ is known and $D_{b}^{bp}$ is often easily obtained (assuming a saturated market for example, or assuming constrained demand, $D_{b}^{bp}$ simply follows from your market share $\mu$):

$$D_{b}^{bp} = \frac{1 - \mu}{\mu} D_{A+b}^{bp}.$$  

Having modeled the process of unbundling both graphically and mathematically, we can have a closer look at what effect unbundling might have on profit. For that purpose the average cost of $A$ and the average cost of $b$ (and therefore the average cost of $A+b$) are assumed to be constant. Had one assumed fixed cost truly fixed in the short term and variable cost per passenger as small as zero, an assumption commonly made in airline revenue management to justify maximizing revenue in order to maximize profit, one would not be able to see the possible cost saving effect of unbundling add-on product $b$. Since cost savings might be the primary reason for unbundling, the assumption of constant average cost made here seems more appropriate. In practice it means that fixed cost need to be scaled down to variable cost (e.g. per passenger) to still account for them in profit calculations.

Under the unbundled pricing circumstances defined (Eq. (1)), the customer flows depicted by the phi-symbols in Fig. 1 and the equations thereafter represent either a gain, a loss or a neutral effect on profit. The sum of the separate effects should be positive in order for unbundling not to negatively affect profit: the total gain $G_{agr}$ should be larger than the total loss $L_{agr}$.

First of all, the share of bundle buyers under pure bundling who remain bundle buyers under add-on bundling (depicted by $\phi_{11}$) represents a gain: these customers now pay more ($p_{A+b}^{bp} < p_{A+b}^{no}$) for the same product ($A+b$) with the same cost ($C_{A} + C_{b}$).

Secondly, the share of bundle buyers under pure bundling who become buyers of only $A$ under add-on bundling (depicted by $\phi_{21}$) represents either a gain, a loss or a neutral effect on profit, depending on the new ticket price $p_{A}^{no}$: when the ticket price is decreased by less than the cost of unbundled product $b$, the cost savings outperform the revenue loss associated with these customers, therefore resulting in a gain; when the ticket price is decreased by more than the cost of unbundled product $b$, the revenue loss associated with these customers outperforms the cost savings, resulting in a loss; when the ticket price is decreased by exactly the cost of unbundled product $b$, this share of customers has a neutral effect on profit.

Furthermore, the share of bundle buyers $A+b$ under pure bundling who become non-buyers under add-on bundling (depicted by $\phi_{31}$) represents a loss: these potential customers did contribute to profit before unbundling (given an economically healthy market price of $A+b$ ($p_{A+b}^{no}$) larger than its cost) but now they do not anymore.

The share of non-buyers under pure bundling who become buyers of only $A$ under add-on bundling (depicted by $\phi_{22}$) represents a gain: these potential customers did not contribute to profit before unbundling; now they do (given an economically healthy market price of $A$ ($p_{A}^{no}$) larger than its cost).

And finally, the share of non-buyers under pure bundling who remain non-buyers under add-on bundling (depicted by $\phi_{32}$) represents a neutral effect on profit: these potential customers did not contribute to profit before unbundling and now they still do not. These effects are mathematically depicted by:

$$G_{agr} =$$

$$\phi_{11} D_{A+b}^{bp} \cdot (p_{A+b}^{no} - p_{A+b}^{bp}) +$$

$$\phi_{21} D_{A}^{bp} \cdot (p_{A+b}^{no} - p_{A}^{bp} + C_{b}) +$$

$$\phi_{22} D_{b}^{bp} \cdot (p_{A+b}^{no} - C_{A})$$

$$L_{agr} =$$

for $C_{b} > p_{A+b}^{bp} - p_{A}^{bp}$

$$\phi_{31} D_{A+b}^{bp} \cdot (p_{A}^{no} - p_{A+b}^{bp} + C_{b}) +$$

$$\phi_{32} D_{b}^{bp} \cdot (p_{A}^{no} - C_{A})$$

$$\{ \text{for } C_{b} > p_{A+b}^{no} - p_{A}^{no} \}$$

(4a)
The sum of these effects should be positive in order for unbundling not to negatively affect profit or for unbundling to be profitable \((G^{ab} > L^{ab})\). Customer research can provide an airline with a distribution of reservation prices from which to forecast the customer flows (the phis) at certain unbundled price settings (although accounting for competition poses some issues). Associated gains and losses can be calculated and summed up in order to check for profitability. Continuous optimization for profit is then at hand. However, the outcome is restricted to a finite number of add-on products \(b\), and to the specific group of customers researched (or, extrapolating, to the specific route, airline or customer population). It is not generally applicable.

**B. Modeling a conventional price change**

A conventional price change can be modeled in the same manner that unbundling was modeled: as an instant in time. Unlike unbundling, however, the customer flows that result from a price change can be forecasted without additional customer research, provided that price elasticity of demand – “defined as [...] the percentage change in quantity demanded divided by the percentage change in price” – is known.

The main difference between a conventional price change and unbundling is that (potential) customers are not offered the option to buy only \(A\) after a price change. The airline does not change bundling strategies but continues to practice pure bundling; only the market price of product bundle \(A+b\) (the airline ticket) is altered. The purpose of modeling a conventional price change here is to compare it to unbundling and exploit its absence of dependence on customer research for solving. A price change can either be a price decrease or increase. To best compare a price change to unbundling only a price increase is modeled here, since unbundling results in a price increase of product bundle \(A+b\) (\(P^{b}_{A+b} < P^{b}_{A} \) from Eq. (1)). One might argue that unbundling can also be compared to a price decrease, since the unbundled ticket price might be lowered (\(P^{b}_{A} \leq P^{b}_{A+b} \) from Eq. (1)). However, fact remains that the lower market price belongs to a downgraded product (\(A\) only), different from product bundle \(A+b\). Still, some work is devoted to price decreases in appendix A. A conventional price increase can be mathematically depicted by:

\[
P^{b1}_{A+b} < P^{b2}_{A+b}
\]

where

- \(P^{b1}_{A+b}\) market price of the airline ticket (product bundle \(A+b\)) under pure bundling before a price change
- \(P^{b2}_{A+b}\) market price of the airline ticket (product bundle \(A+b\)) under pure bundling after a price change

Superscripts \(pb1\) and \(pb2\) stand for pure bundling before and after a price change respectively.

Now price increases can be modeled both graphically and mathematically. Offering a certain airline ticket under pure bundling (product bundle \(A+b\)) brings about a certain market potential. The
customer choice process will result in a number of buyers of (or demand for) the airline ticket – those potential customers whose reservation price $R_{A+b}^{p_{A+b}}$ exceeds the ticket’s market price $p_{A+b}$ – and in a number of non-buyers – those potential customers whose reservation price is less than the ticket’s market price or who find competition to offer a better deal. Again, (potential) customers are modeled as Homo Economici. Like for unbundling, modeling the process of a price increase as an instant in time, will fix market potential (no additional routes are started flying and no potential customers can spontaneously emerge (Eqs. (7a-b)) and customers’ reservation prices (change in income effects are ruled out and no additional wealth can spontaneously emerge). By introducing a price increase, part of your customers continues to buy the product bundle $A+b$ despite its higher market price. Part stops buying your product altogether. Non-buyers before the price change becoming buyers of the bundle $A+b$ is economically ruled out: non-buyers are those potential customers whose reservation price is less than the ticket’s market price or who find competition to offer a better deal; a price increase ($p_{A+b}^{p_{A+b}} < p_{A+b}^{p_{A+b}}$) will not change that. For a price decrease it would be just the other way around: part of the non-buyers starts buying $A+b$ and buyers becoming non-buyers is economically ruled out (see appendix A). A graphical summary can be found in Fig. 2.

![Figure 2. Price increases graphically summarized](image)

Mathematically translating Fig. 2 will result in a system of linear equations (Eqs. (6a-c)):

$$D_{A+b}^{p_{A+b}} = \delta_{11} D_{A+b}^{p_{A+b}}$$

or

$$\begin{bmatrix} D_{A+b}^{p_{A+b}} \\ D_{0}^{p_{A+b}} \end{bmatrix} = \begin{bmatrix} \delta_{11} & 0 \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{bmatrix} D_{A+b}^{p_{A+b}} \\ D_{0}^{p_{A+b}} \end{bmatrix}$$

or

$$\begin{align} D_{A+b}^{p_{A+b}} &= \delta_{11} D_{A+b}^{p_{A+b}} \\ D_{0}^{p_{A+b}} &= \delta_{21} D_{A+b}^{p_{A+b}} + \delta_{22} D_{0}^{p_{A+b}} \end{align}$$

where

- $D_{A+b}^{p_{A+b}}$ demand vector for pure bundling before a price change
- $D_{0}^{p_{A+b}}$ demand for – or number of customers buying – the bundle $A+b$ under pure bundling before a price change
The boundaries of the system are set by (Eq. (7a)) or equivalently by (Eq. (7b)):

\[
D_{A+b}^{pb1} + D_u^{pb1} = D_{A+b}^{pb2} + D_u^{pb2}
\]  

or

\[
\delta_{11} + \delta_{12} = \delta_{22} = 1
\]

The customer flows depicted by the delta-symbols in Fig. 2 and the equations thereafter, can be determined for any price increase without additional customer research. All that is required is a numerical value for price elasticity of demand (price elasticities are often obtained from past data). “Price elasticity is always negative, since price and demand must move in opposite directions”\(^6\). Numerical values for price elasticity of demand are often known in the airline industry. In European and North American markets they typically range from as low as -0.60 (absolutely) to as high as -1.96\(^7\). Lower values are often associated with business travel, higher values with leisure travel. According to Ref. 5 price elasticity of demand, or simply price elasticity, the percentage change in quantity demanded divided by the percentage change in price, is dependent on (1) the presence of close substitutes for a product, (2) the product’s share in the customer’s budget, and (3) the time frame of the analysis. However, since the process of price change was modeled as an instant in time, the third point of dependence drops out. The presence of close substitutes makes demand more elastic: it is easy to switch suppliers which makes customers more price sensitive. In general (although not always), the smaller the product’s share in the customer’s budget, the less elastic demand. Price elasticity is dependent on price and normally exhibits small variations along a demand curve (undesirable for the prospective price setting rules; often only constant values of price elasticity are known). There is, however, a family of demand curves for which price elasticity is constant:

\[
D_{A+b}^{pb} = k \left( \frac{P_{A+b}^{pb}}{\varepsilon_{A+b}} \right)^{\varepsilon_{A+b}}
\]

where

k positive constant

\( \varepsilon_{A+b} \) price elasticity of demand for the bundle A+b (negative constant)

Proof is argued as follows. Since the price elasticity at any price point on any demand curve is mathematically given by the first derivative of that curve multiplied by the price at that point divided by the corresponding demand (Eq. (9)), price elasticity is the same at every point on the demand curve depicted by Eq. (8) and thus constant.
Rearranging Eq. (9) using Eq. (8) leads to

\[ \varepsilon_{A+b} = \frac{dD_{A+b}^{pb}}{dt} \frac{P_{A+b}^{pb}}{D_{A+b}^{pb}} \]

Had the demand curve in Eq. (8) been of any different shape, the rearrangement above would not yield \( \varepsilon_{A+b} \) at every point, meaning price elasticity would not be constant along the curve, exhibiting small variations. Here, constant price elasticities are assumed and thus Eq. (8) applies*. The corresponding family of curves is of familiar shape; Eq. (8) is graphically displayed in Fig. 3. With demand \( D_{A+b}^{pb} \) and price elasticity \( \varepsilon_{A+b} \) known, the positive constant \( k \) can be determined. The assumption of constant price elasticity and the resulting family of demand curves greatly facilitates the derivation of formulas for customer flows resulting from a price change depicted by the delta-symbols in Fig. 2 and the equations thereafter. The share of bundle buyers under pure bundling who remain bundle buyers after a price change (depicted by \( \beta_{11} \)) and the share of bundle buyers under pure bundling who become non-buyers after a price change (depicted by \( \beta_{21} \)) follow from Eqs. (6c), (7b) and (8) (proof can be found in appendix A). The share of non-buyers under pure bundling who remain non-buyers after a price increase (depicted by \( \beta_{22} \)) is equal to one: all non-buyers remain non-buyers since them becoming buyers after a price increase is economically ruled out.

\[ \beta_{11} = \left( \frac{P_{A+b}^{pb}}{P_{A+b}^{pb}} \right)^{\varepsilon_{A+b}} \]

\[ \beta_{21} = 1 - \left( \frac{P_{A+b}^{pb}}{P_{A+b}^{pb}} \right)^{\varepsilon_{A+b}} \]

\[ \beta_{22} = 1 \]

** IV. Optimization: Least Profitable Scenarios

Through comparison to a conventional price change the least profitable scenarios of unbundling (function extrema of the gains-and-losses-equations (4a) and (4b)) can be solved deterministically.

* Equation (8) is preferred over formulas of the form \( \Delta D = \varepsilon \Delta P \) commonly encountered in revenue management, in that the former lends itself for input of price and is still accurate for larger price changes, whereas the latter lends itself for input of price change and is inaccurate for larger price changes. Input of price and larger price changes are both requirements for the prospective price setting rules.

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using the principle of price elasticity. The solution can be optimized dichotomously for profitability – profit guaranteed versus not guaranteed – by setting the unbundled prices such that the least profitable scenarios would still be profitable; profitability would then be guaranteed for any other scenario. The price setting rules this method renders are generally applicable.

To compare unbundling to a conventional price change, the price increase of an airline ticket after a conventional price change needs to be taken equal to the price increase of the product bundle A+b that results from unbundling:

\[
\begin{align*}
&P^{abA}_{b} = P^{b}_{A+b} \\
&P^{abA}_{a} = P^{a|b}_{A+b}
\end{align*}
\]

Furthermore, it follows from Eq. (1) that two unbundling styles can be distinguished: offering a new ticket fare lower than the old fare (a style initiated by European low cost carriers and copied by North American LCCs), \( P^{ab}_{A|b} < P^{b}_{A+b} \), and offering a ticket fare equally high as the old fare (a style initiated by North American incumbents), \( P^{ab}_{A} = P^{b}_{A+b} \). This distinction is made throughout the rest of this paper for reasons of clarity as opposed to mathematical necessity. Mathematically the two styles render the same price setting rules.

A. Unbundling European style

Assuming an unbundled ticket price lower than the old purely bundled ticket price, Eq. (1) reduces to

\[
P^{abA}_{a} < P^{b}_{A+b} < P^{2ab}_{A+b}
\]  

(12)

Remember that the gains and losses induced by unbundling are depicted by Eqs. (4a) and (4b):

\[
\begin{align*}
G^{ab} &= \\
&\varphi^{1}_{11}D^{bb}_{A+b} \cdot \left( P^{ab}_{A+b} - P^{b}_{A+b} \right) + \\
&\varphi^{1}_{21}D^{bb}_{A+b} \cdot \left( P^{a|b}_{A+b} - P^{b}_{A+b} + C_{b} \right) + \\
&\varphi^{2}_{22}D^{bb}_{b} \cdot \left( P^{a|b}_{A+b} - C_{A} \right) \\
C_{b} &= P^{b}_{A+b} - P^{a|b}_{A+b} \\
&\varphi^{2}_{21}D^{bb}_{A+b} \cdot \left( P^{ab}_{A+b} - P^{b}_{A+b} - C_{b} \right) + \\
&\varphi^{2}_{22}D^{bb}_{b} \cdot \left( P^{b}_{A+b} - C_{A} - C_{b} \right)
\end{align*}
\]

This system of linear equations solves least profitable for the extreme values \( G^{ab} \) and \( L^{ab} \); the gain from unbundling should be minimal and the loss maximal in order for it to be least profitable. The system is bounded by Eq. (3b):

\[
\varphi^{0}_{11} + \varphi^{0}_{21} + \varphi^{0}_{22} = \varphi^{0}_{e3} + \varphi^{0}_{e4} = 1
\]

\[\] The price setting rules this method renders depend on price elasticity which might not be deterministic itself. However, assuming price elasticity to be constant results in equation 4.8 as a regulatory relationship on which the rules are based.

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However, it is not economically justified to simply set the smallest of customer flows constituting a gain (depicted by $\Phi_{P1}$, $\Phi_{P2}$ or $\Phi_{P3}$) at 1 and the rest at 0 to obtain the minimum gain; and to set the largest of customer flows constituting a loss (depicted by $\Phi_{L1}$ or $\Phi_{L2}$) at 1 and the other one at 0 to obtain the maximum loss. Through the demand formula in Eq. (8) the customer flows’ extreme values are bounded by values smaller than 1, obtainable by delta-equations (10a-c). These equations were deduced for a conventional price change but they can just as easily be applied to the mathematics of unbundling which proved to be essentially the same as for a price change (that is, concerning the bundle), however as mere boundaries.

From Eqs. (4a) and (4b) repeated above it follows that for the least profitable scenarios we have to distinguish between a ticket price decreased by less than the cost of unbundled product b ($C_b < P_{A+b}^b - P_{A}^b$), wherefore the share of bundle buyers under pure bundling who become buyers of only A under add-on bundling (depicted by $\Phi_{P2}$) constitutes a gain (cost savings outperform the revenue loss); and a ticket price decreased by more than the cost of unbundled product b ($C_b > P_{A+b}^b - P_{A}^b$), wherefore the share of bundle buyers under pure bundling who become buyers of only A under add-on bundling constitutes a loss (the revenue loss outperforms cost savings).

Let us first consider a ticket price decreased by less than the cost of unbundled product b; i.e. the share of bundle buyers under pure bundling who become buyers of only A under add-on bundling constitutes a gain and can be left out from the total loss $L_{rob}^{ab}$. This scenario is least profitable when the total loss $L_{rob}^{ab}$ is maximal thus when the flow of customers that constitute a loss is maximal. The share of bundle buyers A+b under pure bundling who become non-buyers under add-on bundling now is the only share constituting a loss. When unbundling is compared to a comparable price change (Eqs. (11a) and (11b) apply), the share of bundle buyers A+b under pure bundling who become non-buyers under add-on bundling (depicted by $\Phi_{P2}$) is at most equal to the share of bundle buyers under pure bundling who become non-buyers after a comparable price change (depicted by $\Phi_{L2}$). In order for the loss to be maximal the following must hold true (using Eq. (10b)):

$$\Psi_{L2} = \Phi_{L2} = 1 - \left( \frac{p_{rob}^{ab}}{p_{A+b}^{ab}} \right)$$

(13)

The total loss $L_{max}^{rob}$ resulting from this least profitable scenario follows from Eqs. (4b) and (13):

$$L_{max}^{rob} = (1 - ((p_{A+b}^b + b)\delta_{rob}^b)/(p_{A+b}^b + b + p_{A}^b))((p_{A+b}^b + b) - (p_{A}^b + b) + b - (p_{A}^b + b - C_A - C_b))$$

(14)

Furthermore, the total gain $G_{rob}^{ab}$ should be minimal for this scenario to be least profitable. The gain constituted by the share of non-buyers under pure bundling who become buyers of only A under add-on bundling (depicted by $\Phi_{P2}$) is minimal when that same share is minimal: zero.

$$\Phi_{P2} = 0$$

(15)

The share of bundle buyers under pure bundling who remain bundle buyers under add-on bundling (depicted by $\Phi_{P1}$) and the share of bundle buyers under pure bundling who become buyers of only A under add-on bundling (depicted $\Phi_{P2}$), both constituting a gain (the latter because the ticket price is decreased by less than the cost of unbundled product b ($C_b < P_{A+b}^b - P_{A}^b$)), cannot both be zero as well, since Eq. (3b) applies. In order for the gains to be minimal, the share of bundle buyers under pure bundling constituting the largest gain should be zero and the share constituting the smallest
gain should be what is left, which is at most equal to the share of bundle buyers under pure bundling who remain bundle buyers after a comparable price change (depicted by $\varphi_{11}$).

Whether $\varphi_{21}$ should be zero and $\varphi_{12}$ what is left, or $\varphi_{21}$ should be zero and $\varphi_{12}$ what is left, depends on whether the gain per customer that keeps buying the bundle $A+b$ ($P_{A+b}^b - P_{A+b}^A$) is larger than the gain per customer that starts buying only $A$ ($P_{A}^b - P_{A+b}^A + C_b$) or vice versa. If the gain per customer that keeps buying the bundle $A+b$ were largest ($P_{A+b}^b > P_{A+b}^A$ or $C_b > P_{A+b}^b - P_{A}^b$), which is the case when the new price of the bundle is higher than the new base fare by an amount higher than the cost of $b$ (or in practice when the fee charged for the add-on is more than its cost), $\varphi_{21}$ should be zero and $\varphi_{12}$ should be what is left (and follows from Eqs. (3b), (10a) and (13)):

\[
\begin{align*}
\varphi_{21} &= 0 \\
\varphi_{12} &= 1 - \varphi_{21} = \varphi_{11} = \left(\frac{D_{A+b}^A}{D_{A+b}^b}\right)^{\frac{b}{A+b}} 
\end{align*}
\] (17a)

The total gain $G_{\text{min}\{a,b\}}$ resulting from this least profitable scenario would then follow from Eq. (4a) and Eqs. (15), (16a) and (17a):

\[
G_{\text{min}\{a,b\}} = \left(\frac{P_{A}^b}{D_{A+b}^A} \cdot \frac{P_{A+b}^b - P_{A+b}^A + C_b}{D_{A+b}^b} \right) \cdot \left(1 - \left(\frac{P_{A+b}^b}{D_{A+b}^b}\right)^{\frac{b}{A+b}}\right) \cdot \left(P_{A+b}^b - P_{A}^b - C_A - C_b\right)
\] (18a)

This least profitable scenario would still be profitable if the minimum total gain $G_{\text{min}\{a,b\}}$ were larger than the maximum total loss $L_{\text{max}\{a,b\}}$. Combining Eqs. (14) and (18a) this can be mathematically depicted by

\[
\left(\frac{P_{A+b}^b}{P_{A+b}^A}\right)^{\frac{b}{A+b}} \cdot \left(P_{A+b}^b - P_{A+b}^A + C_b\right) \geq \left(1 - \left(\frac{P_{A+b}^b}{P_{A+b}^A}\right)^{\frac{b}{A+b}}\right) \cdot \left(P_{A+b}^b - P_{A}^b - C_A - C_b\right)
\]

which reduces to (proof can be found in appendix B.1)

\[
\left(\frac{P_{A+b}^b}{P_{A+b}^A}\right)^{\frac{b}{A+b}} \geq \frac{P_{A}^b - C_A - C_b}{P_{A+b}^b - C_A}
\] (19a)

Inequality (19a) constitutes the first unbundled price setting rule which applies to a ticket price decreased by less than the cost of unbundled product $b$ ($C_b > P_{A+b}^b - P_{A}^b$) and a fee charged for the add-on higher than its cost ($C_b > P_{A+b}^b - P_{A}^b$).

If on the other hand a fee were to be charged for the add-on lower than its cost, i.e. the new price of the bundle is higher than the new base fare by an amount less than the cost of $b$, the gain per customer that starts buying only $A$ would be largest ($P_{A}^b - P_{A+b}^A + C_b > P_{A}^b - P_{A+b}^A$ or $C_b > P_{A}^b - P_{A+b}^A$), thus $\varphi_{21}$ should be zero and $\varphi_{12}$ should be what is left to arrive at the least profitable scenario (and follows from Eqs. (3b), (10a) and (13)):
The total gain resulting from this least profitable scenario would then follow from Eq. (4a) and Eqs. (15), (16b) and (17b):

\[ G_{\text{min}}^{aob} = \frac{D_{pA}^{aob}}{D_{pA+B}^{aob} + D_{pA+B}^{b}} \times \left( \frac{D_{pA+B}^{b} - D_{pA+B}^{a}}{D_{pA+B}^{b}} \right) \times \left( 1 - \frac{D_{pA+B}^{aob}}{D_{pA+B}^{b}} \right) \times \left( \frac{D_{pA+B}^{b} - C_{A} - C_{B}}{D_{pA+B}^{b}} \right) \]

(18b)

Again, this least profitable scenario would still be profitable if the minimum total gain \( G_{\text{min}}^{aob} \) were larger than the maximum total loss \( L_{\text{min}}^{aob} \). Combining Eqs. (14) and (18b) this can be mathematically depicted by

\[ \left( \frac{D_{pA}^{aob}}{D_{pA+B}^{aob} + D_{pA+B}^{b}} \right) \times \left( D_{pA+B}^{b} - D_{pA+B}^{a} \right) > \left( 1 - \frac{D_{pA+B}^{aob}}{D_{pA+B}^{b}} \right) \times \left( D_{pA+B}^{b} - C_{A} - C_{B} \right) \]

which reduces to (proof can be found in appendix B.2)

\[ \left( D_{pA+B}^{b} - C_{A} - C_{B} \right) \times \left( D_{pA+B}^{b} - D_{pA+B}^{a} \right) > \left( D_{pA+B}^{b} - C_{A} - C_{B} \right) \times \left( D_{pA+B}^{b} - C_{A} - C_{B} \right) \]

(19b)

Inequality (19b) constitutes the second unbundled price setting rule which applies to a ticket price decreased by less than the cost of unbundled product b (\( C_{b} > D_{pA+B}^{aob} - D_{pA+B}^{b} \)) and a fee charged for the add-on less than its cost (\( C_{b} > D_{pA+B}^{aob} - D_{pA+B}^{b} \)). One should notice that Eq. (19b) exactly renders the condition under which a comparable conventional price increase would be profitable.

Let us now consider a ticket price decreased by more than the cost of unbundled product b; i.e. the share of bundle buyers under pure bundling who become buyers of only A under add-on bundling (depicted by \( \varphi_{21} \)) constitutes a loss. For this scenario to be least profitable, again the total loss should be maximal and the total gain should be minimal. The gains are now only constituted by the share of non-buyers under pure bundling who become buyers of only A under add-on bundling (depicted by \( \varphi_{2b} \)) and by the share of bundle buyers under pure bundling who remain bundle buyers under add-on bundling (depicted by \( \varphi_{11} \)). The gain constituted by the share of non-buyers under pure bundling who become buyers of only A under add-on bundling is minimal when that same share is minimal: zero (as for Eq. (15)).

\[ \varphi_{2b} = 0 \]

The gain constituted by the share of bundle buyers under pure bundling who remain bundle buyers under add-on bundling is minimal when that same share is minimal: zero (as for Eq. (16b)).

\[ \varphi_{11} = 0 \]
Since the total gain $G_{unb}^{a,\text{reb}}$ is zero for this least profitable scenario it can never still be profitable; a scenario which should be avoided. This leads us to the third unbundled price setting rule (which should really be practiced as the first):

$$G_b \geq p_{A=\text{reb}} - p_{A=\text{reb}}$$

or

$$P_{A=\text{reb}} \geq p_{A=\text{reb}} - c_b$$

Inequality (20) dictates the ticket price to be decreased by less than the cost of unbundled product $b$.

B. Unbundling North American style

Assuming an unbundled ticket price equal to the old purely bundled ticket price, Eq. (1) reduces to:

$$P_{A=\text{reb}} = p_{A=\text{reb}} < p_{A=\text{reb}}$$

This North American unbundling style is only told apart from the European unbundling style by the non-reduced ticket price. Therefore the price setting rules are identical, though according to Eq. (21), $P_{A=\text{reb}}$ can be replaced by $P_{A=\text{reb}}$. However, Eq. (21) does greatly simplify the equations for gains and losses given by Eqs. (4a) and (4b). Since the ticket price is not reduced, non-buyers before unbundling becoming buyers of the unbundled airline ticket $A$ only (depicted by $\varphi_{2B}$) is economically ruled out: non-buyers are those potential customers whose reservation price is less than the ticket's market price or who find competition to offer a better deal; an equal ticket price ($P_{A=\text{reb}} = p_{A=\text{reb}}$) will not change that. Furthermore, the condition of a ticket price decreased by less than the cost of unbundled product $b$ ($C_b > p_{A=\text{reb}} - p_{A=\text{reb}}$) is automatically satisfied since the ticket price is not decreased at all ($P_{A=\text{reb}} = p_{A=\text{reb}}$). Thus price setting rule (20) is always satisfied. If a fee were to be charged for the add-on higher than its cost ($C_b < p_{A=\text{reb}} - p_{A=\text{reb}}$), the same price setting rule would apply as for a European unbundling style with a ticket price decreased by less than the cost of unbundled product $b$ ($C_b > p_{A=\text{reb}} - p_{A=\text{reb}}$).

‡ The least profitable scenario being improfitable does not mean any other scenario under these conditions would be. Other scenarios might prove profitable; there are just no guarantees in advance.
and a fee charged for the add-on higher than its cost ($c_b > \text{price}_{A+b}$): Eq. (19a). However, $P_{A+b}^{\text{price}}$ can be replaced by $P_{A+b}^{b}$ and thus the first price setting rule is given by Eq. (19c) (proof can be found in appendix B.3):

$$P_{A+b}^{\text{price}} > \sqrt{\frac{P_{A+b}^{b} - c_A - c_b}{P_{A+b}^{b} - c_A}}$$

The second unbundled price setting rule which applies to unbundling North American style and a fee charged for the add-on lower than its cost ($c_b < \text{price}_{A+b}$) is identical to the price setting rule which applies to a European unbundling style with a ticket price decreased by less than the cost of unbundled product $b$ and a fee charged for the add-on lower than its cost: Eq. (19b).

$$\left(\text{price}_{A+b} - c_A - c_b\right) \cdot \left(\text{price}_{A+b}^{b}\right)^{\zeta_{A+b}} > \left(\text{price}_{A+b}^{b} - c_A - c_b\right) \cdot \left(\text{price}_{A+b}^{b}\right)^{\zeta_{A+b}}$$

Even its derivation is identical (except for the fact that $\theta_{25}$ was set equal to 0 for unbundling European style and here is omitted altogether) and can be found in appendix B.4.

It should be noted that although unbundling North American style can be profitable, it will always yield a loss in market share because of the following reason. The higher price of the bundle $A+b$ will in all probability drive customers away. Where by unbundling European style this effect might be compensated by a lower base fare to attract new customers, this is not the case for unbundling North American style; the base fare is not decreased. According to Eq. (8) it can be argued that the share of non-buyers under pure bundling who become buyers of only $A$ after unbundling European style (depicted by $\theta_{25}$) is at most equal to the share of non-buyers under pure bundling who become buyers after a comparable price decrease (depicted by $\theta_{12}$ in appendix A).

V. Applying the Rules

So how do you, as an airline manager, apply the pricing rules deduced in the previous paragraph? If your airline is struggling for profitability and you are seeking to increase ancillary revenue or to cut cost, and if you think it fits your brand, you should unbundle products from your ticket fare. However, caution is advised. Unbundling undoubtedly increases ancillary revenue and can cut costs, but it might just as well negatively affect profitability. Its success depends on your unbundled price settings: the new ticket price and the new fee for the unbundled add-on product you charge compared to your old ticket price. You can choose to do extensive customer research to obtain price settings at which unbundling certain products will be profitable. Customer research is costly and time consuming, however, and the results are not one hundred percent reliable and restricted to the add-on products customers were asked about. You can also choose to go for a cheap, quick and foolproof solution: use the price setting rules that were deduced in the previous paragraph.

If you choose to unbundle a product from your ticket fare and you do not indulge in customer research, you should proceed as follows to guarantee profitability (a decision tree comprising the procedure is depicted in Fig. 5):

1. First of all, you should unbundle product $b$ from your airline ticket $A+b$ such that the price of your airline ticket is either lower or equally high after unbundling; the price of your airline ticket together with the add-on product $b$ (the bundle) should be higher after unbundling ($\text{price}_{A+b}^{gb} \leq \text{price}_{A+b}^{b} < \text{price}_{A+b}^{gb}$).

2. You can either choose to unbundle North American style (a.), with your ticket price equally high after unbundling ($\text{price}_{A+b}^{gb} = \text{price}_{A+b}^{b}$) or European style (b.), with your ticket price lower after unbundling ($\text{price}_{A+b}^{gb} < \text{price}_{A+b}^{b}$). Unbundling North American style can be profitable but it will undoubtedly negatively affect your market share. That is why its path is not depicted in green but in yellow in Fig. 5. By lowering your ticket price...
price you will attract new customers which might compensate the negative effect the higher price of the bundle will have. That is why the European path is depicted in green.

If you still choose not to lower your ticket price but unbundle North American style, you should at least make sure the fee you want to charge for the add-on product b meets certain pricing rules in order to guarantee profitability. If you think it is necessary to charge a fee for the add-on higher than its cost \( (C_b < \frac{p^{PP}_A + p^{PB}_b}{p^{PB}_A + p^{PB}_b}) \) (a.), you should realize that according to Eq. (19c) this fee must be above a certain lower limit to make sure unbundling is profitable, which might be so high it deters customers (the less elastic demand, the higher this lower price limit, *ceteris paribus*):

\[
p^{PB}_A + p^{PB}_b \geq \frac{p^{PB}_A + p^{PB}_b}{p^{PB}_A + p^{PB}_b} \times \frac{C_A - C_b}{p^{PB}_A - p^{PB}_b}
\]

Counter-intuitively, it is not necessary at all to charge a fee for the add-on higher than its cost. If you choose to charge a fee lower than the cost \( (C_b > \frac{p^{PP}_A + p^{PB}_b}{p^{PB}_A + p^{PB}_b}) \) (b.), pricing rule (19b) should be met in order to guarantee profitability:

\[
(C^{a+b}_b - C_A - C_b) \cdot (p^{PB}_A)^{\frac{a+b}{a+b}} > (C^{a+b}_b - C_A - C_b) \cdot (p^{PB}_A)^{a+b}
\]

This pricing rule often reduces simply to \( P^{PB}_A > P^{PB}_b \): the price of your airline ticket together with the add-on product b (the bundle) should be higher after unbundling. Only for very high profit margins \( (C^{a+b}_b > C_A + C_b) \) in combination with certain price inelasticities Eq. (19b) constitutes a contradictory scenario. If we take a look at the shape of the two graphs represented by both sides of inequality (19b) (Fig. 4), we see them intersect at two points between which the inequality is satisfied. For most airline-specific figures the left intersection is at \( P^{PB}_A \) and the right intersection is way up. The inequality is then met by \( P^{PB}_A > P^{PB}_b \) since the upper limit of \( P^{PB}_A \) is already restricted by \( C_b > P^{PB}_A - P^{PB}_b \). Only for very high profit margins (which are uncommon in the airline industry, hence its struggle for profitability) combined with price elasticities absolutely higher than -1.0 (such a combination is also uncommon), is the right intersection at \( P^{PB}_A \) and is the inequality only satisfied in a domain lower than \( P^{PB}_b \) (e.g. this holds for a ticket price of €5000, a corresponding cost of €500 and a price elasticity of -1.2). This is ruled out to begin with since unbundling was defined by \( P^{PB}_A \leq P^{PB}_b \leq P^{PB}_A \) (eq. 1). High profit margins resulting in a contradictory outcome of pricing rule (19b) and low price elasticities (absolutely) asking for a (too) high lower limit of the add-on fee (Eq. (19c)), give rise to the in ancillary revenue literature much discussed thought of unbundling being unsuitable for first and business class travel.
If you choose to unbundle European style and lower your ticket price, you can either decrease your ticket price by more than the cost of add-on product b \((c_b < p^{Pb}_{A+b} - p^{ob}_{A})\) (a.) or by less than its cost \((c_b > p^{Pb}_{A+b} - p^{ob}_{A})\) (b.). Decreasing your ticket price by more than the cost of add-on product b may seem logical. For example, if you are planning to unbundle a “free” meal from your ticket price of €100 where the cost of that meal is €4 and the remaining cost are €90 per passenger, you might opt for a new ticket price of €95 and charge €15 for the meal. Don’t. You should prevent lowering your ticket price by more than the cost of the add-on product; profitability cannot be guaranteed without customer research. That is why its path is not depicted in green but in red in Fig. 5. You should decrease your ticket price by less than the cost of add-on product b \((c_b < p^{Pb}_{A+b} - p^{ob}_{A})\).

If you decrease your ticket price by less than the cost of add-on product b, the only thing left to worry about is whether the price you had in mind to ask for the add-on product is such that profitability is guaranteed. If you are planning to charge a fee for add-on product b higher than its cost \((c_b > p^{Pb}_{A+b} - p^{ob}_{A})\) (a.) – again, like described for the North American unbundling style, this might seem necessary, but surely it is not – you should again realize that according to Eq. (19a) this fee must be above a certain lower limit (furthermore, the same notes as described under this procedure’s third step for the North American unbundling style apply here):

\[
p^{atb}_{A+b} \geq \frac{p^{Pb}_{A+b} - c_A - c_b}{\sqrt{p^{ab}_{A+b} - c_A}}
\]

If you choose to charge a fee lower than the cost \((c_b > p^{Pb}_{A+b} - p^{ob}_{A})\) (b.), again pricing rule (19b) should be met in order to guarantee profitability:

\[
(p^{atb}_{A+b} - c_A - c_b) \cdot \left(p^{Pb}_{A+b}, p^{oa}_{A+b}\right)^{cA+cB} > \left(p^{Pb}_{A+b} - c_A - c_b\right) \cdot \left(p^{Pb}_{A+b}, p^{oa}_{A+b}\right)^{cA+cB}
\]

The same notes as described under this procedure’s third step for the North American unbundling style apply.
Ancillary revenue is considered a solution to the airline industry’s struggle for profitability. Unbundling the traditional airfare undoubtedly increases ancillary revenue and might even cut cost; however, it having a direct effect on demand, unbundling can also decrease profit. To guarantee profitability irrespective of what product is unbundled, airlines should set their prices to meet deterministic pricing rules (19a-c) and before all pricing rule (20). The decision tree model in Fig. 5 and the corresponding procedure enable airline managers to do so in a self-explanatory fashion. The price setting rules are, however, based on a number of assumptions (see Table 2). The methodology behind the model was based on assuring the least profitable scenarios would still be profitable. Not following the corresponding procedure – and setting your unbundled prices differently – would not necessarily mean unbundling negatively affecting profit; there are just no guarantees in advance. That is the reason why the mathematical implications of unbundling being unsuited for first and business class.

**Table 2. Overview of assumptions**

<table>
<thead>
<tr>
<th>Variables assumed to be known</th>
<th>Assumptions made to deduce formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_A^{ob}$</td>
<td>Customers are modeled as Homo Economic</td>
</tr>
<tr>
<td>$e_A^{ob}$</td>
<td>-Unbundling is modeled as an instant in time, therefore</td>
</tr>
<tr>
<td>$C_A^{ob}$, $C_B^{ob}$</td>
<td>-Market potential does not change</td>
</tr>
<tr>
<td>whether it is for a route, a travel class, a whole network, or a combination thereof</td>
<td>-Customer reservation prices remain unchanged (no effects of income changes)</td>
</tr>
<tr>
<td>$p_{A+b}^{b}$</td>
<td>-Price elasticity is taken constant</td>
</tr>
</tbody>
</table>

**VI. Conclusion**

Ancillary revenue is considered a solution to the airline industry’s struggle for profitability. Unbundling the traditional airfare undoubtedly increases ancillary revenue and might even cut cost; however, it having a direct effect on demand, unbundling can also decrease profit. To guarantee profitability irrespective of what product is unbundled, airlines should set their prices to meet deterministic pricing rules (19a-c) and before all pricing rule (20). The decision tree model in Fig. 5 and the corresponding procedure enable airline managers to do so in a self-explanatory fashion. The price setting rules are, however, based on a number of assumptions (see Table 2). The methodology behind the model was based on assuring the least profitable scenarios would still be profitable. Not following the corresponding procedure – and setting your unbundled prices differently – would not necessarily mean unbundling negatively affecting profit; there are just no guarantees in advance. That is the reason why the mathematical implications of unbundling being unsuited for first and business class.
travel – because low price elasticities ask for a (too) high lower limit of the add-on fee for one pricing rule and high profit margins result in a contradictory outcome of another – are subject to further research. Besides, even though profitability of unbundling can be guaranteed, whether it fits a carrier’s strategy, or how it affects its brand, is an entirely different analysis also subject to further research.

Appendix A

Price increases:

\[
D_{A+b}^{p_b^2} = \delta_{11} D_{A+b}^{p_b^4} \rightarrow \delta_{11} = \frac{D_{A+b}^{p_b^2}}{D_{A+b}^{p_b^4}}
\]

\[
\delta_{11} = \frac{k(P_{A+b}^{p_b^2})^{A+b}}{k(P_{A+b}^{p_b^4})^{A+b}} = \left(\frac{P_{A+b}^{p_b^2}}{P_{A+b}^{p_b^4}}\right)^{\epsilon_{A+b}}
\]

\[
D_{A+b}^{p_b} = k(P_{A+b}^{p_b})^{A+b}
\]

\[
d_{21} = 1 - \left(\frac{P_{A+b}^{p_b^2}}{P_{A+b}^{p_b^4}}\right)^{\epsilon_{A+b}}
\]

\[
d_{11} \rightarrow d_{21} = 1 \rightarrow d_{21} = 1 - d_{11}
\]

\[
d_{24} = 0
\]

\[
d_{22} = 1
\]

Price decreases:

\[
\delta_{12} D_{A+b}^{p_b^4} + \delta_{12} D_{A+b}^{p_b^2} = D_{A+b}^{p_b^2} \rightarrow
\]

\[
\delta_{12} D_{A+b}^{p_b^4} = D_{A+b}^{p_b^2} - \delta_{12} D_{A+b}^{p_b^1}
\]

\[
\delta_{11} = 1
\]

\[
\delta_{12} = \frac{D_{A+b}^{p_b^2} - D_{A+b}^{p_b^1}}{D_{A+b}^{p_b^4}}
\]

\[
D_{A+b}^{p_b} = k(P_{A+b}^{p_b})^{A+b}
\]
\[ \varphi_{21} = 0, \varphi_{21} = \varphi_{21} \text{, } \varphi_{21} = \varphi_{21} \text{, } \varphi_{21} = 0, \varphi_{21} = 1 \]

\[ \varphi_{21} = \left( \frac{P_{A+b}}{P_{A+b}} \right)^{s_{A+b}} \]

\[ \varphi_{21} = 1 - \left( \frac{P_{A+b}}{P_{A+b}} \right)^{s_{A+b}} \]

This least profitable scenario would still be profitable if the total gain were larger than the total loss:

\[ \varphi_{21} D_{A+b}^{1} \cdot \left( P_{A+b}^{1} - P_{A+b}^{1} + C_{b} \right) \Rightarrow \varphi_{21} D_{A+b}^{1} \cdot \left( P_{A+b}^{1} - C_{A} - C_{b} \right) \Rightarrow \]

\[ \left( \frac{P_{A+b}^{1}}{P_{A+b}^{1}} \right)^{s_{A+b}} \cdot \left( P_{A+b}^{1} - P_{A+b}^{1} + C_{b} \right) \Rightarrow \left( 1 - \left( \frac{P_{A+b}^{1}}{P_{A+b}^{1}} \right)^{s_{A+b}} \right) \cdot \left( P_{A+b}^{1} - C_{A} - C_{b} \right) \Rightarrow \]

\[ \left( \frac{P_{A+b}^{1}}{P_{A+b}^{1}} \right)^{s_{A+b}} \cdot \left( P_{A+b}^{1} - P_{A+b}^{1} + C_{b} \right) \Rightarrow \left( 1 - \left( \frac{P_{A+b}^{1}}{P_{A+b}^{1}} \right)^{s_{A+b}} \right) \cdot \left( P_{A+b}^{1} - C_{A} - C_{b} \right) \Rightarrow \]

### Appendix B

B.1: Unbundling European style with a ticket price decreased by less than the cost of unbundled product b

\( C_{b} > P_{A+b}^{1} - P_{A+b}^{1} \) and a fee charged for the add-on higher than its cost \( C_{b} < P_{A+b}^{1} - P_{A+b}^{1} \).

\[ \varphi_{21} = 0, \varphi_{21} = \varphi_{21} \text{, } \varphi_{21} = \varphi_{21} \text{, } \varphi_{21} = 0, \varphi_{21} = 1 \]

\[ \varphi_{21} = \left( \frac{P_{A+b}^{1}}{P_{A+b}^{1}} \right)^{s_{A+b}} \]

\[ \varphi_{21} = 1 - \left( \frac{P_{A+b}^{1}}{P_{A+b}^{1}} \right)^{s_{A+b}} \]
B.2: Unbundling European style with a ticket price decreased by less than the cost of unbundled product $b$ 
\( (C_b > P_{A+b}^b - P_{A+b}^{ab}) \) and a fee charged for the add-on less than its cost \( (C_b > P_{A+b}^{ab} - P_{A+b}^b) \).

\[
\begin{align*}
\varphi_{11} &= \frac{\varepsilon_{11}}{\varepsilon_{21}} , \quad \varphi_{21} = 0 , \quad \varphi_{31} = \frac{\varepsilon_{21}}{\varepsilon_{31}} , \quad \varphi_{23} = 0 , \quad \varphi_{33} = 1 \\
\delta_{11} &= \left( \frac{P_{A+b}^{ab}}{P_{A+b}^b} \right)^{\varepsilon_{A+b}} , \quad \delta_{21} = 1 - \left( \frac{P_{A+b}^{ab}}{P_{A+b}^b} \right)^{\varepsilon_{A+b}}
\end{align*}
\]

This least profitable scenario would still be profitable if the total gain were larger than the total loss:

\[
\varphi_{11} D_{A+b}^b \cdot (P_{A+b}^{ab} - P_{A+b}^b) > \varphi_{31} D_{A+b}^b \cdot (P_{A+b}^b - C_b - C_b)
\]

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B.3: Unbundling North American style with a fee charged for the add-on higher than its cost.

This least profitable scenario would still be profitable if the total gain were larger than the total loss:
\[ \varphi_{11} D_{A+b}^b c_b > \varphi_{11} D_{A+b}^b \cdot (p_{A+b}^b - c_A - c_b) \rightarrow \]

\[ \left( \frac{\mu_{A+b}^b}{p_{A+b}^b} \right)^{c_{A+b}} D_{A+b}^b c_b > \left( 1 - \left( \frac{\mu_{A+b}^b}{p_{A+b}^b} \right)^{c_{A+b}} \right) D_{A+b}^b \cdot (p_{A+b}^b - c_A - c_b) \rightarrow \]

\[ \left( \frac{p_{A+b}^b}{\mu_{A+b}^b} \right)^{c_{A+b}} c_b \rightarrow \left( 1 - \left( \frac{p_{A+b}^b}{\mu_{A+b}^b} \right)^{c_{A+b}} \right) \cdot (p_{A+b}^b - c_A - c_b) \rightarrow \]

\[ c_b \cdot \left( \frac{p_{A+b}^b}{\mu_{A+b}^b} \right)^{c_{A+b}} \rightarrow \]

\[ p_{A+b}^b - c_A - c_b - p_{A+b}^b \cdot \left( \frac{\mu_{A+b}^b}{p_{A+b}^b} \right)^{c_{A+b}} \rightarrow \]

\[ c_A \cdot \left( \frac{p_{A+b}^b}{\mu_{A+b}^b} \right)^{c_{A+b}} + c_b \cdot \left( \frac{p_{A+b}^b}{\mu_{A+b}^b} \right)^{c_{A+b}} \rightarrow \]

\[ (p_{A+b}^b - c_A) \cdot \left( \frac{p_{A+b}^b}{\mu_{A+b}^b} \right)^{c_{A+b}} \rightarrow p_{A+b}^b - c_A - c_b \rightarrow \]

\[ \left( \frac{p_{A+b}^b}{\mu_{A+b}^b} \right)^{c_{A+b}} \rightarrow \frac{p_{A+b}^b - c_A - c_b}{p_{A+b}^b - c_A} \cdot (p_{A+b}^b)^{c_{A+b}} \rightarrow \]

\[ p_{A+b}^b > \frac{p_{A+b}^b - c_A - c_b}{p_{A+b}^b - c_A} \cdot (p_{A+b}^b)^{c_{A+b}} \rightarrow \]

B.4: Unbundling North American style with a fee charged for the add-on less than its cost 
\[(c_b > p_{A+b}^b - p_{A+b}^b).\]

\[ \varphi_{11} = \varphi_{11}^{\mathbb{P}}, \varphi_{21} = 0, \varphi_{81} = \varphi_{21}^{\mathbb{P}} \]
\[d_{11} = \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}}, \quad d_{21} = 1 - \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}}\]

This least profitable scenario would still be profitable if the total gain were larger than the total loss:

\[\phi_{21} D_{A+b}^b \cdot \left( P_{s0b}^b - P_{A+b}^b \right) > \rho_{34} D_{A+b}^b \cdot \left( P_{s0b}^b - C_A - C_b \right)\]

\[\left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} D_{A+b}^b \cdot \left( P_{s0b}^b - P_{A+b}^b \right) > \left( 1 - \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} \right) D_{A+b}^b \cdot \left( P_{s0b}^b - C_A - C_b \right)\]

\[\left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} \cdot \left( P_{s0b}^b - P_{A+b}^b \right) > \left( 1 - \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} \right) \cdot \left( P_{s0b}^b - C_A - C_b \right)\]

\[\left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} D_{A+b}^b \cdot \left( \frac{P_{s0b}^b}{P_{A+b}} \right)^{e_{A+b}} > \left( P_{A+b}^b - C_A - C_b \right) + C_A \cdot \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} + C_b \cdot \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}}\]

\[\left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} \cdot \left( \frac{P_{s0b}^b}{P_{A+b}} \right)^{e_{A+b}} > C_A \cdot \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} + C_b \cdot \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}}\]

\[\left( P_{s0b}^b - C_A - C_b \right) \cdot \left( \frac{P_{s0b}}{P_{A+b}} \right)^{e_{A+b}} > P_{A+b}^b - C_A - C_b\]

\[\left( P_{s0b}^b - C_A - C_b \right) \cdot \left( P_{s0b}^b - C_A - C_b \right) \cdot \left( P_{s0b}^b - C_A - C_b \right) \cdot \left( P_{s0b}^b - C_A - C_b \right)\]

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