In 1915 the Minister of the Interior ordered the drawing up of plans for a central drinking water supply in the provinces of South Holland, North Holland and Utrecht. The Government Institute for Water Supply carried out this work and described its findings in minute detail in a report published in 1919. Ground-water always has been — and still is — considered the safest and most reliable source for human consumption and more or less as a matter of course the above mentioned plans were, therefore, based on ground-water abstraction, for which a line of wells had been projected.

In 1919, that is a little over 40 years ago, different catchments with ground-water recovery for public and industrial water supplies were already in operation, some even for many years. Due to lack of knowledge and understanding, the design of a new system, however, still offered many difficulties and ample opportunities for failure. These difficulties were not so much due to the circumstance that in this western part of the Netherlands ground-water generally is salt or brackish. Along the river Lek the drilling of a large number of testholes revealed the presence of fresh water pockets of sufficient magnitude with regard to the proposed amount of abstraction and with a chemical composition of the ground-water otherwise such that it could easily be made fit as source for a potable supply. The real difficulty concerned the lowering of the ground-water table inside the individual wells of the system during operation. At that time submersible pumps had already been applied for many years, but
they were very expensive and rather unreliable, necessitating ground-water abstraction with central pumps and suction lines. With regard to the limited amount of suction head available, this system is only feasible from an economic point of view when during operation the water level in the individual wells does not drop more than 6 to 7 m below ground-surface. It was the duty of the hydrologist concerned to prove that his design satisfied this requirement.

As catchment area for the proposed central drinking water supply a site near Jaarsveld had been chosen. The geo-hydrologic profile here showed what nowadays would be called a two-storied leaky artesian aquifer between depths of 5 to 10 and 100 m below ground surface. At the bottom this aquifer is bounded by a practically impervious layer of clay, at the top overlain by strata of clay and peat and subdivided into two parts by deposits of silty sand and clay roughly between 55 and 75 m below ground surface.

As to confined aquifers without recharge, bounded at top and bottom by fully impervious aquicludes, formulas of the drawdown curves accompanying ground-water abstraction with a single well are already known for many years. Starting from Dupuit’s assumption (1863) that with horizontal aquifers of constant thickness the vertical components of flow can be neglected, these formulas are derived by applying Darcy’s law (1856) and the equation of continuity. For a well in the centre of a circular island (Fig. 1), this gives:

\[ Q = \frac{-2\pi r \cdot H \cdot k}{r} \frac{d\varphi}{dr} \]

Continuity \[ Q = \text{constant} = Q_0 \]

FIG. 1 Artesian well in the centre of a circular island
Elimination of $Q$ from both relations produces the general differential equation:

$$d\varphi = -\frac{Q_0}{2\pi kH} \cdot \frac{dr}{r}$$

By integration between the limits $r = r, \varphi = \varphi$ and $r = R, \varphi = 0$ finally Dupuit's formula is obtained:

$$\varphi = \frac{Q_0}{2\pi kH} \cdot \ln \frac{R}{r}$$

in which $\varphi$ is the steady state drawdown of the artesian water table at a distance $r$ from a well pumped at a constant rate $Q_0$. The radius of the island is represented by $R$, while $k$, the coefficient of permeability and $H$ the saturated thickness of the aquifer are formation constants. When the above mentioned differential equation is integrated between the limits $r = r_1, \varphi = \varphi_1$ and $r = r_2, \varphi = \varphi_2$, the formula of Thiem (1870) is obtained:

$$\varphi_1 - \varphi_2 = \frac{Q_0}{2\pi kH} \cdot \ln \frac{r_2}{r_1}$$

giving the difference in drawdown for 2 points at distances $r_1$ and $r_2$ from the well centre. With good approximation this formula proved true for all geo-hydrologic conditions, for steady as well as for unsteady flow and for every random position of the well. This success initiated the use of Dupuit's formula also for other topographic conditions. There one could not speak of a circular island, and $R$ was called the radius of the sphere of influence, whatever the meaning of this term might be. The fact that in reality the value of $R$ depends on the effective distance to open water at a constant level and with a given situation even varies from one point of observation to another was not always fully recognised. Mixing up steady with near-steady flow also added to the confusion and as regards the value of $R$, formulas have been derived, which actually set back the progress of ground-water hydrology.

Such was the state of hydrologic knowledge when Dr. Ir. J. Versluys, mining engineer of the Government Institute for Water Supply, was called upon to design the ground-water catchment at Jaarsveld. On one
hand Versluys doubted the validity of Dupuit’s formula in the case under consideration, while on the other hand this formula only yields numerical results when the values of the geo-hydrologic constants $R$ and the product $kH$, the so-called coefficient of transmissibility, are known. For both reasons Versluys decided to carry out a test pumping. In both parts of the aquifer fully penetrating wells were set and pumped one by one with various capacities up to 240 m$^3$/hour. The accompanying lowering of the artesian water table was measured each time inside the pumped well and in the aquifer from which abstraction occurred at distances from 10, 40 and 80 m from the well centre. From the result of this and other test pumpings Versluys first drew two important conclusions:

1. in any point of the aquifer, including the well itself, the drawdown of the piezometric surface is linearly proportional to the capacity of abstraction;

2. the actual amount of drawdown depends on the shape and extent of the ground-water body and on the location of the point of observation, but is independent of the presence of existing ground-water flows.

Reduced to an abstraction of 1 m$^3$/minute, that is 1440 m$^3$/day, Versluys measured drawdowns of 0.35 m inside the well and 0.27 m, 0.173 m and 0.10 m at distances of 10, 40 and 80 m from the well respectively. From these measurements he concluded:

3. the drawdown of the piezometric surface decreases linearly with the logarithm of the distance to the centre of the pumped well.

The 3 conclusions together constitute Dupuit’s formula. From the data given, Versluys derived the relation:

$$\varphi = \frac{Q_0}{2\pi \cdot 2650} \cdot \ln \frac{250}{r}$$

in which $\varphi$ and $r$ are expressed in m and $Q_0$ in m$^3$/day.

To use this formula for the calculation of the drawdown due to a number of wells, Versluys otherwise had on the other hand to know their mutual influence. From his first rule he concluded in this respect:

4. the drawdown distribution due to pumping a well system is the sum of the distributions due to pumping the individual wells.

Nowadays this rule is known as the principle of superposition and derived from the circumstance that for the flow of artesian water the Laplace equation is linear.
With this principle of superposition formulated, the calculation of the drawdown due to pumping a straight line of wells is only a matter of mathematics. When the line contains \( n \) wells at constant intervals \( b \) and pumped at capacities of \( Q_1, Q_2, \ldots, Q_n \), the drawdown inside the various wells (neglecting well losses) becomes:

\[
\varphi_1 = \frac{Q_1}{2\pi k H} \ln \frac{R}{r_0} + \frac{Q_2}{2\pi k H} \ln \frac{R}{b} + \frac{Q_3}{2\pi k H} \ln \frac{R}{2b} + \ldots + \frac{Q_n}{2\pi k H} \ln \frac{R}{(n-1)b} \\
\varphi_2 = \frac{Q_1}{2\pi k H} \cdot \ln \frac{R}{b} + \frac{Q_2}{2\pi k H} \cdot \ln \frac{R}{r_0} + \frac{Q_3}{2\pi k H} \cdot \ln \frac{R}{b} + \ldots + \frac{Q_n}{2\pi k H} \cdot \ln \frac{R}{(n-2)b} 
\]

With a central suction line of ample dimensions so that pipe losses are negligible, the drawdown inside the wells is the same for all units:

\[
\varphi_1 = \varphi_2 = \ldots = \varphi_n = \varphi_0
\]

With the total capacity of the system fixed at \( Q \):

\[
Q = Q_1 + Q_2 + \ldots + Q_n
\]

a set of \( n+1 \) linear equations in \( \varphi_0 \) and \( Q_1 \) to \( Q_n \) must be solved. This is a time consuming job but it did not offer any special difficulties to VERSLUYS.

**DEVELOPMENT 1920–1960**

Looking back 40 years, it must be said that in terms of practical results obtained, the work of VERSLUYS is a complete failure. To VERSLUYS it is more or less a matter for congratulation that his scheme for ground-water abstraction never materialized and consequently it never appeared that actual drawdowns were far higher than calculated by him.

In the work of VERSLUYS for the ground-water catchment at Jaarsveld three parts can be distinguished:

1. the carrying out of a test pumping to determine experimentally the relation between the amount of drawdown and the distance to the point of abstraction;
2. the analysis of the test pumping for purposes of interpolation and extrapolation of the drawdowns observed;
3. the application of the method of superposition to calculate the drawdown distribution due to the projected line of wells.
Present hydrologic knowledge can find no fault in the third step of his work, but as regards the first two the following major deficiencies can be distinguished:

1. the test pumping carried out by Versluys was completely insufficient to determine the shape of the drawdown curve with any accuracy or certainty. The test well was situated at a short distance from the river Lek (explaining the otherwise too small drawdown inside the pumped well), resulting in aberrations which even Versluys must have found difficult to accept;

2. when analysing the result of the test pumping Versluys did not take the presence of the river Lek into account. This is the more strange as judging from his second conclusion, Versluys was well aware of the influence of size and shape of the ground-water body on the actual amount of drawdown and zero lowering at a distance of 250 m from a well in an infinite aquifer certainly must have looked strange to him. This neglect had the more serious effects as the final catchment was projected at a far greater distance from the river;

3. with a leaky artesian aquifer the relation between drawdown and distance to the pumped well is not governed by a semi-logarithmic expression but by a Bessel function or a combination of these functions. With the drawdown curves according to both expressions coinciding in the near vicinity of the well, at greater distances to the point of abstraction the Bessel functions give a much larger lowering of the artesian water table.

Versluys may be excused for the last deficiency, but as regards the two first mentioned ones, he certainly can be held liable. Even in his time with a mathematical treatment of ground-water flow problems still to come, far better results could have been obtained.

The development of ground-water hydrology in the years after the work of Versluys for the proposed catchment at Jaarsveld shows many aspects. As first major gain can be mentioned the way to carry out a test pumping. Even with such an important project as supplying a quarter of a million people with ground-water taken from a single catchment near Jaarsveld, Versluys measured the drawdown only in 3 observation holes, to a maximum distance of no more than 80 m from the pumped well and did not pay any attention to the accompanying lowering of the water table.
in the bounding aquifers. Nowadays with such a case, the drawdown would have been measured with about 30 observation holes, set in 4 directions at 90° interval, at distances from 10 to 1000 m from the pumped well and equipped with different screens placed in the various aquifers. The mistake of locating the pumped well close to the river Lek (not only complicating the analysis of test pumping results enormously but even making this well-nigh impossible with complex profiles) would not occur and the well would have been pumped with a far greater capacity as corresponds with a drawdown inside the well of 1.4 m only, making unavoidable natural variations in piezometric level (due to showers, fluctuations in barometric pressure, changes in the level of bounding streams, etc.) negligible.

A test pumping as described above gives a fairly accurate relation between drawdown and distance to the pumped well. When the drawdown curves in all 4 directions are about the same, so that any influence of boundary conditions may be assumed absent, this relation can be used without further analysis to calculate the drawdown caused by a given well system. Notwithstanding all precautions, however, this relation is troubled with different errors due to inaccuracies in observation, natural fluctuations in piezometric level, local variations in geo-hydrologic conditions etc. and furthermore unknown at larger distances from the well. At a large distance from a number of wells however, the drawdown may still be considerable. An analysis of a test pumping is, therefore, still necessary to smooth out the drawdown curve and to calculate the drawdowns for greater distances, up to infinity. Such an analysis, however, is only possible when the formulas for the drawdown curves of wells under various geo-hydrologic conditions are known.

Assuming Versluys had not made the obvious mistake of locating the test well close to the river, then indeed Dupuit’s formula could have been used as a first approximation to analyse the results obtained. Strictly spoken however, this formula only holds true for artesian aquifers without recharge, confined at top and bottom by fully impervious layers. In Jaarsveld the bounding layers in the meanwhile are not impervious, only less pervious, resulting in a recharge from above and from below. For the most simple case of such a leaky artesian aquifer, recharged only from the unconfined aquifer with constant water table above the semi-pervious
layer (FIG. 2), de Glee (1930) gave as derivation of the drawdown formula:

\[
\text{Darcy} \quad Q = -2\pi r \cdot H \cdot k \frac{d\varphi}{dr}
\]

Continuity \[ \frac{dQ}{dr} = -2\pi r \cdot \frac{\varphi}{c} \]

**FIG. 2** Well in a leaky artesian aquifer

in which \( c \) is the resistance of the semi-pervious layer against vertical water movement. Differentiating Darcy's law to \( r \) and substituting the value of \( \frac{dQ}{dr} \) in the continuity equation gives after rearranging terms:

\[
\frac{d^2\varphi}{dr^2} + \frac{1}{r} \frac{d\varphi}{dr} - \frac{\varphi}{kHc} = 0
\]

With the boundary conditions \( r \to \infty, \varphi = 0 \) and \( r \to r_0, Q = Q_0 \) this differential equation has as solution:

\[
\varphi = \frac{Q_0}{2\pi kH} \cdot K_0 \left( \frac{\lambda}{r} \right) \quad \text{with} \quad \lambda = \sqrt{kHc}
\]

and \( K_0 \) a modified Bessel function of the second kind, order zero.

The proper situation in Jaarsveld, where the aquifer in which the test well was set, is recharged from the phreatic water (with constant table) above as well as from the artesian water below (FIG. 3), has been investigated by Huisman and Kemperman (1951). According to their calculations:

**upper aquifer:**

\[
\text{Darcy} \quad Q_1 = -2\pi r \cdot H_1 \cdot k_1 \frac{d\varphi_1}{dr}
\]

Continuity \[ \frac{dQ_1}{dr} = -2\pi r \left( \frac{\varphi_1}{c_1} + \frac{\varphi_1 - \varphi_2}{c_2} \right) \]

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lower aquifer:

DARCY
\[ Q_2 = -2\pi r \cdot H_2 \cdot k_2 \frac{d\varphi_2}{dr} \]

Continuity
\[ \frac{dQ_2}{dr} = -2\pi r \cdot \frac{\varphi_2 - \varphi_1}{c_2} \]

Elimination of \( Q_1 \) and \( Q_2 \) from both sets of equations gives:

\[ \frac{d^2\varphi_1}{dr^2} + \frac{1}{r} \cdot \frac{d\varphi_1}{dr} - \frac{\varphi_1 - \varphi_2}{k_1H_1c_1} = 0 \]

\[ \frac{d^2\varphi_2}{dr^2} + \frac{1}{r} \cdot \frac{d\varphi_2}{dr} - \frac{\varphi_2 - \varphi_1}{k_2H_2c_2} = 0 \]

With the boundary conditions \( r \to \infty, \varphi_1 = 0, \varphi_2 = 0 \) and \( r \to r_0, Q_1 = Q_0, Q_2 = 0 \), these differential equations have as solution:

\[ \varphi_1 = \frac{Q_0}{2\pi k_1H_1} \cdot \frac{1}{\lambda_1 - \lambda_2} \left\{ (\lambda_1 - \alpha_2) \cdot K_0(\sqrt{\lambda_1} \cdot r) + (\alpha_2 - \lambda_2) \cdot K_0(\sqrt{\lambda_2} \cdot r) \right\} \]

\[ \varphi_2 = \frac{Q_0}{2\pi k_1H_1} \cdot \frac{\alpha_2}{\lambda_1 - \lambda_2} \left\{ -K_0(\sqrt{\lambda_1} \cdot r) + K_0(\sqrt{\lambda_2} \cdot r) \right\} \]

with

\[ \frac{\lambda_1}{\lambda_2} = \frac{1}{2} \left\{ a_1 + a_2 + \beta_1 \pm \sqrt{(a_1 + a_2 + \beta_1)^2 - 4a_1a_2} \right\} \]

and

\[ a_1 = \frac{1}{k_1H_1c_1}, \quad a_2 = \frac{1}{k_2H_2c_2}, \quad \beta_1 = \frac{1}{k_1H_1c_2} \]
It is with these formulas, that the analysis of test pumping results in Jaarsveld properly should have taken place. HUISMAN and KEMPERMAN have shown how such an analysis can be carried out with success, provided sufficient observations about the lowering of the artesian water level in both confined aquifers are available.

For the near vicinity of the well, the formulas of the drawdown curves with the different types of artesian aquifer can be written with good approximation in simplified form:

**DUPUIT**

\[
\varphi = \frac{Q_0}{2\pi kH} \cdot \ln \frac{R}{r}
\]

**De Glee**

\[
\varphi = \frac{Q_0}{2\pi kH} \cdot \ln \frac{1.123\lambda}{r}
\]

**Huisman/Kemperman**

\[
\varphi_1 = \frac{Q_0}{2\pi k_1H_1} \cdot \ln \frac{1.123 \cdot L}{r}
\]

with the geo-hydrologic constant \(L\) determined from:

\[
\ln L = \frac{\alpha_2 - \lambda_1}{\lambda_1 - \lambda_2} \ln \sqrt{\lambda_1} + \frac{\lambda_2 - \alpha_2}{\lambda_1 - \lambda_2} \ln \sqrt{\lambda_2}
\]

**Fig. 4** Drawdown of the artesian water table as function of the distance to the point of abstraction

That is to say in all cases of confined aquifers there exists here a linear relationship between \(Q\) and the logarithm of the distance to the point of abstraction. VERSLUYS limited his observations to the near vicinity of the well and even if the concept of a leaky artesian aquifer had been known to him, he would have been unable to choose between the different possibilities. With the 3 drawdown curves in the near vicinity of the well coinciding, at greater distances however the remaining draw-
down is larger to much larger as the recharge from the bounding aquifers is more important (FIG. 4). This again would have given VERSLUYS an unpleasant surprise.

With the results of the test pumping analysed and the lowering of the artesian water table known at any distance from the point of abstraction, VERSLUYS was able to calculate the drawdown distribution due to the proposed straight line of wells by a simple application of the method of superposition. Nowadays the same method is still used when the wells are few in number and must still be applied with a random position of the wells, when the location of the different wells cannot be described with simple mathematical formulas. For a large number of wells of constant capacity equally spaced in a straight line, however, new formulas have been developed which in much shorter time give the most important results. They are treated in the third part of this paper.

Reviewing the development of ground-water hydrology in the past 40 years, it will be clear from the preceding that the most important aspect is the successful application of mathematics to problems of ground-water flow. This application not only provided formulas to describe mathematically the different types of flow, but also assisted greatly in obtaining a better understanding of flow phenomena. In the last decade this application reached a new high in the co-operation between the Netherlands Hydrologic Colloquium and the Mathematical Centre in Amsterdam. The publication of this Colloquium about the steady flow of ground-water to wells (1962) is there to show how much this co-operation has achieved. Outside the Netherlands a similar development can be noticed for which only the names of THEIS, JACOB and especially HANTUSH need to be mentioned as proof.

Nowadays the situation is such that as regards steady flow of ground-water in schematized geo-hydrologic profiles, any problem can be solved. For most cases formulas are available or can be derived, in exceptional cases only recourse has to be taken to numerical methods or model tests. Model tests, especially tests with the glass-plates model developed by DIETZ and SANTING for the Government Institute of Drinking Water Supply still play a major role in the calculation of non-steady flow problems and in problems of steady flow where a larger number of bound-
ary conditions must be investigated or the values of the geo-hydrologic constants vary strongly over short distances. It may be expected, however, that in the near future they will be superseded by the application of electronic computers, which in shorter time are able to give more accurate results.

Summing up, the most important gain in the development of the science of ground-water hydrology is the co-operation between hydrologic practitioners and professional mathematicians. In this respect, however, groundwater hydrology does not differ from any other field of science.

**PRESENT METHODS OF CALCULATION**

With ground-water abstraction by lines of wells, central pumps and suction lines nowadays are superseded by a system where each well is equipped with its own pump (usually of the deep well centrifugal type). With these well pumps each well delivers the same amount of water and

---

**FIG. 5** Infinite straight line of wells in a leaky artesian aquifer of unlimited extent
with an infinite number of such wells in a straight line, the drawdown \( \varphi \) in a random point \((x, y)\) of a leaky artesian aquifer with unlimited extent (FIG. 5) is given by:

\[
\varphi = \frac{Q_0}{2\pi kH} \sum_{n=\infty}^{+\infty} K_0\left(\frac{\sqrt{x^2+(y-n\cdot b)^2}}{\lambda}\right)
\]

in which \( \lambda \) is the characteristic length of the formation:

\[
\lambda = \sqrt{k \cdot H \cdot c}
\]

and the wells are supposed to penetrate the full saturated thickness of the aquifer.

With the interval \( b \) equal to or larger than \( \lambda \), the serie converges rapidly and already a few terms suffice to give accurate results. In most cases, however, \( b \) is small compared to \( \lambda \) and computing \( \varphi \) with the formula above would involve a lot of figure-work. With \( b \) smaller than about \( \frac{1}{2} \) to \( \frac{3}{4} \lambda \), the afore mentioned publication of the Netherlands Hydrologic Colloquium gives as good approximation (fully penetrating wells):

\[
\varphi = \frac{Q_0}{2\pi kH} \left[ \frac{\pi \lambda}{b} \cdot e^{-x/\lambda} - \frac{1}{2} \ln \left(1 - 2e^{-2\pi x/b} \cos \frac{2\pi y}{b} + e^{-4\pi x/b}\right) \right]
\]

This formula looks complicated, but can be simplified in different ways:

1. at greater distances from the line of wells, with larger values of \( x \), the factors \( e^{-2\pi x/b} \) and \( e^{-4\pi x/b} \) rapidly approach zero, reducing the drawdown formula to:

\[
\varphi = \frac{Q_0}{2\pi kH} \cdot \frac{\pi \lambda}{b} \cdot e^{-x/\lambda} = \frac{Q_0}{2b} \cdot \frac{\lambda}{kH} \cdot e^{-x/\lambda}
\]

When the line of wells is replaced by the equivalent gallery, that is a gallery with the same capacity per lineal meter:

\[
q_0 = \frac{Q_0}{b}
\]

the drawdown \( \varphi^1 \) due to ground-water abstraction with this gallery can easily be calculated at:

\[
\varphi^1 = \frac{q_0}{2} \cdot \frac{\lambda}{kH} \cdot e^{-x/\lambda} = \frac{Q_0}{2b} \cdot \frac{\lambda}{kH} \cdot e^{-x/\lambda}
\]
That is to say, at greater distances from the line of wells, the drawdowns equal those of the equivalent gallery. At small distances there exists a difference between both drawdowns, but with \( x > b/2 \), this difference is less than 1% and negligible.

2. at the face of each well, \( x = 0 \) and \( y = r_0 + n \cdot b \), the formula for an infinite straight line of fully penetrating wells gives as drawdown:

\[
q_0 = \frac{Q_0}{2\pi kH} \left\{ \frac{\pi \lambda}{b} - \frac{1}{2} \ln \left(2 - 2 \cos \frac{2\pi r_0}{b}\right) \right\}
\]

\[
q_0 = \frac{Q_0}{2b \cdot kH} \cdot \frac{\lambda}{2\pi kH} - \frac{Q_0}{2\pi kH} \cdot \ln \left(2 \sin \frac{\pi r_0}{b}\right)
\]

With the drawdown at the face of the equivalent fully penetrating gallery equal to:

\[
q_0^1 = \frac{q_0}{2} \cdot \frac{\lambda}{kH} = \frac{Q_0}{2b \cdot kH}
\]

and with \( r_0/b \) small, thus:

\[
\sin \frac{\pi r_0}{b} = \frac{\pi r_0}{b}
\]

the drawdown at the face of the different wells is given by:

\[
q_0 = q_0^1 + \frac{Q_0}{2\pi kH} \ln \frac{b}{2\pi r_0}
\]

3. halfway between two wells, \( x = 0 \), \( y = b/2 + nb \) the drawdown is given by:

\[
q_{b/2} = \frac{Q_0}{2\pi kH} \left\{ \frac{\pi \lambda}{b} - \frac{1}{2} \ln 4 \right\}
\]

which also can be written as:

\[
q_{b/2} = q_0^1 - \frac{Q_0}{2\pi kH} \cdot 0.693
\]

4. according to 2 and 3, in the line of wells the drawdown at the well face is larger and halfway between the wells smaller than that of the
equivalent gallery. Somewhere in this line of wells \( (x = 0) \), at a distance \( w \) from each well, the drawdown must consequently equal the drawdown of the equivalent gallery:

\[
\frac{Q_0 \cdot \lambda}{2b \cdot kH} = \frac{Q_0}{2\pi kH} \left\{ \frac{\pi \lambda}{b} - \frac{1}{2} \ln \left( 2 - 2 \cos \frac{2\pi w}{b} \right) \right\}
\]

from which follows \( W = b/6 \) or:

\[
\varphi_{b/6} = \varphi_0^1
\]

According to the formulas given above, the drawdown due to pumping an infinite straight line of fully penetrating wells in a one-storied leaky artesian aquifer can be split up in two parts, the drawdown \( \varphi^1 \) of a fully penetrating gallery with the same capacity per lineal meter and for the near vicinity of the well corrections \( \Delta \varphi \) to compensate for point abstraction. The magnitudes of these correction factors only depend on the conditions in the near vicinity of the well. As has been shown in the second part of this paper, formation losses here are only a function of the coefficient of transmissibility \( kH \) of the aquifer concerned (formula of THIEM), and are the same for confined or unconfined flow, with or without recharge, for steady as well as for unsteady conditions, etc. The formulas derived above for the additional drawdowns due to point abstraction from a one-storied leaky artesian aquifer, therefore, are valid under all circumstances, independent of geo-hydrologic profile, boundary conditions, etc. With this proposition, however, the split up mentioned above gives a very simple method to calculate the drawdown distribution caused by pumping an infinite straight line of fully penetrating wells at intervals \( b \) with equal capacities \( Q_0 \) under all circumstances:

1. determine the drawdown \( \varphi^1 \) due to pumping a fully penetrating gallery with the same capacity per lineal meter. This drawdown takes into account the general conditions such as geo-hydrologic profile, boundary conditions, etc. The formulas for the drawdown curves of fully penetrating galleries are simple and easy to handle;

2. at both sides of the line of wells, at distances larger than \( b/2 \), the drawdowns equal those of the equivalent gallery;

3. in the line of wells (FIG. 6) the drawdown at the well face is larger by an amount:
FIG. 6 Drawdown curves of line of wells and equivalent gallery

FIG. 7 Fully and partially penetrating well

\[
\frac{Q_0}{2\pi kH} \cdot \ln \frac{b}{2\pi r_0} = \frac{Q_0}{2\pi kH} \cdot 2.3 \cdot \log \frac{b}{2\pi r_0}
\]

halfway between two wells smaller by an amount:

\[
\frac{Q_0}{2\pi kH} \cdot 0.693
\]

while at a distance \( b/6 \) from each well the drawdown is the same as for the equivalent gallery.

With the formulas given above, the well is supposed to penetrate the fully saturated thickness of the aquifer (FIG. 7). In many cases the depth of the aquifer is so large, that full penetration is not justified from an economic point of view. With partial penetration, however, ground-water velocities in the immediate vicinity of the well are higher than accounted for, resulting in an additional loss of head. The influence of partial penetration on drawdown distribution again is limited to the near vicinity of the well and consequently can also be taken into account with a correction factor. The most common situation of a partially penetrating well is shown in FIG. 8 to the left. From the work of De Glee it can be derived that at the face of such a well the additional drawdown due to partial penetration is given with good approximation by:
in which is $p$ is the amount of penetration:

$$p = h/H$$

With a random position of the well screen, as shown in FIG. 8 on the right hand side, an exact formula for the additional drawdown due to partial penetration is given by:

$$\Delta q_0 = \frac{Q_0}{2\pi kH} \cdot \frac{1 - p}{p} \cdot \ln \frac{(1.2 - p)h}{r_0}$$

with $a$ a function of the amount of penetration $p = h/H$ and of the amount of eccentricity $e = \delta/H$. The relation between $a$ and these two parameters has first been given in the publication of the Netherlands Hydrologic Colloquium and is reproduced below:

<table>
<thead>
<tr>
<th>$e$</th>
<th>0</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
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</table>
The formulas given above describe the drawdown distribution due to pumping an infinite line of wells. No doubt they are very interesting, but their practical significance is small as in reality a line of wells always extends over a limited distance only, resulting in smaller drawdowns. In the foregoing it has been shown that with an infinite line of wells the drawdown $\varphi$ can be split up in two parts:

$$\varphi = \varphi^1 + \Delta \varphi$$

in which $\varphi^1$ is the drawdown due to the equivalent gallery and $\Delta \varphi$ a correction to take into account the influence of point abstraction. The magnitude of $\Delta \varphi$ only depends on the conditions in the near vicinity of the well, that is on the value of the coefficient of transmissibility $kH$, the distance $b$ between wells and the diameter $2r_0$ of the well screen (including gravel pack if present). These factors, however, remain the same whether the line of wells is infinite or finite and with a finite number of wells the formulas for the additional drawdowns $\Delta \varphi$, therefore, remain unchanged. With a finite number of wells meanwhile the equivalent gallery also has a limited length only, and consequently the drawdown $\varphi^1$ due to pumping a gallery of infinite length may only partly be taken into account. With a limited number of wells, the drawdown becomes

$$\varphi = \beta \cdot \varphi^1 + \Delta \varphi$$

with $\beta$ always smaller as unity.

Continuing a method of calculation given by Edelman (1946), it has proved possible to determine the value of $\beta$ for points on the line of wells. The value of $\beta$, however, now also depends on the geo-hydrologic profile as well as on boundary conditions. When in a one-storied leaky artesian aquifer (FIG. 5), the line of wells contains $n$ units at interval $b$ and the length $L$ of this line is defined as:

$$L = n \cdot b$$

of which lengths $L_1$ and $L_2$ extend on either side of the point of observation (FIG. 9):

$$L = L_1 + L_2$$

then the value of $\beta$ is given by:

$$\beta = \frac{1}{2} F_1 \left( \frac{L_1}{\lambda} \right) + \frac{1}{2} F_1 \left( \frac{L_2}{\lambda} \right)$$
with \( \lambda = \sqrt{kHc} \)

and \( F_1(u) = \frac{2}{\pi} \int_0^u K_0(u) \cdot du \)

With a confined or unconfined aquifer without gain or loss of water through leaky aquicludes, the value of \( \beta \) with a similar line of wells at a distance \( l \) parallel to open water at a constant level equals:

\[ \beta = \frac{1}{2} F_2\left(\frac{L_1}{2l}\right) + \frac{1}{2} F_2\left(\frac{L_2}{2l}\right) \]

with \( F_2(u) = \frac{u}{\pi} \ln \left(\frac{1}{u^2}\right) + \frac{2}{\pi} \arctan u \)

The values of the function \( F_1 \) and \( F_2 \) are tabulated overleaf.

With ground-water abstraction by means of a straight line of wells, without any doubt the most important drawdown is the maximum drawdown occurring at the face of the centre well(s). Easily and in short time this drawdown can be calculated with the formulas given above, which also provide a clear insight into the composition of the drawdown, the influence of unit capacities and amount of penetration, very important for design purposes. With a limited number of wells, these formulas in the meanwhile cannot be used for a random point outside the line of wells and here the drawdown has still to be found by application of the method of superposition. At greater distances from the line of wells, the amount of figurework can be reduced by arranging the wells in a few sections and concentrating the abstractions in the respective centres of gravity.
### Table for $F_1(u) = \frac{2}{\pi} \int K_0(u) \cdot du$ and $F_2(u) = \frac{u}{\pi} \ln \left(1 + \frac{1}{u^2}\right) = \frac{2}{\pi} \arctan u$

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<th>$u$</th>
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<td>0.618</td>
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At a very great distance the drawdown due to pumping a line of wells even equals the drawdown due to pumping one well in the centre with the combined capacity. At small distances from the line of wells (but larger than $b/2$), the drawdown equals that of the equivalent gallery of finite length, that is

$$\varphi = \beta \cdot \varphi^1$$
As final words, the author of this paper would like to add that the description given above about the development of the various methods for calculation of the drawdown due to ground-water abstraction with a straight line of wells is only meant as an example to demonstrate the general advancement of ground-water hydrology in the past 40 years. This advancement did not come of its own accord, but arose from sheer necessity when wells systems for draining large and deep building sites (as with the Ymuiden lock) had to be designed such that success was assured, when the different aspects of reclaiming the Zuiderzee polders had to be predicted with fair accuracy, when a reliable forecast had to made about the influence of large ground-water catchments for public water supplies, etc. etc. Professor THIJSSE to whom this paper is dedicated, contributed to this advancement in no small way and the author retains pleasant memories of the discussions about the future ground-water table in Amsterdam after the southern Zuiderzee polders have been drained. Ground-water hydrology is an exact science owing to its mathematical origin, but it is also an exacting science as soon after completion of the work under consideration the outcome verifies the prediction. Both aspects must have appealed to professor THIJSSE.