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Shallow foreshore wave height statistics

A predictive model for the probability of exceedance of wave heights

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A predictive model for the probability of exceedance of wave heights

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ABSTRACT:

On shallow foreshores waves are subjected to depth-induced breaking, particularly the highest waves. This results in a profound change in the shape of the wave height distribution. Here, a model for the wave height distribution on shallow foreshores is developed based on information on the total local wave energy ($m_0$), the local water depth (d) and the slope of the foreshore ($\tan \alpha$).

The model for wave height distributions is based on a Composite Weibull distribution which is composed of two Weibull distributions with different exponents. The domain of the Composite Weibull distribution is split into two parts by a transitional wave height. This concept was found to yield a good description of observed shallow foreshore wave height distributions.

The validation of the model proves that the model yields better approximations of measured wave height distributions on shallow foreshores than existing distributions.

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Roman letters:

- $A$ : parameter based on the exponent in the Glukhovskiy distribution (-)
- $d$ : water depth (m)
- $d'$ : ratio of root-mean-square wave height to water depth, $H_{rms}/d$ (-)
- $H$ : wave height (m)
- $H_{1/3}$ : significant wave height defined as the mean of the highest 1/3-part of the wave heights in a wave field (m)
- $H_t$ : scale wave height of the Composite Weibull distribution (m)
- $H_{m0}$ : spectral significant wave height, $H_{m0} = 4\sqrt{\sigma_0}$ (m)
- $H_N$ : the wave height with an exceedance probability of $1/N$ (m)
- $H_{1/N}$ : mean of the highest $1/N$-part of the wave heights in a wave field (m)
- $H_{rms}$ : root-mean-square wave height (m)
- $H_{tr}$ : transitional wave height of the Composite Weibull distribution (m)
- $\bar{H}_S$ : nondimensionalised characteristic wave height, $\bar{H}_S = H_t/H_{rms}$ (-)
- $k_0$ : exponent of the Composite Weibull distribution (-)
- $L_c$ : characteristic wave length (m)
- $L_{0.2}$ : local wave length based on $T_{0.2}$ (m)
- $L_{op}$ : deep water wave length based on peak period $T_{op}$ (m)
- $m_n$ : $n^{th}$ moment of the frequency spectrum (m$^2$)
- $m_0$ : variance of the water surface elevation, i.e. the total wave energy (m$^2$)
- $T_{0.2}$ : wave period based on zero-th and second spectral moment, $T_{0.2} = \sqrt{m_0/m_2}$ (s)
- $T_c$ : characteristic wave period (s)

Greek letters:

- $\alpha$ : slope of the foreshore (°)
- $\alpha_{tr}$ : empirical coefficient of the transitional wave height forecasting function (-)
- $\beta$ : coefficient of the Modified Glukhovskiy distribution (Klopman 1996) (-)
- $\beta_{rms}$ : empirical coefficient of the root-mean-square wave height (-)
- $\beta_{tr}$ : empirical, slope dependent coefficient of the transitional wave height (-)
- $\sigma_{rms}$ : root-mean-square error (-)
- $\gamma_{tr}$ : empirical, slope dependent coefficient of the transitional wave height (-)
- $\kappa$ : exponent of the Modified Glukhovskiy distribution (Klopman 1996) (-)
- $\mu_t$ : mean wave height (m).
- $\psi$ : degree of saturation, $\psi = \sqrt{\sigma_0}/d$ (-)
1 Introduction

1.1 General

Within the framework of the research project “wave propagation over shallow foreshores” attention is given to the wave height statistics. The probability of exceedance of wave heights on shallow foreshores, for instance at the toe of dikes, is important information for the design of coastal protection systems. Here, a new and more accurate method to predict the wave height statistics on shallow foreshores is presented. This report is an extension of the work described in Groenendijk (1998).

In deep water the relatively linear and Gaussian behaviour of waves allows for a theoretically sound statistical description of wave field characteristics. In shallow water the description of wave behaviour is more complicated and the knowledge of the statistical description of wave field characteristics is limited. A special situation occurs when waves propagate over a shallow foreshore. A foreshore is defined as the part of the coast, seaward of a sea-defence work, which influences the wave characteristics. On shallow foreshores depth-induced breaking causes the distribution of wave heights to differ considerably from the Rayleigh distribution, see Figure 1.

![Rayleigh distribution graph](image)

Figure 1: Measured individual wave heights on shallow foreshore with slope 1:250, $H_{avg}/d=0.6$

On a shallow foreshore waves travel considerable distances in shallow water, where waves break due to depth-limitation. This causes the wave characteristics to be mainly locally determined. Therefore, the wave height distribution can be rather accurately described with local parameters (pointmodel): The philosophy behind a pointmodel is based on the assumption that the wave height distribution is mainly determined by the local parameters of the wave field and the water depth regardless of the history of the waves in deeper water. This assumption proves to be valid for shallow water with a reasonably simple bottom topography.

The purpose of this report is to investigate the distribution of individual wave heights on shallow foreshores. Based on laboratory data a Composite Weibull pointmodel is proposed.
to describe shallow foreshore wave height distributions. The Composite Weibull pointmodel is calibrated and validated with 2-D laboratory data (wave flumes). In these experiments normally incident irregular waves on a foreshore with straight and parallel depth contours are considered.

### 1.2 Rayleigh wave height distribution

The short-term statistics of waves in deep water are well described. Longuet-Higgins (1952) showed that the wave heights of Gaussian waves with a narrow banded frequency spectrum obey the Rayleigh distribution:

$$ F_H = \Pr\{H < H\} = 1 - \exp\left(-\frac{H}{\mu_H\sigma_H}\right)^2 $$

in which $H_{rms}$ is the root-mean-square wave height. For the Rayleigh distribution fixed ratios between for characteristic wave heights exist, i.e. $H_{1/3}/H_{rms}=1.41$, $\mu_H/H_{rms}=0.89$ etc., where $H_{1/3}$ is the mean of the highest 1/3-part of the wave heights in a wave field or significant wave height and $\mu_H$ is the mean wave height. When a narrow banded frequency spectrum is assumed all characteristic wave heights are theoretically related to the water surface elevation variance $m_0$, for example, $H_{1/3} = 4\sqrt{m_0}$ and $H_{rms} = 2\sqrt{2m_0}$. For real sea waves the assumption of a narrow banded frequency spectrum is no longer valid. However, Longuet-Higgins (1980) and Tayfun (1990) proved the Rayleigh distribution to give very good approximations of measured wave height distributions in deep water. Hence, the proportionality between various characteristic wave heights are still valid, but the coefficients in the relations between the characteristic wave heights and $m_0$ have to be modified to account for the finite frequency bandwidth. Goda (1979) analysed field data and found that for wind driven waves in deep water that the ratio $H_{1/3}/\sqrt{m_0}$ is approximately 5% smaller than the theoretically predicted $H_{1/3}/\sqrt{m_0}=4$. This means that in deep water, for wind driven waves with a broad banded frequency spectrum the ratio $H_{1/3}/\sqrt{m_0}$ equals approximately $3.8$ and $H_{rms}/\sqrt{m_0}=0.95 \cdot 2\sqrt{2} = 2.69$.

In shallow water the situation changes considerably. When the relative wave height increases, processes like depth-induced breaking, shoaling, triad interactions and bottom friction become relevant. Triad interactions and depth-induced breaking in particular are manifestations of strongly non-linear behaviour of waves, resulting in non-linear spectral components and a strong increase of spectral width. The bound higher harmonics cause the waves to have a definite excess of crest heights and shallow troughs, in contrast to the Gaussian waves in deep water. Thus, the surface elevation in shallow water can no longer be considered a narrow banded linear Gaussian process. This poses a problem for the description of wave statistics in shallow water, since the wave statistics rely heavily on the assumption of a sufficiently narrow banded, linear and Gaussian surface elevation. So instead of the theoretically sound Rayleigh distribution, other distributions are to be used.

### 1.3 Modified Glukhovskiy distribution

The Glukhovskiy distribution (1966) is a pointmodel which takes the influence of depth limited breaking into account by assuming the shape of the distribution to be a function of the relative wave height. Klopman (1996) proposed the following Modified Glukhovskiy distribution:
\[ F_d(H) = \Pr\{H \leq H\} = 1 - \exp\left(-A\left(\frac{H}{H_{rms}}\right)^{\kappa^*}\right) \] (1.2)

In this distribution \(H_{rms}\) is used to normalise the wave heights. Klopmann assumes the relation between \(H_{rms}\) and \(m_0\) to be as for the Rayleigh distribution of a narrow banded Gaussian process: \(H_{rms} = 2\sqrt{2m_0}\). The exponent \(\kappa^*\) is assumed to be a function of the relative wave height parameter \(d' = H_{rms}/d\):

\[ \kappa^* = \frac{2}{1 - \beta d'} \] (1.3)

The coefficient \(\beta\) in Equation 1.3 has been fitted to the laboratory data sets used by Klopmann and Stive (1989); \(\beta = 0.7\) gives a good approximation of measured wave heights (Klopmann, 1996). To assure consistency the second moment of the Modified Glukhovskiy distribution has to equal \((H_{rms})^2\). This yields the following relation between the coefficient \(A\) and \(\kappa^*\):

\[ A = \left[ \frac{2}{\kappa^* + 1} \right]^{\kappa^*} \] (1.4)

The Modified Glukhovskiy distribution as proposed by Klopmann is a pointmodel, yielding a wave height distribution for given local depth \(d\) and wave energy \(m_0\).

A semi-quantitative evaluation of the Rayleigh distribution and Modified Glukhovskiy distribution is carried out below. Figure 2 shows a typical shallow foreshore wave height distribution, measured in a wave flume with a plane shallow foreshore with a slope of 1:100. The straight, dotted line is the Rayleigh distribution and the solid line represents the Modified Glukhovskiy distribution. The wave height distributions are calculated with the measured water surface elevation variance \(m_0 = 1.1E-3\) m\(^2\) and water depth \(d = 0.27\) m.

![Figure 2: Example of calculated wave height distributions; slope 1:100, \(H_{rms} = 0.13\) m, \(d = 0.27\) m](image)

Figure 2 shows that the Rayleigh distribution gives a poor description of the measured wave height distribution. The Modified Glukhovskiy distribution, taking depth induced breaking into account, yields a better approximation of the measured wave heights. However, this
distribution still overestimates the extreme wave heights and underestimates the lower wave heights on shallow foreshores.

### 1.4 Composite Weibull distribution

On shallow foreshores the water depth slowly decreases. This causes the higher waves in a wave field to break without disturbing the distribution of the smaller waves, therefore introducing a change in shape of the wave height distribution. This change in shape is not well described with a wave height distribution containing one shape parameter, such as the Rayleigh distribution and Modified Glukhovskyi distribution. Therefore a Composite Weibull distribution is proposed. It consists of two Weibull distributions, separated by a transitional wave height, $H_{tr}$.

\[
F(H) = Pr\{H \leq H\} = \begin{cases} 
F_1(H) = 1 - \exp\left[-\left(\frac{H}{\bar{H}_1}\right)^{k_1}\right] & H \leq H_{tr} \\
F_2(H) = 1 - \exp\left[-\left(\frac{H}{\bar{H}_2}\right)^{k_2}\right] & H > H_{tr}
\end{cases}
\]  

(1.5)

Waves smaller than the transitional wave height, which are not influenced by the depth, obey the first part of the distribution $F_1(H)$. Waves exceeding the transitional wave height, which are subjected to depth induced breaking, obey the second part of the Composite Weibull distribution $F_2(H)$. The exponents $k_1$ and $k_2$ determine the shape of the distributions and two scale parameters, $\bar{H}_1$ and $\bar{H}_2$, are used to scale the distributions to the wave field of concern. The Composite Weibull wave height distribution is valid for $0 < H < \infty$. In order to obtain continuity the distribution must satisfy $F_1(H_{tr}) = F_2(H_{tr})$. In Figure 3 the Composite Weibull distribution is presented on Rayleigh scale.

![Figure 3: The proposed wave height distribution: The Composite Weibull distribution](image)

Hereafter all wave heights are normalised with $H_{rms}$:

\[
\bar{H}_x = \frac{H_x}{H_{rms}}
\]

(1.6)

in which $H_x$ denotes a characteristic wave height, like $H_{rms}$, $H_{1/3}$ or $H_{m0}$. Doing so, a Composite Weibull distribution, containing the parameters $k_1, k_2, \bar{H}_1, \bar{H}_2$ and $H_{tr}$, is obtained. However, two of the five parameters are determined by the continuity condition.
and the constraint that the normalised $H_{rms}$ of the Composite Weibull distribution has to equal one. The normalised $H_{rms}$ expressed in terms of the parameters of the Composite Weibull distribution, using a transformation and the incomplete gamma functions, yields (see Appendix A.2):

$$\bar{H}_{rms} = 1 = \sqrt{\bar{H}_{1}^{2} \gamma \left[ \frac{2}{k_{1}} + 1, \left( \frac{\bar{H}_{rms}}{\bar{H}_{1}} \right)^{k_{1}} \right] + \bar{H}_{2}^{2} \Gamma \left[ \frac{2}{k_{2}} + 1, \left( \frac{\bar{H}_{rms}}{\bar{H}_{2}} \right)^{k_{2}} \right]}$$ (1.7)

Equation (1.7) together with the normalised continuity condition form a set of two (implicit) algebraic equations with three independent parameters. Now the Composite Weibull distribution contains three independent shape parameters, $k_{1}$, $k_{2}$, and $\bar{H}_{rms}$.

### 1.5 Data for calibration and validation

In order to relate the independent parameters of the Composite Weibull distribution to a certain wave field, measured wave height distributions, corresponding local water depths and spectral moments are necessary. The available shallow foreshore data, obtained from tests performed by WL | Delta Hydraulics, are presented in Table 1.

All tests were performed in the wave flume “Scheldegoot”. This wave flume has a wave maker capable of generating random waves. The wave generator is equipped with a device that absorbs incoming wave energy and so prevents reflection against the wave board and avoids undesired long-periodical waves. At the back of the wave flume a spending beach provides wave damping. The length of this wave flume is 50 m, the width 1.0 m and the depth 1.2 m. An overview of a test set-up of a shallow foreshore with slope 1:100 is given in Figure 4. During a test a number of waves is generated by the wave board at the beginning of the wave flume, using a standard JONSWAP-spectrum. Wave gauges are placed along the shallow foreshore to measure the surface elevation at different water depths. The water depth, $d$, refers to the still water depth. From the measured surface elevations at different water depths statistical wave heights (like $H_{1/3}$, $H_{rms}$ etc.), wave height distributions and spectral moments are determined.

![Figure 4: Shallow foreshore test set-up, slope 1:100.](image)

From the available data two data sets are selected. First a set of tests is selected to use for calibration of the independent parameters of the Composite Weibull distribution. Secondly, a set of tests is selected to validate the proposed pointmodel.
2 Calibration of the model

2.1 Exponents of the Composite Weibull distribution

The shape of the Composite Weibull distribution is determined by the shape parameters \( k_1 \), \( k_2 \), and \( \bar{H}_d \). In this section assumptions about the value of the exponents \( k_1 \) and \( k_2 \) are presented.

The value for \( k_1 \) is based on initial inspection of measured wave height distributions of the calibration data. Measured wave height distributions plotted on Rayleigh scale show a straight line for the lower wave heights. Therefore it is assumed that the first part of the wave height distribution, \( F(H) \), is Rayleigh shaped. This means that the exponent \( k_1 \) equals the exponent of the Rayleigh distribution (\( k_1 = 2 \)).

For further analysis use is made of the parameter:

\[
\psi = \frac{\sqrt{m_0}}{d} \quad (2.1)
\]

in which \( m_0 \) is the variance of the water surface elevation, which equals the zero-th spectral moment and \( d \) denotes the water depth. The properties of the normalised wave height distribution are assumed to be characterised mainly by a relative local wave height, such as \( H_d/d \). Here, instead of \( H_d/d \) the degree of saturation \( \psi \) is used to characterise the wave deformation process on shallow foreshores.

Initial visual fitting of the Composite Weibull distribution to wave height distributions measured on foreshores with different slopes yielded a relation between exponent \( k_2 \) and the degree of saturation as presented in Figure 5.

![Figure 5: Exponent \( k_2 \) versus degree of saturation, for different foreshore slopes](image)

Figure 5 shows a decrease of scatter in \( k_2 \) as the degree of saturation increases. High scatter for low \( \psi \)-values is understandable: When the Composite Weibull distribution is fitted to measured wave height distribution of wave fields with low degrees of saturation the distribution hardly deviates from the Rayleigh distribution. This results in fitting of the
second part of the Composite Weibull distribution to only a few measured wave heights with a low probability of exceedance. This explains the scatter. At the same time, it implies that the value of \( k_2 \) is not important for low \( \Psi \)-values. Since the higher degrees of saturation represent the area of interest, Figure 5 supports an assumption of a constant exponent \( k_2 \). A least squares optimisation method yields \( k_2 = 3.6 \).

Still, the average exponent \( k_2 \) on shallow foreshores (slope 1:20 in Figure 5) is slightly smaller than 3.6 and on milder foreshores \( k_2 \) is slightly higher than 3.6. So one could try to obtain a function for \( k_2 \) which decreases with increasing foreshore slope. Here, for all slopes \( k_2 = 3.6 \) is used.

### 2.2 Transitional wave height

The transitional wave height is the wave height at which the wave height distribution changes its shape. Due to the assumption of two constant exponents the shape of the Composite Weibull distribution totally depends on the normalised transitional wave height \( \bar{H}_r \): Equation 1.7 together with the normalised continuity condition form a set of two (implicit) algebraic equations with three independent parameters. By assuming \( k_1 = 2 \) and \( k_2 = 3.6 \) the Composite Weibull distribution is related to the only shape parameter left, \( \bar{H}_r \). Figure 6 shows how all characteristic normalised wave heights are related to \( \bar{H}_r \) (for example, \( x = 1/3 \) yields \( \bar{H}_{1/3} \)).

![Figure 6: Characteristic nondimensional wave heights related to the \( \bar{H}_r \)](image)

In Table B.1 the relations, as shown in Figure 6, are presented in tabular form. To use the preceding results one still needs to determine \( \bar{H}_r \) for given depth, foreshore slope and energy spectrum \( (m_0, T_p, \text{ etc.}) \). In the following sections \( H_r \) is related to the local water depth, foreshore slope and wave period. Then, in order to scale the wave height distribution to the wave field of concern, \( H_{rms} \) is related to the total wave energy.

### 2.2.1 Depth and foreshore slope

The transitional wave height is the wave height at which the wave height distribution abruptly changes its shape. The change in shape is a result of depth-induced breaking of the waves exceeding the transitional wave height. Therefore the following hypothesis is posed:

\[
H_r \sim d
\]  

(2.2)
With a fit procedure the transitional wave heights are estimated from measured wave height distributions on shallow foreshores with slope 1:20, 1:50, 1:100 and 1:250. The estimated transitional wave heights, nondimensionalised with the water depth, are plotted as a function of the degree of saturation in Figure 7 for each slope.

**slope 1:250**

![Graph of slope 1:250 showing estimated Hs/d against ψ = \sqrt{m_0/d}](image)

**slope 1:100**

![Graph of slope 1:100 showing estimated Hs/d against ψ = \sqrt{m_0/d}](image)

**slope 1:50**

![Graph of slope 1:50 showing estimated Hs/d against ψ = \sqrt{m_0/d}](image)

**slope 1:20**

![Graph of slope 1:20 showing estimated Hs/d against ψ = \sqrt{m_0/d}](image)

Figure 7: Nondimensional transitional wave height versus the degree of saturation for different slopes

In Figure 7 a proportionality between the transitional wave height and the water depth is observed for ψ>0.06 on the gentle foreshores. Although scatter in Hs/d for the steeper
foreshores is observed, the transitional wave height is assumed to be related to the depth via a slope dependent coefficient, $\gamma_r(\alpha)$:

$$H_r = \gamma_r(\alpha) \cdot d$$  \hspace{1cm} (2.3)

For every foreshore slope a coefficient $\gamma_r$ is determined from the average of the estimated values of $H_r/d$, represented by the solid lines in Figure 7. In determining the coefficients only values of $H_r/d$ with $\psi > 0.06$ are used. In Figure 8 $\gamma_r$ is plotted versus the foreshore slope.

![Figure 8: Coefficient $\gamma_r$ versus shallow foreshore slope.](image)

Figure 8 clearly shows that a steep slope leads to a high $\gamma_r$ and therefore a high transitional wave height. This means that for a steep slope less waves deviate from the Rayleigh distribution compared to a gentler slope. This is because part of the waves start breaking at a certain local water depth, but this has not yet resulted in a decrease of wave height at that location (the breaking process needs time/distance from the location at which breaking is initiated.) On a steep shallow foreshore more waves will therefore be Rayleigh distributed, which results in a higher transitional wave height.

Assume that a wave needs approximately one wave length from the point where breaking is initiated to deform and adapt to the changed conditions. Then, instead of the local water depth, the water depth one wave length (for instance $L_{op}$) seaward of the location can be used for the prediction of the transitional wave height at that location. The transitional wave height could be described with $H_r = \gamma d_L$, where $\gamma$ is a constant coefficient and $d_L$ is the water depth one wave length seaward of location. The dependency of $H_r$ on the foreshore slope would be much smaller. This is illustrated with an example.

Suppose one is interested in the wave height distribution at Location B for a foreshore with slope 1:20 and slope 1:100, see Figure 9. To calculate the transitional wave height, one can use Equation 2.3, where $\gamma_r$ is as presented by the dotted line in Figure 8. This yields $H_r = 0.64 - d$ for 1:20 and $H_r = 0.4 - d$ for 1:100. Approximately the same transitional wave heights are obtained when the transitional wave height is calculated at Location B for foreshore slopes 1:20 and 1:100, using $H_r = \gamma d_L$, where $\gamma$ is constant (here: $\gamma = 0.33$). This implies that the transitional wave height related to the water depth at one wave length seaward of the location of interest is less dependent on the foreshore slope. To some extent this approach would undermine the philosophy of a pointmodel, where the wave height distribution is assumed to depend mainly on local parameters, regardless of the history in...
deeper water. However, this clearly shows that there is a physical cause for the dependency of the wave height distribution on the foreshore slope.

![Diagram showing water depth and wave length]

Figure 9: Water depth and wave length seaward of the required location B for different foreshore slopes

Here, the transitional wave height is related to the local water depth via a slope dependent coefficient \( \gamma_r \) as described by Equation 2.3 and presented in Figure 8; \( \gamma_r \) is assumed to be related to the foreshore slope via \( \gamma_r = c_1 + c_2 \tan(\alpha) \). A least squares optimization method yields \( c_1 = 0.35 \) and \( c_2 = 5.8 \). With this slope dependent \( \gamma_r \) transitional wave height of the Composite Weibull distribution can be determined for given depth and foreshore slope.

### 2.2.2 Wave steepness

Figure 7 shows that relating the transitional wave height directly to the water depth overestimates the nondimensional transitional wave height for \( \psi < 0.06 \). The lower values of \( \psi \) represent relatively deep water. In relatively deep water the waves are more limited by the maximum wave steepness than by a limited water depth. By taking the wave steepness into account a better approximation of the measured transitional wave height could be achieved.

Miche (1944) defined a maximum wave height of a regular wave. In deep water the maximum wave height is determined by a maximum wave steepness. In shallow water depth-induced breaking dominates the deformation of the wave height distribution. Therefore in shallow water the maximum wave height is related to the water depth. When it is assumed that the transitional wave height is related to Miche's maximum wave height, the following relation could describe \( H_r \):

\[
H_r = \beta_r \times 0.14 \times L_c \tanh \left( \frac{2\pi d}{L_c} \right) \tag{2.4}
\]

in which \( \beta_r \) denotes a slope dependent coefficient (like \( \gamma_r \)) and \( L_c \) is the local wave length defined by:

\[
L_c = \frac{g T_c^2}{2\pi} \tanh \left( \frac{2\pi d}{L_c} \right) \tag{2.5}
\]

in which \( T_c \) is a characteristic wave period. Here, for \( T_c \) the wave period \( T_{0.2} = (m_0/m_2)^{1/2} \) is used (\( m_0 \) and \( m_2 \) are the local zero-th and second spectral moments).
The empirical coefficient $\beta_r$ in (2.4) is calibrated by comparing estimated and computed values of the transitional wave height. For every slope an optimum $\beta_r$ is determined, using a least squares optimisation method. In Figure 10 the coefficients $\beta_r$ are plotted versus the slope of the foreshore.

![Figure 10: Coefficients $\beta_r$ versus shallow foreshore slope](image)

One can assume that the $\beta_r$ is related to the foreshore slope via $\beta_r = c_3 + c_4 \tan(c)$. A least squares optimisation method yields $c_3=0.46$ and $c_4=9.25$. With this slope dependent $\beta_r$, the transitional wave height of the Composite Weibull distribution can be determined for given depth, foreshore slope and wave period $T_{0.2}$.

Including a wave steepness effect through a Miche-like expression gives an improved fit, but at the expense of an additional parameter: The characteristic wave period. A result of using $T_{0.2}$ as characteristic period in Equation 2.5 is that the forecasting function for the transitional wave height becomes sensitive to higher harmonics. This is because the second order spectral moment $m_2$ weights the energy at the higher frequencies more heavily than lower order spectral moments and therefore introduces a sensitivity to higher harmonics. Whether this sensitivity to higher harmonics is realistic is not verified in this study.

### 2.2.3 Wave energy

The transitional wave height is obtained, using one of the two functions for the transitional wave height described in the sections above. In order to obtain the normalised transitional wave height the root-mean-square wave height is related to the total wave energy $m_0$, scaling the transitional wave height to the wave field of concern.

For a sine-wave the mean square value of the water level elevation $\eta$ is related to the wave height $H$ via:

$$\overline{\eta^2} = \frac{1}{4} H^2$$  \hspace{1cm} (2.6)

Then the variance of the surface elevation of a random wave field, containing sine-waves only, can be assumed to be related to $H_{rms}$ irrespective of the wave height distribution via $m_0 = \frac{1}{4} H_{rms}^2$. To investigate the relation between $H_{rms}$ and the total wave energy on shallow foreshores, measured values of $H_{rms}/\sqrt{m_0}$ are plotted versus the degree of saturation $\psi$ for all slopes.
Figure 11: Ratio \( \frac{H_{rm}}{\sqrt{m_0}} \) measured on shallow foreshores versus the degree of saturation

Figure 11 shows that in shallow water observed values of \( \frac{H_{rm}}{\sqrt{m_0}} \) exceed the assumed value of \( \frac{H_{rm}}{\sqrt{m_0}} = \bar{v} \). This is ascribed to the fact that waves have an excess of crest height and shallow troughs in shallow water.

For real sea waves, with a broad banded frequency spectrum, the root-mean-square wave height in deep water (\( \bar{v} = 0 \)) equals \( H_{rm} = 2.69 \sqrt{m_0} \), see Section 1.2. Therefore it is assumed that

\[
\frac{H_{rm}}{\sqrt{m_0}} = 2.69 + \beta_{rm} \cdot \bar{v} \quad (2.7)
\]

in which \( \beta_{rm} \) is obtained by fitting (2.7) to the measured ratios of \( \frac{H_{rm}}{\sqrt{m_0}} \), as presented in Figure 11. A least squares optimisation method yields \( \beta_{rm} = 3.24 \).

### 2.3 Summary of calibrated pointmodel

For a given water depth, foreshore slope and energy spectrum of the waves a fully predictive pointmodel is obtained. The parameters of the Composite Weibull distribution are obtained from assumptions about the exponents of the distribution \( (k_1 = 2 \) and \( k_2 = 3.6) \), forecasting functions for the transitional wave height (2.3) or (2.4) and a relation between the root-mean-square wave height and the total wave energy (2.7).

Two variations of the pointmodel are used for further analysis: First, the pointmodel based on the Composite Weibull distribution where the transitional wave height depends on \( m_0 \) (Equation 2.7), the depth \( d \) and the foreshore slope \( \alpha \) (Equation 2.3), i.e. \( H_t = \gamma_\alpha(\alpha) \cdot d \) where \( \gamma_\alpha(\alpha) = 0.35 + 5.8 \cdot \tan(\alpha) \) and \( H_{rm} = (2.69 + 3.24 \sqrt{m_0/d}) \cdot \sqrt{m_0} \). Secondly, the pointmodel which also takes the influence of wave steepness into account where the transitional wave height depends on \( m_0 \), the depth \( d \), the foreshore slope \( \alpha \) and the wave period \( T_{0,2} \), i.e. \( H_t \) is determined, using Equation 2.4 and Equation 2.5 in which \( \beta_T = 0.46 + 9.25 \cdot \tan(\alpha) \) and \( H_{rm} = (2.69 + 3.24 \sqrt{m_0/d}) \cdot \sqrt{m_0} \). These two pointmodels are denoted in the following chapter as Pointmodel \(( m_0, d, \alpha \) and Pointmodel \(( m_0, d, \alpha, T_{0,2} \) respectively.
3 Validation of the model

3.1 General performance of the model

The validation of the pointmodel based on the Composite Weibull wave height distribution is carried out by comparing the pointmodel to existing wave height distribution models. The data used for validation are presented in Tables 2 to 6. Note that the data of shallow foreshores with slopes 1:20 and 1:50 are already used in the calibration process. Still, using these data in the validation provides extra insight in the performance of the proposed pointmodel. The validation data of foreshores with slope 1:30, 1:100 and 1:250 are not used in the calibration process.

First, in order to illustrate the performance of the proposed model, some measured wave height distributions and corresponding computed Composite Weibull distributions are presented. The following graphs show two wave height distributions measured on a shallow foreshore with slopes 1:20 and 1:100. From the corresponding measured zero-th spectral moment and water depth, the parameters $H_m$ and $H_{ms}$ have been determined, using (2.3) and (2.7). Hence, the Composite Weibull distribution is obtained.

![Graph showing Composite Weibull Wave height distribution](image)

Figure 12: Computed Composite Weibull Wave height distribution; slope 1:20, $m_p=2.1E-3$ m$^2$ and $d=0.3$ m
3.2 Comparison of wave height distribution models

An indicator of the overall approximation of the measured wave height distribution by a wave height distribution model is the normalised root-mean-square error $\varepsilon_{\text{rms}}$, defined as:

$$
\varepsilon_{\text{rms}} = \sqrt{\frac{1}{i} \sum_{i=1}^{i} \left( \frac{H_{N,\text{comp}}}{H_{N,\text{meas}}} - 1 \right)^2}
$$

in which $i$ denotes the number of tests used to determine the root-mean-square error and $H_N$ denotes a wave height with probability of exceedance equal to $1/N$. For five values of $N$ (i.e. the individual wave heights $H_{50\%}$, $H_{10\%}$, $H_{2\%}$, $H_{1\%}$ and $H_{0.1\%}$) the root-mean-square error is determined for three models: The Rayleigh distribution, the Modified Glukhovskiy distribution (Klopman 1996) and the Composite Weibull distribution (two versions: Pointmodel ($m_0$, $d$, $\alpha$) and Pointmodel ($m_0$, $d$, $\alpha$, $T_{50}$)). In Section 2.3 the two versions of the pointmodel are described. In the figures below the root-mean-square errors of the three models for the five individual wave heights are presented.

Figure 14: Root-mean-square error $H_{50\%}$ for various foreshore slopes
Figure 14 shows that both the Rayleigh distribution and the Glukhovskiy distribution do not give a good prediction of $H_{50\%}$ on shallow foreshores. Figure 2 shows that both distributions underestimate the lower wave heights. This underestimation results in a high root-mean-square error for $H_{50\%}$.

![Graph showing error $E_r$ for different wave height predictions](image)

Figure 15: Root-mean-square error $H_{10\%}$ for various foreshore slopes

Figure 15 shows that the Rayleigh distribution yields a relatively small error for $H_{10\%}$ for all slopes. Figure 2 shows that the Rayleigh distribution gives a relatively good prediction of $H_{10\%}$. This is observed for most of the shallow foreshore wave height distributions, hence the small root-mean-square error for the Rayleigh distribution for $H_{10\%}$. Figures 16, 17 and 18 present the root-mean-square errors for the three models for $H_{2\%}$, $H_{1\%}$ and $H_{0.1\%}$.

![Graph showing error $E_r$ for different wave height predictions](image)

Figure 16: Root-mean-square error $H_{2\%}$ for various foreshore slopes

![Graph showing error $E_r$ for different wave height predictions](image)

Figure 17: Root-mean-square error $H_{1\%}$ for various foreshore slopes
Figure 18: Root-mean-square error $H_{0.1\%}$ for various foreshore slopes.

Since the tests on shallow foreshores with for slopes 1:30 and 1:50 were performed with 500 wave or less, no reliable $H_{0.1\%}$ is obtained. Therefore no root-mean-square error for $H_{0.1\%}$ is presented in Figure 18 for foreshores with slope 1:30 and 1:50.

By averaging the root-mean-square errors for each foreshore slope an overall insight in the approximation of measured wave height distributions by the three models is obtained. In Figure 19 the average root-mean-square error is presented.

Figure 19: Average root-mean-square error versus foreshore slope.

Figure 19 shows that the proposed pointmodels yield the best approximation of the measured wave height distributions. The average reduction in root-mean-square error is about 60% compared to the Rayleigh distribution and about 40% compared to the Modified Glukhovskiy distribution. However, the average reduction in root-mean-square error for the extreme wave heights is less. For example, the averaged reduction in root-mean-square error for $H_{5\%}$ is about 50% compared to the Rayleigh distribution and about 25% compared to the Modified Glukhovskiy distribution.

The best approximation of the measured wave height distributions is obtained when the Miche-like transitional wave height forecasting function (2.4) is used in the Composite Weibull distribution model: Pointmodel $(m_0, d, \alpha, T_{0.2})$. However, as stated before, this forecasting function has the disadvantage of a fourth input parameter $T_{0.2}$. Furthermore the improvements are relatively small compared to the pointmodel with a transitional wave height which is not related to the wave period: Pointmodel $(m_0, d, \alpha)$. Therefore, there is a slight preference to the simpler forecasting function for the transitional wave height (2.3) without the effect of the parameter $T_{0.2}$. Examples of the application of the proposed pointmodel based on the parameters $m_0$, $d$ and $\alpha$ are given in the following section.
3.3 Application of the proposed pointmodel

3.3.1 Recipe

The validation has shown that the Composite Weibull distribution with physically based parameterisations can be used as a predictive model. In this section, a recipe for the application of the Composite Weibull distribution based on the parameters $m_0$, $d$ and $\alpha$ is presented (in Appendix B the mathematics behind this recipe are described):

1. Given slope $\alpha$ and water depth $d$, calculate $H_r = \gamma_r \cdot d$ with $\gamma_r = 0.35 + 5.8 \tan(\alpha)$.

2. Given the total wave energy $m_0$, calculate $H_{rms} = (2.69 + 3.24 \cdot \sqrt{m_0/d}) \cdot \sqrt{m_0}$.

3. Calculate $\tilde{H}_r = H_r / H_{rms}$.

4. Read for calculated $\tilde{H}_r$ the corresponding value of the desired normalised wave height $\bar{H}_x$ in Table B.1.

5. Calculate $H_x = H_{rms} \cdot \bar{H}_x$

In the following section measurements are used to illustrate this recipe.

3.3.2 Example

The wave height distribution, presented on Rayleigh scale in Figure 21, is measured on a shallow foreshore with slope 1:250. The individual wave heights were measured at a water depth $d = 0.15$ m. At that depth the measured water surface elevation variance $m_0 = 5.37E-4$ m$^3$ ($\psi = 0.154$) and the significant wave height $H_{1/3} = 0.097$ m. Using the recipe, as described in the previous section, $H_{0.15\%}$ and $H_{1/3}$ are predicted.

![Figure 20: Measured individual wave heights: slope 1:250, $m_0=5.37E-4$ m$^3$, $d=0.15$, $\psi=0.154$.](image-url)
$H_{0.1\%}$ is determined as follows:

1. For $\alpha=1.250$ and $d=0.15$ m: $\gamma_r=0.373$ and $H_r=\gamma_r \cdot d = 0.056$ m.

2. For $d=0.15$ m and $m_0=5.37E-4m^2$, $H_{ms}=0.074$ m.

3. $\tilde{H}_r = H_r/H_{ms}= 0.757$.

4. Table B.1 (Appendix B) yields $\tilde{H}_{0.1\%} \approx 1.83$.

5. $H_{0.1\%} = H_{ms} \cdot \tilde{H}_{0.1\%} = 0.135$ m. (Measured $H_{0.1\%} = 0.133$ m)

To calculate the significant wave height $H_{1/3}$:

2. Table B.1 yields $\tilde{H}_{1/3} \approx 1.29$.

3. $H_{1/3} = H_{ms} \cdot \tilde{H}_{1/3} = 0.096$ m. (Measured $H_{1/3} = 0.097$ m.)

### 3.4 Range of validity

#### 3.4.1 Ranges used in validation

The point model based on the Composite Weibull distribution has been validated for the following ranges and bottom topography:

1. Range in degree of saturation: $0<\psi<0.15$ (relative wave height $0<H_{ms}/d<0.6$)
2. Range in foreshore slope: $1:2.50<\tan(\alpha)<1:20$
3. Simple bottom topography: A foreshore with straight and parallel depth contours

This section discusses whether the proposed point model can still be used when the foreshore slope or the relative wave height is not in the range for which the model is validated and the foreshore has a more complex bottom topography.

#### Degree of saturation

The proposed model is validated for wave fields obeying: $0<\psi<0.15$, i.e. relative wave height $0<H_{ms}/d<0.6$. For $\psi<0.06$ the model performs just as good as the existing models, for $0.06<\psi<0.15$ the proposed model performs better. Therefore it is concluded that the proposed model can be applied on shallow foreshores for $0<\psi<0.15$.

Few shallow foreshore tests with a degree of saturation exceeding 0.15 are available. This is understandable since on gentle shallow foreshores the wave energy dissipates before extreme degrees of saturation can occur. On steeper foreshores (slope 1:20) the degree of saturation can exceed 0.6. This is caused by the fact that the waves need time/distance from the location at which breaking is initiated to really deform and dissipate energy. On a steep
foreshore this time and distance is limited. For \( \psi > 0.15 \) (\( H_{m0}/d > 0.6 \)) the proposed point model is expected to yield a good description of the wave height distribution, though this needs to be verified for wave energy spectra that deviate considerably from the energy spectra applied in the model tests.

**Foreshore slope**

The foreshore slope influences the wave height distribution via the transitional wave height. In Section 2.2.1 it is assumed that the transitional wave height is related to the water depth via a slope dependent coefficient, see Equation 2.3. However, Figure 7 shows that the scatter increases for steeper foreshore slopes. This implies that the assumed proportionality between the transitional wave height and the water depth becomes less appropriate as the foreshore slope increases. Therefore care must be taken when applying the proposed model on foreshores with slopes steeper than 1:20. Figure 19 shows that the overall performance of the Rayleigh distribution improves for steeper foreshore slopes. For \( \tan(\alpha) > 1:20 \) the Rayleigh distribution can be applied which provides a somewhat conservative prediction of the wave height distribution.

On foreshore slopes gentler than 1:250 it is anticipated that the proposed point model will yield good predictions of the wave height distribution. However, care must be taken when applying the point model on horizontal foreshores, since local wave growth due to wind becomes relatively important on horizontal foreshores. This is a process that is not included in the applied model tests; the effect of wind might require an additional parameter in the point model.

**3.4.2 Shape of the foreshore**

The philosophy behind a point model is based on the assumption that the wave height distribution is mainly determined by the local parameters of the wave field and the water depth regardless of the history of the wave fields in deeper water. This assumption proves to be valid for shallow water with a reasonably simple bottom topography. The data, that are used in the validation of the proposed pointmodel are obtained from measurement on foreshores with straight and parallel depth contours. In coastal engineering practice often shallow bars are found to interrupt the gradual decrease in depth on a shallow foreshores, see Figure 21. The proposed prediction of wave height distributions for such a bottom topography is as follows:

- Seaward of the bar the wave height distribution can be predicted with the proposed model, in which an average foreshore slope must be used, for instance the average slope between two wave lengths, \( L_{opt} \), seaward of the required location and the location itself.
- Landward of the bar the waves propagate into deeper water. Here, as discussed in the following section, the wave height distribution gradually reforms to the Rayleigh distribution. This is induced by the increasing water depth which results in a decrease in degree of saturation. The reforming of the wave height distribution to the Rayleigh distribution can be described with both the proposed point model and the Modified Glukhovsky distribution. However, for a quantitative evaluation of wave height distributions behind a shallow bar additional tests are necessary. For a conservative prediction of the wave height distribution behind a shallow bar the Rayleigh distribution can be used.
- When the water depth starts decreasing again, the waves are once again subjected to depth-induced breaking. The wave height distribution deviates from the Rayleigh
distribution and it is anticipated that the proposed pointmodel based on the Composite Weibull distribution will yield a proper description of the wave height distribution.

Figure 22 shows where the proposed model is anticipated to yield a good prediction of the wave height distribution when waves encounter a shallow bar, while propagating over a shallow foreshore.

Figure 21: Pointmodels on a shallow foreshore with a shallow bar.

**Wave height distributions behind a bar**

To investigate the wave height distributions behind a shallow bar some tests, performed by Kant (1998), are analysed. Kant (1998) measured wave height distributions at several locations in a wave flume with a shallow bar. A schematised impression of the test set-up is presented in Figure 23.

Figure 22: Schematised impression of the test set-up used by Kant (1998)

To investigate the change of the wave height distributions, as the waves propagate into deeper water behind the bar, measured ratios of $H_{110}/H_{1/3}$ are plotted versus the number of wave lengths behind the bar in Figure 24. $L_{0.2}$ is the wave length based on the local $T_{0.2}$ as given by Equation 2.5. The outline of the shallow bar is indicated by the solid line in Figure 24. At approximately two wave lengths behind the bar the water depths is constant.
Figure 23: Reformation to the Rayleigh distribution of wave height distributions measured behind a shallow bar.

Figure 24 shows that the wave height distributions, that strongly deviate from the Rayleigh distribution \( H_{1/3}/H_{1/3} \neq 1.52 \) at the end of the shallow bar, gradually reform to the Rayleigh distribution \( H_{1/3}/H_{1/3} = 1.52 \), the dotted line in Figure 24.

Behind the bar the water depth increases, depth-induced breaking ceases and therefore the degree of saturation decreases. The decrease in degree of saturation behind a bar is shown in Figure 25.

3.5 Summary of validated Pointmodel

The validation shows that the proposed pointmodel yields accurate predictions of the wave height distributions on shallow foreshores. The pointmodel performs well for a relatively simple bottom topography in the following ranges:

1. \( 0 < \psi < 0.15 \) (\( 0 < H_{sobs}/d < 0.6 \)). For \( H_{sobs}/d > 0.6 \) the proposed model is anticipated to yield the best predictions of wave heights on shallow foreshores.
2. \( 1:250 < \tan(\alpha) < 1:20 \). For \( \tan(\alpha) < 1:250 \) the proposed model is anticipated to yield the best predictions of wave heights on shallow foreshores and for \( \tan(\alpha) > 1:20 \) it is proposed to use the Rayleigh distribution.
4 Conclusions

On shallow foreshores waves are subjected to depth-induced breaking, particularly the highest waves. This results in a profound change in the shape of the wave height distribution. The distribution of the lower waves in a wave field is still Rayleigh shaped, since the smaller waves propagate relatively undisturbed towards the coast. The distribution of the higher waves deviates considerably from the Rayleigh distribution, since the higher waves in a wave field break. This change in shape of shallow foreshore wave height distributions is not well described by existing local wave height distribution models.

In this report the Composite Weibull distribution (Groenendijk, 1998) is extended to describe the wave height distributions on shallow foreshores for different foreshore slopes. The main feature of the Composite Weibull distribution is the fact that the distribution is composed of two Weibull distributions with different exponents. The domain of the Composite Weibull distribution is split into two parts by a transitional wave height. This concept was found to yield a good description of observed shallow foreshore wave height distributions.

With empirically estimated values of the two exponents of the distribution and a continuity condition, the Composite Weibull distribution depends on one parameter only, the normalised transitional wave height. Forecasting functions for the transitional wave height have been derived. The simplest forecasting function for the transitional wave height is a slope dependent breaker criterion where the transitional wave height is related to the water depth (2.3). This forecasting function, together with a function for the root-mean-square wave height (2.7), yields a model that predicts the local wave height distribution on shallow foreshores for a given local water depth, foreshore slope and total wave energy.

The validation proves that the model based on Composite Weibull distribution yields better approximations of measured wave height distributions on shallow foreshores than existing distributions.
Acknowledgements

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References


Tables
# Tables

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Table 1: Available shallow foreshore data

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Table 3: Validation data, slope 1:30

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Table 4: Validation data, slope 1:50
### Table 5: Validation data, slope 1:100

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<th>$H_{rs}$ (m)</th>
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Appendix A: Composite Weibull distribution
Appendix A: Composite Weibull distribution

A.1 Composite Weibull distribution

In this appendix expressions for characteristic wave heights of the Composite Weibull distribution are derived, i.e. expressions for the wave height with an exceedance probability of $1/N (H_n)$, the significant wave height ($H_{1/3}$) and the root mean square wave height ($H_{rms}$). The cumulative probability function of the Composite Weibull distribution is given by:

$$F(H) = \Pr\{H \leq H\} = \begin{cases} F_1(H) = 1 - \exp\left[-\left(\frac{H}{H_1}\right)^{k_1}\right] & H \leq H_r \\ F_2(H) = 1 - \exp\left[-\left(\frac{H}{H_2}\right)^{k_2}\right] & H > H_r \end{cases}$$ (A.1)

In order to obtain a continuous distribution the Composite Weibull distribution must satisfy the following condition:

$$F_1(H_r) = F_2(H_r)$$ (A.2)

In Figure A.1 the Composite Weibull distribution is presented on Rayleigh scale.

![Figure A.1: The Composite Weibull distribution.](image)

Differentiation of the cumulative probability function with respect to the wave height $H$ yields the probability density function.

$$f(H) = \frac{d}{dH} F(H) = \begin{cases} f_1(H) = \frac{k_1}{H_1^{k_1}} H^{k_1-1} \exp\left[-\left(\frac{H}{H_1}\right)^{k_1}\right] & H \leq H_r \\ f_2(H) = \frac{k_2}{H_2^{k_2}} H^{k_2-1} \exp\left[-\left(\frac{H}{H_2}\right)^{k_2}\right] & H > H_r \end{cases}$$ (A.3)
A.2 Expressions for characteristic wave heights

A.2.1 Wave height with exceedance probability of 1/N

The wave height with an exceedance probability of 1/N (N>1) is denoted by $H_N$. In coastal engineering practice the wave height with an exceedance probability of 1/N is often denoted by $H_{1/N,100\%}$ (for example, the wave height with an exceedance probability of 1/100 is denoted by $H_{1/100}$). However, in this appendix $H_N$ is used instead of $H_{1/N,100\%}$, in line with the definition of $H_N$. Note that $H_1$ and $H_2$ are the scale parameters of the Composite Weibull distribution and do not denote the wave height with exceedance probabilities 1 and 0.5.

The Composite Weibull distribution is a composition of two separate distributions. Therefore, the determination of $H_N$ depends on the fact whether $H_N$ exceeds $H_N$. First, one assumes that $H_N$ is smaller than the transitional wave height. This means that $H_N$ is determined by the first part of the Composite Weibull distribution $F_1(H)$. For this case $H_N$ is denoted by $H_{N,1}$ and is determined by:

$$Q_{H_1}(H) = \Pr_1\{H \geq H_1\} = \frac{1}{N} = \exp\left(-\left(\frac{H_{N,1}}{H_1}\right)^{k_1}\right)$$  \hspace{1cm} (A.4)

Yielding:

$$H_{N,1} = H_1 \left[\ln(N)\right]^{\frac{1}{k_1}}$$  \hspace{1cm} (A.5)

When $H_{N,1}$ indeed is smaller than $H_N$, $H_N = H_{N,1}$. In Figure A.2 a Composite Weibull wave height distribution with $H_{N,1} < H_N$ is shown.

![Composite Weibull wave height distribution with H_{N,1} < H_N](image)

Figure A.2: Composite Weibull wave height distribution with $H_{N,1} < H_N$.

When $H_{N,1}$ exceeds $H_N$, $H_N$ is determined by:

$$Q_{H_2}(H) = \Pr_2\{H \geq H_2\} = \frac{1}{N} = \exp\left(-\left(\frac{H_N}{H_2}\right)^{k_2}\right)$$  \hspace{1cm} (A.6)

yielding:

$$H_N = H_2 \left[\ln(N)\right]^{\frac{1}{k_2}}$$  \hspace{1cm} (A.7)

Figure A.3 shows a Composite Weibull wave height distribution for $H_N < H_{N,1}$.
A.2.2 Mean of the highest 1/N-part

In coastal engineering the mean of the highest 1/N-part of the wave heights in a wave field is often used to characterise that wave field (N≥1). For example, the significant wave height \( H_{1/3} \) is mean of the highest 1/3-part of the wave heights in a wave field. \( H_{1/N} \) is defined by:

\[
H_{1/N} = \frac{\int_{1/N}^{\infty} H f(H) dH}{\int_{1/N}^{\infty} f(H) dH} = \frac{\int_{H_N}^{\infty} H f(H) dH}{\int_{H_N}^{\infty} f(H) dH} = N \int_{H_N}^{\infty} H f(H) dH
\]  

(A.8)

The determination of \( H_{1/N} \) also depends on the fact whether \( H_N \) exceeds \( H_r \).

Transitional wave height exceeds \( H_{N,1} \)

When \( H_r \) exceeds \( H_{N,1} \), \( H_N \) is given by (A.5) and \( H_{1/N} \) is evaluated via

\[
H_{1/N} = N \int_{H_N}^{H_r} f_1(H) dH + N \int_{H_r}^{\infty} f_2(H) dH
\]  

(A.9)

The first integral can be rewritten into:

\[
\frac{N}{H_N} \int_{H_N}^{H_r} f_1(H) dH = N \int_{H_N}^{H_r} f_2(H) dH - N \int_{H_r}^{\infty} f_1(H) dH
\]  

(A.10)

yielding:

\[
H_{1/N} = N \int_{H_N}^{H_r} f_1(H) dH - N \int_{H_r}^{\infty} f_1(H) dH + N \int_{H_r}^{\infty} f_2(H) dH
\]  

(A.11)

Substitution of the probability density function of the Composite Weibull distribution (A.3) in (A.11) yields:
\[ H_{i/N} = N \int_{H_i}^{H} \frac{H^{k_i}}{H^{k_i}} H^{k_i-1} \exp \left[ -\frac{H^{k_i}}{H_i} \right] dH - N \int_{H_i}^{H} \frac{H^{k_i}}{H^{k_i}} H^{k_i-1} \exp \left[ -\frac{H^{k_i}}{H_i} \right] dH + N \int_{H_i}^{H} \frac{H^{k_i}}{H^{k_i}} H^{k_i-1} \exp \left[ -\frac{H^{k_i}}{H_i} \right] dH \]  

Substitute

\[ t = \left( \frac{H}{H_i} \right)^{k_i} \quad \Rightarrow \quad H = H_i t^{k_i} \quad \Rightarrow \quad dH = H_i^{k_i-1} dt \quad \text{for} \quad i = 1, 2. \]  

(A.13)

When this transformation is applied to (A.12) the following equation is obtained:

\[ H_{i/N} = NH_i \int_{t_i}^{t} t^{k_i} \exp [t_i - t] dt - NH_i \int_{t_i}^{t} t^{k_i} \exp [-t] dt + NH_2 \int_{t_i}^{t} t^{k_i} \exp [-t] dt \]  

(A.14)

With the incomplete gamma functions described in Section A.3, (A.14) is rewritten into :

\[ H_{i/N} = NH_i \left[ \Gamma \left( \frac{1}{k_i} + 1, \ln(N) \right) - \Gamma \left( \frac{1}{k_i} + 1, \left( \frac{H_i}{H_i} \right)^{k_i} \right) \right] + NH_2 \left[ \frac{1}{k_2} + 1, \left( \frac{H_2}{H_2} \right)^{k_2} \right] \]  

(A.15)

**Transitional wave height exceeded by \( H_{N,1} \)**

When \( H_{i} \) is exceeded by \( H_{N,1}, H_{N} \) is given by (A.7) and \( H_{i/N} \) is evaluated via

\[ H_{i/N} = N \int_{H_i}^{H} f_2(H) dH \]  

(A.16)

When the probability density function \( f_2(H) \) from the Composite Weibull distribution is substituted, the following equation is obtained:

\[ H_{i/N} = N \int_{H_i}^{H} \frac{H^{k_2}}{H_2^{k_2}} H^{k_2-1} \exp \left[ -\left( \frac{H}{H_2} \right)^{k_2} \right] dH \]  

(A.17)

which can be transformed with (A.13) into:
\[ H_{U/N} = N H_2 \int_0^\infty \left( \frac{H_2}{H_1} \right)^{k_2} \exp[-t] \, dt \]  
(A.18)

With the incomplete gamma functions (Section A.3) equation (A.18) is rewritten into:

\[ H_{U/N} = N H_2 \Gamma \left[ \frac{1}{k_2} + 1, \left( \frac{H_2}{H_1} \right)^{k_2} \right] \]  
(A.19)

With (A.7) the following equation is obtained:

\[ H_{U/N} = N H_2 \Gamma \left[ \frac{1}{k_2} + 1, \ln(N) \right] \]  
(A.20)

### A.2.3 Root-mean-square wave height

The root mean-square-wave height of the Composite Weibull distribution is

\[ H_{rms} = \sqrt{\int_0^{H_r} H^2 f_r(H) \, dH + \int_{H_r}^{\infty} H^2 f_s(H) \, dH} \]  
(A.21)

\( H_{rms} \) is expressed in terms of the parameters of the Composite Weibull distribution. Using the transformation (A.13) and the gamma functions described in Section A.3, (A.21) is rewritten into:

\[ H_{rms} = \sqrt{H_1^2 \Gamma \left[ \frac{2}{k_1} + 1, \left( \frac{H_1}{H_1} \right)^{k_1} \right] + H_2^2 \Gamma \left[ \frac{2}{k_2} + 1, \left( \frac{H_2}{H_1} \right)^{k_2} \right]} \]  
(A.22)

Normalising all wave heights with \( H_{rms} \):

\[ \bar{H}_s = \frac{H_s}{H_{rms}} \]  
(A.23)

in which \( H_s \) denotes a characteristic wave height, like \( H_u \), \( H_{1/3} \) or \( H_{15s} \), the normalised \( H_{rms} \) is obtained:

\[ \bar{H}_{rms} = 1 = \sqrt{\bar{H}_1^2 \Gamma \left[ \frac{2}{k_1} + 1, \left( \frac{\bar{H}_1}{\bar{H}_1} \right)^{k_1} \right] + \bar{H}_2^2 \Gamma \left[ \frac{2}{k_2} + 1, \left( \frac{\bar{H}_2}{\bar{H}_1} \right)^{k_2} \right]} \]  
(A.24)

### A.3 Incomplete gamma functions

The gamma function is defined by:

\[ \Gamma(a) = \int_0^\infty t^{a-1} \exp[-t] \, dt \quad 0 < t < \infty \]  
(A.25)
This gamma function is a generalisation of the factorial function. The gamma function has the following properties.

\[
\Gamma(a+1) = a \Gamma(a) \quad \text{(A.26)}
\]

\[
\Gamma(a+1) = a! \quad \text{for} \quad a = 1, 2, \ldots, n. \quad \text{(A.27)}
\]

The incomplete gamma function is defined by (Abramowitz and Stegun, 1965):

\[
P(a, x) = \frac{\gamma(a, x)}{\Gamma(a)} = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} \exp[-t] \, dt \quad (a > 0) \quad \text{(A.28)}
\]

in which \( \Gamma(a) \) is the complete gamma function. Therefore \( \gamma(a, x) \) is defined by:

\[
\gamma(a, x) = \int_0^x t^{a-1} \exp[-t] \, dt \quad \text{(A.29)}
\]

The complement of \( P(a, x) \) is defined by:

\[
Q(a, x) = 1 - P(a, x) = \frac{\Gamma(a, x)}{\Gamma(a)} = \frac{1}{\Gamma(a)} \int_x^\infty t^{a-1} \exp[-t] \, dt \quad \text{(A.30)}
\]

yielding

\[
\Gamma(a, x) = \int_x^\infty t^{a-1} \exp[-t] \, dt \quad \text{(A.31)}
\]
Appendix B: Recipe
Appendix B: Recipe

In this appendix a recipe to calculate characteristic wave heights with the pointmodel based on the Composite Weibull distribution is given. Firstly, the recipe together with the applied mathematical formulations is presented. Secondly, a look-up table is presented. Using this table, characteristic wave heights are obtained quickly.

Given the total wave energy $m_0$, the water depth $d$ and the foreshore slope $\alpha$, a wave height with an exceedance probability $1/N$ (here $H_{1/N}$, $H_{1/4}$ and $H_{0.1/4}$) or the mean of the highest $1/N$-part of the waves ($H_{1/10}$, here $H_{1/3}$ and $H_{1/10}$), is obtained as follows:

1. Calculate $H_{ir} = \gamma_{ir} \cdot d$ with $\gamma_{ir} = 0.35 + 5.8 \tan(\alpha)$.
2. Calculate $H_{rms} = (2.69 + 3.24 \cdot \sqrt{m_0/d}) \cdot \sqrt{m_0}$.
3. Calculate $\bar{H}_r = H_{ir} / H_{rms}$

4a. The normalised continuity condition is rewritten into:

$$
\left( \frac{\bar{H}_r}{\bar{H}} \right)^{k_1} = \left( \frac{\bar{H}_r}{\bar{H}} \right)^{k_2}
$$

(B.1)

Equation B.1 together with the constraint that the normalised $H_{rms}$ of the Composite Weibull distribution has to equal unity:

$$
\bar{H}_{rms} = 1 = \sqrt{\frac{\bar{H}_1}{\bar{H}}_1 \cdot \frac{2}{k_1} + 1, \left( \frac{\bar{H}_r}{\bar{H}_1} \right)^{k_1}} + \frac{\bar{H}_2^2}{\bar{H}_2} \cdot \frac{2}{k_2} + 1, \left( \frac{\bar{H}_r}{\bar{H}_2} \right)^{k_2}
$$

(B.2)

form a set of two (implicit) algebraic equations with three independent parameters. In Section 2.1 assumptions about the exponents of the Composite Weibull distribution are proposed: $k_1 = 2$ and $k_2 = 3.6$. For $\bar{H}_r$ (as calculated in step 3), $k_1 = 2$ and $k_2 = 3.6$, values for $\bar{H}_1$ and $\bar{H}_2$ are calculated from (B.1) and (B.2).

4b. Calculate $\bar{H}_{N,1}$ using (see Appendix A.2.1):

$$
\bar{H}_{N,1} = \bar{H}_1 \left[ \ln(N) \right]^{\frac{1}{k_1}}
$$

(B.3)

If $\bar{H}_{N,1} < \bar{H}_r$ then $\bar{H}_N$ equals $\bar{H}_{N,1}$:

$$
\bar{H}_N = \bar{H}_{N,1} = \bar{H}_1 \left[ \ln(N) \right]^{\frac{1}{k_1}}
$$

(B.4)

Else for $\bar{H}_{N,1} > \bar{H}_r$, $\bar{H}_N$ is obtained via
\[ \bar{H}_w = \bar{H}_2 \left[ \ln(N) \right]^{\frac{1}{k_2}} \]  

(B.5)

The mean of the highest 1/N-part of the waves \( \bar{H}_{1/N} \), is obtained as follows:

4c. Using \( \bar{H}_{N,3} \) as obtained in step 4b,

**If** \( \bar{H}_{N,3} < \bar{H}_p \) **then** \( \bar{H}_{1/N} \) **is obtained via** (see Appendix A.2.2):

\[ \bar{H}_{1/N} = N \bar{H}_2 \left[ \Gamma \left( \frac{1}{k_2} + 1, \ln(N) \right) - \Gamma \left( \frac{1}{k_1} + 1, \left( \frac{\bar{H}_p}{\overline{\bar{H}}_p} \right)^{k_1} \right) \right] + N \bar{H}_2 \Gamma \left[ \frac{1}{k_2} + 1, \left( \frac{\bar{H}_p}{\overline{\bar{H}}_p} \right)^{k_2} \right] \]  

(B.6)

**Else for** \( \bar{H}_{N,3} > \bar{H}_p \), \( \bar{H}_{1/N} \) **is obtained via**

\[ \bar{H}_{1/N} = N \bar{H}_2 \left[ \frac{1}{k_2} + 1, \ln(N) \right] \]  

(B.7)

Steps 4a to 4c are followed to obtain Table B.1. This enables one to quickly obtain a certain characteristic wave height \( \bar{H}_p \) for calculated \( \bar{H}_p \).

5. Calculate \( H_s = H_{rms} \cdot \bar{H}_s \), where \( \bar{H}_s \) is \( \bar{H}_{1/3}, \bar{H}_{1/10}, \bar{H}_{2/4}, \bar{H}_{1/6} \) etc..
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WL | delft hydraulics

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2600 MH Delft
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