SOME REMARKS ON THE FUNDAMENTALS OF STRUCTURAL SAFETY

by

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Summary

It is shown that the usual factor of safety is needed almost entirely to account for exceptionally large loads and for a very small part only to account for scatter in strength of aircraft. With present strength requirements comparable load conditions yield inconsistent contributions to the total rate of failure. This results in more structural weight than necessary for the failure rate obtained. The paper advocates to establish the ultimate load as the product of a factor of safety little above unity and the "standard load", which is an exceptionally large load. Non-linearity being the origin of the mentioned inconsistencies the suggested concept particularly applies to high speed aircraft. The paper gives some recommendations for research aiming at the assessment of standard loads and the optimal distribution of the total probability of failure among individual load conditions.

Sommaire

Le communication démontre que le coefficient de sécurité conventionnel est destiné pour la plupart à prévenir la rupture de la structure sous les charges élevées très exceptionnelles et seulement pour une petite partie à compenser les déviations involontaires entre la résistance réelle des avions et leur résistance exigée. Avec les règlements actuels des cas de charge comparables contribuent dissemblablement à la probabilité de rupture, de sorte que le poids de la structure soit plus grand que nécessaire pour la probabilité de rupture acquise. La communication plaide pour la détermination de la charge extrême comme le produit d'un coefficient de sécurité près de l'unité et la "charge standard", une charge très exceptionnelle. La non-linéarité étant l'origine des dissemblances mentionnées la notion suggérée a son importance surtout pour les avions de grande vitesse. Le papier donne quelques recommandations pour les recherches, destinées à la détermination des charges standards et la répartition optimale de la probabilité de rupture entre les cas de charge individuels.
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**List of symbols:**

- \( a \) - constant.
- \( b \) - load level at which \( f = F \).
- \( F_{20} \) - derived effective gust speed.
- \( F \) - factor of safety.
- \( F \) - tolerable probability of failure.
- \( g \) - slope of the Im \( f_{20}-\)curve in the range of exceptionally high loads.
- \( N \) - standard deviation of distribution function of actual strength of aircraft.
- \( \sigma \) - standard deviation of distribution function of actual strength of aircraft.
- ultimate load - load used when designing for static strength.
- limit load - load used when designing for plastic deformation.
Notation

\( a_x \) = Gaussian distribution function of actual strength \( x \) of aircraft.
\( b \) = load level \( x \) beyond which 90% of all failures occur.
\( f_x \) = probability of exceeding the load level \( x \) per hour of flight.
\( j \) = factor of safety needed to compensate discrepancy of actual strength with respect to required strength.
\( k \) = slope of the log \( f_x \)-x-curve in the range of exceptionally large load.
\( n \) = load factor.
\( p \) = scatter factor; 1% of all aircraft fail at loads smaller than \((1-p)\)times the ultimate load.
\( v \) = flight speed.
\( x \) = ratio between a load and the standard load.
\( y \) = load level \( x \) with the maximum contribution to the rate of failure.
\( P \) = probability of failure.
\( \bar{P} \) = tolerable probability of failure.
\( U_{de} \) = derived effective gust speed.
\( \beta \) = slope of the ln \( f_x \)-x-curve in the range of exceptionally high load.
\( \sigma \) = standard deviation of distribution function of actual strength of aircraft.

standard load = load level at which \( f = \bar{P} \).
ultimate load = load used when designing for static strength.
limit load = load used when designing for plastic deformation.
Some remarks on the fundamentals of structural safety.

Prof. dr. ir. A. van der Neut.

I. Introduction.

Present strength requirements establish the level of static strength by means of specified load conditions, called limit loads, which are supposed to occur about once in the life of an aircraft. The ultimate load at which the structure is allowed to fail is the limit load multiplied by the factor of safety. This factor of safety is usually 1.5. The factor of safety is meant to provide precautions against unknown deficiencies of strength as well as against incidents producing excessively severe loads in order to keep the probability of failure below a tolerable amount.

If the limit loads in the various load conditions have equal probability of occurrence equal multiples of these limit loads do not have necessarily equal probability. Consequently the contribution of one load condition A to the rate of failure can be far more than that of another load condition B. This yields that the rate of failure is governed by load condition A. This inconsistency of the present system, which is a consequence of the uniform factor of safety and the emphasis placed on limit loads, means that too much structural weight is spent to the load condition with the smaller probability B. It suggests that equal distribution of the rate of failure over the cases A, and B could be achieved by increasing load A slightly and decreasing load B considerably. This could result in a lighter structure without reducing its safety.

In order to bring the concept of factor of safety in better harmony with the concept of tolerable probability of failure this investigation has been carried out. It yields a proposal for a new method of determining ultimate loads with the main characteristic that the emphasis placed so far on the limit load is shifted to a higher load level of much smaller probability of occurrence.
2. **Probability of failure.**

The factor of safety is meant to keep the probability of failure below a tolerable limit $\overline{P}$.

The probability of failure depends on:

1. the probability of rarely occurring very large loads.
2. the scatter of actual strength of aircraft.

The load spectrum gives the probability per flight hour $f_x$ of those incidents in which the load $x$ is exceeded. Among the very large loads is the load, the probability of which is $\overline{P}$. We will denote this load by $x = 1$, therefore

$$f_1 = \overline{P}$$

and this load will be called "standard load".

If an aircraft fails exactly at the standard load, it has "standard strength". If all aircraft would have exactly standard strength they all would fail when the load exceeds the standard load and the probability of failure would be $\overline{P}$. However some aircraft do have more strength than standard strength and others fail before the standard load is reached. This is partly because the ultimate load, for which the aircraft has been designed, differs from the standard load for one type of aircraft more than for the other. There are however many other causes for this discrepancy due to shortcomings of our knowledge on stress distribution, allowable stresses and due to scatter of size, material properties, etc. and due to margins of safety.

We express the actual strength of an individual aircraft as $x$ times the standard strength for this particular type of aircraft. Let the probability of occurrence of an aircraft of strength $x$ be $a_x$, then the probability of failure per flight hour for the whole collection of aircraft of various types is

$$P = \int f_x a_x \, dx.$$  

This probability $P$ may be greater of smaller than the tolerable limit $\overline{P}$ dependent on the distribution of $a_x$. The distribution function $a_x$ should be such that $P = \overline{P}$. 
3. The factor of safety required for scatter of strength

If we assume that $a_x$ is represented by a Gaussian distribution with the average strength $x = 1$ eq. (2) would yield

$$P > f_x = P,$$

since $f_x$ decreases with increasing $x$. Therefore in order to obtain

$$P = P$$

the average strength should be greater than 1; it has to be $j$. This means that, when we suppose conditions which cause discrepancy between actual strength and design load to be random, the ultimate load for which the structure has to be designed is $j$ times the standard load. The factor $j$ being required in order to account for scatter, this factor may be called "factor of safety required for scatter of strength".

If we assume the frequency distribution of loads in excess of $x$ to be given by

$$f_x = \exp(a - \beta x)$$

and the probability of the actual strength $x$ to be given by

$$a_x = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(x - j)^2\right)$$

the condition (4) is satisfied when (see appendix A)

$$j = 1 + \frac{1}{2} \beta \sigma^2.$$

We replace $\sigma$ by a quantity $p$, which is defined in the following way:

if we count the aircraft, the actual strength of which is smaller than $(1 - p)$ times design load, these aircraft form 1% of the total number of aircraft. This quantity $p$ appeals more to the imagination than $\sigma$.

When $p = 0.20$ this means, that 1 out of every 100 aircraft has failed when 80% of design load has been applied. It seems reasonable to assume that we need not think in figures for $p$ larger than 0.15 to 0.20.

Then (see appendix A) eq. (7) becomes

$$j^2 \frac{\beta p^2}{10,824} - j + 1 = 0.$$  (8)

$j$ proves to depend on $p$ and on the slope of $\log f_x$ versus $x$. 

In order to estimate the magnitude of \( j \) an assumption must be made on \( \beta \) in the range of rarely occurring very large loads in the vicinity of standard load. Since \( f_x \) is in the order of \( 10^{-8} \) per flight-hour, at least in civil aviation, we shall never have sufficient statistical data for establishing \( \beta \). The only way to arrive at numerical conclusions is to make a crude extrapolation from data on the more frequent loads.

Fig. 1 represents \( f_x \) as a function of the derived effective gust speed \( U_{de} \) up to \( U_{de} = 70 \) ft/sec, as obtained from statistical data. Several extrapolations into the range of greater gust speed have been made. In order to establish a \( f_x - x \) curve the axis of \( U_{de} \) has to be transformed into an axis of \( x \). The relation between \( U_{de} \) and \( x \) depends on the characteristics of the aircraft concerned. Therefore the axis of \( x \) could be established only by assuming specified characteristics. It has been assumed that the load factor in the C.A.R. gust case B, where \( U_{de} = 66 \) ft/sec, is 3,33 and that the factor of safety for scatter of strength is \( j = 1,079 \). This yields \( U_{ce} \) at standard load is 103 ft/sec. The reduction of \( U_{de} \) to \( x \) has been made in Appendix B.

The slope of the extrapolated part of the log \( f_x \) versus \( x \)-curve is given by

\[
\frac{d \log f_x}{dx} = k = \frac{\beta}{2,3026}
\]  

(9)

The extrapolations given in fig. 1 correspond to values of \( k \) between 6 and 12. The linear extrapolation of the statistically based log \( f_x - x \)-curve would yield \( k = 8 \).

The factor of safety required for scatter as obtained from

\[
0,2128 \ k \ p^2 \ j^2 - j + 1 = 0
\]  

(10)

for scatter figures \( p \) up to \( p = 0,25 \) have been given in fig. 2a and b. Fig. 2 shows also the value \( x = y \) for which the contribution to the rate of failure \( f_x a_x \) is maximal. The boundary \( x = b \) above which 90% of the failures occur is given in fig. 3 (see Appendix A).

Since the actual log \( f_x - x \)-curve for \( k > 8 \) falls below the assumed straight line when \( x < 0,718 \), the failure rate computed from the straight line formula gives an overestimation and \( j \) will be smaller than given in fig. 2. The part of the curves of fig. 2
in which $j$ is overestimated has been dotted.

Fig. 2 shows that $j$ increases with increasing slope of the
log $f_x$-curve, but even with the steepest slope ($k = 12$) $j$
appears to be a figure only little greater than unity. If we
take $p = 0.20$ and $k = 10$ as reasonable figures the factor
of safety required for scatter is $1.10$, which is indeed a very
small portion of the conventional factor of safety $1.5$.

Usually aircraft do have more strength than is required.
If we assume the mean strength to be $5\%$ in excess of the
required ultimate load and again that $1\%$ of all aircraft have
failed when $(1-p)$ times the ultimate load has been reached we
can establish from fig. 2 the ratio $j$ between ultimate load and
standard load which is needed to make the probability of failure
equal to $\bar{p}$. First we introduce the quantity $\bar{p}$ the scatter factor
with respect to the mean strength, which is

$$\bar{p} = \left( p + 0.05 \right) : 1.05$$

Then we read from fig. 2 the required factor of safety with
regard to mean strength $j$ and find the factor of safety with
regard to the ultimate load $j$ from

$$j = \bar{j} : 1.05$$

The result has been given in fig. 4 (full lines) together
with those for mean strength = ultimate load (dotted lines).
Fig. 4 shows that for the smaller values of $k$ the surplus
strength results in a smaller required factor of safety. However
for the larger values of $k$ and larger values of $p$ the curves
intersect. Beyond the intersection the case of surplus strength
requires a greater factor of safety. This at first glance
surprising result finds its explanation in the fact that for
equal $p$ the standard deviation is larger with increasing surplus
mean strength. Then the more frequent loads at lower load-level
do contribute decisively to the rate of failure. Again we find
that for the representative case $p = 0.20$, $k = 10$ the required
factor of safety is about $1.10$. Therefore the fact that the
mean strength is in excess of the required strength does not
alter our preliminary conclusion that the factor of safety
required for scatter is in the order of $1.1$. 
So it appears that the conventional factor of safety 1.5 is needed almost entirely to account for excessively high loads and for a very small part only to account for scatter of strength.

This conclusion does not imply that we need not worry about scatter.

Discrepancies between actual strength and required strength do have many sources:
ignorance on the magnitude of loads and on their distribution over the aircraft,
ignorance on the stress distribution for given external load,
ignorance on allowable stresses,
ignorance on actual thickness of material,
ignorance on actual material properties,
imperfect simulation of applied loads in static tests
errors of processing not detected by inspection,
misalignments,
initial stresses,
deterioration due to corrosion,
deficient maintenance,
and possibly some other sources.

If we concluded that deficiency of strength due to these reasons does not seriously lower the safety level, and that ignorance, errors and carelessness are not heavily penalized, and if we consequently would not do our utmost to keep all these hazards of deficient strength down, we could be sure that large discrepancies between required and actual strength would have a very great probability. This would change the picture quite considerably, the scatter factor $p$ would be large and $j$ would prove to be a figure for an excess of unity.

Therefore the conclusions that $j$ is very little above unity holds under the restriction that all reasonable efforts are made to banish deficient strength, as we use to do in aeronautics.


The conclusion obtained so far simplifies the problem of safety quite considerably. It means that the decision on the
magnitude of the ultimate load is almost entirely the decision on the standard load, on the magnitude of those very large loads, which have the tolerable probability $P$. In order to obtain the ultimate load this standard load has to be multiplied by a factor of safety. The partial factor of safety required for scatter seems to be about 1,1.

This factor of safety appeared to be affected relatively much by the slope of the log $f_x$-x-curve, and the figure 1,1 was derived from what seemed to be a conservative guess of the slope. However as a consequence of the doubtfulness of this guess design considerations should account for the possibility that the slope may be underestimated. Since steeper slopes would yield greater factors of safety it is reasonable to make allowance for this eventuality by increasing the factor of safety. If $p = 0.20$ is considered to be a reasonable guess the factor of safety needed for scatter and for very steep slope of the $f_x$-x-curve will very certainly be not in excess of 1,2.

This does not change the conclusion that the decision on the magnitude of the ultimate load is almost entirely the decision on the magnitude of the standard load. After having established the standard load the ultimate load is obtained as 1,1 to 1,2 times standard load.

5. **Inconsistencies of present method of establishing ultimate load.**

The usual philosophy that the main problem is to establish the limit load, and that the ultimate load is found as the product of this limit load and an "out- and- out" factor of safety 1,5 proves to be unjustified in view of the ultimate aim to keep the probability of failure below a desired limit.

If one would oppose that this philosophy has proven its reliability during many years the answer is that we possibly had a conservative factor of safety and that our happy experiences do not prove that we did not waste useful load. Apart from this it can be remarked that on the basis of this philosophy we did not offer equal safety to all load conditions. This statement can be illustrated with some examples.

These examples have the common characteristic that the total load is not proportional to the main load parameter.
1. Example: The comparison of gust intensities for up and down gusts, which correspond to ultimate load.
In gust conditions the main parameter is $Uv$, the product of gust and flight speeds.

At limit load the load factor $n = 1 \pm c \ U \ v$.
The ultimate load factor is 1,5 $n$ and can be obtained by a product $(Uv)_{ult}$ such that $1,5 \ n = 1 \pm c \ (Uv)_{ult}$.
This yields in the case of up gust

$$ (Uv)_{ult} = 1,5 \ Uv + \frac{0,5}{c} $$

and in the case of down gust

$$ (Uv)_{ult} = 1,5 \ Uv - \frac{0,5}{c} $$

Therefore the existing requirements do not offer equal safety for up and down gusts. The probability of failure with down gusts is much larger than that of failure due to up gusts.

This inconsistency of the requirements could be removed by giving the limit load conditions less emphasis and considering load conditions which refer basically to ultimate loads.

It is interesting to note that already in 1936 a draft for the Netherlands Airworthiness Requirements accounted for ultimate down gust conditions by taking the factor for initial load equal to 1: factor of safety instead of 1 and by taking the least service weight of the aircraft instead of its gross weight. In this way the full factor of safety was applied to the load parameter $Uv$.

2. Example: Manoeuvring loads on control surfaces.
The control surface limit load is composed of an initial load $S_0$ required for equilibrium in undisturbed flight and the load component $S_1$, which is added in the checked manoeuvre and which is the main load parameter. Again the intensity of the checked manoeuvre load which brings the load up to the ultimate load, depends on the magnitude of the initial load $S_0$; it is

$$ S_{1 \ ult} = 1,5 \ S_1 \ (1 + 0,33 \ S_0 / S_1) $$

Since $S_0 / S_1$ can be as well negative as positive, the magnitude of $S_{1 \ ult}$ and its probability is more or less a matter of hazard.

Again this inconsistency could be removed by considering ultimate load conditions, which refer directly to a certain state of flight.
These two examples have this in common that they apply to load conditions composed of an initial load, which does not vary much and an additional load which has a very wide range of possible magnitudes. In such cases there is a need for an ultimate load condition which represents a highly exceptional state of flight.

The cases mentioned so far could also be placed under the heading of non-linearity. The total load is not proportional to the load parameter: intensity of gust or manoeuvre. It is the type of non-linearity present in prestressed systems.

More in general, if the load is not a linear function of the load parameters the ultimate load has to be established directly from the load parameters in the ultimate conditions.

3. Example: Aero-elastic effects.

An important field of non-linearity is aero-elasticity. Due to the deformation of the structure the aerodynamic load is not a linear function of the load parameters such as dynamic pressure and load factor. It may be that the load of the structure increases more rapidly than the load parameter. In such a case the assessment of ultimate load by taking the product of limit load and factor of safety is not conservative. On the other hand, if the load of the structure increases more slowly than the load parameter, the ultimate load resulting from the product of limit load and factor of safety is overconservative. Therefore when aero-elastic effects are important rationality of design requires that the ultimate load be established by means of a physically defined ultimate flight condition.

6. The limit load criterion.

When ultimate flight conditions are emphasized the importance of limit load is fading, and the question arises whether the limit load could not be cancelled.

This question has two aspects, one in connection with safety, the other one relating to operational requirements.

Indeed failure is the only risk as far as considerations on safety go. Nevertheless there seems to be good sense in worrying about the magnitude of stresses at lower load level. It has been legitimate in the past to take precautions so as to keep the stresses at lower load levels well below the stresses at which the structure fails. For this reason the stress at limit load
was not allowed to exceed the yield limit. The question should be considered whether this is superstition or not.

The requirement relating to limit load had its good reasons at the time when no particular requirements existed on structural fatigue. It was a measure which had the effect of restricting the stresses at load levels which are often reached in service and as such it had its effect of favouring satisfactory fatigue characteristics.

However it was an indirect way of caring for fatigue and not a reliable one even. Now that the stage is reached that direct attention is paid to safe life and fatigue characteristics the need for the limit load as a safeguard against fatigue does no longer exist and the requirement with respect to limit load seems no longer to be a necessity from the point of view of safety of the structure.

There is however the second, the operational aspect. Plastic deformation of structural elements is usually not dangerous but it is a nuisance. The aircraft has to be taken out of service for repair work. Therefore the operational requirement will be that plastic deformation should be exceptional. Take for instance a fleet of 50 transport aircraft, which make in the average 2500 flight hours per annum. If we allow for a probability of those loads which give plastic deformation of $10^{-4}$ per flight-hour, this means that per year about 12 aircraft will have to undergo repair work because of permanent deformation. This is on a fleet of 50 not a low figure. Therefore we may conclude that the probability of exceeding the yield limit load should not be greater than in the order of $10^{-4}$, and that the limit load requirement has to be maintained from the point of view of maintenance.

There is a possibility however that the fatigue requirements would procure in an indirect way compliance with the operational requirement. Fatigue requirements indeed have the effect of keeping the stress level in the normal load range down. If this is true the requirement on limit load would be obsolete.

7. The assessment of standard load.

Since the factor of safety is only little above unity, the main problem is the assessment of the standard load.
The criterion is that the standard load is the load with the tolerable probability $P$. Since $P$ is a very small figure, for civil aircraft in the order of $10^{-8}$ per flight-hour, the assessment of the standard load on the basis of statistical data requires in general an extrapolation of doubtful reliability.

A more reliable way, which has been used quite extensively in the past, is to make use of empirical evidence acquired with aircraft which have flown successfully over a long period and to adjust the load conditions to the lower boundary of the strength of these aircraft. This would not lead to requirements which are identical to present requirements, since those inconsistencies which have been indicated could be avoided.

For instance, when with present requirements the ultimate down gust is smaller than the ultimate up gust, the target when estimating the standard gust speed as well up as down is the standard gust which can be sustained downwise by those aircraft which gave satisfactorily experience. This does not necessarily mean that the present down gust case is satisfactory. Fortunately with most aircraft the available strength exceeds the required downgust strength quite considerably, in many cases due to the landing load requirements.

Therefore, though in general we do not have the ways and means to establish the standard load directly from load statistics, some possibilities for rationalizing our requirements in the sense advocated here do exist. The effort seems to be worthwhile since a reduction of structural weight is the reward.

Though there is little hope that the standard load can be established by direct measurements it seems to be worthwhile to continue these considerations on safety with some further remarks.

8. **Tolerable probability of failure depends on fatality.**

What is to be considered as a tolerable failure rate depends largely on the amount of fatality that attends with failure. Only A. Subsidiary structure might fail without catastrophic consequences. If there is a good chance that the aircraft remains controllable after this type of failure the tolerable probability of failure of such elements is definitely greater.
than that of elements the failure of which is catastrophic. Then the standard load of subsidiary structure could be smaller than that of the main structure.

B. Another example of increased tolerable probability of failure refers to load conditions, in which failure of the aircraft does not necessarily cause the death of the occupants. There seems no reason to believe that some flight conditions could be placed under this heading. But ground load conditions are certainly of this character. Experience has shown that a large percentage of the occupants do survive a failure because of ground loads. Without doubt the present state is to accept a larger probability of failure for ground loads than for flight loads. This is obvious from aircraft accident statistics, where the majority of structural failures is listed under ground loads.

If the probability of failure during flight is assumed to be $10^{-8}$ per flight hour, it seems reasonable to assume that the probability of failure due to ground loads is of the order of $10^{-7}$ per flight hour or even greater.


When dealing with fatigue failure the so-called "fail-safe" structures are left out of consideration, since the occurrence of fatigue cracks is without catastrophic consequences. If however the residual strength of a structure after fatigue is nil or very small structural collapse is the inevitable consequence of fatigue; we have a case of fatigue failure.

Neglecting failures by ground loads and assuming the tolerable failure rate during flight to be of the order of $10^{-8}$ per flight hour, the problem arises how this risk of failure should be distributed between static failure and fatigue failure.

This is a problem with a psychological aspect. If an aircraft is lost when being engaged in a tornado everyone will be very sorry indeed, but this accident will be accepted in the mood that human skill can be overpowered by the violence of nature.

Fatigue failures are appreciated quite differently. They occur without any obvious external cause, they seem to be consequences of deficient engineering, what in fact they usually are. Therefore, whether rationally or not, catastrophic fatigue failures should be the smaller percentage of all structural failures in flight. Assuming a total failure rate of $10^{-8}$ per
flight hour, fatigue failure should be in the order of $10^{-3}$ per flight hour. It can be stated definitely that the state of the art is not up to this target figure. Thinking of structural failures in flight several accidents come to one's mind, which have in common that they are caused by fatigue, whereas examples of static failure are far more difficult to find. May this be so, no one will be inclined to agree that fatigue did not exceed in the last few years the limit of what is tolerable. For the time being the activity of structural engineers is to be focussed on fatigue characteristics.

However when dealing with the factor of safety to static failure we have ultimate static strength in mind and, in our deepest concern with fatigue troubles, we should not shut our eyes for bright hopes of saving structural weight by rationalizing static strength requirements.

10. Optimum design considerations.

The problem has to be considered how to distribute the tolerable probability of failure $\tilde{F}$ among the various load conditions. Failures may occur as a consequence of severe gusts up or down, large accelerations in manoeuvres up or down, or control surface loads. Each of these conditions contributes to the total probability of failure and failure because of any of these conditions has equal catastrophic consequences. Nevertheless they do not necessarily give equal contributions to the total failure rate and from the point of view of minimum weight design they must not give equal contributions.

A distribution based on exact considerations cannot yet be established, but the tendency of the optimum distribution is quite clear. It depends largely on the amount of structural weight required for the particular load condition.

The by far greatest part of structural weight is required for those load conditions in which the aircraft as a whole is accelerated. These are the manoeuvre and gust cases. On the other hand tail surface loads and even aileron loads do add relatively few structural weight.

Therefore a variation of standard control surface loads, which affects the probability of failure by control surface loads in the opposite sense, does not change the structural weight sensibly. However a variation of standard loads in accelerated
flight has a sensible affect on structural weight.

This yields the conclusion: for a given total probability of failure structural weight is minimal when the contribution of control surface loads to the rate of failure is one or more orders smaller than the contribution of accelerated flight to the rate of failure. Then the rate of failure due to manoeuvres and gusts can be $10^{-8}$ per hour.

The same consideration applies to the distribution among up and down loads. The total load in the up gust case is larger than the total load in the down gust case due to the initial load in horizontal flight. Still more this applies to accelerations in manoeuvres, since pulling the control stick is a more impulsive pilot reaction to unexpected events than pushing it. Therefore the structural weight required for up loads exceeds that for down loads.

Consequently in optimum design the tendency should be to choose the standard down load such that its probability is smaller than that of the standard up load. Therefore the standard down gust speed should be greater than the standard up gust speed. This is a fascinating conclusion since present requirements are just the opposite way. Again it demonstrates that gains of structural weight are within our reach, which do not increase the rate of failure.

In chapter 9 the tolerable probability of fatigue failure was for psychological reasons supposed to be of smaller order of magnitude than the tolerable probability of static failure.

If psychological considerations are omitted and equal importance is allotted to failures due to excessively large loads and to failures due to fatigue, the problem how to distribute the tolerable probability of failure between static and fatigue failures is again purely a matter of weight economics. Usually the amount of material which is liable to fatigue is only a small percentage of the structural weight. It is confined to joints, structural elements in the vicinity of cut-outs and members used in the post-buckled stage. Then the optimum weight condition yields that the probability of fatigue failure has to be small compared to the probability of static failure.

11. The military aspect.

The order of magnitude of probability of failure mentioned
so far was meant to apply to civil aviation. With military aircraft the need for saving structural weight is not primarily a matter of economics but basically of safety. Reduction of weight improves flight performances and increases military load, both having a favourable effect on safety. So when deciding on the tolerable probability of failure the totality of safety of mission has to be considered. Again the tendency is clear: the tolerable rate of failure is higher for military than for civil aircraft. Structural engineers can improve safety in military aviation by giving their structures less strength.

However, allowing for a relatively high rate of failure, safety of mission shall have a further increase, if the present scheme of ultimate load is limit load multiplied by the factor of safety is replaced by the scheme, which focusses the attention on what has been called standard loads, and which allots the tolerable probability of failure mainly to the load conditions, which demand the greater part of the structural weight.

Moreover with supersonic aircraft the concept of present requirements becomes obsolete. It is far from realistic to consider an ultimate load condition, where dynamic pressure is 1.5 times the dynamic pressure in a dive, the aerodynamic coefficients are equal to those at limit load and thermal stresses are 1.5 those at limit load. This artificial ultimate load is as far as dynamic pressure and thermal stresses are concerned overconservative. A more realistic approach making use of the suggested concept of standard load could possibly be an effective weapon in the fight at the thermal barrier.

12. Conclusions.

If scatter of actual strength did not exist the ultimate load for which the aircraft had to be designed should be chosen such, that the probability of exceeding this load in flight is equal to the tolerable probability of failure $P$. This load has been called "standard load". Due to scatter of actual strength the ultimate load, which yields the required failure rate $\tilde{P}$, has to be $j$ times standard load, where $j$ is the factor of safety required for scatter of actual strength. It is shown that for
the amount of scatter as present with aircraft structures this factor of safety is about 1,1 to 1,2. The paper advocates to replace the "limit load" by the standard load and to decrease the factor of safety from 1,5 to about 1,1 or 1,2.

It is shown that the present method of establishing the ultimate load yields quite different probabilities of failure for comparable load parameter values, if the total load is not proportional to the load parameter (up against down gusts, checked manoeuvre up against down, aero-elastic effects).

Due to the small order of magnitude of the probability of the standard load statistical data will usually not offer sufficient evidence for its assessment. In stead of unreliable extrapolations past experience may offer the possibility to rationalize strength requirements by adjusting the standard loads to the lower boundary of the available strength of satisfactory aircraft.

The rate of failure is composed of the contributions of individual load conditions. In optimal weight design the distribution of \( P \) among these load conditions should allot the greatest part to those conditions which demand the greater part of the structural weight. Therefore decreasing probability of standard load in the sequence: up gust and up manoeuvre, down gust and down manoeuvre, controls surface loads. For the same reason the contribution of fatigue to the rate of failure should be a minor one, since the amount of material which is critical in fatigue forms a small part of the structural weight.

13. **Recommended research.**

It is recommended that research be done on the following subjects:

1. Statistical data on very rarely occurring loads should be collected, so as to enable a more reliable guess on the magnitude of standard loads.

2. The inconsistency of present up and down gust conditions presents the possibility to establish the standard intensity from the available strength with respect to down gust with aircraft which gave satisfactory experience. Applying this standard gust speed in the case of up-gust the weight
reduction of those parts of the structure for which the up-gust case is critical could be established.

3. Investigations similar to those of item 2 with respect to other conditions, where the total load is not proportional to the main load parameter.

4. The application of optimal design considerations to simplified structural models loaded by 2 load conditions which affect largely different amounts of material, so as to establish the optimal distribution of the probability of failure between the individual load conditions and its effect on structural weight.
Appendix A. The factor of safety required for scatter of strength,  

The standard load is defined as the load which is exceeded with the probability $P$, where $P$ is the tolerable probability of failure. This load varies with the type of aircraft concerned. Measuring the load of an aircraft by its ratio $x$ to the standard load of this aircraft, we can count the number of aircraft of actual strength $x$ in the strength interval $dx$, thereby including any type of aircraft. This yields the relative frequency distribution $a_x$ of aircraft of actual strength $x$. We assume this distribution to be Gaussian and to have $j$ times standard strength as its mean value and $\sigma$ as its standard deviation.

$$a_x = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2} (x - j)^2\right]$$  \hspace{1cm} (A1)

Assuming that scatter of actual strength is symmetrical with regard to the ultimate load, we have that the ultimate load is equal to $j$ times standard load, and $j$ is the factor of safety required for scatter of strength.

The probability of exceeding the load level $x$ is assumed to be:

$$f_x = \exp (\alpha - \beta x),$$  \hspace{1cm} (A2)

which means that $\log f_x$ is a linear function of $x$.

The contribution to the probability of failure by those aircraft which are in the strength class $dx$ is $f_x a_x dx$ and the probability of failure for the whole collection of aircraft is:

$$P = \int_{-\infty}^{\infty} f_x a_x dx.$$  \hspace{1cm} (A3)

The assumption on $a_x$ involves that aircraft of negative strength would exist, which is physically impossible. The assumption on $f_x$ is inconsistent with actual load frequencies for small and for negative values of $x$. However these assumptions, which are introduced for ease of computation, do not affect the integral (A3) importantly, since the contribution of the range of $x$, where $f_x$ and $a_x$ are questionnable, is negligible.

The integrand $f_x a_x$, as established by (A1,2) is again a Gaussian function

$$f_x a_x = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[\alpha - \beta x - \frac{1}{2\sigma^2} (x - j)^2\right] = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[0 - \frac{1}{2} \frac{(x-y)^2}{\sigma^2}\right]$$  \hspace{1cm} (A4)
with the mean value $x = y$ and with the standard deviation $\tau$.

The identity (A4) yields

$$\tau = \sigma$$
$$y = j - \beta \sigma^2$$
$$C = \alpha + \frac{1}{2} \beta^2 \sigma^2 - \beta j.$$  \hspace{1cm} (A5)

Since

$$\frac{1}{\sqrt{2\pi}} \int \exp \left[- \frac{1}{2\sigma^2} (x-y)^2 \right] \, dx = 1,$$

eq. (A3) yields

$$P = \exp \left[ a + \frac{1}{4} \beta^2 \sigma^2 - \beta j \right]. \hspace{1cm} (A6)$$

The object is to establish the required factor of safety $j$ at which the probability of failure is equal to the tolerable probability of failure $\overline{P}$. From the definition of the standard load $x = 1$ follows

$$f_1 = \exp \left[ a - \beta \right] = \overline{P}. \hspace{1cm} (A7)$$

Then from the condition $P = \overline{P}$ and eqs. (A 6, 7) follows

$$\overline{P} = \exp \left[ a - \beta \right] = \exp\left[ a + \frac{1}{4} \beta^2 \sigma^2 - \beta j \right], \hspace{1cm} (A8)$$

hence

\[ j = 1 + \frac{1}{4} \beta^2 \sigma^2 \]

and from (A5)  \hspace{1cm} (A9)

\[ \overline{Y} = 1 - \frac{1}{4} \beta^2 \sigma^2 = 2 - j. \hspace{1cm} (A10) \]

Therefore the required factor of safety depends on the slope $\beta$ of the log $f_x$ versus $x$ curve and the standard deviation $\sigma$. We replace $\sigma$ by a quantity $p$ more appealing to the imagination which is defined in the following way:

Counting the aircraft, the actual strength of which is smaller than $(1-p)$ times design load, these aircraft form 1% of the total number of aircraft.

The relation between $\sigma$ and $p$ follows from the error function

$$\text{erf}(r) = \frac{2}{\sqrt{\pi}} \int_0^r \exp \left[- t^2 \right] \, dt.$$  \hspace{1cm} (All)

The proportion of the number of aircraft, which have failed when the load $(1-p)j$ has been reached, to the total number of aircraft is

$$N = (1-p)j \int_a^\infty a \, dx.$$  \hspace{1cm} (All)

With

$$\frac{x - j}{\sigma \sqrt{2}} = t \quad \text{and} \quad \frac{pj}{\sigma \sqrt{2}} = r,$$  \hspace{1cm} (A12)

(All) yields
\[ N = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp \left[ -t^2 \right] \, dt = \frac{1}{2} \left[ 1 - \text{erf} (r) \right]. \quad \text{(A13)} \]

Putting \( N = 0,01 \) we have \( \text{erf}(r) = 0,98 \) and \( r = 1,645 \). Hence
\[ \sigma = \frac{\beta \tilde{1}}{1,545 \sqrt{2}}. \quad \text{(A14)} \]

Substituting \( \sigma \) from (A14) into (A9) we obtain the quadratic equation for \( j \)
\[ j^2 \frac{\beta \tilde{p}^2}{10,824} - j + 1 = 0. \quad \text{(A15)} \]

This equation enables to establish \( j \) as a function of \( p \) for any given \( \beta \).

Since the assumption on the frequency distribution \( f_x \) according to (A2) can be considered to be valid only in a limited load range, it is necessary to establish the range of \( x \) which gives the major contribution to the integral (A3). The maximum for the product \( f_x a_x \) occurs at \( x = y = 1 - \frac{1}{2} \beta \sigma^2 \), (A16) which is below standard load. Therefore we have to consider the lower boundary of the important load range, denoted by \( x = (1 - q) y = b \), (A17) below which the contribution to the integral (A3) is \( M \), where \( M \) is f.i. 10%.

Then we obtain from (A4, 5, 8)
\[ M = \int_{-\infty}^{\infty} f_x a_x dx = \hat{F} \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2\sigma^2} (x-y)^2 \right] dx. \]

This equation for \( q \) is analogous to eq (A11). It yields
\[ M = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{qy}{\sigma \sqrt{2}} \right) \right], \quad \text{(A18)} \]

\[ M = 0,10 \text{ yields } \text{erf} \left( \frac{qy}{\sigma \sqrt{2}} \right) = 0,80, \quad \frac{qy}{\sigma \sqrt{2}} = 0,9062 \]

from which follows for the lower boundary
\[ b = (1 - q)y = 2 - (1 + 0,55lp)j. \quad \text{(A19)} \]
Appendix B. The $f_x$ curve in the range of exceptionally large loads.

Fig. 1 gives the probability $f$ of encountering during one hour of flight a gust speed in excess of the derived effective gust speed $U_{de}$ at 300 ml/hr flight speed. Statistics cover the gust speed range up to about 70 ft/sec. Several extrapolations into the range of greater gust speed have been made. These extrapolations start from $U_{de} = 66$ ft/sec—the C.A.R. gust case $B$—, where $\log f = -4.55$, and have constant slope in the log $f_x$ versus $U_{de}$ plane.

In order to establish $\beta$ corresponding to these extrapolations the $U_{de}$ scale has to be replaced by a load scale. Since the relation of $U_{de}$ and $x$ depends on the characteristics of the aircraft it cannot be established with general validity.

We assume that the characteristics of the aircraft are such that the load factor in the C.A.R. gust case $B$ is 3.33 which yields the ultimate load factor 5.0. Assuming that the safety factor required for scatter of strength is $j = 1.079$ (corresponding to $p = 0.20$ and $k = 8$ ) the standard load $x = 1$ corresponds to the load factor $n = 5 : 1,079 = 4.64$.

When we assume equal alleviating factors at all gust speeds the factor is

$$n = 1 + K U_{de}$$

From $n = 3.33$ at $U_{de} = 66$ ft/sec follows $K = 0.0354$ ft/sec.

The load factor at standard load ($x = 1$) being 4.64 the corresponding $U_{de} = 103$ ft/sec.

Therefore the relation between $U_{de}$ and $x$ following from (B1) is

$$x = \frac{1+K U_{de}}{1+K 103} = \frac{1+0.0354 U_{de}}{4.64}.$$  \hspace{1cm} (B2)

This load scale has been given in fig.1.

According to (A2)

$$\log f_x = \frac{1}{2,3026} (a - \beta x) = \frac{a}{2,3026} - kx.$$  \hspace{1cm} (B3)

The extrapolations given in fig.1 correspond to the $k$-values 6, 7, 8, 9, 10 and 12.

They apply to $x > 0.718$.  

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**Notes:**

- **Mathematical Notation:** $f_x$ represents the probability density function of gust speed $x$.
- **Decimal Precision:** Numbers are rounded to two decimal places for clarity.
- **Logarithm Use:** Logarithms are used for calculations to simplify the extrapolation process.
Fig. 1 Extrapolation of gust statistics in the range of very large gust speeds.

\[ f = \text{probability per flight-hour of exceeding the "derived effective gust speed" (} U_{de} \text{), with assumed flight speed of 300 ml/hr} \]

standard load

ultimate load, with assumed load factor 5 in the C.A.R. gust case B.
Fig. 2a: Factor of safety required for scatter of strength.

Fig. 2b: Factor of safety required for scatter of strength.
Fig. 3: Boundary $x = b$ above which 90% of all failures occur.

Fig. 4: Comparison of factors of safety needed for strength scatter in the case of mean strength $= 1.05$ times required strength (full lines) and in the case of mean strength $= required strength$ (dotted lines).