Validation of an electroseismic and seismoelectric modeling code, for layered earth models, by the explicit homogeneous space solutions

Niels Grobbe* and Evert Slob, Delft University of Technology, Department of Geoscience & Engineering, the Netherlands

SUMMARY

We have developed an analytically based, energy flux-normalized numerical modeling code (ESSEMOD), capable of modeling the wave propagation of all existing ElectroSeismic and SeismoElectric source-receiver combinations in horizontally layered configurations. We compare the results of several of these modeled source-receiver combinations in a homogeneous medium with explicitly derived homogeneous space Green's function solutions, in order to be able to validate the results of ESSEMOD both in arrival times and amplitudes. Especially the amplitudes are important due to the fact that the main reason seismoelectric phenomena are not yet used in industry, are the weak amplitudes of these phenomena. Here we show that ESSEMOD correctly models the wave propagation of components of the electric field generated by different components of bulk forces, as well as the particle velocity fields generated by a bulk force source and an electric current source. We are capable of validating both amplitudes and arrival times of the results of ESSEMOD for all electroseismic and seismoelectric source-receiver combinations in homogeneous media. Herewith, we reduce uncertainty in our modeling results (also for heterogeneous scenarios) and can get better insights in which parameters affect the amplitudes most. In addition, we show that ESSEMOD is capable of modeling reciprocal source-receivers combinations correctly, implicitly indicating correct modeling of both geometrical configurations (source located above or below the receiver level). ESSEMOD can now be used for comparison with and validation of existing seismoelectric layered earth numerical modeling codes. Afterwards, ESSEMOD can be used for validation of existing seismoelectric finite-element and finite-difference codes.

INTRODUCTION

Since Pride (1994) has derived the system of equations describing the seismoelectric effect in a porous, fluid-saturated medium, quite some research has been carried out regarding this phenomenon. Pride's system of equations is a coupled system of Biot's poroelastic equations and Maxwell's electromagnetic equations. Due to the coupling between Biot's poroelastic equations and Maxwell's electromagnetic equations, the seismoelectric effect can provide us with additional information about for example the porosity and permeability of the medium and information about the pore-fluid content. In recent years, quite some numerical finite-element and finite-difference codes have been developed for modeling the seismoelectric effect in 2D (e.g. Haines and Pride (2006), Zyserman et al. (2010)). Furthermore, Haartsen and Pride (1997) and Garambois and Dietrich (2002) have used an analytically based code to model some 3D seismoelectric source-receiver

combinations in a horizontally layered, radially symmetric medium. We have recently developed an analytically based, energy flux-normalized numerical modeling code (ESSEMOD), capable of modeling the wave propagation of *all* existing ElectroSeismic and SeismoElectric source-receiver combinations in horizontally layered, radially symmetric configurations. ESSEMOD makes use of global reflection coefficients, leading to an efficient numerical scheme due to the fact that explicit computation of the scattering matrices is not required. Fourier-Bessel transformations are used to go back from the horizontal wavenumber-frequency domain to the space-frequency domain. We compare the results of several of these modeled source-receiver combinations in a homogeneous medium with explicitly derived homogeneous space Green's function solutions, in order to be able to validate the results of ESSEMOD both in arrival times and amplitudes. Especially the amplitudes are important due to the fact that the main reason seismoelectric phenomena are not yet used in industry, are the weak amplitudes of these phenomena (e.g. Dean and Dupuis (2011), Thompson et al. (2007)). By validating ESSEMOD with homogeneous space Green's function solutions, we obtain certainty in our modeling results and can get better insights in which parameters affect the amplitudes most. Furthermore, ESSEMOD can then be used to validate existing seismoelectric layered earth numerical modeling codes (e.g. Garambois and Dietrich (2002), Haartsen and Pride (1997)). When compared and validated with these codes, ESSEMOD can be used for testing finite element and finite difference codes, as well as for further investigation of all parameters and coupling effects that play a role in this complex physical phenomenon.

THEORY

Following a similar approach as Haartsen and Pride (1997), the full system of seismoelectric equations can be decoupled into its two propagation modes: the SH-TE mode (indicated below with superscript H) where the horizontally polarized shear wave is coupled to the transverse electric field, and the P-SV-TM mode (indicated below with superscript V) where Biot's fast and slow P-waves are coupled via the vertically polarized shear waves to the transverse magnetic field. We can capture the expressions for both propagation modes separately into the following matrix differential equation

$$\partial_3 \tilde{\mathbf{F}}^{H,V} - \tilde{\mathbf{A}}^{H,V} \tilde{\mathbf{F}}^{H,V} = \tilde{\mathbf{S}}^{H,V} \delta(x_3 - x_3^s), \quad (1)$$

where the tilde denotes that the expressions are given in the horizontal wavenumber-frequency domain. Here, $\tilde{\mathbf{F}}$ denotes the two-way field vector, $\tilde{\mathbf{A}}$ is the two-way operator or system matrix and $\tilde{\mathbf{S}}$ contains the two-way source quantities. The system matrix for the SH-TE system is of size 4×4 , whereas the size of the P-SV-TM system matrix is 8×8 . We follow a

Validation of an electroseismic and seismoelectric modeling code by the explicit homogeneous space solutions

different approach to that of Pride and Haartsen (1996) to obtain the homogeneous space Green's functions. Where Pride and Haartsen (1996) inverted a subset of equations to derive a few homogeneous space-solutions, we model all seismoelectric and electroseismic source-receiver combinations both in ESSEMOD and with homogeneous space Green's function solutions. We make use of energy flux-normalized composition and decomposition matrices. The details of this flux-normalization procedure go beyond the scope of this abstract. One of the benefits is that the transpose of the composition (sub)matrices can be used as the inverse for the decomposition (sub)matrices. Assuming these flux-normalized composition and decomposition matrices are known, we now transform equation (1) to the three-dimensional wavenumber domain (denoted by the breve) using $\partial_3 \iff -jk_3$, where j is the imaginary unit, from which we obtain

$$\begin{pmatrix} jk_3 \mathbf{I} & \tilde{\mathbf{A}}_{12}^{H,V} \\ \tilde{\mathbf{A}}_{21}^{H,V} & jk_3 \mathbf{I} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{F}}_1^{H,V} \\ \tilde{\mathbf{F}}_2^{H,V} \end{pmatrix} = - \begin{pmatrix} \tilde{\mathbf{S}}_1^{H,V} \\ \tilde{\mathbf{S}}_2^{H,V} \end{pmatrix}, \quad (2)$$

which can be solved as

$$\begin{pmatrix} \tilde{\mathbf{F}}_1^{H,V} \\ \tilde{\mathbf{F}}_2^{H,V} \end{pmatrix} = - \begin{pmatrix} \tilde{\mathbf{G}}_{11}^{H,V} & \tilde{\mathbf{G}}_{12}^{H,V} \\ \tilde{\mathbf{G}}_{21}^{H,V} & \tilde{\mathbf{G}}_{22}^{H,V} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{S}}_1^{H,V} \\ \tilde{\mathbf{S}}_2^{H,V} \end{pmatrix}, \quad (3)$$

with the following expressions for the Green's matrices in 3

$$\tilde{\mathbf{G}}_{11}^{H,V} = -jk_3 \tilde{\mathbf{L}}_1^{H,V} \tilde{\mathbf{G}}^{H,V} \tilde{\mathbf{L}}_2^{T,H,V} \quad (4)$$

$$\tilde{\mathbf{G}}_{12}^{H,V} = -2\tilde{\mathbf{L}}_1^{H,V} \tilde{\mathbf{F}}^{H,V} \tilde{\mathbf{G}}^{H,V} \tilde{\mathbf{L}}_1^{T,H,V} \quad (5)$$

$$\tilde{\mathbf{G}}_{21}^{H,V} = -2\tilde{\mathbf{L}}_2^{H,V} \tilde{\mathbf{F}}^{H,V} \tilde{\mathbf{G}}^{H,V} \tilde{\mathbf{L}}_2^{T,H,V} \quad (6)$$

$$\tilde{\mathbf{G}}_{22}^{H,V} = -jk_3 \tilde{\mathbf{L}}_2^{H,V} \tilde{\mathbf{G}}^{H,V} \tilde{\mathbf{L}}_1^{T,H,V}. \quad (7)$$

Here, $\tilde{\mathbf{G}}_{ab}$ denotes a Green's submatrix of the total 8x8 or 4x4 Green's matrix for the P-SV-TM and SH-TE propagation modes, respectively. In equation (4), $\tilde{\mathbf{L}}_{1,2}$ are submatrices (4x4 or 2x2) of the total 8x8 or 4x4 composition matrix, consisting of the eigenvectors of the P-SV-TM and SH-TE propagation modes, respectively. The superscript T denotes matrix transposition (for taking the inverse). The matrix $\tilde{\mathbf{F}}$ is a diagonal matrix (4x4 or 2x2) consisting of the eigenvalues of either the P-SV-TM or the SH-TE system, respectively. The $\tilde{\mathbf{G}}$ is a diagonal 4x4 or 2x2 Green's function matrix, consisting of the scalar Green's functions for the different wavetypes: fast P-wave (Pf), slow P-wave (Ps), vertically polarized shear wave (sv) and transverse magnetic field (tm) for the P-SV-TM mode and the horizontally polarized shear wave (sh) and transverse electric field (te) for the SH-TE mode, respectively. We can now select which source-receiver combination we want to model in a homogeneous space, in order to validate the results obtained from ESSEMOD. We can express all source-receiver combinations in terms of homogeneous space-solutions. When certain field quantities are present in the field vectors of both propagation modes, the SH-TE and P-SV-TM results need to be combined in the end to find the total Green's function solution for these fields. We will start with a comprehensive example and focus on a combined field E_1 due to a pure P-SV-TM source-type: $\hat{f}_3^b$ being the bulk force in the x_3 -direction (depth), where the hat denotes a space-frequency

domain quantity. For the field vector ordering used in ESSEMOD, the expression for the electric field belonging to the P-SV-TM propagation mode due to $\hat{f}_3^b$ reads

$$\tilde{E}_{norm}^{V;\hat{f}_3^b} = -2jk_3 \hat{f}_3^b \sum_{k=1}^4 \tilde{L}_{2,4k}^V \tilde{G}_{kk}^V \tilde{L}_{1,1k}^V \quad (8)$$

The following relation holds between the electric fields for the P-SV-TM mode and the SH-TE mode and the electric field component in the x_1 -direction

$$\tilde{E}_1 = -jk_1 \tilde{E}_{norm}^V / \kappa + jk_2 \tilde{E}_{norm}^H / \kappa, \quad (9)$$

where κ is the radial wavenumber, $\kappa = \sqrt{k_1^2 + k_2^2}$, and where the subscript 'norm' refers to the fact that the field quantities of both propagation modes have been normalized with a factor $-\kappa$. In the case of a pure P-SV-TM source type (like the $\hat{f}_3^b$), the SH-TE contribution to the expression 9 is zero. Using this relation, filling in the correct components of the composition matrices in 8 and transforming analytically back to the space-frequency domain, the necessary homogeneous Green's function solution for $E_1^{\hat{f}_3^b}$ can be obtained. We omit the exact expressions here for brevity. In this way, all possible seismoelectric and electroseismic source-receiver combinations can be expressed in terms of their homogeneous space solution and can be compared with the numerical homogeneous results from ESSEMOD. To further illustrate this, we will look at $v_1^{\hat{f}_1^e}$, the particle velocity in the x_1 -direction due to an electric dipole source in the x_1 -direction, and $v_1^{\hat{f}_1^b}$, the particle velocity in the x_1 -direction due to a bulk force source in the x_1 -direction. For the field vector ordering used in ESSEMOD, the expressions for the particle velocity field belonging to the P-SV-TM propagation mode due to $\hat{f}_1^e$ and $\hat{f}_1^b$, read

$$\tilde{v}_{norm}^{V;\hat{f}_1^e} = \frac{2jk_1}{\kappa} \hat{f}_1^e \sum_{k=1}^4 \tilde{L}_{2,3k}^V \tilde{F}_{kk}^V \tilde{G}_{kk}^V \tilde{L}_{2,4k}^V \quad (10)$$

$$\tilde{v}_{norm}^{V;\hat{f}_1^b} = \frac{2jk_1}{\kappa} \hat{f}_1^b \sum_{k=1}^4 \tilde{L}_{2,3k}^V \tilde{F}_{kk}^V \tilde{G}_{kk}^V \tilde{L}_{2,3k}^V, \quad (11)$$

respectively. Similarly, the expressions for the particle velocity field belonging to the SH-TE propagation mode due to $\hat{f}_1^e$ and $\hat{f}_1^b$, are

$$\tilde{v}_{norm}^{H;\hat{f}_1^e} = -\frac{2jk_2}{\kappa} \hat{f}_1^e \sum_{k=1}^2 \tilde{L}_{1,1k}^H \tilde{F}_{kk}^H \tilde{G}_{kk}^H \tilde{L}_{1,2k}^H \quad (12)$$

$$\tilde{v}_{norm}^{H;\hat{f}_1^b} = -\frac{2jk_2}{\kappa} \hat{f}_1^b \sum_{k=1}^2 \tilde{L}_{1,1k}^H \tilde{F}_{kk}^H \tilde{G}_{kk}^H \tilde{L}_{1,1k}^H, \quad (13)$$

respectively. We can combine the particle velocity fields of both propagation modes into the particle velocity field in the x_1 -direction, via

$$\tilde{v}_1 = -jk_1 \tilde{v}_{norm}^V / \kappa + jk_2 \tilde{v}_{norm}^H / \kappa. \quad (14)$$

Combining equations (10), (12) and (14) as well as (11), (13) and (14), yields again the required analytical homogeneous

Validation of an electroseismic and seismoelectric modeling code by the explicit homogeneous space solutions

Figure 1: (a) Comparison between the analytical Green's function solution (left panel) and ESSEMOD (right panel) for the E_1^{f3} source-receiver combination. The amplitudes are plotted on a logarithmic scale, in order to visualize all generated events. (b) Single trace comparison between the analytical Green's function solution (black dashed line) and ESSEMOD (red line) for E_1^{f3} .

space solutions, which can be directly modeled and compared with the numerical results in a homogeneous medium from ESSEMOD. An overview of the relevant symbols, their physical meaning and the values used for the numerical modeling experiments are presented in Table 1. Here, $\hat{\rho}^E = \eta / j\omega\hat{k}$ and $\hat{\epsilon} = \epsilon_0\epsilon_R + \frac{1}{j\omega}\hat{\sigma}^e - \hat{\rho}^E\hat{\mathcal{L}}^2$. In both numerical schemes (ESSEMOD and the analytical Green's function solutions), all seismoelectric parameters and their mutual relations are used as input, as formulated in Pride (1994).

RESULTS

We have directly modeled the homogeneous space expressions for the source-receiver combinations mentioned above (referred to as 'analytical'), in order to check the results that are generated by ESSEMOD for a homogeneous medium (referred to as 'ESSEMOD'). To simulate a homogeneous medium in ESSEMOD, all medium parameters in the different layers are chosen equal to each other, such that all reflection coefficients are zero. For the modeling geometry we have placed a source at $\mathbf{x}_S = (0, 0, 100)$ m depth, the receivers at 770 m depth and we consider a receiver grid of 51 receivers in both the x_1 - and x_2 -directions with a spacing of 10 m. The source wavelet is the first spatial derivative of a Gaussian with a peak frequency of 30 Hz and an amplitude of 1 GPa/m (for the seismic source types) and 1 GA/m⁻² (for the electrical current source). Figure 1a shows a comparison between the analytical results (left panel) and ESSEMOD (right panel) for E_1^{f3} (looking in the x_2 -direction). We have used the logarithmic scale in order to be able to see most of the generated fields. We can observe that both the timing and the amplitudes of the different wavetypes are almost identical for the analytical case and ESSEMOD. A direct flat EM event is visible at $t=0$ s, which corresponds to the part of equation (8) describing the $\hat{G}_{lm}$ Green's function expression. This is the source-converted direct EM-wave. Around $t=0.2$ s we can observe a hyperbola with an arrival time corresponding to the fast P-wave velocity, a coseismic field. The contribution in the analytical case comes from the part of equation (8) describing the $\hat{G}_{Pf}$ Green's function. Around $t=0.3$ s we observe another hyperbola with a

steeper curvature, meaning a slower propagation velocity. The arrival time of this event, another coseismic field, corresponds to the vertically polarized shear-wave velocity, described in the analytical case by the part of equation (8) dealing with $\hat{G}_{sv}$. All individual contributions of the different wavetypes for different medium parameters or fluid properties can be modeled and checked separately using the corresponding wavetype parts of the analytical Green's function solutions. Using this analysis, it can be seen that the slow P-wave does not contribute in this time window, which makes sense due to its very low propagation velocity. Figure 1b shows a single selected trace from this dataset, showing in red the result obtained from ESSEMOD and black dashed the analytical result. One can clearly see that the phase, amplitude and waveform all match perfectly. However, only one main event is visible in this trace plot, namely the one corresponding to the $\hat{G}_{Pf}$ Green's function. This due to the fact that, as visible in Figure 1a, the other two events have amplitudes that are several orders of magnitude lower than the $\hat{G}_{Pf}$ related event and are only visible due to the logarithmic amplitude scale. In a similar way, Figures 2b and 2e show the similarities for the v_1^{f3} source-receiver combination. One can observe again similar wavetypes arrivals as for the E_1^{f3} case. Now, the flat event at $t=0$ s is known as the coelectric field. Again, an almost perfect match between the analytical results and ESSEMOD can be observed in both figures. Furthermore, Figures 2a and 2d show the results for the v_1^{f3} source-receiver combination. As we now consider a seismic wave quantity due to a seismic source type, one can expect higher amplitudes than in the previous two examples, due to the fact that a complete wavetype conversion is not required to generate the desired field. These higher expected amplitudes can indeed be observed. In addition, now both the fast pressure field and the shear wavefield related arrivals can be observed in the single trace plot (Figure 2d), due to these higher amplitudes. Also, one can observe that in this case the slower, SV-wave related event has a higher amplitude than the fast pressure wave related event. Again, both figures illustrate that both the analytical expressions as well as ESSEMOD generate almost identical results. Finally, Figures 2c and 2f show the results of a reciprocal numerical modeling experiment carried out using

Validation of an electroseismic and seismoelectric modeling code by the explicit homogeneous space solutions

Figure 2: **(a)** Same as (1a) but now for v_1^{fb} . **(b)** Same as (1a) but now for v_1^{Je} . **(c)** Comparison between the homogeneous space results from ESSEMOD for v_1^{Je} (right) and the reciprocal case E_1^{fb} (left). **(d)** Same as (1b) but now for v_1^{fb} . **(e)** Same as (1b) but now for v_1^{Je} . **(f)** Single trace comparison for the reciprocity experiment: v_1^{Je} (red) and the reciprocal case E_1^{fb} (black dashed).

ESSEMOD. The right panel of Figure 2c shows again the result for the v_1^{Je} source-receiver combination, whereas the left panel shows the result obtained for its reciprocal E_1^{fb} . For reciprocity, every receiver at 770 m depth is turned into a source and now there is only one receiver at 100 m depth. Figure 2f shows a single selected trace where the red line represents the result corresponding to v_1^{Je} and the black dashed line shows the reciprocal E_1^{fb} result. Both Figures 2c and 2f display identical results for both scenarios, showing that ESSEMOD is capable of modeling the reciprocal source-receivers combinations correctly (implicitly indicating consistent modeling of both geometrical configurations).

CONCLUSIONS

We have shown that ESSEMOD correctly models the wave propagation of the electric field component in the x_1 -direction due to an f_3^b source type, as well as the particle velocity field component in the x_1 -direction due to both a J_1^e and an f_1^b source type. In addition, it has been shown that ESSEMOD is capable of modeling the reciprocal source-receivers combinations E_1^{fb} and v_1^{Je} correctly (implicitly indicating consistent modeling of both geometrical configurations). We are capable of validating both the amplitudes and arrival times of the results of ESSEMOD for all electroseismic and seismoelectric source-receiver combinations in homogeneous media. Hereby, we reduce uncertainty in the obtained numerical results. From the explicit

Table 1: Overview of the relevant medium parameters used for both modeling schemes.

Property	symbol	Value	Dimension
Porosity	ϕ	0.4	[-]
Pore fluid density	ρ_f	$1.0 \cdot 10^3$	[kg/m ³]
Bulk density	ρ_b	$2.7 \cdot 10^3$	[kg/m ³]
Shear modulus framework of grains	G_{fr}	$9.0 \cdot 10^9$	[Pa]
Pore fluid viscosity	η	$1.0 \cdot 10^{-3}$	[kg/(m s)]
Tortuosity	α_∞	3.0	[-]
Static permeability	k_0	$1.3 \cdot 10^{-12}$	[m ²]
Static electrokinetic coupling	$\mathcal{L}$	$9.07 \cdot 10^{-9}$	[m ² /(s V)]
Dynamic permeability	$\hat{k}$	ω -dependent	[m ²]
Effective fluid density	$\hat{\rho}^E$	ω -dependent	[kg m ⁻³ s ⁻¹]
Magnetic permeability of vacuum	μ_0	$4.0 \cdot 10^{-7} \pi$	[H m ⁻¹]
Velocity of light in free-space	c_0	299792458	[m s ⁻¹]
Dielectric permittivity of vacuum	ϵ_0	$8.85 \cdot 10^{-12}$	[F m ⁻¹]
Relative dielectric permittivity	ϵ_r	14.13	[-]
Bulk electric conductivity	σ^e	$1.2 \cdot 10^{-3}$	[S m ⁻¹]
Eff. electr. permittivity incl. coupling	$\hat{\epsilon}$	ω -dependent	[F m ⁻¹]

expressions we have obtained insight in the strength of the contributions of the four different possible wave types. Once all sources and receivers are validated, ESSEMOD can be used to compare numerical results obtained with other seismoelectric layered earth codes and afterwards also to validate existing seismoelectric finite-difference and finite-element codes.

ACKNOWLEDGMENTS

The research was funded as a Shell-FOM (Fundamental Research on Matter) project within the research program "Innovative physics for oil and gas". The authors are grateful to Jan Thorbecke for his help coding ESSEMOD.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

- Dean, T., and J. Dupuis, 2011, The vibroelectric method — A new tool for near-surface characterization and improved seismic data quality: Presented at the 73rd Annual International Conference and Exhibition, EAGE.
- Garambois, S., and M. Dietrich, 2002, Full waveform numerical simulations of seismoelectromagnetic wave conversions in fluid-saturated stratified porous media : *Journal of Geophysical Research*, **107**, B7, ESE5-1–ESE5-18.
- Haartsen, M., and S. Pride, 1997, Electro seismic waves from point sources in layered media : *Journal of Geophysical Research*, **102**, B11, 24745–24769.
- Haines, S., and S. Pride, 2006, Seismoelectric numerical modeling on a grid : *Geophysics*, **71**, no. 6, N57.
- Pride, S., 1994, Governing equations for the coupled electromagnetics and acoustics of porous media : *Physical Review B: Condensed Matter and Materials Physics*, **50**.
- Pride, S., and M. Haartsen, 1996, Electro seismic wave properties: *The Journal of the Acoustical Society of America*, **100**, no. 3, 1301–1315, <http://dx.doi.org/10.1121/1.416018>.
- Thompson, A., S. Hornbostel, J. Burns, T. Murray, R. Raschke, J. Wride, P. McCammon, J. Sumner, G. Haake, M. Bixby, W. Ross, B. White, M. Zhou, and P. Peczak, 2007, Field tests of electro seismic hydrocarbon detection: *Geophysics*, **72**, no. 1, N9.
- Zyserman, F., P. M. Gauzellino, and J. E. Santos, 2010, Finite element modeling of SHTE and PSVTM electro seismics: *Journal of Applied Geophysics*, **72**, no. 2, 79–91, <http://dx.doi.org/10.1016/j.jappgeo.2010.07.004>.