Metal-Insulator Transition in a 2D Electron Gas: Equivalence of Two Approaches for Determining the Critical Point

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(Received 30 May 2001; published 6 December 2001)

The critical electron density for the metal-insulator transition in a two-dimensional electron gas can be determined by two distinct methods: (i) a sign change of the temperature derivative of the resistance, and (ii) vanishing activation energy and vanishing nonlinearity of current-voltage characteristics as extrapolated from the insulating side. We find that, in zero magnetic field (but not in the presence of a parallel magnetic field), both methods give equivalent results, adding support to the existence of a true zero-field metal-insulator transition.

DOI: 10.1103/PhysRevLett.87.266402

PACS numbers: 71.30.+h, 73.40.Qv

The observation of a metal-insulator transition (MIT) in two-dimensional (2D) electron and hole systems (see a review [1] and references therein) challenges the scaling theory of localization which predicts all electron states to be localized in an infinite disordered 2D system at zero temperature and in zero magnetic field [2]. In fact, the predicted localization of all carriers is beyond experimental verification since, in real 2D systems, it can break down because of any perturbation such as a finite temperature, finite system size, magnetic impurities, etc. To estimate the limiting behavior at zero temperature experimentally, one can follow two principal approaches: Extrapolate the temperature dependence of the resistance to $T = 0$ or analyze temperature-independent characteristics (e.g., the correlation length). Moreover, comparison of distinct criteria is the experimental test to find out whether or not a true MIT exists. The resistance of high-mobility silicon and p -GaAs samples is “flat” at a certain carrier density in a wide range of temperatures [3–5], and it is therefore tempting to extrapolate it to $T = 0$ and identify the corresponding electron density with the critical density for the metal-insulator transition. However, in some of the suggested explanations for the MIT (see, e.g., Ref. [6]), it was stated that, within their models, the “critical” curve is not flat. Therefore, to verify whether or not the temperature-independent resistance corresponds to the critical density, an independent determination of the critical point is necessary. This is why, in this paper, we compare the critical density obtained by a finite-temperature criterion with that obtained by a *temperature-independent* one. Equivalent results from these two distinct methods would justify the extrapolation of the flat resistivity curve to $T = 0$ and strongly support the existence of a true MIT in zero magnetic field.

Methods for determining the MIT point have been described in previous publications. The first criterion is a change in sign of the temperature derivative of the resistivity, $d\rho/dT$ (see, e.g., Ref. [4]). A positive (negative) sign of the derivative at the lowest achievable tempera-

tures is empirically associated with a metallic (insulating) phase. The second criterion is based on a vanishing activation energy, E_a , combined with a vanishing nonlinearity (threshold voltage, V_c) of current-voltage (I - V) characteristics when extrapolated from the insulating phase [7,8]. The activation energy and the threshold voltage are connected via the localization length which is *temperature independent*. While the derivative method deals with the vicinity of the MIT in which the dependence $\rho(T)$ is relatively weak, the I - V method is related to the *insulating phase* with exponential $\rho(T)$. These two methods have not been applied simultaneously to the 2D MIT.

In this Letter, we compare these two independent criteria for determining the metal-insulator transition point in a 2D electron system in silicon metal-oxide-semiconductor field-effect transistors (MOSFETs). We report that in zero magnetic field, both methods yield the same critical electron density $n_c(0)$. Since one of the methods is temperature independent, this equivalence supports the existence of a true $T = 0$ MIT in zero magnetic field. In contrast, in high parallel magnetic fields, where the 2D electrons are fully spin polarized [9], only the I - V method can be used, and it yields the critical density $\approx 1.5n_c(0)$; the derivative criterion does not yield a critical point. This makes uncertain the existence of a true metal-insulator transition in a system of spin-polarized electrons.

Measurements were performed on high-mobility silicon MOSFETs ($\mu_{\text{peak}} = 2.4 \times 10^4 \text{ cm}^2/\text{Vs}$ at 4.2 K), similar to those previously used in Ref. [4]. Data were taken by a four-terminal *dc* technique using an electrometer with symmetric inputs.

The resistivity as a function of temperature in zero magnetic field is shown in Fig. 1(a) for several electron densities on both sides of the metal-insulator transition. The resistivity of the middle (dashed) curve shows virtually no temperature dependence over a wide range of temperatures; this curve separates those with positive and negative $d\rho/dT$ nearly symmetrically (at temperatures above

FIG. 1. Temperature dependence of the resistivity of a Si MOSFET at different electron densities near the MIT in zero magnetic field (a), and in parallel magnetic fields of 4 T (b) and 10 T (c). The electron densities are indicated in units of 10^{11} cm^{-2} . The inset in (c) shows a close-up view of the curve marked by the rectangle.

0.2 K) as reported earlier [10]. The existence of such a T -independent curve at temperatures down to 30 mK clearly shows that the logarithmic corrections to the resistance (which are expected to be very strong in 2D systems with resistivity $\geq h/e^2$ [2]) are absent in this system in zero magnetic field (see also Refs. [4,5]). Assuming that the middle curve remains flat down to $T = 0$, we obtain the MIT critical point at $n_s = 0.795 \times 10^{11} \text{ cm}^{-2}$ which corresponds to a resistivity $\rho \approx 3h/e^2$, as in other experiments on Si MOSFETs [1]. We designate the corresponding electron density as the critical density, n_{c1} .

The MIT point has also been determined by studying the behavior of nonlinear I - V characteristics on the insulating side of the transition. Deep in the insulating state ($n_s < n_{c1}$), a typical low-temperature I - V curve is close to a steplike function: The voltage rises abruptly at low current and then saturates, as shown in Fig. 2(a). The magnitude of the step is $2V_c$. (At higher temperatures the curve becomes less sharp, yet the threshold voltage, V_c , remains practically unchanged.) Closer to the MIT, the I - V curves still show a nonlinear steplike behavior provided that $n_s < n_{c1}$ [see the curve in Fig. 2(b) corresponding to $n_s = 0.743 \times 10^{11} \text{ cm}^{-2}$]. Exactly at $n_s = n_{c1} = 0.795 \times 10^{11} \text{ cm}^{-2}$, the I - V curve is strictly linear [Fig. 2(b)]; at $n_s > n_{c1}$, it becomes *superlinear* which is characteristic of the metallic state [11]. Figure 3 (closed circles) shows that the square root of the threshold voltage is a linear function of electron density (discussed below). Extrapolation of the $V_c^{1/2}(n_s)$ dependence to zero threshold value yields the critical electron density n_{c2} . Note that n_{c2} is equal to n_{c1} with high accuracy.

Not too deep on the insulating side of the transition, and at not too low temperatures, the resistance has an activated form [7,12], as shown in the inset of Fig. 2(a) [13]. Such a form of temperature dependence was interpreted in Ref. [14] as a result of the thermal activation of carriers to the mobility edge, E_c ; in this case, the activation energy is $E_a = E_c - E_F$. Since, in the insulating regime, the I - V curves are strongly nonlinear, we determine the resistivity from dV/dI in the linear interval of I - V curves, i.e., at $I \rightarrow 0$. Figure 3 shows E_a as a function of the electron

density (open circles). In previous detailed studies [7], this dependence was found to be linear near the mobility edge [15]. The present data can also be approximated by a linear function which yields, within the experimental uncertainty, the same critical electron density n_{c2} as the square root of the threshold voltage. The reciprocal slope of $E_a(n_s)$, D^* , can be interpreted [14] as the thermodynamic density of states near the transition point.

The threshold behavior of the I - V curves has been explained within the concept of the breakdown in the

FIG. 2. Current-voltage characteristics at a temperature of ≈ 30 mK in zero magnetic field. (The aspect ratio is equal to 2.4 for this sample.) In case (a), the I - V curve obtained at a higher temperature (211 mK; dashed line) is also shown for comparison; note that the threshold voltage is practically temperature independent. An Arrhenius plot of the resistivity in the insulating phase is displayed in the inset for the following values of B and n_s : 0 T, $0.741 \times 10^{11} \text{ cm}^{-2}$ (circles); 1 T, $0.810 \times 10^{11} \text{ cm}^{-2}$ (squares); 6 T, $0.870 \times 10^{11} \text{ cm}^{-2}$ (diamonds).

FIG. 3. Activation energy and square root of the threshold voltage as a function of electron density in zero magnetic field (circles) and in a parallel magnetic field of 4 T (diamonds).

insulating phase [7,16]. Here we simply outline this concept. The breakdown occurs when the localized electrons at the Fermi level gain enough energy to reach the mobility edge in an electric field, V_c/d , over a distance given by the localization length, L : $eV_cL/d = E_c - E_F$, where d is the distance between the potential probes. The values E_a and V_c are related through the localization length which is temperature independent and diverges near the transition as $L(E_F) \propto (E_c - E_F)^{-s}$ with exponent s close to unity [7]. This corresponds to a linear dependence $V_c^{1/2}(n_s)$ near the MIT, as seen in Fig. 3.

We stress that, in zero magnetic field, both methods—the one based on extrapolation of $\rho(T)$ to zero temperature and the other based on the behavior of the temperature-independent localization length—give the same critical electron density. This adds confidence that the curve with zero derivative $d\rho/dT$ will remain flat (or at least will retain finite resistivity value) down to zero temperature.

It is interesting to compare the $B = 0$ case with that in the presence of a parallel magnetic field. With increasing parallel field, B , the MIT point n_{c2} , determined from the vanishing nonlinearity and activation energy, shifts to higher electron densities, saturating above a critical field, B_{sat} , at a constant value which is approximately 1.5 times higher than that in zero field (see dots in Fig. 4). A similar suppression of the metallic behavior was observed using a resistance cutoff criterion at the level on the order of h/e^2 [17]. As was shown in Ref. [9], in the metallic phase the saturation of the resistance with parallel field signals the onset of full spin polarization of the 2D electrons. Hence, one expects that the 2D system is spin polarized at $B > B_{\text{sat}}$, and that the observed phase boundary shift is a spin effect.

Before discussing the temperature dependence of the resistivity at $B > B_{\text{sat}}$, we note that one cannot assume that the metallic phase is necessarily strictly characterized by positive $d\rho/dT$ [18]: One may have a weakly T -dependent $\rho(T)$ with $d\rho/dT < 0$ and still have a finite

FIG. 4. Dependence of the critical electron density n_{c2} (dots) and of the thermodynamic density of states at the MIT (squares) on parallel magnetic field. The dashed lines are guides for the eye.

resistivity at $T = 0$. The I - V method yields the electron density n_{c2} at which the *exponential* divergence of the resistivity ends, although in principle $d\rho/dT$ may remain negative at this density.

In Fig. 1(b), we show the temperature dependence of the resistivity in a parallel magnetic field high enough to cause full spin polarization ($B = 4$ T). The middle curve corresponds to the critical electron density, $n_{c2}(B > B_{\text{sat}})$, determined by the method of vanishing nonlinearity and activation energy (as shown in Fig. 3 by diamonds). In sharp contrast with the $B = 0$ situation, not only are the $\rho(T)$ curves in the field nonsymmetric about the middle curve, but all of them have negative “insulatinglike” derivatives $d\rho/dT < 0$ in the entire temperature range, although the values of the resistivity are comparable to those in the $B = 0$ case. Moreover, in a strong parallel magnetic field, there is no temperature-independent $\rho(T)$ curve at any electron density: As shown in Fig. 1(c), the curve at the considerably higher density $n_s = 1.335 \times 10^{11} \text{ cm}^{-2}$ compared to $n_{c2} = 1.155 \times 10^{11} \text{ cm}^{-2}$, which could be approximately identified as a flat one in the temperature range used, changes its slope from weakly metallic at $T \gtrsim 0.5$ K to weakly insulating at lower temperatures. The metallic behavior of the resistance as a function of temperature, seen at yet higher electron densities in a parallel magnetic field [Fig. 1(c)], is much weaker than in the absence of field. We therefore conclude that the derivative method does not yield a critical density for the spin-polarized 2D system. Its failure leaves uncertain the existence of a true metal-insulator transition in a parallel magnetic field.

Some theories claim that spin-polarized and unpolarized states are very similar [19]. The authors of Ref. [20] considered the temperature-dependent screening of a random potential and predicted metallic ($d\rho/dT > 0$) temperature dependences of the resistivity for both polarized and unpolarized states. Therefore, one might expect a more or less analogous behavior of $\rho(T)$ curves around the transition

in the two cases. However, this is in contradiction with the experiment: While in zero magnetic field, “metallic” and “insulating” $\rho(T)$ curves are approximately symmetric on both sides of the transition (see above), this symmetry completely disappears in a parallel magnetic field [cf. Figs. 1(a)–1(c)]. An important disagreement between theory [20] and experiment is the dramatic weakening of the metallic temperature dependences in the magnetic field, while, theoretically, the derivative $d\rho/dT$ for the spin-polarized state is expected to be twice as high for the same electron density [20] (for more on this discrepancy, see Ref. [21]). Thus, the properties of the polarized state cannot be deduced from those of the unpolarized state in a straightforward way [22].

Another experimental fact that indicates the effect of polarization is the change in the slope of the dependence of E_a on n_s (Fig. 3) which we link, following Ref. [14], to the thermodynamic density of states at the MIT. In zero magnetic field the inverse slope, D^* , is close to the zero-field density of states, D_0 : $D^* \approx 1.2D_0$ where $D_0 = 2m/\pi\hbar^2$, $m = 0.19m_e$, and m_e is the free electron mass. In a parallel magnetic field, D^* increases and saturates at about $2.7D_0$ (see squares in Fig. 4). This increase is quite surprising since, in the case of full spin polarization, one expects that the density of states should *decrease* by a factor of 2 due to lifting of the spin degeneracy [23]. So, in many respects the behavior of the spin-polarized electron system is peculiar.

In summary, we have compared two principal approaches for determining the critical density for the metal-insulator transition in 2D. In zero magnetic field, both definitions of the critical point are found to be equivalent, strongly supporting the existence of a true $B = 0$ MIT. With increasing parallel magnetic field, the I - V criterion gives the critical point which shifts to higher electron densities and then saturates, which is likely to be a consequence of the spin polarization of the 2D electrons. It is accompanied by the disappearance/weakening of metallic temperature behavior of the resistance so that the derivative criterion cannot be used. The fact that the spin-polarized and spin-unpolarized dilute 2D electron systems behave qualitatively differently poses important constraints on the theory.

We gratefully acknowledge discussions with V. T. Dolgoplov, D. Heiman, M. P. Sarachik, and S. A. Vitkalov. This work was supported by NSF Grants No. DMR-9803440 and No. DMR-9988283, RFBR Grant No. 01-02-16424, and Sloan Foundation.

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- [1] E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, *Rev. Mod. Phys.* **73**, 251 (2001).
- [2] E. Abrahams *et al.*, *Phys. Rev. Lett.* **42**, 673 (1979).
- [3] M. P. Sarachik and S. V. Kravchenko, *Proc. Natl. Acad. Sci. U.S.A.* **96**, 5900 (1999).

- [4] S. V. Kravchenko and T. M. Klapwijk, *Phys. Rev. Lett.* **84**, 2909 (2000).
- [5] A. P. Mills, Jr. *et al.*, cond-mat/0101020.
- [6] Y. Meir, *Phys. Rev. Lett.* **83**, 3506 (1999).
- [7] A. A. Shashkin, V. T. Dolgoplov, and G. V. Kravchenko, *Phys. Rev. B* **49**, 14486 (1994).
- [8] Note that the so-determined critical point was related to a possible formation of the Wigner crystal by V. M. Pudalov *et al.*, *Phys. Rev. Lett.* **70**, 1866 (1993).
- [9] T. Okamoto *et al.*, *Phys. Rev. Lett.* **82**, 3875 (1999); S. A. Vitkalov *et al.*, *Phys. Rev. Lett.* **85**, 2164 (2000).
- [10] D. Simonian, S. V. Kravchenko, and M. P. Sarachik, *Phys. Rev. B* **55**, R13421 (1997).
- [11] At temperatures ≥ 0.2 K, I - V characteristics are nearly symmetric around the critical electron density [see S. V. Kravchenko *et al.*, *Phys. Rev. Lett.* **77**, 4938 (1996)]; however, this symmetry vanishes at lower temperatures where $\rho(T)$ saturates on the metallic side while diverging on the insulating side.
- [12] M. D'Iorio, V. M. Pudalov, and S. G. Semenchinsky, *Phys. Rev. B* **46**, 15992 (1992).
- [13] At lower T , the activated $\rho(T)$ transforms into the Efros-Shklovskii hopping. Note also that, in the immediate vicinity of the transition, $\rho(T)$ dependences are not activated but rather reflection symmetric about the transition and obey the scaling law [see V. Dobrosavljević *et al.*, *Phys. Rev. Lett.* **79**, 455 (1997)].
- [14] C. J. Adkins, S. Pollitt, and M. Pepper, *J. Phys. C* **37**, 343 (1976).
- [15] Deeper in the insulating state, the density of states decreases as compared to its value in the vicinity of the transition, which results in an increase of the slope of the $E_a(n_s)$ dependence.
- [16] The I - V nonlinearity mechanism was also discussed by D. G. Polyakov and B. I. Shklovskii, *Phys. Rev. B* **48**, 11167 (1993).
- [17] V. T. Dolgoplov *et al.*, *JETP Lett.* **55**, 733 (1992).
- [18] See, e.g., X. G. Feng *et al.*, *Phys. Rev. Lett.* **86**, 2625 (2001). Also, it is well known that, in a disordered 3D metal, this derivative can be either positive or negative near the MIT [see, e.g., P. A. Lee and T. V. Ramakrishnan, *Rev. Mod. Phys.* **57**, 287 (1985)].
- [19] Obviously, account should be taken of the spin degeneracy which is equal to $g_s = 1$ ($g_s = 2$) for the polarized (unpolarized) state and involves parameters such as the Fermi energy $E_F \propto 1/g_s$, the density of states $D \propto g_s$, etc.
- [20] V. T. Dolgoplov and A. Gold, *JETP Lett.* **71**, 27 (2000).
- [21] K. M. Mertes *et al.*, *Phys. Rev. B* **63**, 041101(R) (2001).
- [22] Note that the model [20] does satisfactorily describe the low-temperature dependence of the resistance on the parallel magnetic field deep on the metallic side of the transition [see A. A. Shashkin *et al.*, *Phys. Rev. Lett.* **87**, 086801 (2001)].
- [23] We do not believe that the discrepancy is due to a possible shift of the mobility edge when the electron density is varied in the insulating phase. First, for the small deviations of n_s from n_{c2} in this experiment, the slope of E_a vs n_s remains approximately constant, so that the screening ability of the 2D system does not change with n_s , and therefore one expects that the mobility edge does not shift. Second, no shift of the mobility edge is seen in zero magnetic field because we and others [7,14] have found $D^* \approx D_0$.