Design and Optimization

of

Ultra High Speed Permanent Magnet Machine

By

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Thesis

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And was partly in corporation with Aeronamic Co.,

**AERONAMIC B.V.**

In Almelo, the Netherlands.
To my parents
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1-1 Introduction

Electrical machines play an important role in industry as well as in our day-to-day life. There are different types of electrical machines such as DC, asynchronous and synchronous machines. The DC machines are extensively used as a motor in industry, but their use is limited due to widespread use of AC power. The asynchronous machine is the most rugged and the most widely used machine in industry; however, for large power consumption applications they are not very popular. The synchronous machine can have wound rotor, salient pole rotor or permanent magnet rotor. The permanent magnet machines are growing as the most popular machines due to their ease of manufacturing, high power density, speed control and low volume.

The machine that is going to be designed in this thesis is used in a compressor application unit. The motor shaft is connected to the compressor blade shaft. This compressor is used in aerospace industry and high power density and low volume and losses are the major requirements. The nominal power rating of the machine (30 KW) and high rotational speed (150 KRPM) makes the design procedure very critical. Thus, the permanent magnet machine is chosen to be designed for this application. What makes this design different from normal machine design is the very high rotating speed along with the high nominal power. This high speed and power are the definite causes for many losses in different parts of the machine such as stator resistive losses, rotor induced eddy current losses and stator iron losses. Due to the special application requirements, the machine should be designed considering some multiphasic functional areas such as electromagnetics, mechanical dynamics and thermal analyses. However in this thesis only the electromagnetic design is performed and the thermal and mechanical limitations are out of the scope.
1-2 Literature Review

Designing electrical machines has been going with a quite progressive pace since the influence of electrical machines on the industry and daily life was vital. Permanent magnet (PM) machines are the important types of machines due to their high torque/volume and torque/weight ratio compared to wound rotor machines. It is still important to choose the type of PM and its arrangement to achieve a proper performance. There are different PM types and arrangements such as surface mounted and interior bounded. They also can be magnetized in different ways including radially, diametrically, Halbach and ideal Halbach. In [J.O.Tenkorang1995] the Halbach and conventional surface mounted magnets are compared from the torque production point of view. In [Z.Q.Zhu2001] the radially magnetized magnet is compared with Halbach array. And in [K.Atallah1998] and [Z.P.Xia2004] the magnetic design of permanent magnet machines was analyzed using Halbach arrangement.

The winding distribution representation is necessary to design the machine. In order to define the winding distribution by winding factor approach instead of direct Fourier analysis, there are different methods. In [G.R.Slemon1992] the winding factor is the ratio of the MMF of a physical winding distribution to that of a concentrated one. In [R.Richter1967] this factor is defined as the ratio between flux linked of those two types of windings and in [H.Polinder1998], the winding factor is directly the ratio of the equivalent number of turns of a space harmonic of a physical winding to that of a concentrated winding.

After designing the rotor PM type and winding distribution, the next step would be the armature field calculation. The Poisson equations are solved in different regions of the machine. There is a difference between slot-less and slotted machines in Poisson equation in the winding region. [N.Bianchi2005] and [K.atallah1998b] have paid attention to the slot-less modeling while [A.Hughes1977] and [H.Polinder1997] focused on the slotted machine design.

By changing the magnetic field in the winding region, there will be induced voltage in the winding. This voltage is induced due to changing of flux linkage which has different sources such as: flux linkage due to PM, flux linkage due to armature current and slot flux leakage. In [A.M.El-Refai2008], [H.Grop2009], [J.Pyrhonen2008] and [S.R.Holm2007] there is further explanation about the flux linkages and associated inductances.

There are different methods for calculating the electromagnetic torque and mechanical power: the Poynting vector theorem, the Lorentz force and the Maxwell stress method. The pointing vector theorem which calculates the power crossing a defined surface has been explained and used in [F.Deng1997], [F.Deng1998] and [S.A.Sharkh1999] in rotor coordinate system to find the losses in the rotor. When the stator coordinate system is used such as in [B.B.Palit1982], [J.J.Gut1998] and [Z.Xiaojuan1996], the power crossing the surface includes the mechanical power and the rotor losses.

There are different methods to calculate the induced eddy current losses inside the rotor such as volume integrating of induced current density inside the rotor and using the theorem of Poynting Vector. The use of volume integration is explored in the [Z.Q.Zhu1997a] and [V.D.Vee1997] and will not be the method of loss calculation in this thesis due to complicated integrations of existing Bessel functions. The Poynting vector theorem for calculation of losses is used in the [F.Deng1997], [S.A.Sharkh1999] and [Z.Q.Zhu1997b] and will be the interested method for finding losses here in the thesis.
1-3 Design Criteria

In each design procedure at least a criteria is necessary, otherwise there are plenty of designs which are all valid for the purpose. Amongst the criterion for the electrical machine designs some are more important than others; but, it totally depends on the application in which the machine is going to be used. There are criterions such as efficiency, cost, power density, fault tolerance, volume, weight, rotor losses, stator losses, and etc.; but, any of them requires special attention dependent on its own application purpose. From a point of view all the mentioned criterions are important, but to fulfill the design based on all of criterions, it may be impossible or so costly.

The machine which is designed in this thesis is going to be used in the aircraft industry and the criterion which is specified by the industry is the low volume and low rotor induced eddy current losses. The low rotor loss is important in this machine because of the high rotational speed of the machine and the high output power. If the rotor losses are high then the cooling becomes rather difficult and if the temperature rises above the critical value, the permanent magnets start to demagnetize. The stator loss is also important; however, the cooling at the stator side is not as much difficult as the rotor side.

By setting the design criterion to the low rotor losses, it doesn’t mean that other factors such as volume, weight, efficiency, cost and etc. are neglected; of course they are considered, but the more concentration is based on the low rotor losses. Besides, it is simpler to start designing with only one criterion and after that, implementing other criteria which of course might affect the design based on the first criterion.

1-4 Design Methodology

There should be a starting point for any design of electrical machines, whether these points are set by the application or just an assumption based on the experience. In the permanent magnet machine which is designed in this thesis the rotational speed is very high, therefore a retaining carbon fiber sleeve is presumed to keep the magnets in place. The reason to use the carbon fiber sleeve is justified by the mechanical analysis which is not in the scope of this thesis; however, the specified tested sleeve is determined and chosen by the industry to be carbon fiber with the thickness of 4 mm when the tangential speed at the inner surface of the sleeve is 200 m/s. Considering this starting point, the outer radius for the permanent magnets and the whole rotor are determined. Choosing number of poles and permanent magnet type and arrangement is the next step. One parameter that can be changed is the magnet thickness which can significantly attribute to the power, induced voltage in the stator windings, maximum flux density in the slot teeth and stator iron losses. It is also possible to use a shielding cylinder between magnets and carbon fiber sleeve to reduce the amount of induced eddy current losses in the magnets. The shaft material is also a parameter which can slightly change the field of permanent magnets in the machine.

The next step is to choose a proper stator configuration, slotted or slot-less. The difference between these two structures is going to be analyzed later. After choosing the stator
configuration, the winding distributions should be determined. There are different distributions such as single or double layers and number of slots per pole per phase such as 1, 2 and etc. with different short pitch angles. The more the number of slots, the harder to manufacture for this small high frequency machine and the more costly it will become. Therefore the slot-less versus slotted and different number of slots are going to be analyzed and compared in the design optimization chapter.

Not to mention that by choosing a slotted configuration, the maximum flux density in the teeth and of course back iron should be controlled to avoid saturation due to the field of permanent magnets and stator current.

It is assumed that the normal 3-phase voltage source inverter (VSI) is going to be used to feed the machine and the input DC voltage is fixed. The amplitude of the induced voltage in the stator due to the magnet field (no-load voltage) should be equal to the fundamental amplitude of the inverter output voltage. The number of turns in the stator can significantly change the induced voltage in the stator. However, by changing the amplitude modulation of the VSI the two output voltages can always match without increasing the number of voltage harmonics.

By knowing the maximum average power and the maximum induced voltage in the stator, the amplitude of fundamental armature current can be determined. This affects the slot area and therefore winding copper area if there is a limit for the maximum surface current density in the stator windings.

Determining the stator configurations, winding distributions and input voltages, the field due to the armature current can be calculated and subsequently the self and leakage inductances which determine the current time harmonics.

Also, by knowing the current harmonics and stator space harmonics the losses in the stator can be calculated due to current field.

In the graph below all the correlations between different parameters and variables can be observed:
Figure 1-1. Correlations between parameters and variables of the machine.
As is obvious from the figure above, there are lots of correlations between different parameters and variables. However, some dependencies are less significant and can be ignored to find simple relationships. Below you can find these correlations in a text format to realize them better. The stator resistive and iron losses were not presented in the figure above to achieve a clear picture, while they are presented below:

- **No Load Voltage** $\propto$ (Geometry, Speed, Magnets, Winding Distribution, Slot Area and Materials)
- **Current First Harmonic** $\propto$ (No Load Voltage and Power)
- **Maximum Surface Current Density** $\propto$ (Current First Harmonic and Slot Area)
- **Current Harmonics** $\propto$ (Speed, Self Inductance, Leakage Inductance and Voltage Harmonics)
- **Voltage Harmonics** $\propto$ (Inverter Amplitude Modulation)
- **Inverter Amplitude Modulation** $\propto$ (No Load Voltage)
- **Leakage Inductance** $\propto$ (Slot Area and Winding Distribution)
- **Self Inductance** $\propto$ (Conductivities, Materials, Geometry and Winding Distribution)
- **Rotor Losses** $\propto$ (Current Harmonics, Conductivities, Materials, Geometry and Winding Distribution)
- **Stator Iron Loss** $\propto$ (Geometry, Speed, Materials, Conductivities, Magnets and Current Harmonics)
- **Resistive Loss** $\propto$ (Winding Distribution, Slot Area, Materials, Geometry, Magnet, Frequency and Current Harmonics)

According to the figure, the machine is fed by the voltage source inverter as the current harmonics are dependent on the inverter output voltage and machine inductances. The geometry, winding distribution, slot area, materials, conductivities, magnets, speed and power are assumed to be the inputs for the design model. For this machine the power and speed are determined by the industry application and the choice of geometry, materials, conductivities, slot area, winding distributions and magnets should be done in a proper way to achieve low volume and rotor loss.

### 1-5 Thesis Objectives

The objectives of this thesis include designing a high speed permanent magnet machine used in a compressor unit. The designing is based on accurate Maxwell equations representing the governing field equation in the machine. All the governing equations in the machine are going to be derived from which the machine can be designed. Choosing a proper rotor permanent magnet type, number of poles, rotor layers materials and dimensions, stator slots and winding configuration, choose of winding type, rotor losses, stator windings resistive and iron losses are also part of the objectives of the thesis. The next phase of the objectives of the objectives includes the comparison between the designed machines. The slotted vs. slot-less machines with the same winding configurations for each of them are going to be compared. Considering the effect of number of slots on the machine over all
losses and observing the effect of different rotor materials, dimensions, air gap thickness and different shaft speeds on the rotor losses are the final objectives of this thesis. All the results are also verified by Finite Element method.

1-6 Thesis Layout

The thesis is divided into 5 chapters. Chapter one starts with some introduction on the electrical machine design criteria and methodology. Chapter two is the main electromagnetic design of the machine. All the governing equations of the machine are derived in this chapter. This chapter starts with introducing Maxwell equations and then goes for choosing rotor PM type, arrangement, dimensions and number of poles. Then the field of PM is calculated. Physical representing of the winding distributions and armature current field for both slotted and slot-less machines are calculated. Then the stator voltage equations and inductances are introduced and explained. At the end the power and torque of the machine are calculated. Chapter three is devoted to the major machine losses. It starts with rotor induced eddy current losses due to the rotor conductivities and then the resistive losses, including the skin and proximity effects, are calculated for both slotted and slot-less machines. A part of the machine loss is about the stator iron losses which are hysteresis and eddy current losses in yoke and teeth. In chapter four different machines are designed and compared, with the defined criteria, in different aspects such as: slotted, slot-less, different rotor materials and dimensions, different air gap thicknesses and different shaft speeds.
2-1 Maxwell Equations

In electrical machines the magnetic circuits may be formed by ferromagnetic, diamagnetic and paramagnetic materials. Diamagnetic materials (such as copper) have a weak, negative susceptibility to magnetic fields. These materials are slightly repelled by a magnetic field and do not retain magnetic properties when the external field is removed. Paramagnetic materials (such as Magnesium) have a small, positive susceptibility to magnetic fields. They are slightly attracted by a magnetic field but do not retain the magnetic properties after external field is removed [C.Kittel2004]. Ferromagnetic materials (such as Iron) have a large, positive susceptibility to an external magnetic field [R.M.Bozorth1993]. The ferromagnetic materials are divided in to soft and hard materials. Soft materials (such as Iron, Nickel) do not retain the magnetic properties while the hard materials (permanent magnets) retain.

2-1-1 Introduction

In order to solve the magnetic circuits of an electrical machine there are two different possible ways. One way is to use the equivalent magnetic circuit in all parts of the machine by making use of reluctances of different areas. Here the simplified algebraic Maxwell equations are used, and are rather complex in machines where an accurate result is expected. Besides, the equivalent magnetic circuit would be quite large when modeling all the parts of the machine. The
other way is to calculate the differential forms of the Maxwell equations in different regions of
the machine. In this way the magnetic vector potential is first introduced in order to calculate the
magnetic flux density in any region. This method is much more precise than the previous one and
is the basis of field calculation in many Finite Element softwares.
This chapter documents the electromagnetic background for the thesis. Firstly the equations of
Maxwell are listed for a stationary media and then simplified using the Stokes’s theorem. The
equations are further simplified for a quasi-magneto-static situation and by using the Gauss’s
theorem the relation between magnetic vector potential and flux density are achieved. Then for an
instantaneous, linear and isotropic medium the Poisson equation is derived which is the main
leading equation for field calculations though out the thesis.

2-1-2 Maxwell equations in a stationary matter

The Maxwell equations in a stationary matter [H.Blok1975] and [J.V.Bladel1973] are as
follows:

\[
\oint_c \bar{H} \cdot \bar{\tau} \, dl = \iint_s \left( \vec{J}_{ext} + \vec{J} + \vec{\partial_t \bar{D}} \right) \cdot \vec{n} \, ds
\]

(2.1)

\[
\oint_c \bar{E} \cdot \bar{\tau} \, dl + \iint_s \left( \vec{K}_{ext} + \vec{\partial_t \bar{B}} \right) \cdot \vec{n} \, ds = 0
\]

Where \( c \) is the contour around the surface \( s \), \( \bar{H} \) is the magnetic field strength, \( \bar{E} \) is the electric
field strength, \( \bar{B} \) is the magnetic flux density, \( \bar{D} \) is the electric flux density, \( \vec{J}_{ext} \) and \( \vec{K}_{ext} \) are the
external surface and line current densities in the matter, respectively, and \( \vec{J} \) is the induced surface
current density due to induced eddy currents in the matter.
The electric and magnetic flux densities are defined as:

\[
\bar{D} = \varepsilon_0 \bar{E} + \bar{P}
\]

(2.2)

\[
\bar{B} = \mu_0 (\bar{H} + \bar{M})
\]

(2.3)

In which \( \bar{P} \) and \( \bar{M} \) are the polarization and magnetization vectors.
According to the Stokes’s theorem in an arbitrary object with a surface of \( s \) and enclosed contour
of \( c \), for an arbitrary vector of \( \bar{F} \) there is:

\[
\oint_c \bar{F} \cdot \bar{\tau} \, dl = \iint_s (\nabla \times \bar{F}) \cdot \vec{n} \, ds
\]

(2.4)

Therefore it is concluded from equations (2.1) and (2.4) that:
\[ \nabla \times \vec{H} = \vec{J}_{\text{ext}} + \dot{\vec{J}} + \partial_t \vec{D} \]  
(2.5)
\[ \nabla \times \vec{E} + \vec{K}_{\text{ext}} + \partial_t \vec{B} = 0 \]

Assuming the magneto-quasi-static situation, the external line current density and the time variation of electric flux density will be neglected. In this case the Maxwell equations will be:

\[ \nabla \times \vec{H} = \vec{J}_{\text{ext}} + \dot{\vec{J}} \]  
(2.6)
\[ \nabla \times \vec{E} + \partial_t \vec{B} = 0 \]  
(2.7)

Referring to the flux conservation law, there will be:

\[ \oint_s \vec{B} \cdot \hat{n} \, ds = 0 \]  
(2.8)

According to the Gauss’s theorem in an arbitrary object with a volume of \( v \) and enclosed surface of \( s \), for an arbitrary vector of \( F \) there is:

\[ \iiint_v F \cdot \hat{n} \, ds = \iiint_v \nabla \cdot F \, dv \]  
(2.9)

Therefore it is concluded from Gauss’s theorem that:

\[ \nabla \cdot \vec{B} = 0 \]  
(2.10)

By introducing vector \( A \) as the magnetic vector potential, from equation (2.10) it is concluded that:

\[ \vec{B} = \nabla \times \vec{A} \]  
(2.11)

And therefore

\[ \nabla \cdot \vec{A} = 0 \]  
(2.12)

The method used in this thesis to calculate the magnetic flux density in different parts of the machine is associated with the equation (2.11); therefore, by knowing the magnetic vector potential the flux density can be evaluated.

The magnetization vector has two parts: the temporary and permanent parts:

\[ \vec{M} = \vec{M}_t + \vec{M}_p \]  
(2.13)
For an instantaneous, linear and isotropic medium we will have:

\[ \bar{J} = \sigma \bar{E} \]  
\[ (2.14) \]

\[ \bar{M}_r = \chi_m \bar{H} \]  
\[ (2.15) \]

In which \( \chi_m \) is the magnetic susceptibility tensor. This fact transforms (2.3) in to:

\[ \bar{B} = \mu_0 (\bar{H} + \bar{M}) \]
\[ = \mu_0 \bar{H} + \mu_0 \bar{M}_r + \mu_0 \bar{M}_p \]  
\[ (2.16) \]

Using this equation for magnetic field strength and by substituting it in Ampere’s equation (2.6) and according to (2.11), there will be:

\[ \nabla \times \bar{H} = \bar{J}_{ex} + \bar{J} \]
\[ \nabla \times (\frac{\bar{B} - \bar{B}_{rem}}{\mu}) = \bar{J}_{ex} + \bar{J} \]  
\[ (2.17) \]

\[ \nabla \times \bar{B} = \mu \bar{J}_{ex} + \mu \bar{J} + \nabla \times \bar{B}_{rem} \]
\[ \nabla \times (\nabla \times \bar{A}) = \mu \bar{J}_{ex} + \mu \bar{J} + \nabla \times \bar{B}_{rem} \]

By knowing the fact that:

\[ \nabla \times (\nabla \times \bar{A}) = \nabla (\nabla \cdot \bar{A}) - \nabla^2 \bar{A} \]  
\[ (2.18) \]

We will have:

\[ -\nabla^2 \bar{A} = \mu \bar{J}_{ex} + \mu \bar{J} + \nabla \times \bar{B}_{rem} \]  
\[ (2.19) \]

And according to (2.14) and

\[ \bar{E} = -\nabla \bar{A} \]  
\[ (2.20) \]

It can be concluded that:

\[ -\nabla^2 \bar{A} + \mu \sigma \nabla \bar{A} = \mu \bar{J}_{ex} + \nabla \times \bar{B}_{rem} \]  
\[ (2.21) \]
This is called the general Poisson Equation and is the main equation for solving the magnetic vector potential and the flux density in the machine due to different sources such as external stator current, permanent magnetization of the rotor magnets and the induced eddy currents in the machine.

It is assumed that the magnetic vector potential of different sources can simply be added together algebraically (i.e. linear characteristic):

$$\vec{A}_{\text{total}} = \vec{A}_{\text{magnet}} + \vec{A}_{\text{stator current}}$$

(2.22)

### 2-2 Field of Permanent Magnet

In the previous chapter the basics of electromagnetic design were introduced and the leading important Poisson equation was derived. Here, in this chapter, a brief introduction to the use of rotor permanent magnets is discussed and then with the use of Poisson equation, the field of permanent magnets in different parts of the machine is calculated.

### 2-2-1 Introduction

There are different types of permanent magnet (PM) arrangements for the rotor such as surface mounted, interior buried and Halbach arrangements.

Halbach magnetized machines are emerging as a competitive class of permanent magnet brushless machine for a wide range of applications. Halbach PM offers a sinusoidal flux density in the air gap and induced e.m.f without conventional means such as skewing, distributed winding and optimization of magnet pole-arc. Besides, due to self-shielding magnetization, the rotor back iron is not necessary leading to better rotor dynamics. In [J.O.Tenkorang1995] the Halbach and conventional surface mounted magnets are compared for the torque production viewpoint. It was concluded that with the use of magnetic back iron the Halbach array produces much torque than the conventional surface mounted one with the same volume of magnets. Various machine topologies and alternative methods of realizing Halbach magnetized magnets have been described in [Z.Q.Zhu2001]; the radially magnetized PM is compared with Halbach array. It was concluded that the air gap field is a function of magnet thickness for both of the machines and for radially magnetized one there is an optimum thickness and the field doesn’t depend on the number of poles; while in the Halbach machine, the field depends on the number of poles and increases with increase of thickness and pole number. In [K.Atallah1998] and [Z.P.Xia2004] the magnetic design of permanent magnet machines was analyzed using Halbach arrangement. The iron core rotor is beneficial only when Halbach magnetized magnet is relatively thin.

The Halbach PM arrangement is good for high speed applications because of significantly reduction of iron losses. Halbach magnetized magnets can either be fabricated from pre-magnetized magnet segments or manufactured as complete ring magnets, which are subsequently impulse magnetized.
The Halbach magnetized magnets can have several number of segments; the higher the number of segments, the closer the flux density to sinusoidal form. The ideal sinusoidal wave form for the flux density is achieved by the ideal Halbach arrangement.

2-2-2 PM Field Calculations

For the current machine the 2 pole diametrically magnetized ring magnet is chosen. The reason is that such a magnet represents an ideal Halbach performance. The remanent flux density of such a magnet is:

$$B_{rem} = B_{rem,r} \tilde{i}_r + B_{rem,\phi} \hat{\phi} = B_{rem} \sin(\phi) \tilde{i}_r + B_{rem} \cos(\phi) \hat{\phi}$$ (2.23)

That $(r, \phi, z)$ is the rotor cylindrical coordinate system. For calculating the magnetic vector potential and flux density due to rotor permanent magnets, the Poisson equation which was derived in the previous section should be used. In this case the external stator currents will be set to zero and the effect of induced eddy currents field, in the stator iron, on the field of permanent magnets will be neglected. It doesn’t mean that the stator laminations have no electrical conductivity and no eddy current induced losses; but only the effect of induced eddy current in the laminations due to PM field on the main field will be ignored. There will be no induced eddy current in the rotor due to magnet field since its field is synchronous with the rotor layers. Therefore the governing Poisson equation (2.21) will be transformed in to:

$$-\nabla^2 \vec{A} = \nabla \times \vec{B}_{rem}$$ (2.24)

This equation is solved for different regions of the machine and for all the parts except for the magnets the remanence flux density is zero and the equation to be solved gets simpler form that will be the Laplace equation:

$$\nabla^2 \vec{A} = 0$$ (2.25)

In which the $\nabla^2$ is the Laplace operator.

For calculating the rotor permanent magnet field in different parts of the machine, the Maxwell equation should be solved first. The machine is divided in to several concentric circular parts and within each part the material has specific properties. In the table below you can find the different machine parts with their electromagnetic properties along with their major and minor governing equations:
Table 2-1. Governing electromagnetic equations for different parts of machine due to PM field.

<table>
<thead>
<tr>
<th>Regions</th>
<th>Radius Range</th>
<th>Permeability</th>
<th>Major Equations</th>
<th>Minor Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>7- Stator yoke</td>
<td>( R_6 \leq r &lt; R_7 )</td>
<td>( \infty )</td>
<td>( \nabla^2 \vec{A} = 0 )</td>
<td>( \vec{H} = 0 )</td>
</tr>
<tr>
<td>6- Winding</td>
<td>( R_5 \leq r &lt; R_6 )</td>
<td>( \mu_6 )</td>
<td>( \nabla^2 \vec{A} = 0 )</td>
<td>( \vec{H} = \frac{\vec{B}}{\mu_6} )</td>
</tr>
<tr>
<td>5- Air gap</td>
<td>( R_4 \leq r &lt; R_5 )</td>
<td>( \mu_5 )</td>
<td>( \nabla^2 \vec{A} = 0 )</td>
<td>( \vec{H} = \frac{\vec{B}}{\mu_5} )</td>
</tr>
<tr>
<td>4- Sleeve 2</td>
<td>( R_3 \leq r &lt; R_4 )</td>
<td>( \mu_4 )</td>
<td>( \nabla^2 \vec{A} = 0 )</td>
<td>( \vec{H} = \frac{\vec{B}}{\mu_4} )</td>
</tr>
<tr>
<td>3- Sleeve 1</td>
<td>( R_2 \leq r &lt; R_3 )</td>
<td>( \mu_3 )</td>
<td>( \nabla^2 \vec{A} = 0 )</td>
<td>( \vec{H} = \frac{\vec{B}}{\mu_3} )</td>
</tr>
<tr>
<td>2-PM</td>
<td>( R_1 \leq r &lt; R_2 )</td>
<td>( \mu_2 )</td>
<td>( \nabla^2 \vec{A} = -\nabla \times \vec{B}_{rem} )</td>
<td>( \vec{H} = \frac{\vec{B} - \vec{B}_{rem}}{\mu_2} )</td>
</tr>
<tr>
<td>1- Shaft</td>
<td>( 0 \leq r &lt; R_1 )</td>
<td>( \mu_1 )</td>
<td>( \nabla^2 \vec{A} = 0 )</td>
<td>( \vec{H} = \frac{\vec{B}}{\mu_1} )</td>
</tr>
</tbody>
</table>

As can be seen from the table above the shaft region is not assumed to have the infinite permeability of iron; however, it is assumed to have a parametric permeability so that it becomes possible to analyze for different shaft materials such as solid and laminated iron and also air. The permanent magnet region is the only region in which the Poisson equation needs rather more attention unlike the other regions that only the Laplace equation should be solved. There are two more layers above the magnet that are retaining sleeves. The first one is required to shield the induced eddy current field due to armature current (next sections) and the outer sleeve is for retaining the magnets in place for high speeds. One possible option would be to combine the air gap region with the sleeves ones and assume the same electromagnetic characterizes for all of them; nevertheless, it is more accurate to separate these regions from their different relative permeabilities.

This model is rather general which accounts for both slotted and slot less stator configurations. The region 6, winding, is the real winding region only in the slot less machines, but for slotted machine it is the same as air gap and the effect of armature current will be considered by assuming a line current density at the border of air gap and stator slots, which will be discussed in much more details in the next sections. In the slotted machine the slots and the back iron are unified as one region. The reason is that almost all of the flux generated by the rotor PM goes through the slots teeth to the yoke and only a little portion of them pass through the windings in the slots. Therefore, it is reasonable to assume the stator slots and teeth a uniform material with properties of teeth.

The assumptions which are used in this analysis are that the permeability of back iron is assumed to be infinite, the effect of saturation in the iron is neglected and the electrical conductivity of the iron is also neglected so that there will be no induced eddy current in the stator iron due to rotor field.
By disregarding the effect of end regions of machine, the magnetic flux density has radial and tangential components in cylindrical co-ordinate system; therefore, according to equation (2.11), the magnetic vector potential is always in the $z$ direction.

For calculating the magnetic vector potential the Poisson equation should be solved. However, prior to that, the Laplace equation is explained here because the Poisson equation is solved by solving the associated Laplace equation first. The Laplace equation of magnetic vector potential which has only the $z$ component in rotor cylindrical co-ordinate system is defined as:

$$
\nabla^2 \vec{A} = 0 \rightarrow \frac{\partial^2 A_z}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 A_z}{\partial \phi^2} + \frac{1}{r} \frac{\partial A_z}{\partial r} = 0
$$

(2.26)

It’s also common to show the Laplace operator as:

$$
\nabla^2 \vec{A} = \Delta \vec{A}
$$

(2.27)

The Poisson equation in region 2, permanent magnet, is:

$$
\nabla^2 \vec{A} = -\nabla \times \vec{B}_{rem}
$$

(2.28)

That for a diametrically magnetized permanent magnet with the remanent flux density of:

$$
\vec{B}_{rem} = \hat{B}_{rem} \sin(p\phi) \hat{r} + \hat{B}_{rem} \cos(p\phi) \hat{\phi}
$$

(2.29)

And using

$$
\nabla \times \vec{B}_{rem} = \frac{1}{r} \left| \begin{array}{ccc}
\hat{r} & r \hat{\phi} & \hat{z} \\
\frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\
\hat{B}_{rem} \sin(p\phi) & r \hat{B}_{rem} \cos(p\phi) & 0
\end{array} \right|
$$

(2.30)

The Poisson equation of (2.28) will be:

$$
\nabla^2 \vec{A} = \hat{B}_{rem} (1 - p) \cos(p\phi) \hat{z}
$$

(2.31)

Since for the current machine the number of pole pairs, $p$, is assumed to be one; therefore, the Poisson equation in the permanent magnet region will be the same as the Laplace equation. Thus the Laplace equation should be solved in all regions. The Laplace equation is solved by the method of separation of variables [A.Jeffrey1990]; the magnetic vector potentials for different regions are as follows:
Chapter 2  
Electromagnetic Design

\[ \vec{A}_{z,1} = d_1 \left( \frac{r}{R_1} \right) \cos(\phi) \hat{i}_z \]  
\[ \vec{A}_{z,x} = \left[ c_x \left( \frac{R_x}{r} \right) + d_x \left( \frac{r}{R_x} \right) \right] \cos(\phi) \hat{i}_z \quad x = 2, 3, ..., 7 \]

In which \( R_x \) is the outer radius of region \( x \). And by using

\[ \vec{B} = \nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \hat{i}_r & r \hat{i}_\phi & \hat{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix} = \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_z}{\partial r} \hat{i}_\phi = B_r \hat{i}_r + B_\phi \hat{i}_\phi \]  

the magnetic flux density is calculated. The \( c_x \) and \( d_x \) are constants and are calculated form the boundary conditions in different regions. The general boundary conditions for Ampere’s law are:

\[ H_{i,\phi}(r = R, \phi) - H_{i+1,\phi}(r = R, \phi) = -K_s(r = R, \phi) \]  

In which \( K_s \) is the line current density \([A/m]\), and for flux conservation law:

\[ B_{i,r}(r = R, \phi) - B_{i+1,r} = 0 \]

Therefore all the boundary conditions in different borders of the machine are listed in table 2-2:

<table>
<thead>
<tr>
<th></th>
<th>( H_{1,\phi}(r = R_1) = H_{2,\phi}(r = R_1) )</th>
<th>( H_{2,\phi}(r = R_2) = H_{3,\phi}(r = R_2) )</th>
<th>( H_{3,\phi}(r = R_3) = H_{4,\phi}(r = R_3) )</th>
<th>( H_{4,\phi}(r = R_4) = H_{5,\phi}(r = R_4) )</th>
<th>( H_{5,\phi}(r = R_5) = H_{6,\phi}(r = R_5) )</th>
<th>( H_{6,\phi}(r = R_6) = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( B_{1,r}(r = R_1) = B_{2,r}(r = R_1) )</td>
<td>( B_{2,r}(r = R_2) = B_{3,r}(r = R_2) )</td>
<td>( B_{3,r}(r = R_3) = B_{4,r}(r = R_3) )</td>
<td>( B_{4,r}(r = R_4) = B_{5,r}(r = R_4) )</td>
<td>( B_{5,r}(r = R_5) = B_{6,r}(r = R_5) )</td>
<td>( B_{6,r}(r = R_6) = B_{7,r}(r = R_6) )</td>
</tr>
<tr>
<td>2</td>
<td>( B_{1,\phi}(r = R_1) = B_{2,\phi}(r = R_1) )</td>
<td>( B_{2,\phi}(r = R_2) = B_{3,\phi}(r = R_2) )</td>
<td>( B_{3,\phi}(r = R_3) = B_{4,\phi}(r = R_3) )</td>
<td>( B_{4,\phi}(r = R_4) = B_{5,\phi}(r = R_4) )</td>
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<td>( B_{6,\phi}(r = R_6) = 0 )</td>
</tr>
<tr>
<td>3</td>
<td>( B_{1,\phi}(r = R_1) = B_{2,\phi}(r = R_1) )</td>
<td>( B_{2,\phi}(r = R_2) = B_{3,\phi}(r = R_2) )</td>
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<td>( B_{5,\phi}(r = R_5) = B_{6,\phi}(r = R_5) )</td>
<td>( B_{6,\phi}(r = R_6) = 0 )</td>
</tr>
<tr>
<td>4</td>
<td>( B_{1,\phi}(r = R_1) = B_{2,\phi}(r = R_1) )</td>
<td>( B_{2,\phi}(r = R_2) = B_{3,\phi}(r = R_2) )</td>
<td>( B_{3,\phi}(r = R_3) = B_{4,\phi}(r = R_3) )</td>
<td>( B_{4,\phi}(r = R_4) = B_{5,\phi}(r = R_4) )</td>
<td>( B_{5,\phi}(r = R_5) = B_{6,\phi}(r = R_5) )</td>
<td>( B_{6,\phi}(r = R_6) = 0 )</td>
</tr>
<tr>
<td>5</td>
<td>( B_{1,\phi}(r = R_1) = B_{2,\phi}(r = R_1) )</td>
<td>( B_{2,\phi}(r = R_2) = B_{3,\phi}(r = R_2) )</td>
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<td>( B_{6,\phi}(r = R_6) = 0 )</td>
</tr>
<tr>
<td>6</td>
<td>( B_{1,\phi}(r = R_1) = B_{2,\phi}(r = R_1) )</td>
<td>( B_{2,\phi}(r = R_2) = B_{3,\phi}(r = R_2) )</td>
<td>( B_{3,\phi}(r = R_3) = B_{4,\phi}(r = R_3) )</td>
<td>( B_{4,\phi}(r = R_4) = B_{5,\phi}(r = R_4) )</td>
<td>( B_{5,\phi}(r = R_5) = B_{6,\phi}(r = R_5) )</td>
<td>( B_{6,\phi}(r = R_6) = 0 )</td>
</tr>
</tbody>
</table>

There are 13 equations and 13 unknowns; so, the magnetic flux density due to permanent magnet field is calculated for different parts of machine.
2-3 Winding distributions

There are different winding configurations that can be implemented in the machine. There are distributed vs. concentrated windings as well as different number of slots (per pole per phase) and short pitch angle in the distributed winding. The true modeling of them is very important from the analytical ease of calculations.

2-3-1 Introduction

The concentrated winding has higher spatial harmonics rather than the distributed one, and since the rated speed of this machine is so high and because of the high losses due to these high frequency harmonics, the distributed winding is preferred rather than the concentrated one. However, the concentrated winding is easier to manufacture, since the pre-wound coils are just to settle in the slots.

Besides the distributed and concentrated windings, the number of slots, number of slots per pole per phase and the short pitch angle also play a role in the total machine performance and especially in the space harmonics.

In order to observe the effect of the winding distribution on the induced voltage and the armature current field it is necessary to calculate the Fourier-series of a specific winding distribution. To do so, there are two methods: one with direct Fourier analysis and the other one with winding factor approach.

2-3-2 Direct Fourier analysis

The representation of a periodic function $f$ on an interval $[a,b]$ is as follows:

$$f(\phi) = \frac{1}{2} a_0 + \sum_{k=1}^{\infty} \left[ a_k \cos \left( \frac{2k \pi \phi}{b-a} \right) + b_k \sin \left( \frac{2k \pi \phi}{b-a} \right) \right]; \quad a \leq \phi \leq b \quad (2.37)$$

Where

$$a_0 = \frac{1}{b-a} \int_{a}^{b} f(\phi) d\phi \quad (2.38)$$

$$a_k = \frac{2}{b-a} \int_{a}^{b} f(\phi) \cos \left( \frac{2k \pi \phi}{b-a} \right) d\phi; \quad k = 1, 2, \ldots \quad (2.39)$$
Representing a winding distribution by direct Fourier-series has the disadvantage that if a property of the winding such as number of layers or short-pitch angle changes then a complete calculation of series should be done from the beginning which is not time efficient. Therefore another approach for calculation of Fourier-series is presented by winding factor which removes this disadvantage.

2-3-3 Winding Factor:

Several ways of introducing the winding factors have been introduced over the years. In [G.R.Slemon1992] the winding factor is the ratio of the MMF of a physical winding distribution to that of a concentrated one. In [R.Richter1967] this factor is defined as the ratio between the flux linked of these two types of windings, and some other authors introduce the ratio as the one between the surface current densities of the two winding types. In [H.Polider1998], the winding factor is directly the ratio of the equivalent number of turns of a space harmonic of a physical winding to that of a concentrated winding. In whatever way the winding factors are introduced, they have the same meaning since the physical quantity from which they are derived disappears in the derivation and only the equivalent number of turns is left. It is for this reason that the winding factors are simply a tool to simplify the Fourier-series representation of a physical winding distribution. Therefore they are also used to represent the air gap windings in slot-less machines in [K.Sridhar1995] and [K.Atallah1998]. There are four different types of winding factors commonly used today. These are:

- **Distribution factor:** this factor modifies the Fourier coefficient of the winding distribution for the case where the winding is distributed over several slots instead of concentrated in an infinitesimal slot. It is given by:

\[
K_{w,\text{dist},k} = \frac{\sin\left(\frac{k\pi}{2m}\right)}{q \sin\left(\frac{k\pi}{2mq}\right)}
\]

In which \(q\) is the number of slots per pole per phase; \(m\), the number of phases and \(k\), is the spatial harmonic number.

- **Pitch factor:** this factor accounts for the short pitching of the winding; i.e. where the return conductors are located at less than a pole pitch from the go conductors. This factor is written as:
\[ K_{w,\text{pitch},k} = \cos\left(\frac{1}{2} kp\varphi_{\text{pitch}}\right) \] (2.42)

- **Slot factor:** this factor further modifies the Fourier coefficient because of the slotting of the winding (the physical slot width). The slot factor is expressed as:

\[ K_{w,\text{slot},k} = \frac{\sin\left(\frac{1}{2} kp\varphi_{\text{so}}\right)}{\frac{1}{2} kp\varphi_{\text{so}}} \] (2.43)

Where \( \varphi_{so} \) is the slot-opening angle and \( p \) is the number of pole pairs.

- **Skew factor:** when the stator is skewed with respect to the rotor, usually to reduce or eliminate cogging torque, this factor is taken into account. The skew factor is identical in expression to the slot factor:

\[ K_{w,\text{skew},k} = \frac{\sin\left(\frac{1}{2} kp\varphi_{\text{skew}}\right)}{\frac{1}{2} kp\varphi_{\text{skew}}} \] (2.44)

Where \( \varphi_{skew} \) is the skew angle.

Then the total winding factor is the product of those four individual different winding factors:

\[ K_{w,k} = K_{w,\text{dist},k} K_{w,\text{pitch},k} K_{w,\text{slot},k} K_{w,\text{skew},k} \] (2.45)

The winding distribution is expressed in stator co-ordinate system, \((r, \varphi, z)\) which is defined as:

\[ \varphi = \phi + \theta \] (2.46)

And \( \theta \) is the rotor position angle in steady state that is expressed by:

\[ \theta = wt + \theta_0 \] (2.47)

In which \( \theta_0 \) is the position of rotor at time zero (initial position). So the Fourier series of the winding distribution with the winding factor approach when a sinusoidal wave form is assumed for the distribution is expressed as:

\[ n_{aw}(\varphi) = \sum_{k=1,3,...}^{\infty} \hat{n}_{s,k} \sin(kp\varphi) \] (2.48)
where

\[ \hat{n}_{s,k} = \frac{1}{2} N_{s,k} \]  

(2.49)

in which

\[ N_{s,k} = \frac{4}{\pi} K_{w,k} N \sin\left(\frac{k\pi}{2}\right) \]  

(2.50)

And \( N \) is the total number of turns per phase. The winding distributions of other phases, phases \( b \) and \( c \), are the same as phase \( a \) but with phase shifts of \( \frac{2\pi}{3p} \) and \( \frac{4\pi}{3p} \), respectively:

\[ n_{sb}(\varphi) = \sum_{k=1,3,\ldots}^{\infty} \hat{n}_{s,k} \sin(k(p\varphi - \frac{2\pi}{3}))) \]  

(2.51)

\[ n_{sc}(\varphi) = \sum_{k=1,3,\ldots}^{\infty} \hat{n}_{s,k} \sin(kp(\varphi - \frac{4\pi}{3}))) \]  

(2.52)

And when a cosine form is chosen for the winding distribution, it will be:

\[ n_{sa}(\varphi) = \sum_{k=1,3,\ldots}^{\infty} \hat{n}_{s,k} \cos(kp\varphi) \]  

(2.53)

where

\[ \hat{n}_{s,k} = \frac{1}{2} N_{s,k} \]  

(2.54)

in which

\[ N_{s,k} = \frac{4}{\pi} K_{w,k} N \]  

(2.55)

If the cosine form for the winding distribution is chosen it would be easy to separate the triple and non-triple components as follows:

\[ n_{sa}(\varphi) = \sum_{k=\infty}^{\infty} \hat{n}_{s,6k+1} \cos((6k+1)p\varphi) + \sum_{k=0}^{\infty} \hat{n}_{s,6k+3} \cos((6k+3)p\varphi) \]  

(2.56)

In which the first term, the double sided one, represents the non-triple spatial harmonics and the second term, the single sided one, represents the triple spatial harmonics.
2-4 Armature Field

So far, in previous chapters, the field of permanent magnet inside the machine was calculated using the Maxwell equations and specifically the Poisson equation. The winding distribution was also modeled using the winding factor approach. In this chapter the field of armature current of the stator in the machine is calculated with the similar method with the magnet field, solving the Poisson equations in different regions of the machine. The effects of conductivities of rotor layers on the main field are also taken into account in order to calculate the rotor losses which will be discussed in more details in the next chapter.

2-4-1 Introduction

For calculating the magnetic flux density in the machine due to armature current the general Poisson equation (2.21) should be solved in different regions; however, the permanent magnet magnetization is set to zero and therefore the magnets have the same magnetic characteristics as the air.

In calculating the magnetic field in the machine there is a notable difference between the slotted and slot-less stator configurations. In the slot-less stator the windings are located in the air gap and the winding surface current density, $J_s$, is directly used in the Poisson equation as is explained in [N.Bianchi2005] and [K.Atallah1998b]. However, in the slotted structure the solution is a bit different; in the Poisson equation the armature current term is set to zero and the effect of the current field is taken into account in the boundary conditions. An equivalent line current density, $K_s$, is presumed at the border of stator and air gap and the Poisson equations are solved in all regions as is discussed and explained in [A.Hughes1977] and [H.Polinder1997].

The model which is used to calculate the field of armature current in this chapter is so precise, because as it is obvious from the table below, the conductivities of all rotor layers are taken into account along with their permeabilities. Therefore this model is kind of smart as is able to calculate armature field with different materials in the rotor. By considering the conductivity in each rotor layer, the effect of induced eddy current (in that layer) field on the field of other regions is in fact taken into account. The other advantage is that the loss in each layer with conductivity is achievable by using Poynting vector theorem, which will be discussed further in the next chapter. The permeability for each layer is assumed to be a parameter, so that it is possible to simulate different materials with different permeabilities. For the air gap the conductivity is assumed to be zero, so are the ones for the winding region and stator yoke iron. The Poisson equation in the winding region contains the surface current density which is only valid for the slot less stator configuration. In the case of slotted stator, the Poisson equation in this region will simply become Laplace equation, the same as air gap, and the effect of line current density (which is assumed in the border between air gap and stator) will be taken into account in the boundary conditions.

The conductivity in the stator laminations is neglected. There are several reasons for that assumption: firstly, the yoke is mostly laminated to reduce the induced eddy currents effects and the lamination conductivity is very small compared to the solid material. Secondly, it is very
difficult to find the laminated material conductivity with respect to lamination thickness and material. Thirdly, only the stator current time harmonics and permanent magnet space harmonics (which have only the fundamental one in this machine) play a role in the yoke losses, with a little influence of space harmonics of the stator windings. It is also easy to cool down the stator with different methods rather than the rotor which is rotating at ultra high speeds.

The reason to consider the conductivity in the shaft is that different types of shafts could be simulated, such as laminated (with almost zero conductivity), solid and even a hollow shaft. Therefore the aforementioned Poisson equations in different parts of the machine and the electromagnetic characteristics of them are as follows:

<table>
<thead>
<tr>
<th>Regions</th>
<th>Radius Range</th>
<th>Permeability</th>
<th>Major Equations</th>
<th>Minor Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>7- Stator yoke</td>
<td>$R_6 \leq r &lt; R_7$</td>
<td>$\infty$</td>
<td>$\nabla^2 \vec{A} = 0$</td>
<td>$\vec{H} = 0$</td>
</tr>
<tr>
<td>6- Winding</td>
<td>$R_5 \leq r &lt; R_6$</td>
<td>$\mu_6$</td>
<td>$\nabla^2 \vec{A} = -\mu_6 \vec{J}_s$</td>
<td>$\vec{H} = \vec{B}/\mu_6$</td>
</tr>
<tr>
<td>5- Air gap</td>
<td>$R_4 \leq r &lt; R_5$</td>
<td>$\mu_5$</td>
<td>$\nabla^2 \vec{A} = 0$</td>
<td>$\vec{H} = \vec{B}/\mu_5$</td>
</tr>
<tr>
<td>4- Sleeve 2</td>
<td>$R_3 \leq r &lt; R_4$</td>
<td>$\mu_4$</td>
<td>$\nabla^2 \vec{A} = \mu_4 \sigma_4 \partial_i \vec{A}$</td>
<td>$\vec{H} = \vec{B}/\mu_4$</td>
</tr>
<tr>
<td>3- Sleeve 1</td>
<td>$R_2 \leq r &lt; R_3$</td>
<td>$\mu_3$</td>
<td>$\nabla^2 \vec{A} = \mu_3 \sigma_3 \partial_i \vec{A}$</td>
<td>$\vec{H} = \vec{B}/\mu_3$</td>
</tr>
<tr>
<td>2-PM</td>
<td>$R_1 \leq r &lt; R_2$</td>
<td>$\mu_2$</td>
<td>$\nabla^2 \vec{A} = \mu_2 \sigma_2 \partial_i \vec{A}$</td>
<td>$\vec{H} = \vec{B}/\mu_2$</td>
</tr>
<tr>
<td>1- Shaft</td>
<td>$0 \leq r &lt; R_1$</td>
<td>$\mu_1$</td>
<td>$\nabla^2 \vec{A} = \mu_1 \sigma_1 \partial_i \vec{A}$</td>
<td>$\vec{H} = \vec{B}/\mu_1$</td>
</tr>
</tbody>
</table>

As is obvious from the table above, the Permanent Magnets are assumed to have no magnetization with a linear $B-H$ curve. The Ampere-continuity and flux conservation boundary conditions are as listed in the table below.

The boundary condition 6, the Ampere continuity condition between the air gap and stator regions, has this type only for the slotted stator configuration and for the slot less machine it will become $H_{6,\theta}(r = R_6) = H_{7,\theta}(r = R_6)$, in which region 6 is the winding region.

In this chapter the effect of stator time harmonics and the shaft, sleeves and magnet conductivities are all taken in to account. The time harmonics play an important role in the eddy current losses in the rotor parts and so do inside the stator iron.
Table 2-4. The Ampere and flux conservation laws’ boundary conditions due to stator currents.

<table>
<thead>
<tr>
<th></th>
<th>Condition</th>
<th></th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$H_{1,\varphi} (r = R_1) = H_{2,\varphi} (r = R_1)$</td>
<td>7</td>
<td>$B_{1,r} (r = R_1) = B_{2,r} (r = R_1)$</td>
</tr>
<tr>
<td>2</td>
<td>$H_{2,\varphi} (r = R_2) = H_{3,\varphi} (r = R_2)$</td>
<td>8</td>
<td>$B_{2,r} (r = R_2) = B_{3,r} (r = R_2)$</td>
</tr>
<tr>
<td>3</td>
<td>$H_{3,\varphi} (r = R_3) = H_{4,\varphi} (r = R_3)$</td>
<td>9</td>
<td>$B_{3,r} (r = R_3) = B_{4,r} (r = R_3)$</td>
</tr>
<tr>
<td>4</td>
<td>$H_{4,\varphi} (r = R_4) = H_{5,\varphi} (r = R_4)$</td>
<td>10</td>
<td>$B_{4,r} (r = R_4) = B_{5,r} (r = R_4)$</td>
</tr>
<tr>
<td>5</td>
<td>$H_{5,\varphi} (r = R_5) = H_{6,\varphi} (r = R_5)$</td>
<td>11</td>
<td>$B_{5,r} (r = R_5) = B_{6,r} (r = R_5)$</td>
</tr>
<tr>
<td>6</td>
<td>$H_{6,\varphi} (r = R_6) - H_{7,\varphi} (r = R_6) = -K_s$</td>
<td>12</td>
<td>$B_{6,r} (r = R_6) = B_{7,r} (r = R_6)$</td>
</tr>
<tr>
<td></td>
<td>$B_{7,r} (r = R_7) = 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2-4-2 The line current density $K_s$

The line current density $K_s$ which is used in the Ampere-continuity boundary condition is to represent the effect of stator currents in the slot openings at the interface between the air gap and stator. The line current density of phase $a$ of the stator windings is as follows:

$$K_{s,a} (\varphi, t) = \frac{n_{sa} (\varphi) i_a (t)}{R_b}$$ (2.57)

And the total line current density of all 3 phases of the machine is:

$$K_s (\varphi, t) = \frac{[n_{sa} (\varphi) i_a (t) + n_{sb} (\varphi) i_b (t) + n_{sc} (\varphi) i_c (t)]}{R_b}$$ (2.58)

In which the stator currents are defined as:

$$i_a (t) = \sum_{n=1}^{\infty} \hat{i}_a \cos(n \omega t)$$

$$i_b (t) = \sum_{n=1}^{\infty} \hat{i}_a \cos(n \omega t - \frac{2\pi}{3})$$

$$i_c (t) = \sum_{n=1}^{\infty} \hat{i}_a \cos(n \omega t - \frac{4\pi}{3})$$ (2.59)

However, since in a three phase balanced system the even and triple harmonics are filtered out the current waveforms can be rewritten as:

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The winding distribution with cosine format is used, as in (2.53); the reason is that this format can be split into triple and non-triple harmonics which has the same configuration as the current wave forms and make the calculations more comfortable and simpler. The winding distribution of the phase $a$ of the stator windings is:

$$n_{ao}(\varphi) = \sum_{k=-\infty}^{\infty} \hat{n}_{6k} \cos((6k + 1)p\varphi) + \sum_{k=0}^{\infty} \hat{n}_{6k+3} \cos((6k + 3)p\varphi)$$

(2.61)

And by knowing that

$$\cos(x - y) + \cos(x - y - \frac{2k\pi}{3}) + \cos(x - y - \frac{4k\pi}{3}) = \begin{cases} 3\cos(x - y) & k = 3L & L \in \mathbb{Z} \\ 0 & otherwise \end{cases}$$

(2.62)

The total line current density which is assumed to be at the boundary between air gap and stator is made up of time and space harmonics and is defined as:

$$K_s(\varphi, t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} K_{s, 6k+1, 6n+1}$$

(2.63)

In which

$$K_{s, 6k+1, 6n+1} = \frac{3}{2} \frac{\hat{n}_{6k+1} \hat{I}_{6n+1}}{R_p^2} \cos((6k + 1)\varphi - (6n + 1)\omega t)$$

(2.64)

This current density is expressed in the stator system which is the same as locked rotor situation. Nevertheless, as will later on in the rotor loss calculation chapter will be discussed, the current density equation in the rotor system which is the same as synchronous rotating system is of importance. To transform the equation (2.64) in to the rotor system it is necessary to know that for the initial position of rotor equal to zero we will have:

$$\varphi = \phi + \theta = \phi + \omega t$$

(2.65)

Then, the line current density in the rotor system is:
\[ K_s(\phi, t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{3}{2} \frac{\hat{n}_{nk+1} \hat{i}_{nk+1}}{R_6} \cos[(6k + 1)\phi + 6(k - n)\omega t] \] 

(2.66)

When solving the Maxwell equations it is noticeable that all the equations in all regions be solved in a same coordinate system like in the stator or rotor systems. Since all the parameters like line current density, magnetic vector potential, electric field strength and magnetic flux density have the same format, trigonometric, hereafter all the analysis will be in the complex format since it is easier to manipulate. In this case the real parameter will be the real part of its complex format:

\[ A(\phi, r, t) \equiv \text{Re}\{\overline{A}(\phi, r, t)\} \] 

(2.67)

For example the complex format of the line current density in the rotor system will be:

\[ \overline{K}_s(\phi, t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{3}{2} \frac{\hat{n}_{nk+1} \hat{i}_{nk+1}}{R_6} e^{-j[(6k + 1)\phi + 6(k - n)\omega t]} \] 

(2.68)

2-4-3 The surface current density \( J_s \)

The surface current density \( J_s \) which is used in the Poisson equation of region 6 for slotless machines is defined with the same procedure as for the line current density except that it is located in a surface not a line:

\[ \overline{J}_s(\phi, t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{3}{2} \frac{\hat{n}_{nk+1} \hat{i}_{nk+1}}{R_{cw} h_w} e^{-j[(6k + 1)\phi + 6(k - n)\omega t]} \] 

(2.69)

In which \( R_{cw} \) and \( h_w \) are the winding region central radius and width, respectively.

2-4-4 Solution of Poisson Equation for Slotted Machine

To solve the magnetic vector potential and subsequently the flux density, the Poisson equations should be solved in all regions. These equations take simpler forms as the Laplace equations in air gap and stator yoke regions. However, in the shaft, magnet and sleeves regions the Poisson equations need rather more care, since the conductivities are taken into account in these parts. In order to start to solve all these equations, the magnetic vector potential for the shaft, magnet and sleeves regions will be solved first since the solution for other regions have a simple and similar format. All the calculations hereafter are done in the synchronous rotating system.
The assumed solution for the magnetic vector potential in all regions is as follows:

$$\vec{A}_z(r,\phi,t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \vec{A}_{z,6k+1,6n+1}(r,\phi,t) \quad (2.70)$$

In which

$$\vec{A}_{z,6k+1,6n+1}(r,\phi,t) = \hat{A}_{z,6k+1,6n+1}(r) \frac{3}{2} \frac{\hat{n}_{6k+1} \hat{l}_{6n+1}}{R_6} e^{-j[(6k+1)\phi + 6(k-n)\omega t]} \quad (2.71)$$

is chosen in such a way to comply with the line current density format.

The Poisson equation is solved in the complex format for all space and time harmonics separately and the results are summed up together. Therefore the Poisson equation in a region with conductivity will be:

$$\frac{\partial^2 \vec{A}_{z,6k+1,6n+1}}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \vec{A}_{z,6k+1,6n+1}}{\partial \phi^2} + \frac{1}{r} \frac{\partial \vec{A}_{z,6k+1,6n+1}}{\partial r} = \mu \sigma \frac{\partial \vec{A}_{z,6k+1,6n+1}}{\partial t} \quad (2.72)$$

From equations (2.71) and (2.72) we have:

$$\mu \sigma \frac{\partial \vec{A}_{z,6k+1,6n+1}}{\partial t} = -6 j \mu \sigma (k-n) \omega \quad (2.73)$$

In which the right side of the above equation will be named as:

$$\tau_{6k+1,6n+1}^2 = -6 j \mu \sigma (k-n) \omega \quad (2.74)$$

The Poisson equation is solved by the method of separation of variables; thus, the complex solution for the magnetic vector potential for each time and space harmonic will be:

$$\vec{A}_{z,6k+1,6n+1}(r,\phi,t) = \vec{R}_{6k+1,6n+1}(r) \vec{F}_{6k+1,6n+1}(\phi,t) \quad (2.75)$$

By substituting equations (2.75) and (2.74) in to (2.72) there will be:

$$\frac{r^2}{R_{6k+1,6n+1}} \frac{d^2 \vec{R}_{6k+1,6n+1}}{dr^2} + \frac{r}{R_{6k+1,6n+1}} \frac{d \vec{R}_{6k+1,6n+1}}{dr} + \frac{1}{\vec{F}_{6k+1,6n+1}} \frac{d^2 \vec{F}_{6k+1,6n+1}}{d\phi^2} = \tau_{6k+1,6n+1}^2 r^2 \quad (2.76)$$

This equation can be split in two well-known differential equations as follows:
Chapter 2  Electromagnetic Design

\begin{equation}
1) \quad r^2 \frac{d^2 \bar{R}_{6k+1,6n+1}}{dr^2} + r \frac{d\bar{R}_{6k+1,6n+1}}{dr} - \left[ \bar{r}_{6k+1,6n+1}^2 r^2 + (6k + 1)^2 \right] \bar{R}_{6k+1,6n+1} = 0
\end{equation}

\begin{equation}
2) \quad \frac{1}{F_{6k+1,6n+1}} \frac{d^2 F_{6k+1,6n+1}}{d\phi^2} = -(6k + 1)^2
\end{equation}

The first equation is the modified Bessel equation and the solution for this equation is dependent on the space and time harmonics as if for \( k = n \) and \( k \neq n \) the solutions are different as follows:

\begin{equation}
\bar{R}_{6k+1,6n+1} = \begin{cases} 
\bar{a}_{6k+1,6n+1} \bar{I}_{|6k+1|} (r_{6k+1,6n+1} r) + \bar{b}_{6k+1,6n+1} \bar{K}_{|6k+1|} (r_{6k+1,6n+1} r) & k \neq n \\
\bar{a}_{6k+1,6n+1} \left( \frac{r}{R_o} \right)^{|6k+1|} + \bar{b}_{6k+1,6n+1} \left( \frac{r}{R_o} \right)^{|6k+1|} & k = n 
\end{cases}
\end{equation}

In which \( R_o \) is the outer radius of the region and \( \bar{I} \) and \( \bar{K} \) are the complex Bessel functions of the first and second kind for integer order of \( |6k+1| \), respectively. The solution for the second differential equation can be written in many forms, but by choosing a form similar to the vector potential we will have:

\begin{equation}
\tilde{F}_{6k+1,6n+1}(\phi, t) = h_{6k+1,6n+1} e^{-j[(6k + 1)\phi + 6(k - n)\omega t]}
\end{equation}

By combining equations (2.78) and (2.79) for the magnetic vector potential, the solution for region 1 (shaft) will be:

\begin{equation}
\tilde{A}_{z,1,6k+1,6n+1}(r, \phi, t) = \tilde{A}_{z,1,6k+1,6n+1}(r) \frac{3}{2} \frac{\hat{n}_{6k+1} \hat{l}_{6n+1}}{R_o} e^{-j[(6k + 1)\phi + 6(k - n)\omega t]}
\end{equation}

In which

\begin{equation}
\tilde{A}_{z,1,6k+1,6n+1}(r) = \begin{cases} 
\tilde{c}_{1,6k+1,6n+1} \bar{I}_{|6k+1|} (r_{1,6k+1,6n+1} r) + \tilde{d}_{1,6k+1,6n+1} \bar{K}_{|6k+1|} (r_{1,6k+1,6n+1} r) & k \neq n \\
\tilde{d}_{1,6k+1,6n+1} \left( \frac{r}{R_i} \right)^{|6k+1|} & k = n 
\end{cases}
\end{equation}

In which

\begin{equation}
\bar{r}_{1,6k+1,6n+1}^2 = -6j\mu \sigma (k - n)\omega
\end{equation}

For regions 2, 3 and 4 the solution has similar format only a difference in the
\[ \hat{A}_{z,x,6k+1,6n+1}(r) = \begin{cases} \bar{c}_{x,6k+1,6n+1} \hat{I}_{6k+1}((r_x \bar{c}_{x,6k+1,6n+1} r) + \bar{d}_{x,6k+1,6n+1} \hat{K}_{6k+1}((r_x \bar{c}_{x,6k+1,6n+1} r) & \text{if } k \neq n \\ \bar{c}_{x,6k+1,6n+1} \left( \frac{r}{R_x} \right)^{[6k+1]} + \bar{d}_{x,6k+1,6n+1} \left( \frac{r}{R_x} \right)^{[6k+1]} & \text{if } k = n \end{cases} \tag{2.83} \]

In which \( x \) represents the region number with \( R_x \) defined as the outer radius of that region and

\[ \bar{r}^2_{x,6k+1,6n+1} = -6j\mu_x \sigma_x (k-n)\omega \tag{2.84} \]

The complex constants \( \bar{c}_{x,6k+1,6n+1} \) and \( \bar{d}_{x,6k+1,6n+1} \) are defined by the boundary conditions from table 2 (for the slotted machine) which is also valid for the complex values. As can be seen from the solutions for the magnetic vector potential, the constants are different for two cases \( (k = n \text{ and } k \neq n) \); therefore, all the constants found by boundary conditions for all the regions should be solved for these two cases separately.

The Poisson equation for other regions is the Laplace equation because there is no conductivity and external current in those regions. The solution is also independent to the condition if \( k = n \) or \( k \neq n \) and is the same for both situations. The Laplace equation for the magnetic vector potential in other regions including air gap and stator iron will be:

\[ \frac{\partial^2 \bar{A}_{z,x,6k+1,6n+1}}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \bar{A}_{z,x,6k+1,6n+1}}{\partial \phi^2} + \frac{1}{r} \frac{\partial \bar{A}_{z,x,6k+1,6n+1}}{\partial r} = 0 \tag{2.85} \]

And the solution for this equation will be:

\[ \bar{A}_{z,x,6k+1,6n+1}(r,\phi,t) = \hat{A}_{z,x,6k+1,6n+1}(r) \frac{3}{2} \frac{\hat{n}_{6k+1} \hat{I}_{6n+1}}{R_{si}} e^{-j[(6k+1)\phi + (6(k-n))\omega t]} \tag{2.86} \]

In which

\[ \hat{A}_{z,x,6k+1,6n+1}(r) = \bar{c}_{x,6k+1,6n+1} \left( \frac{r}{R_x} \right)^{[6k+1]} + \bar{d}_{x,6k+1,6n+1} \left( \frac{r}{R_x} \right)^{[6k+1]} \tag{2.87} \]

And \( R_s \) is the outer radius of the region \( x \).

The radial and tangential magnetic flux density and magnetic field strength can be found for all regions using:
2-4-5 Solution of Poisson Equation for Slot-Less Machine

In the case of slot less stator the Ampere continuity boundary condition between regions 6 and 7 will become:

\[ H_{6,\phi}(r = R_s) = H_{7,\phi}(r = R_s) \]  

(2.89)

And the Poisson equation in region 6, winding region, will be:

\[ \nabla^2 \vec{A} = -\mu_0 \vec{J} \]  

(2.90)

The assumed solution in all regions for the magnetic vector potential is chosen in a way to have a same format with the surface current density:

\[ \vec{A}_z(r,\phi,t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \vec{A}_{z,6k+1,6n+1}(r,\phi,t) \]  

(2.91)

In which

\[ \vec{A}_{z,6k+1,6n+1}(r,\phi,t) = \frac{3}{2} \frac{3}{R_{cw}} \hat{t}_{6k+1} \hat{t}_{6n+1} e^{-j[(6k+1)\phi + 6(k-n)\omega t]} \]  

(2.92)

The rest of analysis is the same as previous case, slotted machine; therefore the solution for region 1, shaft, will be:

\[ \vec{A}_{z,1,6k+1,6n+1}(r) = \begin{cases} 
\vec{e}_{1,6k+1,6n+1} \hat{t}_{k+1}(\tilde{r}_{1,6k+1,6n+1} \hat{r}) + \vec{d}_{1,6k+1,6n+1} \hat{K}_{k+1}(\tilde{r}_{1,6k+1,6n+1} \hat{r}) & \text{if } k \neq n \\
\frac{r}{R_1} \hat{t}_{k+1} & \text{if } k = n 
\end{cases} \]  

(2.93)
In which

\[ \tau^2_{1,6k+1,6n+1} = -6 j \mu \sigma_1 (k - n) \omega \]  

(2.94)

For regions 2, 3 and 4 the solution has similar format only different in the

\[
\tilde{A}_{z, x, 6k+1, 6n+1}(r) = \begin{cases} 
\overline{c}_{x, 6k+1, 6n+1} \left( \frac{r}{R_x} \right)^{\nu_{6k+1} + 1} + \overline{d}_{x, 6k+1, 6n+1} \left( \frac{r}{R_x} \right)^{\nu_{6k+1}} & k \neq n \\
\overline{c}_{x, 6k+1, 6n+1} \left( \frac{r}{R_x} \right)^{\nu_{6k+1} + 1} + \overline{d}_{x, 6k+1, 6n+1} \left( \frac{r}{R_x} \right)^{\nu_{6k+1}} & k = n
\end{cases} 
\]  

(2.95)

In which \( x \) represents the region number with \( R_x \) defined as the outer radius of that region and

\[ \tau^2_{x, 6k+1, 6n+1} = -6 j \mu_x \sigma_x (k - n) \omega \]  

(2.96)

The complex constants \( \overline{c}_{x, 6k+1, 6n+1} \) and \( \overline{d}_{x, 6k+1, 6n+1} \) are defined by the boundary conditions from table 2 (for the slot less machine) which is also valid for the complex values. In this case also the constants are different for two cases \( (k = n \) and \( k \neq n \)); therefore, all the constants found by boundary conditions for all the regions should be solved for these two cases separately.

The Poisson equation for the winding region in the harmonic format will be:

\[
\frac{\partial^2 \overline{A}_{z, 6k+1, 6n+1}}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \overline{A}_{z, 6k+1, 6n+1}}{\partial \phi^2} + \frac{1}{r} \frac{\partial \overline{A}_{z, 6k+1, 6n+1}}{\partial r} = -\mu_0 \overline{J}_{z, 6k+1, 6n+1} 
\]  

(2.97)

This equation has homogenous and particular answers:

\[ \overline{A}_{z, 6k+1, 6n+1} = \overline{A}_{z, 6k+1, 6n+1}^{\text{hom}} + \overline{A}_{z, 6k+1, 6n+1}^{\text{part}} \]  

(2.98)

The homogenous answer is the Laplace equation solution for this region and the particular solution will be:

\[
\overline{A}_{z, 6k+1, 6n+1}^{\text{part}}(r, \phi, t) = \hat{\overline{A}}_{z, 6k+1, 6n+1}(r) \left( \frac{3 \hat{n}_{6k+1} \hat{l}_{6n+1}}{2 R_{cw} h_w} e^{-j[(6k + 1) \phi + 6(k - n) \omega t]} \right) 
\]  

(2.99)

In which

\[
\hat{\overline{A}}_{z, 6k+1, 6n+1} = \frac{\mu_0 r^2}{(6k + 1)^2 - 4} 
\]  

(2.100)
Therefore the solution in region 6 will be:

\[
\vec{A}_{z,6,6k+1,\phi+1}(r,\phi,t) = \vec{A}_{z,6,6k+1,6n+1}(r) \frac{3}{2} \frac{n_{6k+1}^e}{R_{w}} \frac{e^{-j[\phi(6k+1)\phi + 6(k-n)\omega t]}}{h_w} (2.101)
\]

In which

\[
\frac{\hat{A}}{\hat{A}}_{z,6,6k+1,6n+1}(r) = \vec{c}_{6,6k+1,6n+1} \left( \frac{r}{R_{x}} \right)^{\frac{6k+1}{4}} + \vec{d}_{6,6k+1,6n+1} \left( \frac{r}{R_{x}} \right)^{\frac{6k+1}{4}} + \frac{\mu_{x} r^2}{(6k+1)^2 - 4} (2.102)
\]

The Poisson equation for other regions is the Laplace equation because there is no conductivity and external current in those regions. The solution is also independent to the condition if \( k = n \) or \( k \neq n \) and is the same for both situations. The Laplace equation for the magnetic vector potential in other regions including air gap and stator iron will be:

\[
\frac{\hat{A}}{\hat{A}}_{z,x,6k+1,6n+1}(r) = \vec{c}_{6,6k+1,6n+1} \left( \frac{r}{R_{x}} \right)^{\frac{6k+1}{4}} + \vec{d}_{6,6k+1,6n+1} \left( \frac{r}{R_{x}} \right)^{\frac{6k+1}{4}} (2.103)
\]

And \( R_x \) is the outer radius of the region \( x \).

\section*{2-5 Stator Voltage Equations}

When the field of permanent magnet and armature current are determined in different parts of the machine, then the voltage induced in the windings can be achieved. The induced voltage due to permanent magnet field (no-load voltage) is induced due to changing of flux linkage in the stator windings as the induced voltage due to armature field itself. In this chapter the flux linkages due to magnet and stator current fields plus the slot flux leakage in slotted machine are explained and then the corresponding induced voltages are calculated.

\subsection*{2-5-1 Introduction}

There are different induced voltages in the stator windings due to different sources. The general stator voltage equation of the motor is expressed by:

\[
\vec{e} = R_i \vec{j} + \frac{d}{dt} \vec{A} (2.104)
\]
And in a matrix form it is equal to:

\[
\begin{pmatrix}
e_a \\
e_b \\
e_c \\
\end{pmatrix} = 
\begin{pmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R \\
\end{pmatrix}
\begin{pmatrix}
i_a \\
i_b \\
i_c \\
\end{pmatrix} + 
\begin{pmatrix}
\frac{d}{dt} \lambda_a \\
\frac{d}{dt} \lambda_b \\
\frac{d}{dt} \lambda_c \\
\end{pmatrix}
\]

(2.105)

The total flux linkage is composed of two parts: the flux linkage due to rotor permanent magnets and flux linkage due to stator currents. The flux leakage has mostly four parts: the slot, the air gap, the tooth tip and the end turns flux leakages; however only the slot flux leakage is considered in this thesis. Further explanations on inductance calculations are provided in [A.M.El-Refai2008], [H.Grop2009], [J.Pyrhonen2008] and [S.R.Holm2007]. The voltage equation is then written as:

\[e = R_i + \frac{d\lambda_{s,m}}{dt} + \frac{d\lambda_{s,s}}{dt} + \frac{d\lambda_{s,\sigma}}{dt}\]

(2.106)

In which \(\lambda_{s,m}\) is the flux linkage due to PM, \(\lambda_{s,s}\) is the flux linkage due to armature current field and \(\lambda_{s,\sigma}\) is the slot flux leakage. To calculate the induced voltage in the stator windings, the flux linkage of each phase should be calculated in the first step.

The flux linkage of a coil with a contour of \(c\) and an enclosed surface of \(s\) will be:

\[\lambda = \iint_s B \cdot \hat{n} \, dS\]

(2.107)

by incorporating equation (2.11) into (2.107) we will have:

\[\lambda = \iint_s (\nabla \times \vec{A}) \cdot \hat{n} \, dS\]

(2.108)

And according to the Stokes’s theorem, equation (2.4), the flux linkage will be equal to:

\[\lambda = \oint_c \vec{A} \cdot \hat{t} \, dl\]

(2.109)

As was discussed earlier, the magnetic flux density has only the radial and tangential components in the active region of the machine by neglecting the end regions; therefore, the magnetic vector potential has only the \(z\) component in that region:

\[\vec{A} = A_z(r,\phi)\hat{z}\]

(2.110)
Using the symmetry of magnetic vector potential in distributed windings according to:

\[ A_z(r, \phi + \frac{\pi}{p}) = A_z(r, \phi) \quad (2.111) \]

The total flux linkage of the stator winding for the slotted stator will be:

\[
\lambda = 2 p l_{Fe} \int_0^{\frac{\pi}{p}} n_s(\phi) A(r = R_s, \phi) d\phi \quad (2.112)
\]

In which \( R_s \) is the inner radius of the stator bore that the equivalent line current density is presumed. For the slot less structure, the total flux linkage will be:

\[
\lambda = 2 p l_{Fe} \int_0^{\frac{\pi}{p}} n_s(\phi) A(r = R_{cw}, \phi) d\phi \quad (2.113)
\]

In which \( R_{cw} \) is the winding region central radius.

### 2-5-2 No-Load Voltage for the Slotted Stator

The flux linkage of phase \( a \) of the stator windings due to permanent magnet field will be:

\[
\lambda_{a,m} = 2 l_{Fe} \int_0^{\frac{\pi}{p}} n_{sa}(\phi) A_6(r = R_s, \phi) d\phi \quad (2.114)
\]

In which the \( A_6(r = R_s, \phi) \) is the magnetic vector potential due to rotor permanent magnets in the winding region and according to equation (2.33) it is equal to:

\[
\bar{A}_6 = [c_6 + d_6] \cos(\phi) \tilde{i}_z = \hat{A}_6(r = R_s) \cos(\phi) \tilde{i}_z 
\]

Since the field of the magnets has the triple harmonics, the winding distribution according to (2.53) should be used. Then, by substituting (2.115) and (2.53) in (2.114) for pole pair value of one, the flux linkage of phase \( a \) due to permanent magnet field will be:

\[
\lambda_{a,m} = 2 l_{Fe} \int_0^{\frac{\pi}{p}} \left[ \sum_{k=1,3,...} \hat{n}_{sa} \cos(k\phi) \right] \hat{A}_6(r = R_s) \cos(\phi) d\phi \quad (2.116)
\]
Since the magnetic vector potential is in the rotor coordinate system and the flux linkage in the stator coordinate one, the magnetic vector potential should be transferred to the stator system:

\[
\lambda_{a,m} = 2 l_{Fe} \int_0^\pi \left[ \sum_{k=1,3,...}^{\infty} \hat{n}_{s,k} \cos(k\varphi) \right] \hat{A}_6(r = R_o) \cos(\varphi - \theta) \, d\varphi \tag{2.117}
\]

Since the result of integral, for this type of rotor PM, for all the terms of series is zero except for \(k=1\), the resultant flux density will be:

\[
\lambda_{a,m} = 2 l_{Fe} \int_0^\pi \hat{n}_{s,1} \cos(\varphi) \hat{A}_6(r = R_o) \cos(\varphi - \theta) \, d\varphi \tag{2.118}
\]

And the result of integral would be:

\[
\hat{\lambda}_{a,m} = \pi l_{Fe} \hat{n}_{s,1} \hat{A}_6(r = R_o) \cos(\theta) \tag{2.119}
\]

Then, the induced voltage will be:

\[
e_{a,m} = \frac{d\hat{\lambda}_{a,m}}{dt} = \hat{e}_m \sin(\theta) \tag{2.120}
\]

In which the amplitude of the induced voltage will be equal to:

\[
\hat{e}_m = -\pi l_{Fe} \hat{n}_{s,1} \hat{A}_6(r = R_o) \tag{2.121}
\]

And the induced voltages in other phases will be:

\[
e_{b,m} = \hat{e}_m \sin(\theta - \frac{2\pi}{3}) \tag{2.122}
\]

\[
e_{c,m} = \hat{e}_m \sin(\theta - \frac{4\pi}{3}) \tag{2.123}
\]

### 2-5-3 No-Load Voltage for the Slot less Stator

In the case of slot less machine, the windings are situated in the air gap presenting the surface current density. The procedure to calculate the induced voltage due to permanent magnet field is similar; however with a small difference in the location of winding region:
\[ e_{a,m} = \frac{d\lambda_{a,m}}{dt} = \hat{e}_m \sin(\theta) \]  

(2.124)

In which

\[ \hat{e}_m = -\pi I_F e \omega \hat{n}_{s,1} \hat{A}_6(r = R_{cw}) \]  

(2.125)

And \( R_{cw} \) is the central winding radius and from the previous chapter we have:

\[ \hat{A}_6(r = R_{cw}) = \left[ c_6 \left( \frac{R_9}{R_{cw}} \right) + d_6 \left( \frac{R_{cw}}{R_9} \right) \right] \]  

(2.126)

The induced voltages in other phases also have similar formats.

### 2-5-4 Main Field inductances in the Slotted Stator

In the symmetric, three-phase balanced system the triple terms of the armature current field are zero and the non-triple ones are equal to each other. Therefore the flux linkage matrix due to stator current field can be written as:

\[
\begin{pmatrix}
\hat{\lambda}_{a,s} \\
\hat{\lambda}_{b,s} \\
\hat{\lambda}_{c,s}
\end{pmatrix} =
\begin{pmatrix}
L - M & 0 & 0 \\
0 & L - M & 0 \\
0 & 0 & L - M
\end{pmatrix}
\begin{pmatrix}
i_{sa} \\
i_{sb} \\
i_{sc}
\end{pmatrix}
\]  

(2.127)

In which \( L \) is the self inductance and \( M \) is the mutual inductance between different phases of the machine. The flux linkage of phase \( a \) caused by the currents in all phases, according to equation (2.112) for different space and time harmonics will be expressed by:

\[ \hat{\lambda}_{a,s,6k+1,6n+1} = 2I_F \int_{0}^{\pi} n_{s,a}(\phi) A_{6,6k+1,6n+1}(r = R_9, \phi) d\phi \]  

(2.128)

In which \( A_{6,6k+1,6n+1}(r = R_9, \phi) \) is the magnetic vector potential due to armature currents in winding region, border of stator and mechanical air gap. By substituting (2.56) and (2.86) in stator coordinate system in (2.128) the flux linkage of phase \( a \) will be:

\[ \hat{\lambda}_{a,s,6k+1,6n+1} = 2l_{Fe} \hat{n}_{a,6k+1} \hat{A}_{a,6k+1,6n+1}(r = R_9) \frac{3}{2} \hat{n}_{6k+1,6n+1} \frac{R_9}{R_6} \int_{0}^{\pi} \cos((6k+1)\phi) \cos((6k+1)\phi - (6n+1)\alpha) d\phi \]  

(2.129)
The result of integral will be expressed by:

\[
\lambda_{a,s,6k+1,6n+1} = \pi l_{Fe} \hat{n}_{s,6k+1} \hat{A}_{z,6,6k+1,6n+1} (r = R_o) \frac{3 \hat{n}_{6k+1} R_{6n+1}}{2 R_o} \cos (6n + 1) \omega t \) \tag{2.130}
\]

And by using the current expression according to (2.60) we will have:

\[
\lambda_{a,s,6k+1,6n+1} = \pi l_{Fe} \hat{A}_{z,6,6k+1,6n+1} (r = R_o) \frac{3 \hat{n}_{6k+1}^2}{2 R_o} i_{s,a,6n+1} \) \tag{2.131}
\]

The flux linkage of all phases in a matrix form will be:

\[
\begin{pmatrix}
\lambda_{a,s,6k+1,6n+1} \\
\lambda_{b,s,6k+1,6n+1} \\
\lambda_{c,s,6k+1,6n+1}
\end{pmatrix} = \left[ \pi l_{Fe} \hat{A}_{z,6,6k+1,6n+1} (r = R_o) \frac{3 \hat{n}_{6k+1}^2}{2 R_o} \right] \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
i_{s,a,6n+1} \\
i_{s,b,6n+1} \\
i_{s,c,6n+1}
\end{pmatrix} \tag{2.132}
\]

And the total flux linkage will be:

\[
\tilde{\lambda}_{s,s} = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} L_{s,6k+1,6n+1} \tilde{i}_{s,6n+1} \) \tag{2.133}
\]

2-5-5 Main Field inductances in the Slot less Stator

To calculate the flux linkage in the stator phase windings due to armature field when the windings are situated in the air gap, the procedure is the same as the previous case except in the magnetic vector potential in the central winding region:

\[
\begin{pmatrix}
\lambda_{a,s,6k+1,6n+1} \\
\lambda_{b,s,6k+1,6n+1} \\
\lambda_{c,s,6k+1,6n+1}
\end{pmatrix} = \left[ \pi l_{Fe} \hat{A}_{z,6,6k+1,6n+1} (r = R_{cw}) \frac{3 \hat{n}_{6k+1}^2}{2 R_{cw} h_o} \right] \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
i_{s,a,6n+1} \\
i_{s,b,6n+1} \\
i_{s,c,6n+1}
\end{pmatrix} \tag{2.134}
\]

And the total flux linkage will be:

\[
\tilde{\lambda}_{s,s} = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} L_{s,6k+1,6n+1} \tilde{i}_{s,6n+1} \) \tag{2.135}
2-5-6 Slot Leakage Inductances in the Slotted Stator

Leakage flux of a current carrying slot of an electrical machine can be found using the total magnetic energy stored in the slot. The slot leakage inductance mainly depends on the geometry of the slot and the number of conductors within it. In [R. Richter1967] it has been calculated for the most common slot types and for varying cross section slots. Due to stator and air gap symmetry the self inductances of each phase caused by slot flux leakage ($L_{s\sigma}$) are the same and so are the mutual inductances between phases ($M_{s\sigma}$). In a matrix format they are written as:

\[
\bar{\lambda}_{s,\sigma} = \begin{pmatrix}
L_{s\sigma} & M_{s\sigma} & M_{s\sigma} \\
M_{s\sigma} & L_{s\sigma} & M_{s\sigma} \\
M_{s\sigma} & M_{s\sigma} & L_{s\sigma}
\end{pmatrix}
\begin{pmatrix}
i_s
\end{pmatrix}
\tag{2.136}
\]

In a three phase balanced system, the triple and even time harmonics are cancelled, and therefore, the equation (2.136) can be written as:

\[
\bar{\lambda}_{s,\sigma,6n+1} = \begin{pmatrix}
L_{s\sigma} - M_{s\sigma} & 0 & 0 \\
0 & L_{s\sigma} - M_{s\sigma} & 0 \\
0 & 0 & L_{s\sigma} - M_{s\sigma}
\end{pmatrix}
\begin{pmatrix}
i_{s,6n+1}
\end{pmatrix}
\tag{2.137}
\]

And the total slot flux leakage will be:

\[
\bar{\lambda}_{s,\sigma} = \sum_{n=-\infty}^{\infty} \bar{\lambda}_{s,\sigma,6n+1}
\tag{2.138}
\]

Assuming a uniform current distribution in the conductors (neglecting skin effect) the total slot leakage inductance is:

\[
L_{s\sigma} = N_{snm} N_{sl}^2 \mu_0 L_{Fe} \left( \frac{h}{3b} + \frac{d_{ls}}{b_{so}} \right)
\tag{2.139}
\]

In which $N_{snm}$ is the number of slots in series of each phase, $N_{sl}$ is the number of conductors in each phase layer per slot, $h$ and $b$ are the height and breadth of the winding region, respectively and $d_{ls}$ and $b_{so}$ are the height and breadth of the slot opening. The first term in the self inductance equation is due to the field of conductor itself and the second part is caused by the slot opening which is the same cause for the mutual inductance between phases.

\[
M_{s\sigma} = -N_{snm} N_{sl}^2 \mu_0 L_{Fe} \left( \frac{d_{ls}}{b_{so}} \right)
\tag{2.140}
\]
The only difference is the $N_{mm}$, which is the number of slots that two specific phases are located inside.

The total induced voltage in one phase due to armature field using all harmonics of stator current according to equation (2.61) will be defined as:

$$e = \frac{d\vec{\lambda}_{s,s}}{dt} + \frac{d\vec{\lambda}_{s,\sigma}}{dt}$$  \hspace{1cm} (2.141)

2-6 Power and Torque

There are different methods for calculating the electromagnetic torque and mechanical power: the Poynting vector theorem, the Lorentz force, the Maxwell stresses and the energy methods. In this chapter the simplified Poynting vector method (power balance equation) is used to calculate the power crossing the air gap, neglecting the eddy current losses in the rotor and air friction one in the air gap.

2-6-1 Introduction

The pointing vector theorem calculates the total power crossing the air gap by surface integrating the Poynting vector in the air gap. When this integration is done in rotor coordinates, the power that crosses the air gap is the loss induced in the rotor shielding cylinder due to stator current field as in [F.Deng1997], [F.Deng1998] and [S.A.Sharkh1999]. That is because the coordinate system rotates synchronously with respect to the rotor and the fundamental component of the stator current field doesn’t see the power transferred to the field of the magnets. When this integration is done in the stator coordinate system, the mechanical power associated with the fundamental space and time harmonics can be calculated as in [B.B.Palit1982], [J.J.Gut1998] and [Z.Xiaojuan1996].

Another method to calculate the electromagnetic torque is the Lorentz force method. This method is ideal for slot-less machines since it calculates the force to the conductors located in the magnetic field. The Lorentz force method provides information about the torque ripple which the Poynting vector method doesn’t. On the other hand, the pointing vector method can calculate the induced losses in the shielding cylinder but the Lorentz method doesn’t.

2-6-2 Power and Torque

The instantaneous power crossing the air gap from stator to rotor can be calculated from the power balance equation and is equal to:
\[ P_m = P_e = e_{a,m} i_a + e_{b,m} i_b + e_{c,m} i_c = T_e \omega \] (2.142)

In which \( T_e \) is the machine electromagnetic torque. Formerly, the stator current wave forms were expressed as:

\[
i_a(t) = \sum_{n=1}^{\infty} \hat{i}_n \cos(n\omega t)
\]
\[
i_b(t) = \sum_{n=1}^{\infty} \hat{i}_n \cos(n(\omega t - \frac{2\pi}{3}))
\]
\[
i_c(t) = \sum_{n=1}^{\infty} \hat{i}_n \cos(n(\omega t - \frac{4\pi}{3}))
\] (2.143)

However, since in a three phase balanced machine the even and triple harmonics are removed, the current wave forms can be rewritten as:

\[
i_a(t) = \sum_{m=-\infty}^{\infty} \hat{i}_{6m+1} \cos((6m+1)\omega t)
\]
\[
i_b(t) = \sum_{m=-\infty}^{\infty} \hat{i}_{6m+1} \cos((6m+1)(\omega t - \frac{2\pi}{3}))
\] (2.144)
\[
i_c(t) = \sum_{m=-\infty}^{\infty} \hat{i}_{6m+1} \cos((6m+1)(\omega t - \frac{4\pi}{3}))
\]

By substituting these current waveforms and induced voltages due to rotor permanent magnet, equations (2.120), (2.122) and (2.123), in the power equation and by referring to the theorem that:

\[
\sin(x - y) + \sin(x - y - \frac{2k\pi}{3}) + \sin(x - y - \frac{4k\pi}{3}) = \begin{cases} 
3\sin(x - y) & k = 3L \text{ & } L \in \mathbb{Z} \\
0 & \text{otherwise}
\end{cases}
\] (2.145)

The machine power will be:

\[
P_e = \frac{3}{2} \hat{e}_m \sum_{m=-\infty}^{\infty} \hat{i}_{6m+1} \sin(-6m\omega t + \theta_0)
\] (2.146)

In which \( \theta_0 \) is the initial position of rotor. The average machine power is transferred only via fundamental time harmonic of stator currents:

\[
P_{e,ave} = \frac{3}{2} \hat{e}_m i_1 \sin(\theta_0)
\] (2.147)
And its maximum value will be:

\[ P_{e,ave,\text{max}} = \frac{3}{2} e_m \hat{i}_1 \]  \hspace{1cm} (2.148)

And the maximum electromagnetic torque will be:

\[ T_{e,ave,\text{max}} = \frac{3}{2} \frac{e_m \hat{i}_1}{\omega} \]  \hspace{1cm} (2.149)
3-1 Rotor Losses

Rotor losses are often an important issue in the design of synchronous machines, and in high speed applications they can become the major cause of power dissipation. Therefore, whereas in other kinds of machines a rough estimation of these losses can be accepted, their importance in high speed synchronous machines justifies a greater effort in calculating them more precisely. A method to determine rotor losses based on Poynting vector theorem has been developed and will be presented in this section.

3-1-1 Introduction

The losses inside the rotor are caused by the conductivity of the different parts in the rotor. The shaft iron, magnet and the sleeves, all have certain amount of electrical conductivities which are responsible for the losses. The shaft iron is usually laminated; however there are still induced currents in the laminations. The thinner the laminations, the lower the induced currents. The loss inside the rotor is only due to the field of armature current. The reason is that the field of magnets is rotating synchronously with respect to the rotor and there will be no induced current and subsequently no losses inside the rotor due to the field of magnets. Therefore the only source for losses in the rotor is due to the armature field space and time harmonics. This is also true for
the losses in the magnet and sleeves. However these layers are not laminated in this thesis unlike the rotor shaft iron.

There are different methods to calculate the losses inside the rotor such as volume integrating of induced current density inside the rotor and using the theorem of Poynting Vector. The use of volume integration is explored in the [Z.Q.Zhu1997a] and [V.D.Vee1997] and will not be the method of loss calculation in this thesis due to complicated integrations of existing Bessel functions. The Poynting vector theorem for calculation of losses is used in the [F.Deng1997], [S.A.Sharkh1999] and [Z.Q.Zhu1997b] and will be the interested method for finding losses here in the thesis.

### 3-1-2 Poynting Vector Theorem

The theorem of Poynting in time domain is written as [H.Blok1999]:

\[
-\nabla \cdot \vec{S} = \vec{H} \cdot \partial_t \vec{B} + \vec{J} \cdot \vec{E} + \vec{E} \cdot \vec{J}_{ext}
\]  
(3.1)

Where \(\vec{S}\) is the Poynting vector defined as:

\[
\vec{S} = E \times H
\]  
(3.2)

And by using the constitutive relations for a linear, isotropic matter rotating with a speed significantly lower than the speed of light, in the rotor coordinate system (synchronously rotating rotor):

\[
\begin{align*}
\vec{D} &= \varepsilon \vec{E} \\
\vec{B} &= \mu \vec{H} + \vec{B}_{rem} \\
\vec{J} &= \sigma \vec{E}
\end{align*}
\]  
(3.3)

the theorem of Poynting in the time domain will be rewritten as:

\[
-\nabla \cdot \vec{S} = \partial_t (\frac{1}{2} \mu \vec{H} \cdot \vec{H}) + \vec{H} \cdot \partial_t \vec{B}_{rem} + \sigma \vec{E} \cdot \vec{E} + \vec{E} \cdot \vec{J}_{ext}
\]  
(3.4)

And by introducing the source power density \(p_{source} [W/m^3]\), the energy density in the magnets \(w_m [J/m^3]\), the dissipated power density \(p_{diss} [W/m^3]\) and the external power density \(p_{ext} [W/m^3]\), the equation (3.4) can be written as:

\[
p_{source} = \partial_t w_m + p_{mech} + p_{diss} + p_{ext}
\]  
(3.5)
Since in the rotating rotor system the remanance doesn’t change with time, the mechanical power is set to zero. If the Poynting vector theorem is used in the stator reference system, then the rotor losses plus the mechanical power can be derived from this theorem. Then by having a time average of the whole equation the time derivation of stored magnetic energy density will disappear:

\[
\langle p_{\text{source}} \rangle = \langle p_{\text{diss}} \rangle + \langle p_{\text{ext}} \rangle
\]  

(3.6)

And in the integral form it will be:

\[
-\oint_{S} S \cdot da = \int_{V} \sigma \vec{E} \cdot \vec{E} \, dv + \int_{V} \vec{E} \cdot \vec{J}_{\text{ext}} \, dv
\]

(3.7)

In which \( S \) is the enclosed surface for the volume \( V \). By choosing a proper enclosed surface which doesn’t include the external power density (here the armature current) but includes the parts with conductivity (here the sleeves and magnet ring), the total average dissipated power in the rotor will be:

\[
\langle p_{\text{source}} \rangle = \langle p_{\text{diss}} \rangle
\]  

(3.8)

In the frequency domain and steady state the Poynting vector will be:

\[
\vec{S} = \vec{E} \times \vec{H} = \frac{1}{2} \left( \hat{\vec{E}} e^{j\omega t} + \hat{\vec{E}}^{*} e^{-j\omega t} \right) \times \frac{1}{2} \left( \hat{\vec{H}} e^{j\omega t} + \hat{\vec{H}}^{*} e^{-j\omega t} \right)
\]

(3.9)

\[
= \frac{1}{2} \text{Re} \left\{ \hat{\vec{E}} \times \hat{\vec{H}}^{*} \right\} + \frac{1}{2} \text{Re} \left\{ \hat{\vec{E}} \times \hat{\vec{H}} e^{j2\omega t} \right\}
\]

The average of second part of the equation above is zero if \( \vec{S} \) is a vector of constant length rotating with the speed of \( \omega \). Therefore the complex Poynting vector is defined as:

\[
\hat{\vec{S}} \equiv \frac{1}{2} \hat{\vec{E}} \times \hat{\vec{H}}^{*}
\]  

(3.10)

And, the average dissipated power density will be:

\[
\langle p_{\text{diss}} \rangle = \langle p_{\text{source}} \rangle = \text{Re} \left\{ -\nabla \cdot \hat{\vec{S}} \right\} = \text{Re} \left\{ -\nabla \cdot \left( \frac{1}{2} \hat{\vec{E}} \times \hat{\vec{H}}^{*} \right) \right\}
\]  

(3.11)

And the total power will be:
Chapter 3  

Machine Losses

\[ \langle P_{\text{disc}} \rangle = \langle P_{\text{source}} \rangle = \text{Re} \left\{ -\oint_S \left( \frac{1}{2} \hat{E} \times \hat{H}^* \right) \cdot d\vec{a} \right\} \]  
(3.12)

For taking into the account the effect of space and time harmonics, the average dissipated power will be solved for each time and space harmonics separately and will be summed together to get the total power:

\[ \langle P_{\text{disc}} \rangle = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \langle P_{\text{disc}, 6k+1, 6n+1} \rangle \]  
(3.13)

In which

\[ \langle P_{\text{disc}, 6k+1, 6n+1} \rangle = \text{Re} \left\{ -\oint_S \left( \frac{1}{2} \hat{E}_{6k+1, 6n+1} \times \hat{H}^*_n \right) \cdot d\vec{a} \right\} \]  
(3.14)

In the previous chapter the magnetic vector potential and magnetic flux density due to armature field space and time harmonics were calculated. To calculate the complex pointing vector we have:

\[ \hat{S} = \frac{1}{2} \hat{E} \times \hat{H}^* = \frac{1}{2} \begin{bmatrix} \hat{i}_r & \hat{i}_\phi & \hat{i}_z \\ 0 & 0 & \hat{\hat{E}}_z \\ \hat{\hat{H}}_r & \hat{\hat{H}}_\phi & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \hat{\hat{E}}_z \hat{\hat{H}}_r^* + \hat{\hat{E}}_\phi^* \hat{\hat{H}}_r \\ -\hat{\hat{E}}_z \hat{\hat{H}}_\phi^* - \hat{\hat{E}}_\phi \hat{\hat{H}}_r^* \end{bmatrix} \]  
(3.15)

From previous chapter, the electric field in z direction for each space and time harmonics is equal to:

\[ E_{z, 6k+1, 6n+1} = \frac{\partial \hat{A}_{z, 6k+1, 6n+1}}{\partial t} = -6 j (k-n) \omega \hat{A}_{z, 6k+1, 6n+1} (r) \frac{3}{2} \frac{\hat{n}_{6k+1} \hat{l}_{6n+1}}{R_{sl}} \frac{e^{-j[(6k + 1)\phi + 6(k-n)\omega t + \theta_0]}}{} \]  
(3.16)

And the tangential magnetic field strength will be:

\[ H_{\phi, 6k+1, 6n+1} = -\frac{1}{\mu_0} \frac{\partial \hat{A}_{z, 6k+1, 6n+1}}{\partial r} = -\frac{1}{\mu_0} \frac{3}{2} \frac{\hat{n}_{6k+1} \hat{l}_{6n+1}}{R_{sl}} \frac{\partial \hat{A}_{z, 6k+1, 6n+1}}{\partial r} (r) e^{-j[(6k + 1)\phi + 6(k-n)\omega t + \theta_0]} \]  
(3.17)

Therefore the average dissipated power for each space and time harmonic will be:
\[ \langle P_{diss,6k+1,6n+1} \rangle = \text{Re} \left\{ \frac{-\pi L_f r'}{\mu_0} 6 j(k-n) \omega \left( \frac{3}{2} \frac{n_{6k+1} l_{6n+1}}{R_i} \right)^2 \frac{\tau}{A_{r,6k+1,6n+1}(r)} \left( \frac{\partial \hat{A}_{z,6k+1,6n+1}(r)}{\partial r} \right) \right\} \] (3.18)

In which \( r' \) is the radius for the surface of integration.

### 3-1-3 Total Losses in the Rotor

To calculate the total losses in the rotor the integration surface must be chosen properly. In the chosen enclosed surface there must be no external power, or armature current, but it should have the regions with the electrical conductivity. Thus, to calculate the losses inside the rotor, including losses in the shaft, magnet and sleeves, the integration surface is between the last sleeve layer and the stator winding region, which is chosen in the border of air gap and last sleeve layer here. Then the total average power crossing this surface from stator toward rotor is the total loss induced in the rotor shaft, magnet and sleeves:

\[ \langle P_{diss,rotor,6k+1,6n+1} \rangle = \text{Re} \left\{ -\frac{1}{2} \left( \frac{\hat{E}}{S_T} \times \hat{H} \right)_{4,6k+1,6n+1} \cdot d\vec{a} \right\} \] (3.19)

In which \( S_T \) is the surface in the border of mechanical air gap and sleeve layer.

### 3-1-4 Loss inside the shaft

For calculating the eddy current induced loss inside the shaft the enclosed integrating surface is located between the magnet layer and shaft. Therefore the induced eddy current loss inside the shaft will be:

\[ \langle P_{diss,shaft,6k+1,6n+1} \rangle = \text{Re} \left\{ -\frac{1}{2} \left( \frac{\hat{E}}{S_{Sh}} \times \hat{H} \right)_{3,6k+1,6n+1} \cdot d\vec{a} \right\} \] (3.20)

In which \( S_{Sh} \) is the enclosed surface between magnet and shaft.

### 3-1-5 Loss inside the Magnet

For calculating the losses inside the magnet the enclosed integrating surface is located around the magnet layer, which includes two surfaces: one between magnet and sleeve layer 1
and the other one between magnet and shaft. Therefore the induced eddy current loss inside the magnet will be:

\[
\left\langle P_{\text{diss, magnet}, 6k+1, 6n+1} \right\rangle = \text{Re} \left\{ -\oint_{S_M} \left( \frac{1}{2} \hat{E}_{3,6k+1,6n+1} \times \hat{H}^*_{3,6k+1,6n+1} \right) \cdot d\vec{a} \right\} - \left\langle P_{\text{diss, shaft}, 6k+1, 6n+1} \right\rangle \quad (3.21)
\]

In which the \( S_M \) is the enclosed surface between magnet and sleeve layer 1.

**3-1-6 Loss inside the Sleeve layer 1**

For calculating the losses inside the sleeve layer 1 the enclosed integrating surface is located bounding that layer, which includes two surfaces: one between sleeve layers 1 and 2 and the other one between magnet and the sleeve layer 1. Therefore the induced eddy current loss inside the sleeve layer 1 will be:

\[
\left\langle P_{\text{diss, sleeve1}, 6k+1, 6n+1} \right\rangle = \text{Re} \left\{ -\oint_{S_{sl1}} \left( \frac{1}{2} \hat{E}_{3,6k+1,6n+1} \times \hat{H}^*_{3,6k+1,6n+1} \right) \cdot d\vec{a} \right\} - \text{Re} \left\{ -\oint_{S_M} \left( \frac{1}{2} \hat{E}_{3,6k+1,6n+1} \times \hat{H}^*_{3,6k+1,6n+1} \right) \cdot d\vec{a} \right\}
\]

\( (3.22) \)

In which the \( S_{sl1} \) is the enclosed surface between the sleeve layers 1 and 2.

**3-1-7 Loss inside the Sleeve layer 2**

The loss in sleeve layer 2 is also calculated in the same way by integrating on the surface enclosing that sleeve layer. Therefore, the integration consists of two surfaces: one between sleeve layer 2 and air gap and the other one between sleeve layers 1 and 2. Therefore the induced eddy current loss inside the sleeve layer 2 will be:

\[
\left\langle P_{\text{diss, sleeve2}, 6k+1, 6n+1} \right\rangle = \text{Re} \left\{ -\oint_{S_{sl2}} \left( \frac{1}{2} \hat{E}_{3,6k+1,6n+1} \times \hat{H}^*_{3,6k+1,6n+1} \right) \cdot d\vec{a} \right\} - \text{Re} \left\{ -\oint_{S_{sl1}} \left( \frac{1}{2} \hat{E}_{3,6k+1,6n+1} \times \hat{H}^*_{3,6k+1,6n+1} \right) \cdot d\vec{a} \right\}
\]

\( (3.23) \)
3-2 Ohmic Losses

One of the difficulties in the design of high speed electrical machines is the eddy-current effects in windings. These effects include skin and proximity effects. The AC losses are then higher than the DC losses. In this chapter the DC losses are calculated with especial attention to the Litz wire. Then the AC losses, including skin and proximity losses, are calculated for both the slotted and slot-less machines.

3-2-1 Introduction

The copper loss in the winding can be divided into two parts:

\[ P_{cu} = P_{cu\ skin} + P_{cu\ prox} \]  \hspace{1cm} (3.24)

In which the DC loss is included in both of them. In [J.A.Ferreira1992] it was shown that these losses can be evaluated independently due to their orthogonal behavior. For calculation of skin and proximity effect losses two methods were discussed including rigorous one-dimensional solution in Cartesian co-ordinates and approximate solution. In the approximate solution the change of the magnetic field inside the conductors due to eddy current is neglected while in rigorous solution it’s not. For low values of frequency the approximate solution has the same accuracy as the rigorous one but since the current injected from power electronic has high frequency time harmonics the rigorous solution is preferred rather than the approximate one. Both the skin and proximity effects can be controlled by the use of Litz wires – which include multiple insulated strands, twisted around each other in a bundle. The term “Litz wire” is derived from a German word “Litzendraht” meaning woven wire. It’s constructed of individually insulated magnet wires either twisted or braided in to a uniform pattern. The Litz wire is usually expressed in a format of “n/AWG” in which \( n \) is the number of strands in the bundle and AWG (American Wire Gauge) is defined by:

\[ d = 25.4 * 0.005 * 92^{\frac{36-AWG}{39}} \]  \hspace{1cm} (3.25)

In which \( d \) is the diameter of each strand in [mm].

There are different characteristics for Litz wires such as:

- **Insulation build**: Insulation "build" refers to the thickness or the amount of insulation applied. They are, according to standards, single, heavy, triple and quadruple. The most common insulations for Litz are single and heavy (double) builds Polyurethane-nylon.

- **Twisting Tightness**: The twisting is expressed as Twists per Foot [TPF] and the standard is 12 TPF for most Litz wires. The maximum number of twists in a given length is limited by the size of strands.
- **Served vs. Unserved**: Served Litz wire simply means that the entire Litz construction is wrapped with a nylon textile or yarn for extra strength and protection.

### 3-2-2 DC Losses

There are different factors contributing to the DC resistive losses of the Litz winding in slotted stator of electrical machine. According to [C.R.Sullivan1999] these factors are:

- **Serving Factor**: the serving thickness is fixed for a given number of winding turns in a slot.
- **Strand Packing Factor**: simply twisted Litz bundle consists of several strands woven simply around each other. However there are more complex configurations consisting of sub-bundles where the sub-bundles are made in the simple twisting way.
- **Bundle Packing and Filler Factor**: the way the strands are divided in to bundles and sub-bundles is based on the considerations to decrease the bundle Litz wire skin effect, enhancing the flexibility of overall bundle, resistance to unraveling and packing density.
- **Turn Packing Factor**: the way the turns are packed in a winding and this factor is high for rectangular-cross-section Litz wires.
- **Twist Factor**: the actual length of the conductor which is twisted is higher than a straight one so the resistance is greater.
- **Strand Insulation Factor**: strand insulation thickness is dependent on the diameter of the conductor in the following way [NEMA1997]:

\[
\log_{10} B = X - \frac{AWG}{44.8} - 1.5952 \tag{3.26}
\]

Where \( B \) is the minimum insulation thickness in [mm] and \( X \) is 0.518 for single insulation build and 0.818 for heavy build one. However, this formula only applies to the wire sizes of AWG between 14 and 30 and for the wire size AWG range of 30 and 60 there is a better fit for insulation thickness by [C.R.sullivan1999]:

\[
d_i = d_r \alpha \left( \frac{d_c}{d_r} \right)^\beta \tag{3.27}
\]

Where \( d_i \) is the total conductor diameter including the insulation, \( d_c \) is the copper diameter and \( d_r \) is a reference diameter to make the coefficients \( \alpha \) and \( \beta \) unit less. By choosing the reference diameter equal to 40 AWG \((d_r=0.079 \text{ [mm]})\) the parameters found for single build insulation are \( \alpha = 1.12 \) and \( \beta = 0.97 \); for heavy build insulation they are \( \alpha = 1.24 \) and \( \beta = 0.94 \).

The total Litz wire diameter is:
\[ d_L = \sqrt{\frac{4F_t A}{\pi N_{sl}}} \]  
(3.28)

In which \( A \) is the total area of winding window, \( N_{sl} \) is the number of turns per slot per stator winding layer and \( F_t \) is the turn packing factor.

By assuming \( F_p \) as a factor contributing to serving, bundle packing, filler, strand packing and twist, the outside diameter of individual strand will be:

\[ d_i = \sqrt{\frac{4F_t F_p A}{\pi n N_{sl}}} \]  
(3.29)

Where \( n \) is the number of strands per each Litz wire bundle. From (3.27) and (3.29) the strand copper diameter will be expressed as:

\[ d_c = d_r (1 - \frac{1}{\beta})^{1/\beta} \frac{4F_t F_p A}{\pi n N_{sl}}^{1/2} \]  
(3.30)

Now the DC resistance for each phase neglecting the end regions can be calculated as:

\[ R_{dc} = \frac{8N L_{Fe}}{\sigma \pi d_c^2 n} \]  
(3.31)

In which \( N \) is the total number of turns per phase. The DC copper loss per phase is:

\[ P_{dc} = R_{dc} I_a^2 \]  
(3.32)

Where \( I_a \) is the \textit{rms} amplitude of phase current fundamental harmonic. In this thesis only the resistive losses due to fundamental time harmonic is considered (neglecting also the end regions) because of the following reasons:

- It gives an almost accurate result with simpler calculations,
- The skin effect losses can be ignored (as will be discussed in the next part),
- The approximate solution can be used rather than the rigorous one.

\textbf{3-2-3 AC Losses}

Skin and proximity effects in Litz wire can be divided in to bundle and strand level effects. The bundle level skin and proximity effects can be controlled by the method of twisting and weaving; the bundle level proximity effect is controlled by a simple twisting while the skin
effect is controlled by more complex bundle configurations such as using sub-bundles. So the bundle level eddy current losses can be neglected compared to strand level ones. In the strand level the proximity effect dominates the skin effect for Litz wires with high number of strands. The strand level proximity effect itself consists of internal proximity (due to currents inside the bundle) and external one (the field due to the other bundles currents and the field of permanent magnet). In another division, the strand level proximity effect is divided into current proximity (the field due to current of all other strands in the winding area) and magnet proximity (due to the field of rotor permanent magnet):

\[ P_{cu\,prox} = P_{current\,prox} + P_{pm\,prox} \]  \hspace{1cm} (3.33)

This formula is just to show the division and doesn’t necessarily mean that the proximity effects due to armature current and rotor PM fields are orthogonal. In slotted stator structures, the external field is only due to the currents inside the slot area and the field of rotor permanent magnet doesn’t attribute to the proximity losses. However, in the slot less structure, it is different; the rotating field of permanent magnet has the major influence on the proximity loss and the proximity effect due to adjacent conductors can be neglected. In [J.A.Ferreira1994] the skin and proximity effect losses per unit length of conductor are calculated using the one-dimensional orthogonal approach. The skin effect loss of a single round conductor far from the return conductor is expressed as:

\[ P_{cu\,skin} = F(\gamma)I_{ac}^2 \]  \hspace{1cm} (3.34)

And the proximity effect loss is described as:

\[ P_{cu\,prox} = \int_{l_{wi}} G(\gamma)\tilde{H}_e^2dl \]  \hspace{1cm} (3.35)

In which \( l_{wi} \) is the total length of conductors, neglecting the end regions and \( H_e \) is the amplitude of external magnetic field strength, and:

\[ F(\gamma) = \frac{R_{dc} \gamma ber\gamma bei'\gamma - bei\gamma ber'\gamma}{2 \left( ber'^2\gamma + bei^2\gamma \right)} \]  \hspace{1cm} (3.36)

And

\[ G(\gamma) = \frac{-2\pi \gamma ber_2\gamma ber'\gamma - bei_\gamma bei'\gamma}{\sigma_{cu} \left( ber'^2\gamma + bei^2\gamma \right)} \]  \hspace{1cm} (3.37)

Where \( \gamma \) is a parameter proportional to the ratio of conductor diameter and skin depth.
\[ \gamma = \frac{d_c}{\delta \sqrt{2}} \]  

(3.38)

With the skin depth described as:

\[ \delta = \frac{1}{\sqrt{\pi \mu_0 \sigma_{cu} f}} \]  

(3.39)

As is described in [A.Borisavljevic2010] in high speed machines for a conductor diameter less than a millimeter with frequency smaller than 10 KHz, the skin effect loss change is less than a percent of total DC loss at room temperature; therefore, the skin effect can be ignored, i.e.:

\[ P_{cu,skin} \approx R_{dc} I_{ac}^2 \]  

(3.40)

The proximity loss effect presented above, [J.A.Ferreira1994], is accurate for conductors with the diameter larger than the skin depth. In the high speed electrical machine winding, Litz wire has many strands with diameters considerably much smaller than the skin depth. Therefore it is reasonable to assume that the electrical field is constant in the strand conductor.

The first step to calculate the proximity effect loss is to calculate the power dissipation per unit length of a cylindrical conductor in a uniform field. Using the first term of exact Bessel function [J.Lammeraner1996] the power dissipation per unit length of conductor will be:

\[ P = \frac{\pi \sigma^2 B^2 d_c^4 \sigma_{cu}}{128} \]  

(3.41)

Where \( B \) is the peak radial flux density. This gives the same result when the following approximation is based on the \( G(\gamma) \) function:

\[ G(\gamma) \approx \frac{\pi}{8\sigma_{cu}} \gamma^3 \]  

(3.42)

### 3-2-4 Proximity Effect Loss in Slotted Stator

In the slotted machine the effect of rotor permanent magnet field in the induced eddy current losses in the stator windings is small and the proximity effect is only due to other currents in the winding window area; however, in slot-less machine the proximity effect is mostly due to rotor PM and the effect of other conductors is neglected. The magnetic flux density of the windings is assumed to be trapezoidal in the winding window area; with this assumption, the proximity effect loss will be [C.R.sullivan1999]:
\[ P_{cu,prox} = \frac{\pi^2 \omega^2 \mu_0^2 \sigma_{cu}^2 N_{sl}^2 n^2 d_c^6}{768 b_w^2} R_{dc} I_a^2 \]  

(3.43)

In which \( b_w \) is the breadth of the winding window area. Therefore, the total AC copper losses per phase including skin and proximity effect losses will be:

\[ P_{cu} = F_r R_{dc} I_a^2 \]  

(3.44)

With the factor \( F_r \) defined as:

\[ F_r = 1 + \frac{\pi^2 \omega^2 \mu_0^2 \sigma_{cu}^2 N_{sl}^2 n^2 d_c^6}{768 b_w^2} \]  

(3.45)

### 3-2-5 Proximity Effect Loss in Slot-Less Stator

In the slot less stator, the proximity effect is mostly due to rotor permanent magnet field rather than the neighboring conductors. By assuming the approximate solution, in which the field in the conductor is assumed to be constant, the proximity effect loss will be:

\[ P_{cu,prox} = G(\gamma) \hat{H}_c^2 n l_{strand} \]  

(3.46)

In which \( l_{strand} \) is the length of each strand neglecting the end regions. By using the approximation as stated in equation (3.42) the loss per phase will be:

\[ P_{cu,prox} = \frac{n N L_{Fe} \pi \sigma_{cu} \hat{B}^2 \omega^2 d_c^4}{64} \]  

(3.47)

### 3-3 Stator Iron Losses

In high frequency rotating machines, the iron losses can cause a huge amount of losses. These losses are the hysteresis and eddy current ones. There are different methods to calculate the iron losses; and, the losses are calculated for both slot-less and slotted machines using the maximum flux density in the iron due to magnet and armature fields.
3-3-1 Introduction

The accurate calculation of iron losses in the electrical machines is one of the difficulties for electrical machine designers. There is always a difference between the calculated and measured losses. That is because of the non linear magnetic behavior of the laminations, anisotropic magnetic properties, changing of magnetic properties of laminations due to machine punching and the effect of pressure and temperature.

The prediction of iron losses is very important for high speed machines. There are two different methods for calculating iron losses: the classical and the modern ways.

In the classical methods, the iron losses are divided into eddy current and hysteresis losses [P.C.Sen1997]:

\[ P_{Fe} = P_{Fe,h} + P_{Fe,e} \]  (3.48)

The hysteresis loss has a non linear behavior because it is the energy dissipated in a cycle in which the magnetic field strength and magnetic flux density oscillate in this cycle over the hysteresis loop. The volume density of dissipated power is equal to hysteresis curve area times the oscillation frequency:

\[ P_{Fe,h} = A_{B-H} f \]  (3.49)

In which \( A_{B-H} \) is the area of the magnetization curve \([J/m^3]\) and is a property of the material.

Since the calculation of this area is difficult, empirical equations are usually used, among them is the commonly used Steinmetz equation expressed as:

\[ P_{Fe,h} \propto \omega \hat{B}^s \]  (3.50)

In which \( s \) is the Steinmetz constant and has a value between 1.5 and 2.5.

The eddy current loss has a linear behavior and at low frequencies the effect of eddy current on the magnetic field in the laminations is negligible and the eddy current loss is proportional to \( \omega^2 \), while at high frequencies this effect is not negligible and the loss is proportional to \( \omega^{1.5} \) [R.L.Stoll1974]:

\[ P_{Fe,e} \propto \begin{cases} \omega^2 \hat{B}^2 & \text{for small frequency} \\ \omega^{1.5} \hat{B}^2 & \text{for high frequency} \end{cases} \]  (3.51)

In the modern methods of iron loss calculations the total iron loss is based on the dissipation of magnetic domain walls (Bloch walls) when the magnetic field is changing. This is more accurate and complex than the classical methods because here there is no assumption of homogenous magnetization of laminations. Usually the total iron loss is divided in to three parts: the hysteresis, eddy current and anomalous losses [G.Bertotti1988]:

\[ P_{Fe} = P_{Fe,h} + P_{Fe,e} + P_{Fe,a} \]  (3.52)
The effect of anomalous or extra loss is taken into account as the difference between the calculated hysteresis and eddy current losses and the total measured iron loss. In [F. Fiorillo 1990] this loss is defined as:

\[ p_{Fe,a} \propto \frac{1}{T} \int \frac{dB}{dt}^{1.5} \, dt \]  

(3.53)

### 3-3-2 Calculation of hysteresis and eddy current losses

The properties of the magnetic laminations for the machine are as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity</th>
<th>Lamination thickness</th>
<th>Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon-Steel</td>
<td>$10 \times 10^6 , [S/m]$</td>
<td>0.2 [mm]</td>
<td>4000 $\mu_0$</td>
</tr>
</tbody>
</table>

For the hysteresis loss the volume loss density is roughly estimated by the manufacture’s data [10JNEX900 Si-steel] as:

\[ p_{Fe,h} \approx 11.14 \omega \hat{B}^{1.8} \, [W/m^3] \]  

(3.54)

In [J. Lammeraner 1966] the eddy current loss is explained by assuming the presence of one-dimensional radial magnetic field in laminated sheets where the width of the lamination is much smaller than the length of it. The eddy current loss volume density is derived to be:

\[ p_{Fe,e} = \frac{\hat{B}^2 \sigma f^2}{6} d \frac{F(\zeta)}{\zeta} F(\zeta) \]  

(3.55)

In which $\sigma$ is the electrical conductivity of laminations, $d$ is the lamination thickness and $F(\zeta)$ is expressed as:

\[ F(\zeta) = \frac{3 \sinh(\zeta) - \sin(\zeta)}{\zeta \cosh(\zeta) - \cos(\zeta)} \]  

(3.56)
And \( \zeta \) is the ratio of lamination thickness to the skin depth:

\[
\zeta = \frac{d}{\delta}
\]

With the skin depth defined as:

\[
\delta = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma_i}}
\]

In which the \( \mu_r \) is the relative permeability of the lamination material. It can be shown that for high frequencies the function \( F(\zeta) \) is approximated by \( F(\zeta) \approx \frac{3}{\zeta} \), and at low frequencies by \( F(\zeta) \approx 1 \). Therefore, at high frequencies the eddy current loss is proportional to \( f^{1.5} \) and at low frequencies it is proportional to \( f^2 \).

### 3-3-3 Iron Losses in the Yoke

The maximum amplitude of flux density is calculated separately in each tooth and in the yoke, and then the iron losses in all of them are summed up. It is reasonable to calculate the total maximum flux density in the yoke and teeth by just summing up the maximum flux densities due to the magnet and armature fields.

The maximum flux density in the yoke due to permanent magnets field happens at the \( \varphi = \phi = 0 \) assuming the PM magnetization according to (2.23) and the initial position of rotor. This is independent of number of slots both for slotted and slot-less machines. The maximum of flux density in the yoke is calculated as:

\[
\hat{B}_{pm,\text{y}} = \frac{\lambda_{pm,\text{max,\text{y}}}}{w_{bi} L_{Fe}}
\]

In which \( w_{bi} \) is the yoke width and \( L_{Fe} \) is the active length of the machine. The flux density in the yoke is assumed to have only tangential component and the radial one is negligible. The maximum flux density in the yoke is calculated by assuming that the radial flux in the inner surface of the stator between 0 and \( \frac{\pi}{2} \) (rotor in initial position) goes to the yoke and will have the maximum value at \( \varphi = \phi = 0 \). Using these assumptions the maximum flux in the yoke due to permanent magnets is:
To calculate the maximum flux density in the yoke due to the field of armature current, the procedure is similar to the field of permanent magnets. The only difference is that here the field of higher space harmonics are neglected since they make smaller loops which don’t contribute to the maximum flux density in specific point in the yoke. Therefore, the maximum flux density in the yoke will be:

\[
\lambda_{pm,\text{max},y} = R_e L_{Fe} \int_{0}^{\frac{\pi}{2}} B_{pm, r}(r = R_e, \varphi = \phi) d\phi
\]  

(3.60)

In which the maximum flux is calculated by assuming that at \( \theta = 0 \) all the radial flux at the surface of stator between 0 and \( \frac{\pi}{2} \) goes through the yoke and becomes tangential flux:

\[
\hat{B}_{s, y} = \frac{\lambda_{s,\text{max},y}}{w_{hi} L_{Fe}}
\]  

(3.61)

Then the total yoke iron loss will be:

\[
P_{Fe, y} = V_y (p_{Fe,e,y} + p_{Fe,h,y}) \quad [W]
\]  

(3.63)

In which \( V_y \) is the volume of the yoke.

3-3-4 Iron Losses in the Teeth

The maximum flux density in the tooth happens when the maximum point of air gap field is exactly in front of the tooth. This is also independent of the number of slots. At this point, by assuming that the number of slots is high or the slot pitch angle is rather smaller than the pole pitch angle, the flux under the slot pitch angle go through the tooth. Using these assumptions the flux density in tooth due to permanent magnet field is:

\[
\hat{B}_{pm,t} = \frac{L_{Fe} R_e}{L_{Fe} w_{t}} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} B_{pm, r}(r = R_e, \varphi = \phi) d\phi
\]  

(3.64)
In which the \( w_t \) is the width of tooth and \( \theta_s \) is the stator slot pitch angle.

To calculate the maximum flux density in the tooth due to the armature field, the procedure is similar to the field of permanent magnets. Here, like the previous part, the higher space harmonics of armature field are ignored. Therefore, the maximum flux density in the tooth at \( \theta = 0 \) will be:

\[
\hat{B}_{s,t} = \frac{L_{Fe} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} B_{s,r}(r = R_s, \phi = \phi) \, d\phi}{L_{Fe} \, w_t} \tag{3.65}
\]

The maximum flux density in all teeth is the same; thus the maximum flux density in one tooth is calculated and so the loss in that specific tooth. Then the total losses in all the teeth will be the loss in one tooth times the number of teeth.

The total iron loss in the stator teeth will be:

\[
P_{Fe,t} = N_s \, V_t \left( p_{Fe,e,t} + p_{Fe,h,t} \right) \, [W] \tag{3.66}
\]

In which \( V_t \) is the volume of one tooth. And the total stator iron loss is:

\[
P_{Fe} = P_{Fe,t} + P_{Fe,y} \tag{3.67}
\]
4-1 Power Electronics Drive

4-1-1 Introduction

To supply power to the three phase motor, the normal three-phase voltage source inverter is used. It is also possible to supply the three phase load with three different single phase inverters; however, in this way, 12 switches would be used instead of 6 that makes it more costly. Using three separate inverters has this advantage that they perform independently.

The 6-switch three phase inverter, shown below, consists of three legs; each leg represents a single phase half-bridge inverter.

Figure 4-1. Three-phase inverter, [N. Mohan 2003].
The output voltage of each phase (for example $V_{AN}$) is only dependent on the DC voltage and the switch status, not on the load current. Figure below shows a 3-phase voltage source inverter (VSI) connected to a 3-phase AC motor load. It is assumed that the motor back emf voltages (no-load voltages) are pure sinusoidal (due to ideal Halbach arrangement in the magnets) and the resistances of the stator phase windings are neglected.

![Figure 4-2. Circuit diagram of 3-phase VSI connected to the load, [N.Mohan2003].](image)

For a three-phase balanced load it can be concluded that:

$$v_{AN} = \frac{2}{3}v_{AN} - \frac{1}{3}(v_{BN} + v_{CN})$$ \hspace{2cm} (4.1)

Similar equations are also valid for other phases.

### 4-1-2 Three-phase Voltage Source Inverter (VSI) PWM

In the PWM of the voltage source inverter, three balanced 3-phase sinusoidal control voltages are used and compared with triangular voltage waveform to control the output voltage of the inverter. As is obvious from equation (4.1) the inverter output voltage depends on the output voltages of the three legs which represent three half-bridge single phase inverters. For a half-bridge single phase inverter with a sinusoidal PWM in a linear modulation part the fundamental frequency harmonic is equal to:

$$v_{AN,1} = \hat{v}_{AN,1} \sin(\omega t) = m_d \frac{V_s}{2} \sin(\omega t)$$ \hspace{2cm} (4.2)
In which \( m_a \) is the amplitude modulation ratio. By substituting the (4.2) in to (4.1) the amplitude of fundamental frequency of \( v_{AN} \) is equal to:

\[ \hat{V}_{An,1} = \hat{V}_{AN,1} = m_a \frac{V_d}{2} \]  

(4.3)

The no-load voltage has a pure sinusoidal wave form, as is known from the electromagnetic design in chapter 2, and its amplitude should be equal to the amplitude of fundamental inverter output voltage:

\[ \hat{E}_1 = \hat{V}_{An,1} = m_a \frac{V_d}{2} \]  

(4.4)

Therefore the amplitude modulation can be determined from the equation above, (4.4). By assuming the frequency modulation ratio \( (m_f) \) to be 15, there will be:

\[ v_{AN} = \frac{V_d}{2} + \sum_{k=1}^{m} \hat{V}_{AN} \sin(k \omega t) \]  

(4.5)

In which the voltage harmonics amplitudes versus the modulation ratio are given in figure below. It should be noted that for large frequency modulation ration \( (m_f>9) \) the voltage harmonics are independent of \( m_f \).

Figure 4-3. The normalized harmonic amplitude for the \( v_{AN} \), assuming large \( m_f (m_f>9) \).
Then, by knowing the amplitude modulation ratio from the electromagnetic design of the machine, the harmonics of the $v_{An}$ can be easily determined from the figure above and using the equation (4.1). It should be noted that the 15th harmonic, which is available in each single phase inverter will be removed in the balanced three-phase inverter.

### 4-2 Design Optimization

In the previous chapters and sections the governing electromagnetic equations of the machine were determined, and the correlations between different parameters and variables were realized. To start designing the machine the criteria must be defined, otherwise there are plenty of design options. As mentioned before, in the application presumed for this machine, aerospace application, the low rotor loss along with the low volume is desired. Therefore different types of machines will be designed and optimized to get the favorable machine.

#### 4-2-1 Introduction

There are different parts in the machine that can be changed in order to get a different design. These parts include the number of slots per pole per phase, number of slots, number of phases, number of poles, permanent magnet arrangement, different rotor layers, slotted or slot-less structures. Changing these variables influence the rotor losses, machine total volume, and many other factors such as stator losses, weight and cost.

In the current machine that is going to be designed only a few options are undefined. For example, the numbers of poles are best determined to be two. In this case the switching frequency is low and the losses inside the machine will decrease. The permanent magnet arrangement is also designed to be ideal Halbach (diametrically magnetized) which perfectly reduces the space harmonics due to the magnet field. In [S.R.Holm2002] different arrangements for the magnets are explained and analyzed in detail.

Different rotor structures can also be implemented with different materials. The shaft can be made of solid iron, laminated one or to be hollow. Each of them has some advantages and disadvantages that will be discussed in the next section. A shielding cylinder is required at the surface of the magnet to keep it in place. This shield can also prevent the eddy current losses to go through the magnets to avoid demagnetization. Therefore different rotor layers will be analyzed in the next section to find the best one which has the lowest iron loss. In [F.Zhou2006], [M.R.Shah2009a], [M.Markovic2008], [M.R.Shah2009b], [J.Wang2010] and [S.H.Han2010] the eddy current losses in the rotor are determined and analyzed in detail.

The stator structure is also a part of the machine that can have several configurations. Slotted or slot-less stators are both common in high speed machines. In [N.BianchiJuly2006b], [A.M.El-Rafaie2010], [S.M.Sharkh2009], [P.D.Pfister2010] and [Y.S.Chen1999] the slotted and slot-less configurations for stator are discussed. For high speed applications slot less configuration appears to be proper, the high effective air gap reduces the variations of air gap flux density due to MMF
and PWM and so reduces rotor losses. The slot-less structure is less sensitive to the PM demagnetization due to limited effect of current reaction; however, it has some drawbacks such as low no-load flux density, complex manufacturing process and lower cooling capacity. The cooling capacity in slotted structure is higher because the effective area that the conductors can transfer heat to back iron is higher than the slot-less one. In this section the difference between slot-less and slotted structures will be analyzed from the loss point of view.

Number of slots also plays a rule in the machine performance, the higher the number of slots, the more sinusoidal the field of stator current field and the harder the manufacturing. In [N.Bianchi2006a] the effect of number of slots on the rotor losses is investigated. In this thesis, only the common 6 and 12 number of slots will be compared here, since for higher number of slots it is hard to manufacture for this small high frequency machine.

There are different winding configurations that can be implemented in the machine. The concentrated winding has higher spatial harmonics rather than the distributed one, and since the rated speed of this machine is so high and because of the high losses due to these high frequency harmonics, the distributed winding is preferred rather than the concentrated one. However, the concentrated winding is easier to manufacture, since the pre-wound coils are just to settle in the slots.

### 4-2-2 Design Method

The analytical design has been done in MATLAB 7.7.0 (R2008b). There is a program written as an ‘m.file’ in MATLAB which behaves parametrically. This program represents all the governing electromagnetic equations of the machine which were derived in previous chapters. It also has been written for the steady state performance of the machine since the dynamic behavior is not in the scope of this thesis.

A good advantage of the program is its parametric behavior. That means that there are some parameters as inputs for the program, which the user chooses them, and the outputs are all the electric and electromagnetic data of the machine. This program is written only for a two pole ideal Halbach permanent magnet machine in which the PM is located in the rotor inside the stator. However, all other parameters of the machine still remain as inputs. The list below, perfectly illustrates the inputs and outputs of the program.

The inputs are:

1. Air gap power,
2. Rotational speed in RPM,
3. Active length of the machine,
4. Tangential speed on the outer surface of the magnet,
5. Remanent flux density, conductivity, thickness and relative permeability of the magnet,
6. Conductivity, thickness and relative permeability of the shaft,
7. Conductivity, thickness and relative permeability of the first sleeve layer,
8. Conductivity, thickness and relative permeability of the second sleeve layer,
9. Conductivity, thickness and relative permeability of the air gap,

If slotted:

10. Number of slots,
11. Teeth shoe thickness,
12. Winding window height,
13. Ratio of slot opening and slot top angles,
14. Ratio of slot top and pitch angles,

And if slot-less:

10. Number of slots,
11. Winding window height,
12. Ratio of slot top and pitch angles,
13. Relative Permeability of the winding region,

and

14. Stator yoke thickness, relative permeability and lamination thickness,
15. Total number of turns,
16. Short pitch angle,
17. Turn packing factor,
18. Serving, bundle, twist and filler factor for Litz wire,
19. Number of strands,
20. The inverter input DC voltage.

then the outputs will be:

1. Field of permanent magnet in all the regions of the machine,
2. The induced no-load voltage,
3. Inverter amplitude modulation and voltage harmonics,
4. Litz and strand wire diameters,
5. Maximum surface current density of fundamental harmonic,
6. Geometry data of the machine parts,
7. Electromagnetic torque,
8. Field of armature current in different parts of the machine,
9. Stator main and leakage inductances,
10. Current time harmonics,
11. Eddy current losses in different layers of the rotor,
12. DC resistance of the windings,
13. DC and AC resistive losses inside the windings,
14. Maximum flux density inside the teeth and yoke,
15. Hysteresis and eddy current losses in the yoke and teeth.
Most of the important results are also verified by the Finite Element simulation which is done with COMSOL 3.5a. The COMSOL is connected to MATLAB and this makes it easy to change the parameters in MATLAB and see the results both in MATLAB (the analytical results) and in COMSOL (the FE results).

4-3 Design Results: Optimization 1

The machine that is going to be designed is a 2 pole, diametrically magnetized permanent magnet (ideal Halbach). The slotted and slot-less stators along with different number of slots are going to be designed and compared. Different rotor materials with different thicknesses are also part of the optimization. The results will be verified by the Finite Element simulation. In the first optimization, the machines will be designed to have a same air gap power, rotating speed with same rotor materials and dimensions and nearly same back EMFs. The materials that have been used for the rotor layers are as follows:

<table>
<thead>
<tr>
<th>Region</th>
<th>Material</th>
<th>Conductivity [S/m]</th>
<th>Relative Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 4</td>
<td>Sleeve 2</td>
<td>Carbon Fiber</td>
<td>0.001e6</td>
</tr>
<tr>
<td>Region 3</td>
<td>Sleeve 1</td>
<td>Carbon Fiber</td>
<td>0.001e6</td>
</tr>
<tr>
<td>Region 2</td>
<td>PM</td>
<td>NdFeB</td>
<td>0.667e6</td>
</tr>
<tr>
<td>Region 1</td>
<td>Shaft</td>
<td>Laminated Iron</td>
<td>0</td>
</tr>
</tbody>
</table>

4-3-1 Designed Machine Geometries

In the figure below you can see the designed machine structures for slotted and slot-less stators. For both the slotted and slot-less structures two options for the number of slots are considered, 6 and 12.
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Figure 4-4. Different permanent magnet machine structures which are designed in this thesis, they include slot-less and slotted along with 6 and 12 number of slots.

As is obvious from figure above, the slot-less and slotted machines have the same winding distribution, for the same number of slots; this makes the comparison more realistic. Since these machines are designed for the same power level and rotating speed, the geometries are a bit different from each other. The more data for the geometries of these machines are given in the appendix.

4-3-2 Winding Distributions

For both the slotted and slot-less machines the same winding distributions are chosen. The reason is to have good comparison criteria. Table below shows different possible winding distributions; however only the 4th and the 6th ones are chosen and drawn in the following figure. The reason is that the other types have either higher number of slots that makes the manufacturing hard or have higher space harmonics. In the table below, \( N_s \) is the number of slots,
$n$ stands for the number of layers, $q$ is the number of slots per pole per phase and $\varphi_{pitch}$ is the short pitching angle.

<table>
<thead>
<tr>
<th>Table 4-2. Properties of different winding distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_s$</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

Figure 4-5. Number of conductors of phase $a$ in each slot per slot opening angle for slotted and slot-less machines for the cases 4 and 6 in the table above.

The cosine format for the windings are chosen as was explained in chapter 2. According to the figure above, there is a difference between slotted and slot-less machines’ winding wave forms and that is the wider slot opening angle for the slot-less machine.
4-3-3 Field of Permanent Magnet

When there is no current applied in the stator and only the magnets are magnetized, the magnetic flux density and flux lines for both the slotted and slot-less machines are simulated by FEM according to the figure below:

![Magnetic Flux Density and Flux Lines Simulation](image)

- a) Slotted & Ns=6
- b) Slotted & Ns=12
- c) Slot-less & Ns=6
- d) Slot-less & Ns=12

Figure 4-6. The magnetic flux density and flux lines of permanent magnets inside the machines simulated by FEM.

In the figures below you can find the radial magnetic flux density due to permanent magnets in the middle of mechanical air gap; the analytical results are compared with the FE ones.
As is obvious from the figure above, the analytical and Finite Element method results match perfectly in the slot-less machine. In the analytical method of slotted structure, the effect of slots are neglected and the magnetic flux density is pure sinusoidal.

### 4-3-4 No-Load Voltage

The no-load induced voltages in the winding phase $a$ of the machines, due to permanent magnet field, are presented in the figure below. The FEM results are compared with the analytical results.
As is obvious from the figure above, there is a perfect match between the calculated no load voltages and the Finite Element results. These no load voltages have only the fundamental harmonic (due to the ideal Halbach permanent magnets) and the amplitude of the fundamental harmonic will be used to calculate the amplitude modulation of the VSI.

### 4-3-5 Machine Input Voltage

As mentioned before, the amplitude of the no load voltage is used to calculate the amplitude modulation ratio of the voltage source inverter. Table below shows the amplitude modulation ratio and the maximum of the no load voltage.

<table>
<thead>
<tr>
<th></th>
<th>$m_a$</th>
<th>$E_{no\ Load\ [V]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotted &amp; Ns=6</td>
<td>0.8979</td>
<td>242.5</td>
</tr>
<tr>
<td>Slotted &amp; Ns=12</td>
<td>0.9685</td>
<td>261.5</td>
</tr>
<tr>
<td>Slot-less &amp; Ns=6</td>
<td>0.812</td>
<td>219.2</td>
</tr>
<tr>
<td>Slot-less &amp; Ns=12</td>
<td>0.9294</td>
<td>251</td>
</tr>
</tbody>
</table>

Then the output of the inverter can be calculated, as shown in figure below, independent of the load. Only the first 19 harmonics of the inverter voltage are considered. The figure also contains the normalized harmonic contents of the VSI output voltage; it is normalized to the $V_{dc}/2$. 
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a) Slotted & Ns=6

b) Slotted & Ns=12

c) Slot-less & Ns=6
Figure 4-9. The inverter output voltages for different machines, the left side figures are the output voltage and the right side figures show the normalized harmonic contents.

As can be seen from the figures above, the inverter is working in the linear modulation part with frequency modulation ratio \((m_f)\) of 15.

4-3-6 Electromagnetic Power

Since the average power crossing the air gap is dependent only on the fundamental of the current harmonic and since the power is given as an input to the machine, the current first harmonic can be calculated. Figure below shows the instantaneous electromagnetic power calculated by Finite Element method for different machines.
As can be seen from the figure above the calculated power by the Finite Element method is quite close to the desired value, which is 30 KW. The ripples in the graphs are due to the rough approximations of the FE meshing using Maxwell stress to calculate the power.

4-3-7 Armature Current Field

By setting the magnetization of permanent magnets to zero and feeding the machine with the VSI, the machine main and leakage inductances can be calculated; using the inductances, the current time harmonics are created. Figure below, shows the current ripple due to 11th, 13th, 17th and 19th current time harmonics.
As is apparent from the figure above, the current ripples in the slotted machines are higher than the slot-less ones.

In the figure below you can see the magnetic flux density and the flux lines, simulated by FEM, for different cases when all the existing current harmonics are available and the magnetization of permanent magnets are set to zero. The graphs are shown for $\theta = 2\pi$ to see the effect of rotor conductivities on the main field.

Figure 4-12. The magnetic flux density and flux lines due to armature current simulated by FEM.

The figure below shows the radial magnetic flux density of armature current (with all the existing harmonics) in the middle of mechanical air gap.
Figure 4-13. The radial magnetic flux density in the middle of air gap due to armature current field.

It is to be mentioned that the effect of rotor conductivities are also taken into account and the results are shown when $\theta = 2\pi$, that is equal to $\theta = 0$ for the analytical method; however, for the FEM the machine should start rotating in order to see the effect of rotor conductivities. Therefore after one period the field of armature current is captured. From the figure the effect of slots in the slotted stator on the field of air gap can also be observed. The effect of slots on the field decreases when going outward the slots.

The armature current also induces voltages due to winding main and leakage (in slotted machine) inductances. Figure below shows the induced voltages in the stator windings due to armature currents.
As is obvious from figure above, the slot-less machine doesn’t have a leakage inductance and the induced voltage is only due to main field inductance. The left side figures are the induced voltages due to the fundamental current harmonic and the right side graphs are due to all time harmonics. In both the slotted and slot-less machines there is a perfect match between the analytical and FE results for all the current time harmonics. For the slotted machine, the leakage inductance plays a significant role in the induced voltages and current harmonics.
4-3-8 Rotor Losses

The rotor layers materials have certain amount of electrical conductivities. This is the cause of eddy current loss inside the rotor. The field of permanent magnet doesn’t contribute to the losses, since it is rotating synchronously with the rotor while the field of armature current is the source of eddy current loss. The armature current field has space and time harmonics which rotate with different speeds than the rotor and cause eddy currents. Table below shows the loss in the whole rotor for each space and time harmonic and for different machine types.

<table>
<thead>
<tr>
<th>Table 4-4. Space and time harmonics components of rotor eddy current loss, [W].</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Slotted &amp; Ns=6</td>
</tr>
<tr>
<td><strong>h, k</strong></td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>19</td>
</tr>
<tr>
<td>SUM</td>
</tr>
</tbody>
</table>

b) Slotted & Ns=12

<table>
<thead>
<tr>
<th><strong>h, k</strong></th>
<th><strong>1</strong></th>
<th><strong>5</strong></th>
<th><strong>7</strong></th>
<th><strong>11</strong></th>
<th><strong>13</strong></th>
<th><strong>17</strong></th>
<th><strong>19</strong></th>
<th><strong>SUM</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.29</td>
<td>0.03</td>
<td>0.8</td>
<td>0.27</td>
<td>0</td>
<td>0</td>
<td>1.39</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0.34</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.34</td>
</tr>
<tr>
<td>13</td>
<td>100.3</td>
<td>0.02</td>
<td>0</td>
<td>0.04</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100.36</td>
</tr>
<tr>
<td>17</td>
<td>76.5</td>
<td>0.01</td>
<td>0.03</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>76.55</td>
</tr>
<tr>
<td>19</td>
<td>0.15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.15</td>
</tr>
<tr>
<td>SUM</td>
<td>177.3</td>
<td>0.32</td>
<td>0.06</td>
<td>0.84</td>
<td>0.28</td>
<td>0</td>
<td>0</td>
<td><strong>172.8</strong></td>
</tr>
</tbody>
</table>

c) Slot-less & Ns=6

<table>
<thead>
<tr>
<th><strong>h, k</strong></th>
<th><strong>1</strong></th>
<th><strong>5</strong></th>
<th><strong>7</strong></th>
<th><strong>11</strong></th>
<th><strong>13</strong></th>
<th><strong>17</strong></th>
<th><strong>19</strong></th>
<th><strong>SUM</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0.0273</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0273</td>
</tr>
<tr>
<td>13</td>
<td>4.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4.2</td>
</tr>
<tr>
<td>17</td>
<td>3.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3.2</td>
</tr>
<tr>
<td>19</td>
<td>0.0117</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0117</td>
</tr>
<tr>
<td>SUM</td>
<td>7.44</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td><strong>7.44</strong></td>
</tr>
</tbody>
</table>
As is obvious from the table, in slotted machine with 6 slots the 5th space harmonic has a significant influence in the rotor loss while in 12 slot machine, the space harmonics are negligible. It is to be noticed that although the space harmonics are reduced in 12 slot machine, the losses are higher than in the 6 slot one. It is also noticeable that the losses in the slot-less machine are much lower than the slotted one due to the effect of slots.

Figure below shows the 3-D graphs representing the table above.
Figure 4-15. Space and time harmonics contributions to the rotor loss for different machine types.
4-3-9 Stator Losses

The stator losses include the windings resistive losses and iron losses. The resistive losses due to skin and proximity effects are shown in the table below for the Litz wires with the number of strands equal to 40 for all the machines.

<table>
<thead>
<tr>
<th></th>
<th>DC Resistance (Ohm)</th>
<th>$P_{DC}$ [W]</th>
<th>$P_{AC}$ [W]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotted &amp; $N_s=6$</td>
<td>0.0045</td>
<td>46.2</td>
<td>48.9</td>
</tr>
<tr>
<td>Slotted &amp; $N_s=12$</td>
<td>0.0047</td>
<td>41</td>
<td>43.2</td>
</tr>
<tr>
<td>Slot-less &amp; $N_s=6$</td>
<td>0.0091</td>
<td>113.5</td>
<td>298</td>
</tr>
<tr>
<td>Slot-less &amp; $N_s=12$</td>
<td>0.0107</td>
<td>97.8</td>
<td>255.5</td>
</tr>
</tbody>
</table>

For calculating the iron losses the maximum flux density in the teeth and the yoke are calculated with both analytical and FE methods. The results are represented in the table below:

<table>
<thead>
<tr>
<th></th>
<th>Analytical</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Slotted &amp; $N_s=6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tooth</td>
<td>1.13</td>
<td>1.3</td>
</tr>
<tr>
<td>Yoke</td>
<td>1.15</td>
<td>1.2</td>
</tr>
<tr>
<td>b) Slotted &amp; $N_s=12$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tooth</td>
<td>1.17</td>
<td>1.27</td>
</tr>
<tr>
<td>Yoke</td>
<td>1.15</td>
<td>1.18</td>
</tr>
<tr>
<td>c) Slot-less &amp; $N_s=6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yoke</td>
<td>1.17</td>
<td>1.18</td>
</tr>
<tr>
<td>d) Slot-less &amp; $N_s=12$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yoke</td>
<td>1.19</td>
<td>1.20</td>
</tr>
</tbody>
</table>
By knowing the maximum flux densities in the teeth and yoke, the hysteresis and eddy current losses inside the stator can be achieved. Table below shows the mentioned losses in different parts of the stator:

Table 4-7. The hysteresis and eddy current losses [W] in the teeth and yoke of different designed machines.

<table>
<thead>
<tr>
<th>Loss</th>
<th>$P_{Fe,y}$</th>
<th>$P_{Fe,t}$</th>
<th>$P_{Fe,e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Slotted &amp; Ns=6</td>
<td>151.3</td>
<td>51.7</td>
<td>203</td>
</tr>
<tr>
<td>Loss</td>
<td>$P_{Fe,y}$</td>
<td>$P_{Fe,t}$</td>
<td>$P_{Fe,e}$</td>
</tr>
<tr>
<td>b) Slotted &amp; Ns=12</td>
<td>28</td>
<td>9.6</td>
<td>37.6</td>
</tr>
<tr>
<td>Loss</td>
<td>$P_{Fe,y}$</td>
<td>$P_{Fe,t}$</td>
<td>$P_{Fe,e}$</td>
</tr>
<tr>
<td>c) Slot-less &amp; Ns=6</td>
<td>179.5</td>
<td>61.3</td>
<td>240.8</td>
</tr>
<tr>
<td>Loss</td>
<td>$P_{Fe,y}$</td>
<td>$P_{Fe,t}$</td>
<td>$P_{Fe,e}$</td>
</tr>
<tr>
<td>d) Slot-less &amp; Ns=12</td>
<td>27.7</td>
<td>10.1</td>
<td>37.8</td>
</tr>
</tbody>
</table>

4-3-10 Total Losses

All the losses for different machine types have been calculated neglecting the air friction and power electronics losses. Table below summarizes the machine losses:

Table 4-8. Total machine losses and efficiencies.

<table>
<thead>
<tr>
<th>Loss</th>
<th>Resistive Loss</th>
<th>Iron Loss</th>
<th>Rotor Loss</th>
<th>Total Loss</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slotted &amp; Ns=6</td>
<td>48.9</td>
<td>240.8</td>
<td>126.3</td>
<td>416</td>
<td>98.61</td>
</tr>
<tr>
<td>Slotted &amp; Ns=12</td>
<td>43.2</td>
<td>242.5</td>
<td>178.8</td>
<td>464.5</td>
<td>98.45</td>
</tr>
<tr>
<td>Slot-less &amp; Ns=6</td>
<td>298</td>
<td>121.4</td>
<td>7.44</td>
<td>426.84</td>
<td>98.58</td>
</tr>
<tr>
<td>Slot-less &amp; Ns=12</td>
<td>255.5</td>
<td>120</td>
<td>10.61</td>
<td>386.11</td>
<td>98.71</td>
</tr>
</tbody>
</table>
According to the table above, all the machines have almost the same amount of losses and the efficiencies are also high. Therefore, the choice for the choosing any of the designed machines should be based on other factors such as weight, cost, volume and etc..

4-4 Design Results: Optimization 2

In this section one of the designed machines will be chosen and the optimization will be done on different factors such as rotor layers dimensions and materials, air gap length and total number of strands in the Litz wires.

As was discussed in the previous section, the slot-less machine has a very low rotor loss but a significant stator resistive loss. The disadvantage of slot-less machine is the weak cooling capability. The resistive loss heat can either go toward the yoke or toward the rotor. Since the common surface area of the iron and windings is low compared to the slotted machine, most of the losses will go toward the rotor permanent magnets, which can cause PM demagnetization. In slotted machines, which were designed in the previous chapter, the rotor loss is significant compared to the slot-less machines; but the stator resistive loss is much lower. Due to the presence of slots, the loss heat in the stator can easily go toward outside the machine. Therefore, the slotted machine, with 6 number of slots, will be chosen to optimize the rotor losses by changing the rotor dimensions, materials and air gap length.

4-4-1 Effect of rotor layers materials

Due to the effect of armature space and time harmonics, there will be induced eddy current in the rotor layers. Since the rotor layer has a specific amount of electrical conductivity, there will be ohmic loss inside the layer. There are different possibilities for rotor layers materials in order to reduce the total eddy current loss. In all the cases the machines are designed to have a same air gap power and rotational speed. The maximum flux density and the maximum windings surface current density are kept under the critical values. The total rotor dimension is kept fixed. Therefore by changing the rotor layers, the machine will totally change; however, this change is not significant.

Table below shows different rotor layers:

<table>
<thead>
<tr>
<th>Region</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Laminated iron</td>
<td>Laminated iron</td>
<td>Laminated iron</td>
<td>Air</td>
</tr>
<tr>
<td>2</td>
<td>PM</td>
<td>PM</td>
<td>PM</td>
<td>PM</td>
</tr>
<tr>
<td>3</td>
<td>Carbon fiber</td>
<td>Copper</td>
<td>Aluminum</td>
<td>Carbon fiber</td>
</tr>
<tr>
<td>4</td>
<td>Carbon fiber</td>
<td>Carbon fiber</td>
<td>Carbon fiber</td>
<td>Carbon fiber</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>7.7</td>
<td>7</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>
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The conductivity and relative permeability used for these materials are as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity [S/m]</th>
<th>Relative Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon fiber</td>
<td>0.001e6</td>
<td>1.1</td>
</tr>
<tr>
<td>PM</td>
<td>0.667e6</td>
<td>1.15</td>
</tr>
<tr>
<td>Copper</td>
<td>59.6e6</td>
<td>0.99</td>
</tr>
<tr>
<td>Aluminum</td>
<td>35.5e6</td>
<td>1</td>
</tr>
<tr>
<td>Laminated Iron</td>
<td>0</td>
<td>4000</td>
</tr>
<tr>
<td>Air</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

By changing the layers materials in the modeled machine program, all other parameters remain intact except the current time harmonics. The reason is that the inductances, which are dependent on the rotor conductivities, change and subsequently the current harmonics. The induced eddy current losses per each layer for different cases, as mentioned before, are presented in the table below:

<table>
<thead>
<tr>
<th>Region</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 4</td>
<td>5</td>
<td>3</td>
<td>3.1</td>
<td>5.7</td>
</tr>
<tr>
<td>Region 3</td>
<td>0.3</td>
<td>127.5</td>
<td>164.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Region 2</td>
<td>121</td>
<td>0.5</td>
<td>1.6</td>
<td>108.1</td>
</tr>
<tr>
<td>Region 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>126.3</td>
<td>131</td>
<td>169</td>
<td>114</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 4</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5.7</td>
</tr>
<tr>
<td>Region 3</td>
<td>0.2</td>
<td>137.5</td>
<td>172</td>
<td>0.2</td>
</tr>
<tr>
<td>Region 2</td>
<td>121.4</td>
<td>0.5</td>
<td>1.9</td>
<td>110.5</td>
</tr>
<tr>
<td>Region 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>126.6</td>
<td>141</td>
<td>176.9</td>
<td>116.4</td>
</tr>
</tbody>
</table>

The figure below illustrates the table above for both the analytical and FE results:
According to the figure above, the shaft of the rotor doesn’t have any eddy current loss for different cases. When there is a highly conductive shield between permanent magnets and carbon fiber sleeve, the eddy current loss inside the magnet decreases significantly. However, when this shielding layer is aluminum, the loss in the shield is higher than the case when the shield is made of copper. The reason is the higher electrical resistivity of aluminum with respect to copper. According to the results, for the case when the shaft is hollow, the total rotor loss is the least amongst other cases; but from aerodynamic point of view, it might be a little hard. Cases A and B have almost the same amount of rotor loss; however, the advantage of case B is that the loss inside the magnets are significantly less than case A. That can prevent the heating up of the magnets and subsequently demagnetization. Therefore, for the next part of optimization, case B will be chosen and the optimization will be done on the effects of copper shield and air gap thicknesses on the total rotor losses.

Figure 4-16. The eddy current loss in each region of rotor for different rotor structures.
4-4-2 Effect of shielding cylinder thickness

In this section the effect of shielding cylinder thickness on the rotor eddy current loss will be analyzed. The rotor structure is according to the case B of previous section. The thicknesses of other layers of the rotor remain constant, and only the thickness of copper shield will change. Figure below shows the losses in different layers of the rotor and the total rotor loss for different copper shield thicknesses.

According to the figure above, at the copper shield thickness of 0.8 mm, the copper shield and total rotor eddy current losses are at their minimum values, as is verified by the FE method. However, the loss inside the permanent magnet decreases by increasing the copper shield thickness. The loss inside the carbon fiber doesn’t change significantly, as can be concluded from the graphs.
4-4-3 Effect of air gap thickness

In the slotted machines, the slots contribute to the rotor eddy current loss so much by making space harmonics. The distance between the slots region and the rotor layers plays an important role in the losses in the rotor layers. In this part, the rotor loss is analyzed by changing the air gap thickness. However, it should be noted that by changing the air gap length, the machine will change totally. Therefore, by keeping the air gap power, rotational speed and rotor dimensions constant, the air gap length will be changed to see its effect on the rotor losses. The copper shielding cylinder thickness is at its optimum value, 0.8 mm. Figure below shows the loss distributions in different rotor layers with respect to different air gap length.
According to the figure above, the rotor loss in all the layers always decreases by increasing the air gap length while the surface current density increases. It is to be mentioned that the rotor dimensions for all the cases are kept the same and the stator dimensions increase proportionally; i.e., the ratio between the slot winding window angle and slot pitch angle is constant, and the yoke thickness is also fixed.

As is obvious from the graphs, the resistive loss increases while the iron loss and total machine losses decrease. Therefore, the optimum length for the air gap is achieved by taking into account the value for the maximum surface current density.

### 4-4-4 Optimum number of strands

In this analysis the winding window area and the number of turns are assumed to be fixed. As the number of strands in a fixed winding area increases the eddy current loss decreases but the DC losses increases because of larger insulation area than the copper one; thus, there is an optimum number of strands. So the optimization is based on the resistive losses. As the number of strands increases the costs of the Litz wire increases too. So, it is also possible to optimize the number of strands for minimum cost. From the factors influencing the DC loss, only the strand insulation factor changes with respect to number of strands and the other factors remain constant. For determining the optimum number of strands for the minimum total AC losses, the loss should be minimized with respect to number of strands. In the figure below you can find the total AC and DC losses for different number of strands for slotted and slot-less machines. In this analysis the number of slots for both machines are chosen to be 6 and there is no copper shield between the magnets and the carbon fiber sleeve.
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As is obvious from the figures above, in the slotted machine, the DC loss increases rapidly compared to decrease of AC loss with respect to the number of strands. And the optimum number of strands is found to be 40. In the slot-less machine, the increase of DC loss is not rapid compared to the reduction of AC loss and the optimum number of strands occurs at 661, which is very unreal. Choosing 661 as the number of strands is possible theoretically while makes it impossible from the manufacturing and financial aspects. Choosing the number of strands between 20 and 50, in the slot-less machine, reduces the total loss significantly and is possible to manufacture. In the table below you can find the DC and AC copper losses at the optimum number of strands for both slotted and slot-less machines.
4-4-5 Effect of Speed on the Machine Losses

To see the effect of machine losses and volume with respect to speed, plenty of machines can be designed in any specific speed. However, in this analysis, the power crossing the air gap, the outer magnet surface tangential velocity and subsequently the carbon fiber sleeve thickness, the maximum flux densities in the yoke and teeth and the maximum surface current density are kept constant and the same for different speeds.

Table below shows different machine designed parameters for different speeds:

<table>
<thead>
<tr>
<th>Speed [KRPM]</th>
<th>75</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_{hat} [V]</td>
<td>238</td>
<td>238</td>
<td>265</td>
<td>266</td>
<td>244</td>
<td>245</td>
<td>241</td>
<td>253</td>
<td>253</td>
</tr>
<tr>
<td>B_t [T]</td>
<td>1.19</td>
<td>1.19</td>
<td>1.16</td>
<td>1.18</td>
<td>1.18</td>
<td>1.16</td>
<td>1.21</td>
<td>1.19</td>
<td>1.18</td>
</tr>
<tr>
<td>B_y [T]</td>
<td>1.19</td>
<td>1.19</td>
<td>1.16</td>
<td>1.21</td>
<td>1.18</td>
<td>1.16</td>
<td>1.19</td>
<td>1.15</td>
<td>1.20</td>
</tr>
<tr>
<td>P_{ac} [W]</td>
<td>57</td>
<td>58</td>
<td>51</td>
<td>53</td>
<td>59</td>
<td>58</td>
<td>50</td>
<td>47</td>
<td>48</td>
</tr>
<tr>
<td>P_{Fe} [W]</td>
<td>226.5</td>
<td>226</td>
<td>228</td>
<td>237</td>
<td>236</td>
<td>243</td>
<td>243</td>
<td>257</td>
<td>257</td>
</tr>
<tr>
<td>P_{rotor} [W]</td>
<td>294.4</td>
<td>282</td>
<td>220</td>
<td>200</td>
<td>215</td>
<td>192</td>
<td>131</td>
<td>111</td>
<td>103</td>
</tr>
<tr>
<td>P_{total} [W]</td>
<td>578</td>
<td>566</td>
<td>499</td>
<td>490</td>
<td>510</td>
<td>493</td>
<td>424</td>
<td>415</td>
<td>408</td>
</tr>
<tr>
<td>Volume [cm^3]</td>
<td>632.7</td>
<td>582</td>
<td>514</td>
<td>443</td>
<td>414</td>
<td>384</td>
<td>342</td>
<td>313</td>
<td>294</td>
</tr>
</tbody>
</table>

In this table, $E_{hat}$, stands for maximum no-load voltage; $J$ stands for the maximum fundamental surface current density; $B_t$ and $B_y$, for the maximum flux densities in tooth and yoke, respectively; $P_{ac}$, resistive loss in the windings; $P_{Fe}$, iron loss in the stator core; $P_{rotor}$, loss in the rotor and $P_{total}$, is the total loss in the machine. As is obvious from the table, the maximum surface current density in the windings and maximum magnetic flux density in the tooth and yoke are almost the same for different machines. It is also to be mentioned that for all the machines, the tangential speed at the outer surface of the magnet layer is constant (200 m/s), and therefore the carbon fiber sleeve thickness is the same for all the machines. According to the table, the resistive and iron losses don’t change that much with respect to speed while the induced eddy current loss inside the rotor changes noticeably. The reason why the rotor loss decreases by increasing the speed can
be explained in such a way that the total volume of the rotor decreases by raising the speed value and the total loss in the rotor decreases subsequently. Figure below shows the rotor, stator resistive and iron and total machine losses for different speed values.

![Graph showing machine losses with respect to speed](image)

Figure 4-21. Designed machine losses with respect to speed.

According to the figure above, the total loss is at its minimum value at the speed of 150 Krpm. The rotor loss has a decreasing trend with increasing speed value until 110 Krpm, at which the loss increases; however, after this speed, the rotor loss goes in a decreasing trend. Figure below shows the total volume and overall diameter of the different designed machines for different speeds.
Figure 4-22. Total volume and overall diameter of the designed machines for different speed values.

According to this figure, the total volume and overall diameter of the machine decreases by increasing the speed value. Figure below illustrates the sizes of different designed machines for different speeds.

Figure 4-23. Schematic views of the designed machines for different speed values.
5-1 General Summary

- Due to high speed and high power application of this machine, the permanent magnet rotor is the best solution rather than the induction or wound rotor synchronous machines. The presence of permanent magnet in the rotor makes it possible to design a reliable high power density and high speed machine.
- Number of poles for the machine is adapted to be two in order to decrease higher stator time harmonics.
- Having one pole pair, the ideal Halbach arrangement can be implemented by diametrically magnetizing the magnet ring. In this case the pure sinusoidal field can be achieved in the machine.
- In order to find the magnetic flux density in the machine, the Maxwell equations are solved in different regions of the machine. The equations are in the stationary matter and by assuming the magneto-quasi-static situation, the general Poisson equation is solved in different regions. The magnetic vector potential is first calculated in order to find the magnetic flux density.
- In order to find the field of rotor permanent magnet in the machine, the machine is divided in to seven regions with different relative permeabilities. For a 2-pole ideal Halbach magnet arrangement, the Poisson equations are the same as Laplace equations in all the regions.
- The winding function approach is used instead of direct Fourier analysis in order to represent the physical distribution of the windings in the stator. The cosine form is
chosen, so that it is possible to separate the triple and non-triple spatial harmonics. The effect of distribution, skewing, slots and short pitching are taken into account.

- To calculate the field of armature current in the machine, the Poisson equations are solved in different regions. In this case, also, the machine is divided into seven regions with different relative permeabilities and the effects of rotor layers conductivities are considered by assuming different conductivities for each rotor layer. In the slotted machine an equivalent line current density is presumed in the border of stator slots and air gap while in the slot-less machine an equivalent surface current density is placed in the slots region. The Poisson equations are solved by the method of separation of variables in a complex format for each space and time harmonic and in the rotor reference frame to see the effect of rotor conductivities on the field. The magnetic vector potential and subsequently the flux density are achieved for those harmonics.

- The voltage equations of the machine include the voltages induced due to rotor permanent magnet field, armature current field and slot flux leakage in the slotted machine. The no-load voltage is calculated by finding the flux linkage in the stator windings due to rotor PM field and is pure sinusoidal. For the armature current flux linkage in the stator windings, since the machine is assumed to be balanced and three-phase, the triple harmonic components of flux are zero and the non-triple ones are the same for each winding phase.

- By finding the flux linkages due to armature current field and by knowing the current harmonics, the main field inductances, which are dependent on the time harmonics, are calculated. The slot flux leakages are also zero for triple harmonic components of current and for the non-triple ones, have the same value for each phase. The slot leakage inductance is calculated using the total magnetic energy stored in the slot. The flux leakage has mostly four parts: the slot, the air gap, the tooth tip and the end turns flux leakages; however only the slot flux leakage is considered in this thesis.

- There are different methods for calculating the electromagnetic torque and mechanical power: the Poynting vector theorem, the Lorentz force and the Maxwell stresses. In this thesis the simplified Poynting vector method (power balance equation) is used to calculate the power crossing the air gap, neglecting the eddy current losses in the rotor and air friction one in the air gap.

- The shaft iron, magnet and the sleeves, all have a certain amount of electrical conductivities which are responsible for the rotor induced eddy current losses. A method to determine iron losses based on Poynting vector theorem has been developed. Since the field of magnets is rotating synchronously with respect to the rotor, there will be no induced current and subsequently no losses inside the rotor due to the field of magnets. Therefore the only source for losses in the rotor is due to the armature field space and time harmonics. By using the constitutive relations for a linear, isotropic matter rotating with a speed significantly lower than the speed of light, in the rotor coordinate system, the theorem of Poynting for each space and time harmonic can be achieved. By choosing a proper enclosed surface of integration which doesn’t include the external power density (here the armature current) but includes the parts with conductivity (here the sleeve and magnets), the total average dissipated power in each rotor layer is calculated.
The ohmic losses in the stator windings for high frequency machines (AC losses) are higher than the DC losses. The AC losses include the skin and proximity effects. Using the Litz wires with high number of strands significantly reduced the resistive losses. Some factors affecting the DC losses in the Litz wire are serving, bundle, twist and strand insulation factors. By increasing the number of strands only the strand insulation factor changes. Skin and proximity effects in Litz wire can be divided into bundle and strand level effects. The bundle level skin and proximity effects can be controlled easily and can be neglected compared to strand level ones. In the strand level the proximity effect dominates the skin effect since the skin effect for such a machine and at room temperature is almost the same as DC loss. The proximity effect in the slotted machine is mostly due to the field of adjacent conductors while in the slot-less machine the rotating field of permanent magnet has the major contribution to the proximity effect losses. Using one-dimensional orthogonal approach the power dissipation per unit length of a cylindrical conductor in a uniform field is calculated to find the proximity effect losses in both slotted and slot-less machines.

In high frequency rotating machines, the iron losses can cause a huge amount of losses. These losses are the hysteresis and eddy current ones. The hysteresis loss has a nonlinear behavior because it is the energy dissipated in a cycle in which the magnetic field strength and magnetic flux density oscillate in this cycle over the hysteresis loop. Empirical equations are usually used to calculate the hysteresis loss; among them is the commonly used Steinmetz equation. The eddy current loss has a linear behavior and at high frequencies the effect of eddy current on the magnetic field in the laminations is not negligible. The eddy current loss is explained by assuming the presence of one-dimensional radial magnetic field in laminated sheets where the width of the lamination is much smaller than the length of it. In order to calculate the iron losses, the maximum magnetic flux densities in the yoke and teeth, due to rotor permanent magnet field and armature current, are required. They are calculated separately in each tooth and in the yoke, and then the iron losses in all of them are summed up. To find the maximum flux density in the yoke and tooth due to armature current field, only the fundamental space harmonic is considered since the higher harmonics make smaller loops and don’t contribute to the maximum flux density.

The three-phase voltage source inverter is used to feed the machine with power. The machine is assumed to be balanced and the voltage harmonics of the inverter is calculated. By knowing the voltage harmonics and machine main field and leakage inductances, the current time harmonics are calculated.
5-2 Conclusion

- Designing high speed electrical machines needs considerable amount of attention on different aspects of machine such as electrical power loss, thermal cooling and mechanical limitations.
- Due to high rotational speed of the machine, there will be great amount of electrical losses in the machine. The resistivity loss in the stator, the iron loss in the core and the induced eddy current loss in the rotor, all depend on the frequency of the machine.
- High speed in the air gap also produces air friction loss, which together with other electrical losses in the machine, produce heat in different regions of the machine. Since a considerable ratio of the heat is in the rotor, a good thermal design and cooling system is necessary; otherwise, the magnets will heat up and danger of demagnetization will rise. The loss heat in the stator region can be removed easier than the rotor heat, since the stator cooling is more practical outside the machine.
- Mechanical restrictions play important roles in the machine overall design. In high speed machines the risk of stress, instability and resonance are high. The mechanical, thermal and electrical designs should be mixed to design the whole high speed machine.
- Different stator winding configurations can be implemented in the machine; however, only the 6 and 12 slot machines are considered with short pitching angles equal to 60 and 30 degrees, respectively. Other numbers of slots is also possible but make the manufacturing rather difficult.
- Four different machines are designed: 6-slotted winding distribution with slotted stator, 6-slotted winding distribution with slot-less stator, 12-slotted winding distribution with slotted and 12-slotted winding distribution with slot-less stator.
- In the first optimization, the machines are designed to have a same air gap power, rotating speed with same rotor materials and dimensions and nearly same back EMFs.
- The radial magnetic flux density due to permanent magnets in the middle of mechanical air gap is calculated for all the designed machines. The analytical and Finite Element method results match perfectly in the slot-less machine. In the analytical method of slotted structure, the effect of slots are neglected and the magnetic flux density is pure sinusoidal.
- The no-load induced voltages in the winding phase $a$ of the machines, due to permanent magnet field, are calculated. There is a perfect match between the calculated no load voltages and the Finite Element results. These no load voltages have only the fundamental harmonic (due to the ideal Halbach permanent magnets) and the amplitude of the fundamental harmonic will be used to calculate the amplitude modulation ratio of the VSI.
- The calculated power by the Finite Element method is quite close to the desired value, which is 30 KW. The ripples in the power wave forms are due to the rough approximations of the FE meshing using Maxwell stress to calculate the power.
- Using the main field and leakage inductances, the current time harmonics are created. The current ripples due to $11^{th}$, $13^{th}$, $17^{th}$ and $19^{th}$ harmonics in the slotted machines are higher.
than the slot-less ones; the reason is the presence of slot leakage inductances in the slotted machines.

- By using all the current harmonics, the radial magnetic flux densities in the middle of air gap are calculated for the four designed machines by both analytical and FE methods. There is a perfect match between the analytical and FE results. In the slotted machines the effect of slots on the field are significant and this effect decreases by going outward the slots.

- The armature current also induce voltages due to winding main and leakage (in slotted machine) inductances. The slot-less machine doesn’t have a leakage inductance and the induced voltage is only due to main field inductance. For the slotted machine, the leakage inductance plays a significant role in the induced voltages and current harmonics. In both types of machines there is a perfect match between the analytical and FE results for all the current time harmonics.

- The rotor layers materials have certain amount of electrical conductivities. This is the cause of eddy current loss inside the rotor. The armature current field has space and time harmonics which rotate with different speeds than the rotor and cause eddy currents in the rotor layers. The loss in the whole rotor for each space and time harmonic and for different machine types is calculated. In slotted machine with 6 slots the 5th space harmonic has a significant influence in the rotor loss while in 12 slot machine, the space harmonics are negligible. It is to be noticed that although the space harmonics are reduced in 12 slot machines, the losses are higher than in the 6 slot one. It is also noticeable that the losses in the slot-less machine are much lower than the slotted one due to the effect of slots.

- In the slot-less machines the AC proximity losses are much higher than the slotted machines. And the iron losses in the slotted machines are higher, due to presence of teeth, than the slot-less machines.

- By summing all the losses, the slot-less machine with 12 numbers of slots has the lowest amount of loss, following with the slotted machine with 6 slots. However, all the machines have almost a same amount of losses and the efficiencies are also high. Therefore, the choice for the choosing any of the designed machines should be based on other factors such as weight, cost, volume and etc.

- In the next part of optimization the slotted machine with 6 slots was chosen to optimize the rotor losses by changing the rotor dimensions, materials and air gap length.

- By keeping the air gap power, speed and rotor dimensions fixed, different rotor layers materials are tested. The laminated iron shaft of the rotor doesn’t have any eddy current loss for different cases. When there is a highly conductive shield between permanent magnets and carbon fiber sleeve, the eddy current loss inside the magnet decreases significantly and prevents the heating up of the magnets and the danger of demagnetization. However, when this shielding layer is aluminum, the loss in the shield is higher than the case when the shield is made of copper. The reason is the higher electrical resistivity of aluminum with respect to copper. According to the results, for the case when the shaft is hollow, the total rotor loss is the least amongst other cases; but from aerodynamic point of view, it might be a little hard.
• By choosing the copper shield between the magnet and carbon fiber sleeve, the thickness of copper layer is changed. At the copper shield thickness of 0.8 mm, the copper shield and total rotor eddy current losses are at their minimum values. However, the loss inside the permanent magnet decreases by increasing the copper shield thickness. The loss inside the carbon fiber doesn’t change significantly.

• By keeping the air gap power, rotational speed and rotor dimensions constant, the air gap length will be changed to see its effect on the rotor losses. The rotor loss in all the layers will decrease by increasing the air gap length while the surface current density increases. The resistive loss increases while the iron loss and total machine loss decrease. Therefore, the optimum length for the air gap is achieved by taking into account the value for the maximum surface current density.

• By keeping the winding window area and the number of turns constant, as the number of strands in a fixed winding area increases the eddy current losses decreases but the DC losses increase because of larger insulation area than the copper one; thus, there is an optimum number of strands. In the slotted machine, the DC loss increases rapidly compared to decrease of AC loss with respect to the number of strands. And the optimum number of strands is found to be 40. In the slot-less machine, the increase of DC loss is not rapid compared to the reduction of AC loss and the optimum number of strands occurs at 661, which is very unreal. Choosing 661 as the number of strands is possible theoretically while makes it impossible from the manufacturing and financial aspects. Choosing the number of strands between 20 and 50, in the slot-less machine, reduces the total loss significantly and is possible to manufacture.

• By keeping the power crossing the air gap, magnet outer surface tangential speed, carbon fiber sleeve thickness, maximum surface current density and maximum flux densities in the yoke and teeth, several machines are designed for the speed range of 75 to 150 KRPM. The resistive and iron losses don’t change that much with respect to speed while the induced eddy current loss inside the rotor changes noticeably. The reason why the rotor loss decreases by increasing the speed can be explained in such a way that the total volume of the rotor decreases by raising the speed value and the total loss in the rotor decreases subsequently. The total loss is at its minimum value at the speed of 150 Krpm. The rotor loss has a decreasing trend with increasing speed value until 110 Krpm, at which the loss increases; however, after this speed, the rotor loss goes in a decreasing trend. The total volume and overall diameter of the machine decreases by increasing the speed value.
5-3 Future Research Areas and Recommendations

- In the analytical model, the effect of non-linearity of the magnetic curve for the ironic parts of the machine was neglected. The saturation might impose more strict limitations on the machine design and including this effect can improve the model to a high level.
- The analytical method was programmed in the MATLAB software. This model is built for a 2-pole diametrically magnetized rotor while other parameters and specifications of the machine are variable. In other words the rotor PM is not changeable and an extension to the program would be to further rewrite the program in order to be applicable for all rotor PM types.
- This thesis only dealt with electromagnetic design of the machine and the electrical losses were the main criteria for the design. However, since this is a high speed machine the mechanical limitations such as resonance, instability and stresses play significant roles. By incorporating the mechanical and the electromagnetic designs, a better view of the reality of the designed machine will be achieved.
- In the analysis of this thesis, the air friction loss is neglected. This air friction loss might be very high depending on the speed and rotor dimensions and material. The heat of this loss might go toward the rotor magnets and cause demagnetization. Therefore, another recommendation for a better machine design would be considering the air friction loss in the air gap.
- Thermal design was not in the scope of this thesis. Nevertheless, cooling is one of the main parts of designing high speed electrical machines. A good thermal cooling design of the machine can give us valuable information about the hot spots and temperatures in different parts of the machine and the amount of heat moving through different regions of the machine.
- The voltage source inverter used to feed the machine was assumed to be working in its linear region. Another suggestion for the design is to see the effect of VSI working in the over modulation area.
- The VSI used for the machine has the sinusoidal PWM; checking other modulation techniques and methods for eliminating time harmonics can be of great importance.
- The total loss of the machine was calculated neglecting the losses in the power electronics converter. The loss in the converter might be very high and is nice to calculate it.
- The power of the machine was calculated using the power balance equation. Therefore, the power is the power crossing the air gap and is not exactly equal to the output shaft power since there are rotor and air friction losses in between. To calculate the output mechanical power the Poynting vector theorem in the stator system can be used, which will give more accurate indication of the shaft power.
- For calculating the field of rotor permanent magnets in the slotted machine the effect of slots was ignored. However, to more accurately consider the effect of slots on the field of rotor field, there is a way to assume different relative permeabilities for the radial and tangential fields in the winding slot region.
The iron losses in the teeth and yoke due to armature field were calculated using only the fundamental space harmonic. It seems to be a little difference between the analytical and FE results and would be good to see the effect of higher space harmonics on the maximum flux density.

The slot leakage inductance in the slotted machine was calculated using the total magnetic energy in the slot. The calculation was based in the simple triangular slot while the real slots are not triangular and have more complicated shapes. It is worth to have a look at more precise calculation of slot flux leakage.

In this thesis different slotted and slot-less machines were compared and all of them had distributed windings; however, in high speed applications concentrated windings are also appreciable. Therefore, it is necessary to compare the concentrated winding with distributed one from the losses and other factors points of view.
References:


References


References


References


References


References


Appendix

In this appendix the general data for the designed machines which were used in part 4-3, are presented. The data include the geometries of different parts of the machine along with some information about the rotor layers materials and stator windings specifications. The rotor materials and structure dimensions for all four designed machines are the same, according to section 4-3, and only the stator geometries are different. In the following tables you will find the geometries of the designed machines and not the electrical specifications of them. The electrical properties of the machines, such as voltage, current, losses and etc., are given as the result outputs in the aforementioned chapter.
The designed machine data for slotted stator and $N_s=6$:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
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<td>[S/m]</td>
<td>Conductivity of sleeve 1</td>
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<td>$\sigma_4$</td>
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Table A-1. The designed machine data for slotted stator and $N_s=6$. 
The designed machine data for slotted stator and $N_s=12$:

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<tr>
<td>m</td>
<td>3</td>
<td>-</td>
<td>Number of phases</td>
</tr>
<tr>
<td>q</td>
<td>2</td>
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<tr>
<td>R1</td>
<td>7.7</td>
<td>[mm]</td>
<td>Shaft radius</td>
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<td>[mm]</td>
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<td>$F_p$</td>
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<td>$u_2$</td>
<td>1.15</td>
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<td>Relative permeability of PM</td>
</tr>
<tr>
<td>$u_3$</td>
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<td>Relative permeability of sleeve 1</td>
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<td>Relative permeability of sleeve 2</td>
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<td>[S/m]</td>
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Table A-2. The designed machine data for slotted stator and $N_s=12$. 

111
The designed machine data for slot-less stator and $N_s=6$:

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<td>-</td>
<td>Number of phases</td>
</tr>
<tr>
<td>$q$</td>
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<tr>
<td>$R1$</td>
<td>7.7</td>
<td>[mm]</td>
<td>Shaft radius</td>
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<tr>
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<td>7</td>
<td>[mm]</td>
<td>PM thickness</td>
</tr>
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<td>[mm]</td>
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</tr>
<tr>
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<td>4</td>
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<td>Thickness of second layer</td>
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<tr>
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<td>[mm]</td>
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<td>Stator slot pitch angle</td>
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<tr>
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<td>Number of phase winding turns</td>
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<tr>
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<td>Short pitching angle</td>
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Table A-3. The designed machine data for slot-less stator and $N_s=6$. 
The designed machine data for slot-less stator and $N_s=12$:

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<td>Active length of machine</td>
</tr>
<tr>
<td>$N_s$</td>
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<td>Number of slots</td>
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<td>$p$</td>
<td>1</td>
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<td>Number of pole pairs</td>
</tr>
<tr>
<td>$m$</td>
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<td>Number of phases</td>
</tr>
<tr>
<td>$q$</td>
<td>2</td>
<td>-</td>
<td>Number of slots per pole per phase</td>
</tr>
<tr>
<td>$R1$</td>
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<td>[mm]</td>
<td>Shaft radius</td>
</tr>
<tr>
<td>$w_{pm}$</td>
<td>7</td>
<td>[mm]</td>
<td>PM thickness</td>
</tr>
<tr>
<td>$w_{sl1}$</td>
<td>1</td>
<td>[mm]</td>
<td>Thickness of first layer</td>
</tr>
<tr>
<td>$w_{sl2}$</td>
<td>4</td>
<td>[mm]</td>
<td>Thickness of second layer</td>
</tr>
<tr>
<td>$lg$</td>
<td>1</td>
<td>[mm]</td>
<td>Air gap thickness</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>0.5236</td>
<td>[Rad]</td>
<td>Stator slot pitch angle</td>
</tr>
<tr>
<td>$so$</td>
<td>0.4817</td>
<td>[Rad]</td>
<td>Slot opening angle</td>
</tr>
<tr>
<td>$sow$</td>
<td>10</td>
<td>[mm]</td>
<td>Slot opening width</td>
</tr>
<tr>
<td>$wt$</td>
<td>1.7</td>
<td>[mm]</td>
<td>Plastic tooth width</td>
</tr>
<tr>
<td>$hw$</td>
<td>20</td>
<td>[mm]</td>
<td>Winding window height</td>
</tr>
<tr>
<td>$wbi$</td>
<td>6.5</td>
<td>[mm]</td>
<td>Yoke back iron thickness</td>
</tr>
<tr>
<td>$N$</td>
<td>44</td>
<td>-</td>
<td>Number of phase winding turns</td>
</tr>
<tr>
<td>$\phi_{pitch}$</td>
<td>0.5236</td>
<td>[Rad]</td>
<td>Short pitching angle</td>
</tr>
<tr>
<td>$F_t$</td>
<td>0.8</td>
<td>-</td>
<td>Turn packing factor</td>
</tr>
<tr>
<td>$F_p$</td>
<td>0.8</td>
<td>-</td>
<td>Serving, bundle, strand,… factor</td>
</tr>
<tr>
<td>$u_1$</td>
<td>4000</td>
<td>-</td>
<td>Relative permeability of shaft</td>
</tr>
<tr>
<td>$u_2$</td>
<td>1.15</td>
<td>-</td>
<td>Relative permeability of PM</td>
</tr>
<tr>
<td>$u_3$</td>
<td>1.1</td>
<td>-</td>
<td>Relative permeability of sleeve 1</td>
</tr>
<tr>
<td>$u_4$</td>
<td>1.1</td>
<td>-</td>
<td>Relative permeability of sleeve 2</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0</td>
<td>[S/m]</td>
<td>Conductivity of shaft</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.667e6</td>
<td>[S/m]</td>
<td>Conductivity of PM</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>0.001e6</td>
<td>[S/m]</td>
<td>Conductivity of sleeve 1</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>0.001e6</td>
<td>[S/m]</td>
<td>Conductivity of sleeve 2</td>
</tr>
<tr>
<td>$nn$</td>
<td>40</td>
<td>-</td>
<td>Number of strands in Litz wire</td>
</tr>
<tr>
<td>$dlitz$</td>
<td>3.7</td>
<td>[m]</td>
<td>Diameter of Litz wire</td>
</tr>
<tr>
<td>$dc$</td>
<td>0.469</td>
<td>[mm]</td>
<td>Diameter of each strand</td>
</tr>
<tr>
<td>$B_{rem}$</td>
<td>1.2</td>
<td>[T]</td>
<td>Remanent flux density of PM</td>
</tr>
</tbody>
</table>

Table A-4. The designed machine data for slot-less stator and $N_s=12$. 

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