SCOUR UNDERNEATH SUBMARINE PIPELINES

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1. Abstract

This report deals with the physical experiments on scour underneath submarine pipelines, carried out as part of the MaTS project on the stability of submarine pipelines. Moreover, these experiments, which have been carried out in the laboratory for fluid mechanics of the Department of Civil Engineering of the Delft University of Technology, are the topic of the student thesis of the author, who has been joining the Group of Coastal Engineering as a student assistant. His work has been accompanied by ir. W. Leeuwestein. Prof. dr. ir. E.W. Bijker was the supervisor of the entire project on the stability of pipelines.

After the introduction in chapter 2, in chapter 3 a summing up is given of the previous research on the subject and the theory needed to approach the problem of the scour process underneath pipelines. In the following chapter a description is given of the way in which the experiments have been carried out. Chapter 5 gives the results of the experiments and using the results a functional relation has been tried to find between the scour depth on the one hand and the flow, pipe and sand parameters on the other hand. In chapter 6 the use of physical experiments to predict prototype scour depths is discussed by studying the concerning scale relations. This thesis ends with some conclusions.
2. Introduction

Since the start of the offshore oil and gas exploration in the early fifties the locations of exploration moved to greater distances from the shore and to deeper water. From those locations two means of transport are used to transport the oil (or gas) on shore:

- transport by ship from a reservoir near the exploration platform
- transport by pipeline

In most cases a choice between those two possibilities will be made on financial economical grounds.

If a pipeline is chosen, safety rules for the design of the pipeline have to be made with regard to the preservation of the marine environment, fishermen's gear, and the stability of the pipeline itself. For the Netherlands part of the Continental Shelf those rules are made by the State Directory of Mines. With regard to the location of a pipeline these rules have led to the requirement of the burial of pipelines into the seabottom with their top at least 0.2 m below the bed level.

However, experience with uncovered prototype pipelines have indicated the existence of a natural self-burial mechanism which enables the pipeline to dig itself in 2 or 3 pipe diameters into the bottom [2].

Because the artificial trenching of a pipeline may be hazardous and will be very expensive (± Dfl.500,000/km'), the Netherlands Industrial Council for Oceanology has initiated a research program on the stability of offshore pipelines in their MaTS (Netherlands abbreviation for Marine Technological Research) program.

"Scour underneath pipelines" is part of this study. The objective of this part of the program, which mainly is carried out at the Civil Engineering Department of the Delft University of Technology, is twofold:
1st: to obtain more information on the morphological processes near pipelines and their dependency on wave, current, pipe and sediment parameters

2nd: to predict the depths of scour holes in prototype conditions with the use of the information obtained before

This report mainly deals with the scale tests which have been carried out in the laboratory for fluid mechanics. In these experiments the scour underneath rigidly mounted model pipelines (due to current and/or wave action) has been observed and recorded. Finally a way of approach of the prediction of prototype scour is discussed.
3. Preliminary considerations

3.1. The necessity of physical model experiments.

Three methods can be used to describe the morphological processes near uncovered submarine pipelines:
- Surveying the sea bottom near prototype pipelines
- Extrapolating the results of physical model tests
- Developing a numerical model
Below the advantages and disadvantages of the three methods are discussed.

Surveying prototype pipelines

In fact this is a scale 1:1 test. Tests like these have been carried out on the Dutch part of the continental shelf: the L4 - L7 pipeline (22 km) and the LI0A/F pipeline (4 km). Indeed the results of the surveys show self lowering of the pipelines, respectively [3] and [2], but the great time intervals in which the surveys have been made do not give much information on the local scour process underneath the pipelines. Moreover, the surveys have been carried out under fair weather conditions because it is impossible to carry them out adequately during a storm. Because of the high costs which have to be made for this method only a small number of surveys can be made. Moreover, effects on the scour process of the pipe, water and sand parameters can not be investigated solely.

Physical model tests

Physical model tests can be carried out to investigate the effects of the various parameters on the scour process. In this way the scour process could be described as a function of the relevant parameters. With this function the scale factors should be calculated which can be used to predict the prototype scour process.
This method seems to be very simple and inexpensive but the strong non-linearity between e.g. the water velocity and the sand transport introduces scale effects which hinder the simple calculation of the prototype parameters by multiplying the model parameters by the scale factors. Moreover, most model tests are limited to two-dimensional problems with rigid pipes.

Nevertheless, the model tests can give a lot of information on the effects of the sole parameters on the scour process.

A numerical model

It is possible as well to use numerical models to describe the scour process. Difficulties are met with the numerical description of the water-sediment interaction which is strongly non linear so that a lot of assumptions and linearisations have to be made to make its costs acceptable. Moreover, the reliability of the model will not exceed the reliability of the sand transport formulae. In combination with the results of the physical models, which can deliver the boundary conditions, the numerical model will be at last the most economical way to describe the scour process.

3.2. Theory

In this paragraph an introduction is given of the scour process underneath submarine pipelines and its relevant parameters. Those parameters can be divided into three groups:

a) the parameters which determine the flow pattern around the pipeline
b) the parameters which describe the characteristics of the pipe
c) the parameters which describe the bed material
The bed material underneath a pipeline can be eroded when the critical shear stress of the material is exceeded. The shear stress near the bottom is primarily determined by the flow velocity near the bottom which is a combination of the current velocity \( v_b \) and the orbital velocity \( u_b \). The current velocity near the bottom \( v_b \) is dependent on the average current velocity \( \bar{v} \) and the bottom roughness \( r \). The orbital velocity near the bottom \( u_b \) is dependent on the wave characteristics \( H \) and \( T \) and the water depth \( h \). Combining all this we find the following relevant parameters which determine the flow pattern near the bottom, see Fig. 3.1:

\[
\begin{align*}
\bar{v} & : \text{average current velocity} \\
r & : \text{bottom roughness} \\
H & : \text{wave height} \\
T & : \text{wave period} \\
h & : \text{water depth}
\end{align*}
\]

Fig. 3.1. The parameters which describe the flow pattern around the pipe

The flow pattern near the bottom is obstructed when a pipeline is laid on it. The change in the flow pattern is determined by the pipe characteristics and its location relative to the natural sea.
bottom. Assuming a cylindrical cross section the dimensions of the pipe can be given by its diameter $\phi$. (In some cases two or more pipelines (of different diameters) are brought together). The roughness of the pipe is given by the roughness of the concrete coating, although organisms which will grow on the pipeline can increase the roughness considerably.

When a pipeline is laid uncovered on the sea bed its underside will not be exactly on top of the bed for the whole length of it. In soft soil areas the pipeline may sink a little into the bottom and in areas with mega ripples free spans may occur. As "location parameter" the pipe height $d_0$ is chosen which is the distance between the underside of the pipe and the original sea bed. Another "location parameter" is the direction of the pipeline relative to the direction of the predominant wave propagation and of the predominant (tidal) current.

The angles $\theta_w$ and $\theta_c$ introduce the third dimension into the problem. Altogether we have found the following parameters which indicate the influence of the pipe on the scour process, see Fig 3.2a and 3.2b:

- $\phi$: pipe diameter
- $r_p$: pipe roughness
- $d_0$: pipe height relative to original sea bottom
- $\theta_w$: angle between pipe direction and predominant wave propagation
- $\theta_c$: angle between pipe direction and predominant (tidal) current direction
ad c) Whether the bottom will react on the change of the flow pattern due to the obstruction by the pipeline will depend on the characteristics of the bottom material. For the sandy bottom of the North Sea this can locally be described by its mean grain diameter $D_{50}$, with which other parameters as the fall velocity $w$ and the critical shear velocity $u_{*cr}$ (according to e.g. Shields) can be derived.

Combining the three types of parameters we come to the following relation:

$$\text{scour process} = f \left( \bar{v}, r, H, T, h, \phi, r_p, d_0, \theta_w, \theta_c, D_{50}, t \right) \quad (3.1)$$

In this relation the time parameter $t$ is added because some parameters as e.g. $\bar{v}$, $H$ and $T$ will not be constant in time. Parameters which describe characteristics of the sea water as the dynamic viscosity $\eta$ and the density $\rho_w$, or the bottom material like $\rho_s$ are assumed to be constant. Together with the acceleration of gravity $g$ they may be used to create dimensionless parameters.

If the bottom around the pipeline erodes a scour hole can develop underneath the pipeline. If the pipe is stable, which will be the case for short spans and mild flow conditions, in due time the scour hole will develop
towards a configuration as shown in Fig. 3.3. A two dimensional scour hole can be described with the following parameters:

\[ k_e : \text{maximum depth of scour hole} \]
\[ k_p : \text{depth of scour hole underneath the pipe} \]
\[ L_v : \text{horizontal distance from vertical pipe axis to sea bottom upstream of the pipe (or against direction of wave propagation)} \]
\[ L_a : \text{horizontal distance from vertical pipe axis to sea bottom downstream of the pipe (or in direction of wave propagation)} \]
\[ A : \text{total amount of eroded sediment} \]

Fig. 3.3. Parameters of the scour hole

Because the main interest of this research program is whether a pipe will bury itself and if so, how deep, the scour hole parameter \( k_e \) has been chosen to be the most important to investigate in the physical model experiments. Moreover, the problem has been regarded only in a two dimensional way having the pipe perpendicular to the current direction and/or wave propagation. This leads to the abandonment of the parameters \( \theta_w \) and \( \theta_c \), which leaves us:

\[ k_e = f ( \bar{v}, r, H, T, h, \phi, r_p, d_0, D_{50}, t ) \] (3.2)
For prototype conditions it can be expected that the pipe will not be stable when spans grow and scour holes deepen. Probably the pipeline will bend and touch the bottom again. Because it is very difficult to scale this bending process properly the experiments have been carried out with fixed pipes, in most cases with pipe height \( d_0 = 0 \).

After some experiments with similar pipe diameter but different pipe roughness it was concluded that the pipe roughness did not affect to some extent the value of the maximum depth of the scour hole \( k_e \). The remainder of the experiments has been carried out with the same pipe roughness ( = roughness of P.V.C. ).

Except for the experiments with current in which the development of the scour hole in time has been recorded the maximum depth of the scour hole, \( k_e \), has been measured after the scour hole had reached an equilibrium configuration.

The limitations as stated above leave us with the following relation to be investigated in experiments with current and waves:

\[
 k_e = f \left( \bar{v}, r, H, T, h, \phi, D_{50} \right) \tag{3.3}
\]

For a better understanding of the scour process first experiments have been carried out with only current or waves. In the experiments with only current the pipe diameter \( \phi \) was limited by the water depth \( h \). For theoretical reasons it can be assumed that the pipe diameter may not exceed \( \frac{1}{3} \) times the water depth, otherwise the pipe will affect the water level above it. When using pipe diameters smaller than \( \frac{1}{3} h \) the following relation remains to be investigated:

\[
 k_e = f \left( \bar{v}, r, \phi, D_{50} \right) \tag{3.4}
\]

During the experiments it was observed that the
undisturbed sediment transport upstream of the pipe plays an important part as well in the scour process. The large number of experiments in combination with the difficulty to make reliable measurements of the sand transport have led to the decision to exclude the sand transport of the research on the scour process. In the experiments which will be carried out in the future this parameter should be included.

In the wave experiments the following relation was the objective of the research:

\[ k_e = f (H, T, h, \phi, D_{50}) \]  (3.5)

3.3. Previous investigations

In the last decade already some research with physical models has been carried out on the subject of scour underneath pipelines. Leeuwestein [7] gives a wide scope of all research activities. Here only a brief view on the previous research with its results will be given. Table 3.1 shows the names of the researchers and the limits of the parameters which they have varied in their experiments.

<table>
<thead>
<tr>
<th></th>
<th>h [m]</th>
<th>H [mm]</th>
<th>T [s]</th>
<th>u [m/s]</th>
<th>v [m/s]</th>
<th>D_{50} [\mu m]</th>
<th>b [m]</th>
<th>\phi</th>
<th>d_{o}/\phi</th>
<th>Fixed</th>
<th>r/\phi</th>
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<td>—</td>
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<td>0.52</td>
<td>220</td>
<td>0.50</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>v. Ast &amp;</td>
<td>0.21</td>
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<td>0.16</td>
<td>0.29</td>
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<td>0.50</td>
<td>40</td>
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<td>4.0</td>
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<td></td>
<td>0.50</td>
<td>80</td>
<td>+0.75</td>
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<td></td>
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<tr>
<td>v. Merendonk &amp; v. Roermond</td>
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<td>—</td>
<td>—</td>
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<td></td>
<td>0.50</td>
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<td>Klein</td>
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<td></td>
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<td>50</td>
<td>0</td>
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<td>1.9</td>
<td>0.21</td>
<td>—</td>
<td></td>
<td>150</td>
<td>50</td>
<td>0</td>
<td>x</td>
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<tr>
<td>DHL (a)</td>
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<td>0.10</td>
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<td>0.40</td>
<td>120</td>
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<td>170</td>
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<td>-0.50</td>
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<tr>
<td>DHL (b)</td>
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<td>—</td>
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<td>158</td>
<td>75</td>
<td>0</td>
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</table>

Table 3.1, Previous investigations
Below a short description is given of each research study:

Kjeldsen (1974)

Kjeldsen has carried out experiments with uniform current \( I_5 \). Probably to fulfill scale relations as good as possible, he has used very fine sediment with mean grain diameter \( D_{50} = 74 \, \mu m \). Main objective of his experiment series was to find a functional relation between the dimensionless parameters \( E = \frac{\bar{v}}{\sqrt{2gk_c}} \) and \( F = \frac{\bar{v}}{\sqrt{g\phi}} \).
The results of his experiments are plotted on double logarithmic scale in Fig. 3.4. In fact the parameter \( E \) is a dimensionless Euler number whereas the parameter \( F \) is a dimensionless Froude number. With a regression method he found:

\[
E = 0.972 \, F^{0.80} \tag{3.6}
\]

which is equal to

\[
k_c = 0.972 \left( \frac{v}{2g} \right)^{0.2} \phi^0.8 \tag{3.7}
\]
in which 0.972 is a dimensionless coefficient.

Fig. 3.4.
Results of the experiments by Kjeldsen
Van Ast and De Boer (1973)

Van Ast and De Boer carried out experiments with current, with waves and a combination of both [11]. In the experiments with only current or waves the relative pipe height above the bottom, $d_0/\phi$, has been varied between -0.75 and 1.00 whereupon the corresponding relative scour depth $k_e/\phi$ has been plotted as a function of it, which is shown in Fig. 3.5 for the experiments with current and in Fig. 3.6 for the experiments with waves. Moreover, they developed a method to calculate the scour depth $k_e$ due to current or waves based on continuity of the flow underneath the pipe, but the results of this method do not agree very well with the measured values.

In their experiments with both current and waves they have investigated the relation between the scour depth parameter $k_e/k_{ec'}$, in which $k_{ec}$ is the value of the scour depth due to the corresponding current solely, and the velocity parameter $Q_b/\bar{v}$. Fig. 3.7 shows the results of those experiments, which indicate the decrease of the scour depth when increasing the orbital velocity $Q_b$ for constant current velocity $\bar{v}$.

Fig. 3.7.

Results of the experiments with current and waves by Van Ast and De Boer
Jansen (1981)

In his experiments with current Jansen investigated the flow pattern around an uncovered pipe. Comparison of his laboratory measurements with potential flow obtained with an electric analogon indicated the weakness of the potential theory because of its neglect of the friction of the flow by the bottom and the pipe [41]. From his flow pattern measurements he calculated the discharge upstream of the pipe below the level of half the pipe height, and the discharge underneath the pipe. The ratio of those two discharges has been compared to the relative pipe height above the bottom \( d_0/\phi \), see Fig. 3.8. Finally he measured the scour underneath the pipes from which he concluded an increasing value of \( k_e \) for increasing velocity \( \bar{v} \) with a maximum of 0.4 times the pipe diameter.
Van Meerendonk and Van Roermund (1981)

With their experiments with uniform current the investigators have tried to predict the scour depth $k_p$ underneath the pipe. To do this they have measured the vertical velocity profile underneath the pipe and the sediment transport upstream of the pipe. By assuming the transport capacity underneath the pipe equal to the transport capacity upstream of the pipe in an equilibrium situation, they have tried to find a relation between the mean velocity $v_p$ and the transport capacity $S_p$ underneath the pipe. Knowing the mean velocity and the transport capacity underneath the pipe, they have calculated the corresponding scour depth. These calculated scour depths were not in agreement with the measured scour depths, mainly due to the fact that the mean velocity underneath the pipe was not the relevant velocity parameter to connect with the transport capacity.

Because of this disagreement they have tried another approach, using the criterium of the bottom shear stress $\tau_b$ being equal to the critical shear stress $\tau_{cr}$. To calculate the bottom shear stress the flow pattern around the pipe is needed, which depends among other things on the height of the gap underneath the pipe. Unfortunately this gap height is still unknown and the distribution of the discharge upstream of the pipe into a part which flows over and another part which flows underneath the pipe is difficult to determine for random boundary conditions. So again this method did not lead to the desired prediction of the scour depth underneath the pipe.

The relative maximum scour depth $k_e/\phi$ did increase a little when increasing the mean velocity ($v = 0.17; v = 0.20$ and $v = 0.25$ m/s have been used) but did not exceed the value 0.5 in 1981.
Klein Breteler (1982)

Klein Breteler has carried out wave experiments with varying wave height, wave period, water depth and pipe height above the bottom. He found a standard scour configuration which he used as boundary conditions in a numerical model. In this model he calculates the flow pattern around a pipe assuming potential flow. Comparison of those calculations with the measured flow pattern resulted in only 10% deviation in the areas without eddies.

Remarkable was the fact that for all wave conditions he found a value 0.34 m/s for the maximum positive orbital velocity underneath the pipe, \( Q_p \). Using this value he developed an empirical iterative method to calculate the scour depth \( k_e \) due to waves [6]. It is a great pity he did not vary the pipe diameter nor the mean grain diameter during his experiments.
4. **Description of the experiments.**

4.1. **The small flume**

The first series of experiments has been carried out in the so called "Leidse goot", a small flume in the laboratory for fluid mechanics at the Delft University of Technology. Fig 4.1. shows this flume during an experiment.

This flume, with a cross-section of $0.5 \times 0.5 \text{ m}^2$ and a length of 14 m, has glass windows. On one side of the flume a translating wave board has been situated which generates regular waves of which the period $T$ as well as the wave height $H$ can be adjusted.

In view of the properties of the flume with respect to the wave propagation the maximum adjustable wave period amounts to 2 s and in view of the height of the flume the maximum adjustable wave height is about 80 mm.

If the water depth is sufficient the generated waves can be described by linear or cnoidal wave theory. A period longer than 2 s in this flume leads to two clearly visible disturbances:

- the amplitude of the second-order wave generated by the wave board cannot be neglected anymore with respect to the first order wave
- On the far end of the flume in spite of a wave absorber a part of the incoming wave is reflected.

To prevent reflection as much as possible a permeable slope 1:5 has been installed on which the waves will break.

An inlet in the bottom of the first part of the flume is connected with the pumping system of the laboratory. On behalf of experiments with current the maximum adjustable discharge amounts to $0.120 \text{ m}^3/\text{s}$. Because of the irregularly distributed inflow into the flume it was necessary to place a perforated board in the first part of the flume to obtain a more symmetrical current profile in the direction
perpendicular to the flume's axis. On the far end of the flume the water flows through the permeable slope and over an adjustable spill-way. With the adjustable spill-way and the discharge valve any combination of water height and water velocity can be obtained.

Over a length of about 10 m and a height of about 0.07 m a layer of sand has been placed with $D_{50} = 700 \mu m$. This mean grain size is fairly high relative to the grain sizes found at the North Sea which range from 150 to 250 $\mu m$. The advantage of using this particular grain size is that the movement of a single grain can be observed visually which gives a lot of information about the flow pattern in the vicinity of the pipe. By moving a scraper over the horizontally attached rails on the flume the sand bed can be flattened. The first part of the sand bed has a gentle slope in order to facilitate the establishment of the vertical current profile.

Beyond the flume measuring instruments had been placed to measure the changes of the levels of the local water table and sand bed. A summing-up is made of the measuring instruments used during the experiments:

- Wave gauge
- Laser-Doppler velocity meter
- Bottom profiler
- Several low-pass filters
- X-t recorders
- Micro-propeller current meter

Model pipelines made of PVC or perspex have been used with the following diameters:

25 30 40 50 63 70 80 90 mm

Two pipes with diameter 50 mm were available: a smooth one and another one which had been roughened with the sand from the sand bed. This has been done to investigate the influence of the pipe roughness on the scour process.
The length of the pipes was equal to the width of the flume so that they could be fixed between the glass windows of the flume. In this manner the pipe was always perpendicular to the direction of the current or the propagation of the waves.

4.1.1. Wave experiments

The wave experiments in the small flume have been carried out from February until the beginning of April 1983. The aim of the experiments was to repeat the experiments of Klein Breteler [6] using sand with a mean grain diameter $D_{50} = 700 \ \mu m$ in the present tests instead of $D_{50} = 150 \ \mu m$ to investigate the influence of the mean grain diameter on the scour process. Moreover the influence of the pipe diameter has been a point of research. To make a connection with the experiments of Klein Breteler the same water height $h = 250 \ \text{mm}$ has been used as in his experiments. Besides the pipe diameter the independent wave parameters $H$ and $T$ have been varied during the tests:

<table>
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<th>Wave period $T$</th>
<th>1.0</th>
<th>1.5</th>
<th>1.7</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave height $H$</td>
<td>40</td>
<td>55</td>
<td>80</td>
<td>mm</td>
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</tbody>
</table>

So except for the experiments with the roughened pipe a number of $N_{\phi} \times N_T \times N_H = 8 \times 3 \times 3 = 72$ experiments have been carried out in this series.

In the beginning of each experiment first the sand bed had to be flattened whereupon the model pipe could be fixed between the glass windows in such a manner that the bottom of the pipe just touched the sand bed (pipe height $d_0 = 0$). Before starting the experiment the wave board had been adjusted in order to obtain the desired wave period and wave height at
the spot where the pipe would be fixed ( L = 8.00 m ).

After fixing the pipe the wave board was put into operation and immediately it was visible that sediment eroded underneath the pipe. This process developed very fast in the beginning and within a few minutes it seemed that the gap height underneath the pipe did not increase anymore although sediment was still moving there and back due to the orbital water movement. In fact the development of the scour hole went by very slowly now and one or two hours have been awaited to be sure that an equilibrium scour hole had been formed. In the meantime part of the particles in suspension had settled down both just in front of and behind the pipe where they formed ripples which sometimes propagated gradually from the pipe.

Having an equilibrium scour hole, in five section lines vertical velocity profiles have been measured with the Laser-Doppler velocity meter: at \( x = -2\phi \), \( x = -\phi \), \( x = 0 \), \( x = \phi \), and \( x = 2\phi \) (see Fig. 4.2).

![Diagram](image)

**Fig. 4.2.** Section lines where velocity profiles have been measured in wave experiments

In the first instance the velocity has been measured during thirty seconds at different heights above the sand bed whereupon the mean value of the positive maximum orbital velocity \( \bar{u}_{\text{pos}} \) was taken. Because the velocity gradient just above the bottom is rather strong, the vertical distance intervals near the bed have been
taken smaller than higher up in the the velocity profile. Because this method was time consuming it was replaced by moving the Laser-Doppler velocity meter vertically with a constant velocity of 2 mm/s. In this way in a short time a sufficiently comparable result could be obtained as can be seen in Fig. 4.3.

Fig. 4.3. Two ways of obtaining a velocity profile

After these measurements the wave board was stopped, whereupon the pipe was removed in order to make a recording with the bottom profiler. By choosing the velocity of the instrumentation carrier equal to the transport velocity of the paper role of the x-t recorder and calibrate the movement of the probe of the bottom profiler on a scale 1:1, a plot on a scale 1:1 could be obtained. (Those plots and the measured velocity profiles can be found in the appendices A).

The maximum depth of the scour hole, \( k_e \), has been measured with a point gauge. Those measured values are collected in paragraph 5.1.1 where a more quantitative analysis has been carried out to find a functional relation between this maximum scour depth and the independent variables \( H, T \) and \( \phi \). In this paragraph only a few qualitative remarks are stated:

- Under identical wave conditions the maximum depth of the scour hole, \( k_e \), seems to be of the same order for all pipe diameters
- The maximum depth of the scour hole, $k_e$, increases as the wave period $T$ or the wave height $H$ increases.

- The maximum value of the positive orbital velocity underneath the pipe, $\mathcal{V}_p$, increases as the wave period $T$, the wave height $H$ or the pipe diameter $\phi$ increases.

- The influence of the roughness of the pipe on the scour depth seems to be of minor importance.

- The maximum depth of the scour hole, $k_e$, does not differ much from the values found by Klein Breteler under identical wave conditions; so the influence of the mean grain diameter $D_{50}$ seems to be negligible.

- Except for the small ripple underneath the pipe, the configuration of the scour hole was the same as observed by Klein Breteler.

4.1.2. Experiments with current

The experiments with current in the small flume have been carried out from the beginning of April until half of May 1983. The objective of the experiments was to:

- Investigate the affection of the scour process by the current velocity.

- Investigate the affection of the scour process by the pipe diameter.

- Measure flow velocities around the pipe.

- Investigate the scour process as a function of time.

Pipe diameters ranging from 25 to 70 mm have been used with a current velocity of respectively 0.25 m/s and 0.31 m/s. In all the experiments again in five section lines vertical velocity profiles have been measured.
after formation of an equilibrium scour hole: at \( x = -2\phi \), \( x = -\phi \), \( x = 0 \), and at two interesting section lines down-stream of the pipe, for example just above a ripple or at the deepest part of the scour hole. The velocity profile at \( x = -2\phi \) may be regarded as the undisturbed velocity profile. An example of the results of those measurements is given in Fig. 4.4. All results are collected in the appendices B.

\[
\begin{array}{cccccc}
2.7\phi & 1.3\phi & \phi & -\phi & -2\phi \\
\end{array}
\]

\[ v = 0.25 \]

**Fig. 4.4.**
Section lines where velocity profiles have been measured in experiments with current

Before carrying out an experiment the spill-way at the far end of the flume was adjusted in such a manner that the desired current velocity was obtained at a water depth \( h = 250 \text{ mm} \), which was measured with a point gauge. The current velocity was measured at half the water depth: \( z = 125 \text{ mm} \). Now the water inflow was stopped to flatten the sand bed and to fix a model pipe between the glass windows with its bottom just on the sand bed (pipe height \( d_0 = 0 \)). Thereupon the flow was adjusted again at a water depth of 250 mm. Near the pipe the following phenomena could be observed:

\[ v = 0.25 \text{ m/s} \]

The eddy in front of the pipe transports sediment in upstream direction from underneath the pipe so that a little gap arises. The incoming water can flow underneath the pipe as well now. Because the gap is still very small the water is accelerated sufficiently to exceed the critical shear stress of the sediment underneath the pipe.
Now the current underneath the pipe transports sediment to
the lee side of the pipe. Here the current decelerates and
the particles settle down forming a ripple. This process
continues until the gap underneath the pipe is so wide that
the critical shear stress of the sediment particles is not
exceeded anymore: the scour process has stopped and an
equilibrium scour hole has been formed. After stopping the
inflow and removing the pipe a plot of this equilibrium
scour hole was made, an example of which is given in Fig.4.5.

Fig.4.5. Example of bottom profiler plot after experiment with current

\[ v = 0.25 \text{ m/s} \]

The first part of the scour process with this velocity
is equal to that of \( v = 0.25 \text{ m/s} \), but after a while the ripple
behind the pipe is exposed to pipe induced vortex shedding.
These vortices gradually transport the ripple downstream so
that the slope behind the pipe becomes more gentle. This
apparently reduces the resistance for the current underneath
the pipe which in turn results in an increasing current and
a growing scour depth underneath the pipe.

After a few hours the top of the ripple has moved to a
dozen pipe diameters downstream of the pipe. The maximum
scour depth is not found right underneath the pipe any longer
but just, about one pipe diameter, downstream of the pipe.
The eddies coming off the pipe now attack the slope downstream
of the pipe where they keep on transporting sediment particles,
although the amount of moving particles gets smaller and
smaller. After about six or seven hours only a very slow propagation of the top of the ripple can still be observed.

This situation has been regarded as the equilibrium scour hole because the maximum scour depth and the scour depth underneath the pipe did no longer increase.

The inflow was stopped, the pipe was removed and a plot was made of this equilibrium scour hole with the bottom profiler. An example is given in Fig. 4.6, all plots and current profiles are collected in the appendices B.

Fig. 4.6. Example of bottom profiler plot after experiment with current, \( v = 0.31 \text{ m/s} \)

Especially in the latter experiments it was interesting to investigate the scour process as a function of time. To do this, the probe of the bottom profiler has been bended to make it easier to measure underneath the pipe, too, while continuing the experiment, but only when a sufficiently deep scour hole had been developed, Fig. 4.7.

Fig. 4.7. Bottom profiler with bended probe
By repeating the measurements at certain time intervals a rather good impression could be made of the scour process as a function of time, as can be seen in Fig. 4.8:

Fig. 4.8. Scour depth $k_e$ as a function of time for different pipe diameters $\phi$ (in mm)
Moreover some parameters as $k_e$ and $k_p$ as a function of time can be analysed to make an extrapolation of one of these parameters in the equilibrium situation. In earlier publications, for example van Meerendonk and van Roermund [8], it has been indicated that these parameters can be partly described very well as a function of time using a semi-logarithmic time function.

Finally we can draw the following conclusions from the experiments with current:

- The depth of the scour hole right underneath the pipe, $k_p$, as well as the maximum depth of the scour hole, $k_e$, increases when the pipe diameter or the velocity increases. For $v = 0.25$ m/s the depth of the scour hole right underneath the pipe is the maximum depth of the scour hole.

- The maximum current velocity underneath the pipe increases when the pipe diameter increases.

- The configuration of the scour hole with $v = 0.25$ m/s differs from the one observed with $v = 0.31$ m/s.

All results of these experiments with current can be found in 5.2.1.

4.1.3. Experiments with current and waves

A more general approach of prototype conditions has been tried to achieve by superimposing waves on the current after an equilibrium scour hole due to this current had been formed. Because the number of these experiments was very small, a more qualitative than quantitative approach of the results is discussed in this paragraph.

The most interesting experiment was the one in which after the establishment of an equilibrium scour hole due to the current, a wave with a relatively small wave height was superimposed on the current and when a new equilibrium scour hole had been established the wave height was increased.
This procedure was repeated several times until the maximum possible wave height in the flume was reached.

The effect of superimposing a wave with a small wave height on the current was an increase of the erosion of the scour hole. Further increase of the wave height at last resulted in sedimentation of the scour hole which can be explained by the beginning of sediment transport upstream of the pipe. Apparently this sediment transport at last exceeded the sediment transport capacity underneath the pipe. When the wave height was still very small there was not any transport at all in front of the pipe. A sketch of this phenomenon is drawn in Fig.4.9:

![Fig. 4.9. The influence of the sand transport on the scour process](image)

Anyhow it is clear that as long as no sediment transport is present upstream of the pipe, the scour hole will show continuing erosion when the wave height is increased. If a sediment transport is present upstream of the pipe, it depends on the difference between $S$ and $S_p$ (see Fig.4.9): if the scour hole will erode or not:

- $S > S_p$ sedimentation
- $S = S_p$ equilibrium
- $S < S_p$ erosion
4.1.4. Complementary experiments

Complementary experiments have been carried out which differed from the experiments as described in the first three paragraphs of this chapter. The aim of these complementary experiments was to make a better approach of prototype conditions. First the influence on the scour process of the initial pipe height above the bottom, \( d_0 \), has been investigated in wave experiments. The second experiment was to investigate the influence on the scour process of the sagging of the pipe into its own scour hole. These two experiments will be discussed briefly below.

The pipe height \( d_0 \) above the original bottom

When starting the experiments it was assumed that an initial pipe height above the bottom \( d_0 = 0 \) would be the best defined as well as the most important situation to investigate, because in that situation the greatest depth of the scour hole could be expected. To verify this assumption, some experiments have been carried out with initial pipe heights \( d_0 > 0 \).

Having a pipe height \( d_0 = 0 \), depths of the scour hole have been observed which were greater than the depths of the scour holes found under the same wave conditions with \( d_0 = 0 \). This scour hole was found at the luff side of the pipe. After this had been observed the pipe was lowered step by step which resulted in the vanishing of the scour hole at the luff side of the pipe and finally in the establishment of the well known scour hole when reaching \( d_0 = 0 \) (see Fig. 4.10).

Moreover an experiment has been carried out with a partly buried pipe, \( d_0 < 0 \). In that case the eddies in front of and behind the pipe transported sediment in the direction of the pipe. This sediment formed a slope against the pipe, as can be seen in Fig. 4.11.

So for a scour process to start it is necessary that an initial gap is present.
Fig. 4.10. Influence of the pipe height relative to the original bottom, $d_0$, on the scour process.

VII
$d_0=12.5$ mm

T = 1.7 m
H = 55 mm
h = 250 mm
\phi = 50 mm

Fig. 4.11.
Result of wave experiment with partly buried pipe.
The sagging of the pipe in its own scour hole

A disadvantage of the laboratory experiments is that the fixation of the pipe between the glass windows hinders the sagging of the pipe, due to its submerged weight, into its scour hole. Because it is very difficult, if not impossible, to scale the scour process in the proper way, it would enlarge the scale problems if the sagging of the pipe should be included in the laboratory experiment. In order to scale the velocity of the sagging of the pipe in the proper way it would be necessary to scale for example the bending stiffness of the pipe in the proper way. Moreover, the used flume is not wide enough to carry out such experiments which should rather be done in a basin than in a flume.

Still to get an impression of the effect of the sagging of the pipe, some experiments have been carried out during which the pipe was lowered over a distance of half the gap height after reaching the equilibrium scour hole. This procedure was continued until the scour hole did not erode anymore or until the scour hole was filled up by the caving in of the slopes.

These experiments on sagging pipes have been carried out with waves of which the period was 1.7 s having a wave height of 55 mm. Pipe diameters of respectively 50, 70 and 80 mm were used.

The lowering of the pipe resulted in a deepening of the scour hole whereas the slopes of the scour hole became steeper. At the moment that the flow lines could not follow those steep slopes anymore, the scour hole filled up so that at the end of the experiment the pipe was partly buried. The total sagging of the pipe was about 30 % of the pipe diameter. This meant that the total sagging of the pipe increased with the pipe diameter which was not the case for the depth of the scour hole with initial pipe height $d_0 = 0$. Probably this can be explained by the higher orbital velocities underneath the pipe and the more gentle slopes of the
scour hole when a larger pipe diameter was used. For $\phi = 80$ mm the lowering of the pipe resulted in a total sagging of two times the value of the depth of the scour hole with initial pipe height $d_0 = 0$. However, the complete self burial of the pipe into the sand bed, as shown in some prototype surveys on North Sea pipelines, has not been achieved.

4.1.5. Wave experiments with fine sediment, $D_{50} = 100 \mu m$

As an introduction to the experiments in the Research flume, some experiments have been carried out in the small flume with sand with a very small mean grain diameter: $D_{50} = 100 \mu m$. The procedure of the experiments was the same as in the previous wave experiments, let alone that in these experiments the pipe diameter $\phi$, the wave period $T$, and the maximum orbital velocity near the bottom, $\Omega_b$, have been varied in stead of $\phi$, $T$ and the wave height $H$.

It was remarkable that the depth of the scour hole now was dependable on the pipe diameter for the same wave conditions. The influence of the wave period $T$ seemed to be small but indistinct, for constant $\Omega_b$ and $\phi$.

The choice of $\Omega_b$ as a parameter in stead of $H$ appeared to be a good one because of the fact that the maximum depth of the scour hole, $k_e$, seemed to be linear with $\Omega_b$. The more extensive experiment series in the Research flume will probably give a more decisive answer on that subject.

The results of the experiments in the small flume with $D_{50} = 100 \mu m$ can be found in 5.1.2, where it has been tried to find a functional relation between the maximum depth of the scour hole, $k_e$, and the variables $\phi$, $T$ and $\Omega_b$. The plots of the bottom configuration made with the bottom profiler can be found in the appendices C.
4.2. The Research flume

The Research flume in the laboratory for fluid mechanics is a steel framed structure with glass windows. The dimensions of the flume are:

\[ l = 46.5 \text{ m} \]
\[ b = 0.8 \text{ m} \]
\[ h = 1.0 \text{ m} \]

The flume is equipped with a piston type wave generator which can generate both regular as well as irregular waves, and a pipe system for the adjustment of unidirectional current.

For the experiments with current it was necessary to place a flow rectifier in the first part of the flume in order to rectify the turbulent inflow into the flume. The discharge was controlled by an electric valve, the maximum discharge rate being \(0.5 \text{ m}^3/\text{s}\). For the wave experiments some wire mesh screens were hung into the flume in front of the wave board not only to absorb disturbances on the wave just generated, but also to extinguish the reflected wave from the other end of the flume. On that side of the flume a beach type absorber was installed of which the slope could easily be varied in order to obtain reflection as less as possible. For the experiments with current on the far end of the flume a spill-way was created by placing wooden boards vertically into grooves in the wall of the flume.

Three carriers were placed on rails on top of the flume. One of those was able to ride remote controlled with a velocity that could be adjusted from 0 up to 250 mm/s. Another carrier had been equipped with an elevator which moved vertically with a constant velocity of 4 mm/s. This elevator was used for the Laser-Doppler current meter in order to obtain vertical velocity profiles in a simple way.

A disadvantage of the rails on top of the flume was that they were situated in the cross section of the flume which hindered the installing of the model pipes into the flume. This problem has been solved by making a hatch in one
of the windows of the flume which happened to be made of perspex. Through this hatch the model pipes could be brought into the flume underneath the rails. Another problem was the fixation of the model pipes between the glass windows. To avoid high stresses in these windows when raising the water level, they had been layed in elastic kit on the rims of the steel structure of the flume. Now when raising the water level, the windows are pushed into the layer of elastic kit by the hydrostatic pressure. In this way the flume widens over a few millimeters when raising the water level to the top of the flume. Especially under wave conditions, with a varying water level, it was not sufficient to fix the pipe by simply deviate it slightly from the position perpendicular to the flume's axis because the forces on the pipe due to the wave action were high enough to loosen the pipe. This has been solved by supplying all model pipes with screw thread so that the pipes could be turned tight between the glass windows having a slightly higher water level than the water level during the experiment to follow. After this had been done the water level was lowered slowly to the desired water level for the experiment to follow without causing scour underneath the just fixed pipe.

In Fig. 4.12 a sketch has been made of the set up of an experiment in the Research flume.

4.2.1. Experiments with current

The experiments with current in the Research flume have been carried out from the beginning of May until half of June 1983. The aim of this experiment series was to find the relation between the maximum depth of the scour hole, \( k_e \), the current velocity \( v \) and the pipe diameter \( \phi \). Moreover, this series would make it possible to compare its results with those obtained in the small flume thus introducing another parameter: the mean grain diameter \( D_{50} \).

The bottom of the flume has been covered with a layer of sand (\( D_{50} = 100 \mu m \)) with a thickness of 0.3 m. The length
Fig. 4.12. The Research flume
of the layer was about 15 m. On both sides of the layer a slope 1:7 was created. The last 5 or 6 meters of the flume were kept uncovered to try to decrease the loss of sediment.

The experiments have been carried out with model pipe diameters ranging from 15 up to 180 mm with current velocities of respectively 0.20, 0.30, 0.35 and 0.40 m/s. In all experiments the water height $h$ was 0.66 m. During an experiment a few pipes were used simultaneously by fixing them at such a distance from each other that it was measurable that the current profile in front of a pipe was not disturbed anymore by a pipe upstream of this pipe. In this way it was sometimes possible to carry out an experiment with four pipes simultaneously (with $v = 0.20$ m/s).

The experiments were continued until it was observed that the configuration of the scour hole did not change anymore. For the higher velocities this lasted about 20 hours which made it necessary to continue these experiments through the night. During these periods the scour process could not be followed but one may assume that the ultimate scour hole has developed regularly as indicated in Fig. 4.13:

![Fig. 4.13. Development of the scour hole due to current](image)

The figure shows the scour hole for several time intervals. The configuration of the scour holes have been copied from lines which were drawn on the glass window. Because the scour depth was not always the same broadwise, an average line has been drawn. The various scour processes, observed
at the several velocities, will be discussed below.

\[ v = 0.20 \text{ m/s} \]

At this velocity no or hardly no sediment transport on the undisturbed sand bed was present. Only tunnel erosion was observed underneath the pipe. The maximum depth of the scour hole, situated right underneath the pipe, varied from 20% to 60% of the pipe diameter. Because the flow rectifier had not been installed yet in the first experiments, in some cases the scour hole showed broadwise variations.

It took about 2½ hours until an equilibrium scour hole had been formed (Fig.4.13, line 2).

\[ v = 0.30 ; 0.35 ; 0.40 \text{ m/s} \]

With these velocities some undisturbed sediment transport was observed and on the bottom a ripple pattern developed. In the beginning the scour process developed as described above for \( v = 0.20 \text{ m/s} \), but after about half an hour the ripple behind the pipe eroded whereupon the downstream slope of the scour hole began to propagate. This process went on very slowly and it took about 20 hours before an equilibrium scour hole had been formed, see Fig.4.13, ultimate situation.

The higher the velocity of the undisturbed water, the larger the erosion underneath the pipe. The maximum depth of the scour hole was situated just behind (about 1 pipe diameter) the pipe, having values ranging from 40% up to 80% of the pipe diameter. In the equilibrium situation the slope upstream of and underneath the pipe was smooth whilst it was rippled downstream of the pipe.

Because the motor of the carrier of the bottom profiler was out of order it was not possible to make plots of the scour holes. In stead with 5 point gauges, placed broadwise on the carrier, the maximum depth of the scour hole has been measured. In some of the experiments the maximum depths near to the glass windows differed from the depths in the middle of the flume. This may be caused by the so called "wall effect": the undisturbed velocity near a glass window is less than the velocity
in the middle of the flume due to the friction of the flow by the glass window. Because of this lower velocity the sand transport capacity is lower as well. When the flow reaches the pipe it is accelerated just above and underneath it. Because the accelerating flow underneath the pipe will be relatively little affected by friction, because of its "prop" like distribution, see Fig. 4.14, the transport capacity near the window will be higher than in the middle of the flume which will result in more erosion as well. Indeed this was observed in the flume in most of the experiments: the scour hole near a window was deeper than in the middle of the flume.

![Fig. 4.14. Top view of the wall effect](image)

The scour depths near the glass windows which differed much from the depths in the middle of the flume have not been used for the determination of the average maximum scour depth. The results of this experiment series are collected in 5.2.2.

From these results the desire grew to carry out an experiment with a higher velocity than used in the experiments described above. In view of the loss of sediment to be expected, the carrying out of this experiment has been delayed until the end of the experiments. In this experiment a pipe diameter $\Phi$ 160 mm has been used and an undisturbed velocity $v = 0.67$ m/s. After 40 minutes the velocity has been increased to $0.80$ m/s. These velocities have been measured with a micro propeller current meter in stead of the Laser-Doppler current meter.
because of the high amount of sediment in suspension which made the use of the Laser-Doppler current meter impossible.

At different time intervals photographs of the scour hole near the glass window have been made. From these photographs Fig. 4.15 could be made in which the scour process can be seen as a function of time. Although the earlier described "wall effect" will have played a part in the scour process near the windows, this figure gives a good impression of the nature of the scour process. Remarkable is the fact that the equilibrium scour depth underneath the pipe is reached within a few minutes, whereupon only erosion downstream of the pipe takes place (so called lee erosion).

After carrying out all the experiments with current a question arose:

- Is the velocity, measured at half the water depth, the proper velocity parameter to describe the scour process? Would it not be more likely to use the undisturbed velocity at a height in the order of a pipe diameter above the bottom? That would also make it easier to compare the experiments with current with the wave experiments in which the flow velocity parameter is the maximum orbital velocity near the bottom.

For comparison purpose graphs are made in which the relative current velocity \( v/\bar{v} \) is given as a function of the relative height above the bottom \( z/\delta \). This has been done for prototype conditions as well as for laboratory conditions. Assuming a logarithmic vertical velocity profile, we find:

\[
\frac{v(z)}{\bar{v}} = \frac{\sqrt{g}}{C} \ln \left( \frac{z}{z_0} \right) \tag{4.1}
\]

with

\( \kappa = \) Von Karmann constant = 0.4
\( \bar{v} = \) mean current velocity
\( g = \) acceleration of gravity
Fig. 4.15. Development of the scour hole due to uniform current ($v = 0.67 \text{ m/s}$)
The scour depth underneath the pipe $k_p$ is reached after 5 minutes.
Thereafter only lee erosion takes place.
$\phi = 160 \text{ mm}$
$h = 660 \text{ mm}$
$D_{50} = 100 \mu\text{m}$
C = Chézy coefficient of roughness
z = height above bottom
$z_0$ = elevation at which $v(z)$ equals zero

We assume $z_0 = \frac{r}{33}$ and $C = 18 \log(\frac{12}{r} h)$

with

$h$ = water height
$r$ = equivalent Nikuradse bottom roughness

The bottom roughness is often assumed

$r = 3D_{90} + 1.1\Delta' \cdot (1 - \exp(-25\Delta'/\lambda'))$

with $\Delta'$ the ripple height and $\lambda'$ its wave length.

Combining all this results in:

$$\frac{v(z)}{v} = \frac{\sqrt{g}}{18\log(\frac{12}{r} h)} \ln\left(\frac{33}{r} \frac{z}{r}\right) \quad (4.2)$$

For prototype conditions we assume for example:

$\Delta' = 0.10$ m
$\lambda' = 0.70$ m
so

$r = 3D_{90} + 1.1 \cdot 0.1(1 - \exp(-25.0.1/0.7)) \approx 0.110$ m

$h = 30$ m
$\phi = 1$ m

For laboratory conditions we assume:

$\Delta' = 0.01$ m
$\lambda' = 0.10$ m
so

$r = 3D_{90} + 1.1 \cdot 0.01(1 - \exp(-25 \cdot 0.01/0.10)) \approx 0.011$ m

$h = 0.60$ m
$\phi = 0.10$ m

When we use these values in equation 4.2 we find:

prototype : $\frac{v(z)}{v} = 0.124 \ln(300z) \quad (4.3)$

laboratory : $\frac{v(z)}{v} = 0.155 \ln(3000z) \quad (4.4)$
In Fig. 4.16 both profiles are shown together with a registration of a current profile made with the Laser-Doppler current meter in the Research flume. From this Figure we learn that at $z = \frac{1}{2} \Phi$ the relative velocity in the laboratory is some 20% higher than in prototype.

![Graph](image)

I : prototype logarithmic  
II : laboratory logarithmic  
III : laboratory measurement

Fig. 4.16. The relative current velocity $\frac{v}{\bar{v}}$ as a function of the relative height $\frac{z}{\Phi}$

So in the future we should better choose the velocity parameter at a height in the order of the pipe diameter above the bottom.

From all the experiments with current we can draw the following conclusions:

- The maximum depth of the scour hole increases when the undisturbed velocity $v$ or the pipe diameter $\Phi$ increases

- The velocity $v = 0.20$ m/s only causes tunnel erosion. The higher velocities show the beginning of lee erosion
- The process "tunnel erosion" takes a time in the order of 10 minutes; the process "lee erosion" takes a time in the order of 20 hours

- Unlike the experiments in the small flume with sand with $D_{50} = 700$ μm, for the higher velocities no ripple is found downstream of the pipe. The eroded sediment coming from underneath the pipe gets in suspension and is carried far from the pipe

4.2.2. Wave experiments with $D_{50} = 100$ μm

The wave experiments in the Research flume have been carried out from October until December 1983. The aim was to investigate the relation between the maximum scour depth $k_e$ on the one hand and the maximum orbital velocity near the bottom $\bar{u}_b$, the wave period $T$ and the pipe diameter $\phi$ on the other hand. By comparing the results of this experiment series with the series in the small flume with $D_{50} = 700$ μm the parameter $D_{50}$ could be taken into account as well.

In view of the local character of the scour process under wave conditions, the parameter $\bar{u}_b$ is used in stead of $H$(wave height) as was done in the previous series in the small flume. This has been done as well because the orbital velocity near the bottom makes it easy to relate it eventually to some sediment transport description. In these experiments the following values of the variables have been used:

<table>
<thead>
<tr>
<th>$\phi$ (mm)</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>110</th>
<th>160</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ (s)</td>
<td>1.5</td>
<td>1.7</td>
<td>2.4</td>
<td>3.0</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>$\bar{u}_b$ (m/s)</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Almost all combinations of the above variables have been used in the experiments. In principle the experiments have been carried out as follows:
After flattening the bottom with a scraper the desired wave period $T$ was adjusted whereupon the desired orbital velocity near the bottom was measured with the Laser-Doppler current meter. This was done by varying the wave height. When this had been done it was tried to find other places along the flume's axis with identical registrations of the orbital velocity near the bottom. Especially with the longer wave periods this was difficult, because the second harmonic of the generated wave by the wave board has got a relatively high amplitude which varies the wave height and thus the orbital velocity near the bottom along the flume's axis, see Fig. 4.17. If these places were found it was possible to carry out an experiment with several pipes, provided that one could assume that the disturbance in the orbital velocity and the sand bottom caused by a pipe did not influence those around another pipe. In practice the distance between the pipes was always in the order of 15 times the maximum orbital excursion $a_b = \varphi_b T / 2\pi$, which should guarantee the validity of the assumption made above.

After this procedure the pipes were fixed between the glass windows and the wave generator was put into operation. After about 2 hours an equilibrium scour hole had been developed underneath all the pipes whereupon the wave generator was stopped. The pipes were removed and plots were made of the scour hole with the bottom profiler. These plots are collected in the appendices D.

In 5.1.2. the results of this series are given and a functional relation between $k_e$ and $\varphi_b$, $T$ and $\phi$ has been searched for. In this paragraph only some qualitative conclusions are drawn:

- $k_e$ seems to be practically independent of $T$
- $k_e$ increases when $\phi$ increases
- $k_e$ increases when $\varphi_b$ increases
- for identical wave conditions $k_e$ is larger than in the experiments with coarse sediment, $D_{50} = 700 \mu m$
Fig. 4.17. Variation of the wave height along the flume axis \((T = 3.2 \text{ s}; h = 0.50 \text{ m})\)

4.2.3. Wave experiments with \(D_{50} = 220 \text{ \mu m}\)

After the experiments as described in the previous paragraph had been carried out, the sand in the flume was replaced by sand with \(D_{50} = 220 \text{ \mu m}\). With this sand part of the previous experiments have been repeated. All combinations of the values of the variables \(a, T\) and \(\phi\) as given below have been used in this series:

<table>
<thead>
<tr>
<th>(\phi)</th>
<th>mm</th>
<th>90</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>m/s</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>(T)</td>
<td>s</td>
<td>1.7</td>
<td>3.4</td>
</tr>
</tbody>
</table>

After the experiments plots of the scour holes have been made; they are collected in the appendices E and the results can be found in 5.1.1.

During this series in some cases an optical concentration meter, developed by R. Slot in the Laboratory for Fluid Mechanics [111], was placed downstream of the 160 mm pipe. Measurements have been made of the development of the concentration during a wave period which was obtained by averaging over 64 or 128 wave periods the concentration at equal phase during one period (Fig. 4.18).
Fig. 4.18. Water elevation (top) and concentration in fixed point during a wave period at several stages of the scour process (below).

In the near future this concentration meter, used in combination with a velocity meter, may offer the opportunity to measure the sediment transport directly.

4.2.4. Experiments with current and waves

Unlike the experiments with current and waves in the small flume, where the waves were superimposed on the current after an equilibrium scour hole had been formed by the current, in this series the waves were superimposed at once. The beach type slope at the far end of the flume, installed in order to break the waves, has been used as the spill-way for the current. All experiments have been carried out with a water depth $h = 0.50$ m and a wave period $T = 3.4$ s. The maximum depths of the scour holes are given in table 4.1.

The flow velocities have been measured at a height of 0.20 m above the bottom. The error which is made with respect to the orbital velocity near the bottom will be acceptably small because the relative long wave period implies that the decrease of the maximum orbital velocity towards the bottom is small. The measurements of the maximum scour depths show that the relative increase of the scour depth by superimposing waves is larger when the velocity of the undisturbed current
is smaller.

In view of the small number of experiments and the high
dispersion in the results more investigations on this subject
should be carried out.

<table>
<thead>
<tr>
<th>v</th>
<th>θ_0</th>
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<th>90</th>
<th>160</th>
<th>180</th>
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<tbody>
<tr>
<td>0.20</td>
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<td>28</td>
<td>32</td>
<td>60</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>30</td>
<td>25</td>
<td>90</td>
<td>75</td>
</tr>
<tr>
<td>0.40</td>
<td>0.15</td>
<td></td>
<td></td>
<td>140</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td></td>
<td></td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>m/s</td>
<td>m/s</td>
<td>mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 Scour due to current and waves
h = 0.50 m
D_{50} = 100 μm

4.2.5. Complementary experiments

Just as has been done in the small flume, in the Research
flume some complementary experiments have been carried out
to make a better approach of prototype conditions, such as:

- two directional current
- sagging of the pipe in its scour hole
- gentler slopes underneath the pipe

These three experiments will be discussed below:

Two directional current

In prototype conditions it is to be expected that the
current will be mostly two directional due to the tide. This
has been imitated in the Research flume by turning the direc-
tion of the current after an equilibrium scour hole had been
formed underneath the pipe. This has been carried out with
pipe diameter $\phi = 140$ mm and a current velocity $v = 0.40$ m/s.
Finally this resulted in a symmetrical scour hole with a maxi-
mum depth which did not increase after the turning of the
direction of the current. Only the location of this maximum depth moved from just downstream of the pipe to a position right underneath the pipe (Fig. 4.19).

![Diagram showing flow directions](image)

**Fig. 4.19. Development of the scour hole due to two-directional current**

Because little is known about the morphological time scale of the scour process underneath pipelines, the turning of the direction of the current was carried out after the formation of the equilibrium scour hole due to the unidirectional current. Of course in prototype conditions the tide will not wait for this and will change its direction about every six hours.
Sagging of the pipe

After reaching the equilibrium scour hole during the standard experiments with the pipe diameters $\phi = 160$ mm and a current velocity $v = 0.40$ m/s, the pipes have been pushed down over half the gap height $e_i$. This resulted in further erosion underneath the pipe, especially at the luff side. The increase of the gap height, $\Delta e_i$, was only about 50% of the initial scour depth $e_i$. Any time a new equilibrium scour hole had been formed the pipe was pushed down over half the gap height. At last the flow lines could not follow the steep slope of the upstream part of the scour hole anymore and sedimentation took place (Fig. 4.20). The total sagging of the pipe was almost one pipe diameter.

![Diagram of sagging pipe](image)

Fig. 4.20. The method of pushing down the pipe into its scour hole during experiment with current

Under wave conditions the same procedure has been carried out. Here as well after some sagging of the pipe the stream lines could not follow the steep slope anymore and the eddies, alternatingly formed at both sides of the pipe, filled in the scour hole, see Fig. 4.21. The total sagging of the pipe was less than 50% of the pipe diameter.
These sagging experiments did not confirm the expectation that this sagging is the explanation of the lowering of two or three pipe diameters into the bottom as is suggested by some surveys of prototype pipelines. Probably the steep slopes in the laboratory are a scale effect which hinders the water to flow underneath the pipe. In prototype more gentle slopes are to be expected, due to the larger pipe diameters, which could create the possibility for the scour process to continue.

Before going on with more experiments on the subject of sagging pipes, we should get more information about the influence of this scale effect.

**Gentler slopes underneath the pipe**

In some experiments the two directional current has been imitated by making the scour hole symmetrical artificially, after the equilibrium scour hole had been formed. This was done in order to investigate the influence of the slope upstream of the pipe on the scour process. Meanwhile the pipe was pushed down half the gap height as described in the sagging experiments. The maximum depth of the scour hole obtained with this procedure was almost one pipe diameter as well.
5. Results of the scour experiments

5.1. Results of the wave experiments

5.1.1. Results of the wave experiments in the small flume with coarse sediment, \( D_{50} = 700 \mu \text{m} \)

In this paragraph the results of the wave experiments as described in 4.1.1. are treated quantitatively. The measured values of the scour depths and the wave data of the executed experiments are given below in table 5.1. From this table Fig. 5.1 and Fig. 5.2 have been made in which \( k_e \) is given as a function of \( \phi \) for constant \( H \) and constant \( T \) respectively. Finally the maximum orbital velocity underneath the pipe, \( \omega_p \), is given as a function of \( \phi \) in Fig. 5.3 and Fig. 5.4.

From these figures we can draw the following conclusions:

- \( k_e \) is practically independent of \( \phi \)
- \( \omega_p \) is linear with \( \phi \)
- \( k_e \) increases when \( H \) increases
- \( k_e \) increases when \( T \) increases

The first objective of the experiments was to describe \( k_e \) as a function of \( H, T \) and \( \phi \):

\[
k_e = f(H,T,\phi) \quad (5.1)
\]

If we indeed assume that \( k_e \) is independent of \( \phi \) and we assume a generalized exponential function \( k_e \sim H^x T^y \) we can find with a regression analysis:

\[
k_e \sim H^{1.3} T^{1.5} \quad (5.2)
\]

As described in 3.3. the sediment underneath the pipe is transported by the orbital motion towards both sides of the pipe. In view of the amount of transported sediment and
<table>
<thead>
<tr>
<th>T</th>
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<td>.45</td>
<td>4</td>
<td>8</td>
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<td>8</td>
<td>.44</td>
<td>12</td>
<td>.52</td>
<td>19</td>
<td>.64</td>
</tr>
</tbody>
</table>

Table 5.1 Results of wave experiments in the small flume

$h = 250 \text{ mm}$

$D_{50} = 700 \mu\text{m}$
**Fig. 5.1.** $k_e$ as a function of $\phi$ for constant $H$ (in mm)

- $0: T = 1.7$ s
- $+: T = 1.5$ s
- $\cdot: T = 1.0$ s

**Fig. 5.2.** $k_e$ as a function of $\phi$ for constant $T$ (in s)

- $0: H = 80$ mm
- $+: H = 55$ mm
- $\cdot: H = 40$ mm
Fig. 5.3. $\hat{u}_p$ as a function of $\phi$ for constant $H$ (in mm)
- $\circ$: $T = 1.7$ s
- $+$: $T = 1.5$ s
- $\cdot$: $T = 1.0$ s

Fig. 5.4. $\hat{u}_p$ as a function of $\phi$ for constant $T$ (in s)
- $\circ$: $H = 80$ mm
- $+$: $H = 55$ mm
- $\cdot$: $H = 40$ mm
the depth of the scour hole, the maximum orbital velocity underneath the pipe $\bar{v}_p$ and the wave period $T$ play important parts. Among other things $\bar{v}_p$ will depend on the maximum undisturbed orbital velocity near the bottom $\bar{v}_b$, which can be simply calculated using linear wave theory:

$$\bar{v}_b = \frac{\pi H}{T \sinh(kh)}$$

(5.3)

with $k = 2\pi/L$, the wave number. From equation (5.3) we learn that $\bar{v}_b$ is linear with $H$ and because in practice it is easier to measure $H$ than $\bar{v}_b$, the wave height seems to be a well chosen parameter to describe $k_e$. However, this changes when non sinusoidal waves are concerned in the scour process. As can be seen in Table 5.1 there may occur a great difference between the measured and the calculated value of $\bar{v}_b$, especially for the longer wave periods. In those cases we should use $\bar{v}_b$ as a parameter rather than $H$. This was also the reason for the use of $\bar{v}_b$ as a parameter in the experiments in the Research flume, which have been carried out after this experiment series.

Therefor from here on $\bar{v}_b$ was used as a parameter in stead of $H$. Moreover, the water depth $h$ is included in $\bar{v}_b$ as well. Specifically the maximum orbital velocity during the top of a wave, $\bar{v}_{b_t}$, is used, because it may be assumed representative for the maximum depth of the scour hole.

Meanwhile, the following remarks about the relation between $k_e$ and $\bar{v}_b$ can already be made:

It may be expected that for a certain (small) value of the orbital velocity the scour process will start. When this happens a gap underneath the pipe will be established and this gap will increase when the orbital velocity will be increased. However, there must be a physical limit to the depth of the scour hole, so we may expect the gap height to grow to about one or two pipe diameters, because the disturbance of the flow by a cylinder in potential flow reaches to two pipe diameters from the pipe, see Fig. 5.5 and Fig 5.6.
Fig. 5.5. Expected trend of \( k_e \) as a function of \( \varphi_b \).

Fig. 5.6. Disturbance of uniform flow by a cylinder according to potential theory.

Looking now at the results of the wave experiments, we may assume that these results are situated between the points A and B in Fig. 5.5. The temptation arises to approach the results by a relationship \( k_e = C_1 \varphi_b^z \), in which \( z \) would be equal to the value 1.24 using a regression method, but because of the expected trend of the rest of the curve in Fig. 5.5 the results are approached by a linear relationship:

\[
    k_e = C_2 \varphi_b + C_3 \tag{5.4}
\]

When using the averaged scour depth of all pipe diameters the method of the least squares leads to:

\[
    \bar{k}_e = 0.067 \varphi_b - 0.002 \tag{5.5}
\]

with \( \bar{k}_e \) in m and \( \varphi_b \) in m/s.

This line and the measured values in the experiment series are given in Fig. 5.7.

Because the wave period \( T \) affects \( \bar{k}_e \) as well, it has been tried to find a relation between \( \bar{k}_e \) and the maximum orbital excursion near the bottom \( a_p = \varphi_b T / 2\pi \). With the same reasoning as has been given for the relation between \( \bar{k}_e \) and \( \varphi_b \) the results have been approached by a linear relationship:
\[ \bar{k}_e = C_4 a_b + C_5 \]  \hspace{1cm} (5.6)

Again using the method of the least squares and taking the average scour depth of all pipe diameters, this leads to:

\[ \bar{k}_e = 0.21 \, a_b \]  \hspace{1cm} (5.7)

with \( \bar{k}_e \) and \( a_b \) in m. Fig. 5.8 shows equation (5.7) and the measured values of the experiment series.

---

**Fig. 5.7.** \( \bar{k}_e \) (in mm) as a function of \( a_b \) (in m/s)

---

**Fig. 5.8.** \( \bar{k}_e \) (in mm) as a function of \( a_b \) (in mm)
It was remarkable that unlike the maximum scour depth the maximum orbital velocity underneath the pipe, \( \bar{u}_p \), increased with increasing pipe diameter. The width of the scour hole increased as well with the pipe diameter, see Fig. 5.9.

![Diagram showing different widths of the scour hole](image)

**Fig. 5.9. Different width but constant depth of the scour hole for identical wave conditions**

From the Figures 5.3 and 5.4 we found the linearity between \( \bar{u}_p \) and \( \phi \). For all wave conditions this linearity

\[
\bar{u}_p = C_6 + C_7 \phi
\]  

(5.8)

has been calculated using the method of the least squares:

<table>
<thead>
<tr>
<th>T (s)</th>
<th>H (mm)</th>
<th>( \bar{u}_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>40</td>
<td>( 0.034 + 4.2\phi )</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>( 0.143 + 2.7\phi )</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>( 0.174 + 3.1\phi )</td>
</tr>
<tr>
<td>1.5</td>
<td>40</td>
<td>( \text{not measured} )</td>
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<tr>
<td></td>
<td>55</td>
<td>( 0.156 + 3.2\phi )</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>( 0.162 + 4.0\phi )</td>
</tr>
<tr>
<td>1.7</td>
<td>40</td>
<td>( 0.091 + 4.1\phi )</td>
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<td></td>
<td>55</td>
<td>( 0.142 + 4.3\phi )</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>( 0.323 + 3.8\phi )</td>
</tr>
</tbody>
</table>

with \( \bar{u}_p \) in m/s and \( \phi \) in m.

If we assume the linearity according to equation (5.8), we see that the value of \( C_6 \) is about equal to \( \bar{u}_b \) and \( C_7 \) is
a constant (with average value 3.7), so:

\[ \theta_p = \theta_b + 3.7\phi \quad (5.9) \]

with \( \theta_p \) and \( \theta_b \) in m/s and \( \phi \) in m.

5.1.2. Results of the wave experiments in the Research flume with fine sediment, \( D_{50} = 100 \mu m \)

When we observe registrations of the wave data it becomes clear that the configuration of the orbital velocity gets more and more asymmetrical when using longer wave periods. This is due to the relatively high amplitude of the second harmonic of the generated wave and the higher rate of reflection at the slope at the far end of the flume. Fig. 5.10 gives an example of two different wave registrations in which this phenomenon can be recognized.

![Fig. 5.10. Asymmetrical waves for the longer wave periods: \( T = 1.5 \) s (left); \( T = 4.0 \) s (right)](image)

If we may assume that during the top of a wave the scour underneath the pipe is caused during the acceleration of the water, we can see in Fig. 5.10 that for an asymmetrical wave this duration, \( t_{t}/2 \), is less than \( T/4 \), the equivalent
duration for a symmetrical wave. Fig. 5.11 shows the measured values of \( k_e \) as a function of the period \( T \) for constant \( \omega_b \). In almost all cases a slightly decreasing value of \( k_e \) is found for increasing \( T \). However, it is questionable if one can simply compare scour depths which were caused by waves having the same orbital velocity peak amplitude but different time histories.

Therefore for all wave conditions the value of \( t_c/T \) has been determined and this value has been used to correct the scour depths \( k_e \) by multiplying them by \( 2(1-t_c/T) \). These corrected scour depths, \( k_e^* \), can be found in table 5.2 in which all results of this experiment series are given.

Unfortunately the small intensity of the laser beam of the Laser-Doppler current meter, in combination with the large amount of sediment in suspension, made it impossible to measure flow velocities around the pipe. Nevertheless we can assume that the flow pattern near the pipe is almost the same as has been found in the small flume, although it is difficult to be specific about the values of the maximum orbital velocities underneath the pipes.

If now \( k_e^* \) is plotted as a function of \( T \) for constant \( \omega_b \), we see that for each single pipe diameter the values of \( k_e^* \) can be estimated by a horizontal straight line, which means that \( k_e^* \) is practically independent of \( T \), see Fig. 5.12. In the further course of this paragraph this will be taken into account and if necessary calculations are made with a period averaged, corrected scour depth \( k_e^* \).

For example, \( k_e^* \) has been used to determine the relation between the scour depth and the pipe diameter. In Fig. 5.13 \( k_e^* \) has been plotted as a function of the pipe diameter \( \phi \). For each single value of \( \omega_b \) the relation has been estimated by

\[
 k_e^* = c_\phi \phi^{c_9} \tag{5.10} 
\]

with \( k_e^* \) and \( \phi \) in m. Using the method of the least squares this leads to:
<table>
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<tr>
<th>$d_b$</th>
<th>$T$</th>
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<th>$k_e$</th>
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<td>mm</td>
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</tr>
</tbody>
</table>

Table 5.2 Results of wave experiments in the Research flume

$h = 500 \text{ mm}$

$D_{50} = 100 \mu \text{m}$
Fig. 5.11. $k_e$ as a function of $T$ for different $\phi$ (in mm) at constant $\rho_b$ (in m/s)
Fig. 5.12. $k_e^*$ as a function of $T$ for different $\phi$ (in mm) at constant $a_b$ (in mm)
Fig. 5.13. $K_e^*$ as a function of the pipe diameter $\phi$ for different $a_b$ (in m/s)
<table>
<thead>
<tr>
<th>$a_b$ (m/s)</th>
<th>$k_e^*$ value</th>
<th>$\phi$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>$k_e^* = 0.0615 \phi^{0.57}$</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>$k_e^* = 0.1440 \phi^{0.75}$</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>$k_e^* = 0.1740 \phi^{0.67}$</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>$k_e^* = 0.2319 \phi^{0.78}$</td>
<td></td>
</tr>
</tbody>
</table>

When we average the values found for the exponent of $\phi$, we finally come to the following conclusion:

$$k_e^* \sim \phi^{0.70}$$  \hspace{1cm} (5.12)

with $k_e^*$ and $\phi$ in m.

In Fig. 5.14 $k_e^*$ has been plotted as a function of $a_b$. For reasons, mentioned earlier in 5.1.1., the relation between $k_e^*$ and $a_b$ has been estimated by:

$$k_e^* = C_{10} + C_{11} a_b$$  \hspace{1cm} (5.13)

with $k_e^*$ in m and $a_b$ in m/s. Again using the method of the least squares we can solve $C_{10}$ and $C_{11}$ for the six different pipe diameters:

<table>
<thead>
<tr>
<th>$\phi$ (mm)</th>
<th>$k_e^*$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$k_e^* = 0.0009 + 0.0636 a_b$</td>
</tr>
<tr>
<td>50</td>
<td>$k_e^* = 0.0037 + 0.0806 a_b$</td>
</tr>
<tr>
<td>75</td>
<td>$k_e^* = -0.0083 + 0.2071 a_b$</td>
</tr>
<tr>
<td>110</td>
<td>$k_e^* = -0.0033 + 0.2024 a_b$</td>
</tr>
<tr>
<td>160</td>
<td>$k_e^* = -0.0123 + 0.3272 a_b$</td>
</tr>
<tr>
<td>180</td>
<td>$k_e^* = 0.0074 + 0.2084 a_b$</td>
</tr>
</tbody>
</table>
Fig. 5.14. $k_e^*$ as a function of $U_b$ for different $\phi$ (in mm)
Fig. 5.14 already shows that linearisation of the relation \( \overline{k}_e^* = \hat{a}_b \) will not give astonishing results. Because the correlation coefficient for the four larger pipe diameters were closer to the value 1 than for \( \phi = 25 \) and \( \phi = 50 \) mm, the average has been taken of the tangent of the lines of the four larger pipe diameters, which resulted in:

\[
\overline{k}_e^* = C_{10} + 0.2400 \hat{a}_b \quad (5.15)
\]

with \( \overline{k}_e^* \) in m and \( \hat{a}_b \) in m/s.

So for these wave experiments the assumption

\[
\overline{k}_e^* = f(\hat{a}_b, T, \phi) \quad (5.15a)
\]

has been transformed into

\[
\overline{k}_e^* = C_{12} \hat{a}_b^{1.0} \phi^{0.7} T^0 \quad (5.16)
\]

or

\[
\overline{k}_e^* = C_{12} \hat{a}_b^{1.0} \phi^{0.7} \quad (5.17)
\]

To determine the non dimensionless coefficient \( C_{12} \) in equation (5.17), a general minimalisation procedure for \( \overline{k}_e^* \) has been used: by assuming that the exponents in equation (5.17) are correct, \( C_{12} \) can be calculated according to:

\[
C_{12} = \frac{\sum_{i=1}^{n} \left( \overline{k}_{e_i}^* \hat{a}_{b_i} \phi_{i}^{0.7} \right)}{\sum_{i=1}^{n} \left( \hat{a}_{b_i} \phi_{i}^{0.7} \right)^2} \quad (5.18)
\]

Using the data from table 5.2 \( C_{12} = 0.8571 \) is found. Fig.5.15 shows the measured and the calculated values of \( \overline{k}_e^* \) in one plot. So finally the following relation is found for the wave experiments with \( D_{50} = 100 \) \( \mu \)m in the Research flume:

\[
\overline{k}_e^* = 0.8571 \hat{a}_b \phi^{0.7} \quad (5.19)
\]
Fig. 5.15. Measured values of wave period averaged, corrected scour depths $K_{eg}^*$ as a function of the corresponding values $K_{eb}^*$, according to equation (5.19)
5.1.3. Results of the wave experiments in the small flume with fine sediment, \( D_{50} = 100 \, \mu m \)

In this paragraph the same approach as in 5.1.2. has been taken with regard to the characteristics of the orbital velocity near the bottom. So in table 5.3 the measured value of the scour depth \( k_e \) as well as the corrected value \( k_e^* \) can be found:

<table>
<thead>
<tr>
<th>( \phi , mm )</th>
<th>25</th>
<th>50</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_b )</td>
<td>1.0</td>
<td>.50</td>
<td>40</td>
</tr>
<tr>
<td>( T ) 1.5</td>
<td>.44</td>
<td>35</td>
<td>4</td>
</tr>
<tr>
<td>( T ) 1.7</td>
<td>.49</td>
<td>27</td>
<td>3</td>
</tr>
<tr>
<td>( T ) 1.9</td>
<td>.43</td>
<td>38</td>
<td>1</td>
</tr>
<tr>
<td>( \bar{k}_e )</td>
<td>.10</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>( \bar{k}_e )</td>
<td>.15</td>
<td>1.0</td>
<td>.50</td>
</tr>
<tr>
<td>( \bar{k}_e )</td>
<td>.43</td>
<td>54</td>
<td>6</td>
</tr>
<tr>
<td>( \bar{k}_e )</td>
<td>.48</td>
<td>40</td>
<td>6</td>
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<td>( \bar{k}_e )</td>
<td>.40</td>
<td>55</td>
<td>4</td>
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<tr>
<td>( \bar{k}_e )</td>
<td>.20</td>
<td>1.0</td>
<td>.46</td>
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<tr>
<td>( \bar{k}_e )</td>
<td>.41</td>
<td>74</td>
<td>8</td>
</tr>
<tr>
<td>( \bar{k}_e )</td>
<td>.47</td>
<td>54</td>
<td>14</td>
</tr>
<tr>
<td>( \bar{k}_e )</td>
<td>.43</td>
<td>63</td>
<td>4</td>
</tr>
<tr>
<td>( \bar{k}_e )</td>
<td>mm</td>
<td>m/s</td>
<td>s</td>
</tr>
</tbody>
</table>

Table 5.3 Results of wave experiments in the small flume
\( h = 250 \, mm \)
\( D_{50} = 100 \, \mu m \)
In Fig. 5.16, in which \(k^*_e\) is given as a function of \(T\), again the indeterminacy between \(k^*_e\) and \(T\) is found. So in the rest of this paragraph the period averaged value \(\overline{k}^*_e\) is used in the calculations for the determination of the relations between the scour depth, the pipe diameter and the orbital velocity near the bottom. The same procedures for the calculation of these relations as in the previous paragraph have been used. This resulted in the following:

<table>
<thead>
<tr>
<th>(\theta_b) (m/s)</th>
<th>(\overline{k}^*_e) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>(0.0364 \phi^{0.57})</td>
</tr>
<tr>
<td>0.15</td>
<td>(0.0963 \phi^{0.74})</td>
</tr>
<tr>
<td>0.20</td>
<td>(0.0830 \phi^{0.59})</td>
</tr>
</tbody>
</table>

with \(\overline{k}^*_e\) and \(\phi\) in m.

These estimated lines can be found in Fig. 5.17. Finally the following is obtained, with 0.63 being the average value of the exponent in the equations (5.20):

\[
\overline{k}^*_e = C_{13} \phi^{0.63}
\]  

(5.21)

with \(\overline{k}^*_e\) and \(\phi\) in m.

![Diagram](image_url)

**Fig. 5.17**
\(\overline{k}^*_e\) as a function of \(\phi\) for different \(\theta_b\) (in m/s) and estimated lines according to equations (5.20).
Fig. 5.16. $k_e^*$ as a function of $T$ for different pipe diameters $\phi$ (in mm) at constant $U_b$ (in m/s)
For the relation $\bar{k}^*_e - \bar{u}_b$ was found:

<table>
<thead>
<tr>
<th>$\phi$ (mm)</th>
<th>$\bar{k}^*_e$</th>
<th>$\bar{u}_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$-0.0010 + 0.0050 \bar{u}_b$</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>$-0.0014 + 0.0085 \bar{u}_b$</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>$-0.0017 + 0.0101 \bar{u}_b$</td>
<td></td>
</tr>
</tbody>
</table>

with $\bar{k}^*_e$ in m and $\bar{u}_b$ in m/s. The values of $\bar{k}^*_e$ and $\bar{u}_b$ are plotted in Fig. 5.18:

![Graph showing $\bar{k}^*_e$ as a function of $\bar{u}_b$ for different pipe diameters $\phi$ (in mm)](image)

Fig. 5.18. $\bar{k}^*_e$ as a function of $\bar{u}_b$ for different pipe diameters $\phi$ (in mm)
The tangent of the lines of equations (5.22) seems to be pipe diameter dependent, but because only three pipe diameters have been used the average value of the tangent is taken. This leads to:

$$\bar{k}_e^* = C_{14} (0.0078) \bar{a}_b$$  \hspace{1cm} (5.23)

with \(\bar{k}_e^*\) in m and \(\bar{a}_b\) in m/s. So the outcome of the assumption

$$\bar{k}_e^* = f(\bar{a}_b, T, \phi)$$  \hspace{1cm} (5.24)

is:

$$\bar{k}_e^* = C_{15} \bar{a}_b^{1.0} \phi^{0.63} T^0$$  \hspace{1cm} (5.25)

or:

$$\bar{k}_e^* = C_{15} \bar{a}_b^{1.0} \phi^{0.63}$$  \hspace{1cm} (5.26)

Again the non dimensionless coefficient \(C_{15}\) has been calculated according to equation (5.18) which results in:

$$\bar{k}_e^* = 0.4634 \bar{a}_b \phi^{0.63}$$  \hspace{1cm} (5.27)

It must be said that equation (5.27) is only valid in the area in which the measurements have been carried out.

5.1.4. Results of the wave experiments in the Research flume with \(D_{50} = 220 \mu m\)

This relatively restricted experiment series has been carried out in order to investigate the influence of the mean grain diameter \(D_{50}\) on the scour process. The results are collected in table 5.4 in which one can find again the corrected scour depth \(k_e^*\) and the values of the orbital excursion near the bottom during the top of a wave:

$$a_{b_t} = (\bar{a}_b/\pi) \cdot t_{t}$$ which equals \((\bar{a}_b/\pi) \cdot T/2\) for a sinusoidal wave.
<table>
<thead>
<tr>
<th>$a_b$</th>
<th>T</th>
<th>$t_t/T$</th>
<th>$a_{bt}$</th>
<th>$k_e$</th>
<th>$k^*_e$</th>
<th>$k_e$</th>
<th>$k^*_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.15</td>
<td>1.7</td>
<td>.49</td>
<td>40</td>
<td>10</td>
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<td>mm</td>
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<td>mm</td>
<td>mm</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 5.4 Results of wave experiments in the Research flume

$h = 500$ mm

$D_{50} = 220$ μm

$a_{bt} = (a_b \cdot t_t) / \pi$

When we study the results it appears that the influence of the pipe diameter on the scour depth is less compared with the experiments with $D_{50} = 100$ μm, see Fig. 5.19. For the wave period $T = 3.4$ s this influence seems to be of more importance than for $T = 1.7$ s. Because both the wave period $T$ and the orbital velocity $a_b$, respectively to be found in Fig. 5.21 and 5.20, play important parts in the scour process, $k_e^*$ has been plotted as a function of the maximum orbital excursion near the bottom $a_b$, Fig. 5.22. For the same reason as mentioned in 5.1.1., the functional relation $k_e^* - a_b$ has been estimated by a linear relationship. Using the method of the least squares the following is found for both pipe diameters:
Fig. 5.19. $k_e^*$ as a function of $\phi$ for different $\theta_b$ (in mm) and $T$ (in s).

Fig. 5.20. $k_e^*$ as a function of $\theta_b$ for different $\phi$ (in mm) and $T$ (in s).
Fig. 5.21. $k_e^*$ as a function of $T$ for different $\phi$ (in mm) and $\hat{a}_b$ (in m/s)

Fig. 5.22. $k_e^*$ as a function of $a_b$ for different $\phi$ (in mm)
\[ \phi = 90 \text{ mm} : \quad k^*_e = -0.0005 + 0.26 a_b \]  
\[ \phi = 160 \text{ mm} : \quad k^*_e = -0.0049 + 0.35 a_b \]  
(5.28)

with \( k^*_e \) and \( a_b \) in m. The results of these calculations can be compared fairly well with the relation found with the experiments with \( D_{50} = 700 \mu m \) but are in disharmony with the experiments with \( D_{50} = 100 \mu m \). In the next paragraph this disharmony of the results for the different grain sizes will be discussed.

5.1.5. The influence of the mean grain size \( D_{50} \) on the scour process

Summary of the results of the wave experiments

In this paragraph the four series of wave experiments are compared with each other in order to try to introduce the grain size into the functional description of the scour depth:

\[ k^*_e = f(a_b, T, \phi, D_{50}) \]  
(5.29)

(Because the characteristics of the orbital velocity have not been measured properly in the experiments in the small flume with \( D_{50} = 700 \mu m \), there \( k_e \) is used in the calculations in stead of \( k^*_e \))

For the present the results of the wave experiments are:

<table>
<thead>
<tr>
<th>( D_{50}(\mu m) )</th>
<th>Flume</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>small</td>
<td>( k_e = 0.0005 + 0.21 a_b )</td>
</tr>
<tr>
<td>220</td>
<td>Research</td>
<td>( k^*_e = -0.0027 + 0.30 a_b )</td>
</tr>
<tr>
<td>100</td>
<td>small</td>
<td>( k^*_{sb} = 0.4634 a_b \phi^{0.63} )</td>
</tr>
<tr>
<td>100</td>
<td>Research</td>
<td>( k^*_e = 0.8571 a_b \phi^{0.70} )</td>
</tr>
</tbody>
</table>

It is clear that the grain size may emphasize the influences of other parameters on the scour process: with \( D_{50} = 100 \mu m \) the influence of the wave period \( T \) may be practically neglected; with the larger grain sizes \( D_{50} = 220 \mu m \) and \( D_{50} = 700 \mu m \)
the scour depth seems to be less dependent on the pipe diameter.

It should be possible to describe the scour depth as

\[ k_e = C_16 a_b^x T^y \phi^z \]  \hspace{1cm} (5.30)

in which the grain size is taken into account in the exponents \( y \) and \( z \). However, this seems to be a fairly artificial way to find the best fit curve for all measurements. Therefore a more physical approach has been chosen: the orbital velocity near the bottom is supposed to be the driving force of the scour process while both the fall velocity of the sediment particles, \( w \), and the critical velocity, \( u_{cr} \), resist this. For the present the influence of the pipe diameter is neglected.

In table 5.5 for all wave conditions adjusted in the tests, the dimensionless wave scour parameter \( (a_b)^2 / u_{cr} w \) has been calculated, with

\[ a_b \] = measured maximum orbital velocity near the bottom during a wave top

\[ u_{cr} \] = critical orbital velocity amplitude

with

\[ u_{cr} = \frac{u_{scr}}{\sqrt{f_w/2}} \]

where

\[ u_{scr} \] = critical shear velocity according to Shields

\[ f_w \] = Jonsson's friction coefficient:

\[ f_w = \exp\left(-5.977 + 5.213(a_b/r)^{-0.194}\right) \]

with

\[ a_b \] = orbital excursion near the bottom

\[ r \] = equivalent Nikuradse roughness of the bottom

\[ w \] = fall velocity in still water, calculated with the general formula

\[ \log w = -0.476 \log^2 D_{50} + 2.18 \log D_{50} + 3.19 \]

with \( w \) in m/s and \( D_{50} \) in m.
<table>
<thead>
<tr>
<th>$D_{50}$</th>
<th>$w$</th>
<th>$T$</th>
<th>$a_b$</th>
<th>$a_{bt}$</th>
<th>$f_w$</th>
<th>$u_*$</th>
<th>$u_{cr}$</th>
<th>$(a_b^2)/(u_{cr} \cdot w)$</th>
<th>$k_e^*$</th>
<th>$k_e^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>.008</td>
<td>1.7</td>
<td>.10</td>
<td>24</td>
<td>.0199</td>
<td>.1244</td>
<td>10.1</td>
<td>11</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.20</td>
<td>46</td>
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<td>.1405</td>
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<td>62</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.10</td>
<td>48</td>
<td>.0153</td>
<td>.1416</td>
<td>8.8</td>
<td>19</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.2</td>
<td>.15</td>
<td>67</td>
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<td>.1498</td>
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<td>.1553</td>
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<td>.1385</td>
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<td>.25</td>
<td>127</td>
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<td>.1567</td>
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<td>4</td>
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<tr>
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<td>.18</td>
<td>43</td>
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<td>41</td>
<td>.0371</td>
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<td>8</td>
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<td></td>
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<td>12</td>
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</tr>
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<td>.31</td>
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<td>.0267</td>
<td>.1669</td>
<td>6.5</td>
<td>19</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5 Wave scour parameter $(a_b^2)/(u_{cr} \cdot w)$
with \( w \) in m/s and \( D_{50} \) in m.

Fig. 5.23 shows \( k_e^* \) (\( k_e \) for \( D_{50} = 700 \ \mu m \)) as a function of \( u_b^2/u_{cr}w \) for pipe diameter \( \phi = 90 \ \text{mm} \). (The averaged scour depth of \( \phi = 75 \ \text{mm} \) and \( \phi = 110 \ \text{mm} \) is used for the experiments with \( D_{50} = 100 \ \mu m \) in the Research flume).

Although the result of this approach shows a continually increasing \( k_e^* \) (or \( k_e \)) with increasing values of \( u_b^2/u_{cr}w \), the dispersion is still too high to attach much value to the validity of Fig. 5.23 with regard to prototype predictions. However, the dimensionless parameter \( u_b^2/u_{cr}w \) seems to be a physically relevant parameter to describe the scour depth on which further investigations might appear successful.

Because during the execution of all scour experiments indications have been obtained that the scour depths due to current will be larger than due to waves, the topic of the research program has become the experiments with current rather than those with waves. Therefore no further attention has been paid to extensive testing in order to find the proper relation

\[
k_e = f(a_b, T, \phi, D_{50})
\]
Fig. 5.23. $\kappa_e^*$ as a function of the wave scour parameter $\hat{u}^2/u_{cr} \cdot W$ for different wave periods $T$ (in s)
5.2. Results of the experiments with current

5.2.1. Results of the experiments with current in the small flume

In table 5.6 the results of the experiments with current in the small flume are collected.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$k_e$</th>
<th>$k_p$</th>
<th>$v_p$</th>
<th>$k_e$</th>
<th>$k_p$</th>
<th>$v_p$</th>
</tr>
</thead>
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<td>8</td>
<td>8</td>
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<td>30</td>
<td>10</td>
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<td>30</td>
<td>14</td>
<td>14</td>
<td>.21</td>
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<td>.26</td>
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<tr>
<td>40</td>
<td>13</td>
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<td>.22</td>
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</tr>
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<td>14</td>
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<td>51</td>
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<td>21</td>
<td>21</td>
<td>.36</td>
<td>66</td>
<td>63</td>
<td>.36</td>
</tr>
</tbody>
</table>

Table 5.6 Results of experiments with current in the small flume

$h = 250 \text{ mm}$

$D_{50} = 700 \mu\text{m}$

With these results the Figures 5.24, 5.25 and 5.26 are made in which respectively $k_e$, $k_p$ and $v_p$ are given as a function of $\phi$. For each single parameter the results have been estimated by linear relationships which are formulated below (the method of the least squares has been used):

$$k_e = 0.0029 + 0.27\phi$$

$$v = 0.25 \quad k_p = 0.0029 + 0.27\phi \quad (5.31)$$

$$v_p = 0.0001 + 3.2\phi$$
Fig. 5.24. \( k_e \) as a function of \( \phi \) for different current velocities \( v \) (in m/s).

Fig. 5.25. \( k_p \) as a function of \( \phi \) for different current velocities \( v \) (in m/s).

Fig. 5.26. \( v \) as a function of \( \phi \) for different current velocities \( v \) (in m/s).
\[ k_e = 0.0109 + 0.76\phi \]
\[ v = 0.30 \quad k_p = -0.0124 + 1.04\phi \]  
(5.32)
\[ v_p = 0.12 + 3.2\phi \]

with \( k_e \) and \( k_p \) in m and \( v \) and \( v_p \) in m/s.

In the first instance the results show a linear increase of the three parameters with \( \phi \), but in view of the small number of experiments and the relatively large grain size for the time being no more efforts have been spent on further elaboration of the results. The calculations made with the measured values from the Research flume have showed more reliable results. The experiments in the small flume can still be used to determine the influence of the grain size on the scour process.

5.2.2. Results of the experiments with current in the Research flume

When we have a rapid glance on the results of the experiments with current in the Research flume (table 5.7), we find a relatively high influence of the current velocity on the scour depth.

<table>
<thead>
<tr>
<th>( \phi ) mm</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>63</th>
<th>75</th>
<th>90</th>
<th>110</th>
<th>125</th>
<th>140</th>
<th>160</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m/s )</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>.20</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>11</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>.35</td>
<td></td>
<td>31</td>
<td>30</td>
<td>55</td>
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<td></td>
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<td>130</td>
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<td>.80</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>160</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7 Results of the experiments with current in the Research flume

\[ h = 660 \text{ mm} \]
\[ D_{50} = 100 \mu \text{m} \]
Simple extrapolation of the results to prototype values, (\(v = 1.0 \text{ m/s during storm conditions}\)), leads to extremely high, unrealistic scour depths. For that reason the experiments with the relatively high current velocities \(v = 0.67 \text{ m/s}\) and \(v = 0.80 \text{ m/s}\) have been carried out to verify the hypothesis that the rate of increase of the scour depth with increasing current velocity decreases when arriving at the higher values of the current velocity. The scour depths measured in both experiment will be included in the calculations made in this paragraph. They were:

\[
\phi = 160 \text{ mm} \quad \begin{align*}
    &v = 0.67 \text{ m/s} \quad \ldots \ldots k_e = 0.130 \text{ m} \\
    &v = 0.80 \text{ m/s} \quad \ldots \ldots k_e = 0.160 \text{ m}
\end{align*}
\]

These two values and those from table 5.7 can be found in Fig. 5.27 in which \(k_e\) is given as a function of \(v\) for the different pipe diameters.

---

Fig. 5.27. \(k_e\) as a function of \(v\) for different \(\phi\) (in mm)
Fig. 5.27 shows clearly the importance of the experiments with the relatively high current velocities. Unfortunately the large loss of sediment made it impossible to carry out more "high velocity" experiments.

Fig. 5.28 shows the same values, but here \( k_e \) is given as a function of \( \phi \) in stead of \( v \). For each single value of the velocity \( v \), these functions have been estimated by

\[
k_e = C_{17} \phi^{0.18}
\]

(5.33)

With the method of the least squares the following was found:

<table>
<thead>
<tr>
<th>( v ) (m/s)</th>
<th>( k_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>0.0467 ( \phi^{0.41} )</td>
</tr>
<tr>
<td>0.30</td>
<td>0.1464 ( \phi^{0.64} )</td>
</tr>
<tr>
<td>0.35</td>
<td>0.4409 ( \phi^{0.88} )</td>
</tr>
<tr>
<td>0.40</td>
<td>1.1755 ( \phi^{1.23} )</td>
</tr>
</tbody>
</table>

(5.34)

with \( k_e \) and \( \phi \) in m. Accidentally the averaged value of the exponent is equal to the value found by Kjeldsen [51]:

\[
k_e = C_{17} \phi^{0.8}
\]

(5.35)

In spite of the high dispersion of the values of the exponents in equations (5.34), this seems to be a good estimation for the averaged outcome of the experiments with current with respect to the value of \( C_{18} \) in equation (5.33). Moreover, there are two arguments which contribute to this conclusion:

- In the experiments with \( v = 0.20 \) m/s no undisturbed sediment transport was present and only very local tunnel erosion was observed. This does not seem to be very realistic, so it is questionable if the results of these experiments should be included in the final calculations of the relation between \( k_e \), \( v \) and \( \phi \).
Fig. 5.28. $k_e$ as a function of $\phi$ for different current velocities $v$ (in m/s)
- In the experiment with \( v = 0.40 \) m/s and \( \phi = 90 \) mm, the measured scour depth is so large in comparison to those with the other pipe diameters that it increases the value of the exponent \( C_{18} \) in equation (5.33) considerably. Leaving this value out of the calculations results in a value far smaller than 1.23.

Fig. 5.27 has been used to determine a functional relation \( k_e - v \). Without the "high velocity" values \( v = 0.67 \) m/s and \( v = 0.80 \) m/s the assumption

\[
k_e = C_{19} v^{C_{20}}
\]  
(5.36)

would lead to a value of \( C_{20} \) larger than 1. However, the "high velocity" experiments showed an effect comparable to the relation \( k_e - \phi \), as described in 5.2.1. Because of the high value awarded to the experiments with the 160 mm pipe, the estimation of its relation \( k_e = f(v) \) by equation (5.36) is considered to be one of the more representative for all the experiments with current. For the results obtained with the 160 mm pipe the method of the least squares leads to:

\[
k_e = 0.1741 v^{0.56}
\]  
(5.37)

with \( k_e \) in m and \( v \) in m/s.

When we again assume

\[
k_e = C_{21} v^{C_{20}} \phi^{C_{18}}
\]  
(5.38)

and assume the values \( C_{18} = 0.80 \) and \( C_{20} = 0.56 \) correct, we can calculate \( C_{21} \) according to:

\[
C_{21} = \frac{\prod \left( k_{e1} v^{0.56} \phi^{0.8} \right) }{\prod \left( v^{0.56} \phi^{0.8} \right)^2}
\]  
(5.39)

With the data from table 5.7 and the values of the scour depths for the "high velocity" experiments \( C_{21} = 0.754 \) is calculated.
So finally the following is found:

\[ k_e = 0.754 \nu^{0.56} \phi^{0.80} \]  

(5.40)

with \( k_e \) and \( \phi \) in m and \( \nu \) in m/s.

The assumption that the experiments with the 160 mm pipe would be the more representative for the whole series was confirmed by experiments which were carried out in the Delft Hydraulic Laboratory "De Voorst". Fig. 5.29 shows the results of these experiments which have been carried out with two pipe diameters: \( \phi = 320 \) mm and \( \phi = 500 \) mm.

In the experiment with the 500 mm pipe the velocity was increased gradually from \( \nu = 0.30 \) m/s to \( \nu = 0.90 \) m/s. In the experiment with the 320 mm pipe the current velocity amounted to \( \nu = 1.00 \) m/s. The \( D_{50} \) of the sand used in both experiments was 220 \( \mu \)m. Fig. 5.29 also shows the results of the experiments in the Research flume with the 160 mm pipe.

![Graph showing \( k_e \) as a function of \( \nu \) for \( \phi = 160 \) mm in the Research flume and \( \phi = 320, 500 \) mm at DHL](image)

So far nothing has been said about the grain size used in the experiments. This will be the topic of the next paragraph.
5.2.3. The influence of $D_{50}$ on the scour process due to current

**Summary of the experiments with current**

With regard to the experiments with current, the influence of the grain size $D_{50}$ on the scour process can only be based on two series of experiments:

- with $D_{50} = 700 \mu m$ in the small flume
- with $D_{50} = 100 \mu m$ in the Research flume

Moreover, the results of the experiments at the DHL "De Voorst" can be taken into account, which are also part of the MaTS program.

Anyhow it has become clear that the undisturbed sediment transport upstream of the pipe plays an important part in the scour process. For example this has been observed during the experiments in the small flume with $D_{50} = 700 \mu m$ where in spite of the relatively high grain size (and thus a relatively high critical velocity) scour depths were found in the order of one pipe diameter. However, in those cases the undisturbed sediment transport was nil, leaving all the transport capacity underneath the pipe for erosion.

Using the same velocity during the experiments in the Research flume with $D_{50} = 100 \mu m$, this resulted in the establishment of a certain sediment transport upstream of the pipe. The erosion underneath the pipe equals the transport capacity underneath the pipe minus the incoming transport: $S = S_p - S$.

When this incoming transport is nil, as was observed using $D_{50} = 700 \mu m$, the erosion equals the transport capacity underneath the pipe:

$$D_{50} = 700 \mu m \quad : \quad \text{Erosion capacity} = S_p - S = S_p$$

$$D_{50} = 100 \mu m \quad : \quad \text{Erosion capacity} = S_p - S$$

Indeed it has been observed that the scour process in the small flume using $D_{50} = 700 \mu m$ developed faster than the scour process in the Research flume using $D_{50} = 100 \mu m$, for the same values of pipe diameter $\phi$ and current velocity $v$. 
However, in prototype conditions (e.g. $D_{50} = 150 \ \mu m$; $v = 1.0 \ m/s$), an undisturbed sediment transport can be expected to be present upstream of the pipe, so the experiments with $D_{50} = 700 \ \mu m$ may not be considered representative for the prototype scour process. This leaves us the experiments with $D_{50} = 100 \ \mu m$ in the Research flume and, as a verification, the experiments with $D_{50} = 220 \ \mu m$ at the DHL "De Voorst".

The results of the experiments in the Research flume largely based on the results with the larger pipe diameters can be estimated by equation (5.40):

$$k_e = 0.754 \ v^{0.56} \ \phi^{0.80}$$

or

$$k_e = 1.735 \left(\frac{v^2}{2g}\right)^{0.28} \ \phi^{0.80} \quad (5.41)$$

which is in agreement with Kjeldsen's formula, equation (3.7), although the coefficient 1.735 is non-dimensionless.

For the present this estimation will be sufficient. However, in the near future it will be tried to create a numerical model to describe the scour underneath pipelines. In those models it will be necessary to introduce the undisturbed sediment transport upstream of the pipe as another parameter:

$$k_e = f(v, \phi, S) \quad (5.42)$$
6. Scale relations

6.1. A prototype pipeline on the North Sea

Although the model experiments have been carried out as a scale series in order to find a functional relation between the scour depth and several relevant parameters, it is questionable whether the prototype has been scaled well enough to use the results of the model experiments to predict scour depths for prototype conditions. For example, scale effects may have led to an essentially different flow pattern around the pipe in the model and thus an essentially different scour process.

To investigate this matter the relevant scale relations are discussed. To do so, we first need to know exactly what has to be understood by "prototype conditions". Because the scour due to current action is regarded more important than the scour due to waves, and will be optimal during a storm, the expression "prototype conditions" can be indicated with the values of the following parameters:

\[
\begin{align*}
    h &= 30 \text{ m} \\
    \bar{v} &= 1.0 \text{ m/s} \\
    D_{50} &= 200 \mu\text{m} \\
    \phi &= 1.0 \text{ m} \\
    r_b &= 0.10 \text{ m} \\
    r_p &= 0.005 \text{ m}
\end{align*}
\]

In addition to the values of the parameters the following remarks are made:

- For the vertical velocity profile a logarithmic profile is assumed. For the velocity at half the height of the pipe above the bottom this means that
\[ v_{\phi/2} = \frac{\bar{v} \sqrt{g}}{\kappa \cdot C} \ln(\phi/2z_0) \quad (6.1) \]

with:

\( \bar{v} = 1.0 \text{ m/s} \)

\( g = 9.8 \text{ m/s}^2 \)

\( C = 18 \log(12h/r) = 18 \log(12 \cdot 30/0.1) = 64 \text{ m}^2/\text{s} \)

\( \kappa = 0.4 \)

\( z_0 = r/33 = 0.1/33 \)

\( \phi = 1.0 \text{ m} \)

\[ v_{pr} = v_{0.5} = \frac{1.70^2}{0.4 \cdot 64} \ln(0.5 \cdot 33/0.10) = 0.62 \text{ m/s} \]

- The mean grain diameter \( D_{50} \) found at the Netherlands part of the Continental Shelf ranges from 150 to 250 \( \mu \text{m} \).

- Pipe diameters ranging from 0.30 m up to 1.00 m are used to transport the oil or gas. Because the self burial of the larger pipelines seems more unlikely, \( \phi_{pr} = 1.0 \text{ m} \) has been chosen.

- In the northern part of the Netherlands part of the Continental Shelf, where all gas and/or oil fields are situated, the bottom is relatively flat: \( \Delta r \sim 0.10 \text{ m} \); \( \lambda_r \sim 1.0 \text{ m} \). In the southern part both sand waves (\( \Delta r \sim 6 \text{ m} ; \lambda_r \sim 200 \text{ m} \)) and mega ripples (\( \Delta r \sim 0.5 \text{ m} ; \lambda_r \sim 6 \text{ m} \)) are present.

- The roughness of the pipe is given by the roughness of its concrete coating. When the pipe is uncovered for a longer period, it attracts organisms to grow on it because of its relative warmth due to the transported oil or gas inside it. These organisms can increase the roughness of the pipe up to about 0.03 m.
In contradiction to what can be expected, it is assumed that the pipeline is fixed at a constant height above the original sea bottom and its axis is always perpendicular to the direction of the current.

6.2. **Scaling the scour process**

To determine which relations are important with regard to the scour process, we can review four factors which describe the physical conditions around the pipe:

a) The flow pattern and the pressure field around the pipe, of which the latter can be regarded as the driving force of the flow underneath the pipe.

b) The friction of the flow by
   - the sand bed upstream of the pipe
   - the pipe
   - the gap underneath the pipe

c) The character of the flow, which can be described by the Reynolds number

d) The sand transport capacity
   - upstream of the pipe
   - underneath the pipe

For each of these four factors it is tried to give the functional relations which describe the physical condition:

ad a) If we assume the pressure gradient between the points A and B in Fig. 6.1 as the driving force of the flow underneath the pipe, and assume this gradient proportional with the square of the velocity at the height of the horizontal pipe axis, the reciprode of the pipe diameter and with the

Fig. 6.1.
Parameters concerning the flow underneath the pipe
drag coefficient $c_D$, it follows that

$$I_p \sim \frac{c_D \cdot (v\phi/2)^2}{\phi} \quad (6.2)$$

Proper scaling of this gradient leads to:

$$n_{c_D} \cdot n_{v\phi/2}^2 = n_{\phi} \quad (6.3)$$

ad b) The friction of the flow by the sand bed is a function of the roughness of the sand bed which can be determined by the parameter of Shields: $\mu v^2_*/(\Delta g D)$ with

- $\mu$ = ripple factor
- $v_* = \text{shear velocity near the bottom}$
- $\Delta = \text{relative density } \frac{\rho_s - \rho_w}{\rho_w}$

with

- $\rho = \text{density}$
- $s, w = \text{subscript for respectively sand and water}$
- $g = \text{acceleration of gravity}$
- $D = \text{property of the grain diameter, e.g. } D_{50}$

Assuming $n_{\mu} = 1$,

using sand in the model ($n_{\Delta} = 1$) and defining

$$v_* = \tau/\rho = ghI \quad (6.4)$$

with

- $h = \text{water depth}$
- $I = \text{slope of the water surface}$
proper scaling of the friction of the flow by the sand bed is thus proper scaling of the Shields parameter which leads to:

\[ n_h \cdot n_I = n_D \]  \hspace{1cm} (6.5)

The friction of the flow by the pipe will be dependent on the relative roughness of the pipe: \( r_\phi/\phi \), with

\( r_\phi \) = equivalent Nikuradse roughness of the pipe
\( \phi \) = pipe diameter

Reproduction in a model is found when

\[ n_{r_\phi} = n_\phi \]  \hspace{1cm} (6.6)

For the friction of the flow by the gap underneath the pipe an equal derivation can be given as for the friction by the bed. However, in this case I will not represent the slope of the water level but the pressure gradient over the pipe and \( h \) is the hydraulic radius of the flow underneath the pipe. So again it follows that

\[ n_h \cdot n_I = n_D \]  \hspace{1cm} (6.7)

ad c) The character of the flow around the pipe can best be described with the Reynolds number:

\[ \text{Re} = \frac{v \cdot R}{\nu} \]

with

\( v \) = flow velocity on horizontal pipe axis
\( R \) = hydraulic radius of the pipe
\( \nu \) = kinematic viscosity of the water
This leads to the scale relation

\[ n_v \cdot n_\phi = n_v \]  \hspace{1cm} (6.8)

ad d) Of course the most important factor of all is the sand transport capacity. Most sand transport formulae can be written as

\[ X = f(Y) \]  \hspace{1cm} (6.9)

with

\[ X = \text{sand transport parameter} \]

\[ Y = \text{flow parameter} \]

Although all formulae have been derived for uniform flow, they can still be used to determine scale relations in the case of the scour process underneath pipelines caused by a flow which will not be uniform at all. Equation (6.9) indicates that proper scaling of the sand transport parameter means proper scaling of the flow parameter. In most formulae Shields flow parameter is used:

\[ \frac{1}{Y} = \frac{\mu \cdot v^2}{C^2 \cdot \Delta \cdot D} \]  \hspace{1cm} (6.10)

with

\[ \mu = \mu(C/C_{90}) = \text{ripple factor} \]

with

\[ C = \text{resistance coefficient} \]

\[ C_{90} = \text{coefficient of roughness of the sediment particles } D_{90} \]
v = mean velocity
\( \Delta = \text{relative density } \frac{\rho_s - \rho_w}{\rho_w} \)

s, w = subscript for respectively sand and water

D = grain diameter; in most cases \( D_{50} \) is used

When using sand in the model \( (n_\Delta = 1) \) and assuming \( n_\mu = 1 \), it follows that proper scaling of this flow parameter leads to:

\[ n_v^2 = n_C^2 n_D \]  \hspace{1cm} (6.11)

In fact this is the same relation as found under b) because \( v^2/c^2 = hI \). So for the sand transport capacity upstream of the pipe equation (6.7) is valid as well.

Especially when the transport of sediment in suspension is present, another factor in the sand transport process, the fall velocity \( w \) of the sediment particles, becomes important. In order to obtain a proper reproduction in a model of the vertical concentration distribution, the ratio of fall velocity and transport velocity should be equal to its value in prototype, so

\[ n_w = n_v \]  \hspace{1cm} (6.12)

In the range of grain diameters which is of our interest the fall velocity \( w \) can be written as a function of the grain diameter:

\[ w \sim D_{50}^{1.5} \]  \hspace{1cm} (6.13)

Combining equations (6.12) and (6.13) leads to:
\[ n_v = n_{D50}^{1.5} \]  \hspace{1cm} (6.14)

The resulting scale relations which follow from the four important factors in the scour process, are:

a) pressure field
   \hspace{1cm} (I) \hspace{0.5cm} n_cD \cdot n_v^2 = n_1

b) friction by
   - the bed or the gap
     \hspace{1cm} (II) \hspace{0.5cm} n_h \cdot n_I = n_D
   - the pipe
     \hspace{1cm} (III) \hspace{0.5cm} n_r = n_1

c) flow character
   \hspace{1cm} (IV) \hspace{0.5cm} n_v \cdot n_\phi = n_v

d) transport
   - capacity
     \hspace{1cm} \hspace{0.5cm} n_h \cdot n_I = n_D
   - concentration distribution (V)
     \hspace{1cm} n_v = n_{D1.5}

It will be clear that it is impossible to fulfil (IV), the scale relation concerning the Reynolds number. Whether, and to what extent, the flow field around the pipe will differ from the one in prototype will be discussed below.

A lot of investigators have studied the flow field around a cylinder in uniform flow for different ranges of the Reynolds number. The drag force of the water on the cylinder, \( F_D \), can be written as:

\[ F_D = c_D \cdot \frac{1}{2} \rho v^2 \cdot \phi \]  \hspace{1cm} (6.15)

with

\( c_D = \) drag coefficient
\( \rho = \) density
\( v = \) flow velocity on horizontal cylinder axis
\( \phi = \) cylinder diameter
The drag coefficient $c_D$ is a function of the Reynolds number, as can be seen in Fig. 6.2:

![Graph showing the drag coefficient $c_D$ as a function of the Reynolds number Re.](image)

Fig. 6.2. The drag coefficient $c_D$ for a smooth cylinder in uniform flow as a function of the Reynolds number [9]

Generally the Reynolds number in the model is in the order $10^4$, in prototype its value is about $10^6$. Because the values of $c_D$ differ for both cases, see Fig. 6.2, the flow field around a smooth, infinitely long cylinder is described below for Reynolds numbers ranging from $5 \cdot 10^3$ up to about $10^6$.

$5 \cdot 10^3 < Re < 2 \cdot 10^5$

The vortices shedding off the cylinder mix turbulently so that the wake downstream of the cylinder gets turbulent. For Reynolds numbers close to $5 \cdot 10^3$ this occurs far downstream of the cylinder, but when the Reynolds number is increased up to $2 \cdot 10^5$ the turbulence moves in the direction of the cylinder. The width of the wake will stay more or less constant at a value of about the diameter of the cylinder which is shown in Fig. 6.3:
Re $> 2 \cdot 10^5$

For $Re > 2 \cdot 10^5$ the wake will be grown turbulent up to close to the cylinder. The free boundary layer downstream of point A, the separation point of the laminar boundary layer, will grow partly turbulent. Now the flow outside the boundary layer has got a larger influence on the turbulent boundary layer because of the mixing action. The velocity in the turbulent boundary layer is higher than in the laminar boundary layer. So the turbulent free boundary layer reaches the cylinder again. The relatively short distance between A and the new attach point B of the turbulent boundary layer is known as the separation bubble, Fig. 6.4.

Fig. 6.4. Flow pattern around a cylinder in uniform flow for $2 \cdot 10^5 < Re < 5 \cdot 10^5$
For increasing Reynolds numbers the turbulent free boundary layer will attach to the cylinder. A new turbulent separation point C will be present now. For still higher Reynolds numbers this point C will move down the cylinder whereas point B moves up the cylinder in the direction of point A. The wake downstream of the cylinder will become smaller and smaller and reaches its minimum for \( Re = 5 \cdot 10^5 \) where its width is about half the diameter of the cylinder. For \( Re > 5 \cdot 10^5 \) the separation bubble will disappear because point B has moved up to point A. From there the boundary layer on the cylinder will be turbulent. Point C is moving up the cylinder as well which means that the width of the wake will grow again. Fig. 6.5 shows the wake for \( Re = 5 \cdot 10^5 \).

![Flow pattern around a cylinder in uniform flow for Re = 5 \cdot 10^5](image)

**Fig. 6.5.** Flow pattern around a cylinder in uniform flow for \( Re = 5 \cdot 10^5 \)

The flow fields described above indicate that they may differ considerably in prototype and in the model. Indeed, this has been measured in the almost full scale experiments in the Laboratory "De Voorst" of the DHL where the relative wake length \( L/\phi \) was far smaller than in the experiments in the laboratory for fluid mechanics of the Civil Engineering Department of the Delft Technical University.

So in prototype the flow underneath the pipe will decelerate as soon as it has passed the pipe; in a model with the Reynolds number in the order of \( 10^4 \) the turbulent wake will
keep the jet underneath the pipe steady over a longer distance downstream of the pipe. This may cause erosion of the bed far downstream of the pipe, the so-called lee erosion. Indeed it has been observed that the scour holes in the Research flume in the laboratory for fluid mechanics were more asymmetrical than those in the DHL "De Voorst", see Fig. 6.6.

![Diagram of scour holes](image)

**Fig. 6.6.** Typical configuration of scour holes in the Research flume (left, $Re \approx 10^4$) and in the DHL (right, $Re \approx 10^6$)

Finally some remarks have to be made on the description of the flow patterns for varying Reynolds numbers. These descriptions are given for smooth cylinders in a free uniform flow. However, in prototype the cylinder will not be smooth at all and moreover the vicinity of the bottom will affect the drag coefficient $c_D$: the values of $c_D$ are larger and are reached for smaller Reynolds numbers when a relatively rough cylinder is used as can be seen in Fig. 6.7. The vicinity of the bottom will only enlarge the values of $c_D$. Moreover, it must be taken into account that the kinematic viscosity $v$ in prototype ($T \approx 8^\circ C$) is 1.5 times more than in the laboratory where $T \approx 20^\circ C$. 
Influence of the roughness of the cylinder on the drag coefficient $c_D$ as a function of the Reynolds number [10].

If we consider the sand transport scale relations the most important, in an equilibrium situation the sand transport upstream of the pipe and underneath the pipe should be scaled properly and moreover, they should be equal because of the equilibrium. This means that the ratio of the scale factors of the Shields parameters of both transport capacities should equal 1. First we determine the scale factor of the Shields parameter for the sand transport capacity upstream of the pipe; from equation (6.10) it follows that

$$n_Y^{-1} = \frac{n_u \cdot n_v^2}{n_C \cdot n_\Delta \cdot n_D}$$

(6.16)

and with $n_u$ and $n_\Delta$ both equal to 1:

$$n_Y^{-1} = \frac{n_v^2}{n_C \cdot n_D}$$

(6.17)

The scale factor of the Shields parameter for the sand transport underneath the pipe is obtained as follows:

$$\frac{1}{Y_p} = \frac{\mu \cdot \Delta^2}{C_p \cdot \Delta D} = \frac{\mu h T}{\Delta D}$$

(6.18)
Using equation (6.2) this becomes:

\[
\frac{1}{\frac{\nu}{\nu_X}} \sim \frac{\mu h c_D \nu^2}{\Delta \phi} \tag{6.19}
\]

Again with \(n_{\mu} = 1\) and \(n_{A} = 1\) this means that:

\[
\frac{n_{\nu}}{\frac{\nu}{\nu_X}} = \frac{n_h \cdot n_{c_D} \cdot n_{\nu}}{n_D \cdot n_{\phi}} \tag{6.20}
\]

In equation (6.20) \(h\) represents the height of the water jet underneath the pipe. When we assume this height equal to the scour depth and in the same order of the pipe diameter, it follows that:

\[
\frac{n_{\nu}}{\frac{\nu}{\nu_X}} = \frac{n_{c_D} \cdot n_{\nu}^2}{n_D} \tag{6.21}
\]

The ratio of both scale factors is

\[
\frac{n_{\nu} / \nu}{n_{\nu} / \nu_X} = \frac{1}{n_{c_D} \cdot n_{\nu}^2} \tag{6.22}
\]

and its value should be equal to 1.

The value of the scale factor of the drag force coefficient \(n_{c_D}\) can be taken from the values for \(c_D\) in Fig. 6.2 for respectively \(Re = 10^6\) (prototype) and \(Re = 5 \cdot 10^4\) for the model. The value of the scale factor will be about 0.5. The friction coefficient \(C\) can be calculated according to

\[
C = 18 \log(12R/r) \tag{6.23}
\]

with \(r\) the equivalent Nikuradse roughness of the bottom and \(R\) the hydraulic radius, which can be assumed equal to the water depth.
Using the following values of R and r the scale factor $n_C^2$ can be calculated:

<table>
<thead>
<tr>
<th></th>
<th>h [m]</th>
<th>r [m]</th>
<th>C [$m^k s^{-l}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>prototype</td>
<td>30</td>
<td>0.10</td>
<td>64</td>
</tr>
<tr>
<td>model</td>
<td>0.66</td>
<td>0.02</td>
<td>47</td>
</tr>
</tbody>
</table>

So $n_C^2$ equals $(64/47)^2 = 1.87$.

The ratio of the scale factors of both sand transport capacities now becomes

$$\frac{1}{n_C^2 : n_{C_D}} = \frac{1}{(1.87) \cdot (0.5)} = 1.07$$

Apparently the relatively high roughness of the bottom in the model is reduced by the change in the flow pattern around the pipe, which induces a lower value of the drag coefficient $c_D$.

The scale factor of both flow parameters being close to one, we may assume that the scale factor of the ratio of the sand transport parameters upstream of and underneath the pipe will be close to one as well, which indicates that the relative scour depth $k_e/\phi$ in prototype will be in the same order as has been found in the model.
7. **Conclusions**

The present report deals with the physical model experiments on scour around submarine pipelines, carried out in order to find the functional relations between the scour depth due to current and/or waves and the relevant current, wave, bottom and pipe parameters. The results of the experiments can be used as a scale series for extrapolation to prototype values or provide for boundary conditions in a numerical model that is being developed to describe the scour process.

So far the following conclusions can be drawn:

* Using a numerical model it is possible to determine the detailed flow field around an uncovered pipe. However, when it comes to calculating the sediment transport around the pipe, problems arise due to the unreliability of sediment transport formulae for non-uniform flow. Therefore, physical model experiments are inevitable to understand the scour process and to verify the model with respect to some specific details.

* Generally the scour due to waves is of minor importance compared to the scour due to current. If a certain sediment transport is present upstream of the pipe, superimposing waves on a current results in a decrease of the scour depth.

* For coarse sediment the scour depth due to waves is independent of the pipe diameter and seems to be in the order of magnitude of 25% of the orbital excursion $a_b$. For fine sediment the scour depth due to waves is independent of the wave period and proportional with the pipe diameter to the power 0.70. In both cases a linear relationship between the scour depth and the undisturbed maximum orbital velocity near the bottom was found to be a rough but practical approximation.
Further investigations on scour due to waves should be focussed on the relation between the scour depth and the wave scour parameter $\frac{u_0^2}{(u_{cr} \cdot w)}$.

For the scour depth due to current proportionality is found with the current velocity to the power 0.56 and with the pipe diameter to the power 0.80. As long as a certain sediment transport is present upstream of the pipe, the grain size seems to be of minor importance for the ultimate scour depth.

In a small scale model the transport capacity upstream of the pipe is high relative to prototype conditions due to the relatively rough bottom. The transport capacity underneath a pipe in a small scale model is high relative to prototype conditions due to the larger pressure gradient over the pipe.

Because both transport capacities are equally enlarged in a small scale model, the statement above indicates that scour depths up to 80% of the pipe diameter, as has been found in the small scale experiments with fixed pipes, can be expected in prototype as well. Including the effects of sagging of the pipe and two directional tidal current a total depth of one or two pipe diameters can not be excluded.

One must be aware of scale effects when using the results of the laboratory experiments for predicting prototype scour by extrapolating. Neglecting or ignorance of scale effects may lead to large mistakes. Though by using the results as a scale series the scale effects are included but they vanish when the relevant scale factors approach unity towards the prototype.
Symbols and notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>orbital excursion</td>
<td>m</td>
</tr>
<tr>
<td>a_b</td>
<td>orbital excursion near the bottom</td>
<td>m</td>
</tr>
<tr>
<td>a_b_t</td>
<td>orbital excursion near the bottom during top of the wave</td>
<td>m</td>
</tr>
<tr>
<td>A</td>
<td>total amount of eroded sediment underneath the pipe</td>
<td>m³/m'</td>
</tr>
<tr>
<td>b</td>
<td>width</td>
<td>m</td>
</tr>
<tr>
<td>c_w</td>
<td>wave celerity</td>
<td>m/s</td>
</tr>
<tr>
<td>c_D</td>
<td>drag coefficient</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>Chézy friction coefficient</td>
<td>m⁴/s</td>
</tr>
<tr>
<td>C_i</td>
<td>general constant with numerical index i</td>
<td></td>
</tr>
<tr>
<td>C_90</td>
<td>friction coefficient of sediment particles D_90</td>
<td>m⁴/s</td>
</tr>
<tr>
<td>C_p</td>
<td>friction coefficient underneath the pipe</td>
<td>m⁴/s</td>
</tr>
<tr>
<td>d_o</td>
<td>pipe height relative to original sea bottom</td>
<td>m</td>
</tr>
<tr>
<td>D</td>
<td>grain diameter</td>
<td>m</td>
</tr>
<tr>
<td>D_50</td>
<td>mean grain diameter</td>
<td>m</td>
</tr>
<tr>
<td>e</td>
<td>gap height</td>
<td>m</td>
</tr>
<tr>
<td>E</td>
<td>Euler number</td>
<td>-</td>
</tr>
<tr>
<td>f</td>
<td>function of</td>
<td>-</td>
</tr>
<tr>
<td>f_w</td>
<td>Jonsson's friction coefficient</td>
<td>-</td>
</tr>
<tr>
<td>F</td>
<td>Froude number</td>
<td>-</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity</td>
<td>m/s²</td>
</tr>
</tbody>
</table>
\( h \quad \text{water depth} \quad \text{m} \\
\( H \quad \text{wave height} \quad \text{m} \\
\( I \quad \text{gradient} \quad - \\
\( k = \frac{2\pi}{L} \quad \text{wave number} \quad \text{m}^{-1} \\
\( k_e \quad \text{scour depth} \quad \text{m} \\
\( k_e^* \quad \text{scour depth in equilibrium situation} \quad \text{m} \\
\( k_e^* \quad \text{scour depth in equilibrium situation, corrected for wave asymmetry} \quad \text{m} \\
\( \bar{k} \quad \text{pipe diameter or wave period averaged} \quad \text{m} \\
\( \bar{k}_e \quad \text{scour depth in equilibrium situation} \quad \text{m} \\
\( k_{ec} \quad \text{scour depth in equilibrium situation due to current} \quad \text{m} \\
\( k_p \quad \text{scour depth underneath the pipe} \quad \text{m} \\
\( l \quad \text{length} \quad \text{m} \\
\( L \quad \text{length} \quad \text{m} \\
\( L_a \quad \text{horizontal distance from vertical pipe axis to original sea bottom downstream of the pipe ( or in direction of wave propagation )} \quad \text{m} \\
\( L_v \quad \text{horizontal distance from vertical pipe axis to original sea bottom upstream of the pipe ( or against direction of wave propagation )} \quad \text{m} \\
\( n \quad \text{scale factor} \quad - \\
\( N \quad \text{number} \quad - \\
\( q \quad \text{discharge per unit width} \quad \text{m}^3/\text{sm}^1 \)
\( q_h \) discharge per unit width below the level of half the pipe diameter plus the pipe height relative to the original sea bottom: \( \phi/2 + d_0 \)

\( q_p \) discharge per unit width underneath the pipe \( m^3/sm' \)

\( r \) bottom roughness \( m \)

\( r_p \) roughness of the pipe \( m \)

\( R \) hydraulic radius \( m \)

\( Re \) Reynolds number \( - \)

\( S \) sediment transport (capacity) \( m^3/s \)

\( S_p \) sediment transport (capacity) underneath the pipe \( m^3/s \)

\( t \) time \( s \)

\( t_t \) time interval of top of a wave \( s \)

\( T \) wave period \( s \)

\( u \) horizontal orbital velocity \( m/s \)

\( A \) amplitude of horizontal orbital velocity \( m/s \)

\( u_b \) horizontal orbital velocity near the bottom \( m/s \)

\( A_{bt} \) amplitude of horizontal orbital velocity near the bottom during top of a wave \( m/s \)

\( u_{cr} \) critical orbital velocity \( m/s \)

\( u_{*cr} \) critical shear velocity \( m/s \)

\( u_p \) horizontal orbital velocity underneath the pipe \( m/s \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>current velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>mean current velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_b$</td>
<td>current velocity near the bottom</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_p$</td>
<td>current velocity underneath the pipe</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_x$</td>
<td>shear velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$w$</td>
<td>fall velocity in still water</td>
<td>m/s</td>
</tr>
<tr>
<td>$x$</td>
<td>horizontal coordinate</td>
<td>m</td>
</tr>
<tr>
<td>$X$</td>
<td>sand transport parameter</td>
<td>-</td>
</tr>
<tr>
<td>$y$</td>
<td>horizontal coordinate</td>
<td>m</td>
</tr>
<tr>
<td>$Y$</td>
<td>flow parameter</td>
<td>-</td>
</tr>
<tr>
<td>$z$</td>
<td>vertical coordinate</td>
<td>m</td>
</tr>
<tr>
<td>$z_0$</td>
<td>elevation for zero velocity</td>
<td>m</td>
</tr>
</tbody>
</table>
\( \Delta_r \) ripple height \( \text{m} \)
\( \Delta \) relative density \( \text{–} \)
\( \eta \) water elevation \( \text{m} \)
\( \eta_m \) dynamic viscosity \( \text{–} \)
\( \eta_m \) amplitude of water elevation during top of a wave \( \text{m} \)
\( \theta_c \) angle between pipe direction and predominant current direction \( \text{–} \)
\( \theta_w \) angle between pipe direction and predominant wave propagation \( \text{–} \)
\( \kappa \) Von Kármán constant \( (\approx 0.4) \) \( \text{–} \)
\( \lambda_r \) ripple wave length \( \text{m} \)
\( \mu \) ripple factor \( \text{–} \)
\( \nu \) kinematic viscosity \( \text{m}^2/\text{s} \)
\( \rho \) density \( \text{kg}/\text{m}^3 \)
\( \rho_w \) density of water \( \text{kg}/\text{m}^3 \)
\( \rho_s \) density of sand \( \text{kg}/\text{m}^3 \)
\( \tau \) shear stress \( \text{N}/\text{m}^2 \)
\( \tau_b \) bottom shear stress \( \text{N}/\text{m}^2 \)
\( \tau_{cr} \) critical shear stress \( \text{N}/\text{m}^2 \)
\( \phi \) pipe diameter \( \text{m} \)
\( \sim \) proportional with \( \text{–} \)
\( \approx \) equals about \( \text{–} \)
References


<table>
<thead>
<tr>
<th></th>
<th>Author</th>
<th>Title</th>
<th>Publisher</th>
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