Instantaneous planar pressure field determination around a square-section cylinder based on time-resolved stereo-PIV

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Abstract This paper describes the determination of instantaneous planar pressure fields from TR-PIV on a stationary square-section cylinder, with the face normal to the flow, for $Re_D=9,500$, where $D$ is the chord. The results from this planar pressure imaging (PPI) are compared with mean and fluctuating surface pressure data obtained with a pressure orifice and a surface-mounted microphone in two locations: at the lower surface and at the base of the model. The results for the lower surface of the model show agreement in both mean and fluctuating pressure compared with the pressure orifice and microphone data, and with previous studies. The results for the base show agreement for the mean pressure and give a good prospect for instantaneous planar pressure determination.

1. Introduction

The instantaneous pressure field around a body immersed in a fluid is of great practical importance within aerodynamics since it leads to the determination of important quantities, such as integral aerodynamic loads, for example.

Particle image velocimetry (PIV) has proven its strength as a non-intrusive flow diagnostic tool to characterize instantaneous velocity fields and derived quantities such as vorticity, but it also affords itself to the determination of the instantaneous pressure field. Recent efforts have applied momentum conservation principles on planar velocity data in order to extract more quantities, such as pressure fields and integral aerodynamic forces. The theoretical background for this planar pressure imaging (PPI) and velocimetry-based load determination have been established and its practical implementation for steady (or time-averaged) flows under approximately two-dimensional (2D) flow conditions has been demonstrated (e.g. Baur and Köngeter 1999, Gurka et al. 1999, Oudheusden et al. 2007). A natural extension to these existing PPI strategies is to consider unsteady flows. This can be achieved by time-resolved flow diagnostics such as time-resolved PIV (TR-PIV), cf. Kurtulus et al. (2007).

Pressure dominated flows like bluff-body flows are particularly interesting and challenging test cases for planar methods as PPI due to the complex time-evolving three-dimensional (3D) nature, especially at moderate to high Reynolds numbers.

The object of the present experimental investigation is to apply time-resolved PIV to the square-section cylinder problem, to demonstrate how pressure may be derived from instantaneous planar velocity and acceleration data, and to assess the influence of flow three-dimensionality on the method used. Experimental results obtained from PPI are compared with surface pressure measurements by orifice and microphone, as well as with previous studies.
2. Mathematical formulation

The instantaneous velocity and acceleration fields are acquired by TR-PIV to estimate instantaneous pressure fields using the in-plane pressure gradient components. Two different strategies are compared for the subsequent computation of the pressure fields: (i) direct spatial integration (see Baur & Köngeter 1999, Oudheusden et al. 2007) and (ii) an in-plane Poisson formulation (cf. Gurka et al. 1999). The in-plane pressure gradient components are obtained from the momentum equations as given in equation 1.

\[
\begin{align*}
\frac{\partial p}{\partial x} &= -\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + \mu \nabla^2 u \\
\frac{\partial p}{\partial y} &= -\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) + \mu \nabla^2 v 
\end{align*}
\]

(1)

In the direct approach the pressure gradients are integrated directly from reference point(s) (Dirichlet boundary condition) using a spatial marching erosion scheme (Oudheusden & Souverein 2007), which introduces a dependence on its integration path. The (planar) Poisson approach (equation 2) uses the in-plane divergence of the pressure gradient in combination with Neumann boundary conditions (pressure gradient), next to the use of reference point(s).

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = f
\]

(2)

The right-hand-side for the (planar) Poisson pressure formulation is stated in equation 3, taking into account 3D flow.

\[
f = -\rho \left( \frac{\partial \text{div} \, \text{div}_{xy}}{\partial t} + (\vec{u} \cdot \nabla) \text{div}_{xy} + \left( \frac{\partial u}{\partial x} \right)^2 + 2 \frac{\partial v}{\partial x} \frac{\partial u}{\partial y} + \left( \frac{\partial v}{\partial y} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} \frac{\partial v}{\partial z} \right) + \mu \nabla^2 \text{div}_{xy}
\]

(3)

Unfortunately the current PIV methods only give us the in-plane quantities, where stereo-PIV also gives the out-of-plane velocity component. In case of 3D flow not all quantities are captured to determine the planar pressure gradient. To capture the out-of-plane gradients one must resort to volumetric approaches, for example Tomographic PIV (Elsinga et al. 2006). In this paper terms containing out-of-plane gradients are not taken into account, and the possible consequence of this is to be assessed.

3. Experimental arrangement and procedure

Experiments were performed in a low-speed, open-exit wind-tunnel at the Aerodynamics laboratory at Delft University of Technology. This facility has a transparent test-section of dimensions 0.4x0.4x0.4m³. The turbulence level in the test-section was 0.11% (Tummers 1999). The square-section cylinder of dimensions 30x30mm² spanned the width of the tunnel. In the present experiment, the free-stream velocity was \( U_\infty = 4.7 \text{m/s} \) (\( q_\infty = 13.4 \text{PA} \)) with a cylinder Reynolds number of \( Re_D = 9,500 \). The Karman vortex shedding was observed to occur at a frequency of 20Hz (\( St = f D/ U_\infty = 0.13 \), primary peak in Fig. 5a) and the frequency associated to the vortex formation in the separated shear-layer (Kelvin-Helmholtz instability) was about 500Hz, determined from visual
inspection of the convection of the vortices within the shear-layer.

The square-section cylinder was fitted with a pressure orifice and a microphone in close proximity along the centreline, in order to provide reference values for the mean and the fluctuating surface pressure. Two different orientations of the model were used to have the pressure measurement at either the lower surface or the base.

The mean pressure was measured with a Mensor digital pressure gauge, DGP 2101, connected to the pressure orifice of 0.4\textit{mm} diameter with tubing of about 2\textit{mm} internal diameter. Pressure fluctuations were measured with a Sonion 8002 microphone, and recorded using a National Instruments data acquisition system (PCI-6250, SCXI-1001, SCXI-1520, and SCXI-1314) operating at 100kHz. To convert the microphone signal into pressure values, it is corrected for its sensitivity at the dominant frequency: 20Hz for the lower surface and 40Hz for the base.

As only a comparative study of different pressure measurements is pursued none of the results presented have been corrected for the effects of wind-tunnel blockage. The geometric blockage was 7.5%.

A high-repetition rate stereo-PIV system was used to capture the flow at the symmetry plane of the model. Flow seeding is provided by a Safex smoke generator which provided droplets of about 1\textit{\mu m} in diameter. The symmetry plane is illuminated by a Quantronix Darwin-Duo laser-system with an average output of 80W at 3kHz with wavelength of $\lambda = 527\text{nm}$ (equivalent pulse energy at 2.7kHz: 2x15mJ). The laser-pulse time separation was 100\textit{\mu s}. Images are acquired by two Photron Fastcam SA1 cameras with 1024x1024\textit{pixels} sensor, recording image-pairs at 2.7kHz (cameras running at 5.4kHz). The field of view (FOV) was 76x76\textit{mm} (2.5x2.5D), with digital resolution of 13.5\textit{pixel/mm}. An overview of the experimental arrangement is given in Fig. 1. A total of 2700 image-pairs were recorded spanning a time-interval of 1s for each experimental configuration.

![Fig. 1](image.png)  
**Fig. 1** Wind-tunnel and PIV setup. Note: illumination from the bottom; flow from left to right.

Particle images were processed using DaVis 7.3 software (LaVision GmbH), including stereo- and self-calibration (Wieneke 2005), with a final interrogation window-size 16x16\textit{pixels} and an overlap-factor of 50\% giving a vector grid of 129x129\textit{vectors} with a vector spacing of 0.6\textit{mm} (0.02D). The vector fields were processed with a regional median filter combined with a multiple peak check (included in DaVis 7.3, the regional median filter is based on Nogueira \textit{et al.} (1997). Thresholds
were set at > 2 remove and < 3 (re)insert, and empty spaces were filled by averaging all the nonzero neighbour vectors (< 2% per vector field).

Acceleration terms are determined using central differencing in time, velocity gradients are determined using central differencing in space. Forward/backward differencing is used near the edges of the domain. The pressure field is integrated using a Dirichlet boundary condition (prescribed pressure values from Bernoulli) on the lower edge of the FOV, where the flow is sufficiently steady and free of rotation. For the \( \text{PPI}_{\text{Poisson}} \) approach additional Neumann boundary conditions are enforced on the remaining edges of the integration domain.

4. Results and discussion

Here the results of the stereo-PIV setup and the different pressure determinations and measurements will be discussed. First the velocity results and derived quantities are discussed. Next the pressure fields obtained from PPI are discussed. Then the pressure results from different methods are compared together, and with previous studies. Finally the result of a simple uncertainty analysis is presented.

4.1 Velocity results and derived quantities

Fig. 2 shows the mean velocity and RMS velocity fields, on the left and right respectively. The mean velocity fields (Fig. 2a, 2c, and 2e) appear to be symmetric along the centreline of the wake. A shear-layer can be seen originating from the bottom upstream corner leading to a recirculation zone in the wake. Also it can be seen that the wake is highly three-dimensional, in particular, the out-of-plane velocity shown in Fig. 2e is a significant fraction of the free-stream velocity. The cause(s) of this behaviour are presently unknown, although cellular spanwise cells are well known to occur for bluff body flows (Williamson 1996). The RMS field of the u-component (Fig. 2b) shows a region of high fluctuations (up to 70% of \( U_\infty \)) just under the trailing lower corner. This region doubles in size and halves in intensity one chord down-stream. This region of high fluctuations corresponds to the region where the shear-layer moves up and down, and as the movement of the shear-layer gets larger the fluctuations become more spread. The RMS field of the v-component (Fig. 2d) shows large fluctuations (up to 95% of \( U_\infty \)) associated with the Karman wake, having a maximum at one chord behind the model. These findings are in agreement with the findings of Oudheusden et al. (2005). The RMS field of the w-component (Fig. 2f) shows a sudden increase to 30% of \( U_\infty \) half-way along the model. This indicates that the shear-layer becomes turbulent and results in a turbulent wake.

Fig. 3 shows instantaneous velocity and acceleration components (Fig. 3a, 3c, and 3e and Fig. 3b, 3d, and 3f respectively), the out-of-plane vorticity (Fig. 3g) and the planar divergence (Fig. 3h). The separated shear-layer and its instability are clearly visible, especially in the vorticity field (Fig. 3g). In the shear-layer separate vortices can be identified as part of the Kelvin-Helmholtz instability. These vortices give a typical induced velocity and acceleration field, showing up as small dipoles in the in-plane components (Fig. 3a, 3b, 3c, and 3d). A counterclockwise Karman wake vortex can be seen just next to the lower right corner of the model, showing up as a large dipole in the in-plane velocity fields (Fig. 3a and 3b) and as a collection of positive vorticity (Fig. 3g). The 3D flow in the wake is clearly visible in the w-component, dw/dt-component, and planar divergence fields (Fig. 3e, 3f, and 3h).
4.2 Pressure fields

Fig. 4 shows the PPI results for the direct and the Poisson approaches, on the left and right respectively. The mean pressure and RMS pressure fields are based on 900 pressure fields covering 0.33s (one-third of a complete run). The mean pressure fields of the two approaches agree well, whereas the RMS pressure fields show a clear difference (Fig. 4b and 4e) in particular the direct approach is 50% noisier in the wake than the Poisson approach. This is caused by a ‘memory’-effect in the direct method, this method propagates and accumulates errors along its marching direction. In Fig. 4c the ‘memory’-effect is visualized by pressure-streaks near the lower surface of the cylinder. Both approaches show an asymmetry in the RMS of the pressure fields, which is likely due to the missing out-of-plane terms that were not considered in this analysis.

Despite only 2-3 window-sizes or less across the shear-layer and its vortices, pressure dips are captured, coinciding with the location of (partially) separate vortices of the Kelvin-Helmholtz instability (compare Fig. 3g and Fig 4f). The Karman wake vortex as described before coincides with the large low pressure region next to the bottom right corner of the model (Fig. 4c and 4f).

4.3 Comparison of pressure signals

Fig. 5 shows magnitude spectra for the microphone and the direct and the Poisson PPI approaches, left and right respectively, the spectra are based on the same number of points and the same time-interval (900 and 0.33s respectively; Δf=3Hz). The microphone signal for both cases has a distinct peak: 20Hz at the lower surface corresponding to the Karman frequency and 40Hz at the base, which is double the Karman frequency due to symmetry. The microphone signal is corrected using the microphones sensitivity curve (Sonion A/S) for a single dominant frequency. The spectra obtained at the lower surface shows good agreement between both PPI approaches and the microphone signal for both the peak location and the magnitude (Fig. 5a). The PPI spectra obtained at the base show the same peak location as the microphone signal (40Hz), but also have a signal at the original Karman shedding frequency (20Hz). This extra peak is attributed to the errors introduced by the missing out-of-plane terms in the determination of the pressure gradient.

Fig. 6 shows the actual time signal for PPI and reference pressure (orifice and microphone for mean and fluctuating pressure respectively). Next to the correction for the dominant frequencies the microphone signal was corrected for the time lag of the microphone based on a cross-correlation between the PPI and microphone signal (0.016s and 0.009s for the lower surface and base respectively). The pressure signal for the bottom side (Fig. 6a) shows a good agreement for both the PPI approaches and the microphone with orifice. The relatively large fluctuations due to the Karman shedding are well captured, whereas the small fluctuations for all methods must be considered as noise, as there are no other distinct peaks in the magnitude spectra (Fig. 5). Despite the 3D nature of the cylinder wake, the pressure signal for the base of the model has a fair agreement for the PPI_{Poisson} and the microphone with orifice. This gives hope for future PPI research in unsteady 3D flows.

The mean and RMS pressure from PPI on the lower side are in reasonable agreement with the pressure from the orifice and the microphone, with only a difference of 6% and 9% respectively (see Table 1). These values also show reasonable agreement with previous studies despite the difference in Reynolds number, free-stream turbulence level, and/or wind-tunnel blockage (Bearman & Obajasu 1982, Noda & Nakayama 2003, Nakaguma & Ohya 1984).
In addition the mean pressure from PPI on the back side shows reasonable agreement with the orifice measurement, differing 9% and 7% for the direct and the Poisson approach respectively (see Table 1). Conversely the RMS pressure from PPI on the base differs by 230% and 120% for the direct and Poisson approach respectively (see Table 1). This huge difference can be ascribed to the missing terms in the determination of the pressure gradient. This clearly indicates that for an accurate determination of instantaneous planar pressure the current quasi-2D approach is insufficient.

<table>
<thead>
<tr>
<th>Location</th>
<th>Method/source</th>
<th>Turbulence</th>
<th>Re</th>
<th>St</th>
<th>$(p_{\text{mean}}-p_\infty)/q_\infty$</th>
<th>$\sigma_p/q_\infty$</th>
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<tr>
<td>Lower side</td>
<td>Orifice</td>
<td>0.11%</td>
<td>$9.5\times10^3$</td>
<td>0.13</td>
<td>-1.51</td>
<td>-</td>
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<td></td>
<td>Microphone$^a$</td>
<td>-</td>
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<tr>
<td></td>
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<td></td>
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<td>-</td>
<td>-1.60</td>
<td>0.50</td>
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<tr>
<td>Bearman &amp; Obasaju</td>
<td></td>
<td>&lt; 0.04%</td>
<td>$2\times10^4$</td>
<td>0.13</td>
<td>-1.86$^b$</td>
<td>0.65</td>
</tr>
<tr>
<td>Nakamura &amp; Ohya</td>
<td></td>
<td>0.12%</td>
<td>$1.4\times10^4$</td>
<td>-</td>
<td>-1.65</td>
<td>-</td>
</tr>
<tr>
<td>Back side</td>
<td>Orifice</td>
<td>0.11%</td>
<td>$9.5\times10^3$</td>
<td>0.13</td>
<td>-1.40</td>
<td>-</td>
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<tr>
<td></td>
<td>Microphone$^a$</td>
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<td></td>
<td>-</td>
<td>0.18</td>
<td>0.59</td>
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<td>PPI$^\text{Direct}$</td>
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<td></td>
<td>-</td>
<td>-1.50</td>
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</tr>
<tr>
<td>Bearman &amp; Obasaju</td>
<td></td>
<td>&lt; 0.04%</td>
<td>$1.1\times10^4$; $2\times10^4$</td>
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<td>-1.60</td>
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<td>Nakamura &amp; Ohya</td>
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<td>$1.4\times10^4$</td>
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<td>-1.50</td>
<td>-</td>
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<td>Noda &amp; Nakayama</td>
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<td>0.131</td>
<td>-1.483</td>
<td>-</td>
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</table>

$^a$ corrected for 20 Hz; $^b$ Re undefined; $^c$ corrected for 40 Hz

4.4 Uncertainty analysis

A test was performed to investigate the sensitivity of the pressure integration for capturing the large pressure fluctuations. Using a quarter of the resolution ($64\times64$ vectors instead of $129\times129$ vectors) the pressure signal showed the same agreement for the large fluctuations (Karman shedding at $20\text{Hz}$), whereas the small fluctuations ($> 100\text{Hz}$) increased with 50%.

Based on a simple statistical analysis, the errors on the mean streamwise velocity and RMS of the streamwise velocity were estimated to be 0.3% and 5% of $U_\infty$, respectively. Using a simple linear-error-propagation analysis the errors on the instantaneous streamwise velocity and gradient quantities were estimated to be 1.5% of $U_\infty$ and 18% of the $\sigma_{(\partial u/\partial x)}$ in the wake ($\sigma_{(\partial u/\partial x)}=500s^{-1}$) respectively. The uncertainty on the pressure and pressure gradient depend on general parameters, which vary within the flow. A detailed study of these uncertainties is currently in progress.

5. Conclusions

Mean and fluctuating pressures have been determined from PIV velocity data on and around a square-section cylinder using two PPI approaches and have been compared with a pressure orifice (mean pressure), a microphone (fluctuating pressure), and with previous studies. In the present study only planar velocity info was used, implying that in the in-plane pressure gradients description the out-of-plane terms are not accounted for. The large scale pressure fluctuations and the mean pressure on the lower surface of the model are captured correctly by both PPI approaches and agree well with pressure orifice, microphone and previous measurements. The mean pressure at the base of the model agrees with the pressure orifice measurement and also with previous studies. The differences in the fluctuating pressure in the wake are most likely due to missing out-of-plane
terms for determining the pressure gradient. The time-resolved signal shows a good prospect for future PPI research in unsteady 3D flows. A detailed uncertainty analysis of determining the pressure and pressure gradient with PPI is still to be performed.

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References
Baur T & Köngenter J (1999) PIV with high temporal resolution for the determination of local pressure reductions from coherent turbulent phenomena. 3rd Int. Workshop on PIV, Santa Barbara, 671-676.
Lavision Gmbh: www.lavision.de.
Fig. 2  Mean velocity components and RMS.
   a)-b) Mean u-component and RMS
   c)-d) Mean v-component and RMS
   e)-f) Mean w-component and RMS
Fig. 3  Instantaneous velocity and acceleration components at \( t = 0.01 \)s.

a)-b) \( u \)-component and \( \frac{du}{dt} \)

c)-d) \( v \)-component and \( \frac{dv}{dt} \)

e)-f) \( w \)-component and \( \frac{dw}{dt} \)

g)-h) Out-of-plane vorticity and planar divergence
**Fig. 4** Mean, RMS and instantaneous pressure from PPI.

- a)-c) $PPI_{\text{direct}}$, mean pressure, RMS and instantaneous pressure at $t = 0.01s$
- d)-f) $PPI_{\text{poisson}}$, mean pressure, RMS and instantaneous pressure at $t = 0.01s$

**Fig. 5** Magnitude spectra ($\Delta f = 3 Hz$).

- a) $PPI$ and microphone (corrected for $20 Hz$) at the lower side of the square-section cylinder.
- b) $PPI$ and microphone (corrected for $40 Hz$) at the back side of the square-section cylinder.
Fig. 6  PPI data compared with pressure orifice and microphone data.
   a) Pressure from PPI, microphone and orifice at the lower side of the square-section cylinder.
   b) Pressure from PPI, microphone and orifice at the back side of the square-section cylinder.
   PPI\textsubscript{direct} is omitted for clarity of the graph.