Designing the perfect tender

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Designing the perfect tender

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Abstract

This thesis studies parameters that are assumed to influence the outcomes of tender procedures. These parameters are learning, transaction costs, repetition, complex bids, uncertainty, continued work and irrationality. Government agencies have some degree of freedom when they use tender procedures. They can use this freedom to design the perfect tender, if they know how. This thesis shows that certain parameters can have a significant impact on the outcome of tenders. Learning and transaction costs have a positive impact on the outcome of tenders. Uncertainty, continued work and irrationality can have a negative impact and should be minimized where possible.
Wow, it’s finally coming to and end. The past year has been interesting and most definitely not easy. People may warn you for the rocky road of performing a thesis project, but I never really understood them until a couple months in. Such a large independent project is very different from the guided coursework in previous years. I have learned so much this year.

My thesis project was performed at Grontmij, with supervision as well from TU Delft. At Grontmij I got to be a part of team projects, full of experts on tender management. Not knowing anything about tenders beforehand, this changed quickly. Combining game theory with tender theory and practice was an interesting direction, since there was very little previous work.

This thesis would not exist today without all the help I received along the way. First I would like to thank my thesis advisor, Mathijs de Weerdt, for pointing me in the direction of this assignment and for all of his help during the past year. I also want to thank my supervisors at Grontmij, Coos van Buuren and Stephan Laaper, for their feedback and help in steering me in the right direction. Last but not least, thanks to my colleagues at Grontmij in team Projects for teaching me about tenders and for the great atmosphere to work in.

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Chapter 1

Introduction

This thesis studies government procurement using concepts from game theory and mechanism design. Laws oblige government agencies to use tender procedures in procurement. Those laws restrict government agencies in procurement, because tender procedures must be adhered strictly. Contractors want to win tenders in order to generate business and make money. Since winning is involved a tender can be seen as a game. Using concepts from game theory I analyze tender outcomes.

In the public sector every agency must justify its spending habits. For larger and more expensive projects a tender helps this justification. Tenders must be public, transparent and non-discriminating. There is an open invitation for suppliers to compete and every supplier is treated equally. Winner determination must be a clear process that follows the predetermined rules.

The outcome of tenders is relevant for all involved in a tender: the buyer, the sellers and the general public. Government agencies are the buyer in a tender, they want the best possible contractor to win at the lowest possible price. Contractors are the sellers, they can focus on different objectives to win a tender. Different goals are to generate business, gain high profits or improve their knowledge and experience. Contractors who gain relevant experience will benefit when competing for future assignments. The general public prefers good tender outcomes where good quality works are delivered for reasonable prices. Essentially the public pays for all public spending and their money must be spent wisely.

If one knows which parameter has the most influence on the outcome of a tender, one can design a tender with the exact combination of desired parameters. This tender will yield the best possible outcome. This work considers the following parameters: learning, transaction costs, repetition, complex bidding, uncertainty, continued work and irrationality. The effect of the different parameters on the tender process can be varied in practice. A government agency designs a tender contract and is able to choose which parameters are more prominent.

My contributions are towards both theory and practice. The main contribution of this work is identifying key parameters and their impact on the tender process. This thesis applies game theory to tenders and it shows promising directions in which game theory can be applied to tenders in real life. Knowledge of game theory can help to improve tender
1.1 Research questions

The focus in this thesis is on the contract awarding round in a restricted tender and on the behavior of contractors during this round. The exact rules for winner determination, given a set of bids, can change per tender. This is because the most economically advantageous tender leaves room to define the quality of a project. Weights for every aspect (e.g. price, risk management, sustainability, planning) can vary. The approach is to use concepts from game theory and mechanism design in order to develop a game which models the award round. This game model will be used to analyze the best winner of a tender. The outcome will benefit both suppliers and governments. Government agencies benefit because the winning outcome will be the best possible outcome. The suppliers benefit because they can learn which strategy helps to win a tender. This all leads to the following main question:

**Main question:** How can the contract awarding round from tender procedures be modeled best using concepts from game theory and how can this model help improve tenders such that the best possible outcomes occur?

In order to answer this main question and create the requested model, a number of sub questions is formulated. These sub questions further specify the research direction and are mentioned below. The main question itself is answered in the conclusion in Chapter 7.

**Sub question 1: What literature on game theory and mechanism design related to tenders has been published?**

Game theoretical and mechanism design approaches will be used, so first these theories must be studied in a literature survey. Studying the literature helps to find relevant concepts from these two fields. Chapter 2 addresses this sub question, it discusses game theory, mechanism design and mechanisms closely related to tenders.

**Sub question 2: What are outside limitations on the tender process and how do they influence the model?**

Depending on the size of a project, different laws are applicable. Under all those different laws, the tender process should still be public, transparent and non-discriminating. Regulations are outside influences that determine the course of a tender process. They are discussed in Chapter 3. Not all rules are core properties of the tender process, but they still influence tenders in practice.

**Sub question 3: What are the core properties of the tender process?**

At first a very simple model is created to model the contract awarding round. Incrementally this model is improved and extended to better reflect reality. Key factors of the tender pro-
cess must be identified. Examples of tender properties are the award criterion, the available information, the number of competitors and their strategies. The selected key parameters of the tender process are discussed in Chapter 4, in Section 4.2 specifically.

Sub question 4: How do companies decide on a bidding strategy and which strategies work best?

In a lowest-price tender, contractors should weigh the extra profit of a high price against the extra winning chance of a lower price. When contract awarding is based on both price and quality, different strategies can be applied by contractors when they compose their offer. A supplier can choose to give the minimum quality for the lowest price, or vice versa provide a product of high quality for a higher price. Another choice is to not participate in a tender. This sub question regards the bidding behavior of contractors and how they decide on a strategy. How can the model be used to define the behavior of involved parties? What is the relation between strategies and the chance of winning a contract? An analysis of tender model outcomes can be found in Chapters 4.3 and 5.

Sub question 5: How can a model adhering to the above criteria be empirically evaluated and its results validated?

The aim is to validate the model using empirical evaluation techniques. Experiments help to analyze a tender model. After modeling a tender and empirically evaluating this model the validation of results is an important step. All the experiments from Chapters 4.3, 5 and 6 together form the evaluation of the tender model. Further validation is discussed in Section 7.3.
Chapter 2

Games

2.1 Game theory

The goal of this Chapter is to provide background information on game theory. Mechanism design is a branch of game theory and is discussed in Section 2.2, along with mechanisms related to tenders. Game theory is about playing games, more specifically, it studies strategic decision making. This section starts with a short discussion of the prisoners’ dilemma, a well-known example in game theory. Next, different concepts from game theory are discussed in order to get some basic knowledge of the subject.

2.1.1 Prisoners’ dilemma

The Prisoners’ dilemma is a commonly known example in game theory. In this game two men have been taken prisoner after committing a major crime. They are separated and each prisoner has two options: to be silent or to talk and cooperate with the police. If they are both silent, the police do not have enough evidence on the major crime to convict and they will get charged on a minor offense and spend a year in prison. If both men cooperate they will go to prison for 5 years each. However, if one of them betrays his partner in crime (he talks while his partner stays quiet), the cooperating person will go free and the other gets a long prison sentence of 10 years.

The results from a game are called the pay-off and a pay-off matrix shows possible results depending on the moves each player makes. A pay-off matrix for the prisoners’ dilemma is shown in Table 2.1. Each entry shows the pay-off for the pair (prisoner 1, prisoner 2). The moves for prisoner 1 are shown on the left, the moves for prisoner 2 are shown at the top. This pay-off matrix represents the above story. Higher pay-offs represent a better outcome. Years in prison are represented as a negative pay-off.

2.1.2 Terminology

Players make a move based on the expected outcome of the game for that specific move. A player can describe his valuation of a game outcome numerically. He can do so using utility theory. Each player has his own utility function for any game he plays. His utility function
translates the outcome of a game into a number and is often standardized between 0 and 1. Higher utilities relate to more desirable outcomes for a specific player. If the sum of utilities for every combination of strategies is zero, the game is a zero-sum game. An example is chess, where the winning of one player implies the opponent losing. The pay-off for chess is illustrated in Table 2.2.

Utility can describe how good an outcome is, but not which outcomes will occur often. In other words, which moves will players often choose? This concerns strategies. For the prisoners’ dilemma there are two strategies: either choose Silent, or choose Talk. A dominant strategy is a strategy that will always be better than any other strategy, regardless of what other players do. The dominant strategy in the prisoners’ dilemma is to talk. With this strategy you will get a pay-off of either 0 or −5, which is always better than −1 or −10 respectively.

In game theory the players are almost always assumed to be rational. Rational players are only motivated by maximizing their own utility, considering the information that they have. Players who are not rational are called irrational. It could be that the players only seem irrational because they take other factors from beyond the current game into account. Or players could be irrational because they are inexpert players.

Transaction costs are costs which are necessary for performing a transaction. Making a move in a game can have transaction costs. In game theory these costs are often deemed to be very small and therefore discarded.

In game theory there is a distinction between a single game and a repeated game. In a repeated game a player can apply a simple strategy, where he always chooses the same move in each iteration. The opposite is a mixed strategy, where a player applies different moves in different instances of the game. Players in a repeated game have to consider that their current actions can influence future actions of other players. This is sometimes represented as a player’s reputation. Repeated games are either finitely or infinitely repeated. Each case leads to very different results. The dominant strategy in a finitely repeated game can be
different from the dominant strategy in an infinitely repeated game.

A game has perfect information if all players know all previous moves by all other players. If a game has complete information every player knows the strategies and pay-offs available to the other players, although this does not necessarily include previous moves. Other games have incomplete information. In a game with incomplete information players do not know the strategies and pay-offs for all the other players. Players may want to keep this private information to themselves. It is also possible that not all players have the same information because of the difficulty to describe every relevant bit of information in certain games.

2.1.3 Auctions and strategies

An auction is a type of game where the players are called buyers and sellers. In a typical auction there is a single seller and there are multiple buyers. The buyers compete for the right to buy the object which the seller is selling. The seller with the highest bidding price wins. This type of auction is sometimes called a forward auction. In a reversed auction the roles of buyer and seller are reversed. It is sometimes called a reverse auction. There is a single buyer who wants to buy an object or service while multiple sellers wish to sell it. In a reversed auction the bidder with the lowest price wins. A forward and a reversed auction can be shown to be mathematically equivalent. They differ in terminology, but have the same game properties.

Consider a reversed auction. Sellers are competing for a single buyer to buy an object. Players $i$ can make one move. A player’s move consists of a bid $b_i$ which contains a price. Another move is not competing in the auction game. Each player has a certain valuation for the auctioned object. This is called a player’s type $t_i$. Each player also knows his own reserve price. This is the highest price a buyer is willing to pay, or the lowest price at which a seller will still sell an object.

Utility in auctions is not scaled to values between 0 and 1. Instead, utility is defined as the profit a player makes by competing. Losses translate to negative utility. A seller prefers high prices, far above his type. In a reversed first-price auction, the winning seller has utility $u_i = b_i - t_i$ and a losing seller has utility $u_i = 0$. Players who do not compete also have zero utility. The buyer prefers to pay as little money as possible and she will not pay more than her type. For the buyer holds $u_b = t_b - b_i$, where $b_i$ is the winning bid.

A strategy is how a player decides on which move to play. A player can use different strategies and he considers different factors to select a strategy. These factors are his own type, the opponents’ types and the opponents’ strategies. A player can have some uncertainty about this.

Players can have knowledge and beliefs about the behavior of other players. Knowledge are facts that are known with a 100% certainty. Beliefs are uncertain, usually represented as a probability density function over possible alternatives. Players can believe that opponents apply certain behavior, but they cannot know this for sure if the information is not given.

A player can take into account his beliefs about the strategies of all the other players and determine a strategy based on both his knowledge and beliefs. If all players determine their strategies in this way there is a Bayesian Nash equilibrium [24]. Some equilibria are
2.2 Mechanism design

found by theoretical analysis. Unfortunately, finding an equilibrium is often an NP-hard problem. An easier way to find equilibria is to use experiments. Riley and Samuelson [29] have characterized the Bayes-Nash equilibrium of the first-price sealed-bid auction. The equilibrium expresses the optimal bid as a function of the reservation price of the object, the number of bidders, the bidders’ private values and the distribution of those private values.

2.1.4 Empirical game theory

There are strong theoretical results in game theory. These often regard simpler games. When a game becomes more complicated, a theoretical analysis becomes more difficult. Empirical evaluation can help analyze such complex games. This can be called empirical game theory.

Jordan et al. [19] use regret to measure the quality of a strategy. Regret indicates how much a player could have gained by deviating from his strategy. The analysis assumes that the opponents’ strategies remain constant. In a Nash equilibrium the players have no regret. Every player in the equilibrium is acting in the best possible way and they cannot improve their result by changing strategies.

Since regret indicates the quality of a strategy against certain opponents, it measures how good a player acted in a situation compared to the optimal move. Regret is an individual metric for each player. It can also be used to measure the overall outcome of a game. The maximum of all player’s regrets is called the regret of the game. Both individual regret and game regret will be used in my own experiments.

Jordan et al. [19] consider a restricted version of a first-price sealed-bid auction. Players’ strategies are defined by a factor \( k_i \in [0, 1] \). Players bid \( b_i = k_i t_i \). Using the results from this restricted game, they calculate regret with respect to the true game. In a game with two players they establish that all equilibria are symmetric. They limit their search for Nash equilibria to symmetric strategy profiles.

2.2 Mechanism design

Mechanism design is sometimes called reverse game theory. In game theory the focus is on specific games and the analysis of said games. Strategies for winning a game and properties of the players are considered. In mechanism design the game is not fixed. Different variations of games lead to different properties of those games. Mechanism design focuses not on how to play a game, but instead on the properties of a game. Games can be designed such that they enforce certain behavior in the players, or to exclude manipulation.

Mechanism design is a strategic version of social choice theory, under the assumption that agents maximize their own pay-offs. This means all the agents are assumed rational. This section discusses relevant theory on mechanism design. This discussion of mechanism design is based on Chapter 10 from the book by Shoham and Leyton-Brown [32]. The discussion is built on definitions of desirable properties for mechanisms and on existing mechanisms. Together these definitions lead to results that are impossible to achieve. Because optimal results are impossible, other mechanisms are needed which still adhere to some of the desirable results.
It is desirable that agents are honest and do not try to manipulate a mechanism by lying about their preferences. However, intrinsically agents would like to keep their preferences private. Another good property for a mechanism is efficiency, such that social welfare is maximized. Social welfare is the total utility over all players.

If the best strategy for an agent is to be honest when disclosing preferences, this is called \textit{incentive compatibility in dominant strategies}.

\textbf{Definition 1} (Incentive compatibility in dominant strategies). \textit{For every agent the dominant strategy is to announce his true preferences.}

Mechanisms can be either \textit{direct} or \textit{indirect}. In a direct mechanism an agent can only disclose his preferences. In an indirect mechanism agents have other available actions. Truthfulness is a positive property for direct mechanisms. A truthful mechanism can also be named dominant-strategy truthful.

\textbf{Definition 2} (Truthfulness). \textit{A mechanism is truthful if it is direct and the dominant strategy for every agent is to disclose its actual preferences.}

According to the \textit{revelation principle}, if there exists any (indirect) mechanism that is dominant-strategy truthful then there also exists a direct mechanism that is dominant-strategy truthful. This revelation principle is often used in its contrapositive form: if there is no direct mechanism that is dominant-strategy truthful, then there is not an indirect mechanism that is dominant-strategy truthful. Because of this result, most research has focused on direct mechanisms.

Agents need to use the information they have to decide if they will participate in a game or not. Agents would prefer not to be off worse after participating. An agent wants to be in either the same or in a better position after participating in a game. This is called \textit{individual rationality} and can also be about the expected loss or gain, so that \textit{on average} an agent is not off worse.

\textbf{Definition 3} (Individual rationality). \textit{An agent would never be better off choosing not to participate.}

The Gibbard-Satterthwaite theorem presents a strong negative result. Before presenting this result in Theorem 1, it is necessary to define what a dictatorial mechanism is.

\textbf{Definition 4} (Dictatorial). \textit{If a mechanism is dictatorial, there is a single agent who can choose the winner.}

Being dictatorial is not a positive property for a mechanism, because every agent should have some influence in determining the outcome of a mechanism, not only a single leader. The impossibility result from the Gibbard-Satterthwaite theorem leads to research for alternative mechanisms which are not dictatorial.
Theorem 1 (Gibbard-Satterthwaite). Consider any social choice function, if

1. there are at least 3 outcomes
2. for every outcome there exists a corresponding preference profile
3. it is dominant-strategy truthful

then this social choice function is dictatorial [14].

Another negative result in mechanism design, apart from the Gibbard-Satterthwaite theorem, is the Myerson-Satterthwaite theorem. This impossibility is mostly negative for the operator of a mechanism, because it must sacrifice something. The operator of a mechanism can be included as an agent who is indifferent to the outcome of the mechanism. The operator could run the mechanism at a loss, otherwise it cannot be both Bayes-Nash incentive compatible and individually rational.

Theorem 2 (Myerson-Satterthwaite). No mechanism can be Bayes-Nash incentive compatible, individually rational, and not run a deficit. [23]

Unfortunately these negative results exist which state it is not possible to combine all the desirable properties that you would like. However, by making concessions it is still possible to reach a wide selection of mechanisms which each have some of the desirable properties. The next section discusses a selection of existing mechanisms.

2.2.1 Existing mechanisms

An auction is a type of game. Auctions [20] are widely used in practice: from selling art, or selling second-hand items on eBay, to government procurement of bridges or buildings. A few basic types of auctions are first mentioned, in order to have an understanding of auction mechanisms. After this, combinatorial auctions are discussed. Combinatorial auctions have many applications but also have difficulties when used in practice. A lot of research has been done in combinatorial auctions. At last, matching with contracts will be discussed. This is an alternative for using auctions.

Basic auction types

There are four basic auction types which are widely used: the ascending-bid auction, the descending-bid auction, the first-price sealed bid auction, and the second-price sealed-bid auction. Each of these auction types is discussed below. It is assumed that a single object is auctioned and that bidders have a private valuation for this object. More detailed information on auctions can be found in [20]. In a regular auction, there is a single seller versus multiple buyers. In a reverse auction these roles are reversed and there is a single buyer and multiple sellers.

• Ascending-bid auction In an ascending-bid auction, the price is increased until there is only one bidder left, this bidder wins the object at the final price. This auction type
is also called the English auction. In the ascending auction it is a dominant strategy to remain in the bidding until the price reaches your maximum value. The next-to-last bidder will stop when her value is reached, so the person with the highest value wins the object at (slightly above) the second-highest value. This is the same outcome as in a second-price sealed bid auction.

- **Descending-bid auction** In a descending-bid auction, the price is decreased until the first bidder accepts the price, this bidder wins the object at the accepted price. This auction type is also called the Dutch auction, because it is used in Dutch flower auctions. Each bidder must in advance decide a price which she accepts, depending on no other bidder having yet called out. The strategies in this auction are equivalent to strategies in the first-price sealed bid auction.

- **First-price sealed bid auction** In a first-price sealed bid auction, every bidder can submit exactly one bid, without seeing other bids. The highest bidder wins the object. This highest bidder pays her own bid.

- **Second-price sealed bid auction** In a second-price sealed bid auction, again every bidder submits a single bid, and the highest bidder wins the object. This highest bidder pays the second highest price. This auction type is also known as the Vickrey auction [33]. The dominant strategy in a Vickrey auction is to bid your true value.

For the Vickrey auction it is a dominant strategy to be truthful. In the other three auction types, the mechanism is efficient when all participants are truthful. This leads to the bidder with the highest valuation winning the auction. The mechanisms are also individually rational, if there is no auction fee to be paid.

**Combinatorial auctions**

Often an auction does not revolve around selling one single object, but a variety of objects. Such an auction may be for distributing airport time slots or delivery routes (in a reverse auction). Because assets may complement each other, bidders can have preferences for bundles of items. As a result, bidders would like to bid on combinations of items. Such auctions are called combinatorial auctions. The Combinatorial Auction Problem (CAP) is selecting the winning set of bids, such that every object is sold exactly once or that the revenue is maximized. Combinatorial auctions are often reverse auctions. De Vries and Vohra survey combinatorial auctions in [9].

Consider the following example. A postal service wants to hire subcontractors to deliver packages around the country. In this case, it is important that the winning bids provide coverage such that the whole country receives their packages. In addition, the winning bids should not have overlapping areas, such that every area has only one delivery service. Sandholm provides a detailed application of the combinatorial auction problem in business in [31]. He shows how his application is faster, more structured and more transparent than in-person negotiations.

Combinatorial auctions allow bidders to better express their preferences for different outcomes than is possible in sequential auctions and thus enhance efficiency. However,
preference elicitation is difficult because bidders must determine a value for every subset of objects. The communication cost of sending bids to the auction can be high. With the use of a suitable bidding language, communication complexity can be reduced. Another problem is determining the winner once bids have been received. This problem is NP-hard. A lot of research has been spent developing algorithms for winner determination [7].

**Matching with contracts**

Matching with contracts is another alternative for auctions and negotiations. Contracts are the basic unit of analysis in this model. A contract can be defined in different ways, such that matching with contracts extends existing models. For instance, if a contract is identified by a student and a college, the problem becomes solvable by the Gale-Shapley stable matching algorithm [13]. In this algorithm every college first invites their most preferred set of students to come study. If some students refuse, the college makes new offers to other students to fill up the remaining spots. A solution to the problem is a stable collection of contracts. Another type of contract could be specified by a hospital, a doctor and a wage. Or a contract which specifies a bidder, a bundle of items and a price to be paid. In this case matching with contracts is similar to combinatorial auctions, which were discussed earlier.

Matching with contracts is discussed in [16]. Hatfield and Milgrom present two conditions that restrict the preferences of the colleges: a *substitutes* condition and a *law of aggregate demand* condition. The substitutes condition states that if the college has a wider range of contracts to choose from, weakly more contracts are rejected. The law of aggregate demand condition states that if a college has more contracts to choose from, it admits at least the same amount of students.

Hatfield and Milgrom [16] analyze their general matching with contracts model and come to the following results. When inputs are substitutes, the choices of a profit-maximizing hospital satisfy the law of aggregate demand. Moreover, when the choices of every hospital involved satisfy the law of aggregate demand, the set of doctors employed is the same in every stable allocation and every hospital employs the same amount of doctors in every stable allocation. In this situation it is a dominant strategy for doctors to be truthful, but if the law of aggregate demand fails, then this dominant strategy property does not hold. For these results to be practical, there needs to be a convenient way for hospitals to report their preferences to the mechanism. A way is proposed, called extended assignment valuations, which satisfies both conditions. The new approach reveals similarities between some of the currently most successful auction and matching designs and also among the environmental conditions under which these mechanisms should perform their best. These similarities help understand the limitations of these mechanisms.
Chapter 3

Tenders

This chapter discusses different tender procedures, where the main focus is on the restricted tender. The restricted tender is the most commonly used tender procedure. The economically most advantageous tender process is also discussed. After that the focus shifts to literature on tenders.

When a government agency needs to procure a large project, a tender process is used. Such a large assignment can be from one of three categories: works, services, or supplies. Works are large assignment such as buildings, bridges, or high way construction. Services and supplies are smaller assignments. Moving an office is an example service. Examples of supplies are lease cars, computers, or coffee machines. The monetary value linked to an assignment, together with the category, determines whether or not it is obligated to use a tender process. In this thesis work, the focus will be on the works category.

In a tender, government agencies and contractors are opponents. They both have different motives when they enter a tender. Contractors want to win tenders to gain high profits and government agencies want the best possible contractor to win at the lowest possible price. Government agencies have some freedom in selecting a tender procedure, depending on the project properties, and in the award criteria when they choose to focus on both price and quality. The quality description and constraints can be finalized by the contracting authority.

3.1 Restricted tender

Under European laws there are approximately ten tender procedures which can be used depending on the type of assignment and contract. The restricted tender is the most common procedure. A restricted tender is announced publicly. Companies can apply, followed by a pre-selection. Usually around five companies will be invited to enter for the award round. After this round, the tender is awarded to a single company.

In the selection phase, all interested contractors sent in an application. This pool of contractors must be reduced in order to continue to the next round. The limit for the number of suppliers who will continue is usually five companies. During selection one first checks a number of grounds for exclusion – such as criminal activities – and suitability requirements,
which can be quality demands or financial demands. This filters out unsuitable companies. If there are still too many companies left, there will be a selection to limit the number of participants such that the maximum possible amount of participants remain. This selection must be objective and non-discriminating. After the selection remaining companies can enter an offer.

In the second phase, the contract is awarded. This downsizes the pool of contractors from five competitors to a single winner. Eventually a number of offers has been received, one from each company that has entered. The contracting authority must choose one of the following two awarding criteria to determine the winner: the lowest price criterion or the most economically advantageous tender. The first speaks for itself. The second is more complicated. It is a multi-attribute criterion, which can be finalized in different ways. It is based on the idea that different factors are important, such as price, quality, time, durability, and so on. More about this most economically advantageous tender can be found in [28]. When one supplier remains, the contract can be signed and the tender is complete.

3.1.1 Tender procedures

This section presents some of the alternatives for the restricted tender procedure. Among these alternatives, some procedures are still more popular than others. Certain very project-specific tender procedures are hardly ever used. Below is a selection of possible tender procedures, including the restricted tender.

- **Restricted procedure.** The tender is announced publicly. Companies can enter, followed by a pre-selection. Usually around five companies will be invited to enter for the award round. After this round, the tender is awarded to a single company.

- **Open procedure.** The tender is announced publicly, after which companies can enter for the award round. There is no explicit selection round.

- **Competitive dialogue.** After selection there is space for a bilateral dialogue between procurer and contractor. This procedure is especially useful in situations where the end product is not clearly defined. In this case both sides interact to define the assignment more clearly.

- **Negotiated procedure.** After the public announcement, selection and company applications, there is room for bilateral negotiation. This results in every company entering with their final offer and then the tender is awarded.

All the documents regarding a project end up in the contract. This includes for example the project requirements, how to apply and the winning final offer. Within different contracts there is a distinction between contracts and *integrated contracts*. In the first, a contractor only has to build a work. In the other, a contractor also has to (partially) design a work before construction. This type of contract holds more freedom for the supplier.
3.2 Economically most advantageous tender

The award criterion in a tender is most often the lowest price. With this award criterion the offer with the lowest price wins. Offers only consist of a price. For this price the contractors must execute all the requirements from the project description.

With a lowest price tender there is no way to let companies offer a higher quality. There is another award criterion which focuses on both price and quality. This criterion is called the Economically Most Advantageous Tender (EMAT). With an EMAT tender, the offer with the best quality price ratio wins. This winning offer gives the best value for money.

EMAT awarding is not specifically for restricted tenders. It can also be used in an open procedure tender or in a competitive dialogue or any other procedure with an award round to select the winning contractor.

As an example, imagine a project with two EMAT criteria: life span and risk management. On each criterion, contractors can score insufficient, sufficient or great. Life span is worth 10% of the project value, and risk management 20%. Depending on the scores for each EMAT criterion, players can get a bonus or a penalty. These bonuses and penalties are shown in Table 3.1. Bonuses are subtracted from the offer price and penalties are added to the price. This yields the evaluation price, according to which the offers are then ranked. The offer with the best evaluation price wins the tender.

3.3 Tenders in literature

In literature, contractor selection can mean either selecting viable contractors or awarding the final contract or both. For clarity, in this work contractor selection is the selection of proper candidates and winner selection is the final contract awarding.

Hatush et al. [17] research a systematic multicriteric decision analysis technique for contractor selection and bid evaluation. They conclude that there exists a need for a contractor selection technique that considers multiple criteria. Such a technique is similar to the economically most advantageous tender.

Holt discusses different contractor selection methodologies in [18]. These different methodologies are accompanied by flow-chart to illustrate the decision making process. European law actually leaves little room for choosing a methodology because it dictates the possibilities.
3.3 Tenders in literature

3.3.1 Bidding in tenders

Drew et al. [11] analyze bidding behavior in tenders. They identify three main factors influencing contractor bidding behavior, which are: type of client, type of construction work and size of construction work. The focus is on one of the larger contractors in Hong Kong.

Participants in a tender do not always behave rationally. A form of irrationality is bid manipulation or bid rigging, where one or more players work together against the rest of the players. These players are not only working on maximizing their own utility. They also focus on reducing the utility of certain opponents, so that those opponents never win and stop playing. Porter and Zona [27] find ways to detect bid rigging in procurement auctions. Bid rigging consists of a cartel group where agreements on the bidding price are made in advance. One firm is the designated low cartel bidder, and the rest submit higher bids without any chance of winning.

3.3.2 Transaction costs in tenders

In procurement transaction costs form a considerable amount of the total costs of a project. Transaction costs in a tender arise from the time and effort needed to compile a formal offer. Especially with EMAT tenders an offer that focuses on high quality can take a considerable amount of time to compose. These hours and the resources together form the transaction costs for the sellers. A buyer can also have transaction costs, because a buyer will need time to set up a tender and evaluate and rank all the offers.

The field that studies transaction costs is called transaction economics. Estache and Martimort [12] take transaction costs into account and analyze how these costs affect the regulatory outcome. Transaction costs can influence decisions on different levels. Estache and Martimort research transaction costs in the context of politics and regulatory institutes.

In [6] Constantino et al. focus on minimizing (transaction) costs in a lowest-price tender. Costs are minimized for both the buyer and the sellers. The bidding behavior is not gathered from real data, but simulated by some bidding function. This bidding function should be more representable than a uniform distribution which is used in other works.

Bajari and Tadelis [1] present work about the trade-off between incentives and transaction costs. Incentives are offered to construction companies to finish earlier or deliver some extra quality, so that they will receive more money. This is similar to EMAT tenders, where a bid with higher quality has higher transaction costs and a contracting party can compensate the contractors for this extra work.

3.3.3 Application of game theory in tenders

This section discusses works where a link is made between tender theory and game theory. First, Runeson [30] believes that it is not possible to combine tenders with game theory. Second, other works are discussed that are able to combine game theory with tenders. An example is the work of Perng et al. [26] who have created a serious game on tenders.

Runeson [30] compares tender theory with game theory and points out main differences. The author decides that tendering theory cannot be classified as a game theoretical approach.
Game theory assumes rational agents who receive a pay-off based on their own strategy and the strategies from other players. In game theory players choose an action based on their knowledge about the strategies from other players. In tendering theory the players do not modify their behavior based on their beliefs about the other players. Another difference is in the amount of pricing options or ‘game moves’. In tendering theory there exists an infinite amount of options to choose an offer price from, but in game theory the amount of possible moves should be limited. Finally, game theory models are very specific and limited, in order to have a theoretical analysis.

To summarize, Runeson identifies three main differences between tenders and game theory: rationality, infinite moves and theoretical analysis. All these three points are valid, but I think that they do not prohibit the link between tender theory and game theory. It merely makes the combination more interesting. Rational and irrational players together will lead to unpredictable results. It is indeed true that tender theory has an unlimited amount of possible moves, but this is not uncommon in game theory. Consider for example auctions or negotiations where a price must be decided. In such games there are also infinite possibilities. Lastly, the more complicated a mechanism gets, the more difficult its theoretical analysis will be. There are also games which are very complicated and difficult to analyze only from theory, but such complicated games are still valid games. The fact that tenders are complicated should not lead to the conclusion that game theory is completely useless on tenders.

Now for a more positive note, an example of a tender turned into a game. Perng et al. [26] have created a serious game in which a tender is played. In this tender game there is an EMAT (most economically advantageous tender) tender that focuses only on quality and has a fixed price. Different tender experts are invited to play the game for a varying number of rounds. A few groups of players are formed and the results from the game are analyzed. The authors search for connections, such as between experience and outcome, or group size and the odds of winning. One of their conclusions is that experience helps to improve your result. Larger groups did not lead to worse results for existing players. Nor did profit margin seem to influence the chance of winning for a player.
Chapter 4

Tenders as a game

4.1 Approach

Tenders can be regarded as a game. Because tenders have many complicated restrictions, it is hard to analyze tenders directly with game theory. Games which do have strong theoretical results do not resemble a tender well. Such theoretical results are not easily applicable to tenders. This chapter contains the research approach for the coming chapters in this thesis. After detailing the approach, a basic model is compared to a tender and this model is analyzed.

The reverse first-price sealed-bid auction is chosen as the basic model. There are key differences between a tender and a first-price auction. Discussions with experts help identify the key differences between a tender and the first-price auction. These are discussed in Section 4.2. Each key difference is explained and a motivation is given to its relevance in tenders. Each aspect can be included in the basic model in its own way.

The basic model is discussed and analyzed in Section 4.3. After this, the basic model is extended incrementally. At first, all the key aspects are added to the first-price auction one by one. These models, which are extended with a single parameter, are analyzed and compared to the basic model. Experimental results along with more discussions with experts lead to interesting combinations of aspects for composite models. Not all possible combinations are researched. A selection of possibilities is made. These are composite models with two aspects combined. The models with two aspects indicate future directions for the more extended models. Table 4.2 indicates which game aspects are modeled in which section.

The experiments with the different models are simulations in Java. The simulation environment is detailed for each model. Three points of view are possible during the experiments. These are the point of view of the sellers (contractors), the buyer (contracting authority) and the general public. Contractors want to win a game, they are interested in the best strategies to maximize their own profit. The buyer wants the best contractor to win, which is the seller whose offer the buyer values most. The winner of a tender is not necessarily the best possible outcome. The last point of view is that of the general public. This is an overall viewpoint which considers social welfare. The following research mostly focuses on the point of view of the sellers. Sometimes an analysis of the buyer’s perspective
4.2 Model parameters

A tender can be modeled as a first-price sealed-bid auction with some extra properties. Such a first-price auction can be viewed as a stripped-down version of a tender. Both a tender and a first-price auction are mechanisms with a number of players where in the end one player wins. The winner is decided based on the bids that the players enter. Table 4.1 shows a number of similar and different properties for both a tender and a first-price auction.

A first-price sealed-bid auction is an auction where all players can submit a single bid, the highest bidder wins and pays his own bid. All bids are sealed to ensure the players do not know the bids of their opponents.

There are a number of similarities between tenders and first-price auctions. These are outlined in the top of Table 4.1. In both mechanisms there is a single winner. This winner pays his own bid, and other players do not have to pay. The winner is chosen out of \( n \) players, and is decided by the best price submitted. All bids are sealed, so players do not have information about the bids of the other players. Afterwards the results of the game – the winner and the winning bid – are made public.

There are a number of differences between a tender and a reverse first-price sealed-bid auction. These are shown in the bottom part of Table 4.1. The differences are transaction costs, bid components, repetition, continued work, information and rationality. These six differences form the parameters which define a tender model. Each of the parameters can be added to the basic model. In the coming sections each parameter is discussed. Cases illustrate the influence of parameters in practice. They show how a parameter can be applied or ignored in practice.

<table>
<thead>
<tr>
<th>Property</th>
<th>Tender</th>
<th>First-price auction</th>
</tr>
</thead>
<tbody>
<tr>
<td># Players</td>
<td>1 vs ( n )</td>
<td>1 vs ( n )</td>
</tr>
<tr>
<td># Winners</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sealed bids</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Winning bid</td>
<td>first-price</td>
<td>first-price</td>
</tr>
<tr>
<td>Reversed</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Transaction costs</td>
<td>substantial</td>
<td>negligible</td>
</tr>
<tr>
<td>Bid components</td>
<td>price and quality ( n ) rounds</td>
<td>price 1 round</td>
</tr>
<tr>
<td>Repetition</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Continued work</td>
<td>biased, uncertain</td>
<td>complete and equal</td>
</tr>
<tr>
<td>Information</td>
<td>not always</td>
<td>yes</td>
</tr>
<tr>
<td>Rationality</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Similarities and differences between a tender and a first-price auction
4.2 Model parameters

4.2.1 Transaction costs

**Case 1** (Always losing). *A company always enters in tenders on a certain project type. Sadly, this company never wins. Every tender participation has some small transaction costs. Over the course of losing many tenders, the company has paid these transaction costs many times.*

**Case 2** (Reimburse transaction costs). *A large city wants to build a new city hall. The design of this new building is very important to the city. They use a tender to select a contractor. Contractors who want to enter this tender must submit an offer with a price and also a detailed design for the new city hall. Such a detailed design will cost a lot of time and effort to create. The city decides to compensate the participating companies by reimbursing the transaction costs.*

Tenders have transaction costs. These transaction costs can be quite substantial. In practice the transaction costs will be up to 1% or 2% of the total costs of a project. Transaction costs occur because of the time and resources that are necessary to form an acceptable offer. Especially with EMAT tenders an offer that focuses on high quality can take a considerable amount of hours to compose. These hours form transaction costs for all the sellers. The buyer also has transaction costs, since he has to review and rank all received offers. Sometimes an offer must be accompanied by an extensive initial design, which the contracting party can give some monetary compensation for to reduce the transaction costs. In game theory transaction costs are often discarded because they are deemed to be very small or non-existent. In a first-price auction there are no transaction costs.

4.2.2 Bid components

**Case 3** (Lowest-price tender). *Part of a local highway needs to be repaved, which is a type of assignment that occurs often. Companies decide on a price based on the project requirements. They do not focus on any extras, since the project is very straightforward. The contractor with the lowest price wins the tender.*

**Case 4** (EMAT tender). *A certain government agency holds a tender for a new sewer installation. They prefer a solution where the contractor gives the least amount of nuisance for the inhabitants. The government agency chooses to use an economically most advantageous tender to express their quality preferences. Contractors who use newer techniques, but are a bit more expensive, could still win.*

Tenders often use EMAT awarding so price is not the only factor in an offer. An EMAT offer can consist of a monetary part and an extra part which describes the offered quality. This is often an extra report which describes how a contractor will handle certain quality aspects, such as lifespan, risk, sustainability and maintenance costs. When a contractor offers added value with this quality report, above the basic quality requirements of a project, this contractor’s offer will rate higher than another offer which just meets the basic standards.

These complex offers are multi-dimensional, the price is one dimension and all quality aspects are the other dimensions. There needs to be a means to rank a group of offers in
4.2 Model parameters

Tenders as a game

order to select a tender winner. It is possible to change an offer into a single value, which could represent a grade or be monetary. Pries and Van Reeuwijk [28] discuss ways to translate price and quality into a single value.

4.2.3 Repetition

Case 5 (Repeated tender). There is a tender for the 1-year maintenance of an industrial area. This tender is repeated yearly. Companies who competed before, can easily set up an offer for the following tenders.

Case 6 (Gaining experience). A contractor sees an opportunity to start in a new specialized field and wants to win a certain tender to get started in this field. Because of the experience they can compete in future tenders in this specialization. The company expects a lot of extra revenue from future tenders. The contractor decided to forcibly win the tender by sending a ridiculously low offer, well below costs.

Tenders are repeated, not a single instance. For repeated games, the strategies can be very different from a single game. There are different forms of repetition: limited and unlimited. Both lead to different dominant strategies. Tenders are repeated unlimited. That is, contractors can enter in new tenders as they please. The basic model is only a single instance of a game. Of course the first-price auction can be held multiple times, but this requires new theoretic research into the properties of such an repeated auction.

Because of repetition in tenders, contractors can become more experienced. A contractor can select a project which helps build knowledge and experience which leads to better winning chances in future tenders. Contractors will often need to send in reference projects to demonstrate that they have experience with or knowledge about the specifics of a new project.

4.2.4 Uncertainty

Case 7 (Inexpert contractor). A contractor is new in a line of work. He has some experience with smaller projects, but has not yet won a large tender. He is competing in a tender. The contractor is inexpert and does not yet know how to properly determine the costs for a large project the size of this tender. The contractor’s bid will vary wildly from the bids of the other, more expert, contractors.

Case 8 (Costs inflation). A contractor wins a tender with a certain offer. The construction work takes about a year to complete. Halfway through the project, prices of materials rise and the contractor has unexpected extra costs.

Information in a tender is biased and incomplete. This means uncertainty for all parties involved. Uncertainty can come from the different estimates on the amount of work and resources necessary. This will lead to different parties assuming different costs for the same project. If some parties have worked in the same or a similar area before, they may have additional information about the situation. The information is biased. Contractors have risks which can occur extra costs. They can experience bad weather and stop all construction
work for a week, or materials could suddenly become more expensive. These risks for the contractor result in uncertainty about the true costs of a project.

4.2.5 Continued work

Case 9 (Incomplete contract). The contracting authority has not spent sufficient effort on designing the project requirements. Contractors notice this and send in very low bids. Every time the contracting authority tries to enforce a project requirement, the winning contractor refuses, since it was not clearly included in the contract. Instead, all of the work is seen as continued work beyond the project. The winning contractor can determine his own prices for this extra work. The contracting authority must pay more than expected.

Case 10 (Unforeseen extra work). There is a tender for the construction of a new highway. During construction, the winning contractor finds a vast amount of bombs located in the work site. The contractor could not have reasonably foreseen this problem. This is a risk for the contracting agency and they will have to reimburse the extra costs.

Additional work after winning a tender can produce a lot more revenue for a company. Expectations on continued work after winning a tender may influence the proposed bid for a tender. Continued work can be unforeseen extra work, which the contracting party did not foresee when the project description was made. Not all risks are for the contractor. The contracting authority is responsible for certain risks. They will have to pay extra if these risks occur. It is also possible that the contracting party changed their mind on some small project details. The contractor can bill them extra for this extra work.

4.2.6 Rationality

Case 11 (Negative experience). A contractor does not like to cooperate with a certain contracting authority. This contracting authority often organizes tenders within the contractors specialty field. The contractor always decides to not enter in such tenders. Considering only the possible profit the contractor misses, he is acting irrational.

Case 12 (Cartels). A group of contractors makes price agreements. They decide to let each of them win in turn. The selected winner sends in a serious offer based on his costs. The other contractors in the group send in artificial offers which are strictly higher than the selected winning offer.

In game theory all players are assumed to be rational. The behavior of rational agents is based on the game they play and their own preferences or utility function. Modeling a tender will probably not capture all factors which influence participants in practice. Players in a tender are seemingly irrational, within the limits of a specific model. Factors which are not modeled can still influence the behavior of players in practice. An example is group strategies. Within the basic model, participants are not able to communicate with each other. This makes it impossible for them to consult and apply group strategies. Such a group strategy is to form a cartel, or to force certain opponents out of the market.
4.3 Basic model

This section considers the basic model without any of the parameters included. The basic model is the reversed first-price sealed-bid auction. The first-price auction has been extensively analyzed in literature. A selection of this literature will be discussed. Next, the game design of the basic model is described and relevant strategies are outlined. The theoretical part is followed by an empirical evaluation part. Expectations for the behavior of players in the first-price auction are stated. Simulations are designed to test these predictions.

<table>
<thead>
<tr>
<th>Section</th>
<th>First-price auction</th>
<th>+ transaction costs</th>
<th>+ complex bids</th>
<th>+ repetition</th>
<th>+ uncertainty</th>
<th>+ continued work</th>
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<td>x</td>
</tr>
</tbody>
</table>

Table 4.2: Overview of aspects in the different games.

4.2.7 Selected models

Not all possible models can be analyzed, because there are too many possibilities. Each parameter of the model, can either be added to a model or be left out. With six parameters this means there are $2^6 = 64$ possible models. Only a selection of all those possible models will be analyzed. The first is a basic model, the first-price auction, with none of the parameters added. This basic model is analyzed in Section 4.3. The next selection is all models with a single parameter added. An example is the repeated first-price auction where the ‘repetition’ parameter is added. Six different models can be defined in this way. Each one can be found in Chapter 5. Furthermore a selection is made of other models where multiple parameters are chosen. These models are described and discussed in Chapter 6. Table 4.2 indicates which models are selected exactly and in which sections they occur.
4.3 Basic model

4.3.1 Results from literature

Laffont et al. [21] perform empirical work on the first-price sealed-bid auction. They improve a player’s estimation of the private values of the other players. They introduce the use of a non-linear least squares estimation method for estimating these private values. This estimation method can handle a variety of distributions of private values. In the coming experiments the players use a simpler learning method.

4.3.2 Game model

A buyer wants to procure a certain project. A number of sellers, \( n \), want to do the project. Sellers can have different valuations of the project. Each seller can send in a bid. Sellers can apply different bidding strategies to determine a bid. A bid consists of a price. When all the bids are in, the auction mechanism evaluates and ranks the offers and determines the winner. The lowest offer wins. In case of a tie, one of the sellers with the best price is chosen at random. The outcome of the auction is the winning seller and his winning bid. The buyer must pay the winning bid.

Players can have knowledge and beliefs about parts of the game. If a player has more positive or negative beliefs about a project, he has a bias for lower or higher costs, respectively. Players who consider themselves to be very efficient, will believe they have lower costs. Players are also uncertain about the strategies of the other players. It is possible to let players observe the outcome of a number of auction games. Using their observations, players can form beliefs on the moves of their opponents and estimate their own winning chances.

4.3.3 Strategies

Players want to apply good strategies. Good strategies help players win more often than other strategies do. More specifically, the more money a player makes by winning, the better the strategy is. So a strategy that wins often, but loses a lot of money each win, is not a very good strategy. Utility is measured as the amount of profit a player makes by participating in an auction. Not winning means zero utility. Winning and making profit leads to positive utility. Because players are assumed rational within this model, their aim is to maximize their own utility. Players will want to avoid losing money, because this means negative utility.

Players can use different strategies for competing in a reversed first-price auction. A player can use a simple strategy without any learning or use beliefs on their opponents to decide the price. Each player can have a different cost estimate of the project. Rational players will always bid strictly above their own costs, to avoid negative utility and ensure a profit by winning.

The strategies below are described in pseudocode. Strategy 1 illustrates a very simple strategy in which a player determines his costs and adds a fixed markup to decide on a price. The price, costs plus markup, is then returned to the buyer as an offer. Strategy 2 is more complex and assumes that players learn from observing winning bids. Players use their current beliefs on opponents to estimate a typical winning bid. If a player’s costs are far below
4.3 Basic model

Tenders as a game

this estimated winning bid, he will increase his profit margin and bid higher. Otherwise he applies his original minimum markup. How players learn is shown in Strategy 3. Players observe bids by their opponents and update beliefs on those opponent. They compute a weighted average of the observation and the previous belief. The new observation has a weight of $w$ and the old belief weighs $1 - w$.

Strategy 1 Costs + fixed markup

| Input: Probability distribution $F$, markup $m$ |
| Determine costs $c_i$ based on distribution $F$ |
| return $c_i + m$ |

Strategy 2 Learning

| Input: Probability distribution $F$ |
| Determine costs $c_i$ based on distribution $F$ |
| Compute minimum markup $m$ |
| Estimate winning bid $b_{\text{win}} = \min_{i=0..n} \{ e_i \}$ |
| if $(c_i + m) < b_{\text{win}}$ then |
| Set price $\varepsilon$ below $b_{\text{win}}$ |
| else |
| Set price to $c_i + m$ |
| end if |
| return bid price |

Strategy 3 Observe a bid

| Input: Observation of winning bid $b_i$ by player $i$ |
| Update belief on player $i$ as an estimated bid $e_i = w \times b_i + (1 - w) \times e_i$ |

4.3.4 Expectations

The offer with the lowest price wins the auction, according to the rules. So players would want to send in the lowest possible offer. This could be zero, or even negative in which case the winner would pay for acquiring the project. However, players also have to consider costs when determining the offer price. If a player sends in an offer lower than his costs, the player will lose money by winning. In other words, there is some, perhaps not strict, lower bound on the lowest bids.

Expectation 1. Since rational players never send in an offer below their costs, they will not decrease their bids indefinitely.

Learning from historical bids will motivate players to bid lower than their opponents. There will be, however, a lower bound on the bids, since players bid strictly above their costs.
**4.3.5 Experiment setup**

Players determine a bid price $b_i$ based on costs $c_i$. In order to profit from winning, $b_i > c_i$. To model the variance in the costs for performing a project, players each draw their costs from a normal distribution. Players can have different mean and standard deviation. Unless stated otherwise, the mean is 100 and the standard deviation is 5. A mean of 100 indicates that a seller’s costs are equal to 100% of the project value estimated by the buyer. The project value is assumed to be the same in every round. To determine the significance of results, t-tests will be used with a significance level of 5%.

Utility for the winning seller is $u_i = b_i - c_i$ and for the other sellers $u_i = 0$. When all five sellers are equal, each seller has a 20% chance of winning. Each seller can compute his expected utility: the profit of his bid multiplied by the chance of winning with that bid.

Equation 4.1 shows how to compute regret in a first-price auction. Depending on whether a player wins or loses, different situations occur. If a player wins, he profits since $b_i > c_i$. Perhaps this profit could be higher. Regret in this case is the difference between the winning bid and the second highest bid. If a player loses, his bid might have been too high and winning was possible. He could have won with a lower bid since his costs are below the winning bid. His regret is computed as the difference between his bid and his costs. The last possibility is that a player loses and he could not have won. There is nothing the player could have done to gain higher utility in this case, so his regret is zero. Finally, the regret of the game is the maximum over all the players’ regrets: $r = \max_{i=1..n\{r_i\}}$.

$$\text{regret } r_i = \begin{cases} b_{2nd} - b_i & \text{if player } i \text{ wins} \\ \max \{0, b_{\text{win}} - c_i\} & \text{if player } i \text{ loses} \end{cases}$$ (4.1)

Part of a player’s behavior is how he learns from previous games. A player can learn by observing winning bids. Players can use their observations to form beliefs about the other players’ strategies. Players get to observe only the winning bid and the winning player from an auction. They can model the behavior of each player based on the known bids of that player. Using all the opponent models, players can estimate the chances that they will win a future auction with a certain bid. Bids below all the opponent estimates will have higher chances of succeeding.

**4.3.6 Results**

**Learning speeds** In the auction simulations there are five players competing over a single project. There are five players involved since the last phase in a tender also involves at most five contractors. Each player draws his costs from a normal distribution. These costs are knowledge for the players. Players learn about their opponents by observing winning bids. Players keep track of an estimate for every opponent. Figure 4.1 shows the spread of winning bids for different learning rates. The weights for new observations are 50%, 10%, 5%, 1% respectively.
4.3 Basic model

Tenders as a game

There seems to be a lower bound on the winning bids. This is related to Expectation 1, which stated that players will not keep decreasing their bids indefinitely. The costs for each player were drawn from a normal distribution. The lower bound on winning bids indicates the lowest costs that players typically find. When their costs are too high, the players cannot further lower their bids and the winning bids show an increasing trend.

Notice that Figure 4.1a seems very chaotic. These results come from players who learn with 50% weight each round. Each new observation has a very high influence on the current belief of a player. Any outlying observations significantly influence the beliefs of a player and thus his bids. Figure 4.1d on the other side shows very slow learning. Players who are very slow learners barely update their beliefs. Such players will need many observations before they significantly alter their actions.

A middle ground would be more practical for the players. A good learning strategy would help a player to update his beliefs quickly enough that a few rounds of learning are beneficial. On the other hand it is also good for a player to not act very chaotic. If learning creates too much chaos, a play may just as well disregard learning. In further experiments a learning speed of 10% is used. This ensures fast and stable enough learning. Players will learn for 100 rounds. In practice it will take a lot of time to observe the outcomes of 100 tenders. It would not be a fair comparison if the players in this model receive 1.000 rounds of time to learn from observations.
Three different phases. Players can compete in different simulation rounds: without learning, learning and after learning. At first the players do not learn about their opponents. In the next phase some, or all, players are able to learn from observations. Afterwards, players continue without learning. The learning phase consists of 100 practice rounds. The phases without learning both take 1,000 auction simulations. In these rounds the players do not update any beliefs about opponents. The results are averaged over each phase. Multiple rounds are necessary because of the randomness inside the simulations. Analyzing only a single value is not representative of the model’s properties. Comparing results from before and after learning can help to determine the significance of learning.

In these next experiments the three different phases are applied: before, during and after learning. All players can learn. Figure 4.2 shows an example spread of winning bids. During the first 1,000 rounds the players are not able to learn. They never update an opponent model and there is a lower bound for the winning bids around 99. During learning players bid gradually lower while observing the winning bids of their opponents. After learning the lower bound for the winning bids is around 97. After learning (Figure 4.2b) players often bid lower than before learning (Figure 4.2a). A t-test comparing the two sets of winning bids confirms this: the null hypothesis that $b_{before} \leq b_{after}$ yields a $p$-value < 0.001.

Utility and regret results for the individual players are shown in Figure 4.3. Figure 4.3a shows the average utility for each player during each phase. Comparing the utility before and after learning shows that the variance in utility decreases. When all the players adapt to observations and change strategies, apparently the utility lowers.

Next, regret can indicate whether players perform closer to optimal after learning. Figure 4.3b measures the average regret for each player. Regret is measured for each player and for the complete game. The rightmost bars indicate the latter. The regret values are separated in the three phases. Observing the rightmost entry of Figure 4.3b shows the regret of the game after all players had the opportunity to learn. Regret of the game after learning appears lower than regret before learning. A t-test indicates a $p$-value < 0.001 for the null hypothesis that $r_{before} \leq r_{after}$. This hypothesis can be rejected and $r_{after}$ is significantly lower than $r_{before}$. Players still have regret, but the worst-case regret has improved.
4.3 Basic model

Tenders as a game

4.3.7 Discussion

If all players can learn indefinitely, the game will at some point reach an equilibrium. In this state, players act optimally and they have no reason to alter their behavior any further. Each player has learned about his opponents and can act taking those beliefs into account. The exact properties of the equilibrium depend on the exact settings of the game.

Learning is a useful advantage, especially when other players do not apply learning. In practice, contractors do not use an explicit form of learning where they keep track of...
previous tender outcomes. However, experienced contractors will have beliefs about opponents and the current market. These beliefs come from previous experiences that are used to learn. In tenders, the projects are often large and contractors only have a limited capacity for competing in tenders. It follows that contractors have a limited set of observations from tenders in which they participate. Thus, a contractor who is willing to use a learning strategy, should give sufficient weight to his available observations.

In future experiments regarding learning there could be a focus on the best type of learning or more regarding the effect of learning. The best type of learning for a rational player is the learning which gains the most utility. In this thesis players learn by computing a weighted average based on observations. In literature other forms of learning exist, which may yield better outcomes for learning players. The effect of learning is the extra gain from a learning strategy compared to not learning. The effect of learning can be determined with experiments. Such experiments should also consider the cost of learning: players apply extra effort to learn and this may or may not outweigh the benefits from learning.
Chapter 5

Simple models

In this chapter all the parameters from Section 4.2 are added to a first-price auction in turn. These parameters are transaction costs, complex bids, repetition, continued work, uncertainty and irrationality. Not every aspect can be added trivially to a first-price auction. Each parameter is modeled in the context of tenders and the resulting game models are described. Expectations for the models are generated and experiments designed to test these expectations.

5.1 Modeling transaction costs

Transaction costs represent the time and effort it takes to create an offer. They can also be seen as an entry fee to participate in an auction. Transaction costs can be negated by not asking an entry fee or by reimbursing the costs. The buyer can compensate the sellers for the transaction costs. A first-price auction with transaction costs is studied by McAfee and McMillan [22]. They analyze a first-price auction with entry costs. All bidders must pay a fixed entry cost $k \geq 0$ in order to enter the bidding. This entry cost may represent the cost of preparing a bid or learning what an item is worth. The entry cost which a bidder must pay is a form of transaction costs. Bidders will enter an auction if their expected profits are at least $k$.

McAfee and McMillan show that in a first-price sealed-bid auction with entry costs the optimal number of bidders enter the auction. The number of bidders is optimal if from the point of view of the seller exactly the right number of bidders enter the sealed-bid auction. The seller then has the highest possible expected revenue. They also show that the optimal reserve price is the seller’s own valuation. The seller should not have a reserve price above his own valuation.

5.1.1 Game model

In this game model the players must pay a fixed entry fee to participate. These costs represent the effort that goes into creating an offer. In tenders the transaction costs are usually a small percentage of the total costs, about 1–2%. If the transaction costs are higher, contractors will more often decide not to participate in a tender.
Because of the transaction costs the game is no longer individually rational. This means that players can lose money by playing the game, so sometimes they may be better off choosing not to play. Case 1 illustrates an example of a player who always plays, but never wins and keeps losing money.

Players can decide not to play if the transaction costs are too high. Another reason not to play is if the estimated winning chance is very low. Players have beliefs on their winning chances. If a player’s costs are higher than the project estimate, this player can conclude that his offer is too high and decide not to send in an offer. Another factor in transaction costs is that the buyer of the project also has some transaction costs. These costs are incurred from the effort needed to rank the different bids. These would be higher with complex bids. These costs are not taken into account in this model.

The end result for a player can be one of three options: first, a player did not compete and has zero utility. Second, a player competed and lost, and will have negative utility. Lastly, the winning player will often have gained much and have the highest utility of all the other players, assuming that players only send in profitable offers.

5.1.2 Strategies

Players can choose to ignore the transaction costs and apply a strategy from the basic model. They still must pay the entry costs. They can also take the transaction costs into account and change strategies. Players can either participate or defect. Defecting players do not compete in the game. Participating players can include the transaction costs in the price by adding a small buffer. This markup can equal the transaction costs to compensate for the extra costs. Defecting players can have different motives. Reasons for not playing are transaction costs that are too high, or winning chances that are too low. If the transaction costs are expected to be higher than the profit margin \( b_i - c_i \), players would lose money. Because players have more possible moves they can apply, the set of possible strategies is also larger.

Strategy 4 illustrates a learning strategy which increases both the costs and the price by the transaction costs. Another possibility is applying Strategy 2 and ignoring the transaction costs. This could lead to negative utility by winning. Strategy 5 differs from Strategy 4 in the part where a player chooses not to play. A player defects if he thinks his bid is too high.

---

**Strategy 4 Learning, never defect**

**Input:** Probability distribution \( F \), transaction fee \( f \)

- Determine costs \( c_i \) based on distribution \( F \)
- Compute markup \( m \): markup + \( f \)
- Estimate winning bid \( b_{\text{win}} \)

\[
\begin{align*}
\text{if } (c_i + m) &< b_{\text{win}} \text{ then} \\
&\text{Set price } \epsilon \text{ below } b_{\text{win}} \\
\text{else} \\
&\text{Set price to } c_i + m \\
\end{align*}
\]

**return** bid price
Strategy 5 Defecting in selected situations

Input: Probability distribution $F$, transaction fee $f$
- Determine costs $c_i$ based on distribution $F$
- Compute minimum markup $m$: markup + $f$
  - Estimate winning bid $b_{\text{win}}$
  - if $(c_i + m) < b_{\text{win}}$ then
    - Set price $\varepsilon$ below $b_{\text{win}}$
  - else
    - return ‘Not playing’
  - end if
- return bid price

5.1.3 Expectations

Overall, this game may not yield the best possible outcome. Perhaps a defecting player would have given the best offer. Because of defecting, less players are involved in the game which leads to less competition among players. Less price competition is bad for the buyer, who must pay extra. The buyer could get the same result as in the basic model, or a worse outcome, but not better.

Expectation 3. When one or more players defect, this negatively affects price competition. Overall, the buyer may be worse off.

Expectation 4. Often a player can gain higher utility by defecting.

Expectation 5. If a player always participates (thus never defects), he will often have negative utility. Always participating is not the best strategy.

5.1.4 Experiment setup

Two main experiments are conducted with this model. In the first experiment, the amount of transaction costs compared to the total project size is varied. All 5 players behave similarly. With higher transaction costs, players increase prices and will defect more often. A game where all players defect is called a failed game. In a failed game, the buyer has no contract since there is no winning bid. In the second experiment, there are 5 players who compete against each other. One of the players, Eve, will never defect. Her four opponents will defect if they believe it is the rational choice to do so. The transaction costs in this experiment are fixed at 2% of the project size.

In both experiments the players get the opportunity to learn, and results from before and after the learning phase are compared. Players can defect by sending in an invalid bid. Such an invalid bid contains their costs and it is used to compute utility and regret afterwards. Regret for defecting players is positive when they should have participated.

To test Expectation 4, the result of defecting is compared to the hypothetical result of not defecting. Would a defecting player still defect in hindsight? Would defecting generate more utility than playing? As long as the hypothetical bids are worse than the winning
5.1 Modeling transaction costs

Simple models

5.1.5 Results

Varying transaction costs  What is the effect of transaction costs on the buyer? In the following experiment all players will defect if they believe that is the best action. The transaction costs are varied as a fixed percentage of the estimated project value. The utility for the buyer is measured. Winning bids below the estimated project value give the buyer positive utility, because the project is cheaper than expected. $u_b = 100\% - b_w$. The results from the described experiment are shown in Figure 5.1.

Figure 5.1a shows the utility of the buyer. The results after learning (green line) will represent reality most closely. For tenders where the transaction costs are larger than 2.5% of the project value, the buyer will end up paying more than the estimated project value.
**5.1 Modeling transaction costs**

Players factor in transaction costs into their prices, which makes all prices go up. Interesting to note is that tenders with low transaction costs (0.5% or 1%) can gain extra utility for the buyer. If the buyer ensures a project has such low entry costs, he will be better off because the project turns out cheaper. The buyer can do this by requiring relatively uncomplicated offers or by reimbursing sellers for a part of the entry costs. However, in this experiment the buyer must deal with about a quarter of failed tenders. These failed games are not included in the buyers utility from Figure 5.1a. In reality, the sellers are less willing to defect and most tenders will succeed with a few offers.

As the transaction costs grow larger, more often a situation occurs where none of the players send in a bid. Figure 5.1b shows the relation between the transaction costs and situations where no bids are received. In those situation, the buyer has no offer to accept and the tender fails.

The results in Figure 5.1 actually show the opposite effect of Expectation 3. The reason for players defecting is that they believe their price is too high to win. This means that the players with the lowest prices do not defect. The buyer is not worse off by players defecting. Only if all players defect, the buyer is worse off, since the tender fails.

**Focus on small transaction costs** Figure 5.1a indicates that the buyer should prefer small transaction costs over large or none, since it yields higher utility. This might be partially explained by the observation that a quarter of the tenders completely fail, even with lower transaction costs. In this follow-up experiment the focus is on the effect of small transaction costs (between 0–1%). The results are presented in Figure 5.2. The percentage of failed tenders is not shown, while the utility for the buyer is shown in Figure 5.2a. Regarding the number of failed tenders, this number varies between 21–23% for $0 < f \leq 1$ and is 0% at $f = 0$.

The most interesting values in Figure 5.2a are those after learning (the green triangles). The utility for the buyer is $u = 100\% - b_{win}$. The utility for the buyer shows a decreasing trend as the transaction costs increase. Perhaps the buyer should expect sellers to increase prices to account for the transaction costs. Would sellers have a similar profit margin with or without transaction costs, if you account for the extra fee they must pay? I expect this will be the case.
5.1 Modeling transaction costs  

Figure 5.3: Distribution of winning players in a model with transaction costs. Eve is the only player who always sends in a bid.

Figure 5.4: Average regret, and total utility divided by number of wins. Eve never defects.

Figure 5.2b shows results where the buyer computes utility as $u' = 100\% + f - b_{\text{win}}$. In all cases where $f > 0$ the new utility for the buyer is similar. It is higher than the utility without transaction costs. Sellers bid more aggressively because of the potential loss from the transaction fee. If they decide to enter, they learn to lower their bids. Lowering their bids helps them to win more often, and reduces the risk of losses from the entry costs.

The result before learning (blue line) in Figure 5.2b can be explained by the strategies the sellers follow. Sellers start with an opponent estimate of $100 + f$. They estimate their costs and markup prices up to $100 + f - 1$. This would give the buyer $u' = +1$ since the winning price is typically 99. If a seller has costs too high to bid below 100, they will defect, this avoids negative $u'_b$.

**Eve never defects**  In the next set of experiments Eve never defects. The other players will defect if they believe it is the best move. Figure 5.3 shows how often each player wins before and after learning. Before, Eve wins about half or more of the games. After the learning phase, Eve wins even more games. The winning chances of the other players are roughly equal.
Figure 5.4 compares regret and utility in this experiment. Figure 5.4a shows the average regret for each player. It seems like Eve’s regret is average or below and has very little variance. This indicates Eve performs quite well in hindsight. When Eve evaluates her performance afterwards, she will find that deviating from her original plan is not often very helpful. The other players all show similar regrets. This would be expected, as the other four players have exactly the same behavior. Figure 5.4b shows the total utility divided by the number of wins for each player. Although Eve wins half or more of the games, her utility for a single win is pretty low. Overall, both Eve and the other players end up with similar profits. Although Eve wins more often, she regularly loses at the cost of the entry fee. In the end, all five players have similar profits. T-tests are used to determine the significance of the utility results.

Comparing Eve’s utility results to the opponents results’s with a 2-sample t-test yields a p-value of 0.0208 for the null hypothesis $u_{Alice} = u_{Eve}$. This p-value is below the threshold of 0.05. This result means it is unlikely that Eve’s has the same average utility as the other players. Another t-test with null hypothesis $u_{Alice} \geq u_{Eve}$ yields a p-value of 0.010, which indicates that Alice’s utility is less than Eve’s utility. Eve gains more utility on average.

Players who often defect will win more in a single auction and hardly pay for the transaction costs. In single instances, they do get better outcomes, which is in accordance with Expectation 4. Eve strategy of never defecting seems to be not significantly better or worse than the other strategy. This makes Expectation 5 likely since never defecting is not significantly better than sometimes defecting.

### 5.1.6 Discussion

In practice, contractor experts have said that they have a go or no-go decision depending on the estimated transaction costs of a project. If the transaction costs for a tender are larger than one or two percent of the total project, contractors will decide not to participate in said tender. When contractors receive a (partial) compensation for their entry costs, they can consider this in their decision to participate or defect. Within the previous experiments, the buyer profited from relative small transaction costs. Transaction costs of 1% of the project value or less, were similar or even better than projects without transaction costs.

In the experiments within my limited model, players often decide not to participate in the game. In practice, this rate of defects is much lower. Companies have incentive to participate, because they will go bankrupt if they never receive any work.

In advance Expectation 3 was stated that more defecting players would be bad for price competition. In simulations this was not the case. In literature, the opposite effect was stated: transaction costs may lead to players defecting, but still the optimal number of players participate. Defecting players do not negatively effect the outcome. Intuitively, players who are confident their offer is low enough will participate. Players who think their price is too high will defect. All the cheapest players, who present offers beneficial to the buyer, decide to play.
5.2 Modeling complex bids

**Reimbursing transaction costs**  This section describes a suggestion for a future experiment, where sellers receive a reimbursement for their entry costs. Remember also Case 2, in which contractors received compensation for their effort in designing a new building. Based on the experiment with varying transaction costs, it seems to be appealing to the buyer to secure low transaction costs. To lower these costs, the buyer could decide to reimburse them. This will cost the buyer extra, as each seller must receive the same reimbursement. The buyer should consider whether the extra utility gained outweighs the extra costs. The buyer can also lower transaction costs by being less demanding in the contract. When an offer has less requirements, it will be easier to compose by the sellers. The problem with this approach is that the buyer may not be willing to relax enough requirements to decrease the transaction costs significantly.

As a future experiment, the buyer reimburses sellers for their transaction costs. The buyer can reimburse a part or all of the transaction costs. The costs for the buyer are the costs of the winning offer plus five times the reimbursement. I expect that if the buyer reimburses all of the transaction costs, sellers may abuse this. A seller can send in a bid strategically with little effort and collect the reimbursement. If only a part of the transaction costs are reimbursed, sellers could still defect if the remaining entry costs do not outweigh their expected utility.

**Expectation 6.** *A larger reimbursement by the buyer may cause players to send in fake high-priced bids to collect the reimbursement.*

In addition, if the buyer offers a high reimbursement, she must pay all these extra fees. This lowers the utility of the buyer. I expect that the extra fees do not outweigh the added benefit from lower project price.

**Expectation 7.** *The higher the reimbursement by the buyer, the lower the utility will be for the buyer in a single auction.*

However, in practice contractors will bill the buyer for the transaction costs they have in one way or another. Reimbursement is the most direct way. Without reimbursement, contractors will increase their prices on tenders they win, to account for all the lost costs from tenders they have lost.

5.2 Modeling complex bids

For certain projects it useful to consider both price and quality (Case 4), instead of only focusing on the lowest possible price (Case 3). In an EMAT tender the players send in multi-dimensional bids. Bids contain a price, and also some representation of quality. Players can make a trade-off between price and quality: higher quality also represents higher costs for the player. A high quality offer with a low price will lead to losses for a player. Because of the complex bidding, costs have more influence on a player’s behavior. Costs are not constant for a project, but they depend on the chosen quality.
5.2.1 Literature

In literature, auctions with complex bids are known as multi-attribute auctions. In a multi-attribute auction, the auctioned object has multiple properties which determine its value. Each bid must specify a value for every attribute.

Che [5] first discussed models for government procurement in which firms bid on both price and quality. Che discusses three different auction models: first-score, second-score and second-preferred-offer. In the first-score auction, each contractor submits a sealed bid and the winner must provide the offered quality at the offered price. The first-score model relates best to my model. In the other two models, the winner must provide quality and price related to the second-best, or highest-rejected, bid.

David et al. [8] discuss protocols for multi-attribute auctions. One of their protocols is a variation of the first-price sealed-bid auction. It is based on the model of Che [5]. David et al. perform a number of experiments with this model. They find that the buyer is more motivated to announce a scoring function close to his own utility function as the number of bidders increase.

Bichler [4] performs experimental research on multi-attribute auctions. He compares single-attribute auctions with multi-attribute auctions and finds that utility scores were significantly higher in the multi-attribute auctions. In tenders, this would indicate higher utility for the buyer if an economically most advantageous tender is used instead of an lowest-price tender. Bichler’s experiments also indicate that efficiency was similar between both auction types. He measures efficiency as the percentage of auctions where the bidder with the highest valuation wins the auction. In tenders this translates to the contractor with the lowest costs winning the tender.

If there are multiple attributes to consider, Bichler suggests using a decision support tool. It may not be obvious for the bidder which combination of attributes yields the highest utility. Bichler suggests using such a tool to help in multi-attribute auctions for use in corporate procurement. He thinks buyers in a professional environment will adapt quickly. Less experienced buyers face the risk that their utility function does not correspond to their preferences. In tenders this is a known problem: government agencies who want to use EMAT tendering must be careful in defining the EMAT conditions. If these conditions do not match with their preferences, the tender may not yield the best possible outcome for the agency.

5.2.2 Game model

I propose a model similar to the first-score auction by Che [5]. The buyer solicits bids from $n$ contractors. Each bid defines a quality $q$ and price $p$. Quality may represent multiple factors, such as delivery date, risk management or other standards. For simplicity, quality is modeled as a one-dimensional attribute. Quality in EMAT tenders can also be translated to a single value: the bonus or penalty for a specific bid.

The score of a bid is $s(p, q) = p - V(q)$, where $V(q)$ is the value the buyer has for quality level $q$. This is the bonus the buyer is willing to give in an EMAT tender. The bid with the lowest score wins the auction. All players know $V(q)$ for each $q$ in advance. All sellers
have a cost function \( c(q) \). The costs for a seller increase as the quality level increases. For each quality level \( q \), a corresponding costs level \( c(q) \) is given. The utility for the winning seller is \( u(p, q) = p - c(q) \). All losing sellers have zero utility. The buyer’s utility for a bid is \( u(p, q) = V(q) - p \) which is equal to the score of a bid.

5.2.3 Strategies

Players can determine their cost and the evaluation price for the preferred quality level. They then send in the offer with the best evaluation price. One strategy is to determine costs for all different quality combinations and then pick the offer that a player deems best. This is represented in Strategy 6. Another strategy is to ignore all the quality and just determine the lowest price for the lowest quality.

**Strategy 6** Try all the possible complex bids

**Input:** EMAT bid description, substrategy

```
for each possible quality do
  Determine costs and bid price
end for
Select a single bid to submit, based on expected utility
return best bid \((p, q)\)
```

Players determine costs and prices for all possible quality levels. Based on the cost and price of each bid, players can determine which bid they believe has the highest expected profit. They will send in that bid. Other players might not consider all possible bids. Especially in more extensive EMAT tenders, there could be too many possible bids. Player can also consider a subset of bids. A strategy can be to bid only the lowest price with the minimum quality. Another approach is to focus more on quality and offer bids with the highest quality demands.

5.2.4 Expectations

Not all possible price–quality combinations will be selected by sellers. Since costs generally increase with quality, players must increase prices as quality increases to still make a profit. Players can weigh the quality bonus against the costs increase necessary to reach that quality. If a quality provides a small bonus but a very large costs increase, players may wish to avoid this quality level and select another quality. When quality has a relative small weight compared to the price, players may disregard quality and focus only on providing an offer with the lowest price. Since the quality weight is small, the penalty this yields is also small.

**Expectation 8.** If quality bonus and the costs of quality are similar, players will be indifferent towards choosing a quality level.

**Expectation 9.** If the weight of quality is small compared to the weight of bid price, players will show less preference for a specific quality level.


5.2 Modeling complex bids

5.2.5 Experiment setup

All sellers learn the quality valuation \( V(q) \) of the buyer. This is used as the scoring rule for ranking bids. Each player may consider multiple bids, each with a different quality, and select a single bid to offer the buyer. The buyer evaluates all bids by determining and ranking scores to find the winning bid. The bid with the lowest score wins, this may not always be the bid with the lowest price.

Regret in an auction with complex bids is different from regret in the basic model. Regret measures how much better a player could have done, considering the bids from his opponents. Thus a player must find the best possible bid for that setting. This can be a bid with very different price and quality. Regret is computed in Equation 5.2. Regret is the maximum of all price regrets (see below) over all possible quality options.

\[
\text{regret } r_i = \max_{q \in Q} \{ r'_i(q) \} \tag{5.2}
\]

For the coming simulations, a different form of regret will be used in which only price is considered. This is called price regret or \( r' \). Price regret indicates how much more utility a player could have gained by changing only the price of his bid. This means that the choice of quality remains the same. Price regret is computed more similar to the regret from the basic model. See Equation 5.3.

\[
\text{price regret } r'_i(q) = \begin{cases} 
s_{2nd} - s_i & \text{if player } i \text{ wins} \\
\max\{0, s_{win} - (c_i + V(q))\} & \text{if player } i \text{ loses} 
\end{cases} \tag{5.3}
\]

Similarly quality regret can be defined as the extra utility a player could have gained by switching quality and keeping the price fixed. Quality regret will not be used further on.

5.2.6 Results

Three quality levels In this first experiment all five sellers act similarly by considering all possible bids and offering the most profitable bid. The bids focus only on price and risks. The possible quality levels are -30\%, 0\% and 30\%. A quality level of +30\% induces 30\% extra costs. Figure 5.5 shows the distribution of quality levels among the winning bids. Players mostly select the highest quality and sometimes the lowest quality. Neutral quality is hardly ever selected. Apparently players prefer the most extreme choices available. This
contradicts Expectation 8, since players are not indifferent among quality choices. Instead they express clear preferences by mostly selecting the same quality levels.

Che [5] mentioned that the ‘naive’ scoring rule of following the buyer’s utility over-rewards quality. This effects seems to occur in Figure 5.5. The higher quality levels are selected as winner more often. This effect of overrewarding quality can be countered by subtracting a $\Delta q$ from the score of a bid. $\Delta q$ is a small factor which counteracts the amount in which quality is overrewarded. [5]

**Only two quality levels** In the previous experiment players would prefer a quality +30% bonus over 0% and they would hardly ever bid the -30% quality penalty. If only two of these quality levels were possible, would players express the same ordering of preferences? I expect this will be the case. Again, the chosen quality levels are -30%, 0% and 30%. In turn, each of the three quality levels is left out of the experiment.

**Expectation 10.** Players who have a preference ordering over quality levels, will express the same ordering over a subset of these quality levels.

Figure 5.6 shows the distribution of quality levels among the winning bids. Figures 5.6a and 5.6b seem to be proportional to the results from Figure 5.5 where all three quality levels were included. Only Figure 5.6c stands out, since the proportions of neutral and insufficient quality are the opposite of what you would expect based on results in Figure 5.5. These results contradict Expectation 10. This might be explained by the overrewarding of quality: neutral quality is chosen more often than the lower quality which incurs a penalty.

5.2.7 Discussion

Quality is overrewarded if the buyer uses his own utility function as a scoring rule. This was shown in literature and also empirically shown in experiments. If a buyer does not wish to overreward quality, he should select a scoring function different from his actual valuation. He should lower the rewards for quality to counter the effect. Future experiments can determine how much different the scoring function should be from the quality valuation of the buyer. With the right quality penalty $\Delta q$ quality is no longer overrewarded. In this
case the players are indifferent to quality levels. This assures the set of bids is expected to be distributed evenly over the different quality options. The necessary penalty may depend on the quality $q$.

Expectation 9 has not been tested experimentally in the previous section. Pries and Reeuwijk [28] have shown that the ratio between quality and price should optimally range from 40% to 60%. If price has a weight larger than 60%, a lowest price tender should be used instead of an EMAT tender. If the quality weight is too large, contractors can submit unreasonably large prices for a project, which should be avoided.

The weights of price and quality should be selected carefully in an EMAT tender. Buyers could also decide not to use EMAT tenders at all. Since quality is overrewarded and players mostly bid on the same quality level, a lowest-price tender with fixed quality demands could be sufficient. Players have to use less effort to decide on an offer, since they do not have to decide on quality, but solely on price.

What is the relation between learning and complex bids? Depending on the beliefs of players, they decide on an offer. The distribution of quality levels chosen among the winning bids may be different before and after learning. Learning is useful in the basic model where offers only contain a price. With offers that contain price and quality in an EMAT tender, learning changes. Players should not consider only the price but instead learn from the combinations of price and quality.

5.3 Modeling repetition

Modeling repetition appears to be easy: select a game and play it repeatedly. Repetition also occurs in practice with tender contracts. Some tenders can be repeated each year, consider for example the maintenance contract from Case 5. With repetition the history of previous games usually becomes available to the players. A contractor may want to start in a new field, see also Case 6, and needs to gain experience by winning tenders related to this field. In this way he builds up history. In literature, repeated games have been studied extensively. A repeated game is very simple to describe, but its effect on game outcomes is not always simple.

5.3.1 Game model

The first-price auction is not altered and it is repeated a number of times. The players do not know how many iterations there will be. Players still have to account for costs when determining their bid. When players bid below their costs and have not built a buffer, they could go bankrupt. Players can build such a buffer by winning auctions at a profit.

Not every project has the same size. Players determine their costs as a percentage of the estimated project value. Each project thus has an estimated value. This value is determined by the buyer and is made public at the announcement of the game. Players will have to account for the different project sizes.

This model is not yet very interesting. If a player always loses, he will remain with zero utility. In practice, this is unlikely. When a contractor never wins a tender, he does not have any business and can go bankrupt. To model this difference, players must pay a certain fee
5.3 Modeling repetition

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each round, whether they win, lose or defect. This represents the fixed costs that a company has, for example from employees and offices. Losing repeatedly will give players negative utility, similar to companies who make losses if they never win.

5.3.2 Strategies

Players can choose either a simple strategy which is history-insensitive, or they can use a strategy that changes based on the observations in previous games. Strategy 7 illustrates how a player may use his buffer to win a game. Because the game is repeated players can observe historical outcomes. They can use those observations to learn and improve their beliefs about opponents.

**Strategy 7 Using your buffer**

**Input:** Probability distribution $F$
  - Draw costs $c_i$ from $F$
  - Compute markup $m$

**if** If the winning chances are too low **then**
  - Lower bid price, but stay above costs
  - Optionally: use buffer to lower price and increase winning chances

**end if**

**return** bid price

5.3.3 Expectations

In a repeated auction, players can go bankrupt. Assuming that all players start out with zero money, they can then gain money by winning a tender at a price higher than their costs. Players with more funds could bid below costs at following tenders. This lower price will help them win. Still, submitting prices below costs yield negative utility and players will avoid this. Players lose money due to the fixed costs they must pay every round. These fixed costs will motivate players not to defect. The higher the fixed costs are, the more incentive players have not to defect.

**Expectation 11.** Rational players will always participate and never defect in an auction round.

5.3.4 Experiment setup

A repeated auction can have a different number of rounds. In this model, there is a distinction between the number of auction rounds and the number of simulation rounds. In one simulation, the auction has a number of rounds. The number of simulation rounds is still 1,000, where the average outcomes are reported to eliminate randomness. The number of auction rounds within each experiment will vary. During each auction all players can enter. After each round, the winner and the winning bid are announced. The winning seller
receives his profit, which is added to his buffer. Players can use buffers from previous wins to compensate for losses in following auctions.

### 5.3.5 Results

In an initial experiment, the average utility per game is shown for a varying number of auction rounds. The auction rounds vary from a single round up to 1,000 rounds. Figure 5.7 shows the average utilities. The average utilities converge to a value around 0.43. In further experiments, simulating 1, 10 and 100 auction rounds would probably be representative results.

In the next experiment each player pays a small fee during each round, whether they win or lose or possibly defect. These represent fixed costs and are set to 0.1% of the project size. The average utility is determined for each player by dividing his buffer by the number of rounds. Results are averaged over 1,000 simulations. Figure 5.8 shows the utility results for each player. As the number of rounds increases, the standard deviation in results decreases. The average results do not change significantly as the number of rounds increases.

Expectation 11 is only relevant since there are no transaction costs within this model. Players do have fixed costs every round, whether they defect or participate. Players might
as well participate in every game, since defecting players still pay the fixed costs and have zero chance of winning.

5.3.6 Discussion
Within this model, sellers have no incentive to defect. They are also indifferent to losing repeatedly. In practice companies behave differently. Companies can decide not to compete in a tender. This may be because of transaction costs, or lack of capacity, or perhaps a lack of experience within a certain field. Contractors in practice are not neutral towards losing. Because of fixed costs, such as from maintaining office space and staff, companies gain more incentive to win a tender, the longer they have been losing. A contractor may forcibly win a tender by offering an extremely low price. This contractor may not make a profit on such a project, but at least receive some income.

5.4 Modeling uncertainty
The actual costs of a project will usually differ from the costs estimated in advance. Projects have risks, which can lead to extra costs. Uncertainty can be different among players. For example, certain players may be experts in their fields. Other players can be inexpert because of a lack in experience. Deltas et al. [10] consider auctions with a single inexpert bidder. They start with a pool of fully rational players and add a single player who is less informed but who believes to be informed equally well. This bidder’s information is only slightly different his opponents’ information. The presence of the inexpert bidder can discourage all rational players from participating in the auction.

5.4.1 Game model
In this auction model with uncertainty, uncertainty is modeled in two different ways: first in costs, where the winner’s actual costs are determined in hindsight and second in experience, where players are either inexpert or expert bidders.

In practice, uncertainty is a significant problem in tenders. For a government agency it is near impossible to provide a complete project description to all contractors. There will always be uncertain aspects in a project, such as incomplete information, weather problems, building permit issues, incompetent subcontractors or even explosives found in the ground. Case 8 is an example of this uncertainty where the cost of materials rises. The way players will handle uncertain aspects in practice can vary. Previous work experience in a nearby area may have shown difficulties with archeology findings that caused delays, which leads to a very pessimistic view on the area by this player. This player will expect higher costs than another player who has not encountered such problems.

Uncertainty about a project can be expressed by players randomizing their belief about the costs of the project. In the first-price auction the players already used a randomized strategy to determine their costs for a project. In these strategies the predetermined costs are always correct. In order to better model the players’ uncertainty, the exact costs will only be determined in hindsight. Players can make an estimate in advance and they can
base their offer on this estimate. After winning, the true costs will be randomly determined. The utility of the winning seller is based on his bid and the true costs in hindsight.

Players who are new in a certain area of expertise are more uncertain than experienced players. The new player does not yet know how to estimate the costs of a project properly. This player bids more randomly because of his uncertainty, this also happened in Case 7. He could also have a bias in estimating his costs and think that the costs of a project will be significantly higher or lower.

5.4.2 Strategies

Players with cost uncertainty can use the same strategies as players without this uncertainty. They can also account for uncertainty of costs determined afterwards. Players have a risk in determining their offer, because the real costs are determined after winning. Thus, players can have risk-averse, risk-neutral or risk-seeking behavior.

An inexpert player can have a systematic error where he always bids significantly higher or lower than his opponents. Inexpert players can also have a larger standard deviation in their bidding.

5.4.3 Expectations

Players who encounter cost uncertainty have higher risks. If their costs turn out higher than estimated, they can end up with negative utility. Players could decide to account for this extra risk by increasing their bid prices.

Expectation 12. If a player has more cost uncertainty, his bid price will increase to account for this risk.

In a game with a single inexpert player I expect that this player can greatly influence the outcome. Depending on the cost estimate of an inexpert player, he can bid much lower or higher than his opponents. If there is such a large difference between bids, the inexpert player will either mostly win or hardly ever win. A single player who always or never wins is expected to influence the winning chances of all the other players.

Expectation 13. Inexpert players with extremely low offers will ruin winning chances for the other players.

Expectation 14. Inexpert players with extremely high offers will never win.

Expectation 15. Inexpert players with extremely high offers will positively influence the winning chance of all the other players. All the other players will get proportionally higher winning odds.

5.4.4 Experiment setup

Cost uncertainty is simulated by determining the actual costs of players after the auction. After the auction, the difference in costs is drawn randomly for each player. This value is added to the estimated costs of the player. In this way, the actual costs are related to the
5.4 Modeling uncertainty

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Figure 5.9: Average utility and regret in a model with uncertainty. Players are risk-neutral.

costs estimated in advance. In the following experiments, each player has a value $\Delta c_i$ drawn from a normal distribution with mean zero and standard deviation 3. The actual costs are determined as $\Delta c_i + c_i$. These true costs determine the regret and utility of each seller.

The other form of uncertainty concerns inexpert bidders. A single inexpert bidder is added to the mix of players. The influence of this player on the auction game is analyzed by looking at the distribution of winners among all the players. The winning chance for the inexpert player herself is another result to analyze, based on varying levels of being inexpert.

5.4.5 Results

Uncertainty in costs Figure 5.9 shows average utility and regret in an experiment with cost uncertainty with risk-neutral players. These results should be compared to the results of an auction without uncertainty, which are the results of the basic model in Section 4.3.6.

Figure 5.9a shows a larger standard deviation in the players’ utilities than the basic model in Figure 4.3a. A t-test with null hypothesis $\sigma_{\text{basic}} \geq \sigma_{\text{uncertain}}$ yields a p-value < 0.001. This indicates that the players have significantly larger variance in the model with uncertain costs. The null hypothesis that $u_{\text{basic}} = u_{\text{uncertain}}$ yields a p-value of 0.604, so the average utility is not significantly different. These results make sense since players have the same expected costs, their expected utility does not change. However, they will have more variance in their costs, which directly influences the variance in their utility.

Figures 5.9b and 4.3b show regret in the model with uncertain cost and the basic model. These figures indicate that uncertainty with regard to costs does not impact regret significantly and t-tests confirm this.

The players in this experiment are risk-neutral. The expected change in costs from uncertainty is zero, players who are risk-neutral will not alter their prices. How players adapt to uncertainty, depends on their attitude towards risk. Expectation 12 will apply only when a player is risk-averse. Risk-seeking players may even lower bid prices and for risk-neutral players, nothing will change.
5.4 Modeling uncertainty

A single inexpert bidder  Figure 5.10a shows the influence of a single inexpert bidder. Eve is an inexpert bidder and all her opponents are experts. All the players send in a bid of $c_i + 1$ and there is no learning involved. Because Eve is inexpert, she can be biased in cost estimate. This bias can be both positive and negative. Figure 5.10a indicates that Eve will win most of the games if she believes her costs to be very low. If Eve overestimates her costs, she has little chance of winning. The results show that the lower Eve is bidding, the closer to 100% Eve’s chances of winning are. This confirms Expectation 13. When Eve wins 100%, or close to 100% of the auction simulations, the other players don’t stand a chance. On the other side of Figure 5.10a it shows that sending in really high bids will not help Eve to win an auction. Intuitively, this makes sense. The higher Eve’s bids are, the higher the chance that another player will propose a lower bid. This confirms Expectation 14.

Figure 5.10b shows the winning distribution among players when Eve is bidding very high. With four equal players, each participant would have a winning chance around 25%. With eve always losing, the winning chances of her opponents increase. The opponents now each have a winning chance around 33%. This is in accordance with Expectation 15.

5.4.6 Discussion

In tenders contractors will always have cost uncertainty. As contractors gain experience with certain work or locations, their cost uncertainty decreases. Because of new experience they can better estimate the actual costs of a project. Some uncertainty about the actual costs of a project will always remain.

In practice, there will not be many inexpert bidders who greatly vary in bidding strategies from all other contractors in a field. In the selection phase of a tender, buyers can set experience requirements. If a contractor does not have the minimal required experience, they will be out of the running for that tender. Winning a tender at a great loss by underestimating the costs is a costly mistake. A contractor will be more careful in future tenders to ensure this does not happen again.

Future experiments could focus on the effect of cost uncertainty for the buyer. If risk-averse sellers are more uncertain, they may increase prices and the buyer could have lower
utility. The buyer could avoid high uncertainty for the sellers by carefully creating the project description and demands. Another direction for future experiments is the relation between inexpert bidders and cost uncertainty. Maybe not all players believe that there is cost uncertainty. Players can use different strategies for both types of uncertainty. The beliefs players have on the amount of cost uncertainty and how inexpert opponents are, can influence their strategies. Players may want to protect themselves against risks of inexpert bidders. If the inexpert bidder always bids very low, this may give the players reason to bid lower than they normally would.

5.5 Modeling continued work

Continued work is quite similar to uncertainty. Each project incurs risks. Some risks are for the contractor, which result in higher costs. Some risks are for the buyer, which result in continued work. The example of finding bombs at the work site in Case 10 is similar to Case 8 where materials become more expensive. The difference is that the risk in Case 10 could not have been foreseen by the contractor. The buyer side must pay for the extra costs.

Sometimes a government agency is not careful enough in constructing a contract. Case 9 discusses this situation. The buyer did not state all the relevant project requirements within the contract, so now the winning contractor can bill extra for all demands beyond the contract’s requirements. This makes the extra work very profitable compared to the initial project. Contractors have beliefs about the amount of continued work after winning. In this model the actual amount of continued work is determined afterwards, similar to the actual costs for the winning contractor in the model with uncertain costs.

5.5.1 Literature

A variant of an auction with continued work is an auction with a future. Goeree [15] considers an auction with an aftermarket. The winner of an auction can gain in a future market. All players can enter in this aftermarket. In this model the profits in a future market depend on the winner’s private information and on how others perceive him to be. Players can exaggerate their private information to influence future interactions in a way that is more profitable in the aftermarket. Such exaggeration is called signalling. Signaling behavior by players is different depending on the type of auction. In a first-price auction players signal less than in the second-price auction.

The gains from an aftermarket can be similar to continued work in a tender. The expectations that players have about the amount of continued work can influence their bids. The bids from opponents can indicate what their beliefs about continued work are. A difference with the work from Goeree is that the amount of continued work does not depend on the winner’s private information or beliefs from opponents about the winner. Instead, continued work mostly depends on the risks for the contracting authority.
5.5.2 Game model

In this game, the auction is not really over after winning. The winning player can expect more continued work after winning. Either from unforeseen extra work, or because a project helps build knowledge and experience that leads to more future work. The latter is more applicable in a repeated game.

The amount of continued work is uncertain, and the beliefs about the amount of extra work can differ per player. All players get a probability distribution over the expected amount of work after winning. They can take their expectation on continued work into account when they determine their offer. For example, if a player expects a lot of extra work, he could lower his offered price.

The amount of continued work is modeled in the following way. Every player has a certain belief on continued work. With a chance $p$, there will be some extra work, and thus with chance $1-p$ there will not be any continued work. If there is any continued work, this is modeled with a probability distribution. A risk-neutral player would take the average/mean of this expectation to represent the expected continued work. $p$ times this amount yields the expected gains after winning the tender.

Not all of the expected continued work can simply be subtracted from the current offer. Extra work will also incur extra costs, but the extra work will lead to extra profit. This profit will be around 10% of the extra work. If player expects a certain amount of extra work, this player could subtract 10% of the expected extra money.

5.5.3 Strategies

Players can still use the strategies from the first-price auction and ignore the expected amount of continued work. If players take into account the continued work, they can determine an offer based on costs and then subtract (part of) the expected profit from continued work from the decided price.

5.5.4 Metrics

Continued work $w$ is the amount of continued work. Regret for player $i$ is computed as given in Equation 5.4. A winning player has regret if he could have bid higher and still won. His regret indicates how much extra the player could have gained by bidding higher. The profit from continued work adds to this regret. Losing players have zero regret if they could never have won. If a losing player could have won by bidding lower, his regret indicates how much utility this player could gain by winning.

$$ r_i = \begin{cases} b_{2nd} - b_i + 10\% \times w & \text{if player } i \text{ wins} \\ \max\{0, b_{\text{win}} - c_i + 10\% \times w\} & \text{if player } i \text{ loses} \end{cases} \quad (5.4) $$

5.5.5 Uncertainty versus continued work

The uncertainty parameter is quite similar to this continued work parameter: both have uncertainty, either in costs or in the amount of extra work afterwards. Is it always possible
5.5 Modeling continued work

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To convert either model into the other? A distinction must be made between the general case and the specifics of my model.

In general  Because of costs uncertainty, players can incur extra costs afterwards. Continued work can give players extra profits afterwards. These are opposite, but similar effects. Assume cost uncertainty is measured as the amount of extra costs that is determined after winning. This amount is represented by a random variable $U$. Continued work is represented as the amount of profit continued work gives a player, represented by random variable $W$. $U$ and $W$ can be described by their probability density functions $f_U(x)$ and $f_W(x)$.

Theorem 3. If $f_U(x)$ and $f_W(x)$ are equivalent (one can be transformed into the other), the effects of uncertainty and continued work are also equivalent.

If $f_U(x)$ can be translated into $f_W(x)$, the effect of uncertainty can be translated into the effect of continued work. Players account for $-E[W]$ where they originally considered $E[U]$. This makes all players act the same in both models.

If $f_W(x)$ can be translated into $f_U(x)$, the effect of continued work can be translated into the effect of uncertainty. Players account for $-E[U]$ where they originally considered $E[W]$.

In my model  The above is a general result. In my model the (profit from) continued work is modeled by a different probability distribution than the uncertainty parameter. Uncertainty always occurs in its model and it is modeled by a normal distribution, $U \sim \mathcal{N}(\mu, \sigma)$. The expected extra costs are $E[U] = \mu$. This is illustrated in Figure 5.11a. Continued work has a slightly different probability distribution. There is a certain chance $p$ that continued work occurs at all. If there is any continued work, it is modeled by a normal distribution: $W \sim \mathcal{N}(\mu, \sigma)$. This distribution is shown in Figure 5.11b.

Uncertainty can be seen as a specific instance of continued work. In this instance $p = 1$ since uncertainty always occurs. Thus, uncertainty can be modeled as a form of continued work. The other way around, continued work cannot be translated into costs uncertainty. The uncertainty parameter is modeled to limited to capture the chance that continued work does not occur.
5.6 Modeling irrationality

In most game theory literature all agents or players are assumed to be rational. Modeling rational agents is about finding strategies which generate the most utility. Irrationality is difficult to model. Irrational agents can perform any strategy which is not rational. There exists a variety of such strategies.

Often, a player may seem irrational, but a deeper search will conclude that there is an underlying reason for his behavior. The player seems irrational in a limited model, but if more parameters are taken into account, the player appears rational. For example, a player plays with random bids, instead of a seemingly logical bid. This could be because a player is new to a certain area of expertise and does not yet know the actual costs of a project. This player is not irrational, but he is an inexpert player, like in Case 7. In a limited model, uncertainty can be included to explain the behavior of this player.

Behavior by contractors in practice cannot always be attributed to uncertainty, transaction costs, repetition, continued work or complex bids. The irrationality parameter is modeled as an explanation for all unexplained behavior with regard to the other parameters. Possible other parameters can be included in the model to explain unexpected behavior.

5.6.1 Examples of further parameters

There are many other factors which could influence contractors’ strategies. A selection of such parameters will be discussed. Communication between contractors, restricting laws, lack of capacity or the current market state are possible parameters. It is also possible for a contractor to have had negative experiences with a certain opponent and avoid future tenders with this buyer, see also Case 11.

Basu [3] considers rationalizable strategies, these strategies seem irrational at first, but will yield higher pay-offs eventually. He calls this strategic irrationality. In certain situations, it can benefit a player to apply a seemingly irrational move. Often players act history-insensitive. They will keep believing that all opponents are rational, no matter what they observe. It is possible for players to learn from previous games and predict what the other players will do. A player may assume at first that all everybody is rational, until he observes an outcome which contradicts this belief. Basu uses rationalizable strategies to explain cooperation in repeated games, such as the repeated Prisoner’s Dilemma. The total pay-off over all game rounds can be improved by acting strategically irrational.

If contractors can communicate with each other, it is possible for them to apply group strategies. By working in groups, players can try to manipulate the game outcome to their benefit. Communication between players and group strategies occur in Case 12. Barberà et al. [2] focus on cases where individual and group strategy-proofness coincide. If a game is individual strategy-proof, no single player is able to gain higher utility by manipulating the game. A game is group strategy-proof if no group of players can manipulate the outcome by declaring false information. Individual strategy-proofness is easier to ensure in a game than group strategy-proofness. That is why often the focus is on making sure no single player can cheat. Barberà et al. search for conditions of a game which satisfy both individual and group strategy-proofness.
If players are allowed to communicate in advance, they can make mutually beneficial agreements. It is called coalition-proof if players have no incentive to deviate from these agreements. Peleg [25] shows that almost all equilibria in dominant strategies of finite games are coalition-proof. Peleg also shows that games without dominant-strategy equilibria can still have a coalition-proof equilibrium. Every two-person finite game has at least one coalition-proof equilibrium.

### 5.6.2 Influence of parameters

Which of the previously modeled parameters holds the most influence over the outcome of a tender? One experiment that examines this is to fix the players and their strategies and then vary the model. On a given set of agents this will show the influence of each parameter with regard to the basic model. It is interesting to analyze the spread of winning bids or the average utility per player per game.

I expect that uncertainty will have a large effect on the outcome of the game. Winning bids will vary more wildly. Transaction costs influence the outcome in the number of participating players. When players mostly defect there will be little price competition and the winning bids will be higher on average.

### 5.6.3 Relation to tenders

If one knows which parameter has the most influence on the outcome of a tender, one can design a tender with the exact combination of desired parameters. The effect of each parameter can be made larger or smaller in practice. The buyer designs a tender and is able to choose which parameters are more prominent. The following list explains how a buyer can alter the significance of each parameter within a tender.

- **Transaction costs.** Transaction costs can be reimbursed, to make them smaller. In designing a tender, the buyer should also consider the constraints he is asking from the contractors. When a buyer requires a lot for a basic offer, the transaction costs are higher.

- **Complex bids.** A buyer can completely negate the effect of complex bidding by deciding not to use an EMAT tender. If the buyer uses a lowest-price tender, the sellers will not focus on providing extra quality beyond the basic tender requirements.

- **Repetition.** There will always be some influence because of repetition in a tender. Contractors will want to compete in multiple tenders to remain profitable. A buyer can split up a tender into smaller, similar tenders to increase the influence from repetition. For example, a buyer can split a 10-year maintenance contract into several tenders of 1 or 2 years of maintenance work.

- **Uncertainty.** The effect of uncertainty in costs can be limited by increasing the amount of information the buyer distributed among the sellers. The more information a buyer hands out, the less uncertainty each seller will have in determining his costs. The buyer can weed out inexpert bidders by increasing the experience required to
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complain in the tender. Inexpert bidders will be rejected during the selection phase in the tender.

- Continued work. The buyer makes up the contract which includes all requirements the sellers must adhere to. If the buyer is not very careful in writing up the contract,

5.6.4 Discussion

A future experiment is to measure the influence of each parameter. Create a base experiment with the basic model. Compare its outcomes to that of all the parameters. For example, let each player use the strategy of a fixed markup $x$. Players can learn and alter their markup after each auction round. Let the players continue until an equilibrium occurs: players have zero regret and they do not alter $x$ anymore. Each parameter is expected to yield a different equilibrium. Some outcomes may be closer to the basic model than others. The parameter with the most influence will be the parameter which has an equilibrium where $x$ differs most from $x$ in equilibrium in the basic model.
Chapter 6

Composite models

This chapter contains all the work on models where multiple parameters are included. Not all possible models are researched, as mentioned in Section 4.2.7. In fact, not even all parameters are included in this chapter. The irrationality parameter models all other possible parameters which are not included in this thesis. Irrationality is not considered in any of the complex models within this chapter. Section 5.5.5 showed how the uncertainty and continued work parameters are similar. It is possible to translate one of them into the other. It should be sufficient to select either of these two parameters for the composite models. A lot of the models have the parameter transaction costs. This parameter is the only one that could influence players to defect during a game. Defecting players lead to less competition and this effect is interesting to analyze in combination with different other parameters.

6.1 Transaction costs and complex bids

The combination of transaction costs and complex bids is an interesting one because transaction costs may vary based on the structure of a complex bid. A bid of higher quality will have higher project costs, but will also take more time to construct and thus has higher transaction costs. This model is about the relation between the quality of a bid and its transaction costs. Since transaction costs are often discarded within game theory, there has not been much attention for multi-attribute auctions with entry costs.

6.1.1 Game model

In this game model both transaction costs and complex bids are taken into account. In a tender the transaction costs vary depending on the type of bid a contractor decides on. Bids with a large focus on quality take more time and resources to construct and thus have higher transaction costs.

The amount of transaction costs does not have to be a fixed portion of the expected project value. Instead, transaction costs may increase much faster than the project value. During the design of an EMAT tender, the buyer determines how much work is required to compose a valid offer. The relation between project value, including the quality level, and the transaction costs can be varied. For example $f$ can grow linearly as project value...
Another option is to have $f$ grow exponentially as the project value increases. The relation between the two values can be described as $f = c \cdot v^a$.

The strategies which players can use is the product of those for the individual parameters. Regarding transaction costs, players can decide to account for the fee by increasing the bid price, or not. Players who choose to participate must pay the entry fee. Regarding the complex bids, players can either decide on a single quality level or they can play a mixed strategy based on all possibilities.

I expect that different relations between quality and the transaction costs will influence the strategies of players and thus the outcome of the game. When the transaction costs grow exponentially, players will avoid the higher and thus more expensive quality. Players may become indifferent between quality levels if the relation between $f$ and $q$ reflects the trade-off of quality bonus versus extra effort.

**Expectation 16.** If the transaction costs increase more rapidly than project value, players in an EMAT game will not choose the bids of high quality.

### 6.1.2 Relation to tenders

In a tender, the buyer basically decides how large the transaction costs are. The more requirements the buyer installs on the sellers, the higher the transaction costs become. When a buyer offers a quality bonus which requires an unreasonable amount of extra effort, none of the sellers will apply for this bonus. Instead, they will all focus on low prices. The buyer may as well saved himself the effort of defining an EMAT tender by choosing a lowest-price tender.

Considering only the complex bids, quality is overrated. In tenders, not all contractors focus mostly on high quality. Apparently there is still some disadvantage to offering the highest possible quality. Perhaps the transaction costs can explain the difference between the complex bidding parameter and real world tenders. Bidding high quality yields higher transaction costs than bidding low quality. In practice, the relation between transaction costs and quality level may not be linear. In a model with only quality, a $\Delta q$ was needed to counter the overrewarding of quality. Perhaps the extra transaction costs for higher quality are sufficient as an extra quality penalty.

### 6.2 Transaction costs and repetition

The combination of transaction costs and repetition seems interesting because players can have incentive to wait out a round. This can lead to strategically participating or defecting in certain rounds.

In this composite model the first-price auction is extended with both transaction costs and repetition. This means that an auction with transaction costs is repeated a number of times. Players will start without any buffer. Should they decide to never play, their utility will remain zero. Continually playing and losing, will bankrupt players because their utility keeps decreasing. Players can decide to wait out a game, perhaps because of low winning
chances, and continue playing in the next round. Players will start without any buffer. Winning games helps to grow a buffer.

Players can make a different choice depending on previous profits. If their buffer allows it, a player can decide to always participate. Another option is to ignore the luxury of a buffer and to play based only on the transaction costs parameter. This will lead to defecting if a players believes that his bid is too high.

**Expectation 17.** *Because of the transaction costs, players can benefit from waiting a round by not competing.*

In practice, contractors do not always participate in every possible tender. This may be explained by the transaction costs versus their expected utility. Another reason for contractors to defect is that they are too busy and already have enough work.

### 6.3 Transaction costs and continued work

Players who expect profits from continued work will lower their prices. Players who account for transaction costs will increase their prices. That is why combination of these seems interesting. Depending on the settings used in the experiment, the effect of on or the other parameter will be stronger. For a game model and strategies for each parameter, please refer back to Section 5.1 on transaction costs and Section 5.5 on continued work.

#### 6.3.1 Experiments and results

In order to measure the effect of one parameter compared to the other, two similar experiments are used. First the expected continued work is fixed and the transaction costs are varied. Next the amount of continued work is varied and the transaction costs remain constant.

**Varying transaction costs**

In the following experiment the expectations on continued work is fixed. They are a 80% chance of continued work, with mean 100 and standard deviation 20. The estimate on continued work is 80% of 100 = 80, which is quite large in comparison to the project value of 100%. The transaction costs are varied, similar to the experiment in Section 5.1.5 and results from Figure 5.1.

Results of this experiment are shown in Figure 6.1. Figure 6.1a shows the utility of the buyer. Compared to Figure 5.1a, the buyer is better off. Apparently the effect of decreased prices due to continued work is stronger than the increased prices effect of transaction costs. Figure 6.1b shows the amount of failed tenders for the buyer. These are the games in which no seller sends in an offer. The results in Figure 6.1b show that for small transaction costs (up to about 5%) it hardly ever occurs that all players defect. In contrast in Figure 5.1b, where transaction costs varied and continued work was left out, 25–75% of games failed with transaction costs up to 5%.
6.3 Transaction costs and continued work

(a) Utility for the buyer

(b) No bids received

Figure 6.1: Varying the transaction costs, with fixed beliefs on continued work. Results (utility and failed tenders) for the buyer are shown.

Varying continued work estimates

In the next experiment setup the transaction costs are fixed at 1% of the project value. The beliefs of the sellers regarding expected extra work are varied.

6.3.2 Relation to tenders

In a tender, the transaction costs are mostly decided by the buyer. So is the rough estimate of continued work: the more sure the buyer is of his requirements up front, the less continued work there will be. If a buyer has high standards on the offers, sellers have high transaction costs and may often decide not to participate. The buyer could use expectations on some continued work as an incentive to participate for sellers.
Chapter 7

Concluding

7.1 Conclusions

This thesis studies parameters that are assumed to influence the outcomes of tender procedures. These parameters are learning, transaction costs, repetition, complex bids, uncertainty, continued work and irrationality. This thesis shows that these parameters have a significant impact on the outcome of tenders. This impact can be either positive or negative. Tender procedures offer a certain amount of freedom where government agencies can make small choices regarding the rules of awarding a tender contract. Considering the impact of parameters can help government agencies to design the perfect tender. The perfect tender is a tender with the best possible winner and it maximizes utility and minimized regret for all participants.

This work has revolved around the following main question: How can the contract awarding round from tender procedures be modeled best using concepts from game theory and how can this model help improve tenders such that the best possible outcomes occur?

To answer the main question, tender procedures can be modeled using game theory and by carefully selecting the extent to which parameters are involved, the best possible outcomes will occur. Learning has a positive impact and should be encouraged, as do transaction costs, but only in small amounts. The impact of uncertainty and irrationality should be minimized. The rest of this section will discuss this main result in more detail.

A tender with all the right parameters will maximize utility for both buyer and sellers. The buyer pays the best possible price, which is usually the lowest price. For this price, the buyer receives the best possible product or service. Losing sellers minimize their losses and have minimal regrets. The winning seller maximizes his profit.

Certain parameters have a significantly positive impact on tender outcomes and their influence should be maximized where possible. These factors are learning, repetition, and transaction costs, each will be discussed in turn. From the point of view of a single seller, learning about the other sellers’ bidding behaviors is beneficial. Not learning can be a disadvantage. From the point of view of the buyer, learning sellers are beneficial: the price will lower as competitive sellers learn more. Repetition is a necessary condition to enable learning. In a single game, players have no history to learn about their opponents. In
practice, contractors need more than one project to stay in business and repetition always has a role in tenders. There is plenty of literature on iterative auctions, some of which can be applied to tenders.

It was shown that buyers benefit from creating a tender with small transaction costs. Transaction costs between 0% and 1% give the buyer the most utility. This matches the behavior of contractors in practice. Contractor experts have indicated that they will not participate in any tender with transaction costs higher than 1% or 2% of the project value.

In a model with only transaction costs the sellers defect often. If there are extensive requirements on acceptable offers many games fail completely since all sellers will defect. Sellers defect if they do not want to pay the entry costs. Continued work as an additional parameter to transaction costs can help explain why the amount of defecting contractors is much lower in practice, than expected based solely on entry costs. A contractor’s beliefs on continued work after winning a tender can be incentive for him to lower his price and defect less often. Continued work in combination with transaction costs has a positive impact on tender outcomes for sellers.

The impact of the following parameters should be minimized. Irrationality, uncertainty and continued work can have a negative impact on tender outcomes. Unfortunately, it will often be impossible to avoid these parameters completely in practice.

Irrationality can lead to unexpected results and is defined in this thesis to encapsulate all parameters not otherwise included. Both the buyer and sellers can have negative results from irrational players. Irrational players may bid significantly higher or lower than all opponents or decide to defect. The irrational player will mostly harm himself by never winning or receiving negative utility. A better understanding of the motives of participants in a tender helps to anticipate on their behavior. The participants will seem less irrational.

Uncertainty indicates either an inexpert bidder with a lack of information or bidders with uncertain costs. Inexpert players may seem irrational and behave in unexpected ways. In practice, inexpert players will quickly learn to avoid losses or they will price themselves out of the market and not win any tenders. Contractors with uncertainty about their costs have more risks and may decide on higher prices to account for these risks. This has a negative impact on the buyer’s utility. By carefully constructing the tender contract and providing reliable and relevant information, uncertainty can be minimized.

Continued work in a tender mostly has a negative impact on the outcome for the buyer. Continued work was shown to be very similar to uncertainty with regard to costs. In a tender with very high expectations on continued work, the buyer should foresee this and include the follow-up work in the tender contract. Otherwise the buyer must either hold a new tender or pay the contractor’s price, both of these options are likely to be more expensive.

Complex bids – used in EMAT tenders – may be beneficial in a tender depending on the type of project. In tenders where extra quality is not necessary and there is little freedom in the project design, a lowest price tender should be used instead. If the buyer decides on an EMAT tender, the scoring rule must be determined carefully, to ensure quality is not overrewarded. There is a delicate balance between quality and price, the buyer should be aware of this and design the tender contract accordingly.
7.2 Contributions

The combination of the fields of tender and game theory is very new. This thesis has shown many interesting directions in which game theory can be applied to tenders in practice. Knowledge of this more mathematical field can help in participation in real-life tenders. This thesis makes contributions to both practice and literature. Its main contribution is identifying key parameters in the tender process. Knowledge of these key parameters will benefit both the buyer and seller side of a tender procedure. The proposed game model can be further extended and evaluated in future work. The following section discusses specific suggestions for future work.

7.3 Future work

Many suggestions for future work can be found in the discussion sections from Chapters 4, 5 and 6, a selection of which is repeated here. The suggestions for future work in this section are split up in three main categories. These categories are evaluation and validation of results, the behavior of players and the rules of the games. These three categories indicate the limitations of this thesis. The validation of results could have been better and more extensive. The agents’ strategies could have been more diverse and more models could have been researched.

Further evaluation and validation of the results in this work can be done in different ways. A theoretical analysis can be used, where more literature is studied to find if the conclusions from this work align with conclusions in other research. Consulting tender experts is also an option. Experts can be interviewed or asked to participate in a serious game. Such a serious game could reflect tenders in practice or any of the models proposed in this work. The behavior of tender experts can be compared to the behavior of agents in the auction games from this thesis. In a serious game there could also be a mix of experts and agents playing.

Historical tender data can be used as inputs in experiments. The results from simulations can be compared to the tender outcomes in practice to validate the results. If the tender data comes from tenders similar to a specific model, the outcomes should also be similar. Another point in evaluation is to apply different metrics besides the utility and regret which were mostly used in this work. Social welfare was already mentioned, but not yet analyzed. The point of view of the general public is represented well by social welfare. In future work the social welfare outcomes of simulations could be analyzed further.

Another focus for future work is on the players’ behavior, which are the strategies they apply. In future experiments the players could apply different strategies. These strategies can come from experts but also from related literature. For example, all players have used a single method of learning so far, where they consider the weighted average from historical outcomes. There exists a variety of learning strategies in multi-agent systems. Inspiration from other multi-agent systems could help improve the learning skills of agents in tenders. Other forms of learning may benefit sellers even more than the weighted average used in this work.
Different strategies may improve the simulation outcome of specific models. Players with uncertain costs or expected continued work could have different risk profiles. The sellers could be risk-seeking or risk-averse, instead of having only risk-neutral sellers.

In a model with repetition the selection of strategies can definitely be improved. Literature on repeated games or specifically on iterative auctions can be a guide to more interesting strategies for handling repetition. The players could adapt more to the repetition. The model of the auction with repetition itself can also be extended such that it better represents tenders in practice. In the game an agent would typically have zero utility, or a small negative utility after only losing games. In practice, the stakes are higher for contractors as they cannot exist without any work to perform.

The last general direction for future work is to revisit the rules of the game. This can be measuring the influence of model parameters, or researching new complex models with new combinations of parameters. Perhaps there are other parameters from game theory, which have not been considered, that can also be applied to tenders. Section 5.6.4 outlined an experiment to measure the influence of parameters. In this experiment the influence of parameters is measured by considering their impact on the outcome of games.

Section 5.1.6 already outlined how to analyze the effect of reimbursing transaction costs. In practice, government agencies often face the question if a reimbursement of transaction costs would be beneficial. They must decide to reimburse all, a part, or none of the transaction costs of all the sellers. Do the benefits of reimbursing outweigh the extra costs?

The composite models discussed in Chapter 6 are only a small selection of possible composite models. The models in this chapter can still be evaluated further, as can models with other combinations of parameters. An example is the two-parameter model with complex bids and repetition. This combination is often encountered in literature, it concerns repeated multi-attribute auctions. In repeated multi-attribute auctions each auction concerns goods or services with multiple attributes.

The next step is to consider complex models with three, four or even all the parameters from this thesis included. Such complex models are expected to represent tenders better. A model including all the parameters from this work should represent a tender from real life closely. The outcome of experiments and the behavior of players would relate better to practice, since more influences from practice are accounted for. It would be interesting to compare this last, complete, model to the basic first-price auction model. Is the complex model really a better representation of a tender?


Appendix A

Glossary

**Announce** (*Aankondigen*) – Publicly announce a tender.

**Budget Balance** – The operator of the mechanism makes neither a profit nor a loss.

**Competitive dialogue** (*Concurrentiegerichte dialoog*) – A tender procedure.

**Complex bid** – An offer that consists of both a price and a quality part. The quality part may have other subparts.

**Continued work** (*Meerwerk*) – Extra work for a contractor after a tender has been won.

**Contract awarding** (*Gussen*) – Choosing the candidate who will receive the contract.

**Contracting authority** (*Aanbesteder*) – Party (often from government) that hands out the tender assignment.

**Direct mechanism** – In a direct mechanism an agent can only disclose his preferences.

**Dominant strategy** – A strategy that will always be better then any other, regardless of what other players do.

**Efficiency** – A mechanism selects the choice that maximizes the sum of agents’ utility.

**Economically most advantageous tender** (*EMAT*) (*Economisch meest voordelige inschrijving* (*EMVI*)) – Type of award criterion which focuses on both price and quality.

**Game regret** – The highest regret of a game outcome. This is the maximum of all individual regrets.

**Game theory** – Is about analyzing games.

**Government agency** (*Overheidsinstantie*) – Usually the contracting authority in a tender.

**Grounds for exclusion** (*Uitsluitingsgronden*) – Used in the selection round to exclude (for example) companies with criminal activities.

**Incentive compatibility in dominant strategies** – For every agent the dominant strategy is to announce his true preferences.

**Individual rationality** – An agent would never be better off choosing not to participate.

**Irrationality** – Players who do not behave rational.

**Mechanism design** – Is about creating games, or mechanisms.
Move – A possible action for a player within a game.

Negotiated procedure (Onderhandelingsprocedure) – Tender procedure with a negotiation round between selection and awarding.

Notice (Aankondiging) – Announcement of a tender.

Open procedure (Openbare procedure) – Tender procedure without an explicit selection, but only an awarding round.

Pay-off – Outcome of a game for a specific player.

Pay-off matrix – Matrix depicting the outcome of a game for every player and for every strategy.

Procurement (Aanbesteding) – See tender.

Rationality – Rational players are motivated only by maximizing their own utility.

Regret – A game metric which indicates for a player how much more utility could have been gained in hindsight.

Repeated game – A game that is played multiple times in a row. This could go on infinitely.

Reserve price – The highest price a buyer is willing to pay, or the lowest price a seller is willing to accept for an object.

Restricted tender (Niet-openbare procedure) – Tender procedure with a distinct selection and awarding round.

Selection (Selectie) – Choosing a number of candidates that will continue with the tender.

Selection criteria (Selectiecriteria) – Used in the selection round to narrow down the number of candidates. Includes grounds for exclusion and suitability demands.

Service contract (Diensten) – A type of tender contract.

Social welfare – The sum of utilities of all players.

Strategy – Deciding which move to play in a game.

Supply contract (Leveringen) – A type of tender contract.

Suitability requirements (Geschiktheidseisen) – Used in the selection round to check if suppliers are suitable candidates for a tender.

Tender (Aanbesteding) – A mechanism typically used in government procurement for contractor selection.

Transaction costs – Costs for participating in a tender. These costs are based on the effort that goes into creating a formal offer.

Truthfulness – A mechanism is truthful if it is direct and the dominant strategy for every agent is to disclose its actual preferences.

Utility – Valuation of a certain outcome of a game.

Valuation – Indicates the value which a player assigns to a certain object.

Works (Werken) – A type of tender contract.