On the Natural Frequencies of a Short Circular Cylinder, Including the Effects of Entrained Mass

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Firstly, I wish to thank Jaap Gelling and Lex van Gunsteren for their initial observation that there was some difficulty in predicting the natural frequencies of nozzles with existing semi-empirical formulae.

Secondly, I wish to thank those members of staff at TU that made the experimentation possible, in particular: Jan Snijders for his rapid response to my initial queries and his organisation of details with the workshops and, last but certainly not least, to Hans Weerheim for his diligence in setting-up the instrumentation and test rig - and for finding the time to thoroughly eke out results from the rig.
Summary

This paper contains a summary of recent research work that has been undertaken to quantify the effects of entrained mass on the natural frequencies of oscillation of a short-chord circular cylinder. A brief summary is given of existing exact methods for the prediction of natural vibrational modes of rings and partial rings in air, and of a semi-empirical method which allows for the effect of entrained mass. A short critique of the latter leads directly to the suggestion of a new and perhaps more accurate method of prediction.

There then follows a description of the design of an experimental rig in which the circular cylinder may be clamped in several different configurations and subjected to an external periodic force. A description of the tests in air and water is then followed by an examination of the test results and a detailed appraisal of the accuracy of the suggested calculation method. This examination shows that the semi-empirical formulae, incorporating appropriate attenuation factors, may be used to predict the natural vibrational modes with a high degree of accuracy. Finally, there are suggestions for further experimental research and calculations.
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SECTION 1

INTRODUCTION

This paper has been written as a summary of research work that has been undertaken to quantify and predict the natural vibration frequencies of a short circular cylinder. Of particular importance in this work is the difference in natural frequency obtained when the cylinder is excited in air and when totally submerged in water. In other words, the object of the research is to quantify the effects of the entrained mass of water, i.e. the mass of water adjacent to the solid surfaces, and which oscillates sympathetically.

The origins of this research stem from the desire to predict the natural frequency of vibration of a nozzle; the nozzle itself forms an important thrust augmentation function in many propulsion applications. Of particular interest are marine applications in which the nozzle remains submerged in a rather hostile environment (the sea) throughout its useful life. As can be seen from Figure 1, a nozzle or ducted propulsor consists of a conventional screw propeller which is surrounded by a short-chord ring having a specially designed aerofoil cross section. This is the nozzle, and it may consist of a single-element aerofoil section, as shown, or it may have a slat and/or flap like an airplane wing.

Due to the action of the propeller and the surrounding flowfield, the nozzle is subject to periodic excitation forces that may cause serious damage to the nozzle itself and to the ship. In extreme cases, the vibration can cause severe structural damage and it is not unknown for the nozzle to simply fall off the ship's stern. For the propulsion specialist this means that, in particular, the natural frequencies of nozzle vibration must be selected as far removed from the excitation frequencies as is practicable. In order to achieve this objective he requires a good understanding of the phenomena involved, backed up by accurate mathematical methods for the prediction of the natural frequencies. These methods may consist of simple semi-empirical formulae of known accuracy or may be in the form of comprehensive computer software packages. It is the former that is of foremost interest in this paper. Section 2, below, assesses one such available method in the light of an examination of the underlying principles, and then proceeds to develop an alternative which must be verified.
To determine the effects of entrained mass on the natural frequencies of a nozzle and to assess the accuracy of the new semi-empirical formulae, a special model and test rig have been designed. The model consists of a short circular cylinder which is supported in several different configurations in order to simulate, to some extent, the type and variety of fixtures that connect a nozzle to a ship's hull. The design of the test rig, a description of the test procedure and measuring equipment, and the test results are all presented in Section 3. The Section concludes with a discussion of the results.

In Section 4 is a full comparison between the predicted natural frequencies and those obtained from model tests which show that the semi-empirical formulae, incorporating appropriate attenuation factors, may be used to predict the natural vibration frequencies with a high degree of accuracy. Final conclusions and recommendations for further research are contained in Section 5.
SECTION 2

CALCULATION OF THE NATURAL FREQUENCIES

2.1 Introduction

Nozzle propellers are used extensively in the marine field as propulsors for tugs, trawlers, tankers and icebreakers. Since the nozzle is essentially an appendage protruding from the hull (see Figure 2) it is susceptible to vibrational excitation from the fluctuating pressure field of the screw propeller and from the surrounding flowfield. The ensuing vibrations are undesirable and, apart from discomfort to the ship's crew, can be detrimental to the continued operation of the propulsor. According to Wind [1], fatigue cracks have been known to appear within a few days of operation and the corrosive nature of the surrounding fluid (the sea) will assist in the general deterioration of the weakened structure. In extreme cases, the supporting structure may be weakened to such an extent that the nozzle may fall from the stern of the ship (see Gelling [2] for example).

Of crucial importance in the avoidance of unacceptable vibrational characteristics of the nozzle is a knowledge of the sources of excitation together with an ability to predict the major source of impending trouble: the natural frequencies of the nozzle and its supporting structure. The sources of excitation are known to be of two types:

(1) That due to the periodic pressure fluctuations produced by the rotating propeller, the fundamental frequency of which may be expressed mathematically as:

$$\omega_p = N \Omega \quad \text{Hz} \quad (2.1)$$

where \(N\) is the number of propeller blades, and \(\Omega\) is the propeller rotational speed in rev/sec. Higher order harmonics of equation (2.1) may also be of significance.

(11) That due to the periodicity in the viscous wake shed downstream of the nozzle trailing edge region, i.e. that due to flow induced vibration. This phenomenon is referred to as 'flutter' when applied to the flow about two-dimensional aerofoils or aeroplane wings. Usually, the exciting frequency from this source is too high to cause severe problems in typical marine installations, although Wind [1] indicates that realistic calculations have been made by using an expression given earlier by Blevins [3]:

3
\[ \omega_v = \frac{S_t \cdot V_A}{T_1} \text{ rad/sec} \] (2.2)

where \( S_t \) is the Strouhal number, \( V_A \) is the undisturbed axial flow velocity, and \( T_1 \) is the thickness of the viscous wake as it leaves the nozzle profile.

In order to calculate the natural frequencies of the nozzle structure, recourse must be made to either comprehensive computer programs utilising, for example, finite element techniques or to semi-empirical formulae. In both cases it is imperative that the predicted natural frequencies correspond to those actually measured experimentally either from model- or full-scale tests, preferably both. If or when this has been done, the results have tended to be hidden away as a means of maintaining a "competitive" edge over one's commercial rivals. As a result, there appears to be a complete lack of information in published literature on the measurements taken of the vibrational characteristics of nozzles in both water and air. This paper is the first step in the rectification of this matter.

2.2 Basic concepts and existing methods

2.2.1 Vibration of rings and partial rings

It can be shown, for a full ring of uniform mass and stiffness, that the bending vibrations are the most important. The exact shape of the normal mode of vibration consists of a curve which is a sinusoid on the developed circumference of the ring. These shapes are shown in Figure 3 for the four, six, and eight nodded modes or for two, three, and four full waves along the circumference of the ring. These shapes are taken from den Hartog [4], as is the exact formula for the natural frequencies, viz:

\[ \omega = \frac{n \left( \frac{n^2 - 1}{n^2 + 1} \right) \left( \frac{EI}{\mu R^4} \right)^{1/2}} \text{ rad/sec.} \] (2.3)

where \( n \) is the number of full waves, \( \mu \) is the mass per unit length of the ring, \( EI \) the bending stiffness, and \( R \) the radius.

For a partial ring vibrating in its own plane, the fundamental mode of flexural vibration is illustrated in Figure 4 and the natural frequency is dimensionally the same as equation (2.3), except for the inclusion of a numerical factor, \( f(\alpha) \), which depends on the the central angle \( \alpha \):

\[ \omega = f(\alpha) \left( \frac{EI}{\mu R^4} \right)^{1/2} \text{ rad/sec.} \] (2.4)

The values of the constant \( f(\alpha) \) for the various angles between \( \alpha = 180 \) degrees and \( \alpha = 360 \) degrees are shown in Figure 5. According to den Hartog, natural frequencies calculated from equation (2.4) and Figure 5 may be expected to be of the order of 10% too high since the "clamping" would
normally admit some angular motion.

Finally, from den Hartog, is a formula and figure for the vibration of a partial ring in the direction of the axis of the cylinder (as illustrated in Figure 6):

$$\omega = f\left(\alpha, \frac{EI_2}{C}\right) \left(\frac{EI_2}{\mu R^4}\right)^{1/2} \text{ rad/sec.} \quad (2.5)$$

where $EI_2$ is the bending stiffness in a plane at 90 degrees from $EI$ in equations (2.3) and (2.4), and $C$ is the torsional stiffness which has the form $Gl_p$ for a bar of circular cross section.

2.2.2 Entrained mass

When a flat plate of infinite span oscillates in a plane normal to its chord length, $c$, the mass of fluid adjacent to the plate, inscribed by a circular cylinder, is also considered as part of the accelerated mass (see Figure 7). For a flat plate of negligible thickness this entrained mass is given by:

$$\mu_e = \rho \pi c^2/4 \quad \text{kg/unit length} \quad (2.6)$$

where $\rho$ is the mass density of the surrounding fluid.

To allow for plate thickness, or profile geometry, the entrained mass $\mu_e$ must be reduced from that given by equation (2.6) by an amount corresponding to the volume occupied by the body profile, as illustrated in Figure 8. For typical aerofoil cross sections, this reduction would be of the order of 10% and, in the limit, there would be no entrained mass associated with a body of circular cross section.

2.2.3 Application to annular shrouds

The above concepts have been combined in reference [1] in order to predict the natural frequency of vibration of annular shrouds (nozzles) in air and in water. This is necessary in order to avoid the severe consequences associated with a natural frequency at, or near to, the exciting frequencies due to the propeller and surrounding flowfield. The result of this study is illustrated in Figure 9, with which the following formula for the natural frequency should be used:

$$\omega = \frac{k}{\pi} \left(\frac{EI}{(\mu_1 + \mu_2)R^4}\right)^{1/2} \quad \text{Hz.} \quad (2.7)$$

where $\mu_1$ is the mass per unit length of the nozzle section, $\mu_2$ is the entrained mass per unit length, $R$ is the mean nozzle radius, and $k$ is a
numerical factor which is a function of the angle $\alpha$ defined above. The magnitude of $k$ is obtained from Figure 9, and it should be noted that the total factor $k/\pi$ includes a conversion from rad/sec to Hz. The origins of this approach are apparent from the preceding sub-sections.

Unfortunately, there is the major practical objection to equation (2.7): it simply does not predict the correct natural frequency of a nozzle which is vibrating in air [5], i.e. when $\mu_2$ is zero. Now since the concepts underlying this approach are sound, and on the understanding that the differences between two-dimensional and axisymmetric vibration modes due to entrained mass are incorporated into the numerical factor $k$, then the source of this fundamental error must be found in either Figure 9 and/or equation (2.7).

A comparison of Figures 5 and 9 show, for a specific value of $\alpha$, that the numerical factor $k/\pi$ in equation (2.7) is identical in magnitude to $f(\alpha)$ in equation (2.4) within the range $225^\circ$ to $360^\circ$. Between $180^\circ$ and $225^\circ$, however, there is a considerable discrepancy. It is concluded that Figure 9 has been produced, in part, by reference to the measured vibrational characteristics of a number of nozzles in water and, in consequence, contains some allowance for the different effects of entrained mass in two-dimensional and axisymmetric vibrational modes. It follows logically from this observation that the factor $k$ cannot be used in equation (2.7) for the prediction of the natural frequency of a nozzle vibrating in air outside its range of validity.

In addition to the above observation, there is some question as to the validity of the way in which the magnitude of the entrained mass, $\mu_2$, is calculated. The method adopted in reference [1] is to surround the nozzle profile by a circular section of water, as illustrated in Figure 8, and then to assume that the total volume of entrained fluid takes the form of a toroid that extends over the whole nozzle arc described by the central angle $\alpha$. This is done regardless of the method of attaching the nozzle to the ship's hull. If this assumption is invalid, then the latent errors in the numerical factor $k$ can only be further aggravated. This question is examined below and an alternative calculation method is proposed.

2.3 A new approach

The approach described below was first outlined by the author in reference [6] in the form of an initial prognosis report. As a starting point, it is accepted that the entrained mass associated with the vibration of an infinite span section may be calculated by the methods described in Section 2.2.2 above. However, the extension of this approach to finite-span and cylindrical sections is dealt with in a different manner to that described in
reference [1].

Consider, for example, a finite-span flat plate of length $L$ which is simply supported at both ends as shown in Figure 10. Its fundamental mode of vibration is a sinusoid. Now, since the amplitude of vibration is zero at both ends of the plate, there can be no surrounding mass of fluid which is oscillating sympathetically in these two planes. In addition, the maximum amplitude of vibration at mid span must be less than that of an infinite-span plate which is excited by the same external periodic pressure field. For the finite-span plate, we may express the entrained mass distribution along the surface $S$ by the following expression:

$$\mu_s = \nu_s \sin(\pi S/L) \mu_e \quad \text{kg/m} \quad (2.8)$$

where $\mu_e$ is given by equation (2.6), and $\nu_s$ is an attenuation factor which indicates that the maximum amplitude is less than that of an infinite-span plate. The magnitude of $\nu_s$ may be obtained from experimental observations, and it should be anticipated that it may be expressed as a function of the plate length, $L$.

Consider further a finite-span plate that is clamped at both ends and vibrating as shown in Figure 11. In this case, the displacement curve corresponding to its fundamental mode of vibration may be expressed as follows:

$$\delta_c = 0.5 \nu_c \{1 + \cos(2\pi S/L-\pi)\} \quad (2.9)$$

The corresponding entrained mass distribution along the plate is then given by:

$$\mu_c = 0.5 \nu_c \{1 + \cos(2\pi S/L-\pi)\} \mu_e \quad \text{kg/m} \quad (2.10)$$

where $\nu_c$ is, again, an attenuation factor whose magnitude may be ascertained from experimental observations.

Finally, to extend these ideas to the particular case of a vibrating nozzle, the following methodology is suggested: Since the effect of the entrained mass within the nozzle annulus (see Figure 12) is unknown, and the two-dimensional equations (2.8) and (2.10) are only approximations, then the most effective procedure is to conduct tests on different nozzle clamping configurations, the results from which will yield values of the attenuation factors, $\nu$, that include this effect. The application of the above equations to an annular shroud may then be achieved by dividing the shroud into circular segments which span the various clamping locations (see Figure 13).

The above summarises the new approach suggested for dealing with the entrained mass. In the next Section is a description of the model test rig, testing procedure, and test equipment followed by a discussion of the results.
In Section 5 is a comparison between these results and those predicted by the above approach.
SECTION 3

TEST RIG DESIGN AND MODEL TEST RESULTS

3.1 Introduction

The purpose of the model tests described below is to quantify the effects of entrained mass on the natural frequencies of oscillation of a short-chord axisymmetric body. In other words, tests need to be conducted both in air and in water. The results from these tests should lead to a better understanding of the effects of entrained mass on the vibrational characteristics of nozzles, and may also be used to quantify the magnitude of the attenuation factors identified in the previous Section if the relevant formulae can be shown to be valid.

Immediately following is a brief outline of the design of the test rig, which is then followed by description of the test procedure and equipment. Finally, in this Section, is a presentation and discussion of the experimental results.

3.2 Design of test rig

Since the prime objective of the tests is to find the difference in natural frequencies of a nozzle oscillating in air and in water, it is essential that the nozzle be of light construction. The added mass of entrained water may then be expected to reduce the natural frequencies quite significantly. Secondly, the nozzle cross section should be simple enough in shape so as to allow a realistic appraisal of the accuracy of the formulae presented above. With these design parameters in mind, it was decided to use a flat plate profile for the nozzle with a short chord-diameter ratio (this latter reflecting the current trends in nozzle design). The model, then, consists of a short circular cylinder constructed from steel plate which was bent into circular form and welded together along the chord. This construction method and material selection having the advantage of low cost. The nominal dimensions of the cylinder are 300 x 100 x 1.5 mm.

To facilitate the location of a vibration exciter and to allow for a number of different clamping configurations, a welded frame was manufactured from square steel sections to which the model would be fastened using specially designed clamps. The natural frequency of the frame was calculated to be much higher than those expected from the cylindrical model. Both model
and frame are shown in Plate 1. As to the clamps: The configurations to be tested demanded that clamps be manufactured which would simulate, to some extent, simply-supported connections to the ship's hull. These were made from brass, as shown in Plate 2, and were connected to the model and frame as shown in Plate 3. A single clamp was also manufactured from plastic that would simulate the bulky structure that usually connects the top of a nozzle to the ship's hull. This "clamped box" is 100 mm wide. The whole test rig, illustrating all the above elements, is shown in Plate 4. This Plate shows the rig in an early testing mode with the vibration exciter in position.

3.3 Test procedure and equipment

Initial tests were conducted in air with the test rig standing upright on blocks of foam, as illustrated in Plate 4. The amplitude, frequency and modes of vibration for each clamping configuration were measured via accelerometers placed on the cylinder surface. The accelerometer output was fed via a charge amplifier to an oscilloscope for direct observation. Placed on top of the supporting frame was the exciter, which was driven by an amplified sinusoid from a signal generator. The test procedure consisted, for all clamping configurations, of driving the exciter over a wide range of frequencies so as to identify the fundamental natural mode and frequency of the model. In some instances, higher modes of oscillation were also found and noted. Experience gained from these initial tests proved to be valuable in that a few minor improvements were made to the rig and to the set-up procedure.

For the full comparative tests in water and in air, the model was fitted with eight pairs of strain gauges which were located at 30° intervals around its circumference, one of the pair on the inner contour and the other on the outer contour. The output from each pair of strain gauges was fed via a Wheatstone bridge, an amplifier, and filter to an oscilloscope for direct observation. The rig itself was hung from a horizontal beam by means of four springs, and the beam could be raised or lowered by adjustment of the supporting telescopic uprights (see Plate 5).

For the tests in air, the rig was held in its raised position and the model was excited, as before, thereby permitting an identification of the natural frequencies and modes of vibration. This methodology confirmed the results obtained from the initial tests, any small difference in the measured frequencies being due to the minor improvements made to the rig and to the different added mass effects of the accelerometers and the strain gauges. For the tests in water, the whole rig was lowered into a container of water until the model was totally submerged. A search was then made for the mode and
frequency of vibration that corresponded to that observed in air. This procedure was repeated for all clamping configurations. The complete test rig is shown in Plates 5 and 6.

3.4 Presentation and discussion of test results

Figures 14, 15, 16 and 17 illustrate the clamping configurations for which repeatable and accurate measurements were made of the mode and natural frequency of the model in both air and water. Also illustrated are the mode shapes themselves and the frequencies. An additional configuration was also tested which consisted of four brass clamps spaced 90° apart around the circumference of the cylinder, but has been excluded since the results were inconclusive. In all cases, the exciter was placed above the model and subjected it to periodic forces in a vertical direction, i.e. vertically with respect to the illustrated model orientations. (It should be noted that the tests in water indicated by Figures 14c and 15a produced two measurable frequencies; it was not possible to detect any discernable difference in the mode shapes with the equipment used. Since, in both cases, the higher frequency is approximately double the lower one, it has been assumed that the lower corresponds to the fundamental mode; but see also below).

From Figure 14 it may be observed that mixed modes (i.e. modes that involve normal bending of the model but with locally modified stiffness characteristics due to the clamps) and pure flexural modes of oscillation were observed for a partial ring, this latter corresponding exactly with the form illustrated in Figure 4 except for a 90° shift in the plane of vibration. From the initial tests it was observed that the fundamental frequency in the vertical plane was some 5% lower than that in the horizontal plane. Unfortunately, it was impracticable to induce horizontal oscillations with the current test rig whilst the model was submerged. It should also be noticed from Figure 14 that the difference between the frequencies measured with the model in air and water are quite considerable, which should allow an accurate assessment of the various alternatives for calculating the entrained mass of water.

In Figure 15, it may be observed that the model suspended from a single clamp oscillates in a form similar to that of a ring bending in its own plane, as illustrated in Figure 3. The main difference here being due to the physical presence of the clamp, which introduces a local stiffness characteristic that is different to that of pure normal oscillation. Nevertheless, these modes should be more than adequate for the assessment of the accuracy of the entrained mass corrections presented in Section 2. Once again a significant
reduction in measured frequencies, due to the entrained mass of water, should be observed.

Figure 16 illustrates the fundamental mixed and flexural modes obtained with the model supported by two brass clamps, the latter being somewhat unexpected even though it clearly has its origin in the corresponding flexural mode illustrated in Figure 16. Finally, Figure 17 illustrates the fundamental mixed mode observed with the cylinder held in position by two brass clamps and the clamped box. The reduction in measured frequencies due to the entrained mass is significant.

In the next Section, these results are used to assess the accuracy of the calculation methods proposed in Section 2.
SECTION 4

EVALUATION OF THE ACCURACY OF PREDICTED RESULTS

4.1 Properties of the model

The nominal dimensions of the model were noted above as being 300 x 100 x 1.5 mm, and accurate measurement of the model after experimentation yielded the following:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean diameter</td>
<td>302.3 mm</td>
</tr>
<tr>
<td>chord length</td>
<td>99.5 mm</td>
</tr>
<tr>
<td>thickness</td>
<td>1.5 mm</td>
</tr>
</tbody>
</table>

Taking a mass density for steel of 7.85 g/cm$^3$, these overall dimensions lead to a mass, $\mu$, of 1.178 kg/m distributed evenly around the ring, and to a second moment of area about its bending axis, $I$, of 28.13 mm$^4$. Taking Young’s Modulus for steel as $2.1 \times 10^{11}$ N/m$^2$ and evaluating the expression common to both equations (2.3) and (2.4), we obtain $\sqrt{(EI/\mu R^4)} = 98.0$ rad/sec or 15.6 Hz.

4.2 Accuracy of natural frequency prediction in air

There are two clamping configurations that may be expected to yield natural frequencies close to those predicted by the formulae in Section 2.2.1: The first corresponds to the fundamental mode of flexural vibration illustrated in Figure 14c, which may be compared directly with the mode of oscillation that led to equation (2.4). From the known geometry, the central angle, $\alpha$, is calculated as $321^0$ which leads to a value of the constant $f(\alpha)$ of 0.79 from Figure 5. Substitution of this value, and that of 15.6 Hz obtained above for the expression $\sqrt{(EI/\mu R^4)}$ leads to the following result:

The predicted fundamental frequency for flexural vibrations is 12.3 Hz from equation (2.4), which compares with 12.4 Hz measured experimentally. The agreement is excellent. Evidently the clamping method used in this test was much stiffer than that referred to by den Hartog with respect to an expected 10% overestimation in frequency by equation (2.4).

The second configuration is illustrated in Figure 16a which, because of the symmetry of vibration about the clamping axis, corresponds to a fundamental flexural mode where the central angle, $\alpha$, is $180^0$. From Figure 5, this leads to a value of $f(\alpha)$ of 4.5. Substitution into equation (2.4) yields a predicted natural frequency of 70.2 Hz, which may be compared to the measured experimental value of 69.9 Hz. Again, agreement is excellent.
It is instructive to summarise these results and draw a comparison with those predicted by the formula given by Wind [1]:

<table>
<thead>
<tr>
<th></th>
<th>Wind</th>
<th>den Hartog</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 14c</td>
<td>12.3</td>
<td>12.3</td>
<td>12.4</td>
</tr>
<tr>
<td>Figure 16a</td>
<td>41.2</td>
<td>70.2</td>
<td>69.9</td>
</tr>
</tbody>
</table>

Table 4.1: Comparison of predicted and measured frequencies in air (Hz)

As anticipated in Section 2.2.3, the prediction of Wind grossly underestimates the natural frequency when the included angle, $\alpha$, is 180°.

Unfortunately, the above configurations are the only ones which can be used to test the accuracy of the equations presented in Section 2.2.1. The configuration of Figure 15 for example, in which the model is held by one clamp, shows that the model is vibrating in a mixed (normal) mode. This is easily confirmed by calculating the first normal mode of a ring bending in its own plane, i.e. by using equation (2.3) with $n$ set equal to 2. This mode is illustrated in Figure 3. The calculated frequency is 41.8 Hz (i.e. $2.68 \times 15.6$) which is much higher than the measured value of 26.6 Hz corresponding to Figure 15a. It is perhaps unfortunate that a pure normal mode could not be measured experimentally, but such an undertaking is beyond the scope of the present studies.

4.3 Evaluation of attenuation factors

The objectives of this Section are to determine the values of the attenuation factors, $\nu$, given in equations (2.8) and (2.10) and to examine the effects of different distributions of the entrained mass of water on the accurate prediction of the natural frequencies of vibration. It is clear, however, that the suggested distribution of entrained mass must be integrated over the appropriate plate length in order to arrive at an average total mass, $\mu^1$, distributed uniformly over the plate length. This uniformly distributed mass, $\mu^2$, may then be added to $\mu^1$, the mass per unit length of the nozzle for substitution into the appropriate equations. In other words, this means that there is no way to quantify the actual effects of different distributions of entrained mass when using the equations for natural frequency prediction given by equations (2.8) and (2.10). All that can be undertaken at this stage is an evaluation of attenuation factors which include the average integrated effect of the entrained mass.

In preparation for this, we may calculate the maximum entrained mass, $\mu^e$, corresponding to a torus of water surrounding the whole circumference of the model: Taking the mass density of water as 1.0 g/cm², this leads via equation
(2.6) to a mass, $\mu_e$, of 7.85 kg/m (the effect of the volume occupied by the nozzle profile is negligible). This quantity is some 6.6 times larger than the distributed mass of the model.

In order to develop a suitable methodology for converting $\mu_e$ into an equivalent distributed mass, $\mu_z$, it is advantageous to examine the two flexural modes of vibration first:

### 4.3.1 Flexural modes

We have seen in Table 4.1 that the prediction of frequencies is excellent when the model is oscillating in air. For the calculations in water, we assume initially that the entrained mass is contained within a torus which extends over the arc described by the included angle, $\alpha$. It should then be possible to evaluate the appropriate attenuation factor, $\nu$. The results of these calculations are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Wind</th>
<th>den Hartog</th>
<th>Experiment</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 14c</td>
<td>4.7</td>
<td>4.7</td>
<td>3.8</td>
<td>1.60</td>
</tr>
<tr>
<td>Figure 16a</td>
<td>15.1</td>
<td>33.7</td>
<td>34.8</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of predicted and measured frequencies in water (Hz)

We see immediately that the prediction for a configuration with two diametrically opposed clamps (Figure 16a) is very good and suggests that the arguments proposed above have some validity, even though the attenuation factor seems rather high. This latter observation implies that there may be little advantage in considering a sinusoidal distribution of entrained mass. It should also be noted that the Wind prediction for this configuration is much too low, as expected from above.

For the configuration illustrated in Figure 14c, we see that the predicted frequency is too high which leads, in turn, to an attenuation factor greater than unity. This is unexpected, but may be explained by a consideration of the viscous damping introduced by the large amplitude and low frequency associated with this mode of vibration. Possibly a more qualitative explanation may be forthcoming from an examination of the mixed modes of oscillation:

### 4.3.2 Mixed modes

If, as has been suggested, large amplitudes and/or low frequencies lead to difficulties in regard to the accuracy of the prediction methods, it may be fruitful to examine oscillatory modes that are similar in shape but with
different amplitudes. This is made possible by the measurements taken from four different clamping configurations: These are illustrated, in order of decreasing frequency, in Figures 17, 16b, 14b, and 15b which all exhibit a similar mode shape to that obtained from pure bending, and illustrated in Figure 3. For the vibrations in air, equation (2.3) yields a vibrational frequency of 118.4 Hz when \( n = 3 \), a result that may be compared to the measured values, viz:

<table>
<thead>
<tr>
<th>Fig. 17</th>
<th>Fig. 16b</th>
<th>Fig. 14b</th>
<th>Fig. 15b</th>
</tr>
</thead>
<tbody>
<tr>
<td>148.1</td>
<td>124.0</td>
<td>115.0</td>
<td>97.2</td>
</tr>
</tbody>
</table>

Table 4.3: Measured frequencies in air for four clamping configurations (Hz)

The effects of the different clamping configurations can clearly be seen. An inclusion of the entrained mass in equation (2.3), i.e. by adding an extra mass distribution appropriate to the specific clamping configuration, leads to the following results:

<table>
<thead>
<tr>
<th>Fig. 17</th>
<th>Fig. 16b</th>
<th>Fig. 14b</th>
<th>Fig. 15b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted (Expt.)</td>
<td>55.3</td>
<td>44.8</td>
<td>43.0</td>
</tr>
<tr>
<td>Predicted (Theo.)</td>
<td>44.3</td>
<td>42.7</td>
<td>44.3</td>
</tr>
<tr>
<td>Measured</td>
<td>66.6</td>
<td>53.7</td>
<td>49.3</td>
</tr>
<tr>
<td>Attenuation factor</td>
<td>0.36</td>
<td>0.58</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Table 4.4: Comparison of results in water for four configurations (Hz)

In this Table are the results of calculations using both the measured frequencies in air, as given in Table 4.3, and the calculated frequency of 118.4 Hz for this mode of bending. Clearly, there is no way to predict the effects of entrained mass with equation (2.3) unless one has a prior knowledge of the increased stiffness effects of the particular clamping configuration. Furthermore, a comparison of rows 2 and 3 in Table 4.4 permit the calculation of attenuation factors, which are also given. From these values we see that the attenuation factors increase in value as the frequency of vibration reduces as was also observed with the flexural modes in Section 4.3.1.

On the one hand, we may take the suggestion above that the increase in attenuation factor is due to the viscous effects of the fluid when subjected to large amplitude, low frequency vibrations. On the other hand, the increased stiffness of the supporting structure does lead to a quantifiable increase in the measured vibrational frequencies in air, as seen in Table 4.3. To allow for this difference in stiffness, introduced by the various clamping configurations, we may simply introduce a correction factor in the numerator of equation (2.3) so that the vibrational frequency in air is accurately
predicted. This correction factor must, by implication, be applicable to the vibrational characteristics of the submerged model. Perhaps this prognosis will be more easily explained in two steps:

(1) The normal mode of vibration for the nodal shape under consideration is predicted by equation (2.3) as \[ 7.59\sqrt{\frac{E}{\mu R^4}} \], i.e. as 118.4 Hz. Since the stiffnesses of the four clamping configurations are different from the free oscillations evaluated by den Hartog, correction factors may be evaluated by dividing the appropriate measured frequency in air by 118.4 Hz, viz:

<table>
<thead>
<tr>
<th></th>
<th>Fig. 17</th>
<th>Fig. 16b</th>
<th>Fig. 14b</th>
<th>Fig. 15b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured frequency (Hz)</td>
<td>148.1</td>
<td>124.0</td>
<td>115.0</td>
<td>97.2</td>
</tr>
<tr>
<td>Correction factor</td>
<td>1.25</td>
<td>1.05</td>
<td>0.97</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Table 4.5: Correction for stiffness characteristics

(2) These correction factors may now be used to determine the magnitude of the attenuation factors relating to the entrained mass of water. Firstly, we multiply the frequencies in row 2 of Table 4.4 by the applicable correction factors of Table 4.5, which yield the predicted frequencies in water if the model is considered to be surrounded by a torus of water with a mass distribution, \( \mu_e \), of 7.85 kg/m. (Note: This is reduced to 7.02 kg/m for those configurations with the clamping box). Next, we compare the result to the measured frequency and calculate the attenuation factor:

<table>
<thead>
<tr>
<th></th>
<th>Fig. 17</th>
<th>Fig. 16b</th>
<th>Fig. 14b</th>
<th>Fig. 15b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted (Hz)</td>
<td>55.3</td>
<td>44.8</td>
<td>43.0</td>
<td>35.1</td>
</tr>
<tr>
<td>Measured (Hz)</td>
<td>66.6</td>
<td>53.7</td>
<td>49.3</td>
<td>41.0</td>
</tr>
<tr>
<td>Underestimate (%)</td>
<td>17</td>
<td>17</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>Attenuation factor</td>
<td>0.64</td>
<td>0.64</td>
<td>0.72</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Table 4.6: Comparison of attenuation factors for four configurations

At last, it seems we have a qualitative explanation of the factors contributing to the measured and calculated frequencies: Firstly, if the added stiffness of the supporting structure is quantifiable then the equations of den Hartog may be modified by the simple application of a correction factor. This modification will yield a good estimate of the natural frequency in air (see Table 4.4 for the magnitude of some of these factors). Secondly, to include the effects of entrained mass, we calculate the added mass due to a toroid of water surrounding the exposed surfaces of the nozzle. We may calculate the frequency of the submerged body either by reducing the average entrained mass by some 35% and substitute directly into equation (2.3), or we may use the whole toroid in the calculation and then increase the calculated
frequency by some 15%. Either method should produce the same result for this particular modal shape.

Unfortunately, the difference in attenuation factors is not sufficient to draw any inferences regarding the most likely distribution of entrained mass over the vibrating surfaces of the model. Calculations based on, for example, finite element techniques will undoubtedly lead to some further insights into this problem. However, it should be noted that the area under a sine distribution (in the range 0 to $\pi$) occupies about 64% of the area of its bounding rectangle -- a fact that is surely more than coincidental in regard to the attenuation factors of Table 4.6.

What now needs to be done is check the above prognosis against the remaining experimental results:

4.4 Prognosis evaluation

In order to assess the general validity of the above approach, we may now examine the vibrational modes illustrated in Figures 14a and 15a, both of which oscillate in a mixed mode. If we begin with Figure 14a, for which definite results were obtained in air and water, we obtain the following results:

<table>
<thead>
<tr>
<th>den Hartog (Hz)</th>
<th>Measured in air (Hz)</th>
<th>Stiffness correction factor</th>
<th>Measured in water (Hz)</th>
<th>Attenuation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.8</td>
<td>32.8</td>
<td>0.78</td>
<td>15.3</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 4.7: Evaluation of factors for Figure 14a

In these calculations the frequency calculated in water, with the inclusion of the stiffness correction, was 12.5 Hz. It can be seen that the attenuation factor is a little smaller than those calculated in Table 4.6, as may be anticipated from the difference in the mode of vibration. The same methodology may now be applied to Figure 15a: A stiffness correction is evaluated which corresponds to this clamping configuration and, since the mode of vibration is the same, the attenuation factor from above, i.e. 0.61, may be considered to be valid. The results are as follows:

<table>
<thead>
<tr>
<th>den Hartog (Hz)</th>
<th>Measured in water (Hz)</th>
<th>Stiffness correction factor</th>
<th>Attenuation factor</th>
<th>Calculated frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.8</td>
<td>26.6</td>
<td>0.67</td>
<td>0.61</td>
<td>11.9</td>
</tr>
</tbody>
</table>

Table 4.8: Evaluation of vibrational frequency in water for Figure 15a

The actual measured frequency in water for this configuration was 6.4 Hz
(or 12.6 Hz), and the latter may be deduced as being the correct fundamental mode. Further tests, using water-resistant accelerometers, would undoubtedly show this deduction to be valid.

Finally, then, we may re-examine the results obtained for the flexural modes of oscillation discussed earlier in Section 4.3.1. In these two cases, however, the stiffness correction factor is not relevant since it is already contained in the function $f(\alpha)$ given in Figure 5. The results are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Measured in air (Hz)</th>
<th>Calculated de Hantog (Hz)</th>
<th>Measured in water (Hz)</th>
<th>Calculated in water (Hz)</th>
<th>Attenuation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 16a</td>
<td>69.9</td>
<td>70.2</td>
<td>34.8</td>
<td>$\leftrightarrow$ 34.8</td>
<td>$\rightarrow$ 0.45</td>
</tr>
<tr>
<td>Figure 14c</td>
<td>12.4</td>
<td>12.3</td>
<td>7.5*</td>
<td>6.4</td>
<td>$\leftrightarrow$ 0.45</td>
</tr>
</tbody>
</table>

Table 4.9: Comparison of results in air and water for flexural modes

* This is the higher of the two measured frequencies

To obtain these results, the following procedure has been adopted: Firstly, the frequency in air has been calculated for both configurations as in Section 4.3.1 above. Next, the measured frequency in water was used for Figure 16a in order to evaluate the attenuation factor based on a toroid of water which extends over the whole circumference of the model. This attenuation factor is then used to predict the vibrational frequency of the configuration given in Figure 14c. This process is indicated by the arrows in Table 4.9. (It should also be noted that the frequency in water, for Figure 16a, may also be calculated by taking an included angle of 180°; the predicted frequency of which is 33.7 Hz as given in Table 4.2. However, the attenuation factor of 0.92 may not be used in the calculation for the partial ring of Figure 14c).

Again, we see that the predicted frequency is close to, and slightly lower than, the higher of the two measured frequencies - as was the case with Figure 15a. Further tests with water-resistant accelerometers would help confirm whether or not this higher measured frequency does actually correspond to mode shape given in Figure 14c. Further, it is interesting to note that the calculated frequency of 6.4 Hz in Table 4.9 is identical to the lower measured frequency of the clamping configuration illustrated in Figure 15a. In addition, the measured frequency of Figure 14a in water is almost exactly twice the higher frequency measured for configuration 14c. Further, more detailed tests should produce valuable information on these observations.
SECTION 5

CONCLUSIONS

From the foregoing comparison of predicted and measured results, it may be concluded that the formulae given by den Hartog are more than adequate for the estimate of natural frequencies of vibration of full and partial rings in both air and water --- provided that a stiffness correction factor for the supporting structure is a priori determined. The formula given by Wind is essentially identical to that of den Hartog for flexural oscillations and where it deviates from den Hartog (when \( \alpha \) is less than 225°) it has been shown to be invalid. The Wind formula is not applicable to the normal bending modes of oscillation, a fact which points to a probable misuse of equation (2.7) in his derivation of Figure 9.

For the prediction of natural frequencies of oscillation in water, the equations of den Hartog may be modified to include a distributed mass of entrained fluid which takes the form of a toroid extending over the whole exposed surface of the ring or partial ring. To allow for the effects of different clamping configurations and/or modal shapes, the exact magnitude of this uniformly distributed entrained mass must be derived from the product of an attenuation factor and the mass of fluid within the toroid. An estimate of these attenuation factors has been given above, and may be summarised as follows:

1. Flexural mode 0.45
2. First normal mode 0.61
3. Second normal mode 0.65

These attenuation factors have their origins in the distribution of the entrained mass along the oscillating body surfaces as was originally suggested by the author in reference [6]. Detailed calculations involving, for example, finite element techniques will yield further insight into the exact nature of these attenuation factors.

As to the model tests themselves: The information obtained from this relatively simple model have proved to be very valuable as a first step in the evaluation of the effects of entrained mass on the natural vibration frequencies of a short circular cylinder. The results obtained from the submerged model could be more accurately quantified by the use of waterproof accelerometers, or pressure transducers, in place of the strain gauges used in
the present tests - these devices were not available to the author at the time the tests were undertaken. Complementary tests on a series of finite span flat plates would also produce valuable information regarding the distribution and magnitude of entrained mass along the body surface.
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FIG. 5: THE COEFFICIENT \( f(\alpha) \) IN EQUATION (2.4).
FIG. 6: COEFFICIENTS \( f(\alpha, EI_z/C) \) OF EQUATION (2.5) FOR THE FREQUENCY OF A PARTIAL RING VIBRATING PERPENDICULARLY TO ITS OWN PLANE.

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FIG. 9: THE COEFFICIENT $k$ IN EQUATION (2.7).
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FIG. 11: FUNDAMENTAL FLEXURAL MODE OF A CLAMPED PLATE.

FIG. 12: ENTRAINED FLUID WITHIN THE NOZZLE ANNULUS.
FIG. 13: NOZZLE-HULL CONNECTIONS.

air : 32.8 Hz  
  water : 15.3 Hz

air : 115.0 Hz  
  water : 49.3 Hz

air : 12.4 Hz  
  water : 3.8 Hz

a) 1st mixed mode  

b) 2nd mixed mode  

300φ
nominal

100 mm

c) 1st flexural mode

*) second frequency of 7.5 Hz also detected.

FIG. 14: MODES AND FREQUENCIES WITH CLAMPED BOX.
air : 26.6 Hz  
water : 6.4 Hz

air : 97.2 Hz  
water : 41.0 Hz

*) second frequency of 12.6 Hz also detected

FIG. 15: MODES AND FREQUENCIES WITH SINGLE BRASS CLAMP.

air : 69.9 Hz  
water : 34.8 Hz

air : 124.0 Hz  
water : 53.7 Hz

FIG. 16: MODES AND FREQUENCIES WITH TWO BRASS CLAMPS.

air : 148.1 Hz  
water : 66.6 Hz

FIG. 17: MODE AND FREQUENCY WITH CLAMPED BOX AND TWO BRASS CLAMPS.
Plate 1: Model and frame (model not in test position).

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