Reliability based design of Bottom Founded Offshore Structures

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Reliability based design of
Bottom Founded Offshore Structures

by

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Abstract

The offshore industry and especially the oil and gas production, has faced a remarkable growth in the last decades, causing an increase in the population of offshore structures. The overwhelming majority of these structures are bottom founded and mostly made out of steel. Over these decades much experience has been gained with the design and construction of these steel offshore structures. A number of structural regulations such as the ISO 19902 have been developed based on the industry best practice and currently used worldwide. These are based on the conventional Load and Resistance Factor Design (LRFD) method where the design is guided by the use of partial factors for the load and resistance. These factors were derived based on calibration work on older regulations trying to achieve the same inherent level of safety. This caused the picture about the intended reliability of the ISO 19902 to be vague and as a result the required level of safety is not explicitly defined. However a target annual probability of failure ($P_{f,a}$) for new manned structures has been partially referenced in the standard and used as target value in the current report.

The goal of this report is to investigate the intended reliability when designing bottom founded offshore steel support structures with the ISO 19902 and specifically to research on how the reliability analysis can be engaged in the design phase of offshore steel structures. To this end, the principles of nonlinear pushover analysis, system based and reliability based design are investigated and discussed.

A code has been developed that allows interaction with Matlab and the nonlinear analysis program USFOS. A jacket structure has been designed and optimized based on the ISO 19902 and the achieved reliability level has been estimated. Further a reliability analysis has been performed more explicitly through Monte Carlo simulations accounting for a number of stochastic variables such as the yield stress, the marine growth and the hydrodynamic coefficients and their probability density functions (pdfs).

A sensitivity analysis has been carried out in order to determine which are the parameters of influence to the reliability level. Finally a method of designing with the focus on the desired reliability level is proposed and demonstrated in the examined jacket. The application the reliability based design method offers the potential benefits of reducing the structural steel and ensuring that the required reliability level is achieved.
Acknowledgments

The current report is a result of my personal effort but it would not have been achieved without the valuable contribution of a number of people who helped me and supported me.

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I also need to thank my friends in Greece and in Delft, who walk and on the same path as me and support one another throughout the years.

Lastly but most importantly I truly feel grateful about my family. Their unconditional love, support and encouragement through my life brought me here. Nothing would have been achieved if their presence was not that strong in my life.

Evripidis Apostolidis

Delft, 16 December 2016
List of Acronyms

$C_D$  
drag coefficient

$C_m$  
mass coefficient

$E_m$  
mean value of the environmental load

$E_{10,000}$  
10,000 year return period environmental load

$E_{100,f}$  
factored 100 year return period environmental load

$E_{100}$  
100 year return period environmental load

$F$  
global environmental load

$P_{f,a}$  
annual probability of failure

$P_{f,l}$  
lifetime probability of failure

$R_m$  
mean value of the resistance

$U_c$  
current speed

$V_E$  
coefficient of variation of the environmental load

$V_R$  
coefficient of variation of the resistance

$\beta$  
reliability safety index

$\beta_a$  
annual reliability safety index

$\beta_l$  
lifetime reliability safety index

$\epsilon$  
strain

$\gamma_{f,E}$  
environmental partial action factor

$\sigma$  
stress

**ALARP**  
‘As Low As Reasonably Practicable’

**ALS**  
Accidental limit state

**API**  
American Petroleum Institution

**AUS**  
Northwest cost of Australia

**BIC**  
Bayesian Information Criterion

**BS_{100}**  
100 year return period base shear

**CBA**  
Cost Benefit Analysis
### List of Acronyms

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<th>Description</th>
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<tbody>
<tr>
<td>cdf</td>
<td>cumulative density function</td>
</tr>
<tr>
<td>CoV</td>
<td>Coefficient of Variation</td>
</tr>
<tr>
<td>FLS</td>
<td>Fatigue limit state</td>
</tr>
<tr>
<td>GoM</td>
<td>Gulf of Mexico</td>
</tr>
<tr>
<td>LRFD</td>
<td>Load and Resistance Factor Design</td>
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<tr>
<td>LSF</td>
<td>limit state function</td>
</tr>
<tr>
<td>MSL</td>
<td>mean seal level</td>
</tr>
<tr>
<td>NNS</td>
<td>Northern North Sea</td>
</tr>
<tr>
<td>NS</td>
<td>North Sea</td>
</tr>
<tr>
<td>P_f</td>
<td>probability of failure</td>
</tr>
<tr>
<td>P_r</td>
<td>probability of reliability</td>
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<tr>
<td>pdf</td>
<td>probability density function</td>
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<tr>
<td>PFD</td>
<td>Partial Factor Design</td>
</tr>
<tr>
<td>RP</td>
<td>return period</td>
</tr>
<tr>
<td>RSR</td>
<td>Reserve Strength Ratio</td>
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<tr>
<td>SLS</td>
<td>Serviceability limit state</td>
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<tr>
<td>SNS</td>
<td>Southern North Sea</td>
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<tr>
<td>SVM</td>
<td>support vector machine</td>
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<td>ULS</td>
<td>Ultimate limit state</td>
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<td>WSD</td>
<td>Working Stress Design</td>
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Chapter 1

Introduction

1.1 Motivation

During the last six decades the offshore oil and gas exploration and production has become a remarkably growing field. The reasons behind this growth is the increasing oil and gas demand worldwide coupled with the shortage of oil-land resources caused from economical and political reasons. Currently 30 percent of the world oil and gas production comes from offshore fields and this figure is expected to increase in the future. In order to front this increasing demand, the offshore engineers have been facing the task to build structures which are able to withstand the challenges met in an offshore environment in a safe and economical manner. The main challenges are the extreme environmental conditions as well as the remoteness of the location. Especially the ambiguous nature of the environmental loads renders the structures vulnerable when extreme conditions occur and member failure or system collapse may happen as depicted in Figure 1.1.

Nowadays the support structures used in the offshore industry for a field exploitation can be divided into five categories namely fixed structures, compliant structures, guyed structures, tension leg platforms and anchored floating platforms. Among the population of all the offshore structures the overwhelming majority are fixed structures and most of them made out of steel.

The design and analysis of such structures, placed in ocean environments requires special consideration since they are experiencing environmental loading conditions which are very complicated and contain large uncertainties. Offshore structures are subjected to stochastic loads originating from waves, wind and current. Due to the random nature of these loads the safety of the structures should be addressed in a careful and conscious way. Design of offshore structures is carried out to satisfy the condition that the structure remains safe during its lifetime; it should be able to withstand both operational and extreme loading conditions.

In order to assure this safety condition a number of regulations and codes has been developed over the years, trying to guide in a prescriptive manner the design and analysis of offshore
structures. Currently the American Petroleum Institution (API) Working Stress Design (WSD) and ISO 19902 are used for design of steel offshore jackets all over the world. Based on these codes the required level of safety is achieved by designing using partial safety factors for the stochastic variables, namely the resistance and the loads.

The safety factors for the environmental loads used in the codes were derived from analysis and calibration work for similar offshore structures in the same metocean conditions (Moses and Stahl, 2000). The goal of these codes is to achieve uniform safety levels for structures which are designed based on their methodology. However, from structure to structure some remarkable differences exist which may affect the obtained safety level. These are mainly the differences in the environmental loading due to different field locations and the difference in the structural system selected by the structural engineer. Thus the acquired level of safety (reliability) when designing using the partial safety factor method given by the codes may not be clearly determined.

Moreover the design method given by the codes is performed using linear, i.e. elastic analysis. The stresses are calculated for each individual structural component aiming to respond elastically and not exceed the steel yield stress. This way of designing and analysis is considered as the conventional and widely used among the structural engineers. However other methods of structural analysis and design can be engaged.

1.2 Thesis objective

In light of the aforementioned background the current report aims at providing insight on the system based design and reliability based design of offshore fixed steel structures.

The main research objectives of this study can be summarized in the following points:

- What is intended reliability level in the ISO code.
- Which are the methods of calculating the reliability of a structure.
- How can the reliability of an offshore structure be calculated.
- Which are the parameters of influence to the achieved reliability.
- Is it worthy to engage system and reliability based design into the design of offshore structures.
- What are the advantages and disadvantages when designing in such a way.

1.3 Research approach and outline

The present Chapter constitutes an introduction to the core concepts that will be discussed within the following pages. In Chapter 2 the principles of structural reliability and nonlinear analysis are discussed and the benefit that can be gained when included in the design phase is presented.

In Chapter 3 the design philosophy based on ISO 19902 is presented and the intended reliability levels are described. The methods and the necessary assumptions for calculating the structural reliability of offshore structures are discussed. In Chapter 4 a case study is engaged; a jacket is designed and optimized based on the ISO structural checks, in order to estimate the reliability achieved with the ISO design recipe. In Chapter 5 the reliability of the jacket structure is scrutinized. The reliability of the jacket is calculated with two ways. The first one is based on approximated values for the resistance uncertainty. The second method used is based on Monte
1.3 Research approach and outline

Carlo simulations. The stochastic variables and their distributions are accounted explicitly and a histogram of the resistance is produced. A probability density function is fitted on the obtained histogram and the reliability level is recalculated. The sensitivity of the achieved reliability is checked against a number of parameters of influence. In Chapter 6 a method for reliability based design of bottom founded structures is described and applied in the examined structure. Finally, Chapter 7 provides an overview of the work in terms of conclusions with respect to the obtained results and a short discussion on possible future research for the enhancement of the current study.
Chapter 2

Structural Reliability and Non linear analysis

In the current chapter the principles of structural reliability, non linear analysis and system based design are introduced and their benefits are discussed by the means of examples.

2.1 Structural Reliability Methods

When designing a structure one should account for the actions (loads) that the structure will experience as well as the resistance that it will exhibit. Both actions and resistances are subject to a degree of uncertainty. Thus a probabilistic approach is required in order to meet the objective of the structural design which is to achieve a structure with an acceptable low risk of failure. Herein the term failure as defined by Efthymiou and van de Graaf (1997) is adopted.

Failure
The term failure indicates system collapse as opposed to component failure. The reason for this is that from the point of view of risk to personnel, member yielding or component failure are generally of no consequence. The load level at which system collapse occurs may be substantially higher than the load level where the first member yields or collapses.

Probability of failure and reliability
Having defined the term failure, the probability of failure ($P_f$) is defined as the probability of structural system collapse. In analogy with the probability of failure, the probability of reliability is denoted as $P_r$. Since failure and reliability are the only possible outcomes of a probabilistic assessment and they are mutually exclusive the following identity holds:

$$P_f + P_r = 1$$  \hspace{1cm} (2.1)

The calculation of the probability of failure and consequently the reliability of a structure depends on defining the sources of uncertainties. These sources can be divided into two broad groups.

Type I uncertainties
These are physical uncertainties due to the natural variability of a quantity. They are considered as inherent uncertainties and can generally not be improved or controlled. For example, the uncertainties of the environmental conditions and actions (storms, earthquakes) that the structure will experience through its lifetime belong to this group. They are also called as aleatoric and natural.
2 Structural Reliability and Non linear analysis

Type II uncertainties
In this group the following types of uncertainties can be identified:

- Uncertainties in modeling, stemming from imperfect information about the structure or due to simplifications and approximations in engineering models.
- Statistical uncertainties due to limited data.
- Human error related uncertainties.

They are also known as epistemic uncertainties because they are mostly related with lack of knowledge. These types of uncertainties can be reduced and controlled.

Once all the types of uncertainties have been identified and quantified one can calculate the reliability of the structure. In order to do so, three methods are commonly used as given by Jonkman (2015). In Level III methods the uncertain variables are modeled by their probability density functions (pdfs) and the probability of failure ($P_f$) can be calculated exactly by numerical or analytic integration. In Level II methods the reliability problem is simplified by assuming that the uncertain variables are distributed according to a certain type of distribution (usually the normal or log-normal distribution) and the $P_f$ is calculated by means of the reliability safety index ($\beta$). In Level I methods, which is the conventional way of design based on the current structural codes, each of the uncertain variables is described by one representative value. The reliability is implicitly achieved by an appropriate selection of the partial factors. These methods are more explicitly discussed below and appear in a descending order with respect to the accuracy in the reliability calculation.

2.1.1 Level III methods
The performance of a structure or a system can be described with reference to a set of limit states.

limit state
state beyond which the structure non longer fulfills the design criteria

Let us consider that performance of a system or component is given by only one limit state function (LSF), $G$, expressed as:

$$ G = S - D $$ (2.2)

where,

$S$ = The supply, i.e the resistance term
$D$ = The demand, i.e.the load term

In more general formulations of the reliability problem the function $G$ is termed as the reliability function. It is apparent that for $G>1$ the component or system is safe whereas for $G<1$ is considered as failure. The probability of failure, $P_f$, can be formulated as:

$$ P_f = P(G \leq 0) = P(S - D \leq 0) $$

Let us further assume that the supply, $S$ and the demand $D$ are functions of only one stochastic variable each, and not of many which is usually the case. The pdf for the supply is defined as $f_S(x)$, for the demand as $f_D(x)$ and for the LSF as $f_G(x)$. An illustration of the probability density functions (pdfs) is depicted in Figure 2.1a.
2.1 Structural Reliability Methods

When the supply and demand are independent variables then the probability of failure is expressed as (Vugts, 2013):

$$P_f = \int_{-\infty}^{+\infty} F_S(x) \cdot f_D(x) dx$$  \hspace{1cm} (2.3)

where, $F_S$ is cumulative density function (cdf) of the supply (see Figure 2.1b). The above integral can be calculated either analytically or numerically. The outcome of a Level III method is considered as the true probability since each of the uncertain variables is considered with its exact distribution which is not the case for Level II method as discussed in the following section.

2.1.2 Level II methods

In Level II methods the uncertain variables are modeled by their mean values and their standard deviations and are implicitly assumed as normally or log-normally distributed.

Normally distributed variables

If the stochastic parameters are described with a normal distribution and the reliability function, $G$ is linear, which is the case for equation 2.2, then the mean value ($\mu_G$) and the standard deviation ($\sigma_G$) of the reliability function are defined as:

$$\mu_G = \mu_S - \mu_D$$

$$\sigma_G = \sqrt{\sigma_S^2 + \sigma_D^2}$$  \hspace{1cm} (2.4)

where,

- $\mu_S$ = Mean value of the supply
- $\mu_D$ = Mean value of the demand
- $\sigma_S$ = Standard deviation of the supply
- $\sigma_D$ = Standard deviation of the demand

The probability of failure can be calculated using the cumulative standard normal distribution (Jonkman, 2015):

$$P_f = P(G \leq 0) = \Phi(-\beta)$$  \hspace{1cm} (2.5)
2 Structural Reliability and Non linear analysis

where,

\[ \Phi = \text{The standard normal cumulative distribution function} \]
\[ \beta = \text{The safety or reliability index} = \frac{\mu_G}{\sigma_G} = \frac{\mu_S - \mu_D}{\sqrt{\sigma_S^2 + \sigma_D^2}} \]

Lognormally distributed variables

A lognormal distribution is a distribution where the logarithm of the stochastic variable is normally distributed. Lognormal distributions are normally used when the variables are non-negative values which holds for the supply and demand. In this case the mean value (\( \mu_{\ln G} \)) and the standard deviation (\( \sigma_{\ln G} \)) of the natural logarithm of the reliability function, \( G \) are expressed as:

\[ \mu_{\ln G} = \ln \left( \frac{\mu_S}{\mu_D} \sqrt{\frac{1 + V_D^2}{1 + V_S^2}} \right) \]
\[ \sigma_{\ln G} = \sqrt{\ln \left( \frac{1 + V_S^2}{1 + V_D^2} \right)} \]

where,

\[ \mu_S = \text{The mean value of the non-logarithmized supply variable} \]
\[ \mu_D = \text{The standart deviation of the non-logarithmized demand variable} \]
\[ V_S = \frac{\sigma_S}{\mu_S} = \text{The coeffiecient of variation of the non-logarithmized supply variable} \]
\[ V_D = \frac{\sigma_D}{\mu_D} = \text{The coeffiecient of variation of the non-logarithmized demand variable} \]

The probability of failure can be calculated in the same way as in the normally distributed variables case accounting however for a different safety index \( \beta \).

\[ P_f = P(G \leq 0) = \Phi(-\beta) \] (2.6)

where,

\[ \Phi = \text{The standard normal cumulative distribution function} \]
\[ \beta = \text{The safety or reliability index} = \frac{\mu_{\ln G}}{\sigma_{\ln G}} = \frac{\ln \left( \frac{\mu_S}{\mu_D} \sqrt{\frac{1 + V_D^2}{1 + V_S^2}} \right)}{\sqrt{\ln \left( \frac{1 + V_S^2}{1 + V_D^2} \right)}} \]

2.1.3 Level I methods

In Level I methods all the stochastic variables are characterized by one representative value. The essence is that the representative value of the supply is divided by a partial factor while the representative value of the demand is multiplied by a partial factor. In that way the design values are obtained for which the following must hold:

\[ S_D = \frac{S_k}{\gamma_S} > \gamma_D \cdot D_k = D_D \] (2.7)
2.1 Structural Reliability Methods

where,

\[ S_D = \text{Design value of the supply} \]
\[ D_D = \text{Design value of the demand} \]
\[ S_k = \text{Representative value of the supply} \]
\[ D_k = \text{Representative value of the demand} \]
\[ \gamma_S, \gamma_D = \text{Partial factor of the supply and demand respectively} \]

The partial factors, \( \gamma_S \) and \( \gamma_D \) also termed as partial safety factors, are calculated with the use of Level II methods based on a target reliability level (Vugts, 2013). The representative values are also known as characteristic values that have a prescribed probability of not being violated and are usually expressed as

\[
D_k = \mu_D + k_s \sigma_s \\
S_k = \mu_S - k_D \sigma_D
\]  
(2.8)

where \( k_s \) and \( k_D \) are factors specifying a predetermined number of standard deviations from the mean value. The above described methodology is termed as LRFD or Partial Factor Design (PFD) and is illustrated in Figure 2.2.

![Figure 2.2: Level-I Load and resistance factor design](image)

Level I methods are used in the majority of the structural standards in order to form a guideline for the design. Especially for the case of offshore steel structures, currently two codes are used, namely the ISO 19902 and the API WSD codes. ISO 19902 is a LRFD code and uses the aforementioned procedure. As it was demonstrated above, the LRFD method allows accounting for different partial safety factors for the demand and the supply part. Also the representative values can be chosen somewhat arbitrary since it is the the combination of the representative value and the partial factor that determines the design value. However this is not the case of the WSD codes. The representative values for the demand part are chosen as the best estimates of the true action for the design under consideration and are subsequently applied directly as design action without any factoring as depicted in Figure 2.3. Therefore the safety margins in WSD are inherent in the design resistances only.
However, design situations are usually governed by combinations of different types of actions, each of which justifies use of different partial factor. The method used in WSD can not account for that since no partial factors are used for the action part. On the other hand, LRFD is formatted in such a way that can account for the difference in the uncertainty level each of the action effects, thus engaging the use of different action partial factors. The LRFD is considered as more flexible for including appropriate detail and balance in the formulation of the design situations, and hence is the preferred methodology and used further in the current report.

Having determined the partial factors and consequently the design loads, the next step is to perform structural analysis in order to obtain the design action effects on the structure (stresses, deformations etc). Both ISO 19902 and API WSD are formulated in such a way that the structural analysis performed is linear elastic. The structural checks which are included in both codes are based on elastic response of the structure. Moreover these checks are component based checks, i.e each individual component of the structure should fulfill them. However other methods of structural analysis and design can be engaged as described in the following lines.

2.2 Non Linear analysis and system based design

As discussed above, the methodology of the structural codes is based on linear structural analysis and component based design. This way of structural analysis and design is widely used among the structural engineers. Herein the principles of nonlinear analysis and system based design are introduced and compared with their traditional counterparts.

2.2.1 Linear and Non-linear analysis

The fundamental difference between linear and nonlinear analysis lies in the term stiffness which is a property of a part or a system and relates its response to the applied load as given by the following equation.

\[ F = K \cdot U \]  \hspace{1cm} (2.9)

where

\[ F = \text{is the known vector of nodal loads.} \]
\[ K = \text{is the known stiffness matrix.} \]
\[ U = \text{is the unknown vector of nodal displacements.} \]
2.2 Non Linear analysis and system based design

When a structure deforms under a load its stiffness changes. If this change is regarded as small it makes sense to assume that it remains constant through the loading process. This assumption is the fundamental principle of the linear analysis. If the change of the stiffness (matrix) is not regarded as small it is essential to account for this change into the structural analysis, thus engaging non-linear analysis. There are mainly three sources of non-linear behavior; the material, the geometry and the boundary conditions.

Material nonlinearity

In the linear, elastic theory a linear relation between the stress and the strain is assumed; the strains and stresses developed in the structure are proportional to the acting load. It is also assumed that no permanent deformation occurs and once the acting load is removed the model will return to its original shape and position. Although this simplification is acceptable, if the loads are high enough to cause permanent deformations or if the strains are very large then a non linear material model must be used and plastic behavior must be accounted. In Figure 2.4 a stress-strain diagram, as described by Neal (1977) for typical structural steel in tension is depicted. The relation between stress ($\sigma$) and strain ($\epsilon$) is linear in the elastic range until the upper yield stress is reached at point a. The stress then drops abruptly to the lower yield stress, and the strain then increases at constant stress up to the point b. This behavior being termed purely plastic flow. Beyond b, further increases of stress are required to produce further strain increases, and the material is said to be in the strain-hardening range. Eventually a maximum stress is reached at point c, beyond which the stress decreases due to the formation of a neck in the specimen until rupture occurs at point d.

![Stress-strain relation](image)

**Figure 2.4:** Stress-strain relation

Geometry nonlinearity

Due to the applied load or a combination of loads the structure deforms. For relatively small load values the deformation and consequently the change in the geometry of the structure are considered small and negligible. However as the load increases the change in geometry increases as well, altering in that way the stiffness matrix of the structure. For relatively big changes of the structure’s geometry, one should account for the modified stiffness of the structure. As the deformation evolves the structure’s response becomes more nonlinear. This change in the geometry usually causes the so-called P-∆ effects (Wong, 2009). The deflections in the members of the model may induce secondary moments due to the fact that the ends of the member may no longer be vertical in the deflected position as depicted in Figure 2.5.
2 Structural Reliability and Non linear analysis

**Boundary conditions nonlinearity**

It has already been stated above that as the structure deforms its response becomes more and more non linear. In some cases this change in the geometry can lead to changes in the boundary conditions of the system. This can be caused for example when two bodies come into or out of contact with each other, hence inducing abrupt changes in the stiffness of the system. An example is depicted in Figure 2.6. A beam fixed at the one end is experiencing a tip load, $P$ which induces deformation. Initially the beam and the body $B$ are in no contact, but as the load $P$ increases, the deformation increases as well, until the two bodies get in contact (dotted state in the Figure). This introduces new boundary conditions to the beam analysis which should be accounted in the formulation of the stiffness matrix.

![Figure 2.5: Nonlinear geometry, P-Δ effects](image)

The horizontal load, $H$ induces a deflection, $\Delta$ to the beam. The vertical load $P$ now causes second order moment which is equal to $P\cdot\Delta$.

**Figure 2.6: Nonlinear boundary conditions**

2.2.2 Component-based and system-based design

It has already been stated that the majority of the structural codes is formulated on a component based design, in which the structure is designed in a way that each individual component fulfills a set of specific structural checks under the expected loading conditions. These checks aim at a linear, elastic response of the structure without allowing for any plastic deformations and strains. Based on this concept of design the structure is considered as strong as its weakest component.

The system based design regards the structure as a whole and allows for one or more components to exceed their elastic strength or even fail (e.g. buckle), as long as the overall structural integrity is not impaired. This is achieved in structures which are redundant, i.e. they provide alternative load paths.
## 2.2 Non Linear analysis and system based design

### Redundancy

In structural engineering the term redundancy demonstrates the ability of a structural system to redistribute among its members/connections the loads which can no longer be carried by some other damaged components (Biondini et al., 2008). Unlikely structures exhibiting redundancy, non redundant structures may fail immediately under local damage, such as failure of a load-carrying component.

System based design is strongly related to the nonlinear analysis as it allows for plastic deformations and redistribution of internal forces. Furthermore it finds great applicability in structures where ductile material such as steel is used (Bruneau et al., 2011).

The system based design is usually performed by means of pushover analysis which is a static non-linear method to estimate the ultimate strength of a structure as described in the following lines.

### 2.2.3 Pushover analysis

Pushover analysis is a static nonlinear analysis method. In pushover analysis the structure is subjected to an incremental lateral load until pre-established criteria such as a target displacement of a node or system collapse are met. At every load increment the structure deforms and its stiffness matrix is updated. As an output of pushover analysis is a curve which presents a strength-based parameter against deflection. A typical curve depicts the deflection of one node of the structure against the load factor of the lateral load as illustrated in Figure 2.7.

![Figure 2.7: Typical pushover curve](image)

Such curve is characterized by a first linear response (blue branch) till the first member yield occurs. Following that, the response is non-linear (green branch) and even more members yield till the ultimate load is reached. The ultimate load is located at the point in the curve where after that the deflection is increased without further increase of the load. After the ultimate load is reached the response enters the so called post collapse range (red branch). It becomes evident that pushover analysis employs non-linear analysis, mainly the material and geometry non-linearity. Usually it is performed by means of computer programs which are able to carry out such analysis.
2 Structural Reliability and Non linear analysis

2.3 Benefits from Reliability and Non-linear based design

The notions of reliability based design and non-linear design, as introduced above, can be beneficial when employed in the design phase of a structure. Here these potential benefits are demonstrated with the means of an example. Let us consider a cantilever beam with a point load, $P$ acting at the free end as depicted in Figure 2.8. The cross section of the beam is chosen as a hollow circular section as it is usually used in offshore structures.

![Figure 2.8: Design example of a cantilever beam](image)

The aim of this example is to optimize the beam design firstly using Level I methods and then by means of Level II reliability-based design. This can be achieved by introducing a constrained optimization algorithm:

Minimize the cross section area, $A = \frac{\pi}{4} (H^2 - (H - 2t)^2)$

subject to $F(H, t) > 0$

$0.1 \text{ m} \leq H \leq 1 \text{ m}$

$0.001 \text{ m} \leq t \leq 0.2 \text{ m}$

$H > t$

(2.10)

Here $F$, which is a function of $H$ and $t$, is determined by the selected method of design; for Level I methods it will be formulated based on the LSF whereas for Level II based on the target $\beta$ as discussed below. The constraints for $H$ and $t$ are chosen in the sense to yield a reasonable cross section geometry.

2.3.1 Benefits from reliability based design

For the purpose of demonstrating the benefits from reliability based design the point load, $P$ and the yield stress of steel, $S$ are assumed to be stochastic and normally distributed. Their mean values are denoted $\mu_P$ and $\mu_S$ and their standard deviation $\sigma_P$ and $\sigma_S$ respectively.

Firstly the beam is designed with the use of Level I methods (with safety factors) and after that by means of Level II reliability-based design. For the sake of simplicity let us consider that the safety of the beam is characterized by only one LSF which is related to its bending capacity.

$Z = S - D$

(2.11)

where

$S$ = supply term, i.e. yield stress

$D$ = demand term, i.e. bending stress developed in the cross section

The developed bending stress, $D$ is defined by structural mechanics (Gere, 2004) as:

$D = \frac{P \cdot L \cdot H}{2I_x}$

(2.12)
2.3 Benefits from Reliability and Non-linear based design

where

\[ L = \text{length of the beam} \]
\[ H = \text{diameter of cross section} \]
\[ I_x = \frac{\pi}{64} (H^4 - (H - 2 \cdot t)^4) = \text{cross section’s second moment of inertia} \]

Since the only stochastic variable in the demand term is the tip load \( P \), the mean and standard deviation of the demand are respectively:

\[ \mu_D = \frac{L \cdot H}{2I_x} \cdot \mu_P \]
\[ \sigma_D = \frac{L \cdot H}{2I_x} \cdot \sigma_P \]

The LSF becomes:

\[ Z = S - \frac{32L \cdot H}{\pi (H^4 - (H - 2 \cdot t)^4)} \cdot P \]

The mean and the standard deviation of the LSF can be calculated using equation 2.4 as:

\[ \mu_Z = \mu_S - \mu_D \]
\[ \sigma_Z = \sqrt{\sigma_S^2 + \sigma_D^2} \]

The failure probability is calculated by means of the reliability index \( \beta \) and the cumulative normal distribution.

\[ P_f = \Phi(-\beta) \]
\[ \beta = \frac{\mu_Z}{\sigma_Z} \]

Level I design

As already discussed, in order to design with Level I methods one should first evaluate the characteristic values of the stochastic variables. The majority of the structural codes uses the 95th percentile principle where the characteristic value for the load variables has a 95% probability of non-exceedance and the characteristic value for the resistance variables has a 95% probability of exceedance. This gives a \( k_s \) value\(^1\) of 1.645. Therefore for the normally distributed variables under consideration the characteristic values of the supply and the demand are respectively:

\[ S_k = \mu_S - 1.645 \sigma_S \]
\[ D_k = \mu_D + 1.645 \sigma_D \]

Once the characteristic values have been obtained the design values, \( S_d \) and \( D_d \) can be estimated using the appropriate safety factors, \( \gamma_R \) and \( \gamma_E \) for the resistance and load respectively.

\[ S_d = \frac{S_k}{\gamma_R} \]
\[ D_d = \gamma_E D_k \]

\(^1\)It is calculated using the inverse of the normal cumulative density function (cdf): \( \Phi^{-1}(0.95) = 1.645 \)
Now the LSF can be reformulated by plugging in the design values:

\[
Z = S_d - D_d
= \frac{S_k}{\gamma_R} - \gamma_E D_k
= \frac{\mu_S - 1.645\sigma_S}{\gamma_R} - \gamma_E (\mu_D + 1.645\sigma_D)
\] (2.19)

It is apparent from equations 2.19 and 2.13 that the LSF, Z, is a function of the diameter, H and thickness, t. The constrained optimization algorithm (see eq. 2.10) can be formatted as follows.

Minimize the cross section area, \( A = \pi \left( \frac{H^2}{4} - \left( H - 2t \right)^4 \right) \)

subject to \( Z > 0 \)
\[ 0.1 \text{ m} < H < 1 \text{ m} \]
\[ 0.001 \text{ m} < H < 0.2 \text{ m} \]
\[ H > t \] (2.20)

The numerical values that are assumed for this example are presented in table 2.1. The partial factors \( \gamma_E \) and \( \gamma_R \) are the ones that ISO 19902 suggests. Let the design lifetime of the beam be 20 years. The Coefficient of Variation (CoV) for the tip load and the yield stress are taken as \( CoV_P = 0.25 \) and \( CoV_S = 0.05 \) respectively which are considered reasonable values for offshore structures with the selected lifetime (Efthymiou et al., 1997). The partial load factor \( \gamma_E \) is chosen as varying from 1 to 1.5 whereas for the partial resistance factor a fixed value of 1.05 is chosen.

<table>
<thead>
<tr>
<th>lifetime[ys]</th>
<th>L[m]</th>
<th>( \mu_S )[MPa]</th>
<th>( CoV_S )[-]</th>
<th>( \mu_P )[MN]</th>
<th>( CoV_P )[-]</th>
<th>( \gamma_E )[-]</th>
<th>( \gamma_R )[-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2</td>
<td>355</td>
<td>0.05</td>
<td>1</td>
<td>0.25</td>
<td>1...1.5</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Now the optimization problem can be solved by means of Matlab coding\(^2\). For every value of \( \gamma_E \), based on the specified range, the optimization is performed and an optimized pair of \( H \) and \( t \) is obtained. Then the reliability index \( \beta \) can be calculated using equation 2.16.

For \( \gamma_E = 1.35 \) which is the default environmental partial factor in the ISO 19902 code, the \( \beta \) value associated with the lifetime of the structure is 4.33 which gives a lifetime probability of failure \( (P_{f,l}) = 7.61 \cdot 10^{-6} \). The corresponding \( P_{f,a} \) and the annual reliability safety index \( (\beta_a) \) can be calculated:

\[
P_{f,a} = 1 - \left( 1 - P_{f,l} \right)^{1/\text{lifetime}}
= 1 - \left( 1 - 7.61 \cdot 10^{-6} \right)^{1/20}
= 3.8 \cdot 10^{-7}
\] (2.21)

and \( \beta_a = \Phi^{-1}(P_{f,a}) = 4.945 \)

<table>
<thead>
<tr>
<th>Design method</th>
<th>( \gamma_E )</th>
<th>H[m]</th>
<th>t[mm]</th>
<th>( \beta_a )</th>
<th>( P_{f,a} )</th>
<th>Area[cm(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level I</td>
<td>1.35</td>
<td>1</td>
<td>16.43</td>
<td>4.945</td>
<td>3.8 \cdot 10^{-7}</td>
<td>508</td>
</tr>
</tbody>
</table>

\(^2\)The Matlab built-in function \textit{fmincon} is used which finds the minimum of a constrained multi-variable function.
2.3 Benefits from Reliability and Non-linear based design

Performing this procedure for the whole $\gamma_E$ range, the relation between $\gamma_E$, $\beta_a$ and consequently $P_{f,a}$ can be obtained as depicted in Figure 2.9. It is evident from the graph that the $P_{f,a}$ is very sensitive to the selected $\gamma_E$. Due to this sensitivity it is difficult to capture a target reliability level based on Level I design.

![Figure 2.9: Annual reliability level for $CoV_P=0.25$ and varying $\gamma_E$](image)

In Figure 2.10 the sensitivity of the annual reliability index $\beta_a$ and the corresponding $P_{f,a}$, to varying CoV of the tip load P is plotted for a fixed value of $\gamma_E=1.35$. It is apparent that the use of a single partial factor $\gamma_E$, regardless of the characteristics of the environmental load, leads to non-consistent safety levels.

![Figure 2.10: Annual reliability level for $\gamma_E=1.35$ and varying $CoV_P$](image)

The above mentioned issues can be tackled by means of reliability based design where the base of the design is the target reliability level and the stochastic variables are accounted more explicitly, as demonstrated in the following lines.
Design based on Level II reliability

Firstly a target reliability level should be established. ISO suggests a $P_{f,a}$ for manned structures based on work from Efthymiou et al. (1997) which has a value of $3 \cdot 10^{-5}$. Thus this value is used as the base of the design; we want to achieve a design which has a $P_{f,a} \leq 3 \cdot 10^{-5}$ or in terms of reliability index, $\beta_a \geq 4.013$. The optimization problem introduced in equation 2.10 can be formulated as:

Minimize the cross section area, $A$
subject to $\beta_a \geq 4.013$

$$0.1 \text{ m} < H < 1 \text{ m}$$
$$0.001 \text{ m} < t < 0.2 \text{ m}$$
$$D > t$$

Here the $\beta_a$ is a function of the diameter $H$ and the thickness $t$ (see eq. 2.16). The constrained optimization can be solved again by means of Matlab coding and the results are shown in table 2.3 together with the ones from the previous Level I design. One can see that a reduction approximately 5% in the cross section area can be achieved when the reliability-based design is used.

<table>
<thead>
<tr>
<th>Optimization results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design method</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Level I</td>
</tr>
<tr>
<td>Level II</td>
</tr>
</tbody>
</table>

This procedure is performed for a varying $\gamma_E$ values and a fixed value of $\text{CoV}_P=0.25$ and the results are depicted in Figure 2.11. For each $\gamma_E$ value, the optimized cross section area based on Level I methods is obtained and compared to the optimized cross section area from the Level II based design (see table 2.3). It is clear that the greater the $\gamma_E$ value the lower the ratio. This shows that an overestimated partial load factor can lead to very conservative solutions in comparison to reliability based design.

![Figure 2.11: $\gamma_E$ vs optimized ratio of Areas](image)

$A_{\text{level-II}}$ is the optimized area based on Level II methods and a selected target $\beta_a$. $A_{\text{level-I}}$ is the optimized area based on Level I methods and the selected $\gamma_E$. 

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2.3 Benefits from Reliability and Non-linear based design

It becomes evident that the reliability analysis, when engaged in the design phase, can be beneficial; not only the final product can be more economical, by using less steel, but more importantly a certain level of safety is ensured by choosing a target $\beta$ value as the basis of the design.

2.3.2 Benefits from nonlinear design

It has already been stated that the use of non linear analysis can be beneficial during the design of structures and lead to more economical solutions. This lays to the fact that the non-linear analysis makes full use of material’s strength beyond the elastic limit. In Figure 2.4 a typical stress-strain curve of structural steel is depicted. However, in practice, it is customary to neglect the strain hardening of the material and to utilize mainly the elastic and plastic parts of the stress-strain relationship. To this end, a simple bi-linear approximation is usually adopted. This results in the elastic-perfectly plastic stress-strain model as shown in Figure 2.12.

![Figure 2.12: Perfectly plastic material](image)

Let us now consider the cantilever beam of Figure 2.8. Since here the focus will be on the nonlinear response of the structure, the previously stochastic variables are now deterministic. The tip load $P$ induces a bending stress on the cross section given by equation 2.12. When the bending moment is small, the maximum stress in the cross section is less than the yield stress $\sigma_y$, and therefore the cross section is in the same condition as in ordinary elastic bending with a linear stress distribution, as shown in Figure 2.13b. As the bending moment increases the stresses in the furthest fiber of the cross section increase and at one point they reach the yield stress, $\sigma_y$ (2.13c). At this state the bending moment in the beam is called the elastic moment, $M_{el}$ since the cross section has reached its elastic strength (Gere, 2004).

$$M_{el} = \frac{2\sigma_y I_x}{H} = \frac{\pi H^4 - (H - 2t)^4}{32H} \sigma_y$$

This formula is used for the conventional, elastic design of circular hollow sections. If the bending moment is further increased above the elastic moment $M_{el}$, the strains in the beam will continue to increase and the maximum strain will exceed the yield strain $\epsilon_y$. However, because of perfectly plastic yielding (Figure 2.12), the maximum stress will remain constant and equal to $\sigma_y$, as pictured in Figure 2.13d. Note that the outer fibers of the beam have become fully plastic while a central core (called the elastic core) remains linearly elastic. In that case the bending moment capacity of the beam can be found by integrating the stresses acting on the cross section over the cross sectional area, $A$.

$$M = \int_A \sigma y dA$$
As the bending moment increases still further, the plastic region enlarges and moves inward toward the neutral axis until the condition shown in Figure 2.13e. At this stage the maximum strain in the beam (at the farthest distance from the neutral axis) is perhaps 10 or 15 times the yield strain $\epsilon_y$ and the elastic core has almost disappeared. The bending moment corresponding to this idealized stress distribution, called the plastic moment $M_p$, represents the maximum bending moment capacity and can be calculated using the following equation.

$$M_p = \int_A \sigma y dA = \frac{H^3}{6} - \frac{(H - 2t)^3}{6} \sigma_y$$  \hspace{1cm} (2.25)

![Figure 2.13: Stress states in cross section](image)

The advantage of the nonlinear design can be expressed by means of the shape factor of the cross section which is defined as,

$$f = \frac{M_p}{M_{el}}$$  \hspace{1cm} (2.26)

It is apparent that the shape factor depends solely on the geometrical properties of the cross section. In Figure 2.14 one can see the relation between the shape factor and the ratio $t/R$, with $t$ being the thickness and $R$ the radius of the cross section.

![Figure 2.14: Shape factor of a circular hollow section](image)

Depending on the selected geometrical properties the plastic moment capacity can be up to 1.7 times than the elastic one. This would be the case for a solid circular cross section (not
2.3 Benefits from Reliability and Non-linear based design

hollow). However in practical offshore applications usually a $D/t$ ratio around 60 is selected ($t/R \approx 0.01$) which means that the shape factor is approximately 1.27.

The cantilever beam of the example has already been optimized for Level I methods based on the elastic strength by solving the optimization problem described in equation 2.20 where the limit state function, $Z$, is formulated based on the elastic bending capacity. Solving now the same optimization problem but using the plastic bending capacity instead, a more optimized design is achieved. The results are shown in table 2.4 together with the results from the elastic design which are reproduced. One can see that a 23% reduction in the cross section area can be achieved when the plastic bending capacity is used instead.

| Table 2.4: Elastic versus Plastic strength design results |
|---------------------------------|--------|--------|----------|
| Design method                  | $\gamma_E$ | H[m]  | t[mm]    | Area[cm$^2$] |
| Elastic strength               | 1.35     | 1     | 16.43    | 508          |
| Plastic strength               | 1.35     | 1     | 12.60    | 391          |

The type of nonlinearity used here is the one caused by the material behavior. As discussed earlier the material nonlinearity, coupled with the geometry nonlinearity, are highly dependent with the system based design and pushover analysis, the benefits of which are presented below.

2.3.3 Benefits from System based design

In section 2.2.2 the notions of system based design and pushover analysis have been discussed. The main advantage when employing such methods is that the structure possesses an ultimate strength as a whole and not based on the strength of the individual components. In order to demonstrate these advantages an example of a portal frame with fixed supports and diagonal brace as depicted in Figure 2.15 is used. The assumed numerical values are shown in table 2.5. Since the focus again is on the non-linear system behavior, the values are all deterministic.

<table>
<thead>
<tr>
<th>Table 2.5: Assumed numerical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>L[m]</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>
After performing the pushover analysis, the structural response depicted in Figure 2.16 is obtained where the displacement of node 2 with increasing the load $P$ is plotted. The first yield occurs at load level $1.013 \cdot P$ and its deformed shape is depicted in Figure 2.17a. Based on the elastic theory this would be its ultimate elastic strength. However, the system collapse occurs at a load level $1.474 \cdot P$, with its deformed shape given in Figure 2.17b. Thus an approximate 46% extra strength is gained, indicating how beneficial the system based design can be.
Chapter 3

Design based on ISO

In this chapter the background of the ISO 19902 code regarding the intended reliability level is discussed. A method and a set of assumptions for calculating the structural reliability is presented.

3.1 ISO code definitions

The design philosophy of ISO is to design a structure such that it satisfies a set of limit states:

- Ultimate limit state (ULS)
- Serviceability limit state (SLS)
- Fatigue limit state (FLS)
- Accidental limit state (ALS)

Each of the limit states is verified by means of a set of design situations. In order to determine the design situations, one should firstly identify the exposure level of the structure and then the action and structural models used for the design as discussed in the following sections.

Exposure level

It is a classification system used to define the requirements for a structure based on consideration of life-safety and of environmental and economic consequences of failure (ISO 19900, 1998).

<table>
<thead>
<tr>
<th>Life-safety category</th>
<th>Consequence category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1 High consequence</td>
</tr>
<tr>
<td>S1 Manned non-evacuated</td>
<td>L1</td>
</tr>
<tr>
<td>S2 Manned evacuated</td>
<td>L1</td>
</tr>
<tr>
<td>S3 Unmanned</td>
<td>L1</td>
</tr>
</tbody>
</table>

The notations L1, L2 and L3 used in the above table regard the exposure level of the structure and should not be confused with the reliability levels I, II and III described in section 2.1.
3 Design based on ISO

3.1.1 Action and structural models

In order to design an offshore structure, the engineer should define the model that describes the (environmental) actions and the model associated with the properties and the response of the structure. Each of these two models can be described by a set of characteristics which are:

- deterministic or probabilistic
- linear or nonlinear
- Time independent or time dependent

Deterministic and probabilistic models

The classification between deterministic and probabilistic models depends on each case considered. For instance, deterministic models describing the actions and the structural characteristics are used for the design of bottom founded structures. Probabilistic models are engaged for fatigue related problems or reliability analysis and assessments of structures. Therefore in the current report a probabilistic model for both the action and the structure is used.

Linear and nonlinear models

The actions that act on the structure and its response can be described either by linear or nonlinear relations. In the case of the structural response, the (non-) linearity stems from the material, the geometry and the boundary conditions as described in section 2.2. The (non-) linear action model is usually related with the description of the wave force (e.g. Stokes waves vs linear Airy wave theory).

Time dependent and time independent models

For an action model the distinction between time dependent and time independent is determined based on the scope of the analysis; e.g. for fatigue analysis or motion analysis of floating structures a time dependent action model is needed whereas for design of bottom founded structures a time independent action model should be engaged. Regarding the structural time dependence classification, the dynamic response of fixed offshore steel structures is expected to be small in most of the cases (ISO 19902, 2007). In these cases a quasi-static analysis can be employed.

Quasi-static analysis

Static analysis of a structure subjected to actions that vary slowly in relation to the structure’s fundamental natural period such that the influence of structural accelerations can be either safely neglected or is approximated by using an equivalent quasi-static action.

This classification of the actions and structural models is shown in Figure 3.1 with some typical examples of application. For the case of system reliability analysis of quasi-static responding structures, a probabilistic non linear time independent structural model is required.

As discussed in the previous chapter the fundamental principle of the ISO standard is that the structure is designed with the use of partial safety factors and linear structural analysis. This methodology is applied for each of the limit states. Herein the focus will be on the ULS and ALS and in particular a sub-case of the ALS which is the abnormal environmental actions. The latter case can occur due to the possible exposure to very rare and abnormally severe environmental conditions. The ULS and the abnormal environmental actions are associated with environmental conditions with 100 and 10,000 years return period respectively.
Figure 3.1: Main characteristic of action and structural models with typical examples of applications
3 Design based on ISO

Return period

The term return period (RP) is used to determine the probability of exceedance of an event per year as defined below:

\[
\text{Probability of exceedance per year} = \frac{1}{\text{Return period}}
\]

According to the design procedure from ISO 19902 if a structure satisfies the ULS case then the abnormal environmental case is normally satisfied as well. In the following section, the procedure to design offshore steel structures according to ISO 19902 for the ULS and ALS cases is described.

3.2 Design for the ULS and ALS cases

The basic requirement for satisfying the ULS is that the structure possesses an adequate strength to meet a minimum safety level. This requirement can be met in two ways:

- By explicitly demonstrating that a structure has a certain minimum Reserve Strength Ratio (RSR)
- By implicitly following a partial factor design format, which is the usual design and assessment procedure

3.2.1 Design based on RSR value

All new structures complying with ISO are intended to have a similar margin of safety for the same exposure level. The system reserve strength gives the structure a margin to withstand environmental actions affects that exceed the design action effects and is expressed as RSR as:

\[
RSR = \frac{F_{\text{collapse}}}{F_{100}}
\]  

(3.1)

where,

- \(F_{\text{collapse}}\) is the unfactored global environmental action which, when co-existing unfactored permanent and variable actions are added, causes collapse of the structure;
- \(F_{100}\) is the unfactored environmental load with return period 100 years.

Since the RSR value depends on the ultimate collapse load it can be calculated by means of pushover analysis. However, ISO does not give a specific RSR value for the design of new structures, due to lack of common agreement of the authors on the setting of appropriate RSR criteria. Moreover at the time of writing ISO 19902 few engineers had experience with pushover analysis while the overwhelming majority had extensive experience with component based design. Thus this method can only be adopted if it is approved by the owner and the regulator. Such approval shall include the methodology to be used and the RSR value to be achieved. Due to these difficulties the partial factor design format is the preferred method from ISO.
3.2 Design for the ULS and ALS cases

3.2.2 Design based on partial factor design format

In this case an LRFD method is employed. For quasi-statically responding structures a design action should be calculated given by:

\[ F_d = \gamma_{f,G} \cdot G + \gamma_{f,Q} \cdot Q + \gamma_{f,E} \cdot (E + \gamma_{f,D} \cdot D_E) \]  \hspace{1cm} (3.2)

where,

- \( F_d \) = The design action
- \( G \) = representative permanent actions
- \( Q \) = representative variable actions
- \( E \) = representative environmental actions
- \( D_E \) = equivalent quasi-static action representing the dynamic action effect
- \( \gamma_{f,G} \) = partial action factor for permanent actions
- \( \gamma_{f,Q} \) = partial action factor for variable actions
- \( \gamma_{f,E} \) = partial action factor for environmental actions
- \( \gamma_{f,D} \) = additional partial factor for \( D_E \)

In jacket type of structures, the dynamic effects, described by \( D_E \) have a very small effect in the response of the structure and hence, can be neglected. For the ULS case the representative environmental action, \( E \) is associated with the environmental conditions with 100 years return period. Therefore the 100 year return period environmental load \( (E_{100}) \) and the environmental partial action factor \( (\gamma_{f,E}) \) are determined for the location under consideration. The \( \gamma_{f,E} \) is not a single numerical value for all structures and all environments, but mainly depends on:

- the intended reliability level of the structure
- the characteristics of the long-term environmental conditions at the structure’s specific location.
- The structural system considered.

Thus, \( \gamma_{f,E} \) should be calculated explicitly for each structure that is designed. However, ISO allows a value of \( \gamma_{f,E}=1.35 \) when no other information is available. Now the design action, \( F_d \) can be calculated and subsequently the stresses on the structural members. After that the linear structural checks with their corresponding resistance partial factors are performed. The members are re-sized if they do not fulfill the checks, or in case they are considered as non-optimized. The optimization condition is checked by means of the utilization ratio, \( U \) which is defined as

\[ U = \frac{E}{R} \]  \hspace{1cm} (3.3)

where,

- \( E \) = Factored action effect
- \( R \) = Factored resistance

The optimization ratio is calculated for each member and for each structural check (tension, bending etc). Since the environmental load depends on the members size, an iterative procedure is required in order to obtain an optimized structure. This process is illustrated in Figure 3.2.
3 Design based on ISO

According to this design methodology the structure’s response is linear elastic for the ULS case (with partial factors), as well as for the abnormal environmental condition in the ALS case. The latter case is normally\(^1\) implicitly satisfied based on the ULS design. However, the structure should be checked if it fulfills the structural checks for linear response under the ALS case with all the partial safety factors set to 1.0. This design philosophy is depicted in Figure 3.3.

\(^1\)This holds as long as there is no wave impingement on the topside.
3.2 Design for the ULS and ALS cases

![ISO reliability philosophy for NS structures](image)

Figure 3.3: ISO reliability philosophy for NS structures

The design is based on the 100 year return period load with safety factors and elastic analysis. The response is expected to be linear up to the environmental load with return period 10,000 years. Then for higher load levels, the response is non linear till the system collapses.

It is noteworthy that ISO does not give an explicit value for a target reliability. This means that there is no clear indication for the total system collapse condition of the structure as depicted above, neither in terms of an ultimate load level, RSR value, nor a target probability of failure. The proposed design procedure is based solely on the default value, $\gamma_{f,E} = 1.35$. However adopting a single environmental factor for the design without considering parameters such as the specific metocean characteristics of the structure’s location can lead to non consistent reliability results. This was shown in Figure 2.10 for the simple example of a cantilever beam and is reproduced below. This time the x-axis represents the CoV of the environmental load. This issue is discussed in the following section.

![Reliability level for varying $E_{CoV}$ and $\gamma_{f,E}=1.35$](image)

Figure 3.4: Reliability level for varying $E_{CoV}$ and $\gamma_{f,E}=1.35$
3.3 Reliability level on ISO

As was described section 3.2, the intended reliability of ISO code is achieved either by capturing a target RSR value or by the conventional LRFD format. It was noted though, that no specific RSR value is mentioned in the standard. It is only in the informative annex A of the code that one can find a suggested value of minimum RSR=1.85 for a conventionally framed structure in the North Sea as given by van de Graaf et al. (1994). Contrary, the proposed default value for the $\gamma_{f,E}$=1.35 can be found in the normative annex H of the regulation as was originally introduced by Moses (1981). The fact that a specific value of $\gamma_{f,E}$ is given as a norm in the ISO code whereas the suggested RSR value has a informative role seems irrational (Waegter, 2015).

Moreover ISO does not include in its normative body any specific target reliability values. However in the informative annex A an $P_{f,a}$ depending on the exposure level taken from Efthymiou et al. (1997) is referenced. These values regard new manned installations with L1 exposure level and new unmanned installation with L2 exposure level and are shown in table 3.2.

<table>
<thead>
<tr>
<th>Exposure Level</th>
<th>Target annual probability of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>$3 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>L2</td>
<td>$5 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

These suggested probabilities are obtained from Cost Benefit Analysis (CBA) and are based on the 'As Low As Reasonably Practicable' (ALARP) principle (HSE,1987).

3.4 Probability of failure of offshore structures

The probability of failure has been introduced and discussed in chapter 2.1 by means of a simple example of a cantilever beam. Both the demand and supply term were described by one variable each, $D$ and $S$ respectively, thus forming a two variable reliability model. This was a reasonable assumption for the case of the cantilever beam since the demand was a function of the tip load only and the resistance of the yield stress. However, in the case of an offshore structure the picture is more complex. The demand term (load) stems from permanent, variable and environmental loads with the latter being a function of wind, wave and current actions. Furthermore the supply term (resistance) of the structure is a function of the resistance of each component of the structure. The combination of these component strengths gives the resistance of the structure as a whole. However some reasonable assumptions can be made in order to apply the two-variable reliability model as discussed in the next section.

3.4.1 Reliability Assumptions

Demand term

When a quasi-statically responding structure is subjected to extreme environmental actions, the action effects that are relevant to the system collapse may be reasonably assumed to be proportional to a global action (Vugts, 2013). This global action can be either the base shear or the overturning moment. This means that the multi-variable demand problem can be adequately reduced to a single variable demand model which can be described with a probability density function (pdf), $f_E$. In the current document the related global action is the base shear, and hence
the $f_E$ describes the distribution of the base shear. The expression of this pdf can be calculated with long term statistics and is mainly dependent on:

- The metocean characteristics of the structure’s location
- The lifetime of the structure

**Supply term**

In the same manner as the demand term, the supply term (resistance) of the structure can be related with a global reaction of the structure such as the base shear or the overturning moment. Since the demand term is related to the base shear it is reasonable to assume the same for the supply term. Therefore the pdf of the supply term is defined as $f_R$ and describes the base shear reaction of the structure.

Contrary to the load, the (ultimate) resistance of the structure is assumed to remain unchanged over its service life; there are no aging effects and no degradation. This is considered as a reasonable assumption as long as measures are taken during the design operation and construction of the structure as discussed by El-Reedy (2012).

### 3.5 Reliability calculation

Previously it was defined that both the demand and supply terms can be related to the base shear. An other way of expressing the demand and supply pdfs is by relating them as a multiple of a reference load. Since the design recipe from ISO is based on the 100 year return period environmental load ($E_{100}$) it is reasonable to set as reference load the base shear with return period 100 years. Let us hence define two new variables, $E$, $R$ as:

- $E$: The environmental load associated with the lifetime of the structure and expressed as multiple of the $E_{100}$. For example, $E_{20}=0.8$ means that the environmental load for a structure with lifetime of 20 years is 0.8 times the $E_{100}$.
- $R$: The resistance of the structure expressed as a multiple of the $E_{100}$, i.e. the Reserve Strength Ratio (RSR)

![Figure 3.5: Load and Resistance pdfs](image)

The associated pdfs of $E$ and $R$ are $f_E$ and $f_R$ respectively and are depicted in Figure 3.5. In order to determine these pdfs one should identify the sources of uncertainty that an offshore
structure experiences. The load uncertainty is associated with the randomness in the environmental, permanent and live loads. The resistance uncertainty stems mainly from the randomness in the material, the geometry and the behavior of the foundations. Except from the aforementioned uncertainties, one should also account for the wave force model uncertainty which is related to the hydrodynamic coefficients and the marine growth. An overview of these uncertainties is depicted in figure 3.6.

Figure 3.6: Typical sources of uncertainty in fixed offshore structures

Once the uncertainties are determined they can be used in order to obtain the load pdf \( f_E \) and resistance pdf \( f_R \) as described later in chapter 5. It is referenced in Efthymiou et al. (1997) that if both \( f_E \) and \( f_R \) are approximated with a log-normal distribution the reliability safety index, \( \beta \) can be calculated as:

\[
\beta = \ln \left[ \frac{R_m}{E_m} \sqrt{\frac{1 + V_E^2}{1 + V_R^2}} \right] \sqrt{\ln \left( \frac{1}{1 + V_R^2} \right) \left( 1 + V_R^2 \right)}
\]  

(3.4)

where,

\[ R_m = \text{The mean RSR value} \]
\[ E_m = \text{The mean environmental load} \]
\[ V_R = \text{The CoV of the resistance} \]
\[ V_E = \text{The CoV of the environmental load} \]

Once the above values are estimated, the reliability index \( \beta \) value can be calculated. Note however, that this \( \beta \) value is associated with the lifetime of the structure. The above used formula
is similar to the one used for Level II reliability method in section 2.1.2 where it was noted that it can be used in case both the stochastic variables, namely $E$ and $R$ are log-normally distributed. In case one of the variables is expressed by a different distribution, this method does not hold and one should engage Level III reliability methods as described in section 2.1.1.
Chapter 4

Case study

In order to have a more thorough look into the notions of the previous chapter, a case study of an offshore structure is engaged. An offshore field in the Southern North Sea (SNS) is selected for exploitation and a bottom founded structure is designed. Since the focus of the current report is on the reliability of the offshore structures in extreme conditions, the structure is firstly designed and optimized for the ULS and ALS cases.

4.1 Site conditions and requirements

An offshore structure with a lifetime of 20 years should be designed for the exploitation of the field which is located in the SNS as depicted in Figure 4.1. Typical metocean conditions for the location under consideration are used and are given in appendix A. The ones that are of interest here are associated with the 100 year return period environmental load ($E_{100}$) and the 10,000 year return period environmental load ($E_{10,000}$) as shown in table 4.1. These values regard an omnidirectional environmental load with the same characteristics from every possible direction. The site conditions are presented in table 4.2.

![Figure 4.1: Case study location](image-url)
Table 4.1: Metocean long-term characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Return period [years]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Maximum wave height</td>
<td>[m]</td>
<td>16.85</td>
</tr>
<tr>
<td>Wave period</td>
<td>[sec]</td>
<td>12</td>
</tr>
<tr>
<td>Wind speed</td>
<td>[m/sec]</td>
<td>27.37</td>
</tr>
<tr>
<td>Current speed</td>
<td>[m/sec]</td>
<td>0.93</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum wave height</td>
<td>[m]</td>
<td>21.29</td>
</tr>
<tr>
<td>Wave period</td>
<td>[sec]</td>
<td>13.8</td>
</tr>
<tr>
<td>Wind speed</td>
<td>[m/sec]</td>
<td>30.94</td>
</tr>
<tr>
<td>Current speed</td>
<td>[m/sec]</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Table 4.2: Site conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth</td>
<td>[m]</td>
<td>28</td>
</tr>
<tr>
<td>Tidal range</td>
<td>[m]</td>
<td>3</td>
</tr>
<tr>
<td>Storm surge</td>
<td>[m]</td>
<td>1.2</td>
</tr>
</tbody>
</table>

For such a structure a topside that hosts the staff and the necessary equipment should be accounted. Some reasonable assumptions were made based on the lectures from Bottom Founded Offshore Structures course given in TU Delft (Sliggers and Hoving, 2014). Therefore, the requirements for the topside are given in table 4.3.

Table 4.3: Structure requirements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broad-side length</td>
<td>[m]</td>
<td>30</td>
</tr>
<tr>
<td>End-on side length</td>
<td>[m]</td>
<td>20</td>
</tr>
<tr>
<td>Topside height</td>
<td>[m]</td>
<td>20</td>
</tr>
<tr>
<td>Topside weight</td>
<td>[T]</td>
<td>1700</td>
</tr>
<tr>
<td>Number of conductors</td>
<td>[-]</td>
<td>6</td>
</tr>
<tr>
<td>Diameter of conductors</td>
<td>[inch]</td>
<td>32</td>
</tr>
<tr>
<td>Thickness of conductors</td>
<td>[inch]</td>
<td>1</td>
</tr>
<tr>
<td>Marine growth</td>
<td>[mm]</td>
<td>50</td>
</tr>
<tr>
<td>Characteristic yield stress of piles</td>
<td>[MPa]</td>
<td>430</td>
</tr>
<tr>
<td>Characteristic yield stress of other members</td>
<td>[MPa]</td>
<td>345</td>
</tr>
</tbody>
</table>

4.1.1 Water Depths

In order to determine the loading conditions on the structure, firstly the various water depths need to be defined. These include the mean seal level (MSL) and the maximum water depth (Dmax) as depicted in Figure 4.2.

The MSL is the water depth given in table 4.1 whereas for the Dmax one should account for the deviations in the water level due to tide, storm surge and settlement caused by the structure’s self weight:

\[
D_{\text{max}} = \text{MSL} + \text{half the tidal range} + \text{storm surge} + \text{settlement}
\]

\[
= \text{Highest Astronomical Tide (HAT)} + \text{storm surge} + \text{settlement}
\]

\[
= 28m + 1.5m + 1.2m + 0.3m
\]

\[
= 31m
\]

For the further calculations in the current study Dmax is considered as the design water level in order to determine the extreme wave, current and wind forces on the structure.
### 4.1 Site conditions and requirements

<table>
<thead>
<tr>
<th>Water Level Description</th>
<th>Water Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Still Water Level</td>
<td>HAT</td>
</tr>
<tr>
<td>Highest Astronomical Tide (HAT)</td>
<td></td>
</tr>
<tr>
<td>Mean Still Water Level (MSL)</td>
<td></td>
</tr>
<tr>
<td>Lowest Astronomical Tide (LAT)</td>
<td></td>
</tr>
<tr>
<td>Min. Still Water Level</td>
<td></td>
</tr>
<tr>
<td>Positive Tide Range</td>
<td></td>
</tr>
<tr>
<td>Astronomical Tide Range</td>
<td></td>
</tr>
<tr>
<td>Negative Storm Surge</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.2:** Definition of water levels

#### 4.1.2 Wave field

For each of the ULS and ALS cases a regular wave is considered that acts on the structure with the characteristics given in table 4.1. A regular wave, despite being an abstract representation of a real sea state, it can be usefully employed in the analysis of quasistatically responding offshore structures (Vugts, 2013). In order to describe the regular wave characteristics there are several wave theories which mainly depend on the wave height, \( H \), wave period, \( T \) and the water depth, \( d \). These wave theories have an affect on how the wave height is considered. In linear wave theory the waves are sinusoidal in space and time; they are symmetric with respect to the wave crest and wave trough as well as symmetric with respect to the MSL. In nonlinear wave theory the symmetry with respect to the MSL does not hold. The wave has higher peaks and lower troughs. The symmetry with respect to the vertical line is maintained. Figure 4.3, given in Holthuijsen (2007), illustrates the relationship between the wave theories and the wave and site characteristics.

**Figure 4.3:** Wave theories
Considering the given metocean characteristics for the waves associated with the ULS and ALS one can see that they both fall into the 5th order theory. Therefore the wave crest and trough should be calculated which is performed using the program USFOS. The results are given in table 4.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Return period [years]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave crest</td>
<td>[m]</td>
<td>11.29 15.36</td>
</tr>
<tr>
<td>Wave trough</td>
<td>[m]</td>
<td>5.56 5.93</td>
</tr>
</tbody>
</table>

These wave crests, and especially the wave crest associated with the wave with 10,000 year RP, coupled with the $D_{max}$ are used to determine the height of the structure.

### 4.1.3 Current field

The current speed ($U_c$) that has been given in table 4.1 regards the current speed at the water surface. In order to calculate the $U_c$ over the depth, a power law profile can be used:

$$U_c(z) = U_{c0} \left( \frac{z + d}{d} \right)^\alpha$$  \hspace{1cm} (4.1)

where,

$U_{c0}$ = is the current speed at the water surface  
$z$ = is the vertical coordinate  
$d$ = is the water depth  
$\alpha$ = is an exponent, taken as 1/7

### 4.2 Structure description

Based on the given requirements for the structure as well as the water depth a conceptual design of the structural geometry of the jacket has been performed. The structural configuration is determined from guidance described in Vugts (2013) and Sliggers and Hoving (2014). A jacket type structure has been selected as depicted in Figure 4.4.
The structure consists of 4 legs, 4 piles and 6 conductors. There are three horizontal frames and X braces at the two transverse directions. The height of the structure has been decided on the basis to avoid having wave impingement on the topside. In case a wave hits the topside an abrupt change in the load profile is observed since a bigger area is experiencing the wave load (Haver, 2004). To this end, the structure’s design is such that the elevation of the topside is at a higher level than the level of the maximum wave height which is the one associated with the 10,000 year RP as depicted in Figure 4.5.
Once the site conditions, the metocean characteristic and the structure’s geometry have been defined, the loads on the structure can be calculated. These are stemming mainly from the structure’s and the topside’s self weight as well as from the wave, current and wind. The self weight is applied in a straightforward manner while the environmental loads are calculated as follows.

### 4.2.1 Wave and current induced loads

The force that acts on the members of the structure due to wave and current is calculated according to the Morison’s equation:

\[
F(t) = \frac{1}{4} \rho C_m D^2 \ddot{v} + \frac{1}{2} \rho C_D D v |v|
\]

where

- \( v \) = fluid particle velocity \([\text{m/s}]\)
- \( \dot{v} \) = fluid particle acceleration \([\text{m/s}^2]\)
- \( D \) = Cross section diameter \([\text{m}]\)
- \( \rho \) = mass density of fluid \([\text{kg/m}^3]\)
- \( C_m \) = mass coefficient, [-]
- \( C_D \) = drag coefficient [-]

The mass coefficient \((C_m)\) and drag coefficient \((C_D)\) are taken as 2.0 and 0.7 respectively as given in Sliggers and Hoving (2014). In USFOS this formula is used in order to calculate the
4.3 Structural analysis assumptions

hydrodynamic forces from wave and current. The specified regular wave is 'stepped' through the structure with a selected time increment (e.g. 0.1 sec). On each step the forces on the members are calculated as well as the base shear and overturning moment of the structure. The wave forces at the time (phase) giving either the maximum base shear or overturning moment are kept as the design loadcase.

4.2.2 Wind induced loads

Wind is a phenomenon that varies with time and could cause dynamic response to the structure. As already stated though, offshore steel jackets are considered rather stiff and exhibit a quasistatic response to the environmental loads, including wind. For this kind of structures the wind action can be calculated as (ISO 19902, 2007):

\[ F = \frac{1}{2} \rho_a U_w^2 C_s A \]  

where,

- \( \rho_a \) = the air’s density
- \( U_w \) = the wind speed
- \( C_s \) = the shape coefficient
- \( A \) = the area of the object

4.3 Structural analysis assumptions

In order to perform the structural analysis a number of reasonable structural assumptions have been made. These are related to the modeling of the topside, the conductors and the assumptions regarding the foundations.

4.3.1 Topside

In the structural analysis of the current study a topside has not been modeled as an actual object, rather the loads that are associated with it are transferred to the structure. However its contribution to the structural stiffness could not be entirely omitted. Therefore, a very stiff shell element, representing the topside’ deck has been introduced to the upper part of the structure. In that way the degrees of freedom of the upper nodes are coupled. The high stiffness of the shell element has been achieved by assigning an increased modulus of elasticity and a relative big thickness to the shell element.

Furthermore the center of gravity of the topside is assumed to be at its geometrical center and at an elevation of 10 m above the upper part of the structure as depicted in Figure 4.6a. Since no actual topside is modeled the wind load is transferred to the center of the stiff element as a nodal load and a moment as shown in Figure 4.6b.
4.3.2 Conductors

As per the requirements of the structure, in total 6 conductors are included. The conductors make a limited contribution to the system strength as given in El-Reedy (2012). Therefore they are modeled as purely load attracting members; they only transfer the environmental load acting on them, to the rest of the structure and do not contribute in the overall stiffness. Moreover, the conductors are restrained horizontally but not vertically by the horizontal frames. Hence their own weight is directly transferred to the soil.

4.3.3 Foundation

In the offshore jacket type of structures, the foundation of the structure is achieved through the piles. The piles are driven into the soil and transfer to it all the loads that act on the structure. The pile-soil interaction is a rather complex issue due to the strong non-linearity of the soil’s mechanical properties. This mechanical behavior is also difficult to predict due to the inhomogeneous nature of the soil (Verruijt, 2006). However in the current study a simplified foundation model as given by Vugts (2016) is used. The pile’s end is modeled with a pinned support at at depth of $d_e = 3 \cdot D$, with $D$ being the diameter of the pile as depicted in Figure 4.7.
Furthermore as described in Bomel (2003), the foundation of the structure does not significantly influence the failure mechanism. It is noted that any redistribution of force due to the non-linear soil behavior is assumed to be secondary effect and hence neglected. Therefore, since the focus of the current study is on the reliability and the ultimate strength of steel offshore structures, the foundation model introduced above is considered reasonable.

### 4.4 Structural design and optimization

Once the site conditions, the structural geometry and the assumptions have been defined, the next step is to perform the structural analysis and design the structure is an optimum manner. The structure is modeled in USFOS, the self-weight and the environmental loads are imported and the structural analysis is performed. The optimization is performed with respect to the structural checks for the ULS case as given in ISO 19902, aiming at utilization ratios lower than one as well as minimizing the steel weight required. With this concept a number of practical assumptions have been made as given in the next section.

#### 4.4.1 Structural design assumptions

**Structural groups**

For practical reasons the structure’s components have been divided into 12 major groups based on their function as depicted in shown Figure 4.8 with some remarks regarding the conductors and the topside element. As described earlier, the geometry of the conductors is given by the requirements of the structure. Additionally they are considered as non-structural elements, hence they are not included in the design optimization. The shell element depicted in Figure 4.8l, is modeled solely to represent the contribution of the topside to the system’s stiffness; it is also excluded from the design optimization. Thus the groups that are considered for the optimization are the ones depicted in Figures 4.8a-4.8j. It is assumed that, among the members of each group, the same cross section geometry is used.

**Cross section database**

In order to perform the optimization a database of tubular cross sections should be defined. Here the commercially available cross sections are used where the diameter go up in steps of 2 inches and the wall thickness in steps of 0.125 inch (Tenaris, 2014). Furthermore the cross section database is filtered by keeping only the cross sections for which holds $20 \leq D/t \leq 60$, with $D$ being the outer diameter and $t$ the thickness. The lower limit is set to ensure that steel plates can be cold-rolled into tubulars, whereas the upper limit is set to prevent local shell buckling becoming more dominant than global member buckling (Vugts, 2016).

**Connectivity sequence**

The last practical assumption that has been accounted for the optimization design is the connectivity sequence. The diameters of each member should be smaller than the diameter of the member to which it is connected, in order to allow for welding operations. For example the diameter of the X braces should be smaller than the diameter of the legs. We should also add here that the diameter of the legs should be at least 6 inches bigger than the leg’s in order to achieve the shim connection between the legs and the piles.
Figure 4.8: Structure’s groups
4.5 Structural optimization procedure

In order to perform the structural optimization, a code is developed which allows interaction between USFOS and Matlab. The structural analysis is performed in USFOS, the member’s forces are imported in Matlab where the structural checks are performed and the structure is optimized for every possible attack angle of the environmental load. This is depicted in Figure 4.9 where $E$ is the environmental load and $\theta$ indicates its attack angle; $\theta = 0^\circ$ is for end-on loading whereas $\theta = 90^\circ$ is for broadside loading.

![Figure 4.9: Environmental attack angle](image)

The optimization is performed for every possible attack angle with a step of 5 degrees. Since the hydrodynamic force on the members is highly dependent on their size, an iterative procedure is necessary. An overview of the described procedure is given in Figure 4.10 by means of a diagram. On each step the optimized geometry is stored in order to determine at the end which geometry manages to fulfill the structural checks defined in ISO for every possible attack angle.

After performing the above explained procedure, it turns out that the dominant case for the sizing of the members is for an attack angle of $90^\circ$ (broadside loading). This is considered reasonable since in the broadside loading the area experiencing environmental load is relatively big. Additionally in the broadside loading, the structure demonstrates lower resistance compared to the end-on loading; in the former case the distance between the piles supports resisting the overturning moment is smaller than in the latter case.

The optimized member sizes, based on the environmental load with $90^\circ$ attack angle, are presented in table 4.5. This optimized set of cross sections is checked again for every possible attack wave to verify if the structural checks are fulfilled. As described in section 3.2.2 the checks for the ALS should also be fulfilled with all the partial factors set to 1.0. Therefore the $E_{10,000}$ is also applied on the structure for every attack angle. It is found that only in a few cases (attack angles) the structural checks from ISO are violated. However the utilization ratios in these cases, barely pass the limit of one; the largest is found for an attack angle of $40^\circ$ where the utilization ratio at the piles is 1.015 due to tension-bending in one of the members in the group 'bot peripheral’. These rather small violations of the structural checks can be reasonably considered negligible, hence the cross sections that appear on the table are the final ones and used later on the reliability analysis.
Figure 4.10: Diagram of the structural optimization for ULS design
4.6 Verification of quasi-static behavior

As described in chapter 3, the offshore steel jackets are expected to respond quasi-statically to the environmental loads. Here this condition is checked for the structure under consideration. According to ISO 19902 if the natural period of the structure is less than 1/5 of the peak wave period of the design sea state, then the dynamic effects can be considered negligible. Thus the natural period of our structure should be calculated. This is performed in USFOS accounting for the topside mass, the hydrodynamic added mass and a Rayleigh damping. The first natural frequency was found to be 1.12 sec whereas the 1/5 of the wave period is 2.4 sec. Thus the simplified check of the ISO 19902 is fulfilled and the dynamic response may be neglected.

<table>
<thead>
<tr>
<th>Group</th>
<th>Diameter [inch]</th>
<th>thickness [inch]</th>
</tr>
</thead>
<tbody>
<tr>
<td>legs</td>
<td>66</td>
<td>1.125</td>
</tr>
<tr>
<td>piles</td>
<td>60</td>
<td>1.125</td>
</tr>
<tr>
<td>upper frame</td>
<td>14</td>
<td>0.375</td>
</tr>
<tr>
<td>mid frame</td>
<td>14</td>
<td>0.375</td>
</tr>
<tr>
<td>bot frame</td>
<td>12</td>
<td>0.375</td>
</tr>
<tr>
<td>upper X braces</td>
<td>24</td>
<td>0.5</td>
</tr>
<tr>
<td>down X braces</td>
<td>28</td>
<td>0.625</td>
</tr>
<tr>
<td>upper peripheral</td>
<td>16</td>
<td>0.375</td>
</tr>
<tr>
<td>mid peripheral</td>
<td>16</td>
<td>0.375</td>
</tr>
<tr>
<td>bot peripheral</td>
<td>16</td>
<td>0.375</td>
</tr>
</tbody>
</table>
Chapter 5

Case study - Reliability

In the previous chapter the case study structure has been designed and optimized for the ULS case as per the ISO 19902 design recipe. Herein the achieved reliability level is calculated and its influence on a number of parameters is discussed.

5.1 Definition of uncertainties

As discussed in chapter 3, the reliability of an offshore structure has been related with the environmental load associated with the lifetime of the structure, as well as the structure’s Reserve Strength Ratio (RSR). More precisely, the load and resistance pdfs are of interest for the calculation of the structure’s reliability. In case they are both log-normally distributed, the reliability can be calculated by means of Level II methods, using formula 3.4. If this condition does not hold then Level III methods should be engaged. It is therefore important to describe properly the load and resistance distributions as discussed in the following sections.

5.1.1 Load uncertainty

The load uncertainty stems from the randomness in the environmental, permanent and live loads as depicted in Figure 3.6. However, among them, the uncertainty due to the environmental load is considered predominant and the other two can be neglected as given in Nizamani (2015). The variability in the environmental load acting on an offshore structure arises from the natural variability in the wave, wind and current, hence it is a Type I uncertainty. As discussed in section 3.5 in order to calculate the reliability of a structure the distribution of the global base shear due to the environmental loading associated with the structure’s lifetime should be defined. This distribution must reflect the sea state randomness in the short-term as well as in the long term variations. In order to achieve this condition some certain requirements should be met as described in Vugts (2013):

- A reliable site-specific database, related to the long term and the short term metocean characteristics should be available. This database should describe the joint pdfs of wave, current and wind related to the structure’s lifetime. If the database is short compared to the structure’s lifetime then robust extrapolation is required.

- An accurate model is needed in order to translate these metocean characteristics into a global response such as the base shear.

For the case under consideration, the load uncertainty is described with a lognormal dis-
based on the findings in Efthymiou et al. (1997), where the mean value of the environmental load \((E_m)\) and the coefficient of variation of the environmental load \((V_E)\) are given. These values account for the short term as well as the long term variability in the environmental parameters and are shown in table 5.1. Be reminded that, here the environmental load, \(E\), is the relative base shear load with respect to the 100 year return period base shear \((BS_{100})\); for the SNS location, it is interpreted that the relation between the mean base shear over a lifetime of 20 years and the \(BS_{100}\) is \(E_m=0.84\cdot BS_{100}\).

<table>
<thead>
<tr>
<th>Geographical area</th>
<th>Mean value (E_m)</th>
<th>CoV (V_E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern North Sea (SNS)</td>
<td>0.84</td>
<td>0.212</td>
</tr>
<tr>
<td>Northern North Sea (NNS)</td>
<td>0.81</td>
<td>0.265</td>
</tr>
<tr>
<td>Gulf of Mexico (GoM)</td>
<td>0.79</td>
<td>0.320</td>
</tr>
<tr>
<td>Northwest coast of Australia (AUS)</td>
<td>0.78</td>
<td>0.330</td>
</tr>
</tbody>
</table>

Since these values are not given as an absolute force magnitude, but are normalized with respect to the \(BS_{100}\), they are expected to adequately describe the response regardless of the attack angle considered. For example the relation between the \(E_m\) and the \(BS_{100}\) in the broadside direction is expected to be the same as the one in the end-on direction.

The log-normal parameters in our case are taken from the SNS related values and the obtained pdf is depicted in Figure 5.1.

![Base shear distribution relative to BS\(_{100}\) for the SNS location](image)

**Figure 5.1**: Base shear distribution relative to BS\(_{100}\) for the SNS location

*The values at the x axis are normalized with respect to the BS\(_{100}\)*

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\(^{1}\)Describing the environmental load using a lognormal distribution is also supported by a number of published reports (Moses and Stahl, 2000; Aker Offshore Partner A.S, 1999)
5.1 Definition of uncertainties

5.1.2 Wave force uncertainty

The variability discussed in the previous section stems from the natural variability mainly in the wave characteristics. A further source of uncertainty in the reliability calculations is associated with the potential deficiencies in the wave force model (hydrodynamic coefficients, marine growth etc.), which is not included in the values given in table 5.1. This is a type II uncertainty which has been studied by Digre et al. (1995) where it was determined that it can be adequately represented by a CoV=8% which should be included in the environmental load CoV. For the SNS location this is done as follows:

\[ V_E = \sqrt{0.212^2 + 0.08^2} = 0.227 \]

This way of including the wave force uncertainty in the calculations is also referenced and used in other published reports (Efthymiou et al., 1997; Tromans, 2000), and adopted further in the current report.

5.1.3 Resistance uncertainty

The uncertainty in the resistance is also of great influence to the safety, performance and the structural behavior of a structure. It can cause variations in the system strength that would effect the structural reliability. The resistance uncertainty is due to randomness in geometrical and material properties of the structure. These are mainly related to the (out of) straightness of the members, their diameter and thickness, and their yield strength. In the work of a number of authors, (van de Graaf et al., 1994; Efthymiou et al., 1997) the only resistance-related uncertainty considered is the yield strength \(^2\). It is also stated that the resistance of the structure which was previously related to the BS100 can be described by a log-normal distribution. The mean value of the resistance \((R_m)\) can be calculated by pushover analysis using the mean yield stress whereas the coefficient of variation of the resistance \((V_R)\) is assumed to be half of the yield’s stress CoV (Tromans and van de Graaf, 1992). This is reasoned due to system redundancy and holds in case the strengths of the members are assumed uncorrelated. For the following calculations a lognormal distribution for the yield stress is regarded with CoV=6% as given by DNV (1995). In the previous chapter (see table 4.3) the yield stress of the piles was taken with a characteristic value of 430 MPa and 345 MPa for the piles and the rest members respectively. Assuming that a 95\(^{th}\) percentile principle holds, the mean yield stresses are 477 MPa and 383 MPa respectively and their pdfs are shown in Figures 5.2a & 5.2b

\(^2\)It is assumed that the rest of the uncertainties could be controlled by applying standardized manufacturing, fabrication and installation procedures (Nizamani, 2015)
5 Case study - Reliability

5.2 Reliability calculation

Now that the uncertainties have been defined, the reliability of the structure can be calculated. In order to do so, the RSR of the structure should be determined. This is achieved by means of pushover analysis using the non-linear analysis software USFOS (see appendix B).

5.2.1 Pushover analysis procedure

The principles of pushover analysis were introduced and discussed in section 2.2.3. For an onshore structure such as the case under consideration it finds applicability as follows.

- Firstly the gravitational load, i.e. the topside weight, is applied;
- Then the environmental load, i.e. wind, wave and current is applied on the structure. The load is stepped through the structure with a relative small time step and the maximum base shear among those time steps is identified.
- The forces on the members that are related to the this identified time step act as the reference load-set.
- Following, this reference load-set is increased incrementally. The structure initially responds elastically till some members start to yield.
- From this moment on the structure enters its plastic response until the ultimate load is reached.

In the current report, the pushover curve depicts the displacement of node 203 as shown in Figure 5.3 versus the global environmental load ($F$). Here $F$ is expressed as the ratio of:

$$F = \frac{BS}{BS_{100}}$$

where,

$BS$ = the base shear caused by the environmental load acting in the current step
$BS_{100}$ = the base shear caused by the environment load with RP=100 years

From the definition above and the definition of the RSR (see eq. ——), it is apparent that the maximum $F$ represents the RSR of the structure.

![Figure 5.3: control node for pushover analysis](image-url)
5.2.2 Ductility criterion

It should be noted here that an important assumption has been made in the pushover analysis. According to the ISO 19902 code, when performing non-linear analysis, a ductility criterion should be introduced that will disconnect a member end when the developed plastic strains have reached its tensile strain strength. According to Skallerud and Amdahl (2002), it is suggested to use a fracture criterion of 15% tensile strain for members with yield stress 355 MPa and 10% for members with yield stress 460 MPa. For the further calculations these values are used; when a member’s outer fiber reaches this strain value, the forces are removed from it, and it no longer contributes to the global stiffness.

5.3 Reliability approach I

Here the reliability of the structure is calculated assuming that that the mean value of the resistance \( R_m \) is found with pushover analysis using the mean yield stresses and that the coefficient of variation of the resistance \( V_R \) is simply half the yield stress CoV, thus \( V_R = 0.03 \). It is also assumed that the pdf of the resistance is described by a lognormal distribution. As highlighted in the previous chapter the dominant attack angle, \( \theta \) of the environmental load was found to be around 90°, i.e. broadside loading; for the definition of the attack angle please refer to Figure 4.9. Therefore, a pushover analysis is initially performed for this attack angle. Figure 5.4a depicts the stress state of the structure when the ultimate load is reached. The latter is determined from the pushover curve shown in Figure 5.4b and is found to be RSR=2.494.

![Ultimate stress state and Pushover curve](image)

**Figure 5.4:** Pushover analysis results for \( \theta = 90° \)
5 Case study - Reliability

Since the environmental load, as well as the resistance, are both described with a log-normal distribution the reliability of the structure can be found using equation 3.4 which is reproduced below for convenience.

\[
\beta = \frac{\ln \left[ \frac{R_m}{E_m} \sqrt{\frac{1 + V_E^2}{1 + V_R^2}} \right]}{\sqrt{\ln \left[ \left( 1 + V_R^2 \right) \left( 1 + V_E^2 \right) \right]}} \quad (5.1)
\]

where,

- \( R_m \) = The mean RSR value
- \( E_m \) = The mean environmental load
- \( V_R \) = The CoV of the resistance
- \( V_E \) = The CoV of the environmental load

Be reminded though that the outcome of this equation is a lifetime reliability index since the distribution of the environmental load is associated with the lifetime of the structure which is 20 years. The transformation to annual values can be done in a similar manner as in equation 2.21. The obtained results, by plugging in the calculated values, are presented in table 5.2.

**Table 5.2: Approach I reliability results for \( \theta = 90^\circ \)**

<table>
<thead>
<tr>
<th>Reliability parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>lifetime reliability safety index (( \beta_l ))</td>
<td>4.929</td>
</tr>
<tr>
<td>lifetime probability of failure (( P_{f,l} ))</td>
<td>4.129 \cdot 10^{-7}</td>
</tr>
<tr>
<td>annual reliability safety index (( \beta_a ))</td>
<td>5.485</td>
</tr>
<tr>
<td>annual probability of failure (( P_{f,a} ))</td>
<td>2.064 \cdot 10^{-8}</td>
</tr>
</tbody>
</table>

It becomes evident that the achieved \( P_{f,a} \) is far more conservative than the suggested one by Efthymiou et al. (1997) which has a value of 3 \cdot 10^{-5}. That is an indication of the conservatism that is introduced based on the design recipe from ISO 19902. In order to have a more complete look at the reliability of the structure the same procedure is carried out for all the possible attack angles. A pushover analysis is performed with the attack angle applied in steps of 10\(^\circ\). On each step the RSR is calculated and the results are depicted in Figure 5.5.

Since the geometry of the structure is symmetrical with respect to the x axis, the obtained RSR contour is symmetrical as well. One can notice that the minimum RSR is 2.49 for the attack angle of 90\(^\circ\) or 270\(^\circ\) due to symmetry which is the so-called broadside loading.

As described in the beginning of this section, the \( R_m \) is obtained by pushover analysis with mean yield stresses and the \( V_R \) is taken as half the CoV of the yield stress. It should be borne in mind that this is an assumption from a number of authors and these values could deviate from the true ones. In order to determine what is the influence of these parameters to the achieved reliability a sensitivity analysis should be engaged. Figure 5.6 depicts the relation between \( R_m \) and \( P_{f,a} \) for a fixed value of \( V_R = 0.03 \) whereas in Figure 5.7, \( V_R \) varies and \( R_m \) has a fixed value of 2.49. It becomes evident that the influence of the \( R_m \) is bigger than the \( V_R \). A 20% miscalculation of the \( R_m \) from its true value could lead to an order of magnitude deviation in the achieved reliability. One could argue though, that in the current case this could be neglected since the achieved reliability would still be far more conservative than the suggested one from Efthymiou et al. (1997). However for a different structure which achieves a reliability level
closer to the target value, a deviation of this magnitude is not considered acceptable (Bomel, 2003). This argument is also supported by looking at table 3.2; the difference in the target reliability level between a manned and unmanned structure is of an order of magnitude. The same sensitivity analysis is also demonstrated in Figure 5.8 by means of a surface 3D plot.
5 Case study - Reliability

This sensitivity of the $P_{f,a}$ to the resistance characteristics arises the need to calculate these parameters more precisely. Furthermore the reliability method used here is a Level II reliability where it has been assumed that the resistance is described by a lognormal distribution. In case the resistance is described by a different distribution, one can no longer calculate the reliability using equation 5.1 but a Level III method should be engaged. These issues are treated in the next section.
5.4 Reliability approach II

In this section the reliability of the structure is calculated in a more explicit way. The Monte Carlo method is used in order to determine the distribution of the resistance and its parameters. Monte Carlo is a numerical technique that produces a distribution for an outcome where no exact or analytic solution can be determined (Zio, 2013). In the current report it finds applicability by performing a big number of pushover realizations, each of them with random values for the stochastic variables based on their respective distribution. In this case, the stochastic variable is the yield stress and its distribution is log-normal (see Figure 5.2). A matlab code has been developed that performs an iterative procedure with USFOS. On each iteration a random set of the stochastic variables is generated and given as an input to USFOS. A number of pushover simulations is determined and on each simulation a random yield stress based on its distribution is assigned to the structure’s members and the RSR is calculated. At the end of all the simulations a histogram can be produced and a pdf can be fitted on it. The fitting is performed again in Matlab by checking the goodness of fit of the 20 distributions available in Matlab with the histogram produced. The goodness of fit is based on the Bayesian Information Criterion (BIC)\(^3\). Note that on each realization the members are considered uncorrelated and so is their yield stress. The overview of this procedure is illustrated in Figure 5.9.

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\(^3\)The BIC is a criterion for model selection among a finite set of models. It is considered a reliable way of fitting a set of data to known pdfs. source:wikipedia
5 Case study - Reliability

5.4.1 Monte Carlo analysis

Here the Monte Carlo technique as described previously is performed for the attack angle of 90°. The stochastic variable considered, is the yield stress with a lognormal distribution as given in Figure 5.2. Initially 600 simulations for this attack angle are performed and the obtained $R_m$ and $V_R$ are shown in table 5.3 together with the ones calculated with the precious approach.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Parameter $R_m$</th>
<th>Parameter $V_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approach I</td>
<td>2.49</td>
<td>0.03</td>
</tr>
<tr>
<td>Approach II</td>
<td>2.41</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Comparing the obtained values between the two approaches we should highlight two things:

- The assumption for the $V_R$, being half of the yield stress is reasonable; the calculated value from Monte Carlo simulations is 0.027 which is very close to half the yield stress CoV. This is also supported from Figure 5.7 where the the influence of $V_R$ to the achieved reliability level is not very strong.

- The assumption that the $R_m$ can be calculated using the mean yield stresses should be reconsidered. Figure 5.6 showed us that using the true $R_m$ is crucial in order to achieve a target reliability with precision less than an order of magnitude.

The choice of the number of simulations is based on the required precision of the values of interest. Figure 5.10 illustrates how the $R_m$ and $V_R$ evolves through the simulations. One can see that after a number of 500 simulations both values are stabilized with respect to the third significant number.

![Figure 5.10: Progress of $R_m$ and $V_R$ through the simulations for $\theta = 90^\circ$](image-url)
5.4 Reliability approach II

The outcome of the Monte Carlo simulations is a set of calculated RSRs which can be presented using a histogram. Once this has been generated a pdf can be fitted on it in order to describe the ultimate response of the structure in a statistical manner. The fitting is performed for the 20 built-in Matlab pdfs and it has been determined that the best fit is achieved with the weibull distribution. The corresponding histogram is depicted in Figure 5.11 together with the weibull and the lognormal fitted pdfs.

![Figure 5.11: RSR histogram and pdf fitting for \( \theta = 90^\circ \)](image)

The corresponding reliability level can now be calculated. For the case where the resistance is described with a lognormal distribution, a Level II method can be used in a similar way as in equation 5.1. For the weibull distribution case, the \( P_{f,a} \) is calculated using equation 2.3. The achieved reliability for these two cases is shown in table 5.4.

![Table 5.4: Approach II reliability results for \( \theta = 90^\circ \)](table)

<table>
<thead>
<tr>
<th>Reliability parameter</th>
<th>lognormal</th>
<th>weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{f,l} )</td>
<td>( 9 \cdot 10^{-7} )</td>
<td>( 1 \cdot 10^{-6} )</td>
</tr>
<tr>
<td>( P_{f,a} )</td>
<td>( 4.5 \cdot 10^{-8} )</td>
<td>( 5.1 \cdot 10^{-8} )</td>
</tr>
</tbody>
</table>

It becomes evident that the obtained reliability level from these two distributions is very close. This can be explained considering that both distributions have a good fit on our data. However, in case the outcome of the Monte Carlo simulation can not be fitted with a lognormal pdf, then the reliability levels will differ significantly. As described previously a difference of an order of magnitude or more is not acceptable.

The Monte Carlo technique is performed for the whole spectrum of the omnidirectional load and the obtained \( R_m \) for each attack angle are depicted in Figure 5.12.
5 Case study - Reliability

5.5 Sensitivity of reliability level

At the end of section 5.3 the sensitivity of the achieved reliability to the resistance parameters was discussed. Here the relation between the reliability level and a number of other selected parameters of influence is considered.

5.5.1 Sensitivity to Environmental load parameters

For the reliability analysis performed in the current chapter, it has been assumed that the environmental load is described by a lognormal distribution with $E_m=0.84$ and $V_E=0.212$ for the SNS location. However these are proposed values by Efthymiou et al. (1997) and could deviate from the true ones. The true parameters of the load distribution should be estimated accounting for the points highlighted in 5.1.1. Although the values used here are considered reasonable, it makes sense to check how sensitive the achieved reliability level is to the parameters of the environmental load distribution. In Figure 5.13a the relation between $E_m$ and $P_{f,a}$ for a fixed value $V_E=0.226$ is presented whereas in Figure 5.13b the varying parameter is the $V_E$ and the $E_m$ has a fixed value of 0.84. In both plots the attack angle considered is $90^\circ$.

Based on these Figures one can see that both the parameters have a great impact on the achieved reliability level; an underestimation of the $E_m$ around 20% could result in more than an order of magnitude difference to the $P_{f,a}$. This is also deducted from Figure 5.14 where the influence of both parameters is shown in a 3D plot. Thus, those parameters shall be estimated with care when a reliability analysis is performed and acknowledging their influence to the obtained results.

Figure 5.12: Contour of RSRs for omnidirectional load based on Monte Carlo simulations
The arrows indicate the direction of the environmental load. The attack angle step is $10^\circ$. 

60
5.5 Sensitivity of reliability level

Moreover, as already has been stated, the load is assumed to be described by a lognormal distribution. In case the environmental load assessment yields a different type of distribution then a Level II method of reliability analysis is not valid\textsuperscript{4} and a Level III method should be engaged.

\textsuperscript{4}Unless, of course both load and resistance are normally distributed
5.5.2 Sensitivity to the site characteristics

Sensitivity to the max wave height

Herein the sensitivity of the RSR and the corresponding $P_{f,a}$ to the max wave height, $H_{\text{max}}$ with return period 100 years is examined. In Figure 5.15 this relation is depicted for the attack angle $\theta = 90^\circ$. The result is more than expected since the bigger the wave, the bigger the wave force on the structure and that is depicted with a decaying RSR for increasing values of $H$.

![Figure 5.15: Reliability sensitivity to the max wave height for $\theta = 90^\circ$](image)

One can also observe that a difference of about 1 meter in the max wave height leads to about an order of magnitude deviation in the achieved reliability. This fact highlights the importance of having a reliable database of metocean conditions which can adequately describe environmental characteristics with high return period.

Sensitivity to the wind speed

The next source of environmental loading considered is the wind speed. In Figure 5.16 the RSR and the corresponding $P_{f,a}$ is plotted against the wind speed. It becomes evident that the impact a miscalculation of the wind speed has on the reliability level is negligible. Even for double the design wind speed the deviation in the $P_{f,a}$ is less than an order of magnitude.
5.5 Sensitivity of reliability level

Figure 5.16: Reliability sensitivity to the wind speed for $\theta = 90^\circ$

Sensitivity to the surface current speed

In Figure 5.17 the RSR and the corresponding $P_{f,a}$ is plotted against the current speed. The results here are again expected. With increasing current speed the RSR drops and consequently the $P_{f,a}$ rises. The impact on the reliability may not be as high as the $H_{max}$, however the importance of defining it in a precise manner should not be omitted.

Figure 5.17: Reliability sensitivity to the current speed for $\theta = 90^\circ$
Sensitivity to the water depth

In chapter 4.1.1 the water depth was taken as 31 m. This value was calculated accounting for a number of assumed parameters such as the tide, the storm surge and the expected settlement of the structure. Since these values are taken as best estimates and could deviate from the true ones, it makes sense to check the influence of the site’s water depth to the achieved reliability. This impact is demonstrated in Figure 5.18 where the RSR and the corresponding $P_{f_a}$ are plotted against the water depth. One can see that a potential deviation in the design water depth has a significant influence on the reliability level. A change in the water depth, leads to a change in the environmental load profile acting on the structure, i.e. differences in wave kinematics and differences in the members experiencing direct and indirect loading. Therefore, once again, the importance of a reliable site specific metocean database is highlighted.

![Figure 5.18: Reliability sensitivity to the water depth for $\theta = 90^\circ$](image)

5.5.3 Sensitivity to the wave force model

Here, the second source of uncertainty, namely the wave force model is discussed further. As described under 5.1.2, the uncertainties related to the hydrodynamic coefficients and the marine growth are accounted in the reliability analysis by an extra CoV=0.08 in the $V_E$ (see eq. 5.1.2). This was based on work done by Digre et al. (1995) and used in further studies by Efthymiou et al. (1997) and Tromans (2000). However in a different research performed by Skallerud and Amdahl (2002) this type of uncertainty is accounted in a different manner; a reliability analysis was performed for a jacket structure located in North Sea where the parameters related with the wave force model uncertainty are treated explicitly as stochastic variables. The variables, the assumed pdfs and their parameters for these uncertainties are given in table 5.5 and visualized in Figure 5.19.
Table 5.5: Wave force model and imperfection parameters (Skallerud and Amdahl, 2002)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Mean value</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>drag coefficient ($C_D$)</td>
<td>Lognormal</td>
<td>0.7</td>
<td>0.25</td>
</tr>
<tr>
<td>mass coefficient ($C_m$)</td>
<td>Lognormal</td>
<td>2.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Marine growth</td>
<td>Lognormal</td>
<td>0.05</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Based on these stochastic parameters, the Monte Carlo technique is applied again for the broadside loading and the corresponding histogram together with the fitted pdf is shown in Figure 5.20. Note that the hydrodynamic coefficients and the marine growth are considered uncorrelated with each other, but fully correlated between members. The best fit was achieved with the lognormal distribution with mean value 2.48 and CoV is 0.22. One can see that the obtained $V_R$ is 7 times bigger than the case where it was assumed to be half of the yield stress CoV, namely 0.03. This has an impact on the achieved reliability level which is found to be $P_{f,a} = 7.17 \times 10^{-6}$. This value is two orders of magnitude higher than the one obtained by the previous reliability approach (see table 5.2). The reliability results of the two different approaches and the approach presented in the current section, where the wave force model uncertainty has been considered explicitly, are presented in table 5.6.
These findings arise the question if the uncertainties discussed here should be included as well to the reliability model. The achieved reliability can be subject to significant deviations due to the uncertainty introduced by the wave force model. Therefore it is considered important that these parameters should be employed in a careful manner.

Table 5.6: Reliability results based on different approaches

<table>
<thead>
<tr>
<th>wave force model uncertainty</th>
<th>Approach I</th>
<th>Approach II</th>
<th>Approach III</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean value of the resistance ($R_m$)</td>
<td>extra CoV=0.08</td>
<td>Monte Carlo simulations $R_m=2.41$</td>
<td>Monte Carlo simulations $R_m=2.48$</td>
</tr>
<tr>
<td>coefficient of variation of the resistance ($V_R$)</td>
<td>Half of the yield’s stress CoV $V_R=0.03$</td>
<td>Monte Carlo simulations $V_R=0.027$</td>
<td>Monte Carlo simulations $V_R=0.21$</td>
</tr>
<tr>
<td>annual probability of failure ($P_{fa}$)</td>
<td>$2.1 \cdot 10^{-8}$</td>
<td>$5.1 \cdot 10^{-8}$</td>
<td>$7.17 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>
Chapter 6

Reliability based design

6.1 Introduction

The discussion that was made in the previous chapters about the structural reliability analysis and its sensitivity to a number of parameters laid the basis for a new approach on the design phase of bottom founded structures. This design approach will be confined solely to the desired achieved reliability level, thus not following a design recipe with partial factors such as the one included in the ISO standard. For the purpose of demonstrating this concept let us continue further the design of the jacket considered in the previous chapters and assume the following:

- The meteocean characteristics are considered reliable and are described adequately by a lognormal distribution and its parameters as given in table 5.1.
- The wave force model uncertainty can be sufficiently described by a CoV=0.08 included in the coefficient of variation of the environmental load ($V_E$) as described under section 5.1.2
- The resistance can be also described by a lognormal distribution. The mean value of the resistance ($R_m$) can be initially be estimated by pushover analysis using the mean yield stress values. The true $R_m$ as well as the best fitted distribution can be later calculated by means of Monte Carlo simulations.
- The coefficient of variation of the resistance ($V_R$) can be taken as the half of the yield stress CoV. This is considered a reasonable approximation since its influence to the achieved reliability is weak as discussed under section 5.4.1. Nevertheless this can be exactly calculated after the Monte Carlo simulations.
- The required reliability level is $P_{f,a} = 3 \cdot 10^{-5}$ as suggested by Efthymiou et al. (1997).

Using these assumptions one can define a target RSR value that the structure should possess in order to meet the desired reliability level. Since we have assumed that both the load and the resistance are lognormally distributed, this can be done using equation 5.1 and plugging in the assumed variables as described above. Hence the required minimum RSRs for various geographical locations in order to achieve $P_{f,a} = 3 \cdot 10^{-5}$ are given in table 6.1.

One can notice that the required RSR is site dependent. The reason for this lies in the differences of the $V_E$ values between the locations (see table 5.1); for sea conditions with high inherent randomness the required RSR is increased.

The problem now lays in the method that the engineer should use in order to achieve this value. This is discussed in the following section.
Table 6.1: Target RSR values for $P_{fa} = 3 \cdot 10^{-5}$

<table>
<thead>
<tr>
<th>Location</th>
<th>Target RSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern North Sea (SNS)</td>
<td>1.7</td>
</tr>
<tr>
<td>Northern North Sea (NNS)</td>
<td>1.9</td>
</tr>
<tr>
<td>Gulf of Mexico (GoM)</td>
<td>2.15</td>
</tr>
<tr>
<td>Northwest coast of Australia (AUS)</td>
<td>2.15</td>
</tr>
</tbody>
</table>

6.2 Design procedure

Firstly a pushover analysis shall be carried out for the omnidirectional environmental load. Thus a contour of the obtained RSRS similar to the one presented in Figure 5.12 is produced. Based on this contour, the attack angle that gives the minimum RSR can be determined. This is considered the dominant attack angle since this will define the achieved reliability. The next step is to check the stress state of the structure and identify:

- Which member or group of members initiate the structural collapse
- What is the stress state and the plastic utilization of the rest members and which members are utilized the least.

The cross sections of the members with low plastic utilization ratio can be reduced gradually and check again the response of the structure by performing pushover analysis. It is expected that these changes will cause no significant change in the pushover curve since these members do not initiate the structural collapse. The next step is to start altering the cross sections characteristics of the dominant members and perform again the pushover analysis. The ultimate resistance of the structure should now significantly change. This procedure shall be performed in iterative manner until the RSR reaches the desired value. Once this happens and it has been decided that no further optimization can take place, a Monte Carlo method with sufficient number of simulations should be engaged. The outcome of the Monte Carlo simulations, namely the fitted distribution and its parameters can be used in order to calculate the $P_{fa}$. In case this is considered to be far from the target value the whole procedure should be re-approached by applying different amendments to the members cross sections. However as discussed in the previous sections the obtained reliability level will normally not deviate significantly from the required one. Thus, if it this is the case then the next step is to check the omnidirectional response of the structure and produce again a contour plot, this time for the optimized structure. Once again the RSRs shall be checked and verify that none of them is less than the target one. Following that, the final step is to perform Monte Carlo simulations for the whole spectrum of the attack angles, calculate the reliability for the omnidirectional environmental load and check whether or not the requirement is fulfilled. This process can be visualized in Figure 6.1.

The necessary amendments that should be made in order to achieve the target reliability level are associated with altering the cross sections characteristics, namely, the diameter and the thickness. In the previous chapter it was demonstrated that the design based on the ISO 19902 code leads to a higher RSR value and consequently to a considerably lower $P_{fa}$ value than the target one. Therefore, in order to achieve the target RSR the reasonable amendments that should be made are associated with reducing the diameter and/or the thickness of the structure’s members. The choice of which of these two cross section’s characteristics should be modified and how much depends mainly on whether the member is considered crucial (its failure initiates the structural collapse) or not and the type of its failure (buckling failure, tension failure etc.).
6.2 Design procedure

Preliminary design of the structure

Pushover analysis for the omnidirectional load

Identify dominant attack angle, i.e. attack angle with minimum RSR

Identify crucial and non-crucial members.

Apply amendments to the cross sections.

RSR_{target} achieved and structure optimized?

yes

no

Monte carlo for the dominant attack angle and pdf fitting

Target $P_{f,a}$ achieved?

yes

no

Pushover for the omnidirectional load. Minimum RSR ≥ RSR_{target}?

yes

no

Monte carlo for the omnidirectional load. Is minimum $P_{f,a} \geq P_{f,a,target}$?

yes

no

End of optimization

Figure 6.1: Reliability based design diagram
6 Reliability based design

6.3 Application to the case study

The procedure explained in the previous section, is applied to the case study structure located in the SNS in order to achieve a target $P_{f,a} = 3 \cdot 10^{-5}$. As already stated this can be accomplished by designing the structure with minimum $\text{RSR} = 1.7$ (see table 6.1). Previously it was shown that the dominant attack angle is for $\theta = 90^\circ$ and the corresponding RSR is 2.49. The original stress state and the pushover curve of the structure for this attack angle are depicted in Figure 6.2.

![Figure 6.2: Original structural response for $\theta = 90^\circ$](image)

One can see that the members that are mostly overloaded are the lower X braces; the failure due to buckling of these members initiates the structural collapse. One way to achieve the target RSR, is by reducing the buckling strength of these members by altering the cross section’s characteristics. In Figure 6.3 the sensitivity of the RSR to reduced thickness and reduced diameter of the lower X braces is plotted. Note though, that the reduction of the diameter and the thickness has been applied individually; no combinations of reduced diameter and reduced thickness for each of the groups has been accounted. If such combinations are included in the optimization algorithm the computational time needed will grow exponentially.

It can be deduced that the target RSR=1.7, can be achieved by reducing the thickness of the lower X braces by 35%. The stress state response of the modified structure at the ultimate load and its corresponding pushover curve are depicted in Figure 6.4.
6.3 Application to the case study

From this figure it is apparent that the plastic utilization of the other elements is relatively low, hence the structure can be further optimized. This can be achieved by applying the same concept of sensitivity analysis of the RSR to the reduced diameter and thickness for each of the structural groups; once the optimum reduction has been identified for a group, a sensitivity analysis is performed for the next one based on the modified structure until the optimum structure
6 Reliability based design

is obtained. By doing so, a modified structure that achieves the target RSR in a more optimum way is achieved. The reduced cross sections for each group are shown in table 6.2, together with the original ones. The stress state response of the final modified structure at the ultimate load and its corresponding pushover curve are depicted in Figure 6.5.

Table 6.2: Cross sections of original and modified structure

<table>
<thead>
<tr>
<th>Group</th>
<th>Original structure</th>
<th>Modified structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>legs</td>
<td>66</td>
<td>1.125</td>
</tr>
<tr>
<td>piles</td>
<td>60</td>
<td>1.125</td>
</tr>
<tr>
<td>upper frame</td>
<td>14</td>
<td>0.375</td>
</tr>
<tr>
<td>mid frame</td>
<td>14</td>
<td>0.375</td>
</tr>
<tr>
<td>bottom frame</td>
<td>12</td>
<td>0.375</td>
</tr>
<tr>
<td>upper X brace</td>
<td>24</td>
<td>0.5</td>
</tr>
<tr>
<td>lower X brace</td>
<td>28</td>
<td>0.625</td>
</tr>
<tr>
<td>upper peripheral</td>
<td>16</td>
<td>0.375</td>
</tr>
<tr>
<td>mid peripheral</td>
<td>16</td>
<td>0.375</td>
</tr>
<tr>
<td>bottom peripheral</td>
<td>16</td>
<td>0.375</td>
</tr>
<tr>
<td>weight[T]</td>
<td>562.68</td>
<td></td>
</tr>
</tbody>
</table>
The Monte Carlo technique, when applied to the modified structure, gives $R_m=1.71$ and $V_R=0.035$. The histogram and the corresponding best pdf fitting, here with weibull distribution, are depicted in Figure 6.6. Therefore the $P_{f,a}$ can be calculated through Level III reliability methods and is found to be $P_{f,a}=2 \cdot 10^{-5}$, which is close to the target one.

One can see that a reduction approximately 45% with respect to the steel weight has been achieved. This value however, is subject to further structural amendments and the level of the optimization of the original structure. It does not set a bounder for a minimum or maximum weight reduction. However, it clearly highlights the potential benefit that the engineer can gain by employing reliability analysis in the design phase of bottom founded offshore structures.

In the current section, a method for structural optimization based on a target RSR has been demonstrated. This method is based on sensitivity analysis of the reduced cross section characteristics to the achieved RSR for each of the structural groups. As already stated, combinations of reduced diameter and reduced thickness has not been accounted, due to exponential increase of the computational time needed. A more complete approach to the optimization problem would be by including such possible combinations, acknowledging though the extra cost in computational time.
Chapter 7

Conclusions and Recommendations

In this Chapter, an overview of the work carried out within the framework of the present thesis is presented. This is accomplished through the evaluation of the addressed objectives, as cited in Chapter 1, in accordance with the results obtained in Chapters 4 and 5. Moreover, a critical assessment of the adopted formulation is presented with emphasis on recommendations for future work and enrichment of this study.

7.1 Conclusions

The study that was carried out in the current thesis was focused on the reliability analysis and reliability based design of jacket structures. The current state of design and the method of reliability analysis which is related to non-linear pushover analysis was described and the its potential benefits were discussed. The background of the ISO 19902 standard and its intended reliability was presented. It was determined that the reliability of a structure can be described by two variables, namely the global environmental load and the global resistance expressed as Reserve Strength Ratio (RSR) and their (joint) distributions. In order to investigate these notions in a practical manner a case-study of an offshore bottom founded structure was introduced. The structure was designed and optimized for an omnidirectional environmental load based on the ISO standard and the achieved reliability was calculated. The conclusions drawn from this work may be summarized in the following points:

i. Intended reliability level of ISO 19902

The picture about the intended reliability based on the design recipe of ISO 19902 is considered vague. The proposed design recipe is the conventional Load and Resistance Factor Design (LRFD) design with a default environmental partial action factor \( (\gamma_{f,E}) = 1.35 \). However, no explicit target reliability values are given, either in terms of a required annual probability of failure \( (P_{f,a}) \) or a minimum RSR that the structure should exhibit. It is only in an informative annex of the code that one can find a suggested minimum RSR value of 1.85 for a typical offshore structure in the North Sea (NS) and a suggest value of \( P_{f,a} = 3 \cdot 10^{-5} \). The reliability analysis of the structure that was designed and optimized based on the ISO 19902, yields a minimum RSR=2.49 which corresponds to a \( P_{f,a} = 2 \cdot 10^{-8} \) (see table 5.2). This value is significantly lower than the proposed value, hence indicating the conservatism that is introduced with the conventional LRFD method of the ISO. In order to have a deeper look to the reliability level of the structure a sensitivity analysis has been executed.
ii. Reliability sensitivity analysis

(a) Sensitivity to the environmental load uncertainty

The study was based on the assumption that the environmental load can be described by a lognormal distribution. The sensitivity analysis showed that the achieved reliability is highly dependent on the parameters of the environmental load. The probability density function (pdf) of the environmental load should adequately represent the metocean conditions that are expected to act on the structure during its lifetime. Both the mean value of the environmental load \((E_m)\) and the coefficient of variation of the environmental load \((V_E)\) should be reliably defined, since a miscalculation of the \(E_m\) around 20\% could cause a deviation to the reliability level of an order of magnitude as discussed in section 5.5.1. On the scope of defining the environmental load uncertainty in a more precise manner, a reliable database of the metocean conditions is required that can sufficiently depict environmental loads with high return period. Based on this database the pdf of the environmental load can be defined precisely and verify whether the assumption that it is lognormally distributed is valid. If this does not hold then a Level II method will miscalculate the reliability level, and hence a Level III method is required.

(b) Sensitivity to the resistance uncertainty

In the majority of research performed in the past for reliability analysis of offshore structures the resistance is described by a lognormally distribution and its parameters, namely the mean value of the resistance \((R_m)\) and coefficient of variation of the resistance \((V_R)\). The former parameter is taken from pushover analysis with the mean yield stresses assigned to the members and the latter parameter was assumed as the half of the yield’s stress Coefficient of Variation \((\text{CoV})\). In the current report the resistance uncertainty is performed more explicitly by means of Monte Carlo simulations. The outcome showed that the assumptions about \(R_m\) and \(V_R\) are reasonable. However the pdf fitting procedure could lead to a distribution other than the lognormal. In that case a Level III distribution is required and the reliability level would significantly differ.

(c) Sensitivity to the wave force model uncertainty

The stochastic variables related to the hydrodynamic parameters and the marine growth were included with an extra \(\text{CoV}=8\%\) accounted in the \(V_E\). However when these values where considered explicitly with their individual pdfs, the outcome of the Monte Carlo simulations showed a deviation of two orders of magnitude in the achieved reliability as described under section 5.5.3. Hence a need for more reliable definition of these uncertainties arises.

iii. Reliability based design

A target RSR depending on the site’s location has been proposed that would lead to the required structural reliability level. For the case of Southern North Sea (SNS) the target value is \(\text{RSR}=1.7\), which is lower than the suggested value \((\text{RSR}=1.85)\) referenced in ISO 19902 standard. In order to capture this target value a method has been formulated, based on pushover analysis and Monte Carlo simulations. The application of the proposed method to the case study, lead to a modified structure with approximate 45\% less use of steel. Although this number may be subject to changes depending on the structure and the level of optimization, still it strongly indicates the advantages of reliability based design.
Not only a more economical structure has been achieved but more importantly the desired level of safety is ensured. However this method is difficult to be applied in a context of a structural code recipe. It requires knowledge of non linear pushover analysis as well as engineering judgment in order to decide which amendments should be done.

### 7.2 Recommendations

The aforementioned conclusions with respect to the obtained reliability results are drawn on the basis of a set of assumptions. Therefore, it should be remarked that the present study constitutes an approach on the reliability analysis and design of offshore structures and it also gives rise to further research and investigation topics towards a more integrated study. On this purpose, the research directions that are proposed can be depicted in the following points.

i. **Boundary conditions**

   In the current study the foundations of the examined structure were considered to be pinned at an inflection depth below the sea bed as it is believed that the boundary conditions do not influence the structural failure. However a more detailed modeling of the supports of the structure could be engaged where the site specific soil conditions are accounted. Hence, the importance of the foundation modeling can be quantified and its influence on the reliability level can be highlighted.

ii. **Sources of uncertainty**

   It is recommended to further investigate the sources of uncertainty, namely, the load related, the resistance related, and the wave force model related uncertainties, since their impact on the reliability level is significant. Especially the high influence of the environmental load, highlights the importance of precise description of the metocean conditions. This should be carried with the focus on structure and site specific parameters. Moreover other sources of uncertainty such as ship impact or a freak wave impingement on the topside could be considered.

iii. **Component reliability analysis**

   An individual reliability analysis can be performed for the member or group of members that initiate the structural collapse. A more detailed analysis can be employed considering more local rather than global uncertainties. These could be local dents and imperfections or the residual stresses that might exist. Such an analysis could also include a finer finite element modeling accounting for the structural joints and their contribution to the failure mechanism.

iv. **Other applications of reliability analysis**

   Herein the focus was in the reliability analysis related to the Ultimate limit state (ULS) case. A further study could be in the context of other limit states such as Fatigue limit state (FLS) or Serviceability limit state (SLS). Moreover since the installation and transportation of the offshore structure is usually the dominant criterion for the structure’s geometry it would be of interest to perform a reliability study on this context. Besides the application to jacket structures, the reliability analysis can be performed to other types of structure such as towers and floating structures or even in more civil oriented buildings.
v. Improvement of the proposed reliability based design

The method of reliability based design that was proposed here could be further improved by engaging a more sophisticated algorithm that will optimize the structure in a more automated way. This could be achieved by checking the stress state of the members at the failure state, and employing a computer aided iterative procedure that starts reducing the necessary cross sections and hence optimizing the structure. Furthermore other optimization techniques can be employed like the so-called gradient based optimization, acknowledging though the complexity that is introduced due to the non-linear structural behavior.
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Appendix A

Extreme environmental conditions

Significant wave height

Extreme Value Analysis – Hs
Peaks Over Threshold (threshold = 2)

<table>
<thead>
<tr>
<th>Probability of Non-Exceedance</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Weibull 3 − cutoff: 2.2 confidence interval (5−95%)

<table>
<thead>
<tr>
<th>Return Periods (years)</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
<th>100000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weibull 3, Raw, Truncated (cutoff 2.2)</td>
<td>6.12</td>
<td>7.68</td>
<td>9.09</td>
<td>10.30</td>
<td>11.44</td>
<td>12.51</td>
</tr>
</tbody>
</table>

Extreme Values for: Hs[m]
Number of peaks per year: 73.78
Total number of peaks: 3861
Threshold: 2
Gap treatment: calms
Decorrelation time (days): 0.00
3861 Observations, Min: 2.00 Max: 8.40
Current speed

Extreme Value Analysis – Total_Current Speed
Peaks Over Threshold (threshold = 0.3)

Overall

<table>
<thead>
<tr>
<th>Return Periods (years)</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weibull 2: Raw, Truncated (cutoff: 0.6)</td>
<td>0.74</td>
<td>0.83</td>
<td>1.03</td>
<td>1.12</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Extreme Values for: Total_CurrentSpeed [m/s]
Number of peaks per year: 605.73
Total number of peaks: 18473
Threshold: 3.000000e−001
Gap treatment: calms
Decorrelation time (days): 0.00
18473 Observations, Min: 0.30 Max: 0.86
A Extreme environmental conditions

Wind speed

Extreme Value Analysis – Wind Speed
Peaks Over Threshold (threshold = 10)

<table>
<thead>
<tr>
<th>Return Periods (years)</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
<th>100000</th>
</tr>
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<tbody>
<tr>
<td>Weibull 2: Raw, Truncated (cutoff: 15)</td>
<td>22.22</td>
<td>25.69</td>
<td>27.37</td>
<td>29.28</td>
<td>30.95</td>
<td>32.43</td>
</tr>
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</table>

Fit Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \alpha )</th>
<th>( \sigma )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weibull 2: Raw, Truncated (cutoff: 15)</td>
<td>3.297</td>
<td>13.949</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Extreme Values for: WindSpeed [m/s]
Number of peaks per year: 132.21
Total number of peaks: 6919
Threshold: 10
Gap treatment: calms
Decomration time (days): 0.00
6919 Observations, Min: 10.00 Max: 26.70
Appendix B

USFOS calculation method

USFOS is a numerical tool for ultimate strength and progressive collapse analysis of steel space frame structures. It accounts for nonlinear material as well as nonlinear geometry. The basic idea of the program is to use only one finite element per physical member of the structure. The solution strategy is depicted in figure B.1.

![USFOS solution strategy diagram]

Figure B.1: USFOS solution strategy

The check regarding the yield surface is performed by the plastic interaction function,$\Gamma$ as shown in the following equation:

$$\Gamma = f\left(\frac{N}{N_p}, \frac{Q_y}{Q_{yp}}, \frac{Q_z}{Q_{zp}}, \frac{M_x}{M_{xp}}, \frac{M_y}{M_{yp}}, \frac{M_z}{M_{zp}}\right) - 1$$
In the above notations, \( N \) is the axial force, \( Q \) is the shear and \( M \) is the bending moment. The subscripts \( x,y,z \) denote the axis of loading and the subscript \( P \) denotes the plastic strength. For a tubular cross section where torsion and shear forces are neglected this plastic interaction function is given as

\[
\Gamma = f\left( \frac{N}{N_P}, \frac{M_y}{M_{yp}}, \frac{M_z}{M_{zp}} \right) - 1
\]

\[
= \cos\left( \frac{\pi N}{2 N_P} \right) - \sqrt{\frac{M_y^2 + M_z^2}{M_{yp}}}
\]

The function is defined so that for \( \Gamma = 0 \) full plastification occurs. \( \Gamma = -1 \) represents the initial stress free cross section. For \( \Gamma = 0 \) a plastic hinge is introduced at the member location where the yield surface has been reached. This \( \Gamma \) function is calculated at every iteration for each member of the structure and new plastic hinges are introduced if necessary.

The check regarding instability check is performed by a formulation based on the Current Stiffness Parameter in combination with a Determinant criterion.

**Current stiffness parameter**

The current stiffness parameter, \( S_p \), is defined as

\[
S_p^i = \left( \frac{(\Delta r^i)^T}{(\Delta r^i)^T \Delta R^i} \cdot \left( \frac{(\Delta p^i)^2}{(\Delta p^i)^2} \right) \right)
\]

where,

- \( \Delta r \) = incremental displacement
- \( \Delta R \) = incremental forces
- \( \Delta p \) = relative load increment size at each load step

It is essentially a normalized scalar value representing the slope of the tangent of the equilibrium path at load step \( i \) over the initial slope. Thus it will have an initial value of 1.0. For softening systems it will decrease whereas for stiffening systems (membrane effects) it will increase. The current stiffness parameter gets the value of 0 at the so-called instability point. After the instability point the response enters the post-collapse range where the current stiffness parameter is negative.

**Determinant Criterion**

The determinant criterion checks the determinant of the stiffness matrix. As long as it is positive the structure is considered stable. As the load increases, the structural response becomes more non-linear and for softening systems the determinant will decrease. The instability point is detected once the determinant of the stiffness matrix becomes zero.