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Low-frequency behavior of the turbulent axisymmetric near-wake

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The turbulent wake past an axisymmetric body is investigated with time-resolved stereoscopic particle image velocimetry (PIV) at a Reynolds number \( Re_D = 6.7 \times 10^4 \) based on the object diameter. The azimuthal organization of the near-wake is studied at different locations downstream of the trailing edge. The time-averaged velocity field features a circular shear layer bounding a region of recirculating flow. Inspection of instantaneous PIV snapshots reveals azimuthal meandering of the reverse flow region with a significant radial offset with respect to the time-averaged position. The backflow meandering appears as the major contribution to the near-wake dynamics in proximity of the base, whereas closer to the rear-stagnation point, the shear layer fluctuations become important. For \( x/D \leq 0.75 \), the time-history and probability distributions of the backflow centroid position allow to identify this motion with an irregular precession about the model symmetry axis occurring at time scales in the order of \( 10^3 D/U_\infty \) or higher. The first two modes obtained by snapshot proper orthogonal decomposition of the velocity fluctuations can be related to an anti-symmetric mode of azimuthal wave-number \( m = 1 \) reflecting a radial displacement of the separated flow region, while the third and fourth proper orthogonal decomposition modes are identified with a second mode pair \( m = 2 \) and are representing wake ovalization. Close to the base, a third axisymmetric mode \( m = 0 \) is identified, corresponding to a streamwise pulsation of the reverse flow region. Based on the analysis of the spatial eigen-functions and frequency spectra of the time-coefficients, it is concluded that the anti-symmetric mode \( m = 1 \) is associated with the backflow instability in the very-low frequency range \( St_D = 10^{-4} \) close to separation, whereas more downstream it reflects the fluctuations related to the shear layer development.

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I. INTRODUCTION

Turbulent wake flows past bluff bodies of revolution are of relevance for a variety of engineering systems, particularly in the context of transport industry, and have been the subject of numerous investigations, as discussed in the review of Oertel. A dominant feature of these flows is the large-scale shear layer unsteadiness arising at Reynolds number \( Re > 10^2 \) as a result of growing unstable modes. Notwithstanding the technical relevance of these axisymmetric wake configurations, notably for the high-Reynolds number turbulent regime, they appear to have received much less attention in comparison to nominally 2D configurations, such as the classic case of the circular cylinder wake.

The experiments of Merz et al. on the axisymmetric turbulent wake of a truncated cylinder aligned with the freestream show that the recirculating flow produces strong pressure fluctuations at the base. Further studies conducted by Fuchs et al. indicate that the most intense fluctuations occur in the low-frequency regime corresponding to a diameter-based Strouhal number, \( St_D \approx 0.2 \), and appear in the form of large-scale anti-symmetric (i.e., azimuthal wavelength \( m = 1 \)) oscillations associated with a vortex shedding phenomenon. Such anti-symmetric oscillations have been later found to be a major feature of the wake of other more complex configurations such as truncated cylinders with afterbody.

In a recent investigation, Rigas et al. experimentally characterized the unsteady base pressure field of an axisymmetric truncated model. Based on the analysis of the base flow pressure...
dynamics, the authors identified a second, yet important very-low frequency contribution of the anti-symmetric \( m = 1 \) mode, possibly associated with a rotation of the shedding plane and corresponding instantaneous offset of the base pressure centroid. Spectral analysis revealed that this mode evolves at a normalized frequency \( St_D \approx 0.002 \), which is about two orders of magnitude lower than that associated with the vortex shedding itself. Such low-frequency variations result in the axial symmetry being achieved only by very long-term averaging. An analogous instability has been observed in the wake behind spheres\(^9\) as well as rectilinear bodies (e.g., Ahmed bodies),\(^{10}\) whereby in the latter case it occurs as an irregular shift of the recirculation bubble between two preferred anti-symmetric states. Furthermore, this phenomenon shows similarity with the precession motion experienced by the stagnation point in the wake of turbulent annular jets,\(^{11}\) featuring a comparable characteristic frequency \( (St_D \approx 0.0025, D \text{ being the jet diameter}) \). In this kind of flows, up to 45\% of the fluctuations in the stagnation region have been attributed to such motion.

The existence of such a backflow low-frequency unsteadiness has previously been linked to the persistence in the turbulent regime of the symmetry-breaking mode,\(^{10,11}\) with the prescribed symmetry plane continuously changing its orientation in view of its sensitivity to external perturbations.\(^{12,13}\) Asymmetries in the near-wake flows of axisymmetric bodies have been documented in the literature for subsonic conditions\(^{14-16}\) as well as in the transonic\(^{17}\) and supersonic regime\(^{18}\) and are mainly ascribed to boundary condition effects such as misalignments between wind-tunnel model and free-stream flow. Based on the parametric studies conducted by Wolf \textit{et al.}\(^{19}\) and Klei\(^{20}\) on blunt-based rocket configurations, marked distortions of the near-wake topology can exist in presence of angular misalignments as low as 0.3\(^\circ\). These observations indicate that a careful alignment of the model with respect to direction of the incoming flow is a pre-requisite of any reliable wind-tunnel experiment on such configurations, while furthermore the existence of a low-frequency unsteadiness of the backflow imposes important constraints on the minimum observation-time or computation-time in the context of numerical investigations.

The influence of the recirculating flow unsteadiness on the mean, instantaneous, and fluctuating flow field has been reported for three-dimensional\(^{10}\) and axisymmetric configurations.\(^8,9\) However, the correlation between backflow precession and the shedding phenomenon requires further clarification, both being \( m = 1 \) modes, but occurring at significantly different time scales. Furthermore, as the influence of such an instability on the azimuthal-radial wake topology has been discussed based on either pressure measurements on the base\(^8\) or particle image velocimetry (PIV) measurements\(^9,10\) in proximity of the rear-stagnation point, the streamwise evolution of the low-frequency instability has not been fully characterized yet.

In the current study, the spatio-temporal dynamics of the turbulent axisymmetric wake is investigated in several azimuthal-radial planes. For this purpose, wind tunnel experiments are conducted on the near-wake flow produced by an ogive-cylinder aligned with the free stream flow, at a Reynolds number \( Re_D = 6.7 \times 10^4 \), based on the model diameter. Stereoscopic Particle Image Velocimetry (PIV) measurements are performed in cross-flow planes that are evenly spaced in the streamwise direction, in order to encompass the wake from separation point until beyond the reattachment location. The recordings are performed at two distinct acquisition frequencies to capture the temporal dynamics of the wake fluctuations \( (f_{acq} = 2 \text{ kHz}) \) as well as the long-term instability of the recirculating flow \( (f_{acq} = 100 \text{ Hz}) \). Snapshot proper orthogonal decomposition (POD) (Sirovich\(^{21}\)) is applied to identify the most significant contributions to the near-wake dynamics, which provides a detailed description of their spatial arrangement. The characteristic time scales of the most dominant modes are analyzed in the frequency domain with the spectra of the associated time-coefficients.

II. EXPERIMENTAL APPARATUS AND TECHNIQUES

A. Flow facility and wind tunnel model

The experiments were conducted in the low-speed wind tunnel (W-Tunnel) of the Aerodynamics Laboratories of Delft University of Technology, which has a contraction ratio of 9:1. A square
0.4 × 0.4 m² test-section was used with Perspex walls, allowing optical access for illumination. The tunnel was operated at room temperature and pressure at a free stream velocity of 20 m/s, yielding a Reynolds number $R_e D = 6.7 \times 10^4$ based on the model diameter. The free stream turbulence intensity in these conditions was 0.5%.

The model is a truncated-base cylinder featuring a spherically blunted tangent ogive with nose radius of 3 mm, ensuring a smooth development of the boundary layer up to the junction with the cylindrical main body (see Fig. 1), where Carborundum particles are applied over a 10 mm wide roughness patch to force boundary layer transition. The model was supported by a 3 mm thick vertical plate with sharp edges. The thickness of the support was reduced to 1.5 mm at the junction with the main body in order to limit the development of the horseshoe vortex due to the interference with the incoming boundary layer.22

**B. Measurements apparatus and procedure**

Stereoscopic particle image velocimetry (PIV) measurements were performed in planes perpendicular to the model axis at four streamwise positions, $x/D = \{0.375, 0.75, 1.125, 1.5\}$. The measurement plane at $x/D = 1.5$ is downstream of the mean rear-stagnation point which is located at $x/D = 1.2$.14,23 The flow was seeded with micron-sized droplets by means of a SAFEX smoke generator at a uniform concentration of approximately 5 particles/mm³. Illumination was provided by a Quantronix Darwin Duo Nd–YLF laser (2 × 25 mJ/pulse at 1 kHz). The thickness of the laser sheet in the measurement region was 3 mm. The pulse separation was set to 25 µs, accounting for an out-of-plane particle displacement not exceeding 0.5 mm.

Two Photron FastCAM SA1.1 CMOS cameras (1024 × 1024 pixels 5400 fps, 20 µm pixel pitch) were used for the recordings with the lines of sight subtending an angle of 35° (see Fig. 2). The cameras were equipped with 105 mm focal length Nikkor objectives with aperture set to $f_\# = 2.8$, installed on a lens-tilt mechanism in order to satisfy the Scheimpflug condition. The field of view (FOV) covered a region of approximately $75 \times 75$ mm² (i.e., $1.6 D \times 1.6 D$) centered on the model axis. The resulting optical magnification factor was $M = 0.17$. A total of 5000 double-frame images were acquired for each streamwise position at an acquisition frequency of 100 Hz and 2 kHz accounting for observation-time of 50 s and 2.5 s respectively. The measurement error affecting the instantaneous velocity was estimated to be approximately 1% of the free stream value (Raffel et al.24). The statistical uncertainties relative to the free stream velocity were estimated to be 0.9% and 0.6% for the mean and the RMS of the fluctuations, respectively (Benedict
and Gould\cite{25}, based on RMS fluctuations of approximately 20\% of the free stream velocity. The uncertainty on the out-of-plane component of the velocity was approximately three times higher than the in-plane components, considering the stereoscopic viewing angle (Prasad\cite{26}). The main experimental parameters are summarized in Table I.

A stereoscopic imaging mapping function\cite{27} based on a third-order polynomial was used. An initial estimate of the mapping coefficients was obtained by means of a two-layer calibration plate (LaVision type 10) with equally spaced dots, aligned with the laser sheet within 0.5 mm precision. Residual misalignments between the plate and the laser sheet plane were corrected using a self-calibration procedure based on the disparity between images from left and right view.\cite{28}

Illumination and recording systems were synchronized with a LaVision High Speed Controller hosted by a PC using Davis 8.1 software.

C. Data processing

Image pre-processing was applied to the recordings in order to reduce background light intensity and reflections, especially for the measurements performed in proximity of the model. The intensity of each pixel was normalized with respect to the time-averaged value (Adrian and Westerweel\cite{29}). Further reduction of the time-varying background intensity was achieved by a minimum subtraction over a temporally sliding kernel of 11 images.

An iterative multi-grid cross correlation analysis based on window deformation\cite{30} was used in the velocity vectors calculation. The final interrogation window size was 24 × 24 pixels (2.8 × 2.8 mm$^2$) and the window overlap was set to 75\%, resulting in a vector pitch $p = 0.7$ mm ($p/D = 0.014$). Spurious vectors were detected using the universal median filter.\cite{31} The confidence in the estimation of the particle displacement was assessed by inspection of the signal-to-noise ratio (SNR) of the cross correlation map, whose mean value along the sequence was measured to be above 4 in the separated wake, with a minimum of 1.5 in the shear layer region. Based on the latter values and the relative size of the interrogation window, the present measurements are considered suitable for the description of the large-scale wake dynamics, but not for the analysis of the shear layer fine-scale turbulence.

### Table I. Experimental parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free stream velocity $U_{\infty}$</td>
<td>20 m/s</td>
</tr>
<tr>
<td>Base model diameter $D$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Reynolds number $Re_D$</td>
<td>$6.7 \times 10^4$</td>
</tr>
<tr>
<td>Pulse separation $\Delta t$</td>
<td>25 $\mu$s</td>
</tr>
<tr>
<td>Laser sheet thickness $t$</td>
<td>3 mm</td>
</tr>
<tr>
<td>Field-of-view size $FOV$</td>
<td>$75 \times 75$ mm$^2$</td>
</tr>
<tr>
<td>Magnification factor $M$</td>
<td>0.17</td>
</tr>
<tr>
<td>Acquisition frequency $f$</td>
<td>100–2000 Hz</td>
</tr>
<tr>
<td>Data ensemble size $N$</td>
<td>5000</td>
</tr>
<tr>
<td>Measurements duration $T$</td>
<td>50–2.5 s</td>
</tr>
</tbody>
</table>
D. Proper orthogonal decomposition

Proper Orthogonal Decomposition (POD) is applied to the fluctuating component of the velocity \( u'(x,t) \) to examine the most energetic fluctuations dominating the near-wake dynamics. The fluctuating component of the velocity is obtained by subtracting the average velocity \( U(x) \) from the instantaneous velocity realizations \( u(x,t) \). Snapshot POD\(^\text{21}\) is performed on the fluctuating velocity vector, which is decomposed into a number of spatial functions \( \phi_k(x) \) and temporal functions \( c_k(t) \) as follows:

\[
u'(x,t) = \sum_{k=1}^{N} \phi_k(x) c_k(t),\]

where \( N \) is the total number of snapshots of the velocity field. The functions \( \phi_k(x) \) represent the POD modes and account for the spatial distribution of the fluctuating velocity field, whereas the functions \( c_k(t) \) are referred to as the POD time-coefficients and account for the temporal evolution of the amplitude of the fluctuations. The modes \( \phi_k(x) \) are obtained as the eigen-vectors from the singular value decomposition (SVD) of the autocorrelation matrix \( R \),

\[
R = u'(x,t) \cdot u'(x',t).
\]

The associated eigen-values \( \lambda_k \) represent the energy contribution of the individual modes to the total turbulent kinetic energy of the flow and are sorted in descending order of importance as a result of the SVD, while the eigen-vectors are normalized such that \( ||\phi_k|| = 1 \). The time-coefficients are obtained by projecting the fluctuating field onto the basis of POD modes \( \phi_k \),

\[
c_k(t) = \phi_k(x) \otimes u'(x,t).
\]

In the present work, the spatial eigen-functions \( \phi_k(x) \) are inspected in order to characterize the spatial organization of the dominant fluctuations, whereas their characteristic time scales are retrieved from the spectral analysis of the time-coefficients \( c_k(t) \).

Finally, a reduced order model representation which is obtained using a limited number of modes in combination with the mean flow field can be evaluated as follows:

\[
u_{ROM}(x,t) \equiv U(x) + \sum_{k=1}^{M} \phi_k(x) c_k(t),
\]

where \( M \) is the number of modes relevant for the representation of the flow dynamics \( (M < N) \). The reader is referred to the work of Sirovich\(^\text{21}\) and Berkooz \textit{et al.}\(^\text{32}\) for further details on the mathematical background and procedure of the POD.

III. RESULTS

A. Flow field statistics

The near-wake flow field is examined within cross-flow planes located at different streamwise stations. Non-dimensional spatial coordinates \((x',y',z') = (x/D, y/D, z/D)\) and \( r' = r/D \) and non-dimensional instantaneous and time-averaged velocity components, respectively \((u^*, v^*, w^*) = (u/U∞, v/U∞, w/U∞)\) and \((U^*, V^*, W^*)\), are used throughout the discussion. Similarly, the time is expressed in non-dimensional form as \( t' = tU∞/D \).

The main integral quantities of the boundary layer prior to separation obtained from PIV measurements in the longitudinal plane \( z^* = 0 \) (see Gentile \textit{et al.}\(^\text{23}\)) are provided in Table II along with the associated uncertainty contributions.

To provide a reference for the current discussion, Figure 3 displays the time-averaged streamlines in the vertical plane from the above-mentioned measurements. The separated region extends from the trailing edge up to the rear-stagnation point located at approximately \( x^* = 1.2 \). A secondary stagnation-point can be visualized on the base in proximity of the symmetry axis. The separated flow exhibits a toroidal recirculation region with focus at \((x^*, r^*) = (0.6, 0.33)\). The position of the measurement planes is indicated in green at \( x^* = \{0.375, 0.75, 1.125, 1.5\} \).

The streamwise development of the velocity field is documented by extracting the velocity profile at the various \( x^* \) stations in the plane \( z^* = 0 \). Figure 4(a) illustrates the profiles of the time-averaged
longitudinal velocity component, outlining the streamwise development of the separated shear layer and of the reverse flow region. The turbulent fluctuations of the streamwise and radial velocity components (Fig. 4(b)) indicate a distinct peak associated with the turbulence in the separated shear layer close to separation, which intensifies in the mixing layer region in the longitudinal direction and close to the wake axis in the vertical direction. More details on the streamwise development of the mean flow properties can be found in the work of Gentile et al.\textsuperscript{23}

Also the development of time averaged longitudinal velocity along the wake axis agrees well with the results reported in the literature.\textsuperscript{3,5} More specifically a minimum value $U^* = -0.30$ is herein observed occurring at $x^* = 0.70$, corresponding to approximately 60\% of the rear-stagnation point location (see Fig. 5). Wolf\textsuperscript{12} and Wolf et al.\textsuperscript{14} report slightly higher peak values for the reverse flow ($U^* = -0.32$) at the same streamwise location as in the current study. Merz et al.\textsuperscript{4} report backflow velocities up to $-0.35 U_\infty$. The fluctuations in the longitudinal direction gradually intensify downstream, up to a maximum value of 0.18 $U_\infty$ at $x^* = 1.0$.

The time-averaged velocity field in the cross-flow ($z^*, y^*$) plane is presented in Fig. 6, for the four measurement planes. The projection of the time-averaged position of the shear layer axis that is superimposed onto the color contours has been obtained from the inflection point of the longitudinal velocity profile along the radial direction (Fig. 4(a)).

The contours of the longitudinal velocity define a circular shear layer bounding an inner region of reverse flow. The source-like pattern indicated by the in-plane velocity vectors at $x^* = 0.375$ reflects the outwards motion of the reverse flow approaching the model base in the stagnation regime, whereas for $x^* \geq 0.75$, a sink-like pattern is observed which is associated with the streamlines of the outer flow converging towards the axis. The wake center exhibits a slight shift towards the right, which reflects the high sensitivity of the wake pattern to small misalignments. The circular trace of the shear layer axis contracts in the streamwise direction as the shear layer axis moves inwards from $r^* = 0.48$ at $x^* = 0.375$ to $r^* = 0.34$ at $x^* = 1.5$, reflecting the streamline convergence. The

![FIG. 3. Time-averaged streamlines in the plane $z^* = 0$.\textsuperscript{23} Red dots indicate foci and reattachment points. Gray shading indicates backflow region contoured by red dashed line. Green lines represent stereoscopic PIV measurement planes.](image-url)
FIG. 4. Streamwise evolution of radial velocity profiles: time-averaged longitudinal velocity $U^*$ (a) and RMS of the fluctuating velocity (b).

axial symmetry of the wake topology is also affected slightly by the presence of a thin region of approximately 5% velocity defect resulting from the model support. Previous studies\textsuperscript{15,33} report velocity defects up to 0.15 $U_\infty$ which are identified as an important source of asymmetry of the wake topology and linked to the onset of a preferred azimuthal periodicity.\textsuperscript{34}

The spatial distribution of the turbulence intensity $TI^* = \frac{1}{U_\infty^2} \sqrt{ \langle (u'^2) + (v'^2) + (w'^2) \rangle }$ is displayed in Fig. 7. Consistent with the radial profiles shown in Fig. 4, a general increase of $TI^*$ can be observed when moving away from the model base. A lack of symmetry is observed in the spatial distribution of $TI^*$, which appears to be more pronounced than for the mean flow properties. Based on the axisymmetric pattern rendered by the mean velocity fields (see Fig. 6) and on previous experimental investigations,\textsuperscript{19,20} such asymmetries could be attributed to the presence of angular misalignments of the model with respect to the incoming flow during the experiments.

B. Instantaneous flow field analysis

The azimuthal flow organization and its dynamical evolution are examined by inspection of the instantaneous velocity fields. For this purpose, a set of three uncorrelated snapshots is displayed in Fig. 8 at three selected locations ($x^* = 0.375, x^* = 0.75$ and $x^* = 1.125$). More details on the dynamical behavior can be retrieved from the animations of the same series provided as supplementary material.\textsuperscript{35} At the position closest to the base ($x^* = 0.375$), the instantaneous velocity field exhibits a nearly circular boundary with some small-scale corrugations reflecting the turbulence

FIG. 5. Streamwise distribution of the longitudinal velocity: mean centerline velocity (black) and maximum RMS (red).
FIG. 6. Time-averaged velocity field in cross-flow planes (z∗, y∗). Color contours of out-of-plane component U∗. Vectors plotted every 5th grid-point represent in-plane components W∗ and V∗. Model edge in solid white and mean shear layer axis in dashed white. Total observation-time is ∆t∗ = 2 × 10⁴.

of the separating boundary layer. In contrast, vigorous fluctuations are observed in the innermost portion of the wake (see video 1 of the supplementary material[35]), where the backflow region exhibits azimuthal meandering about half a radius off the wake axis. The in-plane velocity vectors indicate an outwards motion correlated with the maximum value of the backflow velocity. The flow outside the wake appears unperturbed with respect to the free stream conditions and shows no correlation with the inner flow dynamics. The pattern of fluctuations inside the separated flow region reflects an unstable position of the stagnation point on the wake axis; however, the stable position is located away from the wake axis (the reader is referred to Fig. 14 for a schematic depiction of the latter situation). Given the axial symmetry of the problem, this distorted topology does not depend upon the specific azimuthal position. Therefore, the backflow is expected to undergo a free azimuthal meandering in time. Moving downstream (x∗ = 0.75), more significant corrugations of the external wake perimeter are observed associated with a local inrush of high-speed fluid from the outside and eruptions of low speed fluid from inside the wake, which also involve the inner backflow region (see video 2 of the supplementary material[35]). The azimuthal meandering is less evident and the backflow region appears to be confined around the geometric center of the wake reflecting the vanishing effect of the adverse pressure gradient moving away from the base region. In proximity of the rear-stagnation point (x∗ = 1.125), the distortion of the wake perimeter becomes more pronounced, with the interaction between the wake and the external flow in the form of inrush of high-speed fluid and eruptions of low speed fluid extended to almost the entire wake perimeter (see video 3 of the supplementary material[35]).

The backflow meandering appears as a large-scale mode of fluctuations displacing the backflow region with respect to the “nominal” axisymmetric position. Rigas et al. recently discussed the existence of such a mode based on base pressure measurements performed on a similar

FIG. 7. Color contours of the normalized turbulence intensity TI∗ in cross-flow planes (z∗, y∗). Model edge in solid white and mean shear layer axis in dashed white. Total observation-time is ∆t∗ = 2 × 10⁴.
configuration. In order to examine the backflow centroid position in relation to the near-wake topology, a sliding-average operation is applied over a time-interval of $\Delta t^* = 40$ (i.e., 0.1 s, corresponding to 10 snapshots acquired at 100 Hz), to filter small scale fluctuations, while capturing the characteristic time scales (i.e., $t^* \sim 1000$, Rigas et al.\textsuperscript{8}) of the global mode. Following the approach of Grandemange et al.,\textsuperscript{10} the instantaneous position of the backflow centroid is extracted from the sliding-averaged velocity field as

$$z^*_{c(t)} = \frac{\iint u^+(x,t) \cdot y^* dz^* dy^*}{\iint u^-(x,t) \cdot y^* dz^* dy^*},$$

$$y^*_{c(t)} = \frac{\iint u^+(x,t) \cdot z^* dz^* dy^*}{\iint u^-(x,t) \cdot z^* dz^* dy^*}.$$  \(5\)

The short-time averaged position of the backflow centroid thus obtained is superimposed onto the backflow velocity contours (red dot) shown in the snapshots of Fig. 9 along with the reference time-averaged position (yellow dot). The low-pass filtering of the small-scale turbulence that is responsible for the minor corrugations of the wake perimeter (see Fig. 8(a)) returns a smoother velocity field; furthermore, it highlights the radial offset between the instantaneous and the long-time-averaged position of the backflow centroid.

For all streamwise positions examined, the radial offset of the short-time averaged backflow centroid with respect to the mean one appears to be approximately constant over time, whereas the azimuthal position varies with time. More specifically, the radial offset appears to decrease from a maximum of $0.13 \, D$ at $x^* = 0.375$ to a minimum of $0.05 \, D$ attained within the maximum backflow region ($x^* = 0.75$) and to slightly increase again to $0.08 \, D$ in proximity of the rear-stagnation point ($x^* = 1.125$). The corresponding pattern of in-plane velocity vectors shows that the azimuthal
backflow meandering is associated with lateral fluid motions inside the separated wake. In particular, close to the base ($x^* = 0.375$, video 4 of the supplementary material\textsuperscript{35}), the region of maximum backflow exhibits consistently an outward motion (positive radial velocity). The wake further features two weak circulatory regions with little to no backflow. Moving downstream ($x^* \geq 0.75$, videos 5 and 6 of the supplementary material\textsuperscript{35}), the inner backflow appears less coherently organized and no clear correlation between backflow position and in-plane velocity pattern can be inferred.

The two-dimensional probability distributions of the backflow centroid position obtained over an observation-time $\Delta t^* = 2 \times 10^4$ (i.e., 50 s) and presented in Fig. 10 define an approximately circular pattern, reflecting the azimuthal meandering of the recirculation bubble. The occurrence at different azimuths is rather uniform, resulting in the axisymmetric pattern exhibited by the long-term time-averaged velocity field shown in Fig. 6. This result is in good agreement with the works of Rigas \textit{et al.}\textsuperscript{8} and Grandemange \textit{et al.}\textsuperscript{9} who report the axisymmetry of the near-wake topology as a mere long-term statistical property of the turbulent wake. The radial probability distributions outline a shift of the most probable radial coordinate from 0.14 $D$ at $x^* = 0.375$ to $r_c^* = 0.07$ $D$ and $r_c^* = 0.08$ $D$ at $x^* = 0.75$ and $x^* = 1.125$, respectively. This fact indicates that the backflow precession motion reduces in amplitude or even loses coherence when moving from the base region towards the rear-stagnation point, as the large-scale fluctuations become dominated by the concurrent shear layer development. The probability distributions are found to feature a higher probability at $\theta = 0$, which is ascribed to imperfect flow boundary conditions such as small angular misalignments of the model with respect to the free stream flow.\textsuperscript{4,14}

The time-history of the backflow centroid within a time interval $\Delta t^* = 4 \times 10^3$ (i.e., 10 s) is presented in Fig. 11. The centroid position at $x^* = 0.375$ exhibits a general radial offset (0.13 $D$ on
average), with occasional transits across the center-line (Fig. 11, left). When moving downstream, the centroid offset reduces to about 0.06 D. While at the most upstream station this feature can be attributed to the erratic behavior of the reverse flow, in proximity of the reattachment it is more likely ascribed to the global wake distortion associated with the large-scale fluctuations of the shear layer, as it can be inferred from Figs. 8(c) and 9(c). The time-history of the azimuthal coordinate indicates the occurrence of a quasi-regular azimuthal motion at $x^* = 0.375$, with a time period of approximately $\Delta t^* \sim 1000$. The corresponding non-dimensional frequency is in the order of $St_D \sim 10^{-3}$. Although less regular, an azimuthal motion can be also observed at $x^* = 0.75$, with an even longer time scale ($\Delta t^* \sim 3 \times 10^3$) corresponding to a characteristic frequency in the order of $St_D \sim 3 \times 10^{-4}$. Moving further downstream at $x^* = 1.125$, no long time-scale periodicity can be inferred from the time-history of the backflow centroid.

C. Proper orthogonal decomposition analysis

Snapshot POD analysis is conducted in order to examine the large-scale velocity fluctuations dominating the near-wake dynamics. The turbulent kinetic energy spectra illustrated in Fig. 12 indicate that typically between 40% and 50% of the total energy of the flow is captured within the first 10 POD modes, depending on the streamwise station, with a notable faster energy convergence for the most upstream measurement plane ($x^* = 0.375$).
FIG. 11. Time-history of backflow centroid azimuthal and radial coordinates over an observation-time $\Delta t^* = 4 \times 10^3$.

FIG. 12. Energy distribution over the POD modes.

More specifically, the cumulative energy contribution of the first two POD modes $k = 1$ and $k = 2$ (see Section II D for details on the notation) varies between 37% and 23% when moving away from the model base. The energy contributions as well as the spatial distributions of the two modes, which are found to be in azimuthal phase quadrature (see Fig. 13), suggest that they are
dynamically paired. An analogous pairing is observed at \( x^* \geq 0.75 \) for modes \( k = 3 \) and \( k = 4 \), each capturing between 4% and 6% of the energy depending on the position (see Fig. 12). At \( x^* = 0.375 \), a similar energy contribution (i.e., approximately 3% of the total energy) is found for modes \( k = 3 \), \( 4 \), and \( 5 \). The relative contribution of the higher order modes for \( k \geq 3 \) grows moving downstream away from the separation point.

The spatial distribution of the first two POD eigen-functions is examined to assess the spatial coherence of the most energetic fluctuations occurring in the near-wake. The contours shown in

![Spatial eigen-modes](image)

FIG. 13. Spatial eigen-modes \( k = 1 \) (a) and \( k = 2 \) (b) in cross-flow planes \((z^*, y^*)\). Color contours of out-of-plane component \( \sqrt{\lambda} \Phi_{x}^* \). Vectors plotted every 5th grid-point represent in-plane components \( \sqrt{\lambda} q_{x}^* \) and \( \sqrt{\lambda} q_{y}^* \). Base edge in solid white and mean shear layer in dashed white.
Fig. 13 define a dipole distribution characterized by two diametrically opposite regions of excess and defect of streamwise momentum respectively, whereby the linear combination of both modes represents a radial displacement of the backflow region in any azimuthal direction. Such a dipolar organization is identified by Rigas et al. as an anti-symmetric mode of azimuthal wave-number $m = 1$. At $x^* = 0.375$, the highest fluctuations are observed in the innermost portion of the wake and can be ascribed to the precession of the backflow region, whereas more downstream the maxima are attained along the mixing layer region as a consequence of the localized inrush of the outer high momentum flow towards the separated region, as previously inferred from Fig. 8. It can be argued that the anti-symmetric mode $m = 1$ evolves in the streamwise direction reflecting radial displacement of the backflow region close to the separation point and of the entire wake closer to the reattachment. A schematic interpretation of the two situations is shown in Fig. 14. The first one (see Fig. 14(a)) is interpreted as a radial displacement of the backflow region with respect to the time-averaged position in the azimuthal plane (left) which distorts the flow pattern in the longitudinal plane (right) without affecting the outer wake perimeter. The second situation (see Fig. 14(b)) is depicted as a radial displacement of both the shear layer axis and the backflow region in the azimuthal plane and brings a vertical displacement of the entire wake. In both cases, two diametrically opposite regions of defect and excess of streamwise momentum are created, as identified in Fig. 13. While the second situation is often reported in the literature (Wolf et al., Schrijer et al.), being referred to as shear layer flapping, the first one has been only recently documented (Rigas et al.) as a significant contribution of the anti-symmetric mode $m = 1$.

Fig. 15 shows the low-order reconstruction of the velocity field based on the POD modes $k = 1, 2$, obtained at $x^* = 0.375, x^* = 0.75$, and $x^* = 1.125$ for the snapshots shown in Fig. 8, following the methodology described in Sec. II D. The corresponding animations are provided in videos 7-9 of the supplementary material. The reconstructed velocity field gives evidence of the purely displacing effect of the anti-symmetric mode $m = 1$ acting on the inner backflow region at $x^* = 0.375$ and on the whole separated region at $x^* \geq 0.75$. At this location, the in-plane vectors outline the interaction between the flow inside and outside the wake. It can be noticed that both POD spatial eigen-functions and reconstructed velocity field feature the presence of in-plane vortical structures (streamwise vortices) whose presence has been reported in recent studies in relation to the thread vortices ensuing from the symmetry-breaking mode.

The temporal information on the dynamics of modes $k = 1$ and $k = 2$ is retrieved through analysis of the associated POD time-coefficients. Figure 16 shows the scatter plots of the first and second mode time-coefficients obtained for the different measurement planes. Both low and high repetition rate measurements are shown to reveal the role of the wake unsteadiness time scales on the wake topology. In that respect, a remarkable difference can be appreciated between the

FIG. 14. Schematic interpretation of anti-symmetric mode $m = 1$ in the azimuthal and longitudinal plane: inner backflow displacement (a) and global wake displacement (b). Schematics in the longitudinal plane based on POD reconstruction of the velocity field based on first POD mode $k = 1$. Gray streamlines on the right refer to mean flow and red streamlines to the displaced flow field.
FIG. 15. Low-order reconstruction of the instantaneous velocity field based on POD modes \( k = 1 \) and \( k = 2 \) in cross-flow planes \((z^*,y^*)\) for \( t^* = 20 \) (a), 308 (b), and 1268 (c). Color contours of out-of-plane component \( u^*_{ROM} \). Vectors plotted every 5th grid-point represent in-plane components \( w^*_{ROM} \) and \( v^*_{ROM} \). Base edge in solid white and mean shear layer in dashed white. See videos 7-9 of the supplementary material for corresponding animations.  

The power spectral density distributions of the first two POD time coefficients are presented in Fig. 17. Low and high-repetition rate measurements are combined to cover approximately four decades of the frequency spectrum. At \( x^* = 0.375 \) a maximum is found in the very-low frequency range \( St_D = 0.0003-0.0005 \), at downstream stations the power spectral density in this frequency range is averaged-out by the short-term statistics (blue dots).

FIG. 16. Scatter plot of the POD time-coefficients \( c_1(t) \) and \( c_2(t) \) in cross-flow planes \((z^*,y^*)\). Red dots account for \( \Delta t^* = 2 \times 10^4 \), blue dots account for \( \Delta t^* = 10^3 \).
FIG. 17. Power spectral density distribution of the POD time-coefficients $c_1(t)$ and $c_2(t)$ in cross-flow planes ($z^*, y^*$). Solid line indicates data acquired at 100 Hz. Dashed line indicates data acquired at 2 kHz. Frequency indicated by the Strouhal number based on the model diameter $St_D = f D / U_\infty$.

range decreases. A second smaller contribution is observed at $St_D = 0.001$ and is present at all stations. The main very-low frequency peak is ascribed to the backflow centroid dynamics and the time scales are consistent with those inferred from Fig. 11 and with the characteristic frequencies identified by Rigas et al.\textsuperscript{8} and Grandemange et al.\textsuperscript{9} A comparatively smaller contribution reflecting
vortex shedding is further visualized at $St_D = 0.2$ and grows in importance when moving in the streamwise direction reflecting the shear layer growth.

The analysis is concluded with the discussion of the higher order modes with $k \geq 3$. The spatial distributions in Fig. 18 clearly indicate dynamical pairing of modes $k = 3$ and $k = 4$ at $x^* \geq 0.75$, their linear combination allowing for a rotation of the pattern in all azimuthal directions. The two modes feature four regions of excess and defect of streamwise momentum distributed along the external wake perimeter such to form a quadrupole. The in-plane vectors connecting the centers of the red and blue lobes define four in-plane vortical structures with a central saddle point. This alternating inwards and outwards in-plane pattern can be interpreted as a wake ovalization (see videos 10-12 of the supplementary material\textsuperscript{15}). The maxima are attained along the mixing layer and an increase in coherence can be observed departing from the base, which suggests that these two modes represent a higher order mode of fluctuation of the shear layer. A similar spatial organization is identified by Rigas et al.\textsuperscript{8} with a mode of azimuthal wave-number $m = 2$. In the present investigation, such a mode is associated with an inrush of high momentum fluid from two opposite sides of the wake, causing an outward motion of low-momentum fluid in the orthogonal direction with creation of a saddle region in the center.

FIG. 18. Spatial eigen-modes $k = 3$ (a) and $k = 4$ (b) in cross-flow planes ($z^*, y^*$). Color contours of out-of-plane component $\sqrt{\lambda} \phi^*_z$. Vectors plotted every 5th grid-point represent in-plane components $\sqrt{\lambda} \phi^*_x$ and $\sqrt{\lambda} \phi^*_y$. Base edge in solid white and mean shear layer in dashed white.
FIG. 19. Low-order reconstruction of the instantaneous velocity field based on POD modes $k = 3$ and $k = 4$ in cross-flow planes $(z^*, y^*)$ for $t^* = 20$ (a), 308 (b), and 1268 (c). Color contours of out-of-plane component $u^*_{ROM}$. Vectors plotted every 5th grid-point represent in-plane components $w^*_{ROM}$ and $v^*_{ROM}$. Base edge in solid white and mean shear layer in dashed white. See videos 10-12 of the supplementary material for corresponding animations.

The low-order reconstructions of the velocity field based on third and fourth fluctuating modes are given in Fig. 19, and feature a visibly elongated wake perimeter compared to the axisymmetric time-averaged pattern (see videos 10-12 of the supplementary material\cite{note}). The principal axes of the distorted wake are coherent with the inrush and ejection regions, which give the structure of four streamwise vortices when combined. Similar to Fig. 14 for the first mode pair, a schematic interpretation of the second mode pair is presented in Fig. 20.

The power spectral density of time-coefficients from the third and fourth mode is shown in Fig. 21. Despite the larger data scatter with respect to the case $k = 1$ and $k = 2$, a maximum can still be identified in the low frequency regime at $St_D = 10^{-3}$, similar to the contribution of the first mode pair. However, in contrast with the case $k = 1$ and $k = 2$, the energy of this mode does not seem to

FIG. 20. Schematic representation of mode $k = 3$ and $k = 4$ in the azimuthal plane $(z^*, y^*)$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{schematic.png}
\caption{Schematic representation of mode $k = 3$ and $k = 4$ in the azimuthal plane $(z^*, y^*)$.}
\end{figure}
vanish when moving away from the base. This behavior is unexpected and has not being reported in the literature. Its full explanation would require further investigation.

Figure 22 shows the distributions of the spatial eigen-modes $k = 3, 4, \text{ and } 5$ at the most upstream plane $x^* = 0.375$. The color contours of mode $k = 4$ correspond to a quadrupole structure similar to the one identified at $x^* \geq 0.75$. The peaks observed within the shear layer axis however suggest an ovalization mode localized within the backflow region (see video 14 of the supplementary material). Both modes $k = 3$ and $k = 5$ exhibit a central region of excess (defect) of streamwise momentum and two outer and diametrically opposite lobes of defect (excess) of streamwise momentum. The central region can be linked to an axisymmetric fluctuation mode of azimuthal wave number $m = 0$ of the reverse flow (i.e., bubble pumping) in the streamwise direction which appears analogous to the one reported in Rigas et al.\textsuperscript{8} The outer lobes of the two modes however exhibit a phase quadrature relation, suggesting that the $m = 0$ dynamics is accompanied by an additional ovalization contribution. The visualizations in videos 13-15 of the supplementary material give additional details of modes $k = 3$ and $k = 5$. 

FIG. 21. Power spectral density distribution of the POD time-coefficients $c_3(t)$, $c_4(t)$, and $c_5(t)$ at in cross-flow planes ($z^*$, $y^*$). Low repetition rate data indicated with solid line. High repetition rate data indicated with dashed line. $St_D = fD/U_\infty$. 

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FIG. 22. Spatial eigen-modes \( k = 3, k = 4, \) and \( k = 5 \) for \( x^* = 0.375 \). Color contours of out-of-plane component \( \sqrt{\lambda} \phi^* \). Vectors plotted every 5th grid-point represent in-plane components \( \sqrt{\lambda} \phi^*_x \) and \( \sqrt{\lambda} \phi^*_y \). Base edge in solid white and mean shear layer in dashed white.

IV. CONCLUSIONS

The spatio-temporal organization of a turbulent axisymmetric wake was characterized experimentally using time-resolved stereoscopic PIV in different azimuthal planes downstream of the base. Analysis of the time-averaged velocity field indicated reverse flow velocities up to \( 0.3 U_\infty \) at approximately 60\% of the rear-stagnation point location (i.e., \( x^* = 1.2 \)) and turbulent fluctuations with peaks of approximately 0.18–0.19 \( U_\infty \) in the reattachment region.

The instantaneous velocity fields in proximity of the base revealed a pronounced radial offset of the reverse flow region from the wake axis. The centroid of the backflow region was found to undergo a precession motion about the model symmetry axis over an observation-time in the order of \( 10^4 D/U_\infty \). This precession appeared to become less evident when moving downstream towards the rear-stagnation point. Based on the time-history of the backflow centroid position, the characteristic time scales of this motion were found to be in the order of \( 10^3 D/U_\infty \) or higher.

Proper orthogonal decomposition of the velocity fluctuations outlined a major contribution of the mode pair \( k = 1, 2 \), particularly at \( x^* = 0.375 \). The associated spatial eigen-functions returned an anti-symmetric radial displacement of azimuthal wave-number \( m = 1 \) along a varying azimuthal direction. While close to the base, this radial displacement could be entirely ascribed to the backflow azimuthal meandering. It reflected a global motion of the wake associated with inrush of high momentum flow from outside the separated region in proximity of the rear-stagnation point. The frequency spectra of the POD time-coefficients exhibited peaks in the very-low frequency range \( St_D = 0.0003–0.0005 \), weakening along the streamwise direction. Higher order modes revealed the occurrence at \( x^* = 0.375 \) of an axisymmetric pulsation of the recirculation region with azimuthal wave-number \( m = 0 \) (bubble pumping) and of a wake ovalization corresponding to a mode of azimuthal wave-number \( m = 2 \) at \( x^* \geq 0.75 \).