Multipath and Multi-Transmitter Interference in Spread-Spectrum Communication and Navigation Systems

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SUMMARY

Two major limiting effects in radio communication and navigation systems are multipath and multi-transmitter interference. In the case of communication systems, multipath and multi-transmitter interference seriously degrade the bit error probability and hence limit the system capacity. In the case of navigation systems, a receiver estimates the phase or delay of a transmitted signal, which is directly related to the distance to that particular transmitter. The presence of multipath or multi-transmitter interference generally causes errors in the phase or delay measurements, thereby degrading the navigation accuracy.

The aim of this thesis is to analyze the effects of multipath and multi-transmitter interference in direct-sequence spread-spectrum systems and to find ways to improve the receiver performance. The main focus of the thesis is on satellite systems. As an exception to this, one chapter is devoted to the land-based Loran-C system, which can be regarded as a special case of a spread-spectrum navigation system, suffering from similar problems with multipath and multi-transmitter interference.

The first step of the analysis is the examination of the radio channel properties. In chapter 2, an existing narrowband propagation model is extended to a wideband model in order to evaluate the performance of spread-spectrum communications. In chapter 3, it is demonstrated that multipath propagation can significantly decrease the accuracy of delay and phase estimates. For navigation systems, this means that multipath propagation often determines the ultimate position accuracy. For communication systems, multipath propagation can degrade the bit error probability, unless the tracking loop bandwidth is chosen properly. An interesting relation is found between the bandwidth of the spread-spectrum signals and the noise and multipath errors of the code tracking loop. Several measurements are shown to validate and demonstrate the theoretical analysis.
The next step is to find ways to improve the performance. Conventional receivers are often designed to deal with noise only, not with multipath or multi-transmitter interference. If these disturbances are taken into account in the design of the receiver, it is possible to achieve considerable performance improvements. This is demonstrated in chapters 4 and 5, where new GPS and Loran-C receiver structures are described, which are capable of significantly reducing errors caused by multipath, continuous wave interference and cross-rate interference. Prototype GPS and Loran-C receivers have been built which show the feasibility of the new estimation techniques.
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1 INTRODUCTION

Although communication and navigation systems are often thought of as completely different systems, their design actually shows many similarities. In fact, it can be stated that any communication system in principle can also be used for navigation, because the carrier phase and envelope delay of the received signals contain information about the propagation time, and hence about the position of the receiver with respect to the transmitter location.

As the system architectures are very similar, both communication and navigation systems suffer from the same sources of errors. Besides noise, two main causes of degradation are multipath propagation and multi-transmitter (or multi-user) interference. This dissertation addresses these problems for both communication and navigation systems, in particular systems using spread-spectrum modulation.

1.1 Spread-spectrum modulation for communication systems

In the early days of radio communications, there was a strong emphasis on narrowband modulation techniques, i.e., a certain signal was sent with an as small as possible bandwidth, in order to maximize the signal-to-noise ratio at the receiver output. Probably the first to break with this tradition was Goldsmith, who filed a patent in 1924 in which he described a transmission system that used a continuously varying carrier frequency [1]. The idea was to decrease the effects of multipath fading by spreading the transmitted signal over a bandwidth larger than the coherence bandwidth of the channel. This made it possible to avoid unacceptable attenuations, as occur in certain narrow frequency bands due to multipath propagation. Thus, Goldsmith's invention clearly showed one of the advantages of using a technique which became known as 'spread-spectrum' modulation, defined as a means of transmission in which a certain code is used to spread a transmitted signal over a bandwidth that is larger than the bandwidth of the modulating data signal.
During the first few decades of spread-spectrum technology, the research was primarily focused on military applications. The most attractive spread-spectrum properties in this respect were low probability of intercept, antijam capability\(^1\) and the possibility of secure communications. In the first spread-spectrum systems, spreading was accomplished by applying a continuous modulation pattern to the carrier frequency. During World War II, frequency hopping (FH) and time hopping (TH) were invented [1]. In frequency hopping, the carrier frequency is varied over a certain set of frequencies, while for time hopping, the signal is transmitted in bursts with variable duration. Around 1950, direct-sequence spread-spectrum was introduced, where the bandwidth spreading was performed by multiplying the data signal by a spreading code, as shown in figure 1.1. Although all spread-spectrum methods have their advantages and disadvantages, this dissertation primarily deals with direct-sequence spread-spectrum modulation.

\[
\begin{align*}
\text{Bit time} & \quad \text{Chip time} \\
1 & \quad 1 \\
-1 & \quad -1 \\
-1 & \quad -1 \\
1 & \quad 1 \\
p(t) & \quad d(t)p(t) \\
d(t) & \\
\end{align*}
\]

*Fig. 1.1: Direct-sequence spread-spectrum modulation. \(d(t)\) and \(p(t)\) are the data and spread-spectrum code, respectively.*

For many years, spread-spectrum techniques remained exclusively military domain. Only in the late seventies was it realized that bandwidth spreading could be beneficial in civil applications as well [2]. The two most important advantages for civilian use are coexistence with other systems in the same frequency band and the possibility to achieve a high spectrum efficiency in wireless networks. Coexistence

\(^1\) Provided that the narrowband interference power divided by the spread-spectrum signal power is small compared to the processing gain, which is the number of chips or hops per data bit.
with other types of services is possible because of the low spectral density of spread-spectrum systems, which makes it possible to transmit without disturbing existing narrowband transmissions in the same frequency band. Concomitantly, the narrowband transmitters do not disrupt the spread-spectrum transmission, because of the antijam capability. The possibility to achieve a high spectrum efficiency using spread-spectrum techniques has been the main reason for most research in this area during the past decade. The basis for using spread-spectrum modulation in mobile networks is the ability to use Code Division Multiple Access (CDMA). In this counterpart of Time and Frequency Division Multiple Access (TDMA, FDMA), several users can simultaneously get access to the same frequency band by using different spreading codes.

There seems little doubt that CDMA systems can provide higher capacities than existing networks employing TDMA or FDMA techniques [3-7]. However, in order to make a fair comparison, it should be taken into account that conventional FDMA or TDMA systems can be significantly improved by adding such features as dynamic channel allocation [8]. It may be expected that the comparison of optimized CDMA, FDMA and TDMA systems will show capacity differences that are much smaller than the values currently mentioned in several papers [3-7]. Nevertheless, because of its unique characteristics, spread-spectrum CDMA will undoubtedly continue to play a role in future communication systems, perhaps even combined with FDMA and TDMA techniques, as suggested in [9].

While most of the research on wireless communications traditionally focused on specific, separate systems, today most of the emphasis is on the integration of different networks and various types of services. In the end, this should lead to a Global Personal Communications Network. This ultimate aim of telecommunications should make it possible to communicate from anyone to anyone, at any place and at any time [10]. It is clear that satellites will play an important role in bringing communications to 'any place', since land-based networks will never cover the entire earth. Therefore, together with the strong growth of land-based communications, there is also a rapidly increasing interest in mobile satellite communications [11-17]. One of the aims of this dissertation is to analyze the possibilities of direct-sequence CDMA for land-mobile satellite links.
1.2 Spread-spectrum modulation for navigation systems

Historically, visual observations were the basis for navigation. An estimated position was based upon observations of stars, the sun, landmarks or lighthouses. With the introduction of the radio at the beginning of the twentieth century, it became possible to use radio beacons that could overcome two major problems: 1), the limited visual range to landmarks and lighthouses, and 2), the absence of visual observations in bad weather conditions.

Early radio beacons used angular measurements for positioning. During World War I, American mariners measured the direction to two or more 375 kHz transmitters by looking for the null in a rotating loop antenna. The intersection of the different lines of direction would give them a position, which worked up to a thousand miles from the shore [18].

In fact, the method of measuring directions to different radio beacons was a direct translation of the use of visual observations. Later, it became possible to measure the phase of signals from different transmitters, which could be used to estimate the propagation time of the signal, and hence the distance between receiver and transmitter. Knowing the positions of the transmitters, it was then possible to calculate the receiver position.

A problem with using carrier phases is the ambiguity; the measured phase, divided by $2\pi$, only provides a fractional part of a wavelength, while there is no way to directly estimate the integer number of wavelengths between receiver and transmitter. To solve this problem, systems like Omega and Decca transmit at several frequencies; The smallest frequency difference of the different carrier waves determines the largest wavelength ambiguity that can be obtained. A more elegant solution is to apply a special modulation signal with a long repetition time, so that it has a large unambiguous range. If a receiver can estimate the delay of the modulation signal with an accuracy better than half of a carrier wavelength, then it can resolve the ambiguity of the carrier signal.
In order to achieve a large unambiguous range, the modulation signal should have a long repetition time. Conversely, an accurate delay measurement requires that there is a short time between signal transitions or, equivalently, a large signal bandwidth. A modulation type that combines both of these aspects is direct-sequence spread-spectrum modulation. It employs spreading codes with repetition times that can easily be made arbitrarily large in comparison with the chip time, which is the smallest time between signal transitions, and hence determines the accuracy of delay measurements.

1.2.1 GPS

The most prominent example of a spread-spectrum navigation system is the Global Positioning System (GPS), which was designed by the United States Department of Defense in the seventies. GPS consists of 24 satellites, placed in circular orbits with an approximately 12-hour orbital period. Figure 1.2 illustrates the GPS constellation. The spacings between the satellites are arranged such that at least 4 satellites with elevation angles exceeding 5 degrees are in view worldwide, at any time. Range measurements to at least four satellites are sufficient to find a 3-dimensional position, plus the receiver clock offset as compared to GPS time.

Ranging to different satellites is accomplished by measuring the delays of the received spread-spectrum codes. Multiplication of these delays by the propagation velocity gives the corresponding ranges. The positioning accuracy is degraded by a number of error sources, such as the satellite clock, ephemeris, troposphere, ionosphere, geometry, noise, interference and multipath propagation. In order to deny non-NATO military users the use of high accuracy positioning capabilities, an intentional accuracy degradation is implemented by dithering the satellite clock and introducing errors in the ephemeris data. The intentional degradation process, known as Selective Availability, together with the normal error sources, result in a positioning accuracy of about 100 and 300 meters for 95% and 99.99% of the time, respectively [19]. Without Selective Availability, the accuracy would be in the order of 30 meters for 95% of the time. These figures should be taken with some caution, since they represent global averages. This means that for a specific location on earth,
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the accuracy might be significantly better or significantly worse, depending on the local geometry.

![Satellite constellation of the Global Positioning System](image)

*Fig. 1.2: Satellite constellation of the Global Positioning System.*

Although GPS provides an unsurpassed positioning accuracy on a global scale, it is still insufficient for many applications, like the guidance of vessels in harbor approaches and of aircraft during approach and landing. Another important aspect of these types of safety-critical operations is the *integrity*, defined as the possibility to produce timely warnings when the system is not working according to the specifications. During an aircraft precision approach, for instance, an aircraft should be warned within one second whenever some malfunctioning of the navigation system occurs [19]. GPS cannot provide this level of integrity, since it can take up to two hours to detect a satellite failure and warn the users by updating the navigation message.

To enhance both accuracy and integrity, a number of systems have been proposed and implemented. All of them use *monitor* or *reference* receivers, which verify the instantaneous accuracy of all received GPS signals, together with transmitters to
send the integrity and/or positioning information to the users. Since a reference receiver is located in a known position, it can estimate the ranging errors from different satellites. As a large part of these errors only slowly decorrelate in time and distance from the reference receiver, the surrounding users can largely reduce their positioning errors by subtracting the estimated range errors from their measurements. This principle is known as differential positioning. Depending on the user-reference separation, the transmission delay of the range error messages, and the magnitude of remaining errors such as noise and multipath, positioning accuracies in the range of one to ten meters can be achieved. The use of carrier phase techniques makes it even possible to achieve centimeter level accuracy. However, these types of techniques are not well suited for time-critical operations, since the process of carrier ambiguity resolution after occurrence of loss-of-lock or cycle slips takes too much time. An exception to this may be the use of pseudolites, which are additional GPS transmitters, located on the ground and transmitting normal GPS signals plus differential corrections and integrity messages [20].

In addition to GPS, there is currently a Russian satellite navigation system partly in operation, the Global Orbiting Navigation Satellite System (Glonass). Further, a number of augmentations are currently under development, the purpose of which is to provide additional integrity messages, differential corrections and extra ranging signals [21-24]. Combinations of different systems, together forming one navigation system based on satellite technology, are often referred to as Global Navigation Satellite System (GNSS).

One of the most exciting and demanding applications of GNSS is in the field of aircraft navigation. Currently, there is a considerable debate as to whether GNSS could be used instead of the Microwave Landing System (MLS) for precision approach and landing. MLS is a system which is already fully operational. It is designed to meet navigation performance specifications up to the most severe CAT III (Category III) requirements of 60 centimeters vertical and 4 meters lateral errors (2 sigma values) [19]. Although existing satellite navigation systems like GPS and Glonass cannot meet the CAT III specifications at the moment, there seems little doubt that the use of differential GPS - either code DGPS integrated with an inertial navigation system [25] or carrier DGPS augmented by pseudolites [20] - will make it possible to achieve the same performance as MLS. In the end, however, two
important differences remain between MLS and GNSS, both of which tend to be in favor of MLS. The first one is the susceptibility to interference or jamming. As the received power from GNSS satellites is in the order of 70 dB less than MLS near the runway, it is far easier to jam GNSS than MLS. Even though spread-spectrum modulation does provide some level of antijam capability, the received signal power is so low - about -160 dBW for GPS - that one Watt of radiated power is sufficient to completely block the reception of GPS signals within a range of 30 km, as demonstrated in [28]. This great sensitivity means that GPS is not only vulnerable to intentional jamming, but also to unintentional jamming sources such as mobile satellite or radar transmitters that operate closely to the GPS L1 frequency of 1575.42 MHz, or even to the harmonics of television transmitters [29]. For MLS, the interference problem is far less severe: Since the MLS transmitters are located close to the runway, jamming would require about the same effectively radiated power as transmitted by the MLS system itself, which is in the order of 1000 Watt [28].

The second difference between MLS and GNSS is the siting procedure. During the installation of MLS equipment at a certain airport, special care must be taken to avoid multipath signals coming from hangars or other aircraft. Since the MLS antennas are highly directional, it is relatively easy to determine which areas near the runway should be free from obstacles in order to avoid unacceptable multipath levels. For GNSS, there is a similar problem, since there has to be a differential GNSS reference receiver located either at or close to the airport. However, in contrast to MLS, GPS signals arrive from all possible directions, which means that all of the surrounding environment contributes to the received amount of multipath power. If the reference receiver has to meet CAT III requirements based on code measurements, then the standard deviation of the code ranges has to be in the order of a decimeter. As shown in this dissertation, such accuracy requires a signal-to-multipath ratio in the order of 40 dB. Although not impossible, the siting procedure will definitely be more difficult than for MLS.

Besides these two technical differences, there is at the moment also an important political difference between MLS and GNSS; GPS, the inevitable basis for GNSS, is under the control of the United States Department of Defense, which does not give any guarantee on any of the GPS specifications. This makes it very unlikely that non-US aviation authorities will accept the use of GPS instead of MLS. However,
since GPS is available anyway, it could be an attractive approach to use both MLS and GPS, as proposed in [26,27]. Thus, it is possible to benefit from GPS without the need to fully rely on it.

1.2.2 Loran-C

While GPS is currently the most widely used satellite navigation system, the most popular land-based system is Loran-C, which is an acronym of Long Range Navigation. The Loran-C signal modulation can be described as a combination of spread-spectrum time hopping and direct-sequence modulation. Each transmitter sends bursts containing 8 pulses with a time interval of 1 ms. The bursts are transmitted at regular timing intervals, which are different for different groups of transmitters. This means that the signals from different transmitters may overlap for a certain percentage of time, as in any asynchronous time hopping system. By carefully selecting the burst interval times, better known as Group Repetition Interval (GRI), it is possible to minimize the effects of multi-transmitter or Cross-Rate Interference. Within one group or chain of Loran transmitters, the same GRI is used, together with a certain transmission delay to avoid interference from other transmitters from the same chain. Thus, within a chain, synchronous time hopping or, equivalently, time division multiple access is used. Further, there is a direct-sequence component in the signal modulation: The transmitted pulses are multiplied by a phase code, which can take values of \{-1,+1\} and repeats after 16 pulses or 2 GRIs.

Loran-C can provide an absolute positioning accuracy in the order of 400 meters. However, with a repeatable accuracy in the order of 50 meters, Loran-C is even better than GPS with selective availability degradation. The repeatable accuracy is the accuracy with which a user can return to a previously measured position. Thus, errors that are constant in time do not affect the predictable accuracy. Since Loran-C errors are largely caused by groundwave propagation effects, which remain fairly stable in time, the repeatable accuracy is an order of magnitude better than the absolute accuracy. This makes Loran-C a suitable candidate for integration with GPS
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in order to improve availability and integrity [30-32]. The absolute positioning accuracy of GPS can be used to calibrate the slowly time varying Loran-C errors, so that in the case of a GPS outage, Loran-C measurements would be available with an accuracy that is comparable to the accuracy of GPS.

An even more interesting aspect of Loran-C with respect to integrated systems is its ability to transmit differential GPS corrections and integrity messages, as proposed for the Eurofix system [19,30,33]. This capability obviates the need for setting up a new and costly network of transmitters. A disadvantage is the small capacity of the Loran communication channel, in the order of 20 bits per second. However, this should be enough to achieve a differential GPS accuracy of approximately 5 meters, which meets the navigation requirements for vessels during harbor approaches [19,30,33].

1.3 Integration of navigation and communication systems

With the strong growth of radio communication networks during the past decade, we have now arrived at the situation where there are several separate communication and navigation systems that use a similar type of infrastructure, consisting of a number of transmitters on land or in space, covering large parts of a continent, or even operating worldwide. In order to save costs in both infrastructure and receiver equipment, it seems a logical step to integrate communication and navigation services. In fact, this development is already in progress with the introduction of systems as OmniTRACS, which offers two-way messaging and positioning using geostationary satellites [12]. Further, a number of low earth orbit (LEO) satellite networks are currently under development. They are planned to provide both communication and positioning capabilities. Examples of these systems are Iridium, Globalstar, ORBCOMM and Odyssey [13,17]. Each of them consists of a large number of satellites, circling at altitudes in the order of 1000 km. The aim of these LEO networks is to provide global connectivity by operating in conjunction with public switched telephone networks and private mobile radio networks. If the
accuracy of their positioning capabilities is made high enough, they might become serious competitors of - or a welcome supplement to - existing satellite navigation systems such as GPS and Glonass.

In addition to satellite networks, there are currently several land-based mobile radio networks that also could be used for positioning. Consider, for example, the Global System for Mobile communications (GSM), a digital cellular mobile radio network that is operating over a large part of Europe as well as in several countries outside Europe. With this system, a user can generally receive several base stations that transmit data at a rate of about 270 kbit/s [35]. If a receiver is able to synchronize to three or more base stations with an accuracy of less than one percent of a bit time, then it could measure ranges and positions with an error in the order of only ten meters, provided that the base stations are accurately synchronized and that the geometry is good enough, i.e., the base stations should be received from different directions. If the user is able to do active ranging, where the base station sends a signal in response to a user's signal, then the requirement of synchronized base stations can be dropped, at the cost of an increased load of the network. Note that the mentioned accuracy of ten meters can only be achieved when there is a dominant line-of-sight signal. In practice, the line-of-sight path will often be shadowed, resulting in ranging errors of up to hundreds of meters, because the receiver is just tracking multipath signals. However, there are some ways to detect whether a certain signal is from the line-of-sight path or not, for instance, by looking at the signal strength or by comparing the measured position or velocity with those derived from other systems, e.g. odometers. Such an integrated system could make it possible to reject erroneous ranging measurements and use the most accurate ranges to improve the position solution.

Among the applications of integrated communication and navigation, the most obvious are probably in the area of Intelligent Vehicle Highway Systems (IVHS). IVHS is a collective term for all kinds of electronic services to travellers, ranging from route guidance to a form of automatic car driving, called platooning [36]. Many IVHS services depend on knowledge of the vehicle's position, with a required accuracy in the order of 10 meters for functions like route guidance. Positioning is based upon some combination of odometers, compasses, map matching and a form of radiopositioning, which is necessary to provide an initial position and to correct
the slowly accumulating errors from the first three devices mentioned. Many IVHS proposals use GPS as the radiopositioning device. However, since IVHS also depends heavily on communications, in order to provide all sorts of travellers' information, it may be more efficient to use the communication network for positioning as well.

1.4 Multipath propagation and multi-transmitter interference

In general, the performance of any radio channel, whether used for communications or navigation, is limited by the following 7 factors:

1) Transmitted power
2) Antenna gains
3) Additive noise
4) Path loss
5) Shadowing
6) Multipath propagation
7) Multi-transmitter interference

The first four factors determine the maximum signal-to-additive noise ratio at the receiver. The actual signal-to-noise ratio depends on the amount of shadowing and multipath propagation. Multipath can actually have two effects: for multipath delays less than a bit or a chip, the effect is a fading of the received signal amplitude. Larger multipath delays result in inter-symbol interference. Systems for which the signal-to-noise ratio is the only parameter of interest are called noise-limited systems. For many systems, however, multipath and multi-transmitter interference are often more important than the signal-to-noise ratio. In cellular mobile radio systems, for instance, users and base stations of cells with a certain distance separation transmit at the same frequency, thereby causing a certain amount of interference. In communications, this type of interference is known as co-channel or multi-user interference. In this thesis, the more general term 'multi-transmitter
interference' is used, because this also includes interference caused by different transmitters in radionavigation systems.

Besides multi-transmitter interference, multipath propagation usually also has significant effects on the behavior of radio links. Multipath propagation results in a kind of self-interference, since the received signal consists of several replicas of the desired signal, all with different delays, phases and amplitudes. The addition of these signals results in a time-varying amplitude and phase. Further, inter-symbol interference occurs when the multipath delay spread is in the order of, or larger than, the symbol time. All of these effects tend to decrease the system performance.

Compared to narrowband modulation, spread-spectrum modulation has certain advantages with respect to multipath and multi-transmitter interference. The bandwidth spread makes it possible to separate different multipath signals if their relative delay exceeds one chip. This means that deleterious effects of multipath fading can be reduced [37]. However, since most spread-spectrum systems use spreading codes with non-ideal auto-correlation properties, the interference from multipath signals with relative delays of more than a chip is not negligible [37,38]. Similarly, non-ideal cross-correlation properties of different transmitted codes result in multi-transmitter interference. Further, just like narrowband modulation systems, spread-spectrum systems suffer from multipath signals with relative delays smaller than a chip. For spread-spectrum communication systems, multipath and multi-transmitter interference limit the bit error probability, and hence the maximum capacity of a multiple-access system. For navigation systems, the most important parameters are the delay and phase of the direct path signal. As shown in this thesis, the accuracy of delay and phase estimates largely depends on the amount of multipath and multi-transmitter interference.

1.5 Organization of the thesis

The basic objective of this thesis is to present a performance analysis of spread-spectrum communication and navigation systems in the presence of multipath and multi-transmitter interference.
Chapter 2 describes the analysis of a satellite communication system, which uses direct-sequence spread-spectrum modulation. After introducing the channel and the receiver models, the performance is evaluated in terms of bit error and outage probability, throughput and delay. One of the advantages of direct-sequence spread-spectrum over narrowband communications is the possibility to largely separate multipath signals with a delay spacing of at least one chip time. By demodulating and combining several distinct path signals, it is possible to achieve a considerable reduction of the bit error probability.

In order to successfully demodulate a spread-spectrum signal, it is essential to obtain accurate code and carrier synchronization. This is especially important for navigation systems, because these systems derive range and position estimates directly from the measured code delays and carrier phases. Therefore, chapter 3 analyzes the code and carrier tracking errors caused by multipath and multi-transmitter interference. It is shown that the impact of tracking errors on communication systems can be kept fairly low by properly choosing the tracking loop parameters. For navigation systems, however, multipath propagation especially can have a disastrous effect on accuracy. Measurements from the Global Positioning System are used to validate the theoretical analysis and to demonstrate the seriousness of the multipath problem.

In chapter 4, it is explained that the tracking errors caused by multipath are not completely unavoidable: Conventional receivers are simply not designed to deal with multipath. Using maximum likelihood estimation theory, it is possible to design a new receiver that takes the presence of multipath into account. The theoretical tracking error limits of this receiver are derived; they are shown to be a function of the input bandwidth of the spread-spectrum signals. The performance improvement of the proposed receiver over that of a conventional receiver is clearly demonstrated by several measurements, using a prototype GPS receiver.

Although the main portion of the thesis focuses on direct-sequence spread-spectrum modulation in satellite systems, it is possible to use the theory developed in related systems. As an example of this, chapter 5 describes how the receiver concept given in chapter 4 can be modified in order to apply it to the land-based Loran-C navigation system. Loran-C can be regarded as a special type of spread-spectrum
system, which suffers from multipath caused by ionospheric reflections and from the interference of other Loran-C beacons, and different types of transmitters that operate close to the Loran band. A prototype Loran-C receiver demonstrates the feasibility of new estimation techniques which estimate both the desired and the interfering signals. Finally, chapter 6 summarizes and discusses the main results of the work presented in this dissertation.

1.6 References in chapter 1


Introduction


[34] Department of Transportation, United States Coast Guard, 'Specification of the transmitted Loran-C signal', Washington, July 1981.


2 DIRECT-SEQUENCE SPREAD-SPECTRUM MODULATION FOR SATELLITE COMMUNICATIONS

There is no doubt that land-mobile satellite systems will be an important part of future global personal communication systems, complementing the public switched telephone network and cellular networks [1]-[4]. Just as for land based cellular systems, one of the most important requirements of satellite communication systems is bandwidth efficiency. As discussed in the previous chapter, spread-spectrum Code Division Multiple Access (CDMA) is one of the possibilities to achieve high system capacities. In [5] and [6], it is concluded that CDMA yields a better performance than narrowband frequency division multiple access (FDMA) in the case of circuit-switched mobile satellite communications. In [7]-[17], it is pointed out that CDMA can also be beneficial in the case of packet-switched radio networks. However, the analysis of multiple access techniques in [7]-[10] did not take into account the fading characteristics of the radio channel, which is usually an important limiting factor, especially for satellite communications. The influence of the channel characteristics on the performance of direct-sequence CDMA is the main subject of this chapter.

Previous to the performance analysis, descriptions are given of the land-mobile satellite system under consideration, the receiver model and the model of the radio channel. Then, the performance is evaluated in terms of bit error probability, outage probability, throughput and delay. The analysis includes the use of both diversity techniques and forward error correction. Finally, the influence of synchronization errors on the performance parameters is discussed.
2.1 System description

The system under consideration consists of a satellite hub station and a number of mobile terminals on the earth's surface, as depicted in figure 2.1. The hub station may act like a repeater or it may process the signals on-board. On-board processing has the advantage that the relative amounts of noise, shadowing and multipath are reduced. Further, it may be necessary to do on-board processing in the case of a network of several satellites, or in the case of multiple spot beams per satellite, where the hub station has to make certain routing decision for each message. In this chapter, the system performance is analyzed for a single link only. If one would like to apply the following analysis in the case of a repeater hub station, then it should be kept in mind that the relative amounts of noise, shadowing and multipath are, on the average, twice as large.

Fig. 2.1: Land-mobile satellite system.
2.1.1 Channel model

A statistical propagation model for a narrowband land-mobile satellite channel in rural and suburban environments was developed in [18,19,20]. It assumes that the received signal is the sum of a shadowed line-of-sight signal with a lognormal envelope distribution and a sum of multipath signals with a Rayleigh distributed envelope. The resulting probability distribution of the received signal envelope $r$ is given by (see derivation in appendix A):

$$p_r(r) = \frac{r}{b_0 \sqrt{2 \pi \sigma_0^2}} \int_0^\infty \exp \left[ -\frac{(\ln(z) - \mu_0)^2}{2 \sigma_0^2} - \frac{r^2 + z^2}{2 b_0} \right] \frac{I_0(\frac{r z}{b_0})}{z} \, dz \quad (2.1)$$

where $I_0(.)$ is the modified Bessel function of the first kind and zeroth order, $b_0$ is the average scattered power due to multipath, and $\mu_0$ and $\sigma_0^2$ are the shadowing mean value and variance, respectively, of the logarithm of the amplitude.

The phase distribution function $p_\theta(\theta)$ of a lognormally shadowed Rician signal can be found by using equation (4.5.19) of [21], which gives the phase distribution $p(\theta)$ of the sum of a Rayleigh phasor and a phasor with a zero phase and a random amplitude distribution $p(z)$ as:

$$p(\theta) = \frac{1}{2\pi} \int_0^\infty \exp \left( -\frac{z^2}{2b_0} \right) \left[ 1 + G \sqrt{\pi} \exp(G^2) \right] [1 + \text{erf}(G)] p(z) \, dz \quad (2.2)$$

where

$$G = \frac{z \cos(\theta)}{\sqrt{2b_0}} \quad (2.3)$$

and

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt \quad (2.4)$$
By substituting the lognormal probability density function for $p(z)$ in (2.2), the following distribution is obtained for the phase of a shadowed Rician phasor.

$$p_\theta(\theta) = \frac{1}{\sqrt{8\pi^3\sigma_\theta^2}} \int_0^\infty \exp\left(\frac{-z^2}{2b_0} - \frac{(\ln(z) - \mu_0)^2}{2\sigma_\theta^2}\right) \left(1 + G\sqrt{\pi} \exp(G^2)[1 + \text{erf}(G)]\right) dz$$

(2.5)

The phase variance is defined as:

$$\sigma_\theta^2 = \int_{-\pi}^{\pi} \theta^2 p_\theta(\theta) d\theta$$

(2.6)

Note that the mean phase $\mu_\theta$ is zero because $p_\theta(\theta) = p_\theta(-\theta)$. Equations (2.5) and (2.6) are used only to determine the effect of spread-spectrum modulation on the phase variance. The combined effect of phase variation and envelope fading [19] is evaluated by using the following equation (2.7) as an approximation for (2.5).

In [19] it was found that the probability density function of the received signal phase $\theta$ can be approximated by a Gaussian distribution. This approximation can be expected to be less accurate for small signal-to-multipath ratios, something that was not recognized in [19]. The Gaussian approximation is given as:

$$p_\theta(\theta) = \frac{1}{\sqrt{2\pi\sigma_\theta^2}} \exp\left[-\frac{(\theta - \mu_\theta)^2}{2\sigma_\theta^2}\right]$$

(2.7)

where $\mu_\theta$ and $\sigma_\theta^2$ are the mean and the variance of the received signal phase, respectively.

The above model is valid for a narrowband system. In order to apply the model to wideband systems, some modification has to be made. If spread-spectrum modulation is used with a chip time $T_C$ that is less than the delay spread $T_s$ of the
channel, then the multipath power is partially reduced by the correlation operation in the receiver. The envelope and the phase distribution functions remain the same, but the values for $b_0$ and $\sigma_0$ are reduced.

In the following analysis, the wideband radio channel is modeled as a number of paths with relative delays equal to an integer multiple of the chip time, which consist of clusters of signals that have a relative delay much smaller than a chip. The impulse response can be written as:

$$h(t) = \sum_{m=0}^{M-1} r_m \delta(t - \tau_m) e^{j\theta_m}$$  \hspace{1cm} (2.8)

where $r_m$, $\tau_m$, and $\theta_m$ are the gain, delay and phase of the $m$th path, respectively. The first path contains the line-of-sight signal and a part of the multipath signals. Therefore, its path gain and phase are described by (2.1) and (2.7). However, the parameters $b_0$ and $\sigma_0$ will be smaller than for the narrowband model, since only a part of the multipath power is included in the first path. The other paths contain only multipath signals with a Rayleigh path gain distribution and a uniformly distributed phase. The parameters of the various path distributions can be found if the power-delay profile is known. For rural and suburban environments, the average power-delay profile of multipath signals is approximately exponential [22,23]:

$$P(\tau) = \frac{1}{T_s} b_0 \exp\left(-\frac{1}{T_s} \tau\right)$$  \hspace{1cm} (2.9)

where $T_s$ is the delay spread. For a rural environment, a typical value of $T_s$ is 0.65 $\mu$s [23].

The power-delay profile can be used to calculate the average multipath power for each path. For path number $m$, the multipath power $b_{m0}$ is approximated as the power between the delay values $mT_c$ and $(m+1)T_c$, where $T_c$ is the chip duration:

$$b_{m0} = b_0 \left[ 1 - \exp\left(-\frac{T_c}{T_s}\right)\right] \exp\left[-m\frac{T_c}{T_s}\right]$$  \hspace{1cm} (2.10)
Table 2.1 lists the channel parameters of the shadowed Rician probability density function, as given by \([20]\), together with the calculated parameters in the case of spread-spectrum for \(T_s/T_c=6.5\). A small problem that arises when using these values is the fact that the power of the shadowed Rician function is not normalized to one. In fact, the signal power in the case of light shadowing is greater than one, which can result in bit error probabilities that seem to be smaller than theoretically possible. To solve this problem, the signal-to-noise ratio can be normalized by dividing the signal values by the square root of the total shadowed Rician signal power \(C=2b_0+\exp(2\mu_0+2\sigma_0^2)\).

<table>
<thead>
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<th></th>
<th>Light</th>
<th>Average</th>
<th>Heavy</th>
</tr>
</thead>
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<td>(b_0)</td>
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<td>0.126</td>
<td>0.0631</td>
</tr>
<tr>
<td>(\mu_0)</td>
<td>0.115</td>
<td>-0.115</td>
<td>-3.91</td>
</tr>
<tr>
<td>(\sigma_0)</td>
<td>0.115</td>
<td>0.161</td>
<td>0.806</td>
</tr>
<tr>
<td>(\sigma_b)</td>
<td>0.40</td>
<td>0.47</td>
<td>1.55</td>
</tr>
<tr>
<td>(\sigma_b) narrowband</td>
<td>0.14</td>
<td>0.16</td>
<td>1.42</td>
</tr>
<tr>
<td>(\sigma_b) spread-spectrum</td>
<td>0.023</td>
<td>0.018</td>
<td>0.009</td>
</tr>
</tbody>
</table>

2.1.2 Receiver model

In the case of binary phase shift keying modulation, the total received signal is:

\[
r(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} A r_{mk} p_k(t-\tau_{mk}) d_k(t-\tau_{mk}) \cos(\omega_c + \omega_{mk} t + \theta_{mk}) + n(t) \quad (2.11)
\]

where \(m\) and \(k\) denote the path and the user number, respectively. \(A\) is the transmitted signal amplitude, which is assumed to be constant and identical for all users. For user \(k\), \(p_k(t)\) is the spread-spectrum code, \(d_k(t)\) is the data signal, \(\omega_c + \omega_{mk}\) is the
carrier plus Doppler angular frequency, $\theta_{mk}$ is the carrier phase, $\tau_{mk}$ is the time delay and $n(t)$ is white Gaussian noise with a two-sided spectral density of $N_0/2$. The instantaneous path amplitude is denoted as $r$.

The received signal $r(t)$ is converted to baseband and correlated with a particular user code. Assuming that the receiver is able to track the code and carrier phase of path $j$ from user $i$, a signal sample of the correlation output can be written as:

$$z_0 = A r_{ji} d_i^0 + \sum_{k=0}^{K-1} I_k + \eta_i$$  \hspace{1cm} (2.12)

where $\eta_i$ is a zero-mean Gaussian variable with variance $N_0T_b$, (with unit [W], $\eta_i$ and $z_0$ have unit [V]), $d_i^0$ is the current data bit of user $i$ and $I_k$ consists of cross-correlation products from interfering users and multipath signals, which can be written as:

$$\sum_{k=0}^{K-1} I_k = \sum_{k=0}^{K-1} A(d_{k-1}^{-1}X_k + d_k^0 \hat{X}_k)$$  \hspace{1cm} (2.13)

where $d_{k-1}$ and $d_k^0$ are the previous and current data bit, respectively, and

$$X_k = \sum_{m=0}^{M} R_{ii}(\tau_{mi}) r_{mi} \cos\theta_{mi}, \quad \hat{X}_k = \sum_{m=0}^{M} \hat{R}_{ii}(\tau_{mi}) r_{mi} \cos\theta_{mi}, \quad k \neq i$$
$$X_i = \sum_{m=0}^{M} R_{ii}(\tau_{mi}) r_{mi} \cos\theta_{mi}, \quad \hat{X}_i = \sum_{m=0}^{M} \hat{R}_{ii}(\tau_{mi}) r_{mi} \cos\theta_{mi}$$  \hspace{1cm} (2.14)

The partial correlation functions are given by [26]:

25
\[ R_{kk}(\tau) = \frac{1}{T_b} \int_0^\tau p_k(t - \tau) \cdot p_l(t) \, dt \]
\[ \hat{R}_{kk}(\tau) = \frac{1}{T_b} \int_\tau^{T_b} p_k(t - \tau) \cdot p_l(t) \, dt \]  

(2.15)

Knowing the characteristics of the received signal, it is now possible to analyze the performance of the data link. Relevant measures of the performance evaluated in this section are the probabilities of bit errors, packet errors and outages. Further, in the case of packet transmissions, two important parameters are the throughput and packet delay. All these parameters are analyzed for both narrowband and spread-spectrum modulation, including the use of diversity techniques and forward error correction coding.

### 2.2 Bit error probability without diversity

Assuming that the data bits -1 and 1 are equiprobable, the bit error probability \( P_e \) can be expressed as the probability that the correlation output is negative while the transmitted data bit was positive:

\[ P_e = P(z_0 < 0 | d_0^0 = 1) = \int_0^\infty p_e(xA) p_u(x) \, dx \]  

(2.16)

Here, \( p_u(x) \) is the probability density function of the first term of the correlation output \( z_0 \) (2.12) and \( p_e(xA) \) is the bit error probability, given a signal amplitude \( A \) and a path gain \( x \). If the number of interfering terms \( KM \) is large, then the interference can be approximated by Gaussian noise. In that case, \( p_e(xA) \) is given by:

\[ p_e(xA) = \frac{1}{2} \text{erfc} \left( \frac{xA}{\sigma\sqrt{2}} \right) \]  

(2.17)
where \( \text{erfc}(x) \) is the complementary error function, defined as [25]:

\[
erfc(x) \equiv \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt
\]

(2.18)

Note that \( \sigma^2 \) is the sum of the noise variance \( N_0/T_b \) and the variance of the interference \( \sigma_t^2 \), that can be calculated as follows: Since all terms in the summation of (2.13) are independent and symmetrically distributed, the mean is equal to zero. For the same reason, all the cross terms in the calculation of the second moment are zero, so the variance \( \sigma_t^2 \) can be approximated as:

\[
\sigma_t^2 \equiv \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} A^2 E\left[ \left( d_k^{-1} R_{ik}(\tau_{mk}) + d_k^0 \hat{R}_{ik}(\tau_{mk}) \right)^2 \right] E\left[ \left( r_{mk} \cos \theta_{mk} \right)^2 \right]
\]

(2.19)

The second moment of \( r \cos \theta \) can be calculated as follows: If \( m=0 \), then \( r \) has a shadowed Rician distribution, which can be viewed as a Rician distribution with a variable Rice parameter \( z^2/2b_{oo} \). The Rician parameter is the line-of-sight power \( \sigma^2/2 \), divided by the multipath power. In the case of spread-spectrum demodulation, \( b_{oo} \) is the multipath power in the first path, as given by (2.10). The product of this Rician variable \( r \) with the cosine of a uniformly distributed variable gives a Gaussian variable with a mean of \( \sqrt{2} \) and a variance of \( b_{oo} \). Thus, \( E\left( r_{0k} \cos \theta_{0k} \right)^2 = b_{oo} + z^2/2 \), on the condition that \( z \) is constant. This condition can be removed by integrating over the independent lognormal distribution of \( z \):

\[
E\left( (r_{0k} \cos \theta_{0k})^2 \right) = \int_0^\infty \left( b_{oo} + \frac{z^2}{2} \right) p(z) dz = b_{oo} + \frac{1}{2} \exp\left[ 2\sigma_0^2 + 2 \mu_0 \right]
\]

(2.20)

If \( m>0 \), then \( r \) has a Rayleigh distribution. In that case, \( z=0 \) and

\[
E\left( (r_{mk} \cos \theta_{mk})^2 \right) = b_{m0}
\]

(2.21)
The variance of the cross correlations can be calculated as described in [26], assuming that the spread-spectrum code can be approximated by a random sequence:

$$
E\left\{\left[ d_k^{-1} R_{ik}(\tau_{mk}) + d_k^0 \hat{R}_{ik}(\tau_{mk}) \right]^2 \right\} = \frac{2}{3N} \tag{2.22}
$$

where \( N \) is the spreading gain or the number of chips per data bit \( T_b/T_c \). Now, by substituting (2.20)-(2.22) in (2.19), a closed form expression for the interference power is obtained:

$$
\sigma_i^2 \equiv \frac{2KA^2}{3N} \left( b_o + \frac{\exp[2\mu_o + 2\sigma_o^2]}{2} \right) \tag{2.23}
$$

The only thing that is left now in the calculation of the bit error probability is the probability density function \( p_u(x) \) of the first term of the correlation output \( z_o \) (2.12), which consists of a shadowed Rician variable \( r \), multiplied by the cosine of a Gaussian distributed phase. Using [21], the total distribution function of \( r\cos(\theta) \) can be written as:

$$
p_u(x) = 2\int_{\lvert x \rvert}^{\infty} p_r(r) p_\theta \left( \arccos \left( \frac{x}{r} \right) \right) \left| \frac{\partial \arccos(x/r)}{\partial x} \right| dr
\tag{2.24}
$$

$$
= 2\int_{\lvert x \rvert}^{\infty} p_r(r) p_\theta \left( \frac{x}{r} \right) \frac{dr}{\sqrt{r^2 - x^2}}
\tag{2.24}
$$

Substituting (2.1) and (2.7) in (2.24) and using (2.16), the bit error probability \( P_e \) can be expressed as:

$$
P_e = \frac{1}{2} \int_{0}^{\infty} \text{erfc} \left( \frac{xA}{\sigma\sqrt{2}} \right) p_u(x) dx \tag{2.25}
$$

with
\[ p_u(x) = \int_{+\infty}^{-\infty} \int_{0}^{\infty} r \frac{\exp\left[-\frac{(\ln(z) - \mu_0)^2}{2\sigma_0^2} \cdot \frac{(r^2 + z^2)}{2b_0}\right]} {b_0 \sqrt{\pi^2 \sigma_0^2 \sigma_0^2}} \cdot I_0(\frac{rz}{b_0}) \cdot \frac{\arccos^2\left(\frac{x}{r}\right)} {2\sigma_0^2} \cdot \frac{dzdr}{z \sqrt{(r^2 - x^2)}} \]  

Equation (2.26) is valid in the case that the receiver locks on the line-of-sight signal. This requires the bandwidth of the carrier tracking loop to be much smaller than the fading bandwidth of the received signal. If the bandwidth of the tracking loop is larger than the fading bandwidth, then the standard deviation \( \sigma_0 \) of the phase error is approximately zero because the receiver tracks the phase of the total signal. In that case, the distribution function \( p_u(x) \) is equal to the shadowed Rician distribution \( p_r(x) \). However, with a larger tracking bandwidth, the loop noise increases, so there is always a certain phase noise error. Therefore, equation (2.26) provides an upper bound for the bit error probability (by assuming a zero phase error), while a lower bound is obtained by using \( p_u(x) = p_r(x) \), given by (2.1).

Note that for narrowband modulation, \( \sigma^2 \) is equal to the noise power \( N_0 T_b \), while for spread-spectrum modulation, \( \sigma^2 \) is equal to the noise power plus the interference power, given by (2.23). Further, in the case of spread-spectrum modulation, \( b_0 \) in (2.1) and (2.26) should be replaced by the multipath power \( b_{oo} \) of the first arriving path, because the correlation operation in a spread-spectrum receiver largely suppresses multipath signals with relative delays exceeding one chip.

Figure 2.2 shows the narrowband bit error probability for light, average, and heavy shadowing, as a function of the bit energy-to-noise density ratio \( E_b/N_0 \), defined as:

\[ \frac{E_b}{N_0} \triangleq \frac{A^2 T_b}{2N_0} \]  

In the numerical calculation of the bit error probability, equation (2.26) was calculated using the Gaussian integration technique, while (2.25) is evaluated using the Newton-Cotes technique [25]. The difference between this plot and the plots in
[20] is that the effects of phase and envelope fading were calculated separately in [20], while here the combined effect is shown. Furthermore, in [20] an upper bound for the envelope fading was calculated by using an approximation of the complementary error function, while equation (2.25) gives the exact solution.

If figure 2.2 is compared with figure 2 of [20], which shows an upper bound for the bit error probability, it appears that for low signal-to-noise ratios, figure 2.2 gives higher values for the bit error probability. However, this is caused by a mathematical error in the derivation of the upper bound of the bit error probability in [20]; for low signal-to-noise ratios, this upper bound converges to a value of $b_O$ instead of 0.5. The correct upper bound is given by:

$$P_e \leq \frac{1}{\sqrt{8\pi \sigma^2_0}} \frac{\sigma^2}{b_0 + \sigma^2} \int_0^\infty \frac{1}{z} \exp \left( \frac{- (\ln(z) - \mu_0)^2}{2 \sigma^2_0} - \frac{z^2}{2(b_0 + \sigma^2)} \right) dz$$

(2.28)

Figure 2.2: Bit error probability for ideal BPSK and for narrowband BPSK with light, average and heavy shadowing.

For all cases of shadowing, the bit error probability converges to the irreducible error probability $P_{irr}$, caused by phase jitter. A rather complicated expression for this error
is given in [20, equation (27)]. A much easier way to find $p_{irr}$ is to write it as the probability that the received signal in the absence of noise is negative, while the transmitted bit is positive:

$$p_{irr} = \int_{-\infty}^{0} p_u(x) dx, \quad d_f^q = 1$$  \hspace{1cm} (2.29)

Note that the irreducible error probabilities in figure 2.2 correspond with those found in [20].

![Graph showing bit error probability for spread-spectrum modulation](image)

*Figure 2.3: Bit error probability for spread-spectrum modulation with $K=1$ user, $T_o/T_c=6.5$ and code length $N=4095$.*

Next, the effect of spread-spectrum modulation is evaluated by a numerical calculation of (2.25) and (2.26). Comparing figures 2.2 and 2.3, it can be seen that spread-spectrum modulation yields a better bit error probability than narrowband modulation, except for heavy shadowing. The reason for this is that for light and average shadowing, the received signal consists of a strong line-of-sight component, together with a weaker multipath component. Spread-spectrum demodulation enhances the ratio of line-of-sight and multipath power, thereby reducing the
Multipath and multi-transmitter interference

disturbing effects of multipath fading. In the case of heavy shadowing, the direct line-of-sight power is much smaller than the multipath power and the result is an approximately Rayleigh fading signal. Spread-spectrum demodulation now decreases the total used signal power considerably, thereby increasing the bit error probability. In this case, diversity techniques can be used to improve the bit error probability, as demonstrated in the following section.

If the receiver is able to establish synchronization with the phase of the first path, then only the envelope fading has to be considered, which means that \( p_u(x) \) can be substituted by the shadowed Rician distribution \( p_f(x) \) in (2.25). It can be seen from figure 2.3 that the bit error probability for narrowband modulation changes considerably by removing the phase jitter, while in the case of spread-spectrum modulation there is much less change. This is because the phase variance (2.6), which is reduced by approximately a factor of 9 by the use of spread-spectrum modulation, exerts a nonnegligible influence only near the irreducible error probability level, which is less than \( 10^{-9} \) for spread-spectrum modulation.

![Comparison of the bit error probability for narrowband and spread-spectrum modulation](image)

**Figure 2.4:** Comparison of the bit error probability for narrowband and spread-spectrum modulation with average shadowing and envelope fading only for \( K=1 \) user, \( T_g/T_c=6.5 \) and code length \( N=4095 \).
Figure 2.5 shows the results for spread-spectrum modulation with average shadowing and the number of users as a parameter. To maintain a bit error probability of $10^{-3}$, the signal-to-noise ratio has to be increased by about 0.5 dB for $K=100$ users, 1 dB for $K=200$, and 2 dB for $K=400$ as compared to the signal-to-noise ratio for a single user ($K=1$). Note that $E_b/N_0$ is the average signal-to-noise ratio, which does not include interference power from other users. Therefore, increasing $E_b/N_0$ decreases the bit error probability, until the irreducible bit error probability caused by the interference power is reached.

![Figure 2.5: Bit error probability for spread-spectrum modulation with average shadowing for $T_d/T_c=6.5$, $N=4095$ and $K$ as parameter.](image)

Figure 2.6 is similar to figure 2.5, except for a doubled code length value $N$, while the chip time $T_c$ is reduced by a factor of two. In order to keep the same bit error probability, the number of users can be increased more than twice in the case of figure 2.6, because of the reduced cross-correlation power and the reduction in multipath fading. In fact, since the multipath fading parameter $b_{oo}$ (2.10) depends only on the ratio $T_d/T_c$, and the interference power (2.23) depends only on the ratio $K/N$, all plots can be normalized with respect to these two ratios. This means that if the code length $N$ is increased, while $T_d/T_c$ is kept constant, then the number of users can increase proportional to $N$. Further, if $T_d/T_c$ is increased, while $K/N$ is kept
constant, then the multipath fading will be more suppressed, thereby decreasing the bit error probability, provided that there is a dominant line-of-sight signal.

![Figure 2.6: Bit error probability for spread-spectrum modulation with average shadowing for $N=8191$, $T_s/T_c=6.5$ and $K$ as a parameter.](image)

![Figure 2.7: Bit error probability for spread-spectrum modulation with average shadowing for $K=1$, code length $N=4095$, multipath delay spread $T_s=0.65\mu s$ and chip time $T_c$ as parameter.](image)
The effect of a change in the ratio $T_s/T_c$ of multipath delay spread and chip time is clearly visible in figure 2.7. In order to achieve a bit error probability of $10^{-4}$, for instance, about 2 dB less signal-to-noise ratio is required if $T_s/T_c$ is increased from 6.5 to 26.

2.3 Bit error probability with diversity

When spread spectrum modulation is used with a chip time that is less than the delay spread of the channel, there are a number of resolvable paths $M$ that can be used to improve the performance. Two types of path diversity that are considered in the following sections are maximal ratio combining and selection diversity. A detailed description of these techniques can be found in [27,28,22].

2.3.1 Maximal ratio combining

When maximal ratio combining is used, the received signal is coherently correlated with a particular code for $M_d$ different paths, where $M_d$ is the order of diversity. Each path is multiplied by the path gain $r_{mk}$ and all correlation outputs are combined. It is assumed here that errors in the estimates of path gain and phase can be neglected.

The probability density function of the sum of the squared path gains $r_{mk}^2$ is the convolution of the $M_d$ different squared path gain probability density functions. Since the amplitude of the first path has a shadowed Rician distribution $p_r(r)$, the probability density function $p_o(x)$ of a squared shadowed Rician variable follows from a simple transformation:

$$p_o(x) = \frac{p_r(\sqrt{x})}{2\sqrt{x}}$$  \hspace{1cm} (2.30)

All other paths are assumed to have a Rayleigh distributed amplitude, which means that the squared amplitudes have a chi-square probability density function [25]. The
sum of $M_d-1$ independent chi-square variables yields the following probability distribution function:

$$
p_2(x) = \frac{1}{2b_1} e^{-\frac{x}{2b_1}} * \frac{1}{2b_2} e^{-\frac{x}{2b_2}} \cdots \frac{1}{2b_{M_d-1}} e^{-\frac{x}{2b_{M_d-1}}} = \sum_{i=1}^{M_d-1} \frac{2b_i M_d - 4}{M_d - 1} \frac{-x}{2b_i} \prod_{j=1, j\neq i}^{M_d-1} 2b_i - 2b_j
$$

(2.31)

where * denotes convolution.

Now, the probability density function of the sum of all $M_d$ squared path gains can be obtained by a convolution of $p_0(x)$ and $p_1(x)$:

$$
p_{mrc}(\alpha) = \int_0^\alpha p_0(x) p_1(x - \alpha) dx = \int_{0}^{\alpha} \frac{p_r(\sqrt{x})}{2\sqrt{x}} \sum_{i=0}^{M_d-1} \frac{2b_i M_d - 4}{2b_i} \frac{- (x-\alpha)}{2b_i} \prod_{j=0, j\neq i}^{M_d-1} 2b_i - 2b_j
$$

(2.32)

where $\alpha = \sum_{m=0}^{M_d-1} r_{mk}^2$, and $p_r(x)$ is the shadowed Rician distribution, with $b_0$ replaced by $b_{oo}$.

The bit error probability $P_e$ for maximal ratio combining can now be expressed as:

$$
P_e = \frac{1}{2} \int_0^{\infty} \text{erfc} \left[ \sqrt{\frac{\alpha A^2}{2\sigma^2}} \right] p_{mrc}(\alpha) d\alpha
$$

(2.33)
2.3.2 Selection diversity

Ideally, selection diversity selects that path which has the highest signal-to-noise ratio. In practice, however, the signal with the largest amplitude is used since it is difficult to measure an instantaneous signal-to-noise ratio [29].

The cumulative distribution function of the output signal is given as the probability that all $M_d$ path amplitudes are less than some value $x$. Since all path signals are assumed to be independent, the total cumulative distribution function can be written as:

$$P[\eta_0 \ldots \eta_{M_d-1} \leq x] = P(\eta_0 \leq x)P(\eta_1 \leq x)\ldots P(\eta_{M_d-1} \leq x)$$  \hspace{1cm} (2.34)

where $r_m$ is the correlation output for path $m$ and $P(r_m \leq x)$ is a shadowed Rician cumulative distribution function for $m=0$ and a Rayleigh cumulative distribution function for $m>0$, given by:

$$P(x) = \int_0^x \frac{r^2}{b_m^2} e^{-\frac{r^2}{2b_m^2}} dr = 1 - e^{\frac{-x^2}{2b_m}}$$  \hspace{1cm} (2.35)

The probability density function of the output signal can be found by differentiating (2.34) with respect to $x$. The resulting bit error probability for selection diversity is:

$$P_e = \frac{1}{2} \int_0^\infty \text{erfc}\left(\frac{r}{\sigma \sqrt{2}}\right)d \left( \int_0^x p_r(r) \prod_{m=1}^{M_d-1} \left[ 1 - \exp\left(\frac{-x^2}{2b_{m0}}\right) \right] dr \right) dx$$  \hspace{1cm} (2.36)

2.3.3 Diversity results

Figure 2.8 shows the bit error probability for light shadowing, using maximal ratio combining. Again, for the sake of comparison, $P_e$ is also plotted for the ideal case of binary phase shift keying (BPSK), i.e. coherent BPSK without fading and
interference. Without diversity ($M_d=1$), the performance difference with the ideal BPSK plot is small for low values of $E_b/N_0$, about 3.8 dB for $P_e=10^{-3}$. As a result, the gain of maximal ratio combining is limited. In order to keep the bit error probability equal to $10^{-3}$, the signal-to-noise ratio can be reduced approximately 1.3 dB if the order of diversity is increased from 1 to 8.

![Figure 2.8: Bit error probability for light shadowing using maximal ratio combining, $K/N=0.1$, $T_s/T_c=6.5$.](image)

In figure 2.9, for average shadowing, the ratio of line-of-sight and multipath power is smaller and therefore the diversity gain is greater, about 2.2 dB for $P_e=10^{-3}$ and $M_d=8$. In the case of heavy shadowing, plotted in figure 2.10, the irreducible bit error probability due to multi-user interference is very large. Diversity decreases this level from $5.10^{-2}$ to a minimum of $5.10^{-6}$ for MRC with $M=8$. The reason for the poor performance is that the multipath power is dominant, so the use of spread-spectrum without diversity decreases the total used power considerably, thereby increasing the bit error probability. However, if figure 2.10 is compared with the narrowband bit error probability shown in figure 2.2, it can be concluded that spread-spectrum modulation with maximal ratio combining gives a better bit error probability than narrowband modulation. This means that maximal ratio combining is very effective in reducing the effects of multipath fading. The remaining loss in performance is caused by shadowing; compared to light and average shadowing, about 12 dB less signal power is received in the case of heavy shadowing. Therefore,
the bit error probability plots for heavy shadowing always have a separation of at least 12 dB as compared to the ideal BPSK plot.

*Figure 2.9: Bit error probability for average shadowing using maximal ratio combining, $K/N=0.1$, $T_s/T_c=6.5$.*

*Figure 2.10: Bit error probability for heavy shadowing using maximal ratio combining and selection diversity, $K/N=0.1$, $T_s/T_c=6.5$.***
Multipath and multi-transmitter interference

For selection diversity, the bit error probability is calculated for heavy shadowing only, because the probability density function of the first path can be approximated by a Rayleigh probability density function in this case, which removes most numerical difficulties in calculating (2.36). The remaining signal integral was calculated numerically using the Newton-Cotes integration technique [25]. For light and average shadowing, it can be stated that the bit error probability with selection diversity in any case will be worse than the bit error probability with maximal ratio combining, because maximal ratio combining is known to be a superior diversity technique as compared to selection diversity [22], at the cost of a more complex implementation. For heavy shadowing, figure 2.10 shows that selection diversity gives an irreducible bit error probability that is 8 times greater than with MRC for $M=4$.

To show the effect of changes in the parameters, the figures 2.11 and 2.12 depict the bit error probability for different values of $K/N$ and $T_\gamma T_c$, respectively. These figures are calculated for average shadowing, using maximal ratio combining with an order of diversity $M_d=4$. If the relative number of users $K/N$ is increased (figure 2.11), then the bit error probability increases because of the interference power which is proportional to $K/N$. A decrease of the $T_\gamma T_c$ ratio (figure 2.12) causes a decrease of the bit error probability because of the increased multipath rejection.

![Bit error probability for average shadowing using maximal ratio combining with $M_d=4$, $T_\gamma T_c=6.5$, $N=4095$ and $K$ as a parameter.](image)

*Figure 2.11: Bit error probability for average shadowing using maximal ratio combining with $M_d=4$, $T_\gamma T_c=6.5$, $N=4095$ and $K$ as a parameter.*
2.4 Outage Probability

The outage probability $P_{out}$ is the probability that the instantaneous bit error probability exceeds a certain threshold, denoted by $p_o$, and can be written as:

$$P_{out} = P(p_e \geq p_0) = P(x \leq x_0) = \int_{-\infty}^{x_0} xp_u(x) dx$$  \hspace{1cm} (2.37)$$

Here, $x_0$ is the value of the amplitude $x$ at which the instantaneous bit error probability is equal to $p_o$.

The outage probability without diversity is shown in figure 2.13. To achieve an outage probability of $10^{-2}$, for instance, the signal-to-noise ratio has to be 13, 16.5, 19 dB for a threshold $p_o$ of $10^{-2}$, $10^{-3}$ or $10^{-4}$, respectively. Figure 2.14 depicts the outage probability in the case of maximal ratio combining and for a larger relative number of users $K/N$. 

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Figure 2.13: Outage probability for spread-spectrum modulation with average shadowing for $K=1$, $N=4095$, $T_s/T_c=6.5$ and $p_o$ as a parameter.

Figure 2.14: Outage probability for average shadowing using maximal ratio combining with $M_d=4$, $K/N=0.1$, $T_s/T_c=6.5$ and $p_o$ as a parameter.
2.5 Packet success probability

Using the previously defined expressions (2.25), (2.33) and (2.36) for the bit error probability, it is now possible to calculate the packet success probability, which is necessary to obtain the throughput and delay values in the case of CDMA packet transmissions. The packet success probability $P_s$ can be evaluated for slow and fast fading. In the case of slow fading, the path gains are assumed to be constant throughout one packet time, which requires the fading bandwidth to be much smaller than the inverse of the packet time. For fast fading, it is assumed that the path gains are uncorrelated for two consecutive data bits, which means that the fading bandwidth should be large compared to the data bandwidth.

If packets of $N_p$ bits are transmitted, using a forward error correcting code that can correct up to $t$ errors per packet, then the packet success probability in the case of fast fading is:

$$P_s = \sum_{j=0}^{t} P_e^j (1-P_e)^{N_p-j} \binom{N_p}{j}$$  \hspace{1cm} (2.38)

where $P_e$ is given by (2.25), (2.33) or (2.36) for the cases of no diversity, maximal ratio combining or selection diversity, respectively.

In the case of slow fading, the packet success probability becomes:

$$P_s = \int_0^\infty \sum_{j=0}^{t} [P_e(x)]^j [1-P_e(x)]^{N_p-j} \binom{N_p}{j} p(x) \, dx$$  \hspace{1cm} (2.39)

where $P_e(x)$ and $p(x)$ are given by:

No diversity: \hspace{1cm} $P_e(x) = \frac{1}{2} \text{erfc} \left( \frac{xA}{\sigma \sqrt{2}} \right)$ \hspace{1cm} $p(x) = p_r(x)$  \hspace{1cm} (2.40)
Selection diversity: $P_e(x) = \frac{1}{2} \text{erfc} \left( \frac{xA}{\sigma \sqrt{2}} \right)$, $p(x) = p_{sd}(x)$ (2.41)

Maximal ratio combining: $P_e(x) = \frac{1}{2} \text{erfc} \left( \frac{xA^2}{2\sigma^2} \right)$, $p(x) = p_{mrc}(x)$ (2.42)

*Figure 2.15: Packet error probability as a function of $K$ for fast fading, no diversity, $E_b/N_0=10$ dB, $T_sT_C=6.5$, $N=4095$, $N_D=1024$ and $t$ as a parameter.*

Instead of the packet success probability, figure (2.15) demonstrates an example of the packet error probability $Q$, which is defined as:

$$Q = 1 - P_s$$  \hspace{1cm} (2.43)

The packet error probability is drawn as a function of the number of correctable bits $t$. If a maximum of 512 users, for instance, must be supported with a packet success probability of at least 0.99, then the error correcting code must be able to correct 8 errors per 1024 bits.
2.6 Throughput and delay

The previous analysis is applicable especially to circuit switched communications, such as telephony. In these types of applications, bit error and outage probabilities are the main parameters of interest. For packet switched data transmission, the situation is quite different. In this case, the aim is to deliver certain packets of data to a receiver without errors (or with a very small error probability). This is usually accomplished by using some kind of retransmission protocol, whereby packets are retransmitted whenever the receiver detects the occurrence of errors. In such transmission systems, the main parameters of interest are the throughput (often expressed as the number of successfully delivered packets per time slot) and the packet delay. This section investigates the advantages and disadvantages of the use of spread-spectrum CDMA in packet switched communications.

A communication network is considered with such a bandwidth that in the case of perfect Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA), a total number of $N$ users can be accommodated, each transmitting at a bit rate of $I/T_b$, with $T_b$ as the bit duration. Instead of a fixed assignment scheme like TDMA and FDMA, now consider a random access slotted CDMA scheme, where the data sequence is spread by a certain spreading code, consisting of $N$ chips per bit. Further, it is assumed that the total number of users is large enough to get a Poisson distribution function for the offered traffic. Then, the probability $P_{tr}(k)$ that $k$ packets are generated during a certain time slot is given by:

$$P_{tr}(k) = \frac{G^k}{k!} \exp(-G)$$  \hspace{1cm} (2.44)

Here, $G$ is the average number of transmitted packets per time slot.

When a packet is transmitted, there is a certain success probability $P_s(k)$ that it is received correctly. It is assumed that an acknowledgement is sent after successful reception of a packet, so after waiting twice the propagation delay, a user knows if its packet is received or not. Although a receiver itself could estimate whether a packet was successfully received in a single cell or single spot system, acknowledgements are mandatory for multi-cell or multi-spot systems, at least if a
zero packet loss probability is required. When the transmitting user does not receive an acknowledgement for a certain packet, it retransmits that packet after a certain random delay. The steady-state throughput of this transmission system is defined as the average number of successfully received packets per time slot, given by:

$$ S = \sum_{k=1}^{K_{\text{max}}} k P_{tr}(k) P_s(k) $$

(2.45)

Here, $K_{\text{max}}$ is the maximum number of users that can be simultaneously handled by the system, because the number of receivers or available code words is limited.

It may be noted that slotted ALOHA is a special limiting example of slotted CDMA for $K_{\text{max}}=N=1$. In order to make a fair comparison between CDMA and slotted ALOHA, it is desirable to use the same bandwidth for both systems, which gives two options for the slotted ALOHA case: First, the data rate can be chosen equal to the CDMA chip rate. Second, the data rate can be chosen equal to the CDMA data rate, which makes it possible to divide the total bandwidth in $N$ separate ALOHA channels [30]. In this way, a combination of FDMA and ALOHA is made where each user randomly selects a certain frequency band and a certain time slot. The second option can be expected to achieve higher throughput values, because in the first option, the large data rate will cause considerable intersymbol interference because of the relatively large multipath delay spread of the channel. Therefore, the bit error probability and hence the throughput for the first option will always be worse than in the multi-channel ALOHA system, where the data rate is $N$ times smaller.

Assuming that the total amount of traffic is randomly distributed over $N$ channels, the throughput of the multi-channel slotted ALOHA system can be simply calculated as $N$ times the throughput of one narrowband slotted ALOHA channel, with an offered load that is equal to the total offered load divided by $N$. In fact, it is possible to normalize the obtained throughput values for both CDMA and ALOHA by dividing the total system throughput by $N$. Thus, the corresponding throughput per 'channel' is found.
The corresponding average packet delay $D$ is defined as the number of slot times it takes for a packet to be successfully received. Thus, it is the average time duration (in slots) between the packet being offered to the transmitter and the packet being successfully received [17], and is given by:

$$
D = 1.5 + T_d + \left[ \frac{G}{S-1} \right] \left[ \frac{NAT}{2} + 1 + 2T_d \right]
$$

(2.46)

where $G/S-1$ is the average number of retransmissions, $NAP/2$ is the mean retransmission delay and $T_d$ is the propagation delay.

Figures 2.16 and 2.17 show the normalized throughput $(S/N)$ curves of narrowband slotted ALOHA without and with forward error correction, respectively. The results of these and the following figures were obtained for $T_g/T_c=6.5$, $E_b/N_0=20$dB, $N_p=256$ and $t=0$ or $t=10$ in the absence or presence of forward error correction coding. The values of $N$ and $K_{max}$ used were $N=127$ and $K_{max}=2N$. However, as explained earlier, the results are also valid for other values of $N$, as long as $N$ is large compared to one.

![Figure 2.16: Normalized throughput of narrowband slotted ALOHA for: a) light shadowing, slow fading, b) light shadowing, fast fading, c) average shadowing, slow fading, d) average shadowing, fast fading, e) heavy shadowing, slow fading.](image)
Figure 2.17: Normalized throughput of narrowband slotted ALOHA using FEC coding for: a) light and average shadowing, slow fading and fast fading b) heavy shadowing, slow fading, c) heavy shadowing, fast fading.

Figure 2.18: Normalized throughput of slotted CDMA for: a) light shadowing, slow fading, b) light shadowing, fast fading, c) average shadowing, slow fading, d) average shadowing, fast fading, e) heavy shadowing, slow fading.

Figure 2.18 shows that the normalized throughput of slotted CDMA without error correction is less than that of slotted ALOHA. However, when error correction is
applied, then CDMA benefits far more than slotted ALOHA, which results in a larger normalized throughput than narrowband slotted ALOHA for light and average shadowing, see figure 2.19.

![Graph showing normalized throughput of slotted CDMA with different conditions](image)

**Figure 2.19:** Normalized throughput of slotted CDMA, using FEC coding for:
- a) light shadowing, slow fading,
- b) light shadowing, fast fading,
- c) average shadowing, slow fading,
- d) average shadowing, fast fading,
- e) heavy shadowing, slow fading,
- f) heavy shadowing, fast fading.

In figure 2.20, it is demonstrated that the use of maximal ratio combining provides a minor throughput improvement for light and average shadowing, and a considerable improvement for heavy shadowing in comparison with figure 2.18. The same conclusion can be drawn when combined forward error correction and maximal ratio combining (figure 2.21) are compared with the case of just using forward error correction (figure 2.19). The relatively larger improvement for heavy shadowing is caused by the low line-of-sight signal power, which is smaller than the multipath power for heavy shadowing. Thus, if the receiver only tries to demodulate the first
path, it uses only a small fraction of the total received power, assuming that the chip time is smaller than the multipath delay spread. In that case, maximal ratio combining is very useful to make use of all received signal power that is available.

An interesting fact that can be seen in the previous figures is that fast fading yields a higher maximum throughput than slow fading when forward error correction coding is applied, while the performance of fast fading is worse if no error correction is used. This is because, for fast fading, the bit errors are randomly spread over all packets, so each packet has the same success probability, which is relatively low when no error correction coding is used. In the case of slow fading, however, errors appear in bursts. Because of the slow fading, the packet success probability varies, which means that compared to fast fading, a certain portion of the packets has a higher packet success probability, thus yielding a higher throughput when no forward error correction is applied. If error correction coding is used, then the packet success probability is greatly enhanced for fast fading as long as the signal-to-noise plus interference ratio—which is inversely proportional to the offered load—is above a particular threshold value. Beneath that threshold, the packet success probability quickly drops to zero, as can be seen in the figures. For slow fading, error correction coding is less effective, because there are fewer packets with up to $t$ bit errors that can be corrected. Therefore, the maximum throughput in the case of slow fading is less than for fast fading. However, for high values of the offered load, the throughput for slow fading decreases more slowly than for fast fading, because there is always a portion of the packets with a higher success probability than in the case of fast fading.
Figure 2.20: Normalized throughput of slotted CDMA, using MRC ($M_d=4$) and 
FEC for: a) light shadowing, slow fading, b) light shadowing, fast fading, 
c) average shadowing, slow fading, d) average shadowing, fast fading, 
e) heavy shadowing, slow fading, f) heavy shadowing, fast fading.

Figure 2.21: Normalized throughput of slotted CDMA, using MRC ($M_d=4$) and 
FEC for: a) light shadowing, slow fading, b) light shadowing, fast fading, c) 
average shadowing, slow fading, d) average shadowing, fast fading, 
e) heavy shadowing, slow fading, f) heavy shadowing, fast fading.
In figure 2.22, the normalized throughput curves for heavy shadowing when using selection diversity are drawn. It is clear that selection diversity performs worse than maximal ratio combining by at least a factor of two. However, it still considerably improves the throughput as compared to the case of no diversity.

![Figure 2.22: Normalized throughput of slotted CDMA for heavy shadowing, using selection diversity with $M_d=4$: a) FEC, slow fading, b) FEC, fast fading, c) no FEC, slow fading, d) no FEC, fast fading.](image)

Figure 2.23 shows the normalized throughput of slotted CDMA with the number of correctable bits as a parameter. As the error correcting capability increases, the maximum achievable throughput increases to high values. However, the user data throughput is, of course, decreased by the increasing number of bits used for error correction. In [31], it is found that for BCH codes, a maximum net throughput -i.e. the normalized throughput multiplied by the code rate- is obtained for a code rate of approximately $1/2$, which is obtained for $t=18$ and $N_P=255$. Since these figures are very close to case $f$ in figure 2.23, it can be concluded that the maximum net throughput is about 0.4.

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Figure 2.23: Normalized throughput of slotted CDMA for average shadowing, using MRC ($M_d=4$) and FEC, with the number of correctable bits as a parameter:
   a) $t=0$, slow fading, b) $t=0$, fast fading, c) $t=10$, slow fading,
   d) $t=10$, fast fading, e) $t=20$, slow fading, f) $t=20$, fast fading.

Figure 2.24: Normalized throughput of slotted CDMA for average shadowing, slow fading, using MRC ($M_d=4$) and FEC, with $T_s/T_c$ as a parameter:
   a) $T_s/T_c=10^4$ b) $T_s/T_c=10^3$ c) $T_s/T_c=10^2$ d) $T_s/T_c=10$ and 1.
Figure 2.24 demonstrates the influence of the ratio $T_s/T_c$ on the performance of CDMA. It can be seen that the best performance is obtained for a high $T_s/T_c$ ratio. When $T_s/T_c$ is in the order of one or less, then there is practically no longer any benefit in using spread-spectrum to reduce multipath interference, and, therefore, the performance converges to a certain lower bound. When $T_s/T_c$ is increased, then the performance increases up to a certain upper bound, where the multipath interference in the first path, containing the line-of-sight signal, becomes negligible.

The figures 2.25 and 2.26 show the delay versus the normalized throughput. The results were obtained for values of $N_{at}=3$ and $T_d=74$ slots. It can be seen that for narrowband without error correction, the difference between slow and fast fading is considerable, while it becomes negligible when forward error correction is used. In the case of CDMA, fast fading with forward error correction clearly provides the best throughput and delay performance.

Figure 2.25: Normalized throughput-delay curves of narrowband slotted ALOHA for average shadowing: a) slow fading, no FEC, b) fast fading, no FEC, c) slow and fast fading, FEC.
Figure 2.26: Normalized throughput-delay curves of slotted CDMA for average shadowing, using MRC: a) slow fading, no FEC, b) fast fading, no FEC, c) slow fading, FEC, d) fast fading, FEC.

2.7 Influence of synchronization errors

The previous analysis assumed that the influence of code and carrier synchronization on the bit error probability could be ignored. However, this assumption is valid only for specific conditions. It was mentioned in section 2.2 that in order to minimize the influence of carrier phase errors, it is desirable to have a tracking loop bandwidth that is much larger than the fading bandwidth. However, the tracking loop bandwidth cannot be made arbitrarily large, because of noise increase. The upper limit of the tracking loop bandwidth is determined by the maximum tolerable phase jitter, which is related to a certain increase in the bit error probability and to the mean time to lose lock. More details of the behavior of tracking loops in the presence of noise can be
found in many publications, e.g. [32,33]. In [34], a description is given of how the optimum loop bandwidth can be calculated, which minimizes the total effect of both noise and fading on the phase jitter of a carrier tracking loop.

Information on the influence of multipath fading on code tracking is scarce. There is a strong relation, however, with the behavior of bit timing recovery systems. In [35], it is concluded that if the fading bandwidth is small compared to the tracking loop bandwidth, then a noncoherent bit synchronizer tracks the instantaneous position of the maximum eye-pattern opening. For a relatively high fading bandwidth, the loop approximately tracks the weighted mean delay of all incoming multipath signals, i.e., the mean of all delays multiplied by their corresponding path gains. Similar effects can be expected for spread-spectrum code tracking loops. Figure 2.27 depicts the problem of tracking the delay that maximizes the output of a direct-sequence spread-spectrum correlation function. In this simulated example, the input signal consists of two multipath components with a relative spacing of less than one chip. The output correlation function is drawn for the cases of in-phase and out-of-phase signals. It is clear that the delay that maximizes the output correlation varies significantly; for in-phase signals, the delay of the top is somewhere in the middle of both path delays, while for out-of-phase signals, the best delay is smaller than both path delays. This means that in order to minimize loss of signal power caused by tracking errors, the code tracking loop should track the instantaneous delay of the correlation function, which requires a loop bandwidth that is large compared to the fading bandwidth.

The situation is different, however, for timing and ranging applications. In these cases, the key parameter is the delay of the line-of-sight and/or dominant path signal, instead of the delay that maximizes the correlation output. In figure 2.27, for instance, the delay difference between the in-phase and out-of-phase correlation peaks is approximately half a chip, which is a considerable error for a timing or ranging system. In order to provide a better understanding of the conflicting tracking requirements between communication and positioning systems, the following chapter provides a thorough investigation of spread-spectrum tracking errors caused by multipath and multi-transmitter interference.
Fig. 2.27: Correlation function with in-phase and out-of-phase multipath signals. a, b) Line-of-sight and multipath correlation functions, c) correlation function when b) is out-of-phase, d) correlation function when b) is in-phase.
2.8 Conclusions

The performance of a land-mobile satellite channel has been analyzed in terms of bit error probability, outage probability, throughput and delay. The first two measures are especially useful for circuit-switched communications (e.g. voice), whereas throughput and delay are the main parameters of interest in the case of packet-switched (data) communications.

As a first step in the analysis, an existing narrowband propagation model has been extended for wideband applications. Based upon this wideband propagation model and a receiver model, it was possible to derive a closed-form expression for the multipath and multi-user interference, and to calculate all desired performance measures.

The performance of CDMA with or without diversity techniques can be generally expressed as a function of the signal-to-noise ratio $E_b/N_0$, the ratio of delay spread and chip time $T_d/T_C$, the ratio of code length and number of users $N/K$, and the error correcting capability. For a given $E_b/N_0$ and error correcting capability, there are two ways to enhance the performance of CDMA: First, the ratio of delay spread to chip time can be increased by decreasing the chip time, and second, the order of diversity can be increased. The choice between these two measures depends on bandwidth limitations and hardware considerations. A higher order of diversity requires more hardware, while a decreased chip time requires more bandwidth and faster hardware.

Calculations showed that for relatively low signal-to-noise ratios and a low number of users ($K/N=0.1$), CDMA achieves a better bit error probability than narrowband modulation, because the use of spread-spectrum decreases the effects of multipath fading (provided that the chip time is less than the multipath delay spread). For a relatively high number of users, however, CDMA performs worse than narrowband modulation. This is caused by the increased amount of multi-user interference, which is not present in narrowband FDMA/TDMA systems with a single cell or single spot. For multi-cell or multi-spot systems, the situation may be more favorable to spread-spectrum, since narrowband modulation techniques tend to be more vulnerable to inter-cell interference than spread-spectrum modulation.
In the case of packet-switched communications, the capacity of both CDMA and narrowband modulation is limited by multi-user (random-access) interference. Throughput and delay calculations indicate that CDMA can achieve a better performance than the narrowband slotted ALOHA protocol. However, this is only true when there is a dominant line-of-sight path and when forward error correction coding is used. The use of forward error correction is far more beneficial for CDMA than the use of path diversity techniques.

The largest effects of path diversity techniques, especially maximal ratio combining, can be expected in the case of heavy shadowing. This is because a direct-sequence spread-spectrum receiver uses only a small fraction of the available signal power when the delay spread greatly exceeds the chip time. In that case, path diversity techniques are useful in reducing the loss of signal power, as compared to the received signal power for narrowband transmission.

An interesting fact that has shown up in the analysis is that fast fading results in higher maximum throughputs than slow fading when forward error correction is applied, while the performance of fast fading is worse when no error correction is used. The explanation for this is the bursty character of the channel in the case of slow fading, which makes forward error correction coding less effective, simply because there are too many errors in a certain percentage of the packets. Conversely, there are also packets that have significantly fewer errors than in the case of fast fading, where all packets have the same success probability.

2.9 References


3 CODE AND CARRIER TRACKING ERRORS IN GNSS AND COMMUNICATION SYSTEMS

Multipath propagation is almost inevitable in most applications of radio communication and radio navigation, since normally all kinds of possible reflectors are present, such as the earth's surface, buildings, trees or other objects. Further, in the case of asynchronous spread-spectrum modulation, several transmitters simultaneously transmit non-orthogonal signals, which causes a certain amount of multi-transmitter interference. The previous chapter studied the effects of multipath and multi-transmitter interference on the performance of satellite communications, there assuming the receiver was able to synchronize to the desired signal. This chapter focuses on the problems that occur with code and carrier synchronization in the presence of multipath and multi-transmitter interference.

The effects of multipath propagation on spread-spectrum code and carrier tracking have been investigated in a number of papers [1-17]. However, most of the research is focused on specific measurements or specific receiver architectures, which makes it difficult to extrapolate the results to other environments or different types of receivers. The first theoretical analysis of multipath errors was published in [1]. This report studied the effects of a single reflection on code tracking errors for the case of slow fading and a one-chip early-late spacing. Other investigations showed the beneficial effects of smaller early-late spacings [15-17], while in [7-10], it was discovered that receiver antenna movements reduced the variance of multipath tracking errors. Despite all this research, several questions still remain, especially about the exact effects of the early-late spacing, the influence of a finite signal bandwidth and the role of the fading bandwidth. It is the aim of this chapter to provide a general theory which answers all of these questions.

Preceding the analysis of code and carrier tracking errors, a description is given of the channel model and the synchronization systems of a direct-sequence spread-spectrum receiver. Using these models, equations are derived for the synchronization errors caused by multipath and multi-transmitter interference. Both coherent and noncoherent delay lock loops are considered, using an arbitrary early-late spacing.
Several measurements are included in order to demonstrate the seriousness of the problems and to validate the theory.

### 3.1 Channel model

Multipath signals can roughly be divided in three categories: Specular reflections, diffuse reflections and diffraction. The next subsections give a short description of these different categories.

#### 3.1.1 Specular reflection

Specular reflections originate from smooth surfaces that are large in comparison with the first Fresnel zone. The Fresnel zones are surfaces for which the absolute path length difference between the transmitter -via a point on the surface- and the receiver is less than half a wavelength for all points on the surface [18]. On a smooth reflecting plane, in general, the Fresnel zones are ellipses, as illustrated in figure 3.1. In the case of GPS, where the distance from reflector to satellite is usually much larger than the distance from reflector to receiver, the area of the first Fresnel zone can be approximated as:

\[
F \approx \frac{\pi \lambda D}{\sin \gamma}
\]  

(3.1)

where \( \gamma \) is the grazing angle (equal to the elevation angle for horizontal surfaces), \( \lambda \) is the wavelength and \( D \) is the distance from the receiver to the specular point on the reflecting surface.
Fig. 3.1: First Fresnel zone on a reflecting plane.

Specular reflections are characterized by a certain amplitude, phase and polarization, which depend on the material of the reflector and on the grazing angle $\gamma$. For the right-hand circular polarization of GPS, for instance, specular reflections are, in general, elliptically polarized. This means that the reflections can be split into a right-hand circular (RHC) and a left-hand circular (LHC) component. If the grazing angle exceeds the Brewster angle\(^1\), which depends on the reflecting material (e.g. 6 degrees for sea water at 1.5 GHz and less than 1 degree for aluminium), then the LHC component dominates. An ideal antenna would reject the LHC component completely, thereby enhancing the signal-to-multipath ratio (SMR). In practice, however, the rejection of LHC polarized signals by typical GPS antennas is limited to about 10 dB.

3.1.2 Diffuse reflection

When a reflecting surface is not smooth, diffuse relections occur. To determine whether a reflecting surface is smooth, the surface roughness parameter $S$ can be used [18]:

$$S = \frac{4\pi h}{\gamma} \sin \gamma$$  \hspace{1cm} (3.2)

\(^1\) The Brewster angle is the angle of incidence for which the magnitude of the parallel reflection coefficient reaches a minimum.
where $h$ is the height of the surface irregularities. A surface is considered smooth when $S$ is much smaller than one. The skin of an aircraft, for instance, with irregularities in the order of millimetres, can be considered to be smooth. If the reflecting surface is not smooth, diffuse reflections occur. This second type of multipath is characterised by a random amplitude, phase and polarization of the reflected signal. A diffuse reflection can be thought of as a sum of several reflections with different phases and amplitudes, depending on the structure of the surface irregularities and on the geometry of the receiver, transmitter and reflecting surface. As the geometry changes, the phase relation between the various reflections changes, causing a random fluctuation of the total amplitude. Because of the random polarization of reflections from very rough surfaces, an ideal RHC antenna now can reduce the reflection power by 3 dB only, an important difference between this and the case of specular reflections.

3.1.3 Diffraction

The third type of multipath propagation is diffraction. Whenever a radio wave encounters an obstructing object, some of the energy of the wave is diffracted at the edges of the object and becomes bent around the edge. Diffraction is the reason that aircraft suffer from reflections from the earth's surface, even if the reflected wave is shadowed by the aircraft itself [7,12].

3.1.4 Channel description

If the presence of a certain number, say $M$, of multipath signals is taken into account, the received spread-spectrum signal can be written as:

$$x(t) = \sum_{i=0}^{M} a_i(t) p(t - \tau_i(t)) \cos(\omega t + \theta_i(t))$$  \hspace{1cm} (3.3)

where $p(t)$ is the spread-spectrum code and $a_i(t)$, $\tau_i(t)$ and $\theta_i(t)$ are the time-dependent amplitude, delay and phase of the $i$th signal, respectively. Noise is left out
in this equation, since the primary interest is the influence of multipath. The data signal \( d(t) \) is also dropped, because its influence is removed by envelope detection in a noncoherent DLL, or by decision feedback in a coherent DLL. For simplicity of notation, the time dependence of the measured parameters \( a_i, \tau_i \) and \( \theta_i \) is left out in the rest of the report. Note that, in general, all paths can have different frequencies \([\omega + \delta \theta_1(t)/\delta t]/2\), where \( \omega \) is the angular frequency of the line-of-sight signal. The bandwidth spread of these frequencies is called the fading bandwidth, which is a crucial parameter in the analysis of multipath tracking errors. It is inversely proportional to the time required to get a negligible correlation between two multipath errors. Thus a reduction of the multipath error variance requires an averaging time that is much greater than \( 1/\text{fading bandwidth} \).

The fading bandwidth depends on the satellite-receiver-reflector geometry and on the velocity of the receiver. A receiver speed of \( v \) causes a Doppler difference of \([\cos \alpha_r \cos \beta_r - \cos \alpha_\delta \cos \beta_\delta]vf/c \), where \( f \) is the carrier frequency, \( c \) is the speed of light, and \( \alpha_r, \beta_r \) and \( \alpha_\delta, \beta_\delta \) are the differences in elevation and azimuth angles between the receiver speed vector and the vectors from the receiver to the reflector and to the satellite, respectively. If \( v = 100 \text{ m/s} \), for instance, then the Doppler difference can take values up to 1200 Hz. However, if the receiver-reflector geometry is relatively stationary, as in the case of a fixed (reference) receiver, or in the case of reflections on an aircraft from parts of the aircraft itself, then the fading bandwidth is determined by the change of the satellite geometry only, which results in values that are usually much smaller than 1 Hz [8]. A similar effect is caused by ground reflections while the aircraft is at a fixed altitude. However, the aircraft speed together with the roughness of the earth's surface tends to increase the fading bandwidth to typical values in the order of ten Hertz [19].

### 3.2 Receiver model

In order to determine the effects of multipath and multi-transmitter interference, it is necessary to have a model of a direct-sequence spread-spectrum receiver. Generally, there are two different systems that are used in spread-spectrum receivers to estimate
delays and phases; these systems are the coherent / noncoherent delay lock loop (DLL) and the carrier tracking loop.

3.2.1 Delay lock loop

The figures 3.2.a and 3.2.b show the block diagrams of a noncoherent and a coherent delay lock loop, respectively [20-23]. In the case of a noncoherent delay lock loop, the input signal is first down converted to baseband in-phase and quadrature signals and multiplied by an early (pE) and a late (pL) code. The resulting in-phase and quadrature signals are integrated over a certain time $T_p$, that is equal to or smaller than the data bit time. In practice, the signals are sampled, which changes the integration into a summation; as long as the sampling frequency satisfies the Nyquist criterion, this is of no influence on the analysis. The outputs of the integrators are the early and late correlation values, multiplied by the sine or cosine of the carrier phase error and by the data bits. In order to remove the influence of the data and the carrier phase, the noncoherent delay lock loop adds the squared in-phase and quadrature samples for both the early and late signals. Then, the early and late power values are subtracted in order to get a signal value which is proportional to the code delay error, as long as the absolute delay error is smaller than half the early-late spacing. The error signal is filtered by a loop filter, whose output is used to control the frequency of a VCO or NCO (Voltage or Numerically Controlled Oscillator) in such a way that the delay error is driven to zero.

The difference between the noncoherent delay lock loop and the coherent delay lock loop (figure 3.2.b) is that the latter does not square the early and late signals. Instead, it uses estimated carrier phase and data bits to perform a coherent down conversion. So for a coherent delay lock loop, a proper carrier phase estimate from the carrier tracking loop is required to make the system work. At the same time, however, the carrier tracking loop needs an estimate of the code delay in order to track the phase of the incoming signal. Thus the code and carrier tracking loops are strongly dependent, contrary to the noncoherent delay lock loop.
In chapter 4, it is explained how noise tracking errors can be kept small by choosing a small tracking loop bandwidth. However, a small loop bandwidth increases errors caused by dynamical manoeuvres. To solve this problem, many receivers use carrier rate aided tracking [24,25]. This technique makes use of the fact that carrier ranges have a much better accuracy than code ranges, provided that the carrier frequency is
much larger than the chip rate of the spread-spectrum code. In the case of GPS, for instance, the received code frequency is equal to the L1 carrier frequency divided by 1540, apart from a small difference due to ionospheric drift. This means that the desired frequency of the code generator is accurately known, even in cases of severe dynamical manoeuvres, as long as the carrier tracking loop maintains lock. The code tracking loop now only has to track the delay variations caused by the ionosphere. Since these variations are usually very slowly time varying, the tracking loop bandwidth can be made very small - e.g. 0.05 Hz [22] - which greatly reduces the noise errors.

3.2.2 Carrier tracking loop

A block diagram of the carrier tracking loop is drawn in figure 3.3. The input signal is again down converted to in-phase and quadrature signals, which then are correlated with the prompt code. If the delay lock loop is in lock or close to the in-lock state, then the prompt code has approximately the same delay as the incoming code, so there will be a maximum correlation.

Fig. 3.3: Carrier tracking loop.
From the in-phase and quadrature correlation samples, the carrier phase can then be estimated as the (4-quadrant) arc tangent of the quadrature value divided by the in-phase value. The data bits appear in these phase estimates as phase jumps of 180 degrees, which can be detected and corrected prior to the loop filter. Just as for the delay lock loop, the estimated carrier phase errors are used to control a VCO or NCO in order to minimize the phase error.

3.3 Code range errors of a coherent delay lock loop

In order to track the line-of-sight signal delay, the input signal (3.3) is down converted and coherently correlated with an 'early' and a 'late' code. These are replicas of the received spread-spectrum code with a delay of plus or minus \(dT_c/2\) seconds in comparison with the delay of a 'prompt' code, respectively. The parameter \(d\) is often referred to as the early-late spacing. If the loop is in lock, the delay of the 'prompt' code is the desired delay estimate of the input signal. The resulting early and late correlation functions are subtracted to produce the 'S-curve' \(S(\tau):\)

\[
S(\hat{\tau}_o) = \sum_{i=0}^{M} a_i \cos(\theta_i - \hat{\theta}_o) \left[ R(\hat{\tau}_o - \tau_i + \frac{d}{2} T_c) - R(\hat{\tau}_o - \tau_i - \frac{d}{2} T_c) \right] \tag{3.4}
\]

where \(R(\tau)\) is the correlation function of the spread-spectrum code \(p(t)\), and \(d\) is the early-late spacing. For an unfiltered ideal code, \(R(\tau)\) is equal to \(1-|\tau/T_c|\) for \(|\tau|\leq T_c\) and equal to zero elsewhere. \(\hat{\theta}_o\) and \(\hat{\tau}_o\) are the receiver's estimates of the line-of-sight carrier phase \(\theta_o\) and delay \(\tau_o\), respectively. Without multipath, i.e. \(M=0\), the S-curve has a zero point at \(\hat{\tau}_o=0\), with a linear region around it where \(S(\tau_e) = -2a_o \tau_e / T_c\). Here, \(T_c\) denotes the chip time, \(a_o\) is the line-of-sight signal amplitude and \(\tau_e = \hat{\tau}_o - \tau_o\) is the tracking error. Figure 3.4 shows the multipath free S-curve \(S_o(\tau_e)\), normalized with respect to the early-late spacing \(d\), with the condition that \(d \leq 1\). If \(d\) is the often used one chip time, the horizontal lines at \(S_o(\tau_e) = \pm a_o d / T_c\) reduce to points, and the S-curve takes the familiar shape that was used in [1] and [8].
To understand the effects of multipath propagation on code tracking, it is important to distinguish two different cases: the fading bandwidth $B_F$ is large or small compared to the tracking loop bandwidth $B_L$. If $B_F$ is small compared to $B_L$, then the combined $S$-curve of (3.4) is relatively slowly varying, so the DLL can track the instantaneous zero crossing of (3.4). If $B_F$ is large compared to $B_L$, however, then the DLL can no longer follow the fast changes of the zero crossing. As demonstrated in the next section, a coherent DLL has a zero bias in this case, while a noncoherent DLL always ends up with a certain positive bias for relative multipath delays up to $T_c(1+d/2)$.

### 3.3.1 Slow fading

For a small $B_F/B_L$ ratio, the DLL simply tracks that value of $\hat{\tau}_o$ for which the $S$-curve (3.4) is zero while its slope $\delta S(\hat{\tau}_o)/\delta \hat{\tau}_o$ is negative. To do this, it needs at the same time an estimate of the line-of-sight signal phase. The carrier tracking loop provides this estimate, which is equal to the phase of the correlation of the input signal with the estimated 'prompt' code $p(t-\hat{\tau}_o)$.
\[
\hat{\theta}_o = \arg \left[ \sum_{i=0}^{M} a_i \exp(j\theta_i)R(\hat{\nu}_o - \nu_i) \right]
\]

(3.5)

Using (3.4) and (3.5), the time varying delay and phase errors can be calculated for any given set of multipath parameters. To obtain additional insight, however, it is convenient to focus on the case of just one multipath signal, so \( M = 1 \). In this case, the delay error is a kind of deformed sinusoidal function of time. In the following analysis, multipath errors are characterized by three parameters: maximum absolute errors, or maxima and minima, standard deviation and mean. Maximum absolute delay errors occur if the multipath signal has a phase difference of 0 or 180 degrees with the line-of-sight signal. This is because the phase of the sum signal is then equal to \( \theta_o \) - assuming that the signal-to-multipath ratio (SMR) is greater than one; so the multipath component \( \pm a_1[R(\hat{\nu}_o - \nu_1 + dT_c/2) - R(\hat{\nu}_o - \nu_1 - dT_c/2)] \) in equation (3.4) gives a maximal distortion of the multipath free S-curve. To examine these maximum absolute errors, a closed form expression can be derived. The zero point of the combined S-curve is now simply that value of the tracking error \( \tau_e \) for which \( S_o(\tau_e + [a_m/a_o]S_o(\tau_e + \tau_o - \tau_1)) = 0 \). Here, \( S_o(\tau_e) \) is the ideal S-curve from figure 3.4. \( a_m \) is plus or minus the multipath amplitude \( a_I \), corresponding to in-phase and out-of-phase multipath signals. After some algebraic manipulation, the following expression can be obtained:

\[
\begin{align*}
\tau_e &= \frac{a_m \tau_1}{a_o + a_m} & 0 \leq \tau_1 < a \\
\tau_e &= \frac{a_m dT_c}{2a_o} & a \leq \tau_1 \leq b \\
\tau_e &= \frac{a_m}{2a_o - a_m} \left[ T_c \left(1 + \frac{d}{2}\right) - \tau_1 \right] & b < \tau_1 \leq T_c \left(1 + \frac{d}{2}\right) \\
\tau_e &= 0 & \tau_1 > T_c \left(1 + \frac{d}{2}\right)
\end{align*}
\]

(3.6)

where \( a = \frac{(a_o + a_m) dT_c}{2a_o} \) and \( b = T_c \left[ 1 - d \left(1 - \frac{a_o + a_m}{2a_o}\right) \right] \).
under the condition: $d \leq 1$ and $-a_o < a_m \leq a_o$.

Figure 3.5 depicts the maximum and minimum tracking errors as a function of the multipath delay $\tau_i$. The figure is normalized with respect to the signal-to-multipath ratio $SMR=(a_0/a_1)^2$, and with respect to the early-late spacing $d$. The values $a+$, $b+$ and $a-$, $b-$ represent the values of $a$ and $b$ in equation (3.6) for which $a_m$ is $+a_i$ and $-a_i$, respectively. When the signal-to-multipath ratio approaches one ($a_1 \rightarrow a_o$), maximum errors of up to $dTc/2$ can be expected. Since many receivers use $d=1$, these maxima can reach half a chip time, which was demonstrated by the measurements in [9]. Note that if the early-late spacing is one chip time, then $a+=b+$ and $b-=a-$, so figure 3.5 becomes a quadrangle. Special cases of this quadrangle can be found in [1,8,2], while [15] also gives a special case of figure 3.5 for an early-late spacing of 0.1. All those curves were drawn for specific values of the chip time, the early-late spacing, and the $SMR$. Equation (3.6) and figure 3.5 give a general solution, from which all mentioned specific cases can be deduced by filling in the parameter values for $SMR$ and $d$. From equation (3.6), it is clear that an easy way to reduce the maximum multipath errors is to reduce the early-late spacing $d$, since those maxima are proportional to $d$. Unfortunately, filtering of the input signals worsens the multipath errors, which is discussed further on in this chapter. Further, a small early-late spacing increases the acquisition time. An adaptive early-late spacing, as described in [22], avoids this problem.

![Diagram of Maximum and Minimum Delay Errors](image)

**Fig. 3.5: Maximum and minimum delay errors.**

Figure 3.6 shows three specific cases of the maximum and minimum delay errors for a multipath signal with a relative amplitude of 0.5 (SMR=6dB). This figure clearly
demonstrates the advantage of a small early-late spacing and the use of a small chip value. Figure 3.6 shows error envelopes for the GPS P-code \( T_c = 0.1 \) \( \mu s \) and the C/A-code \( T_c = 1 \) \( \mu s \). It can be seen that the difference between the three different error envelopes vanishes for multipath delays smaller than about 0.05 C/A-code chips or 0.5 P-code chips.

![Figure 3.6: Maximum and minimum delay errors for P-code (d=1) and C/A-code (d=0.1 and d=1), SMR=6dB.](image)

If there is more than one multipath signal present, a simple plot like figure 3.5 cannot be drawn, yet the maximum error can still be calculated; now, the maximum error occurs when the instantaneous amplitude \( a_m d \) of the summed multipath S-curves is equal to plus or minus the ideal S-curve, respectively: \( S_o(\tau_e) = -2a_o \tau_e / T_c = \pm a_m d \rightarrow \tau_e = \pm a_m d T_c / 2 \), therefore for multiple multipath signals also the conclusion holds that the maximum tracking errors are proportional to \( d \). Figure 3.5 showed the envelope of the multipath errors. Another important aspect is the mean value of the errors, since this is the remaining error if code ranges are averaged over a time period that is large in comparison with the periodicity of the multipath errors (or exactly equal to a multiple of this periodicity). Plots of these mean errors are drawn in figure 3.7.a and 3.7.b for two different early-late spacings and with the \( SMR \) as a parameter.
It can be seen that a smaller early-late spacing reduces the mean errors considerably, except for small relative multipath delays, where the absolute mean error can be even larger in the case of a smaller early-late spacing. The reason for this is the changed position of the maxima in the mean error curves. The first maximum occurs around a delay value $\tau_1 = a+$ and the second one around $\tau_2 = b+$, where $a+$ and $b+$ are defined in (3.6). For an early-late spacing of one chip time, these points coincide; but for a spacing of 0.1, there are two separate maxima. Since $a+$ is much smaller now
than in the case of $d=1$, the absolute delay errors around $\tau_1 = a +$ can be larger than those of figure 3.7.a. Another effect is that the first maximum in figure 3.7.b is so close to $\tau_1=a-$, that it prevents a negative minimum around $\tau_1=a-$, as in figure 3.7.a.

*Fig. 3.7.c: Standard deviation for a coherent DLL with $d=1$.***

*Fig. 3.7.d: Standard deviation for a coherent DLL with $d=0.1$.***
The corresponding standard deviations of the multipath errors are depicted in the figures 3.7.c and 3.7.d. Note that all error values are given in chip times. To get the resulting errors in meters, the delay errors have to be multiplied by the chip time and the speed of light, which means a multiplication by 293 for the GPS C/A-code or 29,3 for the P-code.

3.3.2 Fast fading

For a large $B_F/B_L$ ratio, the DLL tracks the zero crossing of the time-averaged S-curve. If $B_F$ is also large compared to the carrier tracking loop bandwidth $B_{Lcar}$, then the carrier phase estimate is the phase of the time-averaged sum of correlation values:

$$
\hat{\theta}_o = \arg \left[ E \left\{ a_o \exp(j\theta_o)R(\hat{\tau}_o - \tau_o) + \sum_{i=1}^{M} a_i \exp(j\theta_i)R(\hat{\tau}_o - \tau_i) \right\} \right] = \theta_o \quad (3.7)
$$

In words, the time-averaged sum of the multipath signals is zero, so there is no error in the carrier phase estimate. Note that the signal-to-multipath ratio is assumed to be greater than one, since otherwise the loop may track a multipath signal rather than the line-of-sight signal. It can now be concluded that in the time average of (3.4), the multipath components also reduce to zero, leaving the ideal multipath free S-curve, so the delay estimate is equal to the line-of-sight signal delay. This is even true if $B_F$ is not large in comparison with $B_L$, because the probability density function of the phase differences $\theta_i(t) - \hat{\theta}_o$ in (3.4) is always symmetrical around $\theta_o$, so even with a time-varying estimate $\hat{\theta}_o$, the time average of the multipath components in (3.4) is zero. Figure 3.8 shows a vector representation of the fast fading situation. During a time period of $1/B_L$, the multipath vector rotates many cycles more (or less) than the line-of-sight vector. Because of the relatively small tracking loop bandwidth, a tracking loop only 'sees' the time averaged vector, which is equal to the line-of-sight vector.
3.4 Code range errors of a noncoherent delay lock loop

In a noncoherent DLL, the input signal of (3.3) is again down converted and correlated with an early and a late code, but now the squared envelopes of the resulting correlation values are subtracted to produce the S-curve:

\[
S(\hat{\tau}_o) = \left| \sum_{i=0}^{M} a_i R(\hat{\tau}_o - \tau_i + \frac{d}{2} T_c) \exp(j \theta_i) \right|^2 - \left| \sum_{i=0}^{M} a_i R(\hat{\tau}_o - \tau_i - \frac{d}{2} T_c) \exp(j \theta_i) \right|^2 \tag{3.8}
\]

Just as the coherent DLL, the noncoherent DLL tracks that value of \( \hat{\tau}_o \) for which \( S(\hat{\tau}_o) \) is zero while its slope \( \delta S(\hat{\tau}_o)/\delta \hat{\tau}_o \) is negative.
3.4.1 Slow fading

It is interesting to see that the maximum tracking errors for a noncoherent DLL are exactly the same as for the coherent DLL. In the case of maximum and minimum delay errors, the line-of-sight and multipath signals are in-phase or out-of-phase, which gives a resulting phase that is equal to the line-of-sight carrier phase, provided that the line-of-sight vector is larger than the multipath vector. For this case of a zero phase tracking error, the S-curve of a coherent DLL (3.4) can be rewritten as:

\[
S(\hat{\tau}_o) = \sum_{i=0}^{1} a_i \left[ R(\hat{\tau}_o - \tau_i + \frac{d}{2} T_c) - R(\hat{\tau}_o - \tau_i - \frac{d}{2} T_c) \right] = 0
\]

\[
\Rightarrow \sum_{i=0}^{1} a_i R(\hat{\tau}_o - \tau_i + \frac{d}{2} T_c) = \sum_{i=0}^{1} a_i R(\hat{\tau}_o - \tau_i - \frac{d}{2} T_c)
\]

\[
\Rightarrow \left[ \sum_{i=0}^{1} a_i R(\hat{\tau}_o - \tau_i + \frac{d}{2} T_c) \right]^2 - \left[ \sum_{i=0}^{1} a_i R(\hat{\tau}_o - \tau_i - \frac{d}{2} T_c) \right]^2 = 0
\]  

In (3.9), \( a_I \) should be multiplied by -1 for the case of out-of-phase multipath signals. The last line of (3.9) is equal to the S-curve of a noncoherent DLL (3.8) for this case of a single in-phase or out-of-phase multipath signal. Thus equation (3.9) proves that both coherent and noncoherent delay lock loops yield exactly the same maximum absolute errors, given by (3.6) and shown in figure 3.5. However, the shape of the error curves as a function of time are slightly different. Because of the squared components in the noncoherent DLL S-curve (3.8), it tends to have delay errors larger than or equal to the errors of a coherent delay lock loop. As a result, the mean errors of a noncoherent delay lock loop are larger as compared to the coherent delay lock loop values, see the figures 3.9.a and 3.7.a. This difference is negligible, however, for small early-late spacings; as the early-late spacing becomes small compared to the chip time, then both coherent and noncoherent delay lock loop essentially have the same error patterns for slow multipath fading. It can be seen in the figures 3.9.b, 3.9.d and 3.7.b, 3.7.d that the standard deviations and mean error values are approximately equal for coherent and noncoherent delay lock loops when \( d=0.1 \).
Fig. 3.9.a: Mean error for a noncoherent DLL with $d=1$.

Fig. 3.9.b: Mean error for a noncoherent DLL with $d=0.1$.
Fig. 3.9.c: Standard deviation for a noncoherent DLL with $d=1$. 

Fig. 3.9.d: Standard deviation for a noncoherent DLL with $d=0.1$. 

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3.4.2 Fast fading

For the noncoherent DLL, not only the ratio \( B_F/B_L \) is important to determine if the fading is slow or fast, but also the predetection bandwidth \( B_p \) has to be considered. \( B_p \) is inversely proportional to the time used for correlation before the square envelope operation is performed. \( B_p \) has to be equal or greater than the data bandwidth -50 Hz for GPS- in order to let the signal pass, while \( B_L \) can take values down to 0.05 Hz if carrier-rate aiding is applied [15]. Now, three different situations can be distinguished. First, if \( B_F \) is small in comparison with \( B_L \), it is also small compared to \( B_p \), since \( B_p \) is always greater than \( B_L \). This is the slow fading case. Second, if \( B_F \) is large compared to \( B_L \), but small compared to \( B_p \), then all multipath signals pass the predetection correlation. This is fast fading. Third, there can be very fast fading, for which \( B_F \) is large compared to \( B_p \). In this case, multipath signals have such large frequency differences with the line-of-sight signal that they cannot pass the predetection correlation. Thus the multipath errors reduce to zero, just as the coherent DLL in the case of fast fading. Unfortunately for the noncoherent DLL, values of \( B_F \) that are much greater than \( B_p \) are not very likely in practice (typical values of \( B_p \) for GPS receivers are ranging from 50 Hz when the receiver is in-lock up to 1000 Hz during acquisition, when the Doppler frequency and clock offset are unknown). Normally, \( B_F \) is in the slow fading region for stationary users, while moving users with speeds in excess of several meters per second are normally in the fast fading region.

For fast fading, the resulting S-curve is the time average of (3.8):

\[
S(\hat{\tau}_o) = \sum_{i=0}^{M} \left[ a_i R(\hat{\tau}_o - \tau_i + \frac{d}{2} T_c) \right]^2 - \left[ a_i R(\hat{\tau}_o - \tau_i - \frac{d}{2} T_c) \right]^2 \tag{3.10}
\]

All cross products are filtered out because of their relatively high frequencies, so the resulting S-curve is simply the summation of \( M+1 \) different noncoherent DLL S-curves. Examples of the resulting biases are depicted in figures 3.10.a and 3.10.b.
Fig. 3.10.a: Bias of a noncoherent DLL with $d=1$.

Fig. 3.10.b: Bias of a noncoherent DLL with $d=0.1$.

3.5 Diffuse multipath or multiple reflections

The previous analysis of delay errors primarily focused on the case of specular multipath, caused by a single reflection with a constant amplitude. Although this situation serves as a good model for all situations where there is one dominant multipath signal, it is desired to know the behavior of multipath errors when there
are several multipath signals, or when the multipath amplitude is not constant, as in the case of diffuse multipath. The problem with multiple signals is that the number of parameters - amplitudes, delays and phases - becomes too large to make a general analysis with results that can be compared to the figures in the previous sections. However, it is possible to derive a solution for the case of small early-late spacings and relatively high SMR values. Figure 3.11 depicts the line-of-sight S-curve, together with the multipath S-curves for this situation.

![Line-of-Sight and Multipath S-curves](image)

**Fig. 3.11: Line-of-sight plus several multipath S-curves.**

The delay lock loop tracks that delay value for which the combined S-curve has a zero crossing. Assuming that the multipath amplitude is small compared to the line-of-sight amplitude, it is possible to write the standard deviation $\sigma_\tau$ of the delay errors as the standard deviation $\sigma_m$ of the multipath S-curve amplitude, divided by the derivative of the line-of-sight S-curve.

$$\sigma_\tau = \frac{\sigma_m dT_c}{2a_o} = \frac{dT_c}{2} \sqrt{\frac{1}{2SMR}}$$  \hspace{1cm} (3.11)

Equation (3.11) gives a good approximation of the maximum standard deviation of the delay errors, given a certain SMR value. For SMR values larger than about 6 dB,
equation (3.11) closely corresponds to the maximum standard deviation values found for the single multipath cases, shown in the figures 3.7 and 3.9.

3.6 Carrier range errors

Just as a code tracking loop, the performance of a carrier tracking loop can be affected by multipath, depending on the delays, amplitudes and phases of the multipath signals and on the ratio of the fading bandwidth and the carrier tracking loop bandwidth. It has already been explained in section 3.3.2 that in the case of fast fading, i.e. a fading bandwidth that is large compared to the carrier tracking loop bandwidth, the phase tracking errors reduce to zero, independent of eventual code tracking biases in the case of a noncoherent DLL. For the slow fading case, the equation for the phase estimate $\hat{\theta}_o$ of this loop is given by (3.5) in section 3.3.1. Figure 3.12 depicts the situation for the case of a single multipath signal.

\[ a_t R(\hat{T}_o - T_i) \]
\[ a_o R(\hat{T}_o) \]

Fig. 3.12: Vector diagram showing carrier phase error.

Just as the code tracking error, the carrier tracking error is a quite complicated function to calculate. However, it is possible to give an analytical expression for the amplitude of the carrier phase errors if the assumption is made that the influence of
Code and carrier tracking errors in GNSS and communication systems

code tracking errors on the carrier phase errors is negligible. This assumption is valid for small early-late spacings or for small SMR values, because then the delay error is small compared to the chip time, so the vector lengths in figure 3.12 are influenced insignificantly by the errors in the code delay estimate. In this case, maximum and minimum carrier phase errors occur when the multipath vector is perpendicular to the sum vector, i.e. when \(|\theta_1 - \theta_0| = \pi - \arccos[a_1R(\tau_1)/a_0]\). This results in a maximum phase error \(\theta_e\) given by:

\[
\theta_e = \frac{1}{2\pi} \arcsin \left( \frac{1 - \tau_1/T_c}{\sqrt{SMR}} \right) \text{ [} \lambda \text{]}
\]  

(3.12)

Note that the phase error is given in wavelengths. To get the corresponding L1 and L2 carrier range errors in metres, the results have to be multiplied by 0.19 and 0.24 metres, respectively. Figure 3.13 shows the carrier phase errors for different SMR values. Multipath signals with small relative delays cause the largest phase errors because these signals are hardly attenuated by the correlation operation. Note that the results of figure 3.13 may be inaccurate for low SMR values, together with a large early-late spacing (d=1), because then the previously mentioned assumption about negligible delay errors does not hold.

![Graph showing carrier phase errors](image_url)

*Fig. 3.13: Amplitude of the carrier phase errors.*
Fig. 3.14.a: Standard deviation of carrier tracking errors for $d=1$.

Fig. 3.14.b: Standard deviation of carrier tracking errors for $d=0.1$.

The figures 3.14.a and 3.14.b show the standard deviations of the carrier tracking errors for two different early-late spacings. The errors were calculated using the equations for the coherent DLL given in section 3.3.1. The curves are very similar, except for multipath delays of approximately one chip time. This is because in the case of a one-chip early-late spacing, the influence of code tracking errors on the carrier phase errors is much more important than for the early-late spacing of 0.1 chip, especially when the multipath delay is around one chip time, where the code multipath errors are maximal.
The previous plots were calculated for the case of one specular reflection. In practice, there may be a large number of specular, diffuse or diffracted multipath signals present. It can also happen that there is one dominant diffuse reflection with a time varying phase and amplitude. For all of these cases, the total multipath vector can be modeled by a Rayleigh distributed envelope and a uniformly distributed phase. The sum vector of the line-of-sight signal and the multipath signals then has a Rician distributed envelope and a phase distribution given by [26]:

\[
p(\theta) = \frac{\exp(-\frac{SMR_0}{\sqrt{2\pi}})}{2\pi} \left\{ 1 + \sqrt{\frac{SMR_0}{\sqrt{2\pi}}} \cos\theta \exp\left(\frac{SMR_0}{\sqrt{2\pi}} \cos^2\theta\right) \right\} \left[ 1 + \text{erf}\left(\sqrt{\frac{SMR_0}{\sqrt{2\pi}}} \cos\theta\right) \right]
\]

(3.13)

Where \(\text{erf}(x)\) is the error function [27]. Using this equation, the standard deviation of the carrier phase errors in wavelengths can be calculated as:

\[
\sigma_\theta = \frac{1}{2\pi} \sqrt{\int_0^{2\pi} \theta^2 p(\theta) d\theta}
\]

(3.14)

Note that \(SMR_o\) in equation (3.13) is the signal-to-multipath ratio after correlation with the estimated line-of-sight code. Due to this correlation operation, the power of multipath signals with a certain delay \(\tau_m\) are attenuated by a factor \([R(\tau_m-\tau_o)]^2\), so \(SMR_o\) is always less than the SMR at the input of the receiver, unless the multipath delays are negligible in comparison with the chip time.

Equation (3.14) was calculated numerically using the Newton-Cotes integration technique [27]. From the results shown in figure 3.15, it can be seen that for small SMR values, the phase errors for Rayleigh fading multipath signals are slightly larger than the maximum standard deviations in the case of specular reflections, shown in figures 3.14.a and 3.14.b. This is caused by the time varying amplitude of the Rayleigh fading multipath signal. Since the phase error increases more than proportional when the multipath amplitude increases, the phase error becomes larger as compared to a specular reflection with a constant amplitude. For large SMR values (about 10 dB or more), the phase error is approximately proportional to the multipath amplitude, so for that range there is little difference between diffuse and
specular phase errors. It can be concluded that figure 3.15 provides a useful upper bound for the phase errors, independent of the type of multipath. The upper bound is very tight for SMR above 10 dB and multipath delays that are small compared to the chip time.

![Graph](image)

*Fig. 3.15: Standard deviation of carrier tracking errors in the presence of Rayleigh fading multipath signals.*

### 3.7 Effects of filtering

The previous analysis of multipath tracking errors assumed an infinite bandwidth of the spread-spectrum input signals. In practice, however, the bandwidth of the signals is always limited, which causes a rounding of the sharp edges of the S-curve. Now, there are three effects on multipath errors distinguishable. These effects are illustrated by figure 3.16, which shows a line-of-sight S-curve, together with an in-phase and an out-of-phase multipath S-curve. The combined S-curve has a zero crossing at the intersection of the separate S-curves.
The first effect of filtering is that because of the rounding of the multipath \( S \)-curve edges, the edges of figure 3.5 also become rounded. Second, due to the filtering, the slope of the \( S \)-curve at the desired zero crossing becomes smaller, thereby increasing the multipath errors. In the presence of multipath like drawn in figure 3.16, the delay lock loop tracks that delay value for which the line-of-sight and the multipath \( S \)-curves intersect. The magnitude of the delay error depends on two things: the amplitude of the multipath \( S \)-curve and the slope of the line-of-sight \( S \)-curve. A less steeper slope results in a larger delay error. To calculate the exact deterioration, precise knowledge of the filtered auto correlation function \( R(\tau) \) is required. For a single multipath signal, the maximum tracking error is equal to the amplitude of the filtered multipath \( S \)-curve divided by the slope of the filtered line-of-sight \( S \)-curve, which can be written as:
\[ \tau_e = \lim_{\Delta \to 0} \frac{a_1 \Delta \left[ R\left( \frac{T_c (1-d)}{2} \right) - R\left( \frac{T_c (1+d)}{2} \right) \right]}{2a_0 \left[ R\left( \frac{d-\Delta}{2} \right) - R\left( \frac{d+\Delta}{2} \right) \right]} \] (3.15)

If the signals are unfiltered, then \( R(\tau) = 1 - \sqrt{T_c} \) for \( \tau < T_c \), and the formula for the maximum tracking error reduces to the well-known \( a_1 d T_c / 2 a_0 \) from figure 3.5. In reality, however, the signals are always filtered. Figure 3.17 shows the tracking error \( \tau_e \) divided by \( a_1 T_c / 2 a_0 \) versus the early-late spacing \( d \). The correlation function was filtered by a fourth-order Butterworth filter with a double-sided bandwidth \( B = k / T_c \).

It can be seen that the linear relation between \( d \) and the multipath reduction factor does not hold for early-late spacings smaller than about \( 1/k \). The error reduction factor converges to a value of about \( 1/k \), so there is no use in using an early-late spacing smaller than about \( 1/k \).

![Graph showing multipath reduction factor versus early-late spacing](image)

**Fig. 3.17: Multipath reduction factor versus early-late spacing, for a signal bandwidth of \( k / T_c \).**
The third effect is that for relatively large multipath amplitudes that are outside the linear region of the line-of-sight S-curve, the tracking errors become larger than predicted by equation 3.6. If the amplitude of a multipath signal is equal to the line-of-sight signal amplitude ($SMR=1$), then the multipath errors can even reach maximum and minimum values of plus and minus half a chip time, which is just as large as in the case of an early-late spacing $d=1$. For high $SMR$ values, roughly above $10$ dB, these non-linear effects are very small so the previous analysis can be applied.

### 3.8 Multi-transmitter interference

In code division multiple access systems, specific spread-spectrum codes are used to distinguish between different transmitters. However, the codes that are used do not provide full orthogonality in general, which causes mutual interference. Although the effects of multi-transmitter interference on the bit error probability are well known [28], to the best of the author’s knowledge no previous studies have been conducted to evaluate the effects on the tracking performance. This section shows that spread-spectrum multi-transmitter interference can simply be modeled in the same way as multipath, so it is possible to use the previous multipath analysis with some slight modifications in order to predict the behavior of tracking errors caused by multi-transmitter interference.

Suppose a total number of $K$ transmitters can be received, so the input signal is

$$x(t) = \sum_{i=0}^{K} a_i d_i(t - \tau_i) p(t - \tau_i) \cos(\omega_i t + \theta_i)$$  \hspace{1cm} (3.16)

neglecting noise and multipath. The data signal of the $i$th transmitter is $d_i(t)$. To track the line-of-sight signal ($i=0$), the input signal $x(t)$ is down converted and correlated with $p(t-\tau_0)$. However, besides the desired auto-correlation, at the same time a cross-correlation with the other codes $p(t-\tau_i), i>0$ is obtained. Both auto- and cross-correlation functions consist of triangular pieces, so looking at the sum of auto- and
cross-correlation functions, one cannot distinguish between cross-correlation and multipath; both appear in the same form. However, there are two important differences.

![Graph showing auto and cross correlation peaks]

**Fig. 3.18: Auto and cross correlation functions both appear in the same form.**

The first difference is the presence of different data signals. If these two signals are uncorrelated, then their effect is that after the correlation operation, the interfering cross correlation component is spread over the data bandwidth. In fact, the data bandwidth has the same effect as the fading bandwidth, so the previous multipath analysis also holds for the case of spread-spectrum interference.

When the delay difference between the signals from two GPS satellites is less than one bit time (≈20 ms), however, then the two data signals are certainly not uncorrelated, because of equal preambles and almanacs in the data. In this case, the influence of the data is largely removed, or even completely eliminated, if the correlation is equal to one. If the data signals are correlated, or if the data bandwidth
is smaller than the carrier frequency difference between the transmitters modulo the code repetition frequency, then the latter parameter should be substituted as the fading bandwidth. For the signal-to-multipath ratio, the signal-to-interference ratio $SIR=a_o^2/k_a a_1^2$ can be substituted, where $k_a$ is the attenuation factor due to the correlation; for the GPS C/A-code, $k_a$ is about 0.06. For the GPS P-code, interference can be neglected because of the enormous length of the code, with a repetition time of one week; if the input signal is correlated for one second, then the P-code interference is averaged over about $10^7$ chips, while the C/A-code interference is effectively averaged over 1023 chips only, because of the C/A-code length of 1023 chips. As a result, the signal-to-interference ratio is roughly 40 dB greater for the P-code, causing negligible errors.

A second characteristic of spread-spectrum interference is that cross-correlation peaks can precede the auto correlation peak, as if they were multipath signals with a negative delay. This effect is also observed for long delayed multipath signals. Till now, multipath was modeled as a replica of the auto-correlation peak with a certain positive delay. If the delay exceeded $T_c(1+d/2)$, the resulting multipath errors were zero. Unfortunately, the reality is more complicated; the auto-correlation function of the C/A-code contains unwanted peaks similar to the cross-correlation function with an interfering code. Thus for delays exceeding $T_c(1+d/2)$, there is always a possibility that an unwanted peak distorts the main peak. This unwanted peak again can be modeled as a multipath signal with a certain positive or negative delay that is equal to the delay of the main peak minus the delay of the interfering peak. It can be shown that a multipath signal with positive delay $\tau_I$ gives the same error, but with opposite sign, as a multipath signal with a delay of minus $\tau_I$; independent of the type of loop or fading, $S(\tau_e|\tau_I)=S(-\tau_e|\tau_I)$. In words, the S-curves for $+\tau_I$ and $-\tau_I$ are mirrored versions of each other. Thus if plus $\tau_I$ gives a zero crossing at $\tau_e=\tau_e^{1}$, then minus $\tau_I$ gives a zero crossing at $\tau_e=-\tau_e^{1}$. The conclusion is that all previously shown figures of multipath errors as a function of the multipath signal delay also apply to negative delays by simply multiplying everything by -1.

An interesting fact in the case of two interfering transmitters is that the resulting errors have the same shape, but they are exactly out-of-phase. If the input signal (3.16) is correlated with $p_o(t-\hat{\tau}_o)$ and $p_I(t-\hat{\tau}_I)$ for T seconds, the following output signals are obtained, neglecting the carrier phases:
Multipath and multi-transmitter interference

\[
T \int \left[ a_o p_o(t - \tau_o) + a_1 p_1(t - \tau_1) \right] p_o(t - \hat{\tau}_o) \, dt \\
0 \\
= a_o R_o(\hat{\tau}_o - \tau_o) + a_1 R_c(\hat{\tau}_o - \tau_1) \\
T \int \left[ a_o p_o(t - \tau_o) + a_1 p_1(t - \tau_1) \right] p_1(t - \hat{\tau}_1) \, dt \\
0 \\
= a_1 R_1(\hat{\tau}_1 - \tau_1) + a_o R_c(\hat{\tau}_1 - \tau_o)
\]

(3.17)

Here, \( R_c(\tau) \) is the cross-correlation function of \( p_o(t) \) and \( p_1(t) \). From equation (3.17), it can be seen that the cross-correlation function has a delay of \( \tau_1 - \tau_o \) in comparison with the delay of the auto-correlation function for the first term of (3.17), while the delay is \( \tau_o - \tau_1 \) for the second term. Thus if the correlation function of the first transmitter is distorted by a cross-correlation peak with a positive delay, the second one suffers from exactly the same peak, but now with a negative delay. Also, the SIR values of \( a_o^2/k_o a_1^2 \) and \( a_1^2/k_o a_o^2 \) are different in general, because of differences in received signal powers. For instance, if a certain GPS satellite is received 20 dB stronger than another one, the latter will suffer from an SIR of 3 dB, resulting in C/A-code tracking errors of tens of meters in the case of slow fading. Fortunately, this 'near-far' effect is not a big problem for GPS, because satellites with large received power differences generally also have large Doppler frequency differences, exceeding the tracking loop bandwidth. The situation is slightly different when pseudolites are used, i.e. GPS transmitters that are located on the ground [29]. As users come closer to such pseudolites, the received power can easily become 60 dB stronger than the power of GPS satellites. In order to counteract this near-far effect, pseudolites transmit in a special TDMA format, so that the normal GPS signals are 'jammed' only a minor part of time [29].
3.9 GPS measurements

To verify the theory, several measurements were performed, most of them using a Trimble 4000SST geodetic GPS receiver. This receiver has a noncoherent delay lock loop with a one chip early-late spacing. At the end of this section, some results are shown for a receiver with P-code and C/A-code with a small early-late spacing. To isolate code multipath errors, measured code ranges and carrier ranges were subtracted, leaving the multipath errors, noise, an unknown constant and twice the ionospheric error [3]. The ionospheric error was removed by using simultaneous L1 and L2 carrier range measurements from a reference receiver in a relatively multipath free environment, since the codeless L2 carrier tracking loop was out of lock very often in the presence of strong multipath. The ionospheric error can be estimated as [30]:

\[
I_{L1} = \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2} \left[ R_{ca,L1} - R_{ca,L2} \right] \\
I_{L2} = \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2} \left[ R_{ca,L1} - R_{ca,L2} \right]
\]

(18)

Here, \( I_{L1}, I_{L2} \) and \( R_{ca,L1}, R_{ca,L2} \) are the L1 / L2 ionospheric range errors and the measured L1 / L2 carrier ranges, respectively. Further, \( f_{L1} \) and \( f_{L2} \) are the L1 and L2 carrier frequencies, which are 1575.42 and 1227.6 MHz, respectively.

The unknown range constant was largely removed by setting the mean value of the code measurements to zero. Figure 3.19 shows the measured multipath errors, using a one-second measurement interval without any further filtering of the output data. The main cause of multipath in this case was the Electrical Engineering building of Delft University of Technology. This is an approximately 100 meters tall building with a metal frame structure. The antenna was located about 30 meters from the building.
Multipath and multi-transmitter interference

![Graph showing C/A-code range error vs. Time](image)

*Fig. 3.19: Multipath errors due to the Electrical Engineering building.*

In figure 3.20, a piece of the same measurement is shown together with a curve that was produced by calculating the delay error of a noncoherent delay lock loop for a slowly time varying multipath phase. The delay difference between the reflection and the line-of-sight signal was calculated as in [2] for the case of a vertical reflector:

\[
\tau_1 = \frac{2m}{c} \cos \alpha \cos \eta
\]  

(3.19)

where \( m \) is the perpendicular distance from antenna to the reflector, \( \alpha \) is the difference between the azimuth of the vector normal to the reflector toward the antenna and the satellite azimuth, and \( \eta \) is the satellite elevation. In figure 3.20, \( m \) is 34 meters, \( \alpha \) is 40 and \( \eta \) is 41 degrees, resulting in a delay of 0.13 \( \mu s \). In fact, the delay is time varying because of the time-varying satellite elevation and azimuth angles, while the phase difference \( \theta(t) \) between reflection and line-of-sight signal is linked to the delay by:

\[
\theta(t) = 2\pi f \tau_1(t) + \theta_r(t)
\]  

(3.20)

where \( f \) is the carrier frequency (\( f=1.5 \text{ GHz} \) for L1), and \( \theta_r(t) \) is a phase shift caused by the reflector and the antenna phase pattern. Knowing the time variations of \( \alpha \) and
\( \eta, \tau_I(t) \) can easily be obtained, but an exact calculation of \( \Theta(t) \) is rather difficult, since it requires precise knowledge of the antenna phase pattern and the complex reflection coefficient as a function of the incident angle. Therefore, \( \Theta(t) \) was simply scaled to the measurement. An SMR of 4.6 dB results in a perfect fit of measured and calculated multipath errors, confirming the asymmetrical shape of these errors which causes a non-zero mean. Using the same comparison technique, one can find that the largest error, (-120 meters) shown in figure 3.19, corresponds to an instantaneous SMR of only 1 dB.

![Figure 3.20: Measured and calculated multipath errors.](image)

Figure 3.21 shows multipath errors caused by the Geodetic department building, which is made of concrete and has a height of approximately 30 meters. The GPS antenna was located at a distance of about 75 meters from the building. During this measurement, the specular point at the building disappeared due to the satellite movement, resulting in a gradual decrease of the errors from about 100 to 10 meters peak to peak. Note the amplitude modulation and the phase jumps, which can be caused by two effects: First, most buildings are very rough surfaces for L-band signals, causing a random amplitude modulation of the received reflection. Second, it may be caused by the ripples in the antenna pattern, as explained in [7].
Fig. 3.21: Measured pseudorange errors near the Geodesy building.

Fig. 3.22: Measured and calculated multipath errors, assuming 2 multipath signals.

In figure 3.22, a part of the measurement from figure 3.19 is shown, together with a calculated curve, assuming the presence of two multipath signals with the same
delays but slightly different Doppler frequencies. This causes a certain fading of the resulting multipath amplitude, as the two multipath signals are adding or partially cancelling when their relative phase changes. Note that only the multipath delay was calculated from the known geometry; the rest of the parameters were simply adjusted to get a good fit with the measured curve. Therefore, only the shape of the multipath errors can be verified, showing that multipath from a rough surface - in this case the Electrical Engineering building - indeed can be modeled as a sum of several multipath signals.

Even in apparently multipath-free environments, where standard deviations of about 2 meters or less were measured, the power spectrum of the measured errors showed that the variance of low frequency multipath errors was often larger than the variance caused by noise. Figure 3.23.a is an example of this phenomenon. The standard deviation of this measurement, performed on top of the building of the Geodetic department in Delft, is 2.0 meters. If all peaks in the error power spectrum given in figure 3.23.b that exceed 0.05 meters are attributed to multipath, then it can be calculated that the standard deviation due to multipath only is about 1.4 meters. Since the threshold of 0.05 meters seems rather conservative, it can be expected that multipath dominates the noise in reality.

![Graph](image)

*Fig. 3.23.a: C/A-code range errors on top of the Geodesy building.*
Figure 3.24 shows code range errors, measured near the Electrical Engineering building, together with the corresponding L1 carrier range errors. These carrier range errors were derived by taking a double difference of the GPS satellites 11 and 21, measured simultaneously at two locations. Double differencing is performed by first subtracting simultaneous range measurements to two satellites for each receiver separately. This removes the influences of the receiver clock. Then, the simultaneously measured single differences of two receivers are subtracted to largely remove the effects of troposphere, ionosphere and satellite clock. The resulting double differences practically only consist of the double differenced ranges, noise and multipath. The influence of the time varying range can be eliminated by subtracting a quadratic curve fit from the measured double differences.

The double difference residuals that were measured according to the previously described procedure are approximately equal to the multipath errors of satellite 11 at the receiver near the building, since these errors dominated the noise and multipath of the other three measurements. As can be seen in figure 3.24, maximum carrier range errors of 2 centimetres occur, while the code range errors are limited to about 50 meters. Figure 3.24 shows very clearly that the carrier range errors are zero when the code range errors are maximal, and the reverse, corresponding to calculated curves [13]. This is because for in-phase and out-of-phase multipath signals, the distortion of the tracking curve is maximal, but the phase of the sum vector is equal
to the line-of-sight phase, so the phase error is zero. Maximum carrier range errors correspond to the case where the sum vector is perpendicular to the multipath vector, giving a minimal distortion of the tracking curve and hence a zero code tracking error.

![Carrier and code range errors over time](image)

*Fig. 3.24: Code and carrier range errors.*

An example of satellite interference is depicted in figures 3.25a and 3.25b. Both satellites have approximately the same error curve, but with opposite sign, as predicted in section 2.6. The correlation value between the errors of 3.25a and 3.25b is -0.8, see figure 3.25d. From figure 3.25c, it can be seen that the Doppler difference between the two satellites changes sign in the middle of the measurement. Thus, the second half of 3.25a and 3.25b is a mirrored version of the first part. Since the Doppler difference is much larger than the loop bandwidth most of the time, the fading is fast, so 3.25a and 3.25b depict the bias errors of a noncoherent DLL, while the interference delay gradually changes because of the Doppler difference.
All previous measurements were performed with a C/A-code receiver using a noncoherent delay lock loop with a one chip early-late spacing. The next figure shows measurements from a TurboRogue GPS receiver, which can track both P-code and C/A-code, using a coherent delay lock loop with a 0.1 chip early-late spacing. An interesting phenomenon that can be seen in this figure is that the C/A-code range errors contain a different error pattern than the P-code errors. This is an indication that the dominant multipath signal had a detour exceeding 45 meters (=1.5 P-code chip). In that case, it only affects the C/A-code and not the P-code, as demonstrated by the error envelopes in figure 3.6. Unfortunately, it was not possible to simultaneously measure C/A-code range errors for early-late spacings of 0.1 and 1 chip using the TurboRogue receiver. However, section 4.4 in the next chapter does show such a comparison between different early-late spacings, which demonstrates that multipath can indeed be reduced significantly by reducing the early-late spacing.
Fig. 3.26: Simultaneously measured P-code and C/A-code errors, with the GPS antenna mounted on the fuselage of a Fokker 70 aircraft. Measurement obtained and published with permission from the National Aerospace Laboratory of the Netherlands.

### 3.10 Influence of the satellite geometry

In the previous sections, code and carrier range errors were investigated. In the end, however, only the resulting position errors are of interest. These position errors can only be calculated if the geometry is known. Care should be taken in using the popular Dilution Of Position parameters, since these only provide valid results if the range errors from all satellites have equal variances. If this is the case, the drms (distance root mean squared) position error can be approximated as the standard deviation of the range error multiplied by the PDOP. If there are large differences in the range error variances, then the specific geometry has to be analyzed in order to predict the position errors. However, it is possible to make a few general remarks about this problem: If large errors occur for one satellite and the satellite-receiver vector is approximately perpendicular to the other satellite-receiver vectors, then the
errors of the first satellite directly propagate in the position solution in the same direction as the receiver-satellite vector. The 'bad' satellite can also cause errors in other directions that may even be worse than the range errors themselves, depending on the specific geometry. However, if there are other satellites with smaller errors in approximately the same direction as the 'bad' satellite, then the resulting position errors in the direction of that satellite are reduced.

3.11 Multipath error levels in various navigation applications

The theoretical analysis of multipath errors described in the previous sections will now be used to predict typical multipath error levels in different environments, based on knowledge of the fading bandwidth, multipath delay and the signal-to-multipath ratio. Typical values for these three multipath parameters for circularly polarized L-band signals (such as GPS) in various environments can be found in [19,31,32]. Table 1 summarizes the parameters of these references, together with a prediction of the mean and standard deviation of the GPS code and carrier range errors. The parameters for the DGPS reference site are not extracted from a reference. These figures are only given to illustrate how difficult it is to achieve decimeter accuracy. An SMR of 40 dB is required, while all other environments have SMR values of 20 dB or less. Therefore, the siting of the DGPS reference antenna is the most crucial part in high accuracy differential navigation applications.

In order to calculate the multipath errors in the case of fast fading, it is assumed that the effective SMR is reduced by a factor of $B_F/B_L$. Thus, it is assumed that the multipath Doppler spectrum is approximately flat over a bandwidth of $B_F$. Further, those multipath delay values are used that give the largest errors. This results in P-code errors that are equal to C/A-code errors with a small early-late spacing, since both have the same maximum and minimum multipath errors. However, the P-code is only sensitive to multipath delays smaller than 0.15 μs, while the C/A-code is affected by multipath delays up to more than 1 μs. Thus although the worst-case errors are equal, in practice the P-code errors will be smaller than the C/A-code errors if there are multipath signals with a relative delay larger than 0.1 μs.
Table 3.1: Predicted multipath error levels for various environments, assuming a code $B_L$=0.05 Hz and a carrier $B_L$=15 Hz. The values with a * are for the noncoherent DLL. All other values are approximately equal for coherent and noncoherent DLLs.

<table>
<thead>
<tr>
<th>Environment</th>
<th>SMR [dB]</th>
<th>$B_F$ [Hz]</th>
<th>$\tau$ [µs]</th>
<th>$\sigma$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
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<tbody>
<tr>
<td>Aeronautical, ground reflections</td>
<td>15</td>
<td>30</td>
<td>0-30</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
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<td>Aeronautical, wing reflections</td>
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<td>0.02</td>
<td>0-0.02</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
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<td>0-0.1</td>
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<td>0</td>
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<td></td>
<td>1.5*</td>
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<td>1.5*</td>
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<td>Land mobile, rural/suburban, $v=10$ m/s</td>
<td>5</td>
<td>100</td>
<td>0-0.6</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
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<td>15*</td>
</tr>
<tr>
<td>Land mobile, $v=0$ m/s</td>
<td>5</td>
<td>0.1</td>
<td>0-0.6</td>
<td>30</td>
<td>6</td>
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</table>

3.11.1 Multipath in a differential reference station

A differential reference station is supposed to estimate the range errors which are common to both the reference station and the users (e.g. aircraft), i.e. clock errors, ephemeris errors and atmospheric errors. Unfortunately, noise and multipath errors are generally not common error sources. The resulting variance of the DGPS ranges
therefore consists of the summed variances of the noise and multipath errors from both the reference station and the users.

Since a differential reference station is stationary, it is mainly suffering from slow fading multipath propagation. Suppose the multipath code range errors are required to be less than one meter. Then it can be concluded from the previous analysis that the SMR has to be at least 40 dB in the case of the C/A-code with a one chip early-late spacing. For the P-code, or for a 0.1 chip C/A-code early-late spacing, an SMR of 20 dB or more is required. For CAT III aircraft precision approaches, one meter accuracy is not good enough. In this case, the multipath errors have to be in the order of a decimeter, which requires an SMR of about 40 dB for P-code or C/A-code with a small early-late spacing.

3.11.2 Aircraft multipath

In reference [19], useful information about the aeronautical channel is given. During several hours of flying, the signal-to-multipath ratio and the fading bandwidth were measured, using the Marecs satellite which transmits in the L-band using right hand circular polarization, just like GPS. The receiver antenna was mounted on top of the fuselage of a DC8. For elevation angles ranging from 5 to 35 degrees, SMR values of 10 to 20 dB and $B_F$ values of 10 to 50 Hz were measured, respectively. According to the report [19], the multipath signals are caused by ground reflections. Low SMR values of 10 dB were especially measured during landing manoeuvres. It is important to note that the mentioned SMR values do not incorporate the additional attenuation that occurs when multipath signals with relative delays exceeding one chip time are received by a GPS receiver. In that case, the effective SMR values will increase with 24 dB for the C/A-code [33], while for the P-code the multipath level becomes negligible, on the account of their correlation properties. This advantageous effect is also described in [12,33]. A similar derivation with slightly different notation will now be given for the altitude for which the multipath delay exceeds $T_C(1+d/2)$. To calculate the delay $\tau$ of a ground reflection, the following equation can be used, which assumes the earth surface to be flat:
\[ \tau = \frac{2h \sin \gamma}{c} \]  

(3.21)

where \( h \) is the altitude of the aircraft, \( \gamma \) is the elevation angle and \( c \) is the speed of light. If the altitude is such that the relative delay is less than \( T_c(1+d/2) \), then the ground reflection directly affects the tracking loops. For higher altitudes, the effective SMR is decreased with at least 24 dB for the C/A-code, while the influence of the ground reflection on the P-code becomes negligible. This threshold altitude \( h_{th} \) can be derived from (3.21):

\[ h_{th} = \frac{cT_c(1+d/2)}{2\sin \gamma} \]  

(3.22)

Figure 3.27 depicts the threshold altitude \( h_{th} \) as a function of the elevation angle \( \gamma \) for both C/A-code \((T_c=1\mu s)\) and P-code \((T_c=0.1\mu s)\). The early-late spacing \( d \) is chosen equal to 0.1 chip.

![Threshold altitude vs Elevation angle](image)

*Fig. 3.27: Threshold altitude \( h \) versus the elevation angle.*

In reference [31], multipath measurements from a Boeing 747 are described. Because of the use of a directive antenna - a 16 elements phased-array antenna - reflections from the ground or sea were found to be negligible. However, there was
still multipath present, mainly caused by the wings. An SMR of about 20 dB and a fading bandwidth of 0.02 Hz were reported for those azimuth angles that gave a specular wing reflection. Such a multipath signal would give GPS code ranging errors of about half a meter, assuming a multipath delay of about 0.02 ms. Since it is very unlikely that several GPS signals simultaneously suffer from specular wing reflections, the position error can be expected to be less than half a meter.

### 3.12 Impact of synchronization errors on communications

Section 2.7 gave a brief explanation of the influence of synchronization errors on the performance of a communication system. Using the knowledge obtained from the analysis in this chapter, it is now possible to give some more specific results. In order to illustrate the problem, figure 2.27 is redrawn in figure 3.28.

![Figure 3.28: Correlation function with in-phase and out-of-phase multipath signals. a,b) Line-of-sight and multipath correlation functions, c) correlation function when b) is out-of-phase, d) correlation function when b) is in-phase.](image-url)
The task of a navigation receiver is to estimate the delay of the line-of-sight signal, i.e., it has to track the maximum of signal $a$ in figure 3.28. A communication receiver, however, has to maximize the signal-to-noise ratio. Therefore, it has to track the maximum of the combined line-of-sight plus multipath signals. As explained in this chapter, tracking of the combined signals occurs when the tracking loop bandwidth is large in comparison with the fading bandwidth, i.e., the case of slow fading. Thus for slow fading, the performance of a communication receiver is not affected by synchronization errors, contrary to navigation receivers.

The situation is opposite for fast fading, whereby the tracking loop bandwidth is small in comparison with the fading bandwidth. In this case, a coherent delay lock loop tracks the delay of the dominant path signal, which is generally the line-of-sight signal. It can be seen in figure 3.28 that the instantaneous maximum of the combined signal (c/d) is larger than the maximum value of the line-of-sight signal ($a$), although the difference is not very large; about half a dB, and even less for higher SMR values. This means that a communication receiver experiences only a minor increase in bit error probability. However, many spread-spectrum communication receivers use noncoherent delay lock loops. In the case of fast fading, these loops do not track the line-of-sight delay, but a kind of weighted line-of-sight plus multipath delay, as explained in section 3.4.2. In the case of figure 3.28, this delay would be close to 0.25 chips. Since this delay is close to the zero crossing of the combined correlation function for out-of-phase multipath (c), the result is a dramatic reduction in signal-to-noise ratio.

Figure 3.29 shows the instantaneous correlation output with and without delay errors for an SMR of 5 dB and a relative multipath delay of 0.7 chip. As the relative multipath carrier phase varies from -π to 0, the amplitude degradation gradually decreases from about 3 to 0.5 dB. This amplitude degradation results in an increase of the bit error probability, which can be calculated by integrating the instantaneous bit error probability over all possible multipath carrier phases:

$$P_e = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2} \text{erfc} \left( x(\theta) \sqrt{\frac{E_b}{N_0}} \right) d\theta$$  \hspace{1cm} (3.23)
Here, $x(\theta)$ is the instantaneous correlation amplitude, depending on the multipath carrier phase $\theta$.

![Graph showing the instantaneous correlation output as a function of the multipath carrier phase](image)

*Fig. 3.29: Value of the instantaneous correlation output as a function of the relative multipath carrier phase for SMR=5 dB and a relative multipath delay of 0.7 chip.*

Since the bit error probability is a nonlinear function of the signal-to-noise ratio, the degradation caused by delay errors also depends on the signal-to-noise ratio. This is illustrated by figure 3.30, which shows the bit error probability versus signal-to-noise ratio for a multipath-free channel, together with the case of multipath with and without delay errors. It can be seen that the impact of the tracking errors is considerably larger than the effect of multipath fading. In order to achieve a bit error probability of $10^{-3}$, for instance, about 2 dB extra signal-to-noise ratio is required in order to compensate for the loss caused by the delay errors, while the loss caused by multipath fading only requires about half a dB of extra signal-to-noise ratio.
Fig. 3.30: Value of the instantaneous correlation output as a function of the relative multipath carrier phase for SMR=5 dB and a relative multipath delay of 0.7 chip.

Since the tracking errors depend on both SMR and the relative multipath delay, the degradation in bit error probability also depends on these two parameters. While the previous plot only showed the degradation for one specific SMR and multipath delay, figure 3.31 gives a general picture of the degradation in bit error probability versus SMR and multipath delay. It shows the increase in bit error probability as compared to the bit error probability without delay errors, assuming an $Eb/N_0$ value of 10 dB. For an SMR of 3 dB and a multipath delay of 0.75 chip, the bit error probability is 300 times larger than in the case of no tracking errors. This clearly demonstrates the seriousness of this problem, which should be avoided by using a coherent tracking loop or by choosing the tracking loop bandwidth higher than the fading bandwidth. Of course, the latter option is not always possible, because an increased tracking loop bandwidth increases the noise level.
Fig. 3.31: Bit error probability with delay error, divided by bit error probability without delay error, for $E_b/N_0=10$ dB.

All previous figures were calculated assuming an early-late spacing of one chip. For smaller early-late spacings, the tracking errors and bit error degradation would be smaller. However, many spread-spectrum communication systems use bandlimited signals with a double-sided bandwidth of about $2/T_C$. For these systems, decreasing the early-late spacing does not help, as explained in section 3.7.
3.13 Conclusions

In the analysis of multipath errors, the ratio of fading bandwidth and loop bandwidth turned out to be a crucial parameter; for small $B_p/B_L$ values (stationary users), one suffers from slowly time-varying errors. The magnitude of these errors is equal for coherent and noncoherent delay lock loops, regardless of the early-late spacing, which was proven by a closed-form expression. The time-varying errors generally have a non-zero mean. Thus even a long post-averaging time does not help in eliminating this error source.

For large $B_p/B_L$ values (fast fading), a coherent DLL produces negligible errors, while a noncoherent DLL ends up with a certain bias. Thus for users that mainly suffer from fast fading, like aircraft, a coherent DLL is preferable in order to minimize multipath errors in the pseudoranges. However, it should be noted that if a noncoherent delay lock loop is used with a small (0.1 chip) early-late spacing, then the fast fading errors are very small; for a signal-to-multipath ratio of 20 dB, the errors will be in the order of a decimeter.

The maximum value of code multipath errors is proportional to the early-late spacing $d$, reaching a maximum of $dT_c/2$ if the SMR reaches 1. This means that reducing the early-late spacing ten times also reduces the multipath errors up to a factor of ten. However, this maximum improvement is only achieved for multipath signals with relatively long delays, in the order of half a chip, which corresponds to a detour of 150 meters. For smaller multipath delays, the difference between normal and small early-late spacings decreases until they become equal at a multipath delay of about $dT_c/2$.

Filtering of the spread-spectrum input signals limits the beneficial effects of reducing the early-late spacing. There is little advantage to be gained by making the early-late spacing smaller than the inverse of the double-sided signal bandwidth. This means, for instance, that the proposed Inmarsat additional GPS-like ranging signals will be much more susceptible to multipath than normal GPS signals, because of the approximately ten times smaller bandwidth (2.2 MHz) of the Inmarsat signals [34]. Further, filtering of the spread-spectrum signals causes an additional increase of the multipath errors when the signal-to-noise ratio becomes low.
Multipath and multi-transmitter interference

(< 10 dB). If the SMR approaches one, the errors can reach maximum and minimum values of ±T_c/2, or ±150 meters for the C/A-code, independent of the early-late spacing.

Measurements in severely multipath fading environments showed that multipath (C/A-code) ranging errors up to tens of meters can occur. Even in apparently multipath-free environments, multipath often causes larger errors than thermal noise. Therefore, multipath will often be the main source of errors in differential GPS applications.

Besides multipath, multi-transmitter interference can also cause synchronization errors. Because of relatively large differences in Doppler frequencies, and because of (partially) uncorrelated data, multi-transmitter interference has a similar effect as fast fading multipath. Therefore, a coherent DLL experiences negligible errors, while a noncoherent DLL can give small, but detectable errors. In practice, however, the effects of multi-transmitter interference on synchronization errors are negligible in comparison with the effects of multipath propagation.

3.14 References in chapter 3


Multipath and multi-transmitter interference
4 MAXIMUM LIKELIHOOD MULTIPATH ESTIMATION FOR GNSS AND RELATED SYSTEMS

The previous chapter demonstrated the deleterious effects of multipath propagation on code and carrier tracking loops. Up till now, receivers have hardly used specific counter measures against multipath. Decreasing the early-late spacing - as some manufacturers did - is a step in the right direction. However, carrier range errors are not reduced at all by this technique, while the remaining code range errors can still be considerable.

In this chapter, it is demonstrated that it is possible to largely reduce both code and carrier multipath errors by using a specific receiver structure which simultaneously estimates the parameters of line-of-sight plus multipath signals. The chapter starts with the theoretical background of this new technique, called the Multipath Estimating Delay Lock Loop (MEDLL). An analysis is given of the accuracy levels that can be achieved. This analysis also provides a solution to a flaw in the existing literature on delay lock loop tracking errors. Finally, some implementation issues are discussed, after which a number of measurements are shown which clearly demonstrate the advantages of the MEDLL over a conventional receiver.

4.1 Maximum likelihood estimation of multipath parameters

In the presence of multipath propagation, the received signal at the input of a direct-sequence spread-spectrum receiver can be written as:

\[ r(t) = \sum_{m=0}^{M-1} a_m p(t - \tau_m) \cos(\omega t + \theta_m) + n(t) \]  

(4.1)
where \( p(t) \) is the spread-spectrum code and \( n(t) \) is white Gaussian noise. For a positioning system like GPS, the parameters of interest are the line-of-sight signal delay and phase. However, because of multipath, a conventional delay lock loop is not able to measure these parameters correctly. The MEDLL obviates this problem by taking into account the multipath signals. It estimates amplitudes, delays and phases of both line-of-sight and multipath signals. By separating line-of-sight and multipath signals, the MEDLL can theoretically obtain line-of-sight range measurements that have no errors caused by multipath.

According to maximum likelihood estimation theory, the MEDLL calculates those estimates which minimize the mean square error \( L(a_m, \tau_m, \theta_m) \):

\[
L(a_m, \tau_m, \theta_m) = \int_{t-T}^{t} [r(t) - s(t)]^2 dt
\]

\[
s(t) = \sum_{m=0}^{M-1} a_m p(t - \tau_m) \cos(\omega t + \theta_m)
\]

(4.2)

where \( s(t) \) is the estimate of the line-of-sight plus multipath signals. Equation (4.2) can be minimized by setting the partial derivatives of \( L(a_m, \tau_m, \theta_m) \) to zero. The resulting equations are (see derivation in appendix B):

\[
\hat{\tau}_m = \max_{\tau} \text{Re} \left\{ [R_x(\tau) - \sum_{i=0}^{M-1} \hat{a}_i R(\tau - \hat{\tau}_i) \exp(j\hat{\theta}_i)] \exp(-j\hat{\theta}_m) \right\}
\]

\[
\hat{a}_m = \text{Re} \left\{ [R_x(\hat{\tau}_m) - \sum_{i=0}^{M-1} \hat{a}_i R(\hat{\tau}_m - \hat{\tau}_i) \exp(j\hat{\theta}_i)] \exp(-j\hat{\theta}_m) \right\}
\]

\[
\hat{\theta}_m = \text{arg} \left\{ [R_x(\hat{\tau}_m) - \sum_{i=0}^{M-1} \hat{a}_i R(\hat{\tau}_m - \hat{\tau}_i) \exp(j\hat{\theta}_i)] \right\}
\]

(4.3)

\[
R_x(\tau) = \frac{2}{T} \int_{t-T}^{t} r(t) p(t - \tau) \exp(-j\omega t) dt
\]
In (4.3), $R_x(\tau)$ is the in-phase/quadrature down converted correlation function and $R(\tau)$ is the reference correlation function. Basically, solving the MEDLL equations is like performing a nonlinear curve fit: find a set of reference correlation functions with a certain amplitude, phase and delay which give the best possible fit on the input correlation function. Essentially, a conventional receiver does the same thing, but for one signal only - the line-of-sight signal. When there is multipath present, the MEDLL can improve the curve fit over that of a conventional receiver by choosing $\hat{M}$ larger than one in (4.3), thereby separating line-of-sight and multipath signals. Since the number of multipath signals $M$ is not known in general, this parameter also has to be estimated. One way to do this is to find that value of $\hat{M}$ that minimizes the mean square error given by equation (4.2). Similar to the solution for the other parameters, the following expression for $\hat{M}$ can be obtained.

$$\hat{M} = \max_{M'} \sum_{i=0}^{M'-1} \int_{t-T}^{t} r(t) \hat{a}_i p(t - \hat{\tau}_i) \cos(\omega t + \hat{\theta}_i) \, dt \quad (4.4)$$

This expression can be rewritten as a summation of weighted input correlation functions:

$$\hat{M} = \max_{M'} \text{Re} \left\{ \sum_{i=0}^{M'-1} \hat{a}_i R_x(\hat{\tau}_i) \exp(-j\hat{\theta}_i) \right\} \quad (4.5)$$

Instead of estimating the number of paths by using the previous equation, it is also possible to choose a fixed value for $\hat{M}$ in order to simplify the implementation. In this case, there will be a certain remaining error if $\hat{M}$ is chosen too small or an increased noise error if $\hat{M}$ is chosen too large.

It should be noted that in the definition of the input signal (4.1), all signals are assumed to have the same frequency. However, in practice the signals will have small differences in Doppler frequencies. In order to account for this effect, the maximum likelihood estimates (4.3) could be extended with frequency estimates. However, this is not necessary, because errors in the frequency estimates of multipath signals do not influence the line-of-sight delay and phase estimates;
instead, these errors just reduce the amplitudes of the multipath signals after correlation with a factor that is equal to:

\[
\left\lfloor \frac{T}{\int_0^T \cos((\omega_i - \omega)t)dt} \right\rfloor (4.6)
\]

This means that if the frequency difference \(2\pi/(\omega_i - \omega)\) is large in comparison with the inverse of the correlation time \(1/T\), there will be a negligible amplitude left after correlation. In fact, this is exactly the same effect that caused negligible tracking errors in a coherent delay lock loop in the case of fast fading, as explained in chapter 3. If the frequency difference is not large in comparison with the correlation time, then the maximum likelihood estimation technique will find the remaining amplitude and phase of the multipath component after correlation with the reference signal \(p(t - \tau) \exp(-j\omega t)\).

As an example of the difference between the MEDLL and the conventional DLL in the presence of multipath propagation, the figures 4.1 and 4.2 show two examples of both estimation techniques, using the same input correlation function \(R_x(\tau)\), which was measured with a NovAtel GPS receiver in the vicinity of a tall building. No samples were taken of the leading edge of the correlation function, since all multipath correlation peaks can expected to be in the tail of the line-of-sight correlation function. A conventional delay lock loop only needs two correlation samples around the measured correlation peak to estimate the line-of-sight delay and amplitude. However, because it does not take into account the multipath distortion, its estimates can be highly inaccurate. This also shows in the residuals, that are found by subtracting the measured correlation function and the estimated line-of-sight correlation function, see figure 4.1.
Maximum likelihood multipath estimation for GNSS and related systems

Figure 4.1: Measured correlation function, together with estimated line-of-sight signal and residuals (DLL).

Figure 4.2: Measured correlation function, together with estimated line-of-sight signal, multipath signals and residuals (MEDLL).
The conventional delay lock loop estimate is in fact a special case of (4.3) for $\hat{M}$ equal to one. When $\hat{M}$ is larger than one, then instead of one signal, the MEDLL tries to find several signals that together give a good match with the measured input correlation function. As an example, figure 4.2 shows the estimated line-of-sight and multipath signals for the same input signal as the previous delay lock loop plot. It can be seen that the residuals in this case are much smaller (about 50 times) than the delay lock loop residuals, which is an indication that the estimated MEDLL signal parameters are much more accurate than those from the DLL.

### 4.2 Theoretical accuracy limits

As a result of the MEDLL technique, the code and carrier range errors caused by multipath are greatly reduced. In the end, only noise is the limiting factor. This section tries to find what level of accuracy could ultimately be achieved. First, the MEDLL errors caused by noise are analyzed in the sections 4.2.1 to 4.2.3, taking into account filtering of the incoming spread-spectrum signal. Second, the errors caused by remainders of multipath signals are analyzed in section 4.2.4.

After the receiver’s bandpass filter, the received signal for one particular satellite can be written as:

$$x(t) = \left[ a_o p(t - \tau_o) d(t - \tau_o) + n_p(t) \right] \cos(\omega t + \theta_o) - n_q(t) \sin(\omega t + \theta_o)$$  \hspace{1cm} (4.7)

where $n_p(t)$ and $n_q(t)$ are in-phase and quadrature noise components with a double-sided power spectral density of $N_o/2$ within the pass band of the input filter. In the following analysis, it is assumed that the tracking loop operates in its linear region, which means that the loop can be simply modeled as a linear filter with a certain one-sided noise bandwidth $B_L$. Further, it is assumed that carrier tracking errors and data bit errors have a negligible influence on the code tracking errors. These assumptions are valid as long as the loop signal-to-noise ratio $SNR_L = C/2BN_o$ is above some threshold value, typically about 10 dB [12,14].

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4.2.1 Code tracking loop with infinite signal bandwidth

In a coherent delay lock loop, the input signal \( x(t) \) is correlated with an early and late code, multiplied by the estimated data bits and the estimated carrier signal, which results in the following early and late correlation values:

\[
R_E(t, \tau) = a_o R(\tau + \frac{d}{2} T_c) + n(t + \frac{d}{2} T_c)
\]

\[
R_L(t, \tau) = a_o R(\tau - \frac{d}{2} T_c) + n(t - \frac{d}{2} T_c)
\] (4.8)

In this equations, \( n(t) \) is the remainder of the in-phase noise after the correlation operation. Since this operation can be described as a linear filter, the variance of \( n(t) \) is equal to 2\(B_LN_o\), which is the in-phase noise power in the double-sided loop noise bandwidth of 2\(B_L\) (note that the signal is down converted by multiplying with 2\(cos(\omega t + \theta_o)\), so that the baseband in-phase noise has a double-sided spectral density of \(N_o\)). In order to get an estimate of the delay difference between the received code and the locally generated code, the early and late correlation values are subtracted. Without noise, this results in the S-curve \(S(\tau)\), given by:

\[
S(\tau) = a_o [R(\tau - \frac{d}{2} T_c) - R(\tau + \frac{d}{2} T_c)]
\] (4.9)

The code tracking loop will track that delay for which \(S(\tau)\) is zero, so in the noise free case there will be no difference between the delays of the input signal and the local code. When there is noise present, however, the zero crossing of \(S(\tau)\) will shift to a different position, resulting in a certain delay error. Since the tracking loop is assumed to operate in its linear region, the standard deviation of the delay errors is equal to the standard deviation \(\sigma_n\) of the noise on the S-curve, divided by the slope of the S-curve.

\[
\sigma_\tau = \frac{\sigma_n}{\left| \frac{\partial S}{\partial \tau} (0) \right|} = \frac{\sigma_n}{2a_o / T_c}
\] (4.10)
The noise on the S-curve arises from the subtraction of two noise signals that are present on the early and late correlation functions. If white Gaussian noise is correlated with a pseudo-random noise code, then the autocorrelation function of the random output signal is equal to the autocorrelation function of the code. So two signals with a relative delay of $dT_c$, such as the early and late noise signals, have a cross-correlation value of $R(dT_c)$, which is equal to $1-d$ for unfiltered pseudo-random noise codes. After subtraction of the early and late noise signals, both with noise power $2B_LN_0$, the standard deviation $\sigma_n$ of the noise on the S-curve is related to the input noise by:

$$\sigma_n^2 = 4B_LN_0d$$

(4.11)

By combining the previous equations, and by substituting $C=a_o^2/2$, the following expression for the standard deviation of the delay errors can be obtained:

$$\sigma_t = T_c \frac{B_Ld}{2C/N_o}$$

(4.12)

This result is the same as obtained in [8]. The same equation for $d=1$ was first published in [11], while [12,15] previously mentioned the fact that reducing the early-late spacing would reduce the timing jitter. However, there is one major flaw in all these previous analyses: they all assume an infinite signal bandwidth; this results in a timing error that can be made arbitrarily small by reducing the early-late spacing. It is mentioned in [8] that the early-late spacing should be large enough to keep the early and late samples on the - approximately - linear part of the correlation function, but no exact relation is given between the timing errors, the early-late spacing and the signal bandwidth. The derivation of this relation is the subject of the next section.

4.2.2 Code tracking loop with finite signal bandwidth

The code delay is estimated by finding the maximum of the correlation function. If noise is present, then the maximum correlation value will be at the point where the
differentiated correlation function plus the differentiated noise is zero. Assuming that the differentiated correlation function is linear around the desired zero crossing, the standard deviation $\sigma_\tau$ of the delay error can be written as the standard deviation $\sigma'$ of the differentiated noise signal, divided by the second derivative of the correlation function at $\tau=0$.

$$\sigma_\tau = \frac{\sigma'}{\frac{\partial^2 R}{\partial \tau^2}(0)}$$ \hspace{1cm} (4.13)

The standard deviation of the differentiated noise can be calculated as follows. In the receiver, white Gaussian noise with a double-sided density of $N_0/2$ is down converted, filtered and correlated with a certain spread-spectrum code. The frequency transfer function of the correlation operation is equal to the complex conjugate of the frequency spectrum of the spread-spectrum code, divided by the total correlation time $T$. Thus after down conversion and correlation, the noise power is:

$$\sigma^2 = N_0 \frac{T_c \sin^2(\pi f T_c)}{T}$$ \hspace{1cm} (4.14)

The noise power after differentiation can be found using the rule that differentiation in the time domain is equivalent to a multiplication with $j2\pi f$ in the frequency domain. Further, assuming that the receiver filter can be approximated by an ideal rectangular filter with a double-sided bandwidth $B$, the differentiated noise power can be calculated by integrating the noise spectral density:

$$\sigma'^2 = \int_{-B/2}^{B/2} \frac{4N_0}{TT_c} \sin^2(\pi f T_c)df = \frac{2BN_0}{TT_c}$$ \hspace{1cm} (4.15)

Now the differentiated noise power is known, only the second derivative of the correlation function has to be calculated in order to find the delay error. Without filtering, the correlation function has a triangular shape, with an amplitude $A$ and a width of $2T_c$. The second derivative of this function is:
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\[ \frac{\partial^2 R_0}{\partial \tau^2}(\tau) = \frac{A}{T_c} \left[ \delta(\tau + T_c) - 2\delta(\tau) + \delta(\tau - T_c) \right] \quad (4.16) \]

where \( \delta(\cdot) \) is the Dirac function.

If the signal is filtered with an ideal rectangular filter with an impulse response of \( B \text{sinc}(\pi t B) \), then the second derivative is given by the following convolution:

\[ \frac{\partial^2 R}{\partial \tau^2}(\tau) = \int_{-\infty}^{\infty} \frac{\partial^2 R_0}{\partial \tau^2}(t) B \text{sinc}[\pi(\tau - t)B] dt = \quad (4.17) \]

\[ \Rightarrow \frac{\partial^2 R}{\partial \tau^2}(0) = \frac{2AB}{T_c} \left[ \text{sinc}(\pi T_c B) - 1 \right] \]

Now it is possible to obtain an expression for the code delay error by dividing the noise standard deviation by the second derivative of the correlation function:

\[ \sigma_\tau = T_c \sqrt{\frac{B_L}{2BT_c C / N_0}} \cdot \frac{1}{\text{sinc}(\pi BT_c) - 1} \quad (4.18) \]

Here, \( C = A^2 / 2 \) is the signal power and \( B_L = 1 / 2T \) is the equivalent code tracking loop bandwidth for a total correlation or averaging time \( T \).

The \( 1/\left[ \text{sinc}(\pi BT_c) - 1 \right] \) term is equal to one when \( B \) is an integer multiple of the chip rate \( 1/T_c \). If \( B \) becomes smaller than \( 1/T_c \), then the delay error grows to infinity, which makes sense since the bandwidth does not satisfy the Nyquist criterion in that case. If \( BT_c \) is equal to an integer \( k \) larger than zero, then the delay error can be written as:

\[ \sigma_\tau = T_c \sqrt{\frac{B_L}{2kC / N_0}} \quad (4.19) \]
If this solution is compared with the delay error equation for an early-late spacing of one that was given by [11], it can be seen that the delay error can be reduced by a factor that is equal to $\sqrt{k}$. Another interesting result obtained from this analysis is the comparison between two spread-spectrum codes with different chip rates, but with the same bandwidth. Examples of this are the GPS C/A-code ($1/T_c=1.023$ MHz) and the P-code ($1/T_c=10.23$ MHz); Both are transmitted with a bandwidth of approximately 20 MHz. According to the previous equations, and taking into account the 3 dB higher C/A-code power, the accuracy of the P-code is only $\sqrt{5}$ better than the C/A-code accuracy. If both C/A-code and P-code receivers used a one-chip early-late spacing, then the P-code accuracy would be better by a factor of $10/\sqrt{2}$, which is a factor $\sqrt{10}$ more than in the case of an early-late spacing that is equal to or smaller than one over the signal bandwidth.

The previous analysis provides a lower bound for the code tracking errors, which is reached in the case of Gaussian noise and an infinitely small early-late spacing. It does not show, however, what the degradation is for an early-late spacing that is not infinitely small. In order to find this degradation, equation 4.10 can be rewritten as:

$$\sigma_\tau = \frac{\sigma_n}{\delta S(0)} = \lim_{\Delta \to 0} \frac{\sqrt{2\sigma^2 (1-R(dT_c))}}{2a_0[R(dT_c - \Delta)/2 - R(dT_c + \Delta)/2)]/\Delta}$$

(4.20)

For an infinite signal bandwidth, $R(\tau)=1- \tau / T_c$, and (4.20) reduces to (4.12). For a finite bandwidth and an arbitrary early-late spacing $d < T_c$, equation (4.20) can be evaluated numerically. Figure 4.3 shows the delay error for an early-late spacing equal to one in the absence of filtering, divided by the delay error using a fourth-order Butterworth filter with a double-sided bandwidth $B=k/T_c$. Thus, a noise reduction factor is obtained as a function of the early-late spacing and the signal bandwidth. As can be seen in the figure, the noise reduction factor converges to approximately $\sqrt{k}$, as predicted by the analysis. This minimum reduction value is achieved for an early-late spacing slightly smaller than $1/k$, so there is little advantage in decreasing the early-late spacing below $1/k$.

1 The double-sided bandwidth refers to a baseband signal. Equivalently, the one-sided bandwidth of a modulated RF/IF (Radio/Intermediate Frequency) signal can be used.
Another interesting phenomenon is that for early-late spacings close to one, the performance of small signal bandwidths becomes relatively better than that of high signal bandwidths. This is because for large early-late spacings, filtering has more effect on the decrease of the cross correlation between early and late noise samples than on the decrease of the steepness of the S-curve slope. So if the early-late spacing is one chip, for instance, then the bandwidth should be $2/T_C$ in order to minimize the noise errors.

![Graph showing noise reduction factor versus early-late spacing](image)

*Figure 4.3: Noise reduction factor versus early-late spacing for a double-sided signal bandwidth of $k/T_C$. The noise reduction factor gives the decrease of the noise range errors as compared to the errors of a DLL with a one chip early-late spacing and an infinite signal bandwidth.*
4.2.3 Carrier tracking loop

Under the same assumptions as for the coherent delay lock loop, the carrier tracking loop can be modeled as a linear filter with a noise bandwidth $B_L$, which filters the remaining in-phase and quadrature signals after the input signal has been down converted and multiplied with the PRN code and the data bits. Then, the phase is estimated as:

$$\hat{\theta} = \tan\left(\frac{a_0 R(\tau) \sin \theta + n_q}{a_0 R(\tau) \cos \theta + n_i}\right)$$  \hspace{1cm} (4.21)

If the loop signal-to-noise ratio $C/2N_0B_L$ is above some threshold value, roughly about 10 dB, then the in-phase noise $n_i$ is negligible in comparison with the in-phase signal $a_0 R(\tau) \cos \theta$. Further, the code delay error $\tau$ is assumed to be much smaller than a chip, so $R(\tau)$ is approximately equal to one. In that case, the carrier phase error $\theta_c$ can be written as:

$$\theta_c = \hat{\theta} - \theta \equiv \tan\left(\frac{n_q}{a_0}\right) \approx \frac{n_q}{a_0}$$  \hspace{1cm} (4.22)

Since the variance of the quadrature noise is equal to $2B_L N_0$ and the signal power $C$ is given as $a_0^2/2$, it is possible to obtain an expression for the standard deviation of the carrier phase noise as a function of the carrier-to-noise ratio and the tracking loop bandwidth:

$$\sigma_\theta = \sqrt{\frac{B_L}{C / N_0}}$$  \hspace{1cm} (4.23)

This result is the same as obtained through an alternative derivation in [13].
4.2.4 Accuracy of the MEDLL

In the previous sections, the accuracy of delay and phase estimates in the presence of only noise was analyzed. In the case of multipath estimation, however, the accuracy of the line-of-sight parameters is also affected by the remainders of the multipath signals after subtraction from the input signal. For one multipath signal, the amount of reduction can be calculated as follows. It is assumed that the MEDLL equations are calculated iteratively, estimating delay, phase and amplitude of a certain path signal, after which the estimated signal is subtracted in order to estimate another path. By doing so, the first line-of-sight amplitude estimate will have an error of \( a_I R(\tau_I) + n(0) \), where \( a_I \) and \( \tau_I \) are the amplitude and delay of the multipath signal, respectively, and \( n(0) \) is the correlation noise value at the top of the correlation function. It is assumed here that the influence of phase and delay errors on the the amplitude estimate can be neglected.

The next step is to subtract the line-of-sight estimate in order to get an estimated multipath correlation function. The estimated multipath amplitude is equal to:

\[
\hat{a}_1 = a_1 - [a_1 R(\tau_1) + n(0)] R(\tau_1) + n(\tau_1)
\]  

(4.24)

Thus, the error in the estimated amplitude consists of the amplitude \( a_I \) itself, multiplied by the square of the correlation value \( R(\tau_I) \), plus two noise components \( n(0) \) and \( n(\tau_I) \). When this iterative procedure is repeated several times, the \( i \)th estimate of the line-of-sight amplitude estimate can be written as:

\[
\hat{a}_o = a_o + a_1 R^{2i-1}(\tau_1) + n(0) \sum_{j=1}^{i} R^{2j-2}(\tau_1) - n(\tau_1) \sum_{j=1}^{i-1} R^{2j-1}(\tau_1)
\]  

(4.25)

It can be seen from equation (4.25) that the multipath contribution is suppressed by a factor of \( a_I R^{2i-1}(\tau_I) / a_I R(\tau_I) = R^{2i-2}(\tau_I) \), assuming that the noise components can be neglected. This means that the multipath suppression is directly proportional to the correlation value \( R(\tau_I) \), raised to the power of \( 2i-2 \). For a multipath delay of 0.9 chip, for instance, two iterations give a suppression of 0.01, assuming an unfiltered correlation function, where \( R(0.9)=0.1 \). However, when the multipath delay is only
0.1 chip, then two iterations only give a suppression of $0.9^2 = 0.81$. Thus smaller multipath delays require more iterations to get a certain amount of multipath reduction. For a finite number of iterations, there will always be a certain remaining error. This is illustrated by figures 4.4 and 4.5, which show simulated DLL and MEDLL code and carrier range errors versus the relative delay between multipath and line-of-sight signal. As explained in chapter 3, maximum and minimum code range errors occur when the multipath signal is in phase or out of phase with the line-of-sight signal, while the largest carrier range errors occur for a relative multipath phase of $\pi \cdot \arccos(a_1 R(\tau_f)/a_o)$. These worst case conditions are used in the figures 4.4 and 4.5.

![Graph](image)

**Fig. 4.4: DLL and MEDLL code range errors in the case of one multipath signal, $d=0.1$, SMR=6 dB, 3 meters in vertical scale corresponds to 0.01 GPS C/A-code chip.**

It can be seen that even in the absence of noise, the MEDLL still has a nonnegligible error for multipath delays below one tenth of a chip. For multipath delays close to zero, the DLL and MEDLL performance is approximately equal (by DLL, we mean here a conventional code and carrier tracking loop as described in chapter 3). Note that both DLL and MEDLL carrier range errors are maximal when the multipath delay is zero, because then there is no attenuation by the correlation operation. At the same time, the code range errors reduce to zero for a zero multipath delay, because
the sum of two correlation functions with the same delay cannot be distinguished from a single correlation function with the same delay.

![Graph showing Carrier range error (N) vs. Multipath delay (Tc)](image)

*Fig. 4.5: Amplitude of DLL and MEDLL carrier range errors in the case of one multipath signal, d=0.1, SMR=6 dB.*

### 4.3 Implementation of the MEDLL

The MEDLL has been implemented in a prototype NovAtel GPS receiver. A block diagram of this receiver is depicted in figure 4.6.

After down conversion, the received signal is correlated with a number of complex correlators to get samples of the input correlation function $R_x(\tau)$. In order to reduce the complexity, the receiver employs a conventional carrier tracking loop which tracks the phase of the correlation sample with the highest amplitude. The phase estimates are used to compensate for phase and frequency errors and also to remove data from the input correlation samples. Finally, the correlation samples are averaged over a period of one second or more, after which the MEDLL equations can be solved. The resulting delay estimate can be used to control the code generator in order to keep the line-of-sight correlation peak in the delay window. This control
can be done both closed loop, by keeping the line-of-sight correlation peak in the middle of two correlation samples, or in a quasi open loop manner, where the delay is adjusted only when the line-of-sight peak is shifted one or more samples compared to its desired location. Besides the MEDLL estimates, the processor also calculates a normal DLL estimate of the line-of-sight delay, using an early-late spacing of one tenth of a chip. Thus, the MEDLL performance can be compared with the best possible DLL performance.

An important aspect of the MEDLL is an accurate reference correlation function. Figure 4.7 shows the reference function that is used in our prototype receiver. It was constructed by averaging measured correlation functions over a total averaging time of 400 seconds, in order to obtain a negligible noise level. Further, the measurement was taken at a place with very little multipath contamination. Note that the reference function only covers the main correlation peak. Since we are only interested in
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multipath signals with relative delays up to about one chip time, the reference function only has to be two chips wide.

![In-phase Quadrature](image)

**Fig. 4.7: Reference correlation function**

An interesting fact is that all satellites do not have exactly the same reference function. Because of the auto correlation properties of the Gold codes, there are in fact three different functions, all of which have to be accurately known by the receiver. The value of $R(\tau)$ at $\tau = \pm T_C$ can be -1/1023, 63/1023 or -65/1023 for unfiltered codes, which have a length of 1023 chips [12]. In the case of filtered codes, these values are slightly larger.

The accuracy of the reference function determines the smallest multipath amplitudes that can be estimated by the MEDLL. A relative accuracy of less than one percent, for instance, makes it possible to detect and estimate multipath signals with a relative amplitude larger than 0.01. A possible problem that may arise is the change of the reference pulse shape in time, due to the influence of temperature variations or aging on the receiver filters. If the errors caused by these effects are too large, then some form of shape adaptation has to be implemented, whereby the receiver uses an averaged measured pulse shape to adjust the reference shape.
In comparison with a conventional delay lock loop receiver, the MEDLL requires significantly more hardware and processing power. Considering the hardware, it needs at least twice (and preferably more) the number of correlators of a normal delay lock loop. Further, the signal processor has to calculate the MEDLL equations, which require far more processing time than the calculation of the early minus the late correlation value, as done in a delay lock loop. Therefore, implementation of the MEDLL is only advantageous in high precision DGPS applications, such as aircraft approach and landing. The MEDLL makes it easier to achieve the required accuracy and integrity, because it reduces the error level and it avoids unacceptably large errors that conventional receivers experience because of occasionally occurring strong specular reflections.

4.4 Results for GPS

Several measurements were made in various environments in order to test the prototype MEDLL. First, a relatively multipath free location was used to determine the noise floor and to obtain a good reference correlation function. Figure 4.8 shows DLL and MEDLL errors while the antenna was situated on a flat roof, which acts like a very large ground plane, so multipath from the ground is greatly reduced. After approximately 350 seconds, the antenna was moved to the edge of the building, in order to measure the effect of an increased ground reflection. For the last two minutes, the antenna was moved away from the edge to decrease the multipath level again. The antenna movements are clearly visible in the measurement; For the first 350 seconds, the DLL and MEDLL code range errors have a standard deviation of 14 (\(d=0.1\)), 25 (\(d=1\)) and 13 cm, respectively. The carrier-to-noise density \(C/N_0\) was 46 dB during the measurement, so the theoretical noise limit for the prototype receiver is about 7 centimeters, according to equation (4.12) for an early-late spacing \(d=0.1\). Note that the different errors have been given an offset for a better readability of the plot. Further, the scales of all measurements are relative, since absolute range values are not known.

From 350 to 650 seconds, when the antenna was on the edge of the building, the standard deviation of the DLL and MEDLL errors increased to 98 (\(d=0.1\)), 366 (\(d=1\))
and 27 cm, respectively. This clearly shows the ability of the MEDLL to estimate the multipath parameters and to reduce their influence on the line-of-sight estimates. Further, it demonstrates the advantage of using a small early-late spacing, as mentioned in [8,9,10]. In order to make a fair comparison between a conventional DLL and the MEDLL, a small early-late spacing is used in all of the following measurements. It should be kept in mind, however, that the errors of receivers with a one-chip early-late spacing can be up to ten times larger than for a spacing of 0.1 chip.

![Graph showing DLL and MEDLL code range errors](image)

**Fig. 4.8:** DLL and MEDLL code range errors, measured on top of a tall building, using $B_L = 0.05$ Hz and $M=2$. The standard deviations (sigma) are calculated over the whole time interval. All plots have been given a different offset (+10 m for DLL, d=1, 0 for d=0.1 and -5 m for MEDLL) to improve the readability.

To measure the improvement of the MEDLL under severe multipath conditions, several measurements were taken in the vicinity of buildings. Figures 4.9 and 4.10 show examples of the resulting code range errors in cases of very strong specular
multipath. In the case of figure 4.9, the GPS antenna was located about 30 meters away from the Electrical Engineering building of Delft University, which is an approximately 100 meters tall building with a metal frame structure. In the case of figure 4.10, the antenna was about 50 meters from an apartment building, that was approximately 70 meters tall and made of concrete.

![Graph: Code Range Error vs Time]

**Fig. 4.9: DLL and MEDLL code range errors (DLL errors are those with the largest excursions) near a tall building, using $B_L=0.5$ Hz, $d=0.1$ and $M=1$. The plots have an arbitrary mean value.**

As can be seen, the MEDLL gives an improvement of about a factor of ten. There seems to be no correlation between the DLL errors and the remaining MEDLL errors, which suggests that those remaining errors are caused by other multipath signals. However, calculating the MEDLL estimates for more than one multipath signal did not give a significant improvement. Most probably, the remaining errors are caused by multipath signals with small relative delays, which are difficult or even impossible to eliminate, as explained previously.
Another phenomenon which can be observed in figures 4.9 and 4.10 is that multipath delay errors of a conventional DLL with a small early-late spacing \( d \) can become much larger than \( d/2 \), which corresponds to a range error of 15 meters for \( d=0.1 \). This is an effect of the band-pass filtering of the GPS signals, which changes the ideal triangular correlation function into a smooth shape as depicted in figure 4.7. As explained in chapter 3, due to this filtering the multipath delay errors can reach values up to half a chip (150 meters for the C/A-code) when the signal-to-multipath ratio approaches one, independent of the early-late spacing.

![Figure 4.10: DLL and MEDLL code range errors due to an apartment building, using \( B_L=0.1 \text{ Hz} \), \( d=0.1 \) and \( M=1 \). The plots have an arbitrary mean value.](image)

Figure 4.11 shows how strongly the input correlation function can be affected by multipath. Two measured correlation peaks from the same measurement as that given in figure 4.10 are drawn, together with an estimate of the line-of-sight correlation peak. Only a part of the peaks is drawn, since the prototype receiver does not provide samples of the entire correlation peak. The distortion of the ideal correlation shape caused by in-phase and out-of-phase multipath is clearly visible in figure 4.11. Note that in the case of out-of-phase multipath, the resulting correlation function is almost flat where the leading edge of the line-of-sight peak is supposed to
be. This is an indication that the reflection amplitude is almost equal to the amplitude of the line-of-sight signal, as can be seen in the simulated example of figure 4.12. This figure shows the correlation function for both in-phase and out-of-phase multipath with a SMR of 0 dB. In the case of the measurement of figure 4.10, the MEDLL indeed estimated a signal-to-multipath ratio close to 0 dB at those moments where the DLL range errors reached maximum and minimum values.

![Correlation Function](image)

**Fig. 4.11: Correlation function in the case of in-phase and out-of-phase multipath.**

Figures 4.13 and 4.14 show two examples of relatively moderate multipath levels, measured on top of the Electrical Engineering building. The GPS antenna was in the vicinity of a number of large parabolic dishes, which cause multipath signals with relative short delays. As explained earlier in this chapter, these multipath signals are difficult to estimate and hence their errors are only partially reduced by the MEDLL, which is clearly visible at the beginning of figure 4.14. Another effect that reduces the MEDLL performance is the occurrence of multiple multipath signals with different delays. In general, it is easier to estimate one strong multipath signal than several small ones, because of the larger number of unknown parameters that have to be estimated. Thus, it can be argued that the MEDLL is at its most effective in the presence of specular reflections, as opposed to diffuse multipath situations with many signals with various delays. Nevertheless, in both measurements shown in the
next two figures (measured in a diffuse multipath environment), the MEDLL improved the standard deviation by more than a factor of two.

![Graph showing correlation values vs. delay](image)

**Fig. 4.12:** Simulated correlation function for in-phase and out-of-phase multipath with SMR = 0 dB.

![Graph showing code range error vs. time](image)

**Fig. 4.13:** DLL and MEDLL code range errors, measured on top of the Electrical Engineering building, using $B_L=0.05$ Hz, $d=0.1$ and $M=1$. All plots have an arbitrary mean value.
Fig. 4.14: DLL and MEDLL code range errors, measured on top of the Electrical Engineering building, using $B_L=0.05$ Hz, $d=0.1$ and $M=1$. All plots have an arbitrary mean value.

Since the measurements given in figures 4.13 and 4.14 are quite long, there was a significant change of the ionospheric error. Therefore, a parabolic curve fit was performed on the MEDLL errors in order to estimate the relative ionospheric error, which then was subtracted from the range measurements.

As stated earlier, one of the advantages of the MEDLL is that it also reduces carrier range errors caused by multipath. Unfortunately, this is difficult to prove using the present prototype receiver, since it employs a conventional carrier tracking loop in order to simplify the carrier phase tracking. The MEDLL then only has to calculate carrier range corrections once per second or whenever they are needed. In order to verify these MEDLL carrier range corrections, a second receiver would be necessary. However, by looking at figure 4.15, which shows the MEDLL carrier range corrections together with measured DLL code range errors, it can be concluded that the phase estimation seems to work properly. The estimated carrier range error is maximum or minimum when the code range error is zero and vice versa, just as predicted and demonstrated in [10].
4.5 Conclusions

Conventional direct-sequence spread-spectrum receivers are not designed to work in a multipath fading environment. By using a new receiver architecture - the Multipath Estimating Delay Lock Loop - it is possible to largely avoid the occurrence of code and carrier tracking errors caused by multipath propagation. The MEDLL, which is based on maximum likelihood estimation theory, estimates the signal parameters of both line-of-sight and dominant multipath signals. Test results of a prototype GPS receiver show error reductions up to a factor of ten, as compared to a conventional delay lock loop with a small early-late spacing. The improvement of the MEDLL becomes less for small relative multipath delays. For relative multipath delays
smaller than about 0.05 chip, the difference between DLL and MEDDLL errors becomes negligible. Multipath signals with delays larger than about 0.1 chip can be completely eliminated by the MEDDLL, while a conventional DLL is sensitive to multipath delays up to more than one chip.

The possibility of using very small early-late spacings raised the question of what the ultimate accuracy level would be. Previous literature on this matter did not take into account any bandwidth limitation of the input spread-spectrum signals. The noise analysis presented in this chapter gives a clear expression for the theoretical accuracy limit of code tracking errors as a function of the signal bandwidth. In order to benefit from a small early-late spacing, the double-sided input bandwidth of the spread-spectrum signals should be approximately equal to the inverse of the early-late spacing. An interesting conclusion of this analysis is that the ultimate GPS P-code accuracy is only a factor of $\sqrt{5}$ better than the C/A-code accuracy. This is because both codes are transmitted with the same bandwidth of approximately 20 MHz. Therefore, the C/A-code - with its smaller chip rate - can get relatively more benefit from reducing the early-late spacing, thereby decreasing the difference between C/A-code and P-code accuracy.

The MEDDLL makes it possible to retain high accuracies, even in highly multipath propagating environments. This makes it an ideal solution for many differential GNSS applications that require an accuracy of several meters or less. With conventional receivers, finding a good site with a low level of multipath errors for the differential reference station can be a tedious procedure. Since the MEDDLL can tolerate higher levels of multipath propagation, the siting becomes easier to accomplish. The MEDDLL is also beneficial for carrier phase ambiguity resolution, since the small code range errors reduce the search volume, while the decreased carrier range errors make it easier to reject false solutions. Further, it can be useful for spread-spectrum communication systems, especially for the timing of synchronous CDMA, where multi-user interference can be greatly reduced by using precisely timed orthogonal codes. Because of the timing difficulties, orthogonal codes, such as the Walsh codes, are now only used sometime in the down link (from a base station to the users) [19].
4.6 References in chapter 4


5 LORAN-C SIGNAL PROCESSING

Although the previous chapters primarily focused on satellite systems, a large part of the results can also be applied in related systems, for example, the land-based Loran-C system. This chapter starts with a description of the Loran-C signal structure, indicating the similarities and differences with direct-sequence spread-spectrum systems such as GPS. Then, it is shown how the maximum likelihood estimation techniques discussed in chapter 4 can be modified in order to account for the multipath and interference problems of Loran-C.

5.1 Loran-C signal structure

The Loran-C signal structure has already been partly described in section 1.2.2. The signal consists of series of pulses, with a specific repetition period called the Group Repetition Interval (GRI). Loran transmitters from different chains use different GRI values. Thus, their signals overlap for a certain percentage of time. This type of multi-transmitter interference is called Cross Rate Interference (CRI). Other typical Loran-C problems are skywaves, which are multipath signals, reflected by the ionosphere, and Carrier Wave Interference (CWI), caused by narrowband transmissions close to the Loran-C band of 90-110 kHz. Further, there is atmospheric noise present. The received signal can be written as:

\[
r(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} \sum_{c=-\infty}^{\infty} \sum_{j=0}^{7} a_{mk} p_{kcj} l(t - \tau_{mk} - jT_0 - c \cdot \text{GRI}_k) \cos(\omega_0 t + \theta_{mk}) \\
+ \sum_{l=1}^{L} b_l \cos(\omega_l t + \theta_l) + n(t)
\]  

(5.1)
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\[ = s(t) + n(t) \]

\[ l(t) = (t / t_p)^2 \exp(2 - 2t / t_p) \quad , t \geq 0 \]

\[ l(t) = 0 \quad , t < 0 \]

\[ T_o = 1 \text{ ms} \quad , t_p = 65 \mu \text{s} \]

Here, \( K \) is the number of Loran transmitters, which may have a different group repetition interval \( \text{GRI}_k \). \( M-1 \) is the number of skywaves, each with a different amplitude \( a_i \), delay \( \tau_i \) and phase \( \theta_i \), respectively. The Loran-C signals consist of pulse trains with pulse shape \( l(t) \). These pulses are multiplied by a phase code \( p_{kcj} \), which takes values of \{1,-1\}, and repeats itself every 2 Group Repetition Intervals. There are two different phase code patterns for master and secondary transmitters. That explains the subscript \( k \) for the transmitter number in the phase code \( p_{kcj} \). The subscript \( c \) denotes the GRI number. Even and odd GRI numbers have a different phase code pattern, consisting of 8 values. Within a GRI, the pulse number is denoted as \( j \). Further, the presence of \( L \) carrier wave interference signals is assumed, together with a noise signal \( n(t) \).

Fig. 5.1: Transmitted Loran-C pulse.
Figure 5.1 shows the transmitted Loran-C pulse shape. Throughout the rest of this chapter, instead of the modulated signals, often the equivalent baseband in-phase/quadrature representation is used. This representation is obtained by downconverting the 100 kHz signal, using a reference cosine and sine with a frequency of 100 kHz. Figure 5.2 depicts the in-phase/quadrature representation of the transmitted Loran-C pulse. It is important to remember that both representations contain the same information; the phase (or zero-crossings) of the signal in figure 5.1 is related to the quotient of in-phase and quadrature values in figure 5.2.

![In-phase and Quadrature Representation](image)

*Fig. 5.2: In-phase / quadrature representation of a Loran-C pulse.*

The overall transmission scheme of Loran-C is illustrated in figure 5.3. It shows the in-phase part of a downconverted Loran signal, so phase code reversals are clearly visible as sign changes of the pulses. The signals A and B belong to a master and a secondary station from the same chain. C is from another chain with a slightly larger GRI values. This results in cross rate interference on the signals A and B. In the piece of signal shown in figure 5.3, for instance, it can be seen that the last pulse of C almost interferes with the next pulse of A. As the patterns of A, B and C repeat with their own repetition time of 2GRI, signal C will interfere both A and B -and all other signals with a different GRI- for a certain percentage of time.
Fig. 5.3: Transmitted Loran-C pulse. A and B are a master and a secondary transmitter from one chain. C is a secondary from a different chain.

From the received signal \( r(t) \), a receiver has to estimate the delays \( \tau_{ok} \) and phases \( \theta_{ok} \) of the Loran-C groundwaves. Usually, these estimates are provided by tracking loops, which consist of a certain detector that measures the difference between the desired parameter of the input signal and that of a locally generated signal. A loop filter is used to filter the measured differences and control the local signal in such a way that the difference is driven to zero.

The performance of Loran-C is primarily limited by skywaves, noise, CWI and CRI. Most receivers cope with these disturbances by using the first part of the pulses only to avoid skywaves and by using notch filters to suppress CWI [1-3]. Usually, no special measures are taken against CRI, except for long averaging times, or, equivalently, a loop bandwidth that is small enough to produce an acceptable signal-to-noise plus CRI ratio. However, these methods have three major drawbacks. First, avoiding skywaves by sampling early in the pulse -instead of using the top- reduces the signal-to-noise and the signal-to-interference ratios by an amount in the order of 10 dB, depending on the bandpass filter [2]. Second, it is difficult to ensure that notch filters remove all synchronous or near-synchronous interferences, especially in Europe with its enormous amount of interfering signals [2-3]. Synchronous or near-
synchronous interference means that the interfering frequency is equal to or within a tracking loop bandwidth value from a spectral line of the Loran-C spectrum. This type of interference is only moderately reduced by the tracking loop filter in a Loran receiver, with a reduction of about 10 to 18 dB, depending on the phase code and the CWI frequency [2]. A problem with level sensitive adaptive notch filters is that they simply notch out the strongest signals, which may not correspond to the most dangerous signals for Loran-C. The third problem is that the use of many notch filters may cause pulse distortion, thereby increasing the probability of cycle identification\(^1\) errors. The aim of this chapter is to examine whether advanced estimation techniques can be used to obviate some of the drawbacks mentioned. In order to avoid long and untraceable mathematical derivations, the problems of skywaves, CWI and CRI are treated separately in the following sections.

### 5.2 Skywave estimation

For one specific Loran transmitter, neglecting CWI and CRI, the received signal (5.1) is reduced to:

$$
\begin{align*}
    r(t) &= \sum_{m=0}^{M-1} \sum_{c=-\infty}^{\infty} \sum_{j=0}^{7} a_m P_c j l(t - \tau_m - jT_0 - c GRI) \cos(\omega_c t + \theta_m) + n(t) \\
    &= s(t) + n(t)
\end{align*}
(5.2)
$$

Similar to the estimation of multipath in direct-sequence spread-spectrum systems, described in the previous chapter, it is possible to find maximum likelihood estimates for the Loran-C signal parameters by minimizing the mean square error between the received signal and a set of estimated groundwave and skywave signals. In fact, minimizing the mean square error provides optimum estimates only in the presence of white Gaussian noise. For Loran-C, the noise is not Gaussian due to the appearance of impulsive atmospheric noise. As demonstrated in [4], the effects of

\(^1\) Cycle identification is the estimation of the carrier phase ambiguity, using the measured envelope delay.
this impulsive noise on the maximum likelihood estimates can be accounted for by applying a certain non-linear amplitude transfer function to the input signal. A practical approximation is the use of clipping, which is present anyway in an analog-to-digital converter. By setting the clipping level slightly larger than the measured signal amplitude, the influence of large atmospheric 'spikes' can be largely reduced, since a few 'clipped' pulses only cause a limited effect on the phase and shape of an averaged Loran-C pulse, which is averaged over hundreds or more received pulses.

The resulting maximum likelihood estimates are:

\[
\hat{\tau}_m = \max_{\tau} \text{Re}\{\left[R_x(\tau) - \sum_{i=0, i \neq m}^{\hat{M}-1} \hat{a}_i R(\tau - \hat{\tau}_i) \exp(j\hat{\theta}_i) \exp(-j\hat{\theta}_m)\right]\}
\]

\[
\hat{a}_m = \text{Re}\{R_x(\hat{\tau}_m) - \sum_{i=0, i \neq m}^{\hat{M}-1} \hat{a}_i R(\hat{\tau}_m - \hat{\tau}_i) \exp(j\hat{\theta}_i) \exp(-j\hat{\theta}_m)\}
\]

\[
\hat{\theta}_m = \arg\{R_x(\hat{\tau}_m) - \sum_{i=0, i \neq m}^{\hat{M}-1} \hat{a}_i R(\hat{\tau}_m - \hat{\tau}_i) \exp(j\hat{\theta}_i)\}
\]

\[
R_x(\tau) = \frac{2}{c2 \cdot R_o} \int_{t-T}^{t} r(t) \sum_{c=c1}^{c1+c2-1} \sum_{j=0}^{7} p_{cj}(t - \tau - jT_0 - c \cdot GRI) \exp(j\omega_0 t) dt
\]

\[
\int_{-\infty}^{\infty} l(t) dt
\]

\[
R(\tau) = \frac{1}{R_o}, \quad R_o = \int_{-\infty}^{\infty} l(t)^2 dt
\]

(5.3)

Here, \(R_x(\tau)\) is the input correlation function, which is found by correlating the input signal with a replica of the ideal Loran signal, free from noise and skywaves. This is essentially the same operation that any conventional Loran-C receiver has to perform. For a certain time interval \(T\), all Loran pulses -a total number of \(c2\)- are downconverted and correlated with the phase code and the reference Loran pulse.
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shape \( l(t) \). \( R(\tau) \) is the reference Loran correlation function, which is normalized to one by a division with the pulse power \( R_0 \). Note that correlating the input signal with the ideal Loran pulse shape is equal to matched filtering, which can be achieved by using a narrow bandpass filter with the same spectral envelope as the transmitted Loran signal. The problem with such a filter is that it considerably increases the width of the filtered pulses, thereby reducing the skywave free part of the leading pulse edge. Therefore, conventional Loran receivers always use filters that are wider than the matched filter. This allows for a much simpler way to estimate the groundwave parameters; instead of searching for the maximum of the pulse, the delay can be estimated by searching for a certain point on the leading edge which is free of skywaves. Most Loran-C receivers use this technique [1], at the cost of a decreased signal-to-noise ratio as compared to the matched filter.

The correlation time \( T \) should be large enough for the errors due to noise in the delay and phase estimates of the Loran-C groundwave to be acceptable. However, \( T \) has to be small enough to assure the parameters stay approximately constant, in order to track receiver accelerations. This conflict between noise reduction and dynamical performance is the same as encountered in conventional Loran-C tracking loops. Therefore, \( 1/T \) should be equal to practical Loran-C loop bandwidth values, ranging from about one to tens of seconds.

Since the correlation with the reference Loran pulse shape can be performed by the input bandpass filter, by using a matched filter, obtaining the correlation function \( R_x(\tau) \) becomes remarkably easy. It requires an in-phase/quadrature down conversion of the filtered input signal, which can be combined with the analog-to-digital sampling process by using bandpass sampling [2]. Then, only the correlation with the phase code is left, which is very similar to correlation in a direct-sequence spread-spectrum receiver. Basically, all the Loran pulses from one specific transmitter are coherently added to make one final correlation pulse.

Figure 5.4a shows a simulated example of a part of the input signal after downconversion. To keep the figure legible, a fictitious Loran signal of only 4 pulses is shown. After correlation with the phase code, an output correlation function like that drawn in figure 5.4.b is obtained. Should a matched filter be used, then the correlation function would have a symmetric shape, comparable with the filtered
triangular shape of a direct-sequence correlation function as in the case of GPS. However, in figure 5.4.b, a more wideband input filter was used, similar to conventional Loran-C receivers. As a result, the correlation function shows a close resemblance to the transmitted asymmetric Loran pulse shape $l(t)$.

![Graph showing Loran-C signal after filtering and downconversion.](image)

*Fig. 5.4.a: Loran-C signal after filtering and downconversion.*

Note that the auto correlation function of the Loran pulse codes only has one non-zero value, while the GPS Gold codes, for instance, have a large number of non-zero values. This ideal correlation property is important to Loran for two reasons. First, it provides a complete rejection of long delayed skywaves with delays of one millisecond or more. Second, it prevents false lock points with delay errors of one millisecond or more. With GPS, false lock is possible, but not very likely, since the undesired correlation values are about 24 dB weaker than the desired signal, which gives a signal-to-noise ratio that is normally too low to keep the tracking loops in lock. For Loran-C, the situation is quite different. Because of the much larger dynamic range of the signals, a 24 dB weaker signal from a relatively nearby Loran transmitter would not be difficult to acquire. Therefore, it is desirable to have an ideal auto correlation function in order to avoid the problem of false lock.
An example of the skywave estimation technique is depicted in figure 5.5. In this case, the Loran-C signal from Ejde - about 1340 km away - was used to estimate the parameters of a groundwave plus one skywave. A total correlation time $T$ of 16 seconds was used in order to get a reasonable reduction of noise, CWI and CRI. Then, the delays, phases and amplitudes of groundwave and skywave were estimated by solving equation (5.3). Figure 5.5 shows the received envelope of the correlation function $R_x(\tau)$, together with the estimated groundwave and skywave and the combined envelope of these two. The residuals are calculated by taking the square root of the squared differences between the in-phase and quadrature components of the received and estimated signals. As can be seen, the remaining residuals in the beginning of the burst are very small, in the order of the noise that is present in front of the burst. Just around the top of the groundwave, however, the residuals become larger. This effect is caused by differences of the tails of the transmitted pulses as compared to the ideal Loran pulse shape, which was also noted in [5]. Note that the residuals are larger than the envelope differences, which suggests that there is mainly phase distortion in the trailing edge of the transmitted Loran pulses. Although this pulse distortion makes it difficult to use the trailing edges of the pulses for positioning, the measured residuals can be useful in enhancing the integrity of the measurements. Residuals that are large in comparison with the expected noise level are an indication of unacceptable errors. This type of integrity monitoring was first proposed in [6]. To avoid the problem of the uncertainties in the Loran pulse shape,
other skywave resolution methods may be used which do not need knowledge of the pulse shape. Reference [7] describes some of these methods.

![Graph](image)

*Fig. 5.5: Groundwave/skywave separation.*

Because of the distortion near the top of the transmitted Loran pulses, it is necessary to estimate the pulse delay by using the leading edge instead of the peak. Therefore, the max operation in (5.3) -that finds the delay for which the derivative is zero- should be replaced by a function that searches for a certain point at the leading pulse edge. In practice, this is implemented by finding the quotient of two successive envelope sample values that are closest to some reference value.

Note that because of the downconversion of the input signal, the zero crossings that are used by conventional Loran-C receivers are no longer visible. However, the estimated carrier phase essentially provides the same information. In order to resolve the carrier phase ambiguity, the error in the delay estimate should be less than half a cycle (= 5 μs). If Loran-C is to be used in hyperbolic mode, then the desired time difference between two different transmitters \(a\) and \(b\) (with \(\tau_a < \tau_b\)) is given by:

\[
t_d = \frac{1}{f_o} \left[ \text{Int} \left( f_o ( \hat{\tau}_b - \hat{\tau}_a - \frac{\hat{\theta}_a - \hat{\theta}_b}{2\pi f_o}) \right) + \frac{\hat{\theta}_a - \hat{\theta}_b}{2\pi} \right] 
\]

(5.4)
In equation (5.4), $f_0$ denotes the Loran-C carrier frequency ($f_0=100$ kHz) and \( \text{Int}(x) \) means rounding \( x \) to the nearest integer. Thus the first term of (5.4) gives the time difference between transmitters \( a \) and \( b \) in whole cycles, while the second part adds a fractional part of a cycle.

5.3 Carrier Wave Interference

Although the 90-110 kHz band is reserved exclusively for radionavigation\(^2\), transmissions just outside this band can cause considerable amounts of interference. Because most services occupy bandwidths that are much smaller than Loran-C, the interference they produce is known as Carrier Wave Interference (CWI). In Europe, for instance, the major sources of CWI are communication signals, timing stations and Decca transmitters, which are radiopositioning beacons that transmit carrier waves with an on-off keying period in the order of seconds.

In the case of L CWI signals, the received input signal (5.1) becomes:

$$
r(t) = \sum_{m=0}^{M-1} \sum_{c=-\infty}^{\infty} \sum_{j=0}^{7} a_m p_{cj} l(t - \tau_m - jT_0 - cGRI) \cos(\omega_0 t + \theta_m)
+ \sum_{l=1}^{L} b_l \cos(\omega_l t + \theta_l) + n(t)$$

(5.5)

By applying the maximum likelihood criterion, equations similar to those for the estimation of skywaves (5.3) are obtained, with the addition of some extra equations for the unknown CWI parameters.

\(^2\) The 90-110 kHz band is reserved for Loran-C and related systems as Pulse-8 and Chayka.
\[
\hat{\tau}_m = \max_{\tau} \left\{ \left( R_x(\tau) - \sum_{i=0}^{M-1} \hat{a}_i R(\tau - \hat{\tau}_i) \exp(j\hat{\theta}_i) \right) \right. \\
- \left. \sum_{i=1}^{L} \hat{b}_i \exp[j((\hat{\omega}_i - \omega_0)\tau + \hat{\phi}_i)] \right\} \\
\hat{a}_m = \Re \left\{ \left( R_x(\hat{\tau}_m) - \sum_{i=0}^{M-1} \hat{a}_i R(\hat{\tau}_m - \hat{\tau}_i) \exp(j\hat{\theta}_i) \right) \right. \\
- \left. \sum_{i=1}^{L} \hat{b}_i \exp[j((\hat{\omega}_i - \omega_0)\hat{\tau}_m + \hat{\phi}_i)] \right\} \\
\hat{\theta}_m = \arg \left\{ x(\hat{\tau}_m) - \sum_{i=0}^{M-1} \hat{a}_i R(\hat{\tau}_m - \hat{\tau}_i) \exp(j\hat{\theta}_i) \right. \\
- \left. \sum_{i=1}^{L} \hat{b}_i \exp[j((\hat{\omega}_i - \omega_0)\hat{\tau}_m + \hat{\phi}_i)] \right\} \\
(5.6)
\]
\[
\hat{b}_i = \Re \left\{ \frac{1}{T_1 - T_0} \int_{T_0}^{T_1} \left( R_x(\tau) - \sum_{m=0}^{M-1} \hat{a}_m R(\tau - \hat{\tau}_m) \exp(j\hat{\theta}_m) \right) \right. \\
\left. \cdot \exp[-j((\hat{\omega}_i - \omega_0)\tau + \hat{\phi}_i)] \, d\tau \right\} \\
\hat{\phi}_i = \arg \left\{ \int_{T_0}^{T_1} \left( R_x(\tau) - \sum_{m=0}^{M-1} \hat{a}_m R(\tau - \hat{\tau}_m) \exp(j\hat{\theta}_m) \right) \right. \\
\left. \cdot \exp[-j(\hat{\omega}_i - \omega_0)\tau] \, d\tau \right\}
\]

The equations of (5.6) are in fact an approximation of the true maximum likelihood equations, because the CWI parameters are estimated after correlation of the input signal with the Loran-C phase code pattern. Depending on the CWI frequency and the GRI, the correlation operation decreases the amplitude of the CWI signals by a
certain amount. The disadvantage of this method is that the accuracy of the CWI estimates are reduced (because of the phase code correlation, for instance, the CWI power is reduced by about 10 to 18 dB [2]). Ideally, the CWI parameters should be estimated directly out of the input signal \( r(t) \), instead of using the Loran correlation function \( R_X(\tau) \). However, using \( R_X(\tau) \) has the advantage of a much simpler receiver design. Further, the correlation function \( R_X(\tau) \) only contains those CWI signals that cause the largest problems, i.e. synchronous or near-synchronous signals. All asynchronous signals are largely suppressed by the correlation operation.\(^3\) This means that the most straightforward method to detect CWI is to use the 2 GRI long correlation function \( R_X(\tau) \) (2 GRI is the repetition period of the Loran signal). By calculating the Fast Fourier Transform of \( R_X(\tau) \), it is possible to estimate the frequencies of the most harmful CWI signals. These estimates can be used as initial values in the calculation of (5.6), or as initial values for normal notch filters. Note that this method of detecting CWI is much simpler and just as effective as that proposed in [2,8].

In equation (5.6), \( T_O \) and \( T_I \) define a certain window. Ideally, this window is as long as the complete correlation function, i.e. a length of 2 GRI. However, this is not necessary. By using a part of the correlation function where there are no strong Loran pulses, it is possible to estimate CWI parameters without the need to subtract estimated Loran signals. The calculation of (5.6) can be greatly simplified in this way. To improve the CWI estimates, consecutive phase estimates can be filtered to increase the accuracy of the CWI frequency and phase. For instance, the phase estimates of the previous ten seconds can be used to get a filtered phase and frequency estimate that is \( \sqrt{10} \) times more accurate than the estimates obtained by using only one second of phase estimates.

Figure 5.6.a gives an example of the spectrum of the Loran-C signal after correlation. The spectrum was obtained by calculating the Fast Fourier Transform of the 2GRI long correlation function of a measured signal. There are clearly several strong (near) synchronous CWI signals present, most probably due to Decca

\(^3\) A signal is asynchronous if its frequency difference with a Loran-C spectral line is large in comparison with the tracking loop bandwidth, or the inverse of the total correlation time. The maximum correlation time is determined by the maximum acceleration that the receiver has to cope with.
transmitters. Based on the previously described method, a prototype Loran-C receiver automatically detected a certain number -at the moment up to 30- of the strongest CWI signals outside the 90-110 kHz band, which were estimated and subtracted from the input signal.

Fig. 5.6.a: Loran-C spectrum before CWI cancellation.

Fig. 5.6.b: Loran-C spectrum after CWI cancellation.

The resulting spectrum after CWI cancellation is shown in figure 5.6.b. It can be seen that most dominant CWI signals are significantly reduced. However, some of
the CWI signals are only reduced by a minor amount. This is most probably caused by modulation of the signals. In that case, the CWI cancellation method only cancels a part of the total interfering spectrum; the wider the interfering spectrum is, the less effective is CWI cancellation. There are two ways to solve this problem: First, the modulation can also be estimated, but this will highly increase the complexity of the receiver, since different interfering signals can have different types and speeds of modulation. The second and easiest way is to implement notch filters, where the above described CWI detection method can be used to obtain initial CWI frequency estimates.

5.4 Cross Rate Interference

Next to skywaves and continuous wave interference, cross rate interference (CRI) is the third important source of interference in Loran-C. It is caused by the differences in the Group Repetition Intervals of different Loran chains. This results in an overlap of several Loran signals for a certain percentage of time. A careful selection of the GRI values makes it possible to keep the effects of CRI on Loran-C position determination within acceptable levels, although CRI can still be disturbing in some circumstances. However, the effects of CRI on Loran data modulation are far more severe. It may happen, for instance, that about 5% of the pulses from a specific Loran signal are 'hit' by a stronger cross rate signal. Assuming a random phase difference between the two Loran signals, this would mean that 5% of the pulses would be in error with a probability of 0.5, so the overall pulse error probability would have a lower bound of 0.025. Such a large error probability is clearly unacceptable to most applications. Therefore, Loran-C data transmission schemes use all kinds of coding techniques to decrease the error probability. However, coding is not the only and certainly not the best way to combat CRI. Just as for skywaves and CRI, several signal processing techniques can be used to reduce the influence of CRI.

Neglecting skywaves and CWI, the received Loran signal (5.1) for $K$ different Loran transmitters can be written as:
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\[ r(t) = \sum_{k=0}^{K-1} \sum_{c=-\infty}^{\infty} \sum_{j=0}^{7} a_k p_{kcj} l(t - \tau_k - JT_0 - c GRI_k) \cos(\omega_o t + \theta_k) + n(t) \] (5.7)

One method of dealing with CRI would be to apply maximum likelihood estimation in a way similar to its application in skywave estimation. The receiver would then simultaneously estimate the parameters of several Loran signals. The effects of CRI would be largely reduced by subtracting the estimated CRI signals prior to the estimation of the desired signal parameters. However, this type of estimation technique relies heavily upon knowledge of the shape of the transmitted signals. As shown in section 5.2, only the shape of the leading edges of the Loran pulses is accurately known. This means that groundwaves cannot be completely eliminated, even if their delays, phases and amplitudes are perfectly known. Further, the uncertainty in the shape of the trailing pulse edge make it very difficult to reliably estimate skywave parameters for skywave delays of more than about 100 \(\mu\)s. All these difficulties make a successful application of the maximum likelihood method practically impossible.

However, there are other methods possible to combat CRI, which have the additional advantage of being simpler to implement than the maximum likelihood method. The key to the solution is to get accurate knowledge of the received signal shape, including skywaves and transmitter distortion. This can be achieved by simply using the Loran-C correlation function \(R_x(\tau)\), which is obtained by correlating the filtered input signal with the phase code pattern for a certain time \(T\). If \(I/T\) is chosen equal to practical Loran-C loop bandwidth values (0.01 to 1 Hz), then the level of CRI, CWS and noise on the correlation function will be small enough to get an accurate signal shape, with a signal-to-noise ratio in the order of 20 dB or more. Now, this averaged measured shape from one specific transmitter can be used to cancel the CRI that is caused by that particular transmitter. This is possible because the receiver knows exactly when the next pulse arrives, otherwise it could not perform a coherent averaging. Further, it knows what phase code value the next pulse will have. By multiplying this phase code value (±1) with the averaged pulse shape, a replica of the incoming pulse is obtained. Subtracting this replica from the input signal cancels that specific Loran-C signal. This procedure can be repeated for other CRI signals until
the total interference is reduced to an acceptable level. Mathematically, the
technique can be described as:

\[ r'(t) = r(t) - \frac{p_{nm}}{8n} \sum_{c=0}^{n-1} \sum_{j=0}^{7} p_{cj} r(t - cGRI - mT_o + jT_o) \] (5.8)

where \( r(t) \) is the new input signal, \( p_{cj} \) is the phase code value for the \( j \)th pulse of the
c\( c \)th GRI, and \( T_o \) is the time between two Loran pulses within a GRI. Equation (5.8) shows that the cross rate component is formed by the averaged received Loran pulse,
averaged over the previous \( n \) Group Repetition Intervals, containing a total of \( 8n \)
pulses. This average pulse -the correlation function- is multiplied with the present
phase code value \( p_{nm} \) prior to subtraction from the input signal.

Although the description and the implementation are quite different, essentially the
same type of interference cancellation was proposed independently by [9,10] and
[11]. The main difference between these techniques is that [11] uses a comb filter
structure, where the input signal is averaged over 2 GRI intervals, so all signals of
one chain can together be averaged and subtracted from the input signal. No attempt
is made to use all available pulses in the averaging interval; if this technique uses a
moving average time of 200 GRI\( s \), for instance, then for a certain input sample,
exact 100 previous samples (with delays of -2, -4 ,..., -200 GRI) are averaged and
subtracted from the present input sample. The advantage of this technique is that it
cancels all signals from one specific chain simultaneously. The disadvantage is that
it only works when the different signals have very small differences in Doppler
frequency. For an averaging time of 10 s, for instance, the differences should be very
small compared to 0.1 Hz. Equivalently, the differences in ranges to different Loran
transmitters should stay within a very small fraction of a wavelength (\( \equiv 3 \) km) in the
10 s interval. With a speed of 30 m/s, the maximum range difference can reach
values up to 600 m in 10 s, which is not negligible in comparison with the
wavelength (remember that the maximum differential speed towards two Loran
transmitters is twice the receiver speed). Further, a disadvantage of the 2 GRI
averaging method is that it does not use all of the available signal-to-noise ratio. In
the technique described in [9,10], except for the 2 GRI averaging, the signals are
also correlated with the phase code pattern. The advantage compared to [11] is that
for the same averaging time, the resulting signal-to-noise ratio is 16 times (12 dB) higher, because the phase code correlation averages over 16 pulses in each interval of 2 GRI. Thus, if this technique uses a moving average time of 200 GRLs, then for a certain input sample, all 1600 samples of the previous 200 GRLs are used to make one averaged Loran pulse which is subtracted from the present input sample.

Because of the static design of the prototype receivers described in [9-11], some people got the impression that interference cancellation was not possible in dynamic circumstances. This is absolutely untrue, however. The problem of interference cancellation in a dynamic environment requires the same measures that any conventional receiver has to take, namely some kind of frequency synchronization. This problem is discussed in more detail in the next section.

![Graph](image)

*Fig. 5.7.a: Cross-rate interference in the averaged Lessay signal.*

Figure 5.7.a shows a piece of data after averaging 80 GRL's of signal for the Lessay chain. In this part, no bursts from Lessay or Souston are present, so one would expect to see only noise. However, most of this 'noise' turns out to be caused by cross-rate interference from the Sylt transmitter, which can be seen if the CRI cancellation technique is applied. Figure 5.7.b shows the estimated CRI caused by Sylt, which was obtained by using the above-described method. If this estimated CRI is subtracted from the input signal, then the 'noise' variance decreases by more than
9 dB and the CRI caused by Sylt seems to be completely eliminated (figure 5.7.c). In fact, since the signal-to-noise ratio in the estimated CRI is in the order of 30 dB, it can be expected that the CRI contribution by the Sylt transmitter is reduced by the same amount of approximately 30 dB. However, this is difficult to verify in the measurements, because the residuals are dominated by CRI from other transmitters. Of course, the CRI cancellation procedure can be repeated for all possible CRI sources, but in this specific case Sylt -which was 2 dB stronger than Lessay- was by far the strongest source of CRI. After cancelling Sylt, the remaining Lessay signal-to-noise plus interference ratio of about 7 dB is large enough to get both acceptable navigation and data transmission.

![Signal Value vs Time](image)

*Fig. 5.7.b: Estimated CRI due to Sylt in the averaged Lessay signal.*

A specific problem arises when a certain Loran signal that has to be cancelled is modulated with data by using pulse delay shifts of \( \pm \tau_d \) seconds. In that case, when no attempt is made to incorporate the modulation into the subtracted signal, CRI cancellation will reduce the amplitude to a minimum value of \( \sin(2\pi \tau_d/10^{-5}) \) only, which gives about 5 dB reduction for \( \tau_d = 1 \mu s \). This is because the CRI averaging process only extracts the in-phase part of the received Loran pulses. The quadrature parts, with an amplitude of \( \sin(2\pi \tau_d/10^{-5}) \), average to zero, because the data modulation is normally balanced, i.e. there are equal amounts of phase advances and
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4 To improve the CRI cancellation technique in the presence of data modulation, the estimated modulation has to be applied to the signal replica that is subtracted from the input signal. Since it may be expected that a raw pulse error probability of at least $10^{-2}$ can be achieved, about 99% of the CRI pulses could be cancelled. In 1% of the cases, the receiver uses the wrong data bit value. Thus, it subtracts a Loran pulse that has a quadrature part which is inverted in comparison with the incoming pulse. This results in a quadrature part with approximately twice the original amplitude, while the in-phase part is largely cancelled.

![Signal Value vs Time](image)

*Fig. 5.7.c: Residuals after CRI cancellation.*

5.5 Frequency synchronization

Before discussing the problems of synchronization in Loran receivers, a short description is given of the prototype receiver that was used to generate all the results shown in this chapter. Since the emphasis of this receiver was on verifying the

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4 In fact, this is an approximation, since there is a change in both carrier phase and envelope delay. However, the effect of the envelope delay is negligible in comparison with the change in carrier phase, as shown further on in figure 5.9.
feasibility of new processing techniques related to skywaves, CWI and CRI, no attempt was made to handle receiver dynamics. Figure 5.8 shows a block diagram of the prototype Loran-C receiver. After filtering by a fourth-order bandpass filter with a 3 dB bandwidth of 6.6 kHz, the input signal is sampled by a 12 bit A/D converter, taking in-phase and quadrature baseband samples every 10 μs according to the quadrature bandpass sampling technique, which is well described in [2]. The bandpass sampling process uses a sampling clock of 400 kHz -four times the Loran-C carrier frequency- in order to get a 90-degrees phase shift between two successive samples. This 400 kHz clock is deduced from a 10 MHz atomic reference clock, which assures synchronism between the sampling clock and the incoming Loran-C signals.

![Block diagram of the prototype Loran-C receiver.](image)

**Fig. 5.8: Prototype Loran-C receiver.**

After bandpass sampling, the signal is averaged over a 2-GRI interval for a certain time which is long enough to get a sufficient signal-to-noise ratio. The 2-GRI averaging has the advantage that only 2-GRI-100 kHz samples (≈32000 for GRI=80ms) have to be stored (per chain) for both the in-phase and the quadrature channel. Note that this 2-GRI averaging concept was previously proposed in [12]. As soon as acquisition has been accomplished, there is no longer a need to store the whole 2-GRI interval; it would be sufficient to just collect samples around expected pulse arrival times. After averaging, the 2-GRI interval is correlated with the Loran-C master and secondary phase code patterns to get the correlation functions $R_x(τ)$ that are the basis of all previously described processing techniques.

The main advantage of using an atomic frequency reference in a static environment is the possibility to use the simple 2-GRI averaging technique. This is only possible if all of the received Loran signals remain accurately synchronized with the sampling
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clock during the averaging time interval. In the case of receiver movements, together with the use of a non-ideal sampling clock, the simple 2-GRI averaging is not possible. In order to get the correlation function for all Loran signals, some form of synchronization has to be applied to both the carrier phase and the pulse delay. Basically, there are three ways to achieve this. First, the estimated drift in the estimated carrier phase of a received Loran signal can be used to control the sampling frequency in such a way that phase and delay drift are eliminated. However, since there is usually only one sampling clock, only one particular Loran signal can be synchronized in this way. All other Loran signals will still show a drift in carrier phase and envelope delay because of receiver movements.

A second technique to account for these drifts is to use the estimated phase drift to correct for carrier phase and envelope delay drift in software. If there is a frequency offset of 1 Hz, for instance, then during the correlation operation, a certain pulse that arrives \( t \) seconds after the start of a correlation time interval should be phase shifted by \(-2\pi t\) (plus the additional phase code shift) and delay shifted by \(-10^{-5}t\), prior to adding this pulse to the correlation function. This method has the disadvantage of requiring much processing to do phase rotations and delay shifts. The third and simplest technique to cope with the synchronization problem is to round the estimated phase and delay shifts to a value that corresponds to one sampling clock interval. Thus, carrier phase and envelope delay shifts can be applied very simply by starting to sample a certain pulse one clock interval earlier or later. Of course, this has the disadvantage that the final correlation pulse is in fact an average over a large number of pulses with delay errors that are uniformly distributed over \([-T_s/2, T_s/2]\), where \(T_s\) is the sampling clock interval. However, if \(T_s\) is chosen small enough smaller than or equal to 2.5 \(\mu\)s- then the signal loss and shape distortion because of this averaging can be minimized to an acceptable level, meaning a signal loss smaller than one dB and envelope delay errors less than 1.25 \(\mu\)s.

Knowing the start of the incoming pulses makes it also possible to subtract the estimated Loran signal, which is necessary for CRI cancellation. In this case, the measured correlation function \(R_x(\tau)\) (only the part around the peak, with a width of several hundreds of microseconds) is used to make a replica of a certain Loran signal that can be subtracted from the incoming signal. In the correlation process, the start of a certain pulse in the input signal is accurately known. After multiplication of the
pulse with its known phase code value, it is added to the correlation function. For CRI cancellation, a reverse process can be applied: The averaged pulse shape -i.e. the correlation function- is multiplied with the phase code value and subtracted from the input signal. The modified input signal can then be used for demodulation of other Loran-C signals.

As discussed previously, the use of delay rounding to integer sampling values gives a maximum relative amplitude error of approximately \( \sin(\pi T_s/10^{-5}) \). This is equal to the CRI attenuation that can be achieved when using the cancellation technique. For a sampling rate of 1 MHz, this means an error of 0.31, or a CRI attenuation of 10 dB. In order to improve this, it is necessary to apply phase rotations to individual pulses in both the correlation process and in the signal replica that is used for CRI cancellation.

5.6 Optimum Loran-C data demodulation

Besides position determination, an important feature of Loran-C is the possibility to transmit data. Considering the large coverage area, it seems an attractive and cost-effective idea to use Loran-C for transmitting differential GNSS messages, as proposed in the Eurofix system [13-15]. The most probable modulation type for Loran-C data transmission is pulse position modulation. In this scheme, a delay or advance is applied to the Loran-C pulses. By choosing the delay and advance times to be shorter than half of a carrier period, phase shifts of less than 90 degrees occur, which assures the presence of an average carrier component. For a delay time of 1 \( \mu \text{s} \), for instance, a carrier component is present with a power that is 1.8 dB smaller than that of an unmodulated signal.\(^5\) This is important for the compatibility of conventional Loran-C receivers, which track the average phase of the carrier component. In order to make the average carrier phase equal to the phase of an unmodulated Loran-C signal, the number of delays and advances has to be balanced.

\(^5\) In fact, the loss in average carrier power is limited to 1.3 dB, because only 6 out of every 8 Loran pulses are pulse modulated.
Figure 5.9 shows the in-phase and quadrature components of a 1 μs advanced and a delayed pulse. Because of the advance/delay time, the pulses have a phase difference of ±36 (=360·1 μs/10 μs) degrees as compared to an unmodulated Loran-C pulse. This results in in-phase and quadrature magnitudes of \( \cos(36) = 0.81 \) and \( \sin(36) = 0.59 \), respectively, assuming the pulse amplitude is equal to one. Notice that the pulse delay or advance has relatively much more impact on the carrier phase than on the envelope delay. This means that if modulated Loran pulses are averaged, the resulting pulse shape will have a very minor difference in comparison with the unmodulated pulse shape.

![In-phase / quadrature pulses for 1 ms advanced and delayed Loran pulses.](image)

With the advent of data transmission using Loran-C, an interesting question that comes up is that of what the optimum demodulation technique for this particular type of data modulation is. The answer from maximum likelihood detection theory is to select the symbol that has the minimum mean square error between it and the received signal. This leads to correlating the received Loran pulses with delayed and advanced reference pulses, and selecting that symbol which gives the largest correlation. A problem with this method is that the reference pulse shape is not
known a priori, because of skywaves and transmitter distortion. However, just as for the CRI cancellation technique, it is possible to use the average pulse shape as the reference function.

Using the correlation technique, there is a choice to do the correlations and make decisions for all pulses separately, or to use certain groups of pulses. The latter option is the preferred one, since we are especially interested in the correct receipt of packets of bits, each packet containing a differential correction or an integrity message, for instance.

In general, the received signal after filtering and downconversion can be written as:

\[
 r(t) = \sum_i a_l(t - \tau_i - d_i \tau_d) \cos[\omega t + \theta_i + d_i \theta_d] + n(t) \tag{5.9}
\]

Here, \(i\) is the pulse index number. Each pulse has a certain delay modulation \(d_i \tau_d\) and a phase modulation of \(d_i \theta_d\), where \(d_i\) takes values of \(-1, 1\), \(\tau_d\) is a delay shift, for instance 1 \(\mu s\), and \(\theta_d\) is the corresponding phase shift\(^6\), which is equal to \(2 \pi \tau_d / 10^{-5}\). Now, for a certain number of pulses during the time window \(t_o\) to \(t_f\), the maximum likelihood decoding technique requires the calculation of the correlation value \(R\) for all possible combinations of \(d_i\):

\[
 R = \int_{t_o}^{t_f} r(t) \sum_i l(t - \tau_i) \cos[\omega t + \theta_i + d_i \theta_d] dt
 - \sum_i \cos[d_i \theta_d] \int_{t_o}^{t_f} r(t) l(t - \tau_i) \cos[\omega t + \theta_i] dt
 + \sum_i \sin[d_i \theta_d] \int_{t_o}^{t_f} r(t) l(t - \tau_i) \sin[\omega t + \theta_i] dt
\]

\(^6\) Because of practical limitations, it is not possible to use pure phase modulation. The only possibility is to use pulse position modulation, whereby the phase modulation is directly related to the applied pulse delay or advance.
Note that in (5.10), the pulse delay shift \( d_i \tau_d \) is ignored, since its influence is negligible when compared to the phase modulation \( d_i \theta_d \). A further simplification of (5.10) is made possible by dropping the first term, since \( \cos(d_i \theta_d) \) is a constant value, independent of \( d_i \), which has values of \([-1,1]\). Also, since \( \sin(d_i \theta_d) \) always has the same magnitude, with the sign depending on \( d_i \), it can simply be replaced by \( d_i \). Therefore, the maximum likelihood values of \( d_i \) can be found by maximizing the following correlation value \( R' \).

\[
R' = \sum_i d_i \int_{t_0}^{t_1} r(t) l(t - \tau_i) \sin(\omega t + \theta_i) \, dt
\]  

(5.11)

In words, the input Loran-C pulses should first be downconverted, using a quadrature reference carrier signal. Further, the pulses have to be correlated with the reference pulse shape and the result has to be multiplied with the appropriate phase code value, which is assumed to be present as 180-degrees phase jumps in \( q_i \). Finally, the obtained separate quadrature pulse correlation values have to be correlated with all possible combinations of \( d_i \). The combination that yields the largest correlation is selected as the correct one. This maximum likelihood decoding technique can be applied to both simple coding schemes as proposed in [13], as well as to the more advanced coding methods discussed in [14]. The performance of these types of maximum likelihood decoding techniques, and especially the achievable packet error probabilities under realistic CWI and CRI conditions, is currently being investigated within the Eurofix program.

### 5.7 Conclusions

Theoretically, it is possible to apply maximum likelihood estimation techniques to combat the three major error sources of Loran-C: Skywaves, Continuous Wave Interference and Cross-Rate Interference. In practice, however, the application of maximum likelihood estimation is limited by the uncertainty in the shape of the transmitted Loran-C pulses. This makes it necessary to use only the leading edges of the pulses, while the main advantage of skywave estimation would be the improved
signal-to-noise ratio made possible by using the top of the pulses. However, the
presented estimation technique can still be useful detecting unacceptable skywave
levels and in determining 'safe' groundwave tracking points, free from skywaves.

The uncertainty in the Loran pulse shape has less influence on the detection and
cancellation of Carrier Wave Interference. The only problem that arose here was the
fact that CWI signals are not pure carrier waves in reality. They may have some
modulation pattern, with a bandwidth that is small compared to the Loran-C
bandwidth. Despite the fact that CWI was modeled as being unmodulated, a
prototype receiver managed to suppress the largest interfering signals by more than
30 dB. Further, the developed CWI detection technique has been demonstrated to be
simple and effective in detecting synchronous CWI signals.

The cancellation of Cross-Rate Interference requires accurate knowledge of the
received signal shape, including skywaves and transmitter distortion. An effective
and relatively simple way to obtain this knowledge is to use the averaged received
pulse shape. Multiplying the average pulse shape with the appropriate phase code
value gives a replica of the CRI signal, which can be subtracted from the received
signal. Subsequently, other Loran signals can be demodulated and canceled as well.
CRI cancellation is especially beneficial for Loran-C data transmission, since CRI is
often the main source of errors for this application.

A practical problem that arises when implementing the new estimation and
cancellation techniques is the presence of frequency differences between the various
Loran signals. In order to account for the resulting drifts in phase and delay, some
form of synchronization has to be used. Several possible synchronization techniques
have been shortly discussed in this chapter.

An important property of Loran-C is its ability to transmit data, e.g. differential
GNSS corrections, as proposed for the EUROFIX system [13-15]. For the data
demodulation and decoding, a maximum likelihood detection algorithm has been
proposed. This technique uses the same average pulse shape as the CRI cancellation
method, which makes the implementation of both techniques relatively easy and
efficient.
5.8 References in chapter 5


6 CONCLUSIONS AND DISCUSSION

6.1 Land-mobile satellite communications

Land-mobile satellite systems will be an essential part of the Global Personal Communication Network, mainly because of their ability to provide services to areas that cannot be served economically by terrestrial links. In the design of land-mobile satellite links, the modulation and multiple-access technique play a crucial role. Chapter 2 described the performance analysis of a land-mobile satellite channel, using BPSK direct-sequence spread-spectrum modulation. An existing narrowband propagation model was extended to a wideband model by assuming an exponentially decaying power-delay profile for the multipath signals. Based on this model, a closed-form expression was derived for the interference power of both multipath and multi-user interference, which was used to evaluate the system's performance in terms of bit error probability, outage probability, throughput and delay.

The bit error and outage probabilities are the main parameters of interest in the case of circuit-switched speech communications. It is shown that for a relatively low number of users \(-K/N\) in the order of 0.1, spread-spectrum can tolerate a lower signal-to-noise ratio than narrowband communications for an uncoded bit error probability of \(10^{-2}\) to \(10^{-3}\). However, it can be expected that narrowband modulation can reach a higher system capacity, up to \(N\) users per \(N\) channels, assuming that losses due to call setups can be neglected. In cellular terrestrial systems, narrowband techniques perform much worse because of co-channel interference [1,2]. A similar type of interference is present in multi-spot satellite systems, but the relative amount of co-channel interference is much less than for terrestrial systems [3]. The only reason that spread-spectrum modulation can give a higher capacity in this case are additional factors such as guard bands between narrowband FDMA channels and the voice activity factor [4]. Note that these factors are practical rather than theoretical limitations. It is possible to overcome
them by using more advanced narrowband systems, which will possibly give narrowband techniques a higher capacity than spread-spectrum modulation.

The system capacity issue is quite different for the case of packet-switched data communications, where random access techniques are necessary for transmitting data packets. In this case, the capacity of narrowband systems is limited due to random access interference. For spread-spectrum Code Division Multiple Access systems, the amount of random access interference is essentially the same for both circuit-switched and packet-switched data communications, assuming the same amount of offered traffic. It was shown in section 2.6 that spread-spectrum can achieve a capacity or throughput that is almost twice as much as the capacity of narrowband slotted ALOHA. This is only true if there is a dominant line-of-sight path present. In the case of heavy shadowing, diversity techniques, power control and forward error correction are essential to minimize the performance degradation of a spread-spectrum CDMA system. It should be noted that power control and forward error correction are essential for CDMA anyway. Power control is defined here as maintaining a constant received user power, averaged over several seconds. Thus, no attempt is made to compensate for short time fluctuations due to shadowing or multipath. Only the long-term shadowing effects are eliminated, i.e., the received power differences between light, average and heavy shadowing.

6.2 Tracking errors in communication systems

One of the assumptions in the performance analysis of a land-mobile satellite communication system is that code and carrier synchronization errors can be ignored. An analysis of synchronization errors showed that this assumption is valid only if both the tracking loop bandwidth and the data bandwidth are large or small in relation to the fading bandwidth. In the case of a noncoherent delay lock loop, the same condition as for the loop bandwidth also applies to the predetection bandwidth. In all other cases, synchronization errors cause a certain degradation of
the bit error probability. For instance, if the fading bandwidth is large in comparison with the tracking loop bandwidth, but small in comparison with the data bandwidth, then the tracking loop synchronizes to an average code delay or carrier phase, while the data demodulator experiences different instantaneous delay and phase values. Section 2.2 showed that the resulting carrier phase errors in this case cause a certain degradation of the bit error probability, especially for higher signal-to-noise ratios. Similarly, the code delay errors can also be expected to give a bit error degradation.

The analysis of code tracking errors in chapter 3 demonstrated that the standard deviation of the differences in the delays of instantaneous and average delay values can reach a maximum value of about 0.2 chip for a typical signal-to-multipath ratio of 4 dB. Assuming a linear correlation function, this would result in a signal-to-noise ratio degradation of $(1-0.2)^2$, which equals approximately 2 dB. However, when the spread-spectrum signals are band limited to a bandwidth of about $2/T_C$, then the correlation function decreases less rapidly, which limits the signal loss to less than one dB.

The analysis of the effects of multi-transmitter on synchronization errors shows a close resemblance to that of multipath. The main difference is the presence of different data signals. For uncorrelated data and a tracking loop bandwidth that is small in comparison with the data bandwidth, the signal-to-interference power in the tracking loop is reduced by a factor that is equal to the ratio of the spread-spectrum bandwidth and the tracking loop bandwidth. Since this ratio is usually much larger than the normal processing gain - because the tracking loop bandwidth is usually small compared to the data bandwidth - the effects of multi-transmitter interference on synchronization can be expected to be negligible.

Concluding, multipath propagation and multi-transmitter interference may cause certain synchronization errors, but their effects are relatively small. Further, it is relatively easy to almost completely avoid those errors by choosing the tracking loop bandwidth to be larger than the fading bandwidth.
6.3 GNSS tracking errors

Contrary to communications, the effects of multipath propagation on navigation systems like GNSS are far more severe. The primary reason for this is the different definition of synchronization errors for navigation and communications. For a navigation system, any differences between the estimated delay and phase and the true delay and phase of the line-of-sight signal are regarded as synchronization errors. For a communication system, the best delay and phase estimates are those that maximize the output signal-to-noise ratio. Generally, these values do not correspond to the line-of-sight delay and phase values, as shown in section 2.7. As a result, GNSS multipath tracking errors are negligible only when the loop bandwidth is small in comparison with the fading bandwidth, which conclusion is opposite to that obtained for communication systems.

Unfortunately, for stationary differential GNSS reference receivers, the fading bandwidth is usually so low that a significant amount of multipath remains present. A careful siting of the GNSS antenna has to ensure that the multipath errors are kept at an acceptable level. Further, the use of a delay lock loop with a small early-late spacing —smaller than one chip— can significantly improve the performance over that of conventional receivers with a one-chip early-late spacing. However, as shown in section 3.3.1, a small early-late spacing is only effective against multipath with a relative delay that is larger than the early-late spacing.

For moving users, multipath errors are much less of a problem. Because of the increased fading bandwidth, the input signal-to-multipath ratio is reduced by a factor that is equal to the ratio of fading bandwidth and tracking loop bandwidth. As code tracking loop bandwidth values are in the order of 0.1 Hz, and fading bandwidth values easily reach values of tens of Hz, the resulting multipath levels are at least 20 dB lower than that of stationary receivers. Although this type of fast fading normally gives negligible range errors for a coherent delay lock loop, it can still give significant errors for a noncoherent delay lock loop. Such a noncoherent loop squares the signal after filtering with a certain predetection bandwidth that is much larger than the loop bandwidth. Because of the squaring, a part of the multipath components can no longer be separated from the line-of-sight by simple filtering. Thus, a certain range bias remains, which can take values up to about 40
cm for a 0.1 chip early-late spacing, and up to 4 meters for a one-chip early-late spacing, assuming an input signal-to-multipath ratio of 15 dB. This once again shows the advantage of using a small early-late spacing. However, as discussed in section 3.6 and 4.2.2, a small early-late spacing is effective only if the input signal bandwidth is large enough, approximately equal to the inverse of the early-late spacing.

6.4 GNSS multipath estimation

In many practical applications, the occurrence of multipath propagation is often inevitable. However, this does not mean that synchronization errors due to multipath are unavoidable; Chapter 4 described a new type of spread-spectrum receiver -the MEDLL- that is capable of separating line-of-sight and multipath signals. Ideally, the performance of this receiver is limited by the signal-to-noise ratio only, rather than by multipath. A general expression is found for the code tracking error as a function of the signal-to-noise ratio, tracking loop bandwidth, early-late spacing and signal bandwidth. This expression fills the gap in the existing literature on the relation between the early-late spacing and the signal bandwidth. It is found that the code tracking error is inversely proportional to the square root of the early-late spacing, provided that the signal bandwidth is larger than the inverse of the early-late spacing. For smaller bandwidth values, the tracking error converges to a value that is inversely proportional to the square root of the signal bandwidth. This means, for instance, that the proposed Inmarsat additional GPS-like ranging signals will have a noise level that is about three times larger than that of normal GPS signals, because of the approximately ten times smaller bandwidth (2.2 MHz) of the Inmarsat signals [5].

Except for noise, the performance of the MEDLL is also affected by the delay differences of separate multipath signals. The convergence speed of the iterative MEDLL calculations turns out to be inversely proportional to the relative delay between different multipath signals. Thus, a finite number of iterations gives a certain remaining multipath error. In practice, the difference between MEDLL and
normal DLL multipath errors is negligible for multipath signals that have a delay relative to the line-of-sight signal of less than about 0.05 chip.

A further source of errors is the number of estimated multipath signals. While in reality a continuum of multipath signals may be present, the MEDLL can only estimate a limited number. Although the estimated MEDLL signal parameters generally give a better representation of the input signal than the single set of DLL estimates, there will always be some remaining error if the number of estimated signals is too small. Measurements with a prototype GPS receiver show reductions of the ranging error up to a factor of ten, as compared to a delay lock loop with a small early-late spacing. The largest reduction factors are achieved in environments with high multipath levels, e.g. caused by reflections from buildings. Most probably, this is because strong (specular) multipath situations are better modeled by a limited number of signals than environments with a low level of multipath.

Because of its ability to reduce multipath errors, the MEDLL is beneficial for all differential GNSS applications that require an accuracy of several meters or less. Undoubtedly, one of the most demanding applications is aircraft navigation. The navigation requirements during precision approach and landing are difficult to achieve for current GPS receivers, even in combination with Inertial Navigation Systems. The MEDLL could make differential code GNSS, integrated with INS, a more viable option. Further, the MEDLL is beneficial for differential carrier GNSS as well, since it reduces both code and carrier ranging errors.

6.5 Loran-C multipath and interference cancellation

As a terrestrial navigation system, Loran-C is a good candidate to back up GNSS. Further, it provides the possibility to transmit differential GNSS and integrity messages, which can save a lot of costs in the infrastructure for differential GNSS services.
Loran-C suffers from a number of interfering sources: Multipath or skywaves, Carrier Wave Interference from narrowband transmissions close to the Loran band, and multi-transmitter or Cross Rate Interference from other Loran transmitters. Chapter 5 described how new estimation techniques -similar to the MEDLL- could be applied to cope with all interferences. Further, an optimum demodulation technique was developed for the demodulation of Loran-C data, containing differential GNSS or integrity messages.

A major problem with the application of maximum likelihood estimation to Loran-C is the uncertainty in the received signal shape. This largely reduces the advantage of skywave estimation, although it can still be useful to detect unacceptable skywave levels. The signal shape uncertainty is not a big problem for CRI cancellation and data demodulation, because it is possible to use the averaged received Loran pulses as the reference pulse shape. Measurements from a prototype Loran receiver showed the feasibility of CRI cancellation, which opens the way to more reliable Loran positioning and especially Loran data transmission, as an augmentation to satellite navigation systems.

6.6 Scope of future work

Although many papers have been published on the comparison of CDMA and FDMA/TDMA systems, there is still a need for this type of analysis, especially in the case of packet-switched satellite communications. Future research could include multi-spot interference and different transmission protocols. Another interesting topic is the performance of satellite communications in urban environments. This is not the type of environment satellite systems were traditionally designed for, however, this will change with the advent of Low Earth Orbiting satellite networks. Such systems will be used in a way similar to land-based cellular systems. It can be expected that the satellite link often experiences severe shadowing, especially in urban environments. In this case, the radio channel will exhibit an 'on/off' behavior, contrary to the gradually changing shadowing effects as described by the shadowed
Rician channel model from chapter two. In order to combat the shadowing problems, it is proposed in [6] to use satellite diversity, where the user selects the best satellite. To further analyze the performance of this type of satellite diversity systems, taking into account shadowing effects and multi-spot interference is recommended.

As far as navigation systems are concerned, the main issue at this moment is the realization of new techniques that can enhance their accuracy and integrity. In order to make a decision as to whether GNSS can be used for aircraft approach and landing, for instance, it is necessary to perform a significant number of flight trials, using state-of-the-art technology such as the MEDLL, as described in chapter 4. A practical problem that has to be solved before the MEDLL can be implemented is the optimization in calculation speed and required hardware. A decision has to be made as to the minimum number of correlators required to get an acceptable performance. Further, research is necessary to find a solution to the problem of multipath signals with small relative delays of less than 0.05 chip, since the MEDLL is not capable of significantly reducing the errors caused by those signals.

6.7 References in chapter 6


Conclusions


Multipath and multi-transmitter interference
APPENDIX A: THE SHADOWED RICIAN DISTRIBUTION

This appendix gives the derivation of the shadowed Rician distribution function. This distribution function can be used to describe the amplitude statistics of a lognormally shadowed line-of-sight signal plus a Rayleigh distributed multipath signal. The Rayleigh distribution is given as:

\[ p(r) = \frac{r}{b_o} e^{-\frac{r^2}{2b_o}} \quad (A.1) \]

Here, \( b_o \) is the total amount of multipath power. Since the Rayleigh distribution describes the amplitude statistics of a Gaussian distributed signal, it is especially useful in modeling a large number of multipath signals. When the power differences between the different multipath signals are not too large, then the combined signal will be approximately Gaussian distributed, according to the central limit theorem.

In the case of land-mobile satellite communications, the received signal consists of a line-of-sight and a multipath component. If the line-of-sight signal has a constant amplitude \( z \), then the combined signal has a Rician distributed amplitude, given as:

\[ p(r) = \frac{r}{b_o} e^{-\frac{(r^2+z^2)}{2b_o}} I_0\left(\frac{rz}{b_o}\right) \quad (A.2) \]

where \( b_o \) represents the average scattered multipath power, and \( I_0(x) \) is the modified Bessel function of the first kind and zeroth order:

\[ I_0(x) = \frac{1}{2\pi} \int e^{x\cos(\theta)} d\theta \quad (A.3) \]
However, in practice, the line-of-sight signal amplitude may vary in time instead of being a constant value, because of shadowing effects caused by foliage or other objects. If the shadowing is caused by a large number of objects, such as leaves from a tree, then the total attenuation in dBs will be Gaussian distributed, according to the central limit theorem. If the logarithm of the signal amplitude is Gaussian distributed, then the amplitude itself has a lognormal distribution function, given by:

\[
p(r) = \frac{1}{r\sqrt{2\pi\sigma_0^2}} e^{\frac{-(\ln(z) - \mu_0)^2}{2\sigma_0^2}}
\]  

(A.4)

Here, \(\sigma_0\) and \(\mu_0\) are the standard deviation and mean of the logarithm of the line-of-sight amplitude, respectively.

If a lognormally distributed line-of-sight signal is added to a Rayleigh distributed multipath signal, then the probability density function of the combined signal can be calculated as follows. If the line-of-sight amplitude \(z\) is temporarily kept constant, then \(p(r)\) is a Rician distribution:

\[
p(r|z) = \frac{r}{b_0} e^{\frac{-\left(r^2 + z^2\right)}{2b_0}} I_0\left(\frac{rz}{b_0}\right)
\]  

(A.5)

Now, assuming that the line-of-sight amplitude \(z\) has a lognormal distribution, the combined probability density function can be found using the theorem of total probability:

\[
p(r) = \int_0^\infty p(r, z) dz = \int_0^\infty p(r|z) p(z) dz
\]  

(A.6)

By substituting the Rician distribution \(p(r|z)\) and the lognormal distribution \(p(r)\) in (A.6), the combined distribution is obtained.
\[ p(r) = \frac{r}{b_o} \int_0^\infty e^{-\frac{1}{2b_o} \left( \frac{r^2 + z^2}{2b_o} \right)} I_o \left( \frac{r z}{b_o} \right) p(z) \, dz \]

\[ = \frac{r}{b_o \sqrt{2\pi \sigma_o^2}} \int_0^\infty e^{-\frac{1}{2\sigma_o^2} \left( \frac{r^2 + z^2}{2b_o} \right)} I_o \left( \frac{r z}{b_o} \right) \, dz \]  

(A.7)

This integral cannot be solved mathematically. Therefore, it has to be evaluated numerically. If the shadowing standard deviation is relatively small in comparison with the shadowing mean, then the shadowed Rician distribution can be approximated by the Rician distribution. If the shadowing variance is large in comparison with the multipath power, then (A.7) is approximately equal to a lognormal probability density function. Finally, if the shadowing variance is small in comparison with the multipath power, then the shadowed Rician function can be approximated by the Rayleigh distribution.
APPENDIX B: DERIVATION OF MAXIMUM LIKELIHOOD ESTIMATES

In this appendix, the derivation is given of the maximum likelihood estimates that are the basis of the Multipath Estimating Delay Lock Loop. The maximum likelihood estimates \( \hat{a}_m, \hat{\tau}_m, \hat{\theta}_m \) for the amplitude, delay and phase of the \( m \)th signal are found by minimizing the mean square error function \( L(a_m, \tau_m, \theta_m) \), which is given as:

\[
L(a_m, \tau_m, \theta_m) = \int_{t-T}^{t} [r(t) - s(t)]^2 dt
\]

\[
s(t) = \sum_{m=0}^{M-1} a_m p(t - \tau_m) \cos(\omega t + \theta_m)
\]

This function is minimized by setting the partial derivatives to zero. For the amplitude estimates, the partial derivative is found as:

\[
\frac{\partial}{\partial a_m} L(a_m, \hat{\tau}_m, \hat{\theta}_m) = \int_{t-T}^{t} -2[r(t) - s(t)] \frac{\partial}{\partial a_m} s(t) dt
\]

\[
= -2 \int_{t-T}^{t} r(t) p(t - \hat{\tau}_m) \cos(\omega t + \hat{\theta}_m) -
\]

\[
p(t - \hat{\tau}_m) \cos(\omega t + \hat{\theta}_m) \sum_{m=0}^{M-1} a_m p(t - \hat{\tau}_m) \cos(\omega t + \hat{\theta}_m) dt
\]

The second term contains a squared cosine that can be split into a baseband component and a component at the double carrier frequency. Assuming that the integration time is large compared to the inverse of the carrier frequency, the double frequency component can be neglected. Then, the partial derivative to the amplitude estimate can be written as:

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\[
\frac{\partial}{\partial a_m} L(a_m, \hat{m}, \hat{\theta}_m) = -\text{Re}\{[R_x(\hat{m}) - \sum_{i=0 \atop i\neq m}^{M-1} \hat{a}_i R(\tau - \hat{\tau}_i) \exp(j\hat{\theta}_i)] \exp(-j\hat{\theta}_m)\} + a_m R(0)
\]  

(B.3)

Where \[ R_x(\hat{m}) = \frac{2}{T} \int_{t-T}^{t} r(t) p(t - \hat{m}) \exp(-j\omega t) dt \]  

(B.4)

and \[ R(\tau) = \frac{2}{T} \int_{t-T}^{t} p(t) p(t - \tau) dt \]  

(B.5)

\( R(t) \) is the reference correlation function, which is equal to zero for \( t=0 \). Thus, by setting (B.3) to zero, the amplitude estimate can be found as:

\[
\hat{a}_m = \text{Re}\{[R_x(\hat{m}) - \sum_{i=0 \atop i\neq m}^{M-1} \hat{a}_i R(\tau - \hat{\tau}_i) \exp(j\hat{\theta}_i)] \exp(-j\hat{\theta}_m)\}
\]  

(B.6)

The partial derivative for the delay estimates is:

\[
\frac{\partial}{\partial \tau_m} L(\hat{a}_m, \tau_m, \hat{\theta}_m) = \int_{t-T}^{t} -2[r(t) - s(t)] \frac{\partial}{\partial \tau_m} s(t) dt
\]

\[
= -2 \frac{\partial}{\partial \tau_m} \int_{t-T}^{t} r(t) \hat{a}_m p(t - \tau_m) \cos(\omega t + \hat{\theta}_m) dt
\]

(B.7)

\[
\hat{a}_m p(t - \tau_m) \cos(\omega t + \hat{\theta}_m) \sum_{i=0}^{M-1} \hat{a}_i p(t - \hat{\tau}_i) \cos(\omega t + \hat{\theta}_i) dt
\]

This expression consists of a correlation of the input signal with the estimated \( m \)th signal, minus a summation of \( M \) correlation functions. The \( m \)th correlation function.
of this summation is independent of the delay estimate \( \tau_m \). Therefore, its partial derivative becomes zero, and the whole expression can be rewritten as:

\[
\frac{\partial}{\partial \tau_m} L(\hat{a}_m, \tau_m, \hat{\theta}_m) = \frac{\partial}{\partial \tau_m} \left[ \text{Re} \left\{ [R_x(\tau) - \sum_{i=0}^{\hat{M}-1} \hat{a}_i R(\tau - \hat{\tau}_i) \exp(j \hat{\theta}_i) \exp(-j \hat{\theta}_m)] \right\} \right]
\]

(B.8)

The desired delay estimate \( \hat{\tau}_m \) is that delay value for which (B.8) is equal to zero. Alternatively, \( \hat{\tau}_m \) can be expressed as that delay that maximizes the following expression.

\[
\hat{\tau}_m = \max_{\tau} \left[ \text{Re} \left\{ [R_x(\tau) - \sum_{i=0}^{\hat{M}-1} \hat{a}_i R(\tau - \hat{\tau}_i) \exp(j \hat{\theta}_i) \exp(-j \hat{\theta}_m)] \right\} \right]
\]

(B.9)

The advantage of this latter expression is that it finds the global maximum of the correlation function, while the previous expression might find local maxima, which would cause estimation errors.

Finally, the phase estimate \( \hat{\theta}_m \) can be found by setting the following partial derivative to zero:

\[
\frac{\partial}{\partial \theta_m} L(\hat{a}_m, \hat{\tau}_m, \theta_m) = \int_{t-T}^{t} -2[r(t) - s(t)] \frac{\partial}{\partial \theta_m} s(t) dt
\]

\[
= -2 \int_{t-T}^{t} [r(t) - s(t)] \hat{a}_m p(t - \hat{\tau}_m) [\sin \theta_m \cos(\omega t) - \cos \theta_m \sin(\omega t)] dt = 0
\]

(B.10)

Here, the following trigonometric equality is used to split the carrier component into an in-phase part and a quadrature part:

\[
\cos(\omega t + \theta) = \cos \theta \cos(\omega t) - \sin \theta \sin(\omega t)
\]

(B.11)
Equation (B.10) can be simplified further to:

\[
\sin \theta_m \int_{t-T}^{t} [r(t) - \sum_{\substack{i=0 \atop i \neq m}}^{M-1} \hat{a}_i p(t - \hat{\tau}_i) \cos(\omega t + \hat{\theta}_i)] p(t - \hat{\tau}_m) \cos(\omega t) dt = \cos \theta_m \int_{t-T}^{t} [r(t) - \sum_{\substack{i=0 \atop i \neq m}}^{M-1} \hat{a}_i p(t - \hat{\tau}_i) \cos(\omega t + \hat{\theta}_i)] p(t - \hat{\tau}_m) \sin(\omega t) dt \quad (B.12)
\]

In the summations inside this equation, the terms for \( i=m \) are dropped because they give a zero contribution to the partial derivative (B.10). By dividing both sides of (B.12) by \( \cos \theta_m \), an expression is found for the tangent of the desired phase estimate. Thus, the phase estimate can be obtained by applying the inverse tangent function, preferably the four-quadrant arc tangent function (which looks at the individual signs of the in-phase and quadrature parts to determine in which quadrant the vector lies). Here, this function is called \( \arg(x) \), where \( x \) is some complex number. Now, the phase estimate can be written as:

\[
\hat{\theta}_m = \arg[R_x(\hat{\tau}_m) - \sum_{\substack{i=0 \atop i \neq m}}^{M-1} \hat{a}_i R(\hat{\tau}_m - \hat{\tau}_i) \exp(j\hat{\theta}_i)] \quad (B.13)
\]
APPENDIX C: LIST OF ACRONYMS

BPSK : Binary Phase Shift Keying
CAT : Category
C/A-code : Course/Acquisition code
CDMA : Code Division Multiple Access
CRI : Cross Rate Interference
CWI : Carrier Wave Interference
DGPS : Differential GPS
DLL : Delay Lock Loop
DRMS : Distance Root Mean Squared
FDMA : Frequency Division Multiple Access
FEC : Forward Error Correction
GDOP : Geometric Dilution Of Precision
Glonass : Global Orbiting Navigation Satellite System
GNSS : Global Navigation Satellite System
GPS : Global Positioning System
GRI : Group Repetition Interval
GSM : Global System for Mobile communications
IVHS : Intelligent Vehicle Highway Systems
L1, L2 : 1575.42, 1227.60 MHz GPS carrier frequencies
LEO : Low Earth Orbit
Loran : Long Range Navigation
MEDLL : Multipath Estimating Delay Lock Loop
MLS : Microwave Landing System
MRC : Maximal Ratio Combining
NCO : Numerically Controlled Oscillator
P-code : Precision code
PDOP : Position Dilution of Precision
RMS : Root Mean Squared
SD : Selection Diversity
SIR : Signal-to-Interference Ratio
SMR : Signal-to-Multipath Ratio
SNR : Signal-to-Noise Ratio
TDMA : Time Division Multiple Access
VCO : Voltage Controlled Oscillator
APPENDIX D: LIST OF SYMBOLS

\( A \) : Amplitude  
\( b_o \) : Average multipath power  
\( b_m \) : Average multipath power in path \( m \)  
\( B \) : Double-sided signal bandwidth \( = k/T_c \)  
\( B_F \) : Fading bandwidth  
\( B_L \) : Tracking loop bandwidth  
\( B_p \) : Predetection bandwidth  
\( \beta_{mk} \) : Gain of path \( m \) and user \( k \)  
\( c \) : Speed of light  
\( C \) : Carrier or signal power  
\( d \) : Early-late spacing  
\( D \) : Average packet delay  
\( E_b \) : Bit energy, \( E_b = C T_b \)  
\( d_i \) : Data bit of user \( i \)  
\( \delta(\cdot) \) : Dirac function  
\( G \) : Offered traffic in packets per slot  
\( I_0(\cdot) \) : Modified Bessel function of first kind, zeroth order  
\( k \) : Normalized double-sided signal bandwidth \( = B T_c \), user number (ch. 2)  
\( k_a \) : Cross correlation value  
\( K \) : Number of transmitting users  
\( \lambda \) : Wavelength  
\( M \) : Number of multipath signals  
\( M_d \) : Order of diversity  
\( \mu_0 \) : Shadowing mean  
\( N \) : Processing gain, number of chips per bit  
\( N_{AT} \) : Maximum number of idle slots before retransmission  
\( N_D \) : One-sided spectral noise density  
\( N_p \) : Number of bits per packet  
\( P_e \) : Bit error probability  
\( P_{irr} \) : Irreducible bit error probability  
\( p_o \) : Threshold bit error probability  
\( P_{out} \) : Outage probability  
\( P_d(x) \) : Shadowed Rician probability density function  
\( P_s \) : Packet success probability  
\( P_{tr}(k) \) : Probability that \( k \) packets are generated
Multipath and multi-transmitter interference

\[ p_k(\cdot) \quad : \text{Spread-spectrum code of user } k \]
\[ p_\theta (\theta) \quad : \text{Carrier phase probability density function} \]
\[ Q \quad : \text{Packet error probability} \]
\[ r \quad : \text{Amplitude, path gain} \]
\[ R(\cdot) \quad : \text{Correlation function} \]
\[ S \quad : \text{Throughput} \]
\[ \sigma \quad : \text{Noise standard deviation} \]
\[ \sigma_0 \quad : \text{Shadowing standard deviation} \]
\[ \sigma_\theta \quad : \text{Phase standard deviation} \]
\[ t \quad : \text{Time, number of correctable bits (2.5-2.6)} \]
\[ T_b \quad : \text{Bit time} \]
\[ T_c \quad : \text{Chip time} \]
\[ T_d \quad : \text{Propagation delay} \]
\[ T_s \quad : \text{Delay spread} \]
\[ \tau_{mk} \quad : \text{Delay of path } m \text{ and user } k \]
\[ \theta_{mk} \quad : \text{Phase of path } m \text{ and user } k \]
\[ \omega_c \quad : \text{Angular carrier frequency} \]
\[ \omega_{mk} \quad : \text{Angular Doppler frequency of path } m \text{ and user } k \]
APPENDIX E: RELATED PUBLICATIONS
BY THE AUTHOR

JOURNAL PAPERS AND LETTERS


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CONFERENCE PAPERS


CONTRACT REPORTS


PATENT APPLICATION

APPENDIX F: RELATION TO OTHER WORK

This dissertation is based in part upon previously published results. The table underneath lists the relation between the publications, as listed in the previous appendix, and the different chapters.

The work described in chapter two was performed in cooperation with Ramjee Prasad, Howard Misser and Rogier van Wolfswinkel. Bas 't Hart did a part of the analysis and made the plots as shown in section 3.12. The GPS multipath measurements of chapter 3 were done together with Peter Kranendonk and X. Jin. The prototype MEDLL receiver, that was used to verify the multipath estimation techniques as described in chapter 4, was developed by Pat Fenton and Bryan Townsend, who provided a modified NovAtel GPS receiver, and Jaap Siereveld, who wrote the software for a real-time MEDLL receiver. Chapter 5 is largely based on the results of the graduation work of Hein Andersen, who build a prototype Loran-C receiver and came up with the idea to use the averaged received pulse shape for CRI cancellation.

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SAMENVATTING (SUMMARY IN DUTCH)

Meerwegpropagatie en interferentie in spread-spectrum communicatie- en navigatiesystemen

De prestaties van communicatie- en navigatiesystemen worden voornamelijk gelimeerde door meerwegpropagatie en interferentie van andere zenders. In het geval van communicatiesystemen zorgen meerwegpropagatie en interferentie voor een degradatie van de bitfoutenkans en daarmee van de capaciteit van het systeem. In het geval van navigatiesystemen gaat het vooral om de aantasting van de nauwkeurigheid waarmee de fase en vertraging van signalen gemeten kunnen worden, omdat dit de grootte van de uiteindelijke navigatiefout bepaalt.

Het doel van deze dissertatie is het analyseren van de effecten van meerwegpropagatie en interferentie in direct-sequence spread-spectrum systemen, almede het vinden van nieuwe ontvangerstructuren die de bestaande problemen kunnen reduceren. De dissertatie richt zich vooral op satellietsystemen, maar besteedt ook aandacht aan het terrestriale Loran-C systeem, dat kampt met soortgelijke problemen als het satellietnavigatiesysteem GPS.

De volgende stap is het vinden van technieken om de problemen van meerwegpropagatie en interferentie te verminderen. Bij het ontwerpen van conventionele ontvangers is meestal alleen maar rekening gehouden met de aanwezigheid van additieve ruis. Door ook meerwegpropagatie en interferentie op te nemen in de schattingsmethoden die de basis vormen van de ontvangerstructuren, kunnen aanzienlijke prestatieverbeteringen behaald worden. Dit wordt gedemonstreerd in de hoofdstukken 4 en 5, waar nieuwe GPS en Loran-C ontvangerstructuren beschreven worden die in staat zijn om fouten ten gevolge van meerwegpropagatie, continuous wave en cross-rate interferentie aanzienlijk te verkleinen. Prototype GPS en Loran-C ontvangers zijn gebouwd die de toepasbaarheid van de nieuwe schattingstechnieken aantonen.
ACKNOWLEDGEMENTS

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All colleagues receive my thanks for their support, especially Jaap Siereveld for implementing many ideas into prototype receivers, Edward Breeuwer and Mike Braasch for many stimulating discussions and promoting the MEDLL, Howard Misser, Michel Jansen, Bas’t Hart and Rogier van Wolfswinkel for their contribution to the second chapter, and Hein Andersen for his work on a prototype Loran-C receiver. Appreciation is also extended to Ton Coenen for numerous discussions and creative ideas.

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CURRICULUM VITAE

Richard D.J. van Nee was born in Schoonoord, the Netherlands, on January 17, 1967. From 1979 to 1985, he attended secondary school in Emmen, The Netherlands. He received his M.Sc. degree in Electrical Engineering Cum Laude from Twente University in Enschede, the Netherlands, in 1990.

In June 1990, he joined Delft University of Technology as a PhD student in the Telecommunications and Traffic-Control Systems Group. His work on the analysis of multipath tracking errors and the design of a new spread-spectrum receiver architecture which is much less sensitive to multipath than conventional receivers was rewarded with best paper awards at both the Institute of Navigation GPS conference in Albuquerque, 1991, and the Second Annual Conference on Differential Satellite Navigation Systems in Amsterdam, 1993. He also received an award for the design of new Loran-C techniques at the WGA 22nd Technical Symposium in Santa Barbara, 1993.

From January 1995, Richard van Nee worked as a private consultant to NovAtel Communications, Calgary, working on the design of new GPS receivers for high precision applications. In March 1995, he joined AT&T Bell Labs in Utrecht, where he is working in the area of high speed wireless communications.