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CHAPTER: Introduction

BACKGROUND
Many tasks are taken over by automation. But jobs which require personal contact, like healthcare cannot be fully automated. Because the average age of the population is increasing, the demand for healthcare will increase. To increase the employability of health care workers robots and exoskeletons could be used [1]. Exoskeletons are suits worn by people to increase their strength, or stamina. There are a number of general requirements for exoskeletons. They must be energy efficient, i.e. low friction, since the energy must either be delivered by the human or stored in a limited capacity. And the weight must be low, because the suit must be carried by the human. Passive exoskeletons don’t consume energy, but use springs, dampers or counterweights to apply forces.

An example is the Laevo, a passive exoskeleton that can be worn to reduce the load on people’s lower back [2]. The spring around the hip joint actuates the supportive forces, but also reduces the mobility of the legs. Appendix C this design problem is considered.

A way to apply supportive forces on the body without fixing it’s position is using under-actuation by means of differential mechanisms [3–6].

LITERATURE REVIEW DIFFERENTIAL MECHANISMS
A literature review was done on types of differential mechanisms for exoskeletons, in chapter 2. It was found that differential mechanisms can be composed from transmissions. 16 types of transmissions were categorized and it was found that they had similar properties on the criteria: weight, size, stiffness, stability, noise, backlash and reliability.

VISION
It was found that compliant mechanisms have low friction, wear and backlash, low stiffness and possibly low weight. For many applications the low stiffness of a compliant transmission would be a problem. But in some applications, like the Laevo, were the transmission and spring are in series, the low stiffness can be an advantage. The transmission can be designed to act like a spring: a compliant transmission mechanism. A large range of motion compliant transmission would be needed. But such a system weren’t found.

A new concept type of compliant transmission transmission is proposed: A transmission from two-fold closed-loop tape springs connected by a tendon.

RESEARCH OBJECTIVE
The two research objectives of this thesis are as follows:

Objective 1: Model the behaviour of two-fold closed-loop tape springs.
Objective 2: Model the behaviour of a transmission from connected tape loops.
LITERATURE REVIEW: DIFFERENTIAL MECHANISM TYPES FOR EXOSKELETON: A REVIEW
Differential Mechanism Types For Exoskeleton: A Review

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Abstract

A review was done on types of mechanical transmissions to help designers compose the suitable differential mechanism for an exoskeleton. Methods for number and dimensional synthesis of particular types of differential mechanisms exist, but there is no method for type synthesis. This paper introduces a method to create 2 DOF differential mechanisms from any type of transmission. And a literature review on transmission was done on the criteria: weight, size, stiffness, stability, noise, backlash and reliability. 16 types of transmissions were found that were divided in categories with similar properties. The design method is demonstrated with an example.

Keywords Differential mechanism · Transmission · Type synthesis

I Introduction

The ageing population is placing an increasing burden on caretakers for the elderly. Solutions might come from robotics and exoskeletons technology because they can reduce the workload for caretakers. [13]. An example is the Laevo, lower back supporting passive exoskeleton [32]. However, there are challenges for these technologies, i.a. complicated controllers, high energy consumption and comfort issues[23]. Differential mechanisms can offer solutions for some of these challenges. For example, differential mechanisms are used in robotic graspers [2, 3, 12, 25] and prosthetic hands [20] to create under-actuation to adapt their shape to the grasped object and distributes the contact force, while there is low control complexity and low weight. Bogert used a passive exoskeletons differential mechanisms in order to increase the energy efficiency during walking by transferring power over multiple joints [11]. When an exoskeleton applies forces to the human body it must be well distributed to prevent uncomfortable peak-forces. Because the shape of the body changes unpredictably during motion, using under actuation by means of differential mechanisms is a way to comfortably apply forces to the body.

In literature there are multiple methods to do number synthesis or dimensional synthesis for differential mechanisms. In 1985, Hirose [14] has shown multi-DOF can be created by systematically combining 2 DOF differential mechanisms. Birlgen and Gosselin [6] and Krut [17] derived how forces are transmitted through multi DOF, connected differential mechanisms. There are multiple methods to synthesize different types of 2 DOF differential mechanisms. In a study by Birglen [4], graph theory was used to find all topologies of linkages suitable for 2 and 3 phalanges under-actuated fingers. Chu [9] required all ports to be rotations along the same axis and used graph theory to synthesize 8 gearless differential mechanisms from combinations of predetermined transmission types.
With the methods above many topologies or configurations of the same type can be generated. But there is no systematic method to choose between different types of differential mechanisms for a given set of requirements. Ideally designers could use a method that results in the optimum choice for their application out of all concepts. The aim of this paper is to provide insight in how 2 DOF differential mechanisms can be composed and to give a broad overview of types to guide designers to promising differential mechanism for their performance criteria.

In section II it is derived that differential mechanisms are combinations of transmission mechanisms. Relevant performance criteria for exoskeletons are explained. In section III the results of a literature review on transmissions are presented in a table. These are grouped into categories based on the properties of their components. The proposed design method was demonstrated with an example design in section III.II. And the limitations will be discussed in section IV. Despite these remarks, it is concluded in section V that this overview can help in choosing the right concept.

II Method

II.I Decomposition of differential mechanism

A transmission was defined as a 1 DOF, single-input-single-output mechanism which has a resultant force on it’s ground connection [30]. This means that the output force or moment has changed size and or direction with respect to the input. The ratio between the input and output speed is called the transmission ratio [16]. For a rigid, quasi-static, frictionless mechanism the transmission ratio, R also defines the ratio between input force or torque, $F_{in}$ and output force or torque, $F_{out}$.

Transmission ratio: \[ R = \frac{F_{out}}{F_{in}} \] (1)

It must be noted that whether $R$ is positive or negative depends fully on the chosen coordinate system.

The IFoMM uses the following definition for a differential mechanism: “Mechanism for which the degree of freedom is two and which may accept two inputs to produce one output or may resolve a single input into two outputs [16].” Where the inputs are independent variables and the output dependent variables.

If one of the ports of a differential mechanism is locked, i.e. movement of that port is no longer possible, the differential mechanism becomes a transmission, see fig. 1. From the three possible transmissions formed by locking one port of a differential mechanism at least one will have positive and at least one a have a negative transmission ratio, see fig. 2. A transmission with a negative transmission ratio can be called an inverter.

![Figure 1: Transmission formed by locking a port of a differential mechanism](image1)

![Figure 2: For positive upward input force and positive downward output force, transmission ratio R is graphically depicted: left: Differential mechanism, middle: Transmission right: Inverter](image2)

II.II Differential mechanism synthesis method

Differential mechanisms can be turned into transmissions, but vice versa any transmission can be turned into a differential mechanism by
adding a DOF. This DOF can be added to it’s ground connection (method 1) or in some cases in the transmission chain (method 2). In the first case it must be done so that movement in the direction of the reaction force is introduced. With method 2 a degree of freedom is added in the loop. This additional DOF becomes a port of the differential mechanism.

These methods are illustrated with an example. A 4-bar mechanism is a transmission between \( \phi_1 \) and \( \phi_2 \), see fig. 3. A DOF is added by method 1, i.e. the mechanism is removed from it’s ground support and placed on a pivot, introducing the port \( \phi_3 \), which results in a 2 DOF differential mechanism, see fig. 4. This mechanism is commonly used in under-actuated fingers [5, 18]. Method 2 removes one of the links and replaces it with two links and a single DOF joint. This effectively adds one DOF, \( \phi_3 \). The result is a closed loop 5-bar mechanism, which is also a differential mechanism, see fig. 5.

In case of method 1, if this newly created port is the input where a force or moment, \( F_a \) acts on, then the output forces or moments are \( F_{a1} \) and \( F_{a2} \). Where \( R \) is calculated with eq. (1).

\[
\begin{bmatrix} F_{a1} \\ F_{a2} \end{bmatrix} = \begin{bmatrix} R \frac{1}{R-1} \\ \frac{1}{R} \end{bmatrix} F_a \quad (2)
\]

Birglen and Gosselin [6] showed that if input and output ports are interchanged the force transmission matrix changes to eq. (3a) or eq. (3b).

\[
\begin{bmatrix} F_a \\ F_{a1} \end{bmatrix} = \frac{R}{R-1} \begin{bmatrix} 1 \\ \frac{1}{R} \end{bmatrix} F_a \quad (3a)
\]

\[
\begin{bmatrix} F_a \\ F_{a2} \end{bmatrix} = R \begin{bmatrix} 1 \\ \frac{1}{R-1} \end{bmatrix} F_a \quad (3b)
\]

II.III Criteria

In the previous section it was shown that differential mechanisms can be constructed from combinations of transmissions. Assuming the properties of transmissions carry over on the properties of differential mechanisms the type of differential mechanism can be chosen by selecting the transmissions it is built from.

To compare these transmissions, relevant criteria for exoskeletons were selected. In the proceeding subsections the used performance criteria are explained. The quantitative values of a mechanism’s performance depends on its detailed design, because often there is a trade-off between different performance criteria. To be able to fairly compare transmission mechanisms types, they must have the same scale and application. So, they are applied to an exoskeleton joint where a 20 Nm load must be transferred over a 100° range. There is space for 0.10 m offset from the rotation axis so the 34.9 J of mechanical energy can for example be transported by a 200 N force and a 0.1745 m displacement. The performances were estimated in one of the following four ways: 1. If the concept was an existing product its properties could be found in manufacturer’s specification data. If the design is only found in scientific literature, the data comes from 2. scientific test or 3. theoretical analysis from literature. The references to the sources are given. 4. If the properties were not found in any of these
four ways an estimation was made, recognizable by the absence of reference. If the author lacked knowledge to make an estimation, the spot is left blank. Criteria such as stiffness, weight, dimensions, noise, amount of backlash and the reliability are not quantifiable without making a detailed design. However, general estimations can be made about their relative performance. Like Controzzi, Cipriani and Carrozza [10], a rating on scale from 1-5 was given for each of these criteria where a score 1 is worst and a score of 5 is best.

II.III.I Friction

Therefore energy efficiency is an important criterion because in exoskeletons mechanical energy is scarce since it comes from an energy source and actuators that are carried around adding mass to the system, or it is delivered by the user. Energy is lost due to friction. And in case of fluctuating loading stored elastic energy also contributes to a lower mechanical efficiency [28, 27]. The percentage of the energy lost to friction was defined with:

\[
\eta_{\text{friction}} = \frac{W_{\text{friction}}}{W_{\text{in}}} \cdot 100\%
\] (4)

Where \(W_{\text{in}}\) is the mechanical energy put in the transmission and \(W_{\text{friction}}\) the mechanical energy lost due to friction.

II.III.II Range of motion

Some transmissions have limited range of motion, which is denoted in table 2 by ‘n/a’. For finite range of motion the range is given.

II.III.III Transmission ratio

The transmission ratio like previously defined in eq. (1) was used. The transmission ratio doesn’t have to be constant over the range of motion. Ratio’s defined positive and above 1 although the negative or reciprocal value is also possible with the same transmission mechanism by choosing different coordinates. The minimum and maximum transmission ratio’s for feasible designs over the range of motion were denoted. Some transmissions are not able to change the speed of the input, but are able to reverse it’s motion by directing in a 180° turn. Here the transmission ratio of ’1/-1’ was filled in.

II.III.IV Weight

The main factors contributing to the weight were listed. And the weight of the transmission was estimated for a transmission using examples of applications. A subjective score between 1 and 5 is given indicating relative weight compared to other transmission.

II.III.V Dimensions

The space a transmission needs to produce the same torque or force is compared. Not just the size in rest, but also the space it needs for movement is important. The smaller the space the higher the score on the 1-5 scale.

II.III.VI Stiffness

Two stiffnesses can be defined. \(k_{\text{transmission}}\) is the stiffness between the in- and output port of the transmission.

\[
k_{\text{transmission}} = \frac{dF_a}{du_1 - du_2/R}
\] (5)

Where \(F_a\) is the force on the input port, \(u_1\) is the input displacement and \(u_2\) is the output displacement. A high \(k_{\text{transmission}}\) is considered good, so receives a high rating on the 1-5 scale.

Most transmissions using flexible elements have a stable position. When the output is unloaded and the input is moved away from the stable position the elastic force pushes it back with a force dependent on the displacement. The \(k_{\text{stable}}\) is the rate of increment of this force with displacement.

\[
k_{\text{stable}} = \frac{dF_a}{du_1}
\] (6)

A high \(k_{\text{stable}}\) is considered bad because some of the input work is stored instead of transmitted, so gets low rating. If \(k_{\text{stable}}\) doesn’t exist
because there is no stable position it is denoted with ‘-’.

II.III.VII Noise
Sliding gears or bearing and turbulent fluid flow are sources of noise. In some applications noise is undesirable. So silent transmission score a 5, while noisy systems score a 1.

II.III.VIII Backlash
Backlash can be undesirable if motion is reversed. Therefore much backlash received a low score. Virtual play which is combination of low stiffness and friction was not accounted for in this criterion because these two are subjects considered separately.

II.III.IX Reliability
Reliability was defined as the time a transmission can last without maintenance or replacement. Reliable transmission get a high score. To estimate this typical applications of these transmissions were evaluated.

II.IV Literature search
A search was done to find concepts of differential mechanisms and transmissions to get a varied list of transmission mechanisms. From the differential mechanisms the underlying transmission concepts were derived. Conceptual designs, prototypes and commercial products were used. Also the references of the found sources were used to find more relevant literature. Further specific searches were done to find the transmission’s performances on the criteria. Google scholar, Scopus,com, Espacenet and the TUDelft Library catalogue were searched on the keywords shown in table 1.

II.IV.I Classification
Found examples of transmissions are assigned to one of three general classes: Mechanisms with only solid parts, mechanisms with fluidic parts, mechanisms with both fluidic and flexible parts or mechanisms with flexible parts. This last group is subdivided into fully compliant (fully c) and partially compliant (part). A fully compliant mechanism gains all of it’s movability from the deformation of flexible elements while a partially compliant mechanism also uses regular joints [19].

II.V Application
In order to illustrate this method it is applied to an example problem. The only important requirements for the differential mechanism are its weight, reliability and lack of backlash. The differential mechanism has to be force isotropic, which means it divides the input force evenly over both outputs [17]. The results from the literature review are used to chose the best transmission concept as a building block for the differential mechanism. Method 1 is used for both a positive and negative transmission ratio and method 2 is used. This will result in 3 concepts of differential mechanism of the same type.

III Results
III.I Decision matrix
16 Transmission types were found. They are in listed in the vertical axis of table 2. They are shortly described below:
- Cylindrical gears are gears with teeth on a cylindrical surface [16].
- Bevel gears are gears which have there teeth on a conical surface [16].
- Friction drive are gears which don’t have teeth, but transmit their torque by friction on a line or point contact between the smooth surfaces [16].
- A cam which has a curved surface transmits motions to a follower by exerting normal forces on a line or point contact [16].
- Worm gear have teeth wrapped helically around a cylinder [16].
- Chain and sprocket transmit a rotation from one sprocket to another means a chain consisting
Table 1: keywords searched on

<table>
<thead>
<tr>
<th>Keywords combined with</th>
<th>without the words</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Differential mechanism&quot;</td>
<td>monolithic privacy</td>
</tr>
<tr>
<td>&quot;force divider&quot;</td>
<td>compliant</td>
</tr>
<tr>
<td>&quot;torque splitter&quot;</td>
<td>review</td>
</tr>
<tr>
<td>underactuation</td>
<td>type</td>
</tr>
<tr>
<td>&quot;under actuated&quot;</td>
<td>classification</td>
</tr>
<tr>
<td>force isotropic</td>
<td>gearless</td>
</tr>
<tr>
<td>inverter</td>
<td>mechanical monetary</td>
</tr>
<tr>
<td>transmission</td>
<td></td>
</tr>
<tr>
<td>&quot;summation mechanism&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;self adaptive&quot;</td>
<td></td>
</tr>
<tr>
<td>&quot;mechanical amplifier&quot;</td>
<td></td>
</tr>
</tbody>
</table>

- Linkages are a connection of links held together by joints formed by surface contacts [16].
- A hydraulic cylinder transmission is a connection of two hydraulic actuators forming a closed volume of liquid where port movement results in a proportional movement of other ports.
- A pneumatic transmission is the same as a hydraulic transmission, but the volume is filled with compressible gas instead of liquid.
- A McKibben muscle transmission is a connection of two McKibben muscle actuators, which are air filled bladders contracting from their initial length, $l_{\text{rest}}$ when pressurized [8].
- An air bellows transmission are two connected air bellow actuators, which are air filled bags pushing out a surface when pressurized [26].
- A closed belt transmission is a closed belt strained around pulley. Friction prevents relative movement of the belt and pulleys [31].
- In a cam-tendon transmission the tendon is clamped on both ends so no friction is needed to prevent slipping, but the range of motion is limited [17].
- In a tendon sheath transmission the direction of motion of tendon is diverted by the surface of the sheaths.
- A flexible shaft is structure of cables which is relatively stiff in torsion, but flexible in bending, which makes it possible to divert rotation to a different axis.
- CM no sliding are compliant mechanisms which no sliding between any of its parts. How these concepts scored on the criteria defined in section II is shown in table 2.

### III.II Application

The decision matrix in table 2 shows that concerning weight, reliability and backlash pneumatic actuator depicted in fig. 6 scores best. From eqs. (2) and (3a) it follows that for a force isotropic differential mechanism the transmission ratio of the transmission must be $R = 2$ or $R = -1$. Method 1 adds a DOF to the transmissions ground connection. A pneumatic transmission with $R = 2$ and method 1 results in a differential mechanism as in fig. 7. Using method 1 on a pneumatic transmission with $R = -1$ results in a differential mechanism as in fig. 8. And using method 2 by adding a DOF in the loop to a pneumatic transmission with $R = -1$ or $R = 2$ results in a differential mechanism as in fig. 9.

Figure 6: Pneumatic transmission
### Table 2: Decision-matrix

<table>
<thead>
<tr>
<th>Transmission</th>
<th>Friction [%]</th>
<th>Range of motion</th>
<th>Transmission ratio</th>
<th>Weight</th>
<th>Dimensions</th>
<th>$k_{f \text{transmission}}$</th>
<th>$k_{\text{stable}}$</th>
<th>Noise</th>
<th>Backlash</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only solid parts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical gears</td>
<td>2-6[22]</td>
<td>n/a</td>
<td>1-6[22]</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Bevel gears</td>
<td>3-7[22]</td>
<td>n/a</td>
<td>1-5[22]</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Friction drive</td>
<td>11-30[21]</td>
<td>n/a</td>
<td>1-5[21]</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Cam + follower</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Skrew</td>
<td>&gt; 26[24]</td>
<td></td>
<td>5-75[22]</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td></td>
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<tr>
<td>Chain sprocket</td>
<td>1.4-19[29]</td>
<td>n/a</td>
<td>1-5[29]</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Linkage</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Fluid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydraulic cylinder</td>
<td>4-20[15]</td>
<td>$l_{\text{piston}}$</td>
<td>1-10[15]</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Pneumatic cylinder</td>
<td>4-40[1]</td>
<td>$l_{\text{piston}}$</td>
<td>1-10[15]</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td></td>
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<tr>
<td>Fluid and flexible</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>McKibben muscle</td>
<td>20[8]</td>
<td>0.35$_{\text{rev}}$[8]</td>
<td>1-2</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Air bellows</td>
<td>100$^\circ$[25]</td>
<td></td>
<td></td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td></td>
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<tr>
<td>Flexible part</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closed belt</td>
<td>&gt;1[30]</td>
<td>n/a</td>
<td>1-20[30]</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Cam-tendon</td>
<td>1-30[15]</td>
<td></td>
<td></td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td></td>
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<tr>
<td>Tendon Sheath</td>
<td>43[7]</td>
<td></td>
<td>1/-1</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Flexible shaft</td>
<td>5-15[33]</td>
<td>n/a</td>
<td>1/-1</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td></td>
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<tr>
<td>CM no sliding</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

*material hysteresis is only energy dissipation mechanism

![Figure 7: Differential mechanism formed by adding DOF to ground if R=2](image1)

![Figure 8: Differential mechanism formed by adding DOF to ground if R=-1](image2)

![Figure 9: Differential mechanism formed by adding DOF in the transmission chain](image3)

### IV Discussion

The 1-5 markings the transmission received are somewhat subjective scores. Because if a transmission is designed with focus on optimizing one of the criterion this may result in a better performance on that particular criterion, but this will be at the expense of other criteria. For example the trade-off between stiffness and weight or size can always be made. And eliminating backlash with pre-tension will have the disadvantage in the form of additional...
friction. Therefore these scores should not be interpreted as fixed quantities, but more like general characteristics of a transmission concept. Since the results are not fixed values, but estimates which would be slightly different by an other designer the results should not be followed blindly, but as an advice for auspicious search direction.

V Conclusion

The theoretical analysis revealed that every differential mechanism can be divided into a transmission with an additional degree of freedom. Likewise differential mechanisms can be synthesized by adding a degree of freedom to the transmission. This design methodology was not found in literature.

Some general conclusions can be drawn about the categories of transmissions. Transmissions consisting from only solid parts have a high stiffness, and reliability, but also a high weight, backlash, and large dimensions. Transmissions using a fluid but no flexible parts have a high reliability, no backlash, have relatively high friction. Transmission using fluid and flexible parts have a low weight, low friction, but have a low stiffness and reliability. Transmissions with flexible parts have low friction losses, but a low reliability. Fully compliant transmission without sliding elements have the potential for lowest friction losses. Material hysteresis is the only energy dissipating mechanism. The intrinsic stiffness, $k_{\text{stable}}$ of the mechanism might reduce the energy efficiency.

This paper gives a qualitative comparison between vastly different types of differential mechanisms. The results from this literature review can be used by designers who want to implement a differential mechanism for choosing their concept. So this could helps the design of better exoskeletons of other application.

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Paper: Behaviour of closed-loop two-fold tape springs
Behaviour of closed-loop two-fold tape springs

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Abstract

Keywords rectilinear motion · tape spring · closed-loop · compliant mechanism

This paper investigates the behaviour of closed-loop two-fold folded tape springs (tape loops). Tape loops are large range of motion rectilinear compliant mechanisms. The force deflection behaviour along axial and transverse axis are modelled and experimentally verified with 2 samples. The analytical model was able to predict the transverse behaviour accurately up to deflections of 43%. The axial stiffness was measured at approximately 10 N/m within the 39 mm range of motion. The model predicts the influence of the design parameters on the range of motion and stiffness.

I Introduction

Rectilinear motion mechanisms allow translation in one direction while preventing rotation and transverse translation. Slider joints are a common way to achieve this, but disadvantages of sliders are friction, wear, the need for lubrication and the possibility of backlash. These disadvantages are absent in compliant mechanism. Compliant mechanisms gain there mobility by deformation of flexible parts [9]. Small displacement compliant rectilinear joints are common in planar micro mechanisms. For small motions, parallel leaf springs can form a parallelogram equivalent, which is a linear mechanism for small displacements, but for larger motions parasitic motion occurs [6]. To prevent parasitic motion compound flexures can be used, these consist of two blocks of parallel leaf springs [3], or with beams with lumped compliance hinges [1], connected in series, and opposing direction to compensate for each others parasitic motion. Other in plane linear guide examples are the X-bob a straight line mechanism based on the Roberts mechanisms [7]. However these compliant large range of motion (ROM) rectilinear mechanisms have reduced transverse stiffness for long strokes [10]. If a 3D construction is possible, parallel leaf springs can also be arranged in the out of plane dimension to increase the transverse stiffness [14]. The ROM of these large displacement designs compared to the mechanisms footprint is well below 50% [10]. Moreover a counterforce is felt when translated from its initial fabricated position. This stiffness could be compensated with a pre-stressed negative stiffness structure, but this would use a large area compared to the balanced stroke.

Vehar, Kota and Dennis explored the concept of using closed loop folded tape springs as large ROM compliant mechanisms [15]. The two fold closed loop tape spring (tape loop) is a one degree of freedom rectilinear motion mechanism. Fig. 2 shows how it is constructed and Fig. 4 shows its deformed position. Due to the pre-tension and constant shape it gives no counterforce for axial motion. This linear
motion concept seems attractive for its large ROM and low axial stiffness. Much research is done on the bending moment curve of tape springs [2, 5, 8] and folded tape springs [13, 16, 11]. However the behaviour of a tape loop under loading of axial and transverse forces has not been quantified.

The objective of this paper is to use a insightful model to express the ROM, the force deflection behaviour along the axial and transverse axis and verify the zero force axial behaviour. In Section II a model is proposed and the experimental set-up is presented. The results are in Section III and discussed in Section IV. Finally the conclusions of this paper are in in Section V.

II Method

An analytical model to describe the behaviour of tape loops is proposed and this model is verified with experiments.

II.I Analytical model

A tape loop is formed from a tape spring, a long, straight, thin shell with a transverse curvature profile which is constant along its length $L$, width $b$ and thickness $t$, as shown in Fig. 1.

![Figure 1: Parameter of the un-deformed, tape spring. The thickness, $t$ and transverse curvature are constant along the length, $L$.](image)

Both ends are folded inwards in the same direction as the initial curvature as shown in Fig. 2. The ends are joint by bonding together the ends.

![Figure 2: Assembly of a tape loop from unstressed tape spring](image)

The resulting shape is shown in Fig. 3. It is divided in three types of zones: a: The straight zones are only curved in the $y$-direction just like the initial shape. b: The transition zones have curvature in both directions, and c: the bent zones, are only curved in $x$-direction. The tape loop is clamped to the ground on a straight line of length $l_{clamp}$ in middle of the straight zone. On the opposing straight zone, the output is connected.

![Figure 3: The tape loop is clamped to the grey solid surface. The dotted line marks the opposing clamping line. Three types of zones can distinguished: a: the straight zone, b: transition zone and c: bent zone.](image)

Two types of displacements from the undeformed position are considered: $u$ is the lengthwise displacement along the $x$-axis and $v$ is the transverse displacement, see Fig. 4. Orthogonal forces $N$ or $F$ are applied at the top port indicated by the red arrows. The original diameter of the bent zones is $D_0$ and $R_x$ is the radius of the deformed state. $l$ is the length of the straight and transition zones.
Figure 4: Top: Definition of parameter of undeformed position. Loads \( N \) and \( F \) deform the tape loop. The model predicts both bent sections to deform equally.

To model the behaviour a number of simplifications is made:
- The bent zones have a constant curvature \( 1/R_x \) even when a load is applied.
- Both bent sections describe a 180° arc.
- Elastic energy in the transition zones is constant.
- The straight and transition zones are straight and rigid in bending and shear.
- The in-plane membrane deformations are negligible.

So displacement, \( v \) is only a result of the deformation of the bent zones. The elastic energy in the bent zone is calculated with plate theory.

**Force displacement in y direction**

The radii of the bent zones, \( R_x \) are a result of the transverse load \( N \). \( R_x \) is calculated by finding the minimum in potential energy \( P \), which is the sum of the elastic energy \( U \) and the potential of the force \( V \), which is set at 0 at \( R_x = 0 \).

\[
P = U + V
\]

where:
\[
V = N(D_0 - v) = N2R_x
\]

\( U \) is the elastic bending energy of the 2 bent zones. Each has area \( b\pi R_x \) and a curvature change of \( d\kappa = [d\kappa_{xx}, d\kappa_{yy}, d\kappa_{xy}]^T = \left[ \frac{1}{R_x}, \frac{1}{R_y}, 0 \right]^T \). Using \( \frac{1}{2} d\kappa^T D d\kappa \) for the elastic energy per area this results in:

\[
U = 2 \cdot \frac{1}{2} \pi \cdot R_x \left[ \frac{1}{R_x} \frac{1}{R_y} \right] [D_{11} D_{12}] \left[ \frac{1}{R_x} \frac{1}{R_y} \right] = \pi b (\frac{D_{11}}{R_x} + \frac{R_y D_{22}}{R_y^2} - \frac{2D_{12}}{R_y})
\]

(1)

Where \( R_y = D0/2 \), \( D_{11}, D_{22} \) and \( D_{12s} \) are the bending stiffnesses as in [4]. Solving for \( \frac{\partial P}{\partial R_x} = 0 \) results in:

\[
N = \frac{b\pi}{2} \left( \frac{D_{11}}{R_x^2} \frac{D_{22}}{R_y^2} \right) = k_2 - F_0
\]

(2)

Where the part with \( k_2 = \frac{nbD_{11}}{2R_y} \) depends on \( R_x \) and \( F_0 = \frac{nbD_{22}}{2R_y} \) is a constant force offset. The stiffness at the stable position where \( N = 0 \), is the derivative of the force, \( N \) with respect to \( v \).

\[
k(N = 0) = \frac{D_{22}^{3/2} b}{D_{11} R_y^3}
\]

(3)

In case of an isotropic material \( D_{11} = D_{12} = \frac{E}{1-v^2} \), this can be simplified to:

\[
k(N = 0) = \frac{b\pi^2 E}{12 \cdot (1-v^2) R_y^3}
\]

(4)

If \( t/R_y \) and \( b/R_y \) are held constant if follows that the stiffness increases linearly with scale, \( R_y \).

**Force displacement x-direction**

The length of the straight part of the un deformed tape loop can be calculated with:

\[
l = \frac{L - \pi \cdot D_0}{2}
\]

Where \( L \) is the length of the tape spring, see Fig. 1. The ROM is defined as the maximum displacement \( u \) in one direction from the middle position that doesn’t cause a counterforce because the deformed configuration, has the same shape, hence equal elastic energy.

\[
ROM = \frac{l - l_{clamp}}{2}
\]

(5)
Where \( l_{\text{clamp}} \) is the length of the clamped section as in Fig. 4. At the end of the ROM the clamp reaches the bent section, so the bend section is forced to adopt a smaller radius. This results in a counterforce \( F \). Outside the ROM, where \( |u| > ROM \), \( R_x \) decreases.

\[
R_x = \frac{D_0}{2} - \frac{1}{\pi} (|u| - ROM)
\]

Inserting this in Eq. (1) and taking \( F = \frac{\partial U}{\partial u} \) gives:

\[
F = \frac{1}{\pi} \left\{ \frac{k_2}{\left( \frac{D_0}{2} - \frac{|u| - ROM}{\pi} \right)^2} - F_0 \right\}
\]

(6)

Where the direction of the force is opposing the displacement \( u \).

## II.II Experiments

The force deflection behaviour along the axial and transverse axis is measured to validate the models derived above. Lengths of consumer carpenter tape are used to construct two tape loops. The dimensions are given in Table 1.

**Table 1: Properties of tape springs**

<table>
<thead>
<tr>
<th>( b )</th>
<th>( t )</th>
<th>( L )</th>
<th>( E )</th>
<th>( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 mm</td>
<td>0.118*</td>
<td>242 &amp; 243 mm</td>
<td>207</td>
<td>0.3 GPa</td>
</tr>
<tr>
<td>*measured &amp; manufacturers specifications [12]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ends are spot-welded together with the use of a alignment jig, see Fig. 5. Before welding the ends are chamfered to opposing 1 : 5 slopes so that the overlapped area has minimal influence on the stiffness. At the joint and the opposite side a 4mm hole is punched to allow a bold to pass through to fix the cylindrical clamps. It was chosen to clamp the tape loops on an line contact to allow changes in transverse curvature at the clamp sites. The properties of the tape loop are in Table 2.

**Table 2: Properties of tape loop**

<table>
<thead>
<tr>
<th>( D_0 )</th>
<th>( l )</th>
<th>( l_{\text{clamp}} )</th>
<th>( F_0 )</th>
<th>( k_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 mm</td>
<td>70.7 &amp; 71.2 mm</td>
<td>32 mm</td>
<td>4.77 N</td>
<td>1.22 ( \cdot 10^{-3} ) Nm²</td>
</tr>
</tbody>
</table>

A ‘Testometric DBBMTCL-250kg’ tensile testing machine (tester) with accuracy \( \pm 0.5\% \) of the measured force and resolution 0.001mm, is used to measure the force displacement behaviours. Two experiments are done. Fig. 6 shows the set-up for testing the force displacement behaviour along the \( x \)-direction. A long, flexible, 0.1mm thick, 5mm wide, sheet metal strip is glued to one clamp site to apply a force in \( x \) direction without constraining the \( v \) motion. A weight keeps this sheet steel wire under tension. A displacement in the range \( u = [-55:+55] \) mm is applied, which is just more than the predicted ROM, but not enough to cause permanent deformation. The average stiffness and friction force were calculated from the measurement data.
that the hysteresis increased for each larger displacement cycle. The start of this rise in hysteresis differed by 2 mm to 3 mm. The cycles that were selected for plotting the force displacement plots had a displacement of 0.59% and 0.67% of the initial diameter, see Fig. 9, at 2.45% and 1.49% hysteresis for sample 1 and 2 respectively. After the last cycle a permanent increased local curvature in the middle of the bent section was observed.

### III Results

For both samples the initial few force displacement cycles along the $y$-axis had hysteresis around 1% as can be seen in Fig. 8. After

![Figure 7: The lower aluminium block is the ground. The top aluminium block is connected to the tester sensor and moves up and down, deforming the tape loop. The tape loop is clamped to both top and bottom block.](image)

![Figure 8: The fraction of energy lost relative to the input energy for each incremental cycle. The measurement cycle which is selected for the force displacement plot indicated with a cross.](image)
Figure 9: The displacement is normalized with the initial diameter, $D_0$. Both model and measurement show in increasing stiffness.

Fig. 10 shows the difference $N_{\text{error}} = N_{\text{model}} - N_{\text{measured}}$ between the measurement and model which increases rapidly for large deformations.

Figure 10: The error $N_{\text{error}}$. Up to a deflection of $v = 0.43 \cdot D_0$ the model and measurements are within 0.5N. After $v = 0.5 \cdot D_0$ the error exceeds 1N and quickly grows.

The force deflection behaviour as result of a force $F$ is shown in Fig. 11. The measurement forces are not completely zero as predicted by the model. Within the predicted ROM a linear fit on the measurement has a slope of 7.57N/m and 12.27N/m for sample 1 and sample 2, respectively. The hysteresis loops in Fig. 11 contain 0.11mJ and 0.12mJ respectively. Divided over the total stroke of $2 \cdot 110$mm this is an average friction force of 0.055N and 0.06N for sample 1 and 2.

Figure 11: The two samples had a slightly different length and so a slightly different predicted ROM. The model for a ROM of 39mm.

IV Discussion

The model for the force deflection behaviour in $y$-direction overestimates the force for large deformations. This can be explained by the considering that for larger loads the bent zones are not only loaded by a moment, but also a force, see Fig. 12. This causes the shape to deviate from the constant curvature which is an assumption of the simplified model.

By close inspection of Fig. 13 is can also be noticed that the straight zones aren’t completely straight, but slightly curved in the same direction as the bends. This makes the tape...
loop not a perfect straight line mechanisms, but a lightly curved motion mechanism.

![Diagram of tape loop mechanism](image)

**Figure 13:** During the axial motion test, at a position where \((v > ROM)\) the left straight zone is slightly curved. This causes the lower bent to move to the right with respect to the vertical red line. The dotted box contains the area where the metal strip is glued to the tape loop.

The analytical model predicts \(F = 0\) for \(|u| < ROM\), but a low counterforce is measured. The deviation from the model can partially be assigned to the increased thickness where the strip of spring steel is glued to the tape loop. This destroys the uniform thickness which gives neutral stability. The local thickness increase, over a length of 32 mm and width of 5 mm is \(0.10\text{mm} + 0.12\text{mm} = 0.22\text{mm}\). Assuming that this glued area, highlighted in Fig. 13, is 'half flat' at \(u = ROM\), which introduces the curvature change \(dk_{yy\text{glue}} = -\frac{1}{2}\), this results in a transverse bending energy of \(U_{\text{glue}} = \frac{1}{2}k_{yy\text{glue}}D_{22g}^2 = 0.0077\text{J}\) compared to 0.0012J without increased thickness. Attributing this energy to a constant stiffness would give \(k = 2U_{\text{glue}}/ROM^2 = 8\text{N/m}\) which has the same order of magnitude as the measured stiffness within the ROM.

The hysteresis could be caused by material damping, or even plastic deformation in the material. During axial motion many changes in stress state of the material occur. These could cause local grain boundary slips in the metal which dissipate energy.

Furthermore the analytical models has not been tested with a diversity of tape loop designs, so it is unknown for which parameter space these models are accurate. And also just two samples of the same design might be too few for statistical/empirical "proof".

**V Conclusion**

The proposed model gives an accurate predictions of the transverse force \(N\). Up to a compression 43% the error stays within 0.5N. For further transverse deformations the force is overestimated due to the assumption of constant curvature in the bent zones. The model predicts the initial transverse stiffness and force deflection, but is not valid for very large deformations above 50%. The ROM of the tested tape loop corresponds with the model and was found to be 39mm for a 103mm long tape loop, which is 37.9% of the total un-deformed length in this case. It was tried to fabricate tape loops with zero stiffness in \(\delta_x\) direction by clamping them on a line. However, as discussed, the added stiffness by the strip connection and clamping probably caused a stiffness of around 10N/m. The models predict how the transverse stiffness is influenced by the design parameters. And predicts the ROM from the dimensions of the tape spring. This can help designers who want to implement a tape loop as a rectilinear mechanism.

**References**


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Paper: Closed Loop Two-Fold Tape Spring Transmissions
Closed loop two fold tape spring transmissions

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Abstract

In this paper a new concept for a large range of motion compliant transmission is modelled and experimentally tested. The concept consists of two two-folded closed loop springs connected by a tendon. The non-linear behaviour under loads was modelled by minimizing the total potential energy. The measured stiffness was close to the model and changed during the stroke. The efficiency was measured between 91.7% and 97.6% and the range of motion a 39mm. The optimal configuration for maximal stiffness was found.

Keywords Transmission · compliant mechanism · Two-fold closed-loop tape spring

I Introduction

Tendon transmissions are used for in medical devices [7], exoskeletons [16], prosthetic hands [5] and robotic graspers [11, 12]. They are used to place the actuator away from the joint because of spacial efficiency, faster dynamics, or other practical reasons. If these tendons have to be deflected around a body a guide mechanism must be introduced.

Biological tendon are able to bear high loads, have high lifetime, are spatially efficient and have very low friction. The tendons sheath transmission in human fingers have low friction coefficient of $\mu = 0.03$ for ordinary loading [1] and $\mu = 0.075$ for heavy loading [14]. Unfortunately no engineering counterpart of such low friction tendon transmissions exist.

A well known technical tendon transmission is the Bowden cable system[3]. It is used to control the brakes and derailleur of a bicycle, but also to actuate exoskeletons [13]. The friction coefficient was measured between $\mu = 0.105$ and $\mu = 0.328$ for different types of Bowden cables [4]. This friction also causes quick wear and complex control strategies to handle the direction dependent behaviour [9].

Instead of sliding contacts to guide a tendon around a corner, pulleys can be used as guide mechanisms for tendons [6, 2]. But pulleys have the risk of disengaging when the tendon tension is released or in case of improper alignment. Also, pulley axes have bearings, which require lubrication.

Compliant mechanisms use the elastic deformation of material to allow motion. So there is no need for hinges or sliders. Which gives the advantage that they consist can of one part, there is no friction and there is no need for lubrication [10]. However, in general compliant mechanisms have a relatively low range of motion. Vehar and Kota investigated the use of closed loop tape springs as large range of motion fully compliant mechanisms [17]. The closed-loop two-fold tape springs (tape loops) are compliant rectilinear mechanisms that can be used as tendon guides. Their range of mo-
tion, and force deflection behaviour under axial and transverse loading was modelled [8]. This paper introduces a new transmission concept consisting of multiple tape loops connected by a tendon, see Fig. 2.

Such a transmission has a low stiffness between input and output like the tape loops themselves. For designers it is important to know how such a transmission behaves under loading and how this behaviour is influenced by the design.

The objective of this paper is to introduce, and verify a model for the in plane mechanical behaviour of tape loop transmissions. And we want to measure the transmission’s efficiency.

In Section II a model is proposed to simulates the in-plane mechanical behaviour. A test set-up is presented to verify this model and test the efficiency. In Section III the experimental and simulation results are presented. Section IV discusses the sources of the differences between model en measurements, and in Section V the final conclusions are drawn.

II Method

II.I Concept description

Tape loops are formed by folding a tape spring into a loop and connecting its short edges, as is illustrated in Fig. 1. More details about this can be found in [8].

![Figure 1: Assembly of tape loop from unstressed tape spring](image)

The concept introduced here uses two tape loops attached tangentially at relative angle \( \alpha \), to a circular body with radius, \( R \). They are interconnected with a tendon, see Fig. 2. In this way a tensile force can be transmitted from one tape loop to the next. The tape loops are connected at two places, to the wire and to the body, both on a line of length, \( l_{clamp} \) in the middle of the straight section. The thickness of the tape loop is called \( D_0 \). Multiple of these concepts could be placed in series to get a longer transmission that is able to cover a larger angle, but the presented study is confined to the configuration with 2 tape loops.

![Figure 2: The tape loop transmission diverts a motion with angle \( \alpha \) around the circular body with radius \( R \). The incoming and outgoing forces, \( F_{in} \) and \( F_{out} \) remain parallel to the body. \( f \) is the relative thickness of tape loops. The lightly drawn lines are the deformed configuration.](image)

II.II Metrics

A number of properties of the transmission are investigated. These metrics were normalized with the scale of the mechanism to fairly compare the designs, where the ” ” hat is used for the normalized metric. The range of motion, (ROM) is defined as the maximum displacement one tape loop can make without increasing resistance.

\[
\text{ROM} = l_{\text{straight}} - l_{\text{clamp}} 
\] (1)

Where \( l_{\text{straight}} \) it the length of the straight part of the tape loop and \( l_{\text{clamp}} \) is the length over which the tape loop is clamped, see Fig. 4. The
ROM is normalized with R.
\[ \hat{R} \text{OM} = \frac{R \text{OM}}{R} \quad (2) \]

In compliant mechanisms the amount of stored elastic energy influences the amount of work transferred from input to output. Therefore the efficiency is defined by considering an input displacement path with a same start and end point in order to have equal elastic energy. The efficiency \( \eta \) is defined as:
\[ \eta = 1 - \frac{\text{hysteresis}_{\text{in}}}{W_{\text{flux}}} \quad (3) \]

Where hysteresis\(_{\text{in}}\) is the energy loss in the force displacement cycle. And \( W_{\text{flux}} \) is the total energy lost
\[ \text{hysteresis}_{\text{in}} = \int F_{\text{in}}du_{\text{in}} \quad (4) \]
\[ W_{\text{flux}} = \left| \int_{u_{\text{out}}}^{u_{\text{out}}} F_{\text{in}}du_{\text{in}} \right| + \left| \int_{u_{\text{in}}}^{u_{\text{in}}} F_{\text{in}}du_{\text{in}} \right| \quad (5) \]

Where closed curve integral, \( u_{\text{in}} = 0 \) for a conservative system with efficiency 100%. It is normalized by \( W_{\text{flux}} \) the sum of the input energy during a complete cycle from start point, \( u_{\text{out}} \) to maximum endpoint displacement, \( u_{\text{immax}} \) and back to the start point.

The stiffness, \( k \) of the transmission is defined as the tangent of the input force, deflection curve at zero force while the output port is locked.
\[ k = \frac{dF_{\text{in}}(F_{\text{in}} = 0, u_{\text{out}} = C)}{du_{\text{in}}(F_{\text{in}} = 0, u_{\text{out}} = C)} \quad (6) \]

Where \( F_{\text{in}}, u_{\text{in}} \) are the input forces and displacements, and \( u_{\text{out}} \) the output displacements shown in Fig. 2 and \( C \) is a constant output displacement. \( k \) is normalized as follows
\[ \hat{k} = \frac{k\alpha}{D0} \quad (7) \]

Normalization with \( \alpha \) was done to represent the stiffness per angle. Normalization with \( D0 \) was done because the stiffness of a tape loop with equal proportions scales linearly with size [8].

II.III Modelling

The force deflection behaviour is modelled by defining it as an optimization problem. The mechanisms deformed position is found by finding the local minimum in elastic energy while satisfying the non-linear equality constraint equations imposed by the way the tape loops and tendons are connected.

\[
\begin{align*}
\text{minimize} & \quad P(X) \\
\text{subject to:} & \quad AX \leq b \\
& \quad c(X) = 0
\end{align*}
\]

Where \( P(X) \) is the total potential energy, \( X = [x_1, y_1, x_2, y_2, \ldots]^T \) is the vector containing the node coordinates, which are the border of the straight lines and bends of the tape loops, see Fig. 4. \( A \) and \( b \) are the linear inequality constraints and \( c(X) \) are the equality constraint equations. Matlab R2015a [15] was used to execute the sequential quadratic programming algorithm to solve the optimisation problem using the Jacobians, \( \frac{dP(X)}{dx} \) and \( \frac{dc(X)}{dx} \) to quickly search in the right direction. The next three sections go in more detail about the potential energy and the two kinds of constraint equations.

II.III.1 Potential energy

The objective function is the potential energy, which is the sum of the potential of the external loads, which are assumed constant and the internal elastic energy. The simplification is made that all elastic energy is stored in the tape loop’s half-circle shaped bent sections and all bends maintain a 180° bend.

\[
\begin{align*}
P(X) &= U(X) + V(X) \\
U(X) &= \sum_{i=1}^{n} U_i(X_i) \\
V(X) &= \sum_{i=1}^{n} -F_i \cdot (X_i - X_{0i})
\end{align*}
\]

Where \( F_i \) are the constant external loads, \( X_i \) coordinates and \( X_{0i} \) the original coordinates of node \( i \). Where \( U(X) \) is the strain energy which
is the sum of the strain energy of each node $U_i$. The transverse deflection model of a tape loop from [8] was used. This analytical expression for the elastic energy has the form of Eq. (12). For the simulation half the stiffness parameters were used since only one bent is considered: $F0 = 2.3888N$ and $k_2 = 6.1154 \cdot 10^{-4}N\text{m}^2$. $U_i$ is shown in Fig. 3.

$$U_i = k_2/R_i(X_i) + F0R_i(X_i) \quad (12)$$

Where $R_i$ is the radius of the tape loop at node $i$.

$$R_i = \frac{D_0 + v_l}{2} \quad (13)$$

Where $D_0$ are the are the initial un-deformed bent diameters, see Fig. 4. $v_l$ is calculated from the global coordinates $X_i = (x_i,y_i)$ by transforming it to the local coordinates, $(u_i,v_i)$ which are in the lengthwise and transverse direction of the tape loop

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \cos(\phi_i) & \sin(\phi_i) \\ -\sin(\phi_i) & \cos(\phi_i) \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (14)$$

Where $i = n$ or $i = n + 1$ And where $x_0$ and $y_0$ are the un-deformed, initial, coordinates of the node.

In the initial shape the nodes are at a position of minimum elastic energy. This can be viewed as they are at the bottom of an energy valley. The input and output forces and displacements can pull the nodes out of this minimum.

II.III.II Inequality constraints

At the end of the range of motion of the tape loops, the bend sections have to undergo a transverse deformation, $v_l$ to allow for additional axial motion, $u_l$, see Fig. 4. Under the assumptions, that the bend is circular, this can be expressed as:

$$|u_l| + v_l \cdot \frac{\pi}{4} \leq \frac{\text{ROM}}{2}$$

Which can be written as two linear inequality constraint equations per node.

$$\begin{bmatrix} \frac{\pi}{2} & 2 \\ \frac{\pi}{2} & -2 \end{bmatrix} \begin{bmatrix} u_l \\ v_l \end{bmatrix} \leq \begin{bmatrix} l - l_{\text{clamp}} \\ l - l_{\text{clamp}} \end{bmatrix}$$

By substituting Eq. (14) this was written in terms of $X$ in the form $AX \leq b$ Eq. (8).

II.III.III Equality constraints

The non linear equality constraints are used to describe how two subsequent nodes are connected. The have the form:

$$c(X) = l_1 + l_2 + l_3 + l_4 + l_5 - l_{\text{total}} = 0 \quad (15)$$

Where $l_1$-$l_5$ depend on the node coordinates and $l_{\text{total}}$ is constant. Three different topologies of constraints were used, they are shown in Figs. 5 to 7. With all topologies, the shape of the deformed bent sections is simplified as a 180° circle arc, depicted by a grey circle. And $l_1$-$l_5$ are sections of $l_{\text{total}}$, expressed as function of the node coordinates on the left and right side, $(x_n,y_n)$ and $(x_{n+1}, y_{n+1})$.

The constraint in Fig. 5 describes that the two nodes on a tape loop have to move in a way that the circumference of the tape loop, $l_{\text{total}}$ is constant. It is expressed mathematically
as:

\[ l_1 = \sqrt{(x_n - x_{n+1})^2 + (y_n - y_{n+1})^2} \]
\[ l_2 = \pi \cdot R_n(X) \]
\[ l_3 = 0 \]
\[ l_4 = \pi \cdot R_{n+1}(X) \]
\[ l_5 = l_{straight} - u_n + v_{n+1} \]
\[ l_{total} = 2 \cdot l_{straight} + \pi \cdot D0 \]

Where \( R_{n+1}(X) \) and \( R_{n+1}(X) \) are calculated with Eq. (13).

The other two types of constraints involve a tendon spanned between the midpoint of the clamps of two tape loops. In the configuration of Fig. 6 the tendon stays on the same side. In Fig. 7 the tendon crosses from one side to the other, this is used for the experiments. The length of the tendon is the sum of the sections \( l_1-l_5 \), where \( l_1 \) and \( l_5 \) are straight sections on the tape loop, \( l_2 \) and \( l_4 \) are the part of the tendon in contact with the bent section and \( l_3 \) is the part of the tendon that bridges two neighbouring tape loops.

For both cases the length, \( l_3 \) and angle, \( \theta \) of the tendon spanned between the loops have to be known. They can be found by calculating the tangent line to both circles, using the circle diameters from Eq. (13) and the coordinates of the midpoints of the circles \( C_n \) and \( C_{n+1} \) from the equation below:

\[
\begin{bmatrix}
C_{ix} \\
C_{iy}
\end{bmatrix} = \begin{bmatrix}
x_i \\
y_i
\end{bmatrix} - \begin{bmatrix}
\sin(\phi_i) \\
\cos(\phi_i)
\end{bmatrix} R_i \tag{16}
\]

Where \( i \) is \( n \) or \( n+1 \).

In case the tendon stays on the same side of the tape loops, as in Fig. 6, the other lengths are calculated with:

\[ l_1 = l_{straight}/2 - u_n - \frac{\pi}{2} u_n \]
\[ l_2 = (\phi_n - \theta) \cdot R_n \]
\[ l_4 = (\theta - \phi_{n+1}) \cdot R_{n+1} \]
\[ l_5 = l_{straight}/2 + u_{n+1} - \frac{\pi}{2} u_{n+1} \]
The tendon crosses to the other size of the tape loop as is used in the measurement set-up. And in case the wire crosses from one side of the tape loop to the other side, as in Fig. 7, the following equations apply.

\[
\begin{align*}
    l_1 &= \frac{l_{\text{straight}}}{2} - u_n - \frac{\pi}{2}v_n \\
    l_2 &= (\phi_n - \theta) \ast R_n \\
    l_4 &= (\phi_{n+1} - \theta + \pi) \ast R_{n+1} \\
    l_5 &= \frac{l_{\text{straight}}}{2} - u_{n+1} - \frac{\pi}{2}v_{n+1}
\end{align*}
\]

II.IV Simulation study

The model described above is used to find the best configuration of the transmission concept. A design with \( \alpha = 60^\circ \), \( R = 200 \text{ mm} \), and \( ROM = 40 \text{ mm} \) is taken as the standard configuration. The initial stiffness \( k \) (Eq. (7)) is calculated for the range of input displacements \( u_{\text{in1}} = [-ROM : ROM] \). To do this two load steps were used:

1. \( u_{\text{in}} = u_{\text{in1}} \), \( F_{\text{out}} = 0 \text{N} \) after which \( u_{\text{out}} = u_{\text{out2}} \).
2. \( u_{\text{in}} = u_{\text{in1}} + 1 \text{mm} \), \( u_{\text{out}} = u_{\text{out2}} \).

\( F_{\text{in}} \) after this load step is used to determine the \( k = \frac{F_{\text{in}}}{1 \text{mm}} \). Maximizing the minimum \( k \) in the input displacement range \([-ROM : ROM]\) is used as the objective for the optimization. A grid search for \( \alpha \) and \( ROM \) was done to find the maximum stiffness \( k \). While keeping the dimensions such that the tape loops don’t make contact and the tendons stays in contact with the bend of the second loop, see Fig. 9. To clarify things, this optimization uses the model, which itself uses an optimization algorithm to find the equilibrium position. The results are represented with the normalized metrics, Eq. (7) and Eq. (2).
II.V Experimental set-up

To verify the model and test the efficiency two kinds of experiments were done: the ‘stiffness experiment’ measures the force deflection behaviour of the input of the transmission with the output locked in different positions. The ‘transmission experiment’ tests the efficiency of the transmission using different constant forces on the output. The loads were chosen such that the stresses remain within the elastic region. This means the bends can not be smaller than $D_0/2$ [8].

The vertical input displacement and force are measured using a tensile testing machine, (tester). A constant output force is applied using a weight. The model was tested in a configuration where the tendon crosses diagonally between the tape loops as shown in Fig. 7 and Fig. 10. In this configuration it is possible to have the direction of the tendon at the input parallel to the direction of the tendon at the output. Choosing this direction vertical, it matches both the travel direction of the tester as the direction of gravity, resolving any redirection trouble. Consumer carpenter tapes are used to build tape loops. They are spot-welded start to end. The ground connection at the weld, is clamped on the centreline over a length $l_{\text{clamp}}=32\text{mm}$. On the opposite side the tendon, made from a 0.10 mm thick, 5mm wide, strip of spring steel is glued to the tape loops. With the transmission experiment, a calliper is used to measure $u_{\text{out}}$.

III Results

III.I Experimental results

The stiffness experiment shows in increasing stiffness for larger deflection. Both the model and the experimental results give this behaviour, see Fig. 11. The two measurements from starting, $u_{\text{in}}=0$ and $u_{\text{in}}=10$ deviate from the model. For larger input displacements the stiffness is larger. There is a very small hysteresis loop visible. The experiments was done several times, except for the first load cycle, force deflection curves of the same experimental set-up overlapped.
A clear hysteresis loop is visible in the transmission experiment, see Fig. 12. The vertical displacement $\delta u_{\text{out}}$ of the output is dependent on the weight. The amount of hysteresis measured at the input, the resulting total efficiency, $\eta$, and $W_{\text{flux}}$ are in Table 1.

Figure 12: One cycle of input displacements for the transmission of two tape loops connected at $45^\circ$ for output weights 1.05N, 3.23N and 9.60N. The model predicts a straight line, measurements show a somewhat curved line and hysteresis loop.

Figure 13: For a configuration where $\alpha = 60^\circ$, $R = 200\text{mm}$, $D_0 = 32\text{mm}$, $l_{\text{loop}} = 72\text{mm}$ the force $F_{\text{in}}$ is recorded for input displacements, $u_{\text{in}}$ from different starting positions, while the output was locked, $u_{\text{out}} = 0$. The data is plotted up to the displacement where the normal force on any of the bends is over 10N.

In Fig. 13 it can be seen that the force deflection behaviour differs between between start positions. $k$, which is the slope of the tangent line to at $F_{\text{in}} = 0$ is plotted as a function for the range $[-\text{ROM} : \text{ROM}] = [-40 : 40]\text{mm}$.
Figure 14: Again, $\alpha = 60^\circ$, $R = 200\text{mm}$, $D0 = 32\text{mm}$, $\text{Loop} = 72\text{mm}$. The initial stiffness decreases for larger $u_{in}$.

Fig. 14 shows the stiffness $k$ is lowest at maximum input displacement, $u_{in} = \text{ROM}$. This position was used by the grid search on $\alpha$ and ROM for the highest $k(u_{in} = \text{ROM})$. The results are presented as the normalized metrics below in Fig. 15. Where it can be seen the normalized stiffness $\hat{k}$ goes down for larger $\hat{\text{ROM}}$.

IV Discussion

The overestimation of the force in the stiffness experiment the two of the trails could be explained by the absence of pretension on the tendons. The model assumes tendons with no bending stiffness that closely follows the bends of the tape loops after which they take the shortest path to the next tape loop. Without pretension the tendons in the experiment follow a more gradual curve. Between the experiments the amount of pretension may have varied. A lower pre-tension would lead to a start-up behaviour of the tendons. At positions $u_{in} = 0$ and $u_{out} = 0$ there could be some slack on the tendons.

In the transmission experiment a significant hysteresis was measured. The average friction force $F_w$, which is the hysteresis divided by the total stroke of $F_w = U_{\text{lost}}/(2 \cdot 40\text{mm})$. It is a function of the loading force, see Fig. 16.

Figure 15: With $R = 200\text{mm}$ and $D0 = 32\text{mm}$ the optimum angle $\alpha$. The normalized stiffness still has a higher stiffness for smaller ROM designs. The wrinkles are a result of the resolution of the grid search.

Figure 16: The linear fit through the three measurement points has the equation: $F_w = 0.07 + 0.0173 \cdot F_{out} \text{ N}$

In [8] it was measured that the tape loops had an average friction force of $F_w1 = 0.0555\text{N}$ for axial motion. In the transmission 2 tape loops are used, where the one attached to the input moves more than the other. Considering the ratio of $\frac{F_{out}}{F_{in}} \approx 0.65$ in Fig. 12, and assuming the inverse ratio applies to the displacements $\frac{u_{out}}{u_{in}}$ the effective friction would be, $(1 + 0.65) \cdot F_w1 = 0.09\text{N}$ which is slightly more than the measured friction, but is an explanation for the amount of the hysteresis.

In [4] it was measured that Bowden cables making a $45^\circ$ bend had an efficiency of 80.6%. For low loads the experimental set-up can be viewed as two consecutive $45^\circ$ bends because...
the tendon is deflected twice, see the left picture of Fig. 10. Which would result in an efficiency of $0.806 \cdot 0.806 = 0.65$ for the Bowden cable system. So the efficiency of this transmission is much higher.

The model was validated with a different topology than the use for the optimization, i.e. a different type of constraint was used. But no reason was found to suspect a difference between these topologies. So it is assumed that the experimental verification is valid.

If was found that $k$ decreases with for larger $u_{\text{in}}$. This can be explained with change angle of the connecting tendon during the stroke. A steeper angle means the second bend from the top will be loaded more directly, see Fig. 17.

$$u_{\text{in}} = -40\text{mm} \quad u_{\text{in}} = -20\text{mm} \quad u_{\text{in}} = 0\text{mm}$$

$$u_{\text{in}} = 20\text{mm} \quad u_{\text{in}} = 40\text{mm}$$

**Figure 17:** The bends of the undeformed starting states schematically represented by yellow circles. From left to right is the configuration with increasing input displacement $u_{\text{in}}$.

V Conclusion

A new concept for a tape loop transmission was introduced. A gradient based energy minimization method was used to simulate the tape loop transmission, which contains many geometric non-linearities. The model slightly overestimates the stiffness compared the experiments. But this can be attributed to the initial slack in de tendon. Both in the simulation and in the experiments the stiffness increases for larger deflections. And it was shown how the input stiffness varies during the stroke. For larger input displacement the stiffness decreases. The transmission experiments show that the transmission has significant energy losses. Efficiencies between 91.7% and 97.6% were measured for different loads. This a better efficiency than Bowden cables. The ROM of the tested tape loops is 39mm in both directions from the middle position. The model was used to optimize the design for different ROM’s which showed the trade off between transmission stiffness, $k$ and ROM. However, many other simulations can be done with the model. Designers can use the model to dimension the transmission to their requirements. The transmission could be used for, low stiffness application.

References


A

EXPERIMENTS

A.1. GLASS FIBER REINFORCE COMPOSITE TAPE LOOPS
For transverse loadings tape loops are flexible and have a low strength. But this can be increased by making them larger and from material with a larger elastic strain. Glass fibers have one of the highest elastic strains, so a good construction material for tape springs.

So a first test to make glass fiber tape springs was done. Layers of glass fiber fabric and epoxy resin (*Epoxyresin L + hardener EPH 161*) were wrapped around different diameter PVC pipes, $D = [35, 50, 75]$mm. Thicknesses $t = [0.4, 0.8, 1.2]$mm were tried. Al fibers were laid in the $[0/90]$ directions to keep the number of variables reduced. Although Guest showed interesting bistable behaviour is possible by placing material in the diagonal directions [7]. Before curing the laminate was covered with foil and tightened with tape. Covering with tape was a bad idea since the wrapping resulted in periodic thickness changes instead of the smooth continuous thickness required for tape springs.

The tape springs were folded into tape loops shown in fig. A.1. When unloaded it adopts the shape in fig. A.2. Some tape springs were too thick to allow them to be folded into a tape loop. If tried de-lamination started at the edges resulting in cracking noises and white spots on the tape spring.

![Figure A.1: The 0.8mm thick, $R = 50$mm, $b =$mm tape spring is folded, overlapped and joint to itself using packaging tape.](image)

The transverse force deflection behaviours of the tape loops were tested until failure. Figure A.4 shows the force deflection behaviour, fig. A.3 the setup and fig. A.5 shows a broken sample.
Figure A.2: When released, surprisingly, the tape loop was not straight, but preferred a twisted position. This could be because of the very low torsion stiffness of the material due to the [0/90] fiber layup.

After these tests it was decided not to continue with glass fiber tape springs since there were too many unpredictable variables such as, resin properties, fiber properties, fiber direction, fiber placement in laminate height. Furthermore the production process had to be optimized to get more smooth tape loops. Further experiments were done with steel tape springs.

Figure A.3: The setup for testing the transverse deflection behaviour of the tapeloop.

### A.2. Making Steel Tape Loops

It was chosen to use consumer carpenter tape as source of tape springs. First the ‘Stanley Fatmax Extreme’ model was used because of its size and therefore its strength. The width \( b = 32\text{mm} \) and the thickness \( t = 0.125\text{mm} \). However the transverse curvature of these models turned out not to be constant along its length. So the tape loops produced had an significant and unpredictable axial force. Some tapeloops had multiple stable positions.

For the final experiments carpenter tape measures from a store brand of a Dutch hardware store were used of type 'Gamma 8m'. These have thickness, \( t = 0.118\text{mm} \) and width \( b = 25\text{mm} \).
A.3. Measurement set-up

The first set-up to made use of a nylon wire as tendon, two pulleys to guide the tendon to the horizontal direction, 'Stanley FatMax Extreme' tape loops and pieces of tape to attach the wire to the tapeloops, see fig. A.12. This had the disadvantages:

- The nylon wire showed non-linear elongations when stressed and about 20 – 30% hysteresis when released.
- The tape loops showed had un-predictable axial force deflection behaviour because of the varying transverse curvature.
- The wires were able to slide sideways, see fig. A.13.
- The pulleys friction.

The second measurement set-up used de 'Gamma 8m' type tape loop material which resulted in much
Figure A.5: The glass fiber test sample has failed during the transverse compression test. A brittle fracture at the middle of the bent section is clearly visible. Only one of the two bends was broken. This strength difference could be explained by the production imperfections.

better zero force tape loops. The transission is placed vertically to fit under the vertical operating tensile testing machine. The tape loops are used in a different topology to with the tendon crossing from one side to the other, see fig. A.14.
Figure A.6: The coating is burned and sanded away at the ends of the tape loop to be able to weld on plain steel. An aluminium clamp is used to file the tape springs ends at an 1:5 slope.
Figure A.7: An 3D printed clamp is used to make sure the ends are aligned during spot welding. Before welding an mounting hole has been punched in the middle of the tape springs end.

Figure A.8: The design of the clamping tool is shown here. The tape spring can be aligned by sliding the two parts together and fixing them with the countersunk holds.
Figure A.9: The result after welding. The mounting hole is visible. The welding clamping tool is still on, but it can now be removed. The weld is strong enough to resist loading when it is straight, but it is not strong enough to pass through the bends. So care must be taken to prevent that from happening.
Figure A.10: The tape loop made from the 'Fatmax Extrem' tapes had different diameter bends. 50% of the welds were strong enough. The plastic pipes are taped on to prevent the welds from moving to the bent area because they would not be strong enough to withstand this bending moment.
Figure A.11: The tendon from 0.1mm thick 5mm wide stainless steel strips are glued to the tape loops using a high strength ‘Locktide’ metal glue. This glue has a low film thickness and high shear strength. The strength of a 32x5mm overlap on the tendon was measured above 100N (highest value of available spring scale), before applying the glue tape is used to make sure the tendon was attached over a length of 32mm. Clothespins are used to apply pre-tension during hardening of the glue.
Figure A.12: First test set-up with many disadvantages.

Figure A.13: The wire was able to slide sideways, causing the bent to tilt, making the wire slide sideways even more, i.e. an in-stable system. Two strips of tape on each side of the wire were able to prevent this from happening.

Figure A.14: The set-up to measure the transmission behaviour of the tape loop is shown here. The ground frame is build from 40mm thick aluminium profiles that are bolted to the base of the tensile testing machine.
Figure A.15: This sample shows clear plastic deformation in the middle of the bent section
APPENDIX: MODEL DERIVATION

B.1. STIFFNESS OF TAPE LOOP

The mechanics of anisotropic plate element can be described with the following equation:

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} =
\begin{bmatrix}
A & B \\
B & D
\end{bmatrix}
\begin{bmatrix}
\epsilon \\
\kappa
\end{bmatrix}
\]  

(B.1)

Where \(N\) are the in-plane axial and shear forces per unit length, and \(M\) are the out of plane moments per unit length. \(\epsilon\) and \(\kappa\) are the in-plane deformations and out of plane curvatures respectively. \(A, B\) and \(D\) are the stiffness matrices describing the relation between these two. [8]

When both \(\kappa_x\) and \(\kappa_y\) are unequal to zero, i.e. the Gaussian curvature is unequal to zero, this results in in-plane deformations. Thin shell are relatively stiff for in-plane deformations and compliant in bending. So, non-zero Gaussian curvature causes high elastic energy.

Guest showed that for an orthotropic spring tape with an arclength of \(\beta = \pi\), zero Gaussian curvature deformation have far lower elastic energy than non-zero Gaussian curvature deformations [7]. fig. B.1

**Assumption 1:** The contribution membrane stresses to the total elastic energy is neglection-able for thin shells.

**Assumption 2:** The spring tape is an thin shell

For thin shells the thin shell theory can be applied. So it is assumed that there are no stresses in the \(z\)-direction of the shell.

If membrane deformations \(\epsilon\), are zero, the behaviour can be described by the following equation:

\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} =
\begin{bmatrix}
D_{11} & D_{12} & D_{16} \\
D_{12} & D_{22} & D_{26} \\
D_{16} & D_{26} & D_{66}
\end{bmatrix}
\begin{bmatrix}
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{bmatrix}
\]  

(B.2)

The properties of an orthotropic material which is loaded in plane stress can be derived from 4 engineering constants, which are the ply properties. A combination of multiple plies gives the laminate properties. For a single ply \(E_L\) is the lateral modulus in line with the fibers, \(E_T\) is the transverse modulus perpendicular to the fibers, \(G_{LT}\) is the shear modulus, and \(\mu_{LT}\) and \(\mu_{TL}\) are the lateral contraction coefficient. See fig. B.2.

fig. B.3 shows the the spring tape linear guide concept.

**Assumption 3:** Laminate is symmetric and anti-symmetric A symmetric layup from fibre fabrics are used. This means the laminate is both symmetric and anti-symmetric. So there is no coupling between bending and torsion, so the terms \(D_{16} = 0\) and \(D_{26} = 0\). And there is no coupling between membrane stresses or deformations and bending stresses or deformations.

**Assumption 4:** No torsion occurs

\[
\begin{bmatrix}
M_x \\
M_y
\end{bmatrix} =
\begin{bmatrix}
D_{11} & D_{12} \\
D_{12} & D_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta\kappa_x \\
\Delta\kappa_y
\end{bmatrix}
\]  

(B.3)

The linear guide can be divided into 3 types of zones about which the following assumptions are made:

1. straight zone, where the shell has it's undeformed shape with a constant transverse curvature of \(\kappa_y = 1/R_y\) and a longitudinal curvature of \(\kappa_x = 1/R_x\). Where a positive curvature is curving inward.
Figure B.1: Elastic energy springtaps as function curvatures (Copied from [7])

Figure B.2: Engineering constants with regard to fiber direction. (image copied from: lectureslide Open course ware, Advanced Design and Optimization of Composite Structures I)
2. Bend zone, where the transverse curvature, \( \kappa_y = 0 \) and the longitudinal curvature, \( \kappa_x = 1/R_x \).

3. Transition zone, the part of the shell, connecting the two zones above. The transverse curvature goes from \( \kappa_y = 0 \) at one end to \( \kappa_y = 1/R_y \) at the other end.

**Assumption 4:** The elastic energy in the 'transition zone' stays constant

Only the left bent zone is considered. A downward pointing force, \( F \) acts on the straight line at the boundary of the transition zone to the bent zone. It is schematically depicted in fig. B.4.

**Assumption 4:** The curvature of the bent zone is constant

The potential energy of this curved zone is expressed in a internal, elastic energy, \( E \) and an external potential energy \( B \).

\[
B = 2R_x F \tag{B.4}
\]

\[
E = \frac{\pi b R_x}{2} \left[ \frac{1}{R_x} - \frac{1}{R_y} \right] \begin{bmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} 1/R_x \\ -1/R_y \end{bmatrix} \tag{B.5}
\]

\[
= \frac{\pi b}{2} \left( \frac{D_{11}}{R_x} + \frac{R_x D_{22}}{R_y^2} - \frac{2D_{12}}{R_y} \right) \tag{B.6}
\]

Where the first two terms depend on \( R_x \) and the last term which is negative for positive initial transverse curvature, is the constant elastic energy, independent of the longitudinal curvature, \( R_y \). The derivative of the total energy with respect to the radius \( R_x \) is:

\[
P = E + B = \frac{\pi b}{2} \left( \frac{D_{11}}{R_x} + \frac{R_x D_{22}}{R_y^2} - \frac{2D_{12}}{R_y} \right) + 2F R_x \tag{B.7}
\]

\[
\frac{dP}{dR_x} = \frac{\pi b}{2} \left( -\frac{D_{11}}{R_x^2} + \frac{D_{22}}{R_y^2} \right) + 2F \tag{B.8}
\]

At equilibrium the potential energy is in a local minimum. So the derivative of the potential energy, \( P \) is 0. \( R_x \) can be written as a function of \( F \).

\[
\frac{dP}{dR_x} = 0 \tag{B.9}
\]

\[
R_x(F) = \sqrt{\frac{D_{11}}{D_{22} + \frac{4F R_y}{b \pi}}} \tag{B.10}
\]

It can be noted that if \( D_{11} \) and \( D_{22} \) equal and the loading force \( F = 0 \), the radius \( R_x(F = 0) = R_y \).

Likewise the force can be expressed in terms of radius \( R_x \).

\[
F = \frac{b \pi}{4} \left( \frac{D_{11}}{R_x^2} - \frac{D_{22}}{R_y^2} \right) \tag{B.11}
\]
It can be noted that the force consists of two contributions, a constant part and a part inversely proportional to \( R_x \). This second part could be compensated with a magnet.

**B.1.1. Stiffness**

Considering small displacements \( du \) in the opposite direction to the force, \( F \) from a radius \( R_x \). The stiffness, \( k \) in this direction is:

\[
k(R_x) = \frac{dF}{du} = \frac{dF}{-2dR_x} = \frac{bD_{11}}{2R_x^3}
\]

(B.12)

If the equilibrium position \( R_x \) is expressed as a function of \( F \) and inserted in the equation above this results in:

\[
k(F) = \frac{bD_{11}}{2R_x(F)^3} = \frac{bD_{11}}{2\left(\frac{D_{11}}{\frac{D_{22}}{\frac{D_{11}}{\frac{D_{22}}{R_x^2}} + \frac{4F}{b}}\right)^{3/2}}
\]

(B.13)

\[
= \frac{(D_{22} + 4F)^{3/2}b}{2\sqrt{D_{11}}}
\]

(B.14)

It is interesting to notice that increasing the plate bending stiffness \( D_{11} \) will decrease the stiffness for from a position resulting from a loading force \( F \), because increasing \( D_{11} \) increases the unloaded radius \( R_x \), which has a stronger effect on the stiffness than .

At \( F = 0 \) and assuming \( D_{11} \) and \( D_{22} \) are equal this results in.

\[
k(F = 0) = \frac{D_{11}b}{2R_x^3}
\]

(B.15)

**B.1.2. Maximum force**

This deformed state results in stresses in the material which will lead to failure if the force is increased. An approximation of the maximum force the mechanism can bear will be made.

*Assumption 6: Failure occurs at the bend zone*
Figure B.5: Strength as function of design parameters $R_y$ and $t$. For $\frac{t}{2R_y} > \frac{\text{yield}}{E}$ the maximum force is set to zero because the strip cannot be elastically deformed to a flat plate.

Where the strain state is:

$$
\epsilon_{\text{max}} = \begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_{xy}
\end{bmatrix} = \begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
0
\end{bmatrix} = \frac{t}{2} \begin{bmatrix}
\frac{1}{R_y} \\
\frac{1}{R_y} \\
0
\end{bmatrix}
$$

(B.16)

Where $\epsilon_x$ and $\epsilon_y$ are the strains in $x$- and $y$-direction, which are in this case equal to the principal strains $\sigma_1$ and $\sigma_2$, the first and second principal stresses because there is no shear strain $\epsilon_{xy}$.

**Assumption 7: the material is assumed to be isotropic**

The local stresses are calculated with:

$$
\sigma = Q\epsilon
$$

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2
\end{bmatrix} = \frac{E}{1-\mu} \begin{bmatrix}
1 & \mu \\
\mu & 1
\end{bmatrix} \begin{bmatrix}
\epsilon_1 \\
\epsilon_2
\end{bmatrix}
$$

(B.18)

The Von Mises failure criterion for plain stress is used to determine the maximum stress state at failure.

$$
\sigma_v = \sqrt{\sigma_{\text{yield}}^2} = \sqrt{\sigma_1^2 - 2\sigma_1\sigma_2 + \sigma_2^2}
$$

(B.19)

Where $\sigma_{\text{yield}}$ is the materials yield strength.

Using Matlab the maximum strength is determined as a function of the parameters: thickness $t$, arc length $\beta$, Youngs Modulus $E$, yield strength $\text{yield}$ and initial transverse radius $R_y$.

**B.1.3. Moment-angular deflection**

Bending of a spring-tape in an 180° bent results in a counter-moment, with a specific moment- angular deflection behaviour. Deflection in the same sense as the initial curvature, results in moment which increases first and then reduces to a lower constant moment. If the tip rotation is in the opposite sense as the initial transverse curvature the moment increases to a higher peak moment after which is abruptly drops to a higher constant moment.
Figure B.6: Strength as function of design parameters $R_y$ squared and relative thickness $\frac{t}{h}$. There is a clear optimum relative thickness visible.

Figure B.7: Strength as function of design parameters $R_y$ squared and relative thickness $\frac{t}{h}$. There is a clear optimum relative thickness visible.
Figure B.8: Strength as function of design parameters maximum elastic strain squared, $\frac{\text{yield}^2}{E}$ and Young's modulus $E$. It can be seen the maximum force is approximately proportional with the Young's modulus and approximately proportional with the elastic strain squared. This means materials with a large capacity to store elastic energy allow the strongest design.

**B.2. Transmission Model**

**Constraint equations**

The non linear constraint equations are the mathematical expression for the length of physical sections. The length of these sections must stay constant to prevent the mechanism from adopting infeasible position or orientations. The equations are of the form:

$$c(X) = C$$  \hspace{1cm} (B.20)

Where: $X = [x_1, y_1, x_2, y_2, ...]$ is the coordinate vector, $C$ is the length that stay constant. Three different kind of constraints are implemented: Loop constraints fig. B.11, wire constraints fig. B.9 and cross wire constraints fig. B.10.

$$l_1 = \frac{l_{\text{straight}}}{2} - u_1 - \frac{\pi}{2} v_1$$  \hspace{1cm} (B.21)

$$l_5 = \frac{l_{\text{straight}}}{2} + u_2 + \frac{\pi}{2} v_2$$  \hspace{1cm} (B.22)

The coördinates $(x_n, y_n)$ and $(x_{n+1}, y_{n+1})$ are transformed to the local coördinates $(u_n, v_n)$ and $(u_{n+1}, v_{n+1})$:

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \cos(\phi_i) & \sin(\phi_i) \\ -\sin(\phi_i) & \cos(\phi_i) \end{bmatrix} \begin{bmatrix} x_i - x_0 \\ y_i - y_0 \end{bmatrix}$$  \hspace{1cm} (B.23)

Where $i = n$ or $i = n + 1$ And where $x_0$ and $y_0$ are the undeformed coordinates of the node. The local deformation is used to calculate the radius of the deformed circles.

$$R_n = \frac{D_{0n} + v_n}{2}$$  \hspace{1cm} (B.24)

$$R_{n+1} = \frac{D_{0n+1} + v_{n+1}}{2}$$  \hspace{1cm} (B.25)

Where $D_{0n}$ and $D_{0n+1}$ are the initial un-deformed circle diameters.
Figure B.9: The length of the wire between the two connection points on the tape loops is constant. The length of the 5 sections $l_1$-$l_5$ are expressed as function of the node coordinates on the left and right side, $(x_n, y_n)$ and $(x_{n+1}, y_{n+1})$. The shape of the deformed bent sections is simplified as circular, depicted in grey.

Figure B.10: The wire crosses to the other size of the tape loop.
B.2. Transmission model

Figure B.11: The tape loop is simplified as an inextensional wire spanned around 2 circles.

\[
\begin{bmatrix}
C_{x_n} \\
C_{y_n}
\end{bmatrix} = \begin{bmatrix}
x_n \\
y_n
\end{bmatrix} - \begin{bmatrix}
sin(\phi_n) \\
cos(\phi_n)
\end{bmatrix} R_n \tag{B.26}
\]

Where \( C_{x_n} \) and \( C_{y_n} \) are the global coordinates of the first circle. The coordinates of the second circle center, \( C_{x_{n+1}} \) and \( C_{y_{n+1}} \) are calculated likewise.

\[
\beta = \tan^{-1}\left(\frac{C_{y_{n+1}} - C_{y_n}}{C_{x_{n+1}} - C_{x_n}}\right) \tag{B.27}
\]

\[
\alpha = \sin^{-1}\left(\frac{R_{n+1} - R_n}{\sqrt{(C_{y_{n+1}} - C_{y_n})^2 + (C_{x_{n+1}} - C_{x_n})^2}}\right) \tag{B.28}
\]

\[
\theta = \beta - \alpha \tag{B.29}
\]

\[
l_1 = -u_n - \frac{\pi}{2} v_n \tag{B.30}
\]

\[
l_2 = (\theta - \phi_n) * R_n \tag{B.31}
\]

\[
l_3 = \sqrt{(C_{y_{n+1}} - C_{y_n})^2 + (C_{x_{n+1}} - C_{x_n})^2 - (R_n - R_{n+1})^2} \tag{B.32}
\]

\[
l_4 = (\phi_{n+1} - \theta) * R_{n+1} \tag{B.33}
\]

\[
l_5 = +u_{n+1} - \frac{\pi}{2} v_{n+1} \tag{B.34}
\]

\[
l_1 = -u_n - \frac{\pi}{2} v_n \tag{B.35}
\]

\[
l_2 = (\theta - \phi_n) * R_n \tag{B.36}
\]

\[
l_3 = \sqrt{(C_{y_{n+1}} - C_{y_n})^2 + (C_{x_{n+1}} - C_{x_n})^2 - (R_n - R_{n+1})^2} \tag{B.37}
\]

\[
l_4 = (\phi_{n+1} - \theta) * R_{n+1} \tag{B.38}
\]

\[
l_5 = +u_{n+1} - \frac{\pi}{2} v_{n+1} \tag{B.39}
\]
CONCEPTUAL DESIGN

The motivation to work on a transmission made from tape loops came from the challenge to make the Laevo exoskeleton suitable for walking. The options that were considered, and the choices that were made, are in this chapter.

C.1. BACKGROUND

The Laevo is a wearable passive exoskeleton designed to reduce the loads on a persons lower back during forward bending. It is a light thin mechanism, without motors and power supply, that can on the body. It has 4 contact points: one on the chest, one on the lower back and two on the upper legs. When a person stands straight there are no forces on the contact pads, but if the user bends over the contact force of the chest pad provides a moment around the hip which is caused by the spring elements [2].

The disadvantage if this mechanism is that when the knees move forward during walking it causes the same deformation of the spring mechanism so also a counterforce is felt on the legs and chest. The original challenge of this thesis was increasing the functionality of the Laevo to facilitate walking. It is was summarized in the following problem statement.

**Problem statement:**
Remove the counterforce on the legs during walking while maintaining the back supporting functionality.

The current version of the Laevo uses by approximation a linear torsion spring to reach 20Nm after 50° of the hip flexion. During normal walking the maximum forward hip flexion is around 25° [9]. This results in a 10Nm counter torque on each leg if a normal gait pattern is maintained. As a reference torque the maximum torque during normal walking is used. Lewis and Sahrmann measured hip flexion moments for different postures using force plates [10]. The peak moment during walking in a normal posture was found at 0.7Nm/kg. For a 60 kg person this is 42Nm. Requirements are set relative to the minimum moment of 42Nm.

<table>
<thead>
<tr>
<th>part of max walking moment</th>
<th>[Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Must</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>Plan</td>
<td>&lt;5%</td>
</tr>
<tr>
<td>Which</td>
<td>&lt;1%</td>
</tr>
</tbody>
</table>

C.2. REQUIREMENTS AND CONSTRAINTS

**Functional requirements: Partial Balancing**

The key function of the exoskeleton is supporting the upper-body during forward bending. Viewing the upper-body as an inverted pendulum the required supporting moment would be a sine-shaped moment-angle curve. It was found by the company Laevo that supporting an specific moment curve of the weight
results in a pleasant behaviour. This is a moment characteristic around the hip joint proportional to fig. C.2. Where 0° is the vertical position with leg and torso aligned.

**FUNCTIONAL REQUIREMENT: FOLLOW BODY MOTION**

The Laevo shouldn’t constrain the desired behaviour of the user. To ensure this it must be determined which motion the human makes. In literature the joint angles during normal walking were taking as the requirement. To define the angles a simplified model of the human was drawn, see fig. C.3. The angles set as requirements are in fig. C.4

**FR2a: WALKING MOTION**

The joint angles of the legs during walking are the most important for this design. So they are investigated over time in [?].

**FR2b: WALKING HIP TORQUES**

The counter moment the Laevo exerts during walking should be reduced to an acceptable level.

\[
\int |M_{in}(M_{out} = 0)|M_{out}
\]

As a reference torque the maximum torque during normal walking is used. Lewis and Sahrmann measured hip moments around the transverse axis for different postures using force plates [10]. The moment as a function of percentage of gait cycle is shown in fig. C.7. It can be seen that the maximum moment is in
C.2. REQUIREMENTS AND CONSTRAINTS

Figure C.2: Moment curve around hip joint resulting in pleasant functionality.

Figure C.3: Simplified model of a human's most important way of movement.

Flexion, which is the forward motion, is between 0.3 and 0.8 Nm/kg. For a 70 kg person, this is 21 – 56 Nm. Requirements are set relative to the minimum moment of 21 Nm.

Table C.2: Maximum counter torque requirements

<table>
<thead>
<tr>
<th>part of max walking moment</th>
<th>[Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Must</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>Plan</td>
<td>&lt;5%</td>
</tr>
<tr>
<td>Which</td>
<td>&lt;1%</td>
</tr>
</tbody>
</table>

The current Laevo version 2.0 the maximum counter torque during walking is....

Maximum counter torque

The static maximum hip abduction moment measured in 11 healthy individuals was 1.81 ± 0.35 Nm/kg [11]. So 97.7% of the population of 60 kg people the maximum force is over 60.6 Nm.
<table>
<thead>
<tr>
<th>Angle</th>
<th>Name</th>
<th>ROM</th>
<th>ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ</td>
<td>Knee flexion</td>
<td>-6°±143°</td>
<td>(Protopapadaki, 2007)</td>
</tr>
<tr>
<td>α</td>
<td>Hip flexion</td>
<td>-20°±50°</td>
<td>(Protopapadaki, 2007)</td>
</tr>
<tr>
<td>β</td>
<td>Hip abduction</td>
<td>-5°±10°, -15°±30°</td>
<td>(Kadaba, 1990)</td>
</tr>
<tr>
<td>γ</td>
<td>Hip rotation</td>
<td>-7°±7°</td>
<td>(Kadaba, 1990)</td>
</tr>
<tr>
<td>ε</td>
<td>Pelvic rotation</td>
<td>-7°±7°</td>
<td>(Kadaba, 1990)</td>
</tr>
<tr>
<td>θ₁</td>
<td>Pelvic tilt</td>
<td>-10°±20°</td>
<td>(Kadaba, 1990)</td>
</tr>
<tr>
<td>Κ</td>
<td>Pelvic obliquity</td>
<td>-5°±5°</td>
<td>(Kadaba, 1990)</td>
</tr>
<tr>
<td>θ₂</td>
<td>Lumbar Extension</td>
<td>-4°±4°</td>
<td>(Fitzgerald, 1983)</td>
</tr>
</tbody>
</table>

Figure C.4: Requirements for the joint angles while using the Laev. The angles indicated with * are estimated. The symbols are in fig. C.3.

Figure C.5: Pelvic rotation during normal walking.
Figure C.6: Pelvic obliquity during normal walking (Copied from [?])

Figure C.7: Hip flexion moment as function of gait. (Copied from [10])
Table C.3: Maximum counter torque requirements

<table>
<thead>
<tr>
<th>Part of maximum muscle torque</th>
<th>[Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Must</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>Plan</td>
<td>&lt;5%</td>
</tr>
<tr>
<td>Which</td>
<td>&lt;0.5%</td>
</tr>
</tbody>
</table>

**FR2b: Unconstrained body movements**

**Joint angles**

Spring function of transmission.

Angle-moment curve for equal leg angles bending over

Virtual pivot point

The mechanism must rotate around the axis through the hip joints. Because the space between the two pivot points is taken by the body the mechanism must have a virtual pivot point near the axis of rotation.

**Boundary conditions**

**Weight**

The weight of the exoskeleton is important since it has to be carried around all the time. A light weight design can be accomplished by efficient material usage. Which means the all material is used near maximally. For a single load case this means all material is stressed near its maximum allowable load. Using short and straight load paths reduces the weight. And weight gains can be made by integrating multiple functions in components.

The requirements set up for the weight are in table C.4.

Table C.4: Weight requirements

<table>
<thead>
<tr>
<th>total mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Must</td>
</tr>
<tr>
<td>Plan</td>
</tr>
<tr>
<td>Which</td>
</tr>
</tbody>
</table>

**Strength**

The moments around the hip joint are the highest and have been a challenge to design in a durable way. The present maximum moment exerted by the spring is 20Nm.

Table C.5: Strength boundary condition

<table>
<thead>
<tr>
<th>Loading</th>
<th>[Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Must</td>
<td>&gt;100%</td>
</tr>
<tr>
<td>Plan</td>
<td>&gt;150%</td>
</tr>
<tr>
<td>Which</td>
<td>&gt;200%</td>
</tr>
</tbody>
</table>

**Donning and doffing**

In order to put the suit on and off quickly the is must be possible to fold it open like a jacket. The torso structure must be able to split and the hip belt must be flexible.

**C.3. Concepts choices**

In this section the considered concepts are presented and the reasoning for the choice are briefly explained.
WALKING SOLUTION TYPE
The walking problem can be interpreted in multiple ways:
1. The user uses disengages the spring mechanism.
2. The design is made such that walking straight is possible without adjustments.
3. Walking while bending and walking up stairs are possible without adjustments.

It was chosen for solution 2. More on these concepts below.

1: SWITCH OFF LEG STIFFNESS
Solution type 1 disengages the spring attached to the hip joint. This will either require the use of hands of other body parts, which is bad because it interferes with the users normal behaviour. Or some electronics that try to measure the users intention, which is technologically not feasible in a robust way.

2: WITHOUT ADJUSTMENTS, DIFFERENTIAL MECHANISMS TYPE
A differential mechanism can be used to divide the moment on the hip regardless of their relative position. More on differential mechanisms is in section literature review.

3: EXTENDING SKELETON TO KNEE JOINT
With the current design it is not possible to ideal supporting behaviour in all types of bending: bending using the knee, and bending using the hip, or combinations of these two.

Using the center of mass of a ‘3-link human’, see fig. C.8, and the assumption the center of mass always stays straight above the ankle joint the 3-DOF system is reduced to a 2 DOF system. So with the exoskeleton around two joints, the angle of the upper-body with respect to the vertical can be determined.

![Figure C.8: Schematic model of human to calculate relation between joint angles.](image)

Figure C.9 shows this model in different positions. It can be seen that the angle between the hip and upper-body is not proportional to the absolute lean angle, but depends on the type of bending. Figure C.10 shows a schematic illustration of a concept of exoskeleton around the knees and hip joint.
TRANSMISSIONS IN EXOSKELETONS

There are multiple reasons why transmission are used in exoskeletons. First, in order to increase the effective torque of the actuators, transmissions with high ratios are used. Second, transmissions are also used to place the actuators at a location where its volume is more practical or its weight requires less energy to move [ref required]. Third, transmission are used to couple the motion of multiple joints to create a more energy efficient system. [12] Fourthly, transmissions can be used to distribute actuator torque over multiple joints without constraining its position. This configuration of a transmission is called differential mechanism, which is a form of under-actuation [4]. In this thesis an example design of this last application is made. Table C.6 summarizes the different applications of transmissions in exoskeletons.

Table C.6: Applications of transmissions in exoskeletons

<table>
<thead>
<tr>
<th>Transmission connections</th>
<th>reason</th>
<th>Required properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint-actuator</td>
<td>increase effective torque</td>
<td>high ratio</td>
</tr>
<tr>
<td>Joint-actuator</td>
<td>relocate actuator</td>
<td>covering distance</td>
</tr>
<tr>
<td>Joint-joint</td>
<td>Transfer power between joints</td>
<td>Distance and specific ratio</td>
</tr>
<tr>
<td>Joint-joint-actuator</td>
<td>Actuate multiple joints</td>
<td>Specific ratio</td>
</tr>
</tbody>
</table>
**TRANSMISSION ROUTE**

In the differential mechanisms solution one leg moves forward while the other one moves backward. If the transmission has a ratio \( R = -1 \), an average hip angle of 0 would cause no counterforce. The hip angles and pelvic obliquity angles from literature were added to check if this is true for normal walking.

![Graph](image)

Figure C.11: The summed absolute hip angles during normal walking are not zero, but around 10° and vary by approximately 7° [†].

This small angle fluctuation is not seen as a problem because it will only cause a small counter torque.

To create a differential mechanism for the legs a transmission must be built between both hip joints. This must be done in the space that is available for the exoskeleton, which is a thin space around the body, see fig. C.12. There are two structures connecting the left and right side, the hip belt and torso structure. There are 3 ways to connect these points as shown in table C.7.

![Image](image)

Figure C.12: Physical space available for design of mechanism

<table>
<thead>
<tr>
<th>Route</th>
<th>Loading condition</th>
<th>motion required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrate with hip belt</td>
<td>tensile loading</td>
<td>Must be flexible in bending during donning and doffing</td>
</tr>
<tr>
<td>Torso structure</td>
<td>bending</td>
<td>slight motion in direction of bending load</td>
</tr>
<tr>
<td>Elsewhere, not integrated</td>
<td>no loading</td>
<td>flexible in all direction</td>
</tr>
</tbody>
</table>

**Along hip belt**

Transmission must be flexible along one axis for donning and doffing.

**Along structure**

Structure loaded in bending. Longer route Structure must open up.

The required range of motion of the transmission is determined by the transmission ratio between the transmission. This is represented by fig. C.15. A higher \( R \) means lower ROM. The angular stiffness of 20/50Nm/° is a requirement for the balancing behaviour. This sets the requirement for the stiffness of the transmission as a function of \( R \).
C.4. COMPLIANT TRANSMISSION CONCEPTS

Different solution paths for the functional requirements are explained.

**METHOD: COMBINING FUNCTIONAL ELEMENTS**

\[
\begin{bmatrix}
\frac{dF_{in}}{dF_{out}} \\
\frac{du_{in}}{du_{out}}
\end{bmatrix} = K 
\begin{bmatrix}
\frac{du_{in}}{du_{out}} \\
\frac{dF_{in}}{dF_{out}}
\end{bmatrix} = \begin{bmatrix}
\frac{k - C}{CR} & \frac{R}{C} \\
R & \frac{k}{k - C}
\end{bmatrix} 
\begin{bmatrix}
\frac{du_{in}}{du_{out}} \\
\frac{dF_{in}}{dF_{out}}
\end{bmatrix}
\]  \hspace{1cm} (C.2)

Where \( R \) is the transmission ratio of the transmission element. It is a function of the displacement, \( u_{in} \) and the force \( F_{in} \) on the transmission.

Writing the unknown outputs as a function of the known inputs results in.

\[
\begin{bmatrix}
\frac{du_{out}}{dF_{out}} \\
\frac{dF_{out}}{dF_{in}}
\end{bmatrix} = T 
\begin{bmatrix}
\frac{du_{in}}{dF_{in}} \\
\frac{du_{out}}{dF_{out}}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{R} - \frac{k}{CR} & \frac{k}{CR} \\
\frac{R}{k} & \frac{R}{k - C}
\end{bmatrix} 
\begin{bmatrix}
\frac{du_{in}}{dF_{in}} \\
\frac{du_{out}}{dF_{out}}
\end{bmatrix} \hspace{1cm} (C.3)
\]

The inverse of this equation, which expresses the input in terms of the output is.

\[
\begin{bmatrix}
\frac{du_{in}}{dF_{in}} \\
\frac{du_{out}}{dF_{out}}
\end{bmatrix} = T^{-1} 
\begin{bmatrix}
\frac{du_{out}}{dF_{out}} \\
\frac{du_{in}}{dF_{in}}
\end{bmatrix} = \begin{bmatrix}
\frac{R}{k} & \frac{R}{k - C} \\
\frac{k}{CR} & \frac{k}{CR}
\end{bmatrix} 
\begin{bmatrix}
\frac{du_{out}}{dF_{out}} \\
\frac{du_{in}}{dF_{in}}
\end{bmatrix} \hspace{1cm} (C.4)
\]

Connecting transmissions output to input creates a transmission with transmission matrix which is the product of the transmission matrices \( T_1 \) and \( T_2 \) where the subscripts denote from which.

\[
\frac{du_{in}}{dF_{in}} = T_1 T_2
\begin{bmatrix}
\frac{du_{out}}{dF_{out}} \\
\frac{du_{in}}{dF_{in}}
\end{bmatrix} = \begin{bmatrix}
\frac{R_1}{k_1} & \frac{R_1}{k_1 - C_1} \\
\frac{R_2}{k_2} & \frac{R_2}{k_2 - C_2}
\end{bmatrix} 
\begin{bmatrix}
\frac{du_{out}}{dF_{out}} \\
\frac{du_{in}}{dF_{in}}
\end{bmatrix} \hspace{1cm} (C.5)
\]

Theoretical background A small literature review is done to explore the different principles used in statically balanced compliant mechanisms. These principles can be used as building blocks for the concepts.
TOWARD ZERO STIFFNESS
In order to meet the requirement of a low counter force during motion the stiffness needs to be low. A compliant mechanism with zero stiffness in the direction of the desired motion is called an statically balanced compliant mechanism. Complete balancing of the mechanism isn't required for this design, but using this principle reduces the actuation force.

In order to achieve a statically balanced compliant mechanism, it is considered from an energy perspective. As statically balanced system has a constant potential energy. This energy is the sum of potential energy from external loadings, such as gravity, distributed pressures, and concentrated loadings and its internal elastic potential energy. If there are no external loadings, so only the internal elastic energy is considered. During deformation this elastic energy can be kept constant in two ways: Firstly, the material used has an elastic properties with no stiffness for this particular deformation. Secondly, the deformation causes some material to absorb energy while other material released energy, i.e. positive and negative stiffnesses are added. These two ways to reduce the stiffness of the mechanism are further elaborated in the next subsections.

REDUCING STIFFNESS
Materials with no or very low stiffness in the direction of deformation reduce the actuation force. There exist materials with no stiffness for certain deformation. Liquids for example have no shear stiffness. But since this is the case in all directions, i.e. the material is isotropic no Two groups of anisotropic materials are considered, metamaterials and composites.

METAMATERIALS
Meta materials are materials with properties not occurring in natural materials. Usually it are repeated small structures which properties can be described by unusual material properties. They have been created with negative Poisson ratios [13–15]. This means a tension one axis will result in an expansion of material in an axis perpendicular to it. Combining positive- and negative Poisson ratio materials can result in zero-Poisson ratio material. Materials experiencing axial deformations where the deformation in the transverse direction is constrained are stiffer if Poisson ratio is unequal to zero. So zero-Poisson ratio materials can be beneficial when axial compliance is required [16]. These negative poison ratio materials could have some nice applications in compliant mechanisms. But because of it’s fine internal structure, metamaterials in general have lower strength than normal materials. Meta materials with negative volume compressibility transitions could theoretically could exist for small stress variations [17]. This means the material undergoes contraction when tensioned and expansion when pressurized. For large stress variations these negative compressibility materials would violate thermodynamics laws. Since this material only exist in theory it can’t be applied in the design.
Figure C.15: The stiffness at the hip joint is a function of the transmission ratio between.

**Composites**

Compliant mechanisms in general need high strength and stiffness in some directions to bare the loads. But in the direction of motion a low stiffness is required. Isotropic materials have equal material properties in all directions. In contrast to anisotropic materials which have different material properties in different planes in the material. For compliant mechanisms it can be beneficial designing the material such that it is stiff in the directions where stiffness is required and compliant in the directions where compliance is required.

**Compliant Joint Classification**

**Lumped, Distributed Compliance, Rolling**

**Lumped:**
- low intrinsic stiffness
- high stiffness for tensile loading
- Good approximation of pivots with PRBM
- Low strength in shear and compression
- Low fabrication tolerances, i.e. strength is sensitive for fabrication errors.

**Distributed compliance:**
- High strength for loading shear or compression, because strain is distributed
- Higher intrinsic stiffness relative to lumped compliance.
- Due to distributed deformation infinite DOF system. The system will deform away from the load.

The extreme case of distributed compliance is when the entire mechanism deforms. If the construction material is relatively compliant for certain deformations, but stiff for others, and the deformations are coupled by some geometric constraints, the DOF of the mechanism can be one. Examples of such mechanisms are a slit thin walled tube which has one DOF related to the warping of the cross section. This emerges because the shell bending is more compliant than the membrane deformations. Double curvature (Gaussian curvature) causes induces membrane stresses. An other example is a liquid enclosed in a cylinder. The liquid is stiff in compressing, but has no stiffness in other directions.

**Rolling joint:**
- High stiffness in compressing and shear forces - low strength and stiffness for tensile and out of plane forces - high weight - Some micro slip causing minor energy losses.
- No easy fabrication methods exist
Figure C.16: Using the tendon transmission for higher structural efficiency of the torso structure.

Figure C.17: The wipwap concept can be used to cover straight lines or out of plane bends. The motion is orthogonal to the travel direction of the work.

**FUNCTIONAL SUBSYSTEMS**

A transmission can be build up from multiple functional elements connected in series. For example separate blocks can exist for: Straight line transmission,

Bent in plane
Bent out of plane
Flexible in plane
Flexible out of plane
Elongating / Shortening
Negative stiffness unit

**CONCEPT CLASSIFICATION NEGATIVE STIFFNESS**

Classification of concepts can lead to increased insight. Mapping the design space indicates what was found and what has not been found. A good classification is mutually exclusive and complete in one dimension. Moreover properties can be assigned to classes of designs, which helps searching in a certain area.

For the mechanism to have constant potential energy the energy has to decrease in another location.
Figure C.18: The wipwap in a different configuration

Figure C.19: The wipwap in a different configuration

Figure C.20: A tube with cut slits works as a torsion transmission.
C.4. COMPLIANT TRANSMISSION CONCEPTS

Figure C.21: This concept uses a slit tube with two pivots connected next to the slit. The extending beams make a motion where the endpoints rotate along a virtual pivot axis outside the mechanism.

Figure C.22: Model of transmission element
Decreasing elastic energy is only possible if it was already increased, which means pre-tensions exist in the material.

**IN PLANE PRE-STRESS**
Hair clipp, see fig. C.23.

**MATERIALS WITH IN PLANE PRE-STRESS**
Shrunk leaves.

**STRUCTURES WITH OUT OF PLANE PRE-STRESS**
**MATERIALS WITH OUT OF PLANE PRE-STRESS**

**COUPLING EFFECTS**
If flexible transmission bents when the transmission is blocked, the centroidal axis is the axis that will stay at constant length. If a force is transmitted through the transmission this is done by forces between the rigid bodies. These forces can deform the centroid axis, which was called coupling. A classification is made regarding the orientation of this axis with respect to the central axis the transmission.

Table C.8: Bending of centroidal axis.

<table>
<thead>
<tr>
<th>Direction of forces wrt centroidal axis</th>
<th>Coupling effect on centroidal axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-linear axis</td>
<td>Bending moment yes no</td>
</tr>
<tr>
<td>Perpendicular axis through centroidal axis</td>
<td>no          no</td>
</tr>
<tr>
<td>Skew axis through centroidal axis</td>
<td>no          no</td>
</tr>
<tr>
<td>Perpendicular axis offset from centroidal axis</td>
<td>no       yes</td>
</tr>
<tr>
<td>Parallel axis offset from centroid axis</td>
<td>yes         no</td>
</tr>
<tr>
<td>Skew and offset axis from centroid axis</td>
<td>yes         yes</td>
</tr>
</tbody>
</table>

So it appears that there is no coupling effect if the forces acting between the bodies are act through the centroid axis the transmission.

Transmission with no coupling won't have the tendency to bent or twist when loaded. So it can be flexible without consequences.

On the other hand the advantage of the coupling could be used. If the transmission is integrated with the torso structure it is loaded in bending and torsion. The coupling effect can prevent excessive bending.
C.4. COMPLIANT TRANSMISSION CONCEPTS

Figure C.24: Composite tape springs can be constructed from multiple layers. If the start and end layer are constructed in a staircase like manner where each step is a ply drops and of equal length and thickness the stairs can be stacked together resulting no thickness increase and thus no stiffness increase.

CLOSING THE LOOP
To test the model tape loop samples must be made. A number of requirements:

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Composite thermoplastic</th>
<th>Composite thermoset</th>
<th>Welded carpenter tape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength</td>
<td>best</td>
<td>good</td>
<td>low</td>
</tr>
<tr>
<td>Smoothness</td>
<td>?</td>
<td>?</td>
<td>good</td>
</tr>
<tr>
<td>Predictability</td>
<td>worst</td>
<td>medium</td>
<td>good</td>
</tr>
<tr>
<td>Production effort</td>
<td>hard</td>
<td>hard</td>
<td>easy</td>
</tr>
</tbody>
</table>

CLAMPING OF THE TAPE LOOP
- Full clamping -> Axial stiffness due to transition zone
- Compensate axial stiffness with negative stiffness by locally reducing the width.
- Overlap -> Moderate increased axial stiffness due to transition zone
- Compensate axial stiffness with negative stiffness by locally reducing the width.
- Smooth end to end connection ->
- One piece production with pretension
- Smoothly connecting end using fib
The dimensions and loads can be set as input. This file runs the optimization algorithm to find the stable positions. This file uses the constraints used in the experiment.

**m-file: Coupling_experiment2.m**

```matlab
close all
clear all
clc

% Inputs
global F0 ROM x0 y0 L phi k2 dispcon loads pos0 D EI
lclamp=32e-3; % assumed same length as width

% Geometry: Distance and angles setup
% Gamma tape springs
alpha=45/180*pi; % angle between tape loops

% offset=[Pulley offset in, D0+1 D0-1 D0+2 D0-2 pulley offset out]
offset=[29 34 34 34 34 29]*1e-3; % offset wire from ground
% initial diameter
D=[32 32 32 32 32 32]*1e-3; % Initial diameter tape loops [m]
L1=70.7e-3; % length of straight part of first unloaded tape spring
L2=71.2e-3; % length of straight part of second unloaded tape spring

% measures cartesian coordinates setup 2
Lin=157e-3;
x1=51e-3;
y1=108e-3;
ang1=pi/2;
x2=93.4e-3;
y2=232e-3;
ang2=pi+alpha;
x3=158e-3;
y3=334e-3;
Lout=142e-3; % 142;

phi=[pi/2 ang1 ang2 ang2 -pi/2];
ROM=[Inf L1-lclamp L1-lclamp L2-lclamp L2-lclamp Inf]/2;
ROM=ROM;
% h1hmax=(L1-lclamp)/2-D(4)/sin(alpha);

% dimensions of tape spring
E=207e9;
nuxy=0.3;
t=0.118e-3;
b=25e-3;
```
\texttt{b=ones(1,length(D))*b;}
\texttt{D11=t^3*E/(1-nuxy^2)/12;}
\texttt{D22=D11;}
\texttt{Ry=D/2;}
\texttt{EI=D11*b;}

\% stiffness \texttt{F = -F0 + k2/Rx^2}
\texttt{F0=b*pi/4.*D22./Ry.^2; \% N}
\texttt{k2=b*pi/4.*D11; \% N m^-2}

\% loads
\texttt{tmax=20; \% nr of loadsteps, load is applied in linear steps}

\% displacement constraints , use uin/2, uout/2
\texttt{ux1=0;}
\texttt{uy1=-14.5e-3;}
\texttt{uxn=0;}
\texttt{uyn=0e-3;}
\texttt{dispcon_last1=[ux1,uy1]; \% only first and last node can have displacement constraint}
\texttt{dispcon_last1=[ux1,uy1,uxn];}
\texttt{dispcon_last1=[ux1,uy1,uxn,uyn];}
\texttt{Fxn=0; \% 9.60 3.23 1.03 for transmission experiments}
\texttt{loads_last1=[0 0 0 0 0 0 0 0 0 0 Fxn Fyn]; \% loads vector last loadstep}

\%step 2
\texttt{step2=1; \% for 1 execute step 2, for 0 skip step2}
\texttt{ux1=0;}
\texttt{uy1=-9e-3;}
\texttt{uxn=0;}
\texttt{uyn=0;}
\texttt{dispcon_last2=[ux1,uy1]; \% only first and last node can have displacement constraint}
\texttt{dispcon_last2=[ux1,uy1,uxn];}
\texttt{dispcon_last2=[ux1,uy1,uxn,uyn];}
\texttt{force loads on last node}
\texttt{weight=0;}
\texttt{Fxn=cos(phi(end))*weight; \%[N] \%endpoint load}
\texttt{Fyn=sin(phi(end))*weight; \%[N] \%endpoint load}
\texttt{loads_last2=[0 0 0 0 0 0 0 0 0 0 Fxn Fyn]; \% loads vector last loadstep}

\% calculate initial coordinates (starting point)
\% From coordinates measurement setup 2
\texttt{groundx(2)=x1-cos(ang1)*L1/2;}
\texttt{groundx(3)=x1+cos(ang1)*L1/2;}
\texttt{groundy(2)=y1-sin(ang1)*L1/2;}
\texttt{groundy(3)=y1+sin(ang1)*L1/2;}
\texttt{groundx(4)=x2+cos(ang2)*L2/2;}
\texttt{groundx(5)=x2-cos(ang2)*L2/2;}
\texttt{groundy(4)=y2+sin(ang2)*L2/2;}
\texttt{groundy(5)=y2-sin(ang2)*L2/2;}
\texttt{groundx(1)=groundx(2);}
\texttt{groundy(1)=groundy(2)-Lin;}
\texttt{groundx(6)=groundx(5);}
\texttt{groundy(6)=groundy(5)+Lout;}

\texttt{for \ i=[1:6]}
\texttt{x0(i)=groundx(i)-offset(i)*sin(phi(i));}
\texttt{y0(i)=groundy(i)+offset(i)*cos(phi(i));}
\texttt{end}

\% initial state vector \texttt{pos=[x1 y1 x2 y2 ... xn yn]}
\texttt{pos0(1:2:length(x0)+2-1)=x0;}
\texttt{pos0(2:2:length(x0)+2)=y0;}

\texttt{d0=ones(length(pos0));}
% L=length of constraints
L=[0 0 0 0 0];
{c,ceq,DC,DCeq} =confungrad3(pos0);
L=ceq(1:5);

% keyboard

%% Draw initial shape
figure
plot(x0,y0)
hold on
plot(x0,y0,'.')
plot(groundx,groundy, 'k--','linewidth',1);
plot(groundx([2:3]),groundy([2:3]),'k','linewidth',2);
plot(groundx([4:5]),groundy([4:5]),'k','linewidth',2);

% circle centerpoint
cx1=x0+sin(phi).*(+D/2);% offset-D
cy1=y0-cos(phi).*(+D/2);% offset-D
draw circles
tt=1:101;
for ii=2:length(D)-1
    xc=cx1(ii)+cos(tt/100*2*pi)*D(ii)/2;
    yc=cy1(ii)+sin(tt/100*2*pi)*D(ii)/2;
    plot(xc,yc, 'k')
end
axis('equal')

%% draw energy field around initial shape
valleywidth=3;
for ii=2:length(x0)-1
    width=D(ii)*valleywidth; % width of area to be plotted
    long=2.5*ROM(ii);% length of area to be plotted, just within the range of motion
    gridsize=0.01*ROM(ii);
    %create empty matrices
    P=zeros(floor(long/gridsize),floor(width/gridsize));
    x=zeros(floor(long/gridsize),floor(width/gridsize));
    y=zeros(floor(long/gridsize),floor(width/gridsize));
    pos=pos0;
    for jj=1:floor(long/gridsize)
        for kk=1:floor(width/gridsize)
            x(jj,kk)=x0(ii)+cos(phi(ii))*(-long/2+gridsize*jj)...
                -sin(phi(ii))*(-width/(valleywidth*1.2)+kk*gridsize);
            y(jj,kk)=y0(ii)+cos(phi(ii))*...
                (-width/(valleywidth*1.2)+gridsize*kk)...
                sin(phi(ii))*(-long/2+jj*gridsize);
            pos(1+2*(ii-1))=x(jj,kk);
            pos(2+2*(ii-1))=y(jj,kk);
            P(jj,kk) = summedU(pos);
        end
    end
end
figure(2)
hold on
surf(x*1000,y*1000,P, 'EdgeColor', 'none', 'LineStyle', 'none')
% contour(x,y,P)
end

% plot3(groundx*1000,groundy*1000,ones(1,length(groundx))\+le-4,'k','linewidth',2);
plot3(groundx([2:3])\+le3,groundy([2:3])\+le3,ones(1,2)\+le-4,'k','linewidth',4);
plot3(groundx([4:5])\+le3,groundy([4:5])\+le3,ones(1,2)\+le-4,'k','linewidth',4);
xlabel('x-coordinate [mm]')
ylabel('y-coordinate [mm]')
%save figure for reopening later
savefig('energyplot.fig')

z0=ones(1,length(x0))\+le-3;
plot3(x0*1000,y0*1000,z0,'k','linewidth',2)
plot3(x0*1000,y0*1000,z0,'ko','linewidth',4)

% include circles

\begin{verbatim}
tt=1:101;
for ii=2:length(D)-1
    xc=cx1(ii)+cos(tt/100*2*pi)*D(ii)/2;
    yc=cy1(ii)+sin(tt/100*2*pi)*D(ii)/2;
    plot3(xc*1000,yc*1000,ones(1,length(xc))*1e-2,'k')
end
axis('equal')
\end{verbatim}

% keyboard

%% SOLVE: minimize energy with constraint non linear optimization algritm

options = optimoptions(@fmincon,'Algorithm','sqp');
% use gradients with Sequential quadratic programming
options = optimoptions(options,'GradObj','on','GradConstr','on');
% use analytical gradients
options = optimoptions(options,'TolX',1e-10);
options = optimoptions(options,'TolCon',1e-6);
% options = optimoptions(options,'DerivativeCheck','on');
% options = optimoptions(options,'Diagnostics','on');
lb = []; ub = []; % No upper or lower bounds %

%loadstep 1
pos=zeros(tmax,length(pos0));
posstart=pos0;
[Alin,blin]=linineqconstraint(pos0,phi,ROM); % inequality constraint for end of ROM

for t=1:tmax
    loads=loads_last1*(t-1)/(tmax-1);
    dispcon=dispcon_last1*(t-1)/(tmax-1);
    if step2
        dispconrel=pos(end,[1:2,end-1:end])-pos0([1:2,end-1:end]);
    end
    if EXITFLAG
        EXITFLAG
    end
    \begin{verbatim}
    [pos(t,:),FVAL(t),EXITFLAG(t),OUTPUT,LAMBDA{t},grad(:,t)] =...
    fmincon(@summedU,posstart,[],[],[],[],lb,ub,@confungrad3,options);
    posstart=pos(t,:);
    if EXITFLAG(t)<0||EXITFLAG(t)>2
        disp('optimizer failed')
        break
    end
    \end{verbatim}
end
EXITFLAG

if step2
    dispconrel=pos(end,[1:2,end-1:end])-pos0([1:2,end-1:end]);
    % add displacement in 1st step with new displacement

    \begin{verbatim}
    for t=1:tmax
        loads=loads_last2*(t-1)/(tmax-1);
        dispcon=dispcon_last2*(t-1)/(tmax-1);
        % new displacement constraint is applied in
        % tmax steps relative to old dispcon
        [pos(t,:),FVAL(t),EXITFLAG(t),OUTPUT,LAMBDA{t},grad(:,t)] =...
        fmincon(@summedU,posstart,[],[],[],[],lb,ub,@confungrad3,options);
        posstart=pos(t,:);
        if EXITFLAG(t)<0||EXITFLAG(t)>2
            disp('optimizer failed')
        end
    end
    \end{verbatim}
end
EXITFLAG
end

% All algorithms:
% 1 First order optimality conditions satisfied.
% 0 Too many function evaluations or iterations.
% -1 Stopped by output/plot function.
% -2 No feasible point found.
% Trust-region-reflective, interior-point, and sqp:
% 2 Change in X too small.
% Interior-point and sqp:
% -3 Problem seems unbounded.

%% Animate results over time

h = figure(3);
Frame(tmax) = struct('cdata',[],'colormap',[]);
for t=1:tmax

hold off
plot(pos(t,1:2:end-1),pos(t,2:2:end),'k','linewidth',2)
hold on
plot(pos(t,1:2:end-1),pos(t,2:2:end),'ok','linewidth',2)

% circle centerpoint
Diam=sin(phi).*(-pos(t,1:2:length(pos0))+x0)+cos(phi).*(pos(t,2:2:length(pos0))-y0)+D;
% deformed diameter
cx1=pos(t,1:2:end-1)+sin(phi).*(Diam/2);%
cy1=pos(t,2:2:end)-cos(phi).*(Diam/2);

% plot ground
plot3(groundx,groundy,6e-4*ones(1,length(groundx)),'k','linewidth',2)
plot(groundx([2,3]),groundy([2,3]),'k','linewidth',4)
plot(groundx([4,5]),groundy([4,5]),'k','linewidth',4)

%draw circles
tt=1:101;
for ii=2:length(Diam)-1
   xc=cx1(ii)+cos(tt/100*2*pi)*Diam(ii)/2;
   yc=cy1(ii)+sin(tt/100*2*pi)*Diam(ii)/2;
   plot3(xc,yc,6e-3 *ones(1,101),'color',[0 0 0])
   plot(xc,yc,'color',[255,140,0]/255,'linewidth',3)
   circles(cx1(ii),cy1(ii),Diam(ii)/2,'facecolor','yellow')
end
axis([-.15 0 0 .35 -0.1 0.5]) %('equal')

pause(0.3)
% delete(g);
hold off
Frame(t) = getframe;
end

% movie(Frame,2) %play movie twice

% safe movie to file
myVideo = VideoWriter('simulatiefilm.avi');
myVideo.FrameRate = floor(tmax/5); % set framerate so movie takes approx 5 sec
open(myVideo);
writeVideo(myVideo, Frame);
close(myVideo);

% keyboard
%% Extract Results
for t=1:tmax
   [c,ceq,DC,DCeq]=confungrad3(pos(t,:));
   Constraintforces(:,t)=LAMBDA{t}.eqnonlin; % wire forces
   Constraintforcesxy(:,t)=DCeq*LAMBDA{t}.eqnonlin; % reaction forces on coordinates.
end
% keyboard
% keyboard
F1x=Constraintforces(length(L)+1,:); % 1:2 ratio between node and cable displacement
% F1x=Constraintforces(length(L)+2,:); % constraint force on node1
F1y=Constraintforces(1,:); % Constraint force in 1st wire
v1=(pos(:,2)-pos0(2))*2000;
vout=(pos(:,end)-pos0(end))*2000;

%plot x-displacement and x force
figure
plot((pos(:,end-1)-pos0(end-1))*2000,loads(end)*[0:tmax-1]/(tmax-1))
xlabel('displacement')
ylabel('force [N]')
legend('Model')
title('On end clamped, 2 tapes 90 ° turn')

% plot elastic energy
Pext=(pos*loads'-pos0*loads')'.*([0:tmax-1]/(tmax-1)); % potential of external load
figure(12)
plot(loads(end)*[0:tmax-1]/(tmax-1),FVAL+Pext)
xlabel('Applied load [N]')
ylabel('Elastic energy [J]')
legend('Model')
title('On end clamped, 2 tapes 45 ° turn')

% normal forces at TS guide
for t=1:tmax
N(1,t)=sqrt(grad(3,t).^2+grad(4,t).^2);
N(2,t)=sqrt(grad(5,t).^2+grad(6,t).^2);
N(3,t)=sqrt(grad(7,t).^2+grad(8,t).^2);
N(4,t)=sqrt(grad(9,t).^2+grad(10,t).^2);
Nmax(t)=max(N(:,t)); %[N]
end

figure
hold on
plot(loads(end)*[0:tmax-1]/(tmax-1),N(1,:), 'linewidth',2)
plot(loads(end)*[0:tmax-1]/(tmax-1),N(2,:), 'linewidth',2)
plot(loads(end)*[0:tmax-1]/(tmax-1),N(3,:), 'linewidth',2)
plot(loads(end)*[0:tmax-1]/(tmax-1),N(4,:), 'linewidth',2)
legend('guide1','guide2','guide3','guide4')
xlabel('Applied force')
ylabel('Flexure load')
figure
plot(v1,F1y)
xlabel('Input displacement [mm]')
ylabel('Force [N]')
title('Stiffness of transmission')

figure
plot(v1,vout)
xlabel('Input displacement [mm]')
ylabel('output displacement [mm]')
title('Stiffness of transmission')
This file calculates the potential elastic energy of the bends as a function of the coordinate vector $X$, which is called \textit{pos} in this file.

\textbf{m-file: \textit{summedU.m}}

```matlab
function \[P,dP\] = summedU(pos)
% input position and properties of tape spring element,
% output summed elastic energy
global F0 k2 ROM phi x0 y0 loads pos0
x=pos(1:2:end-1);
y=pos(2:2:end);
% convert xy position to local coordinate system u v
glob=(x-x0)+1i*(y-y0);  % displacements in complex plane
loc=exp(-1i.*phi).*glob; % rotated (local) coordinates
u=real(loc);
v=imag(loc);
% Calculate internal elastic energy and derivatives
D0=sqrt(k2./F0);
D=v+D0;
% if max(abs(u))>ROM/2
% disp('outside of range of motion')
% end
D=v+D0;
U0=k2.*1./D0;
U=k2.*1./D+F0.*v-U0; % internal elastic energy, initial position set to 0
dPdv=-k2.*1./D.^2+F0; % Reaction force=0 for D=D0 or v=0
% use energy peak for
for \[dd=1:length(D)\]
% check for each node if it crossed the centerline
if D(dd)<=0
\[dPdu=0;\]
\[dPdv(dd)=-1000;\] % Steep gradient starting at centerline
\[U(dd)=-1000*v(dd);\] % huge penalty for crossing centerline
end
end
P=sum(U(2:end));
%Potential energy from external loads
if length(loads)~=0
P=P-sum((pos-pos0).*loads);
end
if nargout > 1
% keyboard
% Convert derivatives to global system
\[dPloc=dPdu+1i*dPdv;\]
\[dPglob=exp(1i*phi).*dPloc;\]
\[dPdx=real(dPglob);\]
\[dPdy=imag(dPglob);\]
\[dP(1:2:2*length(dPdx)-1)=dPdx;\]
\[dP(2:2:2*length(dPdy))=dPdy;\]
%first and last node are not connected to tape loop
\[dP(1)=0;\]
\[dP(2)=0;\]
\[dP(end-1)=0;\]
\[dP(end)=0;\]
\[dP=dP-loads;\]
end
```
function [c,ceq,DC,DCeq] = confungrad(loc)

% function gives constraint violations and derivatives thereof as function
% of the state vector, loc.
% there is no inequality constraint c and DC
% the equality constraints are different for the wires and loops.
% Special constraints are used for the first wire input, loops, wire bridge
% on the left side and wire exit on the left side.
% Displacement constraints are added to the constraint vector

global L dispcon x0 y0 D phi
x=loc(1:2:end-1);
y=loc(2:2:end);
c =[]; % Inequality constraints
ceq=zeros(1,length(L)+length(dispcon));
DC= [];
DCeq = zeros(length(loc),length(L)+length(dispcon)); % define size equality constraint vector
% rolling wire constraints

for tt=1:2:length(L)
    % rename variables, use two subsequent nodes
    x1=x(tt);
    x2=x(tt+1);
    y1=y(tt);
    y2=y(tt+1);
    x10=x0(tt);
    x20=x0(tt+1);
    y10=y0(tt);
    y20=y0(tt+1);
    D01=D(tt);
    D02=D(tt+1);
    Lwire=L(tt);
    phi1=phi(tt);
    phi2=phi(tt+1);

    % use function with constraint equations
    if tt==1 % input wire to loop
        [ceq(tt), DCeq(-1+tt*2:2+tt*2,tt)]=Constraineq_wire_left_end1(x1, x10, y1, y10,...
            x2, x20, y2, y20, D01, D02, Lwire, mod(phi1+pi,2*pi)-pi,...
            mod(phi2+pi,2*pi)-pi);
    elseif tt==3 % if the wires two loops on the left side
        [ceq(tt), DCeq(-1+tt*2:2+tt*2,tt)]=Constraineq_num(x1, x10,...
            y1, y10, x2, x20, y2, y20, D01, D02, Lwire,...
            mod(phi1+pi,2*pi)-pi, mod(phi2+pi,2*pi)-pi);
    elseif tt==5 % loop to wire output
        [ceq(tt), DCeq(-1+tt*2:2+tt*2,tt)]=Constraineq_wire_left_end2(x1, x10, y1, y10,...
            x2, x20, y2, y20, D01, D02, Lwire, mod(phi1+pi,2*pi)-pi,...
            mod(phi2+pi,2*pi)-pi);
    else % loop constraints
        [ceq(tt), DCeq(-1+tt*2:2+tt*2,tt)]=Constraineq_num(x1, x10,...
            y1, y10, x2, x20, y2, y20, D01, D02, Lwire, mod(phi1+pi,...
            2*pi)-pi, mod(phi2+pi,2*pi)-pi);
    end
end

% keyboard
end

end

% loop constraints
for tt=2:2:length(L)
    % rename variables, use two subsequent nodes
    x1=x(tt);
    x2=x(tt+1);
    y1=y(tt);
    y2=y(tt+1);
    x0=x0(tt);
    x20=x0(tt+1);
y10 = y0(tt);
y20 = y0(tt+1);
D01 = D(tt);
D02 = D(tt+1);
Lloop = L(tt);
phi1 = phi(tt);
phi2 = phi(tt+1);

% use function with constraint equations
[ceq(tt), DCeq(-1+tt+2:2+tt+2, tt)] = Constraineq_loop(x1, x10,...
y1, y10, x2, x20, y2, y20, D01, D02, Lloop, phi1, phi2);

end

if length(dispcon) == 0
elseif length(dispcon) == 2
    ceq(end-1:end) = [loc(1)-x0(1)-dispcon(1) loc(2)-y0(1)-dispcon(2)];
    DCeq(1, length(L)+1) = 1;  % constraint of node 1 in x dir
    DCeq(2, length(L)+2) = 1;  % constraint of node 1 in y dir
elseif length(dispcon) == 3
    ceq(end-2:end) = [loc(1)-x0(1)-dispcon(1) loc(2)-y0(1)-... 
                      dispcon(2) loc(end-1)-x0(end)-dispcon(3)];
    DCeq(1, length(L)+1) = 1;  % constraint of node 1 in x dir
    DCeq(2, length(L)+2) = 1;  % constraint of node 1 in y dir
    DCeq(end-1, length(L)+3) = 1;  % constraint of end node in x dir
elseif length(dispcon) == 4
    ceq(end-3:end) = [loc(1)-x0(1)-dispcon(1) loc(2)-y0(1)-... 
                      dispcon(2) loc(end-1)-x0(end)-dispcon(3) loc(end)-
                      y0(end)-dispcon(4)];
    DCeq(1, length(L)+1) = 1;  % constraint of node 1 in x dir
    DCeq(2, length(L)+2) = 1;  % constraint of node 1 in y dir
    DCeq(end-1, length(L)+3) = 1;  % constraint of end node in x dir
    DCeq(end, length(L)+4) = 1;  % constraint of end node in y dir
end

if length(dispcon) == 0
elseif length(dispcon) == 2
    ceq(end-1:end) = [loc(1)-x0(1)-dispcon(1) loc(2)-y0(1)-dispcon(2)];
    DCeq(1, length(L)+1) = 1;  % constraint of node 1 in x dir
    DCeq(2, length(L)+2) = 1;  % constraint of node 1 in y dir
elseif length(dispcon) == 3
    ceq(end-2:end) = [loc(1)-x0(1)-dispcon(1) loc(2)-y0(1)-... 
                      dispcon(2) loc(end-1)-x0(end)-dispcon(3)];
    DCeq(1, length(L)+1) = 1;  % constraint of node 1 in x dir
    DCeq(2, length(L)+2) = 1;  % constraint of node 1 in y dir
    DCeq(end-1, length(L)+3) = 1;  % constraint of end node in x dir
elseif length(dispcon) == 4
    ceq(end-3:end) = [loc(1)-x0(1)-dispcon(1) loc(2)-y0(1)-... 
                      dispcon(2) loc(end-1)-x0(end)-dispcon(3) loc(end)-y0(end)-dispcon(4)];
    DCeq(1, length(L)+1) = 1;  % constraint of node 1 in x dir
    DCeq(2, length(L)+2) = 1;  % constraint of node 1 in y dir
    DCeq(end-1, length(L)+3) = 1;  % constraint of end node in x dir
    DCeq(end, length(L)+4) = 1;  % constraint of end node in y dir
else
    disp('dispcon should have length 0 2 3 or 4')
end

end
%% symbolic derivation constraint equations for the loops
clear all
syms x1 x10 y1 y10 x2 x20 y2 y20 D01 D02 Lloop phi1 phi2

%local coordinates
u1=cos(phi1)*(x1-x10)+sin(phi1)*(y1-y10);
v1=cos(phi1)*(y1-y10)-sin(phi1)*(x1-x10);
u2=cos(phi2)*(x2-x20)+sin(phi2)*(y2-y20);
v2=cos(phi2)*(y2-y20)-sin(phi2)*(x2-x20);

R1=(D01+v1)/2;
R2=(D02+v2)/2;
C=R1*pi+R2*pi; % length of the 2 bent sections
l=sqrt((x1-x2)^2+(y1-y2)^2); % distance between nodes

%constraint equation
ceq=C+l-Lloop;
ceq=simplify(ceq);

% derivatives
dCdx1=diff(ceq,x1);
dCdy1=diff(ceq,y1);
dCdx2=diff(ceq,x2);
dCdy2=diff(ceq,y2);
%% symbolic derivation constraint equations with wire crossing

clear all

syms x1 x10 y1 y10 x2 x20 y2 y20 D01 D02 Lwire phi1 phi2

%local coordinates

u1 = cos(phi1)*(x1-x10)+sin(phi1)*(y1-y10);
v1 = cos(phi1)*(y1-y10)-sin(phi1)*(x1-x10);
u2 = cos(phi2)*(x2-x20)+sin(phi2)*(y2-y20);
v2 = cos(phi2)*(y2-y20)-sin(phi2)*(x2-x20);

l1 = -u1 -v1*pi/2;  
l5 = -u2-v2*pi/2;  

R1 = (D01+v1)/2;  
R2 = (D02+v2)/2;

%circle center

cx1 = x1 + sin(phi1)*R1;  
%+l1*cos(phi1);
cy1 = y1 - cos(phi1)*R1;  
%+l1*sin(phi1);

cx2 = x2 + sin(phi2)*R2;  
%-l5*cos(phi2);
cy2 = y2 - cos(phi2)*R2;  
%-sin(phi2)*l5;

c = sqrt((cy2-cy1)^2+(cx2-cx1)^2);  
% distance between centerpoints of circles

Betta = atan2((cy2-cy1),(cx2-cx1));  
% global angle between centers of circles
alpha = asin((R2+R1)/c);  
% angle tangent line wrt centerline circles
theta = Betta-alpha;  
% angle between tangent line and horizontal

l3 = sqrt(c^2-(R1+R2)^2);  
% length tangent line

l2 = R1*(phi1-theta);  
% length of arc section on left circle
l4 = R2*(-theta+phi2+pi);  
% length of arc section on right circle.

%constraint equation

ceq = l1+l2+l3+l4+l5-Lwire;

ceq = simplify(ceq);

% derivatives

dCdx1 = diff(ceq,x1);
dCdy1 = diff(ceq,y1);
dCdx2 = diff(ceq,x2);
dCdy2 = diff(ceq,y2);

% dCdx1 = simplify(dCdx1);
%% symbolic derivation constraint equations tendon single side
clear all
syms x1 x10 y1 y10 x2 x20 y2 y20 D01 D02 Lwire phi1 phi2

% local coordinates
u1=cos(phi1)*(x1-x10)+sin(phi1)*(y1-y10);
v1=cos(phi1)*(y1-y10)-sin(phi1)*(x1-x10);
u2=cos(phi2)*(x2-x20)+sin(phi2)*(y2-y20);
v2=cos(phi2)*(y2-y20)-sin(phi2)*(x2-x20);
l1=-u1 -v1*pi/2;
l5=u2-v2*pi/2;
R1=(D01+v1)/2;
R2=(D02+v2)/2;

% circle center
cx1=x1+sin(phi1)*R1;
cy1=y1-cos(phi1)*R1;
cx2=x2+sin(phi2)*R2;
cy2=y2-cos(phi2)*R2;

% angles
Betta=atan2((cy2-cy1),(cx2-cx1));
alpha=asin((R2-R1)/sqrt((cy2-cy1)^2+(cx2-cx1)^2));
theta=Betta+alpha;
l3=sqrt((cy2-cy1)^2+(cx2-cx1)^2-(R1-R2)^2);
l2=R1*(phi1-theta);
l4=R2*(theta-phi2);

% constraint equation
ceq=l1+l2+l3-Lwire;

% derivatives
dCdx1=diff(ceq,x1);
dCdy1=diff(ceq,y1);
dCdx2=diff(ceq,x2);
dCdy2=diff(ceq,y2);

% dCdx1=simplify(dCdx1);
Function used for optimization. This runs the model with the parameters given as input by the optimization.

```matlab
function [P,F,L,y,uout, Nmax, EXITFLAG] = Coupling_function(alpha, Lloop,Radius, uin, stroke,tmax)

%% Inputs
global F0 ROM x0 y0 l phi k2 dispcon loads pos0 D EI
lclamp=32e-3; % fixed value

% Geometry: Distance and angles setup
% gamma tape loops
% % lengths
Lwirein=300e-3; % length of input wire (pulley-loop clamp)
L1=Lloop; % length of straight part of unloaded tape spring
hh1=Radius*tan(alpha/2)-Lloop/2; % distance from halfway Tape loop to corner
hh2=Radius*tan(alpha/2)-Lloop/2; % distance from halfway Tape loop to corner
L2=Lloop; % length of straight part of unloaded tape spring
Lwireout=300e-3; % length of output wire (pulley-loop clamp)

% offset = [Pulley offsetin, D0+1 D0-1 D0+2 D0-2 pulley offset out]
oopponent = [29 34 34 34 34 29]*1e-3; % offset wire from ground
% % initial diameter
D=[32 32 32 32 32 32]*1e-3; % Initial diameter tape loops [m]
% % dimensions of tape spring
E=207e9;
t=0.118e-3;
nuxy=0.3;
b=25e-3;
b=b*ones(1,length(D))*b;
D11=t^3*E/12/(1-nuxy^2);
D22=D11;
Ry=D/2;
EI=D11*b;
% % stiffness F = -F0 + k2/Ry^2
F0=b*pi/4.*D22./Ry.^2; % N
k2=b*pi/4.*D11; % N m^2

%% convert geometry data
L0=[Lwirein-L1/2 L1 hh1-L1/2 hh2-L2/2 L2 Lwireout-L2/2];
R=hh1/tan(alpha/2); % % angle nth caterpillar wire wrt horizontal
phi=[90 90 90-alpha*180/pi 90-alpha*180/pi 90-alpha*180/pi]/180*pi;
% % angle nth line wrt horizontal
ROM=[1000 L1-lclamp L1-lclamp L2-lclamp L2-lclamp 1000];

%% loads
% tmax=40; % nr of loadsteps, load is applied in linear steps
% Wire pulling at input pulley
Pulleyinput1=[0,ones(1,tmax-1)]/(tmax-1)+0e-3/2;
Pulleyoutput1=[0,ones(1,tmax-1)]/(tmax-1)+0e-3/2;
% % displacement constraints
ux1=0;
uy1=uin;
uxn=0;
uy=0;
dispcon_last1=[ux1,uy1]; % only first and last node can have displacement constraint
% dispcon_last1=[ux1,uy1,uxn,uyn];

% step 2
```
% for 1 execute step, for 0 skip step
step2=1;

% Wire pulling at input pulley
Pulleyinput2=[0,ones(1,tmax-1)]/(tmax-1)*0e-3/2;
Pulleyoutput2=[0,ones(1,tmax-1)]/(tmax-1)*0e-3/2;

ux1=0;
uy1=-stroke;
uxn=0;
uyn=0;

% dispcon_last2=[ux1,uy1]; % only first and last node can have displacement constraint
dispcon_last2=[ux1,uy1,uxn,uyn];
% force loads on last node
weight=0;
Fxn=cos(phi(end))*weight; % [N] % endpoint load
Fyn=sin(phi(end))*weight; % [N] % endpoint load
loads_last=[0 0 0 0 0 0 0 0 0 0 Fxn Fyn]; % loads vector last loadstep

%% calculate initial coordinates (starting point)
groundx(1)=0;
groundy(1)=0;
for i=1:length(L0)
    groundx(i+1)=groundx(i)+L0(i)*cos(phi(i));
    groundy(i+1)=groundy(i)+L0(i)*sin(phi(i));
end

for i=[1 2 3]
    % starting point
    x0(i)=groundx(i)-offset(i)*sin(phi(i));
    y0(i)=groundy(i)+offset(i)*cos(phi(i));
end

for i=[ 5 6 7]
    % starting point
    x0(i-1)=groundx(i)-offset(i-1)*sin(phi(i-1));
    y0(i-1)=groundy(i)+offset(i-1)*cos(phi(i-1));
end

% initial state vector pos=[x1 y1 x2 y2 ... xn yn]
pos0(1:2:length(x0)+2-1)=x0;
pos0(2:2:length(x0)+2)=y0;

% L=length of constraints
L=[0 0 0 0 0];
[c,ceq,DC,DCeq] =confungrad2(pos0);
L=ceq(1:5);

%% minimize energy with constraint non linear optimization algritm
options = optimoptions(@fmincon,'Algorithm','sqp'); % use gradients with Sequential quadratic programming
options = optimoptions(options,'GradObj','on','GradConstr','on'); % use analitical gradients
options = optimoptions(options,'TolX',1e-10);
options = optimoptions(options,'TolCon',1e-6);

lb = []; ub = []; % No upper or lower bounds
% loadstep 1
pos=zeros(tmax,length(pos0));
posstart=pos0;
[Alin,blin]=linineqconstraint(pos0,phi,ROM); % linear inequality constraint for end of ROM
for t=1:tmax
    loads=loads_last*(t-1)/(tmax-1);
dispcon=dispcon_last1*(t-1)/(tmax-1);
    L(1)=L(1)-Pulleyinput1(t);
    L(5)=L(5)-Pulleyoutput1(t);
    [pos(t,:),FVAL(t),EXITFLAG(t),OUTPUT,LAMBDA{t},grad(:,t)] =...
        fmincon(@(summedU,posstart,Alin,blin,[],[],lb,ub,confungrad,options);
posstart=pos(t,:);
    if EXITFLAG~=1
        disp('optimizer failed')
    end
end
if step2
    dispconrel=pos(end,[1:2,end-1:end])-pos0([1:2,end-1:end]);
    % start and end node displacement in 1st step

    % loadstep 2
    for t=1:tmax
        loads=loads_last*(t-1)/(tmax-1);
        dispcon=dispcon_last2*(t-1)/(tmax-1)+dispconrel;
        L(1)=L(1)-Pulleyinput2(t);
        L(5)=L(5)-Pulleyoutput2(t);
        [pos(t,:),FVAL(t),EXITFLAG(t),OUTPUT,LAMBDA{t},grad(:,t)] =...
            fmincon(@summedU,posstart,Alin,blin,[],[],lb,ub,@confungrad,options);
        posstart=pos(t,:);
    end

    %% Extract Results
    for t=1:tmax
        [c,ceq,DC,DCeq]=confungrad2(pos(t,:));
        Constraintforces(:,:,t)=LAMBDA{t}.eqnonlin; % wire forces
        Constraintforcesxy(:,:,t)=DCeq*LAMBDA{t}.eqnonlin; % reaction forces on coordinates.
    end

    % keyboard
    F1x=Constraintforces(length(L)+1,:); % 1:2 ratio between node and cable displacement
    Fly=Constraintforces(length(L)+2,:);

    % normal forces at tape loop nodes
    for t=1:tmax
        N(1,t)=sqrt(grad(3,t).^2+grad(4,t).^2);
        N(2,t)=sqrt(grad(5,t).^2+grad(6,t).^2);
        N(3,t)=sqrt(grad(7,t).^2+grad(8,t).^2);
        N(4,t)=sqrt(grad(9,t).^2+grad(10,t).^2);
        Nmax(t)=max(N(:,t)); % [N]
    end

    %outputs
    P=FVAL; % elastic energy
    uout=norm(dispconrel(end-1:end)); % absolute output displacement after step 1.
end
M-file to find the optimum parameters

```matlab
%% Optimization
close all
clear all
%constants
lclamp=32e-3;
stroke=1e-3;
D0=32e-3;
R=200e-3;
tmax=2;  % nr of step in simulations
n=100;   % nr of experiments in ROM dimension
m=200;   % nr of experiments in alpha dimension
ROMmax=100e-3;

Fly=zeros(n,tmax);
Nmax=zeros(n,tmax);
Exitflag=zeros(n,tmax);
uout=zeros(n,tmax);
uin=zeros(n,tmax);
k=zeros(1,m);
alpha=zeros(1,m);
for t=1:n  % for ROM=Range
    ROM(t)=(t-1)/(n-1)*ROMmax;
    Lloop=ROM(t)+lclamp;
    alphamin1=2*atan2((Lloop-lclamp/2),(R+D0/2))+0.0001;  %
    alphamin2=(D0*2)/(R+D0/2);
    alphamin=min(alphamin1,alphamin2);
    alphamax=160/180*pi;
    k=zeros(1,m);
    for tt=1:m  % alpha range
        alpha(tt)=alphamin+(alphamax-alphamin)*(tt-1)/(m-1);
        if (Lloop-lclamp/2)<=tan(alpha(tt)/2)*(D0/2+R)
            % constraint for tendon staying attached to the tapeloop
            unin(t)=ROM(t)-1e-3;
            % minimum stiffness occurs at end of ROM. -1 to stay within ROM
            [P,Fly(t,:),uout(t,1),Nmax(t,:),Exitflag(t,:)] =...
                Coupling_function(alpha(tt),Lloop,R,uinin(t),stroke,tmax);
            uout(t,:)=uout(t,1)*ones(1,tmax);
            % make matrix for plot
            for i=1:tmax
                if Exitflag(t,i)==1 && Exitflag(t,i)==2
                    Fly(t,i)=NaN;
                end
                if Nmax(t,i)>10;  % use 10N as max force
                    Fly(t,i)=NaN;
                end
            end
    end
    uin(t,:)=uinin(t)*ones(1,tmax)+([1:tmax]-1)/(tmax-1)*stroke;
    k(tt)=(Fly(t,2)-Fly(t,1))/(uin(t,2)-uin(t,1));  % numerical stiffness
    else
        display('infeasible configuration')
    end
end
[kopt(t),I]=max(k);
```
\alpha_{\text{opt}}(t) = \alpha(I);


