SKYLLA: Wave motion in and on coastal structures

Implementation and verification of flow on and in permeable structures

December 1994

TU Delft
Delft University of Technology

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<tr>
<td>a</td>
<td>coefficient in porous media flow friction term</td>
<td>(s/m)</td>
</tr>
<tr>
<td>b</td>
<td>coefficient in porous media flow friction term</td>
<td>(s²/m²)</td>
</tr>
<tr>
<td>c</td>
<td>coefficient in added mass description</td>
<td>(s²/m)</td>
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<tr>
<td>c_M</td>
<td>coefficient in added mass description</td>
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<tr>
<td>D</td>
<td>stone diameter</td>
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<td>g</td>
<td>gravitational acceleration</td>
<td>(m/s²)</td>
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<td>wave height</td>
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<td>reflection coefficient</td>
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</tr>
<tr>
<td>t</td>
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<td>(s)</td>
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<td>T</td>
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<td>u</td>
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<tr>
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<td>ν_t</td>
<td>eddy/turbulence viscosity</td>
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<tr>
<td>ρ_w</td>
<td>specific density of water</td>
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1 Introduction

1.1 Framework for the development of the model SKYLLA

The development of the numerical model SKYLLA started within the framework of the European research project "MAST-G6 Coastal Structures". The objective was to develop a physical-based numerical formulation for water motion on a smooth slope and also on-and-in permeable structures. This formulation led to the development of the numerical model SKYLLA\(^1\). Due to the inspiring results obtained in this European project, The Road and Hydraulic Engineering Division (Rijkswaterstaat) of the Dutch Ministry of Transport and Public Works was prepared to lead the continuation of the development (see contract DWW 743). This further development, outside the European MAST-project, started in 1993. The research mainly concerns the following tasks:

1) Boundary conditions.
2) Rubble mound structures and porous flow.
3) Downward slopes and overtopping.
4) Treatment of turbulence.
5) Treatment of air-entrapment.
6) Treatment of roughness.

The planning of those tasks is described in detail in Klein Breteler and Petit (1993). This report describes the research for Task 2. Activities for Task 1 are described in Petit et al. (1994).

1.2 Considerations for the development of SKYLLA

Numerous coastal structures are studied using small-scale physical models. Physical modelling can be influenced by scale effects. Due to scale effects, various phenomena can be different under prototype conditions than under conditions present in small-scale physical models. In physical models variations in the lay-out of structures are often relatively laborious compared to numerical models. These problems, as well as the complexity of measurements in breaking waves, can be overcome by numerical modelling of the breaking waves on-and-in coastal structures. So, on the one hand the development of a numerical model as a research tool is very important. On the other hand, there are disadvantages such as the simplification and discretisation of the involved physical processes.

Existing one-dimensional models use simplified formulations of, for instance, the free surface. For many applications these simplifications are undesirable. The development of a three-dimensional model, able to simulate the complete breaking of a wave, within the near future seems unrealistic. Therefore, it was decided to develop a two-dimensional (vertical) numerical model that can simulate breaking waves on various types of coastal structures, first for wave motion on smooth impermeable slopes and at a later stage for wave motion

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\(^1\) The model SKYLLA is named after the sea monster from the Greek mythology, being a mistress of Poseidon and living on a rock eating shipwrecked sailors.
on-and-in permeable structures. A proper representation of the wave impact will not yet be included.

1.3 Description of the numerical model SKYLLA

The studies performed within the European MAST-project resulted in the research tool SKYLLA, able to simulate breaking waves on impermeable smooth slopes. These studies are described in Broekens and Petit (1992) and Petit and Van den Bosch (1992) and are summarised by Van der Meer et al. (1992). A brief summary is given below.

The model uses Navier-Stokes equations in two dimensions with a constant turbulence viscosity. The technique for solving these Navier-Stokes equations in two dimensions, is based on the "Volume of Fluid method", see Nichols and Hirt (1980). The fluid is considered as incompressible. The model uses a staggered, non-equidistant grid where for each cell the fluid fraction can vary between zero (empty) and one (full) (Eulerian approach).

The model uses a complex description of the free surface based on an adapted flux-method known as "FLAIR", see Ashgriz and Poo (1991) and is capable of simulating free surfaces that can become multiple-connected whereas air-entrainment can be dealt with. Those two aspects are both essential for the simulation of plunging waves. The entrapped air is modelled as if it were vacuum.

The model includes the option to model the impermeable slopes with "no-slip" or with "free slip" boundary conditions. The choice of the grid does not depend on the lay-out of the slope.

In the previous phase of the development of SKYLLA, incident waves as prescribed by the method by Rienecker and Fenton (1981) were implemented. Both the left-hand side boundary and the right-hand side boundary are weakly reflecting, enabling reflected waves to leave the computational domain with acceptable small disturbance of the wave motion in the computational domain.

1.4 Required program modifications

Many coastal structures are permeable or contain permeable parts. Often, the wave motion inside these structures influences the external wave motion considerably. Therefore, the internal motion cannot be neglected, even in the case that one is mainly interested in the external wave motion. The field of applications of the model SKYLLA can be extended by implementing porous flow. The implementation of permeable structures and porous flow, as well as the verification of the wave motion on and inside permeable structures, will be described in this report.
1.5 Outline

The implementation of the porous flow in permeable structures and the coupling with the external wave motion are described in Chapter 2. To verify whether this implementation works properly for permeable breakwaters, physical model tests have been performed for this purpose. These tests and the results of these physical model tests are described in Chapter 3. The verification of the numerical model with the results of the physical model tests, are described in Chapter 4. The conclusions and recommendations in Chapter 5 will finish this study.
2 Implementation of permeable structures

2.1 Formulae for describing porous flow

For coarse porous media the flow resistance for stationary flow conditions can reasonably well be expressed with the Forchheimer equation. This expression contains a term linear with the flow velocity and a term quadratic with the flow velocity. The first term can be seen as the laminar contribution and the second term as the turbulent contribution. This we can write as:

\[ I = au + bu^2 \]  \hspace{1cm} (1)

where

\[ I = \text{hydraulic gradient} \]
\[ u = \text{bulk/filter velocity} \]
\[ a = \text{dimensional coefficient (s/m)} \]
\[ b = \text{dimensional coefficient (s}^2\text{/m}^2) \]

The Forchheimer equation (Equation 1) is valid for stationary flow. Polubarinova Kochina (1962) added a time-dependent term. This formula is referred to as the extended Forchheimer equation:

\[ I = au + bu|u| + c \frac{du}{dt} \]  \hspace{1cm} (2)

where \( c \) is a dimensional coefficient (s\(^2\)/m). This formula as well as the dependency of the coefficients on the porosity and the particle size can be derived from the Navier-Stokes/Reynolds equation, see Van Gent (1991).

The coefficients \( "a" \) and \( "b" \) are dimensional and contain several parameters. Many empirical and semi-empirical formulae have been derived from measurements. For a literature survey see for instance Hannoura and Barends (1981). Since the expressions for \( "a" \) and \( "b" \), first proposed by Ergun (1952), can also be derived theoretically, these expressions are mostly used. Equation 3 shows the expressions for \( "a" \) and \( "b" \) as given by Ergun and the expression for \( "c" \) as used by Wand and Gu (1991) and Van Gent (1991).

\[
\begin{align*}
a &= \alpha \left(1 - \frac{n}{n^3}\right) \frac{\nu}{g D^2} \\
b &= \beta \left(1 - \frac{n}{n^3}\right) \frac{1}{g D} \\
c &= \frac{1 + \gamma \left(1 - \frac{n}{n^3}\right)}{n g}
\end{align*}
\]  \hspace{1cm} (3)
The non-dimensional coefficients $\alpha$ and $\beta$, however, are not constant. It may well be possible that parameters such as grading, aspect ratio or shape must be implemented in the expressions. The exact appearance of these properties in the friction terms are not known since up to now no full parameter study has been performed on this subject. Based on experiments, the influence of oscillatory flow on the expressions for "$\beta$" and "c" have been implemented in semi-empirical relations, see Van Gent (1993).

For porous flow equations in two-dimensions, the Navier-Stokes/Reynolds equation can be adapted for porous flow. The porosity as well as the friction terms in the Forchheimer equation have to be implemented. See Van Gent (1991) for a derivation of this adapted Navier-Stokes/Reynolds equation for the case of a homogeneous, isotropic porous medium. These expressions (Equation 4) can be implemented in the model SKYLLA.

\[
\frac{1+c_M}{n} \frac{\partial u_f}{\partial t} + \frac{1}{n^2} \left( \frac{\partial u_f^2}{\partial x} + \frac{\partial u_f v_f}{\partial y} \right) = -\frac{1}{\rho_w} \frac{\partial p}{\partial x} - g a u_f - g b u_f \sqrt{(u_f^2 + v_f^2)} + g_x
\]

\[
\frac{1+c_M}{n} \frac{\partial v_f}{\partial t} + \frac{1}{n^2} \left( \frac{\partial u_f u_f}{\partial x} + \frac{\partial v_f^2}{\partial y} \right) = -\frac{1}{\rho_w} \frac{\partial p}{\partial y} - g a v_f - g b v_f \sqrt{(u_f^2 + v_f^2)} + g_y
\]

where

- $u_f$ = filter velocity in x-direction
- $v_f$ = filter velocity in y-direction
- $n$ = porosity
- $\rho_w$ = specific density of water
- $p$ = pressure
- $a$ = dimensional coefficient (s/m)
- $b$ = dimensional coefficient (s^2/m^2)
- $c_M$ = coefficient for "added mass" $(\gamma \cdot (1-n)/n)$
- $g_x$ = gravitational acceleration in x direction
- $g_y$ = gravitational acceleration in y direction

### 2.2 Implementation of porous flow

The computational domain contains cells, of which each can be given a certain porosity and a representative stone diameter. For the non-porous part the porosity is 1 (one) whereas the stone diameter has no physical meaning here. A permeable slope can not be modelled as a smooth slope because it is now modelled as the transition of cells with the porosity 1 (one) and a porosity smaller than 1 (one). Unlike impermeable slopes, the permeable slopes are now modelled as stairs along the borders of cells. The bottom of the computational domain is impermeable.

For the friction coefficients "$a$", "$b$" and "c", the porosity $n$, the stone diameter $D$, and the coefficients $\alpha$ and $\beta$ and $\gamma$ as shown in Equation 3, are input parameters. The influence of oscillatory flow on the coefficient $\beta$ is taken into account as proposed in Van Gent (1993). This requires the additional input parameters $T$, the wave period and the estimated maximum filter velocity.
The porous part is modelled as isotropic. Equations 4 are solved in the permeable part. These expressions have been rewritten and discretised as described in Appendix A. An analysis about the stability of the discretised porous flow equations is described in Appendix B.

2.3 Tests with permeable structures

A number of test cases have been performed to study whether the implementation of permeable structures and porous flow was done correctly. These principle tests, aimed at qualitative aspects, will be proceeded by a quantitative verification using dedicated verification tests, which will be described in the following two chapters. The principle tests are described in detail in Appendix C. Here a short discussion will be given.

A jump occurs in the pressure gradient at the interface of external and internal (porous) flow. For a uniform flow, it has been verified whether this jump occurs in the numerical model as well and whether the size of this jump is near the analytical solution of this phenomenon. A difference of 2.8% occurs, see test case A in Appendix C.

An analytical solution exists for the phreatic surface of a stationary flow through a porous medium, see test case B in Appendix C. A hydraulic head of 1.0 m is computed over a porous block with a length of 6.0 m. Differences between the numerical model and the analytical solution vary roughly between 0.010 m and 0.025 m.

The process of a layer of water entering a porous medium is studied. An analytical solution exists for this one-dimensional process. A layer of water is entering a porous medium, starting with a zero velocity. Due to gravity, the velocity of the layer of water becomes 0.75 m/s when the layer is completely inside the porous medium. This process is simulated numerically. The results are not only qualitatively correct but the difference with the analytical solution for the velocity is limited to 0.0129 m/s.

Some principle tests have been performed to verify phenomenon that cannot be handled using an analytical approach.

The entering of water into a porous medium was computed, using an irregular inflow. It was verified whether any numerical problems would occur. This was not the case.

An oscillatory motion at one side of a porous medium was generated. The porous medium can be characterised with a porosity of 0.4 and a stone diameter of 0.03 m. The width of this part was 1.0 m while the oscillatory motion had a amplitude of 0.15 m and a period of 2.0 s. The wave transmission was 11%. The wave motion was found to be rather realistic, see test case E in Appendix C.

A test was performed to show the discontinuities in the gradients of the free and phreatic surface. The motion of water through a porous block was simulated. All boundaries of the computational domain were closed. The water level at the left-hand side was initially much higher than at the right-hand side. The porous dam had a porosity of $n = 0.5$ and a stone diameter of $D = 0.2$ m. The porous section was 1.0 m wide. Figure 1 shows a graph of the computation. It shows that the free surface at the left-hand side decreases so quickly that the phreatic surface cannot follow these rapid changes (due to porous friction). Near the
nonporous-porous interface and at the surface, water flows from the porous part to the left-hand side (non-porous) part. The phreatic surface has a parabolic shape. In the second graph of Figure 1, it can be noticed that the gradient of the phreatic surface at the right-hand side is smaller than the gradient of the (external) free surface at the right-hand side. Due to the porous friction this gradient is smaller than for flow without friction.

![Figure 1 Discontinuities in slopes of the free and phreatic surface](image)

A principle test with a rubble mound breakwater was performed to verify whether the model would be able to compute a situation in the intended field of application. A rubble mound breakwater was tested with a permeable cover layer and a permeable core. The permeability of the core is lower than for the cover layer. At the left side high and steep waves were generated, leading to a breaking wave in front of a structure. The harbour side boundary was open. Some graphs of the free surface are shown in Figure 2. The figure shows that the model can deal with breaking waves, overtopping and wave transmission with a low-crested structure. No numerical problems occurred although complex shapes of the free surface and the phreatic surface were present. See also test case G in Appendix C.

The conclusion is that permeable parts and porous flow were implemented, leading to rather good results for comparisons with analytical solutions and realistic features for other test cases. Phenomena that are estimated important, for instance pressure jumps at interfaces, the entering process of water into a porous medium and discontinuities in gradients of the surfaces, were simulated with the numerical model. A quantitative verification is still necessary.
Figure 2  Computation of a breaking wave in front of a permeable, low-crested structure
3 Physical model tests

3.1 Description of physical model tests

Dedicated physical model tests were performed for the verification of the numerical model SKYLLA. The tests to study the wave motion on-and-in a Bern breakwater were carried out in the Laboratory of Fluid Mechanics at Delft University of Technology in cooperation with DELFT HYDRAULICS.

A small-scale model of a Bern breakwater was tested in a flume 42 m long, 0.80 m wide and 1.05 m high. The wave generator was compensated for reflections.

The structure was constructed in two steps. A core consisting of coarse granular material was constructed. The section of the wave flume with the core was sealed. This section was filled with water while this volume of water was weighed. This procedure provided the in-situ porosity. After construction of the cover layer, composed of larger granular material, this procedure was repeated. Using the assumption that the porosity of the core before and after the construction of the cover layer was the same, the porosity of the cover layer could be determined. Both porosities appeared to be nearly the same, namely 0.418 and 0.417. Grading curves were produced, see Figures 1 and 2 in Appendix D. The most relevant properties of the material are listed in Table 1 in Appendix D. The level of the crest was 0.95 m; the level of the horizontal berm was 0.80 m and the still water level was 0.75 m, see also Figure 3.

Pressure transducers were positioned in the seaward slope and the core of the structure. These transducers were fixed to thin steel bars attached to the bottom of the flume. Because the steel bars were rather thin, they were free to move slightly in the horizontal direction which may have occurred due to settling of the structure. Therefore, the vertical positions of these transducers were known almost exactly whereas the horizontal positions may have changed a little (in the direction towards the wave generator) but this effects has not been taken into account in the analysis. Ten transducers were positioned as indicated in Figure 3. The two transducers without index did not function. The pressure transducers have an operating pressure range of 350 mbar with a maximum deviation from linearity of 0.06%. The pressure transducers were protected with a small cap with a diameter of 0.02 m, still smaller than the surrounding stones. The signals were recorded with a sampling frequency of 50 Hz after filtering with 25 Hz.

Velocities were measured with an EMF075 simultaneously in the vertical and horizontal direction. A velocity range of 1.0 m/s was used. The maximum error band was 0.6 % in both directions. The transducers reacted on air by giving their maximum voltage (maximum velocity) within a period of 0.20 s. Velocities were measured at every 0.10 m in the x-direction, between the toe of the structure and the area where, due to wave breaking, air was enclosed. Velocities were measured at the levels of 0.35 m, 0.45 m, 0.55 m and 0.65 m above the bottom of the flume. The sampling frequency was 50 Hz.

Polystyrene particles with a density close to the density of water and a diameter of 2 mm were injected to study the velocities in the region where air is enclosed. The particles could be observed with a video camera. However, the particles could not be observed in regions...
with much entrapped air while in the region without entrapped air the LDV-equipment is supposed to provide more accurate data. Therefore, analysis of the velocities of these particles have not been performed.

Surface elevations were recorded with three wave gauges, two in front of the structure and one behind the structure. This set-up provided data on reflections, wave transmission and internal set-up. The maximum error band of these wave gauges is 0.5% of an operating range of 0.75 m. Surface elevations were also recorded by video, making 25 pictures each second. The region above the seaward slope was divided into three partially overlapping sections. In front of each section the video was positioned for a certain period. Since regular waves were studied, the observed surface elevations in each section could be connected to those from the other two sections afterwards. Run-up and run-down levels were measured visually. These were checked by using the video film.

Four series of regular waves were used to reshape the initial profile to a dynamically stable profile. After this reshaping process the actual tests were started up. Regular waves were studied with wave periods of 1.5 s and 2.1 s. For each wave period, four wave heights were tested, varying between approximately 0.10 m and 0.25 m. Wave heights were measured after obtaining a periodic wave field.
3.2 Results of the physical model tests

Four series of regular waves with increasing impact on the dynamic seaward slope were used to reshaped the initial slope. Each series consisted of approximately 1000 waves. After each series, the new seaward profile was measured but not restored. Figure 4 shows the profile development after these four series of waves (measured visually through glass side-wall). The fourth series caused an unexpected increase in crest height of nearly 0.05 m. This reshaped profile was the profile used for all other tests. After this reshaping, the actual tests for validation of the numerical model were performed.

![Berms Breakwater - Profile Development](image)

**Figure 4** Seaward slope after four series of regular waves

For eight waves, several parameters are shown in Table 1. In all tests, the still-water level was 0.75 m. Due to set-up behind the breakwater (internal set-up), an average set-down at the front occurred. The set-up behind the breakwater was measured and used for correction of the water level at the front. The run-up and run-down levels presented in Table 1 are related to this water level at the front (positive is above this average water level). The set-up levels presented are the summation of the measured set-up and the resulting set-down at the front (respectively 2/3 and 1/3 of the presented values).

The run-up levels show that these levels increase almost linearly with increased wave heights. For one wave severe overtopping occurred. The run-down levels vary around the average water level in front of the structure. The reflected and transmitted wave heights were both determined based on the maximum and minimum elevations rather than on energy. Reflection coefficients do not show a clear trend although it seems as if the reflections increase for longer wave periods. The transmission coefficients are very small and tend to increase for
longer wave periods as well. The wave with severe overtopping shows the largest transmission. The internal set-up is larger for higher waves and larger for longer periods.

<table>
<thead>
<tr>
<th>H (m)</th>
<th>T (s)</th>
<th>Run-up (m)</th>
<th>Run-down (m)</th>
<th>( K_r )</th>
<th>( K_t )</th>
<th>Set-up (m)</th>
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Table 1: Measured run-up and run-down levels, reflections, transmissions and internal set-up levels

The analysis of the video images have resulted in graphs with surface elevations in the area above the seaward slope. The surface elevations have been measured at several points of time within one wave period. They will be compared with numerical results. The pressure transducers were recorded simultaneously with the signals from the wave gauges and the steering signal from the wave generator. Because a wave gauge was positioned in the region where the video images were made, those video images could be synchronised with the other signals. Velocity signals measured at different positions at different moments were also synchronised by using the steering signal for the wave generator.

For four waves, figures were produced with surface elevations at a certain moment, while these moments were indicated in the figures with the measured signals from the pressure transducers, see Figures 5 and 6 and the Figures E1-E48 in Appendix E. For each wave the surface elevations are shown at six moments of time whereas the corresponding pressures are printed below these figures. The positions of the pressure transducers are indicated in the graphs with the surface elevations. In some graphs with measured surface elevations, two lines occur at one cross-section. This indicates the area where air is enclosed. The signals from the pressure transducers show that the pressures in the berm do follow the fluctuations of the free surface rather close. This indicates that the pressures inside the berm may be rather close to hydrostatic pressures.

For eight waves, the measured pressures are compared mutually in Figures E49-E64 (Appendix E). A limited number of measured horizontal and vertical velocities are shown in the Figures E65-E78. Upward vertical velocities are positive. Horizontal velocities in the direction towards the crest of the structure are positive. Figures 7 and 8 show respectively horizontal and vertical velocities at three positions above the berm. The figures as well as the time-scale correspond to the Figures 5 and 6.
Figure 5  Measured surface elevations at $t = 1.95$ s; $H = 0.119$ m, $T = 1.5$ s.

Figure 6  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with I corresponds to the snapshot of Figure 5.
Figure 7  Horizontal velocities as function of time; $H = 0.119$ m, $T = 1.5$ s, SWL = 0.80 m

Figure 8  Vertical velocities as function of time; $H = 0.119$ m, $T = 1.5$ s, SWL = 0.80 m.
4 Verification with physical model tests

4.1 Surface elevations

For validation of the numerical model four wave conditions were used: $H=0.119$ m, $T=1.5$ s; $H=0.230$ m, $T=1.5$ s; $H=0.112$ m, $T=2.1$ s and $H=0.217$ m, $T=2.1$ s. For each wave condition comparisons between measured and computed surface elevations above the seaward slope are made for ten points of time within a wave cycle.

The computational domain in the numerical model started at 4 m in front of the toe of the reshaped structure where the waves were generated at this weakly reflecting boundary by applying the method by Rienecker and Fenton (1981) using 16 Fourier-components. This method was adapted to deal with reflected waves as described in Petit et al. (1994). No net transport was allowed through this boundary. At the landward boundary again a weakly reflecting boundary was positioned at 1.5 m behind the crest of the structure. In x and y-direction, 270 and 80 computational cells were used respectively. The computations were performed with a constant viscosity $\nu=0.005$ m$^2$/s. In the discretisation of the equations an up-wind fraction of 0.2 was used. Surface elevations were defined at positions of cells which were filled with fluid for 50%. After an adjustment time of six to eight waves to obtain a periodic computation, data was used for comparison with the measured properties. For the porous media flow friction coefficients the values $a=1000$ and $b_c=1.1$, which were derived from physical model tests (Van Gent, 1993), were applied. The dependency of the coefficient $\beta$ on the flow field, $\beta=\beta_c(1+7.5/KC)$ where $KC=\dot{U}/nD_{50}$, has partially been taken into account by using an estimated characteristic filter-velocity, $\dot{U}$, independent of the position. This led to a constant $\beta$ for each individual computation of which the value depended on the wave condition. A constant value for the added mass coefficient $\gamma$ has been used: $\gamma=0.6 \ (c_M=\gamma(1-n)/n)$.

In Appendix F comparisons between measured and computed surface elevations are shown. For each wave condition five points of time in the first half of the wave cycle and five in the second half are shown. Only the surface elevations above the berm, where the waves are breaking, are shown since in the section in front of the structure only minor differences in wave height occur. Figure 9 shows some comparisons of measured and computed surface elevation for five points of time in the first half of the wave cycles. In the second half of the cycles such comparisons lead to disordered figures and therefore the corresponding computed surface elevations are shown in separate figures (in Appendix F).

For the wave conditions with lower waves, $H=0.119$ m, $T=1.5$ s and $H=0.112$ m, $T=2.1$ s, the comparisons show good agreement in both the first half of the wave cycles and the second half of the wave cycles. For the wave conditions with higher waves, $H=0.230$ m, $T=1.5$ s and $H=0.217$ m, $T=2.1$ s, the comparisons for the first half of the wave cycles show differences but still much smaller than those in the second half of the wave cycles. In this last part of the wave cycle considerable air-entrainment occurs during breaking of these two higher waves. In the figures with measured surface elevations the position of entrapped air is indicated by the area in between the two lines of each surface profile. The comparisons with the computed results become rather complex in this part since the exact position of the free surface is not clear. However, the comparisons indicate that the decrease...
in wave height above the berm faster occurs in the computation than observed from the measurements. It seems as if this overestimated reduction in wave height, possibly due to a too large dissipation, leads to an underestimation of the run-up levels. For all four wave conditions the computed run-up levels are too low compared with the measured run-up levels:

0.07 vs. 0.10 for $H=0.119$ m, $T=1.5$ s;
0.14 vs. 0.21 for $H=0.230$ m, $T=1.5$ s;
0.08 vs. 0.11 for $H=0.112$ m, $T=2.1$ s;
0.18 vs. 0.27 for $H=0.217$ m, $T=2.1$ s.

These run-up values are relative to the average water levels in front of the structure during testing. The choice to define the surface elevations at the positions of cells that are filled with water for 50% instead of another percentage, might influence the computed run-up levels slightly. If for this definition positions of cells that are filled with water for 10% are regarded as surface elevations, the computed run-up levels might increase but not so much that they would fit to the measured run-up levels. The dissipation in the computed breaking process, by the description of the physical processes or by numerical dissipation, is assumed to contribute to a large extend to the deviations in the (underestimated) run-up levels. Run-up levels are also underestimated for the lower waves which do not show severe breaking. Inaccuracies due to the treatment near empty cells are supposed to be relatively small for these cases. Therefore, another modelling aspect might cause inaccuracies as well. For instance, if in the numerical model fluid is more easily transported into the permeable part than in reality, run-up levels will also be underestimated. As will be shown in one of the following sections, comparisons of pore-pressures near this area with infiltration, however, do not indicate that this is the case. Although the model provides a useful representation of breaking waves, the present version is not suitable to predict accurate run-up levels.
4.2 Velocities

Comparisons between measured and computed velocities were made as well. The position closest to the crest without the difficulty of measuring the velocities, due to air-entainment and movement of the wave gauge, was at the position $x=21.6$ m, $z=0.65$ m. Comparisons for both the horizontal $(u)$ and the vertical $(w)$ velocities are shown for several positions in Appendix F. The comparisons clearly indicate that both the horizontal and the vertical velocities for the two wave conditions with the lower waves are computed accurately. Those for the higher waves show also rather good comparisons except for the position $x=21.8$ m, $z=0.55$ m for the waves characterised by $H=0.217$ m, $T=2.1$ s. Except for this condition, the comparisons of velocities at positions close to the permeable layer ($x=21.8$, $z=0.55$ m and $x=22.0$ and $z=0.45$ m) show generally good agreement. Since comparisons of surface elevations for the two higher waves show considerable differences in the area where measurements could not be performed, computed velocities in this region probably correspond also not sufficiently to those in reality.

4.3 Pressures

Also comparisons between measured and computed pressures were made. For the four wave conditions the signals from all pressure transducers are compared to the computed pressures, see Appendix F. The positions of these transducers are shown in Figure 4 and Figure 5. Transducer P4 is positioned in between the transducers P3 and P6. The recorded pressures by transducer P4, however, clearly deviate from those recorded by P3 and P6. Because no decisive physical explanation can be given for these low pressures, estimated to be roughly 50% of the expected pressures for all analysed wave conditions, the signals of transducer P4 are highly questionable. The comparisons with the other transducers are fairly accurate except for the comparisons with signals from transducer P8. Both the internal set-up as recorded by transducer P1 (average level) and the internal wave height are reproduced with a high accuracy. The computed signals at positions just below the breaking waves, transducers P2, P3 and P6, also show good correspondence with the measured signals although these measured signals show more higher-order fluctuations.
5 Conclusions and recommendations

The numerical model, solving the two-dimensional Navier-Stokes equations, for simulating normally incident waves including breaking waves has been extended with porous media flow. The implementation of the combined external and internal wave motion on and inside permeable structures was successful as shown by comparisons with both analytical solutions and physical model tests. The reduction in wave height during breaking is however too large which results in too low run-up levels. The present version of the model is therefore not suitable to predict accurate run-up levels on permeable slopes. Properties such as velocities in the area just before breaking and pore-pressures are however, reproduced with a high accuracy. The treatment of the free surface is supposed to cause the larger part of the inaccuracies. These inaccuracies are to a large extend originated from the modelling of the breaking process. Improvements of this treatment are, however, not easy to obtain. Nevertheless, the model is now capable of providing a detailed flow description of breaking waves on permeable structures. Improvements of the model in relation to a better modelling of turbulence and air-extrusion are expected to increase the accuracy of the model.
References


Appendix A

Implementation of porous flow equations in the SKYLLA code
APPENDIX A

Implementation of porous flow equations in the SKYLLA code

According to Van Gent (1991), the Navier-Stokes equations can in the case of an isotropic homogeneous porous medium be replaced by the following equations:

\[
\frac{1 + c_M}{n} \frac{\partial u_f}{\partial t} + \frac{1}{n^2} \left( \frac{\partial u_f^2}{\partial x} + \frac{\partial u_f v_f}{\partial y} \right) = -\frac{1}{\rho_w} \frac{\partial p}{\partial x} - g a u_f - g b u_f \sqrt{(u_f^2 + v_f^2)} + g_x
\]

\[
\frac{1 + c_M}{n} \frac{\partial v_f}{\partial t} + \frac{1}{n^2} \left( \frac{\partial u_f v_f}{\partial x} + \frac{\partial v_f^2}{\partial y} \right) = -\frac{1}{\rho_w} \frac{\partial p}{\partial y} - g a v_f - g b v_f \sqrt{(u_f^2 + v_f^2)} + g_y
\]

(A-1)

where

- \( u_f \) : filter velocity in x-direction.
- \( v_f \) : filter velocity in y-direction.
- \( n \) : porosity.
- \( \rho_w \) : specific density of water.
- \( p \) : pressure.
- \( a \) : dimensional coefficient (s/m).
- \( b \) : dimensional coefficient (s^2/m^2).
- \( c_M \) : coefficient for "added mass" \((\gamma \cdot \text{(1-n)}/{n})\).
- \( g_x \) : gravitational acceleration in x direction.
- \( g_y \) : gravitational acceleration in y direction.

Conservation of mass is expressed by:

\[
\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} = 0
\]

(A-2)

These expressions will now be rewritten for implementation in the SKYLLA code for the case of non-constant porosity and by redefining the pressure by \( P = p/\rho_w \).

\[
\frac{\partial u}{\partial t} + \frac{n}{1 + c_M} \frac{\partial}{\partial x} \left( \frac{u^2}{n} \right) + \frac{n}{1 + c_M} \frac{\partial}{\partial y} \left( \frac{u v}{n} \right) = -\frac{n}{1 + c_M} \frac{\partial P}{\partial x} + \frac{n}{1 + c_M} v v^2 u
\]

\[
-\frac{n}{1 + c_M} g a u - \frac{n}{1 + c_M} g b u |u| + \frac{n}{1 + c_M} g_x
\]

(A-3)
\[ \begin{align*}
\frac{\partial v}{\partial t} + \frac{n}{1+c_M} \frac{\partial}{\partial x} \left( \frac{\mu}{n} \frac{v}{n} \right) + \frac{n}{1+c_M} \frac{\partial}{\partial y} \left( v \right)^2 &= -\frac{n}{1+c_M} \frac{\partial P}{\partial y} + \frac{n}{1+c_M} v \nabla^2 v \\
- \frac{n}{1+c_M} g a v &= \frac{n}{1+c_M} g b v |u| + \frac{n}{1+c_M} g_y \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0
\end{align*} \] (A-4)

where the sub index "f" has been dropped.

The momentum equations in the x-direction are discretised in a u-velocity point on the staggered grid:

\[ u_{q+1}^{ij} - \bar{u}_{ij} = \Delta t \left( \frac{n}{1+c_M} \frac{p_{q+1}^{i+1/2,j} - p_{q+1}^{i-1/2,j}}{\Delta x_{i+1} + \Delta x_i} \right) \] (A-6)

where

\[ \bar{u}_{ij} - u_q^{ij} + \Delta t \text{DIS}_{ij}[ \frac{n}{1+c_M} \left( \frac{\partial}{\partial x} \left( \frac{\mu}{n} \right)^2 + \frac{\partial}{\partial y} \left( \frac{\mu}{n} \right) + v \nabla^2 u + g a u + g b u |u| - g_z \right) ] \] (A-7)

The discretisation operator \( \text{DIS}_{ij}[.] \) discretises at the velocity point \( u_{ij} \) and time level \( q \).

Since the viscous term should not appear in the momentum equations for the porous flow, the term \( n/(1+c_M)\rho \nabla^2 \) can be replaced by \( n^2 \rho \nabla^2 \). Higher powers than \( n^2 \) may be needed to reduce the influence of the viscous term in the porous domain. Note that outside the porous domain the effect of the viscous term is guaranteed since there \( n=1 \).

For the subroutine TILDE in the SKYLLA code the following discretisation has been used. The upper time index has been omitted:

\[ n_{i+1/2,j} = \frac{\Delta x_{i+1} n_{ij} + \Delta x_i n_{i+1,j}}{\Delta x_{i+1} + \Delta x_i} \] (A-8)

\[ n_{j+1/2} = \frac{\Delta y_{j+1} n_{uj} + \Delta y_j n_{uj+1}}{\Delta y_{j+1} + \Delta y_j} \] (A-9)

\[ r_{ij} = \frac{u_{ij}}{n_{i+1/2,j}} \] (A-10)

\[ s_{ij} = \frac{v_{ij}}{n_{j+1/2}} \] (A-11)
In the x-direction:

\[ r_i^+ = \frac{\Delta y_{j+1} r_{yj} + \Delta y_j r_{yj-1}}{\Delta y_j + \Delta y_{j+1}} \]  \hspace{1cm} (A-12)

\[ r_i^- = \frac{\Delta y_{j-1} r_{yj} + \Delta y_j r_{yj-1}}{\Delta y_j + \Delta y_{j-1}} \]  \hspace{1cm} (A-13)

\[ s_1^+ = \frac{\Delta x_i s_{ij} + \Delta x_{i+1} s_{ij}}{\Delta x_i + \Delta x_{i+1}} \]  \hspace{1cm} (A-14)

\[ s_1^- = \frac{\Delta x_i s_{ij-1} + \Delta x_{i+1} s_{ij-1}}{\Delta x_i + \Delta x_{i+1}} \]  \hspace{1cm} (A-15)

\[ VAV = \frac{\Delta x_i (v_{i+1,j} + v_{i+1,j-1}) + \Delta x_{i+1} (v_{ij} + v_{ij-1})}{2(\Delta x_i + \Delta x_{i+1})} \]  \hspace{1cm} (A-16)

\[ \tilde{u}_y - u_y + \Delta t \left( -n_{i+1/2} \frac{\Delta x_i (u_{i+1,j} - u_{i,j})}{\Delta x_i + \Delta x_{i+1}} + 2 \frac{\Delta x_i (u_{i+1,j} - u_{i,j})}{\Delta x_i + \Delta x_{i+1}} \right) \]  \hspace{1cm} (A-17)

In the y-direction:

\[ r_j^+ = \frac{\Delta y_{j+1} r_{yj} + \Delta y_j r_{yj-1}}{\Delta y_j + \Delta y_{j+1}} \]  \hspace{1cm} (A-18)

\[ r_j^- = \frac{\Delta y_{j-1} r_{yj} + \Delta y_j r_{yj-1}}{\Delta y_j + \Delta y_{j-1}} \]  \hspace{1cm} (A-19)

\[ s_2^+ = \frac{\Delta x_i s_{ij} + \Delta x_{i+1} s_{ij}}{\Delta x_i + \Delta x_{i+1}} \]  \hspace{1cm} (A-20)

\[ s_2^- = \frac{\Delta x_i s_{ij-1} + \Delta x_{i+1} s_{ij-1}}{\Delta x_i + \Delta x_{i+1}} \]  \hspace{1cm} (A-21)
\[ UAV = \frac{\Delta y_{j+1}(u_{\Delta y_{j+1}} + u_{\Delta y_{j-1}}) + \Delta y_{j}(u_{\Delta y_{j+1}} + u_{\Delta y_{j-1}})}{2(\Delta y_{j+1} + \Delta y_{j-1})} \]  
(A-22)

\[ \tilde{v}_y = v_y + \Delta t \left( \frac{-n_{y,1} \frac{1}{2}}{1 + c_M(n_{y,1} \frac{1}{2})} \left[ \frac{r_2 s_2 - r_2 s_2}{\Delta x_i} + \frac{\Delta y_j^2 (s_{y,1} - s_{y}) + \Delta y_{j+1}^2 (s_{y} - s_{y,1})}{\Delta y_{j+1}^2 (\Delta y_j + \Delta y_{j+1})} \right] \right) \]  

\[ g a(n_{y,1} \frac{1}{2}) v_y + g b(n_{y,1} \frac{1}{2}) v_y \sqrt{v_y^2 + UAV^2 - g_y} + \]  

\[ + n_{y,1}^2 \left( \frac{8 (\Delta x_i + \Delta x_{i+1})(v_{i+1} - v_y) - (\Delta x_i + \Delta x_{i+1})(v_y - v_{i-1})}{\Delta x_i + \Delta x_{i+1} + 2 \Delta x_i + \Delta x_{i+1}} \right) \]  

\[ + \frac{2(\Delta y_{j+1} - v_y) - \Delta y_{j+1}(v_y - v_{y,1})}{\Delta y_{j+1}(\Delta y_{j+1} + \Delta y_j)} \]  
(A-23)
Appendix B

Stability of discretised porous flow equations
APPENDIX B

Stability of discretised porous flow equations

To study the stability of the adapted Navier-Stokes equations in the porous medium, the following equation is used:

\[ \frac{\partial u}{\partial t} + \frac{\hat{u}}{n(1 + c_M)} \frac{\partial u}{\partial x} + \frac{\hat{v}}{n(1 + c_M)} \frac{\partial u}{\partial y} = - \left( \frac{a Ng}{1 + c_M} + \frac{b Ng}{1 + c_M} \sqrt{\hat{u}^2 + \hat{v}^2} \right) u \]  \hspace{1cm} (B-1)

This is the momentum equation in x-direction for the case where the porosity is constant. The non-linearities are removed from the equations by considering \( \hat{u} \) and \( \hat{v} \) as constants. Furthermore, the pressure gradient and the term involving the gravitational force have been removed. In the following analysis these velocities are assumed positive.

A partial upwind discretisation of equation B-1 is:

\[ u_{y}^{q+1} - u_{y}^{q} + \mu_{1} (\alpha (u_{y}^{q} - u_{i-1,j}^{q}) + \frac{1}{2} (1 - \alpha)(u_{i+1,j}^{q} - u_{i-1,j}^{q})) + \mu_{2} (\alpha (u_{y}^{q} - u_{y-1}^{q}) + \frac{1}{2} (1 - \alpha)(u_{y+1}^{q} - u_{y-1}^{q})) = - \gamma u_{y}^{q} \Delta t \]  \hspace{1cm} (B-2)

The constant \( \alpha \in [0,1] \) is the upwind fraction.

Modified CFL-numbers are introduced:

\[ \mu_{1} = \frac{\hat{u}}{n(1 + c_M)} \frac{\Delta t}{\Delta x} \]  \hspace{1cm} (B-3)

\[ \mu_{2} = \frac{\hat{v}}{n(1 + c_M)} \frac{\Delta t}{\Delta x} \]  \hspace{1cm} (B-4)

and

\[ \gamma = \frac{Ng}{1 + c_M} \left( a + b \sqrt{\hat{u}^2 + \hat{v}^2} \right) \]  \hspace{1cm} (B-5)

By replacing \( u_{y}^{q} \) by a Fourier component \( D \varphi e^{i(k_{1} \Delta x + j k_{2} \Delta y - q \omega \Delta t)} \), with \( \Omega = -1 \) and after some manipulations, equation B-2 becomes:

B-1
\[ D e^{-i\omega \Delta t} - (1 - \gamma \Delta t - \mu_1 \alpha (1 - \cos(k_1 \Delta x)) - \mu_2 \alpha (1 - \cos(k_2 \Delta y))) + \]
\[ I (\mu_1 \sin(k_1 \Delta x) + \mu_2 \sin(k_2 \Delta y)) \]

(B-6)

In the case where the upwind fraction is zero, this equation simplifies to:

\[ D e^{-i\omega \Delta t} - 1 - \gamma \Delta t - I (\mu_1 \sin(k_1 \Delta x) + \mu_2 \sin(k_2 \Delta y)) \]

(B-7)

This leads to:

\[ |D|^2 - (1 - \gamma \Delta t)^2 + (\mu_1 \sin(k_1 \Delta x) + \mu_2 \sin(k_2 \Delta y))^2 \]

(B-8)

Since the smallest wavelength that can be represented on the grid is \( \lambda_{x_{\text{min}}} = 2\Delta x \) in x-direction and \( \lambda_{y_{\text{min}}} = 2\Delta y \) in y-direction, this leads for \( k = 2\pi / \lambda \) to:

\[ 0 \leq k_1 \leq \frac{\pi}{\Delta x} \]

(B-9)

and

\[ 0 \leq k_2 \leq \frac{\pi}{\Delta y} \]

(B-10)

This means that both \( \sin(k_1 \Delta x) \) and \( \sin(k_2 \Delta y) \) in equation B-6 are in the interval \([0, 1]\). The numerical scheme is stable if \( |D| \leq 1 \). This yields, using equation B-8, the condition for stability:

\[ (\mu_1 + \mu_2)^2 - 2\gamma \Delta t + \gamma^2 \Delta t^2 \leq 0 \]

(B-11)

or

\[ \Delta t \leq \frac{2\gamma}{1/n^2(1+c_k)^2(\bar{u} + \bar{\gamma})^2 + \gamma^2} \]

(B-12)
Appendix C

Test cases for the implementation of porous flow
APPENDIX C

Test cases for the implementation of porous flow

A number of tests have been performed to study whether the implementation of porous flow in SKYLLA gives satisfactory results. Firstly, test case A concerns the jump in pressure gradient at the interface of the porous medium. In test case B, an analytical solution is compared with numerical results for stationary flow through a porous block. This has been done for a layer of water entering a permeable block in test case C. Test cases D, E and F describe principle tests with a layer of water travelling through a porous block with irregular inflow, oscillatory flow in front of a porous dam and discontinuities in slopes of the free and phreatic surfaces. Test case G describes a principle test with a rubble mound breakwater.
Test case A:  
The pressure jump at the interfaces with the porous medium

The momentum equation in x-direction for flow through porous material is given by:

\[
\frac{1+c_M}{n} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{u}{n} \right)^2 + \frac{\partial}{\partial y} \left( \frac{u v}{n n} \right) = - \frac{\partial P}{\partial x} - g (au + bu^2) + g_x
\]  
(C-1)

where \( u \) is the filter velocity in the x-direction. In the case of uniform flow in the positive x-direction, the equation becomes:

\[
\frac{1+c_M}{n} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( \frac{u}{n} \right)^2 = - \frac{\partial P}{\partial x} - g (au + bu^2 + 1)
\]  
(C-2)

Integrating equation C-2 from D-\( \delta x \) to D+\( \delta x \) gives:

\[
P(D + \delta x) - P(D - \delta x) = \left( \frac{1}{n^2} - 1 \right) u^2 - \delta x \left( g (au + bu^2 + 2) + \left( 1 + \frac{1+c_M}{n} \right) \frac{\partial u}{\partial t} \right)
\]  
(C-3)

where \( n \) indicates the porosity of the stone material. By taking the limit for \( \delta x \downarrow 0 \) of equation C-3, the pressure jump becomes:

\[
P^*_D - P^*_D = - \left( \frac{1}{n^2} - 1 \right) u^2 \quad \text{where} \quad P^*_D = \lim_{\delta x \downarrow 0} P(D + \delta x) \quad \text{and} \quad P^*_D = \lim_{\delta x \downarrow 0} P(D - \delta x)
\]  
(C-4)

The same procedure can be used at the left boundary of the porous material:

\[
P^*_B - P^*_B = - \left( \frac{1}{n^2} - 1 \right) u^2
\]  
(C-5)

Outside the stone the pressure is a differentiable function with

\[
\frac{\partial P}{\partial x} = - g
\]  
(C-6)

Inside the stone, the pressure gradient becomes:

\[
\frac{\partial P}{\partial x} = - g (au + bu^2 + 1)
\]  
(C-7)
In a numerical experiment to simulate this phenomenon with a steady flow, the following values were chosen, see also Figure 1: $a=1.0 \text{ s/m}; b=0.75 \text{ m}^2/\text{s}^2; g=2 \text{ m/s}^2; B=1 \text{ m}; D=2.0 \text{ m}; u=1.0 \text{ m/s}$ and $n=0.5$. A uniform grid-size with $\Delta x=0.05 \text{ m}$ was chosen.

A graph of the pressure is shown in Figure 2. This figure shows that the pressure jumps are simulated in steps of $3 \Delta x$. The pressure jumps differ from the value $3 \text{ m}^2/\text{s}^2$ as predicted by equation C-4 and equation C-5. By using equation C-3, $\delta u=3/2\Delta x$ and $\partial u/\partial t=0$, however, the pressure jump becomes:

$$ P(D + \frac{3}{2} \Delta x) - P(D - \frac{3}{2} \Delta x) = 2.4375 \text{ m}^2/\text{s}^2 $$

(C-8)

This value matches the value 2.5065 as calculated by SKYLLA quite well.

The pressure gradients outside and inside the stone, a few cells away from the interfaces match the results of equations C-5 and C-6 perfectly.

Figure 1  Situation sketch.  
Figure 2  Pressure jumps at the interface.
Test case B:

Stationary flow through a porous dam

For the situation sketched in Figure 3, an analytical solution exists in the case of a description of stationary porous flow with only a linear friction term and depth-averaged velocities. In this Dupuit-approximation, the convection terms in the shallow water equations have been neglected as well.

\[ h(x)u(x) - D \quad \text{(C-9)} \]

The pressure is static:

\[ \frac{\partial P}{\partial x} - g \frac{\partial h}{\partial x} \quad \text{(C-10)} \]

The momentum equation:

\[ \frac{\partial P}{\partial x} = -agu \quad \text{(C-11)} \]

By eliminating the pressure and the depth-averaged velocity, an ordinary differential equation for the free surface remains:
\[
\frac{dh}{dx} = -\frac{aD}{h} \tag{C-12}
\]

with boundary condition:

\[h(0) = H_1 \tag{C-13}\]

As is indicated in figure 3, the stone is situated in the section \(0 < x < L\). Inside this section the expression for the phreatic line becomes:

\[h(x) = \sqrt{H_1^2 - aDx} \tag{C-14}\]

By prescribing a value \(h(L) = H_2\), \(D\) can be eliminated from this equation. The resulting phreatic line is given as a function of \(x\):

\[
h(x) = \begin{cases} 
H_1 & \text{for } x < 0 \\
\sqrt{(H_1^2 - (H_1^2 - H_2^2) \cdot x/L)} & \text{for } 0 \leq x \leq L \\
H_2 & \text{for } x > L 
\end{cases} \tag{C-15}
\]

The depth-averaged filter velocity is given by:

\[u(x) = \frac{H_1^2 - H_2^2}{2aLh(x)} \tag{C-16}\]

where \(a\) is the linear contribution in the Forchheimer terms. The quadratic term should be omitted from the flow equations.

In a numerical test, the following parameters were used: \(a = 4.0 \text{ m/s}^2\); \(b = 0.0 \text{ m/s}^2\); \(g = 10.0 \text{ m/s}^2\); \(n = 0.2\); \(L = 6.0 \text{ m}\); \(\Delta x = 0.1 \text{ m}\). The left boundary of the porous block is positioned at \(x' = 2.0 \text{ m}\). The upstream water level \(H_1\) is 2.0 m whereas the downstream water level is kept at a level of \(H_2 = 1.0 \text{ m}\) (no difference with the analytical solution).

The results of the calculation is shown in figure 4. The free surface line resulting from SKYLLA is the \(F = 0.5\) contour. Just outside the porous part a slightly larger difference occurs due to some air-entrapment. The differences between the theoretical solution and the SKYLLA result in the porous part are:

- 0.0233 m at \(x' = 2.0 \text{ m}\)
- 0.0206 m at \(x' = 4.0 \text{ m}\)
- 0.0105 m at \(x' = 6.0 \text{ m}\)
- 0.0234 m at \(x' = 8.0 \text{ m}\)

Reference:

Fig. 4 Comparison of theoretical and calculated phreatic lines.
Test case C:
Layer of water through a porous medium

The test used to check the effect of the Forchheimer terms in the Navier-Stokes equations involves the flow of a layer of water inside a porous medium. The situation is indicated in figure 5.

Fig. 5 Layer of water travelling inside a porous medium.

Since the v-velocities are zero and no variation in the y-direction occurs, the continuity equation becomes:

\[ \frac{\partial u}{\partial x} = 0 \]  \hspace{1cm} (C-17)

This shows that the velocity of the layer of water only depends on time. Therefore the equation of motion of the layer becomes:

\[ \frac{du}{dt} = ng(1 - au - bu^2) \]  \hspace{1cm} (C-18)

The following expression is valid:
\[
\int \frac{du}{1 - au - bu^2} = \int n \, dg \, dt \quad \text{(C-19)}
\]

With the definitions:
\[
u_1 = \frac{-a + \sqrt{a^2 + 4b}}{2b} \quad \text{(C-20)} \quad \nu_2 = \frac{-a - \sqrt{a^2 + 4b}}{2b} \quad \text{(C-21)}
\]
the integrand in the left hand integral of equation C-19 can be written as:
\[
\frac{1}{1 - au - bu^2} = \frac{-1}{b(u_2 - u_1)} \left( \frac{1}{u - u_2} - \frac{1}{u - u_1} \right) \quad \text{(C-22)}
\]
Integration of equation C-18 now yields after some algebra an expression for \( u(t) \):
\[
u_1 - u_2 - \frac{u(t_0) - u_1}{u(t_0) - u_2} e^{-bung(u_1 - u_2)(t-t_0)} \quad \text{(C-23)}
\]
Here, \( u(t_0) \) is the initial velocity at \( t = t_0 \). Note that \( u_1 - u_2 \geq 0 \) which implies that for \( t \to \infty \), \( u \to u_1 \):
\[
u(\infty) = \frac{-a + \sqrt{a^2 + 4b}}{2b} \quad \text{(C-24)}
\]
For the limits where \( a \) or \( b \) vanish is valid:
\[
\lim_{b \to 0} u_1 = \frac{1}{a} \quad \text{(C-25)} \quad \lim_{a \to 0} u_1 = \frac{1}{\sqrt{b}} \quad \text{(C-26)}
\]
In the test described above, the layer of water had already entered the porous material. In the following test, the layer of water has entered the porous medium only partly. Figure 6 shows a sketch of the situation. Here \( x_0 \) indicates the position of the free surface which is outside the stone. Since the velocity remains non-negative in this experiment, the equation of motion becomes:
\[
\frac{\partial}{\partial t} \left( \frac{u}{n} \right) = \frac{\partial}{\partial x} \left( \frac{u}{n} \right)^2 + P - g(au + bu^2 - 1) \quad \text{(C-27)}
\]
C-8
Fig. 6 Layer of water entering the porous medium.

Outside the stone, \( n = 1 \), \( a = 0 \), \( b = 0 \) and inside the stone the porosity has a constant value \( 0 < n < 1 \) and \( a > 0 \) and \( b > 0 \). The continuity equation is:

\[
\frac{\partial u}{\partial x} = 0 \tag{C-28}
\]

Differentiation of equation C-27 shows that on either side of the interface the pressure is a linear function of \( x \):

\[
\frac{\partial^2 P}{\partial x^2} = 0 \tag{C-29}
\]

At the interface, however, the pressure exhibits a discontinuity of which the value can be found by integrating equation C-27 from \(-\delta x\) to \(\delta x\):

\[
P(\delta x) - P(-\delta x) = -\left(\frac{1}{n^2} - 1\right)u^2 - \delta x \left[ g(ua + bu^2 - 2) + \left(\frac{1}{n} + 1\right)\frac{\partial u}{\partial t}\right] \tag{C-30}
\]

Letting \(\delta x\) go to 0 yields:

\[
P^+ - P^- = -\left(\frac{1}{n^2} - 1\right)u^2 \tag{C-31}
\]

where \( P^+ \) is the pressure just inside the stone and \( P^- \) is the pressure just outside the stone.
By introducing the function $P_0(t) = P$ and using the fact that at the free surfaces $P = 0$, the pressure gradients on either side of the stone can be expressed as a function of $P_0(t)$ and $x_0(t)$:

$$
\frac{\partial P}{\partial x} = \frac{-P_0(t)}{x_0(t)} \quad x_0(t) < x < 0 \quad (C-32)
$$

$$
\frac{\partial P}{\partial x} = \frac{-P_0(t) + \left(\frac{1}{n^2} - 1\right)u^2(t)}{\left(\frac{L + x_0(t)}{n}\right)} \quad 0 < x < \frac{L + x_0(t)}{n} \quad (C-33)
$$

The momentum equation outside the stone now becomes:

$$
\frac{du}{dt} = \frac{P_0}{x_0} + g \quad (C-34)
$$

Inside the stone it becomes:

$$
\frac{1}{n} \frac{du}{dt} = \frac{P_0 - \left(\frac{1}{n^2} - 1\right)u^2}{L + x_0} - g(au + bu^2 - 1) \quad (C-35)
$$

These equations can now be used to eliminate the function $P_0(t)$. After some algebra equation C-36 was found:

$$
\frac{du}{dt} = \frac{\frac{g}{n}(L + x_0)(au + bu^2 - 1) - \left(\frac{1}{n^2} - 1\right)u^2 - gx_0}{\frac{L + x_0}{n^2} - x_0} \quad (C-36)
$$

Since outside the porous material the free surface travels with the filter velocity, the following relation can be used:

$$
\frac{dx_0}{dt} = u \quad (C-37)
$$
Equation C-36 and C-37 constitute the set of ordinary differential equations that has to be solved provided \(-L < x_0 < 0\). After the layer has entered the stone \((x_0 > 0)\), equations C-18 and C-37 can be solved.

The set of differential equations was solved using a fourth order Runge-Kutta method where for the time-step \(\Delta t = 0.005\) s was used as well as the following values: \(L = 0.5\) m; \(n = 0.5\); \(g = 4\) m/s\(^2\) (instead of 9.81 m/s\(^2\) to exaggerate the local maximum); \(a = 0.0\) s/m; \(b = 16/9\) s\(^2\)/m\(^2\). In the numerical simulation \(\Delta x = 0.05\) m was used.

At \(t = 0\), the layer of water is just outside the porous material and has a velocity of 0 m/s. The calculation shows that at \(t = 0.435\) s the velocity has a local maximum of \(u = 0.6848\) m/s and the position of the free surface is \(x_0 = -0.2788\) m. At \(t = 0.850\) s, the layer has entered the stone. At that moment the velocity is 0.6447 m/s. After that, the velocity rapidly converges to the stationary value of 0.75 m/s. Figure 7 shows the velocity as function of time as found by solving C-18, C-36 and C-37 (the theoretical solution) together with the velocity as found by SKYLLA.

![Graph showing velocity over time](image)

**Fig.7** Velocity of the entering layer of water.

At \(t = 0.4\) s, the difference between both solutions is 0.0129 m/s.
Test case D:
Layer of water through a porous medium (irregular inflow)

In the previous test (C), it has been verified whether the entering of a layer of water into a porous medium is simulated correctly. Now, a principle test is described where an irregular wave motion is generated above a porous layer. It is verified whether the entering of the porous medium causes severe numerical problems.

A horizontal porous layer with $n=0.2$ and $D=0.03$ m is situated in the middle of the computational domain. An irregular injection of water takes place in the section above this porous layer. The left and right side are closed. Gravitation in the vertical direction causes the water to move through the porous layer and eventually to leave the porous layer into a non-porous section beneath the porous layer. Figure 8 shows six moments of time of the computation. The incoming jet of water varies in time as well as in direction. In the first, third and fifth snapshot the jet, generated in the upper-left corner, increases in strength. The computation was performed without any numerical problems. The figures show that the entering into the porous medium starts at the position where the jet of water hits the porous layer. Later on, the outflow starts just beneath the position where the initial inflow started. Due to the modelling of air as vacuum, air bubbles are also transported downward. The outflow occurs in vertical spouts which seems rather realistic.

Figure 8  Layer of water through a porous medium (irregular inflow).
Test case E:
Oscillatory motion in front of a porous dam

In test case B, an analytical solution has been compared with the numerical results for a stationary flow through a porous dam. A similar computation is now described, although now an oscillatory wave motion is generated at the left hand-side. The right hand-side is closed. It can now be verified whether the model simulates an extra set-up at the right-hand side.

The porous dam has a porosity of \(n=0.4\) and a stone diameter of \(D=0.03\) m. The porous section is 1.0 m wide. The wave period was 2.0 s and the vertical amplitude was 0.15 m. The wave transmission was 11%. Figure 9 shows six moments of time within one wave period. The figures show that within the period that the water moves to the left in the left-hand side section, air entrapment takes place. A leftward motion in the left-hand side section occurs; due to porous friction, the water in the porous medium cannot easily supply the need of water. This causes a steep slope in the free surface near the porous medium which leads to a breaking wave.

Figure 9  Oscillatory motion in front of a porous dam.
Test case F:
Discontinuities in slopes of the free and phreatic surfaces

A test was performed to show the discontinuities in the gradients of the free and phreatic surface. The motion of water through a porous block has been simulated. All boundaries of the computational domain are closed. The water level at the left-hand side is initially much higher than at the right-hand side. The porous dam has a porosity of $n=0.5$ and a stone diameter of $D=0.2\,\text{m}$. The porous section is $1.0\,\text{m}$ wide. Figure 10 shows six graphs of the computation. The first two figures show that the free surface at the left-hand side decreases so quickly that the phreatic surface can not follow these rapid changes (due to porous friction). Near the nonporous-porous interface and the surface, water flows from the porous part to the left-hand side (non-porous) part. The phreatic surface has a parabolic shape. In the third figure, it can be noticed that the gradient of the phreatic surface at the right-hand side is smaller than the gradient of the (external) free surface at the right-hand side. Due to the porous friction this gradient is smaller than for flow without friction. The process seems to be simulated rather realistic.
Figure 10 Discontinuities in slopes of the free and phreatic surfaces.
Test case G:
Rubble mound breakwater

A rubble mound breakwater was tested with a permeable cover layer and a permeable core. At the left side waves with $H=0.45 \text{ m}$ and $T=2.0 \text{ s}$ were generated on a water depth of 0.80 m. The structure is permeable with stone of 0.035 m and a porosity of $n=0.4$. The permeability of the core material was lower than the cover material. The seaward slope was 1:2, the rear side slope was 1:1. The crest of the structure was at a level of 0.90 m and the width of the crest was 1.25 m. The top of the core was at 0.75 m. For the coefficients $\alpha$ and $\beta$ from the expressions for the Forchheimer equation, respectively 1000 and 1.1 were used. The harbour side boundary was open. The snapshots in Figure 11 show calculated surface elevations of a breaking wave in front of this rubble mound structure. The figures show that

- the model can deal with this low-crested structure,
- the model can compute overtopping over this structure,
- the model can compute wave transmission over a low-crested structure,
- the model can compute porous flow,
- the model can deal with a multiple connected phreatic surface (fifth snapshot),
- the model can deal with air entrapment in permeable parts,
- the model can deal with temporarily submerged structures.
Figure 11  Computation of a breaking wave in front of a permeable, low-crested structure.
Appendix D

Description of test material
APPENDIX D

Description of test material

After construction of the core of the Berm breakwater, this part of the wave flume was closed. This section was filled with water. The volume of water was weighed from which the porosity could be derived. After construction of the cover layer, this procedure was repeated. Using the assumption that the porosity of the core before and after the construction of the cover layer are the same, the porosity of the cover layer can be determined. Both porosities appeared to be nearly the same. Grading curves were produced, see Figures 1 and 2. The most relevant properties of the material are listed in Table 1.

<table>
<thead>
<tr>
<th>DESCRIPTION OF TEST MATERIAL</th>
</tr>
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<tbody>
<tr>
<td>MATERIAL</td>
</tr>
<tr>
<td>CORE</td>
</tr>
<tr>
<td>COVER</td>
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</tbody>
</table>

Table 1  Properties of test material.
Figure 1  Grading curve for the core material.

Figure 2  Grading curve for the material in the cover layer.

Dn85 = 0.0195 m
Dn50 = 0.0175 m
Dn15 = 0.0153 m
Dn85/Dn15 = 1.27

Dn85 = 0.0294 m
Dn50 = 0.0266 m
Dn15 = 0.0239 m
Dn85/Dn15 = 1.23
Appendix E

Results of physical model tests
APPENDIX E

Results physical model tests

For four waves, the surface elevations were measured at several points of time within one wave period. For each wave the surface elevations are shown at six moments of time. The corresponding pressures are printed below these figures. The moments of time are indicated with vertical thick lines. The position of the pressure transducers are indicated in the graphs with the surface elevations. However, the positions of pressure transducers P7 and P8 are abusively switched. In some graphs with measured surface elevations, two lines occur at one cross-section. This indicates the area where air was enclosed.

The measured pressures are shown in Figures E49-E64 for eight waves. A limited number of measured horizontal and vertical velocities are shown in the Figures E65-E80. Upward vertical velocities are positive. Horizontal velocities in the direction towards the crest of the structure are positive.

List of figures in Appendix E:

Fig.E1-E12: \( H=0.119 \text{ m} \ T=1.5 \text{ s} \) (surface elevations and pressures).
Fig.E13-E24: \( H=0.230 \text{ m} \ T=1.5 \text{ s} \) (surface elevations and pressures).
Fig.E25-E36: \( H=0.112 \text{ m} \ T=2.1 \text{ s} \) (surface elevations and pressures).
Fig.E37-E48: \( H=0.217 \text{ m} \ T=2.1 \text{ s} \) (surface elevations and pressures).

Fig.E49-E50: \( H=0.119 \text{ m} \ T=1.5 \text{ s} \) (pressures).
Fig.E51-E52: \( H=0.162 \text{ m} \ T=1.5 \text{ s} \) (pressures).
Fig.E53-E54: \( H=0.230 \text{ m} \ T=1.5 \text{ s} \) (pressures).
Fig.E55-E56: \( H=0.284 \text{ m} \ T=1.5 \text{ s} \) (pressures).
Fig.E57-E58: \( H=0.112 \text{ m} \ T=2.1 \text{ s} \) (pressures).
Fig.E59-E60: \( H=0.166 \text{ m} \ T=2.1 \text{ s} \) (pressures).
Fig.E61-E62: \( H=0.217 \text{ m} \ T=2.1 \text{ s} \) (pressures).
Fig.E63-E64: \( H=0.261 \text{ m} \ T=2.1 \text{ s} \) (pressures).

Fig.E65-E68: \( H=0.119 \text{ m} \ T=1.5 \text{ s} \) (velocities).
Fig.E69-E72: \( H=0.230 \text{ m} \ T=1.5 \text{ s} \) (velocities).
Fig.E73-E76: \( H=0.112 \text{ m} \ T=2.1 \text{ s} \) (velocities).
Fig.E77-E80: \( H=0.217 \text{ m} \ T=2.1 \text{ s} \) (velocities).
Figure E1  Measured surface elevations at $t = 0.4 \, T$; $H=0.119 \, m$, $T=1.5 \, s$.

Figure E2  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with [ ], corresponds to the snapshot of Figure E1.
Figure E3  Measured surface elevations at $t = 0.5 \, T$; $H=0.119 \, m$, $T=1.5 \, s$.

Figure E4  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\square$, corresponds to the snapshot of Figure E3.
Figure E5  Measured surface elevations at $t = 0.6 \text{T}$; $H=0.119 \text{ m}$, $T=1.5 \text{ s}$.

Figure E6  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with |, corresponds to the snapshot of Figure E5.
Figure E7 Measured surface elevations at $t = 0.7 \, T$; $H=0.119 \, m$, $T=1.5 \, s$.

Figure E8 Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with ■, corresponds to the snapshot of Figure E7.
Figure E9  Measured surface elevations at $t = 0.8$ T; $H=0.119$ m, $T=1.5$ s.

Figure E10  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\blacksquare$, corresponds to the snapshot of Figure E9.
Figure E11  Measured surface elevations at $t = 0.8 \; T$; $H=0.119 \; m$, $T=1.5 \; s$.

Figure E12  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\downarrow$, corresponds to the snapshot of Figure E11.
Figure E13  Measured surface elevations at $t = 0.4 \, \text{T}$; $H=0.230 \, \text{m}$, $T=1.5 \, \text{s}$.

Figure E14  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\bullet$, corresponds to the snapshot of Figure E13.
Figure E15  Measured surface elevations at $t = 0.5 \, T$; $H=0.230 \, m$, $T=1.5 \, s$.

Figure E16  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\blacksquare$, corresponds to the snapshot of Figure E15.
Figure E17  Measured surface elevations at t = 0.6 T; H=0.230 m, T=1.5 s.

Figure E18  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with ● corresponds to the snapshot of Figure E17.
Figure E19  Measured surface elevations at $t = 0.7 \, \text{T}$; $H = 0.230 \, \text{m}$, $T = 1.5 \, \text{s}$.

Figure E20  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\Box$, corresponds to the snapshot of Figure E19.
Figure E21  Measured surface elevations at t = 0.8 T; H=0.230 m, T=1.5 s.

Figure E22  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with I, corresponds to the snapshot of Figure E21.
Figure E23  Measured surface elevations at $t = 1.0 \; T$; $H=0.230 \; m$, $T=1.5 \; s$.

Figure E24  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\Box$, corresponds to the snapshot of Figure E23.
Figure E25  Measured surface elevations at $t = 0.4 \, T$; $H=0.112 \, m$, $T=2.1 \, s$.

Figure E26  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\bullet$, corresponds to the snapshot of Figure E25.
Figure E27 Measured surface elevations at \( t = 0.5 \, T \); \( H=0.112 \, m \), \( T=2.1 \, s \).

Figure E28 Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with \( \Box \), corresponds to the snapshot of Figure E27.
Figure E29  Measured surface elevations at $t = 0.6 \, T$; $H=0.112 \, m$, $T=2.1 \, s$.

Figure E30  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\MBox{1}$, corresponds to the snapshot of Figure E29.
Figure E31  Measured surface elevations at $t = 0.7 T$; $H=0.112\ m$, $T=2.1\ s$.

Figure E32  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\bullet$, corresponds to the snapshot of Figure E31.
Figure E33  Measured surface elevations at $t = 0.8 \, T$; $H = 0.112 \, m$, $T = 2.1 \, s$.

Figure E34  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\blacksquare$, corresponds to the snapshot of Figure E33.
Figure E35  Measured surface elevations at t = 1.0 T; H=0.112 m, T=2.1 s.

Figure E36  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with 1 corresponds to the snapshot of Figure E35.
Figure E37 Measured surface elevations at \( t = 0.4 \ T \); \( H = 0.217 \ m \), \( T = 2.1 \ s \).

Figure E38 Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with  , corresponds to the snapshot of Figure E37.
Figure E39  Measured surface elevations at $t = 0.5 \, T$; $H=0.217 \, m$, $T=2.1 \, s$.

Figure E40  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\uparrow$, corresponds to the snapshot of Figure E39.
Figure E41  Measured surface elevations at \( t = 0.6 \, \text{T}; \, H = 0.217 \, \text{m}, \, T = 2.1 \, \text{s} \).

Figure E42  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with \( \boxed{\text{I}} \), corresponds to the snapshot of Figure E41.
Figure E43  Measured surface elevations at $t = 0.7 \, T$; $H=0.217 \, m$, $T=2.1 \, s$.

Figure E44  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\square$, corresponds to the snapshot of Figure E43.
Figure E45  Measured surface elevations at t = 0.8 T; H=0.217 m, T=2.1 s.

Figure E46  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with [ ], corresponds to the snapshot of Figure E45.
Figure E47  Measured surface elevations at $t = 1.0 \ T$; $H=0.217 \ m$, $T=2.1 \ s$.

Figure E48  Pressures (Pa) from the eight transducers as function of time (s); The point of time indicated with $\blacksquare$, corresponds to the snapshot of Figure E47.
Figure E49  Pore pressures as function of time; H=0.119 m, T=1.5 s.

Figure E50  Pore pressures as function of time; H=0.119 m, T=1.5 s.
Figure E51  Pore pressures as function of time; H=0.162 m, T=1.5 s.

Figure E52  Pore pressures as function of time; H=0.162 m, T=1.5 s.
Figure E53  Pore pressures as function of time; $H=0.230$ m, $T=1.5$ s.

Figure E54  Pore pressures as function of time; $H=0.230$ m, $T=1.5$ s.
Figure E55  Pore pressures as function of time; $H=0.284$ m, $T=1.5$ s.

Figure E56  Pore pressures as function of time; $H=0.284$ m, $T=1.5$ s.
Figure E57  Pore pressures as function of time; H=0.112 m, T=2.1 s.

Figure E58  Pore pressures as function of time; H=0.112 m, T=2.1 s.
Figure E59 Pore pressures as function of time; H=0.166 m, T=2.1 s.

Figure E60 Pore pressures as function of time; H=0.166 m, T=2.1 s.
Figure E61  Pore pressures as function of time; H=0.217 m, T=2.1 s.

Figure E62  Pore pressures as function of time; H=0.217 m, T=2.1 s.
Figure E63  Pore pressures as function of time; $H=0.261$ m, $T=2.1$ s.

Figure E64  Pore pressures as function of time; $H=0.261$ m, $T=2.1$ s.
Figure E65  Horizontal velocities as function of time; H=0.119 m, T=1.5 s.

Figure E66  Vertical velocities as function of time; H=0.119 m, T=1.5 s.
Figure E67  Horizontal velocities as function of time; $H=0.119$ m, $T=1.5$ s.

Figure E68  Vertical velocities as function of time; $H=0.119$ m, $T=1.5$ s.
Figure E69  Horizontal velocities as function of time; $H=0.230$ m, $T=1.5$ s.

Figure E70  Vertical velocities as function of time; $H=0.230$ m, $T=1.5$ s.
Figure E71  Horizontal velocities as function of time; H=0.230 m, T=1.5 s.

Figure E72  Vertical velocities as function of time; H=0.230 m, T=1.5 s.
Figure E73  Horizontal velocities as function of time; H=0.112 m, T=2.1 s.

Figure E74  Vertical velocities as function of time; H=0.112 m, T=2.1 s.
Figure E75  Horizontal velocities as function of time; $H=0.112$ m, $T=2.1$ s.

Figure E76  Vertical velocities as function of time; $H=0.112$ m, $T=2.1$ s.
Figure E77  Horizontal velocities as function of time; H=0.217 m, T=2.1 s.

Figure E78  Vertical velocities as function of time; H=0.217 m, T=2.1 s.
Figure E79  Horizontal velocities as function of time; H=0.217 m, T=2.1 s.

Figure E80  Vertical velocities as function of time; H=0.217 m, T=2.1 s.
Appendix F

Comparison between physical and numerical results
APPENDIX F

Comparison between measured and computed results

Input data for the computations for verification:

Structure:
- Permeable
- Small-scale Berm breakwater with core
  - Crest of structure : 1.00 m
  - Porosity cover layer : 0.418
  - Stone diameter cover layer : 0.0266 m
  - Porosity core : 0.418
  - Stone diameter core : 0.0175 m
  - Forchheimer friction coefficient $\alpha$ : 1000

Waves:
- Four series of regular waves
  - Wave height : 0.119-0.230-0.112-0.217 m
  - Wave period : 1.500-1.500-2.100-2.100 s
  - Still water level seaward : 0.750 m
  - Still water level harbourside : 0.750 m
  - Forchheimer friction coefficient $\beta$ : flow-dependent: 1.700-1.400-1.550-1.300

Boundaries:
- Two open, weakly reflecting, boundaries
- Waves generated with RF-Wave:
  - Number of Fourier components : 16
  - Net mass transport with $c_M$ : 0.0 m/s
  - $\rho_w$ : 1000 kg/m$^3$

Grid:
- Non-equidistant
  - No. of cells in horizontal direction : 270
  - No. of cells in vertical direction : 80
  - Length of computational domain : 10.0 m
  - Height of computational domain : 2.5 m
  - Horizontal section in front of structure : 6.0 m

Numerical computation:
- Maximum time-step : 0.01 s
- Length of computation : 15.0 s
- Upwind fraction : 0.2
- Eddy viscosity : 0.005 m$^2$/s
Fig. F0  Positions at which measured and computed properties have been compared; P denotes pressures, V denotes velocities.

Positions of pressure transducers:

P1 : x=20.10 m  y=0.65 m
P2 : x=20.35 m  y=0.65 m
P3 : x=20.60 m  y=0.60 m
P4 : x=20.85 m  y=0.55 m
P5 : x=21.10 m  y=0.30 m
P6 : x=21.10 m  y=0.50 m
P7 : x=21.35 m  y=0.35 m
P8 : x=21.35 m  y=0.15 m

Positions for verification of velocities:

V1 : x=21.30 m  y=0.65 m
V2 : x=21.60 m  y=0.65 m
V3 : x=21.80 m  y=0.55 m
V4 : x=21.80 m  y=0.65 m
V5 : x=22.00 m  y=0.45 m
V6 : x=22.00 m  y=0.55 m
V7 : x=22.00 m  y=0.65 m
Fig. F1  Measured (upper) and computed (lower) surface elevations for the first half of the wave cycle; $H=0.119$ m, $T=1.5$ s.
Fig. F2 Measured (upper) and computed (lower) surface elevations for the second half of the wave cycle; $H=0.119 \text{ m}, T=1.5 \text{ s}$.
Fig. F3  Measured (upper) and computed (lower) surface elevations for the first half of the wave cycle; $H=0.230 \text{ m}, T=1.5 \text{ s}$. 

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Fig. F4  Measured (upper) and computed (lower) surface elevations for the second half of the wave cycle; $H=0.230$ m, $T=1.5$ s.

F-6
Fig. F5  Measured (upper) and computed (lower) surface elevations for the first half of the wave cycle; $H=0.112 \, \text{m}, \, T=2.1 \, \text{s}$. 

F-7
Fig. F6  Measured (upper) and computed (lower) surface elevations for the second half of the wave cycle; $H=0.112$ m, $T=2.1$ s.
Fig. F7  Measured (upper) and computed (lower) surface elevations for the first half of the wave cycle; $H=0.217$ m, $T=2.1$ s.
Fig.F8  Measured (upper) and computed (lower) surface elevations for the second half of the wave cycle; $H=0.217$ m, $T=2.1$ s.
Fig. P9-a  Measured (dashed) and computed (lines) velocities; 
u denotes horizontal velocities and w denotes vertical velocities; 
H=0.119 m; T=1.5 s.
Fig. F9-b  Measured (dashed) and computed (lines) velocities; 
$u$ denotes horizontal velocities and $w$ denotes vertical velocities; 
$H=0.230$ m; $T=1.5$ s.
Fig. F9-c  Measured (dashed) and computed (lines) velocities; 
u denotes horizontal velocities and w denotes vertical velocities; 
H=0.112 m; T=2.1 s.
Fig.F9-d  Measured (dashed) and computed (lines) velocities;
u denotes horizontal velocities and w denotes vertical velocities;
H=0.217 m; T=2.1 s.
Fig. F10  Measured (dashed) and computed (lines) pressures; 
$H=0.119 \text{ m, } T=1.5 \text{ s.}$
Fig.F11  Measured (dashed) and computed (lines) pressures; 
$H=0.230 \, m$, $T=1.5 \, s$.  

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Fig. F12  Measured (dashed) and computed (lines) pressures; 
$H=0.112 \text{ m, } T=2.1 \text{ s.}$
Fig. F13  Measured (dashed) and computed (lines) pressures; 
\( H = 0.217 \text{ m}, T = 2.1 \text{ s} \).
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