North Sea temperature modelling with SST forcing

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TITLE: North Sea temperature modelling with SST forcing

ABSTRACT:

For modelling the transport of heat through the free water surface two different approaches exist, viz. a heat flux approach (Neumann boundary condition) and a Sea Surface Temperature (SST) forcing (Dirichlet type boundary condition). In the latter case the model temperature at the top layer is prescribed. Both approaches are applied in 3D shallow water modelling.

In this project have investigated how a heat flux model can be combined with a SST forcing. We have followed an approach known from world ocean modelling, viz. the approach developed in (Barnier et al., 1994). This approach is based on a heat flux forcing in which a correction term is added that makes use of available SST data. The NOAA/AVHRR SST images with weekly composites have been supplied by KNMI.

The simulations, which have been carried out with a special version of TRIWAQ, have shown that it is possible to decrease the 'gap' caused by inconsistencies between remote sensing SST data and meteorological input. By introducing a correction term for the heat flux or by only applying the correction term and neglecting the 'normal' heat flux computed by the meteorological forcing, the differences between the measured climatological sea surface temperature (SST) and the model temperature at the top layer decrease. For a 3D North Sea model the space and time averaged difference between the remote sensing SST and the model SST reduces from about 1.5°C to about 0.7°C.

This project is part of the REST3D project, which aims at the integration of temperature measurements (both sea surface temperature data and in-situ data) and temperature modelling results.

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CONTENTS: TEXT PAGES

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>TABLE</th>
<th>FIGURES</th>
<th>APPENDICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>1</td>
<td>17</td>
<td>1</td>
</tr>
</tbody>
</table>

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Contents

1 Introduction ........................................................................................................................................ 1–1

1.1 Acknowledgement .................................................................................................................. 1–2

2 Temperature boundary forcing ........................................................................................................... 2–1

2.1 Thermal forcing based on world ocean modelling .................................................................. 2–2

2.2 Corrected SST forcing ............................................................................................................... 2–5

2.3 Comparing both approaches ...................................................................................................... 2–6

3 Numerical modelling with simple test basin ................................................................................... 3–1

3.1 Model parameters .................................................................................................................... 3–1

3.2 Day-night variation of remote sensing SST ............................................................................. 3–2

3.3 Model results ........................................................................................................................... 3–2

4 3D North Sea model ......................................................................................................................... 4–1

4.1 Model parameters .................................................................................................................... 4–1

4.2 Model results ........................................................................................................................... 4–2

4.2.1 Sea surface temperatures ................................................................................................ 4–3

4.2.2 Vertical profiles .................................................................................................................. 4–3

5 Conclusions .................................................................................................................................. 5–1

References ......................................................................................................................................... Ref-1

A The heat exchange processes at the sea surface .............................................................................. A-1
List of Figures

Meteorological input for 1994:
3.1 Meteorological data, wind and air pressure
3.2 Meteorological data, air temperature and relative humidity
3.3 Meteorological data, cloudiness

Results for simplified model (8 by 8 in horizontal) for 1994:
3.4 Thermal stratification at INP Mooring station; original heat flux forcing
3.5 Thermal stratification at INP Mooring station; corrected heat flux forcing
3.6 Thermal stratification at INP Mooring station; standard SST forcing
3.7 Thermal stratification at INP Mooring station; SST forcing multiplied by 10
3.8 Difference in temperature compared with remote sensing SST
3.9 Thermal stratification at INP Mooring station; measurements versus model results

Results for 3D North Sea model:
4.1 Remote sensing data versus model results for 21 July 1994
4.2 Remote sensing data versus model results for 4 August 1994
4.3 Remote sensing data versus model results for 18 August 1994
4.4 Remote sensing data versus model results for 15 September 1994
4.5 Remote sensing data versus model results for 26 October 1994
4.6 Evolution thermal stratification; measurements versus model results at INP Mooring station
4.7 Difference in temperature compared with remote sensing SST
4.8 Difference in temperature compared with remote sensing SST
I Introduction

In 1998 a project (‘REST3D’) for integration of satellite remote sensing sea surface temperature data, in-situ data and 3D temperature modelling results has started. The project REST3D aims at the application of a methodology named RESTWAQ (Vos and Schuttelaar, 1995) for temperature modelling at North Sea scale. In the RESTWAQ methodology remote sensing data is combined with in-situ data and model data. The typical 3D behaviour of temperature (due to stratification of sea water) makes that in-situ data and remote sensing sea surface temperature data are more or less complementary.

In the REST3D project a first step towards full integration of temperature measurements and temperature modelling results is carried out. In order to achieve this goal time periods have been selected for which accurate and continuous vertical profile temperature measurements (with a thermistor chain) are available. Such data is only available for two periods at a limited number of locations:

1. For 1989 at six locations in the North Sea (NSP data set from NERC).
2. For 1994 for one location at Oyster grounds in the North Sea (INP mooring data set from DNZ).

The 1989 in-situ data set is clearly more extensive. However, the 1994 remote sensing data set is much more complete. For 1989 only ten images are available, of which some are partially cloud covered. Due to this incompleteness of data, it has been decided to test the model performance for both 1989 and 1994. For 1989 the emphasis is on the thermistor chain in-situ data. For 1994 the emphasis is more on remote sensing sea surface temperature data. The 1994 simulations are also of interest to RWS-DNZ and NIOZ that supplied the INP mooring data for temperature profiles (van Raaphorst et al., 1996).

Within the REST3D project a North Sea temperature model has been calibrated and validated for the 1989 period (1 April to 1 December) with remote sensing data and in-situ data (Vos et al., 1999). A similar validation has also been carried out for the 1994 period (again from 1 April to 1 December), see (De Goede et al., 1999). In the above mentioned projects a heat flux forcing has been applied at the free sea surface. The remote sensing SST data has been used for validation. The numerical results for the 1989 and the 1994 simulation period are in reasonable agreement with the available remote sensing and in-situ data. For the horizontal sea surface temperature distribution during the eight months simulation period the differences are roughly up to 2°C (and locally to 4°C), which is acceptable taking into account the sensitivity of the temperature results on the meteorological data and applied numerical parameters. The stratification is also in good agreement with the measurements. This is the case for both the onset of the thermocline, the thermocline depth and the breakdown of the thermocline at the selected stations.

In order to further improve the accuracy, we will in this project combine the heat flux forcing in TRIWAQ/TEM with a SST forcing. We will make use of NOAA/AVHRR SST weekly composites, which has been supplied by KNMI. The simulations will be carried out
with the so-called TRIWAQ/TEM source code, which has been developed for temperature modelling (Van Kester, 1997).

In Chapter 2 boundary conditions at the temperature sea surface will be discussed. Two possible approaches will be suggested for combining heat flux formulae with a SST forcing. One of these approaches will be selected. In Chapter 3 the approach will be tested for a simple test basin, whereas in Chapter 4 the approach is applied for a 3D North Sea model. For the sake of completeness a detailed description of the heat flux formulae in TRIWAQ/TEM is given in the appendix.

This project is carried out by order of RIKZ as part of the MEETSTRATEGIE 2000+ project, sub project data-model integration (contract number 32992026 dd. 2 August 1999). At RIKZ this project is supported by J.M. de Kok.

1.1 Acknowledgement

KNMI (Hans Roozekrans) is gratefully acknowledged for providing NOAA/AVHRR SST daily images and weekly composites for the 1994 period.
2 Temperature boundary forcing

For the boundary conditions concerning the transport of heat through the free water surface two different approaches exist:

- Calculation of the heat flux through the sea surface by using so-called bulk-formulae (Neumann boundary condition).
- A Sea Surface Temperature (SST) is prescribed at the surface layer of the model (Dirichlet boundary condition).

Both approaches are applied in models in 3D shallow water modelling. For example, the SST approach is used by Pohlmann (Pohlmann, 1996) in a modified version of the shelf sea model of Backhaus (Backhaus, 1985). The bulk formulae approach is applied in many models, e.g. in TRIWAQ/TEM (Van Kester et al., 1997), in (Lane, 1989), (Holt et al., 1999), (Vichi, 1999) and in (Delft Hydraulics, 1999).

A disadvantage of the bulk-formulae approach is that is difficult to obtain accurate meteorological data at a reasonable resolution. This especially holds for the cloud coverage and the relative humidity, which are both essential parameters for the calculation of the heat flux between sea and atmosphere. For example, in (De Goede, 1999) it was shown that inaccurate cloud cover data may yield unreliable model results.

In recent years a large experience has been built up with heat flux modelling. For such models it can now be concluded that if accurate meteorological data is available, then heat flux models are able to produce both horizontal and vertical temperature distributions that are in good agreement with measurements, see (Holt, 1999), (Vos et al., 1999).

In case of a SST approach, the SST data should be available at a reasonable resolution both in space and in time. For the North Sea this is a feasible method, because weekly NOAA/AVHRR SST data is available on approximately a 1 by 1 km grid.

The ideal situation for the simulation of the heat exchange processes through the free surface would be the coupling of a sea model with an atmospheric model. Although progress is being made in this field, this is beyond the scope of our project. For example, the UK Metoffice is using a North Sea model in which the heat flux is calculated by an atmospheric model. Similarly, at KNMI a HIRLAM-WAQUA coupling is used in an operational system. Also in global circulation ocean modelling research is going on coupling of sea and atmospheric models.

In this project we will investigate how a heat flux (bulk formulae) model can be combined with a SST forcing. Two possible approaches will be discussed. The first method follows the approach developed for a world ocean model (Barnier et al., 1994) and is based on a heat flux forcing in which a correction term is added that makes use of available SST data. The second method starts with the SST method, which can be corrected in such a way that it becomes a quasi-heat flux forcing. The latter approach has been proposed by
Uittenbogaard (private communication). In the next sections these two methods will be described in detail.

2.1 Thermal forcing based on world ocean modelling

In this section the thermal forcing for a world ocean model developed in (Barnier et al., 1994) will be described. This approach starts with bulk formulae to determine a heat flux through the sea surface. Next, an estimate of the flux correction is calculated from SST data.

In sea models the advection-diffusion equation for the time evolution of temperature may be defined by

\[
\frac{\partial T}{\partial t} = \text{advection} + \text{diffusion} + \frac{Q_{\text{tot}}}{\rho_c c_{\text{sw}} \Delta z_s}
\]  

(2.1.1)

with \(\Delta z_s\) the thickness of the top layer and \(c_{\text{sw}}\) the specific heat capacity of sea water (3930 J/kg/K). The third term in the right-hand side of Eq. (2.1.1) represents the heat flux through the free surface. For the North Sea the exchange fluxes vary strongly with latitude and time of the year, see e.g. (Lane, 1989).

Alternatively, the third term in the right-hand side of Eq. (2.1.1) can be formulated as a relaxation of the model temperature \(T_s\) toward the climatological sea surface temperature \(T_{\text{sst}}\):

\[
\frac{\partial T}{\partial t} = \text{advection} + \text{diffusion} + \frac{1}{R}(T_{\text{sst}} - T_s)
\]  

(2.1.2)

where \(R\) can be considered as a relaxation constant. Such a formulation is applied frequently in world ocean modelling, see e.g. (Cox and Bryan, 1984). It does not require any meteorological input. Only an estimate of the relaxation parameter \(R\) is required.

In world ocean modelling a coupling of a heat flux model as in Eq. (2.1.1) to a prescribed atmosphere as in Eq. (2.1.2) was first proposed in (Haney, 1971). Following more or less the same approach, in (Han, 1984) this method was applied to perform simulations of the world ocean circulation. The major effect of the method of (Han, 1984) is that the heat flux was updated in such a way that the ocean model was driven toward a state of equilibrium with the prescribed atmosphere. Such an approach is very useful because inconsistencies always exist between the computed heat flux and the measured SST data.

In (Barnier et al., 1994) a slightly different approach was applied in comparison to (Haney, 1971) and to (Han, 1984). The underlying idea was, however, the same. In (Barnier et al., 1994) the heat flux is corrected in such a way that the ocean model is pulled toward a state of “quasi-equilibrium” that is consistent with the prescribed atmosphere. After reaching this state, it is assumed that the difference between the sea surface temperature \(T_{\text{sst}}\) and the model temperature \(T_s\) is small.

We remark that the approach in (Barnier et al., 1994) has a lot of similarities with the calculation of a so-called equilibrium temperatures. We will illustrate this in the following section by comparing the method in (Barnier et al., 1994) with the method for the calculation
of equilibrium temperature as applied for TRIWAQ/TEM, see the appendix in this report and also Appendix A.7 in (Van Kester et al., 1997). These two methods are only applied in a different context. The equilibrium temperature is a theoretical value, representative of the meteorological conditions, which will in reality never be reached.

Following the approach in (Barnier et al., 1994), the heat flux is expanded about between the climatological sea surface temperature $T_{sw}$, yielding

$$Q_{tot}(T_s) = Q_{tot} - \frac{\partial Q_{tot}}{\partial T}(T_{sw} - T_s)$$

(2.1.3)

In Eq. (2.1.3) the heat flux is the sum of two components, viz. a prescribed heat flux (based on meteorological data) and a correction term proportional to the difference between the climatological temperature $T_{sw}$ and the model temperature $T_s$.

In order to compute the total heat flux in Eq. (2.1.3) the derivative of the heat flux with respect to temperature should be computed. In TRIWAQ/TEM the total heat exchange through the free surface is given by

$$Q_{tot} = Q_s - Q_{ev} - Q_{br} - Q_{co}$$

(2.1.4)

with the evaporative heat flux defined by (see also Eq. (A.19))

$$Q_{ev} = f(U_{10}) L_r \rho_{dry}(q_s(T_s) - q_a(T_s))$$

(2.1.5)

the effective back radiation defined by (see also Eq. (A.25))

$$Q_{eb} = \varepsilon \sigma T_s^4 (0.39 - 0.05 \sqrt{e_a})(1 - 0.6 F_c^2)$$

(2.1.6)

and the convective heat flux equal to (see also Eq. (A.26))

$$Q_{co} = \rho_a c_p g(U_{10})(T_s - T_a)$$

(2.1.7)

The solar radiation $Q_s$ is explained in Appendix A, see Eq. (A.3).

Using Eqs. (2.2.4-7) the derivative of the heat flux with respect to the temperature, defined by:

$$K = - \frac{\partial Q_{tot}}{\partial T} = - \frac{\partial Q_s}{\partial T} + \frac{\partial Q_{ev}}{\partial T} + \frac{\partial Q_{br}}{\partial T} + \frac{\partial Q_{co}}{\partial T}$$

(2.1.8)

yields

$$\frac{\partial Q_{ev}}{\partial T} = f(U_{10}) L_r \frac{\partial \rho_{sat}(T)}{\partial T}$$

(2.1.9)

$$\frac{\partial Q_{br}}{\partial T} = 4 \varepsilon \sigma T_s^3 (0.39 - 0.05 \sqrt{e_a})(1 - 0.6 F_c^2)$$

(2.1.10)

$$\frac{\partial Q_{co}}{\partial T} = \rho_a c_p g(U_{10})$$

(2.1.11)

Furthermore, we assume that
\[
\frac{\partial Q_e}{\partial T} = 0 \quad (2.1.12)
\]

For the evaporative flux in Eq. (2.1.9) we use the expression given in Eq. (A.23) and the following approximation for specific humidity of saturated air (cf. Eq. (A.21a)):

\[
q_s(T_s) \approx \frac{\varepsilon e_s}{\rho_{atm}} \quad (2.1.13)
\]

and Eq. (A.12) for the saturated vapour pressure \(e_s\). Substituting Eq. (2.1.13) into Eq. (2.1.9) gives

\[
\frac{\partial Q_{ev}}{\partial T} = \frac{\partial}{\partial T} \left( f(U_{10}) L_v \rho_\infty \frac{e_s(T_s)}{\rho_{atm}} \right) = f(U_{10}) L_v \rho_\infty \frac{\partial e_s(T)}{\partial T} = f(U_{10}) L_v \rho_\infty \frac{e_s(T)}{\rho_{atm}} e_{s(T)} \log(10) \frac{0.03153}{(1 + 0.00412 T_s)} \quad (2.1.14)
\]

In the above described approach it is assumed that the climatological atmosphere is in equilibrium with the sea. Owing to the fact that in Eq. (2.1.3) the heat flux has been expanded about \(T_{sat}\), this variable mainly determines the correction term. Furthermore, this term may be written in the form

\[
Q_2(T_{sat} - T_s) \quad (2.1.15)
\]

and is similar to the one in (Haney, 1971) and in (Han, 1984). However, in the latter two papers it is assumed that the model SST is close to the surface air temperature. Consequently, the correction term is expanded about the air temperature \(T_a\) and not about \(T_{sat}\). In (Haney, 1971) a different interpretation of the model heat flux is given. In that paper Eq. (2.1.15) is written in the form

\[
Q_{sat}(T_s) = Q_2(T_a^* - T_s) \quad (2.1.16)
\]

with \(T_a^*\) the apparent air temperature defined by

\[
T_a^* = T_{sat} + \frac{Q_1}{Q_2} \quad (2.1.17)
\]

with \(Q_1\) the first term in the right-hand side of Eq. (2.1.3). If such a formulation is used, then the advection-diffusion equation (2.1.1) for temperature becomes

\[
\frac{\partial T}{\partial t} = \text{advection} + \text{diffusion} + \frac{1}{R} (T_a^* - T_s) \quad (2.1.18)
\]

where the relaxation time \(R\) reads

\[
R = \frac{\rho_0 c_{pw} \Delta z_s}{Q_2} \quad (2.1.19)
\]
If we assume that $\Delta z = 2\ m$ and $Q_L = 20\ W/m^2$, then we arrive at a relaxation time of about five days. In world ocean modelling the typical relaxation time is in the order of one or two months.

We remark that the SST corresponds to the temperature in the near vicinity (a few cm) of free water surface, which is consistent with the approach described in this section. In case of a Dirichlet SST boundary condition, however, the SST corresponds to the complete top layer of the model, which in general has a thickness of at least 1 or 2 m.

Finally, if the approach of this section is applied with the ‘normal’ heat flux component, viz. the first term in the right hand-side of Eq. (2.1.3), set to zero, then we arrive a quasi-Dirichlet boundary condition. In such a case $Q_L$ can again be considered as a relaxation time.

### 2.2 Corrected SST forcing

In the previous section a combined heat flux/SST method has been described that starts with a heat flux forcing and is corrected on the basis of SST data. The approach that will be described in this section follows a complete different approach. We will start with a SST-Dirichlet boundary condition, which will be corrected in such a way that it becomes a quasi-Neumann boundary condition. This approach has been proposed by Uittenbogaard (private communication) and will now be described in more detail.

Let us assume that the sea surface temperature $T_{SS}$ is the sum of the measured SST and of a temperature deviator $\theta$, defined by

$$T = T_{SS} + \theta$$  \hspace{1cm} (2.2.1)

With the deviator $\theta$ the measured SST is corrected according to an algorithm that in some way uses an upper limit for the heat flux. For the top layer ($k=1$) we define

$$\left(\frac{\partial}{\partial t} + r^{-1}\right)\left(p c_p V \theta\right) = H_{\theta} \{Q_{heat}(T) - Q_{model}(T)\}$$  \hspace{1cm} (2.2.2)

in which $\rho$ is the density of the top layer, $V$ the volume of the top layer and $H_{\theta}$ a heat flux limiter. The variable $Q_{heat}$ represents the heat flux (in Watt) from the air to the water column according to the heat flux formulae in TRIWAOQ/TEM. We remark that $H_{\theta}$ has been multiplied with the horizontal cell area $A$ (in J/s). The heat flux $Q_{model}$ is determined according to the heat balance

$$Q_{model} = \frac{\partial \rho c_p V T}{\partial t} - (D + \Gamma_r) A \left(\frac{\partial T}{\partial z}\right)_{z=\zeta\Delta z} + \left(\frac{\partial U_A x T}{\partial x} + \frac{\partial V_A y T}{\partial y}\right)_{z=\zeta-\frac{1}{2}\Delta z}$$  \hspace{1cm} (2.2.3)

The second term in Eq. (2.2.3) represents the heat loss through molecular diffusion and turbulent mixing through the layer interface of the first and second layer. The third term describes the advective transport. The horizontal diffusion and vertical advection is neglected.
The relaxation time $\tau$ has been introduced to prevent a 'stiff' coupling and may be chosen equal to

$$\tau = \frac{(\Delta z)^2}{(D + \Gamma)_{2-\Delta z}},$$

(2.2.4)

which is in correspondence with typical diffusion time scales for a computational cell.

For the heat flux limiter $H_\theta$ in Eq. (2.2.1) several choices are possible. For example, an infinite-tolerance will yield that differences between $Q_{\text{heat}}$ en $Q_{\text{model}}$ are possible. If $\theta$ is set to zero, then the SST is used as a ‘normal’ Dirichlet boundary condition. On the other hand, in case of zero-tolerance $H_\theta$ changes if e.g. $Q_{\text{heat}}>Q_{\text{model}}$.

It is our aim to apply a finite-tolerance criterion, which becomes active if

$$\left| Q_{\text{heat}}(T) - Q_{\text{model}}(T) \right| \geq \kappa \left( \| Q_{\text{heat}}(T) \| \right)$$

(2.2.5)

with $\kappa$ a tolerance criterion to be specified in some way.

### 2.3 Comparing both approaches

When comparing the above described approaches we have chosen to, at first, test the combined heat flux-SST forcing as developed for world ocean modelling by (Barnier et al., 1994), because of its simplicity (no parameter tuning is required) and the fact that this approach has been applied with success in oceanographic modelling. Owing to its success on ocean scales, we expect that this approach can be applied equally successful on North Sea scales. In the next chapters the approach will be applied to a simple test basin and to a 3D North Sea model.
3  Numerical modelling with simple test basin

In this chapter the heat flux-SST coupling approach described in Section 2.1, viz. the approach developed in (Barnier et al., 1994), will be applied to a simple test basin. The simple test basin consists of 8 by 8 grid points in the horizontal direction, has four closed boundaries and a constant depth of 47.5, which corresponds to the depth at the INP Mooring station. This simplified test basin has already been used in (De Goede et al., 1999). Owing to the fact that temperature processes in the vertical direction are dominant over the processes in the horizontal direction, such a test basin is very suitable for initial testing and for sensitivity analyses.

The 1 April 1994 to 1 November 1994 period will be simulated. This period is one month less than previous long term simulations, see (Vos et al., 1999) and (De Goede et al., 1999) because no remote sensing SST data is available for November 1994.

3.1  Model parameters

The test basin is a very small part of the “real” 3D North Sea Model schematization (NSM), which will be described in Chapter 4.

Some characteristics of the test basin are:

1. model area is a small part of the NSM model area (8 by 8 grid points)
2. 20 equidistant layers in the vertical direction
3. spherical co-ordinates (\(\Delta \lambda=0.125\), \(\Delta \phi=0.08333\))
4. time step of 10 minutes
5. simulation period from 1 April 1994 to 1 November 1994
6. \(\Delta x=8000\) m; \(\Delta y=8000\) m
7. meteorological data supplied by POL (UK)
8. \(C_D\) of 0.002
9. bottom friction via Chézy coefficient (65 m\(^{1/2}\)/s)
10. horizontal viscosity coefficient of 1 m\(^2\)/s
11. horizontal diffusion coefficient of 5 m\(^2\)/s
12. background value for the eddy viscosity of \(7.10^{-4}\) m\(^2\)/s
13. k-ε turbulence model

The heat flux at the water surface is modelled according to a prescription given in Appendix A, see also (Van Kester et al., 1997). The heat flux model required the following meteorological input: wind speed (m/s), air temperature (°C), air pressure (mbar), relative humidity (%) and cloudiness (fraction). For all grid points the same meteorological input values are applied. The meteorological input is plotted in Figures 3.1-3.

Similarly to previous 3D long term simulations a background value for the vertical eddy viscosity of \(7.10^{3}\) m\(^2\)/s is used (Vos et al., 1999).
Concerning the layer distribution, in previous simulations with TRIWAQ/TEM often a non-equidistant forty layer distribution was applied. However, in (De Goede et al., 1999) it has been concluded that the 1994 long term simulation results for an equidistant twenty layer distribution are very similar to the ones for the non-equidistant forty layer distribution. As an illustration, in (Luyten, 1996) an equidistant twenty layer distribution is applied for a 3D North Sea model, whereas in (Holt, 1999) in fact 13 layers are used. Therefore, we have decided to apply twenty equidistant layers. Although in this report only results for such a vertical distribution will be shown, we remark that the same experiments have been carried out with the non-equidistant forty layer distribution as well. The results are almost identical to the ones of the twenty layer simulation.

Weekly SST composites from April 1994 to October 1994 (31 weeks) have been used. From each composite the value at the INP Mooring station has been selected.

3.2 Day-night variation of remote sensing SST

The weekly SST composites are free from cloud cover and are have been processed (Roozekrans et al., 1988). However, in order to use remote sensing SST data in model simulations the weekly fixed SST data should be superpositioned with a day-night variation. Therefore, we have added the following correction:

$$T_{\text{day-night}} = -\cos \left( 2\pi \frac{(t_{\text{hour}} - 4)}{24} \right)$$  \hspace{1cm} (3.2.1)

Thus, the day-night variation has an amplitude of $1^\circ \text{C}$, with a maximum negative correction of $1^\circ \text{C}$ at 4am and a maximum positive correction of $1^\circ \text{C}$ at 4pm.

3.3 Model results

Figure 3.4 contains the thermal evolution for the 1994 period at the centre of the test basin computed with the heat flux forcing as applied in previous projects, see (De Goede et al., 1999). In the upper plot time histories of temperature are shown at layer 1 (top layer), layers 3 and 10 and at layer 20 (bottom layer). Furthermore, it contains the remote sensing SST data for the INP Mooring station (black line). At the x-axis the time in minutes is shown ($T_{\text{end}}=308160 \text{ min.}=214 \text{ days}$). From the lower (and the upper) plot the onset and break down of the thermocline can be seen clearly. Taking in mind that only a small test basin is used, the results are already in reasonable agreement with the upper plot of Fig. 3.9, which shows the measured stratification at the INP Mooring station. The middle plot in Fig. 3.4 contains the total heat flux for July 1994 and the corrected heat flux, see e.g. Eq. (2.1.3). The latter heat flux is, of course, zero because no heat flux correction has been applied in this simulation.

Next, in Figure 3.5 the results are shown for the simulation in which the heat flux correction of Eq. (2.1.3) has been applied. The surface temperatures are apparently closer to the remote sensing SST than in the previous simulation. This is quite obvious, because the correction term forces the model to a state of equilibrium, which means that the difference between the SST and the surface model temperature should become smaller.
Figure 3.6 contains the results for the simulation in with only the correction term, i.e. the second term in the right hand-side of (2.1.3), is used. The first term in the right hand-side of (2.1.3), which represents the 'normal' heat flux forcing, is set to zero. As explained in Section 2.1, this may be considered as a quasi Dirichlet boundary condition with a relaxation time according to (2.1.19). The results show that the sea surface model temperature has the tendency to follow the SST, however, in a 'slow' manner. This can be explained by the fact that the relaxation time is approximately 7 days (assuming that $\Delta t = 47.5/20$ m and $Q_s = 15$ W/m$^2$), see Eq. (2.1.19).

In order to speed up the state of equilibrium (and thus decrease the difference between the $T_{sol}$ and the model temperature $T_s$) we have carried out a simulation in which the correction factor is multiplied by a factor of ten, which corresponds to a relaxation time of approximately 17 hours. The value of ten is a rather arbitrary value. The results are shown in Fig. 3.7. Now, the model sea surface temperature is in close agreement with the SST. In this way, the user is able to accelerate (or decelerate) the speed of the process of reaching a state of equilibrium. The vertical temperature distribution is also in good agreement with the measurements, see Fig. 3.9. We remark that in this simulation no meteorological data is required (only SST data).

Finally, in Fig. 3.8 for all simulations described above the difference $T_{sol} - T_s$ is plotted. The red line represents the situation at the start of this project (no heat flux correction, see Fig. 3.4). It can be seen that the black line (heat flux correction) and the blue line (only SST forcing but multiplied with a factor of 10) correspond to simulations for which the model sea surface temperatures are in better agreement with the measured SST data.
4 3D North Sea model

In the previous chapter the heat flux-SST coupling approach of (Barnier et al., 1994) has been applied with success to a simple test basin. The simple test basin consists of 8 by 8 grid points in the horizontal direction, which represents only a very small part of the 3D North Sea model, which has been used in previous long term temperature model simulations, see e.g. (Vos et al., 1999) and (De Goede et al., 1999). In this chapter we will report on the results of the application of the same heat flux-SST coupling approach to the full 3D North Sea model.

4.1 Model parameters

The model schematization is the North Sea Model (NSM), which is a grid of 105*96 computational elements using spherical co-ordinates. The model goes up to a latitude of 57 degrees and is part of the Dutch Continental Shelf model (DCSM8). The model uses 20 layers over depth. Some characteristics of the model are:

- model area is part of the CSM8 model area
- MMAX=105 en NMAX=96 (47% of the grid points are active (not permanently dry)
- 20 equidistant σ-layers
- spherical co-ordinates (Δλ=0.125, Δφ=0.08333)
- time step of 10 minutes
- simulation period from 1 April 1994 to 1 November 1994
- including salinity and temperature
- average Δx of 8000 m; average Δy of 9000 m
- at northern, western and southern boundary water level conditions (astronomic boundary conditions)
- Meteorological data supplied by POL (UK)
- C_D of 0.002
- depth schematization and friction coefficients of CSM8 model
- bottom friction via Chézy coefficient (varying from 55 to 75 m^1/2/s)
- horizontal viscosity coefficient of 1 m^2/s
- horizontal diffusion coefficient of 5 m^2/s
- background value for the eddy viscosity of 7.10^{-5} m^2/s
- k-ε turbulence model

The heat flux at the water surface is modelled according to a prescription given in Appendix A in (Van Kester et al., 1997). The heat flux model requires the following meteorological input: wind speed (m/s), air temperature (°C), air pressure (mbar), relative humidity (%) and cloudiness (fraction). Only for the wind speed and for the air pressure space varying data has been used. Thus, for the air temperature, relative humidity and the cloudiness values have been applied that are constant for the whole model area (but time varying). The meteorological input is specified every three hours.
For the 3D North Sea model the following three simulations have been carried out:

1. With normal heat-flux forcing (only first term in right-hand side of Eq. (2.1.3); thus no coupling with SST data).
2. Combined heat flux-SST forcing (both first and second term in right-hand side of Eq. (2.1.3)).
3. Only SST forcing with a multiplication factor of ten (only second term in right-hand side of Eq. (2.1.3) and multiplied by a factor of ten).

We remark that similar simulations have been carried out for the simple test basin, respectively the first, second and fourth simulation described in Chapter 3.

### 4.2 Model results

In this section we compare the model results with measurements. Both vertical profiles and SST measurements at several days during the seven month simulation period are considered. The vertical profiles are examined for the INP Mooring station (Figure 4.6). With respect to the SST measurements, the model SST data are evaluated at 21 July; 4 August, 18 August, 15 September and 26 October (Figures 4.1-5, respectively).

For each selected time both SST measurements and model results for the three simulations are plotted. The left upper plot represents the measurements, the right upper plot simulation 1 (with normal heat flux forcing), the left lower plot simulation 2 (combined heat flux-SST forcing) and the right lower plot contains the results of simulation 3 (only SST forcing with a multiplication factor of ten). We remark that SST measurements in these figures represent the uncorrected SST values, see Eq. (3.1.1). So, the results should be interpreted carefully.

For all three simulations we have computed at every time step the average difference between the remote sensing SST data and the model results at the top layer. In Figures 4.7-8 the differences are shown for each month. Furthermore, if these values are also averaged over the simulation period, then we arrive at a difference between the remote sensing SST data and the model results that is both averaged in space and time. In Table 4.1 these values are listed.

<table>
<thead>
<tr>
<th>Simulation no.</th>
<th>Temperature difference ( (T_{\text{sat}} - T_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>simulation 1 (with normal heat flux forcing)</td>
<td>1.5°C</td>
</tr>
<tr>
<td>simulation 2 (combined heat flux-SST forcing)</td>
<td>1.1°C</td>
</tr>
<tr>
<td>simulation 3 (only SST forcing)</td>
<td>0.7°C</td>
</tr>
</tbody>
</table>

Table 4.1. Space and time averaged temperature differences
4.2.1 Sea surface temperatures

In (Vos et al., 1999) it was concluded that for a simulation with a ‘normal’ heat flux forcing, which is comparable with simulation 1, the numerical results were in reasonable agreement with the available remote sensing and in-situ data. The horizontal temperature distribution during whole simulation period was in good agreement with the remote sensing data as well. The differences were roughly up to 2°C (and locally to 4°C), which is acceptable taking into account the sensitivity of the temperature results on the meteorological data and applied numerical parameters. These conclusions are also valid for simulation 1, which can be verified in Figures 4.1-5. As can be seen in Table 4.1 the spatial and time averaged difference between the remote sensing SST and the model surface layer temperature is approximately 1.5°C. The differences are due to the lower variation in the horizontal distribution of the surface model temperature. This may be caused by the fact that some of input parameters of the meteorological forcing are constant in space, see Section 4.1. For simulations 2 and 3 we expect a horizontal temperature distribution that is more in agreement with the SST data, because the correction term is based on (space varying) SST values, see Eq. (2.1.3).

From Figures 4.1-5 we can indeed conclude that the horizontal temperature distribution is in better agreement with the SST data. This is especially the case for simulation 3, in which only a SST forcing is used. In all figures it can be seen that the sea surface model temperatures for simulation 3 are in remarkable agreement with the SST data.

We expect that a combination of simulations 2 and 3, thus a combined heat flux-SST approach in which the correction term is multiplied by a factor of 10 or higher, will lead to even better results. The multiplication of a factor of ten is a rather arbitrary value, which corresponds to a relaxation time of 17 hours. Taking into account this relaxation time and the amplitude of the day-night variation, viz. 1°C, the results are very satisfying.

4.2.2 Vertical profiles

Figure 4.6 contains the thermal evolution at the INP Mooring station of the three simulations and of the measurements. The model results are in good agreement with the measurements. This is the case for the onset of the thermocline, the thermocline depth and the break down of the thermocline. The measurements in Fig. 3.5 show some gaps, because at these periods in time no measurements were available at certain depths.

Similarly to the sea surface temperatures, simulation 3 yields the most accurate results. For simulations 2 and 3 the thermocline depth is in very close agreement with the measurements. We remark that no calibration was required to obtain these results. For simulation 1 the thermocline depth is slightly less compared to the measurements. For simulation 3 it is, however, somewhat surprising that the thermocline breaks down slightly faster than in the measurements, while for simulations 1 and 2 the moment of thermocline break down is in very close agreement with the measurements.
Summarizing, the simulations show that the approach applied in this project, which has been taken from world ocean modelling, is able to decrease the ‘gap’ caused by inconsistencies between remote sensing SST data and model SST.
5 Conclusions

For the boundary conditions concerning the transport of heat through the free water surface two different approaches exist:

- Calculation of the heat flux through the sea surface by using so-called bulk-formulae (Neumann boundary condition).
- A Sea Surface Temperature (SST) is prescribed at the surface layer of the model (Dirichlet boundary condition).

Both approaches are applied in 3D shallow water modelling. For example, the SST approach is used by Pohlmann (Pohlmann, 1996) in a modified version of the shelf sea model of Backhaus (Backhaus, 1985). The bulk formulae approach is applied in many models, e.g. in (Lane, 1989), (Holt et al., 1999) and in a special version of TRIWAQ (Van Kester et al., 1997).

The ideal situation for the simulation of the heat exchange processes through the free surface would be the coupling of a sea model with an atmospheric model. However, this topic is beyond the scope of our project. We have investigated how a heat flux (bulk formulae) model can be combined with a SST forcing. In this project we have combined a heat flux forcing with a SST forcing by using an approach for world ocean modelling, viz. the approach developed in (Barnier et al., 1994). This approach is based on a heat flux forcing in which a correction term is added that makes use of available SST data.

The combined heat flux-SST approach has been examined for a simple test basin of only 8 by 8 grid points in the horizontal direction and for a 3D North Sea model. The simulation period is from 1 April 1994 to 1 November 1994, because for this time frame weekly SST composites are available. The NOAA/AVHRR SST images with weekly composites have been supplied by KNMI. The simulations have been carried out with the TRIWAQ/TEM source code.

The simulations have shown that it is possible to decrease the ‘gap’ caused by inconsistencies between remote sensing SST data and meteorological forcing. By introducing a correction term for the heat flux or by only applying the correction term and neglecting the ‘normal’ heat flux computed by the meteorological forcing, the differences between the measured climatological sea surface temperature (SST) and the model temperature at the top layer decrease. For the 3D North Sea model the space and time averaged difference between the remote sensing SST and the model SST reduces from about 1.5°C to about 0.7°C. However, despite the success of this approach, it may not be concluded that it becomes less essential to use accurate meteorological data. This is due to the fact that the approach in which only a SST forcing is used, can only be applied in case of hindcasting and not in case of forecasting.
References


The heat exchange processes at the sea surface

A.1 Introduction

In literature (Defant, 1961), (Sweers, 1976), (Lane, 1989) there is a great variability of empirical formulations to calculate the heat fluxes across the sea surface. Most formulations differ in the dependency of the exchange on the meteorological parameters such as wind speed, cloudiness and humidity. In this appendix we describe the formulations that have been implemented in TRIWAQ/TEM proposed in the intercomparison study NOMADS. For the physical background of the formulations and definitions we refer to (Gill, 1982).

A.2 Heat exchange at the sea surface

The exchange of heat at the sea surface is the consequence of a number of physical processes. A schematic overview of the heat exchange mechanisms is given in Figure A.1. For the North Sea the four major components, in order of importance, read as follows:

\[ Q_s \quad : \text{solar radiation (short wave), see Section A.3} \]
\[ Q_{ed} \quad : \text{evaporative heat flux (latent heat), see Section A.4} \]
\[ Q_{br} \quad : \text{back radiation (long wave), see Section A.5} \]
\[ Q_{co} \quad : \text{convective heat flux (sensible heat), see Section A.6} \]
\[ Q_a \quad : \text{atmospheric radiation (long wave) see Section A.5} \]
\[ Q_{eb} \quad : \text{effective back radiation (long wave, } Q_{br} - Q_a) \text{, see Section A.5} \]

In our temperature model, meant for simulations of the seasonal thermocline formation on the North Sea, the atmospheric (long wave) radiation is part of the total long-wave radiation flux, the so-called effective back radiation flux. Furthermore, we neglect the relatively small effect of the heat exchange by precipitation \( Q_{pr} \). Therefore the total rate of heat exchange in J/(m² s) is given by

\[ Q_{tot} = Q_s - Q_{ev} - Q_{br} - Q_{co} \quad \ldots \quad (A.1) \]

The change in temperature in the top layer is related to the total heat flux through the free surface by the specific heat capacity of \( c_{pw} \) of sea water (3930 J/kg/K):

\[ \frac{\partial T}{\partial t} = \frac{Q_{tot}}{\rho_0 \ c_{pw} \ \Delta z_s} \quad \ldots \quad (9.4.2) \]

For the North Sea the different exchange fluxes vary strongly with latitude and time of the year, see e.g. (Lane, 1989). The dependence on the geographical position \( \phi \) and the meteorological parameters such as air temperature \( T_a \), cloud cover \( F_c \), atmospheric pressure \( P_{atm} \), relative humidity \( F_{rh} \), and wind speed \( U_{10} \), is specified in the following sections.
A.3 Solar radiation flux

The Earth receives energy from the Sun in the form of radiation. The average energy flux at the mean radius of the Earth is called the solar constant $S$ and has the value

$$ S = 1368 \ J / (m^2 \ s) . $$ \hspace{1cm} (A.2.a)

The energy flux depends on the angle (declination) between the incoming radiation and the Earth’s surface. This declination depends on the geographical position on the Earth and the local time. The Earth axis is not perpendicular to the line connecting the Sun with Earth. This tilting (angle $\delta$) varies with the time of the year and it leads to a seasonal variation of the radiation flux. At June 21, the declination is maximal, 23.5 degrees. Of course, by the rotation of the Earth the solar radiation also varies during the day. Near twelve o’clock local time, the sun elevation above the horizon is maximal. For an overview of the angles used to determine the solar elevation angle $\gamma$, see Figure A.2.

In the heat flux model of POL the temporal and latitude-dependent solar elevation angle $\gamma$ is estimated by:

$$ \sin(\gamma) = \sin(\delta) \sin\left(\frac{\pi \phi}{180}\right) - \cos(\delta) \cos\left(\frac{\pi \phi}{180}\right) \cos(\omega_0 t) , $$ \hspace{1cm} (A.2.b)

with

$$ \delta = \frac{23.5 \pi}{180} \cos(\omega_0 t - 2.95) . $$ \hspace{1cm} (A.2.c)

$\omega_0$ is the frequency of the annual variation, $\omega_1$ the frequency of the diurnal variation and $\phi$ is the latitude. The incoming short-wave solar radiation through a clear sky at ground level is about 0.76 of the flux incident at the top of the atmosphere (Gill, 1982):

$$ Q_{s0} = \begin{cases} 0.76 \ S \ \sin(\gamma), & \sin(\gamma) \geq 0 \\ 0, & \sin(\gamma) < 0 \end{cases} $$ \hspace{1cm} (A.2.d)

with $S$ defined by Eq. (A.2.a). A part of the radiation that reaches the sea surface is reflected by the sea surface. The fraction reflected is called the Albedo $\alpha$, which is dependent on latitude and season. However the spatial variations for the North Sea are small. For the North Sea we use an Albedo coefficient of 0.06, see (Lane, 1989). Cloud cover will reduce the magnitude of the radiation flux that reaches the sea surface. The cloudiness is expressed by a cloud cover fraction $F_c$, the fraction of the sky covered by clouds. The correction factor for cloud cover is an empirical formula. The absorption of solar radiation is calculated (Gill, 1982) as the product of the net downward flux of short wave-radiation in cloudless conditions and factors correcting for reflection and cloud cover:

$$ Q_s = Q_{s0} (1 - 0.4 F_c - 0.38 F_c^2)(1 - \alpha) . $$ \hspace{1cm} (A.3)

For small model areas it is possible to measure the incoming solar radiation and to specify $Q_s$ at input.

A.4 Evaporative heat flux
In order of magnitude, for the North Sea, the heat loss due to evaporation is the second most important heat exchange mechanism, accounting for about 54% of the total heat loss (Lane, 1989). Evaporation is a process that takes place at the boundary surface between water and the air above it. It depends on the conditions in the immediate vicinity of the free surface. Evaporation occurs until the air above the sea surface is saturated. The transport of the water vapour into the air proceeds by turbulent exchange. The turbulent exchange depends on the wind velocity.

Following (Gill, 1982), the equation of state of dry air is approximated by the ideal-gas law:

$$\rho_{\text{dry}} = \frac{P}{RT} \quad \text{(A.4)}$$

with $R$ the gas constant for dry air. Following (Gill, 1982), the equation of state of water vapour is given by the ideal-gas law:

$$\rho_{\text{vap}} = \frac{e}{R_{\text{vap}} T} \quad \text{(A.5)}$$

with $R_{\text{vap}}$ the constant for water vapour. The density of moist air, is given by

$$\rho_{\text{ma}} = \rho_{\text{dry}} + \rho_{\text{vap}} \quad \text{(A.6)}$$

The atmospheric pressure is the total pressure in the moist air. For a mixture of gases the pressure is the sum of the partial pressures:

$$P_{\text{atm}} = P + e \quad \text{(A.7)}$$

By definition (Gill, 1982), the specific humidity of air $q_a$ at sea surface temperature is the mass of water vapour per unit mass of moist air:

$$q_a = \frac{\rho_{\text{vap}}}{\rho_{\text{ma}}} = \frac{\rho_{\text{vap}}}{\rho_{\text{vap}} + \rho_{\text{dry}}} \approx \frac{\rho_{\text{vap}}}{\rho_{\text{dry}}} \quad \text{(A.8)}$$

Using Eq. (A.4), (A.7), (A.8) and (A.9) we arrive at:

$$q_a = \frac{\varepsilon e}{P_{\text{atm}} - (1 - \varepsilon)e} \quad \text{(A.9)}$$

with the state variable $\varepsilon$ equal to

$$\varepsilon = \frac{R}{R_{\text{vap}}} = 0.62197 \quad \text{(A.10)}$$

Starting from Eq. (A.9), substituting the equations of state for dry air and water vapour, using the definition of $q_a$ we can derive for the density of moist air:

$$\rho_{\text{ma}} = \frac{P_{\text{atm}}}{RT[1 + q_a(1/\varepsilon - 1)]} \quad \text{(A.11)}$$

The saturation pressure $e_v$ of water vapour, as function of the temperature $T$ in Kelvin, is approximated by the following relation:

$$e_v = e_v(T)$$
\[
\frac{(0.7859+0.03477(T-273.16))}{(1.0+0.00412(T-273.16))}
\]

\[e_s = 10\] \hspace{1cm} (A.12)

We assume that the relative humidity of the moist air is known. The relative humidity is defined as:

\[
F_{\text{rhum}}(T_a) = \frac{\rho_{\text{vap}}(T_a)}{\rho_{\text{sat}}(T_a)} .
\] \hspace{1cm} (A.13)

Using the ideal-gas law, Eq. (A.5), the unsaturated pressure of water vapour in air at temperature \(T\) may be determined from the relative humidity and the saturation pressure:

\[
e_s(T_a) = F_{\text{rhum}} e_s(T_a)
\] \hspace{1cm} (A.14)

The specific humidity of saturated air \(q_s\) at sea surface temperature is the mass of water vapour per unit mass of saturated moist air:

\[
q_s = \frac{\rho_{\text{sat}}}{\rho_{\text{sat}} + \rho_{\text{dry}}} \approx \frac{\rho_{\text{sat}}}{\rho_{\text{dry}}}
\] \hspace{1cm} (A.15)

Because the ratio between \(e_s\) and the sea surface temperature \(T_s\) both in Kelvin, is nearly unity, the density of vapour at constant vapour pressure is approximately constant:

\[
\frac{\rho_{\text{vap}}(T_a)}{\rho_{\text{vap}}(T_s)} = \frac{T_s}{T_a} \approx 1
\] \hspace{1cm} (A.16)

The evaporation rate is dependent on the relative humidity. At maximal evaporation, the moist air near the water surface obtains a relative humidity of \(I\), before it is exchanged with unsaturated air of higher altitudes. Per unit of volume the mass of water vapour added due to evaporation reads approximately:

\[
\rho_{\text{eva}} = (\rho_{\text{sat}}(T_s) - \rho_{\text{vap}}(T_s))
\]
\[
\approx (\rho_{\text{sat}}(T_s) - \rho_{\text{vap}}(T_a))
\] \hspace{1cm} (A.17)

The moisture is transferred by bodily movement of parcels of air. This entrainment rate \(f(U_{10})\), a net vertical velocity, above the sea surface is a function of the wind speed and the temperature difference (stratification) between the sea surface and the air. For the wind speed function \(f(U_{10})\) usually an empirical linear relation is used. In literature there is a great variety in the coefficients, all based on data fitting, see (Sweers, 1976). In the function \(f(U_{10})\) proposed by POL, following (Gill, 1982):

\[f(U_{10}) = c_e U_{10} ; c_e = 0.0015\] \hspace{1cm} (A.18)

there is no evaporation at zero wind speed; the coefficient \(c_e\), sometimes called Dalton number, is dimensionless. In case of unstable stratification, cold air above a warm sea surface, the entrainment will enhance. In the formulation suggested by Proctor, also given by (Gill, 1982), however the exchange coefficient is independent of the temperature difference between sea surface and air temperature. This independence is not in accordance with the observations for the North Sea given in Table 6 of (Lane, 1989).

The formulation for the latent heat flux implemented in TRIWAQ/TEM, is given by
\[ Q_{ev} = f(U_{10}) L_v \rho_{dry} (q_s(T_s) - q_a(T)) \]  

(A.19)

with \( L_v \) the latent heat of evaporation of water given by

\[ L_v = 2.510^6 - 2.310^3 (T_s - 273.16) \]  

(A.20)

The specific humidity reads

\[ q_s(T_s) = \frac{\varepsilon e_s}{P_{atm} - (1 - \varepsilon) e_s} \]  

(A.21a)

\[ q_a(T) = \frac{\varepsilon e_a}{P_{atm} - (1 - \varepsilon) e_a} \]  

(A.21b)

with \( \varepsilon \) defined in Eq. (A.10). The relation between \( e_a \) and \( e_s \) is given by

\[ e_a = F_{rhum} e_s \]  

(A.22)

with \( e_s \) defined in Eq. (A.12).

Remark 1:
In other formulations for the latent heat flux, e.g. Sweers (1976), the evaporation rate is calculated from the difference in vapour pressure, rather than from density differences such as Eq. (A.16) or relative humidity differences Eq. (A.19). The equivalent latent heat flux then reads

\[ Q_{ev} = f(W) \ L_v \ \varepsilon \rho \frac{(e_s(T_s) - e_a(T_a))}{P_{atm}} \]  

(A.23)

The previous equivalence is based on Eq. (A.17):

\[ \rho_{eva} \approx \left( \rho_{sat}(T_s) - \rho_{vap}(T_a) \right) \]

\[ = \frac{1}{R_{vap}} (e_s(T_s) - F_{rhum}(T_a) e_s(T_a)) \]

\[ = \frac{R}{R_{vap}} \frac{1}{RT_s} (e_s(T_s) - e_a(T_a)) \]  

(A.24)

\[ \approx \varepsilon \frac{\rho_{dry}(T_s)}{P_{atm}} (e_s(T_s) - e_a(T_a)) \]

This leads to the empirical relation Eq. (A.20).

Remark 2:
In TRIWAQ/TEM the effect of evaporation on the volume of water and therefore on the salinity, is neglected.
A.5 Effective infra-red back radiation

The sea radiates heat back to the atmosphere. A part of this back radiation is absorbed and re-emitted by the atmosphere. Therefore the effective infra-red back radiation heat flux $Q_{eb}$ from the sea surface is the back radiation from the sea $Q_{br}$ minus the long wave atmospheric radiation $Q_{aw}$. It is calculated as a product of the radiation by a blackbody at sea surface temperature, the emissivity of water, and a correction factor for cloud cover and the amount of water vapour in the air (Gill, 1982):

$$Q_{eb} = \varepsilon \sigma T_s^4 (0.39 - 0.05 \sqrt{e_a})(1 - 0.6 F_c^2) . \quad (A.25)$$

Remark 3:
In other flux formulations the fluxes due to atmospheric radiation and the back radiation are calculated separately.

A.6 Sensible heat flux

It is assumed that the heat loss of sea water due to convection depends on the entrainment rate $g(W)$, the temperature difference between air and sea and the heat capacity of the air:

$$Q_{co} = \rho_v c_p g(U_{10})(T_s - T_a) \quad (A.26)$$

with for $g(U_{10})$ the wind-speed function proposed by POL, following (Gill, 1982):

$$g(U_{10}) = c_l U_{10} \quad (A.27)$$

Other authors, following Bowen (1926), use the hypothesis that the entrainment rate for heat and moisture are correlated and the ratio between the evaporative heat flux and sensible heat flux is given by the so-called Bowen ratio $\gamma$. Using the Bowen ratio the sensible heat flux is expressed in the latent heat flux:

$$Q_{co} = B Q_{ev} \quad \text{with} \quad B = \gamma \frac{(T_s - T_a)}{(e_s - e_a)} \quad (A.28)$$

For the North Sea convection represents about 8% of the total heat loss (Lane, 1989).

A.7 Equilibrium temperature

The heat balance for a body of water that is well-mixed, with total water depth $H$ and unit area, which only exchanges heat across the free surface, is given by:

$$\frac{\partial (H \rho c_{pw} T)}{\partial t} = Q_{tot} \quad (A.29)$$

with the net heat flux across the surface, $Q_t$, given by Eq. (A.1).

The equilibrium temperature $T_e$ is defined as the SST under stationary meteorological conditions when the net exchange of heat between air and water is zero. The supply in energy is balanced by and equally large loss of energy. The relaxation time to reach
equilibrium depends on the water depth. The equilibrium temperature is a theoretical value, representative of the meteorological conditions, which will in reality never be reached. The heat exchange coefficient $K$ is the derivative of the net heat flux with respect to the temperature:

$$ K = -\frac{\partial Q_{\text{net}}}{\partial T} = -\frac{\partial Q_{x}}{\partial T} + \frac{\partial Q_{\text{ev}}}{\partial T} + \frac{\partial Q_{\text{hr}}}{\partial T} + \frac{\partial Q_{\text{co}}}{\partial T} \quad (A.30) $$

Using the Eq. (A.3), (A.19), (A.25) and (A.26), we obtain

$$ \frac{\partial Q_{x}}{\partial T} = 0 \quad (A.31a) $$

$$ \frac{\partial Q_{\text{ev}}}{\partial T} = f(U_{10}) L_{v} \frac{\partial \rho_{\text{sat}}(T)}{\partial T} \quad (A.31b) $$

$$ \frac{\partial Q_{\text{hr}}}{\partial T} = 4 \varepsilon \sigma T^{3}(0.39 - 0.05 \sqrt{|e_{a}|})(1 - 0.6 F_{c}) \quad (A.31c) $$

$$ \frac{\partial Q_{\text{co}}}{\partial T} = \rho_{a} c_{p} g(U_{10}) \quad (A.31d) $$

For the evaporative flux in Eq. (A.31b) we use the expression given in Eq. (A.23) and the following approximation for specific humidity of saturated air (cf. Eq. (A.21a)):

and Eq. (A.12) for the saturated vapour pressure $e_{s}$.

$$ q_{v}(T_{v}) \approx \frac{e_{s}}{P_{\text{atm}}} \quad (A.21a') $$

A similar approximation has been applied in (Barnier et al., 1994) as well. Substituting Eq. (A.21a') into Eq. (A.23) yields

$$ \frac{\partial Q_{\text{ev}}}{\partial T} = \frac{\partial \left( f(U_{10}) L_{v} \varepsilon \rho_{a} \frac{e_{s}(T_{v})}{P_{\text{atm}}} \right)}{\partial T} $n$  

$$ = f(U_{10}) L_{v} \varepsilon \rho_{a} \frac{\partial e_{s}(T)}{\partial T} \quad (A.32) $$

$$ = f(U_{10}) L_{v} \varepsilon \rho_{a} \frac{e_{s}(T) e_{s}(log10) 0.03153}{(1.0 + 0.00412 T)^{2}} $$

$$ \text{where} \quad \frac{e_{s}(log10)}{1.0 + 0.00412 T} $$
### List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value-Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_s$</td>
<td>Dalton number</td>
<td>0.0015</td>
</tr>
<tr>
<td>$c_{Ht}$</td>
<td>Stanton number</td>
<td>0.00145</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat of air</td>
<td>1004 J/kg/K</td>
</tr>
<tr>
<td>$c_{pew}$</td>
<td>specific heat of sea water</td>
<td>3930 J/kg/K</td>
</tr>
<tr>
<td>$e$</td>
<td>vapour pressure</td>
<td>mbar</td>
</tr>
<tr>
<td>$e_a$</td>
<td>vapour pressure at a given air temperature</td>
<td>mbar</td>
</tr>
<tr>
<td>$e_s$</td>
<td>saturated vapour pressure at a given temperature</td>
<td>mbar</td>
</tr>
<tr>
<td>$F_c$</td>
<td>cloud cover</td>
<td>-</td>
</tr>
<tr>
<td>$f(F_s)$</td>
<td>attenuation function for the solar and atmospheric (long wave) radiation</td>
<td>-</td>
</tr>
<tr>
<td>$F_{rhum}$</td>
<td>relative air humidity at a given temperature</td>
<td>%</td>
</tr>
<tr>
<td>$L_v$</td>
<td>Latent heat of vaporisation</td>
<td>J/kg</td>
</tr>
<tr>
<td>$P_{am}$</td>
<td>Atmospheric pressure</td>
<td>kg/m$^3$/s$^2$</td>
</tr>
<tr>
<td>$q_a$</td>
<td>specific humidity of air</td>
<td>-</td>
</tr>
<tr>
<td>$q_s$</td>
<td>specific humidity of saturated air</td>
<td>-</td>
</tr>
<tr>
<td>$Q_{dr}$</td>
<td>atmospheric radiation (long wave radiation)</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$Q_{nr}$</td>
<td>net atmospheric radiation</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$Q_{br}$</td>
<td>back radiation (long wave radiation)</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$Q_{co}$</td>
<td>heat loss due to convection</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$Q_{ev}$</td>
<td>heat loss due to evaporation</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$Q_{s}$</td>
<td>solar radiation (short wave radiation)</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$Q_{n}$</td>
<td>net solar insolation</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$Q_{tot}$</td>
<td>heat flux through free surface</td>
<td>J/m$^2$/s</td>
</tr>
<tr>
<td>$r$</td>
<td>reflection coefficient for solar and atmospheric (long wave) radiation</td>
<td>-</td>
</tr>
<tr>
<td>$R$</td>
<td>the gas constant for dry air</td>
<td>287.04 J/(kg K)</td>
</tr>
<tr>
<td>$R_{vap}$</td>
<td>gas constant for water vapour</td>
<td>461.50 J/(kg K)</td>
</tr>
<tr>
<td>$T_{out}$</td>
<td>natural background temperature of water in the model area</td>
<td>°C</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (general reference)</td>
<td>°C or K</td>
</tr>
<tr>
<td>$T_a$</td>
<td>air temperature</td>
<td>K</td>
</tr>
<tr>
<td>$T_s$</td>
<td>water temperature at free surface</td>
<td>K</td>
</tr>
<tr>
<td>$u_{10}$</td>
<td>averaged wind speed at 10 m above free surface</td>
<td>m/s</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Albedo coefficient</td>
<td>0.06</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>estimate of Ekman layer depth</td>
<td>m</td>
</tr>
<tr>
<td>$\Delta z_s$</td>
<td>thickness of the surface layer</td>
<td>m</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>emissivity coefficient of water at air-water</td>
<td>0.985</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>state variable ($R / R_{vap}$)</td>
<td>0.62197</td>
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<tr>
<td>$\gamma$</td>
<td>extinction coefficient (measured)</td>
<td>m</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Bowen constant</td>
<td>xx</td>
</tr>
<tr>
<td>$\phi$</td>
<td>exchange coefficient for the heat flux in excess temperature model</td>
<td>J/s/m$^2$/K</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of water</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>density of air</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{dry}$</td>
<td>density of dry air</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{ma}$</td>
<td>density of moist air</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{sat}$</td>
<td>density of saturated air</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{vap}$</td>
<td>density of water vapour</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>reference density of water</td>
<td>1026 kg/m$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann’s constant</td>
<td>$5.67 \times 10^8$ J/m$^2$/s/K$^4$</td>
</tr>
</tbody>
</table>
Meteorological data for 1 April to 1 December 1994

Wind and air pressure

DELFT HYDRAULICS

Fig. 3.1
Meteorological data for 1 April to 1 December 1994
Air temperature and relative humidity

Fig. 3.2
Meteorological data for 1 April to 1 December 1994
Cloudiness (percentages)

DELFT HYDRAULICS

Fig. 3.3
TRIWAQ test basin (20 layers); Normal heat flux forcing
1994 simulation (1-4 to 1-11); VDIFF=7E-5
at INP Mooring station (depth of 47.5 m)

DELT HYDRAULICS
TRIWAQ test basin (20 layers); Combined SST-heat flux forcing
1994 simulation (1-4 to 1-11); VDIFF=7E-5
at INP Mooring station (depth of 47.5 m)

DELFT HYDRAULICS
TRIWAQ test basin (20 layers); Only SST forcing
1994 simulation (1-4 to 1-11); VDIFF=7E-5
at INP Mooring station (depth of 47.5 m)

DELFT HYDRAULICS

Fig. 3.6
TRIWAQ test basin (20 layers); Only SST forcing, multiplied by 10
1994 simulation (1-4 to 1-11); VDIFF=7E-5
at INP Mooring station (depth of 47.5 m)

DELFT HYDRAULICS

Fig. 3.7
TRIWAQ test basin (20 layers);
1994 simulation (1-4 to 1-11); VDIFF=7E-5
Difference with and without corrected heat flux

DELFIT HYDRAULICS
Evolution thermal stratification; 3D model 20 layers
1994 simulation (1 April to 1 November)
Measurements versus model results at IMP Mooring station

DELFt HYDRAULICS

Fig. 3.9
3D North Sea model basin (20 layers), 1-4-1994 to 1-11-1994
Comparison of remote sensing data with model results
Temperature at surface layer at 21 July 1994

DELFT HYDRAULICS

Fig. 4.1
3D North Sea model basin (20 layers); 1-4-1994 to 1-11-1994
Comparison of remote sensing data with model results
Temperature at surface layer at 4 August 1994

Fig. 4.2
3D North Sea model basin (20 layers); 1-4-1994 to 1-11-1994
Comparison of remote sensing data with model results
Temperature at surface layer at 18 August 1994

DELFT HYDRAULICS
3D North Sea model basin (20 layers); 1-4-1994 to 1-11-1994
Comparison of remote sensing data with model results
Temperature at surface layer at 15 September 1994

DELFT HYDRAULICS

Fig. 4.4
3D North Sea model basin (20 layers): 1-4-1994 to 1-11-1994
Comparison of remote sensing data with model results
Temperature at surface layer at 26 October 1994

DELFT HYDRAULICS

Fig. 4.5
3D North Sea model basin (20 layers), 1-4-1994 to 1-11-1994
Difference with Remote Sensing SST
Averaged over model domain

DELFT HYDRAULICS

Fig. 4.7
3D North Sea model basin (20 layers); 1-4-1994 to 1-11-1994
Difference with Remote Sensing SST
Averaged over model domain

DELFT HYDRAULICS

Fig. 4.8