Experimental verification of stress-induced anisotropy

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Helma M.A. Cruts,* Jeroen Groenenboom, Adri J.W. Duijndam, and Jacob T. Fokkema, Delft University of Technology, The Netherlands

Summary

During experiments with a triaxial pressure machine in the laboratory a stress history in rock is induced. While loading and unloading the changes in elastodynamic wave behaviour are measured with ultrasonic transducers. These experiments give insight in the influence of a changing stress state on the wave propagation; the rock becomes anisotropic under differential stress, which expresses itself in an angular dependant change in velocities. A related effect of anisotropy is shear-wave splitting, which is made visible by using single pairs of broad-band shear wave transducers.

With Hooke's law for transverse isotropic media and the elastodynamic wave equation, five relations are derived for the complete determination of the stiffness tensor from wave velocities for transverse isotropic media.

Introduction.

The accurate prediction of the time, the place, and the magnitude of earthquakes is very difficult if not impossible with current techniques. One of the factors that change before an earthquake is the stress situation, which might be measurable through the influence of a changing stress state on the elastodynamic wave propagation. An experiment has been designed to gain insight in the influence of a changing stress state on the propagation of shear- and pressure-waves.

It is thought that, when applying a non-hydrostatic stress, a differential opening and closing of cracks occurs [CRAMPIN ET AL., 1984]. On a large scale the average effect of the cracks is an effective anisotropy of the medium.

Theory

For deformations of an elastic body Hooke's Law holds: the stress is proportional to the strain, according to the relation:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl}, \quad (1)$$

where C_{ijkl} denotes the stiffness tensor. To find operational expressions for the determination of the stiffness constants from wave velocities we need the elastodynamic wave equation, which can be written as:

$$C_{kmij} \partial_m \partial_i u_j - \rho_{kr} \partial_i^2 u_r = \partial_i f_k + C_{kmij} \partial_m h_{ij}, \quad (2)$$

[e.g. DE HOOP, 1991]. In the absence of sources, this wave equation can be written as:

$$C_{kmij} \partial_m \partial_i u_j - \rho_{kr} \partial_i^2 u_r = 0. \quad (3)$$

Plane, harmonic wave-solutions to equation (3) are of the form:

$$u_i = U_i \exp(i(k_j x_j - \omega t)) = U \alpha_i (\exp(i(2\pi/\lambda)(\beta_j x_j - \nu(\beta_j) t))) \quad (4)$$

[HELBIG, 1994], where β_j is a unit vector in the direction of propagation, $U_i = U \alpha_i$ is the displacement amplitude, k_j is the wave vector, and $\omega = 2\pi f$ is the radial frequency. The quantity α_i is a unit vector in the direction of polarisation, while $\nu(\beta_j)$ denotes the propagation velocity.

After substitution of equation (4) into equation (3) we obtain the dispersion relation:

$$C_{ijkl} \beta_i \beta_j \alpha_k - \rho \nu^2 \alpha_i = 0, \quad (5)$$

which can also be written as the tensor equation:

$$(C_{ijkl} \beta_i \beta_j - \delta_{ik} \rho \nu^2) \alpha_k = 0. \quad (6)$$

It is customary to use the 'Kelvin-Christoffel' matrix [HELBIG, 1994]:

$$\Gamma_{ik} = C_{ijkl} \beta_l \beta_j, \quad (7)$$

which allows us to write equation (6) as:

$$(\Gamma_{ik} - \delta_{ik} \rho \nu^2) \alpha_k = 0. \quad (8)$$

This equation has a solution if and only if:

$$\det(\Gamma_{ik} - \delta_{ik} \rho \nu^2) = 0. \quad (9)$$

For any propagation direction this generally leads to three eigenvalues corresponding with three velocities. The polarisation direction is then determined with equation (8).

For transverse isotropic media, where only five independent stiffness constants exist, Hooke's law can be written as:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(c_{11} - c_{12})}{2} \end{bmatrix} \times \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{23} \\ \epsilon_{31} \\ \epsilon_{12} \end{bmatrix}, \quad (10)$$

[HELBIG, 1994].

The complete stiffness tensor for transverse isotropic media can be determined by measuring the velocities of two pure P-waves, one propagating in the direction of the symmetry axis (V_{P3}) and the other in a perpendicular direction (V_{P1}), the velocities of two pure S-waves, one propagating in the direction of the symmetry axis with any polarisation (V_{S3}), and the other propagating in a perpendicular direction and polarised perpendicular to the axis of symmetry and to the propagation direction (V_{SH1}), and the velocity of one quasi P-wave with propagation direction 45 degrees from the axis of symmetry (V_{qP13}). Using these velocities the stiffness constants can be determined with the following five relations:

$$c_{11} = \rho V_{P1}^2, \quad (11)$$

$$c_{12} = c_{11} - 2\rho V_{SH1}^2, \quad (12)$$

$$c_{33} = \rho V_{P3}^2, \quad (13)$$

$$c_{44} = \rho V_{S3}^2, \quad (14)$$

and,

$$c_{13} = -c_{44} +$$

$$\sqrt{4\rho^2 V^4 q_{P13} - 2\rho V^2 q_{P13}(c_{11} + c_{33} + 2c_{44}) + (c_{11} + c_{44})(c_{11} + c_{33})} \quad (15)$$

Shear-wave splitting

One of the effects of anisotropy on the wave propagation is that a shear-wave traversing an anisotropic medium generally splits into two different waves with fixed polarizations and different velocities. The leading faster travelling shear-wave is polarised parallel, and the slower travelling shear-wave perpendicular to the strike of the cracks. This phenomena is called shear-wave splitting. [CRAMPIN ET AL., 1984].

Experiment design

The experiment has been designed in such a way that at least all velocities can be measured for determination of the stiffness tensor for transverse isotropic media. Furthermore, it is possible to perform shear-wave splitting experiments under several loading conditions, by using broad-band shear-wave transducers with a polarization direction of 45 degrees from the main axis.

The experiments are done on a Colton sandstone block of $0.2 \times 0.2 \times 0.2 \text{ m}^3$. To apply a stress field, the block is placed in a triaxial machine. Each of the three axes of this machine can, independently of the other axes, build up a force of 3,500 kN. The maximum stress that can be imposed on the cube is therefore 43.8 MPa. It is not possible to apply a controlled pore pressure to the rock, since it is an open system. Each uniaxial part of the pressure machine has a piston on one side and a pressure plate on the other side which are connected with four tension bars. All plates, except the bottom plate are equipped with spherical seats which allow small rotations of the pressure plates in case the sides of the cube are not parallel.

In each pressure plate three separate broad-band 1 MHz piezo electric transducers are mounted to transmit and receive P-waves and S-waves. The triaxial machine is controlled by a computer, which monitors, besides the pressures, also displacements, measured at the outside of the block. Another computer controls the triggering of the transmitting transducer and stores the received signals (see Figure 1).

Figure 1: Schematic lay-out of the experiment

Pressures.

Measurements on a Colton sandstone cube without applying stresses indicated that the cube contains an initial transverse isotropy with the axis of symmetry in the Y-direction. The stress patterns A, B, C and D as depicted in figure 2, are designed in a way that the transverse isotropy will be maintained. Since cracks tend to open in the direction of the least compressive stress, the stresses in X- and Z-direction must then be kept equal and always larger than the stress in the Y direction. The stresses are kept below 25% of the uniaxial pressure strength ($\pm 46 \text{ MPa}$) to avoid inducing a permanent deformation.

With stress pattern A we try to create an anisotropic stress situation by increasing the stress in the X- and Z-direction, while decreasing the stress in the Y-direction. In B and C the level of stress in the Y-direction is kept constant, while the stress in the X- and Y-directions is increased. With stress pattern D the stress is equally increased in all directions.

Three other stress patterns have been designed which will result in deviation from the original transverse isotropy. With pattern E the stress level in the Y-direction is kept constant while the stress in the Z-direction decreases and the stress in the X-direction increases. In patterns F and G the stresses in both Z- and Y-direction are kept constant and the stress in the X-direction is increased (see figure 2).

Figure 2: Stress patterns

Results

The first experiments are performed by applying stress patterns A, B, C and D. Figure 3 shows the forces as applied by the triaxial machine. First one pressure-wave transducer and one horizontally and one vertically polarised shear-wave transducer were placed on each side of the cube. The first arrival times were picked for the transmitted waves. The traveltimes of the shear waves (two perpendicular polarisations in each direction) are plotted in figure 4.

Figure 3: Stress patterns A, B, C, and D; forces applied by the triaxial machine

Figure 4: Traveltimes for the transmitted shear-waves

It can clearly be seen that the traveltimes change with changing pressures. The difference in velocity between the horizontally and the vertically polarised shear-waves, propagating in the same direction, also changes. This difference is a measure for the anisotropy and is defined as 'Relative Shear-wave Velocity Difference' (RSVD):

$$RSVD = \frac{V_{SH} - V_{SV}}{V_{SH}} \quad (16)$$

For the three main directions the RSVD is plotted in figure 5.

Figure 5: RSVD for the X-, Y- and Z- direction.

As expected the RSVD is nearly zero for the Y-direction, the direction of the symmetry axis. For the X- and Z-direction the RSVD changes with changing stress pattern. It not only depends on the stresses in X- and Z-direction but also on the stress in the Y-direction (the peaks in 'C' are lower than the peaks in 'B'). At increasing hydrostatic pressure all velocities increase, the RSVD in both X- and Y-direction decreases and even gets below the initial value.

The five stiffness constants have been determined by application of equations (10)-(14), the result is plotted in figure 6.

Figure 6: Stiffness constants

The shear-wave splitting experiments are performed with broad-band shear-wave transducers, polarised 45 degrees from the main axis, placed at each side of the sandstone cube. Again stress patterns A, B, C and D are applied. The resulting data for propagation in the X-direction is plotted in figure 7.

The wave indeed splits at a certain anisotropic stress state, which can be seen as an arrival of two different shear-wave velocity sheets. The shear-wave splitting disappears when the rock is loaded equally at all sides. For the horizontally and vertically polarised shear-waves no shear-wave splitting is visible, as can be seen in figure 8.

Figure 7: Diagonally polarised shear-wave propagating in X-direction

Figure 8: Horizontally polarised shear-wave propagating in X-direction

In the second experiment stress patterns A, E, F and G are applied, first with one P, one SH and one SV transducer, and in the last test with a diagonal polarised shear-wave transducer. The normal stresses as applied by the triaxial machine are plotted in figure 9. The RSVD for the X, Y and Z directions is plotted in figure 10. As can be seen in these figures it is also possible to deviate from the original transversal isotropy.

Shear-wave splitting while applying stress-patterns A, E, F and G is plotted in figures 11 and 12. In the Z direction shear-wave splitting is clearly visible, and also in the Y direction some shear-wave splitting can be seen.

Figure 9: Stress patterns A, E, F and G, Forces as applied by the triaxial machine.

Figure 10: RSVD for the X-, Y- and Z- direction.

Figure 11: Diagonally polarised shear-wave propagating in Z-direction

Figure 12: Diagonally polarised shear-wave propagating in Y-direction

Conclusions

With our experiments it is possible to get a quantitative measure for stress-induced anisotropy and to determine the stiffness tensor for transverse isotropic media. Shear-wave splitting can be made visible by using a single pair of broad-band transducers. Both anisotropy and shear-wave splitting depend on the amount and way of loading. Anisotropy and shear-wave splitting appear with differential loading but disappear when the loading is equal in all directions. The results can be interpreted with the existing crack models for anisotropic media.

References

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